LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
School of Engineering Science
Degree Programme in Computational Engineering

Janvier Ukwizagira

ANIMAL SPIRITS IN FINANCIAL MARKETS: AGENT-BASED MODEL

Examiners: Associate Professor Tuomo Kauranne
D.Sc. Matylda Jabłońska-Sabuka

ABSTRACT

Lappeenranta University of Technology

School of Engineering Science

Degree Programme in Computational Engineering

Janvier UKWIZAGIRA

Animal Spirits in Financial Markets: Agent-Based Model

Master's thesis

2016

49 pages, 29 figures, 4 tables

Examiners: Associate Professor Tuomo Kauranne

D.Sc. Matylda Jabłońska-Sabuka

Keywords: Animal spirits, Agent-based model, Financial markets

In this work an agent based model (ABM) was proposed using the main idea from the Jabłońska-Capasso-Morale (JCM) model and maximized greediness concept. Using a multi-agents simulator, the power of the ABM was assessed by using the historical prices of silver metal dating from the 01.03.2000 to 01.03.2013. The model results, analysed in two different situations, with and without maximized greediness, have proven that the ABM is capable of explaining the silver price dynamics even in utmost events. The ABM without maximal greediness explained the prices with more irrationalities whereas the ABM with maximal greediness tracked the price movements with more rational decisions. In the comparison test, the model without maximal greediness stood as the best to capture the silver market dynamics. Therefore, the proposed ABM confirms the suggested reasons for financial crises or markets failure. It reveals that an economic or financial collapse may be stimulated by irrational and rational decisions, yet irrationalities may dominate the market.

Acknowledgements

I would like to express my sincere gratitude to my supervisors D.Sc. Matylda Jabłońska-Sabuka and Associate Prof. Tuomo Kauranne for your continuous support of my thesis. Your patience, motivation, guidance and immense advice helped me from the starting to the writing of this thesis. I could not have imagined having excellent supervisors like you.

Lappeenranta, April 16, 2016.

Janvier UKWIZAGIRA

Contents

List of Symbols and Abbreviations

ABM	Agent Based Model
ACF	Autocorrelation Function
EMH	Efficient Market Hypothesis
JCM	Jabłońska - Capasso - Morale
MSE	Mean Square Error
NYSE	New York Stock Exchange
PACF	Partial Autocorrelation Function
USD	United States Dollar

1 INTRODUCTION

Animal spirits are mostly viewed as psychological forces that pull human decisions towards unpredictable behaviour, Keynes (1936) and Jabłońska (2011). This unpredictability has been considered as a stab in the back to the trading environments. In the trading world, investors are instantly worried about loosing their capital investments due to the risks that might be present. The more the trade relies on an incoming information, the riskier it is as information asymmetries exist and fitfully may cause the trade to go askew. Similarly to other types of markets, financial markets have been found victims of this asymmetric behaviour of information. Therefore, considering the big amount of money which circulates in those markets, they are worth very much attention. Despite that numerous researches have been done on the failure of these markets, the real reasons of failing or proofs of explanation remain obscure, see Claessens and Kose (2013).

When trying to understand the economic and financial collapses, researchers have been building different models from which some of them explain the dynamics of the markets well. Jabłońska (2011) has proposed a dynamic model named Jabłońska-Capasso-Morale (JCM) model which employs most of revealed factors that may lead to market failure. It considers the mean reversion concept, market momentum and farthest neighbor (neighbor with maximal price) effects. The latter model accurately captures the main market price features like spikes and jumps compared to the other proposed models. In this course, an agent based model (ABM) which considers the JCM ideology and maximal greediness of every trader is developed. The historical prices of silver metal, from the 01.03.2000 to the 01.03.2013 were used to test the ability of the proposed ABM to capture the silver market dynamics.

The subdivision of the work is organised as follows. The following section (Section 2) provides a short summary of background of financial markets , agent based modelling, animal spirits and some literature review. In Section 3, an agent based model (ABM) has been developed using an equation based model (JCM) and maximal greediness ideology. The description of the data used in this study, some of the statistical tools used to compare simulations results and the original prices as well as the agent simulator are detailed in this section too. Section 4 discusses the results obtained in two different situations. Firstly, it presents the results obtained when greediness is not maximized on incomplete models. Secondly, the results on the case where the greedy emotions of each market participant are maximized. In the same section, the model performance is analysed by comparing the ABM in two different situations (with and without maximal greediness) and a case prediction using the ABM without maximal greediness. Section 5 tells the results, summary and discussions whereas Section 6 presents conclusions and further work.

2 BACKGROUND AND LITERATURE REVIEW

The place where buyers and sellers trade assets such as securities, commodities and other fungible items, is known as a financial market. Financial markets have been found to have crucial impact on our economy as they channel funds from savers to investors thereby contributing to economic growth. Also, it has been noticed that their activities affect personal wealth, the behavior and the economy of business entities. Together with other innumerable reasons, researchers have got interest in this field to investigate which could be the optimal way of trading in these markets. During a trading event two people, a buyer and a seller, have to make and accept their own decisions in order to have a complete transaction. The seller has to decide whether he (she) accepts or not to sell his (her) asset at a certain price to the buyer who will pay the requested price. As the crucial stage at the market, decision making defies the traders where many observers blame emotional and irrational decisions made by the market participants.

The emotional and irrational decisions were first discussed by John Maynard Keynes (1936) in his book "The General Theory of Employment, Interest and Money", where he described those emotional mindsets as animal spirits. The term "animal spirits" stand for such powerful psychological forces like preferences, instincts, trust and emotions that apparently effect and drive human behavior. The effect of animal spirits, in the existing prices, was seen as contradiction to early classic knowledge which states that market participants are rational according to the efficient market hypothesis (EMH). The EMH says that existing prices always incorporate and reflect all relevant information, see Malkiel ad Fama (1970), Shefrin and Statman (2011). Since Keynes (1936) penned the animal spirits as a reason for economic depression, lack of employment, long term investments, frozen markets and so on, he brought a very start of investigations.

H. Frank (2004) has investigated greed, fear and stock markets dynamics. The intention was to develop a deterministic behavioral stock markets model in which the agents are driven by greed and fear and would be able to comprehend all the facts of stock markets prices. Firstly, all agents admit to a profitable market, however, they panic if the prices change too unexpectedly. The trading activities of agents were fixed at two levels 0.025

and 0.05 called switch regime levels. It is assumed that agents are pretty calm if the average volatility of the last 5 trading periods is below 2.25% and the most recent log price is less than 5% i.e. if the historical prices depict a low volatility. Otherwise agents switch to the active level of 5%.

The price adjustments have followed a log linear function which describes the relationship between the number of assets bought or sold in a given period of time and the price changes caused by the number of submitted orders. Thus excess buying raises the price and excess selling decreases the price. As agents are boundedly rational and their behaviour is persuaded by their emotions, fear and greed emotions were assumed to appear only at two levels though they may raise in turbulent situations. Using the the daily quotes of the Dow Jones index, from 1901 to 2000, the model has shown a significant understanding of the Dow Jones index dynamics. The upward trend depicted by the market data was captured by the model. The model appears to track the main trend in the long run, however, it struggles to produce price variations in the short run. Also, it was able to generate severe crashes as well as uncorrelated price changes and volatility.

Fontana and Marchionatti (2007) have developed an Agent-Based Model that explains a positive theory of animal spirits towards investment. In the reference to Keynes analysis on animal spirits and its connection with entrepreneurial behaviour and the tradition of Marshall and Schumpeter on the entrepreneurship, they define an entrepreneur basically as an innovator, see Emilio (2008). The existence and persistence of irrational behaviour that presses entrepreneurial activities towards investments affect industrial performance. The purpose includes to investigate what determines these animal spirits and to check if it is possible to set an animal spirits rule. During their research the have remarked that political, social and economic atmosphere have a strong influence on entrepreneurial behaviour. As results the internal consistency and validity of the Keynes-Marshall framework were affirmed. For selected variables, a stable behaviour was consistent with the observed facts in investments called cyclical dynamics. Therefore, a positive theory of animal spirits is conceivable in a non-mainstream concept of analysis.

Akerlof and Shiller (2009), in a book entitled "How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism", investigated on how animal spirits manage the economy and discussed how conventional economic theory fails. The conventional economic theory states that agents are strictly rational in contrast to the Keynesian theory that has found that economic oscillations are mainly influenced by "animal spirits". The latter knowledge was confirmed by Akerlof and Shiller after realizing that economic performance is largely mental, though not necessarily rational and emotions like confidence and fairness have a strong presence in economic decision making. They were able to find some other influencing features like corruption and bad faith, money illusion and stories which the rational expectations revolution failed to consider.

Jabłońska and Kauranne (2012) have investigated on animal spirits in population spatial dynamics. The purpose of the study was to explore whether a specific population dynamic model can reproduce the natural fact, originally confirmed by Couzin et al. (2005), that 5% of the population can pull the rest of the whole group towards a specific direction and keep it closer to the subgroup. The population dynamic model proposed in Jabłońska (2011) and Jabłońska and Kauranne (2011) has served as a tool. The main idea behind this model is that, from a population of N particles each particle's movement is energized by three different forces that act on each particle and on the population as a whole.

Firstly, the aggregation forces proposed in Morale et al (2005) indicates that the whole population oscillates around its center of mass. This component justifies the willingness of each individual to stay within a bigger group. Secondly, the momentum effect observed when a sufficiently big subgroup of the whole population has significantly different behaviour which deviates from the population mean. Thirdly, each particle can interact with its neighbors to some extent and intend to minimize the distance to the furthest particle from a $p\%$ range of its closest particles. Finally, each particle can choose to follow its own judgement which is assumed to be random. Therefore, by taking different sizes of an escape subgroup i.e. $p\%$ taken as 4%; 5%; and 8% the simulation results have shown a great evidence that allows to confirm that the size of 5% of the whole group, is the minimum size for the escaping subgroup to pull the rest of the group in the same direction and keep them remain closer to the subgroup. Also they have noticed that any smaller subgroup may influence the whole population movement but is not sufficient to pull and keep it closer to the escape.

3 MODELLING BACKGROUND

Nowadays, most computer simulations in sciences and engineering count on equation-based modeling. However, if one was going to consider this equation-based approach to solve problems from social-economic sciences, the author argues that it would certainly fail to explain some of the system properties as most them can not be formalized mathematically, see Van et al (1998). In social-economics some of agent's internal behaviours can not be easily or explicitly identified while their effect on the global system properties is still significant. For instance, if it was possible to answer to the question, "Why do economies fall into depression?" posed in Akerlof and Shiller (2009), there could not be financial and economic difficulties. It is true that since 2007, the world economy has been experiencing its worst and most dangerous recession after the great depression of 1930, see Dullien et al (2010).

This crisis started in the United States of America in 2008 and now it is crashing European economies. Several observers have shared their opinions about the reasons for the crisis but there are no reliable causes for this disaster that have been universally approved, Dullien et al (2010) and this motivates us to look beyond the available results. To be more specific, we are going to investigate the causes of financial market failure. So far so good, many investigators, including Keynes (1936), Akerlof and Shiller (2009), Jabłonska (2011) and so on, have contributed to the proof of explanation of the problem with respective developments in financial markets. The most recent research findings have shown that irrational and emotional decisions made by traders and market makers might be the cause of financial crisis.

Hence, our focus is on the influence of animal spirits on the price in financial markets. As we have mentioned before, different researches have been published on this invisible target. Therefore, to reach the goal, we have preferred to employ the idea taken from the most recent model which is able to comprehend the financial market dynamics. The latter model considers animal spirits of the involved agents, then mimics the price movements. When it is applied to a financial asset (or security) price, the model proposed in Jabłońska and Kauranne (2012) explains the price dynamics following market momentum and the collective behavior of all market participants. This study aims to investigate whether we can or not observe an improvement on the price movements using a different approach. In addition to the implementation of the main idea in the model selected above, the leading approach assesses each trader's psychology and its effect on the system as a whole then maximize his/her greed emotion with respect to the collective greediness. We will assume that by maximizing everyone's greed, we will be minimizing his/her fear. This can be viewed as a fact that no one goes to the market willing to loose his capital investment.

Our main motive for this investigation originates from the Schelling aggregation model proposed in Schelling (1971), when he was performing an investigation on observed racial segregation in American cities. The Schelling model has depicted how by considering individual agents movements rules, clustering may occur. The latter approach was called agent-based modelling(ABM). The key points for ABM relies on decentralisation for decision-making and on the fact that agents are able to learn, to adapt, agents are autonomous, self-directed and self-contained. Hence, by allowing every agent to make its own decision referring to its greed and fear emotions, we expect different results from what has been obtained using the equation-based simulation approach.

3.1 Equation-based model , Jabłońska-Capasso-Morale model

The Jabłońska-Capasso-Morale (JCM) model was first proposed by Capasso-Bianchi, see Jabłońska (2011), to model animal population dynamics and this model was also used in price herding Capasso et al. (2003), Bianchi et al. (2003) and Capasso et al. (2005). The main idea in this model is that everyone's movement from a population of N agents depends on the location of each agent with respect to the whole population and its interaction with the closest neighbors. The general representation of the model is:

$$dX_N^k(t) = [f(X_t^k) + h(k, X_t)]dt + \sigma dW^k(t) \tag{1}$$

The function $f(X_t^k)$ stands for the forces acting on the whole population, in our context it can be viewed as a group of traders in the spot market where the price is taken as their measure of distance. $h(k, X_t)$ stands for interaction within neighboring individuals in the population group of size N. However, the model above was not able to capture price spikes taken as an important dynamic character of market prices.

Subsequently, considering price as a liquid, Jabłońska (2011) has been able to improve the model written as Equation (1) by adding a momentum component. This momentum term originated from the Burgers' equation commonly known in applied physics like fluid dynamics and in traffic flows. The momentum effect is mostly observed when a particular sufficiently big subgroup of the whole population manifests significantly different behavior that diverges from the total population mean, Jabłońska (2011). Adding this momentum term to the Equation (1), the new model becomes able to reflect the price dynamics even in utmost situations. Below is the general form of the model as presented in Jabłońska and Kauranne (2012).

$$dX_t^k = [\gamma_t(X_t^* - X_t^k) + \theta_t(h(k, X_t) - X_t^k) + \xi_t(g(k, X_t) - X_t^k)]dt + \sigma_t dW_t^k \tag{2}$$

Where k=1,2,3,...,N

- X_t^k is the price of trader k at time t
- X_t vector of all traders' prices at time t
- W_t^k is Wiener process value of trader k at time t
- X_t^* is the population mean at time t
- σ_t is standard deviation of Wiener increment at time t
- γ, θ and ξ are interaction forces
- $N_{p\%}^k$ is the neighborhood of agent k formed by closest $p\%$ of the whole population.

Also $h(k, X_t) = M(X_t) \cdot [E(X_t) - M(X_t)]$ stands for global interaction and $M(X)$ is the mode of the random variable X. $E(X)$ is the classical expected value of the random variable X. The function $g(k, X_t) = \max\limits_{k \epsilon I} \quad \{X_t^k - X_t\}$ where $I = \{k|_{X^k \epsilon N_{p\%}^k}\}$ represents the maximal distance from trader k to its furthest neighbor from its closest $p\%$ of the whole group.

The detailed meaning of each term in the Equation (2) can be found in the subsection 3.3. In addition to the main idea in Equation (2), we will maximize every agent's greediness.

3.2 Greediness maximization

Greediness maximization can be understood as the willingness to do whatever you can in order to earn more than you deserve. If you are a seller, you intend to bid higher price, whereas a buyer's intention is to pay as little as possible. However, this fact is sometimes violated in some computer simulations where prices from both buyer and seller look completely mixed. The latter mixture does not have any justifiable economical reasons. In financial markets, the traders place their bid prices under the common mindset that buyers bid lower than sellers. This mindset holds because after sorting all the bids in either ascending or descending order, the smallest bid from all the sellers is always greater than or equal to the highest bid from all the buyers. Therefore, mixed prices from both buyer and seller remain unjustified. To correct it, we proposed a technique which aims to maximize every trader's greediness. Similarly in an algorithm proposed in subsection 3.3, every trader maximizes his (her) greediness by searching for his (her) best trader.

Firstly, the seller searches for a buyer with the highest bid price. The seller will accept any buyer's price which is greater than or equal to his (her) bid. If the maximal price from all the buyers is less than seller's bid, the seller reduces his (her) price with respect to the difference from the best buyer. The seller's price increment is taken smaller than or equal to the difference price from his (her) best buyer. Secondly, the buyer looks for a seller who has placed the smallest bid price. Depending of the best seller's price, the buyer accepts to pay if the seller's request is less than the price indicated by the buyer. Otherwise the buyer will have to increase the price with respect to the difference from the best seller too. Also, the buyer can not rise his (her) price for more than the gap from the best seller.

3.3 Agent-based model

To be able to develop the ABM, the following trading structure has been proposed. Initially, a population group of N traders, where $N/2$ are buyers and $N/2$ are sellers, will be randomly created around a given market price X^*. $N/2$ buyers and $N/2$ sellers will be put into two different breeds. In our context, a breed stands for a group of traders who share homogeneous behavior towards any price change. The latter differ from one group to another. For instance, fear rises for buyers when the price goes up whereas it decreases for sellers. Normally, buyers tend to bid less prices while sellers look for high bid prices. An agent's price will be its x-coordinate in a plane (x, y) with the origin $(0, 0)$. As we are interested in one variable, which is the price, all traders will be located on a horizontal line. The size of each traders' breed remain the same until the last trading event. It is assumed that at the beginning the sellers have prices which are greater than the present market price whereas buyers have prices which are below the actual market price. At the end of each trading period, every trader displays his/her price. The actual model does not take into consideration the total wealth of the agent, it assumes all agents have equal wealth but their emotions and irrationalities oblige them to bid different prices. Note that the model parameters known as interacting forces together with the standard deviation of the Wiener increment, have been estimated and they are assumed to change over time. For each trader, in order to update his/her price, the next points hold.

- The market price, X^*, is already known from market makers
- The agent interacts with other market participants following the model Equation (2).

- The agent searches for the best trader and moves towards him/her. At each trading period, there are will be the best seller and the best buyer. The best seller is the one with minimum price from all the sellers in contrast to the best buyer who will be the buyer with maximum price from all the buyers. Therefore, you will move towards the best seller if you are a buyer and you will move towards the best buyer if you are a seller.

- For simplicity, we assume that the maximum step size, that can be taken, can not exceed the difference between your and your best trader's actual price. For equal prices, the agent will remain in the same place, display his/her price and wait for the next trading round.

Trader's model

In order to trade, each trader must have his own price, his own neighborhood as well as his own single and same number during all trading events. Note that the neighborhood is taken as the set of prices of 5% closest neighbors. As stated in the subsection 3, the seller will mainly play with three acting forces which are the global mean or market price, market momentum and the closest environment or neighborhood. Depending of how strong each of the three forces is, the trader will have to decide his/her next move direction. In our context, the trader will decide to drop or to raise the price. To maximize the greediness, each trader looks for best trader and moves towards him/her. The detailed trading algorithm can be formulated as follows:

- Initially i.e. $t = 0$, N agents are created with their prices. $N/2$ buyers have prices which are below the the actual market price and $N/2$ sellers have prices which are greater than the actual market price. Let M be the first global mean and w be the first standard deviation of the Wiener increment. The prices of buyers will be randomly uniformly distributed between $(M - |w|)$ and M and the prices for sellers will be randomly uniformly distributed between M and $(M + |w|)$. To play with simulation one can use $(M - n * |w|)$ and M for buyers and M and $(M + n * |w|)$ for sellers $(n = 0, 1, 2, 3, ...)$. For all traders named $k = 1, 2, ..., N$

For each trading period of $t = 0, 1, 2, ..., T$, all traders $k = 1, 2, ..., N$ simultaneously execute all the steps below:

- Let us take the agent named k. k is able to locate his closest neighborhood formed by 5% of the whole population and measure the distance to its furthest neighbor location.

- k feels the intensity of the current market price momentum by identifying the spot with maximum number of traders who have the same price and come from the same breed with k, then k moves towards that spot. i..e k calculates the global interaction generated by market price momentum. Also k reacts to rate of mean reversion level. In short, k calculates the first and the second terms of the Equation (2).

- k generates randomly his/her Wiener increment, which is normally distributed with zero mean and standard deviation σ_t.

- With respect to the Equation (2), k calculates the increment to his (her) price and changes its location accordingly.

- k searches for the best trader. If k is a seller, k looks for the buyer with the maximal price from all the buyers. Depending on the best buyer's price, the seller k will have two options: The buyer has a price which is greater than the price of the seller k, then k sets his price equal to the buyer's price. Otherwise k reduces the price with respect to the difference from the best buyer. If k is the buyer, k hunts for the seller with the minimal price from all sellers. Referring to the best seller price, k also has two options: The best seller's price is less than the price set by k, then k sets its price equal the best seller's price. Otherwise k will have to increase the price with respect to the difference from the best seller.

- k is required to display his (her) price at the end of every trading period.

- Every one prepares for next trading period.i.e. $t = t + 1$.

The detailed algorithm above is given to a group of agents as a set of instructions. The ruling assumption states that everybody from N traders is obliged to cooperate at each trading date and remain in his (her) breed during all cooperations i.e buyer (seller) stays in buyers' breed (sellers' breed). To be more logic, the price is considered to be positive (the trading environment is bounded below from zero) and this excludes negative prices from the market. Whoever who bids a negative price will be required to change his (her) mind and bid any other positive price which might be less than his (her) most recent historical price.

3.4 Some of the model tools used during comparison

- The Mean of a random variable X, also called the expected value of X, is a weighted average of the all possible values that X can take. It is one of the tools used to measure the central tendency of a distribution. Mathematically, the mean of a discrete random variable X, which takes the values x_1, x_2, ..., x_k with respective probabilities p_1, p_2, ..., p_k, is calculated as

$$Mean = \sum x_j p_j, \; j = 1, 2, 3, ..., k \tag{3}$$

- The Standard deviation is a measure of spread which determines how far the values of a distribution are located from the expected value. Using the above random variable X, it is given by

$$Standard\ deviation = \sqrt{\sum x_j^2 p_j - (\sum x_j p_j)^2}. \tag{4}$$

- The Skewness of a given distribution shows the lack of symmetry. The distribution of the random variable X is negatively skewed, positively skewed or unskewed if the skewness coefficient is negative, positive or 0 respectively. For a perfect normal distribution the skewness coefficient is 0. Mathematically, skewness of the distribution of X, is calculated as

$$Skewness = \frac{1}{k} \sum \left(\frac{x_j - \sum x_i p_i}{\sqrt{\sum x_i^2 p_i - (\sum x_i p_i)^2}} \right)^3, \; i = 1, 2, 3, ..., k, \; j = 1, 2, 3, ..., k. \tag{5}$$

- The Kurtosis indicates how the distribution of X is peaked or flat. The kurtosis coefficient will be calculated as

$$Kurtosis = \frac{1}{k} \sum \left(\frac{x_j - \sum x_i p_i}{\sqrt{\sum x_i^2 p_i - (\sum x_i p_i)^2}} \right)^4. \tag{6}$$

  Hence for a normally distributed random variable the kurtosis coefficient is 3. If the kurtosis is greater than 3, the distribution tends to have heavy tails and this means that any random variable from this distribution will contain more extreme elements. Otherwise the distribution will have short tails which implies less extreme values for any random variable taken from such a distribution.

In addition to the moments above, the mean square error (MSE) has been used to compare the proposed model results to the original price. The MSE assesses the average of the squares of the errors of the simulated price from the original price. A model with small MSE will be chosen as the best compared to the remaining cases. The mathematical form of MSE is:

$$MSE = \frac{1}{k} \sum (x_j^{simulated} - x_j^{original})^2 \tag{7}$$

3.5 Data and software

3.5.1 Historical prices of silver metal

As all other precious metals, silver is considered as a form of money and store of value which makes it a marketable asset. In order to analyse the power or strength of the proposed ABM, the historical prices of silver metal have been considered. Similarly, to all tradeable items in financial markets, the market price of silver is not predictable or very hard to forecast. This unpredictability, which played a key reason to considering silver prices in the current study, is analysed through graphical representations of the data together with some statistics such as mean, skewness, kurtosis and standard deviation. A time line plot has been selected to capture all possible patterns including seasonality, trend and cyclic. In addition to the time line plot, graphs of autocorrelation and partial autocorrelation functions and histograms were also used as tools. Even though stationarity in the series is not compulsory as the proposed model incorporates the idea of moving average reversion, the classical time series analysis requires that any observed patterns must be removed from the original series.

To start, the closing prices of silver from NYSE quoted in USD in the period from 01.03.2000 to the 01.03.2013 were plotted in Figure 1. The overall behavior of the silver prices, depicts an upward trend with persistent non-stationary patterns and this can be explained by the average price which is changing over time.

The common behavior of a financial asset's price, mainly characterized by dramatic upward and downward moves, is also observed in these silver historical prices. It can be seen that the data shows existence of long increases followed by dramatic and fairly sharp drops. Figure 2 agrees with the non-stationarity observed in Figure 1. The normalized histogram of the series shows a non-normal distribution plotted against a theoretical Gaussian distribution curve. However, many of the classical approaches expect that the prices are Gaussian or log-normal and therefore the log-returns would follow a normal distribution. Figure 3, of the autocorrelation function (ACF) and the partial autocorrelation function (PACF), avows how the series is not stationary.

The slowly decaying ACF with the second significant lag from PACF confirm the non-stationary process with a significant serial correlation of the same lag. Note that the persistence of high values in the ACF plot represents a long term positive trend found in the time line plot (see Figure 1).

As it was mentioned before, the model employed in this study uses moving average

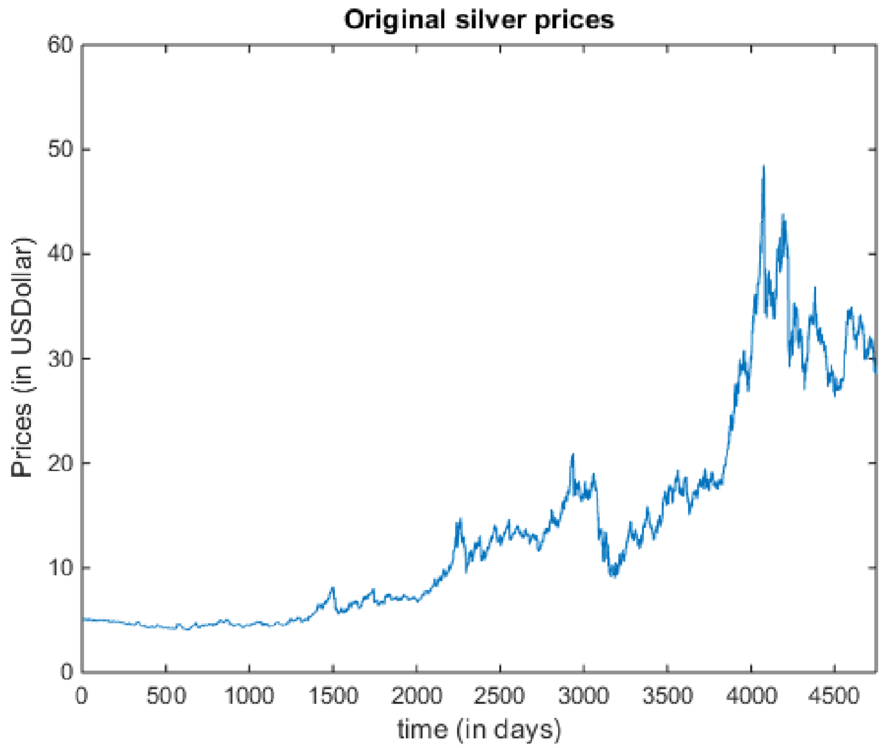

Figure 1: Original daily prices of silver metal

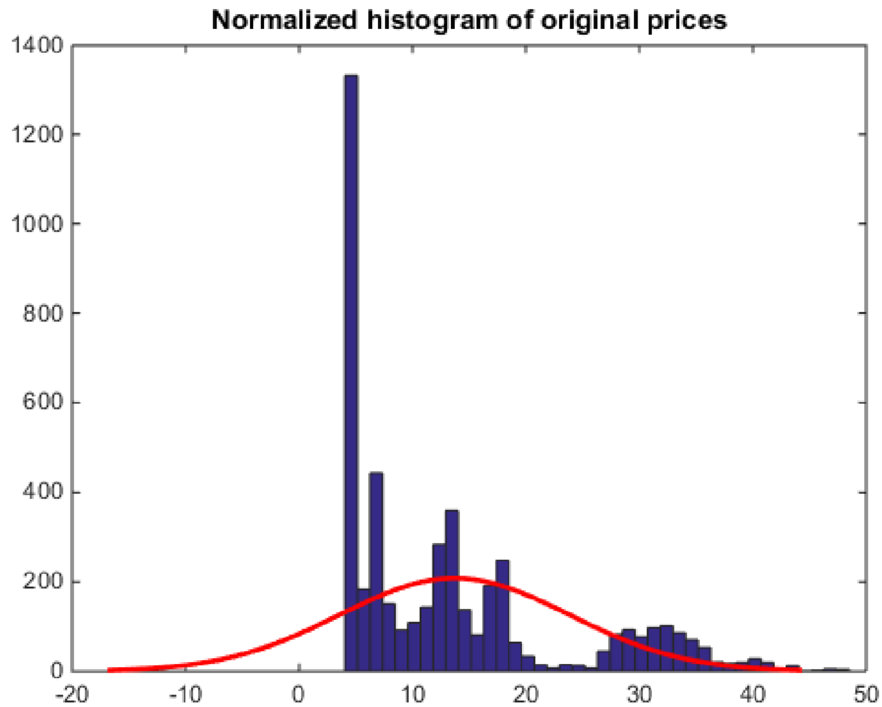

Figure 2: Normal distribution of the original closing daily prices

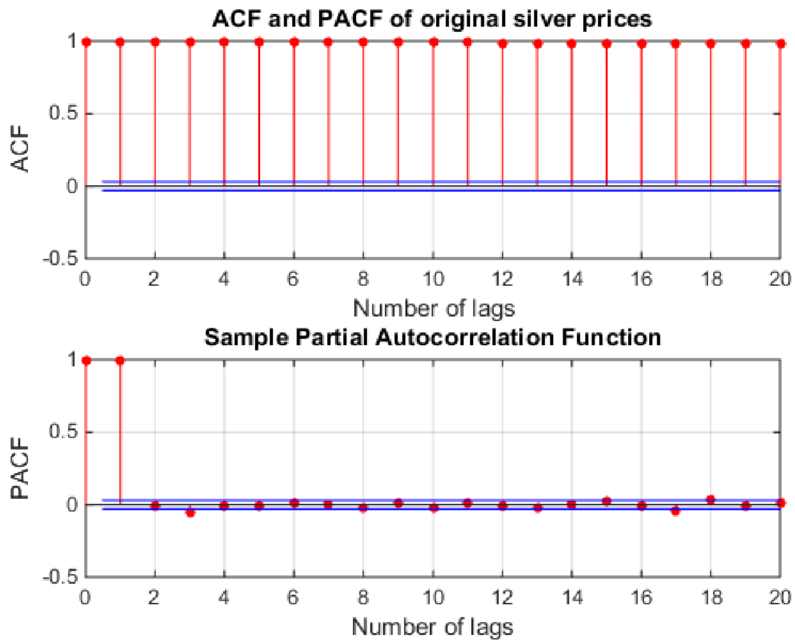

Figure 3: The ACF and PACF of original closing daily prices

reversion concept which exempts us from transforming the series for stationarity requirement, only the original series will be employed and analysed. Nevertheless, to be able to perform any classical analysis on this silver historical prices, the log transformation of the series can be done to make it stationary. The transformed data plotted in Figure 4 seems to follow a stationary series as its values fluctuate all around the mean zero where the small variation is followed by a small variation in the opposite direction and this also holds for big fluctuations which are followed by big ones in the opposite direction too.

The normalized histogram against the so called theoretical normal probability curve (in red) has been plotted in Figure 5. One can notice that the normalized histogram gets closer to a Gaussian curve even though it shows a leptokurtic shape as the highly peaked distribution is more clustered around the mean. The ACF and PACF, in Figure 6, indicate a stationary and a non serially correlated series as all the considerable lags fall in the confidence limits.

Table 1 contains the basic statistics of the original and transformed data. From the original series's row, the kurtosis coefficient is slightly greater than 3 whereas it is more than 4 times greater than 3 for the logged series, which tells us that we do not have a normal distributions either before or after the transformation, but a clear heavy-tailed distribution after transformation. Also the skewness for the logged series is negative in contrast to the skewness from original prices which is positive. This means that more data returns are located below the mean for transformed series, or more prices are clustered above the mean for the original series.

The biggest standard deviation of original prices indicates very significant variations between prices while the standard deviation from the row of transformed prices, which is close to zero, ensures very small changes between returns.

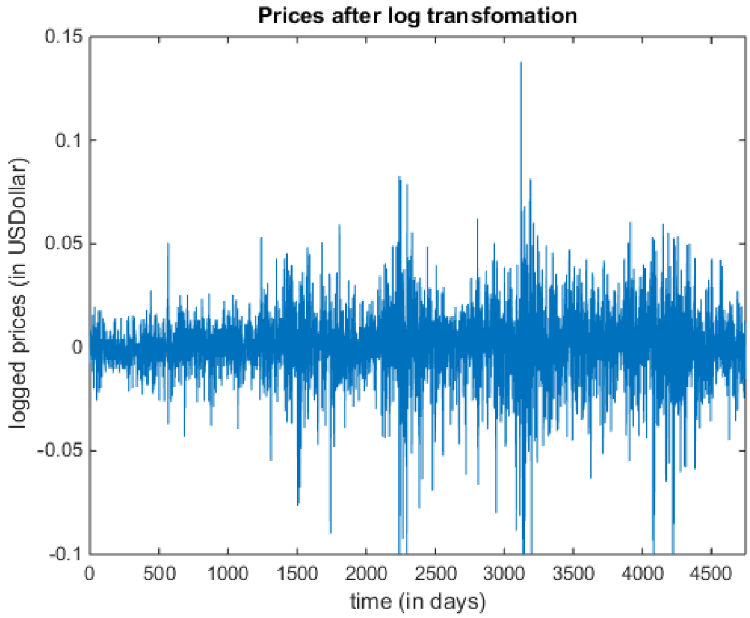

Figure 4: Original prices after log transformation

Table 1: Basic statistics of silver prices

	Mean	Kurtosis	Standard dev.	Skewness
Original series	13.7067	3.2251	10.1364	1.1263
Transformed series	0.00036	14.599	0.01685	-1.3987

3.5.2 A multi-agent system simulator

The purpose of this work is to implement ABM on animal spirits in financial markets. As the ABM deals with some properties of a system that are not simply identified as

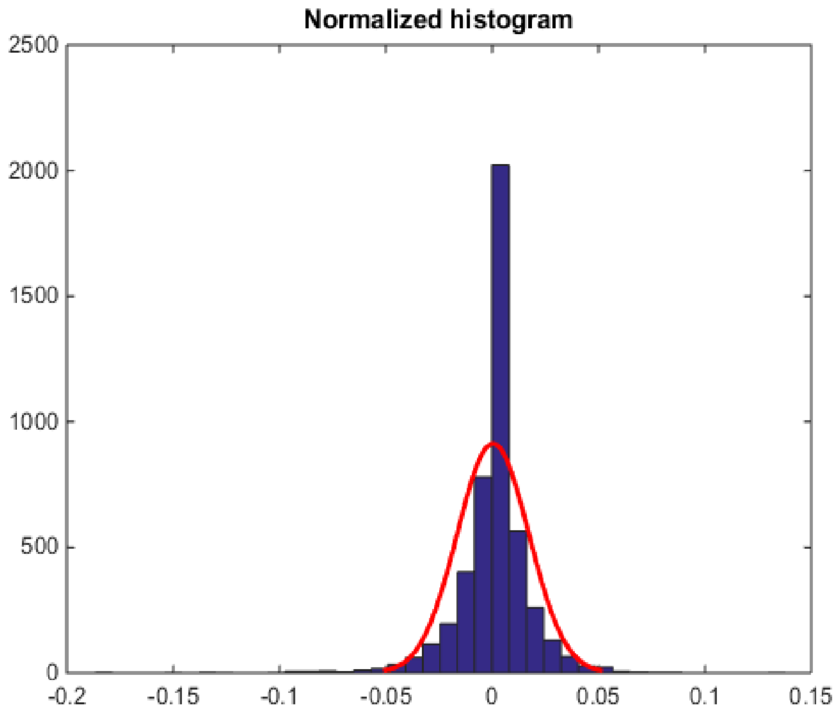

Figure 5: Normal distribution of silver prices after log transformation

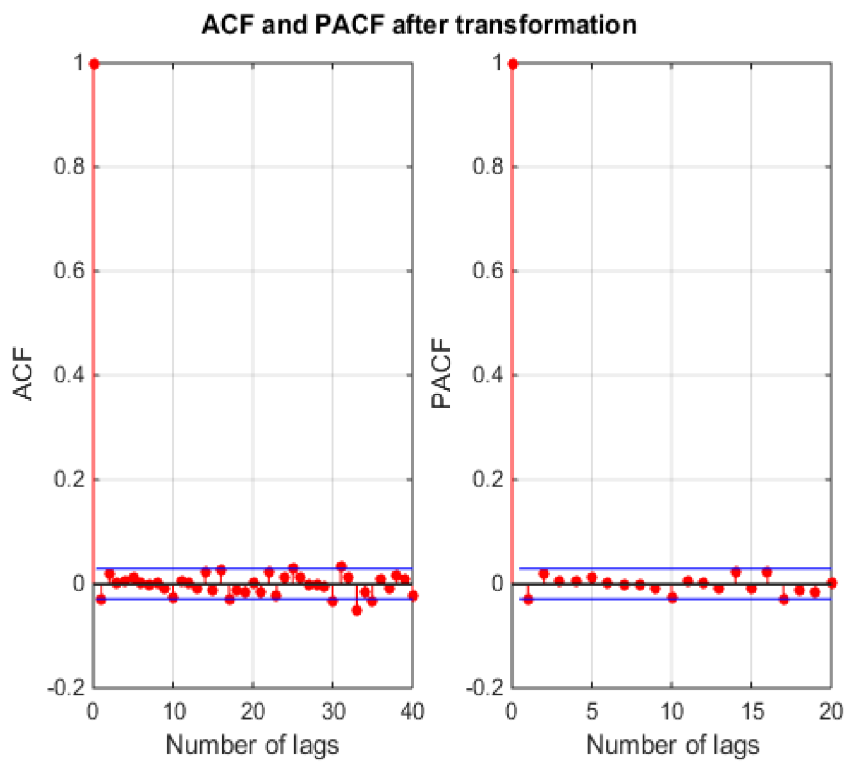

Figure 6: The ACF and PACF after log transformation

functionalities or properties of each component of the system but sometimes emerge from their collective behaviour, see Bandini et al. (2009), so many software tools used in computing may not be suitable for our course. Therefore, to be able to reach our destination, a multi-agent system simulator, which is able to comprehend properties or functionalities of each individual, has to be selected in order to perform the simulations.

NetLogo is a multi-agent programmable modeling environment that simulates natural and social phenomena has been selected for the task. In NetLogo, there are four types of agents named turtles, patches, links, and the observer and the world of two dimensions. The latter is divided into a grid of patches. The turtles are the only type of agents which are able to move in the world which means that a turtle has coordinates whereas the patch is a square piece of ground over which turtles can move. The links are the type of agents that connect two turtles even though they do not have coordinates.

Lastly, the observer, whom we can call an instructor, does not have coordinates either. To get the observer's location well, one can imagine it as it is looking out over the world of turtles and patches.

4 SIMULATION RESULTS

In this section, different simulations were performed using Netlogo and the data obtained were analysed using Matlab. The simulation situations are as described in subsection 3.3. As it has been assumed, everyone in the trading environment does not have to consider his total wealth in order to bid his (her) price. The governing rule is that, whatever is the market price, it is accepted by every trader. This assumption ensures the existence of cooperation between agents hence omitting the reasons to quit or destroy the market with higher (lower) prices because at any market price the trade holds.

The simulations started with an incomplete model which has moving average reversion and random terms (model type I). The second case consisted of the model without momentum effect (model type II). The third case excludes farthest neighbour component from the model (model type III) whereas the last situation was the full model i.e. the ABM with all the terms (model type IV). The market population was composed of 200 traders grouped into two clusters of 100 sellers and 100 buyers.

To perform simulations, we had imported the historical closing prices of silver as the moving average term. To ease the task, alongside the moving average, the model parameters were first estimated using Matlab and then incorporated into the ABM. This process of parameter estimation was not possible with Netlogo as they would require all historical bid prices of all market participants which were not accessible (available). The farthest neighbor, as an interacting force, was analyzed by instructing every trader to search for the trader with the maximum price from a subgroup of neighboring prices of 5% of all traders.

4.1 The model without maximized greediness

4.1.1 Incomplete model of type I

The results presented here were obtained when the model parameters are kept weightless except moving average reversion and random term coefficients. The main leading trading psychology is of JCM model.i.e. greediness is not maximized in this case. The time line plots, combined in Figure 7, depict the evolution of the traders' prices compared to market price. Generally, it indicates an agreement between the agent's price and the market price as all agents agreed on the local and main trends of the market price.

In financial markets, bids are placed in ascending and descending order for sellers and buyers, respectively. The highest bid from the buyers' bids should be always less than or equal to the lowest bid from sellers's bids. This gives an impression that most of buyers' bid prices are below the smallest price from the sellers. However, looking at this plot it is not true. In the same way, we initially had buyers' prices which are less than sellers prices yet from the Figure 7, one can notice that all traders' prices move independently from the agent's breed. In some situations buyers bid higher prices than sellers and this is hardly justified by rational knowledge.

To estimate the market price, we have calculated the average price between sellers and buyers average prices. The resulting series was plotted against the given market price in Figure 8. As it has been said, the simulated price follows the same trend as the given market price. Generally, a sudden change in real market price has implied a sudden change in the simulated price. If the change leads to bigger variation in the original prices, in most of the cases it will be reflected in the simulated prices. The ACF and PACF functions of the simulated series were represented in Figure 9. The persistent high values of the ACF and the first significant lag from PACF prove a serial

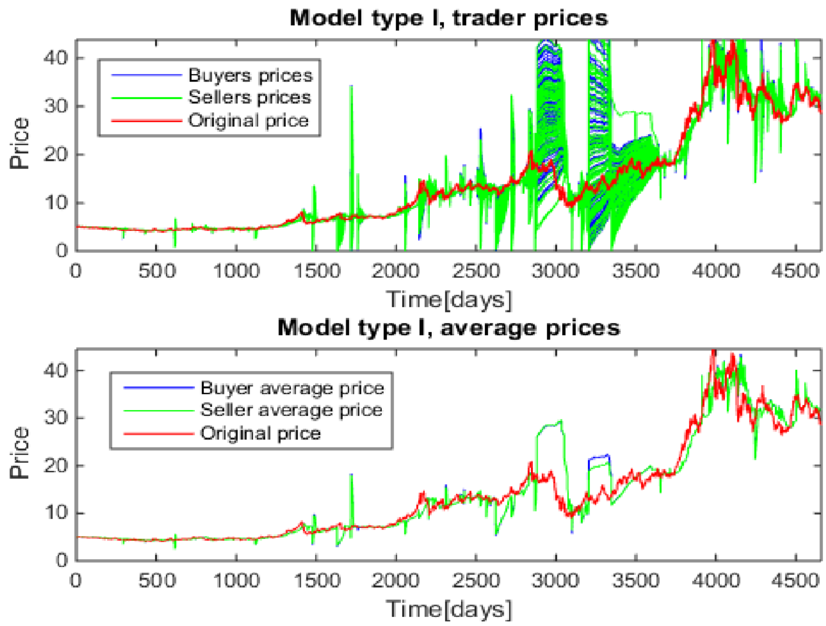

Figure 7: When average is the only acting force on the market

correlation of the first lag and long term positive trend which is clearly observed in the original series.

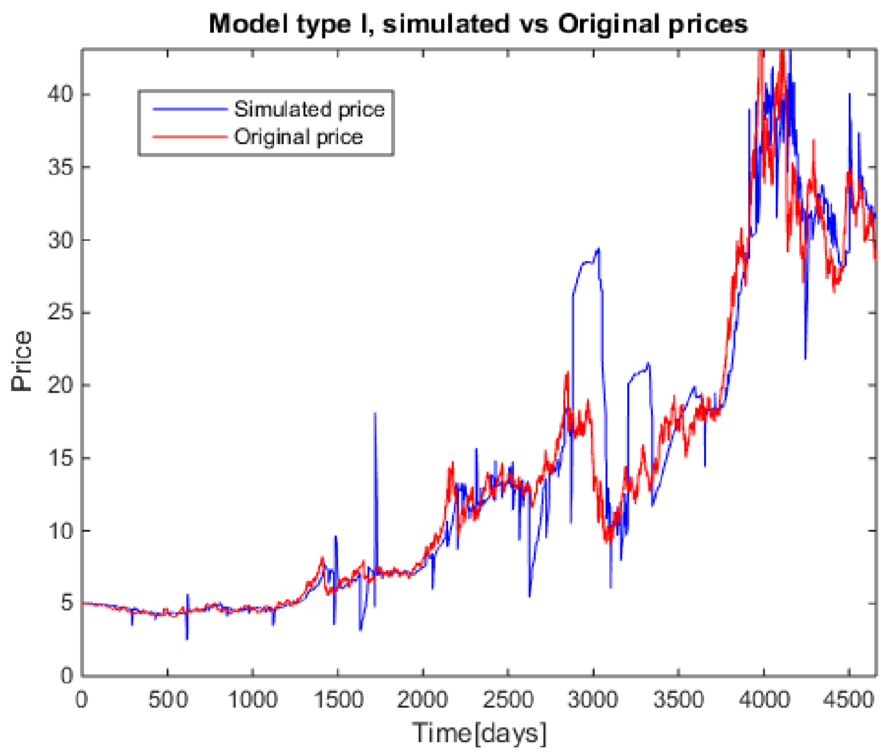

Figure 8: Estimated market price against original market price

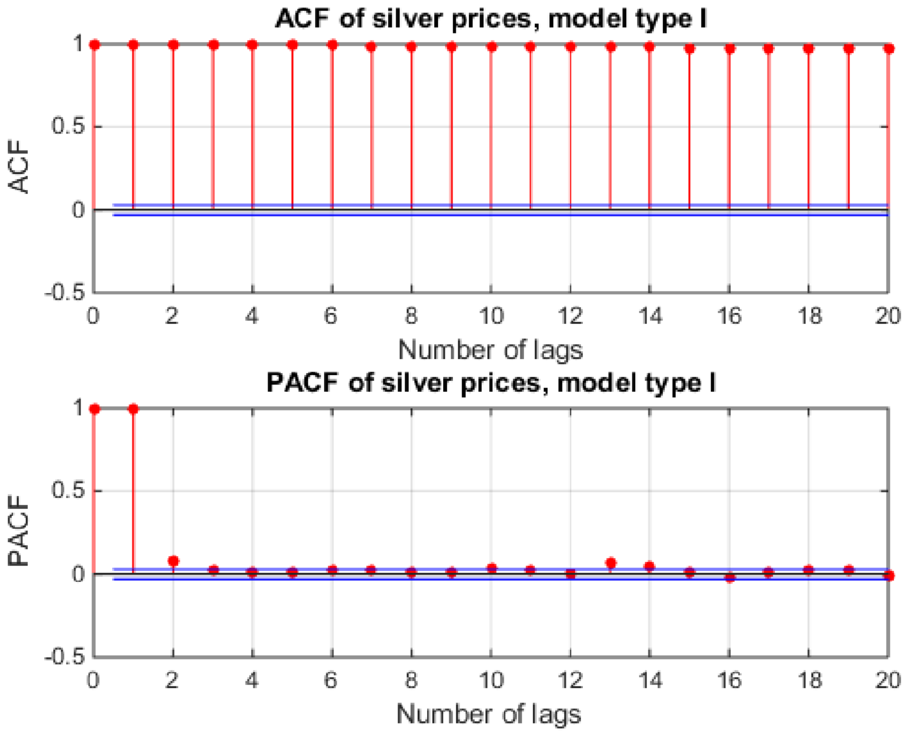

Figure 9: The ACF and PACF for the model type I

4.1.2 Incomplete model of type II

For this type of model, the momentum component was given zero weight and all the other components were left acting. From the Figure 10, the traders' prices successfully followed the market price with reasonable or medium price alterations. Similarly, in the sub-subsection 4.1.1, the agent's price movements can not help to identify his (her) spot (buyer or seller spot) as the bid prices look mixed. However, at the initial date we had two distinct clusters namely buyers' and sellers' breeds with uniform distribution of small prices from buyers and uniform distribution of bigger prices from sellers. This price behaviour remains unobservable in the available results.

Figure 11 shows the simulated price dynamics alongside the original market price. As it can be seen, in Figure 10, the agents' prices move closer to the market price than in the previous case presented in sub-subsection 4.1.1 where any brisk change in the market price strongly implied brisk variations in traders' prices. The overall simulated price, in Figure 11, justifies how the farthest neighbor effect as an interacting force can bring the trader price near the market price specially where there are price spikes. In other words, the power of 5% traders who share the same price, of the whole market participants pulled the rest of traders to the preferred price that in our situation seemed

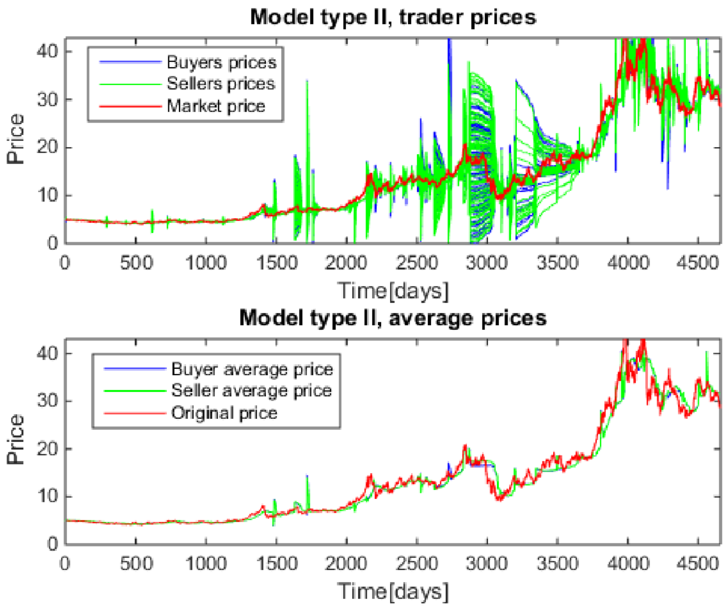

Figure 10: When the momentum component is removed from the model

to move closer to the market price. Figure 12 presents the ACF and PACF functions.

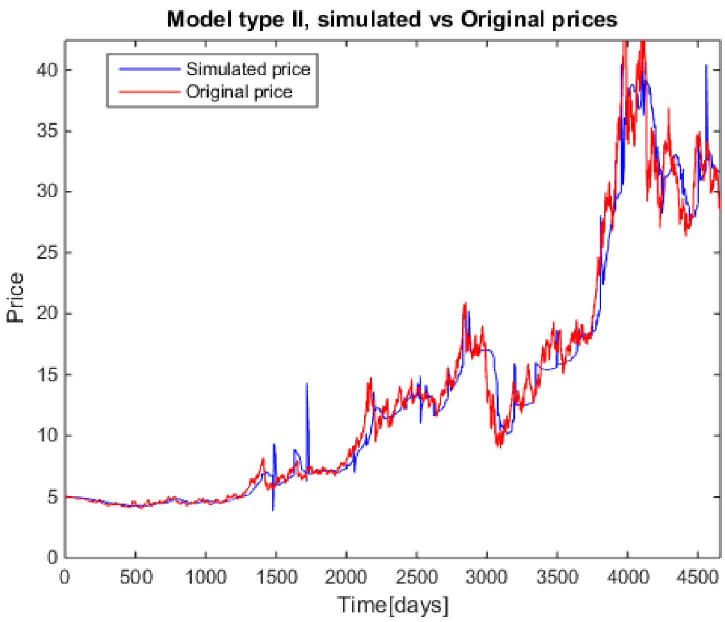

Figure 11: Simulated average price when the momentum component is removed from the model

The plot shows almost a non decaying ACF while the PACF avows a serial correlation between the most recent two values of the series. This keeps emphasising the long run rise of the price and the existence of serial correlation, as it has been found also in the previous model type.

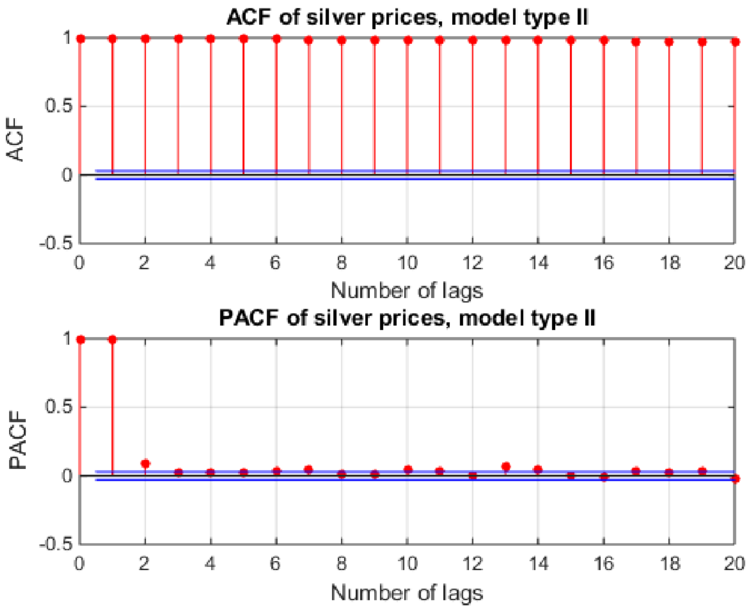

Figure 12: ACF and PACF when the momentum component is removed from the model

4.1.3 Incomplete model of type III

The results presented in this part were obtained when the farthest neighbor interacting force was considered inactive, .i.e. the farthest neighbor component was removed from the complete model. The Figure 13 illustrates the price dynamics when all market participants are individually followed. One can observe the price movements which does not make a very big difference from the previous model cases especially after 2500 days. After this date, the main difference is heavily found at all turning points where there are significant variations compared to the previous results in the model type I and II i.e. the price spikes are wider than in the previous cases. The latter variations may be understood as results of the market momentum effect given its insight by the trader when bidding.

In the reference to the simulation results in 4.1.2, the effect of the market momentum on the market price can be identified by looking at the price spikes in Figure 13 and Figure 14. The ACF and PACF are presented in Figure 15 and agrees on the long term positive trend and the serial correlation of the first lag.

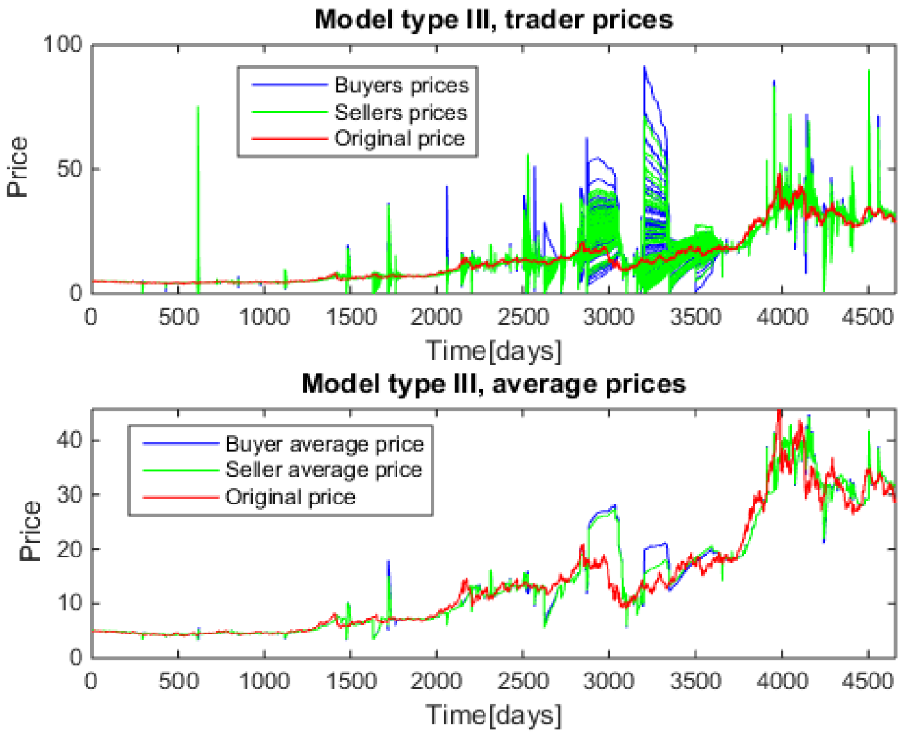

Figure 13: When farthest neighbor component is removed from the model

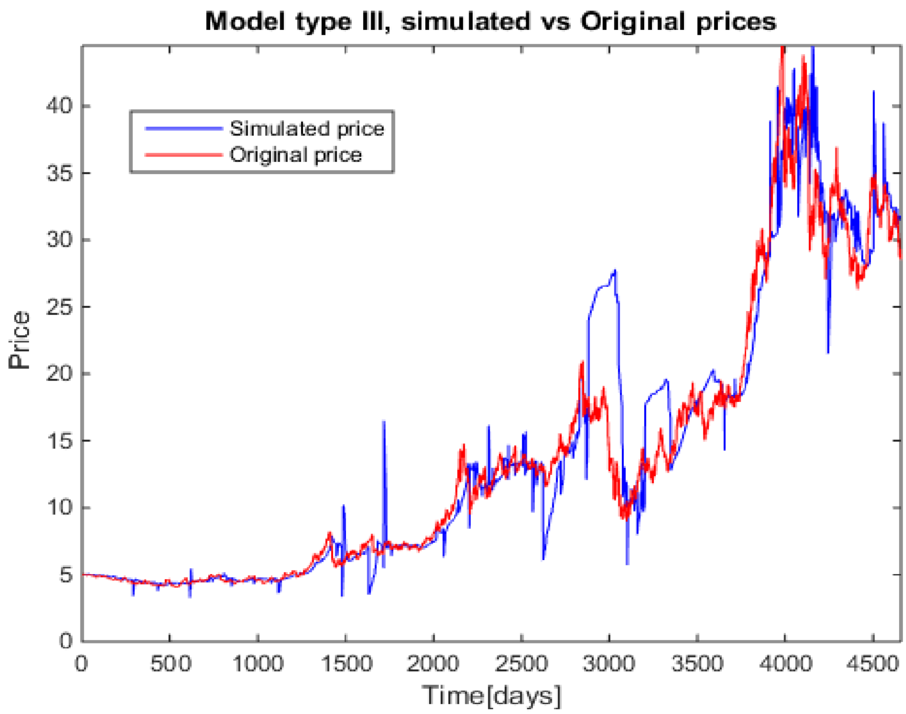

Figure 14: The simulated average price and given market price

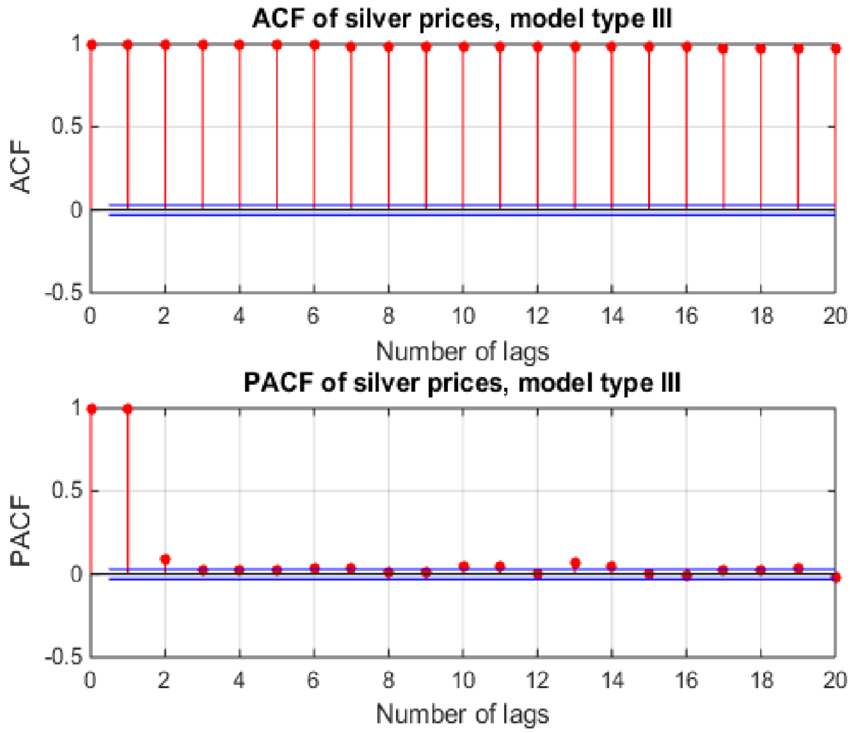

Figure 15: The ACF and PACF for model case III

4.1.4 Model type IV, full model

The full model is the model with all components included. Figure 16 shows the effect of each of the interacting forces acting in the model. The observed price's evolution combines all the other cases discussed above. Mainly the market participants seem to accept the market price even though in some cases they tend to move quite far from it but at the end they come back closer to the market price. As in all other cases discussed before, a big discordance between market price and traders' prices, is mostly observed in the utmost situations.

Figure 18 confirms the non-stationarity of the series justified by the existence of a long term positive trend and serial correlation of the first lag. The time line plot of the average price in Figure 17 proves how the proposed ideology of JCM can help to explain the price dynamics in almost all situations as it reflects all statistical features of the original price. In addition, the main assumption which forces everyone to cooperate is verified and not violated as no chaotic behaviour is appearing in price developments.

Figure 19 presents the simulated price distributions. Compared to the original price distribution, the full model curve(in black) stands as the closest to the market one and

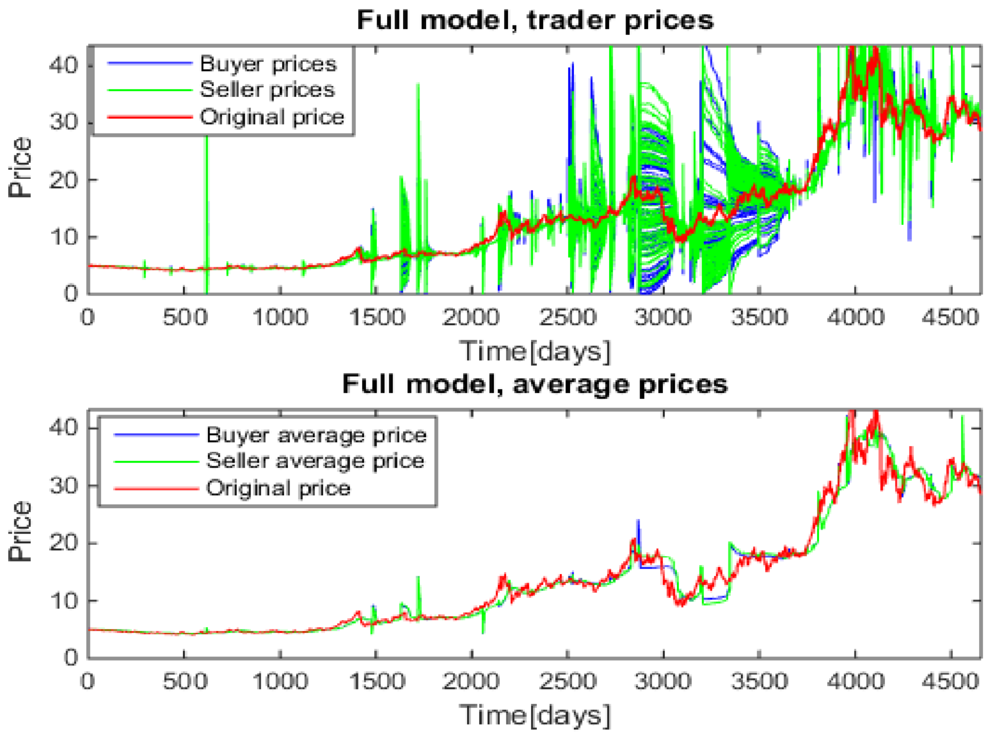

Figure 16: All components are included and acting in the model

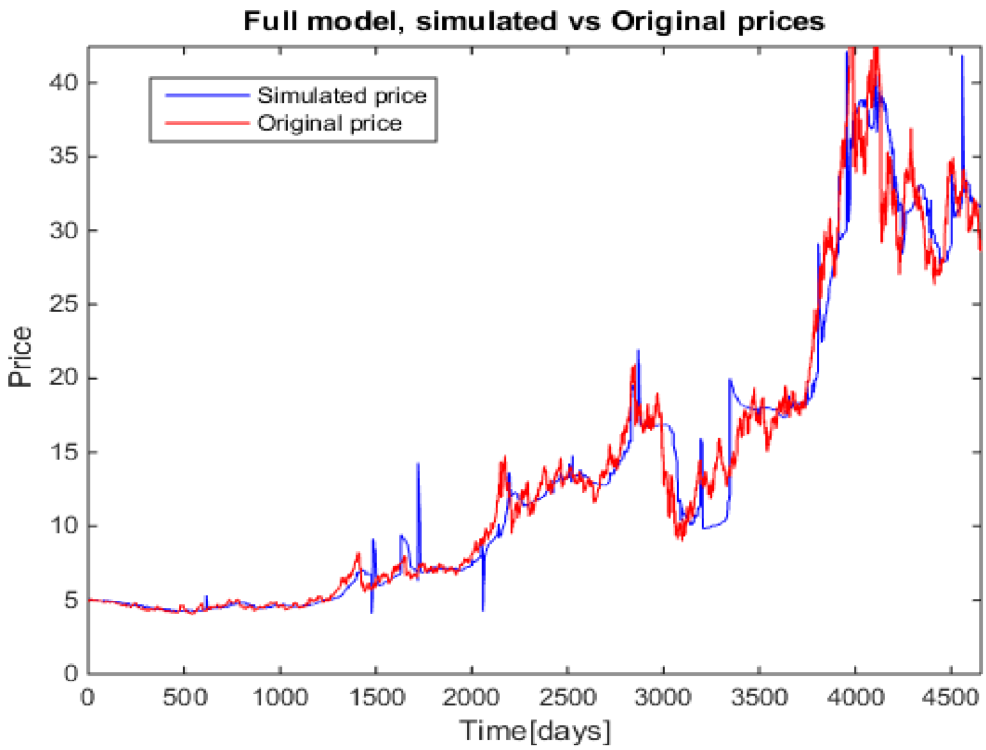

Figure 17: All interacting forces are acting in the model

it is followed by the model without market momentum effect. This has been confirmed in Table 2 where the two cases, type II and full model cases, have the smallest mean squared errors with the full model standing as the best approximate of the original series. From Table 2, we see that with the kurtosis coefficients which are very close to 3, the two simulated model cases (II and IV) nearly tend to follow normal distributions even though they have fewer traders with extreme prices. The positiveness of the skewness coefficients show that many traders bid prices which are slightly greater than the market price.

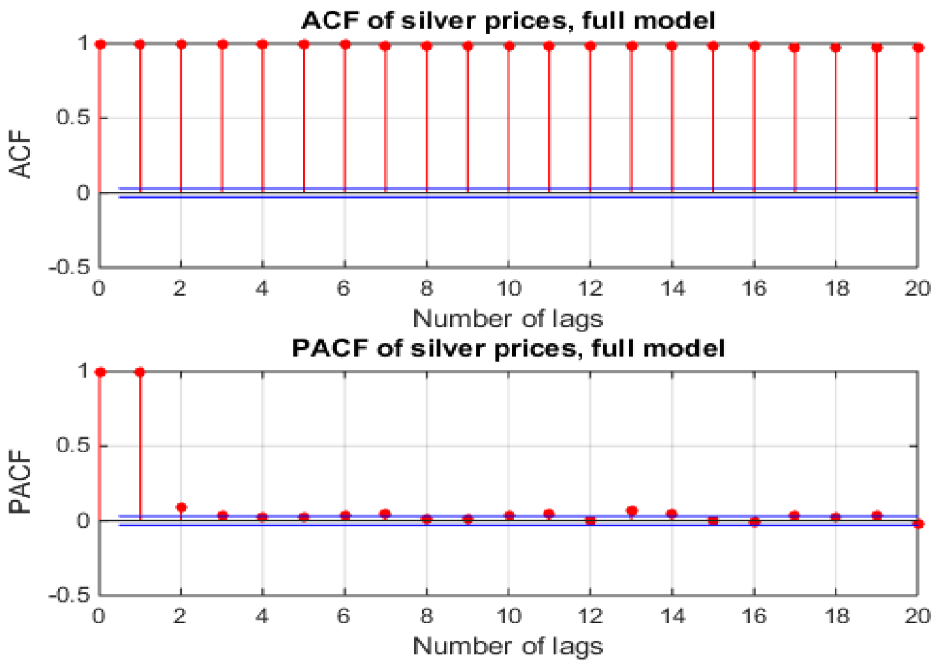

Figure 18: All components are included in the model, ABM without maximal greediness

Table 2: Basic statistics of the simulated prices, ABM without maximized greediness

Model type	Mean	Kurtosis	Standard dev.	Skewness	MSE
Orig. series	13.7067	3.2251	10.1364	1.1263	
Type I	14.3247	2.5912	10.6177	0.9259	0.0018
Type II	13.6592	3.0074	10.1181	1.0827	0.0008
TypeIII	14.2622	2.7134	10.5137	0.9542	0.0010
Full model	13.7151	3.0464	10.0728	1.10872	0.0004

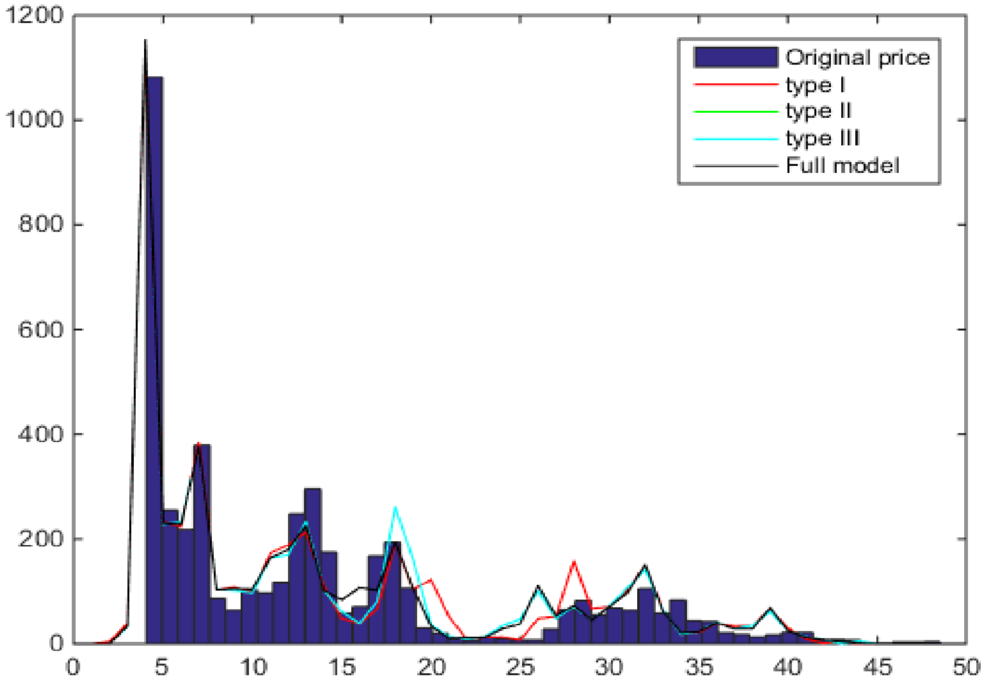

Figure 19: Histograms for ABM without maximized greediness

4.2 The model with maximized greediness

The results obtained on this case were grouped into four main figures with different subplots. The same main features presented in the previous subsection 4.1, the prices simulated without maximized greedy emotions, are the ones that have been analysed here too. Figure 20 contains the price developments for each of the participants after considering his (her) greediness as one of the acting force in each of the four model cases. The overall observed feature is that the sellers move rarely with lower prices than the market price. However, the bids from the buyers are sometimes above or below the market price.

The overall note is that the sellers do not often intend to change their bid prices i.e. they seem to be more greedy than the buyers. On the other hand, the buyers also do not raise their prices many times as their leading price falls below the market price. The later observed movements confirm the most appeared character in the trading environment with one common conception. The sellers bids' prices always move above market price whereas the buyers are always are willing to bid lower prices. When looking at the four time line plots, accompanied with separate average prices plotted against original price, we can not identify the difference between the model case results. This has been reaffirmed by the results presented in Figure 21, where the overall simulated average price is presented. To be able to access this difference, we have used some statistic features summarized in Table 3. Firstly, all the MSE values indicate that the models' results are not that far from the true value with the best average approximate of the original price given by the full model. All the models seem to produce prices that are not normally distributed but with flat and positively skewed distributions.

As it has been confirmed in the previous model cases, the ACF and PACF reveal the same main behaviour, see Figure 22 for all the simulated models. A positive long term trend and a first lag serial correlation are the most interesting features of non-stationary series that have been captured by all the models. Finally, to identify the distribution which can generate any random series of prices which are closer to the series of market price from these four models, Figure 23 has been used to select the histogram with best fit.

From the four distribution curves in Figure 23, the full model corresponding curve fits the histogram better than the others. However, for the other models also, the difference in not really significant.

Table 3: Basic statistics of the ABM with maximized greediness

Model type	Mean	Kurtosis	Standard dev.	Skewness	MSE
Type I	15.1590	2.6407	10.8168	0.8786	0.0021
Type II	15.1376	2.7214	10.9772	0.9542	0.0016
Type III	15.1530	2.6217	10.8707	0.8863	0.0019
Full model	15.0475	2.7639	10.9443	0.9624	0.0015

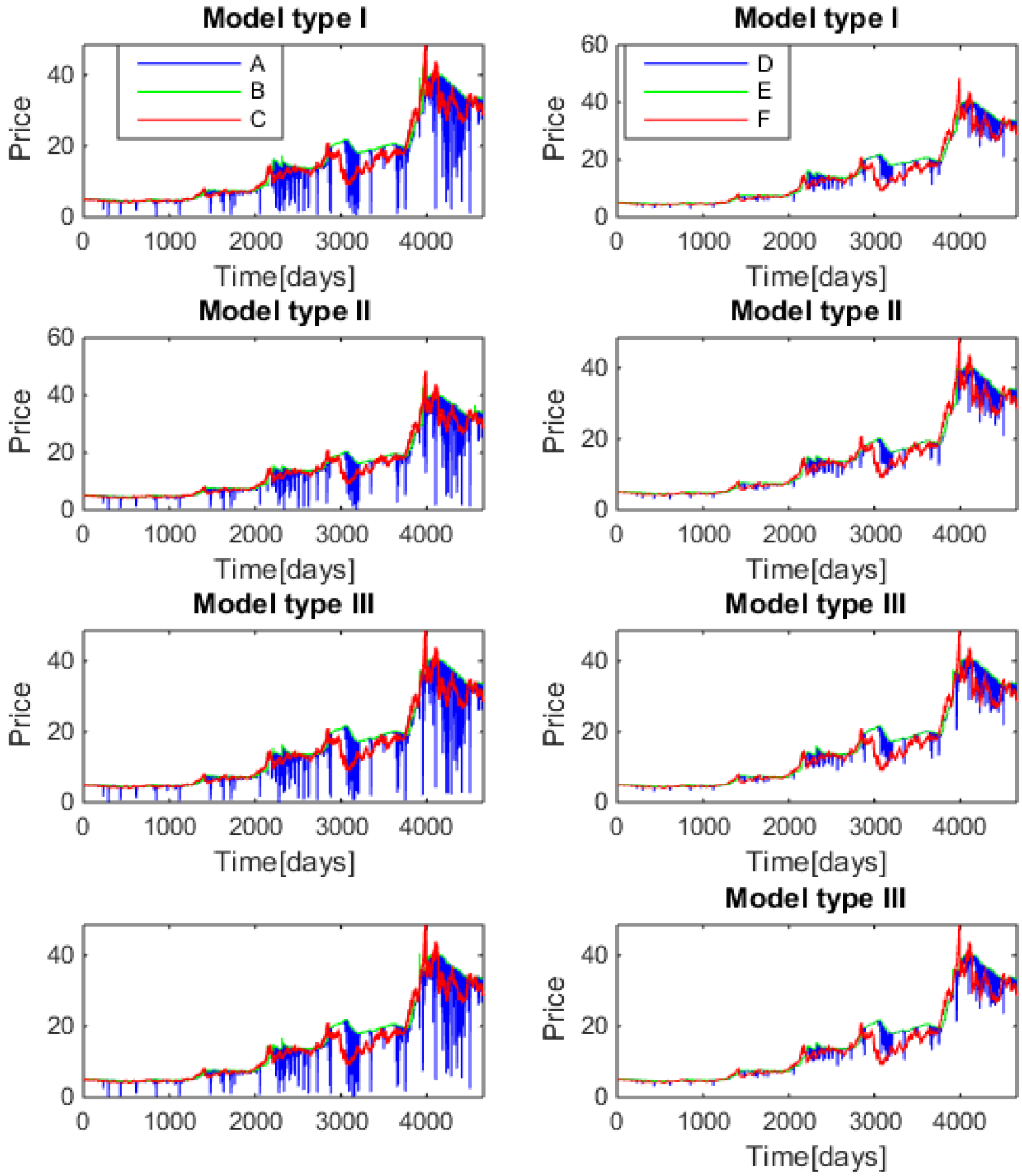

Figure 20: (A) buyer price (B) seller price (C) and (F) stand for market price (D) average buyer price and (E) average seller price

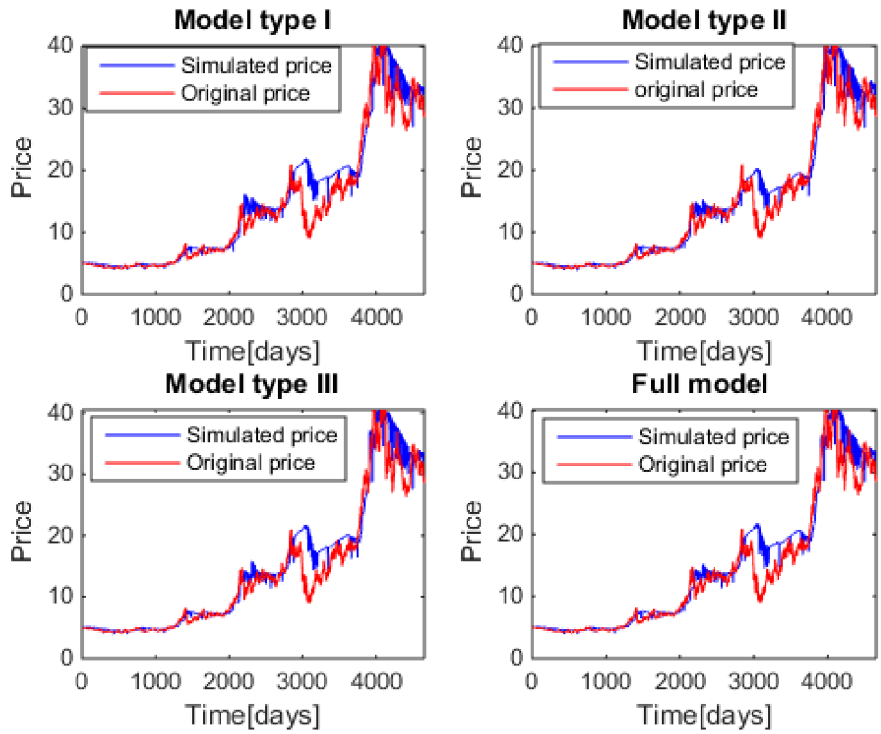

Figure 21: Traders' simulated price with maximized greediness against market price

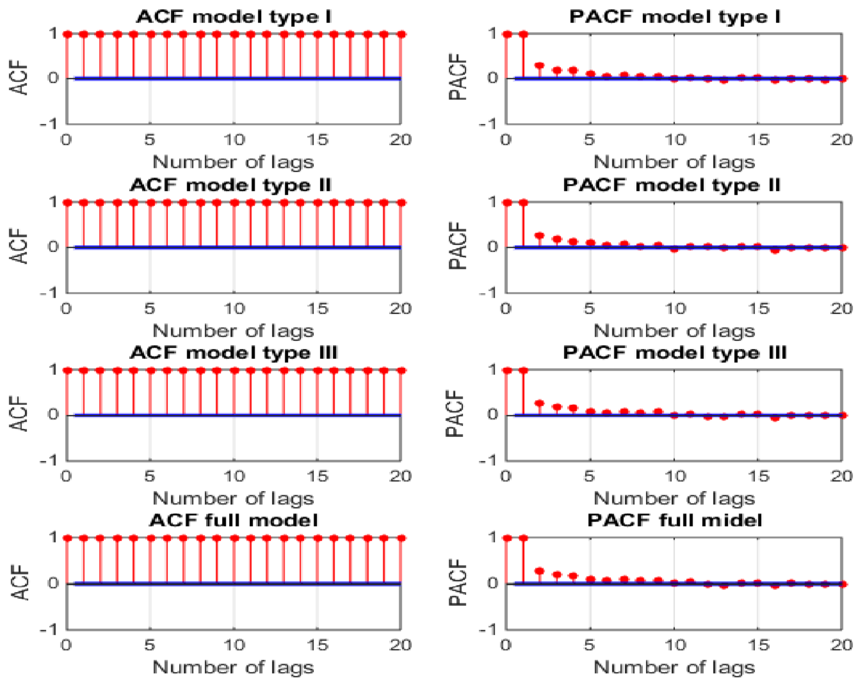

Figure 22: ACF and PACF of the simulated series, ABM with maximized greediness

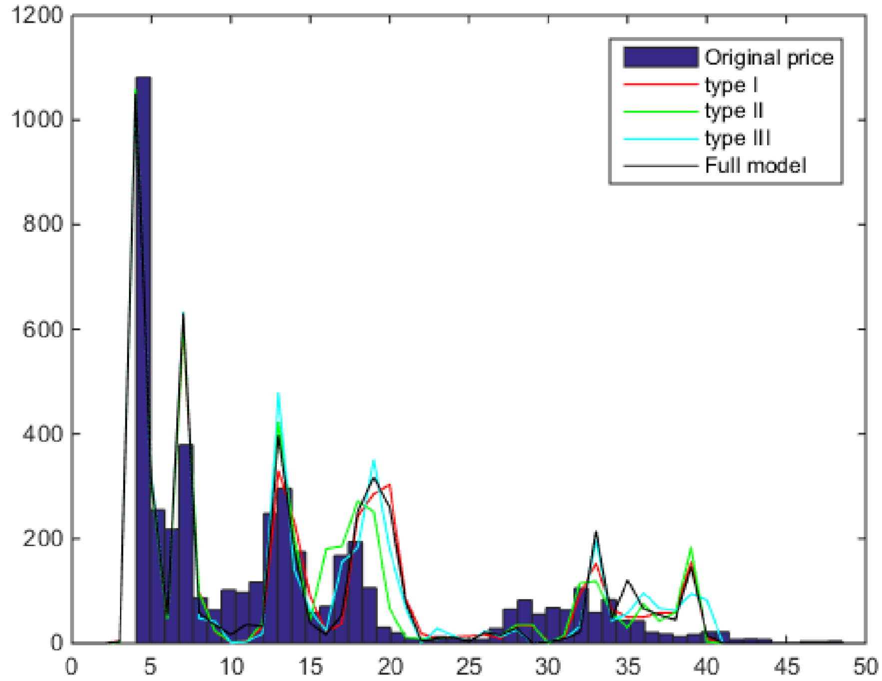

Figure 23: Histograms for ABM with maximized greediness

4.3 ABM performance

4.3.1 Comparison

The performance of the proposed ABM is analysed by comparing the simulation results of the model in two different situations. The first situation refers to look for the price dynamic explanations without the best trader ideology and the second case is where we include the best trader concept. The analysis will be done on the full model cases which stood as the best approximates in each of the two situations (with and without maximal greediness). The Figure 24 compares the price evolution when everyone's greediness is maximized to the case where greediness is not maximized. First of all, the results indicate that the models are able to approximate the market price with small errors and the models are able to mimic the main facts observed in the market price. The series simulated without maximizing every trader's greedy emotions comes as the best approximate of the original series between the two compared to the remaining one.

The kurtosis, skewness and average values indicate that the simulated series, ranked as the best, comes from a distribution, before found as non-Gaussian, which is very close to the distribution of the original market price as they hold almost equal values. Figure

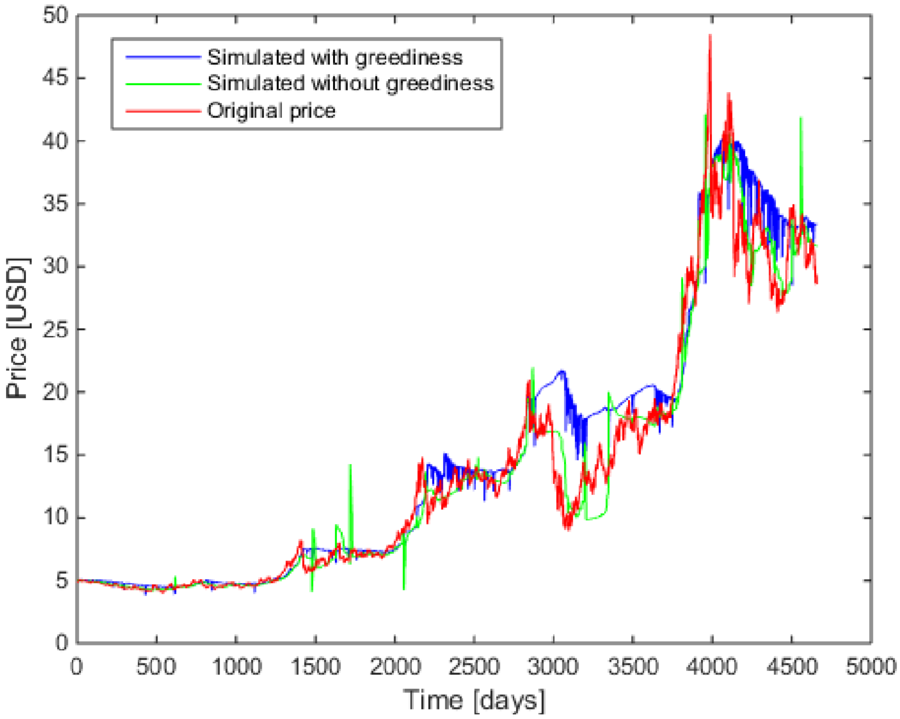

Figure 24: Price dynamics for the full model with and without maximal greediness

25 contains the histogram of original price against distribution curves of simulated prices were plotted. The red curve seems to be the best fitting curve to the original price histogram. The basic statistics from the series simulated in all cases above are grouped in Table 4. From the summary statistics, the mean, standard deviation, skewness and kurtosis coefficients of the full model without greedy emotions are the best approximates of the original series statistics and it was confirmed by its MSE which is very small compared to the one of the full model with maximized greediness. Therefore, the model without maximized greediness can explain the movements of the market price than when greediness is considered as one of acting forces.

Table 4: Basic statistics of the simulated silver prices

	Model type	Mean	Kurtosis	Standard dev.	Skewness	MSE
	Orig. series	13.7067	3.2251	10.1364	1.1263	
Without greediness	Full model	13.7151	3.0464	10.0728	1.10872	0.0004
With greediness	Full model	15.0475	2.7639	10.9443	0.9624	0.0015

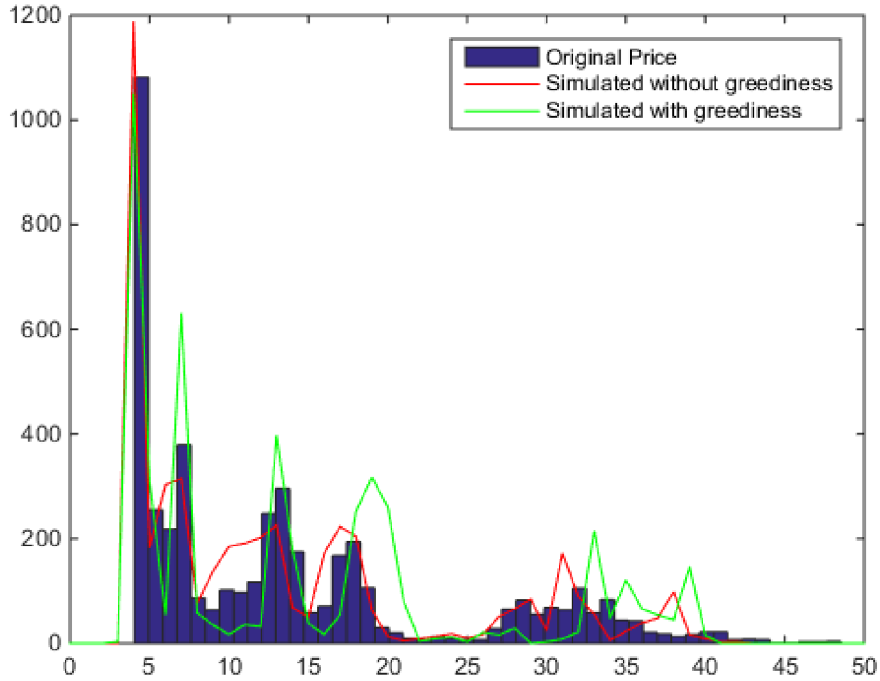

Figure 25: Distribution curves of the two models against the original price distribution

4.3.2 Case prediction without maximized greediness

To forecast using the full model without maximized greediness, we first assumed the closing price from the previous day was known and we initially generated uniformly and randomly distributed prices for sellers and buyers using that closing market price. Differently from the previous simulation results, the market price to be used for the next trading period is the average price from the previous day of all participants' prices. The forecasted window is 4658 days and we have assumed that the model parameters are also known as for a specific security or asset. Figure 26 tells us what would be the market price from each of the participants, for instance, after 1 year if we know the price from yesterday's market.

From Figure 27 Figure 28, the overall forecasted behaviour of the real price is the main trend of the original price. Figure 27 presents the forecasted average buyer and seller prices whereas Figure 28 contains the overall forecasted market price against the real market price. One can see that the ACF and the PACF of the forecasted market price, see Figure 29, reproduces the main properties of the original price: a non-stationary series with long time positive trend accompanied with a serial correlation of the first lag. However, it shows a considerable difference when it comes to bigger unexpected original price variations where the forecasted series fails to agree with it.

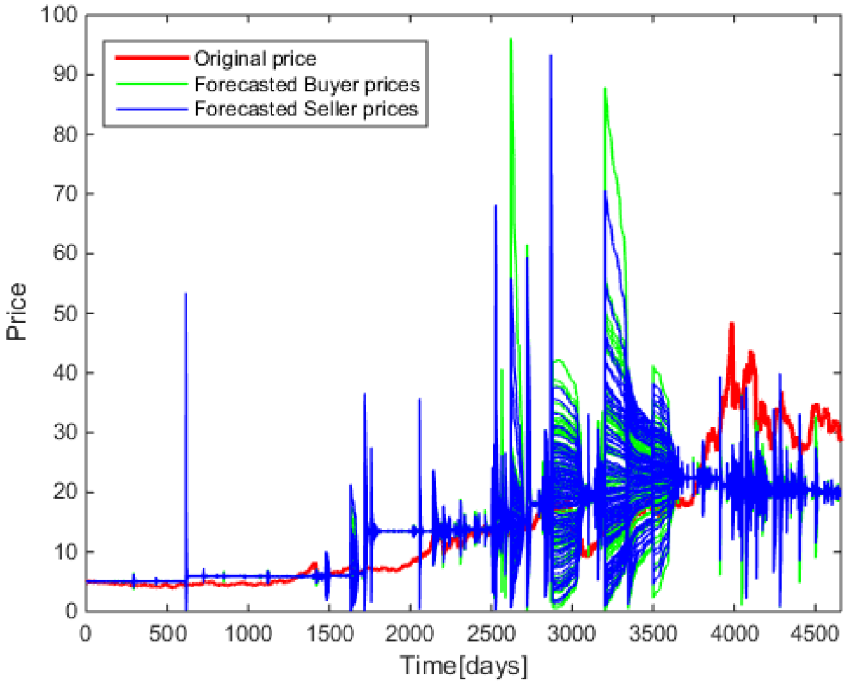

Figure 26: All traders forecasted prices

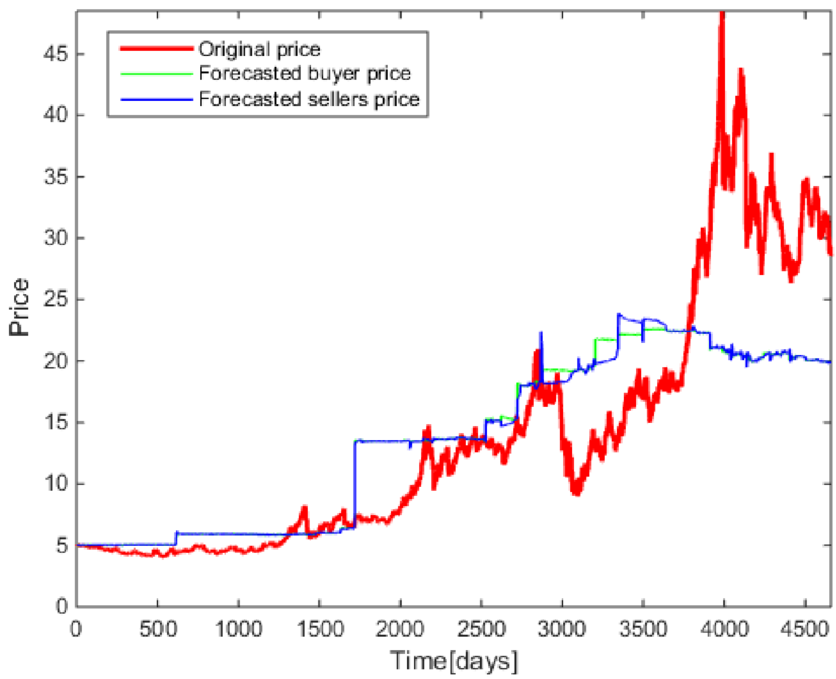

Figure 27: Forecasted buyers and sellers average prices

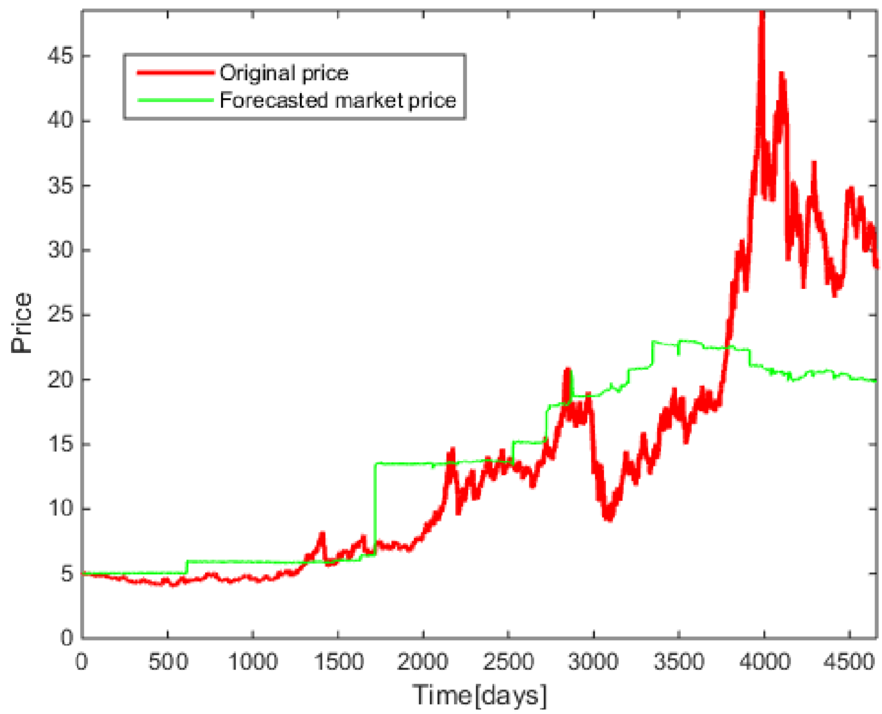

Figure 28: Time line plot of forecasted market price

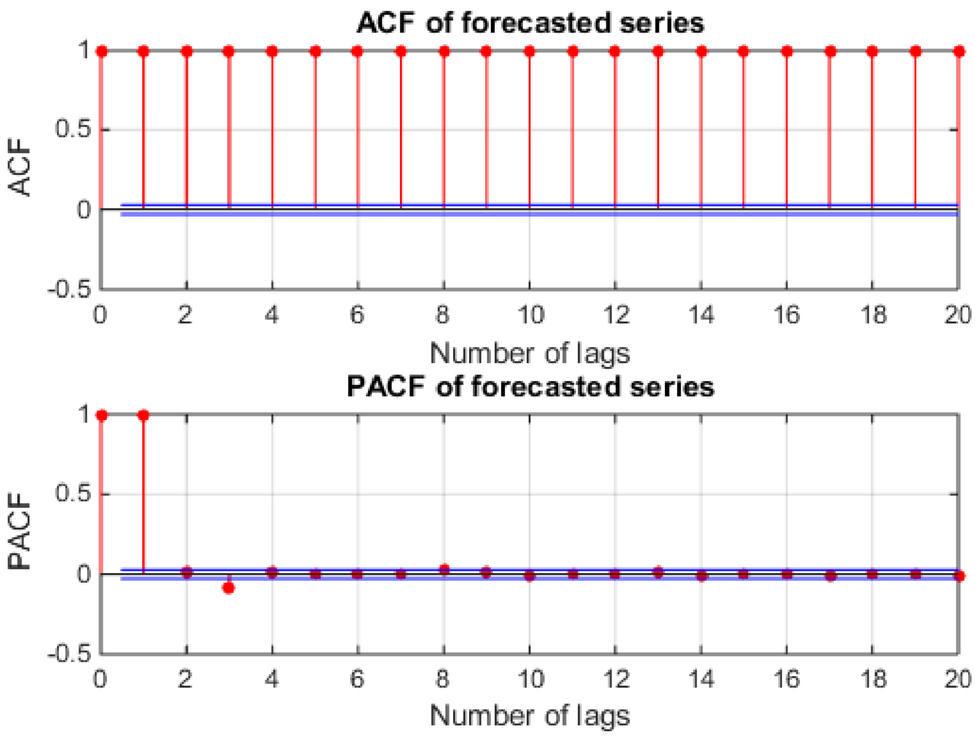

Figure 29: ACF and PACF of forecasted prices

5 RESULTS, SUMMARY AND DISCUSSION

The results discussed above can be summarized and grouped in three main categories. Firstly, the simulations were performed on incomplete and complete models. The next step it was to explain the price dynamics with and without maximized greediness whereas the last part of the results dealt with ABM performance in which comparison of the results, between ABM without maximal greediness and ABM with greediness was done against original prices. In addition to comparison, forecasting the future price of silver metal using ABM without greediness was implemented to weight the strength of the model.

To obtain the results on incomplete and complete ABM models, Netlogo as a multi-agent simulator, was used to simulate the trading world while Matlab software was used to execute the analysis. The market environment was composed of 200 traders, grouped in two distinct breeds, of 100 sellers and 100 buyers. The silver historical price considered in this study was taken on each of the 4658 days. The ABM parameters were first estimated using Matlab and then incorporated into the model. In the full model the farthest neighbour and moving average reversion coefficients were taken equal. The time line plots, histograms, ACF and PACF are the main forms of the obtained results, see Figure 7 to Figure 19. Furthermore, the summary statistics and MSE are grouped in Table 2. All the forms of the results affirmed that the full model stands as the best to explain the original price dynamics, closely followed by the incomplete model of type II i.e. the model without momentum effect. This has been justified by the fact that this full model was able to successfully capture the original price dynamics.

The results acquired, after taking greediness as an additional influencing factor in each of the four models (I, II, III and IV), were gathered in Figure 20 to Figure 23 and Table 3. They have not shown a very big difference between them yet the full model kept winning the title race compared to the incomplete models when they explain the silver price signals. The ABM performance results are assembled in Figure 24, Figure 25 and Table 4 for comparison and Figure 26 to Figure 29 for forecasting.

Reverting to the results in Figure 16 and Figure 17, a clear view of what really happens on the market can be seen. When pressing their bids, sellers and buyers do not know what will be the winning bid price (market price) and this explains the reason why sometimes buyers bid bigger prices than sellers or sellers can bid smaller prices than buyers. This irrational behaviour leads the market till the last trading day and the model successfully kept capturing all the market price dynamics. One the other hand, the results summarized in Figure 20 and Figure 21 show that it is hard to beat the market for everyone. It is clear that the sellers mainly stay above the market price whereas buyers lodge below it. In other words, the more greediness is maximized by every participant the more the trading environment becomes rational. Therefore, it will bring another fact that not only irrational behaviour should be suggested as the reasons for financial crisis because the results have shown that with maximal greediness the model was able to capture all the silver price features. To conclude the two types of models (ABM with and without greediness) were able to capture the main features of the silver price. Nevertheless, the ABM without greediness explained the silver original price better than ABM with greediness.

The forecasting process has assessed the power of the model where it has been found that the ABM is able to follow the main trends of the original price, however, it is still far from forecasting reasonably the future price especially in extreme events. The weaknesses of the model in forecasting the future price of silver may have been affected by the parameter estimation process. Also, the use of model parameter coefficients may be a reason to fail as we did not estimate the farthest neighbour coefficient but instead took it to be equal to the mean reversion coefficient.

6 CONCLUSIONS AND FURTHER WORK

In this work, an agent-based model (ABM) was proposed using the main idea from the Jabłońska-Capasso-Morale (JCM) model and maximized greediness for each of the market participants. To test the ABM, historical prices from the 1^{st} March 2000 to 1^{st} March 2013 of silver metal were used as a tool. The power of the ABM was analysed by simulating the statistical features of later prices when greediness is not maximized and when it is maximized.

Firstly, the original prices were analysed in order to identify any patterns in the prices. With the reference to basic statistics summary and graphical representations, the analysis found that the silver historical prices chosen above follow a non-stationary series. Nevertheless , the preferred JCM's ideology employs the mean reversion concept which has exempted the writer to transform the simulated series for the analysis. The simulations were performed in each of the situations (with and without maximized greedy emotions) and the results were obtained.

In both situations the ABM has been able to capture the market price dynamics with some key differences. The ABM without maximal greediness has followed successfully the original price movements with more irrationalities whereas the ABM when greediness is maximized suggests that the more the market is greedy, the more it backs the rational trading rules. Since ABM in both situations was analysed and the one without maximal greediness stood as the best. Therefore, both rational and irrational behaviours may be suggested as the cause of financial crisis eventhough irrationalities may dominate the market. Forecasting executed on a non-greedy market has shown the ability of the ABM to capture the main price traits.

In the future, the writer suggests to investigate the behaviour of the ABM when the maximized greediness is taken as an additional term, adjusted by a given parameter coefficient, to the JCM. The latter investigation should assess the sensitivity of the new parameter coefficient (the coefficient of the maximal greediness term) to a chaotic behaviour. The starting point should be to estimate the full model parameter coefficients using a multi-agent simulator which requires original prices from each one of the players.

References

[1] George A. Akerlof and Robert J. Shiller: *How Human Psychology Drives the Economy and Why It Matters for Global Capitalism?*, 2009. Princton University press.

[2] Bianchi, A., Capasso, V., Morale, D., Sioli, F.: *A Mathematical Model for Price Herding. Application to Prediction of Multivariate Time Series.* MIRIAM report (confidential), (2003). Milan.

[3] Burton G. Malkiel, Eugene F. Fama : *Efficient Capital Markets: A Review of Theory and Empirical Work*, 1970. The Jounal of Finance, USA.

[4] Capasso, V., Bianchi, A., Morale, D.: *Estimation and prediction of a nonlinear model for price herding. In: "Complex Models and Intensive Computational Methods for Estimation and Prediction"*, (2005). (C. Provasi, Ed.), Padova.

[5] Couzin, I.D., Krause, J., Franks, N.R., Levin, S.A.: *Effective leadership and decision making in animal groups on the move.*, 2005. Nature 433:513-516

[6] Emilio Gregado: *Measuring Entrepreneurship: Building a Statistical System*, 2008: University of Huelva, Published by Springer.

[7] Magda Fontana and Roberto Marchionatti: *Endogenous Animal Spirits and Investment An Agent-based Model.*. WORKING PAPER SERIES. No. 09/2007

[8] Frank H. Westerhoff: *Greed Fear and Stock markets dynamics* , 10.July, 2004. Department of Economics, University of Osnabrueck.

[9] Hersh Shefrin and Meir Statman: *Behavioral Finance in the Financial Crisis: Market Efficiency, Minsky, and Keynes* , 2011: Santa Clara University.

[10] John M. Keynes: *The General Theory of Employment, Interest and Money*, 1936.

[11] Matylda Jabłonska and Tuomo Kauranne: *Animal Spirits in Population Spatial Dynamics*, 2012: Lappeenranta University of Technology, Finland

[12] Matylda Jabłonska: *From Fluids Dynamics to Human Psychology.What Drives Financial markets Towards Extreme Events*, 2011: Doctoral thesis, Lappeenranta University of Technology.

[13] Morale Daniela, Capasso Vincenzo, Oelschlager Karl: *An interacting particle system modelling aggregation behavior: from individuals to populations*, 2005: Journal of Mathematical Biology 50(1), 49-66

[14] Sebastian Dullien, Detlef J. Kotte, Alejandro Márquez, Jan Priewe, *The Financial and Economic crisis of 2008-2009 and Developping countries*, 2010: United Nations conference on Trade and development. Geneva.

[15] Stefania Bandini, Sara Manzoni and Giuseppe Vizzari: *Agent Based Modeling and Simulation: An Informatics Perspective*, 2009. Journal of Artificial Societies and Social Simulation 12 (4) 4

[16] Stijn Claessens and M. Ayhan Kose: *Financial Crises: Explanations, Types, and Implications*, 2013: IMF working paper WP/13/28

[17] Thomas C. Schelling: *Dynamic Models of Segregation*, 1971: Jounal of Mathematical sociology Vol. 1, 143-186. Harvard University.

[18] Van H. Dyke Parunak*, Robert Savit**, Rick L. Riolo***Agent-Based Modeling vs. Equation-Based Modeling: A Case Study and Users Guide* , 1998: *Center for Electronic Commerce USA. **Program for the Study of Complex Systems, Randall Laboratory, Univ. of Michigan.

List of Tables

List of Figures