LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
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INVESTIGATION ON BALANCING OF A HIGH-SPEED ROTOR

Examiners: Professor Jussi Sopanen
D. Sc. (Tech.) Behnam Ghalamchi

ABSTRACT

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Keywords: balancing, unbalance, flexible rotor, rigid rotor, high-speed, bearing

Balancing is an effective way of reducing vibration and reaction forces in rotor's bearings. Two main approaches of modal balancing and influence coefficients method have been used for this aim. However, by considering specific balancing procedure for each rotor and addressing and detecting many of the unbalance sources in manufacturing and assembly this reduction can be ensured in earlier stage.

This thesis provides the theories and backgrounds, in addition of a step by step guidelines for rotor balancing. Then applies those theories and guidelines in order to balance a sample flexible rotor, supported by magnetic bearings, in high-speed. Possible unbalance sources are considered in the text. A simulation analysis featuring the effect of variation of the arrangement in a part of the system components is provided.

The results of the study indicate that the performance of influence coefficient method was great for the magnetic bearing supported rotor. The other imperative result approves the need for accurate assembly procedure of the components to reduce initial unbalance of the rotor.

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Hamed Ghasemi
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TABLE OF CONTENTS

LIST OF SYMBOLS AND ABBREVIATIONS

Greek letters

α	Angular position
ω	Angular frequency (rad/s)
Ω	Rotor service speed (rad/s)

Latin letters

a	Acceleration (m/s^2)
A	Amplitude (m)
c_{ij}	Influence coefficient value for ith balance plane in jth specified speed
C_{min}	Minimum rotor-stator gap for AMB supported rotor (μm)
D	Distance of unbalance plane (mm)
D_{max}	Maximum vibration amplitude in AMB (μm)
d	Distance of correction planes (mm)
e	Mass eccentricity (m)
e_{Per}	Permissible residual specific unbalance (kg·mm/kg)
F	Unbalance force (N)
G	Balance quality grade (mm/s)
$\mathbf{I}_{\text{coeff}}$	Influence coefficient matrix (mm/s/g·mm)
L	Bearing distance (mm)
L_A	Distance of bearing A from center of mass (mm)
L_B	Distance of bearing B from center of mass (mm)
m_u	Unbalance mass (kg)
m	Rotor or component mass (kg)
m_A	Trial mass A
m_B	Trial mass B
$\boldsymbol{M}$	Vector of Correction masses
N	Number of critical speed in operating range
q	Number of vibration measuring planes
r	Correction plane radius (mm)
$\boldsymbol{RU}$	Vector of Residual unbalance

t	Time (sec)
$\boldsymbol{T}_w$	Vector of unbalance Trial mass (g·mm)
U_{Per}	Permissible residual unbalance (g·mm)
$U_{Per,B1}$	Permissible residual unbalance in bearing 1(g·mm)
$U_{Per,B2}$	Permissible residual unbalance in bearing 2 (g·mm)
$U_{Per,HP}$	Permissible residual unbalance for high pressure rotor (g·mm)
U	Unbalance amplitude (g·mm)
v	Velocity (m/s)
$\boldsymbol{v}_{ref}$	Vector of Vibration response without trial mass (mm/s or mm)
$\boldsymbol{v}_{TW}$	Vector of Vibration response to trial mass (mm/s or mm)
$\boldsymbol{V}$	Vibration vector (mm/s or mm)
$\boldsymbol{V}_A$	Vector of current vibration readouts
$\boldsymbol{V}_C$	Vector of theoretical vibration values
$\boldsymbol{V}_{m_A}$	Vibration response to trial mass A
$\boldsymbol{V}_{m_B}$	Vibration response to trial mass B
$\boldsymbol{V}_R$	Vector of residual of vibration
x	Displacement (m)

Abbreviations

1X	First harmonic
AMB	Active Magnetic Bearing
BP	Balance plane
CM	center of mass
DE	Drive End
HP	High pressure
LP	Low pressure
NDE	Non Drive End
QAP	Quadratic Assignment Problem
rpm	Revolution per minute

1 INTRODUCTION

The application of rotating machines is wide-spread, from offices and kitchens to specific industries. Wherever they are working, based on their circumstances and application, some criteria as working conditions are applied to them. Some of these conditions taking care of human safety, partly consider machine safety and reliability and some other should be satisfied since they are vital for machine to operate. Temperature, vibration, noise, pressure are general example of these factors.

The effect of vibration in rotary machines has been studied extensively in recent years. Vibration level in rotating machines is highly important and the vibration magnitude is an indication on how proper a machine is working. Moreover, high level of vibration might affects other parameters such as lubricant temperature, pressure, et cetera. Additionally, it can be followed by machine fault or in the worst condition, damage in the machine. Therefore, tracking and detecting changes in level of vibration while machine is working helps to predict fault/faults.

1.1 Background

It is generally accepted that vibration does not stem from a specific sources, therefore, it is not possible to make an exact recommendation, for vibration reduction, applicable in all the rotary machines. Reasons such as temperature fluctuation, noise disturbances, and mass unbalance can all cause vibration, hence, each should be treated specifically. Not all but the common sources of vibration according to Norfield (Norfield, 2006) can be: “Misalignment, bent shaft, a shaft rubbing on the stator, damaged bearings, turbulence, cavitation, oil whirl, inadequate lubrication, loose mountings, worn gears, stator windings, broken rotor bars and unbalance”. The reason of mentioned sources of vibration can be traced in design problem, non-conformity in manufacturing, assembly faults and sometimes they refer to a machine failure.

Vibration can indicate engineers some information about system’s health. System diagnosis can be done through recording and analyzing of system vibration when it is needed. There is now much evidence to that machines with vibration problem in service were suffering from unbalance forces and misalignment (Gunter, et al., 1976), (EHRLICH, 1999). Thus, by filtering the vibration signal and finding the vibration at needed frequencies up to running speed it is possible to detect the unbalance forces and apply the proper correction; however, it should not be forgotten that amplitude of 1X (first harmonic) can be changed by other faults too.

1.2 Research problem

Unbalance can be referred as a primary reason of vibration in rotary machines. Therefore the sources of unbalance shall be detected and treated in a proper method in order to reduce machine vibration. Performing a right method of balancing with great achievements as its result, demands correct selection of the measuring tools, balancing planes. In addition, prior knowledge about the rotor, the bearings and their behavior is necessary.

The rotating machines, with no exceptions, do need specific balance procedure. Based on the application and characteristic of the rotating machines, individual conditions might be applied on their balance procedure. These conditions satisfy both the human safety as long as the machines are working safely and smoothly. Finding these conditions and appropriate balancing procedure for a specific case provided by a company is the core of the thesis.

1.3 Research motivation

The current thesis is defined and supported by Aurelia Turbines Oy, the machine manufacturer, to review the balancing literatures and help to deep understanding of the subject. Moreover, investigating the reasons behind the high level of vibration and identifying the most common sources of error is demanded. Then, a proper solution for removing or reducing the effect of detected fault shall be provided. In addition, by reviewing related standards, appropriate criteria and clauses will be mentioned. Collecting all these information and making a detailed documentation will be an asset for the company to immune high quality of their product in serial production.

1.4 Research Objectives

In new manufactured or maintained rotary machines, 1st synchronous vibration amplitude is significant compared with other components and dictate the overall vibration of the machine. Unbalance can be addressed as the most known reason for domination of the 1X vibration. Hence, it is important to check unbalance state of these rotors and correct them if needed. Consequently, performing balancing procedure is accepted as last step of manufacturing process and repair of rotary parts in order to decrease unbalance forces and ensure smooth machine operation (Rieger, 1986), (Foiles, et al., 1998). However, the balancing procedures and its needs shall be taken into account in early stages of design. Therefore, following questions will be answered by the present research:

- What are the main sources of unbalance in the considered machine?
- How those fault can be treated to lower the unbalance in rotor?

Balancing procedure is applied on rotating machinery in order to manipulate the effect of vibration on the working quality of the machine. It is worth mentioning that balancing is just a treatment for unbalance sources and it cannot be used to decline vibration due to other mentioned causes. If any fault other than unbalance which cause vibration problem issues does exist in the rotor, it should be removed/repaired first, then a balancing can be done on the rotor (MacKenzie & Rebstock, 1996).

1.5 Structure of the thesis

In this thesis the balancing procedure solely for unbalance mass, as a source of vibration, is studied; the respective balance procedure for mass unbalance is covered and some definitions regarding rotor balancing and vibration measurement are introduced. For involving the reader with the problem and to answer the research questions, by reviewing number of literatures, general theories and backgrounds on rotor characteristic is shortly reviewed and balance criteria and tolerances are defined which all are covered in chapter2. In chapter 3, the rotor characteristic and test results are provided. The related rotor is balanced based on influence coefficient approach and the results are presented in the same chapter. During the test it was observed that there are other sources of unbalance present in the system which could have been removed by taking into account in earlier steps. They are listed at the end of chapter 3. Effect of one of them on rotor unbalance condition in details is investigated in chapter4. In this section residual unbalance from magnet distribution is solitary considered as unbalance source in the rotor. The significance of these unbalance forces are examined by rotor response simulation. At the end, a conclusion including answer to research questions in addition to summarizing highlights of the thesis are the ground for chapter 5.

2 DEFINITIONS AND THEORIES

To address the demands of the thesis, the needed terms and definitions are covered in details in this section. In the beginning of this chapter vibration measurement, measuring tools and their output as vibration readout are shortly reviewed. Then rotor classification for balancing aims is introduced and followed by overviewing of different bearings. Finally, general test and balancing procedures for rotors based on the standards, previous researches and experiences are presented.

2.1 Vibration measurement and vibration content

Vibration can be recorded in three different measures, namely displacement, velocity and acceleration. A periodic displacement can be written in simplest form of equation (1) . By taking the first and second derivatives of equation (1), velocity and acceleration will be in the form of equations (2), (3) respectively. In figure 1, the graph corresponding to this a simple sine wave motion for displacement, velocity and acceleration is shown. Through equation 1 to 3 and the graph in figure 1, it can be seen that displacement is following velocity by 90 degree phase difference and acceleration advanced each of displacement and velocity 180, 90 degree respectively. It should be noted that the max and min in the graph correspond to A for displacement, $A\omega$ for velocity and $A\omega^2$ for the acceleration.

$$x(t) = A\sin(\omega t) \qquad (1)$$

$$v(t) = A\omega\cos(\omega t) = A\omega\sin\left(\omega t + \frac{\pi}{2}\right) \qquad (2)$$

$$a(t) = -A\omega^2\sin(\omega t) = -A\omega^2\sin(\omega t + \pi) \qquad (3)$$

where :

A is displacement amplitude, ω is angular frequency, t is time, x is displacement v is velocity and a is acceleration.

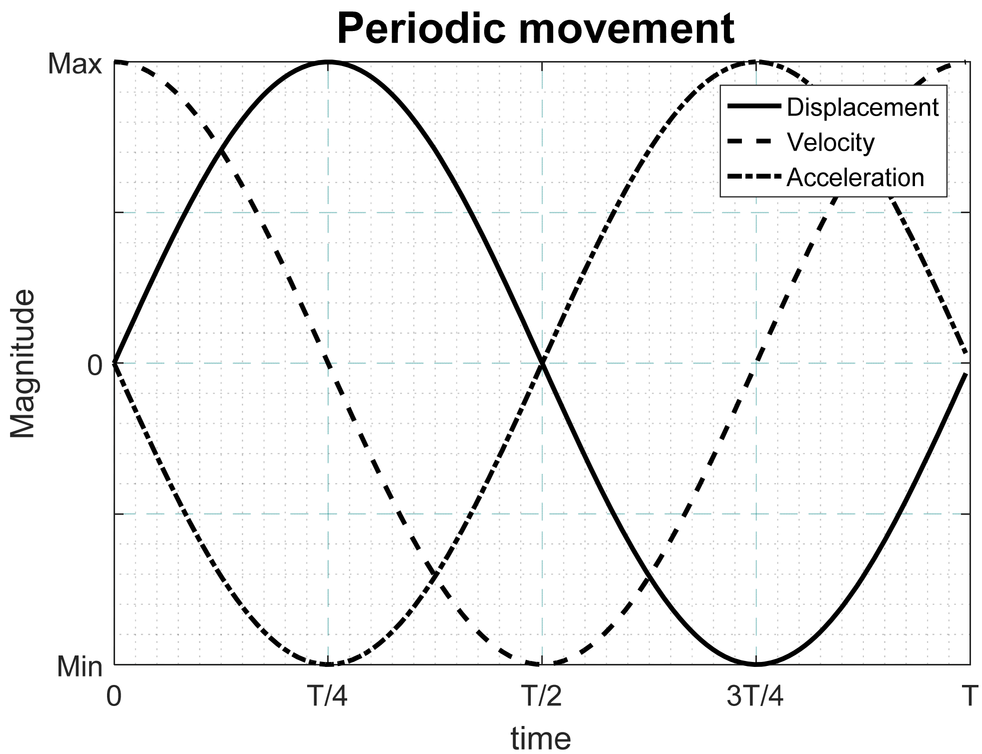

Figure 1. Displacement, Velocity and Acceleration for a simple harmonic motion

There are many types of sensors for measuring vibration. From one aspect, based on measuring unit, they are divided into three main categories of:

1. Displacement sensors
2. Velocity sensors
3. Acceleration sensors

In other aspect, based on assembly method, sensors can be divided into two groups of contact and non-contact. Contact sensors typically attached to bearing casing or other non-rotating parts and measure the vibration level of the stationary part, whereas non-contact sensors usually are mounted close to bearings and attached to bearing casings but data collected from surface of the shaft for radial vibration or other rotary part in axial measurement. Applicable sensors for vibration measurement and their advantages and disadvantages are listed in table 1. An example setup for measuring relative shaft vibration with proximity sensor is shown in figure 2. Other arrangement are also possible.

Table 1. Sensor properties. (Andrés, 2012)

Type of sensor		Advantages	Disadvantages	Measure on	Examples
Contact	Displacement	• Static and dynamic measurement • Exact at low frequencies • No wear • Small and low cost	• Electrical and mechanical noise • Bounded by high frequency • Needs calibration for different material • Not easy to mount • Require external power	Stationary part or casing	• Capacitance • Eddy current
	Velocity	• Easy to mount • Suitable for mid-range frequency • High temperature resistant • No need to external power • Lowest cost	• Low resonant frequency and phase shift • Cross noise • Big and heavy • Require electronic integration for displacement	Stationary part or casing	• Electromagnet linear velocity transducer • Electromagnet tachometer generators
Non-contact	Acceleration	• Easy to mount • Small size • High temperature resistant • Good response at high frequencies	• Sensitive to high frequency noise • Require external power • Require electronic integration for velocity and displacement	Rotary part	• Piezoelectric

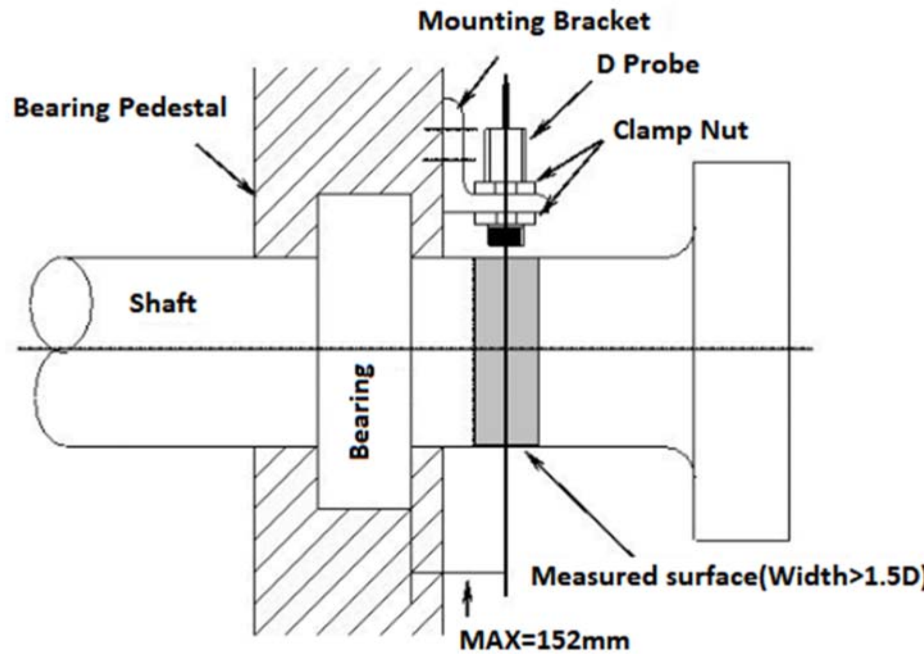

Figure 2. Use of proximity sensor for measuring shaft vibration *(Axis Vibration Transmitter, 2009)*

An important issue regarding the displacement sensor is that the full length of the rotor is not always accessible and it is not possible to mount these type of sensors every location along the rotor axis; while position of measuring plane for these sensors is extremely important. The measurement plane shall be located far enough from the rotor node in all mode shapes within the rotor speed range. Excessive approach of sensors' plane to these node will decrease the accuracy and it is possible an unbalanced rotor be considered within balance criteria. (Darlow, 1987)

Two other factors which might affect the displacement sensors output are mechanical and electrical runout. These sensors working by measuring the gap between the probe tip and the rotary part surface. Since the mechanical runout in the measuring plane does change this gap it will be read out as mechanical vibration by the sensors. On the other hand, the sensor record the mixed changes due to both of the vibration and mechanical runout. Electrical runout is the change of magnetic and/or electrical property of material over the rotor circumference. This material property, if it does exist, can negatively affect the accuracy of eddy current sensors vibration measurement.

It is time to figure out what are the content of vibration recorded by sensors. To answer this question first it is needed to define some new terms. To begin with; unfiltered vibration or direct component is the vibration read through sensors without applying any filter or mathematical operation (Donald, et al., 2003). They can be called raw data since no process is applied on them. They are compound vector of vibration in different frequencies present in the system because of variety of faults. In theory system with no fault should have zero vibration amplitude; however, it is an ideal case an always some faults are present in the system.

Filtering is a process through which unwanted vibrations are removed. Variety of filter can be applied to the raw signal to get the desired output. Figure 3 presents some of the filter types. It can be seen from the figure that low pass filter remove all the signal higher than the defined frequency, whereas high pass filter remove the signals in lower frequency with respect to the predefined frequency. The most common in use filter is narrow band pass in which a narrow band around a central value is defined and only the data inside the band will be passed through the filter. If the central value is set to be equal to rotor running speed, then synchronous or sub synchronous vibration as output signal can be generated. (Muszynska, 2005)

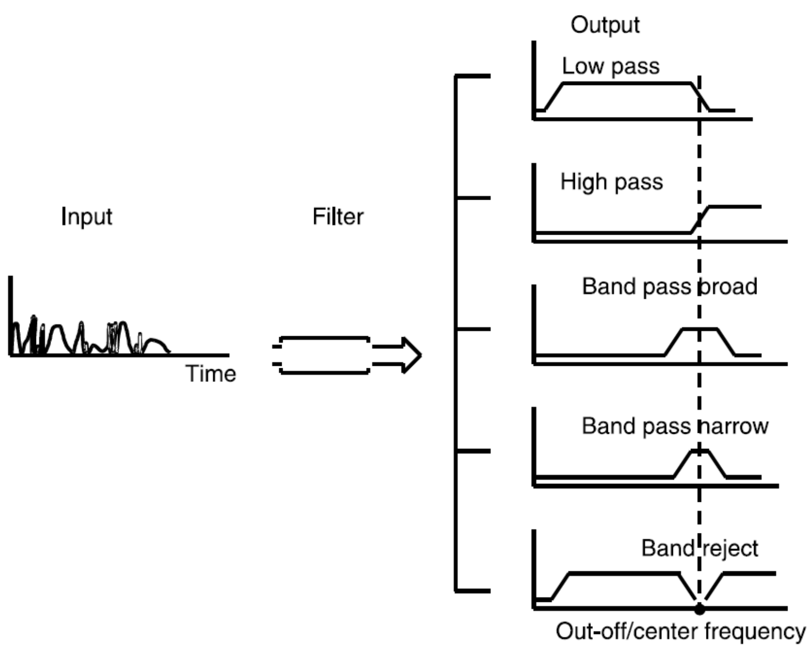

Figure 3. Filter types *(Muszynska, 2005)*

The output of filtering process is called filtered signal. An example unfiltered signal is provided in Figure 4. By applying a band pass filter first and second harmonies are detached from the noisy signal and are drawn in the figure. It can be seen from the figure 4 that although signal contains noise and/or higher harmonies; 1st and 2nd harmonies of the rotation speed are significant.

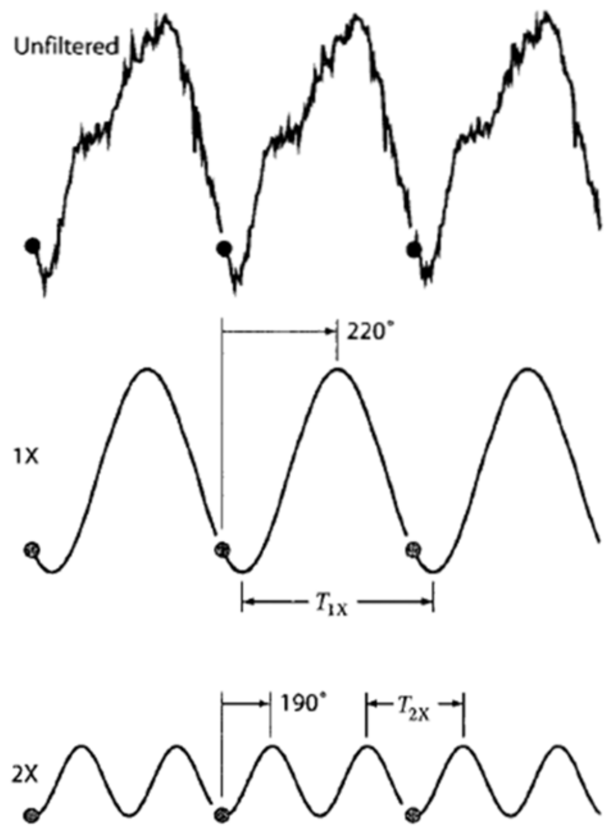

Figure 4. Unfiltered signal and its 1st and 2nd signal component. *(Donald, et al., 2003)*

2.2 Rotor classification

There are different methods for performing balancing on a rotor and its components. Depends on number of parameters, there is no unique balancing procedure for all rotor types. Geometry, mass, rigidity or flexibility, rotor components, service condition, bearing type are some of those effective parameters. Among these characteristics, finding that whether a rotor is rigid or flexible strongly affect balancing procedure. Rotors are mainly divided into rigid and flexible rotors (Rieger, 1986). Other classification are introduced on literature (Norfield, 2006); however, all of them are branches of rigid and flexible rotors. For instance, ISO standard categorize rotor in five classes as (Rieger, 1986), (Rangwala, 2006):

- Rigid rotors: Two plane balancing satisfy balance condition for whole speed range of this rotor category.
- Quasi-flexible rotors: Rotor is in gray area and cannot be considered fully rigid. By special care it can be low speed balanced in such a way that machine work smoothly from standstill to operating speed.
- Flexible rotors: Low speed balancing for this rotor maybe applied as a preliminary balance, but technically these rotor shall be balanced in high speed specifically at critical speed within machine speed range.
- Flexible-attachment rotors: Rotors from each of the previous group with difference in flexibility of the components/the way they are attached.
- Single speed flexible rotors: Branch of flexible rotor with just one selected balancing speed.

Five groups with possible subdivision of quasi-flexible rotors are shown in appendix I.

Before introducing balancing method, it is necessary to become familiar with two main rotor classification of rigid and flexible. Still it should be emphasized that there are some rotors which are in gray area and can be considered in both scenario.

2.2.1 Rigid rotor

This class of rotors have a simple theory and static equilibrium are applicable for them. Rieger (Rieger, 1986) defines rigid rotors as “Rigid rotors are those that can be balanced by addition of suitable correction masses in two axial planes along the rotor”. This primitive definition helps to better understanding of rigid rotor but still more specific information is needed to call a rotor surely rigid. Based on other definitions rotor much stiffer than their support and with service speed[1] well below the critical speed can be considered rigid (Chen & Gunter, 2010). The latter definition is emphasizing that in a rigid rotor, nor first critical speed neither the higher ones affect the rotor dynamic behavior up to their service speed and rotor does not significantly deform while it is rotating. According to Rangwala (Rangwala, 2006) “If analytical methods indicate that potential energy in the bearing is over 80% of the

[1] Service speed or operating speed is rotational speed in which rotor is designed to work.

system's total strain energy, the rotor is considered to be rigid". A rotor is rigid if its service speed be lower than 70% of its first bending natural frequency (ISO 11342, 1998). Based on the author personal experience and as a rule of thumb rotor with maximum service speed:

a) Lower than 60% of its 1st critical speed are rigid,
b) 60% to 70% of service speed are in gray area and depend on the rotor mass distribution and arrangement can be rigid or flexible. For this rotors it is highly recommended to perform flexibility test described in section 2.2.2.
c) 70% and above can be considered flexible.

For a rigid rotor, 2-plane low speed balancing would be the best option to ensure smooth rotor operation to its operating speed; however, position of correction planes along the rotor has a huge effect on the achievable balance quality and amount of mass which is needed to be removed/added from/to the rotor.

2.2.2 Flexible rotor

All the rotors, those do not meet the condition mentioned in section 2.2.1, shall be considered as a flexible rotor. Adams (Adams, 2009) defines flexible rotors as: "A flexible rotor is also definable as one which dynamic bending shape changes with rotational speed, and this speed-dependent dynamic bending may alter the state of balance". Different rotor mode shapes are the examples for rotor bending line changes due to speed change. First three bending modes for a flexible shaft on simple bearing support is shown in figure 5. It can be seen from the Figure 5 that, in this configuration, rotor experience the most deflection in the middle for first bending mode, whereas this point has the lowest deflection on the second mode. In addition, rotor experience maximum deflection at two points along the rotor. For the third mode shape the rotor is getting more flexible with respect to 1st and 2nd bending mode and it vibrate by maximum amplitude at three points.

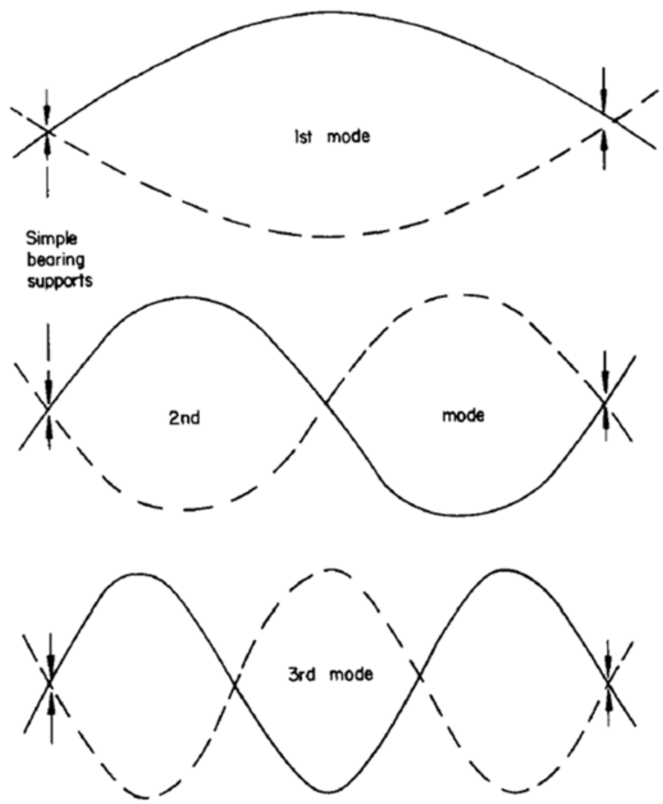

Figure 5. V-mode (banana), S-mode and W-mode shapes of a simply supported shaft. *(Sharp, 1980)*

Annex E, ISO-11342 (ISO 11342, 1998) presented a method to evaluate flexibility of a rotor. It is recommended that rotor with service speed higher than 60% of the 1st critical speed passes the mentioned test (ISO 11342, 1998). Where it is applicable; performing over-speed (see-2.4.1) test before flexibility test is beneficial since unbalance distribution does not change during test due to slight changes in component position. It is a common experience that shrink fitted components or assembly parts may have small changes and cause the rotor unbalance change during early run-up/run-ups. If their effect exist in flexibility test data, the result of the test is not accredited.

Rotor flexibility test

Following procedure can be used to define whether a rotor is rigid or flexible.

a. Mounting rotor in test stand with similar support dynamic properties of the service. The test stand shall be able to rotate the rotor up to its service speed,

b. Rotate the rotor with low sweep rate[2] to service speed. The acceleration shall be small enough that steady state vibration can be measured and recorded at each measurement point for both run up and coast down curve (run-0),

c. The vibration level shall not exceed defined limit based on rotor manufacturer documents or test machine capabilities,

d. By reviewing the results from step (b) following possibilities can be observed:

d.1. Vibration amplitude during run-up and run down change dramatically with change of speed. In this case, it shall be considered that change in rotor vibration may be in result of other parameters like flexible support stiffness, temperature or settling of parts. For determining that whether rotor flexibility is the source of these changes, additional test in d.2d.2 can be performed. (ISO 11342, 1998)

d.2. In spite of some fluctuations, not major change in vibration level is observed with respect to speed. Then, additional test is needed to define whether the rotor is rigid or flexible. Following steps describe the extra test:

d.2.1. Step A: A trial mass is added into the rotor mid-plane relative to bearing which significantly change the vibration level from previous run. Then steps b) & (c) are repeated,

d.2.2. Step B: Previous trial mass is removed and two new trial masses shall be added to the rotor most outer planes in a way that bearing loading condition in low speed does not change. Once more, steps 2 & 3 are repeated,

d.2.3. Changes in vibration with respect to run-0 due to mass m_A and mass set m_B shall be calculated and called $\boldsymbol{V}_{m_A}$, $\boldsymbol{V}_{m_B}$ respectively. Then, equation (4) contributes the result of flexibility test. (ISO 11342, 1998)

$$\frac{\left|\boldsymbol{V}_{m_A} - \boldsymbol{V}_{m_B}\right|}{\left|\boldsymbol{V}_{m_A}\right|} = C \quad \Rightarrow \quad Rotor\ is \begin{cases} rigid & C < 0.2 \\ flexible & C \geq 0.2 \end{cases} \tag{4}$$

2.3 Bearing classification

Bearings play a major role in rotary machines. They provide low friction support to relative contact surfaces in sliding and rotating motion. High demand of bearings in industries and engineers desire to reach higher rotational speeds conducted many researches to develop

[2] Amount of increase/decrease in frequency over certain period of time (Hz/s).

new technologies with higher capacity. Primary purpose of investigations as Harnoy (Harnoy, 2002) consider are: “extend bearing life in machines, reduce friction energy losses and wear, and minimize maintenance expenses and downtime of machinery due to frequent bearing failure”.

Today, variety of bearing concepts are commercialized. They can be categorized based on their type of lubrication, material, shape, load direction, load type and assembly arrangement. Harnoy (Harnoy, 2002) provide a simple classification in his book: rolling-element bearings, hydrodynamic bearings, hydrostatic bearings, and electromagnetic bearings.

2.3.1 Roller bearings

Roller bearings consist of four main parts, namely inner race, roller, cage and outer race. Inner or outer race can be fixed and the other can be moved with moving part depending on the application and the load which is going to be carried by bearing. The rollers let the rings have relative movement with low friction. Ball bearing, cylindrical bearing, needle bearing tapered roller bearing are main roller bearing diversity. Roller bearing has been used for many years in range of industries their characteristics has been improved over the years but they still suffer from temperature limit, power loss and maintenance costs.

2.3.2 Fluid film bearings (hydrostatic & hydrodynamic bearings)

Hydrostatic bearings works with pressurized fluid with a pump directed to bearing whereas in hydrodynamic bearings the oil film is constructed and pressurized by speed of the journal inside the bearings. Similar to roller bearings they have vast usage in rotating machines and are being used for example turbines, compressors and pumps. Temperature, power loss and fluid shearing issue are of the top of reason prevent their usage in very high-speed applications. Zero friction in zero speed and stop/start, no contact and wear, applicability of variety of lubricant and tolerating very high loads are the main reasons for preferring hydrostatic to hydrodynamic bearings (Khonsari & Booser, 2008).

2.3.3 Magnetic bearings

Although magnetic bearings conceptualized in 19^{th} century, they can be called new generation of bearing. It can be predicted that in future, further development of this type of bearing can phase out other type of bearings. Their sophisticated characteristics in inducing very low friction with no contact, wear and without need to any lubrication make them special, however; they are very expensive when consider the whole control system and bearings in compare with other type. Great development of rotor suspension on magnetic bearing in the past decades connected to power electronics and information processing accessibility and development of the theory in control design and rotor dynamics modeling (Bleuler, et al., 2009, p. 9). In addition, they are capable of applying harmonic forces and control rotor behavior to some limits by adjustable stiffness and damping. The idea is based on keeping the rotor in its initial position at zero speed by applying magnetic force equal and opposite of the dynamic forces due to operational malfunction. Principal mechanism of magnetic bearing is shown in figure 6. As the figure 6 illustrates, a displacement sensor measure the changes of position and send it as feedback to controller. The controller apply a magnetic force on mass against of its displacement by adjusting the current. This loop will be continued until the mass backs to its original (predefined in controller) position.

In figure 7, a section view for a sample rotor suspended in magnetic bearing is shown. According to the figure 7, the rotor radial position is monitored with one sensor at each bearing location. In addition, a sensor is recording rotor axial displacement. The output of the sensors will be analyzed and corresponding reaction force will be applied through the magnet bearings. In reality, using one sensor is not sufficient to find the exact position of shaft at each moment. Therefore, two probes with relative angle of 90 degree are used in the same plane. By mixing the output signal of both of the sensors, the exact location of shaft center line in probes' plane is measurable.

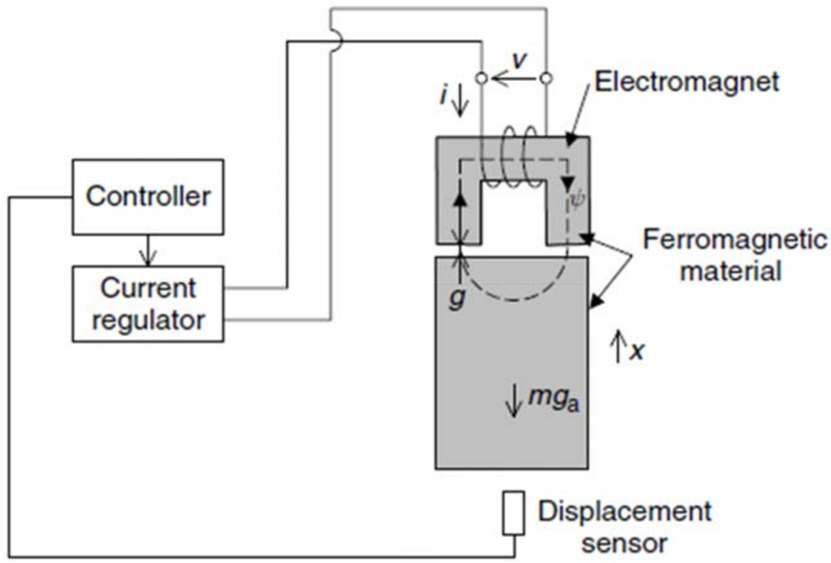

Figure 6. Simple Model for magnetic bearing mechanism *(Chiba, et al., 2005)*

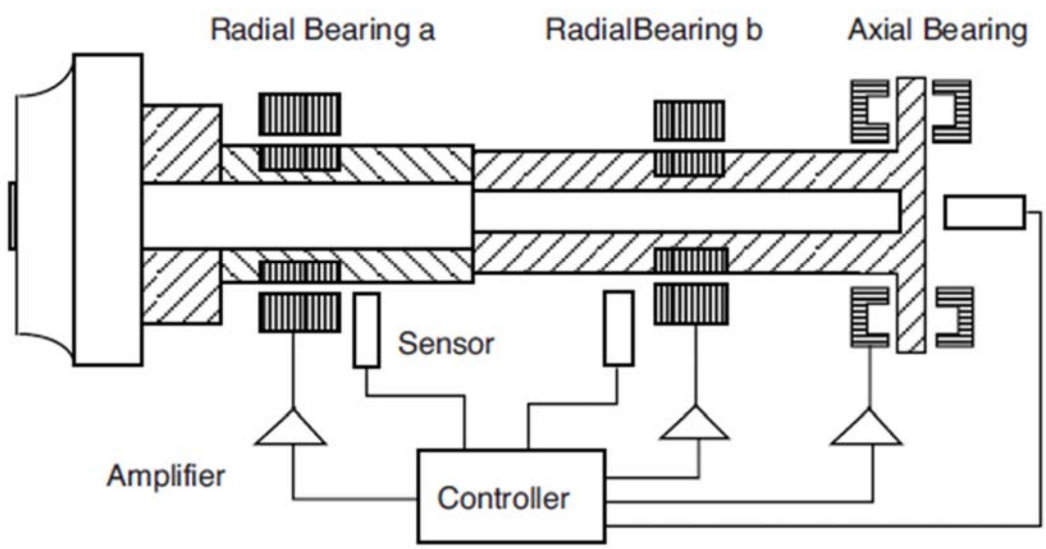

Figure 7. Planar section of rotor hovering in magnetic bearing *(Bleuler, et al., 2009)*

Magnetic bearings can be divided into two group of active and passive. Passive magnetic bearings, rely on their initial design, have fix characteristics whereas in active one, bearings' characteristics can be variable. Stiffness, damping, bearing axis, harmonic forces and excitation forces are important parameters which can be adjusted in need of the project requirements. (Bleuler, et al., 2009)

A comparison between AMB (Active Magnetic Bearing) and conventional bearings is provided in table 2. It should be noted that the characteristics are compared for the same application and size.

Table 2. *AMB and Conventional bearing*

Characteristics	Active Magnetic Bearings	Roller and fluid film bearing
Contact between shaft and bearing	No contact	Direct or indirect contact
Bearing loss	Very low	considerable in high speed
Initial costs	Expensive	Relatively in-expensive
Maintenance	Low maintenance	Relatively high maintenance
Speed limit	No speed limit from bearing aspects speed is only limited by rotor material strength in AMB.	Speed is limited mainly because of temperature, power loss and lubricant issues.
Variable parameter	Wide range of change in stiffness, damping, rotor axis of rotation and unbalance force.	Limited change of stiffness and damping for fluid film bearing and constant parameters in roller bearing
Lubrication	No	Depends on application Yes/No
Wear problem	No contact no wear.	• Common problem in roller bearing • In case of failure in fluid film bearings.
Failure reason	Internal: • Power source • Control system • Interface connections • Magnetic bearing hardware External: • Excessive load.[1]	• Contamination • Misalignment • Assembly fault • Starvation • Brinelling • Fatigue • Over loading and loading direction • Corrosion • Overheating.

[1] According to (Sears & Uptigrove, 1994)

2.4 Balancing Procedure

After being able to determine rigidity or flexibility of a rotor, next step is to get familiar with different balancing approaches for each rotor types. Before beginning to present methods, an example is provided to show how the use of inappropriate method for a rotor will bring unexpected result. Consequently, putting effort to select the best balancing method for a rotor is of the great of importance.

Consider a simple shaft with initial unbalance in the middle plane as it is illustrated in figure 8. The shaft is supported in rigid bearings. Low speed balance correction is applied on the shaft, thus half of the weight of initial unbalance weight is added into two correction planes located in rotor ends in opposite phase angle. Initial balance is applied in the rotor mid-span and the correction planes are located in same distance from their neighbor bearing.

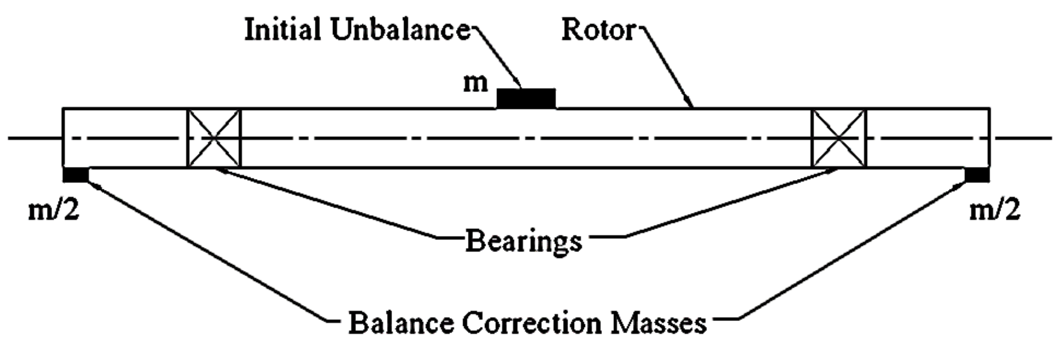

Figure 8. Model of a rotor with initial unbalance (m) and low speed correction in rotor ends (m/2) *(Adams, 2009)*

Although the rotor is perfectly balanced in low speed condition, it can be predicted that rotor will be dramatically vibrate while approaching the first bending mode. An exaggerated schematic of rotor for the first bending mode is presented in figure 9. Arrangement of initial unbalance (m), correction masses (m/2) and rotor mode shape in figure 9 show that all three weights have negative effect on rotor vibration in 1st critical speed whereas by removing added masses (m/2) effective unbalance force corresponding to this mode will be reduced and rotor will experience lower vibration level in the vicinity of 1st critical speed in compare with rotor balanced in low speed at its ends.

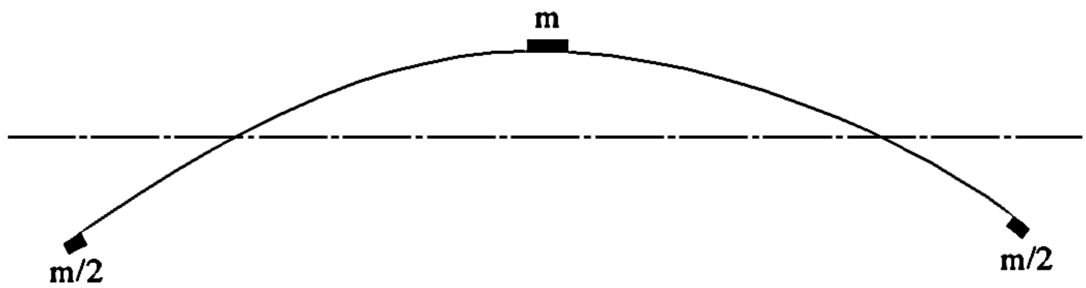

Figure 9. Rotor bending (exaggerated) due to inappropriate unbalance in the first bending mode *(Adams, 2009)*

Although the example is a special case study and unbalance forces in real rotors are randomly distributed along the rotors, it shows that low speed balancing is not necessarily effective for flexible rotors. It can be concluded that flexible rotors shall be tested and balanced in higher speed up to operating speed.

2.4.1 Over-speed test

Where it is defined by manufacturer or related standards, from balancing aspect, over-speed test shall be performed to allow the rotor component move into their final position. Although the position of components will not significantly change after over-speed test, the unbalance distribution along the rotor can be different due to small changes. Consequently, it is advisable to perform over-speed test in early stage of the balancing procedure to guarantee steady condition of the rotor up to its service speed. Rotor with shrink fitted disk, high number of assembly components are subject to over-speed test. Designer of the rotor is the best reference to decide whether over-speed test is needed to be done or not. If it is applicable, over-speed test should be done at 115% of maximum continues speed and rotor should be kept for a while in that speed. Appendix II provides more information on some speed related definition for turbomachinery.

The other reason for over-speed of a rotor is to test the strength of material under higher stress. Increase of 15% rotational speed will increase the stress in material by 22.5%. Some components need to be over-speed tested separately in order to ensure the reliability in sense of material, welded structures and probable deformation. To identify if any changes happened, inspections of the rotors and/or parts are highly recommended before and after the test. In addition, vibration and unbalance of the system before and after the test can be compared to detect the changes.

2.4.2 Rigid rotor balancing

Reducing unbalance forces by adding or removing mass from balance planes in rigid state of rotor (static equilibrium) is defined as rigid rotor balancing. Under normal condition, rigid rotor balancing is done in very low speed even for rigid rotor with high operating speed. For instance, a high-speed rigid rotor with 40000 rpm (revolution per minute) service speed can be balanced in 2000 rpm for whole rotor speed range.

Rigid rotor balancing can be divided into two groups of static and dynamic balancing. Also, unbalance can be categorized as static, couple and static-couple (dynamic). Single plane balancing is suitable for static unbalance and for two other unbalance type 2-plane (dynamic) balancing is needed.

Static balancing is applicable for narrow disks/components where a correction in single plane can remove the unbalance. In addition, rotor with parallel displaced mass axis (figure 10-(a)) with respect to bearing center line or axis of rotation can be corrected by single plane balancing in the plane of center of mass. To define rotor mass axis, consider a rotor is sliced into numerous sections and center of mass for each section is calculated. Then rotor mass axis define as average straight line fitted to these center of masses by taking into account their mass as predominant factor.

In couple unbalance, rotor mass axis has an intersection with rotation axis on the center of mass (figure 10-(b)). This type of unbalance might be happened when two equal unbalances in opposite directions are applied on the rotor. Consequence of these loads is pure moment reaction in bearings and no static unbalance is present. Use of two plane balancing approach and adding two equal weights in opposite phase angle in two correction planes is the best solution to vanish the couple unbalance. The amount of mass is calculated from following equation:

$$m = \frac{U \cdot D}{d \cdot r} \tag{5}$$

where:

U is unbalance amplitude (g·mm), D is distance of unbalance plane (mm), d is distance of correction planes (mm) and r is correction plane radius (mm).

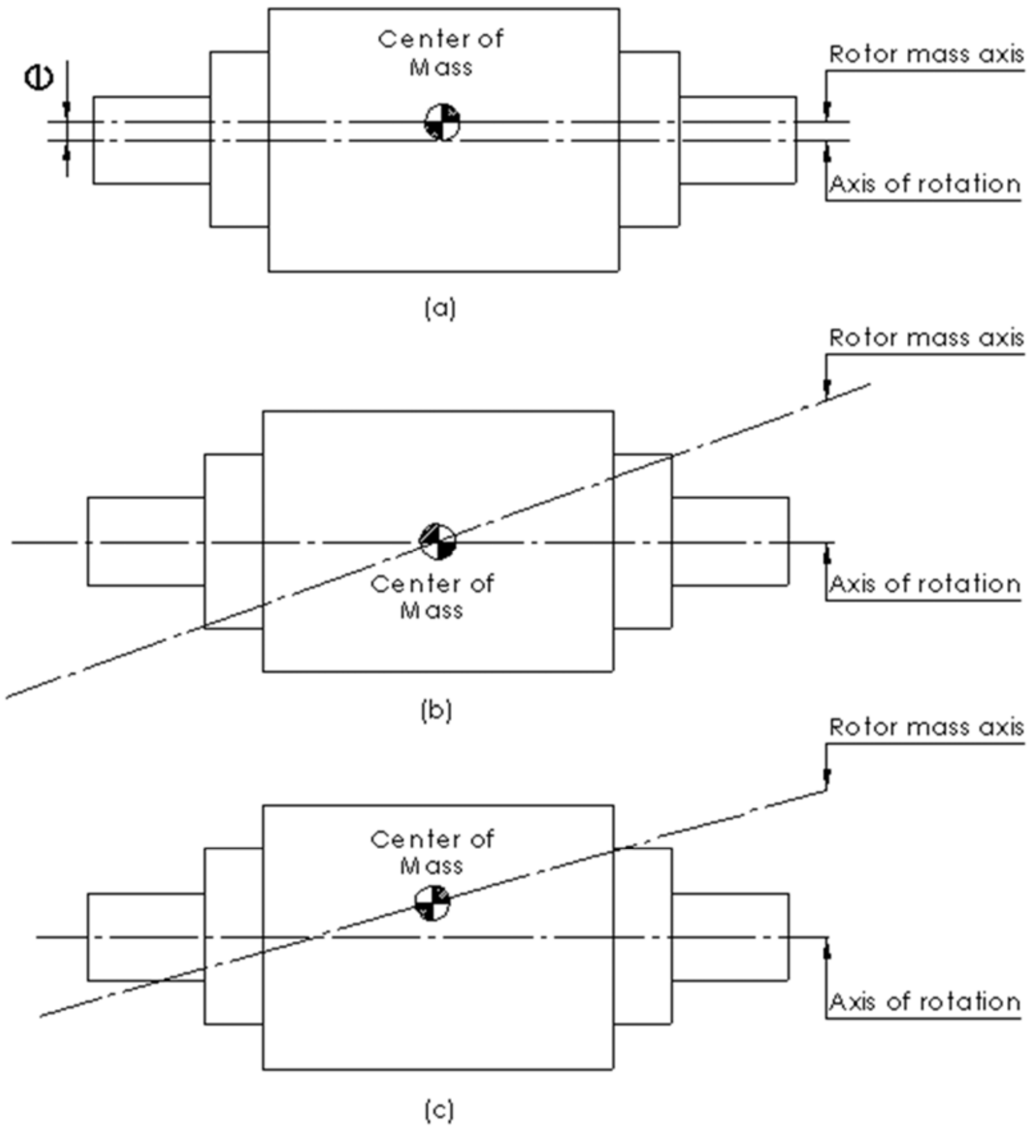

Figure 10. Different low speed unbalance: (a) static unbalance, (b) couple unbalance, (c) static-couple unbalance or dynamic unbalance.

Dynamic unbalance happens in rotor which suffer from both static and couple unbalance simultaneously. As it is shown in figure 10-(c) rotor mass axis intersect axis of rotation in point other than center of mass. Neither single plane balancing nor two plane couple balancing lonely ensures smooth rotor operation in this case. A 2-plane dynamic balancing considering both of static and couple is the solution. This aim is achieved with selecting masses with different in quantity and relative angular position.

Depends on geometry, shape and unbalances distribution of a rotor or components; correction masses calculation can be done in one, two or multiple plane(s). The aim of low speed balance is to make the shaking moments and shaking forces zero in bearings. Equations of (6) and (7) shows mathematical expression for equality of shaking force and shaking moment.

$$\sum F = 0 \tag{6}$$

$$\sum M = 0 \tag{7}$$

Static equilibrium can be used in order to evaluate the reaction forces on bearings and the needed correction masses in selected correction planes. Moreover, prior assembly some of the component are needed to be low speed balanced. By performing accurate assembly procedure and having detail information about unbalance sources, it is possible to correct each unbalance source in its place. This approach helps to having successful low speed balance and result smooth operation of rotors in higher speeds.

Consider an unbalance as eccentrically mounted mass on a rotor. The unbalance force express as:

$$F = m_u \, e \, \omega^2 \tag{8}$$

where:
m_u is unbalance mass(kg), e is mass eccentricity (m), ω is angular frequency (rad/s) and F is unbalance force (N). From equation (8) it can be concluded that amount of unbalance force growth by increasing of rotational speed. This growth helps to get accurate results, since the noises become insignificant in compare with unbalance force. However, the maximum speed for rigid balancing is limited by rigidity of the rotor. If there is no other limitations such as

balance machine capabilities and/or minimum speed limit for settling of components; all speeds lower than half [2] of the 1st critical speed can be selected for low speed balancing.

Rigid Balancing Tolerances

It is possible to decrease unbalance forces by performing balance procedure on a rotor. Balancing is an expensive process and obtaining higher accuracies make higher the expenses. An alter method for reducing unbalance forces is tightening of manufacturing tolerances; but higher accuracy in manufacturing process increase the costs again. Therefore a balance between qualities of products and balancing criteria shall be existed. (Norfield, 2006)

For this aim, number of documents considered the subject and published step by step recommendation to achieve balancing tolerances (Rieger, 1986). Among them, ISO standard 1940-1 (ISO 1940-1, 2003) is the most popular and experimentally created charts and equations which helps to get these criteria by knowing some information about rotor shape and service speed. In ISO 1940-1, balance quality grade is defined as:

$$G = e_{Per} \cdot \Omega \tag{9}$$

where:

G is balance quality grade (mm/s), e_{Per} is permissible residual specific unbalance (kg·mm/kg), Ω is rotor service speed (rad/s). (ISO 1940-1, 2003)

This grade is a value in order to classify machines and calculate permissible residual unbalance. Permissible residual unbalance based on ISO 1940-1 can be computed by:

$$U_{Per} = 1000 \frac{(e_{Per} \cdot \Omega) \cdot m}{\Omega} \tag{10}$$

[2] A safety margin for being sure about the rotor rigidity is considered.

where:

U_{Per} is permissible residual unbalance (g·mm), m is rotor or component mass (kg), Ω is rotor service speed (rad/s), e_{Per} is permissible residual specific unbalance (kg·mm/kg). (ISO 1940-1, 2003)

Now it is needed to find a value for G or e_{Per}. Provided table in appendix III shows experimental recommendation values for G based on the machinery types. Recommended grade of balance quality for balancing of components is one grade tighter than the whole rotor assembly balancing grade. As an example, if it is expected to balance a rotor in G2.5, then the component should be balanced for G1. For components that can be assembled on rotor during rotor balancing, it is recommended that residual unbalance of the rotor is measured before and after assembly. By vector subtraction of results amount of change in residual unbalance of rotor is calculated and should be correct on the last assembled part.

In balancing machine usually vibration or unbalance forces are measured in bearing planes. Then these force are transferred to correction plane and needed mass can be added or removed or where it is possible mass distribution can be changed. Figure 11 is showing a small size typical balance machine with its control system.

Figure 11. Balance machine overview *(Universal Balancing Machines, unknown)*

What it should be noted that here is U_{Per} indicates total permissible unbalance. This value shall be divided into two bearing planes proportional to the distance of each bearing from the rotor center of mass. Figure 12 and figure 13 are showing position of bearing planes relative to center of mass for rotor without overhang mass and rotor with overhang mass respectively.

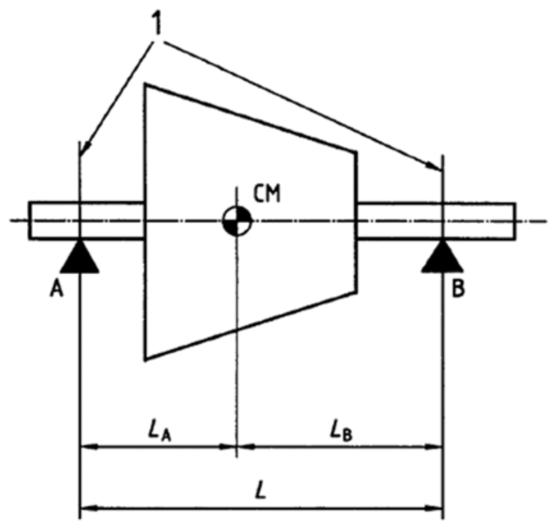

Figure 12. Tolerance planes 1 in bearing A, B and relative position with respect to center of mass (CM) for rotors with CM located between the bearings *(ISO 1940-1, 2003)*.

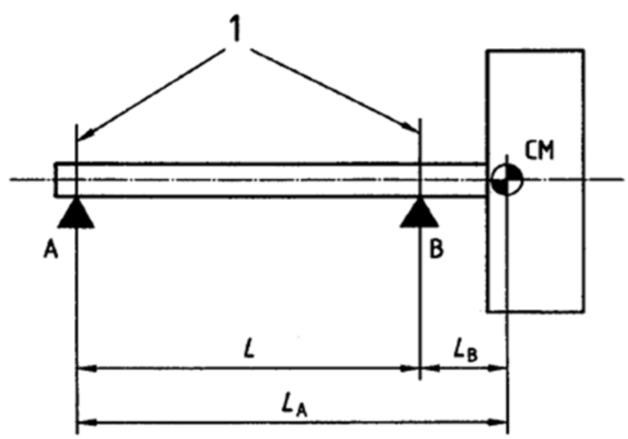

Figure 13. Tolerance planes 1 in bearing A, B and relative position with respect to center of mass (CM) for rotors with CM located outside of the bearings *(ISO 1940-1, 2003)*.

Considering figure 12, figure 13 and below equations permissible residual unbalance in each bearing plane can be calculated. (ISO 1940-1, 2003)

$$U_{Per\ A} = \frac{U_{\mathrm{Per}} \cdot L_B}{L} \tag{11}$$

$$U_{Per\ B} = \frac{U_{Per} \cdot L_A}{L} \tag{12}$$

where:

$U_{Per\ A}$ is permissible residual unbalance in bearing plane A (g·mm),

$U_{Per\ B}$ is permissible residual unbalance in bearing plane B (g·mm),

L is bearing distance (m),

L_A is bearing “A” distance from center of mass (m),

L_B is bearing “B” distance from center of mass (m). (ISO 1940-1, 2003)

As it is mentioned on this Standard following criteria is defined as acceptable range for $U_{Per\ A}$, $U_{Per\ B}$:

- Rotors with CM located between of the bearings

$$0.3U_{Per} \leq U_{Per\ A}, U_{Per\ B} \leq 0.7\ U_{Per} \tag{13}$$

- Rotors with CM located outside of the bearings

$$0.3U_{Per} \leq U_{Per\ A}, U_{Per\ B} \leq 1.3\ U_{Per} \tag{14}$$

Finally, a rotor is in acceptable balance range (it is balanced) based on ISO1940-1 when results from real unbalance test in each bearing do not exceed the dedicated permissible residual unbalance of each bearing calculated from equations (11) and (12).

The above mentioned method is only applicable for low speed balancing of rigid rotors. For flexible rotor another method will be used which is presented in section 2.4.3. In ISO 1940-1, other methods are presented which needs knowledge about the machine dynamics and experience in design. Some of these methods put limits for the force or vibration that a bearing can tolerate (ISO 1940-1, 2003).

In conclusion, what is discussed consider only rigid rotors. Although, according to API 617 (API 617, 2002): “With the use of appropriate procedures, it is often possible to balance flexible rotors at low speed so as to ensure satisfactory running when the rotor is installed in its final environment”, other cases like what is shown in figure 8 illustrates that smooth operation of flexible rotor with only applying low speed balance is related to initial unbalance distribution and position of low speed correction planes. In next section a flexible rotor balancing is described and related classification for flexible rotor balancing procedures are provided.

2.4.3 Flexible rotor balancing (High speed balancing)

High speed balancing can be defined as testing a rotor up to its service speed, measuring force in bearing and try to minimize them for whole speed range by applying correction masses in balancing planes. Where it is defined, maximum speed of test can be extended to over-speed. Over-speed as it is described in 2.4.1 is beneficial for flexible rotor to guarantee reliability, performance and safety issues.

Effectiveness of rigid balancing before high speed balancing is not clear yet. Darlow (Darlow, 1987), Bertoneri, Forte (Bertoneri & Forte, 2015) are of the opinion that low speed balance can be applied to flexible rotor as a first step of balance procedure. In addition, Standards ISO-11342 (ISO 11342, 1998), and API-617 (API 617, 2002) suggest low speed balance prior flexible rotor balancing. Kellenberger (Kellenberger, 1972) in his paper considered a sample rotor that passed rigid balancing. He calculated correction masses for first three critical speeds in three different planes (one plane per mode). The test is repeated with slight change and he added two more balancing planes (totally five) to keep the low speed condition constant while correcting higher modes. His comparison between the result of these two test shows that adding rigid balancing condition in whole procedure of high-speed balance greatly improve the quality of rotor balancing.

In contrast, Sharp (Sharp, 1980) hold the view that without considering dynamic low speed balance, full operation balancing of a flexible rotor is obtainable. Bishop, Parkingson

(Bishop & Parkingson, 1972) in a discussion on Kellenberger (Kellenberger, 1972) provided detail explanation and an example which contradicted the obtained results by Kellenberger. They showed that N-plane balancing methods for a rotor with operating speed higher than "N" critical speeds can be effective and more accurate than the N+2 plane balancing procedure.

In essence, the excellence of none of the mentioned method has not been proved and the result of them is highly dependent to the unbalance distribution, number of correction planes, position of correction planes and rotor mode shapes.

To date, independent of the previous discussion about number of correction planes or primitive rigid balancing of flexible rotor, two main methods for flexible rotor balancing have been introduced. These method categorized as:

- Modal balancing
- Influence coefficients

These methods are on the basis of linear rotor response to unbalance forces. Presence of non-linearity in reality just increase number of balancing runs. In the following of this section, after short review of balancing criteria, these two methods are described. Moreover, advantages, disadvantages are mentioned and then a comparison between them is provided.

Acceptance limits and criteria

For balancing aim, it is imperative that balancing machine simulates the same condition as the service condition for a rotor. But this is not the case in most practices and a range of parameters affect the rotor behavior in balance machine. As a result, the vibration in a balancing machine differs from the vibration of machine in final condition. Thus, vibration limits used for rotating machine health monitoring in operation, cannot be used directly as acceptance limits of vibration in balancing machines.

Section 8.2.5 of ISO-11342 provides a method that enables the users to compare the results from a balance machine with standard vibration limits in service. The method defines the proportion of the balancing test and operation criteria through some coefficients. The coefficients mainly consider the differences of bearings' characteristics and location of vibration measuring planes between the test bench and the operation. These coefficients can be evaluated by rotor dynamic analysis, based on experience or recommended value by annex C of the ISO-11342. (ISO 11342, 1998)

Vibration limits for service condition can be extracted from appropriate part of ISO-7919 and ISO-10816 for rotating shaft and non-rotating parts respectively (ISO 19499, 2007). It should be noted that these standards result tight tolerances and are suitable for fluid film and roller bearings, while AMBs provide a large clearance and it is not needed to consider the tolerances for them as tight as tolerances for fluid film and roller bearings. Some values as guideline for acceptable vibration of AMB machines in service condition is recommended by ISO 14839–2 as it is shown in table 3.

Table 3. Recommended criteria of zone limits *(ISO 14839-2, 2004)*

Zone limit	Displacement D_{max}
A/B	<0.3 C_{min}
B/C	<0.4 C_{min}
C/D	<0.5 C_{min}
NOTE: D_{max} is maximum rotor orbit amplitude C_{min} is the minimum value of radial or axial clearance between rotor and stator.[1]	

According to ISO 14839–2 (ISO 14839-2, 2004), zone limits define as:
“Zone A: The vibratory displacement of newly commissioned machines would normally fall within this zone.
Zone B: Machines with vibratory displacement within this zone are normally considered acceptable for unrestricted long-term operation.
Zone C: Machines with vibratory displacement within this zone are normally considered unsatisfactory for long-term continuous operation. Generally, the machine may be operated for a limited period in this condition until a suitable opportunity arises for remedial action.
Zone D: Vibratory displacement within this zone is normally considered to be sufficiently severe to cause damage to the machine.”

An alternative for previous method is defining limit for residual unbalance of a flexible rotor by criteria given by ISO-1940 (ISO 1940-1, 2003). In ISO-11342 following criteria are recommended as limit for rotor residual unbalance in its speed range:

[1] In machine with AMB supports, rotor has the minimum gap in the back bearings’ position.

- Flexible rotors just affected by the first critical speed are needed to be balanced in a level that modal residual unbalance of the rotor be less than 100% of reference value[2] in low speed and service speed. Residual unbalance in the vicinity of 1st critical speed shall not exceed 60% of the reference value,
- The balance quality of flexible rotors with two critical speed in their speed range shall not exceed 100% of the reference value for low speed and service speed. The criteria for 1st and 2nd critical speed decrease to 60% of the reference value[3] for rotor in this category. (ISO 11342, 1998)

This method is on the basis of equivalent modal residual unbalance. In the latter part of this section, a method for determination of modal equivalent unbalance from ISO-11342 will be provided.

Modal balancing

Modal balancing is balancing of a rotor in its critical speeds within the maximum speed range by applying correction masses considering each mode separately. Gunter, et al. (Gunter, et al., 1976) by considering amplification factor for higher modes showed that a rotor is influenced by only mode shapes in its speed range and contribution of higher modes in rotor response is negligible. Consequently, it is possible to balance a rotor in its speed range by calculating needed modal unbalance correction at each mode. Main consideration of this method according to Gunter, et al (Gunter, et al., 1976) is: "Determined correction corresponding to a mode shape shall not change the achieved balanced condition in previous corrections". For this aim, in theory best place for applying a correction at each mode is the anti-node of that mode and it should be close to the node of other mode shapes. Subsequently, rotor dynamic analysis is used in order to select the best place of trial for putting modal balancing correction masses in modal balancing approach. Bertoneri (Bertoneri & Forte, 2015) performed balancing on a scale rotor with using both influence coefficients and modal balancing approach. Figure 14 shows rotor vibration amplitude versus rotational speed for both methods. It is clear from the figure that, the obtained balance quality through modal balancing is far better than influence coefficient method.

[2] Reference value is defined as acceptance limit for rotor residual unbalance for rigid balancing calculated from ISO-1940.
[3] If one of the critical speed dominates the rotor behavior; less effective mode criteria can be increased to 100% of reference value.

Regrettably, in real condition rotor is mounted in casing and is not accessible for entire length. In addition, it is not always possible to apply mass changes in positions other than correction planes and because of design limitations correction planes are not well distributed. Due to these limitations, the maximum achievable balance quality through modal balancing is reduced. However, Bertoneeri (Bertoneri & Forte, 2015) showed excellence of modal balancing, still due to the mentioned restrictions it is has not been used commercially and it is more applicable for tests conducted in laboratories. For instance, influence coefficient method discards the modal balancing approach for in-situ balancing.

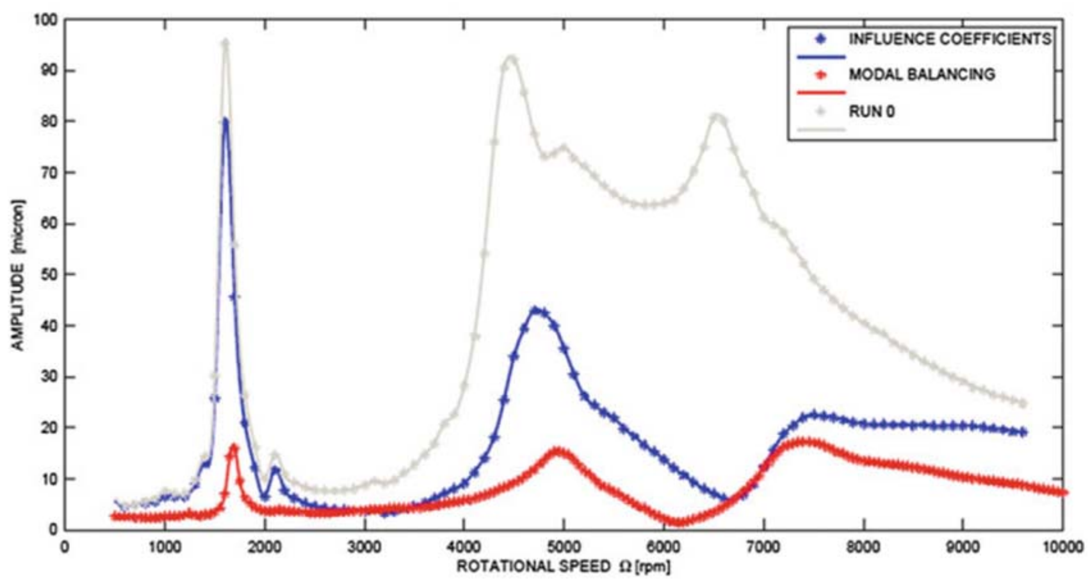

Figure 14. Comparison of final balancing state of a rotor with using two different balancing methods *(Bertoneri & Forte, 2015)*

As it is mentioned before, in practical cases it is not possible to find a plane that just excite one mode without influencing other modes. To some extent, it is applicable to use set of trial masses in order to excite/correct one mode at a time in the rotor. Following example in figure 15 describes the first and second mode shape of a sample rotor with three correction planes. For the 1st mode shape, since the maximum deflection (displacement) of the rotor happens in the middle of the rotor, a correction mass in BP2 (Balance plane) plays a major role in vanishing vibration amplitude caused by this mode. As it is shown in figure 15-(a), a correction mass (m) is added in BP2. Adding m in BP2 will change residual unbalance in low speed. A low speed balance was performed prior high-speed test. For keeping low speed condition unchanged, half of the mass in BP2 is added in BP1 and BP3. The masses in BP1, BP3 have negative effect on the 1st mode bending; however, by using correct proportion of masses in all the three planes, all the vibration caused by the first mode shape can be removed in theory.

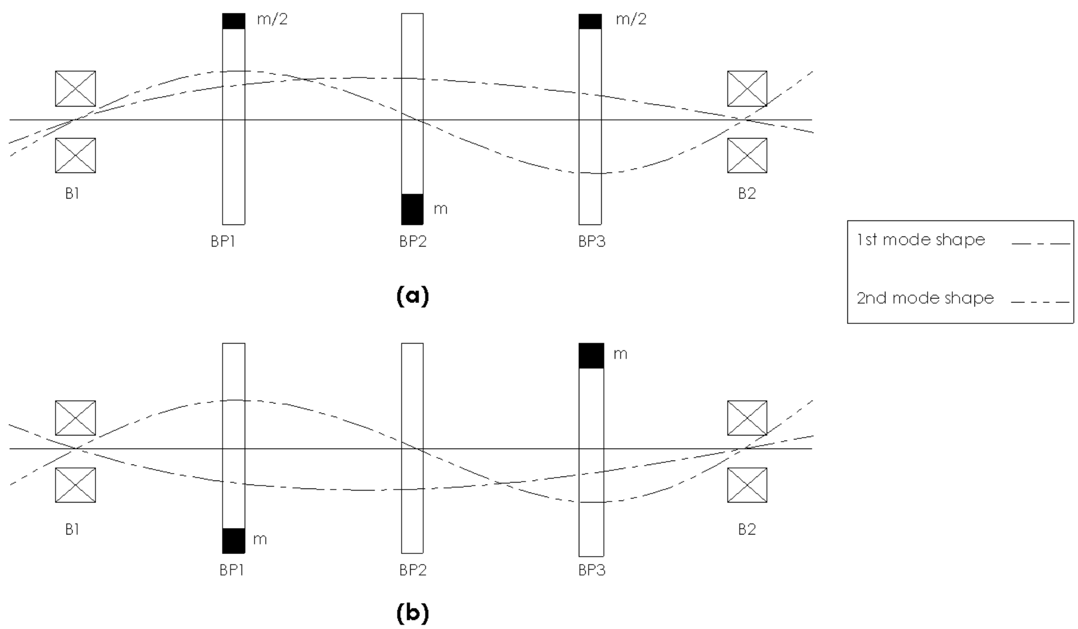

Figure 15. Demonstration of modal balancing approach- Correction of 1st and 2nd mode shapes

For correcting the 2nd mode, two equal mass are added into BP1, BP3 (figure 15-(b)). The masses are added 180 degree apart. The figure 15-(b) indicates that mass in BP1 excite the first mode to vibrate whereas mass in BP3 has the same effect in reverse direction on first mode. Although, this arrangement of corrections keep static unbalance in the same condition, shaking moments are added in bearing planes.

What is done in this example is just based on experiment and familiarity with rotor mode shapes. In the above procedure, if a model of rotor is available and the masses proportion calculated through the model, solely a modal balancing approach is applied. If the trial masses and trial runs are used to determine masses proportion, the method is a simplified combination of influence coefficient and modal balancing methods. In general cases, it is complicated to find trial mass set for every rotor type. Unified approach is a combined method of modal balancing and influence coefficients to find trials set for modal balancing.

Influence coefficient

Influence coefficient method is mostly based on putting trial mass at each plane and measuring its effect on rotor vibration. Then by computing the effect of trial mass at needed speed, like speeds close to critical speeds and operating speed, a combination of correction masses in all planes can be calculated. Equation (15) shows general form of influence coefficients calculation.

$$\boldsymbol{c}_{ij} = \frac{\boldsymbol{v}_{TW} - \boldsymbol{v}_{ref}}{\boldsymbol{T}_w} \tag{15}$$

where:

$\boldsymbol{v}_{TW}$ is vibration vector at defined speed due to trial mass applied in a balance plane, measured in mm/s (mm), $\boldsymbol{v}_{ref}$ is vibration vector at the same defined speed without adding any trial mass, measured in mm/s (mm), $\boldsymbol{T}_w$ is vector of unbalance is applied to the rotor in a balance plane in g·mm, $\boldsymbol{c}_{ij}$ is vector of influence coefficient for a balance plane in specified speed, usually describe in mm/s/(g·mm).

Influence coefficients can be measured for different balancing planes, range of speeds and number of trial masses at each balancing plane. In simplest case, it is common to measure the vibration change in or close to bearing planes to determine the effect of trial mass on the rotor response for each balance plane. If number of measurement speeds equals to number of balance planes influence coefficient of a rotor can be described in a square matrix form. For a rotor with n correction plane and n speed measurements, equations (16), (17) shows the method for calculation of correction masses. What is more, for a square influence coefficient matrix, the solution for correction masses is exact[4].

$$\boldsymbol{V} = \mathbf{I}_{\mathrm{coeff}}\boldsymbol{M} \tag{16}$$

$$\downarrow$$

$$\mathbf{I}_{\mathrm{coeff}}{}^{-\mathbf{1}}\boldsymbol{V} = \boldsymbol{M} \tag{17}$$

[4] Ill condition matrix of influence coefficient an exemption. A simple ill conditioning happens when the balance plane

where the parameters define as:

$\boldsymbol{V}$ is Vector of vibration is needed to be applied to make the existing rotor vibration zero. Elements of this vector are the values of vibration in measuring plane at each speed. The common unit for bearing vibration measurement in balance machine is mm/sec, $\mathbf{I}_{\text{coeff}}$ is influence coefficient matrix. Rows of this matrix are influence coefficient values of each speed and the columns are values of coefficients for each balance plane. Unit of numbers in this matrix are dependent to vibration unit. If vibration is measured in mm/sec then $\mathbf{I}_{\text{coeff}}$ unit is mm/s/(g·mm), $\boldsymbol{M}$ is Vector of correction masses in a balance planes in unit of grams.

Number of speeds in which coefficients are needed to be determined is highly dependent to balance procedure and number of critical speeds. Due to need to record the data in more speed than the balancing planes; it rarely happens to have square matrix of influence coefficients. Consequently, the influence coefficient matrix is not invertible and equation (17) is not applicable any more. The matrix of influence coefficients is now a non-square of "(n×q)" by "m" where "n" is number of speed, q is number of measuring planes and "m" defines number of correction planes and "(n×q)>m". Then the equation (16) is over-determined and least square method can be used for solving a set of equations. Since the exact solution does not exist in this case, vector of residual vibration $\boldsymbol{V}_R$ is defined in form of equation (18):

$$\boldsymbol{V}_R = \boldsymbol{V}_A + \boldsymbol{V}_C \tag{18}$$

where:

$\boldsymbol{V}_A$ is vector of current vibration readouts, $\boldsymbol{V}_C$ is vector of theoretical values for vibration generated by applying computed correction mass in correction planes.

In equation (18), all three vectors contains data in each measuring plane (where the sensors are mounted) and the speed that data are taken. By substituting equation (16) into equation (18), residual vibration equation change to:

$$\boldsymbol{V}_R^{(n\times q)\times 1} = \boldsymbol{V}_A{}^{(n\times q)\times 1} + \mathbf{I}_{\text{coeff}}{}^{(n\times q)\times m}\,\boldsymbol{M}^{m\times 1} \tag{19}$$

The goal of least square method is to minimize sum of square for $\boldsymbol{V}_R$ elements by changing values in vector $\boldsymbol{M}$. By considering residual vibration equal to zero equation (19) can be rewrite in matrix form of equation (20) and performing least square method, correction mass matrix as it is shown in equation (21) is determined.

$$-\boldsymbol{V}_A{}^{(n\times q)\times 1} = \mathbf{I}_{\text{coeff}}{}^{(n\times q)\times m}\ \boldsymbol{M}^{m\times 1} \tag{20}$$

$$\downarrow$$

$$\boldsymbol{M}^{m\times 1} = -\left(\mathbf{I}_{\text{coeff}}{}^{\mathbf{T}}\ \mathbf{I}_{\text{coeff}}\right)^{-\mathbf{1}}\ \mathbf{I}_{\mathbf{coeff}}{}^{\mathbf{T}}\ \boldsymbol{V}_A \tag{21}$$

All elements of matrices of "$\mathbf{I}_{\text{coeff}}$", "$\boldsymbol{V}$" and "$\boldsymbol{M}$" are complex numbers since they represent both the amplitude and phase.

To simplify the influence coefficient method, following step by step procedure can be used to compute influence coefficients and guide to their application for determining correction masses.

1. Rough balancing: Balancing the rotor to some degree and decreasing vibration level of the rotor in order to be able to sweep up whole speed range of balancing. For rough balancing aim, it is possible to use influence coefficients calculated from similar rotor type.
2. Reference run: Recording vibration data for whole speed range and save it as reference run. This run can be used later as input data for calculation of correction masses ($\boldsymbol{V}_A$ in equation (21)).
3. Adding trial mass: Putting trial mass in predefined angular position of a correction plane. It would be much nicer to add the correction mass one at a time in each balance plane. The trial mass should be heavy enough to change vibration level of the rotor significantly, in such wise that noise and small error can hardly affect the results. In other point of view, it should be light enough that does not cause nonlinearity in rotor response. As an estimation for trial mass quantity, it should be able to create 5µm of mass eccentricity. But it is mostly defined experimentally based on rotor behavior, position of correction plane and vibration level of reference run.
4. Performing test: Rotating the rotor from zero to operating speed and recording vibration data for each measuring plane.

5. Influence Coefficient: Subtracting vibration measurement at each bearing when mass is added with corresponding one with no mass (numerator of equation (15)). Dividing calculated vector by unbalance induced due to applying trial mass (equation (15)). The same procedure applies to other plane in order to calculate influence coefficients of every other plane. The result of this step is called influence coefficient of the plane which trial mass is applied.
6. Correction mass: Now, by knowing the rotor's unbalance response and vibration amplitude in the current state (manufactured rotor without mass) correction masses can be calculated. By calculating influence coefficient matrix from step 6 and knowing $\boldsymbol{V_A}$ from step 2 and using equation (21) correction masses in balance planes are calculable.
7. Residual vibration: By importing correction masses calculated in step 6 into equation (19), residual vibration at each speed is evaluated. It should be mentioned that these residual vibration values are just an estimation. Since some errors exist in measurements and calculations the real result after applying correction masses will be to some degree different than the prediction.
8. Achieving balance quality: By applying correction masses determined in step 7 and retesting the rotor, the result can be recorded and compared with the tolerances. If the vibration results exceed the limits, steps of 6, 7 can be repeated over and over until a reasonable result within the criteria is obtained.

Basically, in influence coefficient method no information about the rotor behavior is needed; however, knowing the dynamic of the rotor will be useful. A detailed comparison between influence coefficient method and modal balancing approach is provided in table 4. The main advantage of this method in compared to the modal balancing approach is that there is no need to take care of each mode separately and the correction for all speeds can be determined just in one set of equations. On the other side, the main weakness of this method in compare with the modal approach is for each trial mass rotor shall be rotated up to service speed; whereas, in modal method for each modal trial mass rotor shall be rotated up to the corresponding mode which is going to be excited by trial mass.

Table 4. *Comparison of modal balancing method and influence coefficients method (Darlow, 1987)*

Method characteristic	**Modal balancing method**	**Influence coefficients method**
Main assumptions	• Linear rotor response • Small damping • Planar mode shapes	• Linear rotor response • Superposition theory • non-singular IC matrix
Necessary run before balancing	Number of effective critical speed plus initial run.	Number of correction plane plus initial run.
Maximum speed at each trial run	Up to excited rotor mode shape.	Up to service speed.
Complicated mathematic calculation	Modeling and pre-calculation to find mode shapes and position along rotor axis to excite them.	No prior calculation but complicated calculation for determining correction masses.
Easy to use for operator	Operator shall be deeply familiar with the subject and have knowledge about the rotor and its dynamic.	No special knowledge is needed by using a computer program for calculation.
Adoptability	For each rotor type, vast rotor dynamic analysis is necessary.	Simply can be adopted for new rotor type just by modifying number of correction planes and number of measuring speeds.
Applicability	Because of limited number of balancing planes and not having access to whole rotor length, for applying trial mass it is not easy to excite each mode separately.	As far as correction planes are available the method is useful.

High-speed balance criteria in unit of "g · mm"
As it is discussed earlier, the residual unbalance shall be within a criteria. It is common to define this criteria in unit of "g·mm" as a residual unbalance for a rotor at each speed. The method to calculate this limit is completely covered in previous sections.

Vibration in measuring planes are monitored and recorded in real tests. To transform these vibration to equivalent modal residual unbalance "g·mm" the following procedure with case example from ISO-11342-Annex D (ISO 11342, 1998) is reviewed and explained. In this example the influence coefficient matrix are ready to use and example does not consider the calculation of it. The rotor under study is shown in figure 16. Additional information for the rotor are provided in table 5. In addition, the influence coefficients for each correction plane at rotor critical speeds are listed in table 6 (the coefficients are in (mm/s)/(kg·mm)). The flow of calculation steps are summarized in a diagram shown in figure 17.

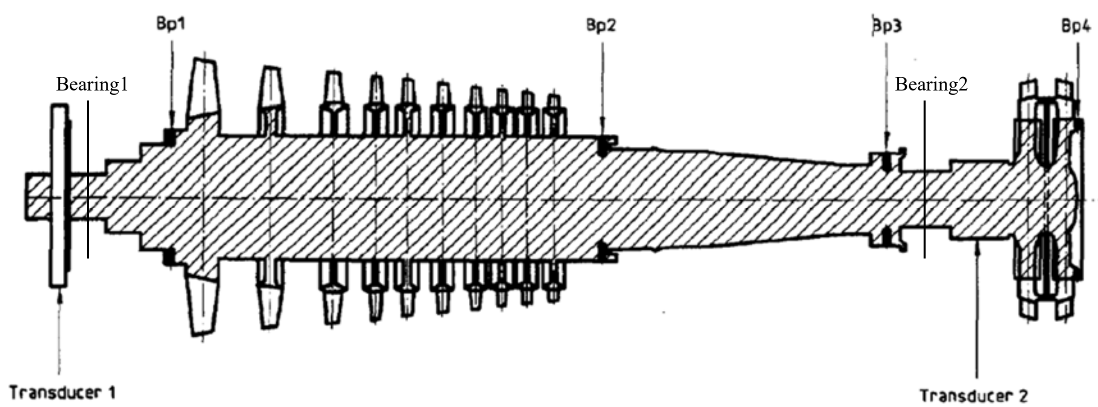

Figure 16. A turbine rotor with four balancing plane and two measuring plane. *(ISO 11342, 1998)*

Table 5. *Turbine rotor data (ISO 11342, 1998)*

Rotor characteristic	**value of parameter**	**unit**
Mass	1625	kg
Operational speed	10125	rpm
1^{st} critical speed	3500	rpm
2^{nd} critical speed	9100	rpm
Balance Grade	2.5	mm/s

Table 6. *Rotor influence coefficients. Stars shows the most sensitive plane for that speed-bearing (ISO 11342, 1998)*

Measuring plane	Balance plane				Speed rpm
	Bp1	Bp2	Bp3	Bp4	
1	0.0594*	0.0330	0.0091*	0.0049	1000
2	0.0022*	0.0227	0.0334*	0.0425	
1	0.249	0.343	0.055	0.360*	3400
2	0.087	0.157	0.102	0.224*	
1	1.99	2.29*	1.56	2.07	9000
2	1.92	1.99*	1.16	0.595	

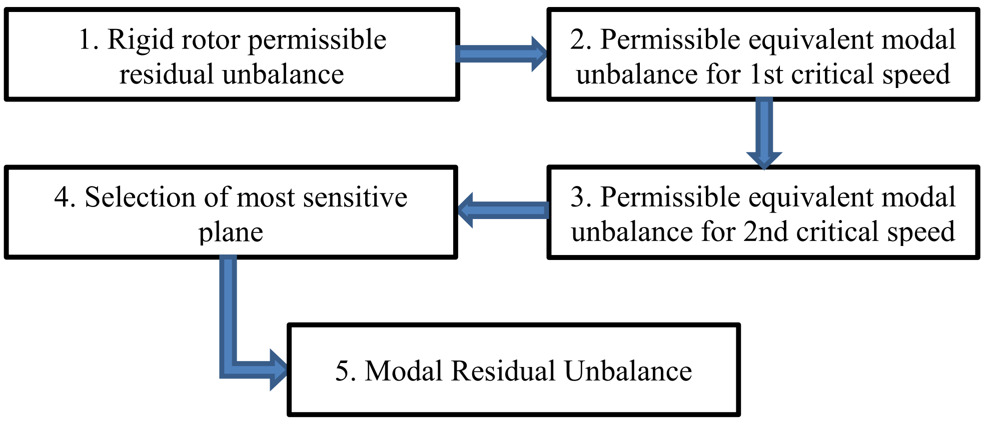

Figure 17. Steps for calculation of modal residual unbalance

1. Permissible residual unbalance for rigid rotor: This rotor shall be balanced according to G2.5 of ISO-1940 (ISO 1940-1, 2003). From equation (10), total permissible residual unbalance is equal to :

$$U_{Per} = 1000\frac{2.5 \cdot 1625}{10125 \cdot \left(\frac{2\pi}{60}\right)} = 3831.5 \quad g.mm$$

Above value shall be divided into 2-plane for low speed. In this case, BP1 and BP3 as closest correction plane to bearings are selected for low speed balancing. Due to high stiffness of bearings, rotor displacement is limited in this area and nodes of mode shapes are located close to bearing position. Thus, by selecting balance plane close to bearings there is less chance of exciting higher mode with correction in low speed. General allocating of permissible residual unbalance for bearing plane is explained in section 2.4.2. These values are transformable from bearings to any correction planes by using equations of (11), (12). When the balance planes are very close to bearing, the values in bearing planes and balance planes are almost the same. Position of rotor center of mass is not provided in standard but from general mass distribution shown in figure 16 and the value provided by ISO-11342 (ISO 11342, 1998) for low speed balance limits (it is half of the total value), it seems that it is located in the same distance from both bearings. As a result, this limit for BP1 and BP3 is 1916 "g·mm".

2. Permissible equivalent modal unbalance for 1st critical speed: Since the rotor is working above its 2nd critical speed, for the 1st mode 100% of tolerance can be considered as permissible residual unbalance.
3. Permissible equivalent modal unbalance for 2nd critical speed: The rotor service speed is close to 2nd critical speed. Allowable residual unbalance for 2nd mode of this rotor shall not exceed 60% of the permissible for rigid area. Therefore the value of permissible equivalent modal unbalance for 2nd mode shape is:

$$U_{Per,2nd\ mode} = 60\% \cdot 3831.5 = 2299 \quad g.mm$$

4. Most sensitive planes: Most sensitive plane at each speed is the plane with highest influence coefficient. The most sensitive plane of the rotor are marked with star in the table 6. BP4 and BP2 have the highest value for 1st and 2nd critical speed respectively.
5. Modal residual unbalance: The values for modal residual unbalance are determined through dividing residual vibration given in table 7 by influence coefficients for most sensitive planes selected in step 4. Calculated values for modal residual unbalance at each speed are given in table 8.

Table 7. *Residual vibration values*[5]

Speed	Measuring plane 1	Measuring plane 2	Unit
1000 rpm	0.01	0.02	mm/s
3400 rpm	0.55	0.22	mm/s
9000 rpm	2.35	1.44	mm/s

[5] For the aim of checking the rotor balance quality, phase angle of the vibration and influence coefficients are not important and they are not included in this example.

Table 8. *Modal residual unbalance*

Speed rpm	Balance plane	Transducer	Residual unbalance g·mm	Permissible g·mm
1000	Bp1	1	$\frac{0.01}{0.0594} \cdot 1000 = 168$	1916
	Bp3	2	$\frac{0.022}{0.0334} \cdot 1000 = 658$	
3400	Bp4	1	$\frac{0.55}{0.360} \cdot 1000 = 1528$	3832
		2	$\frac{0.22}{0.224} \cdot 1000 = 982$	
9000	Bp2	1	$\frac{2.35}{2.29} \cdot 1000 = 1026$	2299
		2	$\frac{1.44}{1.99} \cdot 1000 = 723$	

Now by knowing all above information a general procedure for balancing of different rotors by means of influence coefficient method is provided here.[6]

1. Gathering following information about the rotor:
    - All identity numbers such as: parts, components and assembly package.
    - Mass (kg)
    - Critical dimensions (mm)
      Position of center of mass, bearing distance, position of correction planes, effective diameter of correction planes.
    - Rated speed (rpm)
    - Number of critical speeds before rated speed
    - Critical speeds (rpm)
      All critical speeds within rated speed range plus one critical speed above the maximum allowable speed shall be defined. Then rigidity or flexibility of the rotor should be identified. If it is needed test mentioned in 2.2.2 should be performed.

[6] Depends on the case, change in order of steps and/or remove of some of them is possible.

- Residual unbalance of the components which are separately balanced and relative angular position of them during assembly with respect to the rotor 0-mark.
- Results of runout measurement on assembly parts and coupling surfaces as it is specified in assembly drawings.

2. Finding balance quality from table provided in appendix III.
    - For component balancing the grade should be considered at least one grade tighter than the grade considered for entire rotor assembly.
3. Selection of correction plane for performing rigid balancing.
4. Calculation of permissible residual unbalance by considering selected grade in step 2 and using equation (10).
5. Allocating permissible residual unbalance to bearing planes according to equations of (11), (12).
6. Selection of balancing procedure with respect to the rotor geometry and table1 & 2 of ISO-11342 (ISO 11342, 1998).
7. Performing low speed balance as it is defined in step 6. It is highly important to remember, raise in unbalance result due to assembly component should be corrected only on that component.
8. Try to perform over-speed test where it is defined. During over speed where excess vibration is observed rotor shall be balanced and test shall be repeated. Residual unbalance in low speed after over-speed test shall be checked and corrected if it is significantly changed and residual is out of tolerances.
9. Select high-speed balancing speeds.
10. Performing rough high-speed balance. The resultant run-up, coast down shall be recorded to be used for calculation in next steps.
11. Calculation of influence coefficients as it is described earlier.
12. Calculation of correction masses through recorded run in step 10, influence coefficient calculated in step 11 and equation (21).
13. Rotating the rotor for whole speed range and recording the related curves.
14. Selection of most effective correction planes at each speed by considering influence coefficients.
15. Calculation of residual unbalance in most effective balance planes at each speed.

16. Comparing residual unbalances with permissible residual unbalance. If residual unbalance for all speeds is within criteria rotor is balanced. If they are not in tolerance, step 12-16 shall be repeated. Only thing is needed to be considered is that data of the latest run recorded in step 13 is used to calculate new correction masses.
17. After finishing balancing procedure following data are recommended to be included in final balancing report of the rotor.
    - All information mentioned in step one.
    - Residual unbalance and permissible residual unbalance for low speed.
    - Residual unbalance and permissible residual unbalance at higher speeds.
    - Runout profile of the rotor where it is applicable.
    - Run-up, coast down curve of vibration data (Overall and one-per-revolution component).

2.4.4 Shop verification test[7]

Shop verification test is not part of normal balancing procedure. But in new products it is common way to check the design accuracy by comparing results from test and model. This test is defined in order to verify results from mathematical modeling of a rotor. Performing of this test is highly dependent to the agreement between vendor and purchaser. The test is constructed mainly on defining unbalance force in correction plane and measuring rotor response. Then the result of test shall be consistent with the model data. Test can be performed in balance shop or mechanical test rig, but the differences between dynamic coefficients of bearing casing shall be considered if test bearing support are different than real one. Amplitude of unbalance can be estimated through applying 250µmm mass eccentricity or unbalance from following equation:

$$U_{per} = 6350 \cdot \frac{W}{N} \tag{22}$$

Where, U_{per} is estimated unbalance for unbalance rotor response test (g·mm), N is maximum continues speed (rpm), W is defined as rotor effective load[8] (kg) (API 617, 2002).

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[7] Further information and detail instruction on modeling and performing of this test can be find in API616 (API 617, 2002),API-684 (API 684, 1996)

[8] More information about W can be find in figure 3 & 4 API616- section 4.7.2.7 (API 617, 2002).

The greatest value can be selected as input to be applied for the unbalance response test. When the test is applicable following condition shall be met to approve the design:

1. Critical speed frequencies predicted by model shall not be different more than 5% with the critical speeds detected through the tests.
2. Unbalance peak response from the test data shall not exceed peak data from model.

If each of the above condition is not satisfied, model shall be corrected to be able predict the test results.

3 CASE STUDY

In previous sections, basic definitions and general rotor balancing procedure are provided. For more clarification and showing the way how to use the previous instruction, an example machine will be discussed. In the beginning general aspects and sizes are defined. Then, dynamic of the rotors is reviewed shortly. After that, methods, criteria and balancing procedures applicable to this machine are investigated. Finally, part of the result from experimental high-speed test is presented.

3.1 General characteristic of the machine

The machine consists of two rotors which are called LP (low pressure) rotor and HP (high pressure) rotor. Although they are slightly different, similar balancing approach is applicable for both of them. Both rotors consist of 4 major parts, namely main shaft, turbine impeller, compressor impeller and thrust collar. The compressor impeller is fixed on the shaft by use of shrink fitted joint. The turbine wheel and thrust collar are connected to shaft through hirth coupling joint. Before starting to balance knowing some information about the rotor is necessary. General information for both rotors are given in table 9. In addition, figure 18 give information on some of the important sizes, distances and support of the rotors like bearing, correction planes[1] and position of center of mass.

Table 9*. Aurelia turbine's rotors characteristics*

Parameter	High Pressure rotor	Low Pressure rotor
Mass (kg)	55.22	59.46
Maximum service speed (rpm)	33000	33000
1st critical speed[2] (rpm)	18200	15400
2nd critical speed[2] (rpm)	47000	43000
Number of balance plane	5	5
Main bearing	AMB	AMB
Back-up bearings	Roller bearing	Roller bearing
Number of rows and magnets per plane	10 x 16	10 x 16
Backup bearing clearance (μm)	250	250

[1] It is assumed that correction plane are accessible for balancing aims.
[2] Forward mode is taken into account for critical speed.

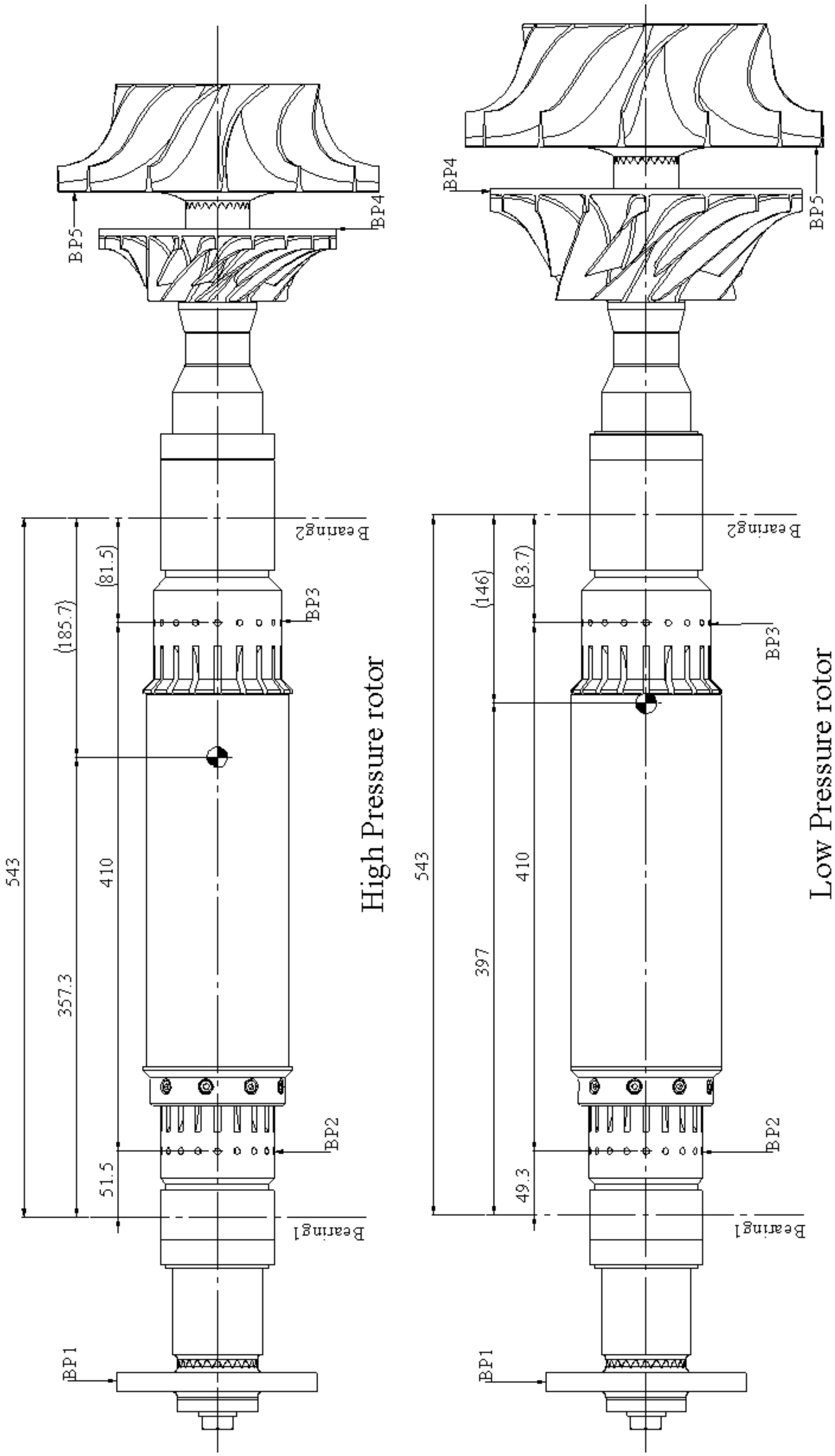

Figure 18. Aurelia HP and LP rotors

In following sections of this chapter just high pressure rotor is considered for investigation and analysis. Similar process can be performed for low pressure rotor.

3.2 Rotor Dynamic[3]

A rotor dynamic analysis has been done for HP-rotor in machine dynamic laboratory of Lappeenranta University. The results shows that rotors pass 1st critical speed (bending mode) in its run up to operating speed. Therefore, rotor is flexible and needs to be balanced in at least 2 plane for reaching the maximum continues speed and keeping the vibration level within specified tolerances.

Figure 19 demonstrate Campbell diagram for HP-rotor. It can be seen that the maximum continues speed (33000 rpm) of the rotor is close to 2nd backward mode (34000 rpm). The backward mode can be excited in rare case of asymmetrical bearing stiffness (Greenhill & Cornejo, 1995). In this case the design is in a way that the magnet bearings provide almost equal stiffness in both vertical and horizontal planes and backward mode will not be excited with the unbalance force.

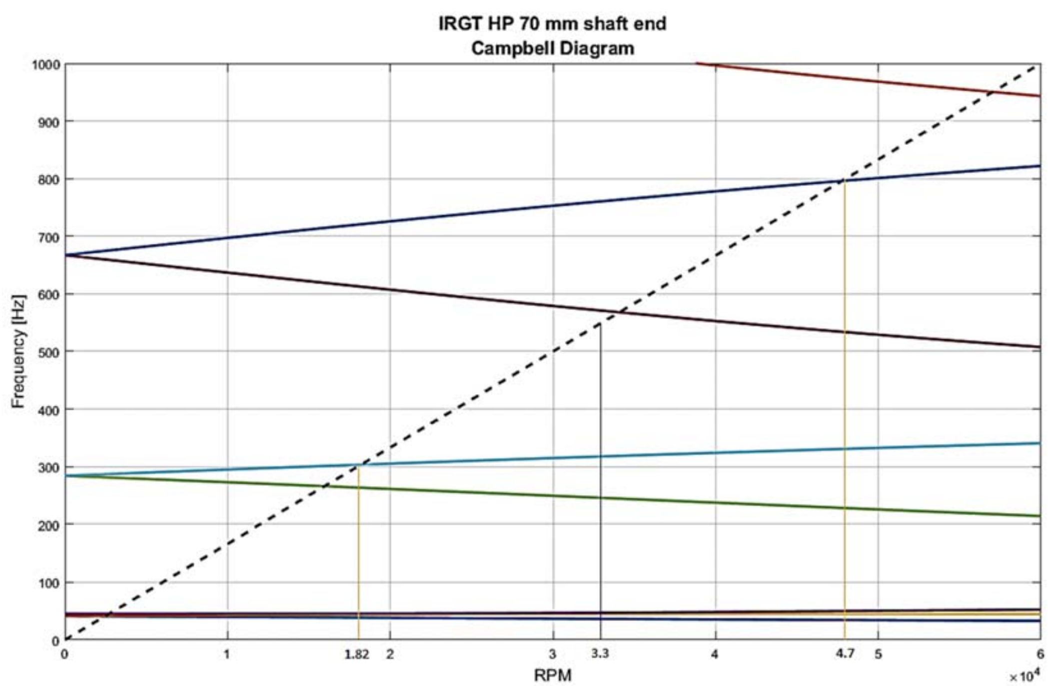

Figure 19. Campbell diagram for HP-rotor

[3] Provided diagrams in this section has been done by machine dynamic laboratory of Lappeenranta University.

Rotor mode shapes and position of correction planes at each mode are shown in figure 20 and figure 21. For both rotors there is potential to have at least 5 balancing planes. These balancing planes are numbered from 1 to 5 in figure 18. Based on the figures BP1, BP3, Bp5 are good choices for applying correction mass on rotor to reduce vibration due to first bending mode. For second mode it seems that BP1 is the best selection among all correction planes.

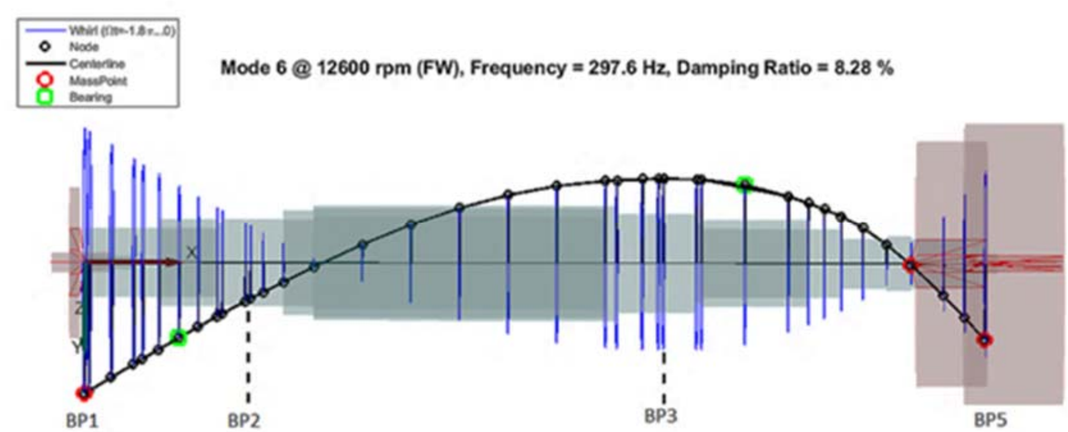

Figure 20. HP rotor first bending mode

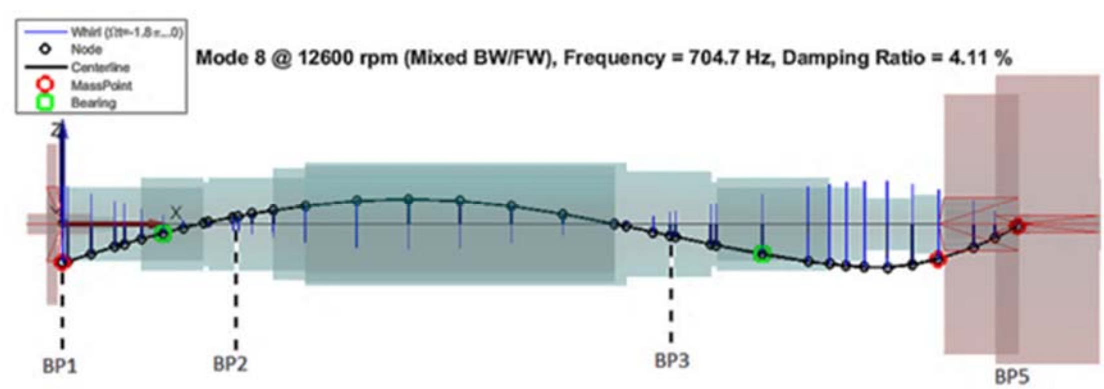

Figure 21. Second bending mode

Although, based on theory this rotor does not need more than 4 correction planes, having access to more correction planes will be an advantage. Having more balance planes will make the balancing much easier by decreasing the degree of over determining of the problem.

3.3 Low speed balancing

Each of the rotor components shall be separately balanced prior assembly on rotor. Residual unbalance for each of the components after balancing shall not exceed permissible residual unbalance for that part according to “G1”. The result of component balancing shall be repeatable within “G2.5”. What is more, 128 magnet pieces are mounted on the shaft. The magnets should be arranged based on their masses in a way that each plane containing of 16 magnets be statically balanced. An investigation on this subject has been done and provided in chapter 4. In addition, residual unbalance of complete rotor containing all above parts, shall be checked. If residual unbalance of the rotor exceed the permissible value, rotor shall be corrected in rigid condition to pass G1 before high speed balancing. However, the final state of low speed can be increased to “G2.5” due to applying correction for high-speed balancing.

Because of light mass of compressor wheel and thrust collar the tolerances are very tight and it is almost impossible to balance them with the required accuracy unless find a highly accurate balance machine which be able to do the required balance. Even though, presence of sort of errors like different axis of rotation during low speed balancing of the components, from their real axis when they are assembled on the rotor, still makes the balancing of these component complicated.

An alternative of separate component balancing is to first start with shaft low speed balance. Then, next component based on assembly order can be mounted on the shaft and the low speed balance state is measured. If a correction is needed, it shall be applied on the latest assembled component. Applying changes in other part of the rotor is prohibited. The procedure is continued in the same manner until all the components are assembled on the rotor[4]. By completing above procedure the rotor is low speed balanced and unbalance from each component is corrected in its plane.

Another issue for performing low speed balancing of these rotors is the position of the bearings. The best practical solution is to mount the rotor on the same condition as it will be in service. Most of low speed balance machine supports have adjustable roller bearings and

[4] It is assumed that there is no restriction to assemble all the components in balancing machine.

rotor is rotated on them for balancing aims. Two possible solutions are applicable in this case:

- Rotating the rotor on the same condition of the service by use of magnetic bearings,
- Rotating rotor from back-up bearing position on balance machine rollers.

The second method is more convenient to be used for low speed balancing and does not affect the results as far as rigid assumption of rotor is valid. The only change shall be applied is to transfer the permissible residual unbalance to this bearing planes. Moreover, in service condition rotor is levitated by magnet bearings and rotor axis of rotation is defined through surface of where the sensors are mounted and it might differ from the back-up bearings axis. Therefore, sensor axis and back-up bearing axis shall be completely coaxial. Otherwise, a huge unbalance will remain for service even if the rotor is perfectly balanced.

3.4 Permissible residual unbalance

From appendix III, balancing grade for both rotors is determined as “G2.5”. The value for total permissible residual unbalance is determined from equation (10). Values for the rotor mass (55.22 kg) and the service speed (33000 rpm) are given in table 9. Service speed for this rotor is far enough from 1st critical speed and first mode shape does not have significant effect on rotor behavior in service speed. Consequently, allowable residual unbalance for 1st critical speed is defined as 100% of the tolerance of rigid state. This value for service speed is same as 1st critical speed. Permissible residual unbalance for the HP rotor in rigid area as an example is calculated as follow:

$$U_{Per,HP} = 1000 \cdot \frac{2.5 \cdot 55.22}{33000 \cdot \left(\frac{2\pi}{60}\right)} = 40 \quad g.mm$$

The above value is total permissible residual unbalance and it is divided proportionally into the bearing planes as:

$$U_{Per,B1} = 40 \cdot \frac{185.7}{543} = 13.7 \quad g.mm$$

$$U_{Per,B2} = 40 \cdot \frac{357.3}{543} = 26.3 \quad gr.mm$$

In addition, vibration limit for in situ balancing and passing acceptance test can be defined from table 3. Considering 250 µm rotor-stator clearance and zone-A for new machine the limit is equal to:

$$\text{Vibration limit} = 0.3\ .\ 250 = 75\ \mu\text{m 0-peak}$$

Then, peak to peak vibration of the rotor in service shall not exceed 150 µm peak-peak. Criteria in whole speed range for both rotors are summarized in table 10.

Table 10. *Permissible residual unbalance for high pressure and low pressure rotors*

<table>
<tr><th>Rotor type</th><th>Speed (rpm)</th><th>Effective plane</th><th>G1 (g·mm)</th><th>G2.5 (g·mm)</th><th>Vibration µm p-p</th></tr>
<tr><td rowspan="4">Low pressure rotor</td><td rowspan="2">Low speed</td><td>Back-up Bearing 1</td><td>4.6</td><td>11.6</td><td rowspan="4">150</td></tr>
<tr><td>Back-up Bearing 2</td><td>12.6</td><td>31.6</td></tr>
<tr><td>1st critical speed</td><td>Most sensitive balance plane</td><td>N/A</td><td>43.2</td></tr>
<tr><td>service speed</td><td>Most sensitive balance plane</td><td>N/A</td><td>43.2</td></tr>
<tr><td rowspan="4">High pressure rotor</td><td rowspan="2">Low speed</td><td>Back-up Bearing 1</td><td>5.5</td><td>13.7</td><td rowspan="4">150</td></tr>
<tr><td>Back-up Bearing 2</td><td>10.5</td><td>26.3</td></tr>
<tr><td>1st critical speed</td><td>Most sensitive balance plane</td><td>N/A</td><td>40</td></tr>
<tr><td>service speed</td><td>Most sensitive balance plane</td><td>N/A</td><td>40</td></tr>
</table>

3.5 Balancing speeds

For rigid rotor balancing, all speed lower than half of the first critical speed of the rotor and above minimum speed of the balance machine are applicable. However, the balance machine shall be able to detect unbalance in range of needed accuracy in that minimum speed. An issue about these rotors is that magnets are freely mounted on rotor and they are not in their final positions. Thus, a lower limit is needed to be defined for minimum balancing speed which put all the magnets in their place. The minimum speed shall be able to create centrifugal force that overcome both mass and force due to magnetic field of the magnet pieces. By knowing the magnet parts properties (provided in table 11), magnetic force and gravity force can be determined. Magnets arrangement and force diagram for a magnet is shown in figure 22.

Table 11. *Aurelia high pressure rotor - Magnet pieces properties*

Properties	Quantity
Mass	40 grams
Effective surface area	580.56 mm^2
Magnetic field strength	0.1 MPa
Magnet position with respect to rotation axis	50.75mm

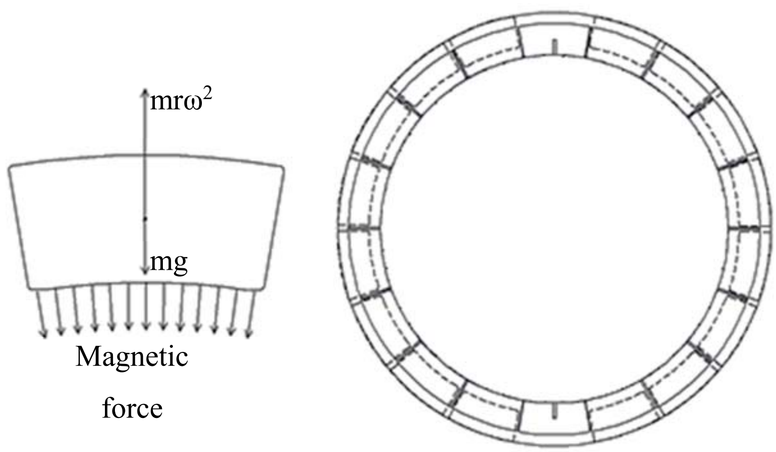

Figure 22. Magnets arrangement and force diagram for a magnet in high pressure rotor.

From the force diagram in figure 22 following calculation are done to derive lowest possible speed for rigid balancing of the rotor:

$$mr\omega^2 = mg + magnetic\ force$$

$$mr\omega^2 = 0.392 + 58.056 = 58.448$$

$$\omega = \sqrt{\frac{58.448}{0.04 \cdot 50.75e - 3}} = 169.682\frac{rad}{s}\ or\ 1620\ rpm$$

In above evaluation, friction between components is not considered. To be ensure that magnets positioning is completely done, 20% higher speed is selected for lower limit speed of rigid rotor balancing. In conclusion, rigid balancing speed for high pressure rotor shall not be less than 2000 rpm. For high speed balancing, depends on the critical speed, balancing speeds can be selected. Following speed in table 12 are planned for low and high-speed balancing of rotors.

Table 12. *Balancing speed of Aurelia turbine rotors*

	High pressure rotor	Low pressure rotor
Balancing speed (rpm)	2000 - 9000	2000 - 7500
	17400	14000
	26500	26500
	33000	33000

3.6 High-speed balancing

The rotors shall be tested up to their service speed and rotor residual unbalance shall not surpass the criteria mentioned in table 10 of section 3.4. If measurements of the rotors' vibration indicate higher level of unbalance in compare to those limits, then flexible rotor balancing shall be performed. As it is discussed earlier, depend on the rotor shape, critical speeds, number of correction planes and position of them, either modal balancing method, influence coefficients method, or combination of them can be used.

In magnetically suspended rotors, in order to control bearing characteristic and rotor-bearing stability, it is common to use non-contact eddy current sensors to measure the shaft vibration. Simultaneously, the output of these sensors are used for system vibration monitoring. Generally these sensors are mounted on rotor casing somewhere close to bearings. Hence, they are not completely fix and are vibrating by casing. Therefore, in this setup they measure shaft vibration relative to the casing they are mounted on. There is a fundamental assumption about magnet bearing systems that they do not allow vibration of rotor transferred to the casing because of their low stiffness and free movement of rotor in them. API 684 (API 684, 1996, p. 58) explained that for the machine with ratio of the bearing casing stiffness to bearing stiffness greater than 3.5, the effect of casing on rotor dynamic can be neglected. In magnetic bearing the support stiffness is much higher than the bearing stiffness and it can be concluded that however the sensors measure the relative shaft vibration, their output is accurate enough to be used for balancing aims since vibration from the sensors is good approximation of the rotor behavior with negligible casing/bearings effect.

It is explained in 3.3 that back-up bearing position on shaft can be used for low speed balancing. The idea is valid only for rigid rotors balancing. Aurelia turbine's rotors are flexible and high-speed balancing cannot be done in that position. Following recommendations can be considered as solution and based on availability can be used for high-speed balancing of rotors.

- In-situ balancing of rotor if at least N [5] correction planes are available in operating condition,
- High speed balancing in a balance machine with magnetic bearing supports,
- Performing a very accurate step by step low speed balancing of the components and rotors.

The first two methods are the most applicable and reliable ones among all options. With considering all of the options, still field balancing is an issue which shall not be forgotten. Having access to at least one correction plane in order to being able to fine tune the vibration level in service is a must, otherwise having good working condition will be challenging.

3.7 Experimental data

High pressure and low pressure rotors had been tested and high-speed balancing by use of influence coefficients method have been applied on them. In the beginning due to high vibration amplitude it was not possible to reach the speeds close to critical speed. So, correction mass based on the data up to highest recorded speed was applied. Then, vibration data for speed up to 1st critical speed were recorded and a correction mass for new data added to the rotor and the rotor was accelerated to 27000 rpm. The result from the tests for NDE (Non-Drive End) and DE (Drive End) bearings are summarized in table 13, table 14 and table 15.

---

[5] N represent number of critical speeds in operating range.

Table 13. *Rotor vibration before applying correction mass*

Low Pressure Rotor				
Speed	ND-end Bearing		D-end Bearing	
[RPM]	Amplitude (µm)	Phase (degree)	Amplitude (µm)	Phase (degree)
3600	19.00	145	3.60	305
6000	11.85	128	3.80	50
7200	10.89	125	5.62	40
9000	10.80	126	8.20	31
9600	10.87	127	9.30	27
12000	17.58	150	17.00	13
13200	30.50	158	29.60	1
13440	36.65	160	35.45	0
13680	45.00	162	43.26	358
13920	58.00	162	55.70	355

Table 14. *Influence coefficients of rotor for BP5 up to 14400 rpm*

Influence Coefficients BP5				
Speed	ND-end Bearing		D-end Bearing	
[RPM]	Amplitude (µm)	Phase (degree)	Amplitude (µm)	Phase (degree)
3600	1.3875	61.536	1.9033	238.34
6000	1.6598	60.485	2.4518	239.55
7200	1.8882	59.926	2.7275	239.14
9000	2.3373	58.805	3.3224	238.91
9600	2.6246	56.723	3.6424	236.97
12000	4.6777	55.487	5.7945	236
13200	7.4748	52.639	8.7892	233.3
13440	9.152	52.852	10.576	236.22
13680	10.997	52.319	12.117	230.85
13920	13.399	53.369	14.573	233.62
14400	25.926	48.584	26.568	229.25

Table 15. *Rotor vibration after applying set of correction masses*

Low Pressure Rotor				
Speed	ND-end Bearing		D-end Bearing	
[RPM]	Amplitude (μm)	Phase (degree)	Amplitude (μm)	Phase (degree)
3600	13.40	136	13.00	182
6000	7.55	91	10.69	155
7200	7.27	76	10.49	147
9000	7.18	64	10.44	140
9600	7.33	59	10.16	138
12000	5.11	60	11.03	127
13200	3.10	86	10.80	115
13440	2.00	110	11.40	109
13680	2.00	160	12.30	102
13920	4.00	295	13.90	92
14400	15.70	209	22.00	66

The final result in table 15 are achieved after applying two correction. Presence of non-linearity and small errors in influence coefficients make the balancing in one shot impossible, especially in higher speed close to critical speed. Figure 23 illustrates vibration curve in NDE and DE bearings for both status before and after balancing.

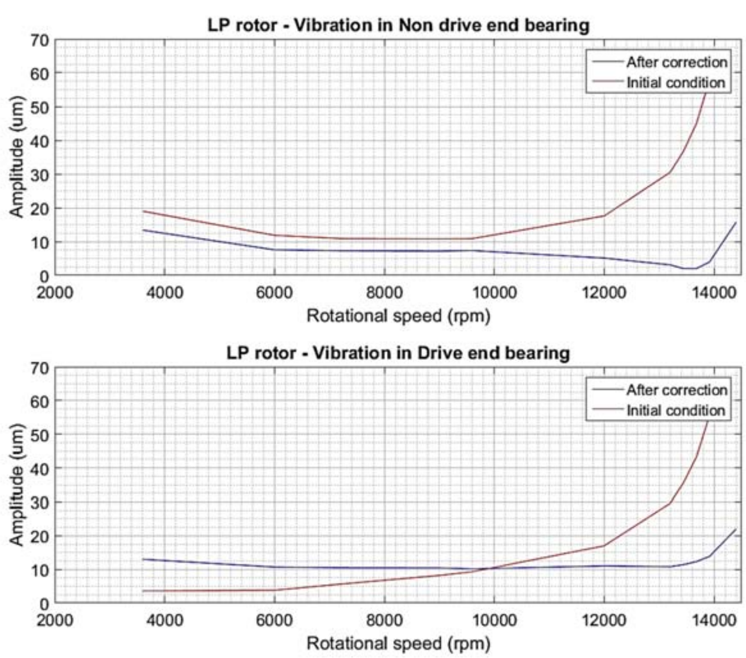

Figure 23. Comparison of vibration in LP-rotor before and after balancing in both bearings.

Similar procedure has been done for high pressure rotor and the influence coefficients has been calculated and provided in table 16. Residual vibration after balancing and initial vibration are presented in figure 24.

Table 16. *Influence coefficients for High pressure rotor*

Influence Coefficients BP5				
Speed	Bearing1		Bearing2	
[RPM]	Amplitude (µm)	Phase (degree)	Amplitude (µm)	Phase (degree)
6000	1.5274	208.68	2.7839	24.125
8400	1.776	205.47	3.2204	24.102
10800	2.4015	206.44	3.7358	23.992
12000	3.8456	211.23	4.0096	30.566
13200	4.8051	210.31	4.7943	29.547
14400	6.4436	209.35	5.9229	26.531
15600	10.218	207.21	8.976	27.802
16800	24.958	207.32	19.569	27.166
17100	40.223	206.92	31.769	26.125
17400	109.77	206.36	79.944	24.98

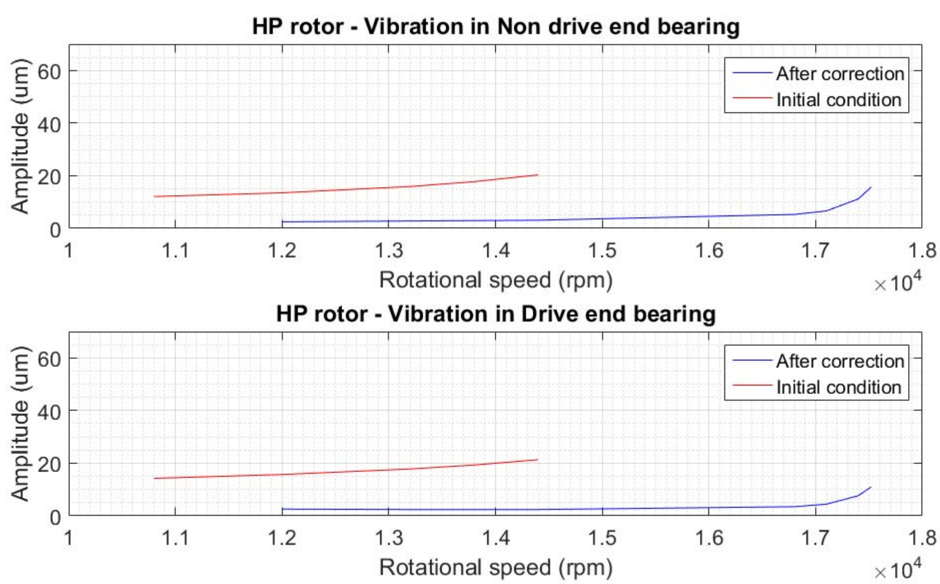

Figure 24. Comparison of vibration in HP-rotor before and after balancing in both bearings.

4 EFFECT OF MAGNETS ARRANGEMENT ON BALANCING

One of the solutions for accurate balancing of rotors as discussed in 2.4.2 is minimization of unbalance due to assembly of parts with prior balancing of those parts. Every turbine rotor, based on the design, might consists of number of blades of the same design. Although they have unique design, by reason of manufacturing tolerances they might have slightly different mass and length. As a consequence, when they are assembled around the rotor, an unbalance force will be induced to the system.

The position of center of mass with respect to same reference will not be consistent as a consequence of different blade length of the same design. This is an important issue for long blades, for instance those are used in wind turbines. For short blades, the residual imbalance force of the arrangement is not affected by the blade length variation considerably, however, the mass difference effect might be significant. In order to reduce this unbalance force the blades are needed to be sorted.

By increasing the number of blades at each row number of combination increase factorially. Number of arrangement for blade some quantity under assumption of “all blades have different masses” is shown in table 17. For small problem it is possible to check all of the variation and select the solution with the lowest residual unbalance. On the other hand, finding the best solution among all possible arrangements in bigger problem seems to be difficult. Mason and Rönnqvist (Mason & Rönnqvist, 1997, pp. 153-167) investigated and modeled this problem through modifying standard QAP (Quadratic Assignment Problem). Applicable method of minimization plus error analysis are reviewed on the same paper. Choi, Kang, and Baek (Choi, et al., 1999, p. 405) used mixed-integer programming for modeling in his work and he recommended heuristic method for solving the problem.

Table 17. *Number of different combination for a known number of blades*

Number of blades	Possible arrangements
4	6
8	5.04×10^{3}
16	1.31×10^{12}
32	8.22×10^{33}
128	3.01×10^{213}

The case study provided in section3 does not contain any assembly blades, however the rotors are containing 128 magnets with different masses and they can be considered as the subject of the arrangement problems. In this section the aim is answer the following questions:

- What is the range of unbalance brought to the system?
- How important is to define position for each piece of magnet prior assembly?

Moreover, it might be considered that, since the rotor will be balanced for low speed condition, there is no need to arrange the magnets. In order to answer above questions and examine the effectiveness of low-speed correction on removing the unbalance caused by magnet arrangements, an analysis of both low-speed and high-speed behavior of the rotor is provided in upcoming part of this chapter.

4.1 Assumptions, modeling and sampling

In the current case study, the magnets radial thickness is very small and only considering their mass is sufficient for this analysis. Based on manufacturing drawing the magnets' masses can vary ±0.5 gram from their nominal mass. The rotor is consisting of 8 row of 16 magnet pieces, totally 128 magnets. Figure 25 illustrates the rotor with its magnets and the balancing planes which used for low speed balancing.

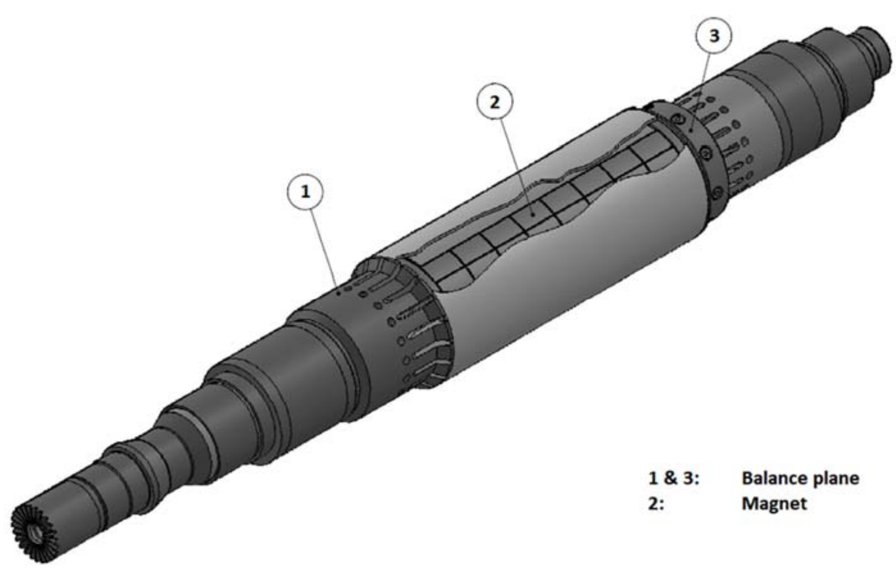

Figure 25. Rotor model showing balancing planes (1&3) and the magnets (2)

For each simulation, 128 random masses are generated within the tolerance limits by using normal distribution. Then, they are divided in 8 groups with 16 magnets per group. After that, they are considered to be assembled around their planes and the corresponding residual unbalance multiplied by the radius of magnets' base is calculated. Next, the calculated force is applied on the center of each row of magnets; nodes number 16, 18, 20, 22, 24, 26, 28, and 30 as they are shown in figure 26. Then, the reaction forces in bearings are calculated for each set. Finally, the unbalance response of the rotor to these unbalances is generated for both states of the rotor when it does and does not pass the low speed balancing. In figure 26, nodes number 5, 41 and 11, 34 refer to mid-points of backup bearings and balancing planes, respectively.

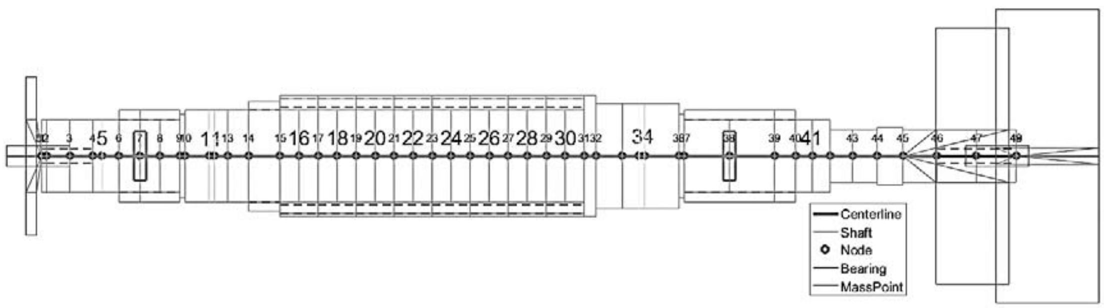

Figure 26. Rotor wireframe finite element model. Nodes 5, 41 show back-up bearings; nodes 11, 34 show low speed balancing planes; even nodes between 16, 30 show magnets' planes center point.

In order to make a conclusion through statistical analysis, the simulation is repeated and responses are recorded for 2000 sets of magnets arrangements. For clear demonstration of the randomly generated magnet weights, first fifty magnet sets mass histogram is shown in figure 27. It is clear from the figure 27 that the generated mass samples follow normal distribution. The nominal mass for each magnet is considered to be 36.25 grams. Histogram of mean values for each set of magnets, as it is shown in figure 28, illustrate the correctness of all randomly generated mass sets.

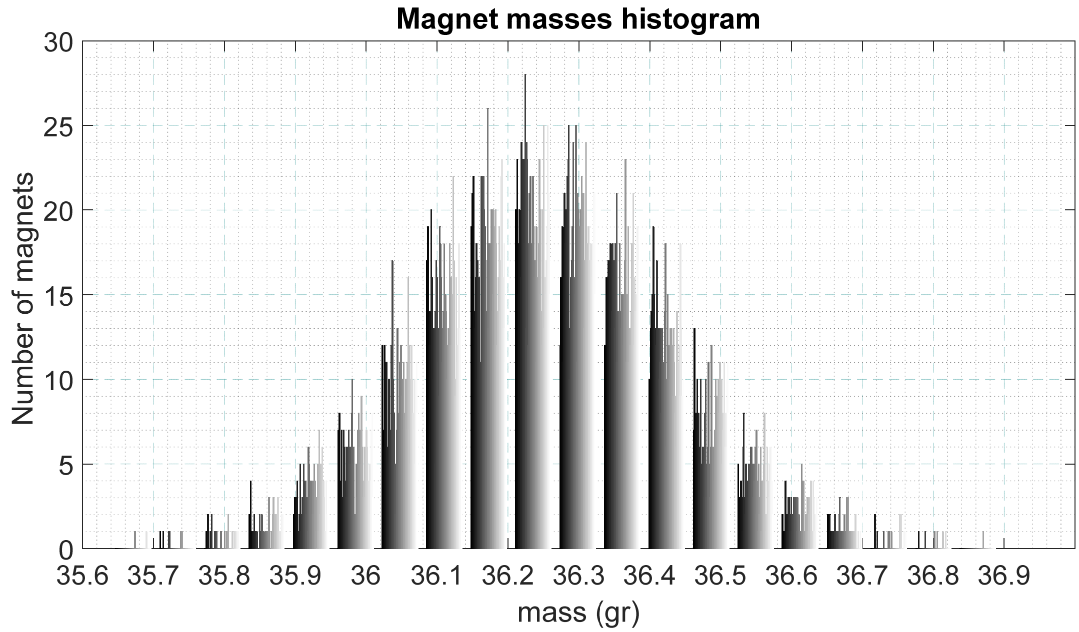

Figure 27.Magnet mass distribution for 50 set of random samples

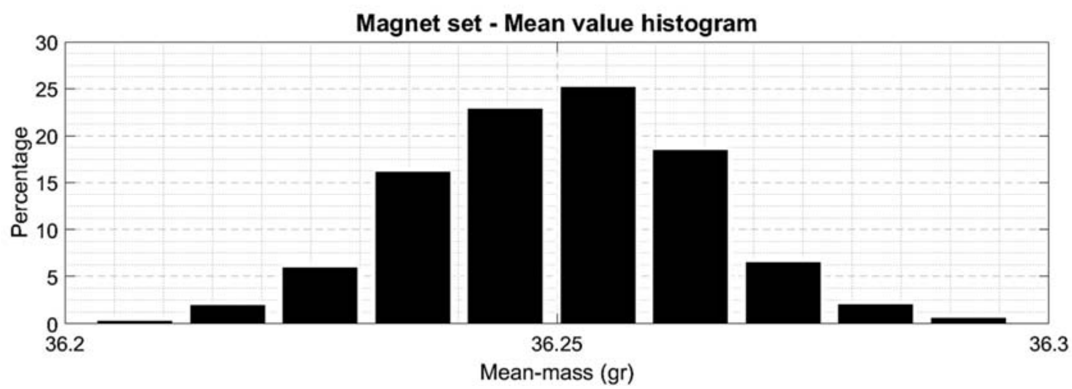

Figure 28. Mean masses histogram for each magnet set

4.2 Residual unbalance and Bearings reaction forces

By knowing the masses around each magnet plane, the residual unbalance can be calculated through equation *(23)*.

$$\boldsymbol{RU} = r \sum_{n=1}^{16} m_n \, e^{i\alpha_n} \tag{23}$$

where:

r is magnet base radius, m_n is n^{th} element of mass vector (g), α_n is n^{th} mass angular position and $\boldsymbol{RU}$ is vector of residual unbalance (g·mm).

The result of equation (23) is a complex number containing both amplitude and angle of residual unbalance at each plane. Number of occurrence of the residual unbalances amplitude per plane is shown in figure 29. It can be seen from the figure that more than 50% of residual unbalances have amplitude of over 25 g·mm which is relatively high with respect to low speed tolerances provided in table 10.

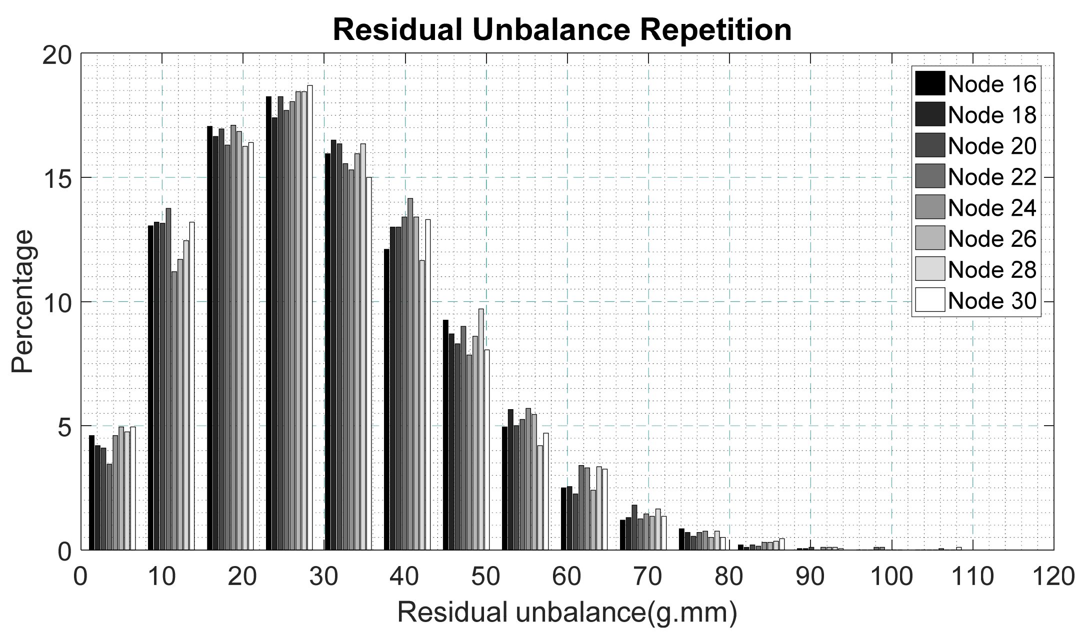

Figure 29. Residual unbalance in magnet planes

By applying these unbalance forces and using static equilibriums (6), (7), the reaction forces in bearings and needed correction masses for low speed correction are achievable. The reaction forces in both back-up bearings (nodes 5, 41 in figure 26) are calculated and the result is shown in figure 30. It can be understood from the figure that, more 90% of the sample sets cause reaction force higher than the tolerance limit for bearing 1 (node 1 in figure 26). The situation in the second bearing (node 41 in figure 26) is slightly better where 75% of data causing the reaction forces exceeding the tolerance. It should be mentioned that the considered tolerance is for whole rotor mass and no downscale is applied to these limits. As a consequence, the evaluated percentage is significant and the result shows dramatic effect of the magnet distributions on unbalance force acting at each magnet node.

In section 4.4, further analysis to check the unbalance significance by magnets arrangement is provided. Rotor unbalance response in high speed is the main tool for comparison and making the conclusion.

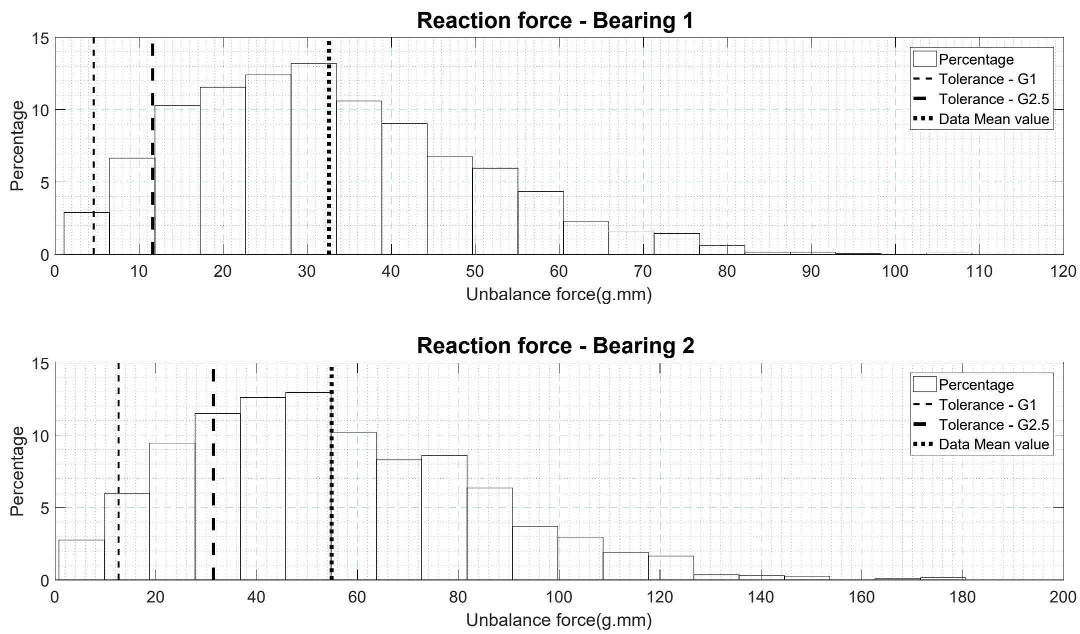

Figure 30. Bearing reaction forces because of different magnet arrangement

4.3 Verification of the results

All the previous analysis are on the basis of the current model. In order to validate the model and find the possible differences, in this section a comparison of the result with the recorded result from the test are provided. Regretfully at the time of the current work it was not possible to do a similar test with the simulation which has been covered by the thesis. Therefore, for making a reference, the test result for turbine disk are used and firstly they are compared with the simulated unbalance response to the same unbalance at the turbine disk. Then the response amplitude of the simulation for unbalance at the turbine disk and magnet sets are compared. In addition, a graph for vibration trend analysis between real data and the simulation will be provided.

To compare the simulation with the test result an exact amount of unbalance shall be added to turbine disk in both cases. The influence coefficient given in table 14 are calculated by vibration difference between two run-up tests in which one of them was done by a trial mass. The trial mass which is used during the test induced a force about 28 g·mm in turbine disk plane. Therefore, for simulation exactly same unbalance is added to turbine disk at the same point and the unbalance response are provided in figure 31.

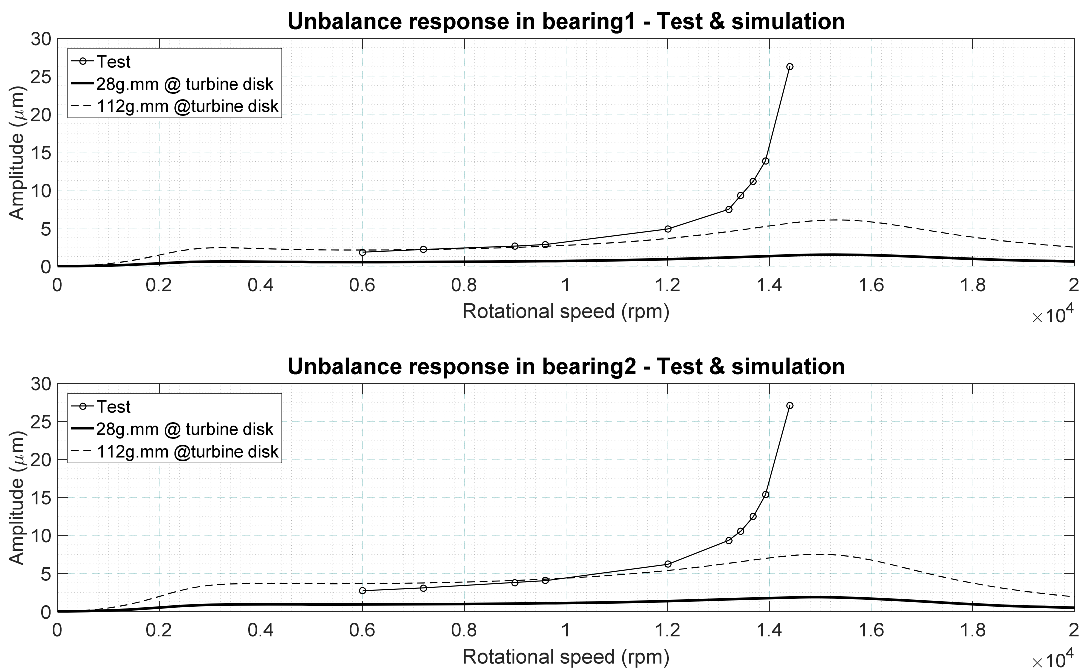

Figure 31. Unbalance response for simulation and test

It can be seen immediately from the figure that both of the test and simulation response have a similar trend, but the model clearly underestimate the response amplitude. By approaching to the 1st critical speed this underestimation error greatly increase. Considering this underrate, collation of the response from the simulation of unbalance in turbine disk and for the worst case scenario of the magnets arrangement can be meaningful to some extent. The unbalance response curves for these two state are provided in figure 32. The diagram illustrates the significance of unbalance forces caused by sample magnets arrangement and their contribution in rotor dynamic within rotor speed range.

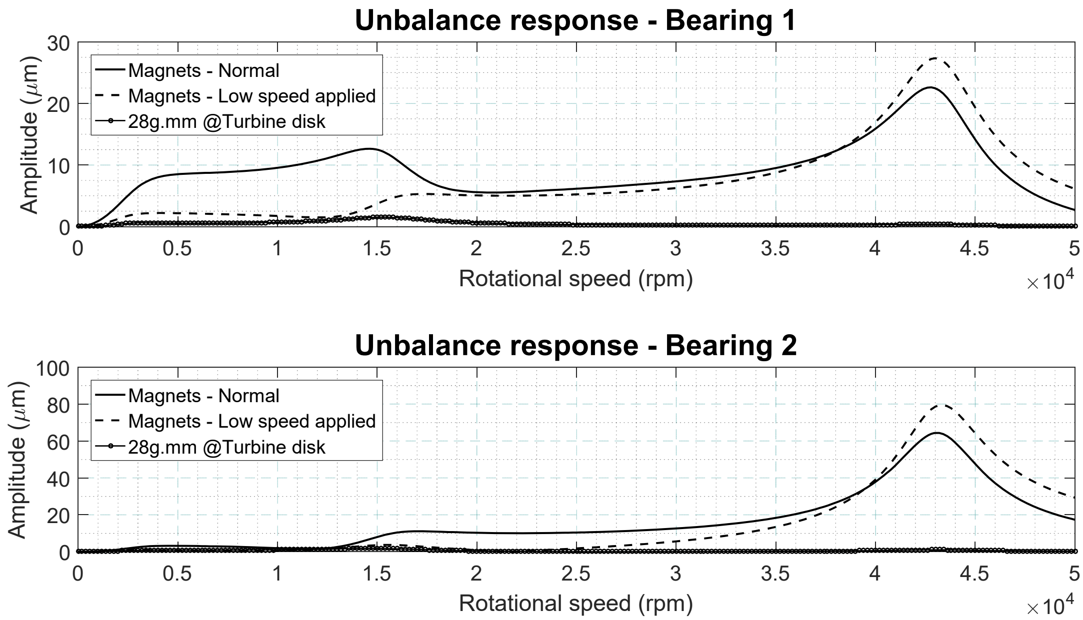

Figure 32. Unbalance response bode plot - Unbalance forces are applied in magnets planes, correction planes and turbine disk

In summary, it seems that the magnet arrangements can seriously influence the rotor unbalance distribution and cause major problem for rotor operation. Although it is possible to remove these effects by performing high-speed balancing on the rotor, it would be much simpler to sort the magnets prior their assembly. Depending on the available data like runout and/or other unbalance sources, different arrangement can be considered to treat all other sources. The simplest method to prevent or diminish this problem is that the magnets can be sorted and assembled in a way that they residual unbalance at each row become equal to zero. Consequently, no excitation will be generated by magnets.

4.4 Unbalance response simulation for magnets arrangement

In order to make final decision about the importance of magnets arrangement, rotor responses to random distributions in high-speed are analyzed in this section. Since it is difficult to compare 2000 of unbalance response curves, rotor response amplitudes in the 1st and 2nd critical speeds, stated in table 9, are considered for this aim. Furthermore, two scenarios are used; in the first one, only unbalance forces from magnets arrangement are applied on the rotor, whereas in the second one, in addition to unbalance forces caused by magnets arrangements, the correction masses in correction planes (node 11, 34 in figure 26) are added to remove the reaction forces in the back-up bearings completely (node 5, 41 in figure 26).

A breakdown of the histogram of the rotor response to randomly distributed unbalance in magnet planes for bearing1 and bearing2 is given in figure 33. These responses are generated in magnet bearing locations. Considering the rotor with no low speed correction and according to the figure, the mean value of the response in the first critical speed for both of the bearings stand just above 4 μm. The mean value for the 2^{nd} critical speed in bearing 1 is about 6.5 μm, whereas for bearing 2 it dramatically increases to approximately 19 μm.

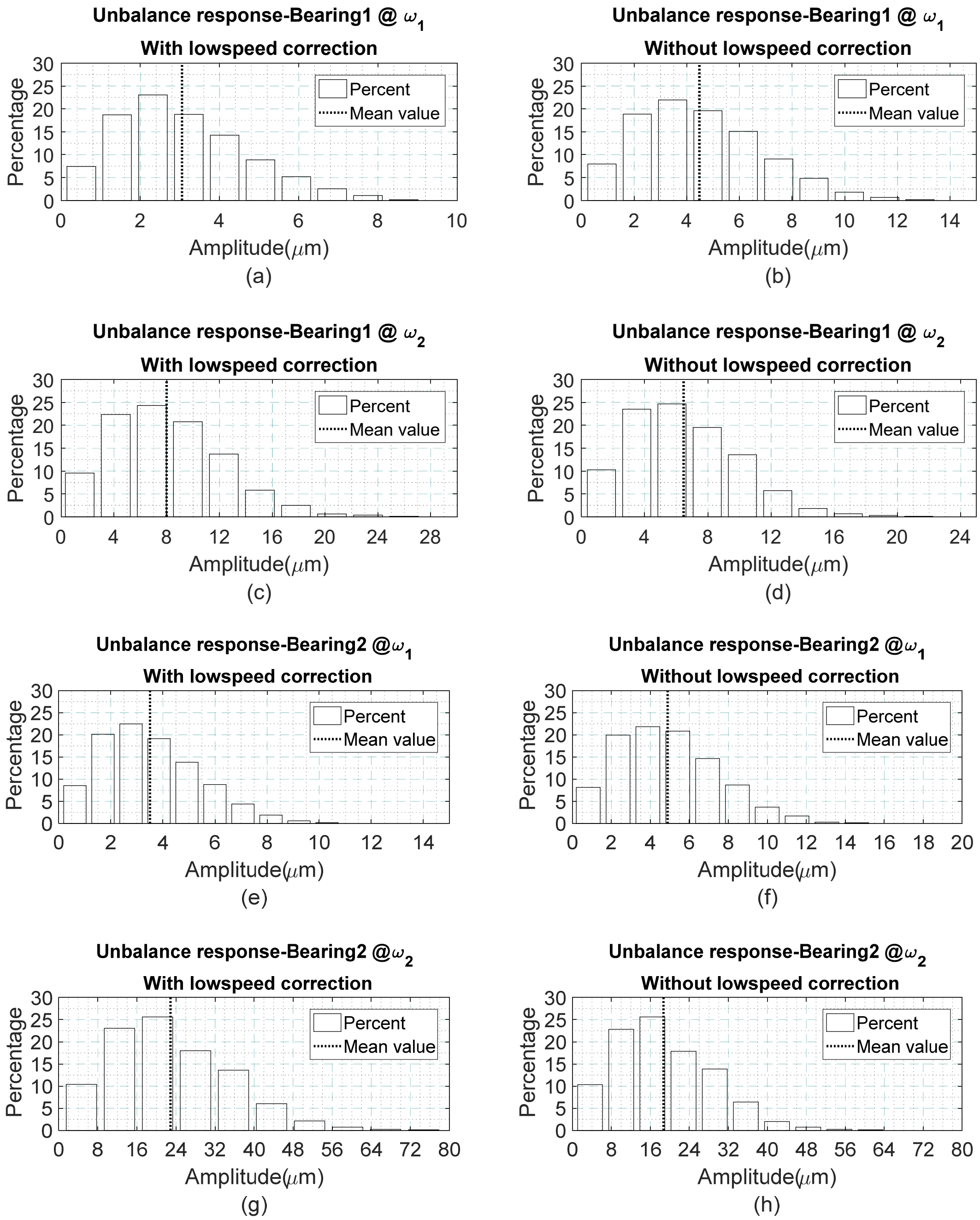

Figure 33. The rotor response to unbalance - Bearing 1: (a) at 1st critical speed with no low speed correction, (b) at 1st critical speed with low speed correction, (c) at 2nd critical speed with no low speed correction, (d) at 2nd critical speed with low speed correction. Bearing 2: (e) at 1st critical speed with no low speed correction, (f) at 1st critical speed with low speed correction, (g) at 2nd critical speed with no low speed correction, (h) at 2nd critical speed with low speed correction.

Looking at the worst case scenarios, it is clear from the figure that for each critical speed, compared to bearing 1, the maximum amplitude of response in bearing 2 are high. Over total simulation, the maximum can be detected for bearing 2 in 2nd critical speed for a relatively high amplitude of almost 64 μm.

Furthermore, to see the effectiveness of low speed balancing, it can be seen from the figure that when the low speed corrections are applied on the rotor, for the first critical speed (a, b, e, f in figure 33), in both bearings the histogram bars are marginally shifted to the left and the mean values are decreased. Although this behavior shows the success of the low speed balancing on the first critical speed, since the mean does not change significantly, a general rule cannot be concluded.

On the other side, for the second critical speed the histogram shows a growth in unbalance response mean value for both of the bearings. It seems that the rotor response in high-speed has been negatively affected by low speed balancing. But similar to bearing 1, no conclusion can be made because the changes are small.

For a better indication of the effect of the low speed balancing on the rotor behavior in high-speed, the unbalance response amplitude in normal condition for each critical speed is subtracted by the corresponding amplitude when the resulting force from correction masses are added to the system. The histogram of the subtracted matrix is presented in figure 34. The figure confirms the previous statements. The low speed balancing of the rotor has a positive impact on 73% and 68% of the samples for the 1st critical speed in bearing 1 and bearing 2 respectively. In addition, the figure illustrates a negative influence for the 2nd critical speed in both bearings with a high percentage of 98%.

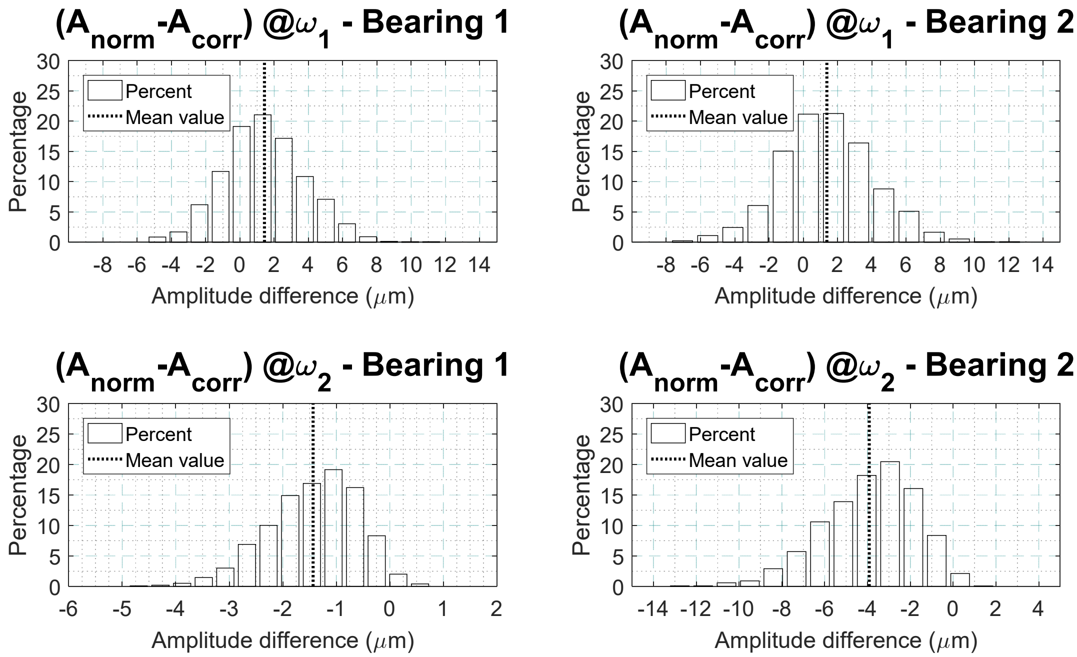

Figure 34. Histogram of Amplitude difference for normal and low speed corrected simulation

To demonstrate the rotor response to unbalance caused by magnets over the speed range, a bode diagram is provided in figure 35. The bode plot features the unbalance response when the maximum change happens for the first and the second critical speeds.

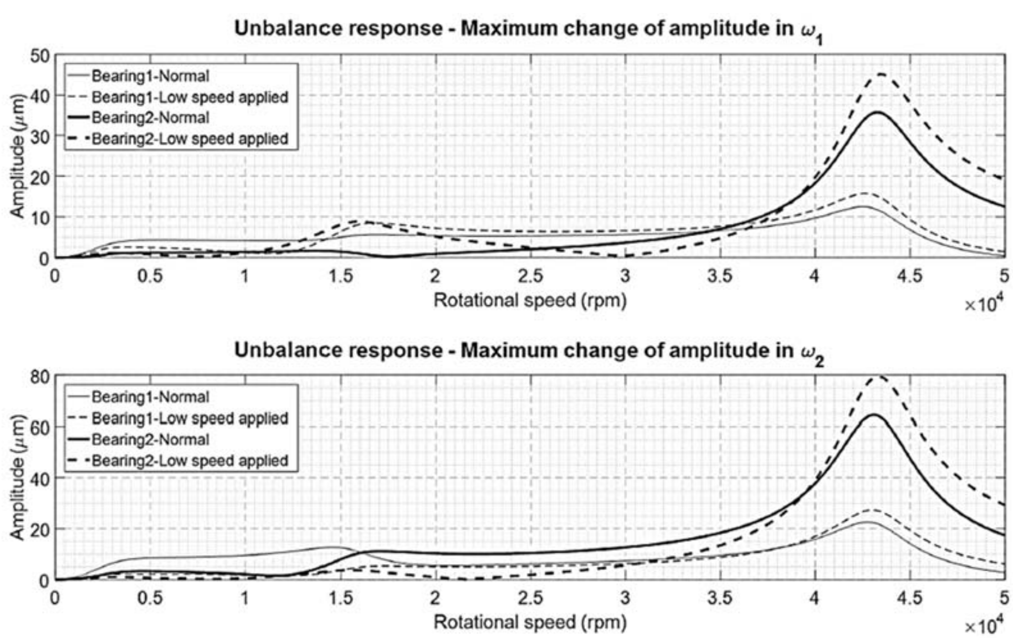

Figure 35. Simulation result with the worst unbalance response

The diagram shows that the rotor likely to vibrate up to 10 µm in the first critical speed in both of the bearings. The vibration magnitude for 2nd critical speed hits 35 µm in bearing 1 when the sample set for creating maximum amplitude in first critical speed is considered. The highest value of the rotor excitation is projected to happen for the worse case in bearing 2 at the 2nd critical speed. However, after applying low speed balancing force in balancing planes it rose from 64 µm to about 80 µm. The input data for these two sample sets and their corresponding bearing reaction forces are provided in table 18 and table 19.

Table 18. *Residual unbalance in magnet planes that cause the highest unbalance response in first resonance speed*

	Residual unbalance – highest amplitude in ω_1		Residual unbalance - highest amplitude in ω_2	
Row #	Amplitude (g·mm)	Phase (deg)	Amplitude (g·mm)	Phase (deg)
1	78.7	-108	35.8	135
2	29.4	-167	80.9	123
3	23.5	-163	8.2	-59
4	23.6	82	95.1	160
5	67.8	-151	40.1	-165
6	35.2	-100	41.2	158
7	41.9	66	9.8	148
8	61.3	8	30.2	41

Table 19. *Bearing reaction force for worse samples*

	Reaction force in bearing planes for the highest amplitude in ω_1		Reaction force in bearing planes for the highest amplitude in ω_2	
Bearing #	Amplitude (g·mm)	Phase (deg)	Amplitude (g·mm)	Phase (deg)
1	25.4	64	90.4	-34
2	79.1	53	173	-35

5 DISCUSSION AND SUMMARY

In the current work, an overview for rotor balancing is provided. Applicable terms, definitions and test for rigid and flexible rotors are presented. Two well-known methods, modal balancing approach and influence coefficients method, are introduced and the positive and negative points of the methods are listed. The influence coefficient method is chosen to be used in order to balance a real rotor. The test results show the successfulness of the influence coefficient method for this rotor balancing. Great reduction in vibration data is obtained; however, further improvement is not possible.

Various reasons can affect the result negatively, so that it prevents achieving lower vibration amplitude. On the first place, presence of non-linearity in the rotor response can have an undesirable effect on the results. The linear rotor response is a fundamental assumption of the influence coefficient method, whereas in reality the rotor will not response to unbalance masses linearly.

The next important parameter can be traced in natural behavior of the magnetic bearings. The magnetic bearings provide variable bearing dynamic coefficients. Active excitation forces in the system cause changes in the stiffness and damping of the bearings. In this case, unbalance forces play the role of excitation forces in the rotor and can change the bearing coefficients. Accordingly, these changes in bearing characteristic will conclude to a change in rotor response. Consequently, the influence coefficients measured by using trial masses include the effect for both, the trial mass and the changes in dynamic properties of the bearing on unbalance response. This phenomenon has not been investigated completely and it can be a valuable topic for further research in the magnetic bearing and rotordynamics field.

Other sources of error can cause inaccuracy in the high-speed balancing procedure, such as: rotor temperature changes, temporary rotor bow, error in sensor readouts, changes in controller parameter and loose part. It should be kept in mind that the condition of the test for all the influence coefficients trial run should be similar. Only the amplitude and phase of the trial mass/masses can be changed from a run to another. A solution to remove all these negative effects is to improve the influence coefficients by statistical analysis and getting mean value out of the confident data. To this aim, number of runs with trial masses are needed. Also these coefficients can be polished over the series production balancing of the rotors.

By using influence coefficient method, the rotor can be balanced and the vibration is brought down to the acceptable level. But the recorded amplitude of initial unbalance is an encouragement for further investigation on the possible sources of unbalance. In chapter 4 of the thesis, an analysis on a specific source of unbalance is done. The aim of the analysis is to check the significance of the magnet arrangement on the rotor response. The analysis has been done by modeling the rotor in MATLAB software. The model used for the analysis was coded in RoBeDyn package in machine dynamic laboratory of Lappeenranta University.

The previous model is modified according to the needs of the new analysis. The magnets' weights are generated through normal distribution. The residual unbalance in magnet planes are imported to the model as unbalance force in magnets' mid planes. The unbalance response of the rotor are generated for both bearings and over 2000 groups of magnets. Then, low speed balancing condition are added to the rotor and the response are regenerated. The result show that the magnets does need to be sorted in order to their unbalance be minimized. Furthermore, it can be concluded from the result that there is not a strong evidence on the efficiency of the low speed balancing on this case.

The sorting of the magnets need to be analyzed and it is not subject of this thesis and further research can be done in other works. In the simplest case the aim of the sorting is to minimize the residual unbalance at each magnet plane. However by knowing the other sources of unbalance, which are caused by the manufacturing and assembly process and are detected by mainly runout measurements, a coupled code with rotor dynamic can make a proper sorting to satisfy the balancing condition for whole rotor. An alternative solution to the sorting is tightening the manufacturing tolerances for magnets to reduce the mass variation. In this way the happening probability of the ill cases will be decreased, however still they might happen. But higher accuracy in manufacturing can impose higher costs to the company.

The random data used in this analysis are generated by normal distribution. Considering the manufacturing process of the magnets, the magnets might be cut out of a cylinder. Thus, each batch consisting of 16 magnets can have less weight variation. But diversity of the weights of the different batches might be considerable. In this case the data for analysis can be generated from uniform distribution. However, if this case happens in reality it is highly recommended to mark the batches separately and assemble each group in one magnet plane.

Balancing makes manufacturers able to certify the smooth operation of their product and guarantee the quality of their final products accordingly. What it is called balancing today, actually is more than just balancing as adding or removing mass to/from the rotor. In common the first dynamic test of rotating machinery can be done in a balance machine which is able to accelerate the rotor to operating speed or even higher. Whatever the test is, for having normal operation in balance machine or later in service the vibration level shall be within acceptable tolerances.

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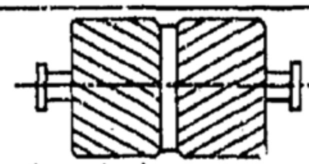
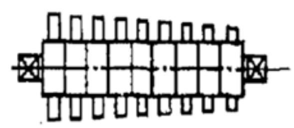
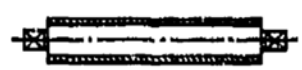
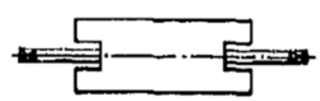
APPENDIX

APPENDIX I

Rotor classification based on ISO Standard (Rieger, 1986)

Class	Description	Example
1	Rigid rotor: unbalance can be corrected in any two (arbitrarily selected) planes and, after that correction, unbalance does not significantly change at any speed up to maximum service speed	 Gear wheel
2	Quasi-flexible rotors: rotors that cannot be considered rigid but can be balanced in a low-speed balancing machine	
2A†	A rotor with a single transverse plane of unbalance (e.g., single mass on a light shaft whose unbalance can be neglected)	 Shaft with grinding wheel
2B†	A rotor with two axial planes of unbalance (e.g., two masses on a light shaft whose unbalance can be neglected)	 Shaft with grinding wheel and pulley
2C†	A rotor with more than two transverse planes of unbalance	 Jet-engine compressor rotor
2D†	A rotor with uniformly distributed unbalance	 Printing-press roller
2E*	A rotor consisting of a rigid mass of significant axial length supported by a flexible shaft whose unbalance can be neglected	 Computer memory drum

*Adapted from ISO Draft Document TC108/SC 1 WG2/N16.

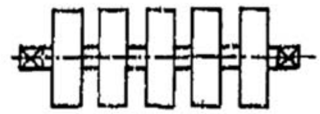
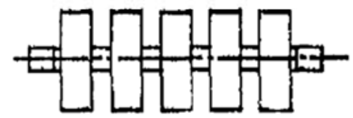
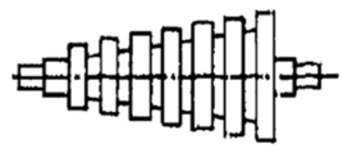
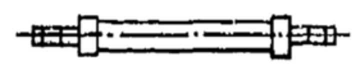
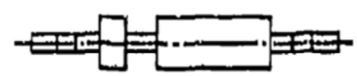
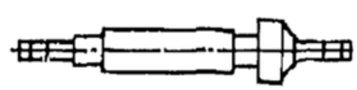
†Rotors where the axial distribution of unbalance is knbwn.

‡Rotors where the axial distribution of unbalance is not known.

It is continued on the next page

Appendix I is continued

Rotor classification based on ISO Standard (Rieger, 1986)

Class	Description	Example
2F‡	A symmetrical rotor, with two end correction planes, whose maximum speed does not significantly approach second critical speed, whose service speed range does not contain first critical speed, and with a controlled initial unbalance	 Five-stage centrifugal pump
2G‡	A symmetrical rotor with two end correction planes and a central correction plane whose maximum speed does not significantly approach second critical speed and with a controlled initial unbalance	 Multistage pump impeller
2H‡	An asymmetrical rotor with controlled initial unbalance treated in a similar manner as class 2F rotors	 Impeller pump. Steam trubine rotor
3	Flexible rotors: rotors that cannot he balanced in a low-speed balancing machine and require high-speed balancing	 Generator rotor
4	Special flexible rotors: rotors that could fall into classes 1, 2, or 3 but have in addition one or more components that are themselves flexible or are flexibly attached	 Rotor with centrifugal switch
5	Single-speed flexible rotors: rotors that could fall into class 3 but for some reason (e.g., economy) are balanced only for a single service speed	 High-speed motor

APPENDIX II

Different rotor speed terms and definitions According to API-616 (API 617, 2002) and API-617 (API 616, 2011):

- "Critical speed: A shaft rotational speed at which the rotor-bearing-support system is in a state of resonance,
- Maximum continuous speed: The highest rotational speed ([rpm]) at which the machine, as-built and tested, is capable of continuous operation. For compressors with variable speed drivers and gas turbines this speed is 105% of the rated speed,
- Maximum allowable speed (rpm): Highest speed at which the manufacturer's design will permit intermittent operation for over-speed and testing transients. The maximum allowable speed is used to establish a trip speed for train components or to establish a speed above the maximum continuous speed for testing,
- Normal speed: The speed corresponding to the requirements of the normal operating condition,
- Rated speed (also known as 100% speed): The highest rotational speed required to meet any of the specified operating conditions,
- Trip speed (in rpm): The speed at which the independent emergency over-speed device operates to shut down a variable-speed prime mover."

APPENDIX III

G value for different machine based on their application (ISO 1940-1, 2003)

Machinery types: General examples	Balance quality grade G	Magnitude $e_{per} \cdot \Omega$ mm/s
Crankshaft drives for large slow marine diesel engines (piston speed below 9 m/s), inherently unbalanced	G 4000	4 000
Crankshaft drives for large slow marine diesel engines (piston speed below 9 m/s), inherently balanced	G 1600	1 600
Crankshaft drives, inherently unbalanced, elastically mounted	G 630	630
Crankshaft drives, inherently unbalanced, rigidly mounted	G 250	250
Complete reciprocating engines for cars, trucks and locomotives	G 100	100
Cars: wheels, wheel rims, wheel sets, drive shafts Crankshaft drives, inherently balanced, elastically mounted	G 40	40
Agricultural machinery Crankshaft drives, inherently balanced, rigidly mounted Crushing machines Drive shafts (cardan shafts, propeller shafts)	G 16	16
Aircraft gas turbines Centrifuges (separators, decanters) Electric motors and generators (of at least 80 mm shaft height), of maximum rated speeds up to 950 r/min Electric motors of shaft heights smaller than 80 mm Fans Gears Machinery, general Machine-tools Paper machines Process plant machines Pumps Turbo-chargers Water turbines	G 6,3	6,3
Compressors Computer drives Electric motors and generators (of at least 80 mm shaft height), of maximum rated speeds above 950 r/min Gas turbines and steam turbines Machine-tool drives Textile machines	G 2,5	2,5
Audio and video drives Grinding machine drives	G 1	1
Gyroscopes Spindles and drives of high-precision systems	G 0,4	0,4

NOTE 1 Typically completely assembled rotors are classified here. Depending on the particular application, the next higher or lower grade may be used instead. For components, see Clause 9.

NOTE 2 All items are rotating if not otherwise mentioned (reciprocating) or self-evident (e.g. crankshaft drives).

NOTE 3 For limitations due to set-up conditions (balancing machine, tooling), see Notes 4 and 5 in 5.2.

NOTE 4 For some additional information on the chosen balance quality grade, see Figure 2. It contains generally used areas (service speed and balance quality grade G), based on common experience.

NOTE 5 Crankshaft drives may include crankshaft, flywheel, clutch, vibration damper, rotating portion of connecting rod. Inherently unbalanced crankshaft drives theoretically cannot be balanced; inherently balanced crankshaft drives theoretically can be balanced.

NOTE 6 For some machines, specific International Standards stating balance tolerances may exist (see Bibliography).