Jukka Kaukonen

SALIENT POLE SYNCHRONOUS MACHINE MODELLING IN AN INDUSTRIAL DIRECT TORQUE CONTROLLED DRIVE APPLICATION

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ABSTRACT

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Jukka Kaukonen

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Synchronous motors are used mainly in large drives, for example in ship propulsion systems and in steel factories’ rolling mills because of their high efficiency, high overload capacity and good performance in the field weakening range. This, however, requires an extremely good torque control system. A fast torque response and a torque accuracy are basic requirements for such a drive. For large power, high-dynamic performance drives the commonly known principle of field oriented vector control has been used solely hitherto, but nowadays it is not the only way to implement such a drive. A new control method - Direct Torque Control (DTC) has also emerged.

The performance of such a high quality torque control as DTC in dynamically demanding industrial applications is mainly based on the accurate estimate of the various flux linkages’ space vectors. Nowadays industrial motor control systems are real time applications with restricted calculation capacity. At the same time the control system requires a simple, fast calculable and reasonably accurate motor model. In this work a method to handle these problems in a Direct Torque Controlled (DTC) salient pole synchronous motor drive is proposed.

A motor model which combines the induction law based "voltage model" and motor inductance parameters based "current model" is presented. The voltage model operates as a main model and is calculated at a very fast sampling rate (for example 40 kHz). The stator flux linkage calculated via integration from the stator voltages is corrected using the stator
The performance of the electrically excited synchronous motor supplied with the DTC inverter is proven with experimental results. It is shown that it is possible to obtain a good static accuracy of the DTC’s torque controller for an electrically excited synchronous motor. The dynamic response is fast and a new operation point is achieved without oscillation. The operation is stable throughout the speed range. The modelling of the magnetising inductance saturation is essential and cross saturation has to be considered as well. The effect of cross saturation is very significant. A DTC inverter can be used as a measuring equipment and the parameters needed for the motor model can be defined by the inverter itself. The main advantage is that the parameters defined are measured in similar magnetic operation conditions and no disagreement between the parameters will exist. The inductance models generated are adequate to meet the requirements of dynamically demanding drives.
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The research work has been carried out during the years 1995 and 1997 in the Laboratory of Electrical Engineering at Lappeenranta University of Technology, where I have worked as a research engineer. The task of writing the dissertation has been performed during last year 1998 at ABB Industry Oy, where I work as a R/D engineer. This year of working experience in the industry in particular proved to be very valuable, since it gave me a wide view and a new practical understanding concerning the large application area of electrical drives.

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Helsinki, March 1999

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LIST OF SYMBOLS

Nomenclature

\( A \) cross area, vector potential
\( B \) flux density
\( c \) coefficient for space vector reduction
\( C \) machine coefficient
\( D \) diameter
\( f \) Magneto Motive Force (MMF), frequency
\( H \) field intensity
\( i \) current
\( I \) current
\( k \) saliency ratio
\( K \) unsaturated saliency ratio
\( L \) inductance, length
\( m \) number of phases
\( N \) number of turns
\( p \) number of polepairs
\( P \) active power
\( \psi_{\text{sis}} \) flux linkage
\( q \) number of slots per phase per pole
\( Q \) number of slots
\( r \) resistance
\( R \) resistance
\( t \) time, instantaneous electric torque
\( T \) temperature, static electric torque
\( u \) voltage
\( U \) voltage
\( W \) weighting factor
\( x \) reactance [pu]
\( X \) reactance
\( \alpha \) angle
\( \beta \) angle
\( \delta \) load angle, length of the air gap
\( \phi \) flux, logical variable of the stator flux linkage error
\( \gamma \) angle
\( \varphi \) angle
\( \kappa \) logical variable of the stator flux linkage orientation
\( \mu \) angle, permeability
\( \theta \) rotor angle
\( \sigma \) angle, conductivity
\( \tau \) time constant, pole pitch, logical variable of the torque error
\( \omega \) angular frequency
\( \xi \) winding factor
\( \psi \) flux linkage
\( \zeta \) angle
\( \Gamma \) characteristic function
\( \Lambda \) permeance

Indices
- \( a \) armature
- \( a, b, c \) phases
- \( AC \) alternating current
- \( act \) actual
- \( base \) base value
- \( c \) intermediate circuit
- \( cor \) correction
- \( cur \) current model
- \( d \) real axis in the rotor oriented reference frame (direct axis)
- \( drift \) stator flux linkage drifting
- \( D \) direct axis damper winding
- \( DC \) direct current
- \( e \) effective, electrical
- \( err \) error signal
- \( error \) error signal
- \( est \) estimate
- \( f \) excitation
- \( FEM \) finite element method
- \( gain \) gain
- \( inc \) incremental
- \( k \) Canay
- \( lin \) linear magnetic conditions
- \( load \) motor load
- \( loss \) losses
- \( m \) magnetisation
- \( max \) maximum
- \( md \) direct axis magnetisation
- \( meas \) measured
- \( min \) minimum
- \( mod \) modelling
- \( mq \) quadrature axis magnetisation
- \( n \) index of sample
- \( no \) load motor no load operation
- \( nom \) nominal
- \( nonlin \) non linear magnetic conditions
- \( offs \) offset
- \( op \) operation point
- \( q \) imaginary axis in the rotor oriented reference frame (quadrature axis)
- \( Q \) quadrature axis damper winding
- \( r \) rotor, reduction
- \( ref \) reference
- \( res \) resultant, reserve
- \( ripple \) ripple of the signal
s  stator, leakage
S  sampling
sA  phase A
sal  saliency
sat  saturation
sB  phase B
sC  phase C
skin  skin effect
stat  static
th  threshold
torq  torque
tot  resultant
u  unsaturation
vol  voltage model
x  real axis in the stator oriented reference frame
y  imaginary axis in the stator oriented reference frame
ψ  flux linkage
σ  leakage

Mathematical symbols
*  complex conjugate
×  cross product
j  imaginary unit \( \sqrt{-1} \)

small letters bold  vector
ALL CAPS BOLD  matrix

Acronyms
AC  Alternate Current
CSI  Current Source Inverter
DC  Direct Current
DFLC  Direct Flux Linkage Control
DSP  Digital Signal Processor
DTC  Direct Torque Control
EMF  Electro Motive Force
FEM  Finite Element Method
GTO  Gate Turn-off Thyristor
ID  identification
IEC  International Electrotechnical Commission
IEEE  Institute of Electrical and Electronics Engineers
IGBT  Insulated Gate Bipolar Transistor
IGCT  Integrated Gate Commutated Thyristor
IM  Induction Machine
ITC  Indirect Torque Control
LCI  Line Commutated Inverter
MMF  Magneto Motive Force
NEMA  National Electrical Manufacturers Association
PC  Personal Computer
PM  Permanent Magnet
PMSM  Permanent Magnet Synchronous Machine
PWM  Pulse Width Modulation
SM  Synchronous Machine
VSI  Voltage Source Inverter
The Song Remains The Same
------------------------------------
(Page/Plant)

I had a dream- crazy dream
Anything I wanted to know
Any place I needed to go

Hear my song- sing along
Any little song that you know
Everything that’s small has to grow.

California sunlight, sweet Calcutta rain
Honolulu starbright- the song remains the same.
1 INTRODUCTION

A modern AC motor drive is a very intelligent system which covers a wide range of different electrotechnical apparatus and a wide scope of electrical engineering skills. A today’s AC motor drive consists of four closely acting main parts: the AC machine, the power electronics, the motor control algorithm and the control hardware, i.e. the signal electronics. In this content only the motor control subsystem is considered. The advances in semiconductors and microelectronics have made the rapid development of AC motor drives possible. Semiconductors used in the switching converters provide the electric energy processing capability and microcontrollers and digital signal processors (DSP) provide the data processing power for complex control algorithms. Many different combinations of AC machines, power converters and control methods are existing. In the following chapters a brief introduction to the evolution of AC drives and their present state is given. For large power, high-dynamic performance drives the commonly known principle of field oriented vector control has been used, but nowadays it is not the only way to implement such a drive. The new control methods are also emerging.

1.1 THE EVOLUTION OF SPEED CONTROLLED AC MOTOR DRIVES

The idea of the vector controlled AC motor drives is based on the ideal control properties of fully compensated DC motors, where the torque and motor magnetic flux can be separately controlled. During the development of the vector controlled AC technology one of the first theoretical problems that appeared was the problem of modelling a multi phase AC winding system in a way that made it possible to separate the torque control from the flux control. The introduction of the space vector in an appropriate reference frame solved the problem.

The importance of synchronous machines in power systems was rapidly increasing during the first decades of the 20th century and therefore intensive research work to develop the synchronous machine theory was done in the 1920’s e.g. in the United States. Synchronous machine models had to be able to analyse power network fault conditions. R.E. Doherty and C.A. Nickle introduced their synchronous machine theory development results at the end of the 1920’s [Doherty & Nickle 1926, 1927, 1928 and 1930]. The theory developed by Doherty and Nickle included the two-axis modelling of the synchronous machine. In the late 1920’s also Park introduced his generalised method for idealised machines [Park 1928, 1933]. The two reaction theory of the synchronous machines is usually referred to as the Park’s two-axis model. Park replaced the measured machine phase quantities by calculatory ones in the rotor reference frame. Park’s two reaction theory actually completed the synchronous machine linear theory. No saturation effects are taken into account and it is clear that the model behaviour differs from the real machine behaviour. The development of a synchronous machine model that takes saturation effects into account has been a challenging problem during the past decades and many different saturation models have been proposed. The two-axis model, however, has normally been the basic method that has been improved.

The space vector theory by Kovács and Rácz [1959], Kovács [1984] for multi-phase AC machines combined the motor phase quantities into a single complex vector variable. The space vector theory - with its simple reference frame changing properties - opened the way for the development of vector control.
The development of electric power switching devices has been a precondition for the development of controlled electrical drives. In 1900 the Mercury-arc rectifier was invented by Cooper-Hewitt. This switching component has been used in the first inverter topologies. The first "Switching tube" inverter circuit was developed by Steinmetz in 1906. After that, in 1925 Prince developed a parallel inverter with natural commutation, and 1928 the parallel inverter with forced commutation appeared which, however, was unable to handle any reactive power, and therefore the use of a synchronous machine as a drive motor was necessary. After reactive power handling - with feedback diodes introduced by Steenback in 1931 - had been made possible and after the development of the cycloconverter family in the beginning of the 1930's, the development of the power electronic frequency converters advanced significantly. The control of motors at that time reminds the control of constant frequency motor drives and the stability as well as the dynamics of the motor drive control were very poor. In these drives the synchronous machine often lost synchronism during load transients and that was one reason why induction machines were preferred.

The first transistor was developed in 1948 but it took still another decade before the first commercial thyristor was introduced. The thyristor made finally the electronic control of electric power circuits possible. Thyristor bridge based controlled DC motor drives developed fast in 1960's. This drive type has been very popular ever since in traction and industry applications. Intensive development work was done to improve also variable frequency AC drive systems and e.g. the pulse width modulation (PWM) technique was introduced in variable frequency AC drives in 1964 [Stemmler 1994].

Felix Blaschke developed the first field oriented vector controlled AC motor drive in the late 1960's [Blaschke 1972]. He was the first one to see that - like in fully compensated DC motor drives - it is possible to control separately the AC motor air gap magnetic flux and torque producing currents. The principle of field orientation applied to AC motor control by Blaschke was called the transvector control. The transvector control was first applied for large synchronous motors by Siemens.

Finns were also active in the field of AC drives in 1970's. Strömberg Oy manufactured asynchronous motors and development engineer Mr. Martti Harmoinen saw that because of its robustness and competitive price the asynchronous motor suited very well for traction drives and industry applications. Thus Strömberg decided to develop a power electronic converter for asynchronous motor control. Mr. Harmoinen's group developed a PWM based variable frequency scalar control for the motor type. The technique was adopted in the Helsinki underground asynchronous motor drives in the beginning of 1980's. These Helsinki underground drives became the first controllable AC traction drive in the world. Strömberg applied the new technology also to paper machine speed controlled drives. AC motor drive technology has been made good use of in many different areas of application and the demand for high dynamic performance necessitated the use of field oriented vector control also in asynchronous machine drives. Substantial development of the microcontrollers and microprocessors made it possible to develop more and more accurate and complicate control algorithms. Vector control methods for AC drive systems have been widely discussed e.g. in [Leonhard 1996a] and [Vas 1990, 1992, 1998].

A new principle for rotating field machine control was introduced in Germany and in Japan almost simultaneously by Depenbrock [1985] and by Takahashi and Noguchi [1986]. Depenbrock's method was called Direct Self Control. According to the principles of this method the stator flux linkage of the AC motor is directly controlled with the stator voltage vector and no current vector control is necessary. A name that describes clearly the method is
the Direct Flux Linkage Control, DFLC. The Finnish team of Professor Harmoinen in ABB Industry Oy, previously Strömberg Oy, developed the first industrial application based on the new control method [Tiitinen 1995]. This control utilises a new powerful digital signal processor (DSP) which had emerged on the market just opportune in the beginning of 1990's. The control method of the commercial product was named the Direct Torque Control - DTC.

1.2 CURRENT AC DRIVES

The power ratings of large variable frequency drives vary from one megawatt up to one hundred megawatts depending on the application. The nominal rotational speeds of these applications vary e.g. from 15 min\(^{-1}\) of a 6 M W cycloconverter fed drive to 18,000 min\(^{-1}\) of a 3 M W high-speed drive equipped with active magnet bearings and solid rotors. A considerable quantity of material concerning large drives has been published by e.g. Leonhard [1996a]; Bose [1997]; Novotny and Lipo [1997] and Vas [1992, 1998].

Table 1.1 shows typical configurations of controlled AC drives and it is constructed according to professor Leonhard’s [1996b] review. Some typical industrial applications are taken into account. The updated table gives a basis for the discussion of different AC drive applications. The table includes the influence of the development of IGBTs and new IGCTs [Buschmann, Steinke 1997].

Table 1.1. Typical configurations of controlled AC-drives according to Leonhard [1996b] (updated).

<table>
<thead>
<tr>
<th>Converters</th>
<th>DC link converters</th>
<th>Current-source converters (CSI)</th>
<th>Cycloconverters with line commutation (GTO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous motor with permanent magnet excitation</td>
<td>Low power (10 kW), very good dynamic performance (servo drives). More than 1 MVA (wind turbines)</td>
<td>Medium power (3 MW), high power density</td>
<td>Medium to high power (4 MW), high speed</td>
</tr>
<tr>
<td>Reluctance synchronous motor</td>
<td>Low to medium power (100 kW)</td>
<td>Medium to high power (5 MW), good dynamic performance (traction drives)</td>
<td>High power (10 MW), low speed, very good dynamic performance</td>
</tr>
<tr>
<td>Squirrel-cage induction motor</td>
<td>Low to medium power (5 MW), high speed, very good dynamic performance (spindle and servo drives)</td>
<td>Medium to high power (5 MW), good dynamic performance (traction drives)</td>
<td>High power (20 MW), subsynchronous operation</td>
</tr>
<tr>
<td>Double fed slip-ring induction motor</td>
<td>Shaft generators on ships (2 MW)</td>
<td>High power (20 MW), limited speed control range</td>
<td>High power (20-40 MW), low speed, good dynamic performance</td>
</tr>
<tr>
<td>Synchronous motor with field and damper windings</td>
<td>8 MVA unit (Siemens)</td>
<td>High power (typ. 40 MW), high speed. The largest: 101 MVA ABB: NASA wind tunnel fan 1998</td>
<td>High power (20-40 MW), low speed, good dynamic performance</td>
</tr>
</tbody>
</table>

AC drive converters can be divided into direct converters and current or voltage source converters with intermediate DC links. Direct converters, such as cycloconverters, connect the phases of the machine instantaneously directly to the supplying network and thus the motor
Phase voltages are determined directly according to the phase voltages of the line and switch controls. DC link converters perform a two-phase frequency conversion. The alternating voltage is first rectified and supplied to the DC link. The DC link filters the harmonics of the voltage or of the current. Depending on the filter type the constructions are called either voltage source inverters (VSI) or current source inverters (CSI). Fig. 1.1a) shows the most typical VSI induction motor drive type with IGB transistors and Fig. 1.1b) represents a synchronous motor cycloconverter drive. Fig. 1.1c) gives an example of a load commutated synchronous machine drive (LCI). VSI has been employed in large industrial asynchronous motor drives, but commercial VSI synchronous motor drives are still being developed.

![Diagram of AC drive configurations]

Fig. 1.1. Different converters for AC drives [Leonhard 1996a],[Bose 1997]: a) VSI induction motor drive; b) cycloconverter drive for a synchronous motor; c) LCI synchronous motor drive. The drives are equipped with rotor position measurement.

Different converters utilise different amounts and different types of power electronic switches. Two- and three-level alternatives of the VSI are the most popular. One inverter leg consists of four components or ten components respectively. The numbers of voltage vectors are $8 \ (2^3)$ or $27 \ (3^3)$ depending on the VSI-type. Fig. 1.2 shows the voltage vectors of the two-level and the three-level VSI types. The three-level VSI type has a much lower relative output harmonic voltage level than its two-level counterpart and thus a remarkably lower switching frequency can be used. This is also necessary in large high voltage drives since present-day large power electronic switches (IGCTs and GTOs) are not capable of withstanding high switching frequencies. The modern largest VSI units produce a maximum output power of about 8 MW. Large voltage source AC converters will probably gain popularity in the future. Also load
Commutated (LCI) converters and cycloconverters will be used in the largest AC drive applications.

![Diagram](image)

**Fig. 1.2.** The voltage vectors of a two-level a) and a three-level VSI b) in a stationary reference frame [Leonhard 1996a].

Because of their high efficiency, high overload capacity and good performance in the field weakening range the synchronous motors have been utilised in large controlled drives, e.g. for ship propulsion and in rolling mills in steel industry. The requirements of a typical high performance metal industry application are shown in Table 1.2.

**Table 1.2. Drive performance requirements of typical dynamically demanding metal industry applications (ABB Industry Oy).**

<table>
<thead>
<tr>
<th>The requirements of motor loadability</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>%T&lt;sub&gt;nom&lt;/sub&gt; (according to the NEMA)</td>
<td></td>
</tr>
<tr>
<td>100 %</td>
<td>continuously, temperature according to class B</td>
</tr>
<tr>
<td>115 %</td>
<td>continuously, temperature according to class F</td>
</tr>
<tr>
<td>125 %</td>
<td>two hours, temperature according to class F</td>
</tr>
<tr>
<td>175 %</td>
<td>one minute, frequently</td>
</tr>
<tr>
<td>200 %</td>
<td>one minute, occasionally</td>
</tr>
<tr>
<td>250 %</td>
<td>10 seconds</td>
</tr>
<tr>
<td>275 %</td>
<td>cut off</td>
</tr>
<tr>
<td>70 %</td>
<td>zero speed</td>
</tr>
</tbody>
</table>

| The requirements of the drive performance | | |
|-------------------------------------------|----------|
| Static accuracy of the torque | ± 1 % of T<sub>max</sub> |
| Torque step response time of rise %T<sub>nom</sub> | 3.5 ... 5 ms (0 ... 70 % n<sub>nom</sub>), 5 ... 12 ms (70 ... 100 % n<sub>nom</sub>) |
| Amplitude of a single air gap torque harmonic | < 3 % pp (1% rms), linear increase allowed | |
| Static speed accuracy | ± 0.01 % of n<sub>max</sub> |
| Dynamic speed accuracy | ± 0.2 ... 0.5 %s of n<sub>max</sub> (error integral during load impulse) |
| Field weakening ratio | 1:5 |

Traditional vector controlled cycloconverter or LCI converter synchronous motor drives have dominated the market. Cycloconverters are suitable for very low-speed drives, e.g. in reversing rolling mills in the metal industry and in mine hoists. The output frequency of the cycloconverter drives is restricted to about 40 ... 50 % of the supplying line frequency. The line current of the cycloconverter includes typical thyristor bridge harmonics and it also produces some subharmonics which are difficult to filter.
An LCI drive consists of a line commutated six pulse thyristor rectifier, a DC-current link and a machine commutated six pulse thyristor inverter. The output frequency is restricted by the duration of the motor bridge commutation. This converter type can thus be used in relatively high-speed drives. At low speeds, when the back electromotive force of the motor is not high enough to commutate the motor inverter thyristors, special methods are needed. The network bridge gives short current pulses so that the thyristors of the motor inverter have time to switch off. The motor torque is thus zero during the commutation. The high reactive power typical for the high current low voltage operation of a thyristor rectifier is another problem related to low speeds and heavy start-ups [Bose 1998 pp. 354-355], [Leonhard 1996a pp. 330-331], [Niiranen 1992 pp. 55-56]. Consequently the main applications of load commutated CSI converters are drives where low speed operation is not needed and the start-up is light. Blowers, compressors, pumps and start-up equipment for gas turbine generators are typical applications.

Vector controlled synchronous motor drives normally utilise position sensors, and, for security, two separate sensors are often installed. Synchronous motor drives for dynamically demanding applications will be equipped with position sensors also in the future. The power ratings of these drives vary typically from one to ten megawatts. Even larger synchronous motors in ship propulsion, where the dynamic demands are much lower, may be operated without a position sensor. Pump and blower drives are also suitable to be operated without a position sensor. The need for a position sensor diminishes the reliability of the drive and increases the costs and thus there seems to be a tendency to develop drives where shaft sensors are not necessary.

1.3 COMPARISON OF CONTROL METHODS - INDIRECT TORQUE CONTROL (ITC), DIRECT FLUX LINKAGE CONTROL (DFLC) AND DIRECT TORQUE CONTROL (DTC)

To understand a DTC or field oriented current vector control a representation of the space vector is required [Kovacs 1959, Leonhard 1996a, Vas 1992]. A symmetrical three-phase stator winding system is considered. These three phases are magnetically displaced by an angle $2\pi/3$ from each other in the space around the air gap periphery. The instantaneous values of the stator phase currents are $i_A(t)$, $i_B(t)$ and $i_C(t)$. The Magneto Motive Force $f_s(\alpha,t)$ in the air gap caused by the stator currents can be defined as a resultant of the three stator phases.

$$f_s(\alpha,t) = N_{se} \left[ i_A(t) \cos \alpha + i_B(t) \cos \left( \alpha - \frac{2\pi}{3} \right) + i_C(t) \cos \left( \alpha - \frac{4\pi}{3} \right) \right], \quad (1.1)$$

where

- $N_{se} = N_s\xi_s$ ~ effective number of turns in series per phase in the stator winding,
- $N_s$ ~ number of turns in series per phase in the stator winding,
- $\xi_s$ ~ winding factor for the fundamental wave,
- $\alpha$ ~ angle from phase sA magnetic axis in stator periphery.

By using the Euler equations Eq. (1.1) it can be rewritten

$$f_s(\alpha,t) = N_{se} \left[ i_{sres}(t)e^{-j\alpha} + i_{sres}^*(t)e^{j\alpha} \right]. \quad (1.2)$$
In Eq. (1.2) $i_{\text{res}}(t)$ the resultant space vector of the stator currents is expressed in a stationary reference frame fixed to stator

$$i_{\text{res}}(t) = i_{sA}(t)e^{j0} + i_{sB}(t)e^{j\gamma} + i_{sc}(t)e^{j2\gamma}. \quad (1.3)$$

The stator current space vector is time dependent and $i^*_{\text{res}}(t)$ is its complex conjugate. $\gamma$ is the phase shift between phases ($\gamma = 2\pi/3$). Usually the stator current vector is reduced with a constant and we get

$$i_*(t) = c[i_{sA}(t)e^{j0} + i_{sB}(t)e^{j\gamma} + i_{sc}(t)e^{j2\gamma}]. \quad (1.4)$$

If $c = \frac{2}{3}$, then a definition of the asymmetrical non-power invariant form of the three-phase space vector is obtained and if $c = \frac{\sqrt{2}}{\sqrt{3}}$, then the symmetrical power invariant form of the three-phase space vector is obtained. In the following the former method where $c = \frac{2}{3}$ is used. Fig. 1.3 illustrates the construction of the space vector of the stator phase currents. It is assumed that the instantaneous values of the phase currents are $i_{sA} > 0$ and $i_{sB}, i_{sc} < 0$.

Fig. 1.3. The space vector $i_*(t)$ of the stator currents $i_{sA}(t), i_{sB}(t)$ and $i_{sc}(t)$.

The magnitude and the angle of the stator current vector vary with the time

$$i_*(t) = |i_*(t)|e^{j\phi(t)}. \quad (1.5)$$

The space vectors of other motor variables such as the voltage vector $u_*$, the flux linkage vector $\psi_*$ etc. can be obtained similarly. For example the voltage space vector $u_*$ is defined as

$$u_*(t) = \frac{2}{3}[u_{sA}(t)e^{j0} + u_{sB}(t)e^{j\gamma} + u_{sc}(t)e^{j2\gamma}] \quad (1.6)$$

It is important to note that the space vector represents only a complex notation of a sinusoidal space distribution and therefore it is not strictly a physical vector.
The basic idea of the field oriented current vector control of AC machines is to control the motor torque using the same technique as in DC machines [Leonhard 1996a, Vas 1990]. The electrical torque $t_e$ of a fully compensated DC machine can be defined as

$$t_e = C \psi_f i_a,$$  \hspace{1cm} (1.7)

where $\psi_f$ is the instantaneous value of the magnetising flux linkage produced by the excitation winding and $i_a$ is the instantaneous value of the armature current. $C$ is in this case a machine dependent coefficient. If the flux linkage of the DC machine is kept constant, its electrical torque can be controlled by adjusting the armature current. Thus the electrical torque is fully controlled and very good dynamic operation can be achieved. For the AC machine the electrical torque can be defined for example as a vector product of the stator flux linkage and the stator current space vectors presented in the stator oriented reference frame

$$t_e = C(\psi_s \times i_s).$$  \hspace{1cm} (1.8)

In the DC machine the magnetising flux linkage and armature current are in space quadrature due to the commutator and the compensating winding, thus Eqs. (1.7) and (1.8) can be considered equal. The current component which produces the magnetising flux linkage (excitation current $i_f$) and the current component which produces the electrical torque (armature current $i_a$) can be controlled separately. This decoupled control of motor flux and electrical torque is the basic idea of the field oriented current vector control of AC machines. The field oriented control is implemented using currents as control variables to control the magnetic state and electrical torque of the machine and thus it is named Indirect Torque Control - abbreviated ITC - in this context.

ITC can be achieved using the space vectors of the electrical variables when the quickly responding stator current space vector is defined in the co-ordinates fixed to the rotating flux linkage vector. Using a moving reference frame fixed to the flux linkage vector it is possible to define the stator current component equal to the armature current of the DC machine. This stator current component is chosen to be the main control input for the torque. It corresponds to the component of the stator current space vector that is in space quadrature to the flux linkage vector. The direct component of the stator current space vector, which in this case is aligned with the flux linkage vector, is used as the control input for the flux of the machine. Fig. 1.4 shows the principle of the field orientation where the axes $(x_\psi, y_\psi)$ correspond to the rotating field oriented reference frame. It can be seen that the flux producing current component is $i_m = (i_{x_\psi} + i_{x_\psi})$ and the torque producing current component is $i_{torq} = (i_{y_\psi} + i_{y_\psi})$. In the case of electrically excited synchronous motors the air gap flux is determined by the armature voltage. The task of the field winding excitation current $i_f$ is to keep the desired armature power factor. In the case of a permanent magnet machine a fictitious excitation current $i_f$ can be assumed. In both cases the machine can operate with the stator current $i_{x_\psi} = 0$. When induction machines are concerned there is no auxiliary excitation and the flux must be controlled by the stator current component $i_{x_\psi}$.
It is possible to implement ITC control using a coordinate system fixed to some of the machine flux linkages or using the rotor reference frame or even the stator reference frame. The idea of field orientation is, however, fully met when the so called stator flux oriented control or rotor flux oriented control is used. In the case of a synchronous motor the air gap flux linkage oriented control is used often. In practice the control is implemented using such a space vector of the flux linkage which is known for its smooth behaviour and which can not change quickly in a transient. Thus the rotor flux linkage oriented control in induction machines and the air gap flux linkage oriented control in synchronous motors are the most used ones. In Fig. 1.5 the principle of the ITC control for AC machines is shown.

The implementation of the indirect torque control has to provide solutions for the following tasks:

- Co-ordinate transformations between the rotating reference frame and the stationary reference frame which calls for trigonometric functions with knowledge of the angle of the co-ordinate system used.
- Acquisition of the various flux linkages from terminal quantities such as voltages, currents, speed and position.
- Impressing the flux producing current component and the torque producing component in the field oriented co-ordinate system.
- Obtaining an electrical torque to close the torque control loop.
- Separate PI control of the flux and torque and, further on, a separate current control of the torque producing and flux producing current components.
- Decoupling of the torque producing and flux producing voltage reference components.
- PWM modulator for switch pulse pattern implementation.
The basic idea of the DFLC method is to adapt directly Faraday’s induction law in the calculation of the machine flux linkage vector $\psi_s$. With a two-level voltage source inverter in contribution with a three-phase winding six different non-zero voltage vectors and two zero voltage vectors can be produced, as introduced in Fig. 1.2. The induction law gives a connection between the voltage vector $u_s$ and the flux linkage vector $\psi_s$,

$$\psi_s = \int (u_s - i_s R_s) dt. \quad (1.9)$$

This equation will later be referred to "the voltage model" of the motor.

According to Eq. (1.9) it is possible to drive the stator flux linkage to any position with the six available non-zero voltage vectors of a three-phase inverter. It can be shown [Takahashi and Noguchi 1986], that the increase of the slip immediately increases the motor torque. On the other hand the flux linkage amplitude should be kept constant, if the motor has to work in the same magnetic working point. These two conditions together with the field orientation give all necessary information to control the power stage switches in order to meet the required flux linkage amplitude and the requested torque. The electric torque $t_e$ of the machine can now be directly calculated as

$$t_e = \frac{3}{2} p \psi_s \times i_s. \quad (1.10)$$

The idea of the torque control in DFLC is based on the fact that the rotor and air gap flux linkages have quite long time constants. For example in induction machines the rotor time constant is typically in the range of 100 ms. Using modern fast DSPs it is possible to make
decisions on the inverter switching states in a few microseconds. With respect to this it is possible to achieve a fast torque change by changing the stator flux according to Eq. (1.9).

The calculated instantaneous torque can easily be compared to the reference value to achieve a fast torque control. At the same time the stator flux linkage is compared to the reference value to ensure a sufficient magnetisation of the machine. According to Takahashi the torque and the stator flux linkage can be controlled using the so called "optimal switching logic table". The optimal switching table is an essential part of DFLC. The name was given by the Japanese inventors. At every modulation instant it selects the most suitable voltage vector to meet the flux and torque control requirements relative to the stator flux linkage orientation.

The optimal switching table is a logic array which has three logical input variables. The logical input for the torque is done by supplying the error of the torque into a three-level hysteresis comparator, and the logical input for the flux linkage is achieved by supplying the flux linkage error into a two-level hysteresis comparator. The third input is a discrete-valued field orientation variable, which can have six different values. These values describe six different switching sectors κ which use different voltage vectors for a certain combination of the flux and torque logical variables. Fig. (1.6) shows the functional principle of the optimal switching table and the comparators for the two-level voltage source inverter. It is important to observe where the stator flux linkage is situated. The flux linkage circle is divided into six segments so that the available voltage vectors are in the middle of each segment. In each segment it is possible to use two voltage vectors in each rotating direction. One vector increases and the other vector decreases the flux linkage length. When two hysteresis controls - one for the flux linkage and the other for the torque - are unified an optimal switching table is found.

Because the flux linkage is controlled inside a hysteresis band its absolute value is either increased 1 or decreased -1. With respect to the torque there are three possibilities: 0 a zero voltage vector is used, 1 flux linkage is turned in positive direction, -1 flux linkage is turned in negative direction.
<table>
<thead>
<tr>
<th>Flux linkage position $\tau$, $\phi$</th>
<th>$\kappa = 0$</th>
<th>$\kappa = 1$</th>
<th>$\kappa = 2$</th>
<th>$\kappa = 3$</th>
<th>$\kappa = 4$</th>
<th>$\kappa = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$u_5$</td>
<td>$u_6$</td>
<td>$u_1$</td>
<td>$u_2$</td>
<td>$u_3$</td>
<td>$u_4$</td>
</tr>
<tr>
<td>-1</td>
<td>$u_6$</td>
<td>$u_1$</td>
<td>$u_2$</td>
<td>$u_3$</td>
<td>$u_4$</td>
<td>$u_5$</td>
</tr>
<tr>
<td>1</td>
<td>$u_3$</td>
<td>$u_4$</td>
<td>$u_5$</td>
<td>$u_6$</td>
<td>$u_1$</td>
<td>$u_2$</td>
</tr>
<tr>
<td>1</td>
<td>$u_2$</td>
<td>$u_3$</td>
<td>$u_4$</td>
<td>$u_5$</td>
<td>$u_6$</td>
<td>$u_1$</td>
</tr>
<tr>
<td>0</td>
<td>not defined</td>
<td>$u_0$</td>
<td>$u_0$</td>
<td>$u_0$</td>
<td>$u_0$</td>
<td>$u_0$</td>
</tr>
</tbody>
</table>

Fig. 1.6. The functional principle of the optimal switching table is based on the three multi-valued logical variables; the torque error, the flux linkage amplitude error and the discrete-valued field orientation.

Fig. 1.7 shows the stator and air gap flux linkages rotating counter clockwise at an angular frequency $\omega$ near nominal speed (no zero voltage vectors are used).

Fig. 1.7. A stator flux linkage vector trajectory as a result of the DFLC control. No unnecessary switchings occur in the DFLC control [Pyrhönen J. 1997].
The dot lines indicate the tolerance band of the stator flux linkage vector length. The torque has its own hysteresis limits. When the upper or lower hysteresis limit is reached a new voltage vector is applied (Fig. 1.7). The stator flux linkage is first integrated using the voltage vector $\mathbf{u}_3$ until the lower limit is reached or until the torque reaches its upper limit. The stator flux linkage is kept inside the hysteresis band and the torque is controlled mainly by changing the flux linkage angle using the voltage vectors transverse to the Stator flux linkage (Fig. 1.7 $\mathbf{u}_3$ and $\mathbf{u}_6$). The transverse voltage vector gives a very fast change in the torque. A large torque step can be achieved in a few milliseconds depending mainly on the subtransient inductance of the machine.

The optimal switching table is an ideal modulator. Every switching transfers the stator flux linkage into the right direction. No unnecessary switchings take place and the dynamic response is extremely good. DFLC is not inductance parameter sensitive. The physical dependence between the voltage and the stator flux linkage is simple and accurate. The only necessary parameter is the stator resistance, which is rather easy to estimate when the current is measured. DFLC controls directly the torque of the motor with the voltage, and no extra current control loops are required. DFLC suits well for different types of rotating field machines and requires in theory no rotor position feedback. However, the direct flux linkage control method, described above, is not capable of working without any extra feedback in the whole motor speed range. Especially at low speeds the integral of the voltage model (Eq. 1.9) contains too much error and the motor stator flux linkage drifts so that it rotates no more origin centred. It seems that up to now the best method to solve this problem is to apply a traditional inductance model that prevents the motor stator flux linkage from drifting. This traditional model of the motor will later be referred to "the current model" of the motor.

A combination of the voltage model and the supervising current model is called the Direct Torque Control Method (DTC). ABB Industry was the first manufacturer that introduced a DTC-based Induction motor drive (ACS600) on the market a few years ago. The DTC-induction motor drive is capable of operating without any rotor position feedback in the whole speed range. The DTC-method is applicable to all rotating field machines, but in the case of a synchronous motor the use of the current model has the disadvantage that a rotor position measurement system must be used. Thus, part of the advantages of the DFLC-method are lost. In Fig. 1.8 the principle of the DTC control for AC machines is demonstrated.

The implementation of the Direct Torque Control has to provide solutions for the following tasks:

- Acquisition of the various flux linkages from terminal quantities such as voltages, currents and position.
- Obtaining the instantaneous electrical torque to close the torque control loop.
- Controlling separately the flux linkage and torque.
- Using the DFLC modulator optimal switching table for the switch pulse pattern implementation.
In a field oriented current vector controlled drive the reference values for the magnetising current component and the torque current component are first calculated. The voltage vector is then selected to produce the required current components. From the physical point of view, the voltage vector selection is not a trivial task, if the saturation of the machine is also considered. The dependence between the current $i$, inductance $L$ and the voltage $u$ under saturation is

$$u = L \frac{di}{dt} + i \frac{dL}{dt}$$  (1.11)

Normally the latter term in Eq. (1.11) is neglected and the first term may have some errors due to erroneous inductance parameters. Because of the parameter dependency of Eq. (1.11) optimal torque control appears to be a difficult task. The current controller must be tuned correspondingly to the motor parameters which are changing due to saturation. Such an adaptation requires a lot of calculation power and may still not be optimal. Inaccurate dependence between the input and output requires higher switching frequency, so that the feedback control can correct the current errors. Table 1.3 still shows a comparison between the current vector control, DFLC and DTC.
Table 1.3. Theoretical comparison of the synchronous motor inner torque control between ITC, DFLC and DTC.

<table>
<thead>
<tr>
<th>DFLC (flux control)</th>
<th>ITC (current control)</th>
<th>DTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance parameter independent</td>
<td>Strongly dependent on inductances</td>
<td>Inductance parameter dependent dep. on speed</td>
</tr>
<tr>
<td>Torque controlled directly by voltage</td>
<td>Torque controlled indirectly by voltage</td>
<td>Torque controlled directly by voltage</td>
</tr>
<tr>
<td>One control loop (control cycle e.g. 25 µs)</td>
<td>Two control loops (control cycle e.g. 100 µs), decoupling circuit needed</td>
<td>One control loop (control cycle e.g. 25 µs)</td>
</tr>
<tr>
<td>Robust hysteresis control (adjusted hysteresis bands are quite complex)</td>
<td>Tuned PI controllers needed</td>
<td>Robust hysteresis control (adjusted hysteresis bands are quite complex)</td>
</tr>
<tr>
<td>Physically accurate control law</td>
<td>Inaccurate control low due to parameter errors</td>
<td>Physically accurate control law</td>
</tr>
<tr>
<td>Over current protection more complicated to implement, because currents are not directly controllable</td>
<td>Over current protection easy to implement, because currents are directly controllable</td>
<td>Over current protection more complicated to implement, because currents are not directly controllable</td>
</tr>
<tr>
<td>Rotor position not required</td>
<td>Rotor position required</td>
<td>Rotor position required</td>
</tr>
</tbody>
</table>

1.4 DTC FOR ELECTRICALLY EXCITED SYNCHRONOUS MOTOR

Usually large synchronous motors in high performance applications are driven with cycloconverters and the power range goes up to 20 ... 40 MW. Cycloconverters, however, are not suitable if Direct Torque Control (DTC) is implemented. A voltage source frequency converter with a fast switching inverter unit is needed for this purpose. DTC suits best for the power area where Insulated Gate Bipolar Transistors (IGBTs) are available. New medium voltage switching device IGCT (Integrated Gate Commutate Thyristor) or GCT can be used for larger units as well. When the inverter unit is built with IGBTs or GCTs switching times are short (typically in the range of a few µs) [Pyrhönen J. 1997].

As mentioned earlier, DFLC cannot be applied to the motor control only without a machine voltage measurement, because the stator flux linkage integration using Eq. (1.9) at low motor speeds is inevitably too erroneous. For instance, when the voltage drop in the switches is larger than the Electro Motive Force of the machine, it is very difficult to maintain the stator voltage integration correct enough. Because of this, especially at low speeds, it is necessary to use a rotor oriented current model for the machine to correct the stator flux linkage $\psi$. When the best properties of DFLC and the traditional current control are combined the DTC-control for all rotating field AC-machines is established. However, using the current model does not change the nature of DFLC; the DTC-system is at any instant capable of producing fast torque changes using the core elements of DFLC. Also DFLC is independent on the saturation of the motor inductances during fast transients, because the torque step is calculated using mainly the voltage integral and the current model inductances are kept as constants during fast transients (required torque step response typically 1-5 ms when the motor operates below nominal speed). The current model acts only as a supervisor that at longer time levels prevents the motor stator flux linkage from drifting out of the centre. No co-ordinate transformations are needed. The current model, however, is calculated in the rotor reference frame, because the rotor of the salient pole synchronous machine is unsymmetrical. To get valid information
from the current model it is necessary to have correct machine parameters as well as knowledge of the position of the rotor.

Fig. 1.9 illustrates the principle of the electrically excited synchronous motor Direct Torque Control (DTC) block diagram (inner control). The two-phase currents of the synchronous motor are measured. The intermediate voltage is measured and the motor voltage is calculated using this knowledge and using the switching states (S1, S2, S3) of the switches. The motor voltage is then integrated in order to get the stator flux linkage. The current model calculates an estimation of the stator flux linkage and this is used to correct the integrated flux linkage. The parameters of the motor model are first initialised in the identification run and estimated on-line during the operation of the drive. A thermal model of the stator resistance $R_s$ is included. The control of the rotor magnetising current is implemented by a combined reaction-$\cos \phi$ control [Pyrhonen, O. 1998]. In the field weakening area the stator flux linkage reference must be reduced and this is done in the special field weakening calculation block. Two hysteresis controls and an optimal switching logic are implemented according to the presentation before.

\[
\psi_s = \int (\psi_{s,d} - \psi_{s,q}) \, df
\]

Fig. 1.9. Principle of the synchronous motor's Direct Torque Control (DTC) [Pyrhonen J. 1997]. The online motor model calculates the feedback signals for the torque and the flux linkage. The voltage model is the main model and the current model acts only as a supervisor that at longer time levels prevents the motor stator flux linkage from drifting out of the centre.
1.5 OUTLINE OF THE THESIS

The main goal of this thesis is to present a synchronous machine model for an industrial Direct Torque Controlled drive. The objective of the work is not to develop the ultimate model to be used for control feedback calculation, but on the contrary the control system demands for a simple, fast calculable and "reasonably" accurate motor model. The demand for a "reasonably" accurate model varies depending on which performance of the torque control loop is wanted. The motor model proposed is a combination of the voltage and current model. The role of both models is presented in different operation stages. Because of the simplicity of the voltage model the current model derivation is emphasised throughout the work. The saturation of the magnetic paths is very important when inductance based current models are used. The initial calculation of the inductance parameters and measurements where the supply unit itself produces the signals used in the parameter measurements is described. Saturation phenomena are studied and inductance models representing the magnetic saturation of the main flux path are proposed. The accuracy and the behaviour of the motor model is tested using a Direct Torque Controlled test drive. The work is divided in three parts:

1) In the first part (chapter 2) the theory of the synchronous motor modelling is described. A method for the motor flux linkage estimation is introduced and problems associated with it are discussed. The physical nature of the saturation effect is emphasised and good physical understanding is necessary. When the phenomena of saturation are known methods for the representation of the main flux saturation may be presented. At the end of the chapter the behaviour of the voltage and current models in load transients is discussed. Finally, a motor model for the DTC drive is proposed.

2) The second part (chapter 3) demonstrates the synchronous motor parameter calculation using standard measurement methods, design programs and finite element calculations. The parameter sets obtained from these different methods are compared and some conclusions concerning the accuracy of the parameter set are made. The reduction factor between the rotor and stator winding systems is handled in its own section, because it is a very essential coefficient for the model and it is often left without any notice. The parameter measurement of the motor model, where the supply inverter is used as a measurement signal generator, is explained. This so called identification run can be performed in the factory or during drive commissioning. At the end of the second part the generation of the inductance models used for the representation of the saturation effects is proposed.

3) In the third part (chapter 4) experimental results are shown for the Direct Torque Controlled test drive system with a 14.5 kVA, four pole salient pole synchronous motor with damper winding and electrical excitation. The stator is supplied with a two-level voltage source DTC inverter and the excitation circuit is supplied from a four-quadrant DC-chopper. The test drive is introduced. The static accuracy of the drive is shown using the torque measurements in static load operation and the dynamics of the drive are proven by load transient tests.
2  MOTOR MODELLING

In the following chapter the basics of the synchronous motor modelling are introduced. At first the history of the synchronous machine modelling is briefly enlighten. The generally known electrical equations of the motor model are presented as the point from which the process of modelling begins. After that different flux linkage estimation techniques are introduced and the problems related to them are discussed. The motor model has as any other mathematical model of the system a set of parameters which are for the electrical machine the resistances and inductances. The motor model parameters are not constant, but they vary according to the operation point of the machine. This variation of the motor model parameters as well as, more specifically, the problem of saturation in the magnetic circuit and the representation of the saturation effect are discussed in detail. The behaviour of the motor different flux estimation methods are examined during the transients. Finally at the end of the chapter a synchronous motor model that may be applied to the DTC drive can be proposed.

The theory of the salient pole synchronous machine is known for almost one hundred years, but the “Song (of the complete machine model problem) Remains the Same” [Led Zeppelin 1973]. During these years many different methods were developed to model the magnetic field conditions in saturated salient pole synchronous machines in order to predict their performance. The purpose of machine modelling in several applications is to find out the behaviour of the machine in different operation states. Machine models can be derived in many different ways depending on the application. For example in steady state analysis phase phasor analysis is adequate, but in transient analysis space vectors and motor differential equations are needed. Also, there are differences between the machine models that are used for transient analysis of the generator-network interaction and the models used to analyse variable speed motor drives. Salient pole generator models are complicated and their main intention is to model the behaviour of the generator during faults when phenomena in the damper windings are dominant. Salient pole motor models which are used in variable speed vector controlled drives are more simple and the main purpose is to find a reasonably accurate motor model that can be implemented in a real time solution.

There are many reasons why the “song remains the same”. The modelling of the salient pole synchronous machine is a very complicated problem. Several factors make it difficult to model the real machine behaviour. Saliency and saliency dependent phenomena are the most difficult. The magnetic paths of the machine are not similar in the different parts of the machine iron core. Salient rotor poles result in flux distortion in the interpolar spaces and the non-uniform air gap. Because of the latter, the peak of the fundamental of the flux density distribution does not coincide with the peak of the resultant Magneto Motive Force (MMF) distribution, unless the resultant MMF distribution is oriented on one of the axes of symmetry (direct or quadrature axis). Stator windings are embedded in slots. This produces a non-sinusoidal MMF distribution in the air gap and MMF harmonics occur. Possible slot skewing and the effects of the end windings lead into a variation of the field density along the length of the laminated stator yoke. MMF distribution of the rotor salient pole is always in the direct axis, but the rotating MMF-wave distribution of the multi-phase stator winding can be in any position, depending on the machine load and power factor with respect to the rotor direct axis. That is why the resultant MMF distribution is quite difficult to determine. The saturation effect is another source of difficulties. It has an effect on all parts of the machine iron volume. The most heavily saturated regions are the stator teeth and rotor pole shoes, while the stator yoke is much less saturated. Salient pole synchronous machines are designed so that saturation occurs even, if the machine operates well below the rated conditions. In the following section
the history of synchronous machine modelling is shortly introduced. The work of Cordon [1994] is used as a basis for the introduction.

Magnetic paths of a salient pole synchronous machine have unsymmetry because of the rotor construction. Thus it is useful to observe the behaviour of a synchronous machine within the two symmetry axis known as direct and quadrature axis. This is generally known as the two reaction theory first introduced by Blondel [Blondel, 1899, 1913]. The basic idea of the theory was to handle the magnetic difference between the direct and quadrature axes. Fundamental propositions of the theory were: “When an alternator supplies a current dephased by angle \( \psi \) with respect to the internal induced E.M.F., the armature reaction may be considered as the resultant of a direct reaction produced by the reactive current \( I \sin \psi \) and a transverse reaction due the active current \( I \cos \psi \).” The theory is proved to be quite accurate for linear steady state conditions. Doherty and Shirley [Doherty, 1918] were the first to use in their research the principle of constant flux linkages at the beginning of transients. They examined the effects of a non-sinusoidal field and developed estimates of initial short circuit currents. Stevenson and Park [Stevenson, 1927] and Wieseman [Wieseman, 1927] developed and applied the theory to graphical field solutions. Although only suitable for infinite permeability conditions, this technique makes a more accurate and convenient field prediction possible in the air gap of the linear salient pole machine. This graphical technique was used by Doherty and Nickle in a series of articles [Doherty, 1926, 1927, 1928 and 1930] which extended the Blondel two reaction method to account for MMF harmonics, torque angle under steady state conditions and single- and three-phase faults.

Magnetic linearity was one of the fundamental assumptions in the works mentioned above. At the same time Park introduced the famous generalised method for idealised machines [Park, 1928 and 1933]. The basic assumption here is that saturation, hysteresis and eddy currents are neglected and a sinusoidal stator MMF distribution is assumed. The method also needs information about the rotor position. Saliency and multi-phase stator windings are considered. This method is very suitable for steady state and transient analysis and has been the basis for salient pole synchronous machine modelling since then. After the research described above was done, the linear theory of the synchronous machine could be developed.

The linear theory assumes that there are no saturation effects in the machine and it is obvious that errors occur due to that assumption. After the development of the two reaction theory, research was focused on the problem how to account for the effects of saturation. Various attempts were made to model the effects of saturation by using the no-load magnetisation curve and the experimental data. Equivalent synchronous reactances were developed, which enable to calculate the effects of saturation on small variations around each operating point [Crary, 1934]. Empirical saturation factors were defined based on test data and guidelines of their use were introduced [Kilgore, 1935]. The saturated synchronous reactance defined from no-load and three-phase short circuit curves was used in the investigation of the power angle and maximum power [Kingsley, 1935].

All methods based on experimental data have similar limitations. They consider saturation only under direct axis excitation and neglect quadrature axis saturation. Saturated quadrature axis reactance defined from the slip test is included in the study of Robertson, Rogers and Dalziel [Robertson, 1937]. Rüdenburg introduced a method for the investigation of a saturated synchronous machine under transient conditions. The method is based on a graphical construction of the no-load curve combined with machine differential equations [Rüdenburg, 1942]. The development continued to focus on the no-load curve until the mid 1960’s, when
the digital computer was invented. The finite difference method of field computation was applied to the saturation problem [Ahamed, 1966]. The development of computing capability made it possible to introduce the finite element method into magnetic field problems in the early 1970’s [Silvester, 1970 and Chari, 1971].

In this work a salient pole synchronous motor model in a variable speed direct torque controlled drive is considered. The assumption that the machine operates under linear magnetic conditions leads to an error in the predicted operation state. It is obvious that the linear model is not sufficient when we try to estimate the motor performance and to make control decisions according to the measured signals and the motor model. Several methods are developed and a lot of papers are written about the problem, how to include the saturation effect into the motor model. Much of the recent work is focused on the introduction of saturation terms into the generalised motor equations. Also data fitting techniques are largely used.

The development of computation capability in microprocessors has made it possible to use different estimators and state predictors based on motor state equations and on the error between the model and measured data. Several methods based on neural networks were proposed. However, it should be remembered that nowadays industrial motor control systems are real time applications with restricted calculation capacity and at the same time the control system has the demands of a simple, fast calculable and reasonably accurate motor model. It is a matter of compromise between the calculation capacity and accuracy. It should be noted that “The song” still remains after this contribution and the purpose of this work is not to develop an ultimate solution. The goal is to develop an improved solution based on the known electrical machine theory. A method to handle these problems in a Direct Torque Controlled (DTC) salient pole synchronous motor drive is proposed.

2.1 ELECTRICAL EQUATIONS OF THE GENERAL MOTOR MODEL

Electrically excited synchronous motors can be divided into different types depending on their rotor structure. In high speed motors (in the case of a synchronous motor high speed is defined over 1500 min⁻¹) cylindrical rotors with two or four poles are used. A synchronous motor with a cylindrical rotor has a uniform air gap and thus the reluctance in the d- and q- axis are equal, in principle. Non-salient synchronous motors are used in a high speed range due to the mechanical strength of the rotor structure. The air gap of salient pole synchronous motors is smaller in the direct than in the quadrature direction and the reluctances in the d- and q- axis are not equal. The reluctance ratio between the quadrature and direct axes is typically 2...2,5. Salient pole synchronous motors are used at low speeds and in high power applications. The DC-excitation current in the field pole winding can be supplied through slip rings, or the excitation energy can be generated in a brushless excitation unit which rotates with the rotor.

The excitation of a synchronous motor can also be generated by using permanent magnets (PM). PM synchronous motors (PMSM) are mainly used in servo drive applications and the power range varies from 100 W to few hundred kW. PMSMs are also used in windmill applications and power goes up to 1 MW. In the near future PM synchronous motors possibly will replace induction motors in low speed high torque applications. Improvement of efficiency and a high power factor can be achieved when PMSM are used. Stator winding usually consists of three-phase windings, but in high power motors a double three-phase winding can be used. Two equal galvanically isolated three-phase windings are embedded to
the stator slots. They can be located in a 30 degree phase shift from each other or they can be on a two layer winding when there is no phase shift between each other.

According to the two reaction theory the quadrature phase model of the salient pole synchronous motor can be developed. It can be said that any rotor construction part of a synchronous motor is a special case of a salient pole rotor and this model is suitable with minor modifications for all synchronous motor types. An electrically excited salient pole synchronous motor equipped with slip rings and damper windings in the rotor is considered in this work. The geometry of the four-pole synchronous motor with the damper winding is shown in Fig. 2.1. In the same figure the direct and quadrature axes are visible to clarify the considered two reaction theory.

**Fig. 2.1. An electrically excited salient pole synchronous motor. Cross section of the geometry. The axes of the two reaction theory are: d - direct axis and q - quadrature axis. The grey area indicates the excitation winding and the black area indicates the damper winding.**

Considering the general equations of a salient pole synchronous motor the basic assumptions were that saturation, hysteresis and eddy currents are neglected and a sinusoidal stator MMF distribution is assumed. The rotor poles have such a shape that the flux density distribution of the rotor excitation is also sinusoidal. When iron losses, end-effects and harmonics due to the stator windings embedded in slots are neglected. We are dealing with a two dimensional magnetic field problem and there is no coupling between the direct and quadrature axis magnetic fluxes. A two-pole three-phase stator winding is assumed, but the model can of course be used for motors which have a larger number of poles.

**Two reaction theory**

The current vector defines the instantaneous magnitude and the angular position of the peak of the sinusoidal distributed MMF wave produced by the three spatially displaced stator windings and we can write

\[
    f_s = \frac{1}{c} N_{se} i_s = |f_s| e^{j\alpha}.
\]

(2.1)

The MMF of the rotor field pole winding can be defined in a similar manner as the MMF of the stator windings Eq. (2.1). Steady state is considered and thus the damper winding effects can be left out. We have to consider that the field pole is supplied by a DC-current and that the
MMF distribution of the field winding is a rectangular wave. In a salient pole motor this rectangular wave, however, creates a sinusoidal magnetic flux density in the air gap. Thus the space vector of the rotor excitation current can be expressed similarly as

\[ i_r(t) = i_t e^{j\theta(t)} , \]  

where \( \theta_t \) is the rotor angle referring to the stationary real axis.

In the definition of the excitation current space vector it is assumed that the MMF distribution is uniform over the pole shoe and the centre of the wave is placed in the centre of the pole shoe. The magnitude of the DC excitation current referred to stator is \( i_t \). The rotor current vector defines the instantaneous magnitude and angular position of the peak of the MMF wave and it can be written

\[ f_{ir} = N_r i_t = |f_{ir}| e^{j\theta_r} . \]  

The resultant MMF is a superposition of the stator and the rotor MMF

\[ f_{tot} = |f_{is}| e^{j\xi} + |f_{ir}| e^{j\theta} . \]  

The pole shoe of the salient pole synchronous motor has such a shape that with rotor excitation the flux density distribution of the field pole winding in the air gap is sinusoidal. The air gap is not uniform in the direct and quadrature direction and the permeance of the magnetic path is much larger in the direct axis direction than in the quadrature axis direction \( (\Lambda_d > \Lambda_q) \). As a consequence of the permeance difference it is obvious that the part of the resultant MMF which lies in the direction of the pole shoe centre line (direct axis) has a much greater impact on the flux production than the part of the MMF which lies on the quadrature direction. It follows that, when the resultant MMF space vector is divided into real and imaginary components \( f_{tot} = f_{tot,d} + j f_{tot,q} \), the flux components can be defined \( \phi_d = f_{tot,d} \Lambda_d \) and \( \phi_q = f_{tot,q} \Lambda_q \). As a consequence of the armature reaction and rotor saliency the flux density in the non-uniform air gap is distorted. Because of the latter, the peak of the fundamental of the flux density distribution does not coincide with the peak of the resultant Magneto Motive Force distribution unless the resultant MMF distribution is oriented on one of the axes of symmetry (direct or quadrature axis). Fig. 2.2 clarifies the situation. It can be concluded that, if we want to observe the behaviour of the magnetic field (by using various flux linkages of the motor) in different operating points, the salient pole synchronous motor has to be considered as a quadrature phase model one phase aligned with the field winding and the other phase aligned with the midpoint of the adjacent poles (the direction of the largest air gap).
In the next section the electrical equations of the salient pole synchronous machine are presented in their natural reference frames. The natural reference frame for the stator windings is a stationary co-ordinate system where the real axis is fixed to the stator phase A (x,y), and for the excitation winding and the damper winding it is the rotating co-ordinate system where the real axis is fixed to the centre of the pole shoe (d,q). Motor electrical equations can be expressed

\[
\frac{d}{dt} \begin{bmatrix} \psi_{sA} \\ \psi_{sB} \\ \psi_{sc} \\ \psi_{d} \\ \psi_{Q} \end{bmatrix} = \begin{bmatrix} u_{sA} \\ u_{sB} \\ u_{sc} \\ u_{d} \\ u_{Q} \end{bmatrix} - \begin{bmatrix} r_s & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & r_t & 0 \\ 0 & 0 & 0 & 0 & r_D \end{bmatrix} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sc} \\ i_d \\ i_Q \end{bmatrix}.
\] (2.5)

As described earlier the quadrature phase model is a useful method when studying salient pole synchronous motors. After the three- to two-phase transformation when the space vector of a variable is calculated and divided into real and imaginary components, the equations can be expressed

\[
\frac{d}{dt} \begin{bmatrix} \psi_x \\ \psi_y \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_f \\ u_D \\ u_Q \end{bmatrix} - \begin{bmatrix} r_s & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & r_t & 0 & 0 \\ 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_x \\ i_y \\ i_f \\ i_D \\ i_Q \end{bmatrix}.
\] (2.6)
The relationship between the motor currents and the flux linkages can be defined by using the various inductances of the motor. Due to saliency the flux linkages are not only functions of the motor currents, but also of the rotor position. When we observe the behaviour of the motor in a stationary reference frame fixed to the stator, the reluctance of the magnetic path varies as a function of the rotor position. This is discussed in detail in the work of Vas [1992].

\[
\begin{bmatrix}
\psi_s \\
\psi_q \\
\psi_D \\
\psi_Q \\
\end{bmatrix} =
\begin{bmatrix}
L_{ss} + \frac{L_{sd} + L_{sq}}{2} - \frac{L_{sq} - L_{sd}}{2} \cos 2\theta \\
\frac{L_{qs} + L_{qs}}{2} - L_{qs} \sin 2\theta \\
L_{ds} \cos \theta_s - L_{qs} \sin \theta_s \\
L_{qs} \cos \theta_s \\
\end{bmatrix}^T \\
\begin{bmatrix}
L_{sd} \sin \theta_s \\
L_{qs} \sin \theta_s \\
L_{ds} \cos \theta_s \\
L_{qs} \cos \theta_s \\
\end{bmatrix}^T \\
\begin{bmatrix}
L_{s0} \\
L_{q0} \\
L_{d0} \\
L_{q0} \\
\end{bmatrix}^T
\]  

(2.7)

It is also important to note that in a motor model expressed in the natural reference frame, even in linear magnetic conditions, there is a mutual coupling between the direct phase and quadrature phase. The motor model is shown in Fig. 2.3.

![Fig. 2.3. Two-axis salient pole synchronous motor model in natural reference frames.](image)

The problem of coupling and the difficulties in the motor equations which are dependent on the rotor angle can be eliminated if the stator equations are expressed in a rotor oriented reference frame instead of a stator oriented reference frame.

**Motor equations in the rotor reference frame**

As a consequence of the difficult motor equations in the stationary reference frame, it is useful to observe the behaviour of a synchronous motor in a rotor oriented reference frame. The reference frame rotates with the rotor angular velocity and it follows that the direct and quadrature magnetic paths are constant in both axes. The calculations are done in the direct and quadrature directions separately. In the direct axis there is the rotor excitation winding (f). The damper winding is replaced by two windings in space quadrature - one in the direct axis (D) and another in the quadrature axis (Q). The stator three-phase winding is also replaced by
two windings in space quadrature - one in the direct axis (d) and another in the quadrature axis (q). The motor model in the rotor oriented reference frame is shown in Fig. 2.4.

Fig. 2.4. Two-axis salient pole synchronous motor model in a rotor oriented reference frame.

The stator variables in the electrical equations must be transformed into the rotor oriented reference frame. This operation needs accurate knowledge of the rotor position. After the transformation we are dealing with space vectors which do not change as a function of time in steady state. The transformation can be done by using the complex notation (Fig. 2.5).

Fig. 2.5. Vector $x$ expressed in stator and rotor oriented reference frames. Superscripts $^s$ - stator and $^d$ - rotor.

The stator voltage equation in the stationary reference frame (stator co-ordinates $^s$) can be written as

$$u^s_i = R_s i^s_i + \frac{d\psi^s_i}{dt}.$$  

By multiplying by $e^{-j\theta}$, we get the stator voltage Eq. in the rotor reference frame

$$u^s_i e^{-j\theta} = u^r_i = R_s i^r_i + \frac{d\psi^r_i}{dt} + j\omega_r \psi^r_i.$$
This can be divided into real and imaginary components and thus we get the direct and quadrature components of the stator voltage.

\[ u_d = R_s i_d + \frac{d\psi_{sd}}{dt} - \omega \psi_{sq}, \quad (2.10) \]

\[ u_q = R_s i_q + \frac{d\psi_{sq}}{dt} + \omega \psi_{sd}. \quad (2.11) \]

Other windings are in their natural reference frames (rotor co-ordinates) and we can write

\[ u_f = R_f i_f + \frac{d\psi_f}{dt}, \quad (2.12) \]

\[ u_D = R_D i_D + \frac{d\psi_D}{dt} = 0, \quad (2.13) \]

\[ u_Q = R_Q i_Q + \frac{d\psi_Q}{dt} = 0. \quad (2.14) \]

The motor voltage Eqs. (2.10-2.14) form the basis for the motor model. The relationship between the motor currents and flux linkages can be defined by using the various inductances of the motor.

\[ \psi_{sd} = \psi_{md} + i_d L_{s\sigma} = L_{md} (i_f + i_d + i_d) + i_d L_{s\sigma}, \quad (2.15) \]

\[ \psi_{sq} = \psi_{mq} + i_q L_{s\sigma} = L_{mq} (i_q + i_q) + i_q L_{s\sigma}, \quad (2.16) \]

\[ \psi_f = \psi_{md} + i_D L_{k\sigma} + (i_f + i_D + i_D) L_{k\sigma} = L_{md} (i_d + i_d + i_d) + (i_f + i_D + i_D) L_{k\sigma} + i_D L_{k\sigma}, \quad (2.17) \]

\[ \psi_D = \psi_{md} + i_D L_{D\sigma} + (i_f + i_D) L_{k\sigma} = L_{md} (i_d + i_d + i_d) + (i_f + i_D) L_{k\sigma} + i_D L_{D\sigma}, \quad (2.18) \]

\[ \psi_Q = \psi_{mq} + i_Q L_{Q\sigma} = L_{mq} (i_q + i_q) + i_Q L_{Q\sigma}. \quad (2.19) \]

Using the voltage and flux linkage equations, the equivalent circuits of the salient synchronous motor can be derived. The motor model described by these equations is shown in Figs. 2.6 and 2.7.

Fig. 2.6. Direct axis equivalent circuit of a synchronous motor. In a case where the excitation current is known, the excitation winding parameters can be replaced by a current source \( i_f = k_F I_f \), where \( k_F \) is called the reduction factor of the excitation current which refers the rotor DC-excitation current to the stator side.
The equivalent circuits of the synchronous motor consists of eight inductance parameters and four resistance parameters. All parameters are referred to the stator voltage level:

**Direct axis:**
- \( L_{md} \): direct axis magnetising inductance
- \( L_{ss} \): stator leakage inductance
- \( L_{qD} \): direct axis damper winding leakage inductance
- \( L_{qf} \): magnetising winding leakage inductance
- \( L_{k} \): Canay inductance

**Quadrature axis:**
- \( L_{mq} \): quadrature axis magnetising inductance
- \( L_{sQ} \): stator leakage inductance
- \( L_{QD} \): quadrature axis damper winding leakage inductance

**Resistances:**
- \( R_s \): stator resistance
- \( R_f \): magnetising winding resistance
- \( R_D \): direct axis damper winding resistance
- \( R_Q \): quadrature axis damper winding resistance

When the excitation current is known, the excitation winding parameters can be replaced by a current \( i_e = k_e I_F \), where \( k_e \) is called the reduction factor of the excitation current which refers the rotor DC-excitation current to the stator side. The direct axis damper winding’s and the field winding’s common leakage inductance \( L_k \) (so called Canay inductance [Canay 1983], [de Oliveira 1989]) is often left out of the study, because the determination of the common leakage inductance is very difficult and inaccurate results will be obtained. The stator leakage inductance is assumed not to be dependent on the rotor position and it is equal in the direct and quadrature direction. In the following it is assumed that the mutual inductances between the direct axis stator winding, the field winding and the direct axis damper winding are equal \( (L_{df} = L_{fd} = L_{Da} = L_{Da} = L_{ID} = L_{DI} = L_{md}) \).

In Fig. 2.8 the vector diagram of the salient pole synchronous motor is given. The machine operates as a motor in a steady state load condition with a load angle \( \delta \). The quadrature phase components are given in the rotor oriented reference frame (d,q). The angle between the rotor
co-ordinates and the stationary reference frame \((x,y)\) is \(\theta\). The rotor is rotating at an angular velocity \(\omega_r = \frac{d\theta_r}{dt}\).

![Diagram of a salient pole synchronous motor](image)

Fig. 2.8. Vector diagram of the salient pole synchronous motor.

**Electric torque production**

Considering a salient pole motor, the mechanism of the electromagnetic torque production differs from the torque production mechanism of smooth air gap motors. It can be shown that, due to saliency, the so-called reluctance torque component will also arise (for example Vas, 1992). Despite of this the instantaneous electromagnetic torque is produced by interaction between the flux linkages and the currents which are in space quadrature. Thus, it applies to all rotating field motors that we can write for the electromagnetic torque vector in the stationary reference frame

\[
t_e = \frac{3}{2} p \psi_s^* \times i_s^* = \frac{3}{2} p \psi_m^* \times i_s^*,
\]

and in the rotor reference frame the modulus of torque is

\[
t_e = \frac{3}{2} p \left| \psi_r^* \times i_r^* \right| = \frac{3}{2} p \left( \psi_{r q} i_q - \psi_{r d} i_d \right) = \frac{3}{2} p \left( \psi_{r q} i_q - \psi_{r q} i_d \right).
\]

By substituting the stator flux linkage Eqs. (2.15) and (2.16) to (2.21) we get

\[
t_e = \frac{3}{2} p \left[ i_{d q} (L_{md} - L_{mq}) + L_{md} i_q i_q + L_{md} i_d i_d + L_{md} i_q i_d \right].
\]

The first term is the reluctance torque due to saliency. The second term is the torque produced by the field and q-axis stator current. The third and fourth terms are damper torques which occur only during transients.
We can also write for the electromagnetic torque

\[ t_e = \frac{3}{2} p \psi_s^* \times i_s^* = \frac{3}{2} p \psi_s^* \times \left( \frac{\psi_s^* - \psi_m^*}{L_{\sigma \sigma}} \right) = -\frac{3}{2} p \frac{1}{L_{\sigma \sigma}} \psi_s^* \times \psi_m^* , \quad \text{where} \quad (2.23) \]

\[ \psi_s^* = \psi_m^* + i_s^* L_{\sigma \sigma} \quad \text{and} \]

\[ i_s^* = \frac{\psi_s^* - \psi_m^*}{L_{\sigma \sigma}} . \]

This expression clarifies the basic idea of the Direct Flux Linkage Control. DFLC is based on the fact that in Eq. (2.23) the air gap flux linkage \( \psi_m \) due to different damping effects in the air gap region has quite a long time constant, typically in the range of 100-500 ms. A large torque step can be achieved by accelerating fast the rotation of the stator flux linkage \( \psi_s \) so that the angle between it and the air gap flux linkage \( \psi_m \) increases fast. If we have a voltage reserve to accelerate the rotation of the stator flux linkage vector then actually the stator subtransient inductance is the main limiting factor during this change.

### 2.2 FLUX LINKAGE ESTIMATION IN DTC

The performance of the high quality torque control as DTC in dynamically demanding industrial applications is mainly based on the accurate estimate of the various flux linkages space vectors. Estimation of the flux linkages of the synchronous motor is based on calculations where flux linkages are calculated from easily measured signals such as currents and voltages. Flux linkages can also be measured directly by using for instance hall sensors assembled in the air gap of the motor, but direct measuring is relatively expensive and unreliable. When faults occur in the flux transducers, the disassembly and stand still costs are considerable. Due to the disadvantage of the direct measurement, flux calculation methods are used in industrial motor drives. Calculation methods are based on the electrical equations of the salient pole synchronous machines and different types of procedures can be specified. Most commonly a current model (motor currents as state variables) or a voltage model (motor flux linkages as state variables) is used. Recent development in the field of microprocessors has made it possible to use more complicated algorithms such as flux estimators [Alaküla 1993], [Das 1996]. When a reliable and fast calculable real time model is needed, a traditional current model or a voltage model or a combination of them is still more appropriate. In the following sections the flux linkage calculation using a current model and the voltage model are studied and their advantages and disadvantages are discussed. Special attention is given to the stator flux linkage, because it is important in the stator side control of a salient pole synchronous motor DTC-drive. The air gap flux linkage is needed in the implementation of the excitation control.
2.2.1 Voltage model

The stator flux linkage estimate can be calculated using the stator voltage equation expressed in the stator oriented reference frame via integration. Stator co-ordination is a natural choice, because of the principle of the Direct Flux Linkage Control (DFLC). Using DFLC there is no need for additional co-ordinate transformations. This integration can be done digitally with a high sampling rate when a powerful DSP is used. Only the stator voltage estimate and measured current vectors \( u_{\text{sest}} \) and \( i_{\text{smeas}} \) are needed in addition to the stator resistance estimate \( R_{\text{sest}} \)

\[
\psi_{\text{sest}} = \int \left( u_{\text{sest}} - i_{\text{smeas}} R_{\text{sest}} \right) dt .
\]  

(2.24)

In practice, when a variable frequency voltage source inverter is used, the voltage waveform consists of pulses. These pulses supplied in the stator windings form the instantaneous voltage vectors. The voltage vectors of the voltage source inverter are considered as discrete values and an analysis using the instantaneous voltage vectors is suitable for investigating the dynamic behaviour of the motor. Stator phase currents are measured. Usually the stator voltage is not measured. Only the intermediate circuit voltage \( U_c \) of the inverter is measured and thus the inverter switching states \( S_A, S_B, S_C \) and a model for the voltage drop in the inverter switches must be used when constructing the voltage vectors for the integration. In 2-level inverters the switching state value is either 1 when connected to the positive potential, or 0 when connected to the negative potential of the intermediate circuit. When 3-level inverters are considered the switching state value is either 1 when connected to the positive potential, \( \frac{1}{2} \) when connected to the neutral point or 0 when connected to the negative potential of the intermediate circuit. From the switching states, the measured intermediate voltage and the model for the voltage drop in the inverter switches we get

\[
u_{\text{sest}}(S_A, S_B, S_C) = \frac{2}{3} U_{\text{smeas}} \left( S_A e^{j\theta} + S_B e^{j2\pi/3} + S_C e^{j4\pi/3} \right) - \frac{2}{3} \left( u_{A,\text{loss}} e^{j\phi} + u_{B,\text{loss}} e^{j2\pi/3} + u_{C,\text{loss}} e^{j4\pi/3} \right)
\]  

(2.25)

where \( u_{A,\text{loss}}, u_{B,\text{loss}} \) and \( u_{C,\text{loss}} \) take into consideration the threshold voltage and the conducting state voltage losses in the power switches and also the switching delay during switch commutation. Eqs. (2.24) and (2.25) can now be rewritten for the stator flux linkage estimate

\[
\psi_{\text{sest}} = \int \left( u_{\text{sest}}(S_A, S_B, S_C) - i_{\text{smeas}} R_{\text{sest}} \right) dt
\]

\[
= u_{\text{sest}}(S_A, S_B, S_C) \Delta t - \int i_{\text{smeas}} R_{\text{sest}} dt + \psi_{\text{sest}} \Big|_{t=0} .
\]  

(2.26)

It can be seen in Eq. (2.26), when considering that the voltage drop in the stator resistance is small in proportion to the stator voltage, that the stator flux linkage moves into the direction of the instantaneous inverter output voltage vector. In the case of non-zero voltage vectors, the velocity is proportional to the intermediate voltage. When a zero voltage is applied, the velocity is small and is proportional to the value \( i \Delta R_c \). In Fig. 2.9 the movement of the stator flux linkage vector is constructed by applying different voltage vectors.
A stator flux linkage estimate based on the integration of the stator voltage estimate is rather easy to implement in discrete form. The calculation can be performed with a very high sampling rate. We can get information of the electromagnetic state of the motor which is valid during very fast transients as well as in a steady state. The calculation method is not sensitive to inductance variations due to saturation effects, since no inductances are present in the expression used in the integration. Instead of inductances, the stator resistance $R_s$ is a critical motor parameter.

In practice, however, the simple and intelligent calculation method described above does have some problems. All the terms in Eq. (2.26) contain some errors and thus at low speeds and especially at zero speed the motor stator flux linkage $\psi_s$ gradually drifts erroneous due to errors in the integration of the stator flux linkage estimate $\psi_{sest}$. At zero speed the stator flux linkage estimation by integration of DC signals is not possible, because even the smallest offset signals cause the estimate to be false and thus the motor stator flux linkage drifts. At higher speeds the error in the integral of Eq. (2.26) is negligible and the motor stator flux estimation is reliable.

In the following the problem of the stator flux linkage drifting and the error sources are discussed. It can be seen from Eqs (2.25 and 2.26) that an error in the integration can be caused by four different factors: 1) an intermediate voltage measuring error, 2) a stator current measuring error, 3) a power switch voltage loss estimation error and 4) a stator resistance estimation error. Measuring errors can be divided into two categories depending on their nature: 1) a gain error and 2) an offset error. Gain and offset errors in the measured intermediate circuit voltage behave in a similar manner. The measured signal is a DC-signal with small ripple and then in practice the effect of the gain and offset error is the same. Two different types of behaviour can be observed when there is an error in the measured intermediate voltage. The first is a constant stable error in the stator flux linkage and the second is an alternating unstable drifting of the stator flux linkage. Let us consider that the measured intermediate voltage $U_{cmeas}$ can be expressed as

$$U_{cmeas} = (1 - k_{gain})U_c + \Delta U_{cofs} = U_c + \Delta U_c,$$

where

$$U_{cmeas} = (1 - k_{gain})U_c + \Delta U_{cofs} = U_c + \Delta U_c,$$ (2.27)
$U_c$ is the actual intermediate voltage and $k_{\text{gain}}$ is the gain coefficient and $\Delta U_{\text{offs}}$ is the offset voltage. $\Delta U_c$ is the total measuring error at a certain time instant. Simulations show that: if $\Delta U_c \geq 0$ ($U_{\text{meas}} \geq U_c$), then a stable stator flux linkage error occurs and if $\Delta U_c < 0$ ($U_{\text{meas}} < U_c$), then an unstable drifting of stator flux linkage occurs. These two different cases are illustrated in Fig. 2.10.

Gain and offset errors in the measured phase currents behave in a similar way. Two different types of behaviour can be observed when there is a gain error in the phase currents. The first is a constant stable error in the stator flux linkage and the second is an alternating unstable drifting of the stator flux linkage. These two phenomena are affected in the same conditions as in the case of an intermediate voltage measuring error. The gain error in the measured currents can be considered as a voltage loss calculation error and further on as a voltage error. Let us consider that the estimated stator voltage $u_{\text{est}}$ can be expressed as

$$u_{\text{est}} = u_{\text{calc}}(S_A, S_B, S_C) - \left(1 - k_{\text{gain}}\right)i_s R_s = u_s + \Delta u_s,$$  (2.28)

where $u_s$ is the actual stator voltage, $u_{\text{calc}}$ is the stator voltage calculated from the measured intermediate voltage and switch states and $k_{\text{gain}}$ is a gain coefficient. $\Delta u_s$ is the total measuring error at a certain time instant. Simulations show that: if $|u_{\text{est}}| \geq |u_s|$, then a stable stator flux linkage error occurs (see Fig. 2.10-1) and if $|u_{\text{est}}| < |u_s|$, then an unstable drifting of stator flux linkage occurs (see Fig. 2.10-2). An offset error in the measured phase current causes a DC-component in the phase current and the integration is unstable.

No accurate models for power switches can be used because they are far too complicated to implement in the motor control algorithms. Power switches are usually described with a simple resistance model ($R_d$) completed with a threshold voltage $u_{\text{th}}$.

$$u_{\text{loss}} = u_{\text{th}} + i_q R_d.$$  (2.29)

The voltage loss estimation error in power switches can be considered similar to the stator resistance error. In the following this error source is considered to be part of the resistance estimation error.
The stability of the DFL control method can be observed via the stator resistance estimate, because it is the most significant variable in the voltage loss calculation and it is very sensitive to errors. Analytically for the induction motor it can be shown, in simulations and also experimentally, that if the stator resistance is estimated too large $R_{\text{est}} > R_{\text{act}}$, the stator flux linkage error term increases in the same direction as the actual value of the flux linkage error and the DFL control is unstable (positive feedback). If the stator resistance is estimated too small $R_{\text{est}} \leq R_{\text{act}}$ the flux linkage error tends to decrease and the DFL control is stable (negative feedback) [Pohjalainen 1987]. The same phenomenon is valid also for the salient pole synchronous motors. The stator voltage integral Eq. (2.24) can be written

$$\psi_{\text{est}} = \int u_{\text{est}} \, dt - R_{\text{est}} \int i_{\text{meas}} \, dt,$$

where the stator resistance estimate can be expressed as

$$R_{\text{est}} = R_s + \Delta R_s.$$  

$R_{\text{est}}$ is the estimated stator resistance and $\Delta R_s$ is the error of the estimated stator resistance. Assuming that the estimated stator voltage and the measured stator current are correct the stator flux linkage error term can be defined as

$$\psi_{\text{err}} = \psi_s - \psi_{\text{est}} = \int u_s \, dt - \int i \, dt - \int u_{\text{est}} \, dt + (R_s + \Delta R_s) \int i_{\text{meas}} \, dt$$

$$= \Delta R_s \int i \, dt = \Delta R_s \int i_x \, dt + j\Delta R_s \int i_y \, dt = \psi_{sx \text{ err}} + j \psi_{sy \text{ err}} = (\psi_{sd \text{ err}} + j \psi_{sq \text{ err}}) e^{j\eta}.$$  

In Fig. 2.11 the stator flux linkage error and the flux linkage drifting are shown in a space vector diagram in case of error in the voltage loss estimation. The voltage losses are assumed to be larger than they really are. For this reason the real stator voltage is larger than the voltage which is used to calculate the stator flux linkage estimate. The given voltage vector is supplied until the stator flux linkage estimate reaches the hysteresis limit. During that time the actual stator flux linkage goes beyond the hysteresis and drifts out of the hysteresis band and the control becomes unstable.
It is assumed that the flux linkage vector is removed from the origin due to the instantaneous error in the stator resistance estimate (time $t_1$) Fig. 2.11. This displacement can be seen in the motor current vector according to the synchronous motor stator flux linkage Eq. (2.15-2.16)

$$i_s' = i_d + j i_q = \frac{1}{L_{sd}} \left[ \psi_{sd} - L_{md} (i_t + i_D) \right] + j \left[ \frac{1}{L_{sq}} \left( \psi_{sq} - L_{mq} i_q \right) \right]$$

$$= \frac{1}{L_{sd}} \left[ \psi_{sdest} + \psi_{sderr} - L_{md} (i_t + i_D) \right] + j \left[ \frac{1}{L_{sq}} \left( \psi_{sqest} + \psi_{sqerr} - L_{mq} i_q \right) \right]$$

$$= \frac{1}{L_{sd}} \left[ \psi_{sdest} - L_{md} (i_t + i_D) \right] + \frac{\psi_{sderr}}{L_{sd}} + j \left[ \frac{1}{L_{sq}} \left( \psi_{sqest} - L_{mq} i_q \right) + \frac{\psi_{sqerr}}{L_{sq}} \right]$$

The stator current space vector moves in the same direction as the flux linkage error term $\psi_{serr}$. The error term which accumulates during one electric cycle can be calculated from Eq. (2.32). It is assumed that the angular velocity of the stator current vector is constant.

$$\Delta \psi_{serr} = (\Delta \psi_{sderr} + j \Delta \psi_{sqerr}) e^{j \theta} = \Delta R, 2\pi \left[ \Delta i_d + j \Delta i_q \right] e^{j \theta}.$$  

For the current components $\Delta i_d$ and $\Delta i_q$ caused by the estimation error we can write from Eq. (2.33) $\Delta i_d = \psi_{sderr} / L_{sd}$ and $\Delta i_q = \psi_{sqerr} / L_{sq}$. Now we can define the difference of the estimation error.
\[
\Delta \psi_{s err} = \Delta \psi_{sx err} + j\Delta \psi_{sx err} = \Delta R_s 2\pi \left( \frac{\psi_{sx err}}{L_{sd}} + j\frac{\psi_{sq err}}{L_{sq}} \right) e^{j\theta}
\] 
\hspace{1cm} (2.35)

In Fig. 2.12 the stator flux linkage of the synchronous machine is shown, when the estimated stator resistance is too large \( R_{s est} > R_{s act} \) or too small \( R_{s est} < R_{s act} \). Results of a DFLC drive simulator are given. Intuitively, it is easy to understand that the control system tends to be stable if the real losses are larger than the estimated losses.

Errors in the measured and estimated signals cause an error in the stator flux linkage estimate and, further on, an error in the torque estimate. It can be shown [Pohjalainen 1987] that the torque estimation error is directly proportional to the error in the intermediate voltage measurement. Also the torque estimation error is directly proportional to the square of the error of the measured current vector. If an error occurs in the stator resistance, the torque estimation error is directly proportional to the estimation error in the stator resistance. It should be noted that the torque estimation error in all different error types is also proportional to an inverse of the supply frequency of the motor. At higher speeds the torque estimation error is negligible, but at zero frequency the torque estimation error increases to infinite values.

When the motor flux linkage drifting out of origin centered occurs, it causes DC-components in the phase variables and the current and flux linkage space vectors rotate eccentrically with a distance defined by the offset error from the origin. An alternating component which varies with the supply frequency of motor arises in the torque estimate.

As described before there are two types of problems in the flux linkage integration and generally speaking the source of the problem is the determination of the stator voltage. The first is a constant integration error caused by errors in the measured intermediate voltages, the motor phase currents, the estimated stator resistance and the estimated voltage losses in power switches. This problem occurs when the stator voltage is estimated to be larger than the actual value. The second problem is the motor flux linkage drifting out of the centre. This problem is...
serious, because it causes an unstable operation of the DFL control and occurs when the stator voltage is estimated to be smaller than the actual value. Error sources affect the flux linkage integration at the same time with different proportional shares. Analytical specification of the effects of different error sources in the calculation is very difficult to carry out. A better understanding can be achieved by using a schematic presentation of the principle of the flux linkage integration which consists of all error sources. This is shown in Fig. 2.13. The controller keeps the estimate origin centred and thus the errors in the integration cause the actual motor flux linkage drift non-origin centred.

![Fig. 2.13. Schematic presentation of the flux linkage integration. The term $\Delta u_s$ is a non predictable voltage estimation error that mainly causes the motor flux linkage error. Coefficients $u$, $v$ and $w$ are used to describe the possible unsymmetry of the stator phase windings.](image)

The stator flux linkage estimation is performed by integrating the phase voltages $u_{\text{est}}$ and $u_{\text{yest}}$, which are calculated from the intermediate voltage and the given switching states of the power switches. Also the voltage losses of the power switches are considered. The total voltage loss can be estimated using the estimated stator resistance and the measured motor phase currents $i_{\text{xmeas}}$ and $i_{\text{ymeas}}$. This loss is also integrated and subtracted from the actual voltage integral. After that, the stator flux linkage estimate is obtained and selection of all control decisions (switch combinations) is done according to this estimate. The actual stator flux linkage is intended to be kept within hysteresis limits and it should be rotated around the origin. If errors are made in the voltage loss estimation, then there is an additional voltage loss $\Delta u_s$. The control system does not recognise this and wrong control decisions are made because all voltage vectors are selected according to the estimate and the estimate is forced to rotate around the origin inside of a certain hysteresis. The problem of the flux integration can be considered as the problem of voltage loss calculation. It can be shown that a hysteresis control as DFCLC which is based on an estimate of a control variable is an oscillating system, even if the estimated control variable is absolutely accurate [Vertanen 1995]. It is also known that additional losses in the system dampen the oscillation. An example for this is the damper winding of the synchronous machine which dampens the oscillations if a synchronous machine starts to dephase. As a conclusion it can be said that, if there is a negative additional
voltage loss $-\Delta u_s$ in the DFL control system (i.e. voltage losses are estimated larger than the actual), the gain of the control system is increased and the control system becomes unstable. On the other hand, if there is a positive additional voltage loss $\Delta u_s$ in the system (i.e. voltage losses are estimated smaller than the actual) the additional positive losses ensure that the control system is stable.

Unstability and drifting problems at low speeds can be avoided if the stator resistance is estimated to be smaller than it deliberately was estimated, but this causes an error in the flux estimate and the zero speed integration problem still exists. If some additional flux linkage correction is implemented, a better control is achieved. The task of this additional correction is to inhibit the motor flux linkage drifting by using the measured phase currents. Motor phase currents can be used as indicators for the flux linkage drifting. One possible method is to apply the motor current model which calculates the motor flux linkages using the measured phase currents and the motor inductance parameters. The accuracy of the voltage integral is sufficient to meet the requirements of a demanding drive, but at very slow speeds some additional correction is needed to adjust the flux linkage calculation. At higher speeds the function of the additional correction is to act as a stabiliser of the control system.

### 2.2.2 Current Model

The flux linkage calculation can be performed applying the commonly known method which calculates various flux linkages of the motor using the measured phase currents and the inductances which are known in advance. This so called current model is very simple to implement. The calculation can be done digitally with a high sampling rate when a powerful DSP is used, and it also fulfils the requirements of a real time control system where simple fast calculable models are needed. The calculation is performed in the direct and quadrature directions separately using the two-axis model of the synchronous machine as explained earlier. All variables have to be transformed into the rotor oriented reference frame. The two-axis current model has a few drawbacks. It contains a set of inductance parameters which must be known. Usually the current model is based on the linear equations of the machine and the saturation of the inductance parameters is taken into consideration by updating the inductance parameters according to the operating point. This causes an error during transients, but in steady state the model is valid. The validation of the current model in steady state is not self-evident. It depends on the accuracy of the saturated value of the inductances. The rotor co-ordinate transformation can also be considered as a drawback, because trigonometric sinus and cosinus functions are needed. This operation also needs accurate knowledge of the rotor position. Since only three of the five motor currents in a two-axis model can be measured, two damper winding currents have to be estimated. The damper windings take part in the motor operation during transients, and so it is obvious that the estimation of the damper winding currents reduce the accuracy of the current model during transients. In the following chapter the current model based on linear magnetic conditions is discussed. After that some equations for non-linear magnetic conditions are presented and the usability of those in the flux linkage calculation is briefly discussed.
Linear magnetic conditions

After the stator phase currents and the excitation current are measured and the damper winding currents are estimated, the various flux linkages of the synchronous motor can be calculated according to Eqs. (2.15 - 2.19) which are repeated here

\[
\psi_{sd} = \psi_{md} + i_d L_{sa} = L_{md}(i_d + i_d + i_d) + i_d L_{sa},
\]
\[
\psi_{sq} = \psi_{mq} + i_q L_{qa} = L_{mq}(i_q + i_q) + i_q L_{sa},
\]
\[
\psi_f = \psi_{md} + i_r L_{fr} = L_{md}(i_d + i_d + i_r) + i_r L_{fr},
\]
\[
\psi_D = \psi_{md} + i_D L_{D_o} = L_{md}(i_d + i_D + i_D) + i_D L_{D_o},
\]
\[
\psi_Q = \psi_{mq} + i_Q L_{Q_o} = L_{mq}(i_q + i_Q) + i_Q L_{Q_o}.
\]

(2.36-2.40)

The damper winding current estimates must be calculated. In the following saturation effects are not taken into consideration. The saturation of the inductance parameters and the temperature dependency of the damper winding resistance cause that damper winding time constants are not constant, but this is often neglected when damper winding current estimators are developed. The air gap flux linkage can be evaluated according to the two-axis theory

\[
\psi_{md} = i_{md} L_{md} = L_{md}(i_d + i_d + i_D).
\]
\[
\psi_{mq} = i_{mq} L_{mq} = L_{mq}(i_q + i_Q).
\]

(2.41) \hspace{1cm} (2.42)

Here \(i_t\) is the excitation current referred to the stator, \(i_t = k_r \cdot I_F\). Here \(k_r\) is a reduction factor and \(I_F\) is the rotor excitation DC current.

Let us first consider the direct axis. The damper winding is short circuited, therefore the direct component of the damper winding voltage equation is

\[
\begin{align*}
\frac{d\psi_D}{dt} &= R_D i_D + \frac{d\psi_D}{dt} = -R_D i_D + \frac{d}{dt}(\psi_{md} + i_D L_{D_o}) = 0.
\end{align*}
\]

(2.43)

The time constant of the direct axis damper winding is defined

\[
\tau_D = \frac{L_{md} + L_{D_o}}{R_D}.
\]

(2.44)

When Eq. (2.41) is substituted into the voltage Eq. (2.43)

\[
0 = R_D i_D + \frac{d}{dt}(L_{md}(i_d + i_d + i_D) + i_D L_{D_o})
\]

and is rewritten, we get

\[
\left[1 + \frac{d}{dt} L_{D_o} + L_{md} \frac{d}{dt}ight] i_D = -\frac{L_{md}}{L_{D_o} + L_{md}} \frac{d}{dt} L_{D_o} + \frac{L_{md}}{R_D} (i_i + i_d).
\]

(2.45)

According to Eq. (2.45) we get a discrete representation for the calculation algorithm of the direct axis damper winding current if the equation is integrated over one sampling interval ((n-
$i_{D(n)} = \frac{\tau_D}{\tau_s + \tau_D} \left[i_{D(n-1)} - \frac{L_{md}}{L_{md} + L_{D\sigma}} \left(i_{f(n)} + i_{d(n)} - i_{f(n-1)} - i_{d(n-1)}\right)\right]. \quad (2.46)$

$\tau_s$ is the sampling interval, $\tau_D$ is the time constant of the direct axis damper winding, $L_{md}$ is the direct axis magnetising inductance and $L_{D\sigma}$ is the direct axis damper winding's leakage inductance.

The current equation of the quadrature axis damper winding can be derived in the same way. According to Eq. (2.46) we get a discrete representation for the calculation algorithm of the quadrature axis damper winding current

$$i_{Q(n)} = \frac{\tau_Q}{\tau_s + \tau_Q} \left[i_{Q(n-1)} - \frac{L_{mq}}{L_{mq} + L_{Q\sigma}} \left(i_{q(n)} - i_{q(n-1)}\right)\right]. \quad (2.47)$$

$\tau_s$ is the sampling interval, $\tau_Q$ is the time constant of the quadrature axis damper winding, $L_{mq}$ is the quadrature axis magnetising inductance and $L_{Q\sigma}$ is the quadrature axis damper winding's leakage inductance. $\tau_Q$ is defined as

$$\tau_Q = \frac{L_{mq} + L_{Q\sigma}}{R_Q}.$$

Now all motor currents are known and according to the motor Eq. (2.36-2.40) the two-axis components of the stator flux linkage can be calculated. The saturation of the inductance parameters is taken into account by updating the inductance parameters according to the operating point. This causes an error during transients but in steady state the model is valid. A complete derivation of the damper winding current estimators can be found in Appendix 1. The modelling of the inductance saturation is discussed in detail in chapter 2.3.

**Non-linear magnetic conditions**

The damper winding current estimators described above are derived using linear equations of the salient pole synchronous machine. Neither do they take into account the saturation of the machine inductance parameters, nor the temperature dependence of the damper winding resistance. The assumption of linear magnetic conditions and constant resistances is not valid when the motor operates at different operating points. Leakage inductances are known to be less saturation dependent than magnetising inductances, and therefore leakage inductances are assumed to be constant and only magnetising inductances are considered as saturation dependent.

A method to include the effects of main flux path saturation in the electrical equations of AC machines can be found in the work of Brown, Kovacs, Vas [Brown 1983] and Melkebeek. These publications are focusing mainly on the modelling of an induction machine or smooth air gap machines in general. Non-linear equations can be applied also to all rotating field AC machines and the equations for salient or non-salient synchronous machines can be found in the publications of Melkebeek [1984], Melkebeek et. al. [1985] and Vas et. al. [1984, 1985, 1986].
The air gap flux linkage can be evaluated according to Eqs. (2.41-2.42), where a resultant magnetisation current exists. It is possible to derive for the time derivatives of the direct axis air gap flux linkages [Vas 1990]

\[
\frac{d\psi_{md}}{dt} = L_{Md} \frac{di_{md}}{dt} + L_{dq} \frac{di_{mq}}{dt}, \quad \text{where}
\]

\[
L_{Md} = L_{md}^d \cos^2 \mu + L_{m1} \sin^2 \mu
\]

\[
L_{dq} = \left( L_{md}^q - L_{md}^d \right) \sin \mu \cos \mu.
\]

Accordingly, we can write for quadrature direction

\[
\frac{d\psi_{mq}}{dt} = L_{Mq} \frac{di_{mq}}{dt} + L_{q1} \frac{di_{md}}{dt}, \quad \text{where}
\]

\[
L_{Mq} = L_{mq}^d \sin^2 \mu + L_{m1} \cos^2 \mu
\]

\[
L_{q1} = \left( L_{mq}^q - L_{mq}^d \right) \sin \mu \cos \mu.
\]

The angle \( \mu \) is the angle of the resultant magnetisation current space vector with respect to the direct axis i.e. \( \mu = \arctan \left( \frac{i_{mq}}{i_{md}} \right) \). These two equations include new inductance parameters. \( L_{Md} \) and \( L_{Mq} \) can be considered as magnetising inductances which include an incremental inductance change during a transient. \( L_{dq} \) and \( L_{q1} \) are cross-coupling inductances in direct and quadrature directions. These cross-coupling inductances take into consideration the cross saturation effect. This causes that the two axes which are in space quadrature are coupled. \( L_{md} \) and \( L_{mq} \) are so called chord slope static magnetising inductances which are usually used in current model calculations. \( L_{md}^d \) and \( L_{mq}^d \) are tangent slope dynamic magnetising inductances.

In linear magnetic conditions i.e. in low flux density levels the tangent slope inductances \( L_{md}^d \) and \( L_{mq}^d \) are equal to the chord slope inductances \( L_{md} \) and \( L_{mq} \). These chord slope and tangent slope inductances are defined from the magnetisation curve in Fig. 2.14. A complete derivation of the time derivatives of the air gap flux linkages can be found in Appendix 2.
Fig. 2.14. Definition of the different inductances from the magnetisation curve.

In the following the damper winding current estimators are calculated using non-linear Eqs. (2.48)-(2.49). Let us again first consider the direct axis. The damper winding is short circuited, therefore the direct component of the damper winding voltage equation is

$$u_D = R_D i_D + \frac{d\psi_D}{dt} = R_D i_D + \frac{d}{dt}(\psi_{md} + i_D L_{De}) = 0.$$  \hspace{1cm} (2.50)

The time constant of the direct axis damper winding can be now defined as

$$\tau_D = \frac{L_{Mdd} + L_{DDe}}{R_D}.$$  \hspace{1cm} (2.51)

When Eq. (2.48) is substituted into the voltage Eq. (2.50)

$$0 = R_D i_D + \frac{d}{dt}(L_{Mdd}(i_q + i_d) + L_{dq}(i_q + i_d) + i_D L_{De})$$  \hspace{1cm} (2.52)

and is rewritten, we get

$$\left[1 + \frac{d}{dt} \frac{L_{De} + L_{md}}{R_D}\right] i_D = - \frac{L_{Mdd}}{L_{De} + L_{Mdd}} \frac{d}{dt}\left(\frac{L_{De} + L_{Mdd}}{R_D}\right)(i_q + i_d) - \frac{L_{dq}}{L_{De} + L_{Mdd}} \frac{d}{dt}\left(\frac{L_{De} + L_{Mdd}}{R_D}\right)(i_q + i_d).$$  \hspace{1cm} (2.53)

According to Eq. (2.53) we get a discrete representation for the calculation algorithm of the direct axis damper winding current if the equation is integrated over one sampling interval \((n-1)\tau_S, n\tau_S\), and the damper current is approximated to change linearly within the sampling period

$$i_{D, n+1} = i_{D, n} + \frac{\tau_D}{\tau_S + \tau_D} \left[\frac{L_{Mdd}}{L_{Mdd} + L_{De}}(i_q + i_d) - i_{D, n-1} + \frac{L_{dq}}{L_{Mdd} + L_{De}}(i_q + i_d) - i_{q, n-1} - i_{d, n-1}\right].$$  \hspace{1cm} (2.54)

$$\tau_S$$ is the sampling interval, $$\tau_D$$ is the time constant of the direct axis damper winding and the inductance parameters are those defined before.
The current equation of the quadrature axis damper winding can be derived in the same way. According to Eq. (2.54) we get a discrete representation for the calculation algorithm of the quadrature axis damper winding current

\[
i_{q[n-1]} = \frac{\tau_q}{\tau_s + \tau_Q} i_{q[n-1]} - \frac{L_M}{L_{Mq} + L_{Qq}} \left( i_{a[n]} - i_{a[n-1]} \right) - \frac{L_{dl}}{L_{Mq} + L_{Qq}} \left( i_{l[n]} + i_{l[n]} + i_{l[n]} - i_{l[n-1]} - i_{l[n-1]} - i_{l[n-1]} \right).
\]

(2.55)

\(\tau_s\) is the sampling interval, \(\tau_Q\) is the time constant of the quadrature axis damper winding and the inductance parameters are those defined before. \(\tau_Q\) is defined as

\[
\tau_Q = \frac{L_{Mq} + L_{Qq}}{R_Q}.
\]

It can be seen that the damper winding current estimators derived from non-linear motor equations differ from the estimators derived from linear equations, mainly because there is a coupling between the direct and quadrature axis estimators. The direct axis damper winding current depends on the quadrature axis damper winding current and vice versa. Also there are four new dynamic inductances \(L_{Md}, L_{Mq}, L_{dq}\) and \(L_{qd}\) in addition to the static inductances \(L_{md}\) and \(L_{mq}\) which are usually used in linear equations. All these inductances have to be defined in the \(i_{md} - i_{mq}\) plane and six inductance surfaces must be known as was introduced by Alaküla [1993].

According to the motor Eqs. (2.36-2.40) the two-axis components of the stator flux linkage can now be calculated. The saturation of the inductance parameters is taken into consideration by updating the inductance parameters according to the operation point. The accuracy of the model is now improved during transients, because the damper winding current estimators also take into account the saturation effect. A complete derivation of the damper winding current estimators can be found in Appendix 3. The modelling of the inductance saturation is discussed in chapter 2.3.

The implementation of a non-linear estimator is much more complicated in a real time application than the implementation of a linear one. That is why it is very relevant to examine, if there is some real advantage in using the non-linear model and if the linear damper winding current estimator is sufficient to meet the current model behaviour requirements in transients. Examination of the non-linear model is performed using a salient pole synchronous motor DTC drive simulator. A schematic block diagram and the flowchart of the developed C-language simulator is illustrated in Appendix 4 [Burzanowska 1990]. A motor model is used where the flux linkages are state variables and the calculation of the various motor flux linkages can be done using Eq. (2.6). All voltage equations are presented in their natural reference frames. The used pu system can be found in Appendix 7. The motor currents can be calculated from Eqs. (2.15-2.19) which can be presented in a form \(i = L^{-1}_{\text{stat}} \psi\) where \(i\) is the motor current matrix, \(L^{-1}_{\text{stat}}\) is the static inductance matrix inverse and \(\psi\) is the flux linkage matrix, all presented in a rotating reference frame fixed to the rotor. Saturation is taken into consideration by updating the inductance matrix \(L^{-1}_{\text{stat}}\) according to the operating point using inductance surfaces \(L_{md}=(i_{md}/i_{mq})\) and \(L_{mq}=(i_{md}/i_{mq})\), which are defined in advance. Leakage inductances are assumed constant. The linear and non-linear damper winding current estimators are used in the Direct Torque Control system and a comparison is made between these two in opposition to the damper winding current calculated by the simulator motor.
model. The motor model simulation algorithm is described in Appendix 5. All six inductance surfaces discussed before are illustrated in Figs. 2.15, 2.16 and 2.17. The static inductance surfaces are measured from the salient pole synchronous machine of our test drive. The dynamic and the cross saturation inductances are calculated according Eq. (2.48) and (2.49).

Fig. 2.15. Static magnetising inductances as a function of the magnetising currents in the direct and quadrature axis.

Fig. 2.16. Dynamic magnetising inductances as a function of the magnetising currents in the direct and quadrature axis.

Fig. 2.17. Cross saturation inductances as a function of the magnetising currents in the direct and quadrature axis.

At first the effect of saturation in the damper winding currents is briefly considered. A current flows in the damper winding only during transients. During transients the flux linkage of the damper winding tries to change and according to Faraday’s law this change of the flux linkage induces currents in the short circuited damper winding. The induced currents try to resist the flux variation and this defines their direction. The direct axis damper winding currents calculated in the rotor reference frame are shown in Fig. 2.18. Both linear and saturated conditions are simulated. Simultaneous torque and stator flux linkage steps are performed.
Saturation of the motor inductances causes that the magnitude of the induced current is higher than under linear magnetic conditions. Also the direction of the step affects the induced current magnitude. Saturation has a minor effect in the duration of the induced current.

![Graph](image)

**Fig. 2.18.** Direct axis damper winding current in a simultaneous torque and stator flux step. Both the linear and saturated conditions are simulated.

In Fig. 2.19 the quadrature axis damper winding currents calculated in the rotor reference frame are shown. The transient is similar as shown in Fig. 2.18. Both linear and saturated conditions are simulated. The influence of the saturation on the quadrature axis damper winding current is much stronger than on the direct axis.

![Graph](image)

**Fig. 2.19.** Quadrature axis damper winding current in the simultaneous torque and stator flux step. Both the linear and saturated conditions are simulated.

In Figs. 2.20 and 2.21 the proposed direct and quadrature axis damper winding current estimators are compared with the damper winding currents calculated from the saturated motor model. Both the linear and non-linear current estimator are illustrated. The transient is similar to those in the previous figures. It is obvious that the linear estimator is erroneous. It can also be seen that the calculation of the non-linear estimator is better, but the benefit of the non-linear calculation is insignificant compared to the complexity of the implementation. It must be pointed out that in the study presented before all motor parameters were assumed to be exactly known. This is not the case in a real drive, where the motor parameters of the equivalent circuits have some calculated or measured values and the accuracy of the
parameters can vary considerably. Especially the parameters of the damper winding can be very inaccurate.

![Graph 1](image1.png)

**Fig. 2.20.** The direct axis damper winding currents calculated by the simulator motor model and the estimated direct axis damper winding currents calculated by the linear and non-linear current estimators. The linear and non-linear estimates are calculated at 1 ms sampling rate and motor model is calculated at 5 μs sampling rate. Rotor reference frame.

![Graph 2](image2.png)

**Fig. 2.21.** The quadrature axis damper winding currents calculated by the simulator motor model and the estimated quadrature axis damper winding currents calculated by the linear and non-linear current estimators. The linear and non-linear estimates are calculated at 1 ms sampling rate and motor model is calculated at 5 μs sampling rate. Rotor reference frame.

As a conclusion of the damper winding estimator study there seems to be no need to use non-linear damper winding estimators. The benefits of the non-linear estimator are not remarkable in relation to the complexity of the implementation. If the sampling rate of the damper winding current integration can be higher (for example 100 μs), then non-linear and also linear model calculation accuracy will improve.

2.3 VARIATION OF THE MOTOR PARAMETERS IN A STEADY STATE
The operating point of the motor in synchronous motor drives can be varied in a wide range. In the constant flux linkage region the motor can be loaded for example up to two and a half times the nominal torque or in the field weakening area the motor can be operating at five times the nominal speed producing half of the nominal torque. This means that the flux linkage of the motor is reduced to one fifth of the nominal flux linkage and the current is correspondingly very large. The reliability of the motor flux linkage calculation depends on the accuracy of the parameters used in the calculation methods. In the previous section two calculation methods have been introduced. The voltage model has only one parameter, the stator resistance. The current model is much more parameter dependent, because it has six inductance parameters and in the damper winding current estimators there are two resistance parameters included in the time constants of the damper winding. Resistances are known to be temperature dependent. Variable frequency of the supply voltage causes variation in the winding resistances also due to the skin effect. All inductances depend on saturation and the effects of saturation are more complex when salient pole synchronous machines are considered. The variation of the motor model parameters can be considered as a control problem or a modelling problem. The effect of parameter variations can be accounted to the controllers or motor models, but this leads to more complex implementations. Before implementation of the parameter models, a good physical knowledge of the phenomenon modelled is required. In the next section the variation of resistances and saturation dependent inductances is considered. Physical phenomena and the principles of parameter models are described.

2.3.1 Motor resistances

Using the voltage integration method to calculate the stator flux linkage no inductances are present in the expression in the integration. Instead of inductances, the stator resistance \( R_s \) is the critical parameter. In the current model there are also damper winding resistances \( R_D \) and \( R_Q \) present in the expression of the damper winding estimators (included to the damper winding time constants). Motor resistances cannot be considered as constant parameters, because they are temperature and frequency dependent. Stator, damper and excitation windings are usually made of copper. The conductivity of copper is \( \sigma = 57 \cdot 10^6 \) S/m (defined at temperature +20 °C) and the temperature coefficient of the resistivity of copper is \( \alpha = 3.81 \cdot 10^{-3} \) 1/K. The resistance of the winding can be defined using winding data as

\[
R_{DC} = \frac{l}{\sigma a J},
\]  

(2.56)

where \( l \) is the length of the winding, \( \sigma \) is conductivity, \( a \) is the number of the parallel conductors and \( J \) is the cross-section area of the conductor or it can be measured using DC measurement. This so called DC resistance is valid when the winding is supplied with a DC. When supplied with AC the alternating field in the conductors produced by AC current causes that the distribution of the current density through the conductor cross-section area is unequal. This phenomenon is known as the skin effect. The resistance of the winding is increased due to the skin effect and the DC resistance defined above is not valid during normal operation of the machine. The AC resistance is somewhat larger than the DC resistance. The increase of the resistance due to the skin effect depends on the supply frequency, on the conductivity of the conductor, on the permeability of the surrounding material and on the shape of the conductor and the slot where the conductor is situated. The higher the frequency of the current
flowing in a conductor is, the more the current is concentrated near the surface of the conductor. When the cross-section of the conductor is decreased the skin effect is also decreased and the value of the AC resistance approaches the DC resistance. In large synchronous machines the stator and excitation windings are made of flat copper bars and the damper winding are made of round copper bars, and thus the skin effect is much more considerable as when round wire windings are used. It is assumed that the resistance can be defined as

\[ R_{AC}(f, T) = R_{DC}(T) + R_{\text{skin}}(f, T), \]  

(2.57)

where \( T \) is temperature, \( f \) is the supply frequency and \( R_{\text{skin}} \) is an approximated increase of the resistance due to the skin effect. \( R_{\text{skin}} \) can be defined for example from motor design data if \( R_{AC} \) is available (\( R_{\text{skin}} = R_{AC} - R_{DC} \)). Usually \( R_{AC} \) is calculated for the motor nominal frequency \( f_{\text{nom}} \). It can be written for the AC resistance

\[ R_{AC} = \left[ 1 + \alpha (T - T_{\text{nom}}) \right] [R_{DC_{\text{nom}}} + R_{\text{skin}_{\text{nom}}}], \]  

(2.58)

where values with subscript “nom” are defined in the nominal operation point. The skin effect in the resistances of the various windings is a very complicated problem and only poor approximate estimates for the resistance variations during supply frequency variation can be found. Usually temperature modelling is adequate.

### 2.3.2 Magnetic saturation effect

The saturation effect depends on the magnetic material used in the stator and rotor magnetic circuits. Magnetic material properties can be described using hysteresis and median magnetising curves. In Fig. 2.22 the hysteresis curve and the median magnetising curve are shown for the typical standard electrical steel. The so called \( B/H \)-curve shows very clearly that saturation exists even at low magnetic field intensities. When the machine operates at nominal flux levels under load, the magnetic flux densities in the stator teeth and rotor pole faces vary between 1.0 ... 1.8 T and then the saturation effect is very strong. It can be seen in Fig. 2.22 that in addition to the non linear phenomenon of the saturation effect we are also dealing with a non-symmetrical phenomenon because of the hysteresis. Usually hysteresis is not taken into account and saturation is considered to be symmetrical using a median saturation curve.
The magnetic paths of the synchronous machine are saturated at nominal flux levels. Saturation appears in all parts of the iron volume. The stator teeth and the rotor pole faces are most heavily saturated. The stator yoke is less saturated because the flux density is much lower in that area. Leakage flux paths are mainly situated in the air. The saturation of the leakage flux path can be considered less dominant.

The effect of saturation in different parts of the flux path differs depending on the rotor structure. Magnetic flux paths are different in non salient and salient pole synchronous machines. The flux path of a non salient pole machine can be considered equal in all excitation directions, but in salient pole machines there is a reluctance difference between the direct and quadrature directions. In salient pole synchronous machines the shape of the rotor pole defines mainly the flux path, because the magnetic flux uses the path of the minimum reluctance. Fig. 2.23 shows the flux plots of the salient pole synchronous machine used in tests when excitation is in the direct axis. The flux plots are obtained from Finite Element Calculations (FEM) calculations. The software package Magnet™ has been used to the calculation of the test motor.

In the case of no load operation of the salient pole machine the flux path is similar as in Fig. 2.23 a). In practice no stator current is flowing. When the machine is loaded it means that in addition to the excitation current also the stator current emerges and the total MMF is a superposition of the stator MMF and the excitation MMF. This means that the magnetic flux turns away from the direct axis towards the direction of the larger air gap (quadrature axis). The magnetic flux does not want to use air as a flux path and the flux density increases in one pole edge and correspondingly the flux density decreases in the other pole edge (see Fig. 2.23. b)). The rotor pole is very heavily saturated in one edge and much less saturated in the other edge.

This unequal saturation of the rotor pole is a phenomenon similar to the well known armature reaction in non compensated DC-machines [De Jong 1982]. The only difference compared to the armature reaction in DC-machines is that in salient pole synchronous machines the stator MMF can be in any position between the direct and quadrature axes (armature MMF of DC-
machine stays always in the quadrature direction). The direct and quadrature axes are magnetically linked together and thus they can not be considered as uncoupled quadrature phase windings as is assumed in the two reaction theory. On the contrary they are magnetically linked together and the direct axis flux saturates the quadrature axis and vice versa. The cross magnetisation effect has been determined first for salient pole machines, but the same phenomenon appears also in uniform air-gap machines. The difference between salient and uniform air gap AC-machines is that the rotor pole causes the cross saturation effect in non uniform air gap machines, but in uniform air gap machines saliency does appear due to the saturated zone in the direction of flux [Kovács 1988]. In the following the physical interpretation of the cross magnetisation phenomenon is studied. In this case a steady state operation is considered, but cross saturation appears also during transients as has been shown in the previous section 2.2.2.

Fig. 2.24 demonstrates the behaviour of the cross saturation effect in salient pole machines. When the resultant MMF \( f_{\text{tot}} = f_d + f_q \) is divided into separate d/q axis components as shown in Fig. 2.24 A), the cross saturation effect described before can be analysed. Let us first consider the case when only the direct axis MMF \( f_{d\text{tot}} = f_d + f_i \) magnetises the machine. The magnetising flux \( \phi_d \) is shown in Fig. 2.24 B) (dotted line). When both \( f_{d\text{tot}} = f_d + f_i \) and \( f_{q\text{tot}} = f_q \) magnetise the machine so that \( f_{d\text{tot}} \) is kept constant, the quadrature axis component has a demagnetising effect on one side and a magnetising effect on the other side of the pole shoe. If linear magnetic conditions are assumed, the magnetising and demagnetising effects are equal (areas \(-\Delta\phi_d\) and \(+\Delta\phi_d\)) and the d-axis magnetising flux is unchanged. Due to saturation the magnetic flux densities are different in saturated and unsaturated conditions. This is shown in Fig. 2.24 C), where the saturation curves of the direct axis flux and fictitious cross magnetising flux are illustrated. (Fictitious cross magnetising curve characterises the effect of the quadrature axis MMF on the direct axis. At low and high saturation levels the effect of cross magnetising disappears because the magnetising and demagnetising areas become equal). Areas \(-\Delta\phi_{dsat}\) and \(+\Delta\phi_{dsat}\) are unequal and \(-\Delta\phi_{dsat} > +\Delta\phi_{dsat}\). This means that the direct axis flux component is decreased due to the quadrature axis MMF although the d-axis MMF is kept constant which is shown in Fig. 2.24 D). We can graphically determine that the direct axis magnetising flux is

\[
\phi_{mdsat} = \phi_{md}(i_{md}) + \phi_{qsl}(i_{mq}) + \phi_{qsl}(-i_{mq}) < \phi_{mdunsat}. \]
It can be seen in Fig. 2.25 how significant the effect of cross saturation is in a salient pole synchronous machine. In the figure the measured saturation curves of the direct and quadrature axis magnetising inductances are shown. The measurements are performed using our test drive. The implementation of the measurements is discussed in more detail in chapter 3. For example, at low values of direct axis excitation i.e. in the field weakening the direct axis magnetising inductance is reduced by 27% as quadrature axis current increases from zero to 1.5 pu. The reduction in the quadrature axis magnetising inductance is even 50% when the direct axis excitation current is increased from 0.5 to 2.0 pu. This means that the cross saturation effect is very dominant when the motor operates at high loads (large load angle) and especially in the field weakening when the direct axis excitation is reduced.
The variation of the motor model parameters can be considered as a control problem [Alaküla 1989] or as a modelling problem. In this work the latter method is considered. Representation of saturation and cross saturation has been the subject of many researchers. A lot of methods were presented to model the inductance variation of synchronous machines in general as well as more specific for power systems stability, for transient studies and for motor drives. One possibility is to solve the complete magnetic field during every step of the calculation using for example the FEM calculation, but such a method is computationally very slow and expensive and can not be used in motor drive models. A much simpler and faster, but less accurate approach is to use the open circuit saturation curves as reference to find the saturation factors for the direct and quadrature axis inductances ($L_{\alpha\alpha} = K_\alpha (X_{\alpha\alpha}) L_{\alpha\alpha\alpha}$, where $X$ is $i$ or $\psi$, $\alpha = d$, $q$ and $L_{\alpha\alpha\alpha}$ is the unsaturated value) [Harley 1980], [Brandwajn 1980]. Analytical expressions for the saturation factors as a function of either the flux linkage or the magnetising current can be derived using curve fitting techniques. There are two main problems when this kind of approach is used. An open circuit saturation curve for the quadrature axis is generally not available for industrial synchronous machines. It is often assumed that the saturation factor for the quadrature axis is equal to the direct axis saturation or the quadrature axis saturation factor is unity i.e. saturation of the quadrature axis is neglected. These assumptions are not adequate when a synchronous machine is considered. Another drawback is that those saturation factors, as defined before for the direct and
quadrature axes, do not take the cross saturation effect into account. A method for appending the cross saturation to the saturation model was among the first ones proposed by De Jong [1982]. The cross saturation in both axes has been taken into consideration by defining the saturation as a function of the resultant MMF \( f_{su} = \sqrt{f_d^2 + f_q^2} \), where \( f_d \) consists of the rotor excitation MMF and the stator direct axis MMF and \( f_q \) consist of the quadrature axis component of the stator MMF. Saturation can also be defined as a function of the resultant magnetising current. The use of the resultant MMF or magnetising current as a saturation level indicator in both axes leads to overestimation of the saturation level in the case of the salient pole machines. This can be explained by the fact, that the resultant MMF does not have the same effect on the direct and quadrature axes, when the resultant MMF is kept constant but the direction of MMF is changed. The saturation effect of the direct axis decreases when the resultant MMF turns from the direct to the quadrature axis.

Stiebler and Ritter [1985] used in their model saturation factors which depend on the direct and quadrature axis magnetising currents with different weights. The weighting factors were constants and dependent on machine properties i.e. the ratio of saliency and windings. Weighting factors take the direction of the resultant MMF into account more reliably than the previous methods. Kamoun and Poloujadoff [1985, 1986] proposed also empirical laws which define the saturation level as a function of the direct and quadrature axis magnetising currents. The functions are derived from measurements and they are quite complicated. The method proposed and used in many of the papers of El Serafi et.al. [1988, 1988, 1991, 1992, 1993] is based on rather simple linear equations. The saturation defined from open circuit characteristics is separated from the cross magnetisation effect. The principle of saturation modelling has been considered detailed and the models are applicable to salient pole synchronous machines, but the definition of the cross magnetisation effect for a machine used in an industrial drive might be rather difficult.

The leakage inductances are known to depend less on saturation than the magnetising inductances, and therefore the leakage inductances are usually assumed to be constant and the magnetising inductances are estimated. In case of an industrial salient pole synchronous machine drive we are looking for a saturation model for the air gap flux linkage. The model has to fulfil the following requirements: 1) Simple and fast calculations. 2) “Reasonable” accuracy. The demand for accuracy depends on the application. 3) The Saturation model should be implemented so that it can be added to the linear current model. 4) The Saturation of the air gap flux linkage is modelled according to the open circuit saturation curves. A linear model is valid in the static state when the values of the inductances are updated according to the operating point of the machine. During transients the validation of the model depends on the damper winding current estimate. 5) The saturation model has to be implemented so that it takes the cross saturation into account.

In order to implement the saturated air gap flux linkage in the current model which is required for the Direct Torque Control, the inductance parameters must be represented as a function of current. The linear stator flux linkage equations become

\[
\psi_{sd} = \psi_{md} + i_d L_{sa} = \left(L_{md}(i_t + i_d)(i_t + i_D) + i_d L_{sa}\right) = L_{md} (i_{md}) + i_d L_{sa}, \tag{2.59}
\]

\[
\psi_{sq} = \psi_{mq} + i_q L_{sa} = \left(L_{mq}(i_q + i_Q)(i_q + i_Q) + i_q L_{sa}\right) = L_{mq} (i_{mq}) + i_q L_{sa}. \tag{2.60}
\]
To provide suitable functions for the variation of these inductances their dependence must be defined, and this is done with respect to the machine currents. In the work of Melkebeek and Willems [1990] it is proposed that the magnetising inductances should be represented as a non-linear function of the direct and quadrature axis MMF, which can also be given as a function of the magnetising currents. The problem of saturation modelling is to find the non-linear functions defined by Eq. (2.61)-(2.62), where machine currents are used as indicators of the saturation level.

\[ L_{md} = f(i_{md}(t), i_{mq}(t)) \]  

(2.61)

\[ L_{mq} = f(i_{md}(t), i_{mq}(t)) \]  

(2.62)

The basic principles of the derivation of the non-linear functions are discussed next. In the last decade there has been a great interest in the saturation and especially cross magnetisation phenomenon. There has been a hard controversy on the nature of the cross magnetising effect in AC machines. Is the cross magnetisation between the quadrature axis windings in the saturated machines merely a mathematical artefact or a physically sound phenomenon? The cross magnetisation is a real physical phenomenon and the reason for that is basically the same as for the armature reaction in the DC-machines. In non salient pole machines cross magnetisation occurs due to the reluctance difference between the “saturated regions” in the direction of the flux and the non saturated region. In salient pole machines cross magnetisation can be considered to be purely a similar phenomenon as that occurring in DC-machines. The only difference is that the armature current can be in any position with respect to the field winding. In addition to this it is also a phenomenon which occurs due to mathematical modelling. A physical fact is that the armature current component which lies in space quadrature to the motor flux causes the cross magnetisation. If we observe the behaviour of the machine in the co-ordinates fixed to the direction of the flux, then the cross saturation effect which has to be considered is a real phenomenon. This is possible in the case of a non-salient pole machine, where the magnetic flux is aligned with the resultant MMF. If we observe the behaviour of the machine in the co-ordinates fixed to any other rotating reference frame or to the static reference frame (for example rotor reference frame as is used in the case of a salient pole machine due to saliency and to the fact that the resultant MMF is not aligned with the produced flux) then there is in addition to the real cross magnetisation also a “mathematical cross magnetisation” due to the modelling technique used. If this “mathematical cross saturation” is not taken into account, it leads to a non physical operation of the machine model. Let us assume that \( L_{md} \) depends only on \( i_{md} \) (resultant excitation in direct axis) and \( L_{mq} \) depends only on \( i_{mq} \) (resultant excitation in quadrature axis), cross saturation is not considered. In the case of a salient pole machine a non physical phenomenon arises, when the quadrature axis current \( i_{mq} \approx 0 \) and the direct axis is saturated. At this point a small reduction in current \( i_{md} \) does not have a large impact on the direct axis flux linkage \( \psi_{md} \), if the direct axis excitation current \( i_{md} \) is slightly reduced and at the same time the excitation current in quadrature axis \( i_{mq} \) is increased, so that \( |i_m| = \sqrt{i_{md}^2 + i_{mq}^2} \) is kept constant. The flux linkage \( \psi_{md} \) does not change remarkably, but the flux linkage \( \psi_{mq} \) increases. In that case \( |\psi_m| = \sqrt{\psi_{md}^2 + \psi_{mq}^2} \) will increase. This is non physical if \( |\psi_m| \) will increase when \( |i_m| \) is kept constant and the angle between \( i_m \) and direct axis will increase.

Irrespective of the nature of the cross magnetisation effect, it is obvious that the additional saturation due to this phenomenon has to be considered. Neglecting this will lead to the
overestimation of the air gap flux linkage. At first a non-salient pole machine is taken into consideration. In the non-salient pole machine the direct and quadrature axis magnetising inductances can be considered equal. This means that the saturation of the magnetic circuit is defined by the magnitude of the resultant MMF, but the saturation is identical regardless of the direction of the resultant MMF. The quadrature axis component of the resultant MMF saturates the direct axis magnetising inductance as much as the direct axis component saturates the quadrature axis magnetising inductance (reciprocal cross saturation). The fundamental component of the MMF and magnetising flux are aligned and their magnitudes are related by a single non-linear characteristic. The amplitudes of the magnetising flux linkage and the current are given by

$$\left| \psi_m \right| = \sqrt{\psi_{md}^2 + \psi_{mq}^2} \quad \text{and} \quad \left| i_m \right| = \sqrt{i_{md}^2 + i_{mq}^2}$$  \hspace{1cm} (2.63)

and the flux linkage components $\psi_d$ and $\psi_q$ may be written as

$$\psi_d = \frac{\left| \psi_m \right|}{\left| i_m \right|} i_d \quad \text{and} \quad \psi_q = \frac{\left| \psi_m \right|}{\left| i_m \right|} i_q.$$  \hspace{1cm} (2.64)

Introducing the saturation factor $K_s$ and the unsaturated magnetising inductances $L^u$, following relationships can be obtained:

$$\frac{\left| \psi_m \right|}{\left| i_m \right|} = K_s L^u \cdot \left| i_d \right| \cdot L^u \quad \alpha = d, q \hspace{1cm} (2.65)$$

$$\psi_{max} = K_s \left| i_d \right| \cdot L^u \cdot i_\alpha = L_{mad} i_\alpha \quad \alpha = d, q \hspace{1cm} (2.66)$$

In a salient pole machine, the MMF and magnetising flux are no longer aligned. The relationship between their amplitudes depends on their respective angular position. On the other hand the angle between the MMF and the flux depends on the position of the resultant MMF. If the resultant MMF is aligned with the direct or quadrature axis then the flux is aligned with the MMF, otherwise the MMF and the flux are apart from each other. This means that the saturation of the magnetic circuit is defined by the magnitude and position of the resultant MMF i.e. it depends on the space vector of the resultant MMF. If $L_d > L_q$, which is the case in an electrical excited salient pole synchronous machine, then the effect of the direct axis MMF on the ability of the quadrature axis MMF flux production is stronger than the effect of the quadrature axis MMF on the direct axis MMF flux production (non reciprocal cross saturation). This new problem can be solved by assuming that the salient pole machine has spatially distributed windings in the direct and quadrature axis with different winding factors. The winding factors for the fundamental harmonic $\xi_{de}$ and $\xi_{qe}$ are selected so that the real and fictitious winding (with the constant air gap equal to the median air gap of the real machine) both have the same non saturated inductances $L_d$ and $L_q$ [Iglesias 1992]. The magnitude of the produced magnetising flux can be determined by

$$\phi = A \left( \sqrt{(N \xi_{de} i_{md})^2 + (N \xi_{qe} i_{mq})^2} \right) = AN \xi_{de} \left( i_{md}^2 + \left( \frac{N \xi_{qe}}{N \xi_{de}} \right)^2 i_{mq}^2 \right),$$  \hspace{1cm} (2.67)
where $N$ is the number of turns in series per phase $(d, q)$ and $\Lambda$ is the permeance of the median air gap machine. The ratio of saliency \( \left( \frac{N \xi_{qe}}{N \xi_{ac}} \right)^2 \) can also be stated using $L_d$ and $L_q$. The inductances are proportional to $N^2$ so we can write \( \left( \frac{N \xi_{qe}}{N \xi_{ac}} \right)^2 = \frac{L_q}{L_d} \). From Eq. (2.67) the resultant magnetising current can be defined which, in case the salient pole synchronous machine saturates the magnetic circuit, is

\[
|j_{sm}| = \sqrt{i_{md}^2 + \frac{L_k}{L_d} i_{mq}^2} = \sqrt{\frac{L_k}{L_q} i_{md}^2 + i_{mq}^2}.
\]  

(2.68)

By using this so called “saturant magnetising current" as an indicator of the saturation level, models for the saturation of the magnetising inductances can be derived

\[
L_{md} = f\left(\frac{j_{sat}}{i_{md}}\right) = f\left(\sqrt{i_{md}^2 + k i_{mq}^2}\right),
\]

(2.69)

\[
L_{mq} = f\left(\frac{j_{sat}}{i_{mq}}\right) = f\left(\sqrt{\frac{1}{k} i_{md}^2 + i_{mq}^2}\right).
\]

(2.70)

The factor $k$ is the saliency ratio which is a function of saturation and can be defined as

\[
k = \frac{L_{mq}^u}{L_{md}} = K \cdot f(i_{md}, i_{mq}),
\]

(2.71)

where $K = \frac{L_{mq}^u}{L_{md}}$ is an unsaturated saliency ratio and $f$ is a characteristic function of the saliency ratio. Magnetising inductances are used, because the stator leakage inductance is assumed to be constant. The saturation functions in Eqs. (2.69) and (2.70) can be derived from the open circuit curves. The characteristic function $f$ is defined so that it can dictate the variation of the saliency ratio, when the machine is saturated.

### 2.4 BEHAVIOUR OF THE MODELS DURING TRANSIENTS

The accuracy of the flux estimation during load transients is a very critical part when a Direct Torque Controlled synchronous motor is used. The optimal switching table ensures that the best available voltage vector which accelerates the stator flux linkage with correlation to the air gap flux linkage is selected and the fastest possible torque response is achieved. This dynamically optimal operation is not self-evident, but it assumes that the stator flux linkage estimate is valid. If this is not the case unoptimal selections are made, the torque response is modest and then the torque reference is not followed. The estimated stator flux linkage amplitude is not so critical when the transient operation of DTC is observed. Instead the angular position of the stator flux linkage vector is the most significant. The amplitude error corresponds to the torque error. A remarkable phase shift between the estimated and the actual
motor flux causes drifting of the actual stator flux linkage, so the control becomes unstable. This means that flux linkage estimation methods with additional time delays (e.g. a method based on additional filtering or flux estimators) cannot be used, because they cause a phase shift during fast transients. In the following sections two flux estimation models, the voltage model and the current model, are examined during load transients. The simulated results are shown. Because this work deals only with the stator side control and specifically the stator flux linkage estimation, the excitation control unit is considered ideal i.e. neither delays nor a lack of ability to increase the excitation current instantly are assumed. The excitation control is implemented using a combined “reaction and unity power factor control” [Pyrhönen O. 1998].

2.4.1 Voltage Model

The behaviour of the voltage model during load transients is studied, using a salient pole synchronous motor DTC drive simulator (Appendix 4). The used pu system can be found in Appendix 7. Error sources of the voltage model are discussed in chapter 2.2.1. The stator resistance is a dominant error source when the voltage integration is performed. It can be assumed that the accuracy of the measurements (i.e. intermediate voltage and phase currents) has been implemented in a way that errors of the measurements are negligible. Assuming that the voltage estimate and the measured current are correct the stator flux linkage estimate error can be calculated using Eq. (2.32).

\[
\psi_{\text{err}} = \psi_s - \psi_{\text{est}} = \int_{t_0}^{t_1} i_s dt - R_s \int_{t_0}^{t_1} i_s dt + \int_{t_0}^{t_1} u_{\text{meas}} dt = \Delta R_s \int_{t_0}^{t_1} i_s dt, \quad \text{where}
\]

\(\Delta R_s\) is the error in the stator resistance estimate, \(t_0\) is the initial time instant before the transient and \(t_1\) is the time instant after the load transient. The stator flux linkage error accumulates during the transients. The stator flux linkage estimate error depends on the resistance error and on the rate of change of the stator current which corresponds to the torque response. The torque response of the DTC controlled synchronous motor can be evaluated using a subtransient flux linkage model of the motor [Vas 1992]. The stator voltage in the stator reference frame can be determined

\[
u_s' = R_s i_s + L_s \frac{d i_s}{dt} + \frac{d \psi_s^-}{dt}, \quad (2.72)
\]

where \(L_s = \frac{L_d + L_q}{2}\) is the average subtransient inductance and \(\psi_s^-\) is the subtransient flux linkage. Physically the subtransient flux linkage is equal to the air gap flux linkage and it is kept constant during the transient by excitation. The time derivative of the subtransient flux linkage is the back Electro Motive Force (EMF). When Eq. (2.72) is transformed into the rotor reference frame it follows

\[
u_s' = R_s i_s + L_s \frac{d i_s}{dt} + j \omega L_s i_s + \frac{d \psi_s^-}{dt} + j \omega \psi_s^- \quad (2.73)
\]

When a fast torque step is performed using a DFMC modulator, it uses only non-zero voltage vectors i.e. the maximum output voltage is used to produce the torque. The average voltage vector during a torque step is in space quadrature with respect to the stator flux linkage vector.
and the maximum acceleration of the stator flux linkage space vector is achieved. This is shown in Fig. 2.26. A simulated torque step from no load to the nominal torque has been performed.

Fig. 2.26. Simulated nominal torque step. The motor speed is 0.5 pu. The stator flux linkage position is in sector 5 and a positive direction of the rotation is assumed. Voltage vectors 1 and 2 are used for the stator flux linkage acceleration (See optimal switching table Fig. 1.6).

It is assumed that during the torque step the resistive voltage drop \( R_s i_s \) and the rotational voltage \( j \omega \psi_s' i_s \) in Eq. (2.73) are negligible when compared to the stator voltage. The subtransient flux linkage is kept constant during the torque step. The derivative of the stator current is obtained

\[
\frac{d i_s'}{dt} = \frac{u_s' - j \omega \psi_s'}{L} = \frac{u_{\text{res}}'}{L},
\]

where \( u_{\text{res}}' \) is the voltage reserve in the operating point. The average stator voltage is in space quadrature with respect to the stator flux linkage. This means that the stator current, which rises towards the same direction with the stator voltage, is also in space quadrature to the stator flux linkage and is used to produce the torque. The time of rise of the stator current which is equal to the torque response can be calculated

\[
\Delta t = (t_1 - t_0) = \frac{L}{|u_{\text{res}}'|} \left| \Delta i_s' \right|.
\]

According to Eqs. (2.32) and (2.75) an error in the stator resistance value causes an error in the stator flux linkage estimate calculated from the voltage integral which is directly proportional to the time of rise of the stator current. On the other hand the time of rise of the stator current is proportional to the voltage reserve on the operating point. The voltage reserve is large at low speeds and a fast response of the stator current can be obtained. In the field weakening range the back EMF is almost equal to the maximum stator voltage of the inverter unit, so the voltage reserve for the torque changes remains small.

Fig. 2.27 illustrates an actual and an estimated stator flux linkage during a nominal torque step. The operating point before the step is the same as in Fig. 2.26. During the transient the flux estimation error is negligible. After the transient, in steady state, the estimation error increases which is obvious as has been explained in chapter 2.2.1. The stator current rises towards the same direction as the applied average voltage vector and the current is nearly in
space quadrature with respect to the stator flux linkage. Excitation control takes care of the necessary magnetisation current of the motor and tries to keep the power factor equal to the unity during the transient.

Fig. 2.27. Simulated nominal torque step. The motor speed is 0.5 pu. The actual stator resistance is 0.05 pu and the stator resistance estimation error is 50%. Torque step response is 1.2 ms. The torque step is performed at the same time instant as in Fig. 2.26. a) Actual and estimated stator flux in stator coordinates. b) Traces of the actual and estimated stator flux and the stator current space vectors in the rotor oriented reference frame.

Fig. 2.28 shows the actual and estimated stator flux linkage direct and quadrature axis components during a nominal torque step. The operating points are a) motor speed 0.1 pu, b) motor speed 0.5 pu and c) motor speed 1.5 pu. The dependency between the voltage reserve and the estimation error can be clearly seen. In the cases a) and b) the estimation error during the transient is negligible, but especially in the quadrature axis flux estimate there is a considerable error in the field weakening and the nominal torque can not be produced.
Fig. 2.28. Simulated nominal torque step. The motor speed is a) 0.1 pu, b) 0.5 pu and c) 1.5 pu. The actual stator resistance is 0.05 pu and the stator resistance estimation error is 50%. The voltage reserve is set to 10% in the field weakening range. In the fast transients the cumulative error is negligible a) and b). In the field weakening the cumulative error in the voltage integral becomes remarkable because of the long transient duration c).
2.4.2 Current model

The behaviour of the current model during load transients is studied using the salient pole synchronous motor DTC drive simulator (Appendix 4). The used pu system can be found in Appendix 7. The main error source of the current model is the inaccurate damper winding current estimation during a transient. The current model and the damper winding current estimation are discussed in chapter 2.2.2. The damper winding current estimator using linear machine equations is adequate, because no remarkable improvement is achieved when a non-linear saturated estimator is used.

The same torque step simulations are performed for the current model as for the voltage model. The motor model and the current model have the same saturated inductance models, because only the transients are studied. The motor model calculates the damper winding current from the motor state equations while linear damper winding current estimators are used in the current model. It can be seen in Fig. 2.29 that during a load transient a flux linkage estimation error occurs. The behaviour of the error is different when comparing it to the voltage model. At low speeds, when a fast torque response can be performed, the damper winding current estimation fails (Fig. 2.29 a and Fig. 2.30 a). In the field weakening an improvement can be noticed. Naturally the torque response is slower. The reaction control of the excitation circuit manages to keep the direct axis airgap flux linkage constant and no damper winding current flows. Only the quadrature axis flux estimation fails. At low speeds the torque response is so fast that a direct axis damper winding current appears. The most important difference between the current model and the voltage model is the behaviour after the load transient. If there is an error in the voltage model estimation, the actual motor flux linkage drifts and the control becomes unstable. Fig. 2.29 b) demonstrates that the current model keeps the flux estimate stable after the damper winding currents have decayed.

![Graph showing the comparison between psisd, psisq, and pu values over time](image-url)
Fig. 2.29. Simulated nominal torque step. The motor speed is 0.5 pu. The torque step response is 1.2 ms. a) The actual and estimated direct and quadrature stator flux components in the rotor oriented reference frame. b) The actual and estimated stator flux x-component in stator co-ordinates.

Fig. 2.30. Simulated nominal torque step. The motor speed is a) 0.1 pu and b) 1.5 pu. The voltage reserve is set to 10% in the field weakening range.
2.5 A MOTOR MODEL FOR THE DTC DRIVE

The performance of a high quality torque control as DTC in dynamically demanding industrial applications is mainly based on the accurate estimate of the various flux linkage space vectors. The tasks of the motor model are: 1) acquisition of the various flux linkages from terminal quantities such as voltages and currents 2) obtaining the electrical torque for closing the torque control loop. When a reliable and fast calculable real time model is needed it should be remembered that nowadays industrial motor control systems are real time applications with a restricted calculation capacity. At the same time the control system demands for a simple, fast calculable and reasonably accurate motor model. A compromise between the computation capacity and the accuracy should be found. In the previous sections the flux linkage calculation by applying the current model and the voltage model as well as the advantages and disadvantages of both models have been discussed. Table 2.1 summarises special features of the current and voltage model.

Table 2.1. Features of the current and voltage models. Advantages are bolded.

<table>
<thead>
<tr>
<th></th>
<th>Voltage model</th>
<th>Current model</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameters</td>
<td>- Stator resistance.</td>
<td>- Inductance parameter set.</td>
</tr>
<tr>
<td></td>
<td>- Damper winding time constants.</td>
<td>- Saturation models of the magnetising inductances.</td>
</tr>
<tr>
<td>parameter models</td>
<td>- Temperature model of the stator resistance.</td>
<td>- Depends on the accuracy of the inductances parameters.</td>
</tr>
<tr>
<td>static accuracy</td>
<td>- Depends on the accuracy of the voltage loss calculation.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- At low motor speed integration is inevitably too erroneous.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- At higher speed accuracy is fair.</td>
<td>- No speed dependence (if the rotor position measurement is accurate enough).</td>
</tr>
<tr>
<td>dynamic accuracy</td>
<td>- Depends on the accuracy of the voltage loss calculation and on the rise time of the stator current.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- At low speed the dynamic accuracy is good due to the fast response of the stator current.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- In the field weakening the dynamic accuracy is poorer due to the slower response of the stator current.</td>
<td></td>
</tr>
<tr>
<td>stability of the calculation</td>
<td>- Underestimation of voltage loss leads to a stable error.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Overestimation of voltage loss leads to unstable drifting.</td>
<td></td>
</tr>
<tr>
<td>implementation</td>
<td>- Easy to implement</td>
<td>- Rather easy to implement.</td>
</tr>
<tr>
<td></td>
<td>- Can be calculated at fast sampling rate (simple calculation).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- stator co-ordinates used.</td>
<td>- Calculation must be done in a slower sampling rate than voltage model (complex calculations).</td>
</tr>
<tr>
<td></td>
<td>- no rotor position needed.</td>
<td>- Rotor oriented co-ordinates.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Accurate rotor position needed.</td>
</tr>
</tbody>
</table>

When all benefits from Table 2.1 are combined, it is easy to conclude that a combination of the voltage model and current model will be the best solution for a reasonably accurate motor model. In DTC the voltage model operates as a main model and is calculated at a very fast sampling rate. The current model is calculated at a much slower sampling rate. The stator flux linkage estimate calculated from the voltage model is corrected using the stator flux linkage estimate which is calculated from the current model. The current model acts only as a supervisor which at longer time levels prevents the motor stator flux linkage from drifting out.
At very slow speeds the role of the current model is emphasised but it never becomes the main model. At higher speeds the function of the current model correction is to act as a stabiliser of the control system. Fig. 2.31 illustrates the proposed motor model of the DTC controlled drive.

At first the measured signals have to be processed. The stator voltage estimate is calculated from the measured intermediate voltage and by using the knowledge of the switching states. The measured phase currents are transformed to quadrature phase components and a coordinate transformation to rotor co-ordinates is performed. The stator current values in addition to the excitation current in the rotor reference frame are used in the current model. The stator voltage estimate and the voltage loss are integrated and, because of the voltage loss this flux linkage component is subtracted from the estimated stator flux linkage. The voltage loss calculation takes the resistive voltage drop, the threshold voltage, the voltage loss during the conducting state and the switching delays into consideration. The current model first estimates the damper winding currents and then the inductance parameters are updated according to the operation point. When all currents of the motor and the inductance parameters are known, the stator flux linkage is calculated. After that a coordinate transformation to the stator co-ordinates has to be performed. The stator flux linkage estimate calculated from the voltage model is compared to the stator flux linkage estimate calculated from the current model and the difference between those is updated to the stator flux estimate of the voltage model. The stator flux correction is a weighted incremental correction. Weighting depends on the operating point of the motor. In the last stage the electric torque calculation is carried out by using the estimated stator flux linkage and the measured quadrature phase current components. The control variables required, i.e. stator flux linkage estimate and the motor’s electric torque estimate, are the outputs of the model and they are used in the hysteresis control in order to define the logical inputs for the optimal switching table.
The basic principle for the motor model is, that the voltage model is calculated in the fast sampling rate and the output of the current model is used to correct the possible error which accumulates into the voltage integral. The current model is calculated at a much slower time rate and the correction is done by adding small increments of the error signal to the voltage integral. The stator flux error signal can be defined using the outputs of the models defined in the stator reference frame

\[
\Delta \psi_x = \psi_{xc} - \psi_{xv}
\]

\[
\Delta \psi_y = \psi_{yc} - \psi_{yv},
\] (2.76)

where \( \psi_{xc} \) and \( \psi_{yc} \) are the stator flux linkage components of the current model and \( \psi_{xv} \) and \( \psi_{yv} \) are the stator flux linkage components of the voltage model. The current model and the error signal are calculated using the sampling time of \( \Delta t_{\text{error}} \) and the correction is made using the sampling time of \( \Delta t_{\text{cor}} \) (\( \Delta t_{\text{error}} > \Delta t_{\text{cor}} \)). Before a new error vector is calculated the stator flux linkage vector rotates ahead. The direction of the error vector is wrong unless it is turned correspondingly to the rotation direction of stator flux linkage. In the time period \( \Delta t_{\text{error}} \) the stator flux linkage vector will turn with the angle \( \zeta = \omega \cdot \Delta t_{\text{error}} \). In Fig. 2.32 the correction of the error signal is presented when the error vector is calculated at an instant \( n \) and an incremental correction is made. The incremental error vector is defined as

\[
\Delta \psi_{\text{sinc}} = \frac{\Delta t_{\text{cor}}}{\Delta t_{\text{error}}} \Delta \psi_s.
\]

The stator flux linkage vector rotation during the calculation delay is not taken into account. There is a phase error between the current model and the voltage model, but the amplitudes are equal. It can be seen that after three incremental corrections the stator flux linkage amplitude is increased. This is not the hoped situation. Only a phase error exists between the models. The correction is made in the wrong direction, because the rotation of the stator flux linkage has not been taken into account.

![Diagram](image)

Fig. 2.32. Correction of the error signal when the error vector is calculated at an instant \( n \) and incremental corrections are made. The stator flux linkage rotation between the calculation intervals is not taken into account. In the zoomed area the principle of the error vector rotating along the stator vector where angle \( \beta \) is kept constant is presented.

This problem can be avoided if the calculation delay is compensated. The correction is performed by keeping the angle \( \beta \) between the stator flux linkage estimate and the error vector
constant during incremental corrections. On the other hand it means that the calculated flux error must be rotated along with the stator flux linkage. The incremental error vector can be turned using the following transformation

\[
\Delta \psi_{x,\text{cor}} = \Delta \psi_x \cos(\zeta) - \Delta \psi_y \sin(\zeta),
\]
\[
\Delta \psi_{y,\text{cor}} = \Delta \psi_x \sin(\zeta) + \Delta \psi_y \cos(\zeta),
\]

where \(\zeta = \omega \cdot \Delta t_{\text{error}}\). In this method the error vector \(\Delta \psi\) is turned using the angle \(\zeta\) which corresponds to the stator flux rotation during the calculation interval of the error signal \(\Delta t_{\text{error}}\).

This type of correction is at first done in a slightly wrong direction, but at the end of the correction interval the direction of the error vector is correct. A better way to implement the turning of the error vector is to perform the correction in the reference frame fixed to the stator flux linkage. This implementation automatically keeps the angle between the error vector and the stator flux linkage constant and the direction of every incremental error vector is correct.

The stator flux linkage error in the stator flux linkage reference frame can be defined

\[
\Delta \psi_x^a = \Delta \psi_x \cos(\alpha) + \Delta \psi_y \sin(\alpha),
\]
\[
\Delta \psi_y^a = -\Delta \psi_x \sin(\alpha) + \Delta \psi_y \cos(\alpha),
\]

where \(\alpha\) is the angle of the stator flux linkage with respect to the stator x-axis. The voltage integral can be corrected according to

\[
\psi_{x(n)} = \left|\psi_{svol(n)}^x\right| + \Delta \psi_{xinc(n)}^x + \Delta \psi_{xcor(n)}^x,
\]
\[
\psi_{y(n)} = \Delta \psi_{yinc(n)}^y
\]

where \(\left|\psi_{svol(n)}^x\right| = \left|\psi_{svol(n)}^y\right|\) is the amplitude of the stator flux linkage, \(\Delta \psi_{xinc}^x = \frac{\Delta t_{\text{cor}}}{\Delta t_{\text{error}}} \Delta \psi_x^w\) and \(\Delta \psi_{yinc}^y = \frac{\Delta t_{\text{cor}}}{\Delta t_{\text{error}}} \Delta \psi_y^w\). The corrected stator flux linkage must be transformed back to the stator co-ordinates and we get

\[
\psi_{x(n)} = \psi_{x(n)}^a \cos(\alpha) - \psi_{y(n)}^a \sin(\alpha),
\]
\[
\psi_{y(n)} = \psi_{x(n)}^a \sin(\alpha) + \psi_{y(n)}^a \cos(\alpha).
\]

Using Eq. (2.79) and definitions \(\cos(\alpha) = \frac{\psi_x}{|\psi_{svol}|}\) and \(\sin(\alpha) = \frac{\psi_y}{|\psi_{svol}|}\), we can write for the voltage integral correction

\[
\psi_{x(n)} = \left|\psi_{svol(n)}^x\right| + \Delta \psi_{xinc(n)}^x \left|\psi_{svol(n)}^x\right| - \Delta \psi_{xcor(n)}^x \left|\psi_{svol(n)}^x\right| + \frac{\psi_{y(n)}^x}{\left|\psi_{svol(n)}^x\right|},
\]
\[
\psi_{y(n)} = \left|\psi_{svol(n)}^y\right| + \Delta \psi_{yinc(n)}^y \left|\psi_{svol(n)}^y\right| + \Delta \psi_{ycor(n)}^y \left|\psi_{svol(n)}^y\right| + \frac{\psi_{y(n)}^y}{\left|\psi_{svol(n)}^y\right|},
\]

and when Eq. (2.81) is rearranged
\[
\psi_{x(n)} = \psi_{x\text{vol}(n)} + \frac{1}{\psi_{\text{vol}}(n)} \left( \Delta \psi_{x\text{inc}(n)} \psi_{x\text{vol}(n)} - \Delta \psi_{y\text{inc}(n)} \psi_{y\text{vol}(n)} \right)
\]
\[
\psi_{y(n)} = \psi_{y\text{vol}(n)} + \frac{1}{\psi_{\text{vol}}(n)} \left( \Delta \psi_{x\text{inc}(n)} \psi_{y\text{vol}(n)} + \Delta \psi_{y\text{inc}(n)} \psi_{x\text{vol}(n)} \right). \tag{2.82}
\]

The stator flux correction shown in Eq. (2.82) corrects the stator flux linkage calculated from the voltage integral in order to make it equal to the current model which is, however, not the best solution. The goal was to develop a combined model where the appropriate features of both models can be utilised. This requirement can be fulfilled by applying a weighted correction

\[
\psi_{x(n)} = \psi_{x\text{vol}(n)} + W \frac{\Delta t_{\text{cor}}}{\Delta t_{\text{error}}} \left( \Delta \psi_{x} \psi_{x\text{vol}(n)} - \Delta \psi_{y} \psi_{y\text{vol}(n)} \right)
\]
\[
\psi_{y(n)} = \psi_{y\text{vol}(n)} + W \frac{\Delta t_{\text{cor}}}{\Delta t_{\text{error}}} \left( \Delta \psi_{x} \psi_{y\text{vol}(n)} + \Delta \psi_{y} \psi_{x\text{vol}(n)} \right). \tag{2.83}
\]

The weighting coefficient \( W (W \in [0...1]) \) determines the importance of the current model in a certain operating point. The weighting coefficient contains also the term \( 1/|\psi_s| \). Using the relative pu values in the calculations below base speed this term is unity and in the field weakening range the weighting factor must be increased by a factor which is proportional to \( 1/|\psi_s| \). The accuracy of the voltage integration improves when the speed of the drive increases, but at very low speeds the current model must be used. The weighting coefficient \( W \) is changed as a function of speed. A possible error in the voltage loss calculation increases when the load of the machine is increased. In the field weakening range the stator current needed to produce a certain torque is higher due to the reduced flux linkage and this also increases the possible voltage loss estimation error. According to these facts we can define a base curve for the weighting coefficient

\[
W(\omega, t_e, \psi_s) = \begin{cases} 
(1 - \frac{\omega}{\omega_{\text{base}}})^k_1 \left( 1 + \frac{t_e}{T_{\text{max}}} \right) & \text{when } \omega < \omega_{\text{nom}} \\
\left( 1 - w \right)^k_1 \left( 1 + \frac{t_e}{T_{\text{max}}} \right) & \text{when } \omega_{\text{nom}} < \omega < \omega_{\text{nom}} \\
\left( 1 - w \right)^k_1 \left( \frac{\psi_{\text{base}}}{\psi_s} \right)^k_2 \left( 1 + \frac{t_e}{T_{\text{max}}} \right) & \text{when } \omega > \omega_{\text{nom}} 
\end{cases} \tag{2.84}
\]

In Fig. 2.33 the weighting factor is given using the coefficients \( k_1 = k_2 = 2, w = 0.9 \) and the maximal torque \( T_{\text{max}} = 3 \) pu. The curves are calculated for the torques \( t_{e1} = 0 \) pu, \( t_{e2} = 1 \) pu, \( t_{e3} = 2 \) pu and \( t_{e4} = 3 \) pu.
The motor model for the DTC drive is now obtained. It combines all the good properties of the voltage model and the current model. The accurate operation of the motor model can be achieved when the so called “weighted correction” is implemented. The incremental correction method is used and the calculation delay is taken into account. The voltage model is very easy to implement and it can be calculated at a very high sampling rate (e.g. 25 µs). The current model is much more complicated, but it can be calculated using a lower sampling rate (e.g. 1 ms). In the voltage model there is only one dominant parameter (stator resistance). The flux linkage estimation using the voltage integral is very sensitive to the error of the stator resistance estimation. The current model includes a set of inductances and damper winding resistances. The most dominant parameters are magnetising inductances. The motor model includes saturated inductance models which take also cross saturation into account. The parameters should be known before the drive can operate. Motor parameter initialisation, measurement and saturation modelling are discussed in the following chapter.

Fig. 2.33. Weighting factor using coefficients $k_1 = k_2 = 2$, $w = 0.9$ and maximal torque $T_{\text{max}} = 3$ pu. The curves are calculated for the torques $t_{e1} = 0$ pu, $t_{e2} = 1$ pu, $t_{e3} = 2$ pu and $t_{e4} = 3$ pu.
3 MOTOR PARAMETER MEASUREMENTS AND MODELLING

3.1 INITIALISATION OF THE MOTOR MODEL PARAMETERS

The following chapter introduces the initialisation of the salient pole synchronous motor model parameters. The traditional synchronous machine parameters given by the machine manufacturer or found by applying standard measurement procedures can be used for the motor model initialisation. In the voltage model there is only one parameter, the stator resistance, which can be derived from the motor data. The current model includes a set of inductance parameters and damper winding time constants. The traditional parameters, however, are seldom directly applicable to a DTC-drive and this applies especially to the inductances which can be accepted, when comparing the machine operating conditions for different traditional inductance parameter definitions. A direct axis synchronous reactance is defined in a no load condition. The transient reactance, subtransient reactances and direct axis transient time constants on the other hand are defined in a three-phase short circuit. The quadrature axis synchronous reactance and especially the quadrature axis subtransient reactance and the transient time constants are defined in more or less miscellaneous operating points. This means that the motor model parameters calculated from the traditional parameters do not correspond to the same magnetic operating condition, so there may arise disagreements on the model parameters. Many leading manufactures use their own design programs which combine motor design theory and experience of many years. Applying modern inverter technology and using a powerful processor it is possible to acquire the needed parameters with the inverter itself. The inverter can be used as a measuring tool during the factory tests or during the commissioning of the drive.

The equivalent circuits of the traditional synchronous machine parameters are based on the analytical study of the symmetrical three-phase short circuit, shown for example in the work of Vas [1992]. It is possible to derive the stator current equations examining the phenomena during a three-phase short circuit. The equivalent circuits of the subtransient reactances $x_d^s$ and $x_q^s$, transient reactance $x_d^t$ and synchronous reactances $x_d$ and $x_q$ and also time constants $\tau_d^t$, $\tau_d^s$ and $\tau_q^s$ can be found. This solution assumes that the mutual inductances of the stator and excitation windings, stator and damper windings and damper winding and magnetising windings are equal $L_{dt} = L_{dD} = L_{tD} = L_{rd}$. The equivalent circuits of the parameters and time constants are shown in Fig. 3.1. These can be used in the initialisation of the current model inductance parameters.
Using the traditional parameter set (subtransient reactances \( x_d \) and \( x_q \), transient reactance \( x_d' \) and synchronous reactances \( x_d \) and \( x_q \)), which is known in advance, the two-axis model parameters can be calculated. The only problem is that there are six inductances in the two-axis model and only five equivalent circuit equations are available. Using these five equations it is impossible to obtain the explicit solution.

The quadrature axis subtransient inductance can be defined according to the equivalent circuit

\[
L_q'' = L_q - \frac{L_{mq}^2}{L_Q}
\]  

(3.1)
where \( L_Q = L_{mq} + L_{Q\sigma} \). The quadrature axis magnetising inductance \( L_{mq} \) is considerably greater than the damper winding’s leakage inductance \( L_{Q\sigma} \). Without making a large error we can assume that \( L_Q \approx L_{mq} \). In that case we get for the subtransient inductance

\[
L''_q = L_q - \frac{L_{mq}^2}{L_Q} \approx L_s\sigma + L_{mq} - L_{mq} = L_s\sigma.
\]  

(3.2)

The estimate can be adjusted by using a correction factor \( k_{47\sigma} \), so that \( L_s\sigma = k_{47\sigma} \cdot L''_q \), and is based on experience. Typically factor \( k_{47\sigma} \) is around 0.7 ... 0.8.

Other parameters needed in the current model can be initialised based on the equivalent circuits given in Fig. 3.1. The equations derived from those equivalent circuits are

\[
L_{s\sigma} = k_{47\sigma} \cdot L''_q,
\]

\[
L_{md} = L_d - L_{s\sigma},
\]

\[
L_{mq} = L_q - L_{s\sigma},
\]

\[
L_{\sigma\sigma} = \frac{L''_d L_{md} - L_{sd} L_{md}}{L''_d - L''_d},
\]

\[
L_{Q\sigma} = \frac{L_{mq}^2}{L_q - L_{mq}},
\]

\[
L_{D\sigma} = \frac{L''_d L_{md} - L_{sd} L_{md} - L_{sd} L_{md}}{L_{s\sigma} L_{s\sigma} + L_{s\sigma} L_{md} + L_{md} L_{s\sigma} - L_a L_{s\sigma} - L_a L_{md}}.
\]

When all the inductance parameters of the two-axis model are known, the resistances of the damper windings \( R_D \) and \( R_Q \) can be calculated.

\[
R_D = \frac{\left( L_{D\sigma} + \frac{L_{md} L_{s\sigma}}{L_{md} + L_{s\sigma}} \right)}{\frac{L''_d}{L''_d}},
\]

(3.4)

\[
R_Q = \frac{\left( L_{Q\sigma} + \frac{L_{mq} L_{s\sigma}}{L_{mq} + L_{s\sigma}} \right)}{\frac{L''_q}{L''_q}}.
\]

The calculation of the direct and quadrature axis damper winding current estimates needs four terms which can be defined as

\[
k_{mod d1} = \frac{L''_d}{L''_d + L_{s\sigma}}, \quad k_{mod d2} = \frac{L_{md}}{L_{md} + L_{D\sigma}},
\]

(3.5)

\[
k_{mod q1} = \frac{L''_q}{L''_q + L_{s\sigma}}, \quad k_{mod q2} = \frac{L_{mq}}{L_{mq} + L_{Q\sigma}}.
\]

Using Eqs. (3.3)-(3.5) the current model can be initialised. The success of the initialisation will depend on the accuracy of the available traditional motor parameters. In the following
chapter different methods to determine traditional synchronous machine parameters are
discussed. A comparison is made between parameters obtained from a standard measurement
procedure, from manufacturer data, from measurements using a DTC drive as a measuring
equipment and from FEM calculations.

3.1.1 Standard measurements

Laboratory tests for synchronous machines are defined in standards IEC 34-4 “Methods for
determining synchronous machine quantities from tests” and IEEE Std 115-1983 “Test
procedures for synchronous machines”. Standard measurements have been carried out to find
the test motor parameters. The test motor is an electrically excited salient pole synchronous
machine: 14.5 kVA, 400V/21A, 50 Hz/1500 rpm, excitation 10.5 A/ 55 V, cosφ = 0.8 (cap).
The measurement consists of: a DC resistance measurement, a no load test, a steady state
short circuit test, a slip test, a short circuit test for excitation winding, a sudden three-phase
short circuit test and the measurement of the load curves (V-curves). The test report of the
measurements can be found in Appendix 6. The measured machine parameters are
summarised in Table 3.1.

Table 3.1. Traditional parameters of the test machine which are measured using standard measurement
procedures. Test machine: 14.5 kVA, 400 V/21 A, 50 Hz/1500 rpm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>result [pu]</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>stator resistance $r_s$</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>excitation winding resistance $r_f$</td>
<td>0.0083</td>
<td>referred to stator side</td>
</tr>
<tr>
<td>reduction factor $k_{r_i}$</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>direct axis synchronous reactance $x_d$</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td>quadrature axis synchronous reactance $x_q$</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>direct axis transient reactance $x_{d_t}$</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>direct axis subtransient reactance $x_{d_s}$</td>
<td>0.105</td>
<td></td>
</tr>
<tr>
<td>quadrature axis subtransient reactance $x_{q_s}$</td>
<td>-*</td>
<td>* can not be measured</td>
</tr>
<tr>
<td>excitation winding time constant $\tau_{d_0}$</td>
<td>236 ms</td>
<td></td>
</tr>
<tr>
<td>direct axis transient time constant $\tau_d$</td>
<td>54 ms</td>
<td></td>
</tr>
<tr>
<td>direct axis subtransient time constant $\tau_{d_s}$</td>
<td>24 ms</td>
<td></td>
</tr>
<tr>
<td>quadrature axis subtransient time constant $\tau_{q_s}$</td>
<td>-*</td>
<td>* can not be measured</td>
</tr>
</tbody>
</table>

3.1.2 Motor design calculations

Many leading manufactures use their own design programs which combine motor design
theory and experience of many years. Calculation is based on the known AC motor theory and
the famous work of Richter (Elektrische Maschinen: Band I and II) is usually referred to
[Richter, 1963, 1967]. In addition to the known AC machine theory the experience of many
years of manufacturing and testing is used effectively in the calculations. This means that
different types of coefficients and experimental data curves based on experience are added to
the calculation algorithms. These calculated parameters do not necessarily correspond to the
same magnetic operation condition. In Table 3.2 the traditional parameters of the test motor
are listed. Parameters are given by the machine manufacturer.

Table 3.2. Traditional parameters of the test machine motor data. Test machine: 14.5 kVA, 400V/21A, 50
Hz/1500 rpm.
### Parameter motor data [pu] Note

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>stator resistance ( r_s )</td>
<td>0.048</td>
</tr>
<tr>
<td>excitation winding resistance ( r_l )</td>
<td>0.00793(^\text{pu})</td>
</tr>
<tr>
<td>reduction factor ( k_n )</td>
<td>4.63</td>
</tr>
<tr>
<td>direct axis synchronous reactance ( x_d )</td>
<td>1.196</td>
</tr>
<tr>
<td>quadrature axis synchronous reactance ( x_q )</td>
<td>0.475</td>
</tr>
<tr>
<td>direct axis transient reactance ( x_d' )</td>
<td>0.129</td>
</tr>
<tr>
<td>direct axis subtransient reactance ( x_d'' )</td>
<td>0.09</td>
</tr>
<tr>
<td>quadrature axis subtransient reactance ( x_q'' )</td>
<td>0.109</td>
</tr>
<tr>
<td>excitation winding time constant ( \tau_{do} )</td>
<td>284 ms</td>
</tr>
<tr>
<td>direct axis transient time constant ( \tau_d )</td>
<td>31 ms</td>
</tr>
<tr>
<td>direct axis subtransient time constant ( \tau_d'' )</td>
<td>6 ms</td>
</tr>
<tr>
<td>quadrature axis subtransient time constant ( \tau_q'' )</td>
<td>8 ms</td>
</tr>
</tbody>
</table>

#### 3.1.3 Measurement using a DTC motor drive

The DC stator resistance and subtransient inductances in the direct and quadrature directions have been measured. The measurement method is presented more closely in chapter 3.3. In this chapter only the results are given. The DC resistance is measured using the DC magnetisation of the machine. During the subtransient inductance measurement the motor is supplied with a short voltage pulses (duration of the pulses is less than the direct axis damper winding time constant \( \tau_d \)) and the direct axis subtransient inductance can be calculated from

\[
L_d'' = \frac{u_d dt}{di_d} = \frac{d\psi_d}{di_d} \approx \frac{\Delta \psi_d}{\Delta i_d}. \tag{3.6}
\]

Similarly, when the stator is supplied with short voltage pulses (the duration of the pulses is less than the quadrature axis damper winding time constant \( \tau_q \)) aligned with the quadrature axis of the motor the quadrature axis subtransient inductance can be calculated from

\[
L_q'' = \frac{u_q dt}{di_q} = \frac{d\psi_q}{di_q} \approx \frac{\Delta \psi_q}{\Delta i_q}. \tag{3.7}
\]

In any other position between the direct and quadrature axes the measurement can be performed on a similar way. In Fig. 3.2 the subtransient inductance is measured as a function of the rotor angle from the direct to quadrature axis. The rotor is turned manually to different positions using 10 degree steps.
The motor magnetising inductances in the direct and quadrature directions are also measured. The measurement is based on the flux linkage obtained from the voltage integral and the phase currents measured. The inductance measurement using the voltage integral has to fulfil a few conditions. The motor electrical angular velocity has to be high enough to ensure that the voltage integral is reliable. The excitation control has to be set to \( \cos \phi = 1 \) control. The inductance measurement has to be done after all transients are settled and the damper winding currents are zero. This ensures that a possible error in the damper winding parameters does not affect the magnetising inductance measurement. At first no load saturation curve of the direct axis magnetising inductance is measured. The motor is magnetised solely by the excitation winding and thus the stator direct axis current is zero during no load. Thus the effect of a possible error is eliminated in the stator leakage inductance. It is difficult to measure the quadrature axis magnetising inductance in no load conditions. In order to enable the measurement of the quadrature axis magnetising inductance the motor must be loaded. The magnetising inductances and the stator leakage inductance can be calculated from the stator flux linkage Eq. (3.8) defined in the rotor reference frame.

\[
\begin{align*}
L_{md} &= \frac{\psi_{sd} - i_d \cdot L_{q\sigma}}{i_d + i_q + i_D} \quad \text{at} \quad \{\theta_b = 0\} \\
L_{mq} &= \frac{\psi_{sq} - i_q \cdot L_{q\sigma}}{i_q + i_Q} \quad \text{at} \quad \{\theta_q = 0\} \\
L_{q\sigma} &= \frac{\psi_{sd}}{i_d} - L_{md} \quad \text{at} \quad \{\theta_b = 0, \quad i_b = 0 \text{ and } L_{ind} = L_{ind\text{load}}\}.
\end{align*}
\]

In Fig. 3.3 the measured no load saturation curves for the direct axis magnetising inductance and stator leakage inductance are shown. Also the stator leakage inductance seems to saturate about 50 %. The accuracy of the stator leakage measurement is rather poor and this result should be noted sceptically. The stator leakage inductance is measured with the excitation winding short circuited and the motor is driven as a reluctance synchronous motor at no load. The no load curve of the magnetising inductance was measured beforehand.
Inductance measurement in loaded conditions are carried out by adjusting the stator flux linkage reference so that the d-axis resultant magnetising current $i_{md} = i_d + i_t + i_D$ is set to a specific value and then by varying the torque reference which affects the q-axis resultant magnetising current $i_{mq} = i_q + i_Q$. The measurement is repeated over the operation range of $\psi_{ref} = 0.3 \ldots 1.3$ pu and $\theta_{ref} = 0 \ldots 2.5$ pu. At each set point the inductances are calculated by the control software. As a result we obtain the direct $L_{md} = f(i_{md}(t), i_{mq}(t))$ and quadrature axis magnetising inductances $L_{mq} = f(i_{md}(t), i_{mq}(t))$ as a function of $i_{md}$ and $i_{mq}$. The stator leakage inductance is assumed constant during the measurement. Figs. 3.4 and 3.5 show the measured inductance saturation surfaces and from those the effect of the magnetic cross coupling is clearly seen.

**Fig. 3.3.** Measured no load saturation curves for the direct axis magnetising inductance and the stator leakage inductance.

![Graph showing measured no load saturation curves for direct axis magnetising inductance and stator leakage inductance.](image)

**Fig. 3.4.** a) Direct axis magnetising inductance and b) quadrature axis magnetising inductance surfaces as a function of the direct axis magnetising and quadrature axis magnetising currents.
The traditional parameters of the test motor are compared in Table 3.3. The DTC drive measurement corresponds to the nominal operation point. Comparison shows that there are differences between the parameters and the time constants, especially all the transient values and the reduction factor differ very much.

Table 3.3. Comparison of the traditional parameters of the test motor. Test machine: 14.5 kVA, 400V/21A, 50 Hz/1500 rpm. *not measured, ‡ no load value, † operation point not specified.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Motor data</th>
<th>Standard measurement</th>
<th>DTC measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>stator resistance ( r_s )</td>
<td>0.048</td>
<td>0.048</td>
<td>0.051</td>
</tr>
<tr>
<td>excitation winding resistance ( r_f )</td>
<td>0.00793</td>
<td>0.0083</td>
<td>‡</td>
</tr>
<tr>
<td>reduction factor ( k_{\text{ri}} )</td>
<td>4.63</td>
<td>4</td>
<td>3.96</td>
</tr>
<tr>
<td>direct axis synchronous reactance ( x_d )</td>
<td>1.196*</td>
<td>1.19*</td>
<td>1.066</td>
</tr>
<tr>
<td>quadrature axis synchronous reactance ( x_q )</td>
<td>0.475*</td>
<td>0.56*</td>
<td>0.439</td>
</tr>
<tr>
<td>direct axis transient reactance ( x_d' )</td>
<td>0.129</td>
<td>0.33</td>
<td>‡</td>
</tr>
<tr>
<td>direct axis subtransient reactance ( x_d'' )</td>
<td>0.09</td>
<td>0.105</td>
<td>0.125</td>
</tr>
<tr>
<td>quadrature axis subtransient reactance ( x_q'')</td>
<td>0.109</td>
<td>‡</td>
<td>0.194</td>
</tr>
<tr>
<td>excitation winding time constant ( \tau_{\text{rp}} )</td>
<td>284 ms</td>
<td>236 ms</td>
<td>†</td>
</tr>
<tr>
<td>direct axis transient time constant ( \tau_d )</td>
<td>31 ms</td>
<td>54 ms</td>
<td>†</td>
</tr>
<tr>
<td>direct axis subtransient time constant ( \tau_d'' )</td>
<td>6 ms</td>
<td>24 ms</td>
<td>†</td>
</tr>
<tr>
<td>quadrature axis subtransient time constant ( \tau_q'' )</td>
<td>8 ms</td>
<td>‡</td>
<td>†</td>
</tr>
</tbody>
</table>

3.1.4 Finite element method (FEM)

The software package Magnet™ is used to evaluate the test motor measurements by finite element calculations. FEM calculations are executed in three phases. First the no load saturation curves of the direct axis and quadrature axis magnetising inductances are calculated. Finally the direct and quadrature axis magnetising inductances are calculated when the machine is operating in different loaded conditions. The cross section of the test machine is shown in Fig. 3.6. The design data of the test machine can be found in Appendix 8. The machine has a 3-phase stator winding, the number of slots \( Q = 24 \), the number of slots per phase and per pole \( q = 2 \), the stator stack length \( l = 140 \) mm, the stator inner diameter \( D = 196 \) mm and the number of turns in series per slot \( N_s = 56 \). The excitation winding is a four pole winding. The number of turns/pole \( N_t = 220 \). The direct axis magnetising inductance
$L_{md} = f(i_f) \big|_{i_d=i_q=0}$ is calculated using different values of the rotor excitation current. The quadrature axis magnetising inductance $L_{mq} = f(i_q) \big|_{i_d=i_q=0}$ is calculated using different values of the stator excitation, when the current distribution is purely in the quadrature axis. When the loaded operation points are calculated $L_{md} = f(i_d, i_f, i_q)$ and $L_{mq} = f(i_d, i_f, i_q)$, the measured stator and excitation currents are used to develop the exact stator phase current distribution corresponding to the measured data. The electrical and mechanical torque is measured and used to ensure the result of the FEM calculation.

Fig. 3.6. The cross section of the test machine. The stator winding system and the direct and quadrature axes are also shown.

**Definition of the current distribution:** Measured currents are calculated based on the quadrature axis winding system observed in the rotor oriented reference frame and they have to be transformed to the 3-phase system. A steady state operation is considered. At the instant when the quadrature phase system corresponds to the 3-phase system the direct axis of the rotating rotor reference frame is aligned with the direct axis of the stationary reference frame fixed to phase A of the stator (see Fig. 3.6). At that time

$$i_x = i_d, \quad i_y = i_q \quad \text{and} \quad I_F = \frac{i_f}{k_r}. \quad (3.9)$$

A transformation to a three-phase system in that case is

$$i_a = i_d$$

$$i_c = \frac{1}{2}(-\sqrt{3}i_q - i_x) \quad (3.10)$$

$$i_b = -(i_a - i_c)$$

When the co-ordinates, the rotor position and calculation time instant are selected as shown in Fig. 3.6 and described above, the calculated phase currents $i_a, i_b, i_c$ and excitation current $I_F$ can be used to represent the measured operating point for the FEM calculation model.
**FEM calculation:** The non-linear two dimensional field problem is solved using the magnetic vector potential $\mathbf{A}$, which is defined $\mathbf{B} = \nabla \times \mathbf{A}$. The magnetic flux density distributions in the air gap is calculated in every measured point.

In Fig. 3.7, the flux plot calculated by FEM is shown. The machine operates at nominal load and with a nominal flux.

![Flux plot](image)

**Fig. 3.7. The flux plot of the test machine at nominal load and with a nominal flux.**

**Fourier analysis of the flux density distribution:** The flux density distribution in air gap of the machine can be obtained from the result of the FEM calculation. The measured magnetising inductances are based on the space vector theory and the space vector theory assumes a sinusoidal flux distribution. In order to carry out the comparison between the measured and calculated magnetising inductances, only the fundamental harmonic of the flux density distribution must be considered. Fig. 3.8 shows the air gap flux density and the fundamental harmonic of flux density distribution for the no load nominal flux operation.

![Flux density and harmonic](image)

**Fig. 3.8. Air gap flux density and fundamental harmonic of the air gap flux density distribution. No load nominal flux operation.**
Calculation of the magnetising inductances: The amplitude $\hat{B}$ and the phase shift $\gamma$ of the fundamental harmonic can be found as a result of the Fourier analysis. The amplitude of the air gap magnetic flux can be calculated by surface integration over one pole pitch

$$\hat{\Phi} = \frac{3\tau_p}{\pi} \int_0^{\tau_p/2} \hat{B} \cos \left( \frac{x}{\tau_p} \right) dx dL' = \frac{2\hat{B}}{\pi} \tau_p L', \quad (3.11)$$

where $L' = l + 2\delta$ is the effective length of the machine, $\delta$ is the length of the air gap, $\tau_p = \frac{\pi D}{2p}$ is the pole pitch, $D$ is the inner diameter of the machine and $p$ is the number of pole pairs. The magnetising flux linkage of phase A is then

$$\hat{\psi}_{mA} = \frac{3}{2} \frac{2}{\pi} \hat{B} \tau_p L' \xi_1 N, \quad (3.12)$$

where $\xi_1$ is the winding factor of the fundamental harmonic and $N$ is the number of turns in series. Now the space vector of the magnetising flux linkage defined in the rotor oriented reference frame can be found

$$\psi_m = \frac{2}{3} \left[ \psi_{mA}(t) e^{j0} + \psi_{m_b}(t) e^{\frac{2\pi}{3}} + \psi_{m_c}(t) e^{\frac{4\pi}{3}} \right] = \hat{\psi}_{mA} e^{j\gamma}. \quad (3.13)$$

This is due to the fact that the space vector is defined to be equal to the peak value of the phase variable and the instants of calculation were defined for the moment when the direct axis of the rotating rotor reference frame is aligned with the direct axis of the stationary reference frame fixed to phase A of the stator. The magnetising flux linkage can be divided into direct and quadrature axis components as

$$\psi_{md} = \psi_m \cos \gamma$$
$$\psi_{mq} = \psi_m \sin \gamma \quad (3.14)$$

The direct and quadrature components of the stator current and excitation current were inputs into the FEM calculation and thus the magnetising inductances can be finally calculated

$$I_{md FEM} = \left. \frac{\psi_{md}}{i_d + i_q + i_D} \right|_{i_D = 0}$$
$$I_{mq FEM} = \left. \frac{\psi_{mq}}{i_d + i_Q} \right|_{i_Q = 0} \quad (3.15)$$

Comparison of FEM-calculations vs. Measurements: Fig. 3.9 a) shows a comparison between the measured and FEM calculated $L_{md}$ behaviour, when there is excitation only on the direct axis. The difference between FEM - calculation and measurement is its maximum 10 %. In Fig. 3.9 b) the quadrature axis magnetising inductance is shown. No comparison can be made, because the machine has no auxiliary winding in the quadrature direction.
The results of the FEM-calculations for the motor under load are most interesting. Fig. 3.10 a) shows the motor direct axis magnetising inductance at the nominal flux linkage and at different loads. There is a difference between the two results but the phenomenon of saturation clearly appears in both results. Fig. 3.10 b) shows the motor quadrature axis magnetising inductance at the nominal flux linkage and at different loads. There is a difference between the two results at small loads, but at high loads the results are nearly equal. The measured values are found to be in agreement with the calculated ones and the inductance measurement performed by the inverter itself is found to be reliable.
At first a non salient synchronous machine is considered. A multi phase winding produces a peak value of the resultant Magneto Motive Force (\( \hat{f}_s \) is the amplitude of the MMF) depending on the number of turns per phase \( N \), on the winding factor for fundamental harmonic \( \xi_1 \), on the pole pairs \( p \) and on the number of phases \( m \)

\[
2\hat{f}_s = m \frac{4}{\pi} \frac{\xi_1 N}{2p} \hat{i}_s.
\]

(3.16)

For a three-phase machine this follows...
\[
2 \hat{f}_s = \frac{4}{\pi} \frac{\xi_{is}}{2p} N_s \hat{i}_s = \frac{4}{\pi} \frac{\xi_{is}}{2p} \sqrt{2} I_s = \frac{6\sqrt{2}}{\pi} \xi_{is} N_{sp} I_s.
\] (3.17)

\(N_{sp}\) is the number of turns in series per pole pair in the stator winding. A single phase rotor winding is used in the excitation circuit and it is supplied with a DC supply. The peak value of the fundamental harmonic MMF is in this case
\[
2 \hat{f}_r = \frac{4}{\pi} \xi_{ir} N_{rp} I_F.
\] (3.18)

\(N_{rp}\) is the number of turns in series per pole pair in the rotor winding. When the Magneto Motive Forces in Eqs. (3.17) and (3.18) are equal, we can define the stator current, which produces the same MMF from the stator side as the excitation current from the rotor side.

\[
I_s = \frac{\xi_{ir} N_{rp} I_F}{\xi_{is} N_{sp}} \frac{\pi}{6\sqrt{2}} \frac{4}{\pi} \frac{2}{3\sqrt{2}} \xi_{ir} N_{rp} I_F.
\] (3.19)

The ratio between the stator current and the rotor excitation current is called the reduction factor and can be defined as

\[
\frac{I_s}{I_F} = k_r = \frac{\sqrt{2}}{3} \frac{\xi_{ir} N_{rp}}{\xi_{is} N_{sp}}.
\] (3.20)

Equal Magneto Motive Forces in (3.17) and (3.18) do not produce an equal flux density in the air gap, because of the unequal stator and rotor leakage fluxes. Therefore, the unequal parts of the stator MMF and rotor MMF affect the air gap.

It can be shown that the reduction factor defined for the non salient synchronous machine is equal with that for the salient pole synchronous machine. It is assumed that the pole shoe of the salient pole rotor has such a shape that it produces a sinusoidal distributed air gap flux. The excitation winding is on the direct axis and the reduction factor is needed only in the direct axis calculation.

The reduction factor is used in the current model when the measured excitation current and the rotor resistances are referred to the stator voltage level. Also the excitation current reference calculated at the stator voltage level is referred to the excitation winding voltage level for the excitation unit current control. The importance of the reduction factor is clearly seen when the interaction between the stator and the excitation circuit is considered. An erroneous reduction factor affects the current model flux estimation, which directly affects the excitation control and torque.

When the rotor resistances are referred from the rotor single phase winding system to the three-phase stator winding system or vice versa, the rotor single phase DC current may be replaced by a three-phase equivalent current which produces an equal flux density in the air gap. Power losses in the real winding and the three-phase equivalent winding must be equal
\[
P_F = R_F I_F^2 = 3 R_{Flv} I_{Flv}^2.
\] (3.21)

For the Magneto Motive Force of the single phase DC winding and the three-phase equivalent it can be written Eqs. (3.16) and (3.17)
\[
\hat{f}_{\text{ev}} = \hat{f}_r \rightarrow \frac{3}{2} \frac{4}{\pi} \frac{\xi N_{\text{np}}}{2} = \frac{4}{\pi} \frac{\xi_1 N_{\text{np}}}{2} I_F. \tag{3.22}
\]

From Eq. (3.22) we can write for the equivalent three-phase current
\[
I_{\text{ev}} = \frac{\sqrt{2}}{3} I_F. \tag{3.23}
\]

Eq. (3.21) is now rewritten
\[
P_F = R_F I_F^2 = \frac{2}{3} R_{\text{ev}} \left( \frac{\sqrt{2}}{3} I_F \right)^2 = \frac{2}{3} R_P I_F^2. \tag{3.24}
\]

The equivalent three-phase resistance can be defined as
\[
R_{\text{ev}} = \frac{3}{2} R_F. \tag{3.25}
\]

The number of phases is now equal. The equivalent three-phase resistance can be referred to the stator voltage level
\[
R_f = \frac{3}{2} R_F \left( \frac{\xi_1 N_{\text{sp}}}{\xi_{\text{st}} N_{\text{np}}} \right)^2 = \frac{1}{3 k_n^2} R_F. \tag{3.26}
\]

The equivalent three-phase rotor current rms value is defined in Eq. (3.23) \( I_{\text{ev}} = \frac{\sqrt{2}}{3} I_F \). The amplitude of the equivalent three-phase rotor current is \( \hat{i}_{\text{ev}} = \frac{2}{3} I_F \). When the space vector of the equivalent three-phase rotor current is constructed, the amplitude of the rotor current space vector \( \frac{2}{3} I_F \) is obtained. In Fig. 3.11 the rotor current space vector is constructed, when the equivalent three-phase rotor current in phase A is at its positive maximum and the currents at phases B- and C are half of their negative maximum.

![Fig. 3.11. Equivalent three-phase rotor current space vector.](image-url)
The Magneto Motive Forces produced by the rotor current space vector in the rotor voltage level and the reduced rotor current in stator voltage level must be equal

\[
\frac{2}{3} I_F \xi_{r1} N_{rp} = |i_f| \xi_{s1} N_{sp},
\]

(3.27)

where \( |i_f| \) is the reduced rotor current space vector at the stator voltage level. The ratio between the amplitudes of the current space vectors gives for the reduction factor

\[
\frac{|i_f|}{I_F} = \frac{2}{3} \frac{\xi_{r1} N_{rp}}{\xi_{s1} N_{sp}} = k_r,
\]

(3.28)

The difference between the DC reduction factor and the space vector reduction factor is \( \sqrt{2} \).

We can write for the resistances

\[
R_i = \frac{2}{3} \frac{1}{k_r^2} R_F.
\]

(3.29)

In Eq. (3.30) the reduction calculation for the space vector presentation is summarised, using the current model excitation winding resistance and the measured DC excitation current.

\[
i_f = k_r I_F = \sqrt{2} \cdot k_n I_F
\]

\[
R_i = \frac{2}{3} \frac{1}{k_r^2} R_F = \frac{1}{3} \frac{1}{k_n^2} R_F
\]

(3.30)

The excitation current of the synchronous machine is measured, so no parameters of the excitation winding are needed. Instead of parameters, the reduction factor \( k_r \) must be known, because the measured excitation current must be referred to the stator. The reduction factor can be initialised based on the plate values. In Fig. 3.12 a synchronous motor, which is running at its nominal operating point with nominal excitation current \( i_{fnom} \) and nominal power factor \( \cos(\phi_{nom}) \), is represented as a space vector diagram in the rotor reference frame.

---

**Fig. 3.12.** Synchronous motor vector diagram. The motor is running at its nominal operating point.
The stator quadrature axis flux linkage can be defined

\[\psi_{sq} = \psi_s \cdot \sin(\delta_s) = i_{\text{nom}} \cdot \cos(\delta_s + \phi_{\text{nom}}) \cdot L_q .\] (3.31)

Using Eq. (3.31) it is possible to iterate the load angle \(\delta_s\), when we know the nominal stator flux linkage, the stator current, the power factor and the quadrature axis magnetising inductance. Those values can be obtained from the rating plate and motor data. The iteration can be realised by using for instance the Newton-Rapson method

\[\delta_{s(n+1)} = \delta_{s(n)} - \frac{f(\delta_s)}{f'(\delta_s)} .\] (3.32)

The first guess for the load angle \(\delta_{s0}\) is the value corresponding to \(\cos \phi = 1\).

\[\delta_{s0} = \arctan\left(\frac{i_{\text{nom}} \cdot L_q}{\psi_s}\right) .\] (3.33)

The stator direct axis flux linkage can be defined as

\[\psi_{sd} = \psi_s \cdot \cos(\delta_s) = -i_{\text{nom}} \cdot \sin(\delta_s + \phi_{\text{nom}}) \cdot L_d + i_l \cdot L_{md} .\] (3.34)

where \(i_l\) is the excitation current referred to the stator. From Eq. (3.34) we get

\[i_l = \frac{\psi_s \cdot \cos(\delta_s) + i_{\text{nom}} \cdot \sin(\delta_s + \phi_{\text{nom}}) \cdot L_d}{L_{md}} .\] (3.35)

Finally the nominal excitation current \(I_F\) from the rating plate is known and the reduction factor can be written

\[k_i = \frac{i_l}{I_F} .\] (3.36)

### 3.3 IDENTIFICATION RUN

The traditional synchronous motor parameters and rated values which are always available can be used to initialise the motor model parameters. These parameters can be adjusted using the measurement capability of the DTC drive to get a more accurate motor model. The capability of the motor control to determine the machine parameters by itself is an important feature that ensures that the parameter set is defined under the same magnetic conditions, so no disagreement between the parameters will occur. In this chapter a measurement procedure for the machine model parameter identification is proposed. Parameter identification is an automatic sequence of measurements and it is executed with the machine at standstill and at no load running. Because of the automatic operation it is called identification (ID) run. The ID-run can be performed in a factory test bench or during a drive commissioning. The ID-run measures the following motor parameters, which are listed in Table 3.4.

Table 3.4. Parameters measured by the identification run.
In the following the basic principles for the measurements are introduced.

### 3.3.1 Stator resistance measurement

The stator resistance can be estimated by a method introduced by Luukko et al. [1997]. The motor is supplied at standstill with a DC-current. If $i_s$ is constant and $i_t$ is zero $\psi_s/dt = 0$.

The estimated stator flux linkage is defined as

$$\psi_{\text{est}} = \frac{1}{t_1} \left( \int (u_s - R_{\text{est}} i_s) dt \right) = -\int (u_s - R_{\text{est}} i_s) dt,$$

where the estimated stator resistance $R_{\text{est}}$ is defined as a sum of the actual resistance $R_s$ and an error term $\Delta R_s$:

$$R_{\text{est}} = R_s + \Delta R_s. \quad (3.38)$$

If the estimated stator voltage is assumed to be equal to the actual stator voltage and the measured current equal to the actual current, the difference between the estimated and the actual flux linkage is

$$\psi_{\text{est}} - \psi_s = -\int_{t_1}^{t_2} (u_s - R_{\text{est}} i_s) dt - \int_{t_0}^{t_1} (u_s - R i_s) dt = -\int_{t_0}^{t_2} \Delta R_s i_s dt = \Delta R_s i_s (t_0 - t_1). \quad (3.39)$$

From this equation the error term $\Delta R_s$ can be calculated. As the actual flux linkage $\psi_s$ is not known, two estimates, $\psi_{\text{est}1}$ and $\psi_{\text{est}2}$ are needed. Then

$$\psi_{\text{est}2} - \psi_s = \Delta R_s i_s (t_0 - t_2),$$

$$\psi_{\text{est}1} - \psi_s = \Delta R_s i_s (t_0 - t_1). \quad (3.40)$$

Subtracting the latter equation from the former, an estimate to the error term $\Delta R_s$ is obtained

$$\Delta R_s = \frac{\psi_{\text{est}2} - \psi_{\text{est}1}}{i_s (t_2 - t_1)}. \quad (3.41)$$

Now, the actual resistance is

$$R_s = R_{\text{est}} - \Delta R_s. \quad (3.42)$$
The error term $\Delta R_s$ can effectively be minimised using an iterative method. Starting with $R_{\text{sest1}} = 0$ gives a rough error estimate $\Delta R_{\text{sest1}}$ (Eq. (3.41)) in a few milliseconds. Keeping the DC-current and setting $R_{\text{sest2}} = R_{\text{sest1}} + \Delta R_{\text{sest1}}$ and calculating $\Delta R_s$ again adjusts the error term to a new and better value. Iteration can be stopped, when $\Delta R_s < \epsilon$, where $\epsilon$ is the desired uncertainty of the stator resistance.

### 3.3.2 Subtransient inductance measurement

The subtransient inductance measurement is based on the method for induction machines introduced by Vas [1993], and the same measurement is applicable to synchronous motors.

The direct-axis voltage equation of an electrically excited synchronous machine at standstill is

$$u_d = R_i i_d + L_{sd} \frac{di_d}{dt} + L_{md} \frac{d(i_D + i_l)}{dt}.$$  \hspace{1cm} (3.43)

The equivalent magnetising current of the rotor is defined as

$$i_{md} = \frac{\psi_D}{L_{md}} = i_d + \frac{L_{D}}{L_{md}}(i_D + i_l),$$

$$= i_d + (1 + \sigma_D)(i_D + i_l) \hspace{1cm} (3.44)$$

where $L_D = L_{md} + \frac{L_{D\sigma}}{L_{D\sigma} + L_{\sigma}}$ and rotor leakage factor $\sigma_D = \frac{1}{L_{md}} \frac{L_{D\sigma} - L_{\sigma}}{L_{D\sigma} + L_{\sigma}}$.

Substituting $(i_D+i_l)$ from Eq. (3.44) to Eq. (3.43) gives

$$u_d = R_i i_d + \left(L_{sd} - \frac{L_{md}}{1 + \sigma_D} \right) \frac{di_d}{dt} + \frac{L_{md}}{1 + \sigma_D} \frac{di_{md}}{dt}.$$  \hspace{1cm} (3.45)

The total leakage factor is defined as $\sigma = 1 - \frac{L_{nd}^2}{L_{sd} L_D}$. Now the voltage equation is

$$u_d = R_i i_d + \sigma L_{sd} \frac{di_d}{dt} + (1 - \sigma) L_{sd} \frac{di_{md}}{dt}.$$  \hspace{1cm} (3.46)

By defining the direct-axis subtransient inductance $L_{d}^{''}$ as

$$L_{d}^{''} = L_{sd} - \frac{L_{md}^2}{L_D}$$  \hspace{1cm} (3.47)

the voltage equation becomes

$$u_d = R_i i_d + L_{d}^{''} \frac{di_d}{dt} + (1 - \sigma) L_{sd} \frac{di_{md}}{dt}.$$  \hspace{1cm} (3.48)
$\frac{di_{md}}{dt}$ in the voltage Eq. (3.48) can be defined using the voltage equation of the direct axis damper winding

\[ r_{p0}i'_{d} + \frac{d\psi_{d}}{dt} = r_{p0}i'_{d} + \frac{d\psi_{d}}{dt} = 0. \]  
(3.49)

Substituting $i_{md}$ defined in Eq. (3.44) into Eq. (3.48) the voltage equation of the direct axis damper winding can be rewritten ($i_{q} = 0$ during measurement) as

\[ \tau_{d} \frac{di_{md}}{dt} = i_{d} - i_{md}. \]  
(3.50)

Using the voltage equation of the damper winding based on the time constant Eq. (3.50) can be expressed as

\[ u_{d} = R_{d}i_{d} + L_{d} \frac{di_{d}}{dt} + (1 - \sigma) L_{sd} \frac{i_{d} - i_{md}}{\tau_{d}}. \]  
(3.51)

It can be seen from (3.51) that, if the motor is supplied with a short voltage pulse (duration of the pulses is less than the direct axis damper winding time constant $\tau_{d}$), the subtransient inductance $L_{d}'$ is the predominant inductance. Thus the direct axis subtransient inductance can be calculated from

\[ L_{d}' = \frac{u_{d} dt}{di_{d}} = \frac{d\psi_{sd}}{di_{d}} = \frac{\Delta \psi_{sd}}{\Delta i_{d}}. \]  
(3.52)

Similarly, the quadrature axis subtransient inductance $L_{q}'$ is defined as

\[ L_{q}' = L_{sq} = \frac{L_{nd}^{2}}{L_{Q}}, \]  
(3.53)

which is the predominant inductance in the quadrature axis, if the stator is supplied with short voltage pulses (duration of the pulses is less than the direct axis damper winding time constant $\tau_{q}$), aligned with the quadrature axis of the motor. Here $L_{Q} = L_{mq} + L_{Qe}$. The quadrature axis subtransient inductance can be calculated

\[ L_{q}' = \frac{u_{q} dt}{di_{q}} = \frac{d\psi_{sq}}{di_{q}} = \frac{\Delta \psi_{sq}}{\Delta i_{q}}. \]  
(3.54)

Before the measurement of the direct axis subtransient inductance the rotor has to be turned to the direct position with respect to the stator co-ordinate fixed to the phase A of the stator winding. Turning is achieved by using DC magnetisation. The stator and the rotor are DC magnetised and the rotor will turn to the direct position. The measurement can be performed using a direct axis voltage pulse. The excitation winding is short circuited during measurement. The stator current and the stator flux linkage are triggered two times during the current rise. The possible orientation of the magnetic flux at the beginning of the measurement has to be considered. The triggering of the current and flux linkage is done from the middle of the current rise. Fig 3.13 explains the measurement.
In Fig. 3.14 the measuring connection is shown. The measurement is done using a direct axis voltage pulse after the rotor is turned to the direct position.

Before the quadrature axis subtransient inductance can be measured the rotor has to be turned to the quadrature position with respect to the stator co-ordinate fixed to phase A of the stator winding. The turning is implemented by using a DC magnetisation. The direction of the stator DC magnetisation is changed to the quadrature position. The stator and rotor are DC magnetised and the rotor will turn to the quadrature position. The measurement can be performed by a direct axis voltage pulse. The stator current and stator flux linkage are triggered two times during the current rise. In Fig. 3.15 the measurement connection is shown. The measurement is done using a direct axis voltage pulse after the rotor is turned to the quadrature position.

**3.3.3 Reduction factor measurement**

In principle the reduction factor $k_t$ can be determined easily using a measurement, in which an excitation current step is performed and the excitation current and stator current are measured...
before and after the step. The air gap flux linkage should be constant during the measurement.

The measurement is rather difficult, because a part of the MMF (produced either by the rotor or by the stator excitation current) produces a leakage flux, another part of the MMF affects the iron and the rest of the MMF affects the air gap. In order to obtain the current information which actually affects the air gap the no load curves and stator and rotor leakage inductances $L_{n\sigma}, L_{r\sigma}$ must be known. In Fig 3.16 the defined no load curves using the stator and rotor excitation are illustrated. The air gap line must be defined and from these the reduction factor can be calculated.

![Fig. 3.16. No load curves using stator and rotor excitation. The shared part A of the rotor excitation current affects the air gap and the shared part B affects the iron. The shared part A' of the stator excitation current affects the air gap and shared part B' affects the iron.](image)

In practice it does not cause any remarkable error when the reduction factor is measured by an excitation current step and the stator and excitation currents are measured in both operating points. Fig. 3.17 clarifies the execution of the measurement, while the motor is running at no load.

![Fig. 3.17. Measurement of the reduction factor using an excitation current step.](image)

The reduction factor is the ratio of the stator current difference to the rotor current difference, when the air gap flux linkage is kept constant.
The motor magnetising inductances in the direct and quadrature directions are measured. The idea of the measurement is presented before in section 3.1.3. The measurement is based on the flux linkage obtained from the voltage integral and the measured phase currents. The inductance measurement using the voltage integral has to fulfil a few conditions. The motor electrical angular velocity must be high enough to ensure that the voltage integral is reliable. The excitation control must be set to \( \cos(\phi) = 1 \) control. The no load saturation curve of the direct axis magnetising inductance is measured. The motor was magnetised solely by the excitation winding and thus the stator direct axis current is zero during no load. In this method we eliminate the effect of a possible error in the stator leakage inductance. The stator leakage inductance is measured when the excitation current is controlled to zero and the motor is driven as a reluctance synchronous motor with no load for a short period. The quadrature axis magnetising inductance can not be measured in no load conditions. In order to measure the quadrature axis magnetising inductance the motor is accelerated with different torque references using the motor inertia as load. At each measuring point inductances are calculated by the control software. As a result we obtain the direct \( L_{md} = f(i_{md}) \) and quadrature axis magnetising inductance data \( L_{mq} = f(i_{mq}) \) as a function of \( i_{md} \) and \( i_{mq} \). The data are stored and used later on for inductance surface calculations (explained in section 3.4).

### 3.3.5 Measurement procedures

In Fig. 3.18 the principle of the ID-run is demonstrated. The ID-run consists of a DC-magnetisation part and of a six stage no-load run. ID-run measurement procedures are explained in Table 3.5.
### Table 3.5. ID-run measurement procedures

<table>
<thead>
<tr>
<th>Stage</th>
<th>Measurement</th>
<th>time duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC magnetisation</td>
<td>stator resistance, subtransient inductances</td>
<td>15 - 30 s</td>
</tr>
<tr>
<td>Acceleration</td>
<td>First point of the quadrature axis magnetising inductance</td>
<td>&lt; 2 s</td>
</tr>
<tr>
<td>1</td>
<td>reduction factor</td>
<td>20 s</td>
</tr>
<tr>
<td>2</td>
<td>stator leakage inductance</td>
<td>10 s</td>
</tr>
<tr>
<td>3</td>
<td>second point of the quadrature axis magnetising inductance</td>
<td>&lt; 10 s</td>
</tr>
<tr>
<td>4</td>
<td>saturation curve for the direct axis magnetising inductance</td>
<td>30-60 s</td>
</tr>
<tr>
<td>5</td>
<td>third point of the quadrature axis magnetising inductance</td>
<td>&gt; 2 s</td>
</tr>
<tr>
<td>6</td>
<td>fourth point of the quadrature axis magnetising inductance</td>
<td>&gt; 5 s</td>
</tr>
</tbody>
</table>

After the sixth stage the motor is decelerated until zero speed is achieved. The motor parameters of the current model are calculated again. Now more accurate results are achieved. The identification run is completed.

### 3.4 GENERATION OF THE MAGNETISING INDUCTANCE MODELS

As the industrial salient pole synchronous machine drive is concerned we are searching for a saturation model for the air gap flux linkage, which has to fulfil the following requirements:

1) Simple and fast to calculate. 2) A “reasonably” accurate. 3) The saturation model should be implemented so, that it can be added to the linear current model. 4) The saturation of the air gap flux linkage is modelled according to the open circuit saturation curves. The linear model is valid in steady state when the values of the inductances are updated according to the operating point of the machine. In transients the validation of the model is depend of the damper winding currents estimates. 5) The saturation model has to be implemented so that it takes the cross saturation into consideration. The problem of saturation modelling is to find the non-linear functions defined by Eqs. (2.61)-(2.62), where the machine currents are used as the indicators of the saturation level.

\[
L_{\text{nd}} = f(i_{\text{nd}}(t), i_{\text{mq}}(t))
\]

\[
L_{\text{mq}} = f(i_{\text{nd}}(t), i_{\text{mq}}(t)).
\]

It is proposed in Eqs. (2.69) and (2.70) that by using this so called “saturant magnetising current” as an indicator of the saturation level, models for saturation of the magnetising inductances can be derived

\[
L_{\text{nd}} = f(i_{\text{nd}}^\text{sat}) = f\left(\sqrt{i_{\text{nd}}^2 + ki_{\text{mq}}^2}\right),
\]

\[
L_{\text{mq}} = f(i_{\text{mq}}^\text{sat}) = f\left(\sqrt{\frac{1}{k}i_{\text{nd}}^2 + i_{\text{mq}}^2}\right).
\]

Factor \( k \) is a saturation dependent saliency ratio and it can be defined as

\[
k = \frac{L_{\text{mq}}}{L_{\text{nd}}} = K \cdot f(i_{\text{nd}}, i_{\text{mq}}),
\]
where $K = \frac{L_{mq}}{L_{md}}$ is the unsaturated saliency ratio and $\Gamma$ is a characteristic function of the saliency ratio. The saturation functions in the Eq. (2.69) and (2.70) can be derived from open circuit curves. The characteristic function $\Gamma$ is defined so that it can dictate the variation of the saliency ratio when the machine is saturated.

In non-salient pole machines the direct and quadrature axis magnetising inductances can be considered equal $L_{md}(i_{md}) = L_{mq}(i_{mq})$. This means that the saturation of the magnetic circuit is defined by the magnitude of the resultant MMF and saturation is identical regardless of the direction of the resultant MMF. The quadrature axis component of the resultant MMF saturates the direct axis magnetising inductance as much as the direct axis component saturates quadrature axis magnetising inductance (reciprocal cross saturation). So it can be stated that saturation of the direct and quadrature axis magnetising inductances can be modelled using an open circuit saturation curve. An open circuit saturation curve of our test machine assuming non-salient rotor construction is shown in Fig 3.19.

\[L_{md} = L_{mq}\]

\[\text{[pu]}\]

\[i_m \text{ [pu]}\]

**Fig. 3.19.** Open circuit saturation curve of the fictitious non-salient synchronous test machine.

If the open circuit saturation curve is known, then the inductance surface can be generated using the resultant magnetising current $|i_m| = \sqrt{i_{md}^2 + i_{mq}^2}$ as an indicator of the saturation level. This model takes cross saturation into account. The magnetising inductance surfaces generated from the open circuit saturation curve are represented as a function of direct and quadrature axis currents $L_{md} = L_{mq} = f(i_{md}, i_{mq})$ in Fig. 3.20.
In a salient pole machine, the MMF and magnetising flux are no longer aligned. The relationship between their amplitudes depends on their respective angular position. On the other hand the angle between the MMF and the flux depends on the position of the resultant MMF. If the resultant MMF is aligned with the direct or quadrature axis then the flux is aligned with the MMF, otherwise the MMF and the flux are apart from each other. This means that the saturation of the magnetic circuit is defined by the magnitude and position of the resultant MMF i.e. it depends on the space vector of the resultant MMF.

The magnetising inductance saturation surface of the non salient pole synchronous machine is reconsidered. The inductance surface is generated by calculating the values of the magnetising inductance from the open circuit saturation curve \( L_{md} = f(i_{md},0) \) as a function of the resultant magnetising current \( \left| i_m \right| = \sqrt{i_{md}^2 + i_{mq}^2} \). The inductance surface is a paraboloid in which the open circuit saturation curve explicitly defines the shape of the saturation surface. On the other hand the inductance surface may be realised by projecting the open circuit saturation curve to a direction of the resultant magnetising current \( i_m \). This can be carried out by rotating a \((i_{md},i_{mq})\)-plane by an angle \( \alpha \). Fig. 3.21 clarifies the situation.

**Fig. 3.20.** Magnetising inductance surface of a non salient pole synchronous machine.

**Fig. 3.21.** Projection of the open circuit saturation curve to the direction of the resultant magnetising current.
If \( L_d > L_q \) as it happens to be the case in an electrical excited salient pole synchronous machine then the effect of the direct axis MMF on the ability of the quadrature axis MMF flux production is stronger than the effect of the quadrature axis MMF on the direct axis MMF flux production (non reciprocal cross saturation). As the inductance surface is concerned, it means that the level of the inductance surface \( L_{md} = f(i_{md}, i_{mq}) \) of the salient pole machine rises while the resultant magnetising current \( i_m \) turns from the direct axis to the quadrature axis. Fig. 3.22 shows the difference between the direct axis magnetising inductance surfaces of a salient and a non salient pole machine.

![Fig. 3.22. The difference between the direct axis magnetising inductance surfaces of the salient and non salient pole machine. The white plane describes the properties of a cylindrical rotor machine.](image)

The behaviour of the salient pole saturation surface corresponds to the resultant magnetising current \( |i_{sat}| = \sqrt{i_{md}^2 + \frac{L_q}{L_d}i_{mq}^2} = \sqrt{i_{md}^2 + ki_{mq}^2} \), which in the case of a salient pole synchronous machine saturates the magnetic circuit (see par. 2.3.3). The coefficient \( k \) depends on the resultant magnetising current vector \( i_m \). The inductance surface is a paraboloid, where the open circuit saturation curve defines the shape of the surface. The rise of the surface level, when compared to the non salient pole inductance surface, is defined by the ratio of saliency \( L_{mq}(i_m)/L_{md}(i_m) \).

### 3.4.1 Direct axis magnetising inductance model

The direct axis magnetising inductance surface can be generated by projecting the open circuit saturation curve to the direction of the resultant magnetising current \( i_m \) and by adding a “saliency-offset” to the value which is obtained from the projection. Fig. 3.23 shows the idea of the saliency-offset.
The saturation estimation model of the non salient pole machine can be used also for a salient pole machine, but in addition a saliency-offset term must be added to the value obtained from the non salient pole model. It is obvious that the accuracy of the model depends on the determination of the saliency-offset. The offset is not constant. It depends on the resultant magnetising current vector $i_m$ i.e. the amplitude and the angle of the vector. The saliency-offset $L_{sal \_off}$ can be defined as a function of the resultant magnetising current vector

$$L_{sal \_off} = k_d \left( L_{md}^u - L_{md} \right), \quad (3.56)$$

where $\left( L_{md}^u - L_{md}(i_m) \right)$ takes the shape of the open circuit saturation curve and the amplitude of the resultant magnetising current vector into account and coefficient $k_d = f \left( \angle i_m \right)$ takes the angle of the resultant magnetising current vector into account. $L_{md}(i_m)$ is the direct axis magnetising inductance calculated from the open circuit saturation curve using the resultant magnetising current $|i_m| = \sqrt{i_{md}^2 + i_{mq}^2}$ as an indicator of the saturation level. $L_{md}^u$ is the unsaturated value of the direct axis magnetising inductance. Factor $k_d$ is the saturation dependent saliency ratio and it can be defined as

$$k_d = \sqrt{\frac{L_{mq}^u}{L_{mq}^m}} = \sqrt{K \cdot \Gamma \left( \angle i_m, i_{mq} \right)}, \quad (3.57)$$

where $K = \frac{L_{mq}^u}{L_{mq}^m}$ is the unsaturated saliency ratio and $\Gamma$ is a characteristic function of the saliency ratio. The function of factor $k_d$ can be considered to be the same as the function of the factor $k$ in Eq. (2.69) i.e. it takes the non reciprocal cross saturation into account. The characteristic function of saliency $\Gamma$ describes the behaviour of the saliency ratio when the machine is saturated. The quadrature axis magnetising inductance saturates when the motor is loaded. It means that the saliency ratio decreases when the resultant magnetising current turns from the direct to the quadrature axis i.e. the quadrature axis current component arises during the loaded operation. It follows that the influence of the quadrature axis MMF to the direct
axis ability to produce the flux decreases. The function $\Gamma = \left( \frac{\alpha}{\pi/2} \right)^2$ can be used for the characteristic function of saliency $\Gamma$, where $\alpha$ is the angle of the resultant magnetising current and $\alpha \in \left[ 0 \ldots \frac{\pi}{2} \right]$. We get for the direct axis magnetising inductance

$$L_{mdsal} = L_{md}(i_m) + k_d \left( \angle i_m \right) \left( L^u_{md} - L_{md}(i_m) \right).$$

(3.58)

The accuracy of the proposed direct axis magnetising inductance saturation model is examined next. The inductance surface is generated using Eq. (3.58) and using the measured open circuit saturation curve which has been shown in Fig. 3.19. The generated inductance surface model is compared to the measured one. In Fig. 3.24 the measured direct axis magnetising inductance surface is represented again as a function of the direct and quadrature axis magnetising currents.

![Fig. 3.24. Measured direct axis magnetising inductance surface of the test synchronous machine.](image)

In Fig. 3.25 the generated direct axis magnetising inductance surface model is shown. The inductance surface is generated by calculating the resultant magnetising current $|i_m| = \sqrt{i_{md}^2 + i_{mq}^2}$ for the different d/q-magnetising current values and by using linear interpolation to calculate the so called “non salient inductance” from the open circuit saturation curve. After that the “saliency offset” is added to “the non salient inductance” value. The saliency ratio factor $k_d$ is calculated using the non saturated saliency ratio and the characteristic function $\Gamma$ which has been mentioned above.
In Fig. 3.25, the error surface between the measured and modelled inductance surface is shown. The error is given in percents

\[
\Delta L_{md} = \frac{L_{md\,\text{meas}} - L_{md\,\text{mod}}}{L_{md\,\text{meas}}} \cdot 100\%.
\]  

(3.59)

It follows from the definition of the error calculation that the current model flux linkage calculation fails according to

\[
\begin{align*}
\Delta L_{md} < 0 & \Rightarrow \text{saturation underestimation} \Rightarrow \text{actual flux} < \text{estimated flux} \\
\Delta L_{md} > 0 & \Rightarrow \text{saturation overestimation} \Rightarrow \text{actual flux} > \text{estimated flux}
\end{align*}
\]
The same error surface is projected to the $i_{\text{ind}}$-$L_{\text{ind}}$ plane in order to define the extreme values exactly in Fig. 3.27.

![Fig. 3.27. Error surface projection to the $i_{\text{ind}}$-$L_{\text{ind}}$ plane.](image)

It can be concluded from the error surface that the generated model underestimates the saturation when the motor operates deep in the field weakening range ($i_{\text{ind}} < 0.7$). In the nominal flux area ($0.95 < i_{\text{ind}} < 1.2$) and over flux area the model overestimates the saturation. However, the accuracy of the model can be considered satisfactory. The maximum error remains under 5%.

### 3.4.2 Quadrature axis magnetising inductance model

The modelling of the quadrature axis magnetising inductance saturation surface is a much more complicated problem than the modelling of the direct axis inductance surface. The starting point for the modelling - the open circuit saturation curve when there is excitation only in the quadrature direction (ref. Fig. 3.19) - is practically difficult to measure. In the next chapter the measured quadrature axis inductance surface is considered using the load test and the applicability of the modelling method to the direct axis magnetising inductance. In Fig. 3.28 the measured quadrature axis magnetising inductance surface is shown.

![Fig. 3.28. Measured quadrature axis magnetising inductance surface. The saturation effect occurs most strongly when the resultant magnetising current is in the middle of these two axes (45°).](image)
It can be seen that the direct axis magnetising current $i_{md}$ cross saturates the quadrature axis magnetising inductance. When observing the surface in the direction of the resultant magnetising current $i_{m} = i_{md} + j i_{mq}$ it is noticed, that the saturation effect of the direct axis magnetising current increases when the resultant magnetising current moves away from the quadrature axis. The strongest saturation effect is obtained when the resultant magnetising current is in the middle of these two axes and it weakens when the resultant magnetising current moves toward the direct axis. Also it can be noticed that the level curves are hyperbolas with respect to the origin. In the case of the direct axis magnetising inductance saturation they are ellipses. The non salient model and the saliency offset for the quadrature axis can be written

$$L_{mqal} = L_{mq} (i_{m}) + k_{q} \left( \angle i_{m} \right) \cdot \left( L_{u} - L_{mq} (i_{m}) \right),$$  \hspace{1cm} (3.60)$$

where $k_{q}$ is

$$k_{q} = \sqrt{\frac{L_{md}}{L_{mq}}} = \frac{1}{K} \Gamma \left( \angle (i_{md}, i_{mq}) \right)$$

is the saturated saliency ratio and $\Gamma$ is a characteristic function of the saliency ratio. The function of factor $k_{q}$ can be considered the same as that of factor $k$ in the Eq. (2.70) i.e. it takes the non reciprocal cross saturation into consideration. The characteristic function of saliency $\Gamma$ describes the behaviour of the saliency ratio when the machine is saturated. The function $\Gamma = \left( 1 - \frac{\alpha}{\pi/2} \right)^2$ can be used for the characteristic function of saliency $\Gamma$, where $\alpha$ is the angle of the resultant magnetising current and $\alpha \in [0..\pi/2]$. This model assumes that the open circuit saturation curve is known and the level curves are elliptical. The saturation surface of the quadrature axis magnetising inductance can not be modelled using this method. The saturation function $L_{mq} = f(i_{md}, i_{mq})$ for the quadrature axis magnetising inductance has to represent the inductance level curves as hyperbolas and this leads to a far too complicated problem. A much more simple model can be used if the level curves are presented as straight lines [Väänänen, 1997].

The saturation function of the quadrature axis magnetising inductance for the $i_{md}, i_{mq}$ - plane is based on the fact, that the function $L_{mq} = f(i_{md}, i_{mq})$ is constant for a certain $i_{md}, i_{mq}$ -plane trace i.e. level curves. We can write for the level curve

$$f(i_{md}, i_{mq}) = \text{constant},$$  \hspace{1cm} (3.61)$$

which with the different values of the constant will generate a set of curves. It is essential that, using Eq. (3.61) the shape of the level curves can be represented with sufficient accuracy.

$$L_{mq} = f(i_{md}, i_{mq}) = g(f(i_{md}, i_{mq})) = g(x)$$  \hspace{1cm} (3.62)$$

The original two-variable saturation function $f(i_{md}, i_{mq})$ is transformed to a single-variable function $g$ with the argument $x = f(i_{md}, i_{mq})$. The saturation function $g(x)$ can be defined by measurement using different $i_{md}$-$i_{mq}$ values. The function $f(i_{md}, i_{mq})$ represents the level curves with arbitrary $i_{md}$-$i_{mq}$ pairs and in the case of a straight line the level curve it is defined as

$$f(i_{md}, i_{mq}) = i_{md} + i_{mq},$$  \hspace{1cm} (3.63)$$

Fig. 3.29 clarifies the saturation curve and the surface model generation.
We get for the quadrature axis magnetising inductance

\[ L_{mq_{sat}} = g(i_{md} + i_{mq}) \]  
(3.64)

The accuracy of the proposed quadrature axis magnetising inductance saturation model is examined next. The inductance surface is generated using Eq. (3.64). The generated inductance surface model is compared to the measured one. In Fig. 3.30 the measured quadrature axis magnetising inductance surface is presented again as a function of the direct and quadrature axis magnetising currents.
Fig. 3.31. Generated quadrature axis magnetising inductance surface.

In Fig. 3.32. the error surface between the measured and modelled inductance surfaces is shown.

Fig. 3.32. Error surface between the measured and modelled inductance surfaces.

In Fig. 3.33 the same error surface is projected to the $i_{mq}-L_{md}$ plane in order to define the extreme values exactly.
Although the accuracy of the model is quite poor, it behaves in a similar way as the actual saturation surface. The total estimation error of the stator flux linkage using these saturation models is a combination of the direct and quadrature axis models. The direct axis flux linkage is dominant when the motor operates in a constant flux range below the base speed, because the load angle does not exceed very high values (usually load angle less than 40°). In the field weakening range the load angle is very often high even at low loads and the quadrature axis flux linkage becomes dominant. On the other hand the voltage model is dominant and accurate in the field weakening range, which reduces the importance of the saturation models. The total estimation error can be defined when the estimated inductances $L_{\text{mdest}}$, $L_{\text{mqest}}$ are defined as a sum of the actual values $L_{\text{md}}$, $L_{\text{mq}}$ and the error terms $\Delta L_{\text{md}}$, $\Delta L_{\text{mq}}$

$$L_{\text{mdest}} = L_{\text{md}} + \Delta L_{\text{md}}$$
$$L_{\text{mqest}} = L_{\text{mq}} + \Delta L_{\text{mq}}.$$  

(3.65)

We can write for the stator flux linkage estimation error

$$\Delta \psi_{sd} = L_{sd} i_d + L_{sd} (i_d + i_t + i_D) - L_{mdest}(i_d + i_t + i_D) = \Delta L_{md} i_d,$$
$$\Delta \psi_{sq} = L_{sq} i_q + L_{mqest} (i_q + i_Q) - L_{sq} (i_q + i_Q) = \Delta L_{mq} i_q,$$

(3.66)

$$|\Delta \psi_s| = \sqrt{(\Delta L_{md} i_d)^2 + (\Delta L_{mq} i_q)^2}.$$

The most important feature of the saturation models is that using Eqs. (3.58) and (3.64) the saturation of the direct and quadrature axis magnetising inductances can be represented with adequate accuracy which ensures a sufficient accuracy of the current model. The saturation models are constructed in a way that they will behave as real inductances and non physical operation will not exist. The motor model combines the current model and the voltage model incorporating the good properties of both and the valid stator flux linkage estimate will be obtained in any operation point. The performance of the motor model proposed is verified in the next chapter. Experimental results are given.
4 EXPERIMENTAL RESULTS

4.1 A DTC CONTROLLED TEST DRIVE

The laboratory test drive consists of a 14.5 kVA salient pole synchronous motor, a torque sensor, a DC-machine with a DCS500 drive and a DSP-controlled stator and rotor power electronic units, Fig. 4.1. The power electronic units include the necessary software to work as a digital oscilloscope and thus it is possible to make current measurements. All variables are pu values and the used pu system can be found in Appendix 7. The stator winding of the synchronous motor is supplied by an industrial DTC inverter unit and the excitation winding is supplied by a four-quadrant DC-chopper with a fast current hysteresis control. However, there is a delay in the communication between the DTC stator inverter and the excitation unit.

The belt transmission is applied in the test drive, because experiment results are needed in the field weakening area. Also it is easy to change the gearing ratio when using the belt transmission. The performance measurements are carried out with a test application, the loading of which is a DC machine operated either speed or torque controlled. The PI control of the speed control has to be adjusted sluggish because of the above mentioned belt transmission. Therefore, the speed control of the loading DC machine cannot keep the rotational speed of the test application constant in all tests. An excitation control of a DTC controlled synchronous motor introduced by Pyrhönen, O. [1998] is used.

![Diagram of the test drive](image_url)

*Fig. 4.1. Arrangement of the test drive. The laboratory test drive consists of a 14.5 kVA salient pole synchronous motor, a torque sensor, a 55 kW DC-machine with a DCS500 drive and a DSP-controlled stator and rotor power electronic units. Both drives have a pulse encoder. The belt transmission has been applied the test drive, because there is a need of experiment results in the field weakening area.*
Table 4.1 shows the nominal values of the electrically excited salient pole synchronous motor and the DC-machine used in the test drive.

### Table 4.1. Nominal values of the test motor and the loading DC-machine as given by the supplier.

<table>
<thead>
<tr>
<th>Electrically excited salient pole synchronous motor</th>
<th>DC-machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power $P_n$</td>
<td>$P_n$</td>
</tr>
<tr>
<td>14.5 kVA</td>
<td>55 kW</td>
</tr>
<tr>
<td>Voltage $U_{sn}$</td>
<td>$U_{sn}$</td>
</tr>
<tr>
<td>400 V</td>
<td>600 V</td>
</tr>
<tr>
<td>Current $I_n$</td>
<td>$I_n$</td>
</tr>
<tr>
<td>21 A</td>
<td>100 A</td>
</tr>
<tr>
<td>Excitation current $I_{ex}$</td>
<td>$U_{ex}$</td>
</tr>
<tr>
<td>10.5 A</td>
<td>180-110 V</td>
</tr>
<tr>
<td>Frequency $f_n$</td>
<td>$f_n$</td>
</tr>
<tr>
<td>50 Hz</td>
<td>3.1-2.1 A</td>
</tr>
<tr>
<td>Speed $n_n$</td>
<td>$n_n$</td>
</tr>
<tr>
<td>1500 rpm</td>
<td>1500 rpm</td>
</tr>
<tr>
<td>Power factor $\cos \phi$</td>
<td>0.8 cap.</td>
</tr>
</tbody>
</table>

The data of the torque and speed measuring system are given in tables 4.2 and 4.3. The torque-speed transducer and the measurement unit are supplied by Vibro-meter SA. The torque transducer is inductive and it is mounted on a measuring shaft. The conditioner unit (ICT 610) supplies the torque transducer with a voltage at 8 kHz carrier frequency. The signal transmission is contactless, i.e. transmitted by rotary transformers which are connected to the signal conditioner unit by a shielded 4 pole cable. The output signal of the torque transducer is first amplified, demodulated and filtered and then brought out to the display unit (PDG 762). The sampling frequency is 1 kHz. The output signal (5 V at full scale display) is also available on the BNC output. The speed measuring module (PDC 753) is basically a frequency/analogue (F/A) converter. It converts speed proportional signals from optical transducers into analogue values. The frequency divider is an 8-bit frequency divider.

### Table 4.2. Data of the torque and speed transducer.

<table>
<thead>
<tr>
<th>Torque transducer with photo electric speed pick-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>TG-20/BP and ML 103</td>
</tr>
<tr>
<td>Torque $T_s/T_{max}$</td>
</tr>
<tr>
<td>200/400 Nm</td>
</tr>
<tr>
<td>Sensitivity</td>
</tr>
<tr>
<td>0.3495 mV/Nm (at 10 V excitation)</td>
</tr>
<tr>
<td>Accuracy class of rated torque</td>
</tr>
<tr>
<td>0.5 %</td>
</tr>
<tr>
<td>Linearity of rated torque</td>
</tr>
<tr>
<td>0.25 - 0.5 %</td>
</tr>
<tr>
<td>Speed $n_{max}$</td>
</tr>
<tr>
<td>9000 rpm</td>
</tr>
<tr>
<td>Pole wheel</td>
</tr>
<tr>
<td>60 teeth</td>
</tr>
</tbody>
</table>

### Table 4.3. Data of the torque and speed measuring unit.

<table>
<thead>
<tr>
<th>Measurement unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque signal conditioner type</td>
</tr>
<tr>
<td>ICT 610</td>
</tr>
<tr>
<td>Carrier frequency</td>
</tr>
<tr>
<td>8 kHz</td>
</tr>
<tr>
<td>Frequency range</td>
</tr>
<tr>
<td>0 to 1600 Hz slope 18 dB/oct.</td>
</tr>
<tr>
<td>Accuracy class</td>
</tr>
<tr>
<td>0.1 %</td>
</tr>
<tr>
<td>Response time</td>
</tr>
<tr>
<td>appr. 300 µs 10 to 90 % of f. s. d.</td>
</tr>
<tr>
<td>Torque display unit</td>
</tr>
<tr>
<td>PDG 762</td>
</tr>
<tr>
<td>Digital counter and speed display unit</td>
</tr>
<tr>
<td>PDC 753</td>
</tr>
<tr>
<td>Minimum measurable speed</td>
</tr>
<tr>
<td>5 rpm</td>
</tr>
</tbody>
</table>
4.2 STATIC TORQUE ACCURACY

The drive static torque accuracy is measured. It defines the accuracy of the motor model because an error in the stator flux linkage estimate is directly proportional to the error of the motor electrical torque. The motor model has been introduced before. The combined current/voltage model’s inductance parameters, saturation models, stator resistance and reduction factor are measured by the inverter itself using the identification run (Fig. 3.18 and Table 3.5). The shaft torque is measured and the actual electrical torque is calculated taking friction losses into account. Fig. 4.2 shows a comparison between a motor model with constant inductance parameters and a motor model with saturated inductances. It can be seen how remarkable the error difference is between those two models. This proves that without some kind of saturation model it is not possible to achieve an acceptable motor drive torque controller.

![Graph showing static torque error as a function of load torque.]

Fig. 4.2. Static torque error as a function of the load torque $T_{\text{load}}/T_{\text{nom}}$. $T_{\text{nom}}$ is the nominal torque of the test motor.

The static torque error measurement results are illustrated in Fig. 4.3 and 4.4. The error is represented as a function of the motor speed. We can get an idea of the accuracy of the combined model when the weighting of the current model and the voltage model change while the motor speed is increased. At zero speed and at very low speeds the current model is the dominating one. When the motor speed is higher than 0.5 pu, the voltage model is practically the only model and the task of the current model is to act as a stabilizer of the voltage model. Torque estimate pu errors are defined by comparing torque error to the nominal torque or torque reference. Load tests are performed over a speed range of 0 to 1 pu using loads 0.6 to 2.5 pu. Two different definitions for the torque error are used, because various institutes define the torque error in the different way.
It is very complicated to generate saturated inductance models so that good static accuracy is achieved also at very high loads. Motor model parameters measured by the supply unit behave well at motor torques less than 1.6 pu. The torque estimation error is less than 10 % at a speed range from 0 to 1 pu when compared to the motor nominal torque or less than 5 % when compared to the torque reference. At motor torques larger than 1.6 pu the accuracy of the motor model is poor at low speeds but improves when the voltage model becomes more reliable. The voltage model behaves well at higher speeds at any load. At low speeds when the current model is dominating the motor magnetic path saturates more than the inductance models estimate. The estimated stator flux linkage is larger than the actual one and the estimated motor electrical torque becomes larger than the actual. The torque error achieves its largest values in the middle of the speed range. This is the most difficult part of the motor model operation because the stator flux estimate is a combination of the two separate models and at that point neither the current model nor the voltage model is dominant. Both models are erroneous and the errors are summed. It should be remember that less than 13 % torque error at 2 pu torque in the whole speed range is a very good result. Traditional current controlled
drives typically can have 30\% static torque error and even more due to an erroneous inductance model. In practice, the drive, despite of this, is a high performance drive because the speed controller compensates the error by changing the torque reference. If the results achieved are compared with the demands of table 1.2 it is noticed that the 1\% (of $T_{\text{max}}$) static torque accuracy is a difficult challenge for the manufacturer. In practice this can be achieved by building the motor inductance models using laboratory tests where the actual torque of the machine is measured and where the inductance models are updated respectively. The model proposed here is easy to generate and it is applicable as a general inductance model to speed controlled drives. It is, however, not accurate enough for torque controlled drives where a very high torque must be reached with 1\% accuracy.

4.3 DYNAMIC PERFORMANCE OF THE MOTOR MODEL

The DTC system provides the fastest possible dynamical torque steps. The use of a transverse voltage vector gives a very fast change in the torque. A large torque step can be achieved in a few milliseconds. This procedure is automated using the so called optimal switching logic table. The so called optimal switching table is no more optimal if the stator flux linkage space vector estimate is erroneous. In the worst case the control can become unstable and dephase. The dynamic performance of the test drive which at the same time means the dynamic test of the motor model is done. The dynamic performance test consists of the following parts: 1. Start with nominal torque to no load, 2. dynamic torque response, 3. start to nominal load, 4. flystart to nominal load, 5. zero speed operation, 6. reverse of the operating mode i.e. from motoring to generating with nominal load and 7. load impact test. In Fig. 4.5 the trace of the measured stator current space vector and the stator flux linkage estimate space vector at the start with the nominal torque are represented. No oscillation or drifting occurs. The stator flux linkage estimate is initialised properly because no oscillations due to additional magnetising currents from the stator side occur.

![Figure 4.5](image_url)
Fig. 4.6 shows the torque response of the same start as above with nominal torque. The estimated torque is calculated using the measured stator current and calculated stator flux linkage. The torque response time is about 2 ms for the test drive.

![Torque Response](image)

**Fig. 4.6.** The estimated ($\tau_e$) torque for the prototype drive at the start with the nominal torque (the estimated electrical torque is calculated using the measured stator currents and the calculated stator flux linkage).

A ramped motor start is shown in Fig. 4.7. The motor torque is nominal after acceleration.

![Motor Start](image)

**Motor start**

($n_{ref} = 0.3$ pu, $T_{load} = T_{100\%}$)

**Fig. 4.7.** Ramped motor start. The speed control, $n_{ref} = 0.3$ pu. $i_x$ and $i_y$ are stator current components in the static stator reference frame.
In Fig. 4.8 a flystart is shown. The motor torque is nominal after acceleration. A successful flystart to the nominal load torque without any oscillations may be considered as a very difficult transient in which the motor flux linkage calculation must be well initialised and estimated during the start and acceleration. After acceleration the static state is achieved and the operation is very stable.

![Motor flystart graph](image)

**Motor flystart**

\(n_{\text{start}} = 0.05 \text{ pu, } n_{\text{ref}} = 0.4 \text{ pu, } T_{\text{load}} = T_{100\%}\)

Fig. 4.8. Motor flystart. The load torque is nominal. Speed control. The flystart is performed from speed 0.05 pu and the motor is accelerated to speed 0.4 pu.

Fig. 4.9 shows zero speed operation at 0.8 pu torque.

![Zero speed operation graph](image)

**Zero speed operation**

Fig. 4.9. Zero speed operation.
In Figs. 4.10 and 4.11 the motor operation mode is reversed from motoring to generating i.e. the direction of the rotation is reversed while the motor electrical torque is kept constant. Tests with two different speeds are performed. The first one from 0.1 to -0.1 pu and the second one from 0.01 pu -0.01 pu. Unfortunately, the control of the DC machine fails around zero speed and the load decreases a little while the direction of the rotation is reversed.

Fig. 4.10. Motor operation mode reversed from motoring to generating, \( n_{\text{ref}} = 0.1...0.1 \) pu.

Fig. 4.11. Motor operation mode reversed from motoring to generating, \( n_{\text{ref}} = 0.01...0.01 \) pu.
Load impact tests are performed. Figs. 4.12 and 4.13 shows torque steps with a very good response. New operating points are achieved without any oscillations. This can be seen especially from Fig. 4.13 were the direct and quadrature axis currents are represented.

**Torque step**

\[ n_{rot} = 0.5 \text{ pu} \]

**time [s]**

Fig. 4.12. Load impact test. The stator current components are given in the static stator reference frame.

**Torque step**

\[ n_{rot} = 0.5 \text{ pu} \]

**time [s]**

Fig. 4.13. Load impact test. The stator current components are given in the rotating rotor reference frame.

Finally a torque step from the positive nominal torque to the negative nominal torque is performed.
4.4 SUMMARY OF THE MEASUREMENTS

The performance of the test drive and the motor model proposed is verified in static and dynamic operation conditions. According to the test results it is possible to obtain sufficient static accuracy of the DTC’s torque controller for an electrically excited synchronous motor. The better the parameter estimation is the better is the static accuracy and stability of the drive at steady state. By controlling the torque of the motor via the stator flux linkage the fastest torque response will always be achieved. The dynamic response is fast and a new operation point is achieved without oscillation. A stable operation is achieved through the whole speed range.

A combination of the voltage model and current model seems to be the best solution for a reasonably accurate motor model. In DTC the voltage model operates as a main model and is calculated at a very fast sampling rate. The stator flux linkage calculated from the voltage model is corrected using the stator flux linkage from the current model. The current model acts only as a supervisor that at longer time levels prevents the motor stator flux linkage from drifting out of the centre. At very slow speed ranges the role of the current model is emphasised. At higher speeds the function of the current model correction is to act as a stabiliser of the control system.

The modelling of the main inductance saturation is essential and cross saturation has to be considered. The effect of cross saturation is very significant. The DTC inverter can be used as a measuring equipment and the parameters needed for the motor model can be defined by the inverter itself. The main advantage is that the parameters defined are measured in the similar magnetic operating conditions and no disagreement between the parameters will exist. The inductance models generated are adequate to meet the requirements of dynamically demanding drives.
5 CONCLUSIONS

The performance of such a high quality torque control as DTC in dynamically demanding industrial applications is mainly based on the accurate estimate of the various flux linkages’ space vectors. Nowadays industrial motor control systems are real time applications with restricted calculation capacity and at the same time the control system has a demand for a simple, fast calculable and reasonably accurate motor model. A motor model for the Direct Torque Controlled (DTC) salient pole synchronous motor drive is proposed. The drive meets the requirements.

The flux linkage calculation by a current model and the voltage model and their advantages and disadvantages have been studied. Special attention is paid to the stator flux linkage, because it plays a very important role in the stator side control of a salient pole synchronous motor DTC-drive. The main advantages of the voltage model are: minimal sensitivity to inductance parameter variations, easy to implement, no rotor position needed, can be calculated at a high sampling rate and good dynamic accuracy. The disadvantages are: erroneous at low speeds and a voltage loss overestimation can cause unstable drifting. The main advantages of the current model are: no speed dependence, stable calculation method. The disadvantages are: inductance parameter sensitivity, dynamic accuracy poor due damper winding current estimation, accurate rotor position needed.

In this work a motor model combining the voltage model and current model is introduced. The stator flux linkage calculated via integration from the stator voltage is corrected using the stator flux linkage from the current model. The current model acts only as a supervisor that at longer time levels prevents the motor stator flux linkage from drifting out of the centre. At very slow speeds the role of the current model is emphasised but it newer becomes the main model. At higher speeds the function of the current model correction is to act as a stabiliser of the control system.

The goal was to develop a combined model where good features of both models can be utilised. This requirement can be fulfilled if weighted correction is used. The weighting coefficient determines the importance of the current model in a certain operation point. The accuracy of the voltage integration improves when the speed of the drive increases, but at very low speeds the current model must be used. The weighting coefficient is changed as a function of speed. A possible error in the voltage loss calculation increases when the load of the machine is increased. In the field weakening range the stator current which is needed to produce a certain torque is higher due to the reduced flux linkage and this also increases the possible voltage loss estimation error. According to these facts a base curve for the weighting coefficient can be defined.

The current model contains a set of inductance parameters which are to be known. The validation of the current model in steady state is not self-evident. It depends on the accuracy of the saturated value of the inductances. The parameter measurement of the motor model where the supply inverter is used as a measurement signal generator is presented. This so called identification run can be performed in the factory or during drive commissioning. Inductance measurements during ID-run have been verified using FEM-calculations. The measured values are found to be in agreement with the calculated ones and the inductance measurement performed by the inverter itself is found to be accurate enough to meet the requirements of the Direct Torque Controlled drive. The generation of inductance models
used for the representation of the saturation effect is proposed. The inductance models take cross saturation into account. The modelling of the main inductance saturation is essential and cross saturation has to be considered. The effect of cross saturation is very significant. The DTC inverter can be used as a measuring equipment and parameters needed for the motor model can be defined by the inverter itself. The main advantage is that the parameters defined are measured in similar magnetic operation conditions and no disagreement between the parameters will exist. The inductance models generated are adequate to meet the requirements of a dynamically demanding drive.

Experimental results prove the performance of the electrically excited synchronous motor supplied with a DTC inverter. It is shown that it is possible to obtain good static accuracy of the DTC’s torque controller for an electrically excited synchronous motor. The dynamic response is fast and a new operation point is achieved without oscillation. A stable operation is achieved through the whole speed range.

In the near future the author will concentrate in his work on the practical implementation of full scale prototype using the proposed motor model algorithms and there will be a lot of challenges to conquer during that project. As was said in the beginning of this thesis “The song remains the same” after this contribution, too. A considerable number of research topics have come up during the work of our team.
REFERENCES


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APPENDICES

Appendix 1

Derivation of the damper winding current estimators (linear equations)

1 Direct axis damper winding current estimator

The direct axis air gap flux linkage can be evaluated according to the two-axis theory

\[ \psi_{md} = i_{md}L_{md} = L_{md}(i_f + i_d + i_D) . \]  

(1)

The damper winding is short circuited, therefore the direct component of the damper winding voltage equation is

\[ u_D = R_Di_D + \frac{d\psi_D}{dt} = R_Di_D + \frac{d}{dt}(\psi_{md} + i_DL_{D\sigma}) = 0 . \]  

(2)

The time constant of the direct axis damper winding is defined as

\[ \tau_D = \frac{L_{md} + L_{D\sigma}}{R_D} . \]  

(3)

When Eq. (1) is substituted into the voltage Eq. (2)

\[ 0 = R_Di_D + \frac{d}{dt}(L_{md}(i_f + i_d + i_D) + i_DL_{D\sigma}) \]  

\[ \Rightarrow \]  

(4)

\[ 0 = i_D + \frac{d}{dt} \left( \frac{L_{md}}{R_D}(i_f + i_d + i_D) + i_DL_{D\sigma} \right) \]  

\[ \Rightarrow \]  

(5)

\[ 0 = i_D + \frac{d}{dt} \left( \frac{L_{md}}{R_D}(i_f + i_d) + i_DL_{D\sigma} + \frac{L_{md}}{R_D} \right) \]  

\[ \Rightarrow \]  

(6)

\[ \left[ 1 + \frac{d}{dt} \frac{L_{D\sigma} + L_{md}}{R_D} \right] i_D = -\frac{d}{dt} \frac{L_{md}}{R_D} (i_f + i_d) \]  

\[ \Rightarrow \]  

(7)

\[ \left[ 1 + \frac{d}{dt} \frac{L_{D\sigma} + L_{md}}{R_D} \right] i_D = -\frac{L_{md}}{L_{D\sigma} + L_{md}} \frac{d}{dt} \frac{L_{D\sigma} + L_{md}}{R_D} (i_f + i_d) \]  

(8)

According to Eq. (8) we get a discrete representation for the calculation algorithm of the direct axis damper winding current if the equation is integrated over one sampling interval \((n-1)\tau_s, n\tau_s\), and the damper current is approximated to change linearly within the sampling period

\[ i_{D(n)} = \frac{\tau_D}{\tau_s + \tau_D} \left[ i_{D(n-1)} - \frac{L_{md}}{L_{md} + L_{D\sigma}} (i_{f(n)} + i_{d(n)} - i_{f(n-1)} - i_{d(n-1)}) \right] \]  

(9)

\(\tau_s\) is the sampling interval, \(\tau_D\) is the time constant of the direct axis damper winding, \(L_{md}\) is the direct axis magnetising inductance and \(L_{D\sigma}\) is the direct axis damper winding's leakage inductance.
2 Quadrature axis damper winding current estimator

The quadrature axis air gap flux linkage can be evaluated according to the two-axis theory

\[ \psi_{mq} = i_{mq} L_{mq} = L_{mq} (i_q + i_Q). \]  

(10)

The damper winding is short circuited, therefore the quadrature component of the damper winding voltage equation is

\[ u_Q = R_Q i_Q + \frac{d}{dt} \psi_Q = R_Q i_Q + \frac{d}{dt} (\psi_{mq} + i_Q L_{Q0}) = 0. \]  

(11)

The time constant of the quadrature axis damper winding is defined as

\[ \tau_Q = \frac{L_{mq} + L_{Q0}}{R_Q}. \]  

(12)

When Eq. (10) is substituted into the voltage Eq. (11)

\[ 0 = R_D i_D + \frac{d}{dt} (L_{md} (i_t + i_d + i_D) + i_D L_{D\sigma}) \]  

\[ \Rightarrow \]  

(13)

\[ 0 = i_Q + \frac{d}{dt} \left( \frac{L_{mq}}{R_Q} (i_q + i_Q) + i_Q \frac{L_{Q0}}{R_Q} \right) \]  

\[ \Rightarrow \]  

(14)

\[ 0 = i_Q + \frac{d}{dt} \left( \frac{L_{mq}}{R_Q} i_q + i_Q \frac{L_{Q0} + L_{mq}}{R_Q} \right) \]  

\[ \Rightarrow \]  

(15)

\[ \left[ 1 + \frac{d}{dt} \frac{L_{Q0} + L_{mq}}{R_Q} \right] i_Q = - \frac{d}{dt} \frac{L_{mq}}{R_Q} i_q \]  

\[ \Rightarrow \]  

(16)

\[ \left[ 1 + \frac{d}{dt} \frac{L_{Q0} + L_{mq}}{R_Q} \right] i_Q = - \frac{L_{mq}}{L_{Q0} + L_{mq}} \frac{d}{dt} \frac{L_{Q0} + L_{mq}}{R_Q} i_q \]  

(17)

According to Eq. (17) we get a discrete representation for the calculation algorithm of the quadrature axis damper windings current if the equation is integrated over one sampling interval \((n-1)\tau_s, n\tau_s\), and the damper current is approximated to change linearly within the sampling period

\[ i_{Q(n)} = \frac{\tau_Q}{\tau_s + \tau_Q} \left[ i_{Q(n-1)} - \frac{L_{mq}}{L_{mq} + L_{Q0}} (i_Q(n) - i_Q(n-1)) \right] \]  

(18)

\( \tau_s \) is the sampling interval, \( \tau_Q \) is the time constant of the quadrature axis damper winding, \( L_{mq} \) is the quadrature axis magnetizing inductance and \( L_{Q0} \) is the quadrature axis damper winding’s leakage inductance.
Appendix 2

Derivation of the magnetising flux linkage time derivatives (non linear equations)

The direct axis air gap flux linkage can be defined using the direct axis resultant magnetising current

\[
\psi_{md} = L_{md} \cdot i_{md} = L_{md} \cdot (i_d + i_q + i_D) \quad (1)
\]

\[
\psi_{mq} = L_{mq} \cdot i_{mq} = L_{mq} \cdot (i_q + i_Q) \quad (2)
\]

The saturation level can be determined by the amplitude of the magnetising current space vector

\[
|i_m| = \sqrt{i_{md}^2 + i_{mq}^2}. \quad (3)
\]

The chord slope inductances depend on \( |i_m| \) and can be defined as

\[
\psi_{md} = L_{md} \cdot \left| i_m \right| \cdot i_{md} \quad (3)
\]

\[
\psi_{mq} = L_{mq} \cdot \left| i_m \right| \cdot i_{mq} \quad (4)
\]

Let us first consider the direct axis. The direct axis air gap flux linkage time derivative can be written as

\[
\frac{d\psi_{md}}{dt} = \frac{d(L_{md}i_{md})}{dt} = L_{md} \frac{di_{md}}{dt} + i_{md} \frac{dL_{md}}{dt} \quad (5)
\]

Eq. (5) consists besides of the direct axis magnetising current time derivative also of the direct axis magnetising inductance time derivative. For the latter we can write using the chain rule of the differential calculation

\[
\frac{dL_{md}}{dt} = \frac{dL_{md}}{d[i_m]} \cdot \frac{di_m}{dt} = \frac{\frac{d\psi_{md}}{dt}}{\left| i_m \right|} \cdot \frac{di_m}{dt} = \frac{i_m \left| \frac{d\psi_{md}}{dt} \right| - \psi_{md} \left| \frac{di_m}{dt} \right|}{\left| i_m \right|} \cdot \frac{di_m}{dt} \quad (6)
\]

In Eq. (6) \( L_{md}^d = \frac{d\psi_{md}}{d[i_m]} \) is the dynamic tangent slope inductance and \( L_{md}^s = \left| \psi_{md} \right| \left| i_m \right| \) is the static chord slope inductance.

The equation of the direct axis magnetising inductance time derivative contains also the time derivative of the magnetising current space vector. The magnetising current space vector includes both the direct and quadrature axis magnetising currents, thus there will be a term which contains the cross-coupling of these two axes.

\[
\frac{di_m}{dt} = \frac{\frac{1}{2} \left( i_{md}^2 + i_{mq}^2 \right)^{\frac{1}{2}}}{\frac{1}{2} \left( i_{md}^2 + i_{mq}^2 \right)^{\frac{1}{2}}} \cdot \left( 2i_{md} \frac{di_{md}}{dt} + 2i_{mq} \frac{di_{mq}}{dt} \right) = \left( \frac{i_m}{i_m} \right) \frac{di_{md}}{dt} + \left( \frac{i_m}{i_m} \right) \frac{di_{mq}}{dt} = \cos \mu \frac{di_{md}}{dt} + \sin \mu \frac{di_{mq}}{dt}
\]

\( \mu \) is the angle of the magnetising current space vector with in correlation to the direct axis. The magnetising currents in both axes can be defined as
\[ i_{md} = |i_m| \cos \mu \]
\[ i_{mq} = |i_m| \sin \mu \]  \hspace{1cm} \text{(8)}

When Eq. (6), (7) and (8) are substituted to Eq. (5) we get

\[
\frac{d\psi_{md}}{dt} = \frac{d(L_{md} i_{md})}{dt} = L_{md} \frac{di_{md}}{dt} + i_{md} \frac{dL_{md}}{dt} = L_{md} \frac{di_{md}}{dt} + i_{md} \frac{L_{md}^d - L_{md}}{|i_m|^2} \frac{dj_{m}}{dt} \\
= L_{md} \frac{di_{md}}{dt} + i_{md} \frac{L_{md}^d - L_{md}}{|i_m|^2} \left( \cos \mu \frac{di_{md}}{dt} + \sin \mu \frac{di_{mq}}{dt} \right) \\
= \left( L_{md}^d \cos^2 \mu + \sin^2 \mu L_{md} \right) \frac{di_{md}}{dt} + \sin \mu \cos \mu \frac{L_{md}^d - L_{md}}{|i_m|^2} \frac{dj_{m}}{dt} \\
= \left( L_{md}^d \cos^2 \mu + \sin^2 \mu L_{md} \right) \frac{di_{md}}{dt} + \sin \mu \cos \mu \frac{L_{md}^d - L_{md}}{|i_m|^2} \frac{dj_{m}}{dt} \\
\]

\[
\frac{d\psi_{mq}}{dt} = \frac{d(L_{mq} i_{mq})}{dt} = L_{mq} \frac{di_{mq}}{dt} + i_{mq} \frac{dL_{mq}}{dt} = L_{mq} \frac{di_{mq}}{dt} + i_{mq} \frac{L_{mq}^d - L_{mq}}{|i_m|^2} \frac{dj_{m}}{dt} \\
= L_{mq} \frac{di_{mq}}{dt} + i_{mq} \frac{L_{mq}^d - L_{mq}}{|i_m|^2} \left( \cos \mu \frac{di_{mq}}{dt} + \sin \mu \frac{di_{md}}{dt} \right) \\
= \left( L_{mq}^d \cos^2 \mu + \sin^2 \mu L_{mq} \right) \frac{di_{mq}}{dt} + \sin \mu \cos \mu \frac{L_{mq}^d - L_{mq}}{|i_m|^2} \frac{dj_{m}}{dt} \\
= \left( L_{mq}^d \cos^2 \mu + \sin^2 \mu L_{mq} \right) \frac{di_{mq}}{dt} + \sin \mu \cos \mu \frac{L_{mq}^d - L_{mq}}{|i_m|^2} \frac{dj_{m}}{dt} \\
\]

The time derivative of the quadrature axis air gap flux linkage can be derived in the same way, and we get

\[
\frac{d\psi_{mq}}{dt} = L_{Mq} \frac{di_{mq}}{dt} + L_{qdl} \frac{di_{md}}{dt}, \text{ where} \hspace{1cm} \text{(10)}
\]

\[ L_{Mq} = L_{mq}^d \sin^2 \mu + L_{mq}^d \cos^2 \mu \]

\[ L_{qdl} = (L_{mq}^d - L_{mq}) \sin \mu \cos \mu \]
Appendix 3

Derivation of the damper winding current estimators (non linear equations)

1 Direct axis damper winding current estimator

The direct axis air gap flux linkage time derivative can be evaluated as

$$\frac{d\psi_{md}}{dt} = L_{Md} \frac{di_{md}}{dt} + L_{dq} \frac{di_{mq}}{dt}.$$  \hspace{1cm} (1)

The damper winding is short circuited, therefore the direct component of the damper winding voltage equation is

$$u_D = R_D i_D + \frac{d\psi_D}{dt} = R_D i_D + \frac{d}{dt}(\psi_{md} + i_D L_{Do}) = 0.$$ \hspace{1cm} (2)

The time constant of the direct axis damper winding is defined as

$$\tau_D = \frac{L_{Md} + L_{Da}}{R_D}.$$ \hspace{1cm} (3)

When Eq. (1) is substituted into the voltage Eq. (2)

$$0 = R_D i_D + \frac{d}{dt}\left(L_{Md}(i_t + i_d + i_D) + L_{dq}(i_q + i_Q) + i_D L_{Do}\right) \Rightarrow \hspace{1cm} (4)$$

$$0 = i_D + \frac{d}{dt}\left(\frac{L_{Md}}{R_D}(i_t + i_d) + \frac{L_{dq}}{R_D}(i_q + i_Q) + \frac{L_{Do}}{R_D} i_D\right) \Rightarrow \hspace{1cm} (5)$$

$$0 = i_D + \frac{d}{dt}\left(\frac{L_{Md}}{R_D}(i_t + i_d) + \frac{L_{Da} + L_{Md}}{R_D} i_D + \frac{L_{dq}}{R_D}(i_q + i_Q)\right) \Rightarrow \hspace{1cm} (6)$$

$$\left[1 + \frac{d}{dt}\frac{L_{Da} + L_{Md}}{R_D}\right] i_D = -\frac{L_{Md}}{L_{Do} + L_{Md}} \frac{d}{dt}\left(\frac{L_{Do} + L_{Md}}{R_D} (i_t + i_d)\right) - \frac{L_{dq}}{L_{Do} + L_{Md}} \frac{d}{dt}\left(\frac{L_{Da} + L_{Md}}{R_D} (i_q + i_Q)\right) \Rightarrow \hspace{1cm} (7)$$

According to Eq. (8) we get a discrete representation for the calculation algorithm of the direct axis damper windings current if the equation is integrated over one sampling interval \((n-1)\tau_s, n\tau_s\), and the damper current is approximated to change linearly within the sampling period

$$i_{D(n)} = \frac{\tau_D}{\tau_s + \tau_D} \left[ i_{D(n-1)} - \frac{L_{Md}}{L_{Md} + L_{Do}} (i_{t(n)} + i_{d(n)} - i_{t(n-1)} - i_{d(n-1)}) - \frac{L_{dq}}{L_{Md} + L_{Do}} (i_{q(n)} + i_{Q(n)} - i_{q(n-1)} - i_{Q(n-1)}) \right]\hspace{1cm} (8)$$

\(\tau_s\) is the sampling interval. The inductances are defined in Appendix 2.
2 Quadrature axis damper winding current estimator

The quadrature axis air gap flux linkage time derivative can be evaluated to be

$$\frac{d\psi_{m_q}}{dt} = L_{M_q} \frac{di_{m_q}}{dt} + L_{qd} \frac{di_{md}}{dt}. \quad (10)$$

The damper winding is short circuited, therefore the quadrature component of the damper winding voltage equation is

$$u_Q = R_Q i_Q + \frac{d\psi_Q}{dt} = R_Q i_Q + \frac{d}{dt} (\psi_{m_q} + i_Q L_{Qd}) = 0. \quad (11)$$

The time constant of the direct axis damper winding is defined as

$$\tau_Q = \frac{L_{M_q} + L_{Qd}}{R_Q} \quad (12)$$

When Eq. (10) is substituted into the voltage Eq. (11)

$$0 = R_Q i_Q + \frac{d}{dt} \left( L_{M_q} (i_q + i_Q) + L_{qd} (i_d + i_D) + i_Q L_{Qd} \right) \Rightarrow \quad (13)$$

$$0 = i_Q + \frac{d}{dt} \left( \frac{L_{M_q}}{R_Q} i_q + \frac{L_{Qd}}{R_Q} i_D + i_Q \frac{L_{M_q}}{R_Q} \right) \Rightarrow \quad (14)$$

$$0 = i_Q + \frac{d}{dt} \left( \frac{L_{M_q}}{R_Q} i_q + \frac{L_{Qd} + L_{M_q}}{R_Q} + \frac{L_{qd}}{R_Q} (i_d + i_D) \right) \Rightarrow \quad (15)$$

$$\left[ 1 + \frac{d}{dt} \frac{L_{Qd} + L_{M_q}}{R_Q} \right] i_Q = -\frac{d}{dt} \left( \frac{L_{M_q}}{R_Q} i_q + \frac{L_{qd}}{R_Q} (i_d + i_D) \right) \Rightarrow \quad (16)$$

According to Eq. (17) we get a discrete representation for the calculation algorithm of the quadrature axis damper winding current if the equation is integrated over one sampling interval ($\tau_s$, $n \tau_s$), and the damper current is approximated to change linearly within the sampling period

$$i_{Q(n)} = \frac{\tau_Q}{\tau_s + \tau_Q} \left[ i_{Q(n-1)} - \frac{L_{M_q}}{L_{M_q} + L_{Qd}} (i_{q(n)} - i_{q(n-1)}) - \frac{L_{qd}}{L_{M_q} + L_{Qd}} \left( i_{d(n)} + i_{D(n)} - i_{d(n-1)} - i_{D(n-1)} \right) \right]$$

$\tau_s$ is the sampling interval. The inductances are defined in Appendix 2.
Appendix 4 [Burzanowska 1990]

The schematic block diagram and the flowchart of the PC-based C-language DTC-simulator.

The simulations of the DTC-control and the motor models are coded in a C-language simulation program which has been first developed for asynchronous motors. The necessary modifications are made to change the motor model from an asynchronous motor to a synchronous motor and the special features of the synchronous motor has been added (for example excitation circuit and control). Fig. 1. shows the block diagram of the simulation program and Fig. 2 the flowchart of the program. During the simulation the differential equations of the motor and the processes are solved numerically using the trapezoidal integration. The method is numerically stable and gives accurate results when short time steps are used. The time step for the numerical integration is 5 $\mu$s.

Fig. 1. Variable-speed DTC inverter-motor drive simulator.
Fig. 2. Flowchart of the variable-speed DTC inverter-motor drive simulator.
Appendix 5

Motor model simulation algorithm.

Inputs of the synchronous motor simulator are the stator voltage space vector \( u_s \), the excitation voltage \( u_f \) and the mechanical angular velocity \( \omega_{\text{mech}} \) and outputs are the stator current space vector \( i_s \), the excitation current \( i_f \), the currents of the damper winding \( i_D \) and \( i_Q \) and the electrical torque of the motor \( t_e \). As parameters the simulator uses all the inductances and resistances of the salient pole synchronous machine equivalent circuit.

The simulation algorithm of the salient pole synchronous motor is the following:

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Initialisation of the flux linkage and current variables. First calculate the inductance inverse matrix ( L_{\text{stat}}^{-1} ).</td>
</tr>
<tr>
<td>1</td>
<td>Calculation of the stator flux linkage derivatives using supplied stator voltage ( u_s ). The calculation is performed in the reference frame fixed to the stator.</td>
</tr>
</tbody>
</table>
|      | \[
|      | \frac{d\psi_s^{\text{re}}}{dt} = \omega_b \cdot (u_s^{\text{re}} - i_s^{\text{re}} \cdot r_s) 
|      | \frac{d\psi_s^{\text{im}}}{dt} = \omega_b \cdot (u_s^{\text{im}} - i_s^{\text{im}} \cdot r_s) 
|      | \] |
| 2    | Calculation of the damper winding and the excitation winding flux linkage derivatives in the reference frame fixed to rotor. |
|      | \[
|      | \frac{d\psi_f}{dt} = \omega_b \cdot (u_f - i_f \cdot r_f) 
|      | \frac{d\psi_D}{dt} = -\omega_b \cdot i_D \cdot r_D 
|      | \frac{d\psi_Q}{dt} = -\omega_b \cdot i_Q \cdot r_Q 
|      | \] |
| 3    | Numerical integration of the flux linkages using calculated derivatives in phase 2. |
|      | \[
|      | \psi_{s,k+1}^{\text{re}} = \psi_{s,k}^{\text{re}} + \frac{1}{2} \left[ \frac{d\psi_{s,k-1}^{\text{re}}}{dt} + \frac{d\psi_{s,k}^{\text{re}}}{dt} \right] \cdot T 
|      | \psi_{s,k+1}^{\text{im}} = \psi_{s,k}^{\text{im}} + \frac{1}{2} \left[ \frac{d\psi_{s,k-1}^{\text{im}}}{dt} + \frac{d\psi_{s,k}^{\text{im}}}{dt} \right] \cdot T 
|      | \psi_{D,k+1} = \psi_{D,k} + \frac{1}{2} \left[ \frac{d\psi_{D,k-1}}{dt} + \frac{d\psi_{D,k}}{dt} \right] \cdot T 
|      | \psi_{Q,k+1} = \psi_{Q,k} + \frac{1}{2} \left[ \frac{d\psi_{Q,k-1}}{dt} + \frac{d\psi_{Q,k}}{dt} \right] \cdot T 
|      | \psi_{f,k+1} = \psi_{f,k} + \frac{1}{2} \left[ \frac{d\psi_{f,k-1}}{dt} + \frac{d\psi_{f,k}}{dt} \right] \cdot T 
|      | \] |
| 4    | Update inductances \( L_{\text{m}}(\psi_{s},\psi_{mq}) \) and \( L_{\text{mq}}(\lambda_{s},\lambda_{mq}) \), calculate the inverse of the inductance matrix, \( L_{\text{m}}^{-1} \). |
| 5    | Calculation of the motor currents according to the matrix equation |
|      | \[ i = L_{\text{stat}}^{-1} \psi \] |
| 6    | Calculation of the electric torque of the synchronous motor |
|      | \[ t_e = \psi_s^{\text{re}} \cdot i_s^{\text{re}} - \psi_s^{\text{im}} \cdot i_s^{\text{im}} \] |
| 7    | Return to phase 1. |
Appendix 6

Standard synchronous machine measurement test report.

General

The measurements are carried out using methods presented in the standards IEC 34-4 "Methods for determining synchronous machine quantities from tests" and IEEE Std 115-1983 "Test procedures for synchronous machines". The machine parameters are given in per unit values. The base values per unit presentation are the rated values of the test machine. The time constants are given in seconds. The supply unit is a 125 kVA synchronous generator, the DC supply for the excitation is a six pulse thyristor rectifier. The AC stator side measuring unit is a NORMA 6100 D Power Analyser and the DC excitation side measuring unit is a NORMA 5155 D Power Analyser. The Tektronix 2430A Digital Oscilloscope is used for transient measurements. The measured data are acquired using a GPIB bus to PC for post processing.

Rated machine data

\[ S_n = 14.5 \text{kVA} \quad U_n = 400/231 \text{V} \quad I_n = 21 \text{A} \quad I_t = 10.5 \text{A} \]
\[ f = 50 \text{Hz} \quad n_n = 1500 \text{rpm} \quad \cos \varphi = 0.8 \text{cap} \]

The reduction factor calculated from the motor winding data:

\[ k_{ii} = 4.637 \]

Resistances, reactances and time constants (from machine data):

\[ x_d = 1.196 \quad x_q = 0.475 \quad r_s = 0.048 \quad \tau_{d'} = 0.284 \text{s} \quad \tau_{q''} = 0.008 \text{s} \]
\[ x_d' = 0.129 \quad x_d'' = 0.109 \quad R_f = 5.63 \Omega \quad \tau_{d} = 0.031 \text{s} \quad \tau_{q'} = 0.006 \text{s} \]
\[ x_d'' = 0.090 \]

Measurements:

DC resistances

\[ R_f(20^\circ \text{C}) = 5.86 \Omega \quad R_c(20^\circ \text{C}) = 0.53 \Omega \]

per unit values

\[ r_t = \frac{1}{3 \cdot k_{ii}^2} \cdot R_f \cdot \frac{1}{Z_b} = 0.0083 \quad r_s = \frac{0.53}{Z_b} = 0.048 \]
No load and steady state short circuit test

- no load test with rotor excitation
- no load test with stator excitation, excitation winding short circuited
- steady state stator short circuit

In Fig. 1 the no load curves (rotor excitation $u_d(i_d)$ and stator excitation $u_s(i_s)$) and the steady state stator short circuit curve $i_s(i_t)$ are shown. All values are per unit. (base values: $I_s = 21$ A, $I_t = 10.5$ A and $U_s = 400$ V).

![Diagram](image_url)

Fig. 1. The no load curves (rotor excitation $u_d(i_d)$ and stator excitation $u_s(i_s)$) and the steady state stator short circuit curve $i_s(i_t)$.

Using the no load curve $u_s(i_s)$ and the short circuit curve $i_s(i_t)$ the direct axis unsaturated $x_{du}$ and saturated synchronous $x_d$ can be calculated.

$$x_{du} = \frac{AC}{BC} = \frac{1}{0.70} = 1.43$$
$$x_d = \frac{DF}{EF} = \frac{1}{0.88} = 1.136$$

The same parameters can be applied by using the stator excitation no load curve $u_s(i_s)$.

$$x_{du} = \frac{u_{sair}}{i_{sair}} = \frac{1}{0.72} = 1.38$$
$$x_d = \frac{u_s}{i_s} = \frac{1}{0.84} = 1.19$$

The reduction factor $k_n$ obtained from rotor excitation no load curve $u_d(i_d)$ and stator excitation no load curve $u_s(i_s)$.

$$k_n = \frac{i_s}{i_t} \bigg|_{(u_s=1)} = \frac{2 \cdot 0.72}{0.36} = 4$$

, coefficient 2 is equal to rated current ratio $i_u/i_t = 2$. 
Slip frequency test

The slip frequency test is carried out by a driving synchronous machine using a DC machine. The slip is adjusted to 5% of the synchronous speed of the tested synchronous machine. The excitation circuit is open and the stator is supplied with nominal voltage. The stator current and voltage alternate as a function of the rotor position. The direct and quadrature axis reactances can be calculated when there is the slip $s$

$$X_{ds} = \frac{u_{\text{max}}}{i_{\text{min}}} \quad X_{qs} = \frac{u_{\text{min}}}{i_{\text{max}}}.$$  

Then the quadrature axis reactance $x_q$ can be calculated

$$x_q = x_d \cdot \frac{X_{qs}}{X_{ds}}.$$  

Fig. 2 illustrates the measured current and the voltage waveforms.

![Fig. 2. The measured current and the voltage waveforms during slip frequency test.](image)

Quadrature axis reactance

$$X_{ds} = \frac{650}{4.0} = 162.5 \ \Omega \quad X_{qs} = \frac{620}{8.1} = 76.5 \ \Omega$$

$$x_q = 1.19 \cdot \frac{76.5}{162.5} = 0.56$$
Excitation winding short circuit

From the excitation winding short circuit test the excitation winding time constant \( \tau_{d_0} \) can be defined. The speed of the synchronous machine is nominal and the excitation is supplied from the rotor. The sudden short circuit of the excitation winding is performed. The excitation winding time constant \( \tau_{do} \) is defined as a time when the stator voltage is decayed to 63 % of the initial value (exactly 1-1/e = 0.6321). In Fig. 3 the sudden short circuit of the excitation winding is shown.

![Graph showing the decay of stator voltage](image)

**Fig. 3. Sudden short circuit of the excitation winding.**

It can be obtained for the excitation winding time constant \( \tau_{d_0} = 0.236 \) s.

Sudden three-phase short circuit

The synchronous reactance \( x_d \), the transient reactance \( x_d' \) and the subtransient reactance \( x_d'' \) and also the transient time constant \( \tau_d \) and the subtransient time constant \( \tau_{d'} \) characterise the behaviour of the synchronous machine in the sudden three-phase short circuit. The current flowing during the short circuit can be represented by using a fitted exponential equation if it is assumed that the excitation current is constant during the short circuit. The resistive voltage drop is small when compared to the reactive voltage during the fast transient, so the resistive voltage drop can be neglected.

\[
i_k = \frac{\mu_s}{x_d} + \left( \frac{\mu_s}{x_d} - \frac{\mu_s}{x_d} \right) e^{\frac{-\tau}{\tau_d}} + \left( \frac{\mu_s}{x_d} - \frac{\mu_s}{x_d} \right) e^{\frac{-\tau}{\tau_{d'}}} = i_e + i_{e_1} e^{\frac{-\tau}{\tau_d}} + i_{e_2} e^{\frac{-\tau}{\tau_{d'}}},
\]

where

- \( i_k \) = short circuit current
- \( \mu_s \) = phase to earth voltage before short circuit
- \( \tau \) = time after short circuit
Fig. 4 shows the measured three-phase short circuit current with reduced excitation. The excitation current is $i_e = 0.86$ pu and the stator voltage before short circuit is $u_s = 1.16$ pu.

Fig. 4. Measured three-phase short circuit current with reduced excitation.

Fig. 5 shows the short circuit current in the semi logarithm scale. The subtransient short circuit current $i_s'''$, the transient short circuit current $i_s'$, the steady state short circuit current $i_s$ and the transient time constant $\tau_d$ and the subtransient time constant $\tau_d''$ can be defined using the graphical methods.

Fig. 5. Short circuit current in the semi logarithm scale.
The reactances can be calculated from Eqs.

\[ X_d = \frac{u_v}{i_s}, \quad X'_d = \frac{u_v}{i'_s}, \]

where

- \( u_v \) = phase to earth voltage before short circuit
- \( i_v \) = subtransient short circuit current
- \( i'_s \) = transient short circuit current

The subtransient time constant \( \tau_d \) is defined as a time when the current \( (i_v - i'_s) \) decays 63\% of its initial value. Correspondingly the transient time constant \( \tau'_d \) is defined as a time when the current \( (i'_s - i_s) \) decays 63\% of its initial value.

Table 1 Direct axis Subtransient and transient reactances and time constants.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage before short circuit ( u_v )</td>
<td>1.21 pu</td>
</tr>
<tr>
<td>Steady state short circuit current ( i_s )</td>
<td>1.42 pu</td>
</tr>
<tr>
<td>Transient short circuit current ( i'_s )</td>
<td>3.70 pu</td>
</tr>
<tr>
<td>Transient short circuit ( X_d )</td>
<td>0.33 pu</td>
</tr>
<tr>
<td>Transient time constant ( \tau'_d )</td>
<td>0.054 s</td>
</tr>
<tr>
<td>Subtransient short circuit current ( i'_s )</td>
<td>11.4 pu</td>
</tr>
<tr>
<td>Subtransient reactance ( X'_d )</td>
<td>0.105 pu</td>
</tr>
<tr>
<td>Subtransient time constant ( \tau_d )</td>
<td>0.014 s</td>
</tr>
</tbody>
</table>

Fig. 6 shows the measured envelope curve of the short circuit current and the calculated curve using the parameters defined above. It can be noticed that the subtransient time constant defined is too short because the exponential fit decreases too rapidly.

![Fig. 6. The measured envelope curve of the short circuit current and the calculated curve using the parameters defined above.](image-url)
The subtransient time constant can be adjusted so that the exponential fit curve and the measured curve will agree. In Fig. 7 the measured envelope curve of the short circuit current and the calculated curve is shown using $\tau_d'' = 0.024$ s.

![Graph showing measured and exponential fit curves for short circuit current.](image)

**Fig. 7. The measured envelope curve of the short circuit current and the calculated curve ($\tau_d'' = 0.024$ s).**

**Summary of the measurements**

Table 2 Measured parameters of the test machine.

<table>
<thead>
<tr>
<th>parameter</th>
<th>meas.</th>
<th>data</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>stator resistance $r_s$</td>
<td>0.048</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>excitation winding resistance $r_f$</td>
<td>0.0083</td>
<td>-</td>
<td>* referred to stator side</td>
</tr>
<tr>
<td>reduction factor $\lambda_d$</td>
<td>4</td>
<td>4.63</td>
<td></td>
</tr>
<tr>
<td>direct axis synchronous reactance $x_d$</td>
<td>1.19</td>
<td>1.196</td>
<td></td>
</tr>
<tr>
<td>quadrature axis synchronous reactance $x_q$</td>
<td>0.56</td>
<td>0.475</td>
<td></td>
</tr>
<tr>
<td>direct axis transient reactance $x_d'$</td>
<td>0.33</td>
<td>0.129</td>
<td></td>
</tr>
<tr>
<td>direct axis subtransient reactance $x_d''$</td>
<td>0.105</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>quadrature axis subtransient reactance $x_q''$</td>
<td>*</td>
<td>0.109</td>
<td>* can not measure</td>
</tr>
<tr>
<td>excitation winding time constant $\tau_d$</td>
<td>0.236 s</td>
<td>0.284 s</td>
<td></td>
</tr>
<tr>
<td>direct axis transient time constant $\tau_d'$</td>
<td>0.054 s</td>
<td>0.031 s</td>
<td></td>
</tr>
<tr>
<td>direct axis subtransient time constant $\tau_d''$</td>
<td>0.024 s</td>
<td>0.006 s</td>
<td>* corrected value</td>
</tr>
<tr>
<td>quadrature axis subtransient time constant $\tau_q''$</td>
<td>*</td>
<td>0.008 s</td>
<td>* can not measure</td>
</tr>
</tbody>
</table>
Load curves of the test machine

![Graph showing load curves with various cosines of phase angle (cos φ) values.]

Fig. 8. Load curves of the test machine.
Appendix 7

Used per unit system

Generally accepted principles of the per unit system used in a mathematical description of electrical machines are shown. Per unit (pu) system is the dimensionless relative value system defined in terms of base values. A pu quantity \( x_{\text{pu}} \) is defined as an absolute physical value \( X_{\text{act}} \) in SI-unit divided by its base value \( X_B \):

\[
x_{\text{pu}} = \frac{X_{\text{act}}}{X_B}
\]

The motor nominal values are chosen to be the basic values in the pu system.

**Nominal values of the test machine**

| Power \( S_n \) | 14.5 kVA | Frequency \( f_n \) | 50 Hz |
| Voltage \( U_n \) | 400 V | Speed \( n_n \) | 1500 l/min |
| Current \( I_n \) | 21 A | Power factor \( \cos(\phi) \) | 0.8 cap. |
| Excitation current \( I_f \) | 10.5 A | Reduction factor \( \hat{k}_m \) | 4.637 |

The pu system is referred to the following base quantities:

\[
U_B = \sqrt{\frac{2}{3}} \cdot U_n \quad \text{Amplitude of a nominal stator phase voltage [V]}
\]

\[
I_B = \sqrt{2} \cdot I_n \quad \text{Amplitude of a nominal stator phase current [A]}
\]

\[
\omega_B = \omega_n = 2 \cdot \pi \cdot f_n \quad \text{Stator nominal angular velocity [Hz]}
\]

\[
p_B = p \quad \text{Number of pole pairs}
\]

where \( U_n \) is a nominal RMS value of a motor main voltage

\( I_n \) is a nominal RMS value of a motor phase current

\( f_n \) is a nominal value of a motor synchronous frequency

\( \omega_n \) is a nominal value of a motor angular velocity

From these so-called derivative base quantities are determined as follows:

\[
I_B = \frac{1}{\omega_B} \quad \text{Time}
\]

\[
Z_B = \frac{U_n}{\sqrt{3} \cdot I_n} = \frac{U_B}{I_B} \quad \text{Impedance}
\]

\[
S_B = \sqrt{3} \cdot U_n \cdot I_n = \frac{3}{2} U_B \cdot I_B \quad \text{Power}
\]

\[
\psi_B = \sqrt{3} \cdot U_n \cdot \omega_n = \frac{U_B}{\omega_B} \quad \text{Flux linkage}
\]

\[
T_B = \frac{\sqrt{3} U_n \cdot I_n}{\omega_n} = p \cdot \frac{S_B}{\omega_B} = \frac{3}{2} p \cdot \psi_B \cdot I_B \quad \text{Torque}
\]

\[
L_B = \frac{\psi_B}{I_B} = \frac{Z_B}{\omega_B} \quad \text{Inductance}
\]

\[
C_B = \frac{1}{\omega_B \cdot Z_B} \quad \text{Capacitance}
\]
Appendix 8

Hitzinger synchronous machine data sheets and design data (documents are obtained from manufacture).

1. Design of stator winding:

The stator winding is designed with twelve winding ends and is also designed for low harmonic produced in generator operation. The twelve winding ends are connected via a selector switch to change in three positions:

a) parallel connection of two stator groups (400/231 V)

b) zick zack connection (700/400 V)

c) series connection (800/462 V)

At any of these three positions it is possible to use the star or delta connection of the resulting stator winding on the 6 terminals of the output.

The stator insulation material is Lack/Glasseide for high voltage resistance.

There is a special combination of pitching the stator winding and drilling of stator. To get low harmonic contents and excellent results in any possible stator connection the stator is drilled by two slots.

The calculated winding factors are:

<table>
<thead>
<tr>
<th>Harmonic No</th>
<th>Winding factor Phase - neutral</th>
<th>Winding factor Phase- phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.79881</td>
<td>0.69179</td>
</tr>
<tr>
<td>3</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>5</td>
<td>0.04281</td>
<td>0.03707</td>
</tr>
<tr>
<td>7</td>
<td>0.03058</td>
<td>0.02648</td>
</tr>
<tr>
<td>9</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>11</td>
<td>0.07262</td>
<td>0.06289</td>
</tr>
<tr>
<td>13</td>
<td>0.06145</td>
<td>0.05321</td>
</tr>
<tr>
<td>15</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>17</td>
<td>0.01259</td>
<td>0.01090</td>
</tr>
<tr>
<td>19</td>
<td>0.01127</td>
<td>0.00976</td>
</tr>
<tr>
<td>21</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>23</td>
<td>0.03475</td>
<td>0.03008</td>
</tr>
<tr>
<td>25</td>
<td>0.03195</td>
<td>0.02767</td>
</tr>
<tr>
<td>27</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>29</td>
<td>0.00738</td>
<td>0.00639</td>
</tr>
</tbody>
</table>

1. Technical data:

Attached data sheets are calculated for (400 V stator connection a) parallel connected groups. The number of windings per slots is 56; two groups are internally connected in series and two groups are connected via the selector switch in parallel. The total number of windings per phase is 2x2x28 = 112 windings per phase (231 V).

The internal values (Kontrolldaten) are for this connection (400/231 V).

Notice that the maximum output current is depending on the selected connection of stator winding:

a) at parallel connection (400/231 V) nominal current is 21 A
b) at zick zack connection (700/400 V) nominal current is 10.5 A
c) at series connection (800/462 V) nominal current is 10.5 A

In delta connection the current in the winding must not be reduced so the line can be by a factor of 1.732 higher at the lower line voltage.

2. Rotor winding:
The rotor winding is also with lack/Glasseide insulation for high voltage resistance. There should be no problem when feeding with a thyristor controlled DC-converter.

The number of turns per pole is 220 windings. The four polewindings are connected in series to obtain the most symmetric magnetic conditions.

There is an extra support fixing the rotor winding for high speed operation.

3. Damper winding (squirrel-cage winding)

The squirrel cage winding is of copper material diameter 5.0 mm, on each pole there is six bars. As an endplate copper in the same shape as the laminated pole material is used. The thickness of the copper end plate is 1 mm per side.

This squirrel cage winding allows also limited asynchronous operation, for example as a motor but only for short time period of some seconds depending on the stator current.

In case of asynchronous operation high voltages is induced at the pole main winding. Parallel resistance must be used maximum 8 times pole resistance (8x5.6 = 44 Ohms maximum)!