Finance

Bachelor's Thesis

THE LINKAGES BETWEEN THE UNITED STATES AND FINNISH STOCK MARKETS – A BIVARIATE GARCH APPROACH

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1 Introduction

1.1 Background

The research on the linkages between the international stock markets is current once again, due to at least four contributing factors. First, the end of the highest point in the recent boom and bust cycle of the real economies has been witnessed fairly recently on a global scale. Second, the echoing shocks in the world's stock markets caused by the collapses in banking in addition to some other elements of bad news have been widely noted. Third, the removal of obstacles to financial transactions around the world keeps on continuing. Fourth, the spread of market sophistication, owing to the removal of these obstacles and perhaps resulting to even more interdependence, keeps on manifesting itself in a multitude of ways.

There are different methodologies to estimate the impact of shocks through the system of international stock exchanges, one of them being the GARCH analysis. A typical GARCH estimation is usually done between two major stock markets, like the bivariate GARCH test on the United States and Japanese stock markets by Theodossiou and Koutmos (1994). Also, the study of multiple major stock markets is popular, see e.g. Kanas (1998) and Caporale, Pittis and Spagnolo (2006). In addition to this, a third segment is the research on the interactions of major stock markets with emerging markets, usually concentrating on the emerging markets from the Central and Eastern European region; like the work of Kasch-Haroutounian and Price (2001); see also Li (2007) for the study of the Chinese markets and the United States.

However, as is apparent from the above, most of the research focuses on discovering the dynamics between major stock markets; or between major and emerging stock markets. Arguably, there is a real niche demand for the study of major and minor stock market interactions strictly within the developed countries. It can be highly beneficial to understand these relationships, because these developed markets offer sophisticated derivatives instruments that are competitively priced, yet are not so well known as their counterparts on the bigger markets, and as such can cater to people interested in taking advantage of these things by hedging or betting against interesting scenarios. Naturally, the often stated reasons; such as asset allocation and diversification come to play as well. Thus, the United States and Finnish stock markets are selected for testing with the GARCH models.

1.2 Objectives

This thesis aims to study the linkages between a major and a minor stock market. Specifically, this study aims to answer the following key questions.

Q1:

Are there any linkages between the two stock markets?

Q2:

What is the direction and importance of any such linkage?

Q3:

How does the magnitude of these linkages compare to the magnitude of the inner dependencies within the markets themselves?

1.3 Methodology

Descriptive statistics are calculated for the series in addition to diagnostic tests. ARCH effects tests are performed before any model estimation takes place and again after a model estimation, but this time for the resulting residuals. ARCH, Univariate GARCH and bivariate GARCH tests are then executed. Popular variations of these tests are run with frequently used test parameters and settings.

1.4 Limitations

Aggregate indices are used to represent both of the selected stock markets. The Finnish stock market is very small and has a few key companies that can significantly contribute to the index value at any given time.

1.5 Structure

This study is structured as follows. First, selected methodology is presented in detail. Second, the data is characterized. Third, empirical test results are listed along with the appropriate diagnostic tests. Finally, conclusions are made.

2 Methodology

2.1 ARCH

All of the methods covered in this thesis are based on the ARCH specification. Engle (1982) proposes the ARCH(1) model as

$$y_t = \beta_1 + \beta_2 x_{2t} + u_t \, , \quad (1)$$

$$u_t \sim N(0, \sigma_t^2) \, , \quad (2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \, , \qquad (3)$$

where y_t is the dependent variable of daily returns, β_1 and β_2 are parameters, x_{2t} is an independent variable of daily returns, u_t is the error term or innovation, $N(\cdot)$ is a normal distribution function, σ_t^2 is the daily variance, α_0 and α_1 are parameters, u_{t-1}^2 is the previous error term that is squared. All further methods rely on the assumption made in formula (2) if nothing else is advised.

These equations can be represented as

$$y_t = \beta_1 + \beta_2 x_{2t} + u_t \, , \qquad (4)$$

$$u_t = v_t \sigma_t \, , \qquad (5)$$

$$v_t \sim N(0,1) \, , \qquad (6)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \, , \qquad (7)$$

where σ_t is the standard deviation or volatility of daily returns.

The mean equation can be shown in the simplest form as

$$y_t = \beta_1 + u_t \ . \tag{8}$$

Notably, all further methods rely on the formula (8) as their default mean equation if not otherwise instructed.

Alternatively, the mean equation can have an autoregressive process AR(1) as

$$y_t = \beta_1 + \beta_2 y_{t-1} + u_t \ . \tag{9}$$

The ARCH(q) specification is written as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + ... + \alpha_q u_{t-q}^2 \ , \tag{10}$$

where u_{t-q}^2 is the last squared error term.

It must be noted that ARCH(q) has a number of limitations. First, it can be difficult to determine the value of q. Second, the value of q might be required to be extremely large to capture all of the dependencies. This would then result in a substantial conditional variance that would not be parsimonious. This issue could be circumvented by having only two parameters with the last one including the lagged terms that would be linearly declining. Ultimately, the non-negativity constraints might be easily violated when there are too many parameters to be estimated. The GARCH model overcomes many of these problems.

2.2 Univariate GARCH

2.2.1 GARCH

Bollerslev (1986) and Taylor (1986) present the GARCH(1,1) model as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 , \tag{11}$$

where β is a new parameter and σ_{t-1}^2 is the previous variance term.

2.2.2 GARCH-M

Engle, Lilien and Robins (1987) specify the GARCH(1,1)-M model as

$$y_t = \beta_1 + \delta \sigma_{t-1} + u_t , \tag{12}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \text{ ,} \tag{13}$$

where δ is the risk premium and σ_{t-1} is the previous standard deviation.

Alternatively the mean equation can be expressed as

$$y_t = \beta_1 + \delta \sigma_{t-1}^2 + u_t \text{ .} \tag{14}$$

The use of a customized GARCH model over the basic GARCH model is only beneficial if the added or modified elements can be actually observed as real features of the selected data series. In the case of a GARCH-M model, this would require that the selected stock market exhibits features that can be expressed mathematically by adding a GARCH term in the mean equation.

2.2.3 EGARCH

The Exponential GARCH model, due to Nelson (1991), can be constructed in many ways. It can be as

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] + \alpha_2 \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta \ln(\sigma_{t-1}^2) \text{ ,} \tag{15}$$

where $\ln(\cdot)$ is the natural logarithm function and α_2 is a new parameter.

Another version is simply

$$\ln(\sigma_t^2)=\alpha_0+\alpha_1\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}}+\beta\ln(\sigma_{t-1}^2)+d\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}}, \tag{16}$$

where the parameter d replaces α_2 while the equation changes somewhat.

The EGARCH model is an asymmetric model. It recognizes the leverage effects if they exist in the data. This is done by allowing the shocks to enter the variance equation by two routes as terms $\alpha_1\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}}$ and $d\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}}$ according to formula (16). Negative shocks in the second term retain their sign.

2.3 Bivariate GARCH Models

2.3.1 VECH

Bollerslev, Engle and Wooldridge (1988) assert the VECH parameterization around a

MGARCH(p,q)-M model

$$y_t = \beta_1 + \delta H_t \omega_{t-1} + \epsilon_t \,, \tag{17}$$

$$VECH(H_t) = C + \sum_{i=1}^{q} A_i VECH(\epsilon_{t-i} \epsilon'_{t-i}) + \sum_{j=1}^{p} B_j VECH(H_{t-j}) \,, \tag{18}$$

$$\epsilon_t | \psi_{t-1} \sim N(0, H_t) \,, \tag{19}$$

where y_t is an $N \times 1$ mean vector, β_1 is an $N \times 1$ vector of constants, H_t is the conditional variance-covariance matrix, ω_{t-1} is a vector of value weights, ϵ_t is an $N \times 1$ innovation vector, $VECH(\cdot)$ is a column stacking operator of a lower portion symmetric matrix, C is a $\frac{1}{2}N(N+1) \times 1$ vector, A_i, $i=1,\ldots,q$, and B_j, $j=1,\ldots,p$, are $\frac{1}{2}N(N+1) \times \frac{1}{2}N(N+1)$ matrices and ψ_{t-1} is the previous information set. The assumption in the formula (19) is made also for the remaining methods.

If $p=1$, $q=1$ and $N=1$, then the above can be described as

$$VECH(H_t) = C + A\, VECH(\epsilon_{t-1} \epsilon'_{t-1}) + B\, VECH(H_{t-1}) \,, \tag{20}$$

where H_t is a 2×2 matrix, C is a 3×1 vector, A and B are 3×3 matrices and ϵ_{t-1} is a 2×1 vector.

The contents of these matrices and vectors can be visualized as

$$H_t = \begin{bmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{bmatrix}, \tag{21}$$

$$C = \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix}, \tag{22}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{23}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}, \tag{24}$$

$$\epsilon = \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}. \tag{25}$$

An example of the VECH operator can be provided as

$$VECH(\epsilon_t \epsilon_t') = VECH\left(\begin{bmatrix}\epsilon_{1t}\\ \epsilon_{2t}\end{bmatrix}\begin{bmatrix}\epsilon_{1t} \epsilon_{2t}\end{bmatrix}\right) = VECH\begin{pmatrix}\epsilon_{1t}^2 & \epsilon_{1t}\epsilon_{2t}\\ \epsilon_{2t}\epsilon_{1t} & \epsilon_{2t}^2\end{pmatrix} = \begin{bmatrix}\epsilon_{1t}^2\\ \epsilon_{2t}^2\\ \epsilon_{1t}\epsilon_{2t}\end{bmatrix}. \quad (26)$$

The conditional variance can be seen as three equations in the bivariate case;

$$h_{11t} = c_{11} + a_{11}\epsilon_{1t-1}^2 + a_{12}\epsilon_{2t-1}^2 + a_{13}\epsilon_{1t-1}\epsilon_{2t-1} + b_{11}h_{11t-1} + b_{12}h_{22t-1} + b_{13}h_{12t-1}, \quad (27)$$

$$h_{22t} = c_{21} + a_{21}\epsilon_{1t-1}^2 + a_{22}\epsilon_{2t-1}^2 + a_{23}\epsilon_{1t-1}\epsilon_{2t-1} + b_{21}h_{11t-1} + b_{22}h_{22t-1} + b_{23}h_{12t-1}, \quad (28)$$

$$h_{12t} = c_{31} + a_{31}\epsilon_{1t-1}^2 + a_{32}\epsilon_{2t-1}^2 + a_{33}\epsilon_{1t-1}\epsilon_{2t-1} + b_{31}h_{11t-1} + b_{32}h_{22t-1} + b_{33}h_{12t-1}. \quad (29)$$

2.3.2 Diagonal VECH

Bollerslev, Engle and Wooldridge (1988) introduce the diagonal VECH model as

$$y_{it} = \beta_i + \delta \sum_j \omega_{jt} h_{ijt} + \epsilon_{it}, \quad (30)$$

$$h_{ijt} = c_{ij} + a_{ij}\epsilon_{it-1}\epsilon_{jt-1} + b_{ij}h_{ijt-1}, \quad (31)$$

$$i, j = 1, \ldots, N. \quad (32)$$

Again, the mean equation can be expressed in its simplest form as

$$y_{it} = \beta_i + \epsilon_{it} \; . \tag{33}$$

The mean equation can also be viewed as

$$y_{it} = \beta_{1i} + \beta_{2i} y_{it-1} + \epsilon_{it} \; . \tag{34}$$

The above mentioned restricted variance formula, which essentially is the diagonal VECH model, is still quite problematic. Albeit, it no longer has a large set of parameters when compared to the basic VECH model. However, it still lacks positive definiteness. For more information about the positive definiteness problem and GARCH see Laurent et al. (2006). A solution to this problem is the BEKK parameterization.

2.3.3 BEKK

Engle and Kroner (1995) present the BEKK model, which is as follows in the bivariate case

$$H_t = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B \; , \tag{35}$$

where C, A and B are 2×2 parameter matrices.

The matrix notation can be expressed as

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ & c_{22} \end{bmatrix}' \begin{bmatrix} c_{11} & c_{12} \\ & c_{22} \end{bmatrix} \quad (36)$$

$$+ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \epsilon_{11,t-1}^2 & \epsilon_{11,t-1}\epsilon_{22,t-1} \\ \epsilon_{22,t-1}\epsilon_{11,t-1} & \epsilon_{22,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (37)$$

$$+ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}. \quad (38)$$

The H_t matrix is always positive definite due to the quadratic nature of the terms on the equation's RHS.

2.4 Model Estimation

The parameter estimates for the models are acquired by maximizing the log likelihood function using maximum likelihood. The log likelihood function can be described as[1]

$$l(\theta) = \frac{-TN}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} (\ln|H_t| + \epsilon_t' H_t^{-1} \epsilon_t) , \quad (39)$$

1 See alternative representations in Appendix 1

where $l(\cdot)$ is the log likelihood, θ is the parameter vector to be estimated, N is the number of stock markets.

The log likelihood function presented in formula (39) is highly non-linear in θ. Thus, its maximization requires an iterative method. The hill climbing technique is applied with different methods. The following iteration algorithms are used: BFGS by Broyden, Fletcher, Goldfarb and Shanno described in Press et al. (1988) and BHHH by Berndt, Hall, Hall and Hausman (1974). These algorithms essentially compute the multiplying matrix G.

When using the BHHH algorithm, the function being maximized has the general form

$$F(x)=\sum_{t=1}^{T} f(y_t, x) , \qquad (40)$$

where $F(\cdot)$ is the log likelihood function.

The BHHH algorithm then chooses

$$G=J^{-1} , \qquad (41)$$

where J is

$$J = \sum_{t=1}^{T} \left[\frac{\partial f}{\partial x}(y_t, x)' \frac{\partial f}{\partial x}(y_t, x) \right]. \tag{42}$$

The nature of the hill climbing technique is that after n iterations

$$G = -H^{-1} \vee G \approx -H^{-1} , \tag{43}$$

where H is called the Hessian, which is a matrix of second derivatives.

Under fairly general conditions $-J$ will have the same asymptotic limit as the Hessian H, when $-J$ is divided by T.

3 Data

The data consists of daily closing-price observations from MSCI Finland (PI) and MSCI USA (PI). The data ranges from 12/30/1988 to 12/30/2008 for Finland and from 12/30/1988 to 12/31/2008 for the United States. These series are then transformed into natural logarithmic returns starting from the very beginning of 1989.

In order to avoid empty cells and entries with zeros, some date rows are completely wiped out from the data when needed[2]. In the case of multivariate estimation additional removal is also necessary, because in certain situations a market can be pricing on a particular day while another market is not pricing at all. Thus, at this stage these resulting series will be subsets of their original natural logarithmic returns. However, the process does not stop here. The series are also rounded, whenever appropriate, this determined by complexity,

2 Penzer (2007) highlights a different solution to deal with these issues

so that they may adequately complete the estimation sequences.

The data series are acquired from Thomson Datastream Advance Version 4.0 SP4b. These series are edited and processed using various software[3].

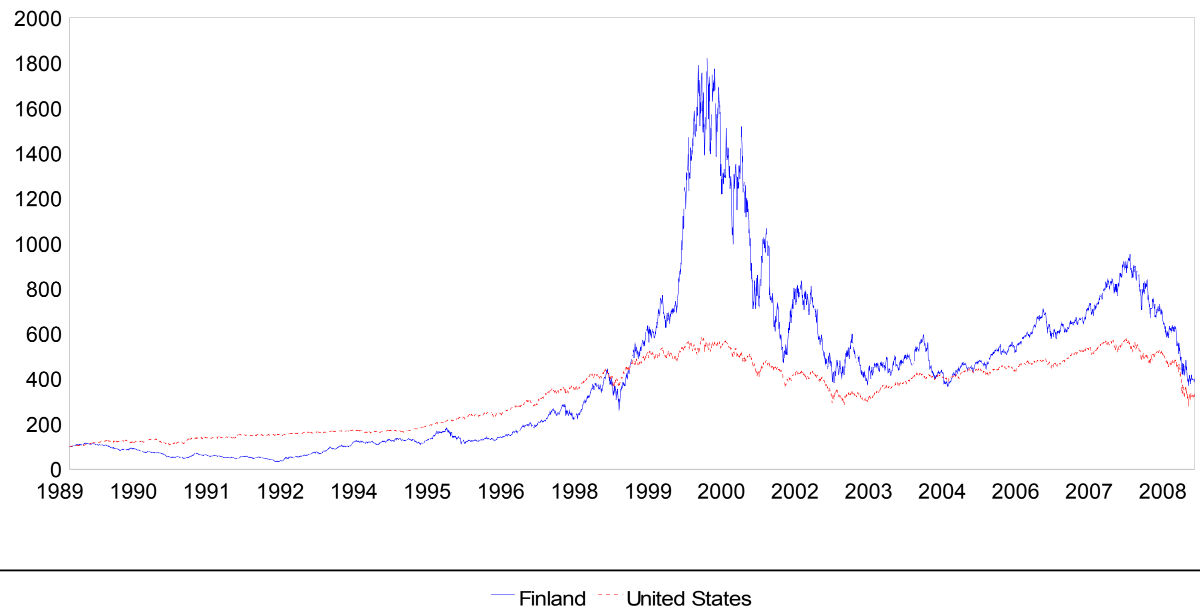

Figure 1. Original Price Series Rebased to 100, Finland and United States

Notes: Data with 3 decimal places is used.

Figure 1 represents the price series rebased to 100[4] and Figures 2 and 3 show the natural logarithmic returns[5] for Finland and the United States.

It should be noted that due to the index construction methodology of these MSCI indices, the influence of single highly capitalized stocks is not limited by index rules. This means that stocks like Nokia can have a substantial impact on the changes of the MSCI Finland index. A solution to this problem would be to use an index like OMX Helsinki Cap. However, as the goal is to model the whole market strictly on the basis of market capitalization, these limitations are not considered.

3 See Appendix 2 for a complete listing of software used to edit, process and test the data
4 See Appendix 3 for the original price series
5 See Appendix 4 for the percentage returns and reduced natural logarithmic returns

The use of the MSCI indices, to depict the selected stock markets, offers many benefits. These start all the way from the shared index construction standards, long pricing histories and denominations in local currencies[6].

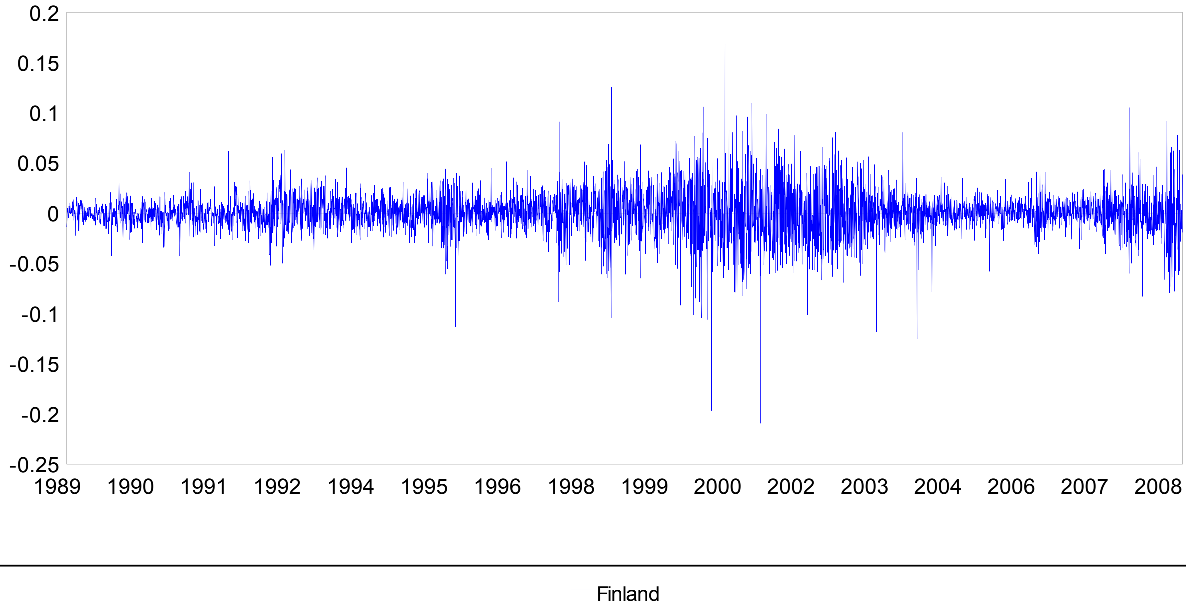

Figure 2. Natural Logarithmic Returns, Finland

Notes: Data with 6 decimal places is used.

6 It is also noteworthy to mention that it can be problematic to collect indices from different index providers and form a new group out of them for later use. This is because as a group they might lack their previous poise and integrity.

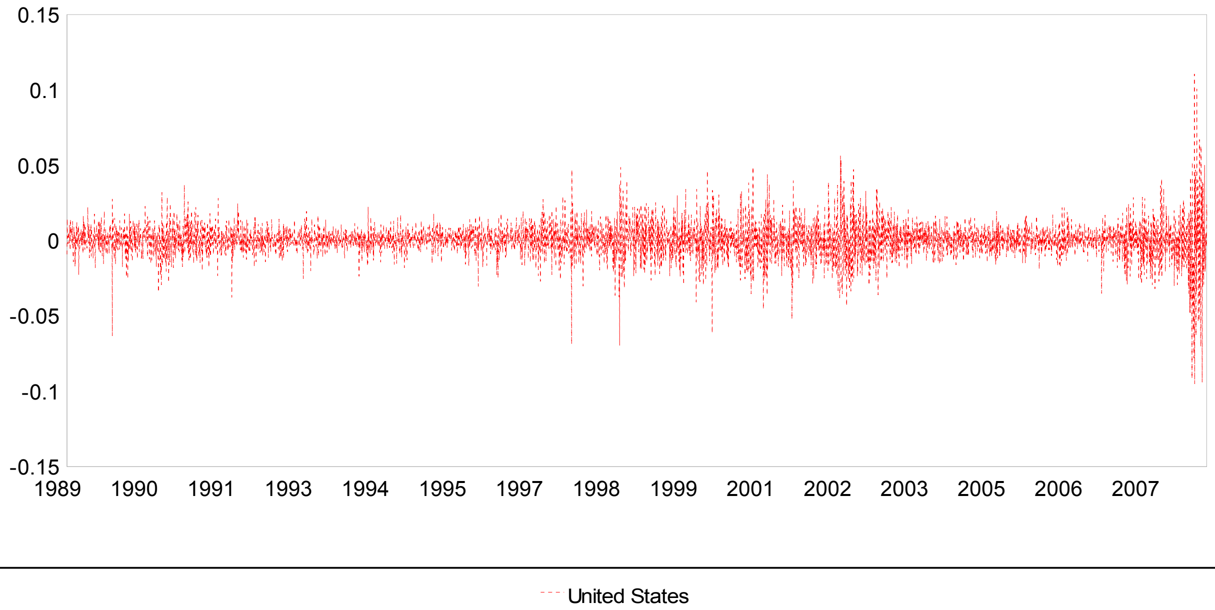

Figure 3. Natural Logarithmic Returns, United States

Notes: Data with 6 decimal places is used.

4 Empirical Results

4.1 Preliminary

Descriptive statistics and diagnostic tests are presented in Table 1. The series differ in the amount of observations when the data is in the form suitable for tests. The United States data has a lower mean, but a higher median. The data for Finland is more volatile. Both of the series have negative skewness and high kurtosis. Jarque-Bera indicates that the series are not normally distributed. The series are also autocorrelated and the Ljung-Box statistics, which are denoted by $Q(\cdot)$ and visible on the diagnostic tests panel, all reject the hypothesis of linear independence. Thus, these series present typical features, which are usually found in time series data relating to stock markets that consist of returns data[7].

Table 2 shows the test results for the ARCH effects tests. All tested lags indicate the

7 In this work the word 'return' refers strictly to the change in asset prices that excludes dividends and occurs on a preselected interval.

presence of ARCH effects as is evident from the strong 1% significance levels reported for all of the $F(\cdot)$ and $LM(\cdot)$ tests. This applies to both markets.

Table 1. Descriptive Statistics and Diagnostic Tests on the Series

This table represents the results of descriptive statistics and diagnostic tests. The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland	United States
Panel A. Descriptive Statistics		
Observations	5015	5047
Mean	0.000274	0.000239
Median	0.000507	0.000563
Maximum	0.168625	0.110426
Minimum	-0.209307	-0.095137
Standard Deviation	0.021003	0.011225
Skewness	-0.360321	-0.281135
Kurtosis	10.72809	13.37325
Jarque-Bera	12588.21***	22694.77***
Panel B. Diagnostic Tests		
$AC(1)$	0.034**	-0.045***
$AC(2)$	-0.030***	-0.066***
$AC^2(1)$	0.137***	0.220***
$AC^2(2)$	0.123***	0.380***
$PAC(2)$	-0.031***	-0.068***
$PAC^2(2)$	0.107***	0.349***
$Q(1)$	5.9164**	10.060***
$Q(2)$	10.364***	31.802***
$Q(6)$	25.880***	45.335***
$Q(12)$	40.068***	82.972***
$Q(30)$	88.320***	142.98***
$Q^2(1)$	93.790***	244.30***
$Q^2(2)$	170.34***	975.25***
$Q^2(6)$	426.47***	2603.7***
$Q^2(12)$	718.06***	5216.1***

(Continues on the next page)

***(Table 1.** Continued)*

$Q^2(30)$	1722.0***	9122.3***

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. $AC(1)$ is the first-order autocorrelation of the series. $AC^2(1)$ is the first-order autocorrelation of the squared series. $PAC(2)$ is the second-order partial autocorrelation of the series. $PAC^2(2)$ is the second-order partial autocorrelation of the squared series. $Q(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the series, distributed as $\chi^2(1)$. $Q^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared series, distributed as $\chi^2(1)$. Data with 6 decimal places is used.

Table 2. ARCH Effects Tests on the Series

This table represents the results of ARCH effects tests. The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland	United States
$F(1)$	95.51764***	256.9275***
$LM(1)$	93.76873***	244.5716***
$F(2)$	77.35802***	497.2536***
$LM(2)$	150.1712***	831.1576***
$F(6)$	47.48205***	276.2944***
$LM(6)$	269.9177***	1248.818***
$F(12)$	29.46850***	174.6043***
$LM(12)$	331.0808***	1482.256***
$F(30)$	17.50076***	84.01432***
$LM(30)$	477.6836***	1684.555***

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. $F(1)$ is the first-order F-statistic. $LM(1)$ is the first-order Lagrange multiplier statistic. Data with 6 decimal places is used.

4.2 ARCH Results

The ARCH(1) test results are realized in Table 3. They show high coefficients for squared lagged error terms, meaning high linkage to previous spikes observed in the data. This is visible on the row containing the parameter α_1. The $Q(\cdot)$ tests on the other hand, are located on the lower portion of Table 3 and display reduced levels of significance for some of the lags when comparing the results to the same tests performed for the original raw data in Table 1. This suggests that the ARCH(1) test manages to capture some of the patterns found in the original data. Still, this improvement can be considered rather weak.

Table 3. ARCH(1), BFGS

This table represents the results of ARCH(1) tests based on formulas (2), (3) and (8). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_t	σ_t^2	y_t	σ_t^2
β_1	0.0005316729**		0.0003977177**	
	(0.0002546658)		(0.0001366120)	
α_0		0.0002687308***		0.0000844077***
		(0.0000076534)		(0.0000023567)
α_1		0.4760464749***		0.3650643107***
		(0.0321779700)		(0.0306622990)
Panel B. Diagnostic Tests				
LL	12580.70814317		15801.97552410	
$Q_s(1)$	14.092***		0.3855	
$Q_s(2)$	14.494***		9.8847***	
$Q_s(6)$	26.091***		12.192*	
$Q_s(12)$	41.976***		42.750***	
$Q_s(30)$	83.006***		78.377***	
$Q_s^2(1)$	2.3227		3.3279*	
$Q_s^2(2)$	13.868***		401.76***	
$Q_s^2(6)$	95.205***		670.65***	
$Q_s^2(12)$	188.61***		1575.8***	
$Q_s^2(30)$	542.44***		2297.0***	

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. Data with 6 decimal places is used.

4.3 Univariate GARCH Results

Table 4 has the first GARCH test results. They show that previous variance is highly influential in determining the amount of current variance as per the coefficient entries for the parameter β. The diagnostic tests prove that the GARCH(1,1) specification works fairly well with these series. Moreover, when comparing the segments where the $Q(\cdot)$ tests are reported in Table 3 and 4, it is fairly clear that the ARCH(1) model does not explain the data very well, at least when ranked against the GARCH(1,1) model.

Table 4. GARCH(1,1), BFGS

This table represents the results of GARCH(1,1) tests based on formulas (2), (8) and (11). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_t	σ_t^2	y_t	σ_t^2
β_1	4.23697e-04**		4.69399e-04***	
	(2.09817e-04)		(1.14250e-04)	
α_0		1.23014e-06***		7.66584e-07***
		(3.30687e-07)		(1.98372e-07)
α_1		0.05400***		0.05737***
		(0.00558)		(0.00655)
β		0.94537***		0.93676***
		(0.00538)		(0.00774)
Panel B. Diagnostic Tests				
LL	13315.95762664		16534.63125116	
$Q_s(1)$	44.925***		0.0463	
$Q_s(2)$	44.987***		0.9220	
$Q_s(6)$	50.465***		11.337*	
$Q_s(12)$	73.830***		29.752***	
$Q_s(30)$	98.630***		43.911**	

(Continues on the next page)

***(Table 4.** Continued)*

$Q_s^2(1)$	3.2391*	0.0012
$Q_s^2(2)$	4.1232	1.5180
$Q_s^2(6)$	4.7627	1.7222
$Q_s^2(12)$	6.9815	5.6076
$Q_s^2(30)$	23.742	16.659

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. Data with 6 decimal places is used.

The results of the ARCH effects tests on standardized residuals of the previous GARCH(1,1) estimation are reported in Table 5. These results indicate clearly that the ARCH effects are less visible than before. Again, this is a consequence of far fewer test results that are significant or as strongly significant as before. Only four tests leave traces of ARCH effects.

Table 5. ARCH Effects Tests on Standardized Residuals of GARCH(1,1)

This table represents the results of ARCH effects tests based on the estimated standardized residuals of the GARCH(1,1) model. The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland	United States
$F(1)$	4.971564**	5.76E-05
$LM(1)$	4.968619**	5.76E-05
$F(2)$	3.298556**	0.789371
$LM(2)$	6.592381**	1.579187
$F(6)$	1.193109	0.295740
$LM(6)$	7.158430	1.776282
$F(12)$	0.756517	0.479346
$LM(12)$	9.085323	5.760439

(Continues on the next page)

*(**Table 5.** Continued)*

$F(30)$	0.847108	0.532343
$LM(30)$	25.44175	16.01829

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. $F(1)$ is the first-order F-statistic. $LM(1)$ is the first-order Lagrange multiplier statistic. Data with 6 decimal places is used.

Next, a variant of the previous GARCH(1,1) test is presented in Table 6. Here, the Finnish series has a significant lagged mean of the first order, marked as β_2, while the same coefficient for the United States series is not significant at all. The performance of this version of the GARCH test, in explaining the changes in series, is improved as indicated by the diagnostic tests.

Table 6. AR(1)-GARCH(1,1), BFGS

This table represents the results of AR(1)-GARCH(1,1) tests based on formulas (2), (9) and (11). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_t	σ_t^2	y_t	σ_t^2
β_1	3.89347e-04***		4.7238e-04***	
	(6.28486e-06)		(1.1349e-04)	
β_2	0.10227***		-3.6784e-03	
	(1.03639e-04)		(0.0150)	
α_0		1.18862e-06***		7.6589e-07***
		(4.02535e-09)		(1.8541e-07)
α_1		0.05490***		0.0574***
		(4.29477e-04)		(6.4817e-03)
β		0.94469***		0.9368***
		(6.22409e-04)		(7.3169e-03)
Panel B. Diagnostic Tests				
LL	13336.04965113		16531.38866096	
$Q_s(1)$	0.0680		0.2177	
$Q_s(2)$	0.9741		1.1044	

(Continues on the next page)

***(Table 6.** Continued)*

$Q_s(6)$	6.4161	11.638*
$Q_s(12)$	24.486**	30.092***
$Q_s(30)$	44.042**	44.115**
$Q_s^2(1)$	2.6985	0.0033
$Q_s^2(2)$	4.2347	1.5305
$Q_s^2(6)$	4.7577	1.7379
$Q_s^2(12)$	6.6440	5.6304
$Q_s^2(30)$	23.692	16.687

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. Data with 6 decimal places is used.

Table 7 depicts the results of GARCH-in-Mean estimation. The added GARCH term is not found to be significant in either of the cases. This is evident as presented on the row beginning with the parameter δ. A model with this exact specification does not add any value over a basic GARCH(1,1) approach while making use of the selected data.

Table 7. GARCH(1,1)-M, BFGS

This table represents the results of GARCH(1,1)-M tests based on formulas (2), (13) and (14). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_t	σ_t^2	y_t	σ_t^2
β_1	2.80449e-04*		0.0003818708**	
	(1.66023e-04)		(0.0001619635)	
δ	0.67835		1.3222247643	
	(0.72629)		(1.7346963728)	

(Continues on the next page)

*(**Table 7.** Continued)*

α_0	1.23130e-06***	0.0000007728***
	(2.40636e-08)	(0.0000001910)
α_1	0.05407***	0.0576168926***
	(3.45245e-04)	(0.0065332229)
β	0.94531***	0.9364473033***
	(0.00109)	(0.0075639576)
Panel B. Diagnostic Tests		
LL	13316.25782254	16534.92342618
$Q_s(1)$	44.974***	0.0501
$Q_s(2)$	45.034***	0.8679
$Q_s(6)$	50.582***	10.835*
$Q_s(12)$	74.356***	29.468***
$Q_s(30)$	99.626***	43.339*
$Q_s^2(1)$	3.2065*	0.0029
$Q_s^2(2)$	4.1204	1.5320
$Q_s^2(6)$	4.7496	1.7242
$Q_s^2(12)$	6.9461	5.5547
$Q_s^2(30)$	23.569	16.674

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. Data with 6 decimal places is used.

The EGARCH results reside in Table 8. All of the coefficients are reported to be significant. It can be concluded that it is advantageous to use the EGARCH specification for this set of data. The Unites States series in particular works well with EGARCH. Notably, a negative entry for the parameter d implies a heightened level of volatility just after bad news.

Table 8. EGARCH(1,1), BFGS

This table represents the results of the EGARCH tests based on formulas (2), (8) and (16). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_t	σ_t^2	y_t	σ_t^2
β_1	1.4400e-04***		0.000242920**	
	(2.2655e-10)		(0.000111023)	
α_0		-0.1396***		-0.248165168***
		(7.2941e-11)		(0.028281905)
α_1		0.1279***		0.111239632***
		(6.7919e-03)		(0.010331257)
β		0.9947***		0.982550651***
		(6.1489e-04)		(0.002506460)
d		-0.0263***		-0.088990669***
		(4.2423e-11)		(0.008152098)
Panel B. Diagnostic Tests				
LL	13342.95872093		16605.78063815	
$Q_s(1)$	+		0.1268	
$Q_s(2)$	+		0.8116	
$Q_s(6)$	+		9.8719	
$Q_s(12)$	+		29.013***	
$Q_s(30)$	+		45.514**	
$Q_s^2(1)$	+		0.7658	
$Q_s^2(2)$	+		0.9477	
$Q_s^2(6)$	+		1.4036	

(Continues on the next page)

*(**Table 8.** Continued)*

$Q_s^2(12)$	+	5.6638
$Q_s^2(30)$	+	17.342

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. + indicates that a particular diagnostic test was not performed. It is important to note that convergence was not reached when estimating the Finnish stock market using the BFGS algorithm. The estimated coefficients and standard errors presented here for Finland are actually from a second test where the iteration was started with BHHH and after 50 iterations switched to BFGS. Therefore, the reported estimations are not identical and as such they are not entirely comparable as the alterations in the use of the root finding algorithm might theoretically result in finding a different root than the root found otherwise. Data with 6 decimal places is used.

4.4 Bivariate GARCH Results

Table 9 presents the bivariate Diagonal VECH results. The parameters a_{11} and b_{11} refer to the Finnish coefficients of the squared lagged error term and the lagged variance term, respectively. The results reported in the table are in-line with the univariate tests mentioned earlier.

Table 9. MV(2)-GARCH(1,1), Diagonal VECH, Simple Variances, BFGS

This table represents the results of bivariate diagonal VECH tests based on formulas (31), (32) and (33). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_{1t}	h_{1t}	y_{2t}	h_{2t}
β_1	3.64677e-04*		4.75426e-04***	
	(2.16225e-04)		(1.28870e-04)	

(Continues on the next page)

*(**Table 9.** Continued)*

c_{11}	1.25186e-06***	
	(3.40063e-07)	
c_{22}		7.78886e-07***
		(1.99344e-07)
a_{11}	0.05443***	
	(0.00573)	
a_{22}		0.05801***
		(0.00685)
b_{11}	0.94498***	
	(0.00559)	
b_{22}		0.93616***
		(0.00781)
Panel B. Diagnostic Tests		
LL	29006.17278854	
$Q_s(1)$	40.460***	0.0006
$Q_s(2)$	40.666***	0.8134
$Q_s(6)$	45.938***	13.750**
$Q_s(12)$	66.001***	29.666***
$Q_s(30)$	90.365***	45.573**
$Q_s^2(1)$	2.9510*	0.0003
$Q_s^2(2)$	3.9819	3.5705
$Q_s^2(6)$	4.6466	4.4830
$Q_s^2(12)$	8.0140	7.3602
$Q_s^2(30)$	25.747	16.672

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. Data with 4 decimal places is used.

The first GARCH BEKK results are expressed in Table 10. The diagonal elements a_{11}, a_{22}, b_{11} and b_{22} capture the markets' own ARCH and GARCH effects, respectively. The off-diagonal elements a_{12}, a_{21}, b_{12} and b_{21} show the cross-market effects: shock and volatility spillover, respectively. All of these coefficients are found to be significant.

When reviewing these results it is good to keep in mind the sector orientation and balancing of the MSCI Finland index. As Nokia is heavily weighted on the index there might be some sort of dependence to other technology stocks that can offer both cross-market shocks as well as volatility spillover. Moreover, Finnish stock returns might reflect other global markets and it is difficult to say the true news impact of the Finnish market in regard to causing shocks in the United States. Furthermore, due to the size of the Finnish stock market and again perhaps its construction, it is more volatile and prone to shocks than the Unites States stock market. The diagnostic tests show that there is some room for improvements within the model, at least for the Finnish side.

Table 10. MV(2)-GARCH(1,1), BEKK, BFGS

This table represents the results of bivariate BEKK tests based on formulas (33), (36), (37) and (38). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_{1t}	h_{1t}	y_{2t}	h_{2t}
β_1	0.000289166		0.000418025***	
	(0.000211939)		(0.000127704)	
c_{11}		0.000894923***		
		(0.000144607)		
c_{21}				-0.000231407
				(0.000210917)
c_{22}				-0.000962335***
				(0.000124743)
a_{11}		0.186587047***		
		(0.008319767)		
a_{12}		0.026340426***		
		(0.005197695)		
a_{21}				-0.069804276***
				(0.018745257)

(Continues on the next page)

*(**Table 10.** Continued)*

a_{22}			0.225893311***
			(0.013131285)
b_{11}	0.980120173***		
	(0.001611951)		
b_{12}	-0.004190125***		
	(0.001066009)		
b_{21}			0.023011691***
			(0.005650564)
b_{22}			0.967862354***
			(0.003907633)
Panel B. Diagnostic Tests			
LL		29121.21949238	
$Q_s(1)$	39.277***		0.0014
$Q_s(2)$	39.730***		0.6421
$Q_s(6)$	45.465***		13.567**
$Q_s(12)$	66.431***		29.823***
$Q_s(30)$	93.302***		45.810**
$Q_s^2(1)$	18.217***		0.0054
$Q_s^2(2)$	25.324***		3.0455
$Q_s^2(6)$	27.434***		3.8275
$Q_s^2(12)$	31.691***		6.5935
$Q_s^2(30)$	45.970**		13.747

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. Data with 4 decimal places is used.

Figures 4 and 5 visualize the standardized residuals from the previous multivariate BEKK estimation[8]. Figure 6 finalizes the results of the estimation by drawing the conditional variances and covariances[9]. Increased conditional variance is clearly visible during times

8 See Appendix 5 for the corresponding residuals and squared standardized residuals
9 See Appendix 6 for the conditional correlations and conditional standard deviations

of trouble in the stock markets, e.g. during the late 2008. An examination of the conditional covariance adds to the understanding of the dynamic shifts that have occurred in the stock markets during the last twenty years.

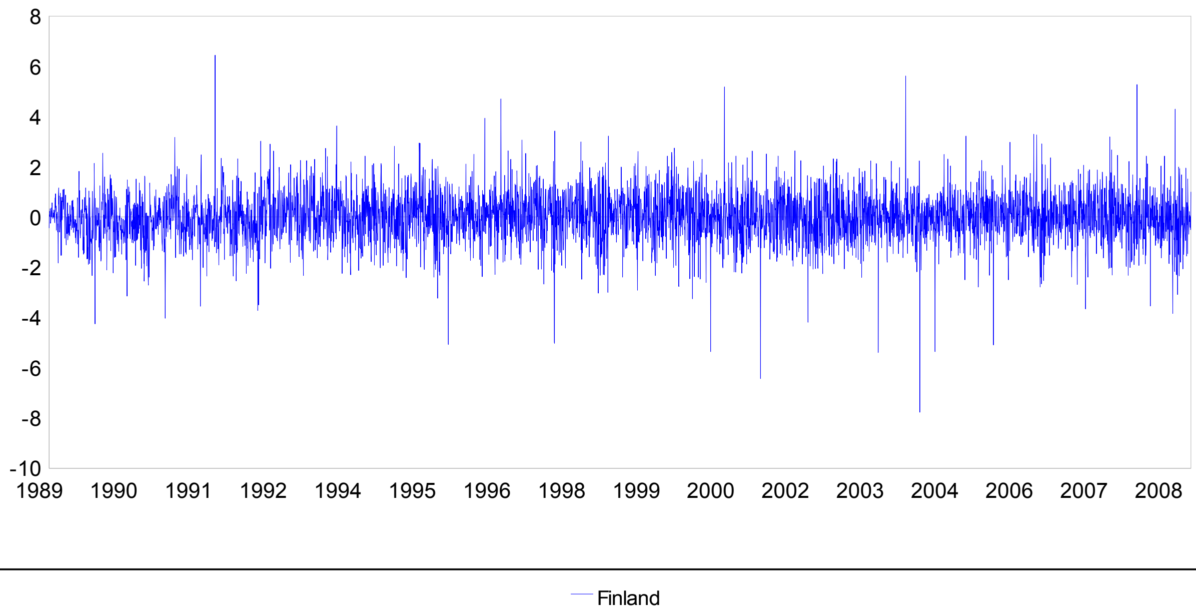

Figure 4. Standardized Residuals, Finland, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

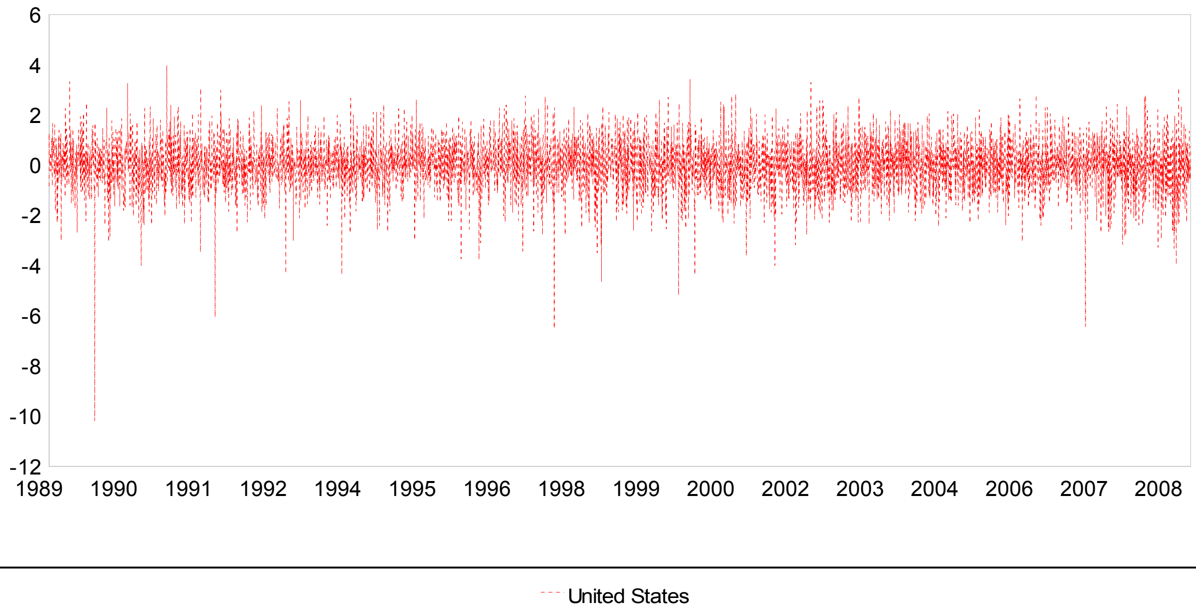

Figure 5. Standardized Residuals, United States, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

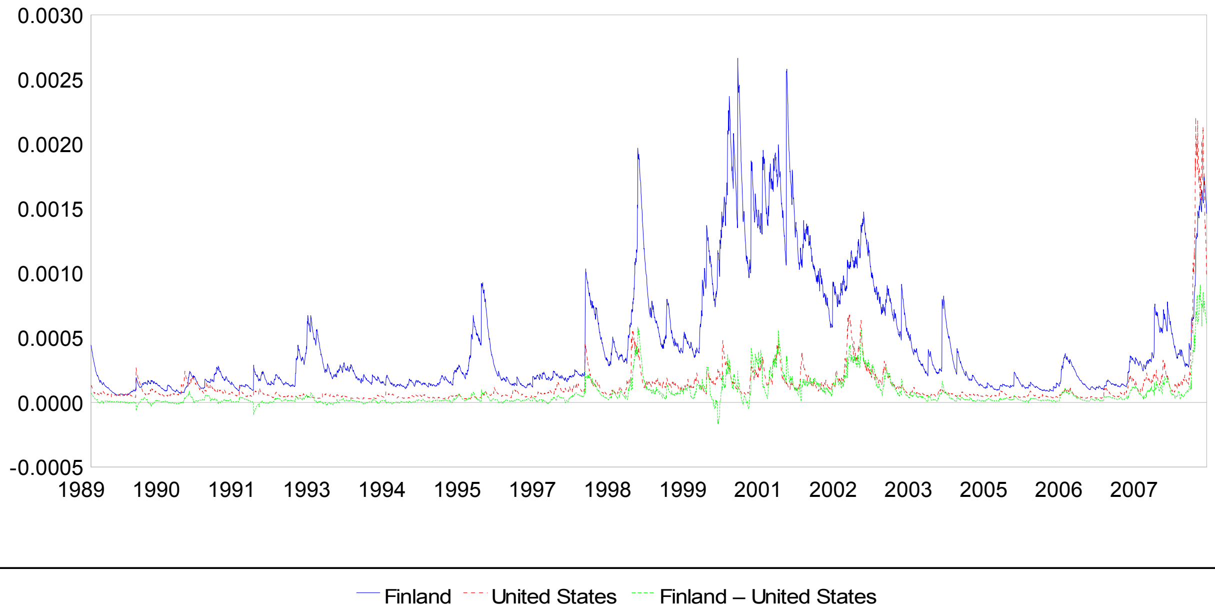

Figure 6. Conditional Variances and Covariances, Finland and United States, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

Table 11 describes another set of BEKK estimation results. These results are similar to the previous findings, only offering a slight enhancement in the form of β_2 that is found significant in both cases.

Table 11. MV(2)-AR(1)-GARCH(1,1), BEKK, BHHH

This table represents the results of bivariate BEKK tests based on formulas (34), (36), (37) and (38). The underlying data is from MSCI Finland (PI) and MSCI USA (PI) indices. This data is modified to form new data series of natural logarithmic returns, ranging from 1989 to 2008, which are then used in the calculations.

	Finland		United States	
Panel A. Parameter Estimates				
	y_{1t}	h_{1t}	y_{2t}	h_{2t}
β_1	0.000285013		0.000458457***	
	(0.000199137)		(0.000118770)	
β_2	0.097286590***		-0.103924787***	
	(0.012194847)		(0.015199303)	
c_{11}		0.000905158***		
		(0.000087626)		
c_{21}				-0.000098453
				(0.000145596)
c_{22}				0.000934508***
				(0.000072623)
a_{11}		0.193520355***		
		(0.005542799)		
a_{12}		0.024686103***		
		(0.003740546)		
a_{21}				-0.053200558***
				(0.009790162)
a_{22}				0.221622291***
				(0.008194657)
b_{11}		0.979409421***		
		(0.000935673)		
b_{12}		-0.004201983***		
		(0.000766305)		
b_{21}				0.017569528***
				(0.002739593)

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***(Table 11.** Continued)*

b_{22}		0.969801311***
		(0.002249708)
Panel B. Diagnostic Tests		
LL	29166.24752560	
$Q_s(1)$	0.0079	42.327***
$Q_s(2)$	1.8976	43.508***
$Q_s(6)$	7.5497	59.417***
$Q_s(12)$	23.269**	77.686***
$Q_s(30)$	45.022**	94.408***
$Q_s^2(1)$	13.750***	0.1425
$Q_s^2(2)$	22.115***	3.7228
$Q_s^2(6)$	23.803***	4.5376
$Q_s^2(12)$	26.893***	7.9055
$Q_s^2(30)$	41.369*	15.278

Notes: *, ** and *** indicate the levels of significance at 10%, 5% and 1%, respectively. Standard errors are marked in parentheses. LL is the log likelihood. $Q_s(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the standardized residuals, distributed as $\chi^2(1)$. $Q_s^2(1)$ is the heteroscedasticity-consistent Ljung-Box statistic of the squared standardized residuals, distributed as $\chi^2(1)$. Data with 4 decimal places is used.

5 Conclusions

The Finnish stock market is discovered to be relatively well linked to the United States stock markets. Shocks and volatility spillover occur, transmitted from Finland to the United States and vice versa. The fundamental reasons for these linkages are obscure and require more testing, but clear patterns emerged to prove the existence of the linkages between these two markets.

These cross-market effects or linkages are still much smaller than the interconnections, which are based on the markets' own historical data. The conditional variances in both stock markets seem to be powerfully connected to the previous levels of variance in their own home markets. Additionally, these conditional variances are also influenced by home market shocks, whose contribution is far greater than that of the international shocks. Lagged mean variables that portray the domestic market's price movement in the conditional mean equation are also proved significant in most of the cases.

There is still potential in studying these interactions by selecting multiple much narrower time spans and ranking them. Also, selection of other international series to see the origination of some of the shocks first hand would be useful. A system to capture daily shocks from the data in an orderly manner and distribute this information or the direction of the shocks using signal variables, would be tremendous. Intra-day dynamics would be an interesting topic with added data points and matching news terms. Although, to introduce intra-day data would spell trouble and require further modifications to the models and testing in general. Lastly, using multiple indices to explain the same market or market segments could improve the research greatly.

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Appendix 1

An alternative way of expressing the log likelihood function as

$$L=\sum_{t=1}^{T} L_t \ , \qquad (44)$$

$$L_t=\frac{N}{2}\ln(2\pi)-\frac{1}{2}\ln|H_t|-\frac{1}{2}\epsilon_t^{'}H_t^{-1}\epsilon_t \ , \qquad (45)$$

where L is the joint log likelihood, L_t is log likelihood of observation t.

For clarity, this can also be represented as

$$L(\theta)=\sum_{t=1}^{T} L_t(\theta) \ , \qquad (46)$$

$$L_t(\theta)=\frac{N}{2}\ln(2\pi)-\frac{1}{2}\ln|H_t(\theta)|-\frac{1}{2}\epsilon_t^{'}(\theta)H_t^{-1}(\theta)\epsilon_t(\theta) \ . \qquad (47)$$

Appendix 2

Table 12. Software

Here is a complete listing of the software used for the data and tests that made it into this paper.

	Data Acquisition	Data Formatting	Data Modifications	Variable Creation	Graphs	Descriptive Statistics	ARCH Effects Tests	ARCH Tests	Univariate GARCH Tests	Bivariate GARCH Tests
EViews 5.1 Standard Edition - Jan 10 2007 build		✓		✓		✓	✓		✓	
Microsoft Office Excel 2003 (11.8033.6568) SP2		✓	✓							
OpenOffice.org Calc 2.4.1					✓					
RATSData for Windows (v. 6.35)		✓								
Thomson Datastream Advance Version 4.0 SP4b	✓									
WinRATS (v. 6.35)				✓				✓	✓	✓

Appendix 3

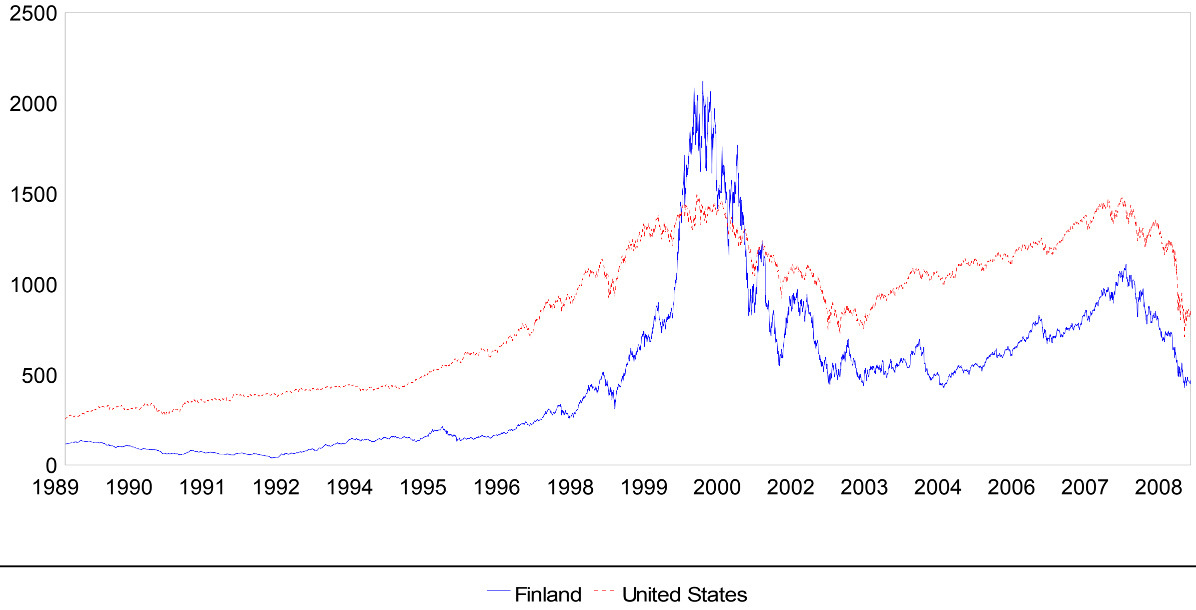

Figure 7. Original Price Series, Finland and United States

Notes: Data with 3 decimal places is used.

Appendix 4

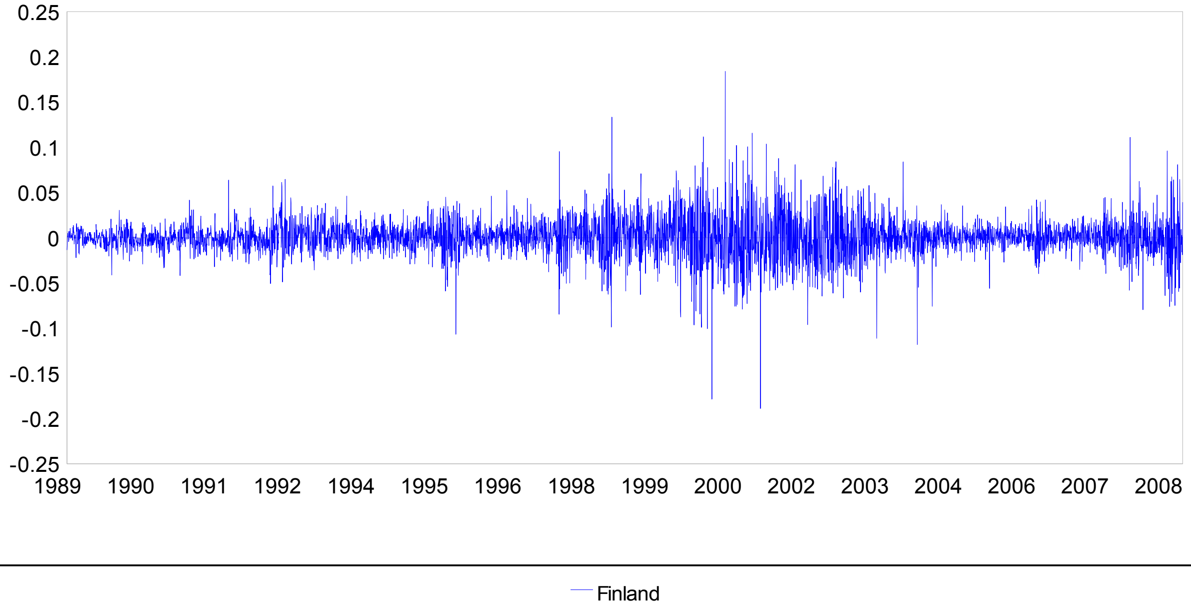

Figure 8. Percentage Returns, Finland

Notes: Data with 6 decimal places is used.

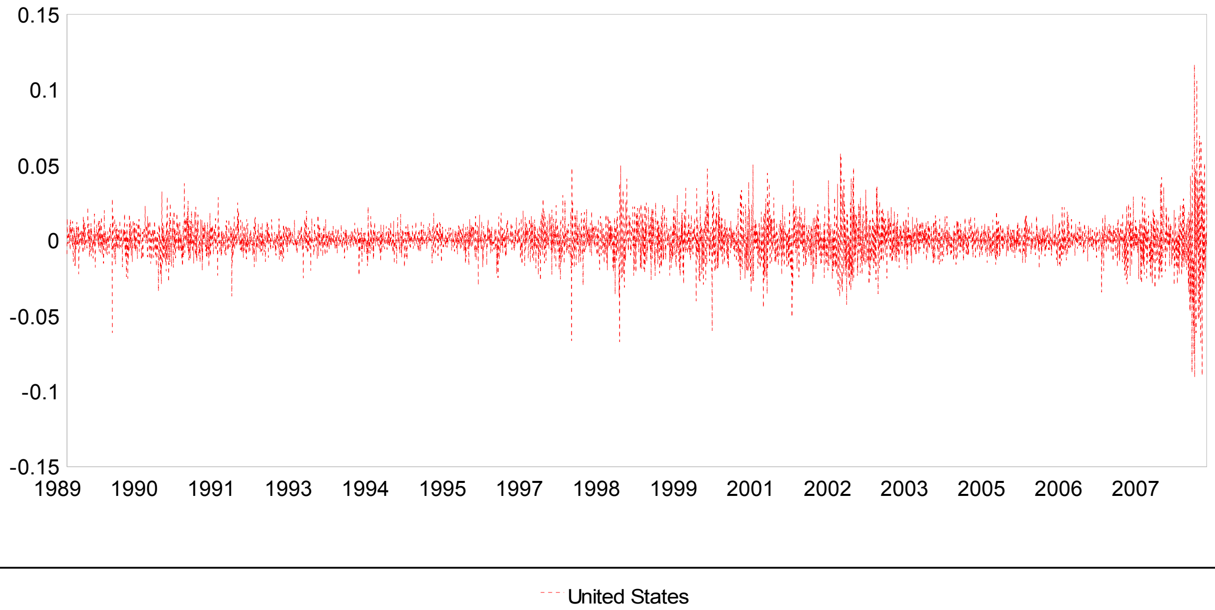

Figure 9. Percentage Returns, United States

Notes: Data with 6 decimal places is used.

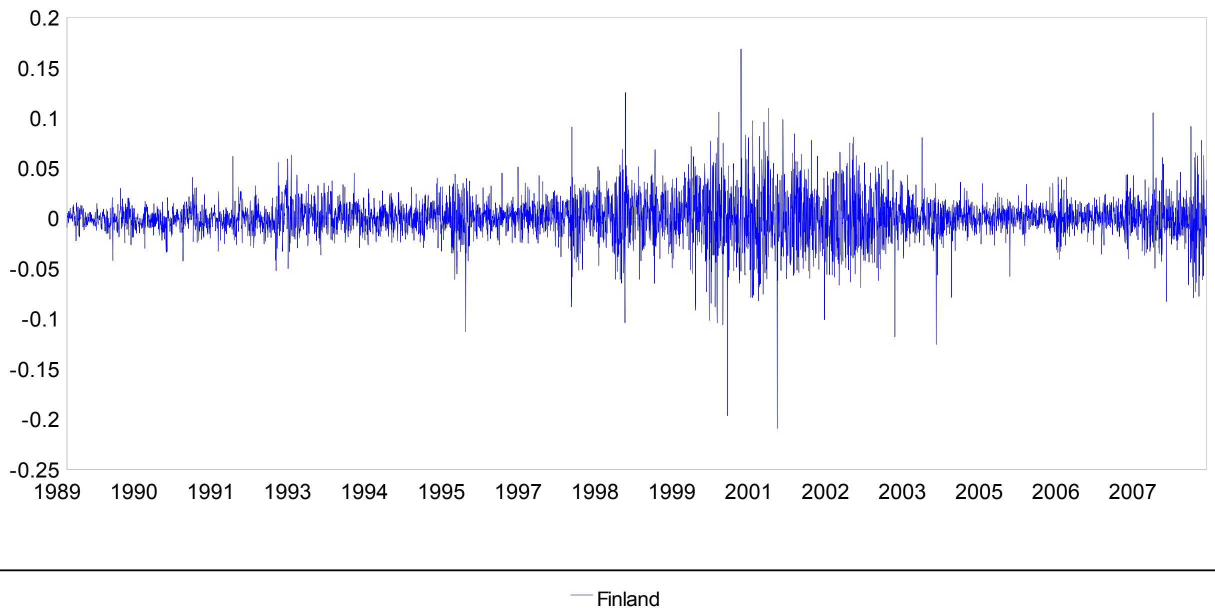

Figure 10. Natural Logarithmic Returns, Reduced, Finland

Notes: The series are reduced to the subsets of their former individual series in order to avoid missing data and zero entries in the case of simultaneous estimation of multiple series. After this, the resulting new individual series are rounded to an appropriate level, so that they will be able to perform the estimation steps adequately. Data with 4 decimal places is used.

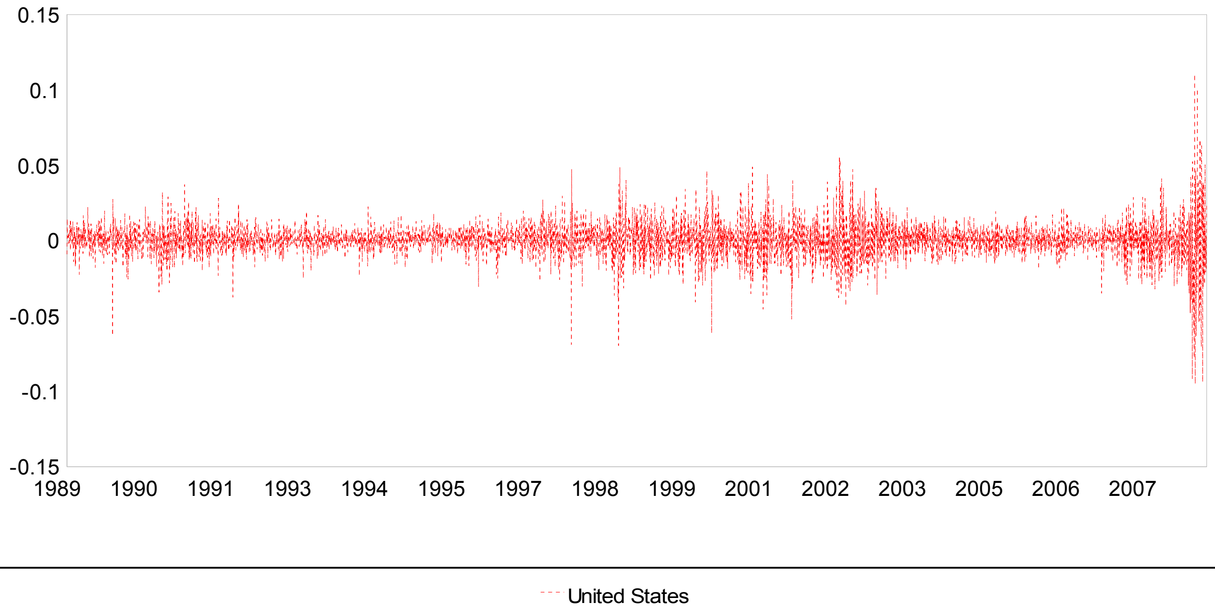

Figure 11. Natural Logarithmic Returns, Reduced, United States

Notes: The series are reduced to the subsets of their former individual series in order to avoid missing data and zero entries in the case of simultaneous estimation of multiple series. After this, the resulting new individual series are rounded to an appropriate level, so that they will be able to perform the estimation steps adequately. Data with 4 decimal places is used.

Appendix 5

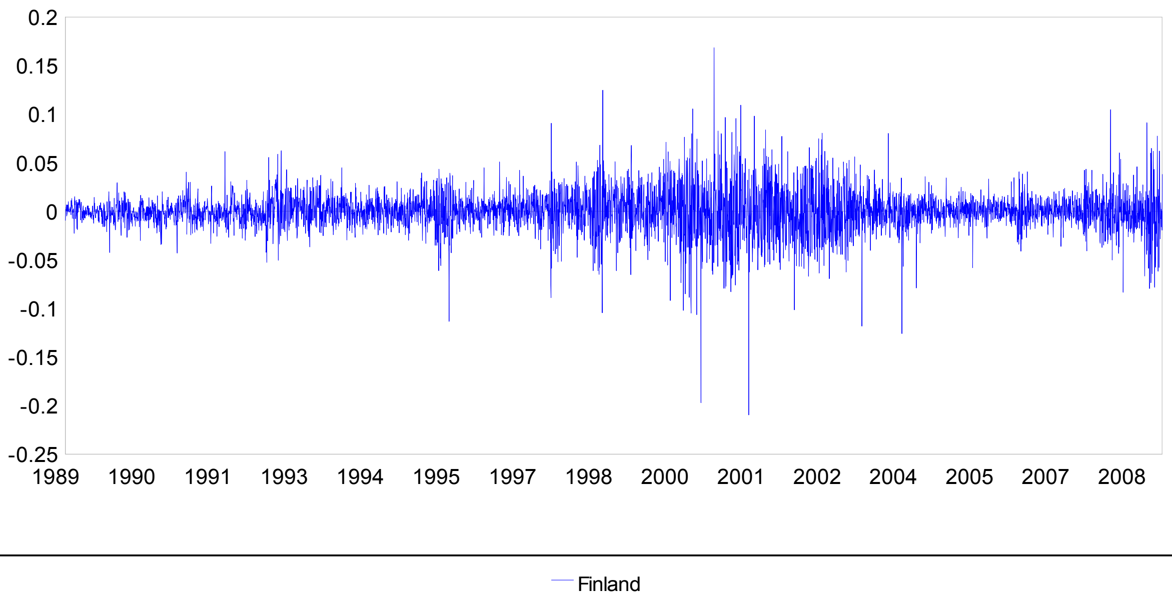

Figure 12. Residuals, Finland, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

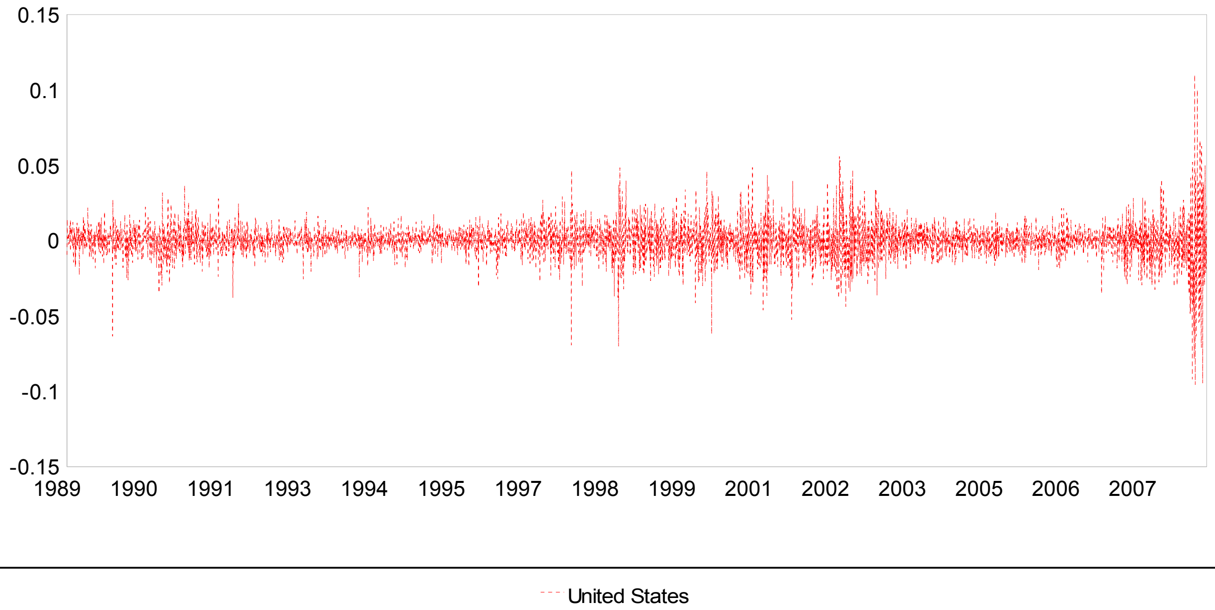

Figure 13. Residuals, United States, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

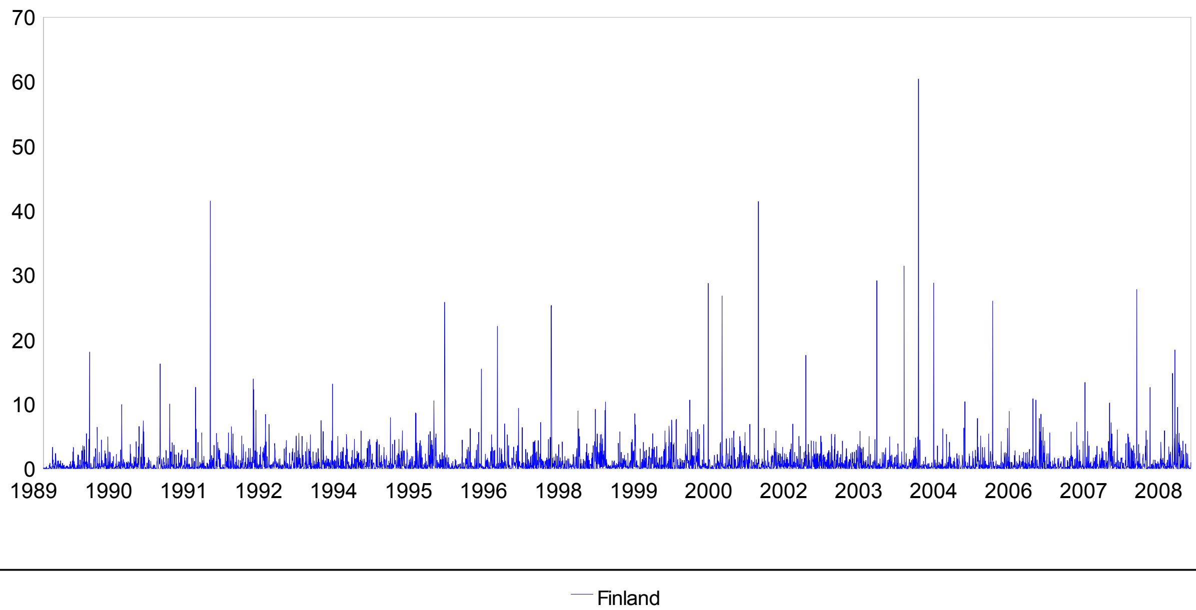

Figure 14. Squared Standardized Residuals, Finland, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

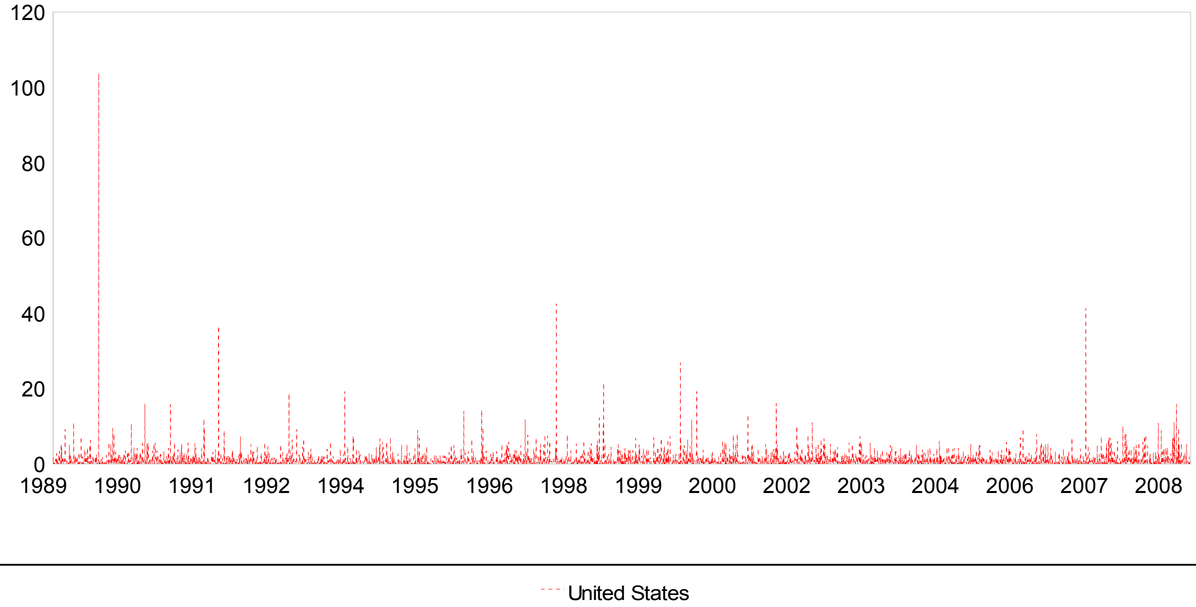

Figure 15. Squared Standardized Residuals, United States, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

Appendix 6

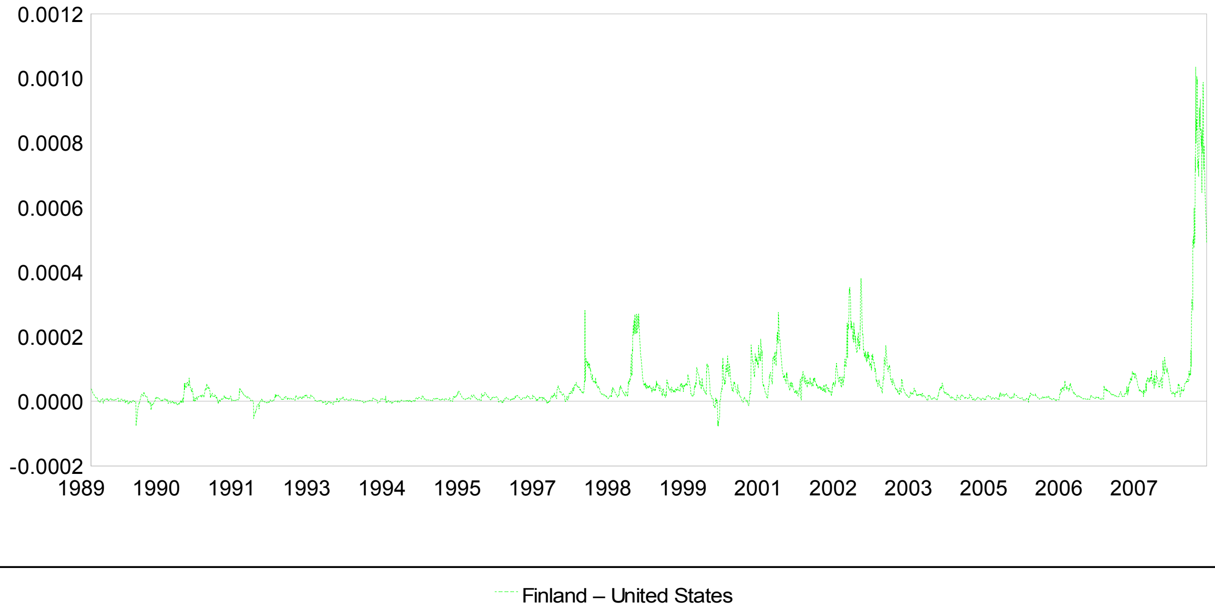

Figure 16. Conditional Correlation, Finland and United States, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.

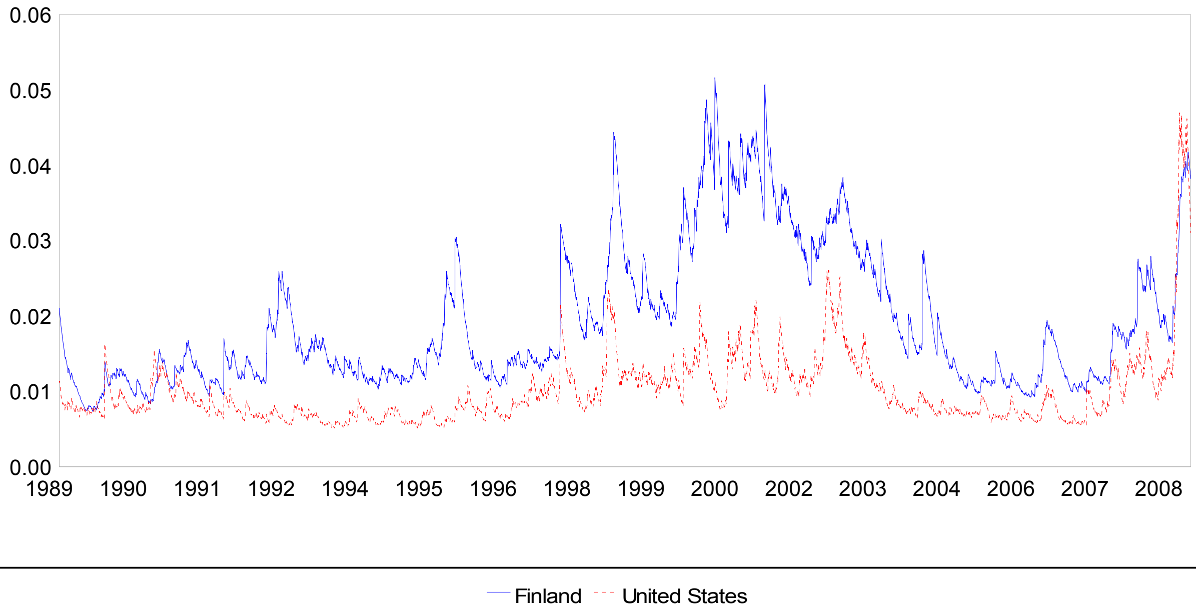

Figure 17. Conditional Standard Deviations, Finland and United States, MV(2)-GARCH(1,1), BEKK, BFGS

Notes: Data with 4 decimal places is used.