LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
School of Business and Management
Master's Degree Program in Strategic Finance and Business Analytics

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How Is Credit Risk Estimated? Discussion and Evidence based on Cross-Industry Sample Utilizing the KMV Model

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ABSTRACT

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Title:	How Is Credit Risk Estimated? Discussion and Evidence based on Cross-Industry Sample Utilizing the KMV Model
Faculty:	School of Business and Management
Master's Programme:	Master's Degree Program in Strategic Finance and Business Analytics
Year:	2016
Master's Thesis:	Lappeenranta University of Technology, 82 pages, 13 figures, 13 tables and 4 appendices
Examiners:	Professor Mikael Collan Dr. Azzurra Morreale
Keywords:	Credit risk, KMV model, risk modeling, decision-making

The purpose of this thesis is to focus on credit risk estimation. Different credit risk estimation methods and characteristics of credit risk are discussed. The study is twofold, including an interview of a credit risk specialist and a quantitative section. Quantitative section applies the KMV model to estimate credit risk of 12 sample companies from three different industries: automobile, banking and financial sector and technology. Timeframe of the estimation is one year. On the basis of the KMV model and the interview, implications for analysis of credit risk are discussed.

The KMV model yields consistent results with the existing credit ratings. However, banking and financial sector requires calibration of the model due to high leverage of the industry. Credit risk is considerably driven by leverage, value and volatility of assets. Credit risk models produce useful information on credit worthiness of a business. Yet, quantitative models often require qualitative support in the decision-making situation.

TIIVISTELMÄ

Tekijä:	Ilari Lyytikäinen
Tutkielman nimi:	Kuinka luottoriskiä mallinnetaan? Analyysiä ja tuloksia eri toimialoilta KMV-työkalua hyödyntäen
Tiedekunta:	Kauppatieteellinen tiedekunta
Pääaine:	Rahoitus
Vuosi:	2016
Pro gradu –tutkielma:	Lappeenrannan teknillinen yliopisto, 82 sivua, 13 kuvioita, 13 taulukkoa ja 4 liitettä
Tarkastajat:	Professori Mikael Collan KTT Azzurra Morreale
Avainsanat:	luottoriski, KMV-malli, riskien mallintaminen, päätöksenteko

Tutkimuksessa käsitellään luottoriskin mallintamista. Tutkimus on kaksijakoinen käsittäen sekä tutkimushaastattelun että kvantitatiivisen osion. Kvantitatiivisessä osiossa mallinnetaan luottoriski kahdelletoista eri yritykselle kolmelta eri toimialalta: autoteollisuus, finanssiala sekä teknologiateollisuus. Luottoriski mallinnetaan vuodeksi eteenpäin. KMV-mallin sekä tutkimushaastattelun pohjalta esitetään ehdotuksia liittyen luottoriskin analysointiin ja -työkalujen käyttöön

KMV-työkalu tuottaa yhteneväisiä tuloksia nykyisten luottoluokitusten kanssa. Erityisesti kuitenkin finanssialan luottoriskin määrittäminen on haasteellista korkean velkaantumisasteen johdosta ja edellyttää KMV-mallin muokkaamista. Luottoriskiin vaikuttavat pääasiassa yrityksen velkaantumisaste, omaisuuserien arvo sekä arvon volatiliteetti. Luottoriskityökalut tuottavat hyödyllistä informaatiota yhtiön luottokelpoisuudesta kuitenkin usein työkalujen tuottama informaatio yksinään ei ole riittävää, vaan lisätietoa yrityksen laadullisista tekijöistä tarvitaan, jotta luottoriski voidaan määrittää perusteellisesti.

ACKNOWLEDGMENTS

This Master's thesis has required more effort than I initially thought. I felt sometimes that I was spending countless hours achieving very little results. However, I have learned a lot during the process not only about the topic that my thesis covers, but patience and perseverance.

The writing process taught me, what it feels like to concentrate on something for such a long time. Overall this was quite an educational experience. I would like to thank my supervisor, Professor Mikael Collan for his valuable comments and advice on the thesis. His comments were very helpful for the writing process.

Finally, and most of all I would like to thank my family and friends for the support that I have got from them. Now the project has been completed, and it is time to set focus on new challenges.

In Espoo, January 19, 2016

Ilari Lyytikäinen

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1 Introduction

1.1 Background and the Motivation of the Study

The objective of this thesis is to focus on credit risk estimation and modeling. The thesis aims to answer following type of questions: how credit risk is estimated? What kind of different methods and tools do exist for the estimation of credit risk currently? What are the most important factors affecting credit risk?

Generally risk management is one of the primary areas in the field of finance and the very backbone of the companies. Over the recent years managing credit risk has become even more important. Recently consultancy firm McKinsey conducted a study and reported that the amount of debt has soared substantially over the past years (McKinsey 2015). This brings up new challenges to proper management of credit risk now and in the future, as the level of debt is higher than before.

Historically risk management has developed a long way since the days of the portfolio theory by Harry Markowitz in the 1950s. Nowadays it forms one of the core areas of finance. Dowd (2002) states three main contributing factors for such a rapid development of risk management in theory and practice. These three contributing factors are: 1) volatile environment 2) growth in trading activity and 3) advances in information technology.

Firstly, corporations operate now in a more volatile environment than ever before. Exchange, interest rates and stock markets can move unpredictably in any direction at a quick pace. Secondly, the growth in trading action has increased due to the globalization and IT development. Investments move across country boarders now in seconds, since more investors have tools and better access to investing. Finally, rapid IT development has actually established enough computational power and speed for demanding calculations that were not possible before, which has led to new innovations and possibilities in the field of modeling

and assessing risk. All these causes among many others have boosted the implementation of different risk management systems across companies worldwide.

Assessment of credit risk constantly receives attention in the business newspapers, as the three leading credit rating agencies: *Fitch, Moody's* and *Standard & Poor's* (*S&P*) give credit ratings to countries and companies. These credit assessments are frequently used as a basis component for different investment calculations and investment decisions, in order to adjust the calculations to correspond to the riskiness of the underlying investment. As a result lowering credit ratings typically have long-term consequences for the underlying organizations and countries alike, when the cost of debt increases. As an example, when Standard & Poor's announced downgrading of Finland's credit rating from AAA to AA+, downgrading received the attention of many leading international news agencies[1][2][3].

This thesis is useful for those interested in understanding how credit risk has been assessed previously in the past, and is currently estimated in today's business. For an individual investor in-depth understanding of credit risk is especially essential, when considering investing in financial securities such as bonds, derivatives or structured products. Similarly, for corporations the understanding of credit risk is critical, particularly when forming long-term partnerships with other companies or using derivatives for hedging purposes. Misinterpretation or the lack of understanding credit risk often tends to result in unfavorable consequences.

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[1] Reuters 10.10.2014 "*Update 2-Finland loses one of euro zone's last top credit ratings*."

[2] The Wall Street Journal 10.10.2014 "*S&P Cuts Finland's Credit Rating One Notch to AA-Plus*"

[3] BloombergBusiness 12.10.2014 "*Finland's Lost AAA Rating Prompts Premier Plea for Action*"

1.2 Purpose of the Study

The purpose of the study is to focus on credit risk assessment in business-to-business (B2B) context. By definition credit risk refers to the probability that a party of the transaction will not be able to fulfill contractual liabilities, for example, in the case of a default, or because of other financial problem.

The main research question of the study is stated as the following: "*How is credit risk estimated in the modern B2B context?"* In order to cover the main research question in more detail, the question has been divided into multiple sub-research questions:

1. What are the main factors of credit risk?
2. What kind of different credit risk models for B2B do currently exist?
3. How are credit risk models used in practice or in practical decision-making?

The main research question is covered in the literature review and by applying a current credit risk model to three different industries. Out of each industry four companies are selected. The estimation of credit risk and yielded results are discussed afterwards. Simultaneously attention is paid to sub research questions by considering different factors affecting credit risk of these firms, and what kind of implications does credit risk estimation have on decision-making process, and what should be taken into account in the actual decision-making situation.

Additionally, different factors contributing to credit risk, are examined based on the earlier studies conducted in the field of credit risk. Furthermore, factors are discussed in the qualitative section of the study, as the factors have a critical role in the outcome. The second question is explored in the literature review (Chapter 2), as this chapter covers the main major credit risk assessment tools from the early 19th century up to the present time. Third sub-research question is covered in the interview section of the study, and discussed further in the results section.

For the literature review a great amount of articles was covered from many well-known journals, for example, Journal of Banking & Finance, Journal of Finance

and Journal of Financial Economics. These articles were found primarily by using different online databases. Typical keywords used were: Credit risk, assessment of credit risk, default risk and CreditMetrics. In addition to articles, technical documents of different risk measurement tools were examined for the literature review.

1.3 Methodology

This study relies upon two different methodological approaches: quantitative and qualitative, since in most cases credit risk is not purely determined by only single type of approach**.** Quantitative numbers often need qualitative support in order to assess credit risk. The objective of the quantitative section is to quantify credit risk in pure numbers relying on existing credit risk model. The section shows how credit risk is estimated for 12 different sample companies.

The employed model is based on the KMV credit measurement tool originally developed by KMV Corporation. The KMV model forecasts the actual probability of default, the Expected Default Frequency (EDF), for each firm utilizing market and historical information. In other words the Expected Default Frequency is a firm-specific value, which can be used to rank different firms in terms of their probability of default. The model is discussed more in detail in chapter 3 and chapter 4.

Qualitative section contains a semi-structured interview of a credit risk professional, who has gained experience of credit risk in B2B setting. The qualitative part aims to shed light on questions related to the practical use of credit risk models, and what implications results have for decision-making. Furthermore, factors contributing to credit risk are examined in this part.

1.4 Focus of the Study

The focus of this study is on credit risk, however, there exist different types of other risks that are closely related to the credit risk, such as market and operational risk, see Figure 1. These other types of risk will not be discussed to a great extent, since they are out of the scope of this study. The thesis is particularly focused on estimating credit risk.

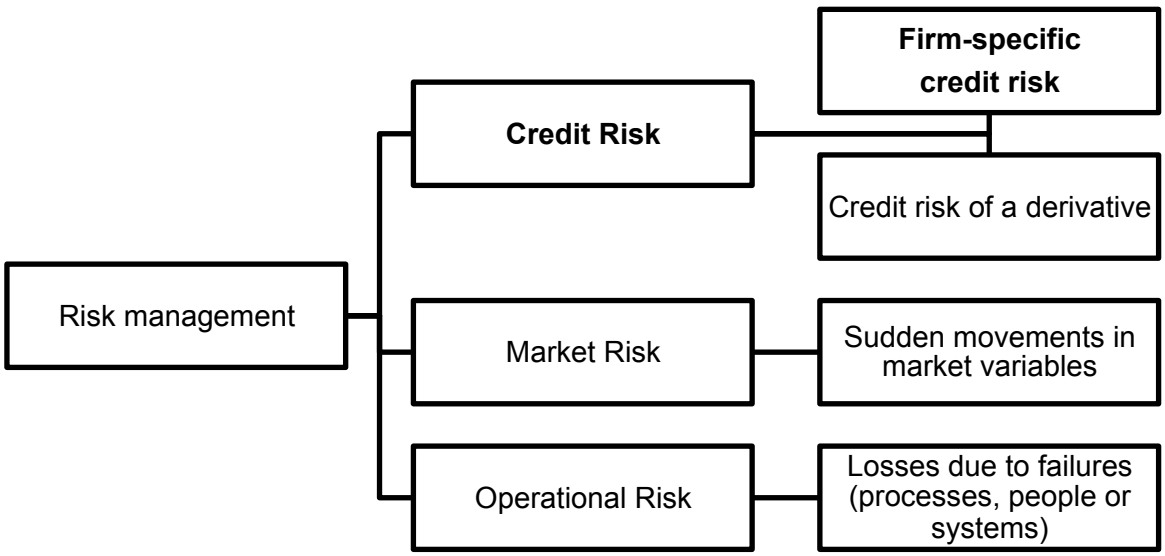

Figure 1. Focus of the study in bold.

Furthermore, the study aims to cover credit risk only in Business-to-Business context, leaving out retail credit risk in Business-to-Consumer setting (B2C). In B2B context, the focus is on credit risk, to which firms are exposed in their every day operations. Typically credit risk reflects in a company's credit rating.

Moreover, as there exist multiple tools for credit risk estimation, certain methods are left out from the thesis. Credit scoring and prediction methods based on the use of artificial neural networks are left outside the scope of this thesis. This allows a more comprehensive focus on the remaining tools covered in the study.

Traditionally, the understanding of credit risk has been critical for banking and financial sector. In the banking and financial sector credit risk usually makes the

largest amount of risk. Additionally, companies who do not realize their receivables or receive a payment soon after the business transaction has been completed are often exposed to credit risk. Therefore, industries with long terms of payment often need to consider credit risk in their risk management.

Credit risk modeling depends on the type of the underlying asset, for example, modeling credit risk is different for bonds versus derivatives, since in reality these assets behave differently (linear versus non-linear relationship) and these characteristics need to be taken into account in the modeling part. Covering all the different types of assets would take a substantial amount of time, and be out of the scope and focus. Therefore, the focus is more on credit risk rising from traditional asset types such as loans and bonds, leaving out the modeling of credit risk for derivatives.

Finally, certain assumptions need to be made in the actual quantitative modeling part of the study as in the most cases of modeling real world. Models are only representations – usually simplifications of the actual underlying phenomena. In the quantitative section of the study the aim is to apply the KMV model to 12 internationally and publicly traded companies from three different industries: 1) Automobile 2) Banking and Financial Sector and 3) High Technology. Companies and limitations are discussed further in chapter 4: Research Methods and Data.

1.5 Structure of the Study

The structure of the study is set as follows. The second chapter considers briefly different types of risks and relationships between them. The focus is then set on credit risk how credit risk has been measured previously from the beginning till this day. Furthermore, different firm characteristics and macroeconomic factors influencing a firm's credit risk are discussed. Third chapter aims to explain theoretical framework: how all the models and theories are connected to one another. Additionally, the basis of the KMV model: the Merton model is clarified more in detail.

Fourth chapter lays a basis for the actual research part. Methodology is explored and assumptions regarding the employed model are discussed. Fifth chapter discusses the results by combining results from both parts: quantitative and qualitative, and analyzes reasons behind the obtained results. Finally, we conclude our study by summarizing, what we have covered, and what still needs to be examined in the future in the final chapter 6. The following Figure 2 summarizes the structure of the study and the main context of each chapter, and more importantly, what is to follow.

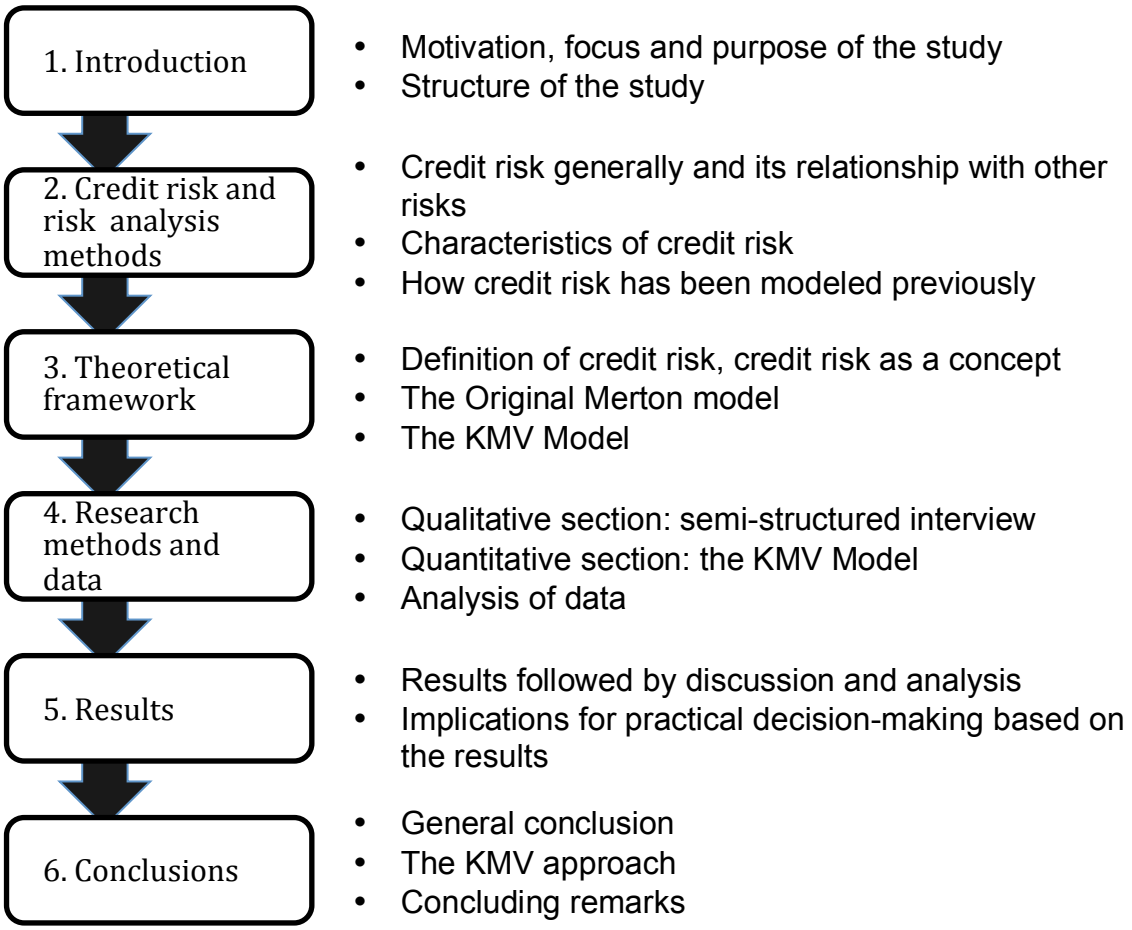

Figure 2. Structure of the study.

2 Credit Risk and Risk Analysis Methods

In this chapter different types of risk are briefly discussed first, and their relationship to the credit risk is considered. The discussion is followed by factors of credit risk at the macroeconomic level and at the individual firm-specific level. Furthermore, characteristics of credit risk that need to be taken into account when modeling credit risk are discussed. The final section of the chapter two focuses on the development of credit risk models up to this day in a chronological order. This chapter forms a literature review of studies on how credit risk has been measured and modeled over the past decades.

2.1 Credit Risk and Different Types of Risk

Risks can be classified into different groups in a number of different ways. Marrison (2002) divides risks into three different categories in the following way: 1) market risk 2) credit risk and 3) operational risk. Credit and market risk are often difficult to differentiate from one another; Jarrow and Turnbull (2000) even argue that credit and market risk are inseparable. Credit risk can be defined as a probability that a borrower will default on any type of debt by failing to make required payments. Market risk is caused by the changes in market factors, for instance, changes in market prices, interest rates and exchange rates. However, these two types of risks are often related to one another, and separating them from one another can be a challenging assignment. Third risk type, operational risk refers to risks arising from a firm's daily operations, for example, a probability of employees going on a strike or a vital machine breaking down.

Another way of classifying risks is by the source of uncertainty. J. P. Morgan and Reuters (1996) divide risks into the following categories:

1. Credit risk
2. Operational risk
3. Liquidity risk
4. Market risk

In this classification liquidity risk refers to the inability of a firm to fund its illiquid assets - for instance, not being able to purchase raw materials due to the lack of funds.

2.1.1 Relationship Between Different Kinds of Risks

Over the course of time, some of risks have become more critical relative to others, and others have become non-existent. For instance, operational risk such as a machine breaking down can be replaced faster nowadays than in the past, assuming there are spare parts available. Failure to deliver goods to a customer on time, and the consequent obligation to compensate customer, can be transferred to an insurance company to be taken care of. On the other hand social media and technology have made corporations more careful about their reputation, since they can lose their brand value, customers and, consequently, profits after even one moment under a negative spotlight.

Secondly, new technology has made information move more easily and made it inexpensive. This should have increased fluctuations in stock prices after bad or good item of news, since more investors than before have access to information immediately. However, different types of risks depend greatly on various factors, for example, industry, customers and markets. Liquidity risk can increase, if customers are in financial trouble and face difficulties delivering payments on time. Market risk can become critical after a central bank's announcement to raise interest rates or a government's announcement to restrict cash flows.

2.1.2 Credit Risk Factors at the Macroeconomic Level

Credit risk is affected by macroeconomic factors. Carling, Jacobson, Lindé and Roszbach (2007) discovered in their study that macroeconomic variables such as yield curve, output gap (= actual gross domestic product - potential gross domestic product) and consumers' expectations have significant explanatory power for firms' default risk. Additionally, common firm-specific financial ratios were found to

be good proxies in explaining credit risk. The findings of the study have also been confirmed then by Bonfim (2009) and Fama (1986), who note that firms' default rates are dependent on current macroeconomic conditions. During the periods of high economic growth, firms tend to take on more debt. Bonfim points out that this becomes an apparent problem once the growth slows down. The role of macroeconomic factors contributing to credit risk is also taken into account by Wilson (1997), who laid down the basis for CreditPortfolioView tool by McKinsey.

When assessing the economy in general, Bangia, Diebold, Kronimus, Schagen and Schuermann (2002) also underscore that the rating migrations are linked to macroeconomic conditions and asset quality. Furthermore, Nickell, Perraudin and Varotto (2001) studied transition probabilities on industry, country and stage of the business cycle, and have found similar results that a business cycle is the foremost factor in explaining credit rating transitions. There is a higher probability of credit downgrade and default during economic slowdown, as can be seen in the Figure 3.

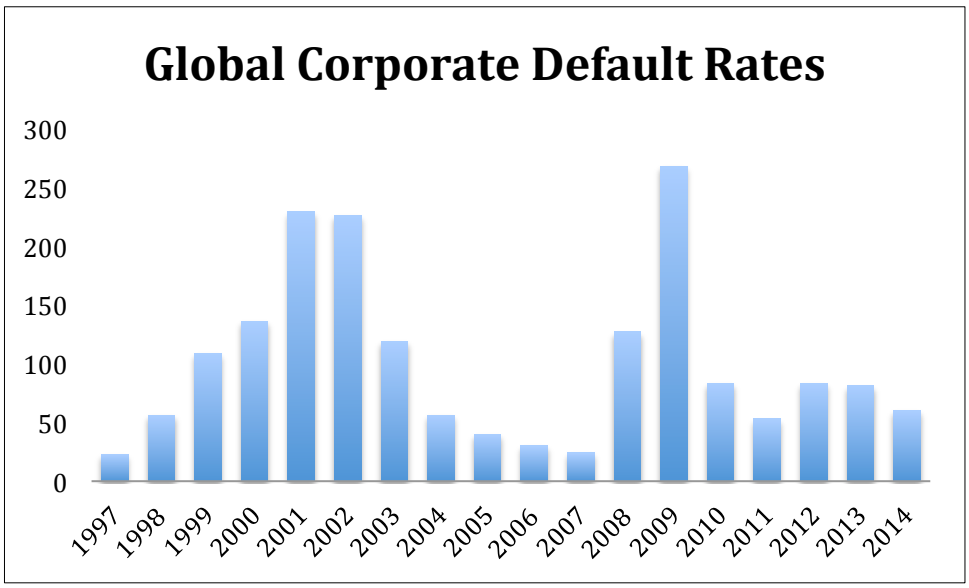

Figure 3. Global Corporate Default rates over the period of 1997 – 2014 based on Standard & Poor's (2015).

At the turn of the 21st century, the global economy was suffering from the Dot-Com boom and bubble. Many companies had been overvalued in comparison to their

profit making capability. This resulted in a wave of defaults, as can be seen from the Figure 3 above. Another wave of defaults occurred at the end of the decade in the 21st century, when the global economy was hit by the financial crisis. Gersbach and Lipponer (2003) suggest that increase in defaults and credit risk is not only driven by the increase of default probability, but also by changing default correlations at the time of negative macroeconomic shock, meaning that more firms tend to undergo financial problems during that moment. Furthermore, Vishny and Shleifer (1992) note that a company can carry up to 40% of more debt in a boom in comparison to a recession. Therefore, companies tend to take on more debt during a boom.

2.1.3 Firm-specific Factors Contributing to Credit Risk

In addition to macroeconomic factors at the aggregate level, certain firm-specific characteristics have been found to be contributing factors in explaining credit risk. Bonfim (2009) conducted a study covering more than 30,000 firms and the results indicated that several factors, including financial structure, profitability, liquidity, recent sales performance and investment policy had an influence on firms' credit risk. More importantly, the study also discovered that firms, which had had financial problems in the past, were more likely to have them in the future as well. Therefore, when assessing credit risk of a firm, past payment history should be taken into account. Past payment history, along with loan terms, borrower characteristics, economic conditions and legal constraints, were previously already found to be contributing factors toward the likelihood of default by Lawrence, Smith and Rhoades (1992).

From the perspective of capital structure Hackbarth, Miao and Morellec (2006) urge firms to adjust their capital policy choices to economy's business cycles, since operating cash flows depend on current economic conditions, hence firms should adjust their capital policy more during a boom and less during a recession. Furthermore, Hackbarth et al. (2006) found that firms also tend to pay as much as 120 basis points more for debt capital during the time of recession.

2.2 Credit Risk Measurement

Currently, there are a number of models to measure and analyze credit risk. The choice of the specific model depends on the number of factors:
i) What is the type of risk a practitioner is trying to model?
ii) What is the underlying security in question?
iii) What kind of assumptions one is ready to make, when deploying that model?

Every model comes with its own advantages and shortcomings. There simply does not exist just one perfect model yet, hence the third factor - assumptions – plays usually the most critical role in choosing the correct model, since in many cases, this factor also gives basis to the shortcomings of that model.

Saunders and Allen (2002) underline the importance of data input and fundamental model assumptions: different models produce very different results based on the data employed and fundamental assumptions made regarding the model. This should be kept in mind, when comparing results of different credit risk models. In the following sub-sections, a brief history of credit risk measurement is covered, as well as a brief introduction of a couple of the most present credit risk models employed by corporations today. For a more thorough analysis and explanation of current credit risk models, as well as a mathematical basis, for example, see Saunders and Allen 2002; Crouhy, Galai and Mark 2000; and Altman and Saunders 1998.

Measuring credit risk is difficult for a number of reasons. The nature of credit risk in comparison to other types of returns is different. As can be seen in Figure 4, market returns are more widely scattered around relative to the credit returns, which are concentrated highly in a certain area. Typically, there is no upside potential in credit returns (excluding certain derivatives), whereas market returns are dependent on market conditions, which can move in either direction.

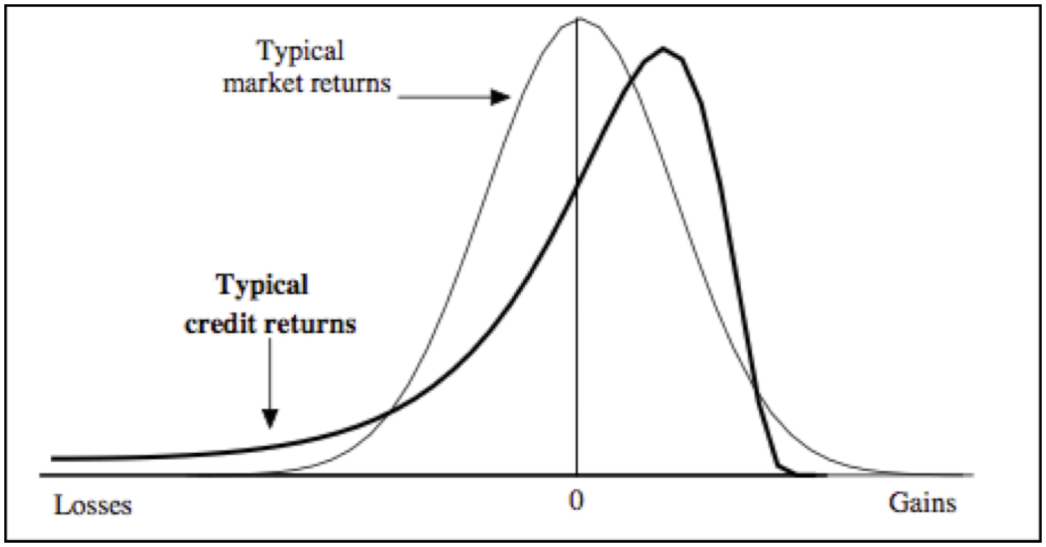

Figure 4. Comparison of market returns versus credit returns. Source: CreditMetrics JP Morgan (1997).

Furthermore, models need to take into account the behavior of the underlying financial instrument. Altman and Saunders (1998) and Hull (2012) emphasize subtle differences between loans and over-the-counter (OTC) instruments. The value lost by an OTC instrument depends usually on the current status of that specific instrument. OTC instrument can either be a liability or an asset to the financial institution. Consequently, the value lost and credit risk is determined by the current status of the contract. Secondly, typically amount lost due to a loan is greater in quantity than in the case of the OTC contract.

2.2.1 History of Credit Risk Measurement

Before this day measurement of credit risk has developed a long way. Altman and Saunders (1998) pointed out in their article that 20 years ago most financial institutions relied entirely on a subjective analysis or individual “banker” system to assess credit risk related to corporate loans. Back then information was gathered from different characteristics of a borrower and used to evaluate borrower’s ability to pay back their debt. The characteristics under the scrutiny of bankers were: borrower character (reputation), capital (leverage), capacity (volatility of earnings),

and collateral. The model is known as the 4 “Cs” of credit (Figure 5). The assessment of whether or not to grant credit was made on the basis of the 4 “Cs”. However, the credit judgment was mostly subjective, based on the opinion of an expert - in this case, a banker.

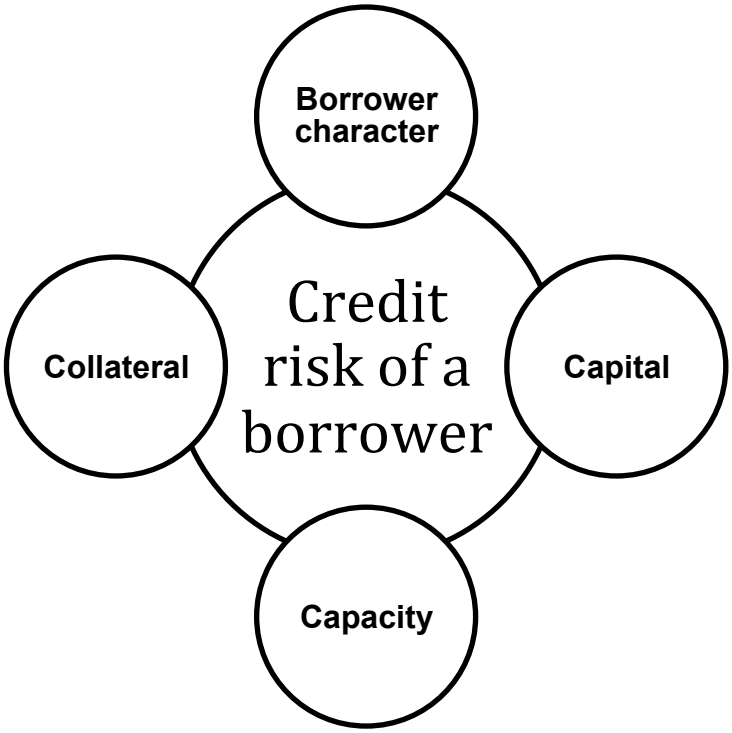

Figure 5. The “4Cs” of credit model.

Interestingly, Sommerville and Taffler (1995) showed that bankers’ or experts’ opinions are prone to bias when making a subjective judgment on a borrower’s credibility in the context of a less developed country. In such a context, bankers tend to be overly pessimistic about the credit risk than what the objective models actually predict, and are less tempted and likely to grant a loan. This perception could indicate that bankers are better aware of risks as a whole, than what models can actually predict, since objective models only capture certain dimensions of credit risk based on which input variables are employed.

Furthermore, Sommerville and Taffler (1995) pointed out that multivariate credit scoring systems (= systems that utilize more variables than one variable in measuring credit risk) tend to outperform systems that are based on subjective expert judgments. However, even though the models had a higher overall

predictive power, banker judgment could outperform the model when taking into account type I and type II errors. This perception could indicate that bankers (subjective judgments) could form a more comprehensive credit assessment on a borrower's credit worthiness.

Since then, the move has been away from subjective systems towards more objective based credit systems. The development towards more objective credit systems has been partly enabled due to the increase in computational power, and, consequently, better ability to model credit risk in more detail. Therefore, most present credit risk models such as CreditMetrics, CreditPortfolioView, CreditRisk+ and the KMV framework were developed in the late 90s or at the turn of the 21st century.

First objective credit systems were accounting based credit-scoring systems. Most of them univariate in nature, measuring only one variable at a time, for instance, key accounting ratios related to profitability or liquidity. Afterwards, the specific key accounting ratio of a borrower was compared to industry or group average ratios. The worth a borrower's credit was based on this interpretation. (Altman 1968; Altman & Saunders 2000)

Clearly such a system based on only one ratio at a time, and neglecting other characteristics of a borrower, led sometimes to faulty interpretations regarding a borrower's creditworthiness. This ultimately led to a need for a more sophisticated credit measurement, and consequently the development of multivariate models, of which one of the first measures was *Z-score*, tool based on discriminant analysis. (Altman 1968; Altman & Saunders 2000)

The final discriminant function, *Z-score*, as presented in its original form by Altman (1968):

$$Z = 0{,}012x_1 + 0{,}14x_2 + 0{,}33x_3 + 0{,}006x_4 + 0{,}999x_5 \qquad (2.1)$$

where;

x_1= Working capital / Total assets

x_2 = Retained Earnings / Total assets

x_3 = Earnings before interest and taxes / Total assets

x_4 = Market value equity / Book value of total debt

x_5 = Sales/Total assets

Z = Overall Index

Altman (1968) found that combining financial ratios into one multivariate framework, accounting for several different ratios, resulted in a better default prediction. In his original research, the *Z-score* was successful to predict bankruptcy accurately in 94 percent of the initial sample. However, the research was conducted on publicly held manufacturing corporations, as there was enough public information available on those corporations. Therefore, the study did not take into account other industries or corporations of smaller size. Ten years later Altman, Haldeman and Narayanan (1977) improved the Z-score model to count for two more variables. This new model, also known as the “Zeta” model performed even better in the prediction of company default than the original Z-score.

Multivariate accounting based credit-scoring models brought many important features and aspects into modeling credit risk that neither of the previous systems (experts judgment or univariate approaches) could employ. However, Altman and Saunders (1998) have criticized multivariate credit models for at least three reasons. The first criticism points towards the fact that multivariate models use accounting data, which is based on book values rather than market values. Book values reflect historical value at a certain point of time T, disregarding sudden movements in the actual market value, even though sudden movements in value can often be of critical importance in borrower’s conditions. Furthermore, book values are only reported at certain intervals, since firms tend to report their book values on a yearly or quarterly basis. Secondly, the multivariate approach

assumes that the world is linear. Assumption that there is a linear relationship between different variables often does not hold true. Finally, accounting based credit models can only be tenuously linked to theoretical models. Hence, a number of more practical new approaches were later proposed.

In recent years, credit risk modeling has developed, rapidly becoming a central factor in the risk management systems at institutions (Altman & Saunders 1997). As a consequence, multiple financial institutions and consulting firms have found the growing market, and are marketing their own credit risk models to other corporations (Lopez & Saidenberg 2000). Currently, there exist multiple models for credit risk measurement, see Figure 6.

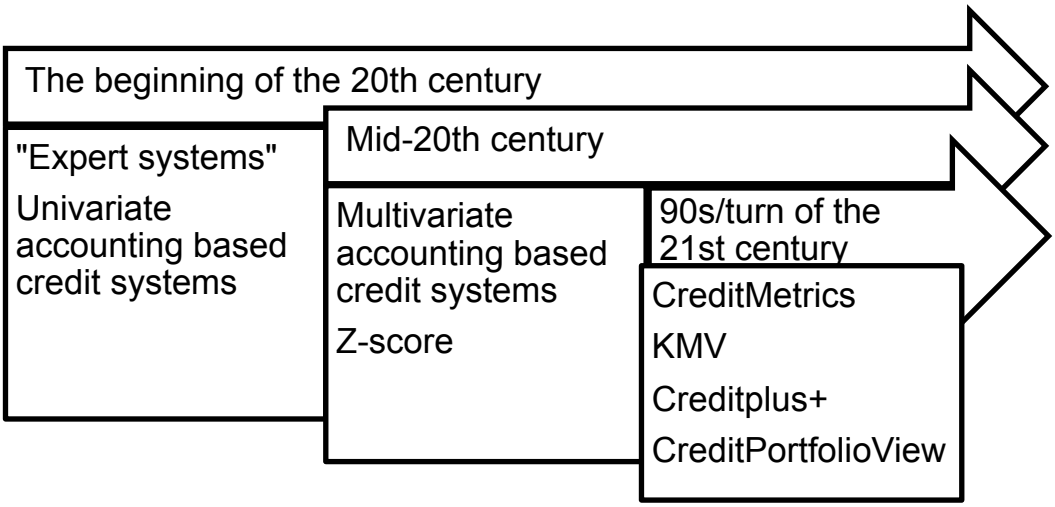

Figure 6. Development of credit risk models over the past century.

Altman and Saunders (1998) state multiple forces that have driven the increase of credit risk measurement: 1) worldwide increase in the number of bankruptcies 2) trend towards disintermediation by the higher quality and largest borrowers 3) more competitive margins on loans, 4) declining value of real assets in many markets and 5) dramatic growth of off-balance sheet instruments with risk exposure.

Federal Reserve System Task Force on Internal Credit Risk Models, FRSTF, (1998) divides credit risk models into two separate groups based on how credit risk is measured. Certain models define credit losses as loan defaults, meaning

there are only two different final outcomes: either default or non-default. The second group of models views credit losses as rating migrations. This approach assumes that a credit instrument may decline (improve) in value if its rating is downgraded (upgraded). The first set of models is known as default models, and the second group of models is referred to as mark-to-market or multi-state models.

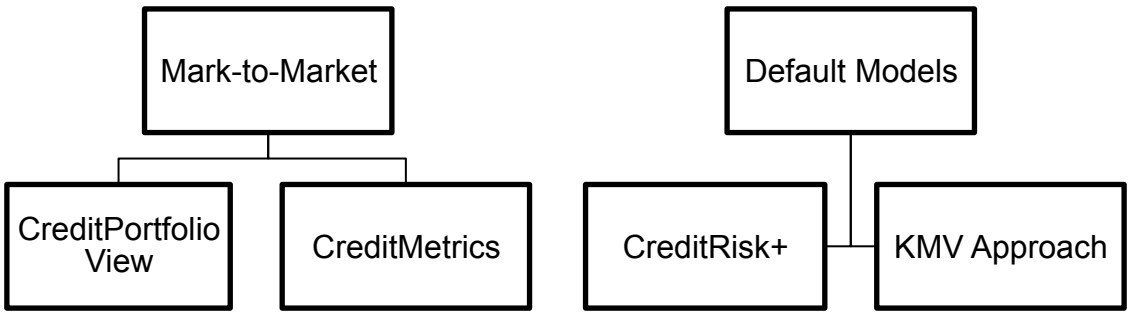

Figure 7. Classification of the latest credit risk models: Mark-to-Market and Default Models. Source: FRSTF (1998).

Saunders and Allen (2002) highlight the difference between the two classes of models. Default models only assume default or no-default neglecting the possible credit migration, whereas mark-to-market type of models can track the changes in market value instantly. Consequently, mark-to-market models are able to capture changes in credit quality more accurately in comparison to default models.

Despite the classification the common underlying purpose of models of either group is to forecast the probability distribution function of losses that may rise from any corporation's credit portfolio. Typical for the distribution function is that it is not symmetric. The actual shape of credit loss distribution is comparable to the distribution of credit returns, see Figure 4. The shape of the distribution is based on the assumption that credit defaults or rating changes generally do not happen often. Secondly, there is usually a cap on returns of debt instruments. (Lopez & Saidenberg 2000)

2.2.2 Value at Risk (RiskMetrics)

RiskMetrics is primarily used to measure market risk. However, both risk measurement tools RiskMetrics and CreditMetrics share the same origin, and therefore both have been included in this review. Value at Risk is firmly connected to CreditMetrics. Value at Risk often abbreviated as *VaR* is defined as the maximum potential change in the value of the underlying portfolio at a certain likelihood (%) over a certain time period *t*. As a risk measurement tool it is one of the most widely spread and utilized among corporations. The Basel Committee uses VaR figure as a standard to set the minimum amount of capital to be held against market risks. (Marrison 2002)

Most modern banks disclose their VaR figures on a routinely basis. When the Basel II accord came into effect, banks have been required to publish their VaR figures as a part of their risk management. Furthermore, banks are reporting individual VaR figures more commonly for each type of risk: equity, interest rate and foreign exchange (Pérignong & Smith 2008).

Typically, VaR figure is shown as a distribution of the following shape, see Figure 8.

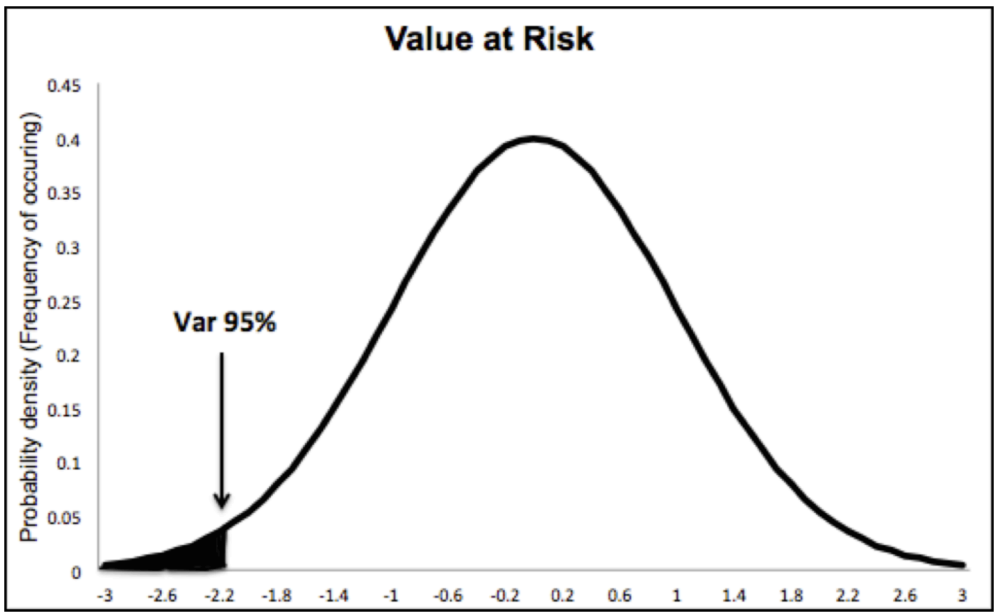

Figure 8. Value at risk graphical presentation, Var at 95 %.

In practice VaR answers the question: "*what is the value that can be lost at a certain probability?*" For instance, if a company's 5 % VaR figure is 10 million USD over one day, it means that in an undesirable scenario, there is a 5 % probability that a corporation *can lose* 10 million USD or *more* over that day.

Value at Risk can be calculated in three different ways: 1) Parametric VaR 2) Historical simulation and 3) Monte Carlo simulation. All the different types of measurements are based on market risk factors, which are derived from the security prices currently traded on the market. Each method has its own advantages and disadvantages when calculating VaR. Therefore; a correct type of method depends on the type of the underlying portfolio. (Marrison 2002)

VaR has been often criticized for not being an accurate tool for risk management (Taleb 2012), especially in times of high market volatility. VaR neglects the losses that occur beyond the VaR level, even though the losses could be very critical to a company's survival (Yamai & Yoshiba 2005). However, as Taleb (2012) points out these highly unlikely events often reshape the markets and, are those events that are critical and should really be avoided if a company is to be successful in the long-term.

Artzner, Delbaen, Eber and Heath (1999) have criticized VaR as a measure of risk due to its mathematical properties. Furthermore, Danielsson, Embrechts, Goodharrt et al. (2001), Leippold, Trojani and Vanini (2006) have criticized VaR for its potential destabilizing effects on the economy. Even though after all the criticism VaR has received, it still remains as the main tool for measuring market risk (Pérignon, Deng & Wang 2008).

All the criticism could explain why in practice, VaR is actually utilized carefully as Pérignong et al. (2008) found that commercial banks over sample period of 1999 – 2005 had a tendency towards conservatism when calculating their VaR. The VaR figure was usually set to be higher as it was supposed to, indicating higher requirement for economic capital. The study claimed two possible motives for

overstating VaR: i) banks want to be very cautious when they are measuring their market risk, and/or ii) they did not take diversification effect (when VaR is measured across multiple business lines and risk categories) into account.

2.2.3 Credit Value-at-Risk and CreditMetrics

CreditMetrics was first published in 1997 by JP Morgan, and was first introduced in CreditMetrics Technical Document. The approach is based on credit migration analysis meaning the probability of a financial instrument moving from one credit quality to other, including the possibility of default within a certain time horizon, typically a year. For example, a bond rated BBB at the beginning of the period dropping by one rating to BB a year from now. Typically credit migrations are presented in a transition matrix, and the data is based on historical average migrations, how financial instruments of that rating have typically behaved over the specific time horizon.

As with Value at risk figure, CreditMetrics also produces a distribution of value for an instrument at the risk horizon. In its original form CreditMetrics applied to bonds and loans, which behave in the same way. However, in the case of derivatives, the model needs readjusting, since derivatives such as swaps or forwards typically behave differently. (Crouhy et al. 2000)

Crouhy et al. (2000) point out several challenges with CreditMetrics. Firstly, the portfolio distribution is not normal and instead it is highly skewed, and secondly, measuring diversification effect is more complex for credit risk than it is for market risk, since multiple simplifying assumptions have been made on correlations of the asset returns. Finally, CreditMetrics does not take into account the effect of market risk, since forward values and exposures are taken from deterministic forward curves. The only uncertainty included in the model is the probability of credit migration. Consequently, credit risk is analyzed independently of market risk.

Unlike VAR, CreditMetrics model seeks to construct the volatility of value due to credit quality changes, since this tool cannot use daily observations to build the estimation. Another distinct difference between these models is that modeling credit risk does not require that returns be normally distributed, which is the case with RiskMetrics and a major shortcoming of that model. (CreditMetrics 1997). The following Figure 9 summarizes the main idea of CreditMetrics.

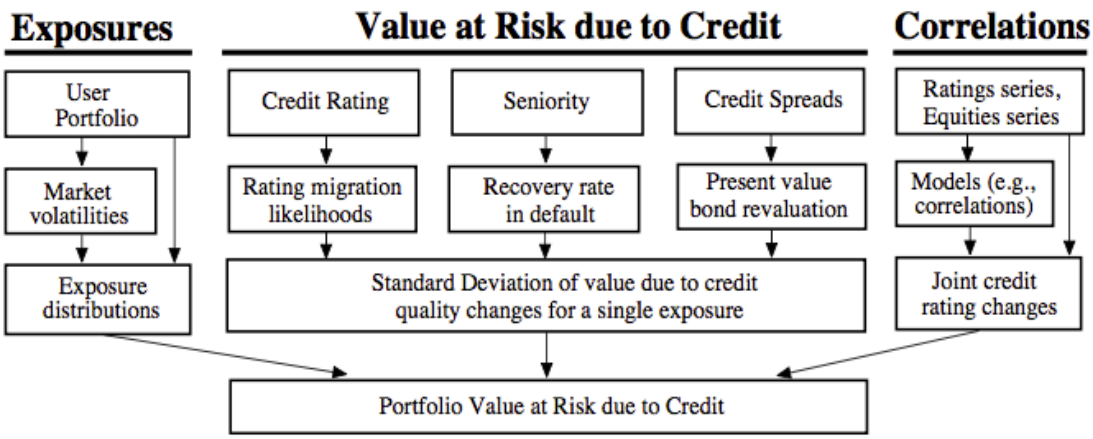

Figure 9. CreditMetrics framework as originally presented by JP Morgan. Source: CreditMetrics (1997).

For a more detailed and complete description of CreditMetrics, see CreditMetrics Technical Document (1997) and for a practical example see Crouhy et al. (2000).

2.2.4 Option Pricing / Structural Approach by KMV

As an alternative, KMV, later bought by Moody's, a firm specialized in credit analysis, proposed option pricing or structural approach to credit risk modeling. The proposed approach does not depend on the rating of the obligor and a possible credit migration from one rating to another as CreditMetrics does. Instead the model relies on "Expected Default Frequency," abbreviated as *EDF.* (Crouhy et al. 2000)

KMV's model assumes that each issuer is specific and creditworthiness or default risk is characterized by three different factors: 1) asset return distribution, 2)

capital structure and 3) default probability. The probability of default for each issuer is derived from the model originally proposed by Merton (1974). The EDF can be determined for any firm, and is best suitable for firms traded publicly, since the model incorporates public information in its modeling process. (Crouhy et al. 2000)

Both aforementioned approaches CreditMetrics and KMV have the same base theory behind them, Merton'asset value model originally proposed by Merton in 1974 (Crouhy, Galai & Mark 2000).

2.2.5 CreditRisk+ by Credit Suisse Financial Products

The development of CreditRisk+ model began in the early 90s, and the Credit Suisse Group published the final version in December 1996. CreditRisk+ risk measurement tool is applicable to credit risk rising from all types of instruments: consumer and retail loans, bonds and derivatives.

Unlike CreditMetrics, CreditRisk+ assumes that a default will either happen or not. Default is assumed to be a continuous variable. From this perspective CreditRisk+ can be considered to be a default model. The tool does not consider the effects of rating downgrades or upgrades as the CreditMetrics approach does. The major advantage of this approach is the small data input relative to other approaches, although as a downside the model is not as accurate as the CreditMetrics, which is based on VAR-type of approach of credit upgrades and downgrades. (Saunders & Allen 2002)

Since, Creditrisk+ approach assumes that default is a continuous variable, it is best suitable for modeling credit risk of instruments/portfolios that are held till the end of their maturity. If instruments are traded actively and require marking-to market, other measures of credit risk are more suitable for measuring credit risk of such type of instruments. (Saunders & Allen 2002)

As CreditRisk+ requires relatively little data input to be calculated the model suffers from certain disadvantages. Firstly, the approach does not account for market risk. Secondly, since the model only assumes default or no-default for each borrower, it does not take into account the effect of rating change. On the last note CreditRisk+, like CreditMetrics and the KMV approach, does not deal with nonlinear products, which is considered to be a limitation for the model. (Crouhy 2000)

2.2.6 CreditPortfolioView

CreditPortfolioview approach for risk measurement, originally proposed by McKinsey company (1997), is based on the assumption that credit risk as a whole is determined by the current state of the economy, for instance, during a recession there is a tendency towards higher credit risk, whereas during a boom, credit risk tends to be smaller. This assumption is similarly backed by the studies of Wilson (1997) and Nickell et al. (2001).

This basic underlying assumption established by Wilson (1997), and later McKinsey forms a basis for CreditPortfolioView. The model takes a different interpretation on diversification in comparison to other models, as it assumes that credit risk is driven by the state of the economy. Furthermore, the model calculates macroeconomic state for every country individually. The approach assumes that different sectors react differently to macroeconomic changes; for instance, some industries, such as construction or leisure, are more sensitive to changes in macroeconomic environment, rather than the retail industry.

As a downside Crouhy et al. (2000) note that the approach requires a lot of data input in order to forecast accurately for each sector in each country. If compared to Creditrisk+, the model is more dependent on data availability. In certain cases there are difficulties to find enough data for each country and sector. This makes the approach very dependent on available data.

2.2.7 Differences and Similarities between the Models

Previously, there have been a couple of comparative studies on the most present credit risk models (CreditMetrics, CreditPortfolioView, CreditRisk+ and the KMV approach). For instance, International Swaps and Derivatives Association (ISDA) and the Institute of International Finance (IFF) examined how the models performed in measuring credit risk in 25 commercial banks from 10 different countries. Models were created to measure different markets: corporate bonds and loans, middle markets, mortgages and retail credits. Most importantly, models provided consistent outputs when similar inputs were used. Some of the tested models provided almost identical outputs. Most of the dissimilarities related to the calculation of correlation, valuation and the treatment of cash flows. This study is line with a similar study of Gordy (2000), who compared CreditRisk+ and CreditMetrics approaches. The study found that primarily discrepancies between the models were based on different assumptions on distributional assumptions and functional forms.

Parameter estimation has received attention of Koyluoglu and Hickman (1999) as well, who highlight the importance of parameter estimation in measuring credit risk. In their study they found that differences in key parameter estimation played a crucial role in the similarity of outputs provided by the models. When they adjusted key parameter values of three models (CreditMetrics, CreditPortfolioView and CreditRisk+) results were more similar than before the adjustment. As a basis of this they stated that instead of comparing how models perform in measuring credit risk – given certain assumptions, the focus should be more on parameter estimation: how the models estimate their key parameters (correlation and volatility). There is even a saying “GIGO”, *garbage in, garbage out* meaning that if your data is not correct, the results produced by the model are not accurate either.

Moreover, Nickell et al. (2001) focused on measuring credit risk for a large portfolio of Eurobonds denominated in dollars by employing CreditMetrics and The KMV approach. As a conclusion they found that both models failed in estimating credit risk of the portfolio. Saunder and Allen (2002) remind that the comparison of

different credit risk models is not beneficial, if the models do not measure credit risk accurately.

However, as Lopez and Saindenberg (2000) emphasize testing accuracy of credit risk models is more challenging and complicated, because of the lack of observations. Credit risk models typically have longer forecast horizons, usually a year - in comparison to that of market risk models - typically a day. Models that are used to test for accuracy require a lot of data inputs (observations). As the horizon of credit risk models is typically long, the lack of observations makes testing for the accuracy of the model difficult.

3 Theoretical Framework

Conceptually credit risk is broad. The traditional definition is set by the Bank for International Settlements (1999), which defines credit risk as: "*the potential that bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms*". However, this definition leaves some room for interpretation: what is the degree to which the borrower will fail to meet the obligations, completely or only to a certain degree? How likely this is to happen? Therefore, different credit models take quite a different approach on credit risk. Table 1 summarizes different views on credit risk taken by different models.

Model name	Approach taken on credit risk	How does the approach emerge in practice?
CreditMetrics	Credit migration analysis	Probability that the underlying asset (loan/bond) moves from one credit quality to another based on historical transitions.
The KMV Model	Default probability and loss distribution	Incorporates both public and historical information, taking into account default, however, based on the output of the model, a letter credit rating can be derived.
CreditRisk+	Focuses on default only	Requires relatively little data. The model does not consider credit migrations.
CreditPortfolioView	Focuses on default only	Incorporates the current state of economy by taking into account macroeconomic variables that affect credit cycles.

Table 1. Present credit risk models

When a default of a debtor occurs, creditor is almost certainly to suffer losses. On the other hand credit migration does not certainly cause any losses to the creditor. The degree and severity of losses at the time of the default depend on the type of the loan and its contractual characteristics (subordinated debt versus non-subordinated debt), as certain types of loans have better terms in the case of default.

As a concept credit risk is similar to bankruptcy risk, and certain models such as Z-score are employed originally to predict bankruptcy of a firm. However, credit risk is a wider concept in comparison to bankruptcy risk, as it includes the probability of credit migration together with the probability of default (default risk). Further, credit risk is widely affected by current market risk factors, and as a result the outcome of credit risk models depends on market variables as well.

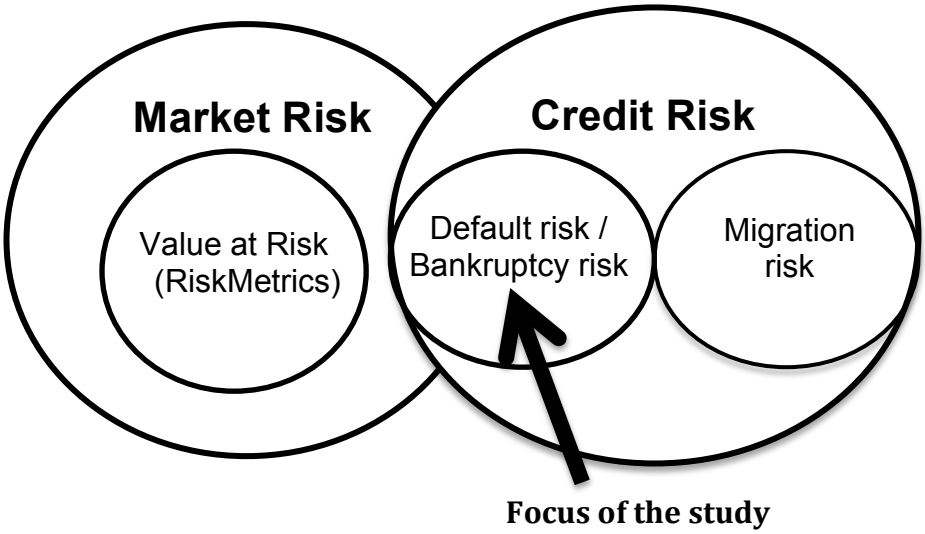

Figure 10. Theoretical framework relationship of credit and market risk and credit risk presented in more detail.

Many of the credit risk models have their basis on similar models that have been proposed to measure market risk originally, for instance, value at risk developed by JP Morgan forms a basis for many credit risk models such as CreditMetrics and CreditRisk+. All the models aim to generate a probability distribution, of which a certain percentile over a time period *t* is calculated.

What is common for all the aforementioned credit risk models is that they are best suitable for measuring credit risk of traditional instruments, such as bonds or loans. Crouhy (2000) argues that measuring credit risk is currently too challenging to more complex instruments, such as derivatives, since this type of model needs to take into account the current state of the underlying instrument.

Typical for credit risk is that it does not happen independently from other types of risks. In most cases it correlates highly with other types of risks and is affected by them – most commonly by market risk. To give a simple example on a general level, when interest rates rise (market variable), this phenomenon puts a pressure on investments financed with debt finance (assuming that the original loan is tied to interest rates) making the investment more risky from the perspective of a creditor, and increasing credit risk simultaneously.

As was discussed previously in previous chapters, when we discussed different types of models for modeling credit risk. Credit risk can either refer to:

i) risk that a firm's outstanding credit rating decreases, for example, previously a firm rated AAA is downgraded to AA. This leads to increase in credit risk, since a firm is closer to neglecting its payments.
ii) risk that a firm's is unable to meet its contractual liabilities to its creditors at the due date.

In practice credit risk is realized, when a firm is unable to meet its liabilities. Some of the models previously discussed modeled credit risk due to the movements of a firm's credit rating (CreditMetrics and CreditPortfolioView). Other group of models assumed that credit risk is modeled from default or no-default standpoint.

The KMV model is based on option pricing approach originally developed by Merton in 1974. The KMV approach is best suitable for publicly traded firms, since the value of equity is then market determined. The model incorporates public information in default prediction, as the public market information can be regarded as a good proxy of a firm value: the value of equity (market price x outstanding stocks) is a forward looking in nature, as investors form the value of a firm based

on future cash flows and expectations. Simultaneously, the KMV model exploits historical information by using information provided by balance sheet on debt value. (Crosbie & Bohn 2003)

3.1 The Merton Model

Merton (1974) revolutionized measuring of credit risk by applying the option based approach on equity of a firm, and assumed that a firm's equity is a European call option on the underlying assets that can be exercised at the maturity T. In this sense equity can be considered as an option from the option perspective, since equity owners have the residual claim on the assets, when all the other obligations have been fulfilled. As a result the compensation to equity holders at the maturity T can be defined as:

$$V_E = \max(V_A - X, 0) \tag{3.1}$$

where:

V_E is the market value of equity

V_A is the market value of assets

X is the exercise price

However, the original Merton model assumed that a firm was financed only with two types of liabilities: a single class of debt and a single class of equity. Debt can be considered to equal to the exercise price X. Hence, mathematically value of equity can be expressed in the following way:

$$V_E = V_{\square} N(d1) - e^{-rT} X N(d2) \tag{3.2}$$

where:

$$d1 = \frac{ln\left(\frac{V_A}{X}\right) + \left(r + \frac{\sigma_A^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d2 = d1 - \sigma_A\sqrt{T}$$

r = risk-free interest rate

By looking at the previous equation, it can be shown that equity and asset volatility are related to one another by the following mathematical expression:

$$\sigma_E = \frac{V_A}{V_E} \Delta\sigma_A \qquad (3.3)$$

These two equations form the core of the KMV approach. By solving them simultaneously, asset value and asset volatility implied by the equity value, equity volatility and liabilities could be obtained. Asset value and asset volatility are key variables needed in the calculation of Distance-to-Default.

Furthermore, the classic Merton approach (1974) assumed that a single liability (zero-coupon bond) promises a continuous fixed coupon flow and has an infinite maturity. In addition to that the model takes a view that no other payments are made by the company.

3.2 The KMV approach

Later Vasicek and Stephen Kealhofer have modified the original Merton model to better suit the needs of credit risk estimation. The modified version takes the model closer to the real world application, and assumes that the firm's equity is a perpetual option, and on a liability side the model can incorporate more than one type of liability, currently in total five classes of liabilities: short-term, long-term, convertible, preferred equity and common equity.

In addition to multiple liabilities, the KMV model extends the original assumptions of the Merton model in multiple ways such as:

1) Any and all classes of liabilities can make fixed payments (dividends and coupon payments)
2) Default is regarded as a company-wide event in comparison to obligation-specific default.

Kealhofer (2003) emphasizes the main differences between the original Merton and the KMV approach, stating that the main objective of the Merton Model is the valuation of the company's debt, based on the company's asset value and volatility, whereas the KMV approach is focused more on the relationship between the company's equity characteristics and its asset characteristics. The KMV approach exploits this relationship as stated in the Equation 3.3. When asset value and volatility are known, given the company's default point, the KMV model can be used to calculate credit risk of a company. Default prediction is the primary focus of the model.

More importantly, the KMV approach assumes that there are three main elements that determine the default probability of a firm. These factors are known as: 1) firms capital structure; 2) the volatility of asset returns 3) the current asset value. Value of assets is the measure of the discounted future free cash flows produced by the firm. This factor considers the future of the firm and its prospects regarding the firm's industry and the economy. The volatility of asset returns reflects the uncertainty or risk surrounding the asset value, and is influenced by the type of business. Capital structure illustrates the firm's contractual liabilities that it must meet at a certain point of time in the future.

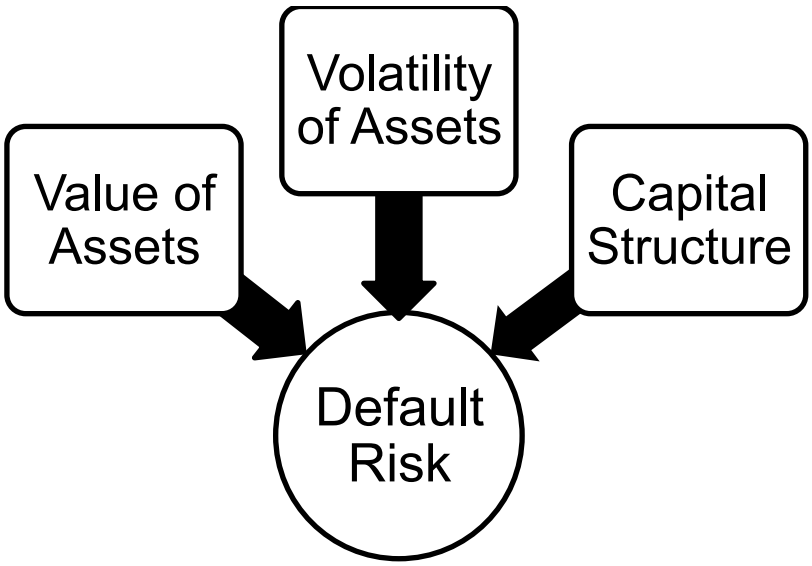

Figure 11. Three main components determining default probability according to the KMV approach, source: Crosbie and Bohn (2003).

The underlying assumption of the KMV model is that a corporation defaults, if the value of assets falls below the book value of liabilities (number 4 in the following Figure below). The “default zone” is indicated by the black colored area (EDF) in Figure 12, and mathematically this can be expressed as:

$$p_t = Pr[V_a^t \leq X_t | V_A^0 = V_A] = Pr[\ln V_A^t \leq \ln X_t | V_A^0 = V_A] \qquad (3.4)$$

where:

p_t is the probability of default by time t.

V_A^t is the market value of the firm′s assets at time t.

X_t is the book value of the firm′s liabilities due at time t.

A more comprehensive graphical description of the model can be found below in Figure 12.

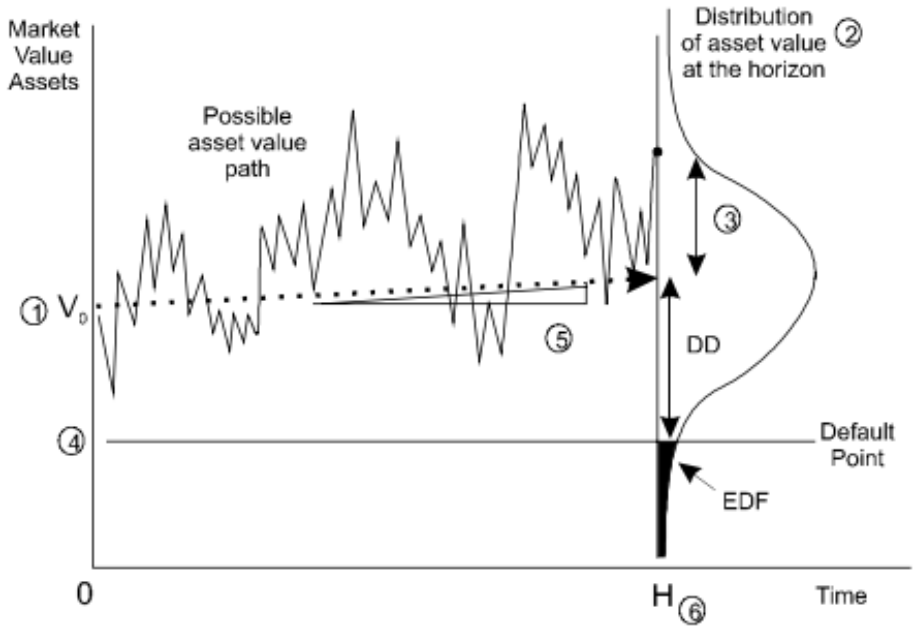

Figure 12. Six variables affecting the default probability of a firm. Source: Crosbie and Bohn (2003).

As seen in Figure 12 originally graphed by Crosbie and Bohn (2003), there are in total six variables that drive the default probability of a firm:

1. The current asset value
2. The distribution of the asset value at time H
3. The volatility of the future assets value at time H
4. The level of default point, the book value of the liabilities
5. The expected rate of growth in the asset value over the horizon
6. The length of the horizon, H.

In order to calculate default probability, Distance-to-Default measure must be first calculated. Distance-to-default (DD) is defined as:

$$DD = \frac{ln\frac{V_A}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}} \qquad (3.5)$$

Distance-to-Default value tells us, how many standard deviations a company is away from the default point (number 4 in Figure 12). Once we have calculated Distance-to-Default, it can be used to track down the number of companies, who went bankruptcy within a certain time period T from now, given the same distance-to-default. In practice this is conducted by comparing Distance-to-Default to a private database of firms defaulted over the past years, provided by Moody's.

Kealhofer (2003) highlights the importance of using a relevant database gathered from historically realized defaults. KMV Company has been gathering these data from all the publicly traded companies and their defaults since year 1973 in the United States. Comparison of that database and outcomes based on the normal distribution of asset value are significant in terms of probability of default. If the distribution of asset value is assumed normal, however, there could be significant differences between theoretical probabilities and realistic probabilities.

4 Research Methods and Data

4.1 Empirical Methodology

Empirical part consists of two different methodologies: qualitative and quantitative. Qualitative part consists of a semi-structured interview, which aims to shed light on questions arising from determining credit risk from the practical point of view. Furthermore, in this section certain characteristics of firms are discussed that have an effect on a firm's credit risk.

Quantitative section applies the KMV approach covered earlier to three different sectors in order to derive credit risk for each firm operating in one of these sectors: automobile industry, banking and financial sector and technology. The aim of these two different methodologies is to complement one another in order to form a more complete and valid illustration of determination of credit risk.

4.2.1 Semi-Structured Interview

Qualitative research was conducted as a semi-structured interview. The purpose of the semi-structured interview was to approach research questions from three main themes: credit risk modeling, firm characteristics and decision-making based on credit models. For a more detailed explanation of themes and actual interview questions, see Appendix 1.

Typical for a semi-structured interview is that the themes, around which the interview takes place are known in advance, but the questions are not known precisely (Hirsjärvi, Remes & Sajavaara 1997). This was the circumstance with the underlying study as well. Themes including some example questions of each theme were sent to an interviewee a couple of weeks beforehand. This allowed preparation for the interview.

Interview questions were open form questions. Leaving scope for answers, no precise pre-answers were given beforehand. According to Hirsjärvi et al. (1997) this can be considered as both, an advantage and disadvantage. On one hand the interview provides a lot of information on the research questions. However, on the other hand there is a risk of losing focus on the themes, and missing the most important points from the perspective of the research questions. Undoubtedly, there is a trade-off between the amount of information collected, and risk of losing focus with the semi-structured interview.

Bernard (1988) points out that semi-structured interview is great in the sense that the interviewer can obtain a lot of information on the research questions, and simultaneously there is a likelihood that new leads can arise during the interview. Leads can take the consideration of research questions to a new light, and yield answers that would not have been obtained otherwise.

The semi-structured interview took place on Wednesday 4, November 2015. Total time of the interview was approximately one hour. The interviewee was a credit professional, who has a sound background in the field of credit risk and risk management. The professional has been working in the relevant field for more than 20 years. Over the course of his professional career, he has gained experience from credit issues of Finnish companies of all sizes from different industries.

4.2.2 Modeling Credit Risk

The quantitative section of the study utilizes the KMV model developed by KMV Corporation (2003) to predict the actual probability of default. Expected default frequency (EDF), is calculated for selected firms from three different industries. The EDF is a firm specific value, and can be used to determine relative default risk of a company in comparison to other companies. Moreover, after the calculation of Distance-to Default, this value can be tracked to any rating system in order to

derive the equivalent rating of the obligor, for example, AAA rating or BB+ rating. For a more thorough explanation, see for instance Crosbie and Bohn (2003).

The KMV model was chosen for the study, as it combines information from both sources from historical values (balance sheet: liabilities) and future expectations (market value: stock price). Risk measures based on market information have been found to be more reliable measures than measures relying only on financial statements (Hillegeist, Keating, Cram and Lundsted, 2004). Therefore, by combining information from both types of sources, we aim to reach more reliable and consistent results.

Before applying the KMV model Merton's option-pricing model was used to estimate asset value and volatility for each selected firm. Given liabilities, equity value and volatility, asset value and volatility were calculated using solver in Excel to solve for two equations simultaneously. First initial guesses were given to asset value and volatility, after which Solver was used to reach final solutions. Two equations (asset volatility and asset value) were solved simultaneously, by minimizing the sum of squared differences between model values and observed values in order to reach values that satisfy the following conditions:

Observed equity value = equity value given by Black-Scholes
Observed equity volatility = equity volatility given by Black-Scholes

For a further reference on how asset value and volatility of asset was estimated using the Merton model, see Löffler and Posch (2007) and Appendix 3.

4.2.3 Assumptions

As stated previously under the methodology chapter, we assume that a default point lies somewhere between the current or short-term liabilities and long-term liabilities. Default point is given by the equation:

Short-term liabilities + (1/2) long-term liabilities = default point of a firm (4.1)

As suggested in the original KMV methodology document by Crosbie and Bohn (2003), liabilities are based on book values. However, in practice certain sectors such as banking and financial industry could have a different default point, as the sector is tightly regulated by regulators (Crosbie & Bohn 2003).

Furthermore, in order to calculate the probability of default in practice, we need to make certain assumptions on the shape of the distribution of the value of assets at the horizon *H* (Fig. 12). For simplicity, we assume that the distribution follows a normal distribution, although in practice this assumption does not hold. By assuming the shape of distribution to be normally distributed, we can calculate default probabilities for firms, given their Distance-to-Default. Finally, the expected asset growth is assumed to be 5,0 % and risk-free rate 2,0 % for each company included in the study.

4.3 Data

In order to explore the practical nature of the KMV approach in more detail, three different industries were chosen: 1) automobile industry 2) financial industry and 3) high-tech industry. Out of each industry four different companies were selected to represent each industry. All the selected companies are publicly listed and traded across several stock exchanges.

In order to calculate the market value of each company, Thomson One Database was used to collect stock prices from the past year from the time period of

6.10.2014 – 6.10.2015. Market capitalization graph for each selected company over the last year can be found in Appendix 2. The book value of debt was obtained from the balance sheets of the companies based on financial year 2014 reported values. When necessary, exchange rate conversions have been conducted. All the values are reported in euros.

Three different industries were chosen, since each industry is assumed to exhibit different characteristics from the other two. Automobile industry is very capital intensive, whereas financial industry is considered as highly leveraged, and high-technology industry is traditionally considered more volatile in terms of stock price in comparison to the other two, since the industry is based mostly on high expectations regarding future innovations and cash flows.

We expect automobile and banking industry to be quite constant in terms of asset volatility. However, the recent events of automobile industry (the German car maker incident with the gas emissions) might have had an effect on the industry volatility in general. On a general level high-technology industry is expected to be more volatile relative to the other two industries, since the industry is based mostly on expectations, and expectations can change quickly. Banking industry is traditionally highly leveraged due to its business model.

The following twelve companies were chosen to represent each sector, see Table 2 below. All the selected companies are traded publicly and internationally, and have operations across different countries in more than one continent.

Automobile	Financial sector	Technology industry
Volkswagen	Deutsche Bank	SAP
Tesla Motors	Danske Bank	Siemens
Mercedes-Benz	Commerzbank	Ericsson
BMW	Nordea	Nokia

Table 2. Sample of companies from three industries.

Further, the selected companies have currently received the following credit ratings by three major credit rating agencies. Credit ratings are found in Table 3 below.

	Moody's	Standard & Poor's	Fitch Ratings
Financial sector			
Deutsche Bank	A3	A	A+
Danske Bank	A2	A-	-
Commerzbank	Baa1	A-	A+
Nordea	Aa3	Aa-	AA-
Automobile			
Volkswagen	A3	A-	-
Tesla Motors		B-	
Mercedes-Benz	A3	A-	A-
BMW	A2	A+	-
Technology industry			
SAP	A2	A	
Ericsson	Baa1	BBB+	
Nokia	Ba2	BB	
Siemens	Aa3	A+	

Table 3. Credit ratings of the represented companies as of 2014.

It is expected that our results will yield similar results with those of the credit rating agencies, since the current situation has not changed dramatically from the year 2014, when credit ratings were updated last time.

4.3.1 Automobile Industry

Automobile industry has been reshaped over the recent weeks, since the car manufacturer Volkswagen admitted to cheating on Diesel emissions. Volkswagen had implemented software in its vehicles that modified the gas emission to meet the necessary standards during the test assessment. Subsequently, the stock price plunged by almost a quarter, wiping out 15,6 billion euros from the market value on the day when the news broke out (Bloomberg 21.09.2015). At the very bottom, the stock was valued at 60% of its initial value prior to the scandal (Bloomberg 02.10.2015). Undoubtedly, this incident had a significant influence on the market value of Volkswagen, and consequently on the value of its assets.

This event is expected to reflect in the credit risk as well, since the company bears now less equity relative to debt than before. Secondly, value of assets has decreased now, as the company is facing lawsuits by several governments, regulators and individuals. Volkswagen is forced to make necessary reserves to cover the upcoming expenses. Finally, as a consequence of such a high plunge in the stock price and upcoming lawsuits, the uncertainty surrounding the value of assets has most likely increased, bringing pressure on downgrading the current credit rating.

The Diesel gas emission scandal that took place at Volkswagen did not just hit the Volkswagen group, but the whole car manufacturing industry was shaken by it. Of all the carmakers, German carmakers were worst hit by the scandal. On the other hand, surprisingly the electric car manufacturer, Tesla Motors gained from the scandal in the short-term, as its share price increased. In the Figure 13, stock price changes (%) of car industry are presented, when the scandal hit the market.

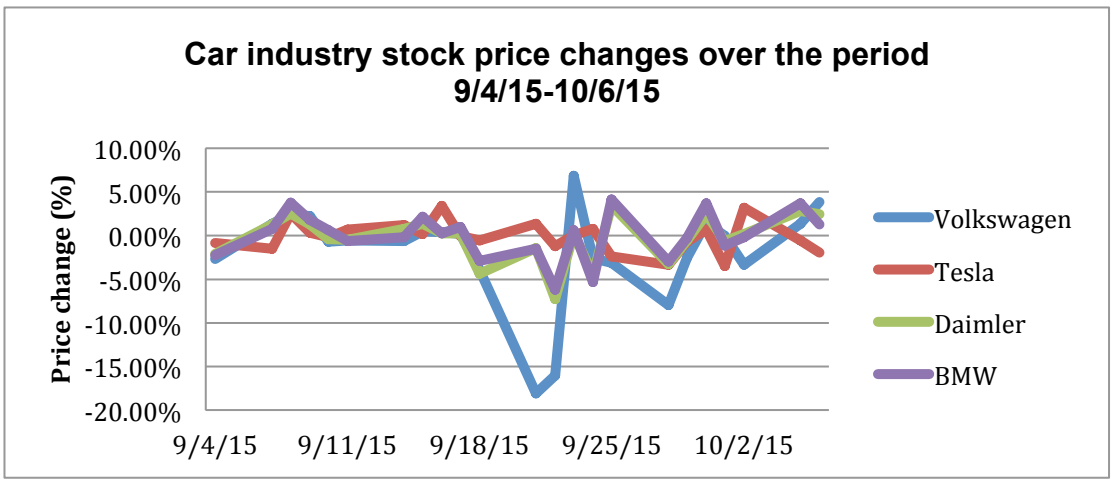

Figure 13. Price changes over the period of 4.9.2015 – 6.10.2015.

In terms of debt-to-equity ratios automobile industry exhibits three times the amount of debt in relation to equity, as can be seen in the Table 4. The only exception, which stands out is the electric car manufacturer, Tesla Motors that relies on debt financing almost twice as much as the other car companies. However, this is typical for a young innovative company, which has aggressively

financed its growth with debt. Arguably, it could be assumed that Tesla Motors exhibits higher risk due to the higher debt-to-equity ratio.

	Volkswagen	Tesla Motors	Mercedes-Benz	BMW
Total liabilities	261,02	4,88	145,05	117,37
Equity	90,19	0,91	44,58	37,22
Debt-to-Equity ratio	2,89	5,35	3,25	3,15
Default point	195,86	3,49	106,01	88,22

Table 4. Debt-to-Equity Ratios: automobile industry. Note that all the values are based on historical values and reflect the historical performance.

From the perspective of applying the KMV approach, calculation of default point is crucial. Default point for each firm can be seen in Table 4. Volkswagen has the highest default point, since it is the largest company of the four manufacturers, and Tesla Motors has the lowest respectively.

Furthermore, volatility of equity across different time frames (1, 3 and 5 year periods) was calculated, and daily standard deviation was annualized (multiplied by SQRT(250), assuming that there are 250 operating days a year) to correspond volatility at a yearly level. The volatility across different time frames is presented in Table 5. Mercedes-Benz and BMW are less risky in comparison to the other two firms from the standpoint of volatility of equity returns. Tesla Motors seems to be the riskiest firm in terms of equity volatility, however, when taking into account data from the recent year only, the level has decreased. Generally, in the long-term the volatility of equity of Tesla Motors has decreased from the past.

Time interval		Volkswagen	Tesla Motors	Mercedes-Benz	BMW
1 year estimate	Daily volatility	2,45%	2,46%	1,91%	1,94%
1 year estimate	Annualized volatility	38,70%	38,97%	30,17%	30,73%
3 year estimate	Daily volatility	1,84%	3,27%	1,61%	1,55%
3 year estimate	Annualized volatility	29,03%	51,70%	25,51%	24,46%
5 year estimate	Daily volatility	2,03%	3,40%	1,90%	1,89%
5 year estimate	Annualized volatility	32,03%	53,68%	29,97%	29,86%

Table 5. Volatility of equity of automobile industry based on different time samples.

4.3.2 Banking and Financial Sector

Arguably, as the news of Volkswagen emission scandal broke out, market values of the German banks (Deutsche Bank and Commerzbank in our study) were affected by it. Since the German car-manufacturers are one of the largest clients of the aforementioned banks. Secondarily, banks were most likely affected by the contagion effect of decrease in market value of several other industries (for instance, subcontractors) that are dependent on the German car industry.

As was already formerly discussed, the default point for banks is typically different from the default point for other firms in the KMV approach. There are multiple reasons for that. First, the Basel accords set certain thresholds for the minimum capital requirements that banks must hold as a reserve against the different types of risks that they are facing in their operations. Basel II set the minimum total capital threshold to 8 %, and Basel accord III is expected to make adjustments to the current capital requirements (Bank for International Settlements 2013).

	Deutsche Bank	Danske Bank	Commerzbank	Nordea
Total liabilities	1635,48	27,19	530,04	639,51
Equity	68,35	2,50	26,96	29,84
Debt-to-Equity ratio	22,34	10,87	18,68	21,43
Default point	-	-	-	-

Table 6. Debt-to-Equity ratios of banking and financial sector.

Secondly, typical for banking and financial sector are the high leverage ratios as can be seen in Table 6. Debt-to-Equity ration for each bank is at least 10,0 and the maximum ratio is more than 22,0. Debt-to-Equity ratio increases as the banks take in more deposits than withdrawals. As the banks' primary business model is taking in deposits and granting loans, the high leverage is natural for this industry.

Thirdly, banking industry exhibits different characteristics in relation to the other two industries from the standpoint of the use of derivatives. The use of derivatives is rather more a rule of thumb than an exception for the industry. Typical for the use of derivatives in the industry is that they are off-balance sheet instruments, not directly observable from the balance sheet.

For these aforementioned reasons use of liabilities in the KMV approach could be biased for measuring credit risk of the financial sector. The inability of Distance-to-Default approach in its current form to predict bankruptcy of financial firms is also recognized, and discussed in the paper by Chan-Lau and Sy (2006). Therefore, instead of using short-term and (1/2) x long-term liabilities for calculating the default point of a financial firm, Distance-to-Capital figure is used, as suggested by Chan-Lau and Sy (2006). The only difference between the different measures is that the distance-to-capital measure has a different default point. Mathematically Distance-to-Capital can be expressed as:

$$DR_T = \frac{ln\left(\frac{V_t}{\lambda L_t}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \tag{4.2}$$

Distance-to-Default $\lambda = 1$

Distance-to-Capital $\lambda = \frac{1}{1 - PCAR_i}$

Where the $PCAR_i$ component stands for the required capital adequacy ratio set by the regulators. In this case the required capital adequacy ratio is set by the Basel framework to be 4,00 %. The percentage equals to the minimum common equity capital ratio in 2014. (Bank for International Settlements 2013)

As for automobile industry, volatility of equity was calculated for banking and financial sector as well. Volatilities are shown in Table 7. Commerzbank and Deutsche Bank seem to have been more volatile in terms of equity across the 5-year period in comparison to their Nordic counterparts. Currently, Danske Bank seems to be the least risky of the sample based on the volatility of equity over the past year.

Time interval		Nordea	Danske Bank	Commerzbank	Deutsche Bank
1 year estimate	Daily volatility	1,72%	1,46%	1,77%	1,94%
1 year estimate	Annualized volatility	27,27%	23,13%	27,99%	30,64%
3 year estimates	Daily volatility	1,41%	1,49%	2,52%	1,70%
3 year estimate	Annualized volatility	22,32%	23,52%	39,86%	26,91%
5 year estimates	Daily volatility	1,68%	1,90%	2,97%	2,19%
5 year estimate	Annualized volatility	26,52%	30,11%	46,89%	34,57%

Table 7. Volatility of equity of banking and financial sector across different time periods.

4.3.3 Technology Industry

Supposedly, technology industry can be expected to be more volatile in comparison to the other two industries, since the business is considered to be highly dependent on new innovations and expectations. From the perspective of

debt to equity ratio Siemens has the most leverage from technology industry. In contrast to Siemens SAP and Ericsson are the least leveraged.

	Siemens	Nokia	SAP	Ericsson
Total liabilities	73,37	12,39	18,91	15,84
Equity	31,51	8,61	19,60	15,53
Debt-to-Equity ratio	2,33	1,44	0,96	1,02
Default point	54,98	9,84	13,73	13,31

Table 8. Debt-to-equity ratio of technology companies.

As for automobile and banking industry, volatility of equity across several time periods for technology firms is similarly presented in the following Table. Undoubtedly, Nokia seems to have been the most volatile in terms of equity over the past years across all time periods. Most likely this is a consequence of the business model change that the company has undergone from cell phones to networks.

Time interval		SAP	Siemens	Ericsson	Nokia
1 year estimate	Daily volatility	1,52%	1,41%	1,73%	2,23%
1 year estimate	Annualized volatility	24,05%	22,29%	27,29%	35,21%
3 year estimate	Daily volatility	1,28%	1,22%	1,49%	2,57%
3 year estimate	Annualized volatility	20,31%	19,37%	23,59%	40,63%
5 year estimate	Daily volatility	1,38%	1,37%	1,74%	2,97%
5 year estimate	Annualized volatility	21,89%	21,70%	27,43%	46,97%

Table 9. Volatility of equity of technology firms.

The other three companies seem to be equally volatile in terms of volatility of equity across all time frames. Interestingly, volatility of equity has not changed dramatically from 5-year period to 1-year period for Ericsson, SAP and Siemens.

5 Results

In the following sections the results from the KMV model and the semi-structured interview are discussed. The aim of this chapter is to present results obtained from the KMV model. These results are discussed and analyzed together with the results provided by the semi-structured interview.

5.1 Automobile industry

Asset value and volatility were calculated by using the Merton option pricing approach. The results for these figures are reported in the following 10 among with Distance-to-Default and default point.

	Volkswagen	Tesla Motors	Mercedes-Benz	BMW
Asset value	307,22	31,64	214,97	169,02
Asset volatility	6,49%	33,08%	10,22%	9,81%
Value of equity	51,38	30,35	72,84	53,98
Default point	195,86	3,49	106,01	88,22
Distance-to-Default	7,68	6,84	7,35	7,08

Table 10. Results of automobile industry

Surprisingly, Volkswagen has the highest Distance-to-Default followed by Mercedes-Benz and BMW, and finally Tesla Motors. Based on the obtained results, it seems that the emission scandal is not realized in the credit ratings. None of the companies above receives a significant probability of default, if the distribution of value of assets at the horizon H is assumed to follow a normal distribution. This was tested, by applying a NORMdist. fuction to Distance-to-Default figures. The function gave a probability of default of 0,00% to all companies. However, in reality the distribution of value of assets exhibits fatter tails based on the empirical evidence, meaning that the probability of default is higher than zero percent.

Based on the results in Table 10, Volkswagen seems to be the least risky of the automobile industry from the perspective of asset volatility. Tesla Motors on the other hand is the most risky of the sector. Multiple reasons can explain the

obtained results. Tesla Motors represents high-tech electric vehicle manufacturer, and seems to be valued accordingly. Based on balance sheet values, value of equity is valued at 0,91 billion dollars in comparison to that of the market value 30,35 billion USD (26,86 billion €). In contrast to other car manufacturers relatively high amount of equity to debt increases total asset volatility in this model.

Secondly, typically new companies, such as the one that Tesla Motors embodies, are more uncertain in contrast to traditional companies. This point also came up during the interview: certain companies have already established a good track of record, and therefore can be considered less uncertain; Volkswagen and other car manufacturers have existed longer than Tesla Motors, and consequently there is more information available about them in contrast to Tesla Motors. During the interview information regarding payment history, and especially on-time timeliness of payments came up to be one of the main factors in determining credit risk. As other car companies have existed longer, they have a longer track of payment records, which most likely lowers credit risk.

Thirdly, Tesla Motors represents new kind of car industry. Electric cars have not existed for such a long time. As a consequence the value of these assets might not be as reliable as other cars' in a secondary market. Over the course of the interview, it was mentioned that the availability of a secondary market greatly decreases credit risk of the underlying firm, as the creditor can liquidate and sell the asset in the worst-case scenario. For financial firms, who finance new firms, it is important that there is a decent secondary market for the assets for these reasons.

In spite of the aforementioned statements and high volatility of assets, Tesla Motors received quite a high Distance-to-Default number. High Distance-to-Default number might be explained by the fact that the value of assets is high – there is room for a loss of value before the firm goes bankrupt. However, by taking a closer look at the value of assets most of it consists of equity, which is somewhat based on market participants' expectations about the future. It should be noted that there is a probability that equity could lose its value quickly. Over the past years volatility

of equity, as reported previously in Table 5, has been changing for Tesla Motors from 53,68 % to 38,97 %. Therefore, Tesla Motors might have received a higher Distance-to-Default number than originally anticipated due to high expectations that markets are laying on it. Higher amount of equity relative to debt could explain the higher volatility.

Surprisingly, Volkswagen received the highest Distance-to-Default number of all the car companies. The loss of equity value did not result in significant differences. Volkswagen is still in the same group in terms of Distance-to-Default number with the other traditional car manufacturers. The company was estimated to have the lowest volatility of assets, meaning that the value of assets did not change significantly over one year time period.

5.2 Banking and Financial Sector

Similarly, as with the automobile industry, the Merton model was used to estimate value of assets and asset volatility for the Banking and Financial sector. The Merton model estimated volatility of assets in financial sector to be lower relative to other industries. These estimates are in line with Crosbie and Bohn (2003). Based on the obtained results financial sector characterizes lower asset volatility in comparison to the other two industries, as can be seen in Table 11. In terms of asset volatility German banks are not as volatile as their Nordic counterparts. This could be due to many different reasons such as a more geographically diversified portfolio of assets or operations in less risky locations.

In terms of asset value the Merton model estimated Deutsche Bank to be the largest of the selected group with the asset value of 1637,79 € billions followed by Nordea, Commerzbank and Danske respectively. On a general level the low equity/total asset ratio might also explain the low asset volatility of the industry, since the Merton model tends to estimate overall volatility of assets towards volatility of debt, which is presumably lower than volatility of equity in this case.

As was already formerly discussed defining the default point for this sector is challenging, therefore different default measures were calculated to give an overview of credit risk at different default points. Chan-Lau and Sy (2006) suggest the use of Distance-to-Capital. Distance-to-Capital ratios are reported with Distance-to-Default figures in Table 11.

	Nordea	Danske	Deutsche Bank	Commerzbank
Asset value	669,48	460,93	1637,79	530,73
Asset volatility	1,74%	1,38%	0,65%	0,59%
Value of equity	42,64	27,50	34,70	11,18
Default point	639,15	442,19	1635,48	530,04
Distance-to-Capital	3,16	3,67	1,63	1,77
Distance-to-Default	5,51	6,63	7,91	8,69

Table 11. Results of Banking and Financial sector.

Predictably different distance measures yielded contradictory results. Based on the Distance-to-Default figure Commerzbank is furthest away from default followed by Deutsche Bank, Danske and Nordea respectively. However, when the model was calibrated to take into account the regulatory capital requirements as required by the Basel framework, Danske and Nordea performed strongest from the group. In terms of Distance-to-Capital Danske and Nordea are furthest away from default, while Commerzbank and Deutsche Bank fall in another group.

Furthermore, results obtained by Distance-to-Capital measure are also supported by the total capital ratios reported by each bank of the sample. Bank for International Settlements requires each bank to calculate, and report their total capital ratio to their exposure to unexpected events such as credit losses. The higher the total capital ratio, the better the capacity of a bank to absorb credit losses (Bank for International Settlements 2010). From the perspective of the total capital ratios Nordea and Danske have the highest ratio relative to their risk exposures. On the basis of total capital ratios this could indicate that Distance-to-Capital is indeed better suitable for estimating credit risk in the financial sector than Distance-to-Default.

	Nordea	Danske	Deutsche Bank	Commerzbank
Total capital	30,049	22,44	68,29	31,47
Total risk exposure amount	145,48	116,02	396,65	215,18
Total capital ratio	20,66%	19,34%	17,22%	14,63%

Table 12. Reported capital ratios. Source: Balance sheets.

Based on Distance-to-Capital Danske seems to have the highest credit rating followed by Nordea, Deutsche Bank and Commerzbank respectively. Lower Distance-to-Capital of the German banks could be arguably because of higher exposures to countries with higher risk. In this case as the volatility of assets is low for the whole industry, the low Distance-to-Capital of German banks is mainly driven by high liabilities, or either by low asset value. Their Nordic counterparts have relatively lower liabilities or higher asset value.

However, these results should be interpreted carefully, as the default point only takes into account liabilities that are reported on the balance sheet. This leaves out off-balance sheet liabilities that most likely contribute to the overall credit risk. Hence, banks are required to report their outstanding derivatives in a more detail in their annual reports. Nevertheless, the KMV model is able to estimate ordinal credit rating for the banking sector.

5.3 Technology Industry

The results from technology industry are presented in the following Table 13.

	SAP	Siemens	Ericsson	Nokia
Asset value	89,74	142,40	45,07	37,95
Asset volatility	19%	11%	18%	24%
Value of equity	71,21	70,48	29,54	25,80
Default point	13,73	54,98	13,31	9,84
Distance-to-Default	10,01	9,02	7,01	4,12

Table 13. Results of technology industry: asset value and volatility, value of equity, default point and Distance-to-Default.

Based on the Distance-to-Default number of the KMV approach, SAP and Siemens performed best from technology industry followed by Ericsson and finally Nokia. The overall Distance-to-Default number of SAP and Siemens is better than any of the companies of the car industry (see Table 10 in the previous section). From the perspective of asset volatility Siemens is the least volatile (11,00 %) followed by Ericsson and SAP with 18,00 % and 19,00 % respectively. Nokia has the highest volatility of assets (24,00 %), and this most likely reflects in its Distance-to-Default figure. Furthermore, most of Nokia's asset value consists of equity, which tends to increase overall asset volatility in the model.

As with the automobile industry, none of the companies from technology industry received a significant probability of default, if the values are fit to the normal distribution. Our obtained results are consistent with those of the credit rating agencies. Siemens and SAP both had received best credit rating from this group in 2014 from the credit rating agencies, whereas Nokia's rating was the lowest. Initially Ericsson was placed in between SAP and Nokia (Table 3).

There could be multiple reasons behind Nokia's Distance-to-Default figure and credit rating. Nokia has gone through a lot of changes lately. The changes in its business model have laid pressure on its stock price in both directions. The company gave up on its cell phone division, which initially formed the core of its business model, and sold its map division to a group of car manufacturers. After structural changes the new focus is on networks.

During the interview the type of industry also received attention. Certain industries are more stable, meaning that cash flows do not differ from one year to another (for instance, retail business versus tourism), whereas in certain industries cash flows tend to be more uncertain. In fact, this could be realized as a higher credit risk. In the KMV approach these companies usually have a higher volatility of assets.

In comparison to Siemens, Nokia has a very focused product portfolio: cell phones (earlier) and networks (currently), whereas Siemens offers many different types of

devices. The demand for these devices is arguably overall higher. From this standpoint Siemens is more diversified in terms of product portfolio than Nokia. Therefore, its volatility of assets is most likely lower than Nokia's, and subsequently Distance-to-Default is increased.

In the interview it was mentioned that certain upcoming investment possibilities might have an impact on firm's current credit worthiness. If the sector is considered growing, it is easier for firms to get funding. This has likely posed a challenge for Nokia, since cell phone industry is highly competitive and considered a saturated market. On the other hand firm's current strategy: networks is considered to be a developing sector with growth potential in the future. However, the results of this strategy do not seem to have realized in the KMV model.

5.4 Implications for Practical Decision-making

The KMV model can estimate credit risk by taking into account many different components, such as general market expectations, current debt capacity and past performance. Therefore, it can be argued to perform better than the traditional credit risk models, which rely primarily on historical information. Nonetheless, the estimates by the KMV model should not be blindfolded trusted.

Specifically in practice or practical decision-making, credit risk models are regularly accompanied by a greater qualitative analysis of the underlying firm's credit risk. Typically, there is need for a broader in-depth analysis, as there happens sudden changes in the company or its markets. In the interview it was denoted that such situations that need a greater in-depth analysis are often surprising changes in a firm's financial situation, for instance, the lack of liquidity or another dramatic change. Generally credit risk models only predict credit risk for one year in advance, and credit risk is estimated on a yearly basis. This underlines the importance of a broader analysis. Similarly, the major leading credit rating agencies analyze firms and countries in a broader context, and publish their

qualitative analysis to the public simultaneously, as letter credit ratings are published.

Furthermore, in the interview it was mentioned that at certain times the estimation of credit risk alone does not suffice to form a complete assessment of the underlying firm's credit risk. In these circumstances, it is often essential to consider other features of the firm such as: balance sheet in general, market outlook (competitors and demand for products), and what investment opportunities exist there for the firm (what the firm could invest in, if it was given credit).

As was discussed in the interview, certain balance sheet items are given a more cautious emphasis than others in the estimation of credit risk. Typically intangible assets such as goodwill are treated more carefully - in comparison to tangible assets - specifically when goodwill amounts to a relatively large sum on the balance sheet. Moreover, another balance sheet item that typically receives the attention is receivables: how rapid is the turnover of the receivables, as this influences the underlying firm's liquidity, and consequently its ability to pay back its credit on time.

More comprehensive analysis of other features affecting credit risk comes in question particularly at times of unexpected changes in the firm's financial state or environment, or when a firm is applying for a credit over a longer period of time. Usually credit risk models do not yield reliable estimates over longer time horizons, as credit risk models typically only estimate risk for one year from now. When the estimate horizon increases, the models become more inaccurate, therefore ordinarily other sources of information are needed to complement the assessment of credit risk.

More importantly, the KMV approach makes a number of different assumptions that need to be taken into account, when considering the estimates of the model in the actual decision-making situation. In practice firms can settle their debts with their creditors, or suggest a new payment schedule. A Firm is not immediately bankrupt, when the value of its assets falls below the default point. More

importantly, it is also of creditors' interest that a company will pay its obligations rather later than never. Secondly, based on the studies the value of assets is not normally distributed. In reality the distribution of value of assets at the horizon T exhibits fatter tails (extreme values are more likely) meaning that the results of the model could diverge significantly from real values.

Thirdly, the KMV model provides easily applicable, simple and understandable tool for measuring credit risk. As a concept Distance-to-Default and Distance-to-Capital are clear to understand even for someone, who is not very familiar with the field. However, as a downside the KMV model only produces a single number, which illustrates credit risk and from which credit rating is derived, consequently ignoring the uncertainty around the figure. In this sense credit risk estimation tools that provide the whole distribution of probable outcomes, and show potential losses as Xth percentile of the distribution are more useful. On the other hand, these kinds of tools require more understanding about the field in general.

6 Conclusion

6.1 Discussion

The object of this thesis was to focus on credit risk estimation with a practical example of credit risk estimation using one of the covered credit risk measurement tools. Simultaneously, the aim was to discuss different factors affecting credit risk of the selected twelve firms from three different industries on the basis of the KMV model and the semi-structured interview. The selected industries for modeling credit risk were automobile, banking and financial sector and technology industry.

The main research question was stated: **"How is credit risk estimated in the modern B2B context?"** The question was examined by covering a number of different journal articles, and by applying the KMV model to the selected sample of companies. Currently, there are multiple methods for estimating credit risk. In the literature review of the thesis we covered the development of different techniques for assessing credit risk over the past century, beginning with the subjective individual (a banker's) assessments on a company's credit risk, and concluding with the most developed models: CreditMetrics, the KMV approach, CreditRisk+ and CreditPortfolioView currently available for modeling credit risk.

The literature review (Chapter 2) answered to the sub-research question: "**What kinds of different credit risk models for B2B do currently exist?"** Based on the literature review evidently credit risk can be modeled and estimated in many different ways (see previous paragraph). Different models take a different approach on estimating credit risk, therefore occasionally yielding distinct results from each other. We presented an overview on how current credit risk models view credit risk in Table 1 in chapter four.

The sub-research question: "**What are the main factors of credit risk?"** was covered by studying a number of articles focusing on characteristics of credit risk. Factors were also discussed in the interview. Credit risk is driven by many different

factors, of which most significantly by certain firm-specific characteristics and macro-economic conditions. In general firm- specific characteristics such as a higher leverage and greater uncertainty (volatility) of future cash flows contribute to the underlying firm's credit risk. Based on the earlier literature and the interview, the past payment history or changes in payment schedule tend to predict credit risk: companies with fixed on-time payment plans are more likely to have a lower credit risk. From this perspective historical payment schedule seems to be a good proxy for estimating credit risk.

From the macroeconomic perspective, the current state of the economy is a significant cause contributing to credit risk. When the economy is in recession, more companies are to suffer financial losses or default, and consequently the credit risk tends to be greater. Certain industries are more affected by the state of the economy than others due to the nature of cash flows.

The final sub-research question: **"How are credit risk models used in practice or in practical decision-making?"** was considered in the interview. In general credit risk models provide their users with a good overview of credit risk of the underlying business. However, estimates of the models should not be blindfolded trusted, especially when a sudden change occurs in a company's financial situation or business environment. This highlights the need for a broader qualitative analysis. As a result, qualitative information regarding credit risk is often used together with quantitative analysis.

6.2 The KMV Model

In the practical section of the thesis credit risk was assessed for twelve companies by employing the KMV model. The KMV model was selected, as it combines both historical information (balance sheet based) and future expectations (market value), unlike most of the other models.

The KMV model was used to estimate Distance-to-Default for all the selected companies. Additionally, Distance-to-Capital was estimated for banking and

financial sector due to the nature of the sector. The ordinal ranking by credit risk was generally similar to that of the previous credit ratings given by the major credit rating agencies.

Volkswagen was not affected significantly by the emission scandal, as we initially expected. The company still received the highest Distance-to-Default figure of the automobile industry. Unpredictably, Tesla Motors received relatively high Distance-to-Default due to the high value of equity, as markets valued the company at a high value, and it reflected on the company's credit worthiness subsequently. This rises up a question, whether a high credit rating is correct, as it is based mostly on market expectations in the model.

Precise letter ratings were not obtained; as we did not have an opportunity to use the historical database covering companies default rates and probabilities of default, given their Distance-to-Default. It would be interesting to compare obtained results to the actual historical database.

Assessment of credit risk for companies in banking and financial sector presented a challenge, since the industry is characterized by a relatively high level of leverage. This is in line with Crosbie and Bohn (2003). For this reason, Distance-to-Capital was used instead of Distance-to-Default in the KMV model, as suggested by Chan-Lau and Sy (2006). The selected method yielded more consistent results with the existing credit ratings and those based on the Basel framework. However, this sector keeps receiving the continuous attention of the regulators. Therefore, it is anticipated that new tools for assessing credit risk will be developed and come into effect with the latest Basel accord.

6.3 Final Concluding Remarks

In the light of the covered literature and the study, measuring credit risk is a challenging assignment. Models rely on many different assumptions depending on each specific model. Therefore, it is crucial to take into account these assumptions (for instance, default point, interest rates and assumed growth rate of assets) in

the decision-making situation. Typically, credit risk models yield best possible estimates during normal market conditions, and tend to underestimate risk at times of unexpected events.

In general, credit risk models aim to quantify credit risk for a more understandable form, and give a good overview on a firm's creditworthiness for a year from now. However, under certain circumstances other qualitative characteristics of a company need to be reviewed. Usually this comes to analyzing items of the balance sheet, considering a firm's strategy and investment policy, as these affect a firm's future cash flows and its long-term profitability. Quantitative results are frequently accompanied with qualitative analysis on the firm's future aspects.

For additional research, it would be interesting to test our sample firms using other credit risk models covered in this thesis and compare results. Would the results be similar or different? Furthermore, as we obtained the results from over a relatively tranquil period in the market, it would be interesting to test how the KMV model performed over more turbulent market conditions such as a market crash. Would the model yield similar results, or would the results distinct from the previous? How would such an event have an effect on credit risk?

We also chose not to include artificial neural networks used for credit scoring in the study. As an additional research topic, it would be intriguing to test and see, what kind of results those models could produce, given the same sample. Moreover, as this research only contained a single interview, several interviews should be conducted using the same question form in order to gather a wider and more in-depth perspective on the topic.

Credit risk is definitely a subject that keeps attracting the attention in the future as well. New models will be proposed, as the computational power increases and programs develop. In our study we focused only on firm-specific credit risk (Figure 1), restricting our attention to credit risk, which typically relates to the cost of debt, when a company issues new debt. In the meanwhile, however, we neglected credit risk of certain derivatives to which some firms are exposed primarily today.

The latter field of credit risk modeling is still to develop to a great extent. Finally, to conclude understanding of credit risk is important for corporations and individuals alike - without a proper understanding - consequences could be severe.

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APPENDICES

Appendix 1 Semi-Structured Interview Questions

Appendix 2 Sample Companies Market Capitalization

Appendix 3 Merton Model for Estimating Asset Value and Volatility

Appendix 4 KMV Model for Estimating Distance-to-Default

Appendix 1 Semi-Structured Interview Questions

Firm characteristics

1. How would you describe: what are the three most important dimensions of a firm, when determining its credit worthiness? (for instance: industry, leverage or management)
2. How is the role of firm industry relevant to your decision to grant a loan?
3. What is the role of the type of assets (tangible versus intangible assets) in the assessment of credit risk?

Credit Risk Modeling

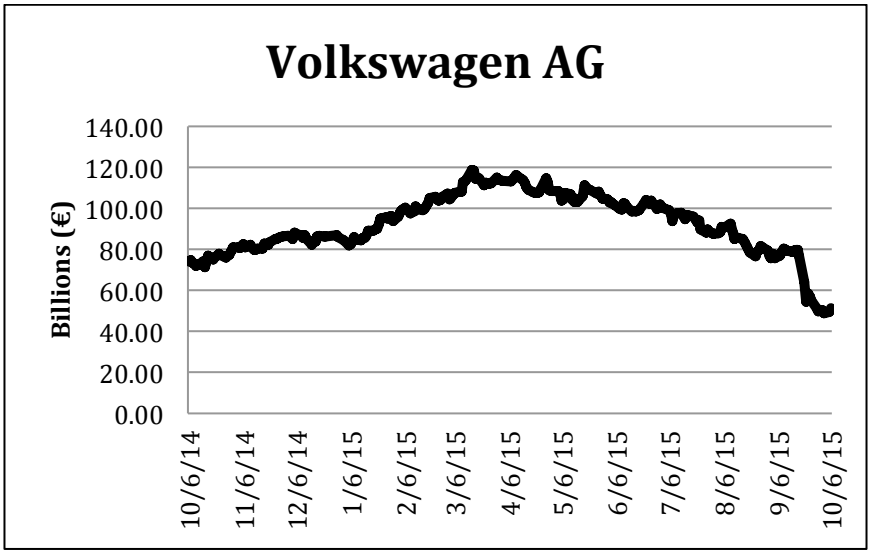
1. Which model(s) do you employ in determination of credit risk?
2. How do you consider the state of the economy when determining credit risk?
3. What is usually the time span of credit risk determination?
4. How often is the information updated?

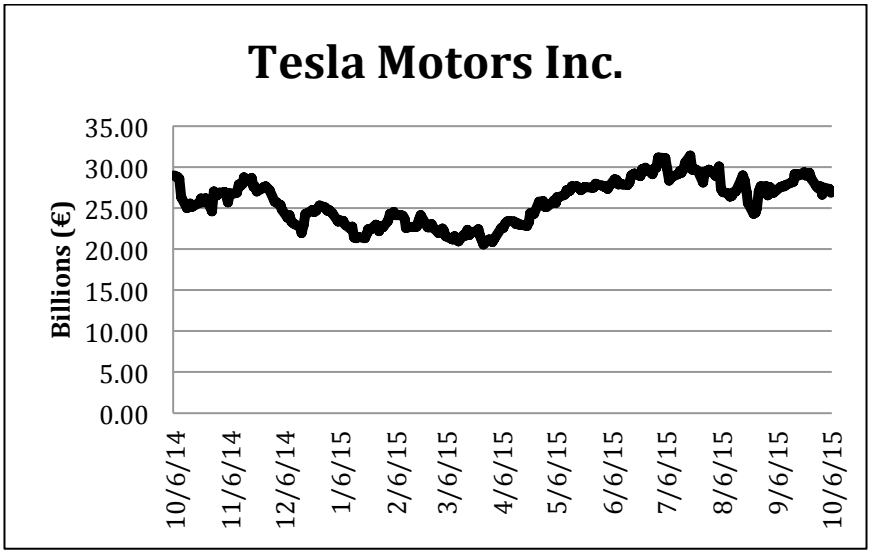
Decision-Making Based on Credit Risk Model

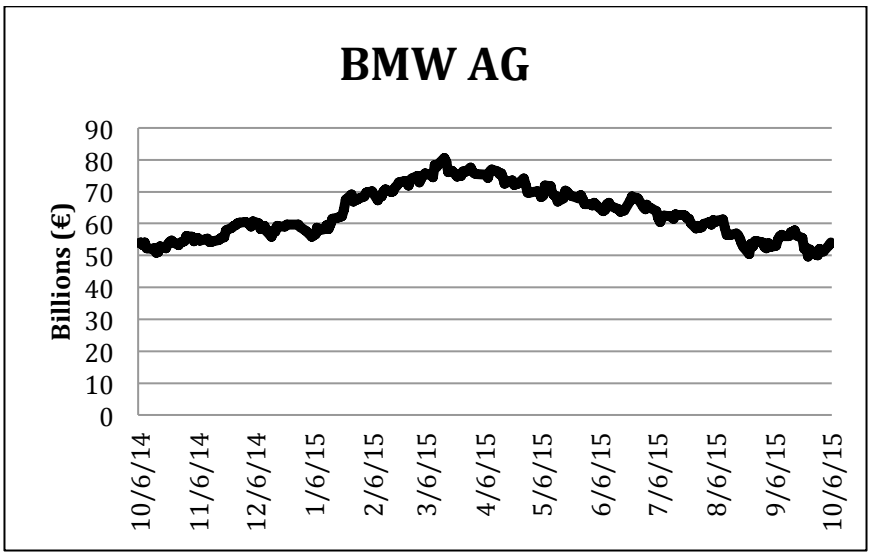
1. What is the role of information that is currently available on the firm when determining credit assessment?
2. How do you interpret the past performance of the firm affecting its credit worthiness?
3. If a firm has neglected its debt payments, does this have an effect on its probability of getting a new loan? Would it be possible for such a firm still to get a loan? And how would this happen in practice?
4. How reliable do you consider the results of credit risk determination?
5. When determining credit risk do you take into account anything else that is relevant regarding the underlying risk?

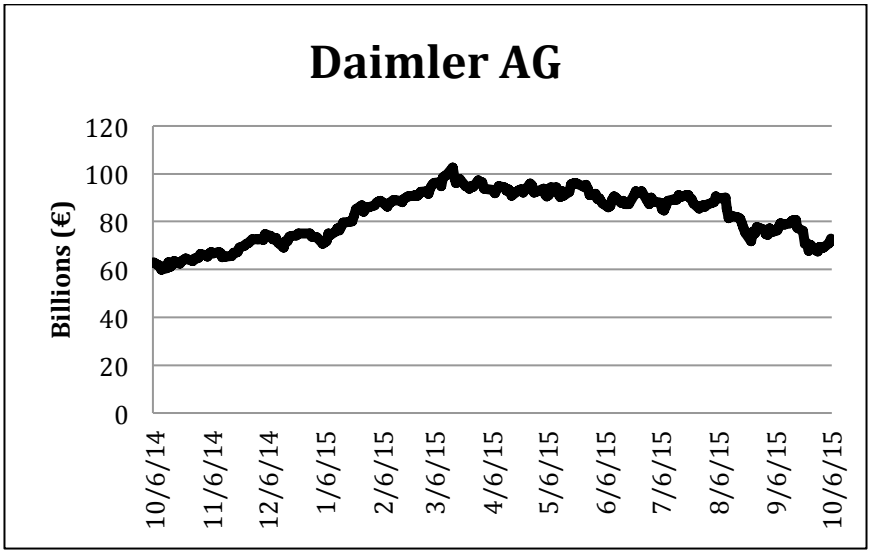
Appendix 2 Market Capitalization of the Sample Companies

Automobile Industry

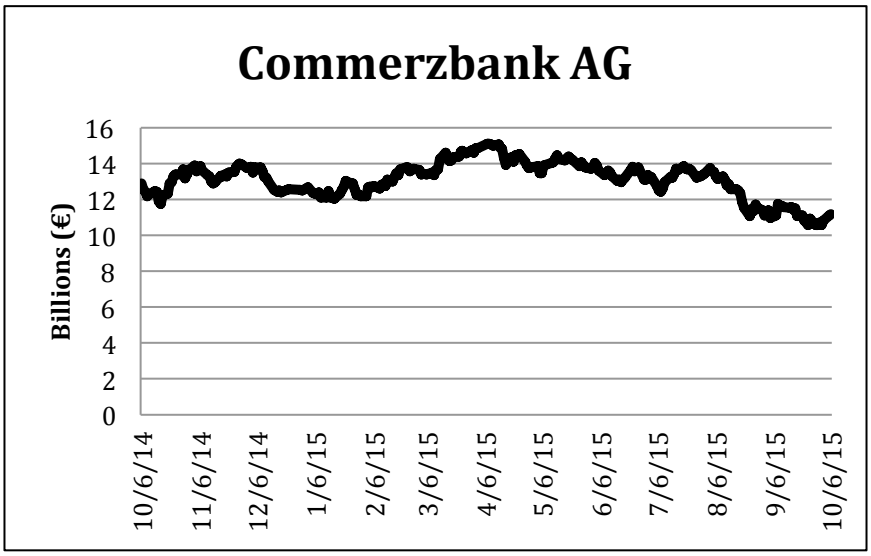


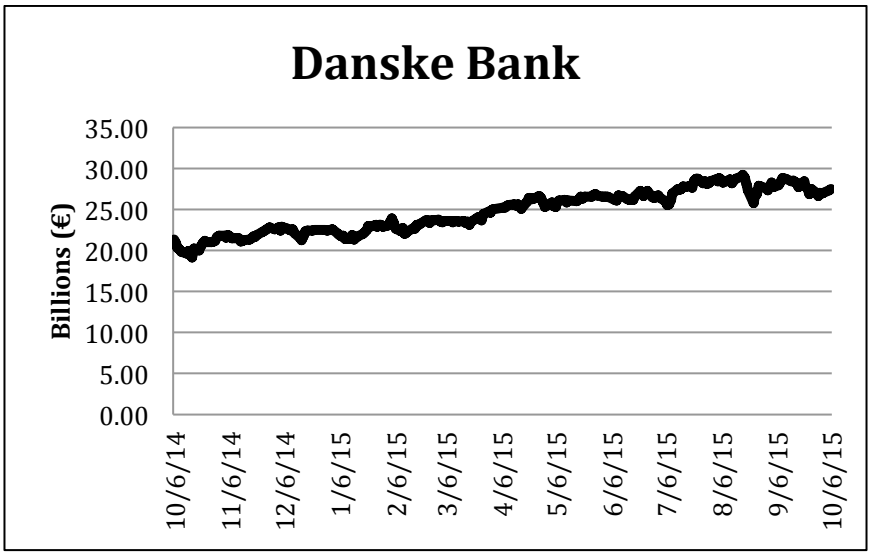


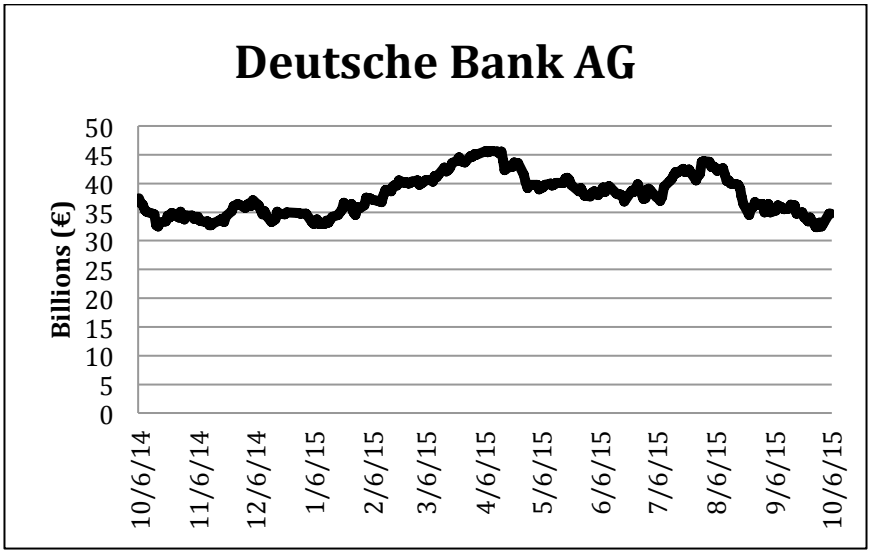


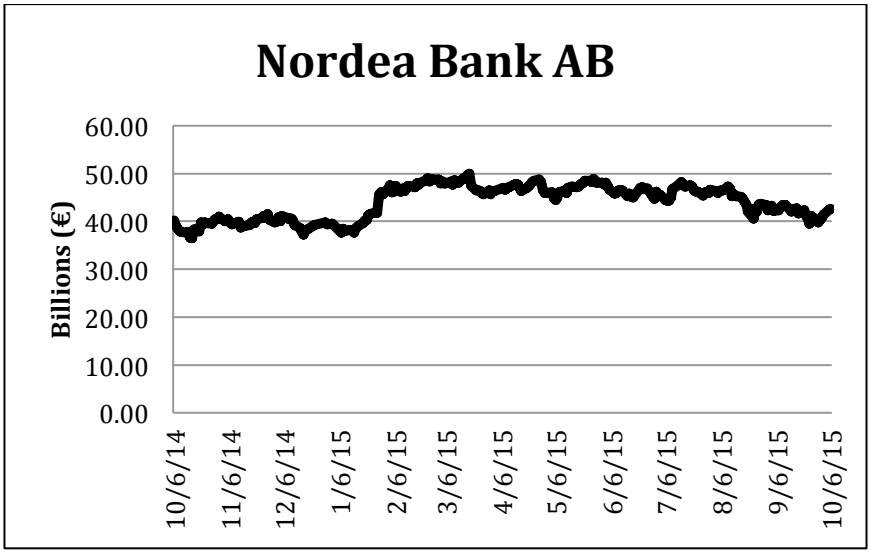


Banking and Financial Sector

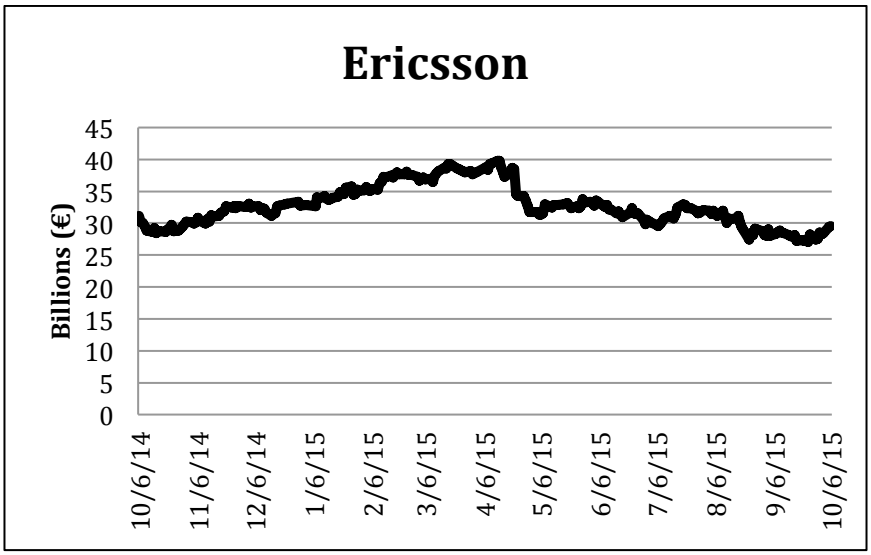


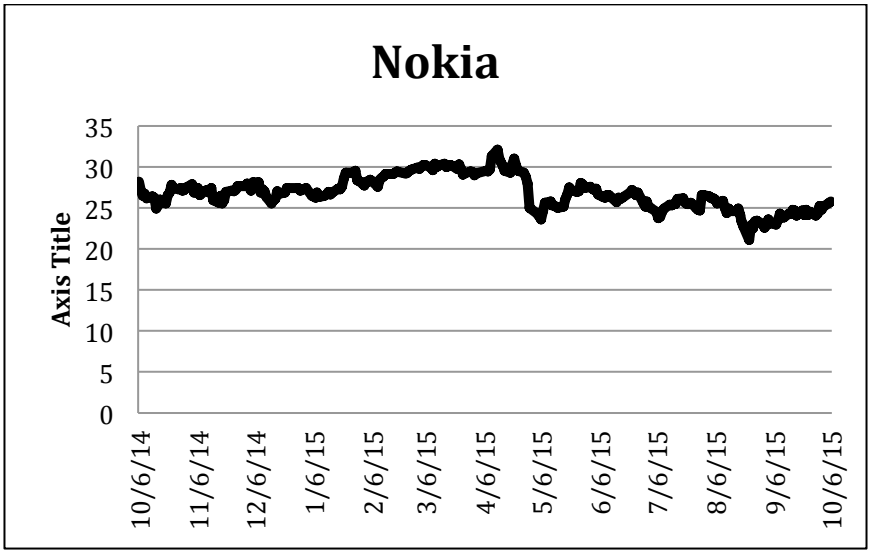






Technology Industry

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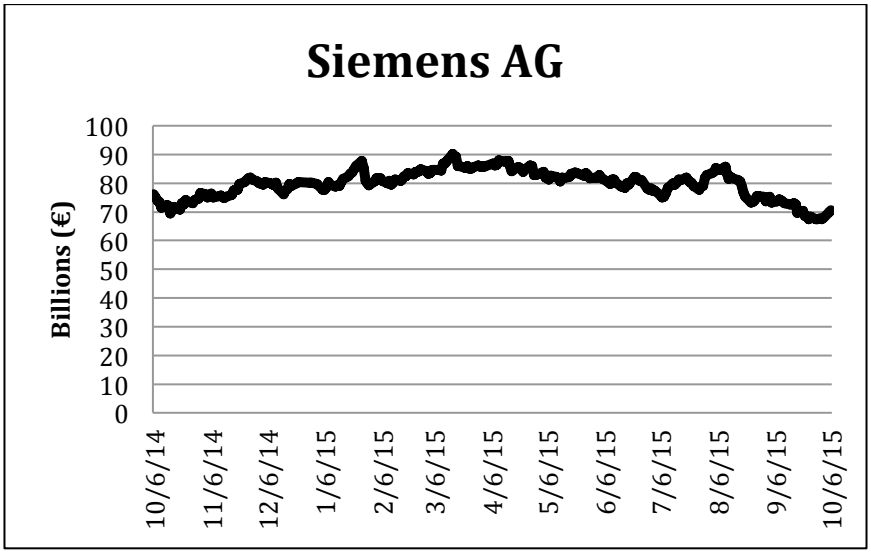
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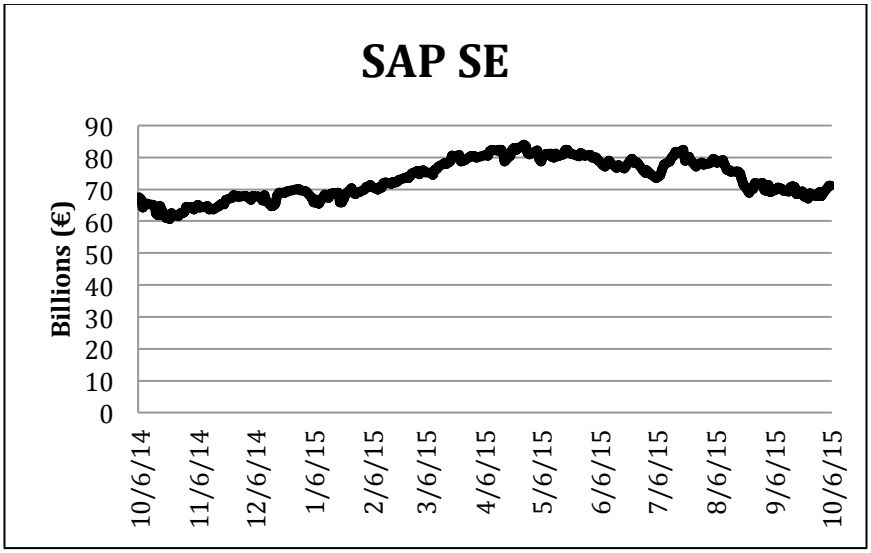
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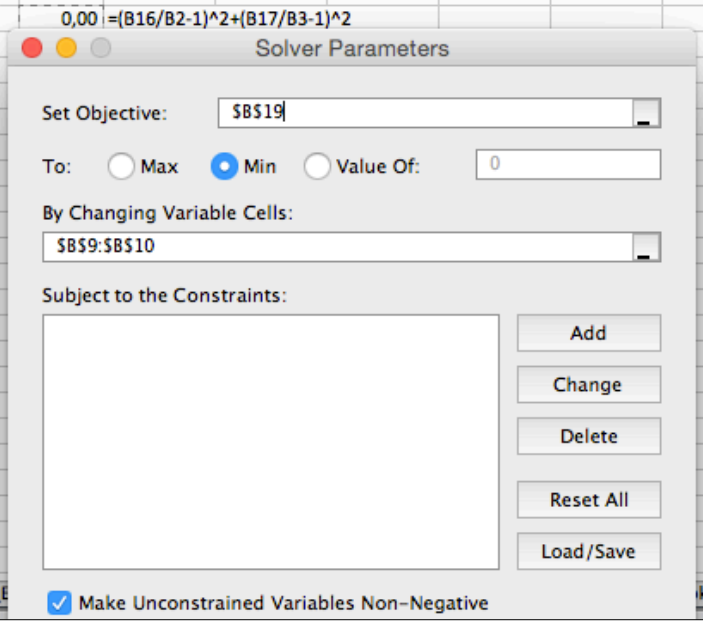
Appendix 3 The Merton Model for Estimating Asset Value and Volatility

	A	B	C
1	**Data/Assumptions**		
2	Equity Value	29,54	
3	Equity volatility	0,27	
4	Liabilities	15,84	
5	Risk-free rate	0,02	
6	Horizon (T-t)	1,00	
7			
8	**Unknowns**		
9	Asset value	45,07	initial assumption/guess B2 + B4 (=Equity + Liabilities)
10	Asset volatility	17,89%	initial assumption/guess = B3*B2/B9 (= Equity volatility * Equity/Total Assets)
11			
12	**Model values from Black-Scholes formulae**		
13	d1	6,05	=ln(B9/B4)+(B5+B10^2/2)*B6)/(B10*B6^0.5)
14	d2	5,87	=B14-B10*B6^0,5
15	Equity value	29,54	=B9*NORMSDIST(B14)-B4*EXP(-B5*B6)*NORMSDIST(B15)
16	Equity volatility	0,27	=(B9/B16)*B10*NORMSDIST(B14)
17			
18	**Objective:minimize dviation data - model**		
19	Squared rel. Errors	0,00	=(B16/B2-1)^2+(B17/B3-1)^2



Appendix 4 The KMV Model for Estimating Distance-to-Default

	A	B	C
24			
25			
26	**Model Values**		
27	Asset value	45,07	
28	Asset volatility	17,89%	
29	Horizon (T-t)	1	
30	assumed expected growth	0,05	
31	Default point	13,309	
32			
33	Numerator	1,2537	=LN(B27/B31)+(B30-B28^2/2)
34	Denominator	0,1789	=B28*B29
35	Fraction (Distance-to-Default)	7,0095	=B33/B34
36			
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$$DD = \frac{ln\frac{V_A}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}$$