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MASTER'S THESIS

IMPACT OF UNSYMMETRICAL LOADS IN DISTRIBUTION NETWORKS

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Abstract

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Impact of unsymmetrical loads in distribution networks

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Keywords: Distribution networks, voltage unbalance, unsymmetrical load, transformer.

Since it is virtually impossible to balance loads in three-phase system, unbalance in a varying degree exists almost in all distribution networks. The aim of the thesis is to analyze the impact of this unbalance subject to different configurations of distribution system and winding connection of the supplying transformer. Also impact of the voltage unbalance on the equipment is investigated.

In order to make the investigation more visual, the following calculations have been conducted:

- Unsymmetrical load in four-wire star connected network
- Unsymmetrical load in four-wire star connected network with broken zero conductor (or three-wire network).
- Unsymmetrical load when the supplying transformer is so-called zigzag transformer.

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Lappeenranta, May 2009 Mansur Gapuev

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Abbreviations and symbols

Roman letters

a	Operator
B	Capacitive susceptance
e	Electromotive force
$\mathbf{J}$	Jacobian matrix
I	Current
$\mathbf{I}$	Current matrix
I_{A0}, I_{B0}, I_{C0}	Zero-sequence currents
I_{A1}, I_{B1}, I_{C1}	Positive-sequence currents
I_{A2}, I_{B2}, I_{C2}	Negative-sequence currents
k_0	Coefficient unsymmetry by zero-sequence
k_2	Coefficient unsymmetry by negative-sequence
l	Line length
L	Inductance
r_l	Per-kilometer resistance
r_k	Short circuit resistance
R	Resistance
R_k	Resistance of the transformer
P	Active power
Q	Reactive power
S	Apparent power
U	Voltage
$\mathbf{U}$	Voltage matrix
U_{A0}, U_{B0}, U_{C0}	Zero-sequence voltages
U_{A1}, U_{B1}, U_{C1}	Positive-sequence voltages
U_{A2}, U_{B2}, U_{C2}	Negative-sequence voltages
x_l	Per-kilometer reactance
x_k	Short circuit reactance

X	Reactance
X_k	Reactance of the transformer
$\mathbf{Y}$	Bus admittance matrix
y_{ij}	Elements of admittance matrix
Z	Impedance
Z_k	Impedance of the transformer

Greek letters

ϕ	Angle between voltage and current
δ	Angle between beginning and end.

Subindexes

add	Additional
eqv	Equivalent
GRD	Grounded
L	Line voltage
L	Load
n	Nominal
p	Phase voltage
D	Delta connection
S	Star connection

Acronyms

AC	Alternating current
cos (φ)	Power factor
D,Δ	Delta connection
DC	Direct current
DS	Distribution system

HV	High voltage
LV	Low voltage
MV	Medium voltage
N,n	Neutral
NR	Newton-Raphson
NEMA	National Electrical Manufacturers Association
PE	Protective earth
SWER	Single wire earth return systems
SCADA	Supervisory for Control And Data Acquision
SCR	Silicon controlled rectifier
THD	Total harmonic distortion
TT	Direct connection of a point with earth, direct connection with earth, independent of any other earth connection in the supply system
TN-S	PE and N are separate conductors that are connected together only near the power source
TN-C-S	Part of the system uses a combined PEN conductor, which is at some point split up into separate PE and N line.
TN-C	A combined PEN conductor fulfills the functions of both a PE and N conductor
Y	Star connection
Z	Zigzag connection

Introduction

Electricity is one of the most important components of the development of the modern society. We cannot imagine our life without electric lighting, habitual electric devices, electric heating and conditioning. Some present-day devices need qualitative electricity for correct and long work. In simple words, power quality can be characterised by sinusoidal voltage source, without waveform distortion, variation in amplitude or frequency.

A distribution system's network carries electricity from the transmission system and delivers it to consumers. Typically, the network includes medium-voltage (less than 50 kV) power lines, electrical substations and usually pole-mounted transformers, low-voltage (less than 1000 V) distribution wiring and sometimes electricity meters.

One of the main characteristics of distribution system is reliability. Reliability characterises the ability of the system to withstand to one or another anomalous situation. Anomalous situation can be different in type, seriousness, time of action. One of the most widespread anomalous situations in distribution systems is unsymmetrical situations which often happen because of unsymmetrical loads, short circuits in one or two phase and some other reason, about which will be said later.

In the thesis the all above-mentioned issues are investigated.

1.1 The history of evolution of distribution systems

At the very beginning of electricity generation, direct current (DC) generators were connected to loads at the same voltage, as at the time it was not known the efficient way to transform the level of DC voltage. The voltages had to be significantly low with such systems because it was difficult and dangerous to distribute high voltages to small loads. As we know, the losses in a conductor are proportional to the square of the current, the length of the conductor, and the resistivity of the material, and are inversely proportional to cross-sectional area.

Early transmission networks were already from copper conductors, which is one of the best economically and technically feasible conductors for this application. To decrease the current while keeping power transmission constant requires increasing the voltage which, as previously mentioned, was problematic. This meant in order to keep losses to a reasonable level the Edison system needed thick cables and a lot of local generators to provide with electricity large area. . Because of above-listed reasons, the transmission/distribution systems were not extended from the point of generation and were within about 1.5 miles (2.4 km).

The most considerable changes in the electricity generation and transmission occurred after the adoption of alternating current (AC) following the War of Currents. Power transformers, installed at substations, enabled to raise the voltage from the generators and reduce it to supply loads. The higher voltage the lower current was necessary in the transmission and distribution lines and consequently

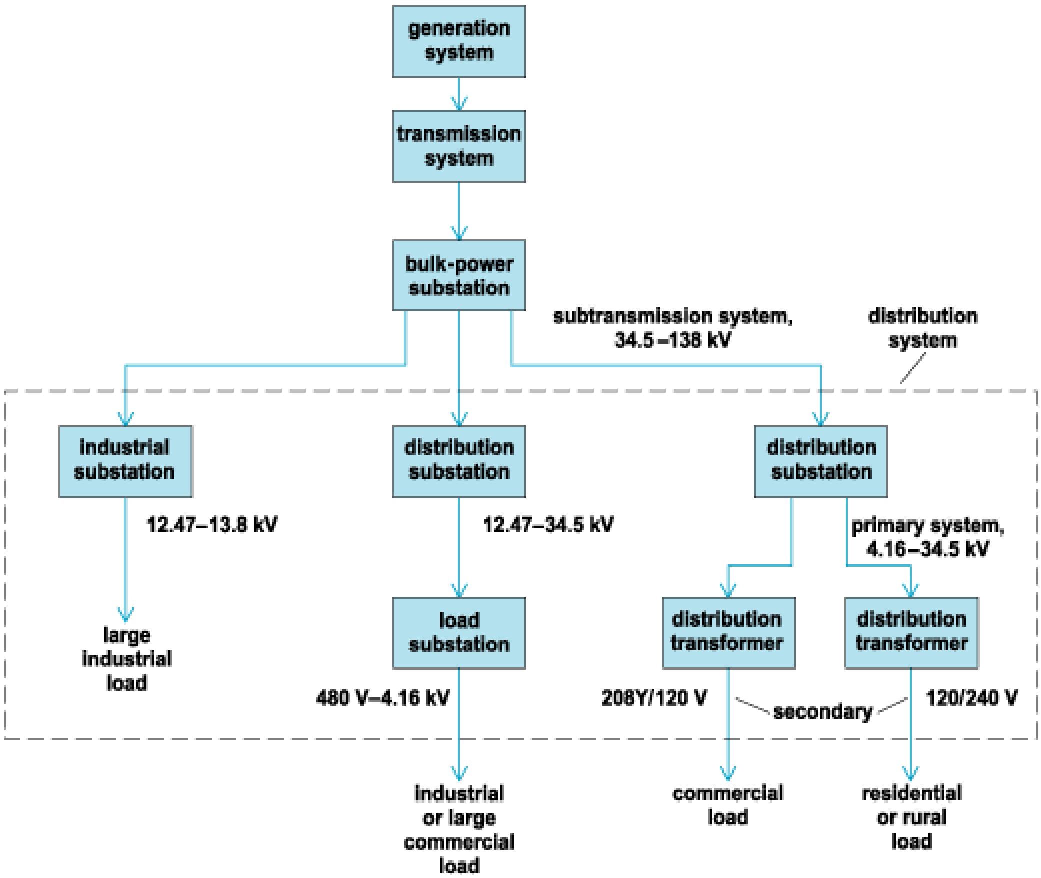

Fig.1.1. Overview of the power system from generation to consumer's switch

the size of conductors required and distribution losses incurred. This made it more economic to distribute power over long distances with acceptable losses. The ability to transform to extra-high voltages enabled generators to be located far from consumers with transmission systems to interconnect generating stations and distribution networks. Early distribution systems in North America used a voltage of 2200 volts corner-grounded delta. Gradually this was increased to 2400 volts. As cities grew, most 2400 volt systems were upgraded to 2400/4160 Y three-phase systems, which also benefited from better surge suppression due to the grounded neutral. Some city and rural distribution systems continue to use this range of voltages, but most have been converted to 7200/12470Y.

European systems used higher voltages, generally 3300 volts to ground, in support of the 220/380Y volt power systems used in those countries. In the UK, urban systems progressed to 6.6 kV and then 11 kV (phase to phase), the most common distribution voltage.

1.2 Distribution network configuration

Distribution networks can be divided into two types - radial and interconnected. The difference between them is that the radial network leaves the station and passes through the network area without any connection to other supply. It is more typical for long rural lines with isolated load areas. Interconnected networks have multiple connections to other points of supply and can be mainly met in urban areas.

The interconnected model is more desirable in the areas with important customers, such as, for example, hospitals or industry. In case of fault situations or required maintenance, the out-of-order area can be separated from undamaged part by opening the switches. Operation of these switches may be by remote control from a control centre or by a lineman.

Distribution networks are usually performed in the form of overhead lines with traditional utility poles and wires and, increasingly, underground construction with cables and indoor substations. Although, underground distribution is significantly more expensive than overhead construction, they are used when it is not eligible to use the overhead lines (For example, in urban areas with expensive cost of the land). Distribution feeders emanating from a substation are generally controlled by a circuit breakers which will open when a fault is detected. Automatic Circuit Reclosers may be installed to further separate the feeder thus minimizing the impact of faults.

The main characteristics of electricity supply to customers are listed below:

- AC or DC - Virtually all public electricity supplies are AC today. Users of large amounts of DC power such as some electric railways, telephone exchanges and industrial processes such as aluminium smelting usually either operate their own or have adjacent dedicated generating equipment, or use rectifiers to derive DC from the public AC supply
- Voltage, including tolerance (usually +10 or -15 percentage)
- Frequency, commonly 50 & 60 Hz, 16-2/3 Hz for some railways and, in a few older industrial and mining locations, 25 Hz. [1]
- Phase configuration (single phase, polyphase including two phase and three phase)
- Maximum demand (usually measured as the largest amount of power delivered within a 15 or 30 minute period during a billing period)
- Load Factor, expressed as a ratio of average load to peak load over a period of time. Load factor indicates the degree of effective utilization of equipment (and capital investment) of distribution line or system.
- Power factor of connected load
- Maximum prospective short circuit current
- Maximum level and frequency of occurrence of transients
- Earthing arrangements - TT, TN-S, TN-C-S or TN-C

Different types of earthing arrangements are shown in the figure 1.2

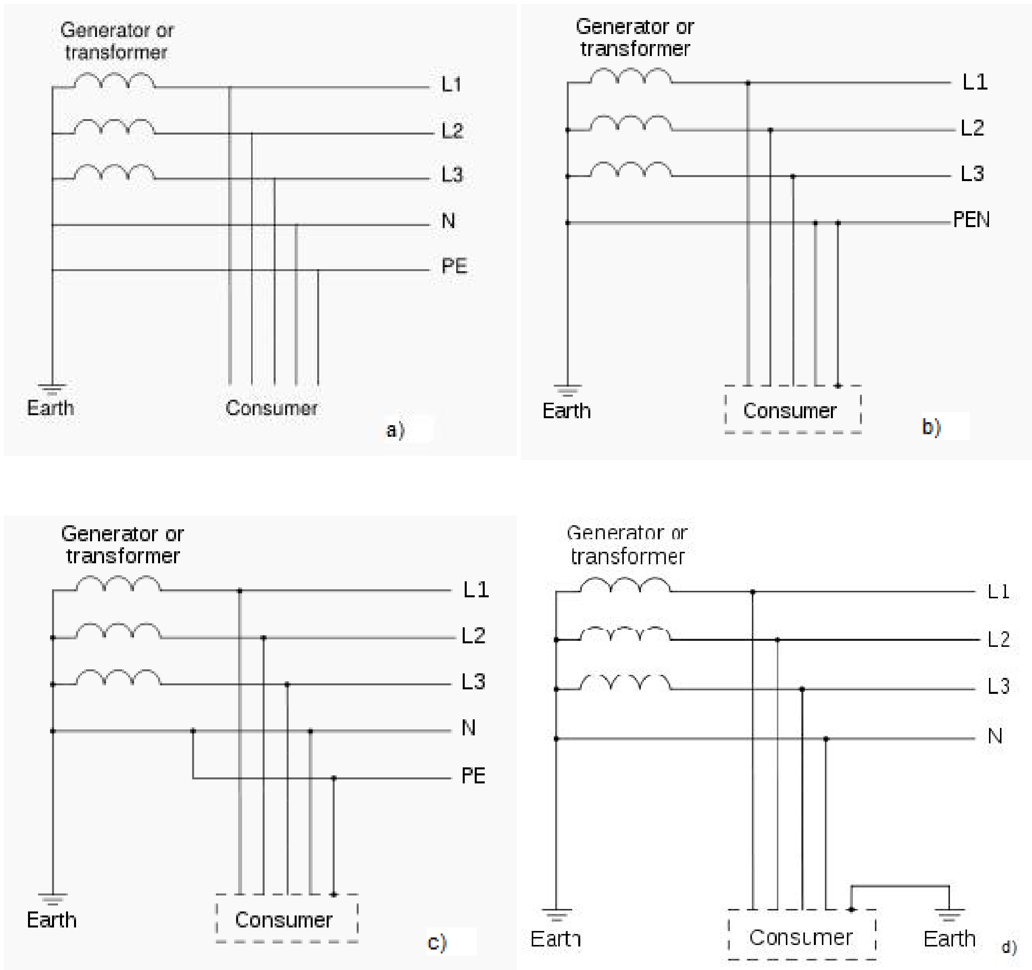

Fig.1.2. Earthing arrangements. a) **TN-S:** separate protective earth (PE) and neutral (N) conductors from transformer to consuming device, which are not connected together at any point after the building distribution point. b) **TN-C:** combined PE and N conductor all the way from the transformer to the consuming device. c) **TN-C-S earthing system:** combined PEN conductor from transformer to building distribution point, but separate PE and N conductors in fixed indoor wiring and flexible power cords. d) **TT**, the protective earth connection of the consumer is provided by a local connection to earth, independent of any earth connection at the generator.

1.3 Different types of distribution systems used worldwide

In different countries, in process of evolution of distribution systems, there have appeared some differences among them. There are such differences between European and North American distribution systems, between British and Norwegian and so on. Of course, every DS has its own advantages and disadvantages as compared to the others [6].

In **the North American** distribution system in a MV network, a neutral conductor is used, which is earthed at the distance of 300m. As well, the branch lines are usually single-phase or two phase.

There are circuit breakers on the main lines, on the branch-offs, there are fuses and sectionalizers that automatically open the circuit when a fault occurs.

And, another thing is transformer. The transformers are single-phase construction, and they are coupled between the phase and the neutral.

The British distribution system differs from the American system for example, by the fact that neutral point of the supplying transformers is earthed through a resistance. Distribution systems of low rated distribution transformers are three phase units. In the traditional British system, there are two medium-voltages in the same geographic region: 33 kV and 11 kV. Underground cabling is more common than in America, although less common than in the western continental Europe. The phase voltage in the low-voltage side is 240 V. Residential areas built up with single-family houses and terrace houses are often supplied with a sturdy three-phase low-voltage cable, from which short, single phase service lines leave at fixed connections.

In **the traditional Norwegian** distribution system the neutral point of the secondary winding is not earthed, there is no neutral conductor in the distribution line, and the 230 V equals to the phase to phase voltage. The frames of the load equipment are earthed, although this practise is gradually disappearing.

In Finland, in installation inside buildings, the neutral from the transformer substation has traditionally been used as the return conductor of the single phase loads. Traditionally, the neutral has also been connected to the conductive frames of devices (neutral as protective earthing), although from 1990 this practise has been replaced by so-called five conductor system, in which a separate protective conductor from the main distribution board of the building is coupled to the frames of the devices.

The five conductor system enables indicating of a low current earth contact.

North American and European power distribution systems also differ in that North American systems tend to have a greater number of low-voltage, step-down transformers located close to customers' premises. For example, in the US a pole-mounted transformer in a suburban setting may supply 1-3 houses, whereas in the UK a typical urban or suburban low-voltage substation would normally be rated between 315kVA and 1000kVA (1MVA) and supply a whole neighbourhood. This is because the higher voltage used in Europe (415V vs. 230V) may be carried over a greater distance with acceptable power loss. An advantage of the North American setup is that failure or maintenance on a single transformer will only affect a few customers. Advantages of the UK setup are that the transformers may be fewer, larger and more efficient, and due to diversity there need be less spare capacity in the transformers, reducing power wastage. In North American city areas with many customers per unit area, network distribution will be used, with multiple transformers and low-voltage busses interconnected over several city blocks.

Rural Electrification systems, in contrast to urban systems, tend to use higher voltages because of the longer distances covered by those distribution lines. 7200 volts is commonly used in the United States; 11 kV and 33 kV are common in the UK, New Zealand and Australia; 11 kV and 22 kV are common in South Africa. Other voltages are occasionally used in unusual situations or where a local utility simply has engineering practices that differ from the norm.

In New Zealand, Australia, Saskatchewan, Canada and South Africa, single wire earth return systems (SWER) are used to electrify remote rural areas.

1.4 Summary

In the process of engineering of new electrical transmission and distribution networks it is essential to choose the appropriate system design philosophy in order to correspond to the local social and economic conditions. It applies also to reinforcement or replacement of the outdated networks, although in this situation there exist some previous groundwork.

Changing from one practice of building of distribution systems (for example, from UK type to USA), even if better in some parameters, is seldom economically justified, at least in the short term.

2 Theory (symmetrical components) of load flow calculation for unsymmetrical situations.

As it was said previously, one of the reasons of deterioration of the quality of electrical supply is unsymmetrical situations. Unsymmetrical situations are undesirable disturbance in work of distribution systems which occur mainly because of unsymmetrical loads or short circuit. Below unsymmetrical situations produced by unsymmetrical loads, consequences of unsymmetrical situations and the ways to avoid them will be described.

2.1 Sources of voltage unsymmetry and the ways to reduce it

The main sources of voltage unsymmetry are arc steel-smelting furnaces, traction substations of alternating current, electric welding machines, single-phase thermal electric installations or any powerful single-phase, two-phase or three-phase unsymmetrical consumers of electric power, including domestic. For example, the summary load of some factories contain 85....90% unsymmetrical load. Thus, coefficient of unsymmetry by zero-sequence (k_{0U}) a nine-storied inhabited building can amount to 20%, which on the substation busses (the point of common joining), can exceed normally admissible 2%.

The main ways to avoid the unsymmetry are

- Uniform distribution of loads in the phases
- Application of the symmetric installations

2.2 Consequences of unsymmetrical situations

The main disadvantage of unsymmetrical situations is that they bring to unsymmetry of voltage. Unsymmetrical load currents flowing through elements of electrical supply cause unsymmetrical voltage drop. As a consequence, there appears an unsymmetrical system of voltages on the leads of the electrical receiver. Deviation of voltage of overloaded phase can exceed admissible level, when the deviation of voltages of the rest can be within normal limits. Besides the deterioration of the voltage, under unsymmetrical situations the conditions of work of electrical receivers and the most of elements of distribution network become much worse, also the reliability of the whole system decreases.

Unsymmetrical situations influence considerably on the mode of operation of **asynchronous motors**, the widespread three-phase electrical receivers, for which the particular significance has a voltage of negative sequence. Resistance of negative sequence of electric motors is equal to resistance of braked motor; consequently, it is in 5-8 times less then resistance of positive-sequence. Therefore, even not large unsymmetry of the voltages causes considerable currents of negative sequence. They superimpose on the currents of positive-sequence and produce the additional heating of the stator and the rotor (especially massive part of the rotor), which, in turn, results in rapid ageing of the isolation and decrease of available power of the motor (decrease of the efficiency). For example, the lifetime of the completely loaded asynchronous motor, working under unsymmetry of the voltage about 4% shortens in two times. Under unsymmetry of voltage 5%, available power decreases to 5-10%.

Under unsymmetry of the voltages, in **synchronous machines** besides the rise of additional losses of active power and heating of the stator and the rotor, there may arise dangerous vibrations as a result of appearance sign-changing rotating moments and tangential forces, pulsating with double frequency of the network. When the unsymmetry is considerable, the vibration may be dangerous, in particular if the durability of the details is not sufficient and there are defects in the welded connection. When the unsymmetry of the currents does not exceed 30%, the dangerous overstrains, as a rule, do not appear.

In case of presence of direct-sequence and negative-sequence currents, the summary currents in separate phases of the **distribution networks** increase, which brings to increase of the losses, which is usually not permissible in view of heating. The currents of zero-sequence always flow through grounding electrode. It dries out and increases the resistance of grounding elements. It can be inadmissible from the point of view of working the relay protection, as well as because of strengthening impact on low-frequency communication settings and means of the railway blockage.

The voltage unsymmetry noticeably worsens the mode of operation of the **multiphase gated rectifiers**: the rippling of the rectified voltage considerably increases the conditions of work of the thyristor converters also deteriorate.

Under unsymmetrical voltages, **condenser installations** load by reactive power irregularly from each phase, which makes impossible using rated condenser power. In addition, in this case condenser installations strengthen already existing unsymmetry, as the output of the reactive power into network in phase with the least voltage will be lower, than in the other phases (proportionally to the square of the voltage on the condenser installation).

Unsymmetry of the voltage also influenced monophase electric receivers. For example, if the phase voltages are not equal, incandescent lamps, connected to the phase with higher voltage have bigger luminous flux, but considerably lower lifetime as compared with lamps, connected to the phase with lower voltage. Unsymmetry of voltages also complicate functioning of the relay protection, leads to errors in operation of the electricity meters and so on.

The general influence of unsymmetrical voltages on the electrical machines, different devices, lamps, conductors, transformers is considerable decrease of their lifetime.

2.3 Theory of symmetrical components for unsymmetrical situations

The theory of symmetrical components allows comparatively simplify computation of unsymmetrical situations. The essence of this theory is that any unsymmetrical three-phase system of vectors (currents, voltages) can be represented as three symmetrical systems. One of them has a positive sequence of phase interlacing ($\mathbf{A_1} \rightarrow \mathbf{B_1} \rightarrow \mathbf{C_1}$), the other has negative ($\mathbf{A_2} \rightarrow \mathbf{C_2} \rightarrow \mathbf{B_2}$). The third system is called zero-sequence system and consists of three equal vectors, coinciding in phase ($\mathbf{A_0}, \mathbf{B_0}, \mathbf{C_0}$) [8].

Thus, for each phase one can write

$$\begin{aligned} \mathbf{A} &= \mathbf{A_1} + \mathbf{A_2} + \mathbf{A_0} \\ \mathbf{B} &= \mathbf{B_1} + \mathbf{B_2} + \mathbf{B_0} \\ \mathbf{C} &= \mathbf{C_1} + \mathbf{C_2} + \mathbf{C_0} \end{aligned} \qquad \mathbf{(2.1)}$$

The system of quantities of positive sequence

$$\mathbf{A_1;\ B_1 = A_1}\,a^2\mathbf{;\ C_1 = A_1}\,a. \qquad \textbf{(2.2a)}$$

The system of quantities of negative sequence

$$\mathbf{A_2;\ B_2 = A_2}\,a\mathbf{;\ C_2 = A_2}\,a^2. \qquad \textbf{(2.2b)}$$

The system of quantities of zero sequence

$$\mathbf{A_0 = B_0 = C_0} \qquad \textbf{(2.2c)}$$

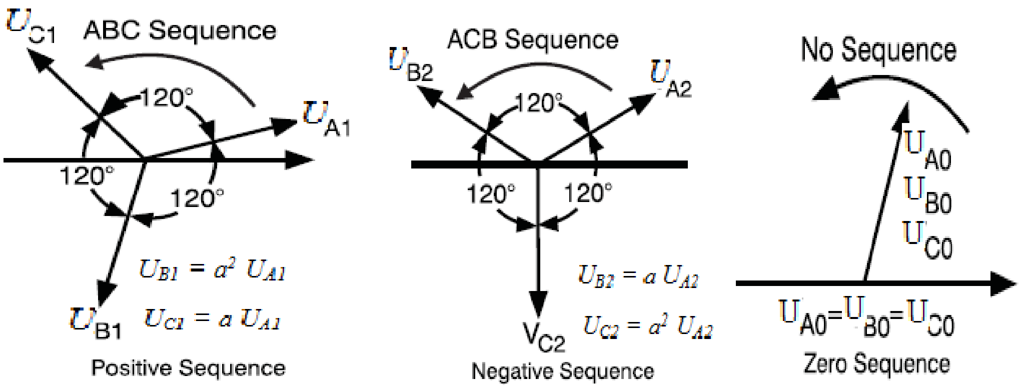

Fig. 2.1. Symmetrical components.

Multiplication the vector by a means its rotation to 120^0 contraclockwise. The rotation the vector to 240^0 can be represented by multiplying it by a^2.

where a is operator, $a = e^{j120^0}$, or in complex form

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \qquad \textbf{(2.3)}$$

In complex number theory, we defined j as the complex operator which is equal to √-1 and a magnitude of unity, and more importantly, when operated on any complex number rotates it anti-clockwise by an angle of 90^0

I.e. $j = \sqrt{-1}$

For the operator, these equations hold true

$$\begin{aligned} &a^2 + a + 1 = 0 \\ &a^3 = e^{j2p} = 1 \\ &a^4 = a^3 a = a \end{aligned} \qquad \textbf{(2.4)}$$

The first equation from (2.4) is shown on the fig. 2.2

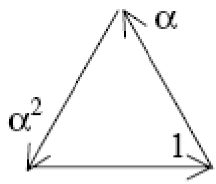

Fig. 2.2.Phasor addition

From the equations **(2.2)** it follows, that when we use the method of symmetrical components, it is enough to calculate the values for any single phase, for example **A**, after which it is not difficult to determine the symmetrical components for the rest two phases and the whole values of respective phase values, that is:

$$\begin{aligned} \mathbf{A} &= \mathbf{A_1} + \mathbf{A_2} + \mathbf{A_0} \\ \mathbf{B} &= \mathbf{A_1}a^2 + \mathbf{A_2}a + \mathbf{A_0} \\ \mathbf{C} &= \mathbf{A_1}a + \mathbf{A_2}a^2 + \mathbf{C_0} \end{aligned} \qquad \textbf{(2.5)}$$

Thus, instead of one unsymmetrical circuit, one calculates three, but considerably more easier, which makes the whole calculation significantly simpler.

The symmetrical components of the phase **A**, for example, can be derived if one knows the whole values of the phase quantities. The equation for determination the component $\mathbf{A_1}$ can be obtained by multiplication the second and third equations of the system **(2.5)** by a and a^2 respectively and following summation of all equations of this system. As a result, we will get

$$\mathbf{A}_1 = \frac{\mathbf{1}}{\mathbf{3}}\left(\mathbf{A} + a\mathbf{B} + a^2\mathbf{C}\right) \qquad \textbf{(2.6a)}$$

Similarly, the equation for determination the component $\mathbf{A_2}$ can be obtained by multiplication the second and third equations of the system **(2.5)** by a and a^2 respectively and following summation of all equations of this system. As a result, we will get

$$\mathbf{A}_2 = \frac{\mathbf{1}}{\mathbf{3}}\left(\mathbf{A} + a^2\mathbf{B} + a\,\mathbf{C}\right) \qquad \textbf{(2.6b)}$$

The equation for determination $\mathbf{A_0}$ can be obtained by summation the all three equations of the system **(2.5)**

$$\mathbf{A_0} = \frac{\mathbf{1}}{\mathbf{3}}(\mathbf{A} + \mathbf{B} + \mathbf{C}) \qquad \textbf{(2.6c)}$$

By application of the equations **(2.6),** it is not difficult to determine the symmetrical components of given system of vectors and graphical way as it is shown on the figure 2.3.

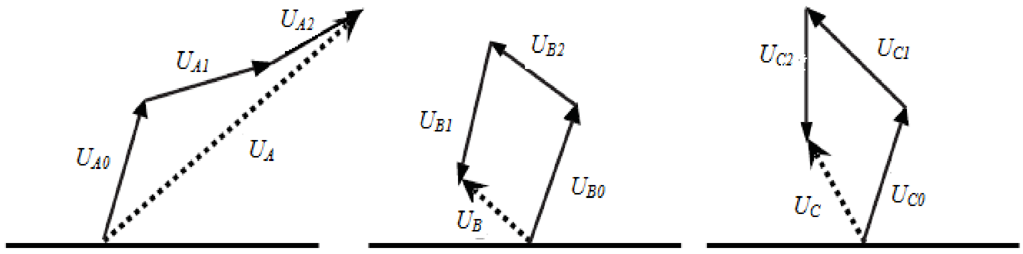

Fig.2.3. Graphical construction for the determination of symmetrical components

Geometrical vector sums of the positive and negative sequences of three phases, as for any balanced systems, are equal to zero. As opposed to this, the system of quantities of zero-sequence, as it follows from **(2.2)** is not balanced, that is

$$\mathbf{A_0} + \mathbf{B_0} + \mathbf{C_0} = \mathbf{3\ A_0} \neq 0 \qquad \textbf{(2.7)}$$

All above mentioned equations hold true for currents and voltages under unsymmetrical situations in any three-phase electrical installations.

Unsymmetrical currents, flowing in phases of the circuit, cause unsymmetrical voltage drop in the resistances of the phases, which can be decomposed into symmetrical components. The voltage drop of positive sequence is caused by the current of positive sequence; the voltage drop of negative sequence is caused by the current of the negative sequence and so on, that is, the current of each sequence creates the voltage drop of respective sequence.

For different sequences the resistances of the elements of three-phase circuit can differ by values.

2.4 Summary

As one can see the voltage unbalance is very dangerous on the one side and cause great losses in different equipment on the other. Also it worsens the mode of operation of some instalments such as multiphase gated rectifiers. So the rate of unbalance should be kept in acceptable ranges. To do this, the level of unbalance should be calculated by one or another method.

The solution of unbalanced electrical circuits is considerably easy with the method of symmetrical components and in the case of extended networks it is the only acceptable method. It is very powerful analytical tool which is used by a great number of computing programs.

The unbalance can be avoided if to distribute the loads in the phases in the appropriate way. Also there exist some balancing instalments to level out the unbalance.

3 Mathematical equations for calculating the load flow (currents, voltages, losses)

Mathematical equations for calculating the load flow can be used both in manual and automatic calculations of the state of the electric networks. The load flow calculations are fulfilled in order to keep the system running in a stable and safe state and are used to determine possible or optimal choice of the network's components (transformers' voltage regulators, automatic control settings of the machine regulators). The determining inputs are usually the voltages and/or currents and/or the active/reactive power at the consumer's port. Conductors - overhead lines and cables – are important elements, so, on the one hand, the reason of such calculations to find out, whether they will withstand such a state in normal conditions, and, from the other hand, to find out their influence on the load flow. In order to carry out load flow calculations in a simple way, it is common practice to use as few circuit elements as is possible for the given task. In the case of low voltage lines in most cases an ohmic resistance will do and even for high voltage lines in most cases the longitudinal impedance is taken into consideration.

As it said above, the power flow calculations are conducted to find out the best solutions for constructing and maintenance of the electric networks. During the load flow studies there used both initial data and some special methods for finding out one or several unknown parameters of the networks.

For different elements of power energetic there is a different set of initial parameters [7].

- **Power plants**
  - Supplied active power Pg
  - Terminal voltage U, to be maintained at the plant
  - Reactive power generation and consumption capacity (Q_{max}, Q_{min})
- **Lines**
  - Impedances of the equivalent circuit (R, jX, G, jB)
- **Transformers**

- Short-circuit impedance *(R_k, jX_k)*

Ø **Compensation devices (compensators)**

- Impedance (*R, jX)*

Ø **Loads**

- Active and reactive power *(P, jQ)*

Besides the constant parameters of network, there are varying amount of controller data:

Ø **On-load tap-changer data (position, number and size of the steps)**

- Is stepping automatic; if so, on what criterion?

Ø **Control principles of compensators**

Ø **Power of interconnectors between subsystems**

- Regulating power plants

Ø **Control principles for DC links (Finland-Sweden, Finland-Russia)**

Also, in the calculations, there used following control parameters:

Ø method, convergency criterion, number of iterations, blockings (Tap changers, compensators)

At the beginning of this chapter, the most common equations for single-phase and three-phase circuits will be reviewed and after that load flow equations are described.

3.1 Short review of the simplest equations

In a balanced three phase system, knowledge of one of the phases gives the other two phases directly. However this is not the case for an unbalanced supply. In a star connected supply, it can be seen that the line current (current in the line) is equal to the phase current (current in a phase). However, the line voltage is not equal to the phase voltage. The line voltages are defined as

$$\begin{aligned} U_{RY} &= U_R - U_Y, \\ U_{YB} &= U_Y - U_B, \\ U_{BR} &= U_B - U_R. \end{aligned} \tag{3.1}$$

Figure 3.1(a) shows how the line voltage may be obtained using the normal parallelogram addition. It can also be seen that triangular addition (Fig3.1 (b)) also gives the same result faster.

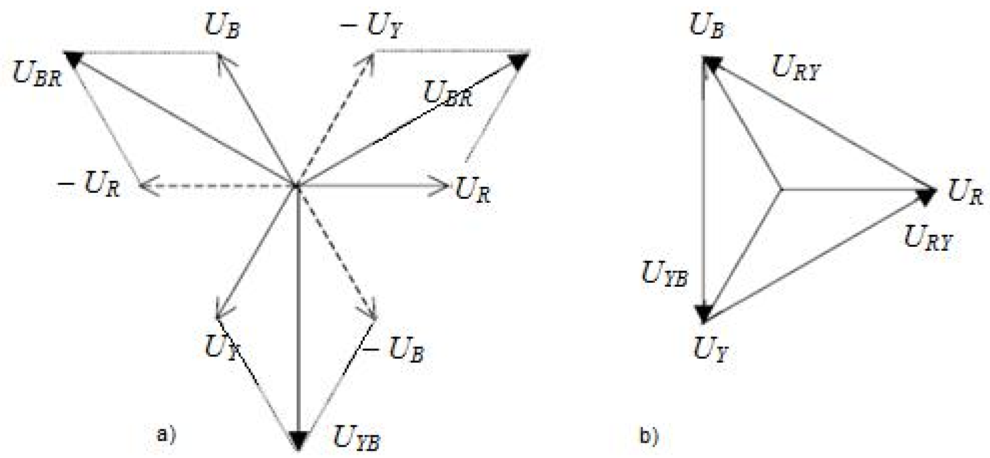

Fig 3.1. Parallelogram (a) and triangular (b) additions.

For a balanced system, the angles between the phases are 120^0 and the magnitudes are all equal. Thus the line voltages would be 30^0 leading the nearest phase voltage. Calculation will easily show that the magnitude of the line voltage is √3 times the phase voltage.

$$I_L = I_P,\ \ |U_L| = \sqrt{3}\,|U_P|\ ,\ \ |I_L| = \sqrt{3}|I_d| \qquad (3.2)$$

Similarly in the case of a delta connected supply, the current in the line is √3 times the current in the delta.

It is important to note that the three line voltages in a balanced three phase supply is also120^0 out of phase, and for this purpose, the line voltages must be specified in a sequential manner. i.e. U_{RY}, U_{YB} and U_{BR}. [Note: U_{BY} is 180^0 out of phase with V_{YB} so that the corresponding angles if this is chosen may appear to be 60^0 rather than 120^0].

A balanced load would have the impedances of the three phase equal in magnitude and in phase. Although the three phases would have the phase angles differing by 120^0 in a balanced supply, the current in each phase would also have phase angles differing by 120^0 with balanced currents. Thus if the current is lagging (or leading) the corresponding voltage by a particular angle in one phase, then it would lag (or lead) by the same angle in the other two phases as well (Figure 3.2(a))

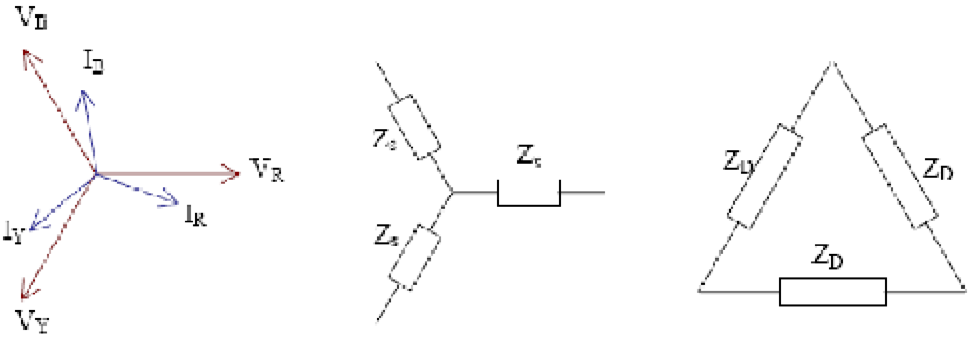

Fig. 3.2 Phasor diagram (a), star connection (b) and delta connection (c)

The balanced load can take one of two configurations – star connection, or delta connection. For the same load, star connected impedance and the delta connected impedance will not have the same value. However in both cases, each of the three phases will have the same impedance as shown in figures 3.2(b) and 3.2(c). It can be shown, for a balanced load (using the star delta transformation or otherwise), that the equivalent delta connected impedance is 3 times that of the star connected impedance. The phase angle of the impedance is the same in both cases. $Z_D = \surd 3\ Z_{star}$.

Note: This can also be remembered in this manner. In the delta, the voltage is √3 times larger and the current √3 times smaller, giving the impedance 3 times larger. It is also seen that the equivalent power is unaffected by this transformation.

Three Phase Power

In the case of single phase, we learnt that the active power is given by

$$P = U\, I \cos f \qquad \textbf{(3.3)}$$

In the case of three phases, obviously this must apply for each of the three phases. Thus

$$P = 3\ U_p\ I_p \cos f \qquad \textbf{(3.4)}$$

However, in the case of three phases, the neutral may not always be available for us to measure the phase voltage. Also in the case of a delta, the phase current would actually be the current inside the delta which may also not be directly available.

It is usual practice to express the power associated with three phase in terms of the line quantities. Thus we will first consider the star connected load and the delta connected load independently.

For a balanced **star** connected load with line voltage U_L and line current I_L,

$$U_{\text{star}} = \frac{U_{\text{L}}}{\sqrt{3}}, \; I_{star} = I_L \qquad \textbf{(3.5)}$$

$$Z_{\text{star}} = \frac{U_{\text{star}}}{I_{\text{star}}} = \frac{U_{\text{L}}}{\sqrt{3} I_{\text{L}}}$$

$$S_{star} = 3U_{star}I_{star} = \sqrt{3}U_LI_L \qquad \textbf{(3.6)}$$

Thus,

$$P_{star} = \sqrt{3}U_LI_L \cos f, \qquad \textbf{(3.7a)}$$

$$Q_{star} = \sqrt{3}U_LI_L \sin f \qquad \textbf{(3.7b)}$$

It is worth noting here, that although the currents and voltages inside the star connected load and the delta connected loads are different, the expressions for apparent power, active power and reactive power are the same for both types of loads when expressed in terms of the line quantities.

Thus for a three phase system (in fact we do not even have to know whether it is a load or not, or whether it is star-connected or delta-connected)

Apparent Power	$S = \sqrt{3}U_LI_L$	**(3.8a)**
Active Power	$P = \sqrt{3}U_LI_L \cos f$	**(3.8b)**
Reactive Power	$Q = \sqrt{3}U_LI_L \sin f$	**(3.8c)**

3.2 Load flows in radial and simple loop networks

In radial networks the phase shifts due to transformer connections along the circuit are not usual important because the currents and voltages are shifted by the same amount.

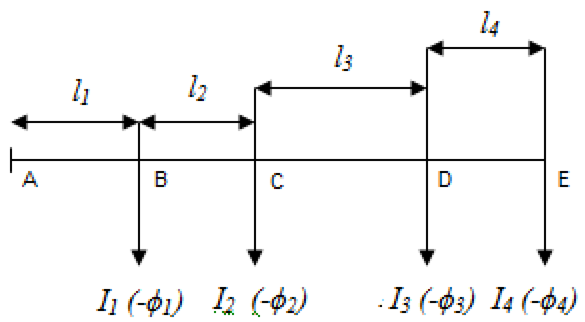

Fig.3.3. Feeder with several load tappings

In figure 3.3 one can see a distribution feeder with several tapped inductive loads (or laterals) and fed at one end. The total voltage drop in this situation is determined by the next way. At first, we determine the current in AB

$$AB = (I_1\cos(\phi_1) + I_2\cos(\phi_2) + I_3\cos(\phi_3) + I_4\cos(\phi_4) -$$
$$j\,(I_1\sin(\phi_1) + I_2\sin(\phi_2) + I_3\sin(\phi_3) + I_4\sin(\phi_4))$$

The currents in the other sections of the feeder are obtained by the same way. And now, it is not difficult to determine the voltage drop from the equation

$$\Delta U = RI\cos(\phi) + XI\sin(\phi) \qquad \textbf{(3.9)}$$

for each section. That is

$$R_1\,(I_1\cos(\phi_1) + I_2\cos(\phi_2) + I_3\cos(\phi_3) + I_4\cos(\phi_4))$$
$$+ R_2\,(I_2\cos(\phi_2) + I_3\cos(\phi_3) + I_4\cos(\phi_4) + R_3\,(I_3\cos(\phi_3) + I_4\cos(\phi_4)$$
$$+ R_4\,(I_4\cos(\phi_4) + X_1\,(I_1\sin(\phi_1) + I_2\sin(\phi_2) + I_3\sin(\phi_3) + I_4\sin(\phi_4))\ \textit{and so on}$$

If the resistance per loop metre (the term loop meter refers to single phase circuit and includes the go and return conductors) is *r* ohms and reactance per loop metre is *x* ohms, we have

$$\Delta U = r\,(I_1 \cdot l_1 \cos(\phi_1) + I_2 \cos(\phi_2) \cdot (l_1+l_2) + I_3 \cos(\phi_3) \cdot (l_1+l_2+l_3)$$
$$+ I_4 \cos(\phi_4) \cdot (l_1+l_2+l_3+l_4)) + x\,(I_1\, l_1 \sin(\phi_1) + I_2 \sin(\phi_2) \cdot (l_1+l_2)$$
$$+ I_3 \sin(\phi_3) \cdot (l_1+l_2+l_3) + I_4 \sin(\phi_4) \cdot (l_1+l_2+l_3+l_4))$$

Load flows in closed loops. In a closed loop in order to avoid the circulating currents, the product of the transformer transformation ratios round the loop should be unity and the sum of the phase shifts in a common direction round the loop should be zero. This is illustrated in the figure 3.4.

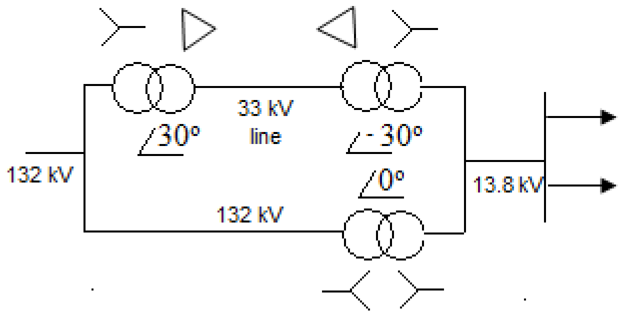

Fig.3.4. Loop with transformer phase shift.

In this example

$$\left(\frac{33}{132}\angle 30^o\right)\cdot\left(\frac{13.8}{33}\angle -30^o\right)\cdot\left(\frac{132}{13.8}\angle 0^o\right) = 1\angle 0^o$$

In practice the transformation ratios of transformers are often changed by means of tap-changing equipment. This results in the product of the ratios round the loop being no longer unity, although the phase shifts are still equal to zero. An undesirable effect in circulating current set up around the loop

Frequently the out-of-balance or remnant voltage represented by the auto-transformer can be neglected. If this is not the case, the best method of calculation is to determine the circulating current and consequent voltages due to the remnant voltage acting alone, and then superpose these values on those obtained for operation with completely nominal voltage ratios.

3.3 Load flow calculations for large systems

The most widespread methods of solving large systems are Gauss-Seidel, which has been used for many years and is simple in approach and Newton-Raphson methods, which although more complex has certain advantages.

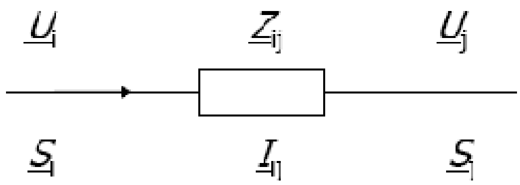

Fig. 3.5. Single node

From Ohm`s law

$$\underline{I}_{ij} = \frac{\underline{U}_i - \underline{U}_j}{\underline{Z}_{ij}} \tag{3.10}$$

$$S_i = U_i \times I_{ij} \qquad S_j = U_j \times I_{ij} \qquad S_h = I^2_{ij} \times Z_{ij} \tag{3.11}$$

To find the voltage in different nodes we can use Kirchhoff’s 1st Law

$$\underline{I}_i = \underline{Y}_{i0}\underline{U}_i + \sum_j \underline{Y}_{ij}\left(\underline{U}_i - \underline{U}_j\right) \tag{3.12}$$

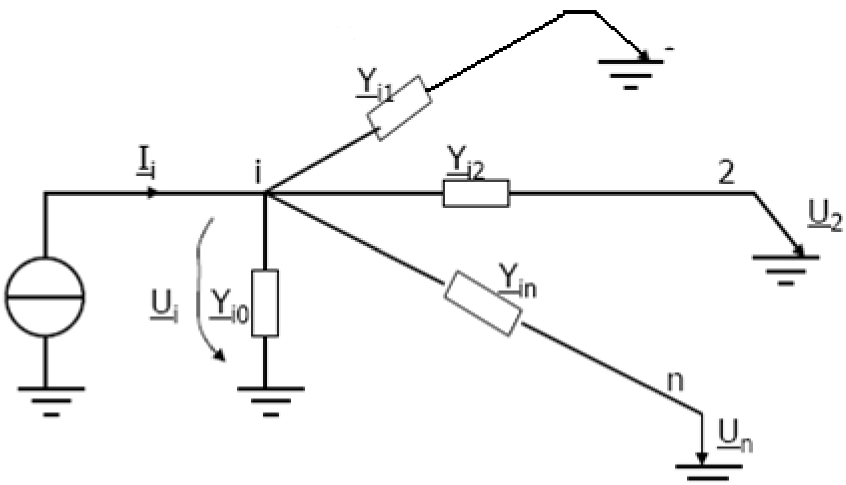

Fig.3.6. To load flow calculations

After grouping, we obtain the following equation

$$\underline{I}_i = \left(\underline{Y}_{i0} + \sum_j \underline{Y}_{ij} \right) \underline{U}_i - \sum_j \underline{Y}_{ij} \underline{U}_j \tag{3.13}$$

And, if to consider all nodes

$$I = \begin{bmatrix} \underline{I}_1 \\ \dots \\ \underline{I}_n \end{bmatrix} = \begin{vmatrix} \underline{y}_{11} & \cdots & \underline{y}_{1n} \\ \dots & \dots & \dots \\ \underline{y}_{n1} & \cdots & \underline{y}_{nn} \end{vmatrix} \begin{bmatrix} \underline{U}_1 \\ \dots \\ \underline{U}_n \end{bmatrix} = YU \tag{3.14}$$

Where **Y** is the bus admittance matrix, diagonal element y_{ii} is self-admittance and equal to sum of the admittances from the node i

$$y_{ii} = y_{i0} + \sum_j y_{ij} \tag{3.15}$$

and y_{ij} is mutual admittance, that is, admittance between nodes i and j, with (–) sign.

As powers are known, but currents and voltages are unknown, powers can be expressed with currents and voltages in next way

$$\underline{S}_i = \left(\underline{Y}_{i0}^* + \sum_j \underline{Y}_{ij}^* \right) U_i^2 - \sum_j \underline{Y}_{ij}^* \underline{U}_j^* \underline{U}_i \tag{3.16}$$

There are two iterative ways to solve voltages: Gauss-Seidel method and Newton-Raphson method.

Gauss-Seidel method

The first iteration

$$\underline{U}_i = \frac{\dfrac{P_i - jQ_i}{\underline{U}_i^*} + \sum_j Y_{ij} U_j}{Y_{i0} + \sum_j Y_{ij}} \tag{3.17}$$

And we continue with the new values for voltage until the difference in voltages between the consecutive iterations is small enough. The disadvantage of this method is that it converges slowly.

Gauss-Seidel acceleration factors

In order to speed up the convergence, the correction in voltage is multiplied by the constant ω

$$U^{p+1} = U^{p} + \omega\,(U^{p+1} - U^{p}) = U^{p} + \omega \Delta U^{p} \tag{3.18}$$

Which depends on the concrete network and usually equal to 1,6.

Newton-Raphson method

In this method, we assume f(x) = 0, and then make initial guess x_0 and find Δx_1 such that $f(x_0 + \Delta x_1) = 0$. From the Taylor series

$$f(x_0) + f'(x_0)\Delta x_1 = 0 \tag{3.19}$$

$$\Rightarrow \begin{bmatrix} f_1(x_{10}\ldots x_{n0}) \\ \ldots \\ f_n(x_{10}\ldots x_{n0}) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \ldots & \frac{\partial f_1}{\partial x_n} \\ \ldots & \ldots & \ldots \\ \frac{\partial f_n}{\partial x_1} & \ldots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \ldots \\ \Delta x_n \end{bmatrix}$$

$$[f(x_{i0})] = [J][\Delta x] \tag{3.20}$$

Where ***J*** is the Jacobian matrix

Then the process is repeated with the value $x_1 = x_0 + \Delta x_1$ if there are several equations $f_i(x_1 \ldots x_n) = 0$ i = 1...n

$$[\Delta x] = [J]^{-1}[f(x_{i0})] \tag{3.21}$$

For load nodes we have the following power equations:

$$
\begin{aligned}
P_i &= \left(G_{i0} + \sum_j G_{ij} \right) U_i^2 - \sum_j U_i U_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) \\
Q_i &= -\left(B_{i0} + \sum_j B_{ij} \right) U_i^2 - \sum_j U_i U_j \left(G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij} \right)
\end{aligned}
\qquad \textbf{(3.22)}
$$

At the beginning, we make guesses for voltages (absolute value and angle). After that we calculate P_i and Q_i with the above equations, and compare them with the actual initial data (P, Q) → mismatch (ΔP and ΔQ). Then corrections in voltages are calculated (absolute values and angles) by applying Newton-Raphson method so that ΔP and ΔQ converge as much as possible. And this process is repeated until ΔP and ΔQ are small enough.

This method is mathematically difficult, but converges fast. So, it is the most common method.

For nodes we have the following power equations:

$$
\begin{aligned}
&\underline{S}_i = \underline{U}_i \underline{I}_i^* \qquad \Rightarrow \underline{S}_i^* = \underline{U}_i^* \underline{I}_i \\
&\underline{I}_i = \left(\underline{Y}_{i0} + \sum_j{}' \underline{Y}_{ij} \right) \underline{U}_i - \sum_j{}' \underline{Y}_{ij} \underline{U}_j \qquad \underline{S}_i^* = \left(\underline{Y}_{i0} + \sum_j{}' \underline{Y}_{ij} \right) U_i^2 - \sum{}' \underline{Y}_{ij} \underline{U}_j \underline{U}_i^* \\
&\rightarrow \begin{cases} P_i = \left(G_{i0} + \sum_j{}' G_{ij} \right) U_i^2 - \sum_j U_i U_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) \\ Q_i = -\left(B_{i0} + \sum_j B_{ij} \right) U_i^2 - \sum_j U_i U_j \left(G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij} \right) \quad \delta_{ij} = \delta_i - \delta_j \end{cases}
\end{aligned}
\qquad \textbf{(3.23)}
$$

Or alternative representation:

$$
\begin{aligned}
&U_i = |U_i| \angle \delta_i \qquad U_j = |U_j| \angle \delta_i \Rightarrow P_i - jQ_i = \sum_j |U_i| |y_{ij}| |U_j| \angle (\Phi_{ij} + \delta_j - \delta_i) \\
&\begin{cases} P_i = \sum_j |y_{ij}| |U_i| |U_j| \cos (\Phi_{ij} + \delta_j - \delta_i) \\ Q_i = -\sum_i |y_{ij}| |U_i| |U_j| \sin (\Phi_{ij} + \delta_j - \delta_i) \end{cases}
\end{aligned}
\qquad \textbf{(3.24)}
$$

In the NR method for load flow studies, correction in voltages will be done by the mismatch ΔP and calculations will be conducted in the next order

1. Linearization of node equations

$$
\begin{bmatrix} \Delta P_1 \\ \cdots \\ \Delta P_{n-1} \\ \hline \Delta Q_1 \\ \cdots \\ \Delta Q_{n-1} \end{bmatrix}
=
\left[\begin{array}{c|c}
\dfrac{\partial P}{\partial \delta_1} \cdots & \cdots \dfrac{\partial P}{\partial U_k} \\
\cdots & \cdots \\
\cdots & \cdots \\ \hline
\cdots & \cdots \\
\cdots & \cdots \\
\dfrac{\partial Q}{\partial \delta_1} \cdots & \cdots \dfrac{\partial Q}{\partial U_k}
\end{array}\right]
\begin{bmatrix} \Delta \delta_1 \\ \cdots \\ \Delta \delta_{n-1} \\ \hline \Delta U_1 \\ \cdots \\ \Delta U_{n-1} \end{bmatrix}
$$

where n is number of nodes. **(3.25)**

2. Selection of initial values U_{i0}, δ_{i0} Calculation of mismatches (actual – calculated)

$$\Delta P_i = P_{li} - P_i \qquad \textbf{(3.26a)}$$

$$\Delta Q_i = Q_{li} - Qi$$ where P_{li} and Q_{li} are loads. **(3.26b)**

3. We find the inverse for the Jacobian matrix and solve the corrections for angles and voltages
4. We substitute new values to voltages and angles and calculate the new partial derivative matrix
5. We calculate the new power mismatches. If the mismatches are more than given tolerance, we return to item 3

Equations for partial derivatives

$$\frac{\partial P_i}{\partial \delta_i} = \sum_j U_i U_j \left(G_{ij} \sin \delta_{ij} \quad B_{ij} \cos \delta_{ij}\right) \qquad \frac{\partial P_i}{\partial \delta_j} = \quad U_i U_j \left(G_{ij} \sin \delta_{ij} \quad B_{ij} \cos \delta_{ij}\right)$$

$$\frac{\partial P_i}{\partial U_i} = \sum_j 2\left(G_{ij} - G_{i0}\right) U_i - \sum U_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}\right)$$

$$\frac{\partial P_i}{\partial U_j} = -U_i \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}\right) \qquad \frac{\partial Q_i}{\partial \delta_i} = \sum_j U_i U_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}\right)$$

$$\frac{\partial Q_i}{\partial \delta_j} - U_i U_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}\right)$$

(3.27)

$$\frac{\partial Q_i}{\partial U_i} = -\sum_j 2\left(B_{ij} + B_{i0}\right)U_i - \sum_j U_j\left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right)$$

$$\frac{\partial Q_i}{\partial U_j} = -U_i\left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right) \tag{3.28}$$

For each node, there are P_i, Q_i, U_i and δ_i

$$S_i = U_i I_i^* \quad \Rightarrow \quad S_i^* = U_i^* I_i \tag{3.30}$$

$$I_i = \sum_j y_{ij} U_j \qquad S_i^* = P_i - jQ_i \tag{3.31}$$

$$\Rightarrow P_i - jQ_i = U_i^* \sum_j y_{ij} U_j \tag{3.32}$$

And the complex parameters

$$U_i = |U_i|\angle\delta_i \qquad U_j = |U_j|\angle\delta_i \qquad y_{ij} = |y_{ij}|\angle\Phi_{ij}$$

$$\Rightarrow P_i - jQ_i = \sum_j |U_i||y_{ij}||U_j|\angle(\Phi_{ij} + \delta_j - \delta_i) \tag{3.33}$$

$$\begin{cases} P_i = \sum_j |y_{ij}||U_i||U_j|\cos\left(\Phi_{ij} + \delta_j - \delta_i\right) \\ Q_i = -\sum_j |y_{ij}||U_i||U_j \sin\left(\Phi_{ij} + \delta_j - \delta_i\right) \end{cases} \tag{3.34}$$

3.4 Summary

Although it is virtually impossible to expound the all load flow calculation methods in one chapter, main principles of solution for simple radial and loop networks were described and two methods of load flow calculations in large system were also investigated. If the load flows in simple network can be solved manually, in large system application of computers simplifies the calculations.

4 Case study calculations

The main aim of these case study calculations is to visually analyse and show the essence of processes occurring in three phase radial distribution system. Also we will try to summarise and, where it is needed, to apply the information from the previous chapters. I will examine three-phase network, when one or two phases overloaded or underloaded. The connection on distribution transformer also will be changed. First study will be with delta-star connection with zero conductor. The next case is when the zero conductor is broken out. The third case will be delta-zigzag connection on the transformer`s secondary. This connection is used in order to minimise the impact of unsymmetry, so we will examine how much it is justified. The study will be conducted by method of symmetrical components for unsymmetrical situation. Also for first case I will make comparative calculations by method of neutral displacement.

4.1 introduction to the calculations

As it said before, the calculations are conducted in order to examine the impact of unsymmetrical loads on one or two phases on the currents and voltages in the rest. Also I will examine the impact of the zero conductor on this unsymmetry. The third case calculation is conducted to find out how the zigzag connection on the transformer's secondary reduce this unsymmetry.

At the beginning we should find the impedances of all components of the network. For the transformer the resistance on the secondary is equal to

$$R_k = r_k \cdot \frac{U^2}{S_n} \quad \textbf{(4.1)}$$

Where r_k is short circuit resistance of the transformer. U is the rated voltage in the transformer's secondary and S_n is total power of the transformer.

The reactance of the transformer on the secondary is equal to

$$X_k = x_k \cdot \frac{U^2}{S_n} \quad \textbf{(4.2)}$$

Where x_k is short circuit reactance of the transformer.

For the conductors we have:

$$R_l = r_l \cdot l \quad \text{and} \quad X_l = x_l \cdot l. \qquad \textbf{(4.3)}$$

where r_l linear resistance and x_l are linear reactance of the line and l is the length of the line.

Concerning the neutral conductor, as in some types of unsymmetrical situation there may be high currents in the neutral current, we will use the same type of conductor as for phases.

To define the impedances of the loads we can use next equations

$$Z = \frac{U_p^{\ 2}}{S_n} \qquad \textbf{(4.4)}$$

Where $U_p = U/\sqrt{3}$. And $R_L = Z \cdot \cos(\varphi)$ and $X_L = Z \cdot \sin(\varphi)$ where $\cos(\varphi)$ is the power factor of the load.

4.2 Unsymmetrical situation when we have four-wire delta-grounded star connection.

Let`s assume that we have four-wire radial distribution system. The connection on the transformer`s secondary is star with grounding of neutral point. And let`s assume, that we have unsymmetrical situation, for example, one phase is underloaded or overloaded. The method of calculation for the both cases is the same, so in this calculation we will assume that one of the phases is underloaded, but in MathCad calculations we will set the block of different values for the load power. Further, the conductor is a cable AMKA 3x70+95 (r_l = 0.15, x_l = 0.03), the distance from the transformer to the load is $l = 300$ m. Power factor of the loads cos (φ) is assumed to be 0,9. The neutral conductor can be different from the phase one, but in these calculations in order to simplify we will assume that it is the same as phase conductor.

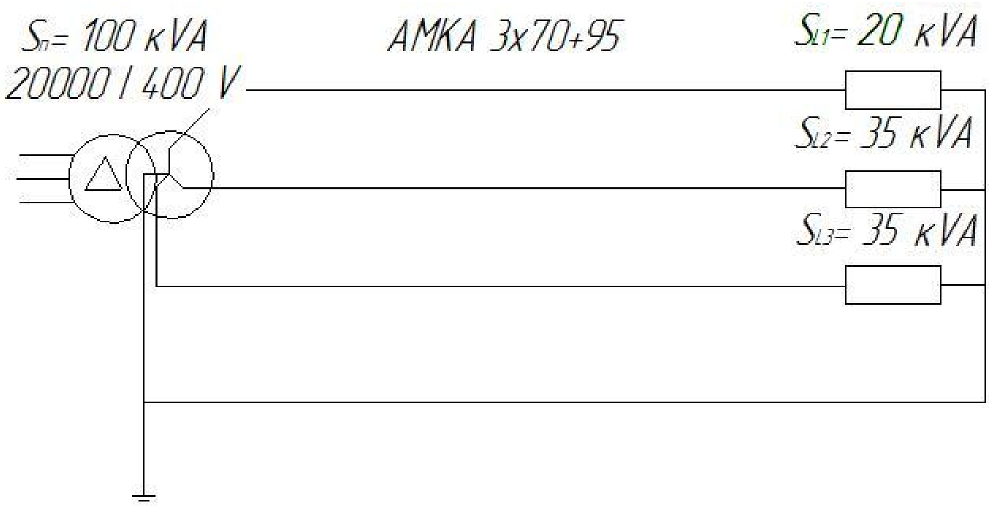

Fig 4.1 Four-wire distribution network with unsymmetrical load

The rated power of the distribution transformer is 100 MWA. Also we will assume that the system behind the transformer is infinite bus.

Our task is to define the phase and neutral currents, voltages at the beginning of the phases (at the transformer) and load voltages. All the calculations are conducted by method of symmetrical components for unsymmetrical situations.

To do this, we replace the impedance of unsymmetrical load by two, one of which is equal to the load impedances on the other phases and the second is the difference between them (fig 4.2) [1]

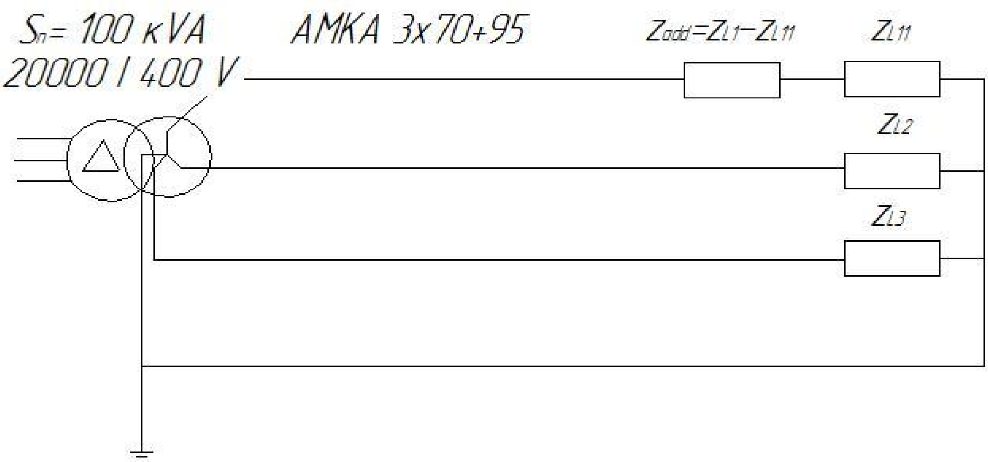

Fig.4.2. The network after replacing unsymmetrical load by two ones, one of which is equal to the loads in the other phases.

Let`s calculate parameters of the elements of the network and make equivalent circuit for first case.

$$R_k = r_k \cdot \frac{U^2}{S_n} = 1.75 \cdot \frac{400}{100 \cdot 10^3} = 0.028 \text{ (Ohm)}$$

$$X_k = x_k \cdot \frac{U^2}{S_n} = 3.6 \cdot \frac{400}{100 \cdot 10^3} = 0.058 \text{ (Ohm)}$$

And the total impedance $Z_k = R_k + X_k$ $Z_k = 0.028 + j\,0.058$ (Ohm)

For conductors $Z_l = l \cdot (r_l + j \cdot x_l)$ $Z_l = 0.15 + j\,0.03$ (Ohm)

For the load of phase A:

$$Z_{L1} = \frac{U_p^{\ 2}}{S_{L1}} = \frac{230^2 V}{20 \cdot 10^3 kVA} = 2.645$$

$R_{L1} = Z_{L1} \cdot \cos(\varphi) = 2.381$,

$X_{L1} = Z_{L1} \cdot \sin(\varphi) = 1.153$

$Z_{L1} = 2.381 + j1.153$ (Ohm)

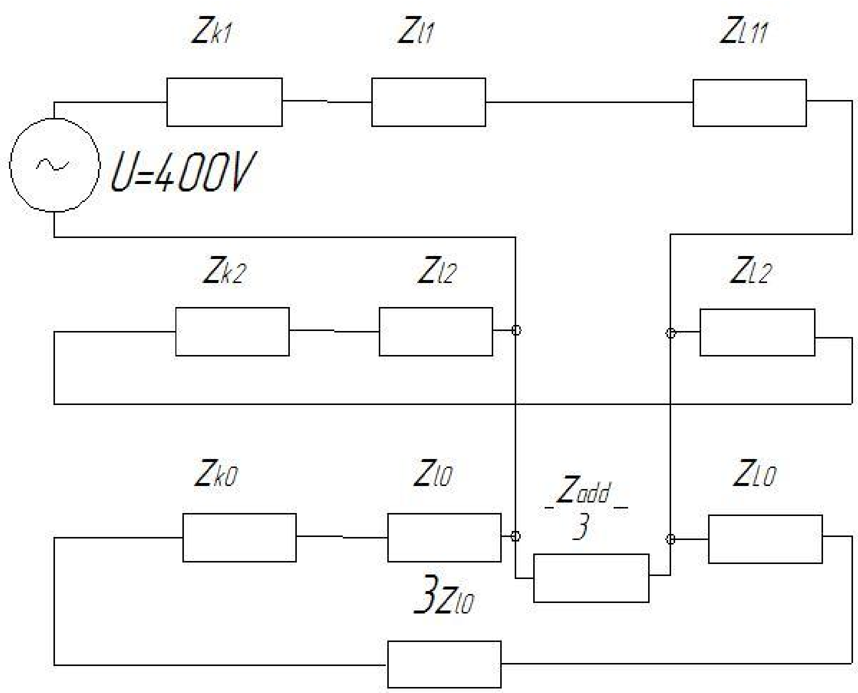

Fig 4.3 Equivalent circuit

For the loads of phases B and C

$$Z_L = \frac{U_p^{\ 2}}{S_{L1}} = \frac{230^2 V}{35 \cdot 10^3 kVA} = 1{,}511$$

$$R_L = Z_L \cos(\varphi) = 1.36$$

$$X_L = Z_L \sin(\varphi) = 0.659$$

$$Z_L = 1.36 + j0.659 \text{ (Ohm)}$$

Now we should determine the additional impedance

$$Z_{add} = Z_{L1} - Z_{L11} = 2.381 + j \cdot 1.15 - 1.36 + j \cdot 0.659 \text{ (Ohm)}$$

$$Z_{add} = 1.02 + j \cdot 0.494 \text{ (Ohm)}$$

Let`s find impedances of each sequence and after that the equivalent impedance of the circuit respectively to the place of unsymmetry for the special phase (A).

The positive-sequence impedance is equal to the sum of impedances of each element of the network:

$$Z_1 = Z_{k1} + Z_{l1} + Z_{L11} \qquad \textbf{(4.5)}$$

The negative sequence is frequently the same as positive

$$Z_2 = Z_{k2} + Z_{l2} + Z_{L2} \qquad \textbf{(4.6)}$$

For zero-sequence impedance we have some differences. For transformers the value of zero-sequence impedance depends on the way of connection of the windings and embodiment. For connection delta- star with grounding, we can assume that it is equal to the positive-sequence impedance. For the cable we can assume that $Z_{0l} = 3.5 \cdot Z_{1l}$, and for load we will assume the impedance to be the same as for positive and negative-sequences. The zero-sequence impedance of the zero conductor is tripled because currents of all three phases flow through this conductor. Thus

$$Z_0 = Z_{k0} + 3.5Z_{l0} + Z_{L0} + 3Z_{l0} \qquad \textbf{(4.7)}$$

So, the equivalent impedance

$$Z_{eqv} = \frac{\dfrac{\dfrac{Z_{add} \cdot Z_2}{3}}{\dfrac{Z_{add} + 3 \cdot Z_2}{3}} \cdot Z_0}{\dfrac{\dfrac{Z_{add} \cdot Z_2}{3}}{\dfrac{Z_{add} + 3 \cdot Z_2}{3}} + Z_0} \qquad \textbf{(4.8)}$$

And now the positive-sequence current can be calculated from the equation [2],

$$I_1 = \frac{jU_f}{Z_1 + Z_{eqv}} \qquad \textbf{(4.9)}$$

where $U_f = U/\sqrt{3}$. The negative-sequence current is equal to

$$I_2 = -I_1 \frac{Z_{eqv}}{Z_2} \qquad \textbf{(4.10)}$$

And zero-sequence current

$$I_0 = -I_1 \frac{Z_{eqv}}{Z_0} \qquad \textbf{(4.11)}$$

And knowing the symmetrical components we can calculate the phase values of the currents and respectively voltages.

$$\begin{aligned} I_a &= I_0 + I_1 + I_2 \\ I_b &= I_0 + a^2 I_1 + a I_2 \\ I_c &= I_0 + a I_1 + a^2 I_2 \end{aligned} \qquad \textbf{(4.12)}$$

The current in the zero conductor equals to the sum of the phase currents:

$$I_z = I_a + I_b + I_c \qquad \textbf{(4.13)}$$

And the phase voltages are equal to the multiplication of the phase impedances to respective phase currents

$$\begin{aligned} U_a &= I_a (Z_{k1} + Z_{l1} + Z_{L1}) \\ U_b &= I_b (Z_{k2} + Z_{l2} + Z_{L2}) \\ U_c &= I_c (Z_{k2} + Z_{l2} + Z_{L3}) \end{aligned} \qquad \textbf{(4.14)}$$

And the voltages at the loads

$$U_{L1} = I_a \cdot Z_{L1}$$
$$U_{L2} = I_b \cdot Z_{L2} \qquad \textbf{(4.15)}$$
$$U_{L3} = I_c \cdot Z_{L3}$$

By substituting our values for transformer, line and load impedances, we get

$$Z_1 = 1.538 + j0.747 \text{ (Ohm)}$$
$$Z_2 = 1.538 + j0.747 \text{ (Ohm)}$$
$$Z_0 = 2.453 + j0.822 \text{ (Ohm)}$$
$$Z_{eqv} = 0.251 + j\,0.117 \text{ (Ohm)}$$

And the symmetrical components of the currents

$I_1 = 50.561 + j\,104.679$, $I_2 = -8.422 - j16.814$, $I_0 = -4.094 – j\,11.736i$

And the phase currents

$$\boldsymbol{I_a} = 38.044 + j\,76.129 \text{ (A)}$$
$$\boldsymbol{I_b} = 80.086 – j\,106.983 \text{ (A)}$$
$$\boldsymbol{I_c} = -130.483 – j\,4.501 \text{ (A)}$$
$$\boldsymbol{I_z} = -12.353 - j\,35.355 \text{ (A)}$$

The root-mean-square meanings of these currents

$I_A = 84.868$ A $\quad I_B = 133.638$ A $\quad I_C = 130.561$ A $\quad I_Z = 37.451$ A

The phase voltages

$$\boldsymbol{U_a} = 2.793 + j\,225.086 \qquad U_A = 242.002 \text{ V}$$
$$\boldsymbol{U_b} = 203.092 - j\,104.761 \qquad U_B = 228.519 \text{ V}$$
$$\boldsymbol{U_c} = -197.359 - j104.371 \qquad U_C = 223.257 \text{ V}$$

The voltages at the loads

$$U_{L1} = 2.793 + j\,225.086 \qquad U_{L1m} = 225.103 \text{ V}$$
$$U_{L2} = 179.422 – j\,92.765 \qquad U_{L2m} = 201.984 \text{ V}$$
$$U_{L3} = -\,174.529 - j\,92.087 \qquad U_{L3m} = 197.333 \text{ V}$$

The current unbalance factor of the negative sequence is defined as

$$k_{2I} = \frac{|I_2|}{|I_1|} \quad k_{2I} = 0.162 = 16.2\ \% \tag{4.16}$$

The current unbalance factor of the zero-sequence is equal

$$k_{0I} = \frac{|I_0|}{|I_1|} = 0.107 = 10.7\% \tag{4.17}$$

Now we will make the comparative calculation for the same situation but this time with the method of neutral displacement. As we know, under unsymmetrical situation, the neutral point of the three phases can shift. We should calculate the displacement voltage (U_{n0}), after which we will be able to define the load voltages and consequently, the currents flowing in the phases.

$$U_{n0} = \frac{\frac{U_a}{Z_a} + \frac{U_b}{Z_b} + \frac{U_c}{Z_c}}{\frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_c} + \frac{1}{Z_{l0}}} \tag{4.18}$$

Where

$$\begin{aligned} Z_a &= Z_{k1} + Z_{l1} + Z_{L1}, \\ Z_b &= Z_{k2} + Z_{l2} + Z_{L2}, \\ Z_c &= Z_{k2} + Z_{l2} + Z_{L3} \end{aligned} \tag{4.19}$$

And $U_a = U, U_b = a^2U, U_c = a \cdot U$

In our case we get the voltages at the phases

$$\begin{aligned} U_{An} &= U_a - U_{n0} \\ U_{Bn} &= U_b - U_{n0} \\ U_{Cn} &= U_c - U_{n0} \end{aligned} \tag{4.20}$$

And the phase currents

$$I_a = \frac{U_{An}}{Z_a},\ I_b = \frac{U_{Bn}}{Z_b},\ I_c = \frac{U_{Cn}}{Z_c},\ I_0 = \frac{U_{n0}}{Z_0} \tag{4.21}$$

From the above equations we get

U_{n0} = - 6.546 + j 1.381 | U_{n0} | = 6.69 V

U_{An}= 236.546 - j1.381 | U_{An}| = 236.55 V

U_{Bn} = -108.454 - j 200.566 | U_{Bn}| = 228.011 V

U_{Cn} = -108.454 + j 197.805 | U_{Cn}| = 225.586 V

For the currents

I_{an} = 74.625- j 36.72 |I_{an}| = 83.174 A

I_{bn} = -108.305 - j 77.804 |I_{bn}| = 133.355 A

I_{cn} = -6.513 + j 131.775 |I_{cn}| = 131.936 A

And the current in the zero conductor

I_{n0} = - 40.193 + j 17.242 | I_{n0}| = 43.736 A

If to compare the results of two methods of calculation, we can see that the calculating error does not exceed 5%. The biggest difference has phase ***a***: for current it is 2% and for voltage 2.3%

$$\frac{I_a - I_{an}}{I_a} \cdot 100\% = \frac{84.868 - 83.174}{84.868} \cdot 100\% = 2\% \qquad \textbf{(4.22)}$$

$$\frac{U_a - U_{An}}{U_a} \cdot 100\% = \frac{242 - 236.55}{242} \cdot 100\% = 2.3\% \qquad \textbf{(4.23)}$$

The error can be result both of calculating inaccuracy and the assignment of initial data. For example, the zero-sequence impedance for cables of different types can mainly vary from 2.5 to 4.5. As well as zero-sequence impedance of loads can be different depending on the character of the load. But in the ranges of the given task, I consider that the received results are satisfying.

In order to check the equations for symmetrical components, I took three different values of load power for special phase (phase ***a*** in my case). The results for the first case (when the power of the phase ***a*** is less than the powers of the other two phases) I brought above. In this case the voltage at the phase ***a*** is higher than the voltages at the other two phases and current is lower.

The second case when the power of phase *a* is the same as for the phases *b* and *c*. This case we have symmetrical situation and as expected we have only positive component and the negative and zero components are equal to zero. The main result of symmetrical loads, that there is no voltage displacement and no current in the zero conductor.

The third case is when the load of one phase is more than the loads of the rest. In this case we have the opposite situation than in the first case. The current is higher than in the other two phases and the voltage is lower. Also in this case we have current in the zero conductor. For both first and third cases the voltage at the underloaded phases can be higher than the nominal phase voltage.

There are two vector diagrams below for the situation, when all three loads are equal (that is, balanced loads) and for the above described unsymmetrical situation.

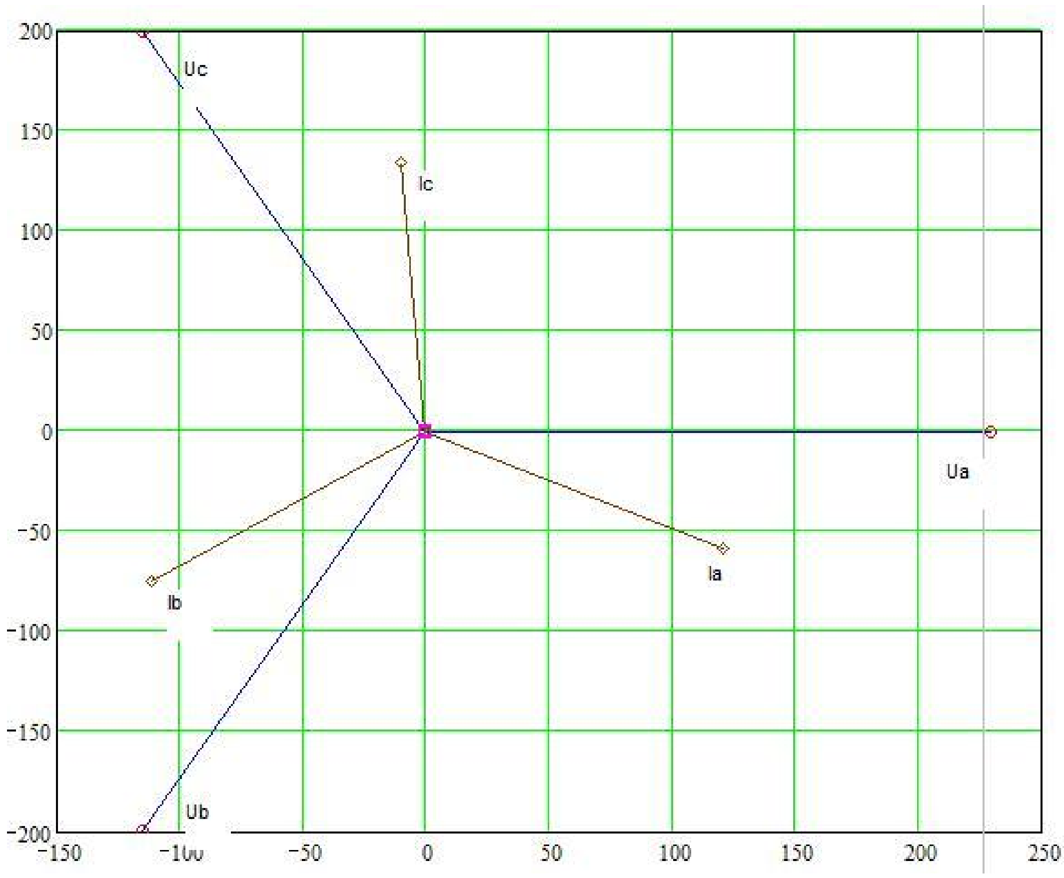

Fig 4.4. The vector diagram of voltages and currents in the balanced situation

From the first diagram we can see, that the neural point of voltages and currents corresponds to zero. The distribution of the vectors is symmetrical and the angle between phases is 120°. And as follows from the second figure, the neutral point is shifted to the left and there appears zero conductor current which balances the current difference in the phases.

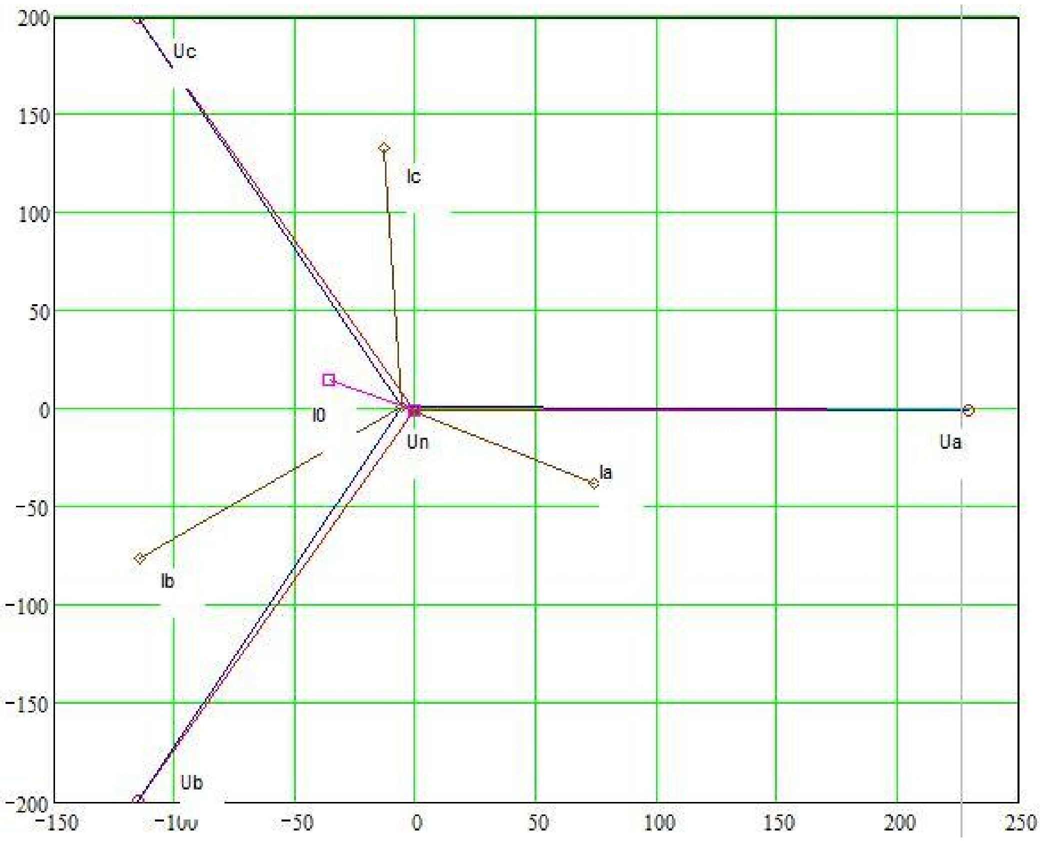

Fig 4.5. The vector diagram of voltages and currents in the unbalanced situation

4.3 Unbalanced situation with the broken zero conductor

In the four-wire network the neutral conductor required when we have unsymmetrical situation. In this case the sum of the phase current flow through it.

$$I_a + I_b + I_c = I_0 \quad \textbf{(4.25)}$$

Let`s have a look to what it brings about. In this case we have the network shown in the figure 5.6. When the neutral conductor is broken or when we have three-wire network under unbalanced loads, there is no way to zero-consequence currents. In this case we have only positive-sequence and negative-sequence components of the current and voltage.

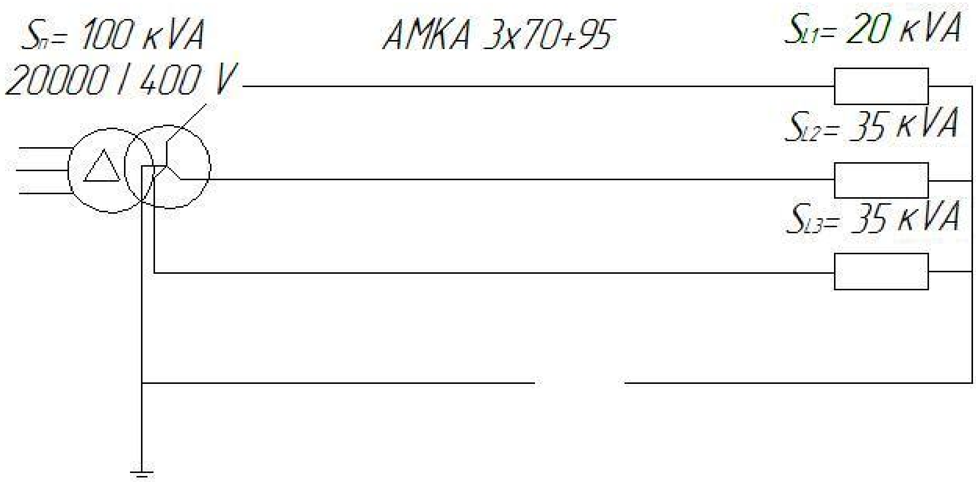

Fig.4.6. Unbalanced situation when neutral conductor is broken.

In this case we also replace the impedance of the unsymmetrical load by, one of which is equal to the impedances of the loads in other two phases.

But now, because we do not have path for zero-sequence current, the equivalent circuit will look in other way.

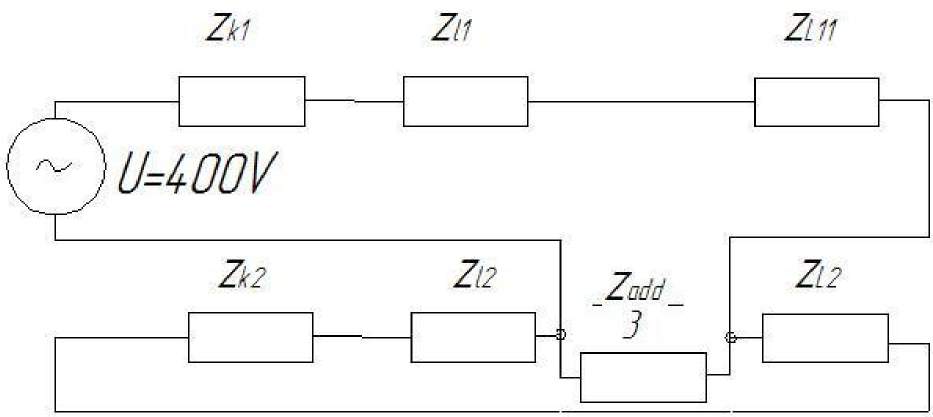

Fig.4.7. the equivalent circuit when there is no way for zero-sequence currents

The positive-sequence impedance is equal, as before, to the sum of impedances of each element of the network:

$$Z_1 = Z_{k1} + Z_{l1} + Z_{L11}$$

The negative sequence is also the same as positive

$$Z_2 = Z_{k2} + Z_{l2} + Z_{L2}$$

In this case the equivalent impedance

$$Z_{eqv} = \frac{\dfrac{Z_{add} \cdot Z_2}{3}}{\dfrac{Z_{add} + 3 \cdot Z_2}{3}} \qquad \textbf{(4.26)}$$

And now the positive-sequence and negative-sequence components of the current can be calculated from the equations,

$$I_1 = \frac{jU_P}{Z_1 + Z_{eqv}} \qquad I_2 = -I_1 \frac{Z_{eqv}}{Z_2}$$

Where $U_p = U/\sqrt{3}$.

In case of absence of zero-sequence, the equations for calculating phase currents look in next way

$$I_a = I_1 + I_2$$

$$I_b = a^2 I_1 + a I_2$$

$$I_c = a I_1 + a^2 I_2$$

For the phase voltages we get

$$U_a = I_a Z_a$$

$$U_b = I_b Z_b$$

$$U_c = I_c Z_c$$

And the voltages at the loads

$$U_{L1} = I_a \cdot Z_{L1}$$

$$U_{L2} = I_b \cdot Z_{L2}$$

$$U_{L3} = I_c \cdot Z_{L3}$$

By substituting the numerical data we get

$$Z_1 = 1.538 + j0.747 \text{ (Ohm)}$$

$$Z_2 = 1.538 + j0.747 \text{ (Ohm)}$$

$$Z_{eqv} = 0.279 + j\,0.135 \text{ (Ohm)}$$

The positive and negative components of the currents

$$I_1 = 49.932 + j\,102.88i \quad |I_1| = 95.441 \text{ A}$$
$$I_2 = -\,9.051 - j18.612 \quad |I_2| = 23.508 \text{ A}$$

The phase currents

$$\boldsymbol{I_a} = 40.881 + j\,84.268 \text{ (A)}$$
$$\boldsymbol{I_b} = 84.809 - j\,93.446 \text{ (A)}$$
$$\boldsymbol{I_c} = -125.759 + j\,9.036 \text{ (A)}$$

Verification of the calculations

$$\boldsymbol{I_a} + \boldsymbol{I_b} + \boldsymbol{I_c} = -0.069 - j\,0.142 \approx 0$$

The root-mean-square meanings of these currents

$$I_A = 93.661 \text{ A} \qquad I_B = 126.194 \text{ A} \qquad I_C = 126.084 \text{ A}$$

The phase voltages

$$\boldsymbol{U_a} = 0.023 + j\,266.331 \quad U_A = 266.331 \text{ V}$$
$$\boldsymbol{U_b} = 200.248 - j\,80.41i \quad U_B = 215.789 \text{ V}$$
$$\boldsymbol{U_c} = -\,200.202 - j\,80.02 \quad U_C = 215.601\text{V}$$

The voltages at the loads

$$U_{L1} = 0.162 + j\,247.733 \qquad U_{L1m} = 247.733 \text{ V}$$
$$U_{L2} = 176.929 - j\,71.24 \qquad U_{L2m} = 190.733 \text{ V}$$
$$U_{L3} = -\,177.022 - j\,70.561 \qquad U_{L3m} = 190.566 \text{ V}$$

The current unbalance factor of the negative sequence is

$$k_{2I} = \frac{|I_2|}{|I_1|} \qquad k_{2I} = 0.181 = 18.1\%$$

Also for this situation I consider three cases: when the power of the load of phase ***a*** is less than the load of the other two phases (or we have two overloaded phases), when the loads of all three phases are equal (symmetrical situation), and when one phase is overloaded.

For second case, when we have the symmetrical loads, there is no neutral displacement, no neutral currents and correspondingly in this case we do not need the neutral conductor. In ideal, the need in neutral conductor is always tried to be as less as possible.

In case when we have one underloaded (two overloaded) or overloaded phases the situation is radically different. The voltage of overloaded phases much lower than the voltages of other phase(s), and at the same time the voltage(s) of underloaded phase(s), depending on the situation, can be higher than the nominal voltage of the transformer's secondary.

Below I brought vector diagrams for the first and third cases. From these figures and from the previous calculations one can see that in case when phase ***a*** is underloaded, we have neutral displacement to the left and the vector designating the current of phase ***a*** is shorter than the others, when in third case vice versa.

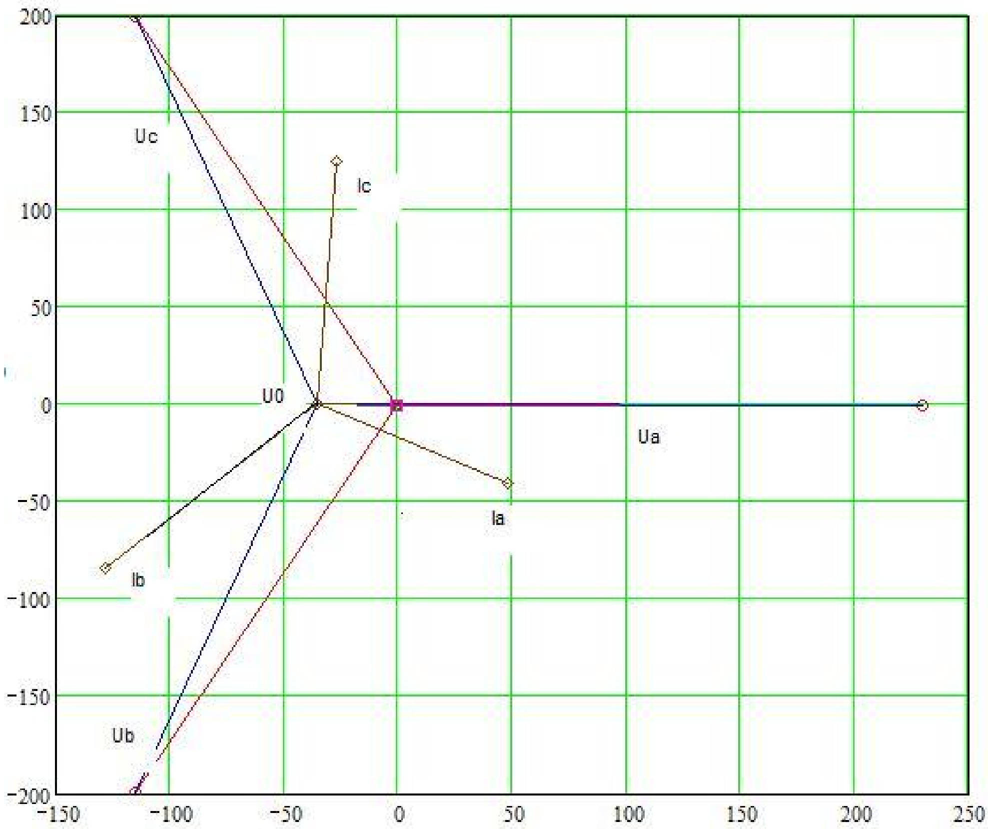

Fig.4.8 The vector diagram of voltages and currents in the unbalanced situation with the broken neutral conductor (underloaded phase *a*).

If to compare the result for the same unsymmetrical situation with neutral conductor and without it, we can see that in second case this has bigger consequences on the phase values of the voltages and currents.

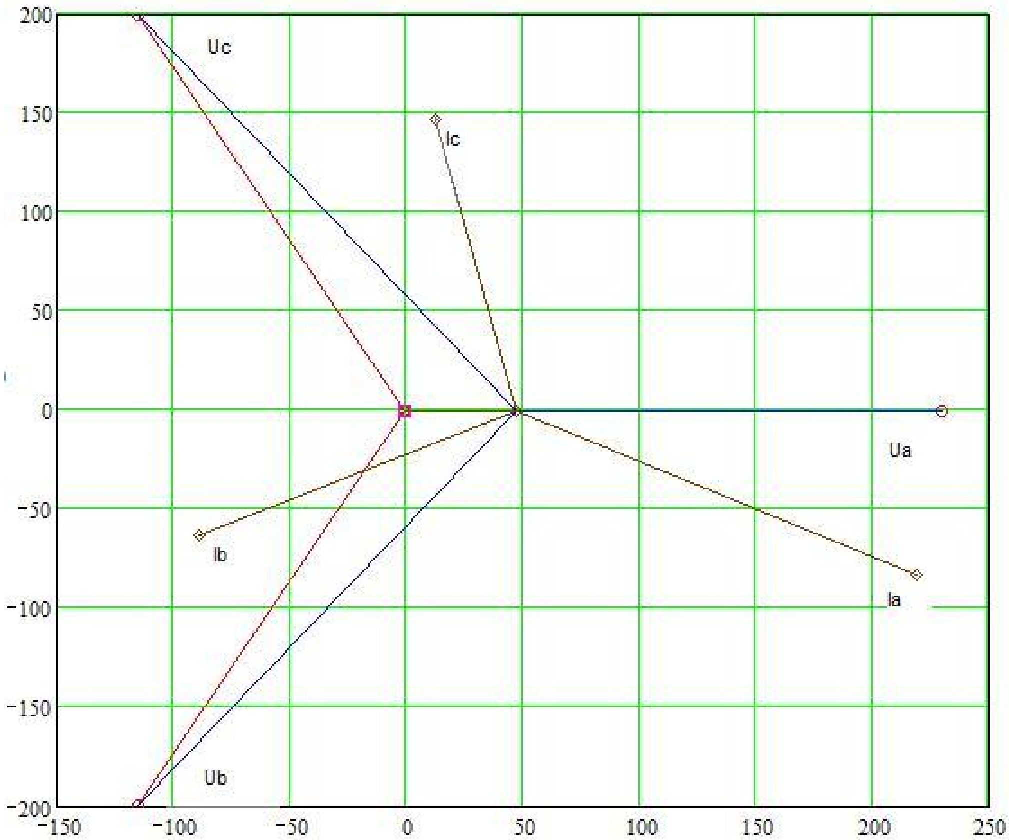

Fig.4.9 The vector diagram of voltages and currents in the unbalanced situation with the broken neutral conductor (overloaded phase *a*).

The comparative values for these two cases are given below in table 4.1.

Table 4.1. Phase values of voltages and current with neutral conductor and without

	With neutral	Without neutral	Abs. difference	Rel. difference
I_a, A	84.868	93.661	-8.79	10.3%
I_b, A	133.638	126.194	7.44	5.5%
I_c,A	130.561	126.084	4.48	3.4%
U_a,V	242.002	266.331	-24.33	10%
U_b,V	228.519	215.789	12.73	5.5%
U_c,V	223.257	215.601	7.65	3.4%

One can notice that in case of absence of neutral conductor the unsymmetry is bigger, especially for voltages. In case with neutral conductor the difference in the currents` value in the overloaded and normal phases are higher than in case with broken zero conductor but the difference between phase voltage values smaller than in case with no zero conductor.

4.4 Unbalanced situation when supplying via zig-zag transformer

The zero-sequence impedance of the zig-zag transformer is very law as compared with positive-sequence and negative-sequence impedances. Positive and negative-sequence impedances are approximately two times as much as impedances of respective transformer with connection Δ/Y_{GRD}. Also the zero sequence currents flowing through zig-zag transformer`s coils, creates magnetizing forces which are directed opposite to each other. It eliminates the impact of zero sequence currents.

Taking into account all above mentioned, for the phase voltages were got followig values

$$U_a = 210.945 + j106.512 \quad U_A = 236.31 \text{ V}$$

$$U_b = 205.125 + j\,106.351 \quad U_B = 231.056 \text{ V}$$

$$U_c = 200.727 + j\,104.071 \quad U_C = 226.102\text{V}$$

The root-mean-square meanings of these currents

$$I_A = 82 \text{ A} \qquad I_B = 132.144 \text{ A} \qquad I_C = 129.131 \text{ A}$$

A zigzag connection may be useful when a neutral is needed for grounding or for supplying single-phase line to neutral loads when working with a 3-wire, ungrounded power system. Due to its composition, a zigzag transformer is more effective for grounding purposes because it has less internal winding impedance going to the ground than when using a wye-type transformer.

It is not efficient to use zigzag configurations for typical industrial or commercial loads, because they are more expensive to construct than conventional wye-connected transformers. But zigzag connections are useful in special applications where conventional transformer connections aren't effective

5 Analysis of impact of unsymmetrical loads (depending on vector group of mv/lv transformer) and quality of voltage

As we could see from case study calculations, the vector group of supplying transformer inpacts to the value of the inbalance. In this chapter different types of connection of transformers primary and secondary and respectively, advantages and disadvantages of such connections in distribution networks will be discribed.

As well as we will analyze the physical processes occuring in the windings of the transformers that bring about to leveling the unbalance or, on the contrary, to its strengthening.

Also at the end of the thesis the voltage quality issues are investigated and the consequences of voltage distortion and higher voltage drop are described.

5.1 Transformers and their role in distribution systems

The transformer that connects the high voltage primary system (4.16kV to 34.5 kV) to the customer (at 480 volts and below) is usually referred to as a "distribution transformer". These transformers can be either single-phase or three-phase and range in size from about 5 kVA to 500 kVA [10(power distribution engineering]. With given secondary voltage, distribution transformer is usually the last in the chain of electrical energy supply to households and industrial enterprises.

There are 3 main parts in the distribution transformer:

1. Coils/winding – where incoming alternate current (through primary winding) generates magnetic flux, which in turn develop a magnetic field feeding back a secondary winding.
2. Magnetic core – allowing transfer of magnetic field generated by primary winding to secondary winding by principle of magnetic induction.

First 2 parts are known as *active parts.*

3. Tank – serving as a mechanical package to protect active parts, as a holding vessel for transformer oil used for cooling and insulation and bushing (plus auxiliary equipment where applicable)

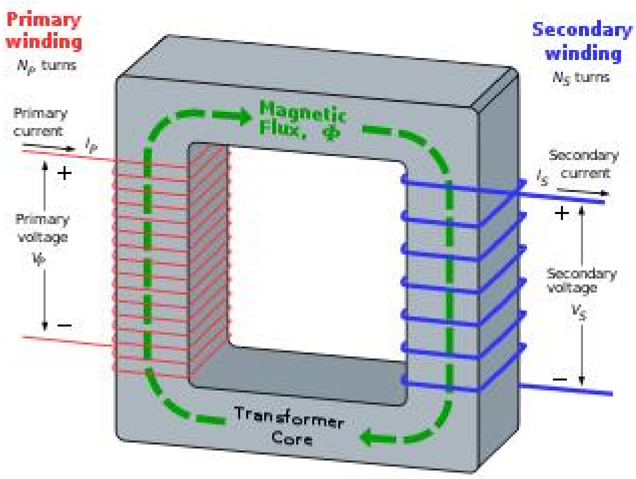

Fig 5.1. Schematic view of the single-phase distribution transformer

Distribution Transformers are usually fulfilled from copper or aluminum conductors and are wound around a magnetic core to transform current from one voltage to another. Distribution transformers come in two types- dry-type and liquid. The Dry Type Distribution Transformers are usually smaller and do not generate much heat and can be located in a confined space at a customer's location. The liquid type usually have oil which surrounds the transformer core and conductors to cool and electrically insulate the transformer (see also Oil Filled Transformers). The liquid distribution transformer types are usually the larger and need more than air to keep them from overheating thus in this type of transformers oil insulator is often used.

The winding connections of the transformers depend on the character of load supplying by them and usually wye-delta, delta-wye, delta-delta or wye-wye (wye can be grounded).

In table 5.1 there shown some of the standard kVAs and voltages for distribution transformers. [4]

Table 5.1 Standard distribution transformer kVAs and voltages

kVAs		High voltages		Low voltages	
Single-phase	Three-phase	Single-phase	Three-phase	Single-phase	Three-phase
5	30	2400/4160Y	2400	120/240	208Y/120
10	45	4800/8320Y	4160Y/2400	240/480	240
15	75	2400/4160Y	4160Y	2400	480Y/277
25	112.5	4800/8320YX	4800	2520	240X480
37.5	150	7200/12,470Y	8320Y/4800	4800	2400
50	225	$12470Y_{GRD}/7200$	8320Y	5040	
					4160Y/2400
75	300	7620/13200Y	7200	6900	4800
100	500	$13200Y_{GRD}/7620$	12000	7200	12470Y/7200
167		12000	12470Y/7200	7560	13200Y/7620
250		$13200/22860Y_{GRD}$	12470Y	7980	
333		13200	13200/7620		
500		$13800/23900Y_{GRD}$	13200Y		
		13800	13200		
		$14400/24940Y_{GRD}$	13800		
		22900	22900		
		34400	34400		
		43800	43800		
		67000	67000		

5.2 Analysis of impact of unsymmetrical loads depending on vector group of mv/lv transformers.

A vector group determines the phase angle displacement between the primary (HV) and secondary (LV) windings..

The phase windings of a three-phase transformer can be connected together internally in different configurations, depending on what characteristics are needed from the transformer. For example, in a three-phase distribution system, it may be necessary to connect a three-wire system to a four-wire system, or vice versa. Because of this, transformers are manufactured with a variety of winding configurations to meet these requirements.

Different combinations of winding connections will result in different phase angles between the voltages on the windings. This limits the types of transformers that can be connected between two systems, because mismatching phase angles can result in circulating current and other system disturbances.

Transformer nameplates carry a vector group reference such at Yy0, Yd1, Dyn11 etc. This relatively simple nomenclature provides important information about the way in which three phase windings are connected and any phase displacement that occurs

Winding Connections

HV windings are designated: Y, D or Z (upper case)

LV windings are designated: y, d or z (lower case)

Where:

Y or y indicates a star connection

D or d indicates a delta connection

Z or z indicates a zigzag connection

N or n indicates that the neutral point is brought out

Phase Displacement

The digits (0, 1, 11 etc) relate to the phase displacement between the HV and LV windings using a clock face notation. The phasor representing the HV winding is taken as reference and set at 12 o'clock. It then follows that:

Digit 0 means that the LV phasor is in phase with the HV phasor

Digit 1 that it lags by 30 degrees

Digit 11 that it leads by 30 degrees etc

All references are taken from phase-to-neutral and assume a counter-clockwise phase rotation. The neutral point may be real (as in a star connection) or imaginary (as in a delta connection)

Table 5.2.Phase shift depending on connection of windings

Phase shift (deg)	Connections		
0	Yy0	Dd0	Dz0
30 lag	Yd1	Dy1	Yz1
60 lag	Dd2	Dz2	
120 lag	Dd4	Dz4	
150 lag	Yd5	Dy5	Yz5
180 lag	Yy6	Dd6	Dz6
150 lead	Yd7	Dy7	Yz7
120 lead	Dd8	Dz8	
60 lead	Dd10	Dz10	
30 lead	Yd11	Dy11	Yz11

When transformers are operated in parallel it is important that any phase shift is the same through each. Paralleling typically occurs when transformers are located at one site and connected to a common busbar (banked) or located at different sites with the secondary terminals connected via distribution or transmission circuits consisting of cables and overhead lines.

Under unsymmetrical load, when the currents in the phases are not equal to each other, the voltage drops are different. The character and magnitude of the change of secondary voltage of the transformer depend on the way of connection of the primary and secondary windings as well as on the character and magnitude of the load. Let`s examine the impact of the unsymmetrical load on the secondary voltage under different types of connection of the windings.

Delta-delta connection

Let`s assume that the load is included between two terminals *a* and *b* (Fig 5.2). As the phase *ab* is included parallel to two tandem phases *ac* and *ba*, than when the total impedances of windings of all three phases are equal and magnetizing component is insignificant, distribution of the load current I_2` to all phases of primary and secondary windings will meet the figure 5.2. that is, in phase *ab* the current is equal to 2/3 $I_2$`, where I_2` is load current, and the current in phases *ac* and *cb* will be equal to 1/3 $I_2$`, as impedance of two series connected phases *ac* and *cb* is twice as much as the impedance of phase *ab* [3].

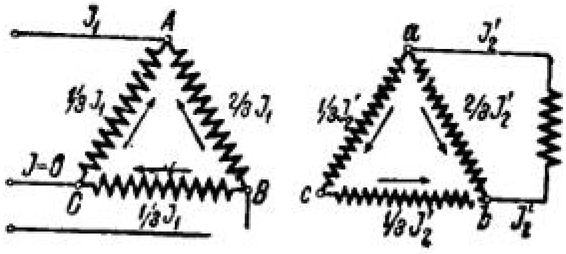

Fig.5.2. Unbalance under Δ/Δ connection.

The currents in the secondary will be balanced by the currents in the primary, which means that in the primary phases the current will be distributed exactly as in the secondary phases, that is in the phase *AB* it will be equal to 2/3 I_1, where I_1 is the line current flowing to the node *A*, and in the phases *AC* and *CB* it will be equal to 1/3 I_1. The directions of the currents are such that in the line coming to the node *C* there is no current. Consequently, the potential of the terminal *C* will not change under load. That is, the point *C* of the potential diagram will stay at the same place where it was, when the potentials of the terminals *a* and *b* will shift to the same direction with respect to the position under no-load operation.

In the figure 5.3 there shown the potential diagram assuming that the secondary current is corresponding in phase with primary voltage between terminals *AB*. In this picture triangle *ABC* is potential triangle of primary voltages and *abc* is potential triangle of secondary voltages under load.

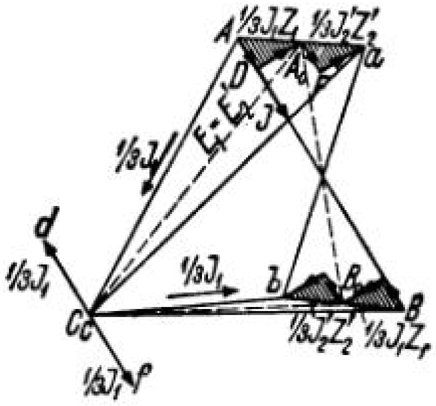

Fig.5.3. Potential triangles under unsymmetrical load

From this figure we can see that under unsymmetrical load, which is under consideration, the voltages of the loaded and one of the adjacent phases decrease, and the voltage of the other adjacent phase increases.

Star-star and delta-star connections

As in previous example, let`s assume that the load is included between two conductors of the secondary. In this case the current will flow only in the phases, adjacent to these conductors, and consequently in the conductors of the primary, conjugated to the mentioned. Thus, the current in the phase *oc* of the secondary will be equal to zero, and in the phase *OC* of the primary will flow only magnetizing current.

By replacing the star-connection by delta-connection, one will get the identical system, which we examined before.

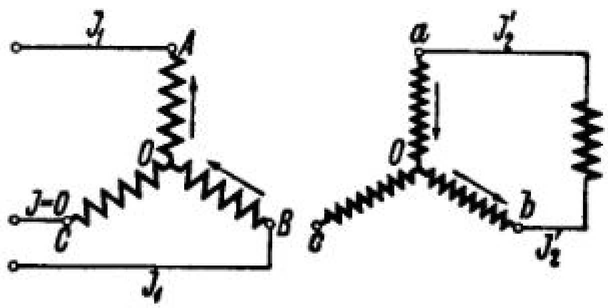

Fig.5.4. Unbalance under star-star connection

Therefore, the construction of the points, responding to the potentials of terminals *a* and *b* under load will not differ from drawn in the figure 5.3.

The phase voltages in this case are not equal either. Voltage of one loaded phase is higher, and the voltage of the second phase is less than the voltage of third loaded phase.

For the delta-star connection, all the same and the above-mentioned processes take place.

Zig-zag transformer

The secondary winding of each phase of zig-zag transformer consists of two coils. One coil is located on one core and the second one on the second core, and the end of the first coil, for example, x_1, is connected to the end of the second coil, for example, y_2.

Thus, the zigzag transformer contains six coils on three cores. The first coil on each core is connected contrariwise to the second coil on the next core. The second coils are then all tied together to form the neutral and the phases are connected to the primary coils. Each phase, therefore, couples with each other phase and the voltages cancel out. As such, there would be negligible current through the neutral pole and it can be tied to ground.

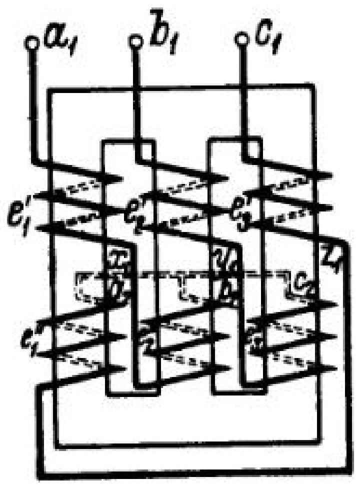

Fig. 5.5. Zig-zag transformer

If one phase, or more, faults to earth, the voltage applied to each phase of the transformer is no longer in balance; fluxes in the windings no longer oppose. Zero sequence (earth fault) current exists between the transformer's neutral to the faulting phase. Hence, the purpose of a zigzag transformer is to provide a return path for earth faults on delta-connected systems. With negligible current in the neutral under normal conditions, engineers typically elect to under size the transformer; a short time rating is applied (i.e., the transformer can only carry full rated current for, say, 60 s).

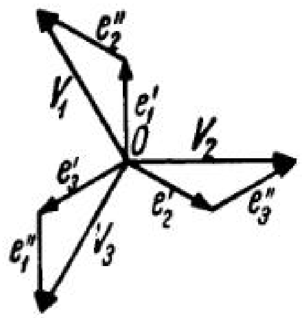

Fig.5.6. Vector diagram of zig-zag transformer.

When the coils of the transformer are connected in zig-zag, the vector diagram will look like in the figure 5.6. From this figure we can see, that the phase voltages U_1, U_2, U_3, will be equal to the geometrical sum of the voltages of the respective coils, and

$$U_1 = U_2 = U_3 = \sqrt{3}\, e_1 = \sqrt{3}\, e_2 \qquad \textbf{(5.1)}$$

In consequence of location of the secondary winding on two cores, unsymmetrical load lies on all phases in more or less equal degree reducing the phase voltage difference. Also, as it said in chapter 4, the effect of elimination of the magnetazing forces by zero sequence currents occur, which reduce the impact of the unsymmetrical situation

5.3 The quality of voltage and its impact on consumers

Defined in terms of magnitude (amplitude) and duration (length), voltage events - appearing in the form of sags, swells, impulses, and total harmonic distortion - can affect equipment performance in different ways. Typically, the ultimate impact of such events is determined by the sensitivity of the equipment on the branch circuit.

- **Voltage sags**

  A sag is a period of low voltage. Minor sags occur frequently, sometimes without disturbing equipment performance. Major sags, on the other hand, always disturb equipment performance. Sags occur for many reasons, including voltage drop caused by long runs of wire, switching loads, poor wiring, and overloaded branch circuits.

- **Voltage swells**

  A swell is a period of high voltage. Swells have serious impact on equipment function; however, they are not as common as sags. Both minor and major swells affect equipment performance.

- **Impulses**

  An impulse is a short burst of energy that lasts for less than a cycle. Impulses range in magnitude from twice the nominal voltage to several thousand volts. Not every impulse has an impact on equipment performance. However, when impulses occur repeatedly over time (or when the energy level is very high), an impulse can cause equipment degradation or even immediate failure.

- **Harmonics**

  AC voltage is a sine wave that repeats 50/60 times per second (Hertz = cycles/second). This is the fundamental frequency. Harmonics are alternate frequencies that distort the sinusoidal waveform. Total harmonic distortion (THD) is measured as a percentage of the fundamental frequency.

Equipment runs well on voltage that is a clean (or slightly distorted) sine wave. High levels of distortion may cause equipment problems. On single-phase branch circuits, levels of THD greater than 5% to 8% should be investigated.

5.4 Effects of voltage distortion.

Electrical equipment is designed to work at nominal voltage (+/-10%). Although equipment may not fail the first time an event occurs, excessive stress from repeated voltage events can cause damage over time. When voltage is outside of equipment design specifications, for example, equipment has to work harder, run hotter, or insulation may have to withstand extreme voltage levels. For instance, a refrigerator is designed to operate between 108VAC and 132VAC - that is, a typical voltage range for a nominal 120V piece of equipment. If voltage runs consistently below 108V on the circuit powering the refrigerator, the compressor motor will run hotter, reducing its operation and service life.

Sags are the most prevalent power quality issue for equipment. Momentary sags may not affect the refrigerator referenced above, but they will cause problems for more sensitive equipment, such as computers. The greater the voltage sag, the greater the likelihood of damage. Similarly, the greater the number of sags occurring, the greater the chance of failure or damage.

Although voltage swells occur less frequently than sags, even relatively minor swells can damage equipment. Therefore, they require immediate attention. The longer a swell's duration, the more extensive the damage will be. An example here would be a large motor creating voltage sags by drawing high inrush currents. When the motor is stopped abruptly, voltage swells are generated. Left uncorrected, these sags and swells will lead to computer disruptions and frequent hardware replacement in the facility.

THD can produce excessive heat, generate electro-magnetic interference in communications circuits, and cause electronic controls to fail. Non-linear loads, such as PCs, copying machines, and variable-frequency drives, create harmonic currents that distort the voltage sine wave. The more electronic devices on a circuit, the greater the likelihood of severe voltage distortion. A good example of such a problem involved a hospital technician who tested a circuit for two days before installing patient monitoring equipment. One instance of voltage harmonics, amounting to 5.2% THD, was noted. Recognizing this low level of THD wouldn't cause a problem, the technician installed the patient monitoring device. Within hours, the device failed. The technician reviewed new data to find a THD event reaching 10.2%. Further investigation using a circuit analyzer and long-term recorders found there were several non-linear loads plugged into the same branch circuit as the patient monitoring device. When certain combinations of these loads were on simultaneously — along with the new equipment — excessive harmonics flowed, causing a distorted voltage waveform and sporadic shutdown of the device.

Besides overheating, the other major effect of current distortion on an electrical system is the creation of voltage distortion. This distortion will have minimal effect on a distribution system, but unlike current distortion, it isn't path dependent. So harmonic voltages generated in one part of a facility will appear on common buses within that facility. High-voltage distortion at the terminals of a nonlinear load doesn't mean high distortion will be present throughout the system. In fact, the voltage distortion becomes lower the closer a bus is located to the service transformer. However, if excessive voltage distortion does exist at the transformer, it can pass through the unit and appear in facilities distant from the origin.

The effect on loads within the facility could be detrimental in certain cases. For example, extreme voltage distortion can cause multiple zero crossings for the voltage wave. For equipment where proper sequencing of operations depends on a zero crossing for timing, voltage distortion can cause misoperation. Most modern electrical equipment uses an internal clock for timing sequencing so it's unaffected by multiple zero crossings.

Voltage distortion appears to have little effect on operation of nonlinear loads connected either phase-to-phase or phase-to-neutral.

On the other hand, 5th harmonic voltage distortion can cause serious problems for 3-phase motors. The 5th harmonic is a negative sequence harmonic, and when supplied to an induction motor it produces a negative torque. In other words, it attempts to drive the motor in a reverse direction and slows down its rotation. So the motor draws more 60-Hz current to offset the reverse torque and regain its normal operating speed. The result is overcurrent in the motor, which either causes protective devices to open or the motor to overheat and fail. For this reason, removing 5th harmonic current from systems powering 3-phase loads is often a high priority in industrial facilities.

System harmonic voltage distortion is caused by the flow of harmonic currents through system impedance, namely inductive reactance. For each frequency, at which harmonic current is flowing, there is a corresponding inductive reactance associated with system and thus a voltage drop at that frequency. The factors that affect the system inductive reactance are the generator, transformer, series line reactors or current limiting reactors, and circuit conductors. Fig. 1 When the load current consists of fundamental current and 5th harmonic current, there will be a voltage drop across the system impedance at both the fundamental and 5th harmonic frequencies. The presence of any harmonic voltage causes distortion of the system voltage. The voltage will be the least distorted nearest to its generating source and will become more distorted nearer to the load, as the harmonic current flows through larger amounts of impedance. If the load in Fig. 1 draws harmonic current, the highest level of voltage distortion will be present at the load terminals, less voltage distortion at the transformer secondary terminals and less distortion yet at the generator and its associated source impedance.

Effect on other equipment

The major problem with voltage distortion is that other loads supplied by a dis torted voltage source may not operate properly or efficiently. When an electrical load is supplied with distorted voltage, then it receives harmonic voltage in addition to the fundamental frequency voltage. If a load is supplied with a harmonic voltage, then it will also draw harmonic current (for each frequency of applied voltage). Any non-linear load, drawing distorted current, will cause harmonic voltage distortion. The magnitude of voltage distortion will depend on the magnitude of harmonic currents flowing from source to load and the circuit impedances that this current flows through between the source and load. Nema STD MG-1, part 30.1.2 explains that motor efficiency is reduced, electrical losses increased, and motor temperature is increased when the motor supply voltage is distorted.

In some cases, the voltage distortion may be so severe as to cause parts on the voltage waveform to touch the zero volts axis at more points than at 0 and 180 degrees. Referred to as multiple zero crossings, this type of voltage distortion can cause mis-operation of zero cross sensitive circuits such as those used in SCR controllers and electronic timing circuits.

5.5 Higher voltage drop and power losses

As it said before, almost all electrical equipment is designed to operate at certain defined voltage. But economically unreasonably to serve every customer on a power distribution system with a voltage specified on the nameplate of its equipment. The main reason is that voltage drop exists in each part of the power system and the further electrically the load the lower the voltage on its terminals. So, there exists dilemma: if the limits of voltage provided by the power company are too broad, the cost of appliances and other utilization equipment, especially computers will be high because they will have to be designed to operate satisfactorily within these limits. On the other hand, if the voltage limits required for satisfactory operation of the utilization equipment are too narrow, the cost of providing power within these limits will be excessively high. [1]

If to speak more exact higher voltage drop is undesirable because it causes overheating of the equipment, shortens the lifetime of the insulation. If the voltage drop is higher than the allowable range, some electrical devices may not work correctly. For some machinery there is lower limit, after reaching it by voltage, this machinery is automatically switches off from the network.

5.7 Summary

Transformers are one of the key elements of the electric energy system. The vast majority of processes, occurring in the networks impact on the work of the transformer and vice versa, the embodiment of the transformer has an impact of the mode of operation of the whole network. In this chapter were described unsymmetrical situations due to different vector groups of the transformer. Also, at the ending paragraphs of the thesis, the voltage quality and the consequences of its distortion were described.

6 Conclusion

The diploma thesis has been related to unsymmetrical situations in distribution systems. In order to make the description of the work more understandable and comprehensive, the thesis was began from common issues, different types of distribution systems used in different countries and the typical distribution network configuration was described. The main part of the work is specification how the method of symmetrical components can be applied to the **system with unsymmetrical loads.** In this case, although the method of symmetrical components is the same, the equations which describe the boundary conditions are a bit different from those, which are used for calculating short circuits. Also to make the description of the method clear, in fourth chapter there were conducted some case study calculations, results of which have totally corresponded to the equations described in last chapter.

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