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Master's Thesis

Optimizing Bollinger band parameters: Individual stock and portfolio approach

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ABSTRACT

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The focus of this thesis is to study and optimize the moving average length (N) and standard deviation multiplier (K) parameters for Bollinger bands. The standard parameter values for Bollinger bands are 20 and 2 for N and K respectively, however the theoretical background behind Bollinger bands is rather lacking and thus the standard parameter values are not supported by scientific evidence.

The performance analysis of different parameter combinations is achieved by simulating a trading strategy with all different possible parameter combinations covering the parameter value of N from 5 to 50 in increments of 1 and the parameter value of K from 1.0 to 3.0 in increments of 0.1. The optimized parameter values are then tested out of sample and compared to the standard parameter values.

Optimizing the parameter values gives better results in many cases, however occasionally the standard parameters will perform better. Change in volatility is seen as an important factor when determining how well historically optimized Bollinger bands will perform in the future.

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Tämän tutkielman kohteena on tutkia ja optimoida liukuvan keskiarvon (N), sekä keskihajonnan (K) parametrien arvot Bollinger nauhoille. Standardi arvot parametreille ovat N=20 ja K=2 mutta teoreettinen tausta Bollinger nauhojen tukena on valitettavasti melko puutteellinen, joten tieteellisiä todisteita parametrien arvoille ei ole esitetty.

Eri parametriyhdistelmien suoritusanalyysi saadaan aikaan simuloimalla kaupankäyntistrategia kaikilla eri parametriyhdistelmillä niin, että parametri K:n arvot ovat välillä 5–50 ja parametri N:n arvot välillä 1.0–3.0. Optimoidut parametriarvot testataan näytteen ulkopuolella olevalle datalle.

Parametriarvojen optimointi antaa parempia tuloksia monissa tapauksissa, mutta standardi parametrit toimivat joissain tapauksissa paremmin kuin optimoidut parametrit. Volatiliteetin muutos nähdään tärkeänä tekijänä määritettäessä, kuinka hyvin historiallisesti optimoidut arvot toimivat tulevaisuudessa.

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My journey in Lappeenranta University of Technology was truly one of the greatest things I have experienced in my life. I was able to meet wonderful people, study exchange semesters in Russia and Indonesia and the skillset acquired from the university ultimately took me to Belgium and Ireland. The next stop will be Sweden and only God knows where life will take me after that. One could say that the future seems quite bright.

After a many sleepless nights the thesis finally done and I can proudly tell my parents and friends that I will graduate. Some may have lost faith, but never my parents. I have to give a huge thank you to my beloved mother and father for always believing and supporting me in all aspects of life. Thank you

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1. Introduction

Technical analysis can be regarded as endeavoring to predict future price movements based on historical data. The analysis focuses mainly on past prices, volume and recognizable price patterns, such as double tops and double bottoms. For decades the scientific community, professionals and amateur traders have pursued for superior trading strategy hidden behind the vast amounts of easily available market data. On the contrary, fundamental analysis attempts to determine the security's intrinsic value using quantitative and qualitative factors such as financial information, dividend yield, market conditions and company's management capabilities. In the world of academics, technical analysts and fundamentalists often clash with each other contemplating on the superiority of the methods. The world of technical analysis is vast, sometimes complex and difficult to understand, and lacks a generally accepted definition for technical analysis. Ciana (2011) summarizes the full meaning of technical analysis below:

> *"Technical analysis is the extraction of information from market data into objective visualizations through the use of mathematics with an emphasis on investor behavior and supply and demand to explain the current and anticipate the future path of the financial markets"* (Ciana, 2011, p.3).

Technical analysis can be positioned between a scientific field, such as econometrics, and application. Interpretation of technical analysis methods and technical indicators are more often than not subjective and lack strict rules or laws. On the contrary, econometric models follow usually a much stricter set of rules. However, technical analysis can be roughly divided into two different categories, *charting* and *statistical* analysis. Charting refers to technical analysis methods where security price charts

are the main source of data and information and the utilization of mathematical models is less applied. Chart reading can be seen more as an art form than a robust technical analysis model because of the subjective nature of the analysis method. Statistical technical analysis, being a more mechanical method, is mostly based on mathematical models and theories and thus can be seen as a more robust and scientific method. Bollinger bands are included in the latter category since the bands are structured using moving average and volatility measures that are derived from historical price data rather than just price charts.

Even though Malkiel (1996) famously compares technical analyst to an alchemist trying to turn scrap metal into gold, there are multiple studies which show support for technical analysis either by demonstrating that stock price movements are predictable to some degree or by simulating profits with technical indicators. Such literature includes Jegadeesh and Titman (1993), Chan et al. (1996), Neely et al. (1997), Leigh et al. (2002), Enke and Thawornwong (2005), Atsalakis and Valavanis (2009) and Kazem et al. (2013). On the other hand, Hoffmann and Shefrin (2014) argue that individual investors who rely on technical analysis are prone to make poor decisions, have poor portfolio management, high transaction costs and earn lower returns than investors who do not use technical analysis. McLean and Pontiff (2016) study shows that once profitable technical analysis methods might decay after publication because more and more people start applying the method on the market.

Proponents of technical analysis argue that it is counterproductive to analyze company financials since the current market value already reflects all publicly available information. Thus, it makes more sense to analyze the possible future price movements and investor behavior and try to predict where security prices are heading. Furthermore, fundamental analysis is arguably much more cumbersome task as the financial data companies release is often more or less superficial and parts of the specific information is undisclosed in fear of losing one’s competitive edge

to competitors. The advantage for technical analysis is that the market data does not lie, you don't necessarily have to make any assumptions and the data is the same for everyone. Whether the technical analysis method works or not can be debated, but the underlying market data is always correct and indisputable, assuming the data has been collected from reliable sources in a suitable manner. According to Gerig (2015) around 55% of the trades in The United States and 40% trades in Europe were at the time of publication executed by high frequency trading algorithms, machines. Computers are the perfect candidate for utilizing technical indicators as the indicators do not necessarily require any objective interpretation. The increasing amount of machine trading and automation will arguably expand the utilization of technical analysis and trading rules in the future.

One of technical analysis most widely recognized tool is Bollinger bands which were developed by John Bollinger in the 1980's. Traditional Bollinger bands are generated using a 20 day (simple) moving average as a middle line, which is then shifted plus minus 2 standard deviations (of the underlying asset calculated from the same 20 day moving average window) above and below the middle line. The upper and lower band thus creates a "channel" for the stock price and if the asset price moves outside the bands, a buy or sell signal is created. The standard deviation is a measure of volatility, so as the volatility of the underlying asset increases (decreases), the bands will automatically converge (diverge). Nowadays Bollinger bands are built into many financial information systems such as Bloomberg Terminal, Thomson Reuters Eikon and InFront terminal to help investors and traders make decisions on buying, selling and market timing. Evidence for the popularity of Bollinger bands can be seen on Ciana's (2011) Bloomberg study of technical analysis indicators places Bollinger bands as the third most preferred option just after relative strength index (RSI) and moving average convergence divergence (MACD). Academic literature regarding Bollinger bands is somewhat mixed, studies conducted by Leung and Chong (2003), Balsara et al. (2009), Kannan et al. (2010), Butler and Kazakov (2012) and Coakley et al. (2016) show evidence that Bollinger band trading strategy can yield excess profits.

Others, such as Lento et al. (2007), Fang et al. (2014) and Chen et al. (2018) argue that Bollinger bands cannot be used profitably once transaction costs are taken into consideration or that once profitable trading strategy has lost its effectiveness on modern day era.

1.1. Objectives and methodologies

This thesis analyses whether the Bollinger band parameters could be optimized based on historical and portfolio performance on stocks traded in North American markets. Traditionally, Bollinger bands are structured using parameters of 20 and 2 for *N* and *K*, which represent the moving average length and standard deviation multiplier respectively. Definitions for the parameters are presented in the methodology chapter on page 39. The standard parameter values are based on Bollinger's (2002) analytical studies of different asset classes so that around 95 percent of the asset price movement would stay within the bands. However, as this study shows, stock returns are not normally distributed around the mean but rather fat-tailed and leptokurtic, which implies that the standard parameters of 20 and 2 might not be the optimum ones to capture 95 percent of the price movement. Studies made by Fama (1976, p.21) and Andersen et. al. (2001) present similar results for stock return distributions as shown in this thesis. This study attempts to optimize the parameter values based on historical performance. The tested values for *N* and *K* range from 5 to 50 for *N* in increments of 1 and from 1.0 to 3.0 for *K* in increments of 0.1. Since the theoretical background behind the supposed effectiveness of Bollinger band trading strategy is rather lacking, a more computationally heavy brute force approach is used in this study to perform the optimization. The simulation model built to perform this study tests all the possible parameter combinations on the given range and calculates the performance which is measured as an annual rate of return.

Data for this study consists of daily adjusted closing prices for 60 stocks from North American stock markets and was gathered from Yahoo Finance, InFront and Reuters for a 10 year time period of 1.1.2006 – 31.12.2016. The selected period of 10 years of data is a convenient round figure and long enough to have two different learning periods and out of sample testing period. The selected time period also included different kind of market events such as 2008 financial crisis and couple of smaller market declines. Longer time period would have been interesting but the simulation model in Matlab turned out to be so heavy that running the model for 20 years of data would have taken multiple hours with a standard computer. The first seven years of the data is used for learning purposes and the remaining three years for out of sample performance measures. The parameters are optimized based on past performance using two different length historical data sets of 7 and 3 years prior the 3 year hold-out period. To get a more comprehensive view, the parameters are also evaluated on portfolio level by creating 6 different stock portfolios based on industry sectors. Overall the simulation goes through 966 different parameter combinations for each stock. The performance of the parameter combinations are then compared to the traditional Bollinger band parameters of 20 and 2 as well as to a simple buy and hold strategy, however, the main focus of the study is on the relative performance between the parameter combinations rather than absolute performance over buy and hold strategy. Finally, a sensitivity analysis is performed in the purpose of checking how critical the parameter determination actually is and how much the trading strategy performance varies when the parameters are altered slightly.

This thesis seeks to answer the following three research questions:

1. Is it plausible to optimize Bollinger band parameters using historical data and what factors affect the parameter value optimization?
2. Can optimized Bollinger band parameters yield robustly better returns than the generally proposed parameters of 20 and 2?

3. How sensitive the performance of Bollinger bands is to small changes in parameter values in relation to profitability?

1.2. Structure of the study

The structure of the thesis is as follows: Chapter two dives into the theoretical background behind Bollinger band trading strategy by introducing some key concepts which technical analysis is based on. To give reader a better understanding of the subject, theory and implications of two other related technical analysis methods, price channels and moving average envelopes are presented in chapter three, and compared to Bollinger bands. Chapter four will discuss the recent literature and examine the academic work of other researchers in the field. Chapter five presents the data and methodology used in this study. Results are presented in chapter six with further analysis of the subject. Chapter seven will summarize the main findings and conclude this thesis with suggestions of future research.

2. Theoretical background

This chapter and the rest of the thesis will describe the stock market direction by using the phrases of bull and bear market. The thesis will follow Pagan and Sossounov's (2003) definition of bull and bear market where the market is said to be in a bull (bear) phase if the general stock market prices are trending upwards (downwards) a minimum time period of four months. In the event of price movement beyond 20 percent up or down in less than four months, the minimum time window constraint is disregarded.

2.1. Dow Theory

The field of technical analysis was born in the early 20th century when Dow theory was introduced for the first time. Charles Dow, a man behind the famous Dow Jones Industrial Average index (DJIA), developed the theory that described stock market movements surprisingly well considering that Dow had used a relatively small data sample. Due to Dow's early death, he never had the chance to publish the theory but much of the work was done by William Hamilton who took the ideas from Dow's letters and introduced them as the famous Dow theory. (Edwards & Magee, 2001)

Hamilton's (1922) and Rhea's (1932) work of compiling Charles Dow's thoughts to a presentable format is rather vast and the books discuss the subject much more detailed than what is presented in this thesis. However, to give reader a better understanding of how technical analysis came to be and how stock prices move and behave, Dow theory is an excellent way to approach the subject. Hamilton's and Rhea's work can be summarized to a six basic tenets that define the Dow theory.

1. The stock market reflects all information

   Dow Theory and efficient market hypothesis agree on the basic idea that all information is included in market prices and any new information is quickly and efficiently distributed across the entire market. However, Dow theory does not suggest that every market participant has the same amount of information, but only that the average market prices reflect all available information and no one party can manipulate the market since the market is always bigger than the manipulator. (Hamilton, 1922 p.41-43)

2. Three trends of the market

   The stock market has three kinds of price trends that are distinguished by time horizon. Primary trends are major up or downward movements that can last several years and usually include a price gain or decline of over 20 percent. Primary trends are disturbed by a secondary swing in the opposite direction. These can be seen as smaller, usually around one third of the size of preceding primary trend, temporary corrections or recoveries that usually have a time horizon measured in weeks or months. Third and the shortest time horizon trends are the minor trends that can be considered as a daily fluctuation and can be ignored in terms of the Dow Theory. (Hamilton, 1922 p.27)

3. Three phases of the price trend

   There are three distinctive phases in a primary price trend starting with the accumulation phase where a small group of knowledgeable investors starts to load a stock. In this phase the market prices do not move that much because of a minority group of investors. In the second phase larger groups, including technical analysts, will catch up and market prices will start to move more rapidly. Final third phase is the excess where the market starts to overheat, discussions about possible bubbles will start to appear and the stock prices will start to incline

slower. These same three phases can also be found in bear markets in a very similar manner. (Hamilton, 1922 p.27-28, 37)

4. Volume confirms the trend

   Market trends are confirmed by increases or decreases in volume such that in a bullish primary trend you should expect to see a significant increase in volume. Secondary swings or corrections should be accompanied by a decrease in volume and activity. Similarly in a bear market, volume increases with the primary trend and decreases in times of recovery swings. The event where a secondary swing is supported by a large volume, it might mean that a primary trend is actually changing direction. (Hamilton, 1922 p.136-137)

5. Trends in indices confirm each other

   No one index can confirm where the primary trend is heading in the future but it takes multiple indices to confirm the trend direction. The stock market likes to move in unison so that generally all the stocks and industries are gaining or losing. Surely there will always be outliers and some stocks or industries might lag behind weeks or even months but generally speaking there is a one direction for the whole market. (Hamilton, 1922 p.138-140)

6. Trends persist until definite a reversal

   Investors should be patient with determining when a primary trend is going to an end and ultimately changing the direction of movement. No trend is going to last forever and all bull and bear markets have their ends, but even though indicators show that trend might be changing, it might be better to be patient than immediately start unloading the position. (Hamilton, 1922 p.273-276)

Even though the tenets above have been introduced about a hundred years ago, they are still relevant in modern days' ever changing stock market. The principles behind Dow theory lay a strong foundation for today's technical analysis techniques, which

often use indicators such as volume or trendline to forecast future events. Surely in present day stock market where for example volume and market timing decisions are largely made by machines (see Gerig, 2015), the Dow theory is somewhat lacking but it still gives an overall picture on what technical trading is fundamentally based on.

2.1. Mean Reversion

Mean reversion in stock prices is defined as a tendency for the prices to revert back towards their long term trend line (Balvers et al. 2000). Contrarian investment strategies, going against the general market sentiment, are often based on mean reversion which would imply that the stocks that have performed comparatively poorly in the past (losers) are mispriced in the market and thus should start to appreciate in value in the future. Correspondingly, stocks that have performed better than average in the past (winners) are expected to underperform the losers in the future. The principle behind Bollinger bands can be thought to work much of the same way as mean reversion. When stock prices move towards the upper band, the price is considered to be high, or the stock to be overbought, on a relative basis and prices should start advancing towards the lower band (Bollinger, 1992). Multiple studies, such as Fama and French (1988), Cecchetti et al. (1990), Balvers et al. (2000) and Gropp (2004) show support for the mean reverting characteristics of stock prices at least in longer time horizons. Jegadeesh (1991) however concludes that after the year 1926 mean reversion in The United States and United Kingdom stock markets is only statistically significant in January. Spierdijk et al. (2012) study the mean reversion process on stock indices in 19 different OECD countries in the 20th century. Results suggest that the speed in which the mean reversion occurs is highly time varying and the highest speed mean reversions were found in times of high economic uncertainty such as Black Monday in 1987. Bali et al. (2008) report somewhat similar results stating that the speed in which mean reversion occurs is much faster in times where there has been a sharp downfall in the market and also faster for smaller

stocks rather than larger ones. The methodology used in the study makes it highly interesting in terms of Bollinger bands. Bali et al. (2008) predictive regression model supports mean reversion in shorter time horizon as well, showing that the expected return for the upcoming months is higher the smaller the lowest daily return on the past 4 months has been. This result supports the argument that when there is an extreme event in the market and the daily stock prices drop sharply penetrating below the lower Bollinger band, the stock prices have a tendency to start moving back up again. This kind of stock price movement should make the Bollinger band trading rule profitable if the parameters for the bands are correctly estimated.

2.2. Support and resistance

Dow theory's secondary and minor trends can be explained by support and resistance levels. A price that follows a bullish primary trend but fluctuates up and down marking new higher local highs and lows is said to reflect support and resistance levels. A support level is a price level where buying power overcomes the selling power and a stock price starts to incline after a decline. Resistance level is the opposite of support and indicates a price level where selling power overcomes the buying power and stock price starts turning downwards again. In figure 1, support levels are indicated by the even numbers 2, 4 and 6 and resistance levels are indicated by the uneven numbers 1, 3 and 5.

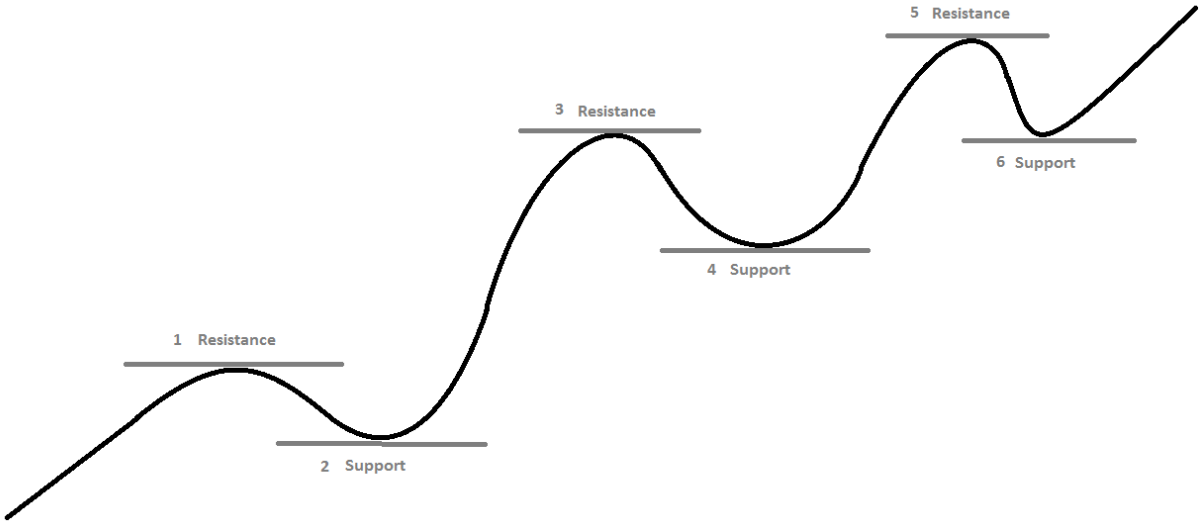

Figure 1: Draft of how bullish stock price advances and fluctuates from resistance to support

When it comes to the trend, support and resistance levels offer a strong confirmation whether the trend continues or reverses. For a bullish trend to continue, each resistance level should be at a higher price level than the previous one. Correspondingly each support level should be on a higher price level than the previous. If the succeeding level is not higher than the previous, there is a strong chance of trend reversal. The levels can also be explained by investor's psychological behavior. If we divide the market to three different groups, buyers, sellers and neutrals, and imagine on being on a support level where prices start to move upwards, the buyers would surely be happy but also dissatisfied that their position is not bigger. The sellers, however, would be on the losing side and hoping the prices would turn back down so they could exit the position at a minimal loss. The neutrals would feel that they are missing out and are waiting a good moment to get in and take a position. All three groups would be on the buy side when the prices start to drop and thus a new support level would be achieved. Similarly, a resistance level could be

explained by the exiting long positions and new short positions entering the market. (Murphy, 1999, p.55-65)

Following Stern (2007) and Bhandari (2012) method of calculating support and resistance levels for a stock price one can find that the principle is rather similar as in Bollinger bands. *H*, *L* and *C* represent the stock's previous day high, low and closing price respectively. S_1, S_2, R_1 and R_2 are the respective first and second support and resistance levels.

$$Pivot\ (P)\ =\ \frac{(H+L+C)}{3}$$

$$1st\ Resistance\ (R_1)\ =\ 2P-L$$

$$1st\ Support\ (S_1)\ =\ 2P-H$$

$$2nd\ Resistance\ (R_2)\ =\ P+(H-L)$$

$$2nd\ Support\ (S_2)\ =\ P-(H-L)$$

The difference between the previous day's high and low can be thought to represent volatility. As the volatility increases, the first and second support and resistance levels will move further away from the pivot point, much in the same way as Bollinger bands will diverge with increasing volatility. Bollinger bands provide dynamic support and resistance levels for the stock price and thus when the stock price tags the upper band, a resistance level, it is to be expected that the prices would start depreciating. Support and resistance level effectiveness can be tested with a "bounce analysis", i.e. how often the price will bounce off the level and not penetrate below (above) the estimated resistance (support) level. Zapranis et al. (2012) argue that support levels tend to have a stronger bounce effect than resistance levels and conclude that

support levels work as an efficient estimator of a trend reversal. However, the trading rules used in the study based on bounce and penetration of the level failed to generate excess returns even when transaction costs were not taken into consideration. Osler (2000) reports similar results on his study on the foreign exchange market, stating that prices tend to bounce more often than penetrate the support and resistance levels published by 6 different technical analysis providers.

3. Price channels, envelopes and bands

Price channels, envelopes and bands are three distinctive technical analysis methods that can be grouped together since they all focus on either mathematical or visual price patterns. The theory behind these three different technical analysis methods relies heavily on the concepts of Dow theory's trend and support and resistance levels introduced earlier. A short introduction to price channels and envelopes is made to give the reader a better understanding of how Bollinger bands are distinguished from other types of bands that may look very similar at first glance.

3.1. Price channels

Price channel consists of two parallel lines that are drawn on top of a stock price chart to form a tunnel for the price. The channels are plotted usually against two local lows and local highs, which reveal the trend of the channel, however determining the correct low and high points is arguably difficult and subject to one's own interpretation. Alternatively the channel can be drawn using just two local lows to form a lower line, which is then shifted a fixed distance upwards to form the upper line. The trend can be bullish, bearish or horizontal and will determine on which side the investor will be trading, i.e. going long or short on the stock. The two parallel lines form support and resistance levels and when the price penetrates the line, there is said to be a breakout which usually causes the trend to change. Bullish price channel is presented in figure 2 where the breakout and trend reversal can be seen. This upward trend would create a profit potential for an investor who would take a long position on the stock, but deciding the timing would be arguably rather difficult. One option would be to take a long position as early as possible once one thinks that the upward trend has been confirmed. Another possibility would be to wait until the price reaches a new local low, in other words, when the price moves closer to the lower line, and hope that the price will bounce back up towards the upper line and follow the

trend. The third possibility would be to wait until there is a clear breakout and take an opposite position in hopes of that trend direction will change from bullish to bearish or vice versa.

Figure 2: Price channel modeled on top of Microsoft stock price chart 27.10.2008 – 30.6.2010

Unfortunately there is no real math behind price channels, but they are open to traders own interpretation of the assumed trend direction. One could argue that price channels are more of an art form than anything else. However, when it comes to Dow theory, the price channels seem to have a connection with the concepts of primary and secondary trends as well as the different phases of the trends.

3.2. Envelopes

Moving average envelopes (for definition see Leung and Chong, 2003) can be considered a more complex form of the traditional price channel presented above. Envelopes are typically plotted over a stock price chart in a way that the envelope forms an upper and lower bound for the stock price to move within. When the stock price penetrates above or below the upper or lower bound respectively, the stock is considered to be either overbought or oversold, and a signal is generated. This signal is based on the idea of mean reversion which suggests that stock prices tend to revert back towards the long time average after price swings. One of the most often used price envelopes is the moving average envelope, where a simple moving average is calculated and shifted a fixed percentage above and below itself to form the bounds that offer support and resistance levels. (Schwager, 1996, p.79-82)

Moving average envelope is presented by the blue lines in figure 3 where a 20 day simple moving average is plotted and then shifted 5 percent up and down to form a price envelope. The stock price tends to stay within the bounds but with more aggressive movements the bound is pierced and the stock price usually tends to revert back towards the simple moving average line.

Figure 3: Moving average envelopes (20,5) on top of Microsoft stock price chart 27.10.2008 – 30.6.2010

Choosing the correct moving average length and the percentage of how it should be shifted is hard. Arguably a trader with a long investment horizon should choose a longer moving average than a trader with shorter investment horizon but nevertheless the selection is not a simple problem. One view would be to choose a percentage so that the envelope would contain about 95 percent of the historical stock prices. This method would ensure that only major price movements would push the stock to break the bounds and generate a buy or sell signal, thus increasing the odds that mean reversion would occur. The proposed method, however, has major flaws as well. Stock market volatility is not constant and some time periods are more volatile than others. This results in a situation where in times of low volatility, the envelope could end up being too wide apart and in times with high volatility too narrow. The width of the envelope would need to be adjusted to match the prevailing stock market conditions which would then again result in difficulties determining the correct moving

average length and envelope width. Previous literature by Jacinta Chan and Zainudin (2016), Modell and Lynngård (2017) and Leung and Chong (2003) has shown support for a profitable use of moving average envelope based trading rules.

3.3. Bollinger bands

Bollinger bands work very similarly to price envelopes but contain the added benefit of automatically adjusting the width of the bands with the respective changes in volatility. In other words, Bollinger bands catch the stochastic nature of volatility. In times of high volatility, the upper and lower bands diverge from the moving average and similarly in times of low volatility the bands converge. Bollinger bands are displayed in figure 4, in which the adjustment to volatility is clearly visible.

Figure 4: Bollinger bands (20,2) on top of Microsoft Stock price chart 27.10.2008 – 30.6.2010

Bollinger bands can be used for different purposes such as volatility visualization or signaling but the normal usage would be to generate buy and sell signals either solely or with the help of an oscillator (see for example Edwards & Magee, 2001) such as relative strength index, momentum or moving average convergence/divergence indicator. When used solely, a trader would be taking a long position when the stock price penetrates or moves close to the lower band and short position when the price penetrates or moves close to the upper band. When the price penetrates the band, it is being viewed as a signal of overbought or oversold and it is to be expected that the price would start to return towards the mean or the middle band. To work, such strategy would then need to have the band parameters so that the bands would be wide enough to capture only the more extreme events where the price movement overreacts and then reverts back towards the mean. However, the Bollinger bands will incur the same nature of problems as the price envelopes when choosing the

parameters for calculation. The standard parameters, proposed by Bollinger (2002), are moving average length of 20 and a standard deviation, a proxy for volatility, multiplied by 2. Unfortunately Bollinger's proposed standard parameters do not have much of a scientific justification but are more or less arbitrary. In fact, much of the theoretical framework for Bollinger bands is missing and the framework is mainly based on real life observations and tests conducted by Bollinger himself. His justification for the standard parameters is the argument that, on average, they tend to work best over all markets (Bollinger, 2002). Others, such as Butler and Kazakov (2012) have argued that the correct band parameters for a given stock should be the ones that yield the best results and profits when simulated using historical data. Alternatively, correct parameters could be viewed to be so that the bands would capture about 95% of the price observations and the remaining observations could be considered as extreme events that have a low probability of happening. These extreme events, large price fluctuations from the mean, are the events that drive the returns on Bollinger bands trading strategy.

The theoretical framework for Bollinger bands is relatively flexible, unlike most academic frameworks which are usually more rigid and absolute. The framework is based on price patterns, mean reversal and support and resistance levels. However, more recent research done by Oleksiv (2008) tries to provide a more thorough statistical framework for Bollinger bands trading strategy. Oleksiv makes three assumptions about the data in order to justify the use of Bollinger bands. Assumption 1 assumes that the data is stationary, in other words, the joint probability distribution is constant over time. Stationarity in order of two implies that the mean and variance are constant as shown below where p_t is the asset price at time t:

$$E(p_t) = m$$

$$Cov(p_t, p_{t+i}) = E(p_t, p_{t+i}) - m^2 = C_i$$

Transferring this to Bollinger bands would mean that the simple moving average and standard deviations are constant over time:

$$E(p_t) = m \approx \frac{1}{N}\sum_{i=0}^{N-1} p_{t-i} = SMA_t = constant$$

$$\sigma_t = \sqrt{E[p_t - E(p_t)^2]} = \sqrt{\frac{\sum_{i=0}^{N-1}(p_{t-i} - m)^2}{N}} = constant$$

If the data is not stationary the simple moving average cannot be considered as a valid estimator of the mean. This also implies that if the mean estimate is incorrect, the assumed extreme events where the price moves outside the bands could not be statistically justified and thus the optimal parameters could not be justified either. The assumption of stationarity can be however weakened to only consider the data as locally stationary, where the mean and variance are only constant for a given interval of time. Locally stationary data would thus rationalize the usage of optimal parameters but only for a given interval of time and the optimum would need to be changed over time. (Oleksiv, 2008)

Assumption 2: probability distribution of returns is symmetrical

Bollinger bands upper and lower bands are plotted equal distance from the mean, simple moving average. For this to be statistically accurate, the probability distribution around the price mean should be symmetrical. In other words, the stock returns should not be skewed as skewness would then stress the stock price more on one side of the mean and thus the probability for the stock price to break outside the bands would we skewed as well. The assumption of two standard deviations around the mean to cover 95% of the events to be true would then imply that the bands for the skewed data should not be plotted equal distance from the mean but actually nonsymmetrically as show below: (Oleksiv, 2008)

$$Upper\ band = SMA_t + K_1\sigma$$

$$Lower\ band = SMA_t - K_2\sigma$$

$$K_1 \neq K_2$$

Assumption 3: Probability distribution function is known

In order to estimate the optimal parameter for standard deviation multiplier, the probability distribution function should be known to make sure that 95 % of the price movements would stay within the bands. If the probability function is not known, the estimation becomes a much more calculation heavy problem as multiple parameters would need to be tested to check how much of the data stays within the bands. (Oleksiv, 2008)

If it is assumed that stock returns are normally distributed with a mean of *m*, a plus-minus 2 standard deviation area from the mean would then contain 95 % of the daily stock movements and the remaining 5 % could be expected to be extreme events or outliers. This would back the argument for the standard deviation parameter to be 2 but as Fama (1976, p.21) and Andersen et. al. (2001) have shown, stock returns are fat tailed and right skewed, not normally distributed. In other words, extreme events are more probable to happen than what the normal probability distribution suggests. Figure 5 shows the normal distribution and a distribution of daily S&P 500 index returns over the period of 2000-2017. As one can see the stock price returns are slightly fat tailed, right skewed, leptokurtic and do not follow a perfect normal distribution.

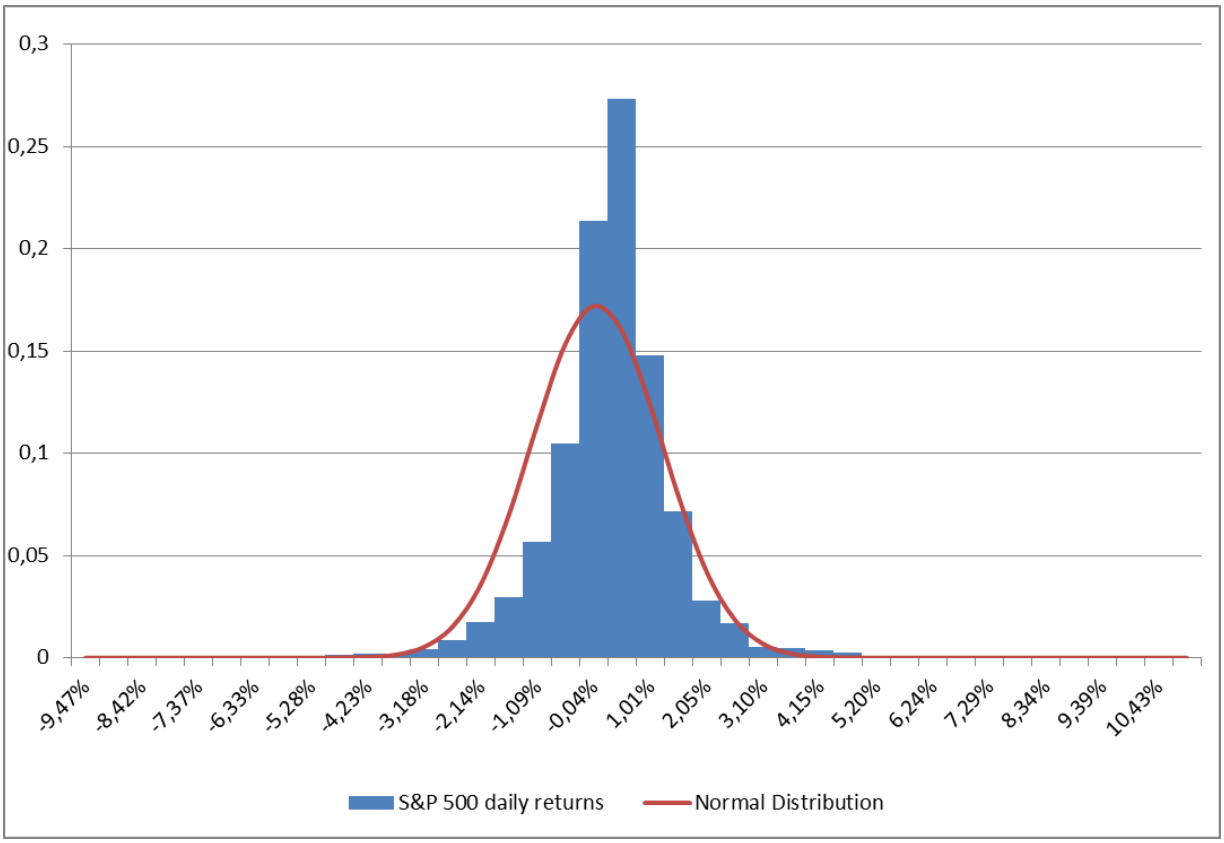

Figure 5: S&P 500 logarithmic daily return distribution and normal distribution from time period 1.1.2000 – 31.12.2017

Stock prices' right skewness implies that the Bollinger bands upper band should be slightly further away from the moving average than the lower band. However, practical tests have shown that even though the traditional Bollinger bands are plotted symmetrically around the mean, they still manage to capture around 90 percent of the stock price movements which is quite close to the suggested 95 percent target level. Liu et al. (2006) study results show even better results with capturing a minimum of 94% of the price action within the bands with standard Bollinger band parameters.

4. Literature review Bollinger bands

Leung and Chong (2003) study focused on the profitability possibilities of trading strategies based on Bollinger bands and moving average envelopes with a few different moving average lengths. Parameters chosen for the trading rules were 10 days, 20 days, 50 days and 250 days, 3% and 5% for the moving average envelopes and 2 standard deviations for Bollinger bands. The study was conducted using eleven major stock indices around the world and a fifteen year time period covering January 1985 to December 2000. Leung's and Chong's results show that moving average envelopes tend to work better than Bollinger bands on shorter moving average lengths but when it comes to longer moving average lengths, the Bollinger bands tend to outperform the envelopes slightly. Overall, most of the trading strategies were profitable but there was no comparison between the profitability and a market return so it is hard to tell whether the suggested trading strategies would have outperformed a simple buy and hold strategy. The results also show that there is a steady decline on a number of generated trading signals with the increasing moving average length and envelope width. Bollinger bands with a moving average length of 10 generated around 40 to 60 signals annually but when the length was increased to 250 the number of signals dropped to only a few per year. Interestingly, the profitabilities of different length Bollinger bands were quite close to each other and no distinctive trend between the parameter values and profitability could be found, although a 50 day length bands performed slightly worse than the rest. The conclusion of the study suggests a trading strategy based on Bollinger bands for longer time periods and moving average envelopes for a more short term investing tool.

A comprehensive study of technical trading rules presented by Balsara et al. (2009) focused on the differences between the regular and contrarian versions of the moving average crossover rule, channel breakout rule and Bollinger band breakout rule.

Regular breakout rules assumed that the stock price will continue to move on the current direction, i.e. when the stock price breached the upper (lower) band a buy (sell) signal was generated. Contrarian rules assumed the opposite and generated a reveres signal compared to the regular rules. It is to be noted that the “contrarian rules” on Balsara et al. (2009) study is what most of the other academic research, including this thesis, consider as “regular rules”. The trading rule analysis was done with around 3500 different stocks that were included in S&P 500, NASDAQ Composite and DJIA indices and covered a time period from 1990 to 2007. Results of the study suggest strong evidence supporting the contrarian version trading rules for all of the three different methods. Even at 1% confidence level, the majority of the contrarian strategies managed to generate excess returns compared to buy and hold strategy. Bollinger band moving average lengths used on the study were 20, 40, 60, 120, 150 and 180 and the results show clear diminishing trend on the returns when increasing the length. Strikingly, the study suggests that Bollinger band trading strategy works best on strong bear markets even in absolute terms and not just relatively, however, there is no mention whether short selling was allowed or not which makes interpreting the results slightly harder. Overall the results of the study show strong support for mean reversion and support and resistance levels since the contrarian rules tended to work better than the normal ones.

A rather new study by Coakley et al. (2016) is closely related to this thesis’ subject but was conducted in foreign exchange (FX) market rather than stock market and tested the profitability of several different technical trading rules of which one being Bollinger bands. The study used data from 22 different currencies quoted in US dollars from a time period of 1996 to 2015. Coakley et al. (2016) argue that more traditional technical analysis methods such as moving average rule and channel breakout rule do not generate statistically significant excess returns after data snooping bias is taken into account. However, more modern trading rules such as Bollinger bands and moving average convergence/divergence rules are robustly profitable according to the study. Different Bollinger band parameters were also

studied although the results do not give a clear picture of how the profitability fluctuated with different parameter combinations. That being said, the optimized, best performing, parameters for all but one currency were 5 and 1 for moving average and standard deviation respectively which differs greatly from the standard 20 and 2. It is understood that FX market behaves differently than stock market and that volatility on stock market is normally greater than on FX market (see Busch et al. (2011) & Andersen et al. (2007)) but still one could argue that the study provides some support for the idea that Bollinger band parameters can be optimized to function better and yield larger returns than the standard ones.

Kabasinskas and Macys' (2010) Bollinger band optimization study attempts to find the optimized parameters in the Baltic stock market for short term investing. The data used in the study is unfortunately very limited containing only 2 stocks being Baltika AS and Klaipedos Nafta AB, and only three different parameter combinations for Bollinger bands were considered. For both stocks, parameter combination of N=10 and K=1.8 generated the largest returns, beating the two other combinations of N=20, K=2 and N=5, K=1.6. Parameters of 20 and 2 worked well for Baltika stock, which during the time period the study covers was much more volatile from the two, but poorly for Klaipedos Nafta. On the other hand, parameters of 5 and 1.6 worked relatively well for Klaipedos Nafta but more poorly for Baltika. Although the data sample is limited, the results of the study suggest that larger parameter values seem to work better for stocks that are more volatile.

On the contrary, there are plenty of studies that show no support for achieving excess returns with Bollinger bands trading strategy. Lento et al. (2007) study argues that traditional Bollinger band trading does not outperform a simple buy and hold strategy even when transaction costs are not taken into account. Fang et al. (2014) research conducts that Bollinger bands have lost their capability of providing predictive power and excess returns in more recent years. Results of the study suggest that the trading

strategy was working rather well until 1983 but has lost its effectiveness immediately after that year. A very recent study conducted by Chen et al. (2018) points out the importance of transaction costs by showing that even when Bollinger band trading rule manages to achieve excess returns per se, the transaction costs from a high number of trades will wipe out the excess returns entirely.

5. Data and methodology

Stock market data for this thesis was gathered from InFront, Reuters and Yahoo Finance. The data consists of daily stock quotes for 60 different US stocks from a time period of 1.1.2006 – 31.12.2016 that are adjusted for both stock splits and dividends. At the time when the data was gathered, the year 2016 was the most recent full year and the 10 year time period was long enough to have a longer and shorter learning period as well as few years of out of sample testing set. Although for example 20 years of data would have been more desired than 10 years, it turned out that the Matlab simulation model was so computationally heavy that it would have been taken multiple hours to run the model with a standard computer, and thus 10 years was decided to be sufficient. The time period was divided into three different sub periods where the first two were used for learning purposes and optimizing the trading algorithm and the third for testing how the optimized trading algorithm works in practice out of sample. The first time period covers the time between 1.1.2006 and 31.12.2013 and the second time period covers the time between 1.1.2011 and 31.12.2013 and the third one between 1.1.2014 and 31.12.2016. For convenience's sake, we are going to name these periods to 1st and 2nd learning period and hold-out period respectively. The two learning sets overlap in order to have the most recent data before the out of sample testing included in the model. S&P 500 index development and the determined time periods are presented in figure 6. One can see that the market has behaved very differently on the two learning periods. The first learning period includes the 2008 financial crisis, during which the S&P 500 index fell more than 50% in value. After the year 2010, there have been few smaller selloffs where the index has dropped quite significantly, mainly in 2011-2012 and 2015-2016, but the index has recovered rather quickly after the declines. The distinctive natures of the two learning periods should yield interesting results on how the Bollinger band parameters perform on different types of market conditions. Overall the data consists of 166,324 stock quotes and all the testing and data handling was done with Matlab.

Figure 6: S&P 500 index development over period 03.01.2006-30.12.2016 and the determined learning and hold-out periods used in the study.

In order to test the trading algorithm performance in different sectors, the stocks were divided into six different sectors, or portfolios, which are finance, Information technology, healthcare, Industrial, consumer goods and services and energy. The grouping to different sectors was made with the help of the Global Industry Classification Standard (GICS). The different portfolios and their significance will be discussed more in the results chapter.

Annualized returns for the stocks and portfolios per the three different sub periods are presented in table 1 below. More detailed descriptive statistics are moved to Appendix 1 in order to save space. In the 1st learning period, most of the portfolios manage to receive around 10 – 15% annualized return with just a simple buy and hold strategy where at the start all the stocks have an equal weight and no balancing is done during the time period. However, the outlier here is the finance portfolio that has an annualized return of just 0.46% over the whole 7 year time period. Few companies here stand out with very high annualized returns, which are Amazon.com Inc., 35.49%, Apple Inc., 34.01% and Cabot Oil & Gas, 32.12%. On the other side, companies with very poor annualized returns are Citigroup Inc., -25.95%, Bank of America Corporation, -12.29% and Electronic Arts Inc., -11.36%. It will be interesting to see how the trading strategies with Bollinger bands work with these companies specifically even though the above returns are only from the learning period. The annualized returns on the 2nd learning period are on a significantly higher level as expected. All of the portfolios yield returns of over 12% per annum with Healthcare ranking the highest at 25.07%. Best performers on the period are Cabot Oil & Gas, Abbots Laboratories and Moody's Corp. The annualized returns for the hold-out period are much more scattered ranging from around -4% to 25% on portfolio level. Information technology portfolio seems to perform the best at the hold-out period with a simple buy and hold strategy with an annualized return of 25.42% and Energy portfolio being the worst performer with an annualized return of -3.90%. Stand out companies at the hold-out period are NVIDIA Corporation, 91.57%, Electronic Arts Inc., 51.10%, Coca-Cola, 37.15% and on the poor performance side Southwestern Energy, -34.76%. Here again, it will be interesting to see how the Bollinger band trading strategies work with these companies especially. One could argue that it will be difficult to outperform a buy and hold strategy that yields an annualized return of around 20-30%. Then again the stocks that yield a negative return when buy and hold strategy is applied could perform better with a trading strategy based on Bollinger bands.

Table 1: Annualized returns for stocks and portfolios over the three sub periods

	Company	1st learning 1.1.2006-31.12.2013	2nd learning 1.1.2011-31.12.2013	Hold-out period 1.1.2014-31.12.2016
Finance	Citigroup Inc.	-25.95%	2.17%	4.82%
	JPMorgan Chase & Co.	8.50%	13.19%	16.94%
	Bank of America Corporation	-12.29%	3.57%	12.42%
	The Goldman Sachs Group, Inc.	5.96%	2.19%	12.16%
	Morgan Stanley	-4.49%	4.46%	12.71%
	American Express Company	10.16%	29.67%	-4.71%
	Moody's Corp	4.73%	45.15%	8.30%
	NASDAQ OMX Group	1.81%	19.48%	21.62%
	U.S. Bancorp	7.76%	17.08%	11.39%
	Wells Fargo	8.42%	15.62%	10.08%
	Average	**0.46%**	**15.26%**	**10.57%**
Information Tech	Oracle Corporation	17.93%	7.58%	1.94%
	QUALCOMM Incorporated	9.68%	15.89%	-0.99%
	Accenture plc	18.47%	21.79%	15.55%
	Intel Corporation	3.69%	11.49%	15.60%
	NVIDIA Corporation	3.74%	1.36%	91.57%
	Activision Blizzard, Inc.	15.28%	14.13%	27.12%
	Electronic Arts Inc.	-11.36%	11.88%	51.10%
	Cisco Systems, Inc.	4.54%	5.14%	14.47%
	Apple Inc.	34.01%	20.75%	15.87%
	Microsoft	7.50%	13.30%	21.93%
	Avergage	**10.35%**	**12.33%**	**25.42%**
Healthcare	Johnson & Johnson	9.61%	17.30%	11.26%
	Pfizer Inc.	9.05%	24.68%	5.81%
	Abbott Laboratories	26.82%	55.85%	2.46%
	Baxter International Inc.	11.47%	14.16%	7.48%
	Lilly (Eli) & Co.	3.39%	18.63%	16.44%
	Aetna Inc	17.33%	32.10%	23.69%
	Amgen Inc	5.85%	29.14%	10.46%
	Boston Scientific	-9.65%	16.51%	22.01%
	Merck & Co.	11.41%	16.26%	9.36%
	Mylan N.V.	11.98%	26.07%	-3.60%
	Avergage	**9.73%**	**25.07%**	**10.54%**
Industrial	The Boeing Company	13.00%	30.08%	7.31%
	United Technologies Corporation	13.36%	15.67%	1.74%
	Lockheed Martin Corporation	16.95%	34.34%	23.24%
	3M	11.80%	20.30%	11.74%
	Eaton Corporation plc	16.06%	16.99%	-0.62%
	Caterpillar Inc.	9.73%	0.98%	4.60%
	Deere & Co.	17.88%	5.30%	7.44%
	General Electric	0.82%	19.20%	8.25%
	Rockwell Collins	8.81%	10.21%	9.91%
	Dover Corp.	15.54%	20.13%	0.42%
	Avergage	**12.39%**	**17.32%**	**7.40%**
Consumer	Ford Motor Co.	11.47%	-2.07%	-3.36%
	Coca-Cola	9.84%	10.47%	37.15%
	Pepsi	8.08%	11.34%	11.57%
	Amazon.com, Inc.	35.49%	29.36%	23.51%
	Altria Group Inc.	19.53%	22.61%	26.42%
	Macy's, Inc.	8.81%	30.38%	-9.92%
	Costco Wholesale Corporation	15.83%	22.09%	13.24%
	The Procter & Gamble Company	8.15%	11.42%	4.74%
	Kellogs	8.38%	9.67%	9.72%
	McDonald's Corp.	20.43%	11.63%	11.67%
	Avergage	**14.60%**	**15.69%**	**12.47%**
Energy	Total SA	5.52%	9.45%	0.26%
	Exxon Mobil Corporation	10.99%	13.57%	-0.10%
	Chevron Corporation	15.53%	14.39%	2.33%
	Valero Energy	2.78%	27.50%	14.04%
	ConocoPhillips	10.61%	15.51%	-7.01%
	Williams Cos.	16.81%	37.56%	-0.78%
	Southwestern Energy	11.13%	1.09%	-34.76%
	Cabot Oil & Gas	32.12%	56.27%	-15.28%
	Schlumberger Ltd.	9.88%	4.01%	0.34%
	Helmerich & Payne	27.41%	20.84%	1.93%
	Avergage	**14.28%**	**20.02%**	**-3.90%**

This thesis focuses on the research around the Bollinger band parameters and how they could be calibrated to achieve higher profits compared to the commonly used parameters of moving average of twenty and a standard deviation multiplied by a factor of two. The idea behind Bollinger band is relatively simple and it is based on the *N* period moving average, also called as the middle band, of the time series. Simple *N* period moving average at time *t* is defined as:

$$SMA_{N,t} = \frac{1}{N}\sum_{i=0}^{N-1} p_{t-i}$$

Where p_t is the stock price at time *t*

Upper and lower bands are calculated based on the middle band and on the volatility of the time series. Time series volatility at time *t* is measured as an *N* period standard deviation:

$$\sigma_t = \sqrt{\frac{\sum_{i=0}^{N-1}(p_{t-i} - \bar{p})^2}{N}}$$

where:

$$\bar{p} = \frac{1}{N}\sum_{i=0}^{N-1} p_{t-i}$$

The lower band is obtained by subtracting the standard deviation multiplied by a factor of *K* from the middle band. The upper band is obtained similarly by adding the standard deviation to the middle band.

$$BB_{lower,t} = SMA_{N,t} - K\sigma_t$$

$$BB_{upper,t} = SMA_{N,t} + K\sigma_t$$

Figure 7: Bollinger Bands (20,2) on top of Microsoft stock price chart 31.12.2013 – 31.12.2015

Figure 7 shows the completed Bollinger bands built over Microsoft stock price chart. The parameters in this example are the commonly used moving average of 20 and a standard deviation multiplied by 2. The orange line represents the stock price, red line is the middle band and the two blue bands are the upper and lower band. Volatility increases and decreases can be easily seen from the chart by looking at the width of the bands. As the volatility increases the bands diverge from the middle band and when the volatility is lower the bands converge towards the middle band. The more the bands diverge, the more difficult it is for the price to breakout from the bands and generate a buy or sell signal for the trader. When the stock price is trading above the upper band, the stock is considered as overbought and when the price is trading below the lower band the stock is considered oversold. The buy and sell signals can be defined as follows:

$$BUY_t = (p_{t-1} < BB_{lower,t-1}) \ \& \ (p_t > BB_{lower,t})$$

$$SELL_t = (p_{t-1} > BB_{upper,t-1}) \ \& \ (p_t < BB_{upper,t})$$

Buy signal is generated when the stock price breaks the lower band from below and a sell signal is generated when the price penetrates the upper band from above. In the context of this thesis a buy signal means going long on the stock and sell signal means selling the stock short. By going short on the stock the trader will profit even when the prices are going downwards during a bear market.

The parameters used for the calculations of the Bollinger bands are definitely not trivial as can be interpreted from the equations and chart above. The moving average should be short enough to catch the price movement and generate a buy or sell signal as early as possible but long enough so that small price movements do not generate superfluous and unneeded signals. The multiplication factor for the standard deviation will also have a large effect on how much price movement is needed to generate a buy or sell signal. Traders with a long-term view may want to increase the length of the moving average and increase the width of the bands. On the other hand active traders, looking to profit from short-term price swings, may want to choose shorter moving average and narrower bands. One thing that should be kept in mind is that the measure of volatility is also depended on the length of the simple moving average. When the length of the moving average increases it is highly likely that the standard deviation will increase as well and vice versa. This is demonstrated in figure 8, where the same Microsoft stock chart has been plotted with two sets of Bollinger bands with different lengths of moving average keeping the standard deviation multiplier two. The orange line represents the stock price, red and green lines represent the simple moving averages of 10 and 50 respectively, when blue and purple lines are the corresponding upper and lower bands. The Bollinger band with a

shorter moving average length clearly reacts much more quickly to changes in the underlying asset price. Also, the bands with a shorter moving average length are generally much closer to the stock price curve, except in the events where there is a big price swing and a sudden increase in volatility.

Figure 8: Bollinger Bands (50,2) and (10,2) on top of Microsoft stock price chart 31.12.2013 – 31.12.2015. Orange line represents the stock price. Red and blue lines are the middle, upper and lower bands respectively for the Bollinger band (20, 2). Green and purple lines are the middle, upper and lower bands for the Bollinger band (50, 2).

To test how the parameters change the performance of Bollinger bands, a trading simulation model was built in Matlab. The simulation model is essentially a multilayer FOR loop that tests every possible combination of parameters *N* and *K* within a given range for a prescribed group of stocks. It was determined that the parameter *K*, the multiplier for standard deviation, was sufficient to be tested in a range of 1.0 to 3.0 in .1 increments. Considering the hold-out period's relative shortness of three years, the

moving average length was deliberately kept relatively short as well and a range of 5 to 50 days was chosen in 1-day increments. It is to be noted that Bollinger himself suggests in his 2002 book a parameter combinations of (10, 1.9), (20, 2.0) and (50, 2.1), for *N* and *K* respectively. Altogether the trading model covers 966 different parameter combinations for each stock. Each Bollinger band combination was then fitted over the given stock price chart and buy and sell signals were generated in a manner stated earlier above. A profit (or loss) was then simulated for each band using the generated buy and sell signals and mapped in a table as a return on investment.

The returns and profits in the trading model are calculated in the following principle. When the first buy signal is generated the model takes a long position on the stock at a market price for that day. For simplicity sake, the daily market price for a given stock is assumed to be the closing price for the day. Long position is closed at a market price when a sell signal is generated and at the same time, an equal size short position is taken on the stock. Positions will thus change from long to short to long and so on. Any open positions will be closed on the last day of the simulation period. Returns presented in this thesis are annualized over time period t as follows:

$$Annualized\ return = (1 + cumulative\ return)^{(365/t)} - 1$$

6. Results

For the purpose of studying the Bollinger band parameters for different portfolios, the 60 stocks were divided into six different portfolios that represent common industry sectors. These sectors were chosen to be finance, Information technology, healthcare, Industrial, consumer goods and services and energy and the stock allocation was made with the help of the Global Industry Classification Standard (GICS). Studies such as Gopikrishnan et al. (2000), Doherty et al. (2003) and Arouri et al. (2011) have suggested that stock returns and/or volatility actually correlate between stocks when the stocks have been divided into different sectors or clusters. Given this information, there might be a possibility to assign optimal Bollinger band parameters based on the industry sectors. Heatmaps on Figure 9 present how the total portfolio profit changes with different parameter combinations, in other words, how the portfolio return changes if all of the stock in a given portfolio would use the same parameter combination for *N* and *K*. Starting with the consumer portfolio, the heatmap does not really suggest that any parameter combination would work better than others. Slightly higher profits for the portfolio can be achieved using parameters of (1, 1.6) for *N* and *K* respectively but the heatmap does not really give very strong evidence to support this. Consumer portfolio had also the lowest annual return standard deviation which should, in theory, equal lower parameter values. Heatmap for energy portfolio suggests that better profits can be achieved using parameters around 8 and 1.6. The bigger yellow spot around that area means that multiple different parameter combinations have been working in the past. Optimal parameters for finance portfolio are significantly greater than for other portfolios being around 30 and 2 for *N* and *K* respectively. Given that the finance portfolio had the largest return standard deviation on the 2nd learning period, the larger parameter values make sense in order to avoid superficial signals. Healthcare portfolio heatmap suggests that there are two different parameter combination areas that have yielded relatively higher returns in the past. The parameter combinations that have performed the best

are around 18 and 1.1 as well as 16 and 1.7 for *N* and *K* respectively. Industrial portfolio's best performing parameters were around 18 and 1.2, but the heatmap shows that larger parameter combinations have performed rather adequately as well. In a similar manner, the best-performing parameters for information technology are populated in two different areas, being around 6 and 1.0 as well as 12 and 2.3. All of the heatmaps, except the one for consumer portfolio, suggest that there are indeed some parameter combinations for each given portfolio that perform substantially better than most. Majority of the yellow, high return areas, are situated around the bottom left corner meaning that lower parameter values seem to have worked better in the learning period rather than larger ones. Additionally, one can see that long moving average length combined with a narrow band width, as well as, short moving average length combined with wide band width perform poorly. This is well in line with the Bollinger band theory, which suggests that in order to capture only the extreme events, around 95 % of the stock price would need to stay within the bands. If a long moving average length is combined with narrow bands, the stock price can easily penetrate the bands since the moving average will not react fast enough to price fluctuations. Vice versa, if a short moving average length is combined with wide bands, the stock price will never penetrate outside the bands since the moving average reacts fast to price fluctuations a the width of the bands is so wide.

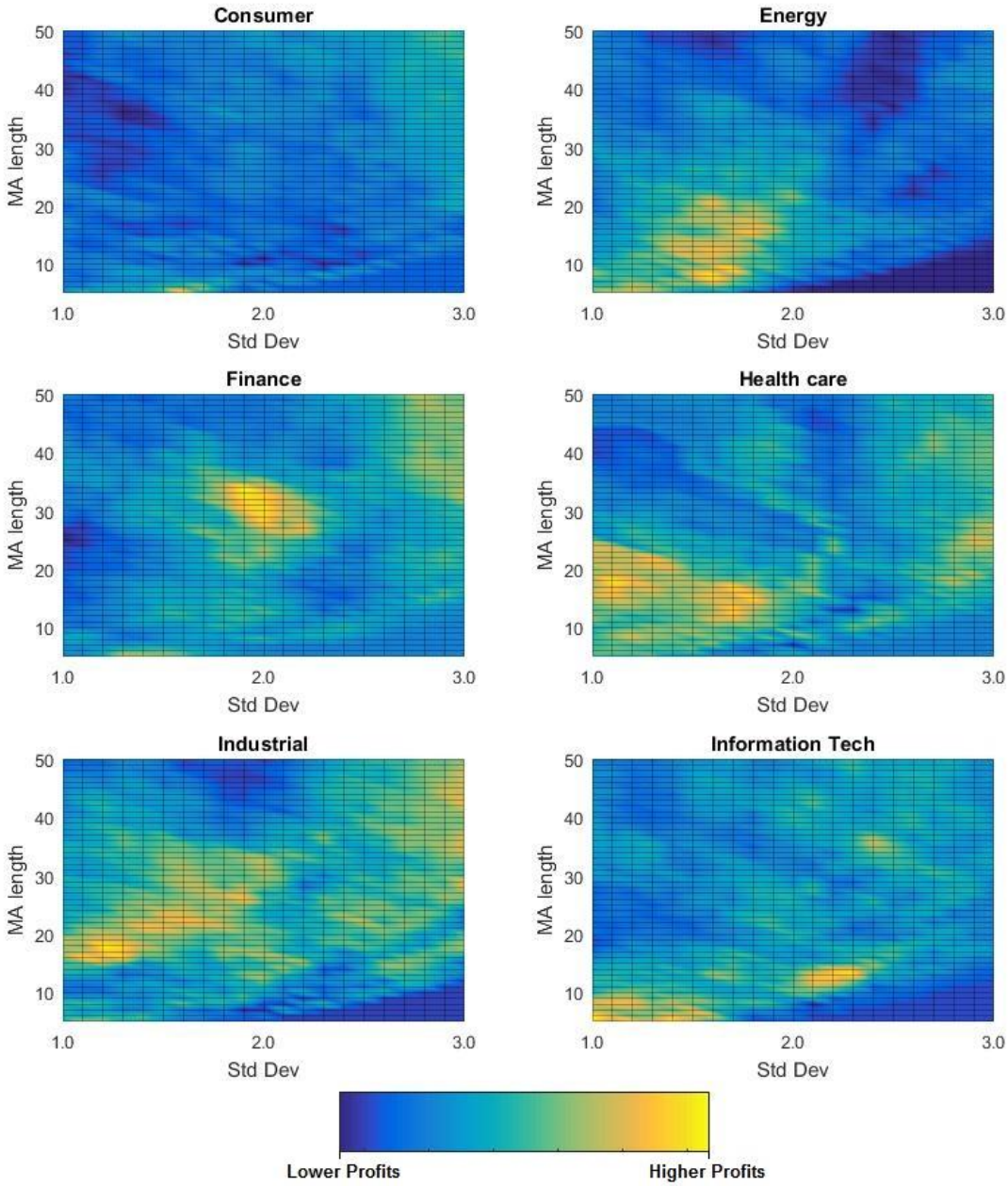

Figure 9: Heatmaps for returns over each portfolio for 2nd learning period. Bright yellow color means higher returns and darker blue areas represent the lower returns. Returns presented on the heatmaps are not comparative to each other but only within the given portfolio. The optimized values for portfolios are: Consumer (5, 1.6), Energy (8, 1.6), Finance (34, 1.9), Healthcare (18, 1.1), Industrial (18, 1.2) and Information Technology (6, 1.0).

Heatmaps for portfolio returns on the 1st learning period are presented in figure 10. Comparing the two different heatmap sets from the different learning periods, one can see that there are quite a lot of differences. Where the heatmaps from the 2nd learning period showed bigger yellow areas for high returns, the heatmaps from the 1st learning period show much more precise areas for the highest returns. This suggests that the parameter values had a much bigger effect on the profitability in the 1st learning period than on the 2nd one. Some similarities between the figures 9 and 10 can be found as well. Just like on the 2nd learning period, very short moving average lengths combined with very wide bands do not seem to perform well. Similarly combining a very long moving average length with narrow bands is not recommended. The different nature of the two periods seems to have a large effect on how precisely the Bollinger band parameters need to be chosen. Looking at the tables on appendix 1, the volatility on the first period was on a much higher level than on the second one. Also the first period covers the 2008 financial crisis whereas the second period had only relatively minor market declines. In essence, based on the figures 9 and 10 alone, it seems that high market volatility requires more exact parameter determination than low volatility conditions. Also the length of the investment horizon does seem to have an effect on how precisely the parameters need to be determined.

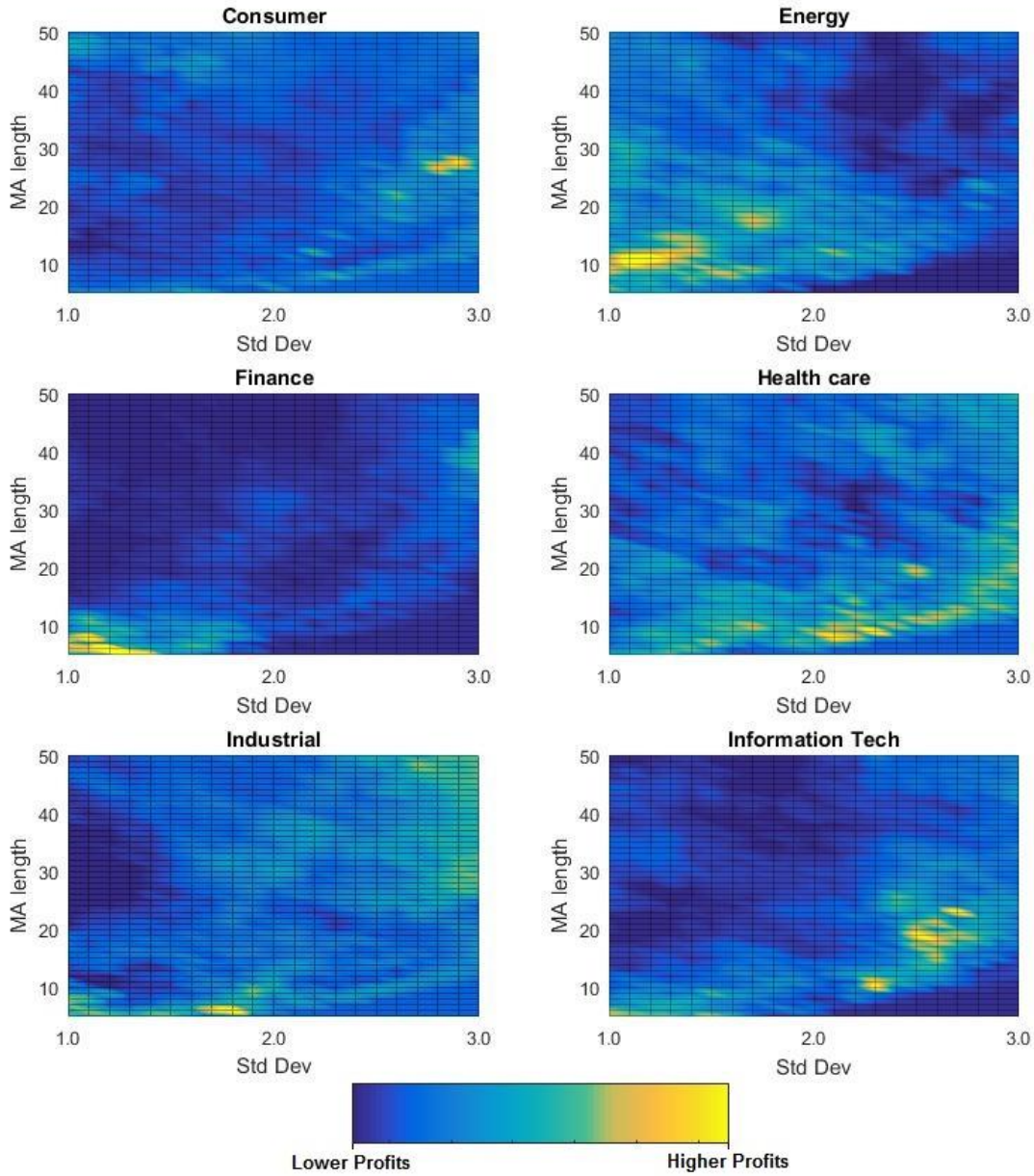

Figure 10: Heatmaps for returns over each portfolio for 1st learning period. Bright yellow color means higher returns and darker blue areas represent the lower returns. Returns presented on the heatmaps are not comparative to each other but only within the given portfolio. The optimized values for portfolios are: Consumer (27, 2.9), Energy (11, 1.2), Finance (6, 1.2), Healthcare (9, 2.3), Industrial (6, 1.8) and Information Technology (23, 2.7).

Table 2 presents the results when Bollinger band parameters were optimized using the longer seven-year 1st learning period. Starting with the total results over all stocks, Bollinger band parameters optimized for portfolios yielded the best results with an annual return of 4.854%, being 0.559 percentage points greater than the result using parameters 20 and 2. Parameters optimized individually for each stock performed just slightly better than the 20 and 2 parameters and 0.532 percentage points worse than the portfolio parameters. Overall, none of the Bollinger band trading strategies was able to beat a buy and hold strategy which yielded an annual return of 10.417% on average for all stocks. The results on individual stock and portfolio level reveal the large fluctuations on the returns between each strategy and parameter combinations. Looking at the finance portfolio, parameters of 20 and 2 and optimization on portfolio level (6, 1.2) were able to outperform the buy and hold strategy. Returns for parameters optimized on individual stock level are almost exactly the same as for buy and hold strategy on average. Interestingly, the moving average length for the optimized parameters is very short both on individual stock and portfolio level. Likewise, the band width is significantly narrower than for all stocks on the finance portfolio except for Moody's Corp for which the optimized band width parameter is 2.6. Returns for the technology portfolio have similar nature as for finance. Again, even though optimized parameters performed better on the learning period than the standard ones, they did not manage to outperform the parameters of 20 and 2 on the hold-out period. However, the parameters optimized on portfolio level (23, 2.7) yielded almost exactly the same return as the parameters of 20 and 2. Annual buy and hold return for NVIDIA Corporation was around 91%, the return for the optimized parameters (23, 2.7) was around 22% and around -1% for the standard parameters. The results for NVIDIA Corporation show that if the stock price is gaining sharply, Bollinger bands will not work nearly as well as a buy and hold strategy. Electronic Arts stock behaved similarly, large buy and hold profit outperformed Bollinger bands greatly. Parameters optimized on portfolio level (9, 2.3) yielded the best return for Bollinger bands in healthcare portfolio and the standard parameters of 20 and 2 yielded the smallest return. However, the bands were not able to outperform buy and hold strategy. Interestingly the optimized parameters on portfolio level were 9

and 2.3 for *N* and *K* respectively, which are relatively short in terms of moving average length and wide in term of band width. As seen on the heatmaps earlier, short moving average length combined with wide bands usually does not yield the best results but the healthcare portfolio seems to be an exception even though the when comparing the 1st learning period to the hold-out period, volatility actually decreased on the latter period. Individually optimized parameters performed the best for Industrial portfolio, although again underperforming buy and hold strategy. Within the portfolio, there is a lot of variation on the optimized parameter values. Optimized moving average lengths were over 30 for 4 out of the 10 stocks and under 7 for 4 as well. Rather surprisingly the parameters optimized on portfolio level performed almost as well as the individually optimized parameters even though there was so much variation on the parameter values on the individual level. Parameter optimization for the consumer portfolio was not able to outperform the standard parameters in the hold-out period. Parameter optimization for Amazon.com Inc., Altria Group Inc. and McDonalds Corporation was not successful as the Bollinger bands with the optimized parameters did not generate any buy and sell signals on the hold-out period. As no positions were opened for these three stocks, the return is simply zero. Looking at the optimized parameter values for the three stocks, one can see that all of the combinations have a relatively short moving average length and relatively wide band width. The last portfolio is the energy portfolio where the individually optimized parameters performed the best, outperforming even the buy and hold strategy. Return volatility on the hold-out period was the greatest on the Energy portfolio by a large margin. There are also large variations on the individual stock level within the portfolio. Bollinger bands with parameters 20 and 2 returned almost 43% annually when applied to Total SA stock but when the strategy was applied to Williams Cos. stock the return was negative 30% annually. For both stocks the buy and hold return was around zero percent but the return volatility for Williams Cos. (58.42%) was more than double of Total SA (26.14%) on the hold-out period.

Table 2: Trading simulation results when parameters optimized with 1st learning period. BB (20,2) represents the results using Bollinger bands (BB) with standard parameters of 20 and 2. Column BB Opt shows the results when the parameters are optimized on individual stock level and the respective parameter values are shown in column Opt (N,K). Column BB Portf shows the results using optimized parameters for the respective portfolio and the parameter values for each portfolio are shown on the average row. Column B&H is the result for buy and hold strategy. Difference columns represent the difference between the two methods in percentage points.

	BB (20,2)	BB Opt	BB Portf	B&H	Diff BB Opt BB(20,2)	Diff BB Portf BB(20,2)	Diff BB Opt B&H	Opt (N,K)
Citigroup Inc.	6.141%	1.397%	20.117%	4.825%	-4.744	13.975	-3.427	(6 1.7)
JPMorgan Chase & Co.	8.645%	17.771%	19.426%	16.944%	9.126	10.781	0.826	(6 1.6)
Bank of America Corp	14.235%	10.385%	10.385%	12.424%	-3.851	-3.851	-2.039	(6 1.2)
The Goldman Sachs Group	23.436%	6.283%	14.849%	12.160%	-17.154	-8.588	-5.877	(7 2)
Morgan Stanley	23.872%	-3.989%	9.910%	12.712%	-27.861	-13.962	-16.700	(5 1.3)
American Express Company	0.587%	20.795%	11.963%	-4.714%	20.208	11.376	25.508	(6 1.5)
Moody's Corp	16.601%	11.558%	4.176%	8.303%	-5.043	-12.425	3.254	(12 2.6)
NASDAQ OMX Group	8.028%	12.765%	1.248%	21.624%	4.737	-6.780	-8.859	(10 1.2)
U.S. Bancorp	22.171%	22.196%	9.752%	11.385%	0.025	-12.419	10.811	(7 1.6)
Wells Fargo	18.797%	5.495%	9.044%	10.081%	-13.302	-9.753	-4.586	(6 1.7)
Average Finance	**14.251%**	**10.465%**	**11.087%**	**10.574%**	**-3.786**	**-3.164**	**-0.109**	**(6 1.2)**
Oracle Corporation	10.489%	5.200%	1.425%	1.936%	-5.288	-9.064	3.265	(26 2.4)
QUALCOMM Incorporated	8.587%	-9.049%	-9.156%	-0.995%	-17.636	-17.743	-8.054	(37 2.8)
Accenture plc	30.969%	-1.027%	16.176%	15.548%	-31.997	-14.794	-16.575	(6 1)
Intel Corporation	-4.939%	3.745%	11.438%	15.600%	8.684	16.376	-11.855	(7 1.2)
NVIDIA Corporation	-1.456%	21.962%	21.962%	91.569%	23.419	23.419	-69.607	(23 2.7)
Activision Blizzard. Inc.	2.673%	9.964%	18.629%	27.122%	7.290	15.955	-17.158	(12 1.1)
Electronic Arts Inc.	17.912%	-16.615%	9.970%	51.101%	-34.527	-7.942	-67.716	(5 1.5)
Cisco Systems. Inc.	28.136%	4.637%	19.679%	14.471%	-23.499	-8.457	-9.833	(7 1.4)
Apple Inc.	13.050%	12.366%	12.366%	15.874%	-0.684	-0.684	-3.508	(25 2.8)
Microsoft	13.135%	25.656%	13.976%	21.932%	12.520	0.841	3.724	(5 1.1)
Average Technology	**11.856%**	**5.684%**	**11.646%**	**25.416%**	**-6.172**	**-0.209**	**-19.732**	**(23 2.7)**
Johnson & Johnson	-2.202%	-7.281%	5.503%	11.259%	-5.079	7.705	-18.541	(11 2.3)
Pfizer Inc.	10.933%	20.614%	-3.520%	5.813%	9.681	-14.452	14.801	(6 1.4)
Abbott Laboratories	-7.977%	-10.639%	2.239%	2.460%	-2.661	10.216	-13.099	(9 1.2)
Baxter International Inc.	6.382%	-3.252%	20.895%	7.478%	-9.635	14.513	-10.730	(5 1.1)
Lilly (Eli) & Co.	9.351%	13.251%	15.191%	16.439%	3.900	5.840	-3.188	(9 1.9)
Aetna Inc	0.401%	9.448%	9.448%	23.689%	9.047	9.047	-14.242	(9 2.3)
Amgen Inc	15.986%	4.077%	-0.938%	10.459%	-11.909	-16.924	-6.382	(27 3)
Boston Scientific	-7.844%	-9.867%	1.253%	22.006%	-2.022	9.098	-31.873	(20 2.5)
Merck & Co.	8.090%	16.407%	12.371%	9.364%	8.317	4.281	7.043	(10 1.7)
Mylan N.V.	-19.404%	-8.965%	-26.556%	-3.596%	10.438	-7.152	-5.370	(43 2.3)
Average Healthcare	**1.371%**	**2.379%**	**3.589%**	**10.537%**	**1.008**	**2.217**	**-8.158**	**(9 2.3)**
The Boeing Company	8.656%	4.581%	5.074%	7.315%	-4.075	-3.583	-2.733	(27 2.7)
United Technologies Corp	-1.668%	3.737%	-2.473%	1.738%	5.406	-0.805	1.999	(30 2)
Lockheed Martin Corp	-1.786%	-1.247%	7.824%	23.242%	0.539	9.610	-24.489	(10 2.5)
3M	-11.234%	10.581%	-7.660%	11.744%	21.816	3.574	-1.163	(32 2.9)
Eaton Corporation plc	8.069%	1.336%	-1.738%	-0.616%	-6.733	-9.807	1.952	(48 2.7)
Caterpillar Inc.	5.390%	-4.346%	19.662%	4.603%	-9.736	14.272	-8.949	(7 1.7)
Deere & Co.	0.909%	13.052%	13.052%	7.436%	12.143	12.143	5.616	(6 1.8)
General Electric	2.792%	12.670%	9.596%	8.252%	9.877	6.804	4.418	(6 1.7)
Rockwell Collins	5.726%	4.918%	4.408%	9.910%	-0.809	-1.318	-4.993	(34 2.1)
Dover Corp.	-1.056%	10.561%	-0.796%	0.424%	11.617	0.260	10.137	(5 1.2)
Average Industrial	**1.580%**	**5.584%**	**4.695%**	**7.405%**	**4.005**	**3.115**	**-1.821**	**(6 1.8)**
Ford Motor Co.	10.544%	-0.391%	-0.391%	-3.357%	-10.935	-10.935	2.966	(27 2.9)
Coca-Cola	-9.925%	-22.305%	-6.240%	37.145%	-12.381	3.685	-59.450	(45 1.7)
Pepsi	27.666%	10.546%	-0.892%	11.566%	-17.120	-28.558	-1.020	(22 1.3)
Amazon.com. Inc.	1.478%	0.000%	-12.494%	23.513%	-1.478	-13.972	-23.513	(11 2.9)
Altria Group Inc.	-6.813%	0.000%	8.661%	26.422%	6.813	15.474	-26.422	(6 2)
Macy's. Inc.	0.623%	-5.067%	-4.448%	-9.918%	-5.689	-5.071	4.851	(12 2.2)
Costco Wholesale Corp	-0.668%	7.691%	5.498%	13.239%	8.359	6.167	-5.548	(21 2.4)
The Procter & Gamble Comp	4.050%	-6.003%	1.309%	4.739%	-10.054	-2.742	-10.742	(44 2.7)
Kellogs	0.610%	10.249%	-3.375%	9.725%	9.639	-3.985	0.524	(23 3)
McDonald's Corp.	9.168%	0.000%	7.777%	11.668%	-9.168	-1.390	-11.668	(6 2)
Average Consumer	**3.673%**	**-0.528%**	**-0.459%**	**12.474%**	**-4.201**	**-4.133**	**-13.002**	**(27 2.9)**
Total SA	42.626%	37.711%	20.548%	0.259%	-4.915	-22.078	37.452	(13 2)
Exxon Mobil Corporation	0.704%	4.375%	9.113%	-0.105%	3.671	8.409	4.479	(48 2.7)
Chevron Corporation	-8.249%	2.718%	1.878%	2.331%	10.967	10.127	0.387	(13 2.3)
Valero Energy	17.461%	12.852%	14.895%	14.038%	-4.610	-2.567	-1.186	(33 1.2)
ConocoPhillips	-13.249%	-17.629%	-14.388%	-7.012%	-4.380	-1.139	-10.618	(13 2.4)
Williams Cos.	-30.290%	16.231%	-0.051%	-0.776%	46.520	30.238	17.007	(5 1)
Southwestern Energy	-37.583%	-29.839%	-31.831%	-34.757%	7.744	5.752	4.917	(11 1.3)
Cabot Oil & Gas	-21.700%	-8.450%	-12.617%	-15.280%	13.250	9.083	6.830	(11 1.1)
Schlumberger Ltd.	-3.789%	-2.293%	-3.666%	0.341%	1.496	0.123	-2.634	(49 2.9)
Helmerich & Payne	-15.525%	7.789%	1.804%	1.935%	23.314	17.329	5.854	(12 2.1)
Average Energy	**-6.959%**	**2.346%**	**-1.431%**	**-3.903%**	**9.306**	**5.528**	**6.249**	**(11 1.2)**
AVERAGE ALL	**4.295%**	**4.322%**	**4.854%**	**10.417%**	**0.026**	**0.559**	**-6.095**	

Results, when the optimization was performed using the 2nd learning period, are resented on table 3 below. Looking at the total result, Bollinger band parameters optimized on portfolio level outperformed both the standard parameters and individually optimized parameters but underperformed compared to the buy and hold strategy. Also, the individually optimized parameters outperformed the standard parameters of 20 and 2. Overall the results suggest that the parameter calibration using the shorter 2nd learning period succeeded better than when using the longer 1st learning period. Also, it is important to point out that the descriptive statistics in Appendix 1 show that the volatility measures on the hold-out period were much closer to the volatility on the 2nd learning period rather than on the 1st learning period. However, the buy and hold return, which can be interpreted as an overall market direction over the period was actually a slightly closer match between the 1st learning period and hold-out period rather than the 2nd learning period and hold-out period. The results suggest that the change in volatility is somewhat negatively correlated to the performance of the bands, in other words, the more the volatility measures differ from the learning period to the out of sample period the worse the parameter optimization works and vice versa. Looking at the five best-performing stocks on table 3 when parameters are optimized on individual level (BB opt column) one can find General Electric, Altria Group Inc., Merck & Co., US Bank Corp and Apple Inc. The amount of the volatility measures had changed from 2nd learning period to the hold-out period was very small for all of these five stocks. Correspondingly, finding the stocks that had very large differences in volatility measures between the two periods, one can find Williams Cos., Southwest Energy, and Abbott Laboratories. These three happen to be also the worst performers when the Bollinger band parameters were optimized on individual stock level.

The variation between each of the optimized parameter values on table 3 is so large that no conclusions can be made on what would be the best parameter combination in general. Large moving average length and wide bands seem to work well with some stocks like US bank Corp (42, 2.7) and Cisco Systems (48, 2.7) but poorly for

others, such as Pfizer Inc. (42, 3.0). Then again short moving average length and narrow bands work well for Helmerich & Payne (7, 1.6) and Bank of America Corporation (5, 1.6) but poorly for Baxter International Inc. (6, 1.0) and Kellogs (6, 1.0). The lack of generalization on the parameter values would also support the argument for parameter optimization.

The length of the learning period does also have a large impact on how well the optimized Bollinger band parameters perform out of sample. Based on the results of this study, a suggested learning period length should be kept relatively short, possibly limiting the maximum length of the learning period to match the estimated length of the investment period. If one thinks about how the Dow theory describes the phases of the stock market, the primary trend should last several years before a reversal of a trend direction. Thus, keeping the learning period and investment period relatively short, it is more likely that the primary trend of the market will continue moving to the same direction as earlier. When the primary trend has reversed, a new parameter optimization should be performed with the most recent data that includes the newly reversed trend. Looking at the results on tables 2 and 3 for Citigroup Inc., Bank of America Corporation, Electronic Arts Inc. and Boston Scientific, one can see that the profit with parameters optimized on individual stock level are much higher when the 2nd learning set was used to perform the optimization. All four stocks had an overall price decline during the 1st learning period, but overall price increase during the 2nd learning period and hold-out period (see Appendix 1). This result supports the argument that when the primary trend is heading to the same direction on the learning period and out of sample, the optimized parameters perform better than if the trend direction is different for the two periods.

Table 3: Trading simulation results when parameters optimized with 2nd learning period. BB (20,2) represents the results using Bollinger bands (BB) with standard parameters of 20 and 2. Column BB Opt shows the results when the parameters are optimized on individual stock level and the respective parameter values are shown in column Opt (N,K). Column BB Portf shows the results using optimized parameters for the respective portfolio and the parameter values for each portfolio are shown on the average row. Column B&H is the result for buy and hold strategy. Difference columns represent the difference between the two methods in percentage points.

	BB (20,2)	BB Opt	BB Portf	B&H	Diff BB Opt BB(20,2)	Diff BB Portf BB(20,2)	Diff BB Opt B&H	Opt (N,K)
Citigroup Inc.	6.141%	11.951%	17.008%	4.825%	5.810	10.867	7.127	(26 1.7)
JPMorgan Chase & Co.	8.645%	13.606%	12.696%	16.944%	4.961	4.050	-3.338	(26 2.2)
Bank of America Corporation	14.235%	20.316%	16.415%	12.424%	6.080	2.180	7.892	(5 1.6)
The Goldman Sachs Group	23.436%	-5.155%	7.931%	12.160%	-28.591	-15.505	-17.315	(46 1.2)
Morgan Stanley	23.872%	3.691%	3.691%	12.712%	-20.181	-20.181	-9.020	(34 1.9)
American Express Company	0.587%	-5.258%	8.947%	-4.714%	-5.845	8.360	-0.545	(14 1.9)
Moody's Corp	16.601%	15.618%	7.122%	8.303%	-0.983	-9.479	7.315	(9 2.2)
NASDAQ OMX Group	8.028%	0.000%	13.354%	21.624%	-8.028	5.327	-21.624	(20 2.9)
U.S. Bancorp	22.171%	25.901%	11.703%	11.385%	3.730	-10.467	14.516	(42 2.7)
Wells Fargo	18.797%	13.286%	23.080%	10.081%	-5.511	4.284	3.204	(23 2.3)
Average Finance	**14.251%**	**9.396%**	**12.195%**	**10.574%**	**-4.856**	**-2.056**	**-1.179**	**(34 1.9)**
Oracle Corporation	10.489%	10.748%	14.309%	1.936%	0.260	3.821	8.813	(23 2.3)
QUALCOMM Incorporated	8.587%	12.163%	11.527%	-0.995%	3.575	2.940	13.157	(12 1.9)
Accenture plc	30.969%	10.452%	-1.027%	15.548%	-20.518	-31.997	-5.096	(39 2.6)
Intel Corporation	-4.939%	19.650%	3.021%	15.600%	24.588	7.959	4.049	(24 2.5)
NVIDIA Corporation	-1.456%	17.080%	-7.596%	91.569%	18.536	-6.140	-74.490	(7 1.5)
Activision Blizzard, Inc.	2.673%	-0.210%	-7.484%	27.122%	-2.884	-10.157	-27.332	(23 1.9)
Electronic Arts Inc.	17.912%	6.552%	9.650%	51.101%	-11.360	-8.263	-44.549	(5 1)
Cisco Systems, Inc.	28.136%	15.598%	-4.248%	14.471%	-12.539	-32.385	1.127	(48 2.7)
Apple Inc.	13.050%	24.476%	13.002%	15.874%	11.426	-0.048	8.602	(13 2.3)
Microsoft	13.135%	15.003%	26.705%	21.932%	1.868	13.570	-6.929	(5 1.3)
Average Technology	**11.856%**	**13.151%**	**5.786%**	**25.416%**	**1.295**	**-6.070**	**-12.265**	**(6 1)**
Johnson & Johnson	-2.202%	0.083%	-9.461%	11.259%	2.285	-7.259	-11.176	(20 1.7)
Pfizer Inc.	10.933%	0.987%	9.963%	5.813%	-9.946	-0.969	-4.826	(42 3)
Abbott Laboratories	-7.977%	-14.661%	-1.875%	2.460%	-6.684	6.103	-17.122	(15 1.6)
Baxter International Inc.	6.382%	-7.217%	5.723%	7.478%	-13.599	-0.659	-14.695	(6 1)
Lilly (Eli) & Co.	9.351%	2.959%	18.457%	16.439%	-6.392	9.105	-13.480	(35 1.1)
Aetna Inc	0.401%	-8.953%	4.986%	23.689%	-9.353	4.585	-32.642	(13 1.7)
Amgen Inc	15.986%	1.286%	3.078%	10.459%	-14.700	-12.907	-9.173	(25 2.9)
Boston Scientific	-7.844%	1.661%	1.661%	22.006%	9.505	9.505	-20.345	(18 1.1)
Merck & Co.	8.090%	25.979%	-1.396%	9.364%	17.889	-9.485	16.615	(12 1.7)
Mylan N.V.	-19.404%	-0.971%	18.082%	-3.596%	18.432	37.486	2.625	(17 1.7)
Average Healthcare	**1.371%**	**0.115%**	**4.922%**	**10.537%**	**-1.256**	**3.550**	**-10.422**	**(18 1.1)**
The Boeing Company	8.656%	4.581%	4.079%	7.315%	-4.075	-4.577	-2.733	(28 3)
United Technologies Corp	-1.668%	3.731%	0.731%	1.738%	5.399	2.399	1.993	(31 2)
Lockheed Martin Corp	-1.786%	2.371%	11.567%	23.242%	4.158	13.353	-20.871	(41 3)
3M	-11.234%	21.977%	1.758%	11.744%	33.211	12.992	10.233	(23 2.7)
Eaton Corporation plc	8.069%	-5.741%	8.443%	-0.616%	-13.810	0.374	-5.125	(17 1.2)
Caterpillar Inc.	5.390%	-4.697%	-4.697%	4.603%	-10.087	-10.087	-9.301	(18 1.2)
Deere & Co.	0.909%	9.437%	20.856%	7.436%	8.528	19.947	2.000	(21 1.1)
General Electric	2.792%	30.440%	2.112%	8.252%	27.648	-0.680	22.188	(9 1.9)
Rockwell Collins	5.726%	13.441%	18.953%	9.910%	7.715	13.227	3.531	(5 1.3)
Dover Corp.	-1.056%	-0.361%	6.116%	0.424%	0.695	7.173	-0.784	(36 2.3)
Average Industrial	**1.580%**	**7.518%**	**6.992%**	**7.405%**	**5.938**	**5.412**	**0.113**	**(18 1.2)**
Ford Motor Co.	10.544%	7.929%	7.929%	-3.357%	-2.615	-2.615	11.286	(5 1.6)
Coca-Cola	-9.925%	-10.371%	-9.995%	37.145%	-0.446	-0.070	-47.516	(9 2.2)
Pepsi	27.666%	5.725%	7.874%	11.566%	-21.941	-19.792	-5.841	(49 2.8)
Amazon.com, Inc.	1.478%	-9.130%	62.949%	23.513%	-10.608	61.470	-32.643	(19 1.4)
Altria Group Inc.	-6.813%	28.231%	-12.931%	26.422%	35.044	-6.118	1.808	(13 2.8)
Macy's, Inc.	0.623%	-0.220%	-3.521%	-9.918%	-0.842	-4.144	9.698	(6 1.5)
Costco Wholesale Corp	-0.668%	3.996%	-6.360%	13.239%	4.664	-5.691	-9.244	(30 1.9)
The Procter & Gamble Comp	4.050%	-6.751%	-4.689%	4.739%	-10.801	-8.740	-11.490	(48 1.4)
Kellogs	0.610%	-5.157%	2.837%	9.725%	-5.767	2.227	-14.882	(6 1)
McDonald's Corp.	9.168%	10.052%	12.269%	11.668%	0.884	3.102	-1.616	(7 1.7)
Average Consumer	**3.673%**	**2.430%**	**5.636%**	**12.474%**	**-1.243**	**1.963**	**-10.044**	**(5 1.6)**
Total SA	42.626%	-1.212%	19.264%	0.259%	-43.838	-23.362	-1.471	(11 2.2)
Exxon Mobil Corporation	0.704%	1.637%	13.053%	-0.105%	0.934	12.350	1.742	(34 2.2)
Chevron Corporation	-8.249%	2.718%	-4.693%	2.331%	10.967	3.555	0.387	(13 2.3)
Valero Energy	17.461%	-7.490%	18.571%	14.038%	-24.951	1.109	-21.528	(6 1.8)
ConocoPhillips	-13.249%	-1.243%	6.276%	-7.012%	12.006	19.525	5.768	(9 1)
Williams Cos.	-30.290%	-15.006%	-15.006%	-0.776%	15.284	15.284	-14.229	(8 1.6)
Southwestern Energy	-37.583%	-28.993%	-23.601%	-34.757%	8.590	13.982	5.764	(20 1.6)
Cabot Oil & Gas	-21.700%	17.377%	9.747%	-15.280%	39.077	31.448	32.657	(9 1.7)
Schlumberger Ltd.	-3.789%	-11.161%	-1.476%	0.341%	-7.372	2.313	-11.502	(9 2.2)
Helmerich & Payne	-15.525%	21.878%	11.445%	1.935%	37.403	26.970	19.943	(7 1.6)
Average Energy	**-6.959%**	**-2.149%**	**3.358%**	**-3.903%**	**4.810**	**10.317**	**1.753**	**(8 1.6)**
AVERAGE ALL	**4.295%**	**5.077%**	**6.481%**	**10.417%**	**0.781**	**2.186**	**-5.341**	

In three out of the six portfolios, Finance, Industrial and Energy, at least one of the Bollinger band trading strategies actually outperformed buy and hold strategy as well. The fact that a trading strategy based solely on Bollinger bands is able to outperform a buy and hold strategy even when the general stock market is on the uptrend gaining around 10% per annum is proof that there is a possibility to gain excess returns with Bollinger bands. It should be understood that the signals generated by Bollinger bands are not meant to be used solely to make buy and sell decisions but should be used in conjunction with other technical indicators or oscillators, such as relative strength index (RSI), moving average convergence divergence (MACD) and rate of change (ROS), that can give confirming signals when making buy and sell decisions. When choosing oscillators or other technical indicators, one should be careful to avoid multicollinearity traps where two or more oscillators are derived from the same data and thus are collinear to each other. An example of multicollinearity would be choosing two oscillators that both measure trend, linear regression and MACD. Both of these oscillators would more often than not give the trader the same signals, providing a false sense of security and confirmation. Oscillators used in conjunction with Bollinger bands should be chosen so that all measure different concepts, for example one that measures trend and other that measures volume.

6.1. Sensitivity analysis

Table 4 and 5 below show the sensitivity analysis of the optimized parameters on the hold-out period over all the 60 stocks included in the study. The percentage values on the tables show how much the profit would change on average in the hold-out period if the optimized parameter values of *N* and *K* are changed +/- 2 and +/- 0.2 respectively. Looking at the standard deviation parameter, it is clear that even small deviations from the optimized parameter change the performance of the Bollinger

bands significantly. Table 4 reveals that changing the value of the optimized standard deviation parameter by 0.1 translates to a 3.95% and 29.78% changes in overall returns on average when the parameters are optimized using the 1st learning period. Changing the parameter value by 0.2 translates to 18.26 and -9.63% shift in overall performance. Table 5 shows that the overall performance is even more sensitive to small changes in the value of the standard deviation parameter, decreasing the overall performance by a minimum of 19.17% when the parameter optimization is based on the 2nd learning period. Similarly, changing the value of the moving average length parameter has a big impact on the performance of the Bollinger bands. Increasing and decreasing the moving average length by 1-day from the optimized value changed the overall performance by 42.47% and -11.50% respectively as seen on table 4. When the optimization was done using the 2nd learning period, 1 day change in the length of the moving average translated to a -0.38% and -42.12% changes in overall performance. Shifting the optimized value of *N* by 2 days would change the performance by -10.96% and -62.38% when the parameters are optimized using the 1st learning period. When the optimization was based on the 2nd learning period, the same 2 days shift in moving average length would change the overall performance by 8.33% and -47.55%.

The sensitivity analysis on the tables 4 and 5 reveals also that the optimization based on the 2nd learning period was a lot better than optimization on the 1st learning period. Looking at the signs of the percentage changes on table 5, one can see that 7 out of the 8 cases presented would cause the overall profitability of the parameters to decrease from the optimized values. When the optimization was done using the longer 7 year period, 4 out of the 8 cases would actually increase the performance from the optimized parameter values. Overall, when looking at the sensitivity analysis, it is safe to say that the parameter values have a large impact on the performance of the Bollinger bands and thus optimizing the values is highly recommended.

Table 4: Sensitivity analysis of parameter values – 1st learning period. Table represents how much the total profit changes in hold-out period in percentage points when the optimized Bollinger band parameters are changed. MA +/- # presents the parameter *N* change in increments of 1. SD +/- # presents the parameter *K* in increments of 0.1. Opt is the optimized parameter based 1st learning period.

	SD -0.2	SD -0.1	Opt	SD +0.1	SD +0.2
MA +2			-10.96%		
MA +1			42.47%		
Opt	18.26%	3.95%	**Opt**	29.78%	-9.63%
MA -1			-11.50%		
MA -2			-62.37%		

Table 5: Sensitivity analysis of parameter values – 2nd learning period. Table represents how much the total profit changes in hold-out period in percentage points when the optimized Bollinger band parameters are changed. MA +/- # presents the parameter *N* change in increments of 1. SD +/- # presents the parameter *K* in increments of 0.1. Opt is the optimized parameter based 2nd learning period.

	SD -0.2	SD -0.1	Opt	SD +0.1	SD +0.2
MA +2			8.33%		
MA +1			-0.38%		
Opt	-31.62%	-23.73%	**Opt**	-19.17%	-47.39%
MA -1			-42.12%		
MA -2			-44.75%		

Figures 11 and 12 present the similar heatmaps that were presented on the portfolios earlies, but now on the individual stock level. The two examples presented are Abbot Laboratories and General Electric and the heatmaps cover the profitability in the 2nd learning period. The two examples were chosen to show the difference in the

parameter sensitivity on individual stock level. Looking at figure 11, Abbott Laboratories, one can see that there is a rather large high return area, illustrated by the yellow color, around the parameter values of 20 and 1.2 for *N* and *K* respectively. The actual optimized parameter values for Abbott Laboratories are 15 and 1.6 for *N* and *K* respectively. The heatmap shows that changing the optimized parameter values slightly would not cause a huge change in the performance of the bands. However, heatmap for General Electric on figure 12 shows a completely different result. The optimized parameter value for General electric was 9 and 1.9 for *N* and *K* respectively. Even a small change on the optimal values would change the overall performance significantly and decrease the profitability by a large amount illustrated by the small bright yellow spot on the heatmap.

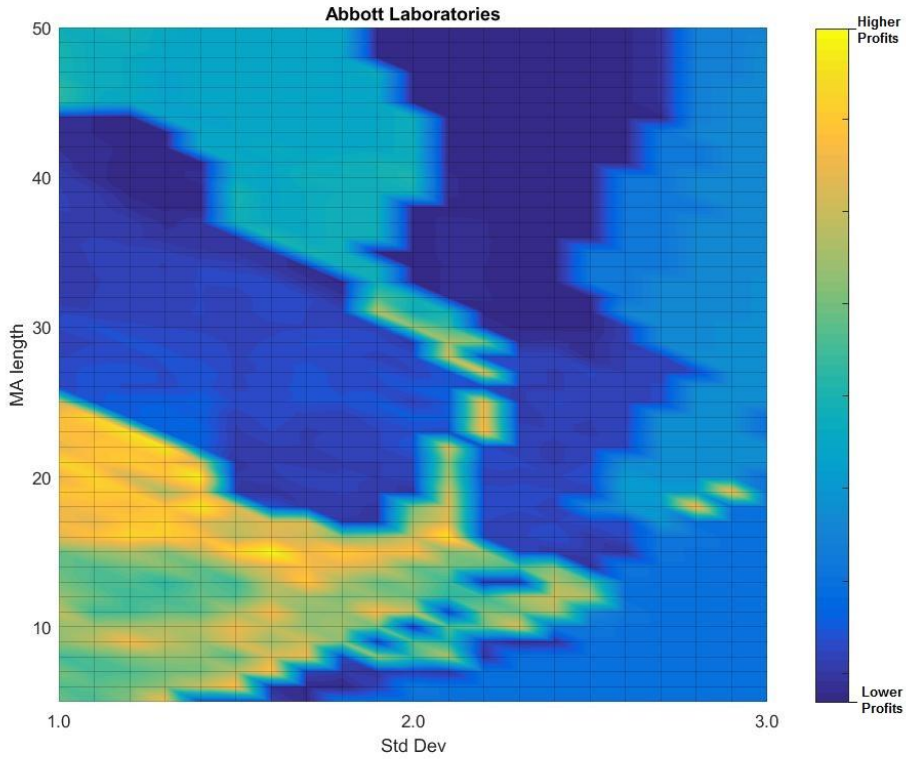

Figure 11: Return comparison for different Bollinger band parameter combinations. Abbott Laboratories – 2nd learning period. The optimal value is at 15 and 1.6.

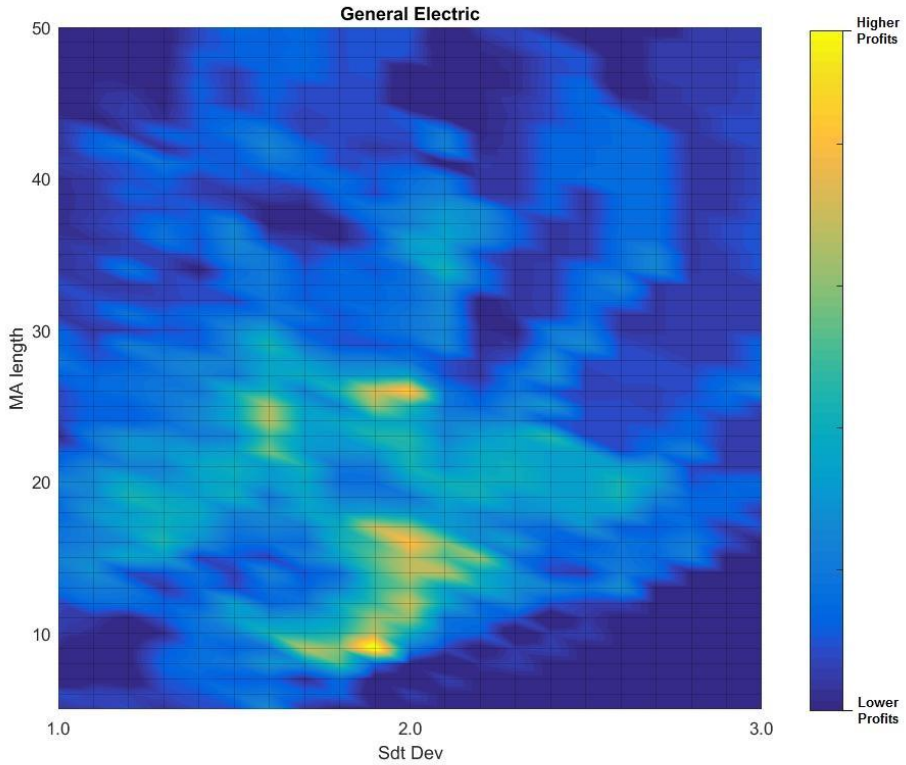

Figure 12: Return comparison for different Bollinger band parameter combinations. General Electric – 2nd learning period. The optimal value is at 9 and 1.9.

Overall, the sensitivity analysis reveals that the parameter values have a big impact on the profitability and performance of the Bollinger bands trading strategy. On individual stock level, some stocks are more sensitive than others to small changes in parameter values but the overall results show that on average, the sensitivity is high and thus some kind of parameter value optimization would be highly recommended.

6.2. Discussion

The aim of this empirical study was to answer the three research questions presented in chapter 1. The first research question was: *Is it plausible to optimize Bollinger band parameters using historical data and what factors affect the parameter value optimization?* Given the results of the study, a parameter optimization is preferred and can lead to better results than using the standard parameters of 20 and 2 for *N* and *K* respectively, however in some cases the optimization will lead to worse results than using the standard parameters. The parameter optimization seems to be affected by the length and volatility of the learning period. Based on the results of the study, a suggested approach would be to use a relatively short learning period with the most recent available data so that the current market conditions would be incorporated on the optimization model. By relatively short learning period length, it is meant that the learning period length does not exceed the length of the investment period. Of course, history does not always repeat itself but if there are no signs that the current market conditions will change in the future, it is better to use only the more recent data for the optimization. It would be highly interesting to develop an adaptive parameter optimization method that would adjust itself to the market conditions. As the change in volatility seems to be an important factor in how well the optimized Bollinger band signals work, an optimization model that would adapt to changes in volatility could be beneficial. An interesting approach would be to include a CBOE volatility index (VIX), which measures the implied volatility of the S&P 500 index for the next 30 days, to the optimization model so that the changes in VIX would change the length of the learning period. Another approach would be to assign three different levels for the volatility of the underlying stock, high medium and low, and build the optimization model so that the parameters for each volatility level would get optimized differently. The problem with this approach is that VIX itself is highly volatile, so some smoothing might be required.

The second research question was*: Can optimized Bollinger band parameters yield robustly better returns than the generally proposed parameters of 20 and 2?* The findings of this study support the argument for parameter optimization as the overall results for the optimized parameters outperformed the standard parameters of 20 and 2. Although the results vary greatly between each stock, the optimized parameters did generate greater profits than when the parameters were not optimized on average. The fact that the portfolio optimization outperformed the individually optimized parameters suggests that there is a benefit of grouping stocks by industry and assigning certain optimized parameters for each industry. An interesting idea for a study would be to test the co-movements of different stock returns and volatility measures and grouping stocks into portfolios based on these measures and then performing the Bollinger band parameter optimization. Further research regarding the length of the optimal learning period would be beneficial as well.

The third and final research question in this thesis was: *How sensitive the performance of Bollinger bands is to small changes in parameter values in relation to profitability?* As presented by the sensitivity analysis, the performance of the Bollinger bands is highly sensitive to the assigned parameter values. The heatmaps presented earlier for each portfolio and for two individual stocks show that some cases are much more sensitive than others, but on average the sensitivity measures are very high. However, in most of the cases, there is a certain “area” of parameter values close to each other that all yield relatively better results than other parameter values that are significantly different than the optimal values. The results of the sensitivity analysis support the argument for optimizing the parameters as the performance of the Bollinger bands is highly depended on the assigned parameter values.

7. Summary

This thesis studied the optimization of Bollinger band parameter values with a data collected for 60 stocks traded in North American stock markets from a time period between 1.1.2006 - 31.12.2016. First seven years of the data was used as a learning set for the optimization algorithm which was built using Matlab, and the last 3 years of the data was used for out of sample test and performance measures. The learning set was further divided into two different length overlapping data sets in order to examine how different market conditions affect the parameter optimization and the performance of the optimized parameter values. The stocks were divided into six different stock portfolios based on industry sectors based on Global Industry Classification Standard (GICS). The assigned portfolios were finance, information technology, healthcare, industrial, consumer and energy, and each portfolio contained 10 rather well known stocks from the given sector.

The empirical test results show evidence that the parameter values should be optimized to maximize the effectiveness of a Bollinger band trading strategy. Overall the results show that when the parameters were optimized both on individual stock level and on a portfolio level, the optimized parameters outperformed the standard Bollinger band parameters of 20 and 2 when applied to out of sample data. However, even without the consideration of transaction costs, the Bollinger band trading strategy did not manage to outperform a buy and hold strategy on average but only in few individual cases. Considering that the trading strategy based on optimized Bollinger bands was able to yield a profitable outcome, a more research should be done in conjunction with other technical oscillators to see whether the trading strategy could be improved further.

The study was able to show that a change in market volatility will have a major impact on how well the optimized Bollinger band parameters perform. The changes in market volatility between the learning period and out of sample period seem to be negatively correlated to the performance of the optimized parameters. A trader using the optimized parameters can expect the parameters to perform rather well if the market volatility will remain somewhat constant over time, but if the market volatility will start to increase or decrease greatly, the benefits achieved with the optimization will start to deteriorate. Unfortunately the study failed to achieve a generalization regarding the optimal parameter values due to large variations in the optimized values of the parameters. However, the optimized parameter values for stocks that were grouped to portfolios based on industry sectors were somewhat similar within the given portfolio. Rather surprisingly, optimizing the parameters in the portfolio level yielded the best results suggesting that the stocks within the same industry have quite similar daily price fluctuations and volatility measures. The results clearly show that a large moving average length should not be combined with narrow bands and a short moving average length should not be combined with wide bands.

The final research question was to study how sensitive the performance of the Bollinger band is to small changes in the values of the parameters. The results suggest that even a very slight change from the optimized parameter value will have a relatively big impact on the performance. When the parameters were optimized using the shorter learning period, changing the optimized parameter value of the moving average by 1 or the standard deviation multiplier by 0.1, the performance of the bands would decrease by around 20 percent on average when the performance was measured as an annual rate of return. The sensitivity was noticed to vary greatly between each stock and portfolio, some stocks and portfolios were much more sensitive to small changes in parameter values than others.

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Appendices

Appendix 1: Descriptive statistics for the data

Table 6: Descriptive statistics 1st learning period. Mean column present the mean stock price, min and max columns the minimum and maximum stock prices respectively. Std dev column represent the annual return volatility for the given stock and portfolio. B&H return column shows the buy and hold return for the given stock and portfolio.

	Company	Mean	Min	Max	Std dev	B&H return	N
Finance	Citigroup Inc.	152.719	9.937	497.106	67.37%	-25.95%	2013
	JPMorgan Chase & Co.	35.028	13.201	52.966	47.63%	8.50%	2013
	Bank of America Corporation	20.725	2.962	45.269	63.75%	-12.29%	2013
	The Goldman Sachs Group, Inc.	138.057	46.582	220.135	43.54%	5.96%	2013
	Morgan Stanley	30.172	8.192	63.552	62.51%	-4.49%	2013
	American Express Company	44.132	8.910	85.811	42.85%	10.16%	2013
	Moody's Corp	38.021	13.708	74.584	43.74%	4.73%	2013
	NASDAQ OMX Group	25.742	14.376	45.610	46.02%	1.81%	2013
	U.S. Bancorp	24.345	7.457	37.245	41.51%	7.76%	2013
	Wells Fargo	25.675	6.702	40.925	51.85%	8.42%	2013
	Average				**51.08%**	**0.46%**	
Information Tech	Oracle Corporation	22.495	11.069	36.268	30.89%	17.93%	2013
	QUALCOMM Incorporated	41.374	23.993	66.746	32.68%	9.68%	2013
	Accenture plc	40.234	20.459	76.419	28.22%	18.47%	2013
	Intel Corporation	16.815	9.159	24.351	31.31%	3.69%	2013
	NVIDIA Corporation	15.310	5.475	36.688	52.49%	3.74%	2013
	Activision Blizzard, Inc.	10.780	4.900	17.666	37.75%	15.28%	2013
	Electronic Arts Inc.	29.843	10.945	61.400	41.42%	-11.36%	2013
	Cisco Systems, Inc.	18.349	11.353	28.407	32.98%	4.54%	2013
	Apple Inc.	34.875	6.512	90.615	36.40%	34.01%	2013
	Microsoft	22.842	12.176	35.317	29.07%	7.50%	2013
	Average				**35.32%**	**10.35%**	
Healthcare	Johnson & Johnson	52.555	35.748	86.191	16.41%	9.61%	2013
	Pfizer Inc.	16.255	8.475	28.143	24.05%	9.05%	2013
	Abbott Laboratories	12.685	6.666	35.514	33.51%	26.82%	2013
	Baxter International Inc.	25.946	15.432	37.675	23.22%	11.47%	2013
	Lilly (Eli) & Co.	33.591	19.580	51.122	23.23%	3.39%	2013
	Aetna Inc	38.006	16.248	66.254	45.52%	17.33%	2013
	Amgen Inc	58.244	34.971	108.462	27.14%	5.85%	2013
	Boston Scientific	10.401	4.970	26.480	39.51%	-9.65%	2013
	Merck & Co.	29.807	15.367	44.852	27.52%	11.41%	2013
	Mylan N.V.	20.352	5.770	44.500	34.65%	11.98%	2013
	Average				**29.48%**	**9.73%**	
Industrial	The Boeing Company	61.236	23.448	124.701	30.52%	13.00%	2013
	United Technologies Corporation	60.655	30.436	103.503	25.83%	13.36%	2013
	Lockheed Martin Corporation	69.378	43.445	134.270	24.40%	16.95%	2013
	3M	69.271	33.673	127.593	23.86%	11.80%	2013
	Eaton Corporation plc	33.925	11.584	67.826	33.46%	16.06%	2013
	Caterpillar Inc.	59.889	17.261	98.046	35.48%	9.73%	2013
	Deere & Co.	55.662	20.251	84.922	38.22%	17.88%	2013
	General Electric	18.249	5.120	29.576	33.64%	0.82%	2013
	Rockwell Collins	50.105	24.014	70.355	28.06%	8.81%	2013
	Dover Corp.	37.577	15.277	74.182	31.81%	15.54%	2013
	Average				**30.53%**	**12.39%**	
Consumer	Ford Motor Co.	7.925	1.001	14.934	51.33%	11.47%	2013
	Coca-Cola	51.218	28.587	71.269	30.50%	9.84%	2013
	Pepsi	53.560	35.961	77.932	18.30%	8.08%	2013
	Amazon.com, Inc.	139.278	26.070	404.390	43.72%	35.49%	2013
	Altria Group Inc.	17.289	8.862	33.356	20.22%	19.53%	2013
	Macy's, Inc.	24.833	4.599	47.839	46.93%	8.81%	2013
	Costco Wholesale Corporation	57.890	30.197	111.842	24.26%	15.83%	2013
	The Procter & Gamble Company	51.227	33.740	75.815	18.57%	8.15%	2013
	Kellogs	41.840	27.968	60.110	17.97%	8.38%	2013
	McDonald's Corp.	55.188	22.510	90.436	19.93%	20.43%	2013
	Average				**29.17%**	**14.60%**	
Energy	Total SA	39.296	26.792	54.431	31.52%	5.52%	2013
	Exxon Mobil Corporation	62.176	41.762	88.773	26.42%	10.99%	2013
	Chevron Corporation	67.615	36.028	107.907	28.50%	15.53%	2013
	Valero Energy	27.859	10.444	56.547	44.99%	2.78%	2087
	ConocoPhillips	39.400	19.306	65.317	31.80%	10.61%	2013
	Williams Cos.	15.347	4.559	31.456	40.92%	16.81%	2013
	Southwestern Energy	32.502	12.400	51.650	51.44%	11.13%	2013
	Cabot Oil & Gas	13.765	4.422	39.358	47.98%	32.12%	2087
	Schlumberger Ltd.	63.266	30.363	94.687	40.54%	9.88%	2013
	Helmerich & Payne	36.204	10.672	71.894	53.62%	27.41%	2013
	Average				**39.77%**	**14.28%**	

Table 7: Descriptive statistics 2nd learning period. Mean column present the mean stock price, min and max columns the minimum and maximum stock prices respectively. Std dev column represent the annual return volatility for the given stock and portfolio. B&H return column shows the buy and hold return for the given stock and portfolio.

	Company	Mean	Min	Max	Std dev	B&H return	N
Finance	Citigroup Inc.	38.329	22.525	52.067	39.52%	2.17%	754
	JPMorgan Chase & Co.	38.002	24.283	52.966	30.62%	13.19%	754
	Bank of America Corporation	10.164	4.748	15.235	42.67%	3.57%	754
	The Goldman Sachs Group, Inc.	124.013	81.079	169.038	30.09%	2.19%	754
	Morgan Stanley	19.797	11.450	29.636	44.18%	4.46%	754
	American Express Company	54.720	38.566	85.811	23.52%	29.67%	754
	Moody's Corp	42.734	24.163	74.584	32.75%	45.15%	754
	NASDAQ OMX Group	25.064	18.609	37.677	30.25%	19.48%	754
	U.S. Bancorp	27.573	17.666	37.245	23.83%	17.08%	754
	Wells Fargo	29.211	19.359	40.925	27.28%	15.62%	754
	Average				**32.47%**	**15.26%**	
Information Tech	Oracle Corporation	29.536	22.947	36.268	27.52%	7.58%	754
	QUALCOMM Incorporated	52.954	39.934	66.746	25.14%	15.89%	754
	Accenture plc	57.638	41.486	76.419	25.08%	21.79%	754
	Intel Corporation	19.724	15.725	24.351	23.27%	11.49%	754
	NVIDIA Corporation	13.874	10.559	23.828	39.80%	1.36%	754
	Activision Blizzard, Inc.	12.379	9.813	17.666	26.66%	14.13%	754
	Electronic Arts Inc.	19.175	10.945	27.990	38.84%	11.88%	754
	Cisco Systems, Inc.	16.739	11.528	23.173	29.13%	5.14%	754
	Apple Inc.	61.001	40.523	90.615	28.14%	20.75%	754
	Microsoft	25.643	20.012	35.317	23.19%	13.30%	754
	Average				**28.68%**	**12.33%**	
Healthcare	Johnson & Johnson	62.120	47.366	86.191	13.51%	17.30%	754
	Pfizer Inc.	19.952	13.428	28.143	18.40%	24.68%	754
	Abbott Laboratories	18.536	8.854	35.514	46.76%	55.85%	754
	Baxter International Inc.	29.896	22.972	37.675	20.44%	14.16%	754
	Lilly (Eli) & Co.	37.821	26.809	51.122	17.94%	18.63%	754
	Aetna Inc	44.437	28.601	66.254	27.32%	32.10%	754
	Amgen Inc	70.677	42.233	108.462	22.42%	29.14%	754
	Boston Scientific	7.303	4.970	12.380	32.08%	16.51%	754
	Merck & Co.	34.407	24.141	44.852	18.38%	16.26%	754
	Mylan N.V.	26.236	16.160	44.500	26.74%	26.07%	754
	Average				**24.40%**	**25.07%**	
Industrial	The Boeing Company	71.892	49.215	124.701	23.99%	30.08%	754
	United Technologies Corporation	75.561	57.704	103.503	21.61%	15.67%	754
	Lockheed Martin Corporation	79.057	53.950	134.270	17.87%	34.34%	754
	3M	84.911	61.015	127.593	19.86%	20.30%	754
	Eaton Corporation plc	45.335	28.268	67.826	29.82%	16.99%	754
	Caterpillar Inc.	78.708	58.945	98.046	28.34%	0.98%	754
	Deere & Co.	73.046	53.402	84.922	25.59%	5.30%	754
	General Electric	17.657	12.142	24.982	22.80%	19.20%	754
	Rockwell Collins	54.816	40.304	70.355	22.13%	10.21%	754
	Dover Corp.	49.910	32.773	74.182	27.11%	20.13%	754
	Average				**23.91%**	**17.32%**	
Consumer	Ford Motor Co.	10.664	7.211	14.934	30.66%	-2.07%	754
	Coca-Cola	59.725	48.474	71.269	22.67%	10.47%	754
	Pepsi	61.857	50.651	77.932	13.92%	11.34%	754
	Amazon.com, Inc.	238.384	160.970	404.390	32.59%	29.36%	754
	Altria Group Inc.	25.304	17.366	33.356	15.19%	22.61%	754
	Macy's, Inc.	31.879	18.582	47.839	29.31%	30.38%	754
	Costco Wholesale Corporation	79.597	56.283	111.842	17.92%	22.09%	754
	The Procter & Gamble Company	59.141	48.328	75.815	14.78%	11.42%	754
	Kellogs	48.186	40.187	60.110	14.24%	9.67%	754
	McDonald's Corp.	77.888	59.195	90.436	14.44%	11.63%	754
	Average				**20.57%**	**15.69%**	
Energy	Total SA	39.758	30.285	50.269	25.14%	9.45%	754
	Exxon Mobil Corporation	72.088	56.131	88.773	18.54%	13.57%	754
	Chevron Corporation	89.796	70.471	107.907	20.49%	14.39%	754
	Valero Energy	25.118	13.377	45.077	39.28%	27.50%	772
	ConocoPhillips	48.240	36.966	65.317	20.99%	15.51%	754
	Williams Cos.	22.248	11.632	31.456	30.80%	37.56%	754
	Southwestern Energy	36.254	25.820	49.000	33.06%	1.09%	754
	Cabot Oil & Gas	23.174	9.775	39.358	40.96%	56.27%	772
	Schlumberger Ltd.	69.945	51.475	86.892	29.78%	4.01%	754
	Helmerich & Payne	49.407	31.165	71.894	37.88%	20.84%	754
	Average				**29.69%**	**20.02%**	

Table 8: Descriptive statistics hold-out period. Mean column present the mean stock price, min and max columns the minimum and maximum stock prices respectively. Std dev column represent the annual return volatility for the given stock and portfolio. B&H return column shows the buy and hold return for the given stock and portfolio.

	Company	Mean	Min	Max	Std dev	B&H return	N
Finance	Citigroup Inc.	48.973	34.348	60.468	25.99 %	4.82 %	756
	JPMorgan Chase & Co.	59.273	48.904	85.696	21.74 %	16.94 %	756
	Bank of America Corporation	15.688	10.922	23.015	26.82 %	12.42 %	756
	The Goldman Sachs Group, Inc.	173.640	137.716	241.625	22.61 %	12.16 %	756
	Morgan Stanley	31.407	20.993	43.103	27.13 %	12.71 %	756
	American Express Company	74.402	49.764	91.109	21.64 %	-4.71 %	756
	Moody's Corp	93.301	69.052	109.402	22.43 %	8.30 %	756
	NASDAQ OMX Group	50.829	32.270	70.309	19.94 %	21.62 %	756
	U.S. Bancorp	40.746	35.752	51.698	18.69 %	11.39 %	756
	Wells Fargo	48.166	39.928	56.104	19.67 %	10.08 %	756
	Average				**22.67 %**	**10.57 %**	
Information Tech	Oracle Corporation	38.776	33.162	44.360	20.46 %	1.94 %	756
	QUALCOMM Incorporated	60.930	40.687	74.086	27.31 %	-0.99 %	756
	Accenture plc	93.488	71.383	124.106	19.83 %	15.55 %	756
	Intel Corporation	29.918	21.149	36.992	23.04 %	15.60 %	756
	NVIDIA Corporation	31.513	14.658	116.951	35.43 %	91.57 %	756
	Activision Blizzard, Inc.	28.303	16.119	45.198	29.02 %	27.12 %	756
	Electronic Arts Inc.	57.373	21.540	85.560	31.08 %	51.10 %	756
	Cisco Systems, Inc.	25.164	19.040	30.870	20.60 %	14.47 %	756
	Apple Inc.	101.426	66.432	126.932	24.00 %	15.87 %	756
	Microsoft	45.622	31.725	62.552	23.52 %	21.93 %	756
	Average				**25.43 %**	**25.42 %**	
Healthcare	Johnson & Johnson	98.598	78.070	121.332	14.83 %	11.26 %	756
	Pfizer Inc.	29.792	24.853	35.569	18.33 %	5.81 %	756
	Abbott Laboratories	40.489	33.075	48.766	20.99 %	2.46 %	756
	Baxter International Inc.	39.218	31.634	48.761	18.66 %	7.48 %	756
	Lilly (Eli) & Co.	68.303	45.689	85.407	23.24 %	16.44 %	756
	Aetna Inc	98.536	62.818	133.674	25.35 %	23.69 %	756
	Amgen Inc	140.646	102.169	170.849	25.86 %	10.46 %	756
	Boston Scientific	17.107	11.370	24.480	25.58 %	22.01 %	756
	Merck & Co.	53.556	44.350	63.529	20.54 %	9.36 %	756
	Mylan N.V.	50.454	34.140	76.060	37.91 %	-3.60 %	756
	Average				**23.13 %**	**10.54 %**	
Industrial	The Boeing Company	126.504	103.523	154.315	21.85 %	7.31 %	756
	United Technologies Corporation	100.684	81.068	115.927	17.50 %	1.74 %	756
	Lockheed Martin Corporation	192.687	131.771	264.108	16.89 %	23.24 %	756
	3M	147.515	112.718	176.404	16.19 %	11.74 %	756
	Eaton Corporation plc	60.355	44.445	71.133	24.31 %	-0.62 %	756
	Caterpillar Inc.	79.483	54.988	99.313	24.36 %	4.60 %	756
	Deere & Co.	81.390	69.100	102.342	22.31 %	7.44 %	756
	General Electric	25.993	21.702	31.911	18.28 %	8.25 %	756
	Rockwell Collins	82.737	69.096	95.708	18.28 %	9.91 %	756
	Dover Corp.	68.886	50.781	84.954	24.60 %	0.42 %	756
	Average				**20.46 %**	**7.40 %**	
Consumer	Ford Motor Co.	12.851	10.349	15.125	23.20 %	-3.36 %	756
	Coca-Cola	124.882	64.328	214.999	32.42 %	37.15 %	756
	Pepsi	91.164	69.794	106.901	13.94 %	11.57 %	756
	Amazon.com, Inc.	503.404	286.950	844.360	31.84 %	23.51 %	756
	Altria Group Inc.	49.651	29.542	67.487	16.26 %	26.42 %	756
	Macy's, Inc.	47.689	28.294	66.875	30.54 %	-9.92 %	756
	Costco Wholesale Corporation	131.183	98.244	160.551	16.67 %	13.24 %	756
	The Procter & Gamble Company	76.398	63.795	87.245	14.10 %	4.74 %	756
	Kellogs	65.141	51.471	84.611	17.22 %	9.72 %	756
	McDonald's Corp.	99.520	81.848	126.910	15.85 %	11.67 %	756
	Average				**21.20 %**	**12.47 %**	
Energy	Total SA	47.157	37.037	62.109	26.14 %	0.26 %	756
	Exxon Mobil Corporation	81.258	63.365	92.545	19.50 %	-0.10 %	756
	Chevron Corporation	95.951	64.318	117.847	23.34 %	2.33 %	756
	Valero Energy	53.219	39.727	68.507	31.20 %	14.04 %	774
	ConocoPhillips	54.090	30.793	78.389	32.93 %	-7.01 %	756
	Williams Cos.	34.245	10.081	53.022	58.42 %	-0.78 %	756
	Southwestern Energy	23.566	5.150	48.930	60.26 %	-34.76 %	756
	Cabot Oil & Gas	27.820	14.902	41.057	36.61 %	-15.28 %	774
	Schlumberger Ltd.	81.555	58.717	109.790	24.69 %	0.34 %	756
	Helmerich & Payne	66.225	39.286	102.458	40.24 %	1.93 %	756
	Average				**35.33 %**	**-3.90 %**	