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Sami Moisio

A SOFT CONTACT COLLISION METHOD FOR REAL-TIME SIMULATION OF TRIANGULARIZED GEOMETRIES IN MULTIBODY DYNAMICS

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Science, not art.

Preface

The research for this dissertation was carried out between 2006 and 20012 at the Lappeenranta University of Technology. The research was funded by projects such as the GRASP which is a part of the European Communitys' Seventh Framework Programme and the doctoral program of Concurrent Mechanical Engineering. I would also like to thank the Research Foundation of Lappeenranta University Of Technology for the support grant i recieved.

There are many people who, more or less knowingly, have contributed to the process of writing this dissertation. The ways in which people have influenced this dissertation are plentiful and some even quite surprising. Starting from my childhood friends wondering how things work, to my sons doing "crash tests". I can not mention everyone in this preface but I will try to do justice to at least some of them by mentioning them here.

First of all I would like to thank Professor Heikki Handroos for making this dissertation possible. Without the funding he has arranged this would not have happened.

Secondly I would like to thank Professor Paulo Flores and Professor Josep Maria Font Llagunes for their work as reviewers and as opponents. It is an honor to get such distinguished opponents.

A great influence to this dissertation has been Dr. Asko Rouvinen. He threatened to "fire that hippie" plenty of times at the beginning of my studies (to my knowledge less freqently lately) and I am very glad that he did not do so. The citation by Asko "this is science, not art" has been imprinted into my brain forever. Also I would like to thank Dr. Pasi Korkealaakso for advice and guidance during the making of this dissertation. The help and opinions have been invaluable.

I would like thank Professor Aki Mikkola and his research team. They have been very inspirational in their enthusiasm and dedication. It has been a pleasure to watch your genuine enthusiasm. Especially I would like to thank Aki for all the years of cooperation. Checking papers "as a team" was always a pleasure. Also I would especially like to give my thanks to Marko Matikainen for the critical opinions.

It would be a disgrace for me not to thank the people at MeVEA. Although this dissertation might have been finished sooner without MeVEA, it has been a pleasure to work with everybody.

Last but very certainly not least I would like to thank my beloved wife Anja and my two sons Benjamin and Sebastian. Without them life would be very boring. They make my life living instead of existing.

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Sami Moisio

Abstract

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When modeling machines in their natural working environment collisions become a very important feature in terms of simulation accuracy. By expanding the simulation to include the operation environment, the need for a general collision model that is able to handle a wide variety of cases has become central in the development of simulation environments. With the addition of the operating environment the challenges for the collision modeling method also change. More simultaneous contacts with more objects occur in more complicated situations. This means that the real-time requirement becomes more difficult to meet. Common problems in current collision modeling methods include for example dependency on the geometry shape or mesh density, calculation need increasing exponentially in respect to the number of contacts, the lack of a proper friction model and failures due to certain configurations like closed kinematic loops. All these problems mean that the current modeling methods will fail in certain situations. A method that would not fail in any situation is not very realistic but improvements can be made over the current methods.

Keywords: collision, simulation, real-time, triangularized geometry collision, penalty method

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Tiivistelmä

Sami Moisio
Tunkeumallinen reaaliaikainen kontaktimalli kolmioiduille grafiikoille
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Mallinnettaessa koneita niiden aidossa toimintaympäristössä, kontaktit muodostuvat keskeiseksi tekijäksi simuloinnin tarkkuuden kannalta. Kun mallia laajennetaan sisältämään toimintaympäristö kasvaa yleisen kontaktimallin merkitys simulaattoriympäristöjen kehityksessä. Ympäristön lisäyksen myötä myös mallinnuksen haasteet muuttuvat, useampia kontakteja useampien kappaleiden välillä monimutkaisemmissa tilanteissa. Tämän vuoksi reaaliaikaisuus muodostuu yhä suuremmaksi ongelmaksi. Yleisiä ongelmia nykyisissä kontaktin mallinnusmenetelmissä ovat, riippuvuus mallinnettavasta geometriasta ja verkotuksen tiheydestä, laskentatarpeen kasvu eksponentiaalisesti suhteessa kontaktien määrään, kunnollisen kitkamallin puuttuminen sekä ongelmatilanteet erilaisissa konfiguraatioissa kuten kinemaattisten ketjujen tapauksessa. Jotkut mallit myös luottavat primitiivigeometrioiden käyttöön kontaktia mallinnettaessa, joka tekee niistä epätarkempia ja hankalampia määrittää. Tämän vuoksi on tärkeää, että kontaktimalli perustuu kolmioituun geometriaan. Kaikki nämä ongelmat tarkoittavat, että nykyiset mallinnusmenetelmät epäonnistuvat jossain tapauksissa. Menetelmän kehittäminen joka ei epäonnistuisi missään tapauksessa ei ole kovin realistinen, mutta nykyisiä malleja voidaan parantaa merkittävästi.

Hakusanat: kontakti, reaaliaika simulointi, kolmioitujen geometrioiden kontakti

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CONTENTS

a	Contact area radius rigid sphere-plane
$\boldsymbol{A}^i$	Rotation matrix from the body i reference frame to the global reference frame
$\widetilde{\boldsymbol{A}^i \bar{u}^{P_i}}$	Skew-symmetric matrix of body i
$\widetilde{\boldsymbol{A}^j \bar{u}^{P_j}}$	Skew-symmetric matrix of body j
A_{stiff}	Contact area rigid sphere-plane
C	Viscose damping coefficient
$\boldsymbol{C}_q$	Jacobian matrix
c_r	Coefficient of restitution
d	Distance vector in the direction of $n^{P_{ij}}$
$\dot{d}$	Velocity in the direction of $n^{P_{ij}}$
E	Young's modulus
E_k	Kinetic energy
F_C	Contact Force
$\boldsymbol{F_C}^i$	Total contact force including normal and tangential forces applied to body i
F_C^j	Contact Force applied to body j
F_F	Flores collision normal force
$F_{Floresmod}$	The modified Flores normal force with L_r rigid sphere-plane
F_{HC}	Hunt-Crossley collision normal force
f_n	Normal force magnitude i
$\boldsymbol{F_n}$	Normal force produced by the soft contact
F_{LN}	Lankarani-Nikravesh collision normal force
F_{nHertz}	Hertzian contact normal force
$F_{nViscose}$	Hertzian contact normal force with viscous damping
$F_{nTriangularized}$	The modified Flores normal force with L_r rigid sphere-plane
F_{mod}	The modified Popov normal force rigid sphere-plane
F_{modFr}	The modified Popov normal force with L_r rigid sphere-plane
F_{popov}	The normal force according to Popov rigid sphere-plane
f_t	Tangential friction force magnitude i
$\boldsymbol{F_t}$	Tangential force represented by friction

χ_{HC}	Hunt-Crossley hysteresis damping factor
K	Hertzian generalized stiffness parameter
K_p	Generalized stiffness
L_c	Collision area circumference
L_{sec}	Collision area circumference sector length
L_r	Collision area circumference correction factor
m	Mass
$\boldsymbol{M}$	mass matrix
m_1	Mass of body 1
m_2	Mass of body 2
n	Hertzian contact exponent
$\boldsymbol{n}^{P_{ij}}$	Normal vector of the contact
$\boldsymbol{P_i}$	Contact point i
$\boldsymbol{q}$	vector of n generalized coordinates
$\boldsymbol{Q}^e$	vector of generalized external forces
$\ddot{\boldsymbol{q}}$	vector of accelerations of n generalized coordinates
$\boldsymbol{Q}^C$	vector of generalized contact forces
$\boldsymbol{Q}^\nu$	quadratic velocity vector that includes velocity dependent inertia forces
R_i	Radius of sphere i
R_j	Radius of sphere j
$\boldsymbol{R}^i$	Center position vector of body i
$\dot{\boldsymbol{R}}^i$	Velocity vector of body i
$\boldsymbol{r}^{P_i}$	Position vector of contact i in the global reference frame
$\dot{s}$	Relative velocity
$\boldsymbol{s}$	Distance vector between contact points
$\dot{\boldsymbol{s}}$	Relative velocity between contact points
$\boldsymbol{s}^T$	Transpose vector of s

$\bar{\boldsymbol{u}}^{P_i}$	Position vector of the contact i within the body reference frame
Δv_1	Change of velocity applied to 1
Δv_2	Change of velocity applied to 2
v	Velocity
x_0	Stribeck velocity
z	Bristle deflection
$\boldsymbol{\lambda}$	vector of Lagrange multipliers
$\delta \boldsymbol{W}^{C^i}$	virtual work of the contact forces
$\delta \boldsymbol{q}^{P^i}$	virtual displacement of the position vector of the contact location
υ	Poisson's ratio
δ	Penetration depth of the colliding objects
$\dot{\delta}$	Relative velocity of the colliding objects
$\dot{\delta}^-$	initial contact velocity
σ_0	Stiffness coefficient of the contacting surfaces
μ_s	Static friction coefficient
μ_d	Dynamic friction coefficient
σ_1	Friction damping coefficient
$\tilde{\boldsymbol{\omega}}^i$	Skew-symmetric matrix of the body i angular velocity
$\tilde{\boldsymbol{\omega}}^j$	Skew-symmetric matrix of the body j angular velocity
χ_{LN}	Lankarani-Nikravesh hysteresis damping factor
χ_F	Flores hysteresis damping factor

CHAPTER 1

Introduction

When modeling machines in their natural working environment collisions become a very important feature in terms of simulation accuracy. The internal contacts of a mechanism have always played a crucial part in modeling mechanisms but it has been possible to make specialized solutions in order to solve for them like Shabana et al. [66] did for wheel/rail contacts or Moisio et al. [52] for tactile sensor simulation. By expanding the simulation to include the operation environment, the need for a general collision model that is able to handle a wide variety of cases has become central in the development of simulation environments. With the addition of the operating environment the challenges for the collision modeling method also change. More simultaneous contacts with more objects occur in more complicated situations. This means that the real-time requirement becomes more difficult to meet. Common problems in current collision modeling methods include for example dependency on the geometry shape or mesh density, calculation need increasing exponentially in respect to the number of contacts, the lack of a proper friction model and failures due to certain configurations like closed kinematic loops. All these problems mean that the current modeling methods will fail in certain situations. A method that would not fail in any situation is not very realistic but improvements can be made over the current methods.

Generally, contact modeling can be divided into collision detection and collision response algorithms [53]. Collision detection is only used to find out if bodies are colliding and where, whereas collision response resolves how the bodies react to the collision. The two tasks are closely tied together since the collision detection algorithm has to provide the collision response with the correct data.

The collision detection can be divided into two areas: collision detection between triangles and providing the temporal memory of the previous collision states to the collision response algorithm. Traditionally the contact detection only handles the detection of intersecting objects but in this dissertation providing the temporal memory for the collision response was also assigned to the collision detection. As the collision detection needs to provide the appropriate information to the collision response the addition of the temporal history to it is natural. The existing collision detection algorithms have not been interested in the collision history of the objects as most of them have not been directly linked to a collision response model. Collision detection methods have only been focusing on the computational efficiency of the models. On the other hand the collision response models have not had the collision history available and therefore the information has not been used. In a sense the collision history has been in between the two segments of collision detection and collision response.

The collision response model can also be divided into three areas of interest: the collision normal force model, damping model and the friction model [26]. Each of these areas present different problems for the collision response model. All of the different areas of collision response have been widely investigated. Basically collision response modeling determines how the two colliding objects behave during the contact period. The objects can penetrate or the collision can be instantaneous depending on the modeling method. Various different properties need to be taken into account during the collision period such as colliding object geometry, initial velocity and material properties [40]. All of the different properties add complexity to the model and taking them all into account becomes very complicated.

1.1 Real-time collision modeling

The real-time requirement is a major challenge for a collision detection algorithm especially in a case with triangularized geometries. The triangularized geometry data needs to be solved very efficiently to stay within the real-time requirement. This usually means some sort of a sorting algorithm like collision trees. Collision detection is considered such an important aspect of physics simulations that software libraries have been constructed to perform collision detection alone [10, 36, 51, 60, 11]. In the context of this thesis collision detection refers to the process of comparing two rigid-bodies, detecting whether they penetrate and then providing the collision response with the required data. In order to meet the real-time requirement the collision detection is divided into two phases: a broad-phase and a narrow-phase. In the broad-phase the number of intersections is reduced by using various culling techniques [19]. The narrow-phase then identifies which pairs of objects are intersecting and in another step calculates the collision information for the collision response. The open source or commercial collision detection libraries usually focus only on detecting the collisions whereas they should accommodate the collision response method with the exact information needed for the calculations. Therefore the collision response method would also limit the selection of collision detection algorithms. In Fig. 1.1 is presented a typical collision tree advancing from broad phase to narrow phase methods. The trunk level can be for example to detect if two vehicles are in close proximity, the branch level could be to see if the bumpers of the vehicles collide and the primitive level could be the triangle-triangle tests for the bumpers. Typical methods for the different levels are for example a spatial partitioning method for the trunk level, bounding volumes for the branch level and for example triangle-triangle detection on the primitive level.

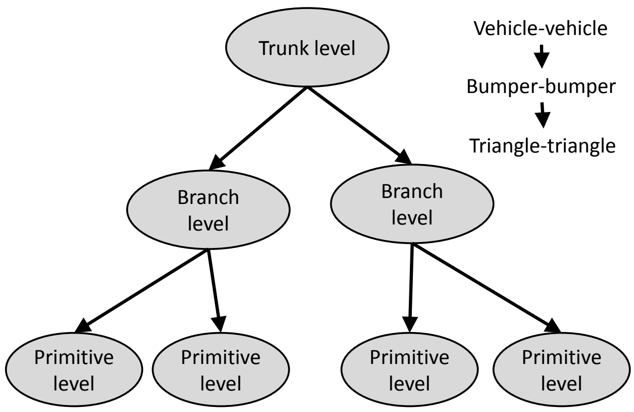


Figure 1.1. Typical collision tree advancing from broad phase to narrow phase.

The real-time collision response poses an equally challenging problem as the collision detection. The challenges arise from the demand for fast calculations and stability under different configurations. Regarding fast calculations the different collision response methods produce a wide range of results in different configurations. Some methods depend on the configuration and for example can fail when kinematic loops occur. Also some methods increase in complexity at a worst case scenario of $O(m^4)$ where m is the number of contact points [16]. This makes the use of such algorithms very difficult in multibody-multicontact scenarios. A general collision modeling method should be able to handle any simulation configuration without failing. In real-time simulations often the iteration in order to find an exact collision instance is not possible especially with methods that require for solving the entire system. This also limits the use of some algorithms. In general the requirement for a real-time solution that would be as independent of the simulated scenario as possible poses some very serious demands on the collision response modeling method. The stability of the collision response method also is challenged by the real-time requirement. Calculating more complex environments increases the time of a single calculation cycle requiring the increase in time-step size in order to keep the simulation real-time. The increase in the time-step in turn causes stability issues especially with very stiff systems such as collisions. All these aspects need to be considered in order to make an effective collision response method.

1.2 Scope of the work and outline of the dissertation

The motivation of this study is to present a novel method for solving the collisions of triangularized geometries in a real-time simulation environment. The method is a penalty based soft contact model. The soft contact method allows the objects to deform very locally around the collision enabling the formation of a proper collision surface between two rigid bodies. This enables for a more accurate description of the colliding forces over the contact period compared to the non-penetrating contact models. The non-penetrating models are more accurate in terms of energy conservation but for cases such as holding torque around the contact or friction the penalty methods are better.

The proposed method presents solutions to some of the most common problems in soft collision modeling such as determining the normal directions and defining the normal force magnitude.

Chapter 2 discusses Real-time simulation environments and Multibody Systems (MBS) in general and reviews the need for developing a collision modeling method to further develop the current simulation environments.

Chapter 3 discusses the collision modeling in multibody simulations. The chapter covers the main areas of collision modeling: collision detection and response methods.

Chapter 4 discusses the common issues related to geometry based collision modeling.

Chapter 5 discusses the main areas of collision modeling from the perspective of the developed method. Section 5.1 covers the collision detection as well as the solution for providing the temporal history to the collision response. Section 5.2 discusses the proposed collision response method covering the common problems of penalty methods: collision point placement, collision normal definitions, collision normal force magnitude and the friction modeling.

In chapter 6 some numerical experiments using the proposed method are described. The experiments range from simplified mathematical examples, to more complicated scenarios. The simplified models are used to validate the proposed model against Hertzian collision models and the more complex models show the functionality of the model in a general case.

1.3 Contribution of the dissertation

The following original contributions are presented in this dissertation:

1. Introduction of a method for providing temporal history, in a triangle-triangle collision case. The temporal history can be utilized in many ways, mainly through the possibility of parameter integration. In the method presented in this study the temporal history is utilized in the dynamic friction model and the collision penetration calculations.

2. Introduction of a triangularized collision model with a dynamic friction model. The dynamic friction models have not been utilized in triangularized collision models because of the lack of temporal history.

3. Introduction of a novel method for placing collision points, solving the normal directions in the collision points and solving the penetrations at the collision points. The proposed method simplifies the problematic areas leading to the model performing consistently regardless of the collision scenario.

4. Introduction of a novel method for solving the collision normal force magnitude based on the collision area circumference. The method utilizes the collision circumference data from the collision detection to determine the

geometric stiffening of the collision normal force method. The method is based on the Hertzian contact theory and is a modification of it. The use of the collision area circumference eliminates the need to know what kind of geometry is colliding in calculating the normal force magnitude.

CHAPTER 2

Real-time simulation environments

Real-time simulations are growing in popularity as its' possibilities are gradually becoming clear for the industry. Initially the development of different simulators has originated from the military in the form of different flight simulators. An excellent example of saving material and operation costs as well as improving the operator safety. Recently the medical community has started developing a number of different simulators in order to train doctors and other personnel [23]. For example different surgical procedure simulators have been developed which is also an excellent example of how to benefit from simulations. Instead of risking lives by training on patients the surgeon can be made familiar to the operation using an advanced simulator environment the same way as an astronaut will train on a simulator.

2.1 Training simulators

Recently the heavy industry has also developed an interest in simulator training. When a machine operator can be trained to use a machine with a simulator instead of a real machine a number of benefits can be gained. The real machine is not tied up in training but can be used in production the whole time. With a real machine the training times tend to be as short as possible in order to keep the machines in production. On the other hand, with a simulator, the training can be done in a stress free environment using as much time as is needed in order to get the feel for the machine.

In simulator training a driver is also allowed to make mistakes. With a real machine making mistakes has to be carefully avoided because it usually results in expensive repairs. A relatively minor damage to a machine can easily cost tens of thousands of euros which pays for a lot of simulator training hours. Being able to make mistakes also makes it possible to test the limits of the machine as well as to show the consequences of incorrect actions. Most modern machines have some sort of safety equipment ensuring operator safety. With a simulator the driver can actually try out how the safety measures work and what kinds of situations are potentially dangerous. Visually seeing the actual situation and being in it helps the driver to prevent such conditions with the real machine. Simulator training also ensures the safety of the environment in addition to the safety of the driver and machine. For example in a harbor environment it is virtually impossible to fence off a safe area for the trainee to practice in.

One of the efficiency aspects of using a simulator is that the trainee can question the methods being used currently. New ways of operating the machine can be tested to see if they might be better than the methods currently being used. In the same way the trainer can show the trainee that the methods he is using are ineffective. Due to the availability of different quality/quantity meters in a simulator environment the trainer can show the trainee the better method of operating the machine. This feature is especially useful for the already experienced machine operators. By presenting factual information on the better operating method the driver learns the method much easier.

Sometimes a machine is so difficult to handle that some people just cannot learn to operate the machine properly. Very much like driving a car most people can learn to operate the machine but not everybody can be good at it. This aspect is very cost ineffective when training the operators using real machinery. Using simulator training the unsuitable operator trainees can be cut out at a much earlier phase of the training without wasting money. Usually a company buying a machine also needs to train the operator for the machine in question. Using simulators the training can be started at the same time as the machine is ordered. This means that as soon as the new machine is available the operator is already efficient in using it. Also in the hiring process suitable candidates can be screened using a short simulator training period in the same manner as other psychological tests.

2.2 Production development simulators

The benefits of a simulator for production development purposes can offer huge advantages. For example it is possible to make just one prototype to verify the final version of the simulated construct. This has been possible using offline simulators as well but the operator feedback has been possible only using the prototype. The latest simulation technologies enable the user feedback already before a single part of the real machine has been manufactured. Using a real-time simulator with a large scale environment the usability of a new construct can be tested including the user feedback. Using a motion platform and other immersive visualization techniques the feedback can be very accurate.

One area that can benefit tremendously from real-time simulation environments is the software development of machines. Modern simulators can be verified to function within 10% accuracy of the real machines. This means that the control software can be developed using a verified simulator environment without the need for large test facilities or hardware. The effect of, for example different acceleration ramps, can be tested quickly and easily.

Simulators can also be used as measuring devices in cases where some property is very difficult to measure. For example the joint forces are very difficult if not impossible to measure in machines. Using a simulator the joint forces can be monitored during the operations and the effect of the operator on the durability of the machine can be evaluated. Some new developments have been proposed where the stresses of the real-time simulation model could be evaluated in real-time.

2.3 Real-Time Multibody Systems (MBS) a brief overview

A common method for creating dynamic simulation environments is Multibody System Dynamics (MBS). A Multibody System consists of rigid and flexible bodies and joint constraints that connect the bodies. A Multibody simulation also usually has external components such as actuators and external forces. The multibody simulation approach uses numerical methods to solve nonlinear equations of motion with respect to time. Using modern computer technology large mechanisms can be modeled accurately using the MBS approach. Shiehlen [61] presents a good review of multibody system dynamics as well as Shabana [63] and Yoo [71]. Also several books have been written regarding MBS over the years [56, 31, 65, 64, 55, 62].

A well known form of equations of motion for constrained multibody systems applying Lagrange multiplier technique can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}_q^T\boldsymbol{\lambda} = \boldsymbol{Q}^C + \boldsymbol{Q}^e + \boldsymbol{Q}^\nu \tag{2.1}$$

where $\boldsymbol{q}$ is the vector of n generalized coordinates, that define the position and orientation of each body in the system, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{Q}^e$ is the vector of generalized external forces, $\boldsymbol{Q}^C$ is the vector of generalized contact forces, $\boldsymbol{Q}^\nu$ is the quadratic velocity vector that includes velocity dependent inertia forces, $\boldsymbol{C}_q$ is the Jacobian matrix of the constraint equations and $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers. In Eq. 2.1 the contact forces are substituted to the vector of generalized contact forces. When contact forces and their locations for bodies are solved, the generalized contact forces can be extracted using the principle of virtual work. The relationship between the contact forces and the generalized contact forces can be defined as follows:

$$\delta \boldsymbol{W}^{C^i} = \int_{V^i} \delta \boldsymbol{r}^{P^{i^T}} \boldsymbol{F_C}^i = \delta \boldsymbol{q}^{i^T} \boldsymbol{Q}^{C^i} \tag{2.2}$$

where $\delta \boldsymbol{q}^{P^i}$ is the virtual displacement of the position vector of the location of the contact and $\boldsymbol{F_C}^i$ is contact force including normal and tangential forces for body i.

The different multibody formulations are not discussed in detail in this dissertation as there is no development proposed to MBS methods themselves. The significant factor in the different formulations is the method in which the external forces can be applied. In order to utilize the advantages of the penalty based collision method external forces need to be applied efficiently and accurately to the system.

Multibody system dynamics can be applied to create real-time simulation environments such as [44]. The real-time simulations present challenges for the multibody formulation effectiveness. The equations need to be efficient in order for the simulation to remain real-time. Different formulations have been presented that are dedicated especially for real-time simulations [13, 24]. These formulations enable the creation of larger simulation environments while remaining within the real-time requirement.

The real-time MBS methods enable to create the mechanism models for the simulation environments but in order for the environment to be interactive a proper collision modelling method is required. An extensive simulation environment requires the collision modeling to be accurate, efficient and robust enough to handle a wide variety of different configurations. For example operator training on a simulation environment without propoer collision modeling is quite useless. This need has lead to the development of the collision modeling method presented in this dissertation.

CHAPTER 3

Collision modeling in multibody simulations

The collisions in multibody simulations have been researched extensively throughout the existence of MBS methods. Proper collision models are required as the models become more and more complex encompassing larger environments and a wider range of events. A multitude of different methods have been presented based on different requirements and assumptions. For example Gilardi [26] and Drumwright [17] have made recent surveys on different models. The characteristics of each model are defined by the initial conditions that are set on what the model should be like. Whether the collision should be allowed to penetrate or not for example changes the principles of the modeling completely. The friction model requirement adds more restrictions and demands on the modeling method. Different collision models perform differently depending on the problem in question, some better some not so well, all depending on what they were designed for.

The previous research on collision modeling can be divided into three categories: collision detection, collision response models and friction models. Each of these categories encompasses a huge amount of previous research relating to them which is why only the very relevant work is referred to in this work. Therefore for example the analytical and impulse based collision response methods are only covered here superficially as the penalty method was chosen as the basis for the new modeling method.

The terminology in contact modeling is quite complex [9]. Some terms have been misused and misunderstood quite often. Also a wide variety of terms can be used to implicate the same thing. The terms get more complicated when incorporating a contact model to a multibody simulation environment as well as when adding friction terminology. In multibody simulation a rigid body is a non-deformable body whereas in contact modeling a rigid body is a body that cannot be penetrated. The use of the rigid body term in connection to a non-penetrating contact can cause misunderstandings. The deformation of a body can take place in two ways: the equations of motion description of flexibility on the body level or as a local deformation in the contact equations. The deformation in the contact equations does not change the shape of the object but it is used in order to account for the local deformations in the colliding objects caused by the collision alone. This will also cause the two rigid bodies to penetrate each other. The flexibility on the body level can be defined for example by using the floating frame of reference method [65] to incorporate the body flexibility. This body level flexibility will change the shape of the object. A body that can change shape can be called a flexible, soft or compliant body. Quite surprisingly the only term used for bodies that do not change shape is rigid body. The definitions of the terms for allowing two colliding bodies to interpenetrate are a lot more complex. A number of different terms can be used to express the same thing depending on the context. Non-penetrating contacts can be called rigid contacts, rigid body contacts, discrete contacts or colliding contact. Penetrating contacts can be called soft contacts, continuous contacts or non-colliding contacts. Also the terminology in the contact event is somewhat confusing. The terms collision, contact and impact are all used slightly differently depending on the source [53, 9, 26]. The term contact is usually referred to as the bodies are interpenetrating or touching each other for an extended period of time. Collision is used to describe the state before the contact where the objects come to contact. The term impact refers to the entire collision event starting from the instant the object come to touch each other to the instant they depart. Collision and impact are often used with a similar meaning of describing the entire collision event. The terms collision and contact are not discernable by anyone who has not been dealing with collision models. Therefore the use of the terms can be confusing. In order to clarify the meaning of the terms in this study the terms rigid and flexible body are used to describe the overall body deformability of objects. The terms soft and rigid contact are used to describe whether the two colliding objects are allowed to penetrate. The terms collision and impact are used to refer to the entire collision event and the term contact as the state where the two bodies are touching each other. The friction modeling also has some terminological problems. The term dynamic friction can be used in conjunction to sliding states in friction modeling or it can be used to express if the friction model needs temporal history. The accurate terms would be dynamic friction and dynamic friction models. The same applies to the term static friction. The term can be used to express the frictional force in a static case as well as a reference to a static friction model. These two should not be confused.

3.1 Collision detection

In computer graphics the geometries are often presented in a triangularized form. This is due to the fact that a triangle always forms a plane whereas for example a quad is not always a plane. The planarity helps solve reflections and lighting and also aids in the collision detection. The use of parametric surfaces is not common in simulations unless the application requires exact precision. Although it would be more accurate to present shapes and surfaces analytically it is not always possible and almost never easy. In most cases a triangularized assumption of the form does not pose a problem. In some cases for example rolling pin joints this can present a problem and another approach is recommended. In a general case the triangluarized geometries offer freedom of form and a flexible method for representing geometries.

In itself the collision detection algorithm is usually a very straightforward mathematical procedure. What makes it difficult is the real-time requirement. The description of a triangularized shape consists of a vast amount of data which makes the collision detection very time-consuming. Simple straight surfaces are very simple and efficient to calculate, but even a simple curving surface can cause problems. Due to the vast amount of data required to describe a triangularized shape, it is impossible to perform primitive collision testing for all triangles in real-time. Therefore, it is necessary to use other methods to exclude bodies and body parts that for certain are not in contact with each other from the primitive level collision testing. Various collision detection hierarchies, such as trees, or spatial partitioning techniques are normally used for this purpose [19, 20]. The collision detection is not a main focus of this study and therefore the collision detection algorithms are not extensively covered here. For surveys on different collision detection methods see [43, 39].

In addition to the intersection queries the collision detection algorithm is tasked to provide the necessary data for the collision response algorithm. In the case of penalty methods this usually means the positions, penetration depths and normal directions. Some methods also require some more complex calculations such as solving for a point inside the intersecting volume and other inside/outside calculations.

Temporal history in the collision response has not been previously used in a triangle-triangle collision case. Some spring based friction models have been using temporal history on a solid mesh vertice but a model where the temporal history of a contact in a general triangularized case could be utilized has not been presented before. Tang et al. [68] report a model utilizing face-vertice and edge-edge collisions pairs that track the collision path although the tracking model is not presented in and therefore can not be evaluated.

Without the temporal history the collision states are solved separately without linking the current state to the previous state leading to assumptions in the collision response modeling. In order to utilize integrable variables such as used for example in dynamic friction models the temporal history has to be enabled. Also the normal force models can utilize the integrability of parameters.

3.2 Collision response

Collision response models can be divided into three different categories: analytical [3, 25], impulse [45, 50, 29] and penalty methods [34, 37, 42, 53, 21, 30, 46, 70, 16]. The analytical and impulse methods are using the rigid contact assumption whereas the penalty methods are soft contacts. In the impulse based approach contacts between bodies are considered as a collision at a specific instant in time without the need to solve contact forces meaning that the change in the object velocities is applied directly to the bodies over one time-step. The method is fast and easy to implement but problems arise with steady contacts in static configurations as well as with multibody-multicontact cases [26]. Analytical methods are based on the use of constraints to handle contacts. In contrast to impulse based methods the method is stable in steady contacts but, however, due to simultaneous solving of all contacts it is also computationally expensive and the addition of frictional forces is troublesome as it can lead to unsolvable configurations [4, 58, 7, 16, 18, 1]. Penalty methods allow for small penetrations in the colliding objects and therefore can be categorized as soft contacts. The collision is represented by placing temporal spring-dampers at the contact points (see Fig. 3.1). The contact force is then applied to the model as an external force which does not affect the dimensions of the equations of motion. Brogoliato et al. [7] presented a comprehensive survey of the different methods and recently Drumwright and Shell [17] performed an evaluation of some of them quantifying the performance with respect to robustness and speed.

Analytical and impulse methods give accurate descriptions for contacts and are often used when no interpenetrations are allowed between the contacting bodies.

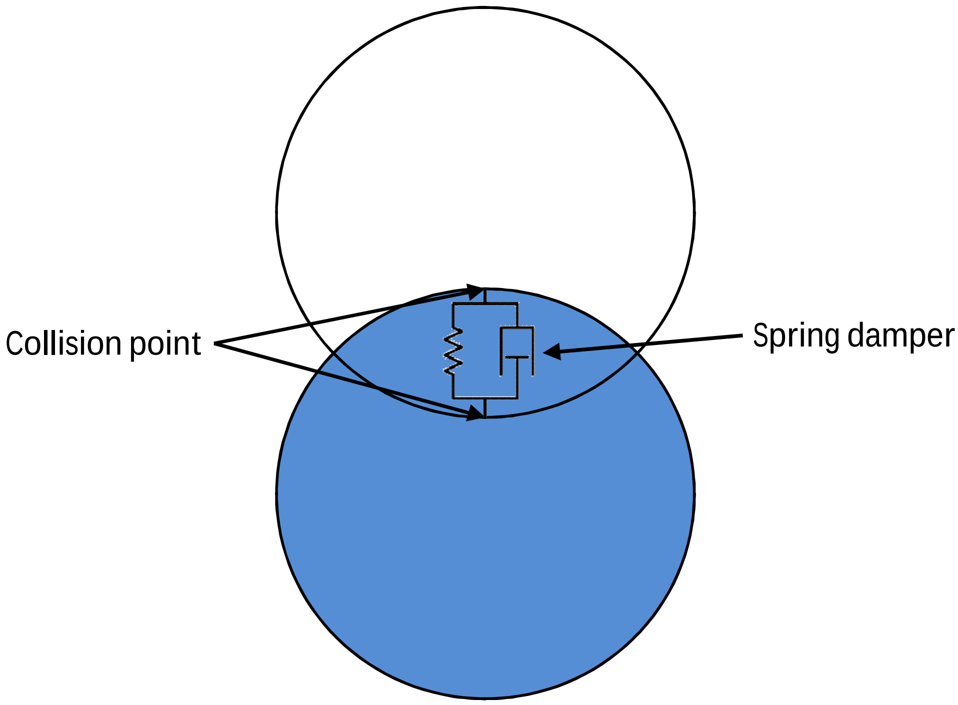


Figure 3.1. A temporal spring damper positioned between two collision points.

These methods allow larger time-steps compared to penalty methods with stiff springs. However these methods lead to complicated equations especially in multibody-multicontact situations with friction. In terms of energy conservation the analytical and impulse based methods are more accurate than the penalty based methods.

In a soft contact where two rigid bodies collide the deformation of the bodies requires always some sort of an assumption. In a real collision both bodies deform locally or in a larger scale to form a unified collision surface. This is not the case when dealing with rigid bodies. Solving the deformation of the bodies is currently too time consuming in order to calculate in real-time. Therefore the soft contact approach has been introduced where the local deformation is taken into account by letting the objects penetrate each other. It enables the creation of a collision surface without modeling the actual deformation of the bodies. A proper collision surface offers possibilities for modeling holding torque around the colliding surface as well as making the collisions more realistic in comparison to non-penetrating point contacts. For example the surface pressure can be a decisive factor in some applications. In order to solve for surface pressure between two rigid bodies they need to be allowed to interpenetrate.

3.2.1 Analytical and impulse based methods

The analytical and impulse methods appear very similar when comparing their general assumptions. They both are rigid contacts where the collision takes place instantly. This assumption gives both models some similar characteristics. The basic property of non penetration makes the formulation of a friction surface and holding torque around the colliding surface difficult as the contact is always occurring at one point in time and space. Some models account for the friction around the contacting point in the formulations but they are not accurate in general cases. Also modeling stiction around the collision point is not possible.

As the collision is instantaneous, integration of parameters through temporal history becomes impossible. Even at resting contact the discrete methods come out of contact after each contacting time-step. This will break the integration. This means that the dynamic friction models cannot be used in the discrete collision models.

In large scale real-time simulations the collision modeling method should not be connected to the dynamical model complexity. Multibody-multicontact situations will often cause analytical collision response modeling methods to become excessively slow as the system matrices will become too big. In the same manner multiple simultaneous collisions can cause problems for the impulse based methods as the impulse propagation order is compromised. For example if a walking robot has both feet on the ground a closed loop is formed between the robot legs and the ground can lead to inconsistencies in the impulse propagation order. By separating the equations of motion from the collision modeling the possibility for parallelization and further speed optimizations improve greatly.

3.2.2 Penalty methods

There are two types of penalty methods, normal force methods and geometry based models. The normal force methods only deal with a single force model that will account for the entire collision occurrence such as [37, 33, 46, 21] and do not formulate multiple collision points or other geometry related aspects.

The geometry based models such as [42, 53, 30, 34, 16] take into account the geometry formulating collision points, normal directions and then constructing the colliding force based on that data. Normal force models are only applicable to certain limited collision cases. For example the Hertz model can only be used with perfectly elastic spheres. The geometry based models on the other hand are usually applicable to general cases.

The first modern penalty methods for multibody simulation were initially presented by [42, 37, 46] which were all normal force models. In order to incorporate the penalty method for simulation models that would also account for the colliding object geometry the first geometry based models were presented by [42, 53]. Since then many others have used the formulation [30, 34, 16] in geometry based simulations with different initial assumptions. The basis for all of the penalty methods is that spring-damper elements are placed at the contact point/points. The main difference between the several geomery based models is where to place the collision points, how to define the normal direction of the force and how to define the normal force magnitude. At the same time those are the main problems of the penalty methods in general. Different methods range from placing contact points to points of deepest penetration to assigning each penetrated vertex a force element to creating collision points based on the interpenetrated volume.

3.3 Normal force models

A normal force model determines how the collision surface normal force behaves during the collision. Normal force models deal only with the amount of force generated by the collision. The effect of the geometry on the collision is included in the normal force model but it is assumed that the geometry is of a certain type such as two externally colliding spheres.

The modern normal force models can be considered to initiate from Heinrich Hertzs model in 1880's [33, 40] when Hertz studied the behavior of two frictionless spheres colliding. The normal force model forms the basis for any collision model as it also effects the friction in the model through the static friction relation. The normal force model can be made to include the damping properties of the colliding objects as well. The initial Hertz model does not include any damping in the system meaning that the colliding spheres were considered as perfectly elastic. The inclusion of the damping in the normal force is a natural process but it should not be forgotten that the tangential friction in the collision can also add a damping component in the collision model. Therefore all of the damping in the system cannot be included in the normal force model. In the mathematical models dealing with wither frictionless models or models that have no tangential displacement have damping only in the normal direction.

A short review of some of the different models based on the Hertz theory is presented here. Good comparisons on the different models have been presented for example by [26, 21, 47]. The initial Hertz model states that the normal force F_{nHertz} for two colliding frictionless spheres can be calculated as follows

$$F_{nHertz} = K\delta^n \tag{3.1}$$

in which K is the generalized stiffness parameter, δ is the penetration depth and n is the exponent that is usually 3/2. In the case of two colliding spheres i and j the stiffness coefficient K can be expressed as

$$K = \frac{4}{3(\sigma_i + \sigma_j)} \left[\frac{R_i R_j}{R_i + R_j} \right]^{\frac{1}{2}} \tag{3.2}$$

where σ_i and σ_j are

$$\sigma_l = \frac{1 - \upsilon_l^2}{E_l}, (l = i, j) \tag{3.3}$$

and υ_l and E_l are Poisson's ratio and Young's modulus. The Hertz model does not include any damping and therefore the collision is perfectly elastic. A perfectly elastic collision is not very realistic so a spring-dashpot model with a velocity based voiscous damping was introduced.

$$F_{nViscose} = K\delta^n - \dot{\delta}C \tag{3.4}$$

in which C is the voiscous damping coefficient and $\dot{\delta}$ is the relative velocity of the colliding objects in the normal direction. This model presents a problem at zero penetration where the velocity based damping term already has a force. This means that the model is discontinuous. The model itself is very simple to implement and has been widely used despite of the discontinuity. This model is however very unstable in some cases. For example if a very light object hits another object with a high velocity the collision will easily be unstable as the discontinuity is large. The damping force is in no way limited and if the velocity is large in relation to the object kinetic energy the collision will result in a situation where the object is repelled from the contact at an increased velocity breaking the energy balance.

To introduce a better damping model that was not discontinuous Hunt and Crossley [37] presented a model that included a nonlinear viscous-elastic damping. The normal force F_{HC} was presented as

$$F_{HC} = K\delta^n + \chi_{HC}^{\delta^n \dot{\delta}} \tag{3.5}$$

in which the hysteresis damping factor can be expressed as

$$\chi_{HC} = \frac{3K(1 - c_r)}{2\dot{\delta}^-} \tag{3.6}$$

where c_r is the coefficient of restitution and $\dot{\delta}^-$ is the initial contact velocity. The Hunt-Crossley model is very stable in practical applications as it does not have discontinuities and the force does not develop very rapidly. It is probably the most used version of the Hertzian contact models. It is easily implemented and is fairly robust. Lankarani and Nikravesh [46] also presented a model with a modified hysteresis damping factor

$$\chi_{LN} = \frac{3K(1 - c_r^2)}{4\dot{\delta}^-} \tag{3.7}$$

leading to a collision force F_{LN} which is very similar to the Hunt and Crossley model.

$$F_{LN} = K\delta^n[1 + \frac{3(1 - c_r^2)}{4} \frac{\dot{\delta}}{\dot{\delta}^-}] \tag{3.8}$$

The model presented by Lankarani and Nikravesh does not include the effect of permanent or plastic deformation. They also proposed a model that does include the effect but it is more complicated than the model proposed by Flores et al. [21]. The revised Lankarani-Nikravesh model assumes that the shape of the colliding object as well as the collision motion is such that the maximum penetration and the permanent indentation can be estimated at the beginning of the contact which is not possible in a general case.

The model presented by Flores [21] the normal force includes a hysteresis damping including the permanent deformation. The hysteresis damping factor is than expressed as

$$\chi_F = \frac{8K(1-c_r)}{5c_r\dot{\delta}^-} \tag{3.9}$$

and the normal force F_F model takes then the form

$$F_F = K\delta^n[1 + \frac{8(1-c_r)}{5c_r}\frac{\dot{\delta}}{\dot{\delta}^-}] \tag{3.10}$$

The Flores model performs better with low coefficients of restitution whereas the Lankarani-Nikravesh perfoms better with high restitution collisions. In general applications it is very important to use a model capable of dealing with low coefficients of restitution. If a surface geometry model is used in order to represent a volumetric object the material properties alone do not represent the properties of the object. For example a hollow metal cube does not have the coefficient of restitution of a solid metal cube. A common object in a simulation environment is for example a shipping container. It is usually represented as a single rigid body that does not have the same material properties related to collision as a solid metal cube as it behaves completely differently. In a simulation environment the material properties are usually related to the geometry as well as the material of which the object is made of. For example a shipping container has a very low coefficient of restitution whereas a solid cube has a relatively high restitution. Therefore in general applications it is necessary to be able to represent the lower coefficients of restitution as well. It should be remembered that high damping also means stiffness in the system that can cause instability.

3.4 Friction models

Friction has been a topic of interest for a very long time. The earliest studies on friction can be considered to originate from Leonardo da Vinci. Amontons [2] first published friction laws that were based on the unpublished ideas of da Vinci in 1699. Since then the friction models have made huge leaps in terms of understanding how friction forms from the Coulomb model [12] to the recent LuGre [8] and Bliman and Sorine models [6]. Some surveys on different friction models are provided for example in [2, 57, 38]. Also the some penalty based collision response models have proposed different friction models incorporated into the model such as dopico et al. [15].

Friction in itself is a very complex phenomenon. In a general case with two colliding objects a multitude of aspects affect the amount of friction between the two objects ranging from lubrication to surface roughness. In specific cases under laboratory conditions the friction properties of a certain collision can be acquired [48]. Outside of the laboratory circumstances though, the situation is completely different. It is nearly impossible to verify and measure the frictional properties of naturally occurring collisions in a general environment. In a simulated case the amount of variables affecting the friction are limited. Unless specifically defined there is no source of variation for example lubricating film on top of objects. On the other hand in a real case the number of affecting factors is usually unknown. This leads to the generalization of parameters in the simulation case. In a general collision algorithm a best guess approach usually results in the best results due to the generalization of parameters. Verification measurements can be done using some simplified simulation models as in [41] but the circumstances always change for a general case. This means that even a calibration through a simplified model will result only in a base line definition of the parameters. Even with inaccurate parameters for the friction, the results can be acceptable in comparison to a real case due to the varying conditions in the real collisions.

The friction models can be divided into two categories: Static friction models and dynamic friction models [2]. The static friction models do not have any memory of previous states whereas the dynamic friction models have some knowledge of prior states for example through integration. The dynamic friction models are a step further from the more primitive static friction models in that they can represent the friction in a more complete way. The static friction models suffer for example from discontinuities at zero sliding velocity leading to creep. For example in robotic grasping using a force closure grasp with friction the relative velocity in between the graspable object and the robot gripper is near zero the whole time of the grasp. This leads to the graspable object slipping away from the robot gripper over time with the static models. Using a dynamic friction model these problems can be avoided.

Static friction models represent the more traditional models and they are defined as static as they have no temporal memory. This means that they are functions of normal load and sliding velocity alone. The dynamic models on the other hand use the temporal history in order to model the more complex phenomenon, for example stress and strain. The static models are a combination of the classical models of friction; Coulomb, voiscous, static friction (stiction) and the Stribeck effect. These phenomenon can be used in order to describe friction quite well. The major problem with these models is that they are discontinuous at zero sliding velocity. Some methods have been introduced in order to relax the discontinuity over zero but they all result in other problematic situations. For example a case where an object collides perfectly or near perfectly along the surface normal the coulomb friction can result in the object bouncing sideways instead of bouncing directly upwards (see Fig 3.2). This can be relaxed using a step function to cross the zero velocity which in turn causes increased sliding at near zero velocities. Another problem at zero velocity is the application of stiction. The force that is required to initiate sliding motion is greater than the Coulomb friction. This leads to a model where an external load has to be applied that exceeds the stiction limit. In order to determine this, a condition has to be applied at zero sliding velocity stating that the external force has to exceed the stiction force or otherwise the friction force equals the external force. This condition increases the discontinuity at zero. Also, in terms of modeling, the external load would also have to be known exactly. In a simple linear sliding case this is easily done but in a general colliding case this will lead to increased calculations and assumptions. In a case with multiple collision points and normal as well as tangential directions the resolution of the sum of the forces resisting the external load will be very difficult to determine.

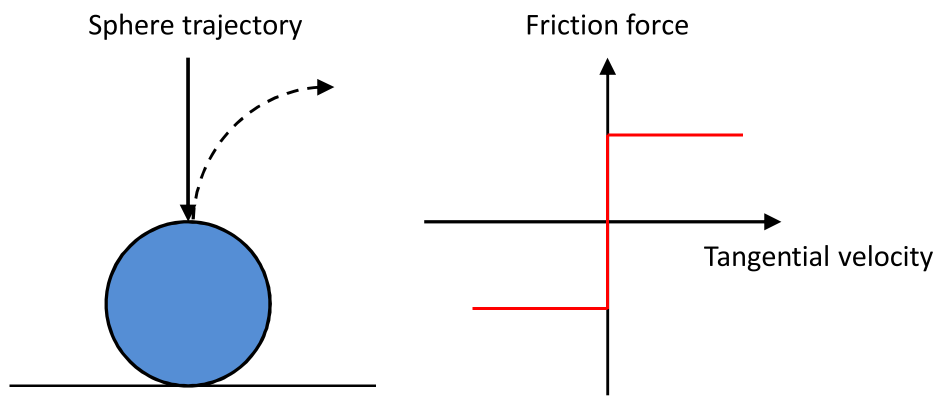


Figure 3.2. Zero tangential velocity collision.

In order to correct the errors in the static models the dynamic friction models have been proposed. The static models cannot express phenomenon such as presliding displacement, frictional lag, varying break-away force and stick-slip. The presliding displacement is a phenomenon that relaxes the zero velocity crossover. It basically means that before sliding occurs the object displaces minimally as the contacting surfaces deform. This feature allows for a friction force even without sliding velocity allowing static holding. Frictional lag represents the difference in the friction force between the increasing and decreasing sliding velocities. It presents therefore hysteresis behavior introducing damping into the system. Varying break-away force means that the break-away force needed to initiate sliding in the system is dependent on the rate of change of the applied external force. Slow force application results in a higher break-away force. The introduction of the phenomenon results in a model that enables real-world scenarios far better than the simpler static friction models.

In some collision response models the use of dynamic friction models is deemed unnecessary or impossible. There are applications where the dynamic friction models are not needed but for general applications the importance of a dynamic friction will become clear very quickly. If an object is represented as a perfect CAD model of a mechanism the surfaces can be perfectly flat. In a real part the unevenness of the object surface can cause the object to form lock. The surface roughness cannot usually be modeled into a real-time simulator, by means of geometry, meaning that the friction has to be able to include the effects of the surface roughness. For example a robot trying to grasp a cube. If a robot gripper is depicted by two flat surfaces and the cube is a perfect cube the grasp on the cube has to be secured by friction. This method is called a force closure grasp with friction (see Fig 3.3). By using a static friction model the cube will eventually slip out of the grip if left in place whereas the dynamic models can hold the cube indefinitely.

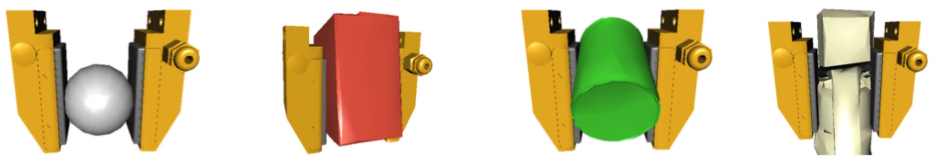


Figure 3.3. Robot fingers grasping objects.

In the proposed model the LuGre model [8] was chosen. Comparative studies

[22] show that the LuGre model is advantageous in comparison to the Bliman-Sorine model [6]. One of the biggest advantages of the LuGre model is that it is of a lower order than the Bliman-Sorine model.

The use of the LuGre model requires integration of parameters over time which causes a problem. The collision tracking or collision history is difficult to create when using triangularized geometries. The LuGre model can be easily adapted to be used in continuous collision methods (penalty) but using it with discrete collision methods (impulse, analytical) is impossible as the collisions occur instantaneously making integration of parameters impossible. Also the addition of friction to the collision equations may result in the equations becoming unsolvable. Some models such as presented by Armstrong [2] and Yamane [70] use springs to model the static friction and then the sliding motion is done using a voiscous friction. This is not an accurate friction description.

In [4, 58, 16] is explained how the addition of friction complicates the equations required to solve the collision in the analytical and impulse based methods. In some cases adding stiction to the impulse and analytical models makes the equations such that they do not always have a solution. Using a penalty method for determining the collision response these problems can be avoided. In this study friction is not addressed in terms of integrability to the collision modeling since it is not a problem as such in the penalty method. The friction force can be easily added as a tangential component to the normal force.

The LuGre model [8] captures the dynamic behavior of the contact surface using the first order differential equation for bristle deflections, which can be written as follows:

$$\dot{z} = \dot{s}_t - \sigma_0 \frac{\mid \dot{s} \mid}{g(\dot{s}_t)} z \tag{3.11}$$

where z is bristle deflection, $\dot{s}_t$ is the tangential velocity, s is the relative velocity and σ_0 is the stiffness coefficient of the contacting surfaces. In (3.11), $g(\dot{s})$ is used to capture the Stribeck effect [2] in order to describe stick-slip phenomena, and can be calculated as follows:

$$g(\dot{s}) = \alpha_0 + \alpha_1^{-(\frac{\dot{s}_t^T \dot{s}}{\dot{x}_0^2})} \tag{3.12}$$

where $\dot{x_0}$ is the Stribeck velocity and the parameters α_0 and α_1 are defined as follows:

$$\alpha_0 = f_n \mu_d \tag{3.13}$$

$$\alpha_1 = f_n(\mu_s - \mu_d) \tag{3.14}$$

where f_n is contact normal force , and μ_s and μ_d are the static and dynamic friction coefficients, respectively. Using state variables of friction and adding a viscous term, the friction force can be written as follows:

$$f_t = \sigma_0 z + \sigma_1 \dot{z} + c_t \dot{s} \tag{3.15}$$

where σ_1 is the friction damping coefficient and c_t is the tangential voiscous damping. More on the parameter selection and model verification can be found for example in [41].

CHAPTER 4

Geometry based penalty methods

Basically the geometry based formulations can be categorized into three categories: single collision point models, multiple collision point models and the volumetric models. The single point models use for example only the deepest penetration point as the collision point [53, 42](also the normal force models can be categorized as deepest point contacts). They are usually fairly simple and quite often fail to produce reliable results. In very specific cases the single point models can give better results than the multiple point methods but in some cases they fail. The multiple collision point methods are more advanced and sometimes fairly complex [34, 16]. They formulate the collision using multiple collision points to represent the collision surface. The multiple point methods are far superior in terms of robustness to the deepest penetration models but they suffer from the problems related to the deformation assumption. The volumetric models try to define a collision volume and calculate the collision from the penetrated volume [30, 32, 68, 28]. These methods usually require quite extensive calculations and assumptions.

The penalty methods all proceed in a succession that can be determined as an algorithm shown in algorithm 1. For each collision the contact points are determined after which the collision event is resolved in each individual collision point. As a result the sum of all the collision forces can be applied to affect the colliding bodies. The differences in the models all constitute to the collision point formulation, normal determination and the normal force magnitude. It should be noted that even the volumetric and deepest penetration models formulate collision point/points and proceed according to the algorithm.

Basically all of the developed geometry based methods suffer from the fact that

Algorithm 1: Formulating a penalty based collision

Data: Data from collision detection
Result: Collision forces in all collision points
for *Collision* **do**
  Determine the collision point amount and locations;
  for *Collision points* **do**
    1. Determine collision point normal directions;
    2. Determine collision point penetration;
    3. Determine collision point normal force magnitude;
    4. Determine collision point tangential friction;
  end
  Apply collision forces to colliding bodies;
end

they do not know where the collision point came from. They all solve the forces based on a single collision configuration solved at a single time-step. This leads to situations where guesses or assumptions have to be made in order to determine to which direction and how hard the collision forces should push the objects. With complex geometries the guesses often fail and end up producing unsatisfactory results.

The lack of temporal knowledge on the previous collision states also has lead to the use of static friction models instead of dynamic friction models. The static models fail in many situations such as friction based robotic grasping. More on the different friction models can be found in section 3.4.

The different penalty methods are not discussed here separately due to the great number of different variations. Instead the different properties of the penalty methods are discussed in sections 4.1 and 4.2 linking them to exemplary models. The problems presented in section 4.2 lead to the need for a new collision modeling method that addresses the problems of normal direction, normal force magnitude and friction. More on these problems and how they are solved in the proposed method can be found in section 5.

The following sections provide detailed information on the advantages and disadvantages of the penalty based collision models.

4.1 Advantages of penalty methods

There are several advantages in the use of a penalty method in comparison to using analytical or impulse based methods. The main advantages of the penalty method are the ease of implementation, independency of the equations of motion and the effectiveness of the solution. One of the less utilized advantages of the penalty methods is the continuity of the solution. The proposed method utilizes this continuity in order to implement a dynamic friction model but it could possibly also be used in order to stabilize the penalty method.

4.1.1 Calculation efficiency

In multibody-multicontact environments the penalty method is faster than the analytical or impulse based methods [16]. In a large scale model with a lot of articulated bodies and colliding objects the equations of motion matrices become excessively large in the analytical methods. Also in the impulse based methods the impulse propagation between the articulated colliding objects can lead to large scale LCP problems. In penalty methods the increase in calculation demand is nearly linear whereas in analytical and impulse based methods the increase can be as high as $O(m)^4$ where m is the number of collision points.

The algorithms for solving the penalty forces are quite effective but the number of collision points can increase rapidly with complex geometries. Especially with dense meshes and complex geometries. Depending on the penalty method it is possible that the geometry mesh can be made very coarse without losing accuracy in the collision response.

4.1.2 Penetration

Non-penetrating contacts are problematic in some simulation cases due to the fact that they cannot form a collision surface and therefore the creation of a proper friction patch or surface pressure is also impossible. For example rotation around the collision normal cannot be modeled accurately around a single point. Some assumptions can be made to emulate friction around the point but it can result in unrealistic behavior. Enabling the bodies to penetrate by using a soft contact enables a more accurate modeling of the rotational hold as well as friction in general. By allowing the objects to penetrate the damping behavior of the friction will also be included. As there is no tangential displacement in the non-penetrating contacts the damping occurring from the friction cannot be included but with penalty methods it can be included.

4.1.3 Continuity

The penalty method is sometimes also called continuous contact and the non-penetrating contacts are called discrete collisions. This means that using the penalty method the collision is spread out over several time-steps, so the bodies remain in contact during the collision period which enables the possibility of parameter integration. Without the possibility of parameter integration for example the use of a dynamic friction model is impossible. Through the use of integration over the collision event other features could also be incorporated into the model such as an energy balance constraint.

4.1.4 Independency of equations of motion

The penalty method does not require any assumptions on whether the modeled system is articulated or not. In some cases solutions with articulated bodies become excessively slow (LCP/NCP-based methods for example) due to large matrices or even unsolvable due to linkages between joint constraints and collision forces. The penalty method performs consistently regardless of the modeled system. The dynamics modeling method can be freely chosen as long as external forces can be applied to it in a reasonable manner.

As the penalty method is not connected to the system matrices, the solution can be separate from the model. This makes parallelization very easy to implement. Different collisions can be calculated simultaneously as the collisions do not require information of eachother.

4.1.5 Ease of implementation

The collision forces produced by the penalty method are very easy to implement in an MBS model. The forces can simply be added as external forces enacting on the colliding bodies. This applies to both normal and tangential components. The friction can be added just by applying tangential components to the normal direction and adding it to the force affecting the colliding body. This is a very simple and effective procedure producing good results in friction modeling.

4.1.6 Independence of mass

One of the not so well known advantages of penalty methods over analytical and impulse methods is the independence of mass. Especially in the impulse method the collision outcome is highly dependent on the mass ratio of the two colliding objects. The penalty method is dependent on mass only indirectly through the stiffness problem of the contact but the effect is more manageable in comparison to the discrete methods. The easiest way to show the problem is to consider the linear momentum conservation law:

$$m_1 \Delta v_1 = m_2 \Delta v_2 \tag{4.1}$$

in which m_1 and m_2 are the masses of the colliding objects and Δv_1 and Δv_2 are the changes in the object velocities. Assuming that the m_1 is very large and m_2 is very small and solving for Δv_2 gives us

$$\Delta v_2 = \frac{m_1}{m_2} \Delta v_1 \tag{4.2}$$

from which can easily be seen that if the ratio between m_1 and m_2 is very large the change in object velocity Δv_2 can become unnaturally high. This constitutes a numerical instability very much in the same way as the stiffness problem with penalty methods only that the mass ratio problem cannot be solved by making the time-step smaller.

Another example can be made using a static stack of boxes. If all the boxes in the stack are of similar weight none of the methods have problems with the stack. But when one box in the middle of the stack is made significantly lighter than the rest the impulse and analytical methods can cause the whole stack to explode. The penalty methods produce the same amount of force as before and the stack remains stable.

4.2 Disadvantages of penalty methods

The penalty methods have several advantages over the other modeling methods as well as some disadvantages. As with any simulation method it is important to recognize the problematic areas in the method in order to avoid problems.

The penalty methods suffer from two main problems; stiffness of the system and the assumption on the deformation during the collision. As the two colliding objects interpenetrate the rigid body assumption is broken. How the penetrated volume is handled is the separating factor between the different models. The stiffness of the model is something all the penalty methods suffer from and there is no definite solution to it.

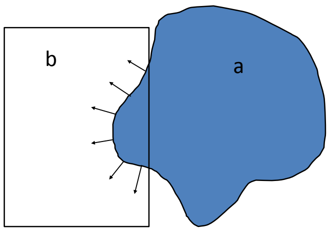
4.2.1 Excessive penetration

The penalty method assigns a collision force on the colliding objects pushing them away from each other over the collision duration. This can be a problem with high velocity collisions as the force is not strong enough to keep the objects from penetrating excessively. Moore [53] introduced a model where the collision response was divided into two parts, collision and contact. This was done in order to avoid problems with excessive penetrations during the collision as well as avoiding problems with static contacts. Recently [16] and many others employ the same main principle of dividing the collision response. This is not a desired feature as it always requires some sort of a transition between the two different methods. The excessive penetration usually results from a situation where a real object would be deformed permanently to a great extent. In practical terms the object would be broken beyond repair. In a simulation it should not be necessary for the collision model to produce unrealistic collisions. Therefore it is acceptable to recognize that such collisions usually result in a failure in the simulation environment as they would result in failure in reality as well. Permanent large scale deformation should be modeled using other methods.

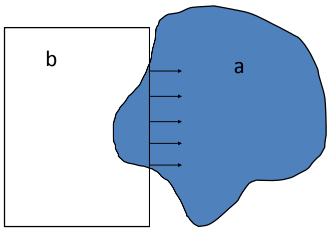
4.2.2 Deformation assumption

Two rigid bodies cannot penetrate each other so by allowing the bodies to intersect we are breaking the initial rigid-body assumption and thus creating an error in the simulation. This in turn causes some problems in the modeling of the collision. Determining how the interpenetrated objects behave during the collision requires an assumption on how the bodies deform leading to differences in collision point location, normal direction and normal force magnitude definitions between the different models. A collision surface forms as two bodies would deform forming a unified surface. As that is not the case with rigid bodies the definition of how the collision surface forms is essential in how the collision model works. It can be assumed for example that one of the objects is infinitely stiff and that the deformation takes place in only one of the objects. In figure 4.1(a) is shown a case where the body a is assumed to be infinitely stiff. The black arrows represent the normal directions on the body. The same objects colliding in the same way are shown in figure 4.1(b) with the difference that the body b is assumes to be infinitely rigid. As can be seen the normals and collision surface are completely different in both cases. The reality would be somewhere in between the two as seen in figure 4.1(c). The problem arises when making the assumption on how the collision surface forms and where the normals point.

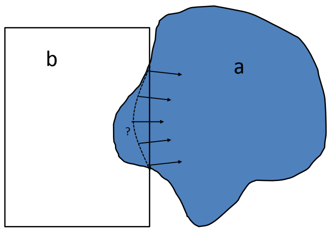
These problems are model dependent as each model solves them differently.



(a) Collision surface if body a is infinitely stiff



(b) Collision surface if body b is infinitely stiff



(c) Collision surface if neither body is infinitely stiff

Figure 4.1. Intersection between two colliding objects

4.2.3 Entry-exit error

In real-time simulations it is not possible to search for an exact collision instant by going back in simulation time or by reducing the time-step. This usually causes the system to slow down considerably and therefore become non real-time. This could be done to some smaller models but as the solution should be general this is not considered as an option. It can be argumented that it is possible but in practical solutions it is not. The discrete nature of the simulation causes a small error in the penetration when entering and exiting the contact (see Fig. 4.2). This small error in turn causes some error in the energy balance of the system. As seen in the figure 4.2 the initial contact is detected when the bodies are already intersecting. Also it can be seen that the force calculated at the beginning of the last time-step in contact affecting the body even after the bodies no longer intersect. The error can be reduced by making the time-step smaller but computational efficiency mostly prohibits the use of sufficiently small time-steps. This means some method for stabilizing the error needs to be considered. The developed method uses integration in order to solve for the penetration between the objects. This eliminates some of the error in the entry phase but causes a small level error in the surface as well as increasing instability at higher time-step sizes.

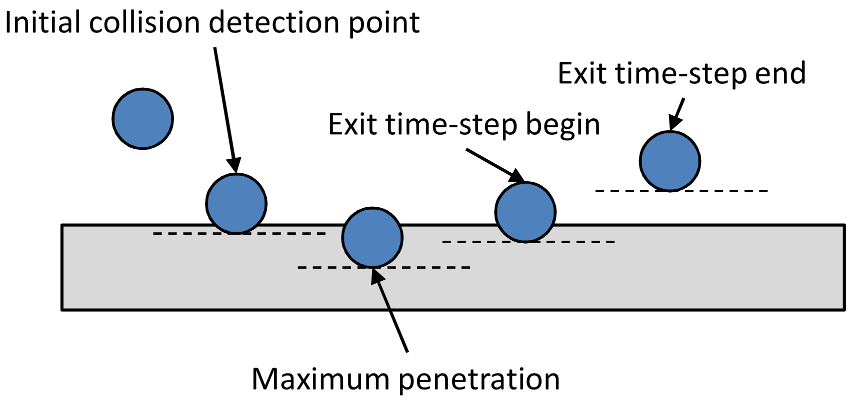


Figure 4.2. Entry and exit error in discrete simulations.

4.2.4 Stiffness and energy balance

One of the disadvantages of the penalty method is the requirement for a small time-step. The spring dampers are usually very stiff and therefore have high nominal frequencies. This problem can be alleviated by for example making the spring damper elements nonlinear but it cannot be removed. When modeling otherwise stiff systems the small time-step requirement is not a limiting factor as the other elements also require a small time-step. On the other hand for example for large systems with many complex colliding geometries the collision detection takes time. In such cases the extended time-step would be advantageous. A common problem with real-time collisions in such cases is that the collision detection takes such a long time that the required time-step is too much in order for the collision system to remain stable. The time-step requirement is not only problematic with collisions but with all stiff components such as hydraulics in the model.

With very stiff systems the discrete change in the force amount results in errors in the energy conservation of the system during integration. All penalty methods suffer from this problem and it is very difficult to solve. This can be seen by the illustration in Fig. 4.3 where the beginning and end states of a collision time-step are shown. The force effecting throughout the time-step is calculated at the beginning of the time-step. This means that the energy balance of the system will never be exactly correct. The worst case scenario is a light weight object colliding at a fast pace, producing a large penetration over a single time-step to an object with little kinetic energy. For example it could be that the time-step begins in a position where the collision is not yet detected. Then during the next time-step the object penetrates the opposing object at maximum pace producing a considerable intersection. Then solving the forces using the penetration produces such a high force that the object is expelled in an unrealistic manner. One way of relaxing this problem is to make the contact force softer and thus making the discrete changes in the model smaller. This is not always realistic and therefore not possible but however required in some applications. This also creates a problem when trying to incorporate realistic material properties to a collision response. For example in a practical application a steel spheres Young's modulus needs to be made smaller than the realistic value would be, in order to avoid instability. Through the use of a small time-step this error can be minimized but it remains none the less.

We can take for example a one dimensional test case where a mass colliding with a spring is used to demonstrate the situation. If at time-step t_0 the penetration δ_0 of a body is 0.001m and the stiffness coefficient K = 1e7 N/m the resulting force F_0 = 10 000 N. If on the time-step δ_1 penetration δ_1 is 0.002 m the resulting force F_1 = 20 000 N. The work done by the collision during one time-step based on the calculation of the force at time-step t_0 is W = 10 J. If the work is calculated by integrating the work during the entire time-step updating the force the resulting work W = 15 J (see Fig. 4.5 for reference). These values are purely fictional and relate to no specific case but show the problem underlying the method. This is a serious issue with penalty methods but nonetheless they should not be shunned as the overall energy balance will be better than this calculation example shows. The error in the work done by the collision leads to an interesting feature in the penalty method. In the penetration phase of the collision the work done by

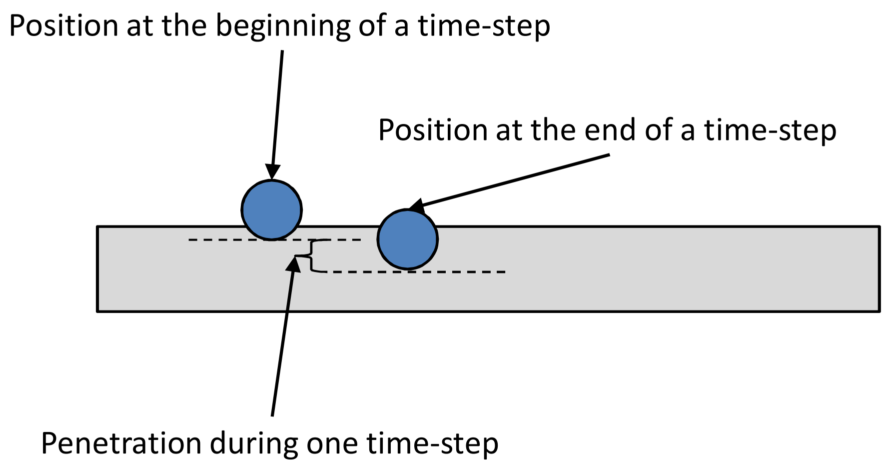


Figure 4.3. Discretization error in penalty methods.

the collision is less than it should be and on the exit phase the work is too much. This leads to the alleviation of the problem of light stiff objects while keeping the overall energy balance better. The energy accumulation asymmetry can be seen in figure 4.4 where the blue line represents the realistic collision force produced by a linear spring and the red line represents the force produced by a discretisized system.

The major practical problem of the method is not actually the overall energy balance of the entire collision event but the single step error with light stiff objects. This happens when the error in the work done by the collision during one time-step is very high in relation to the kinetic energy in the colliding object resulting in an extremely high acceleration in the colliding object. This can be shown using the same example only defining the mass of the colliding object as m = 0.1 kg and the initial velocity as v = 2 m/s. Calculating the kinetic energy of the object using the classical formula.

$$E_k = \frac{1}{2}mv^2 \tag{4.3}$$

The kinetic energy E_k = 0.2 J. And by enacting a work of 10 J on it during the first time-step would mean that the object would bounce off at a significantly

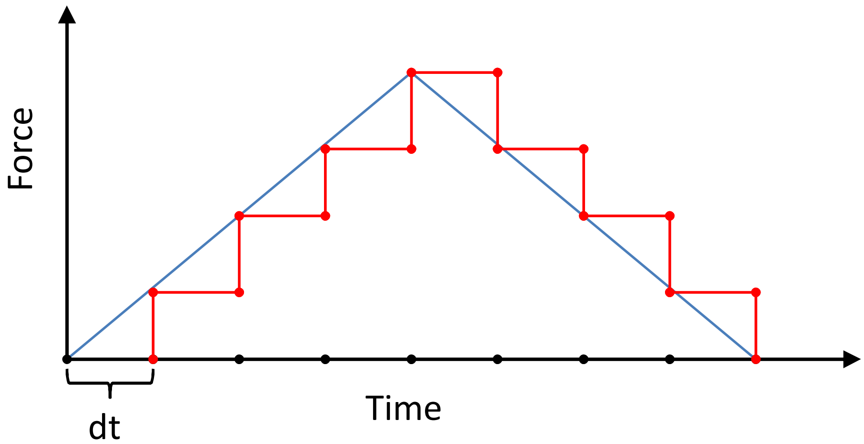


Figure 4.4. Energy accumulation asymmetry.

greater velocity. If a spring-damper based collision model was considered the situation would be far worse still. In the entry phase the velocity related damping would increase the normal force increasing the work done to the object. Using the formula 3.4 and a viscous damping coefficient C = 1e4 Ns/m(typically one or two decades lower than the stiffness) the force would be increased to 30000N and the work done by the collision would be 30 J. This is one major reason why the spring-damper model is not a good option for collision models especially as the damping force is discontinuous in relation to the penetration.

The model stability can be tested using the experiment in section 6.1. By changing the simulation time-step and the coefficient of restitution the sensibility to the discretization error can be observed. At first a coefficient of restitution was chosen as being 0.05 as it is most disadvantageous in terms of stability as the damping force causes a peak in the collision force. The Flores model produces reasonable results only up to time-step size 0.0046 s while the Hunt-Crossley model still produces reasonable results at time-step 0.02 s. Using a higher c_r = 0.5 the Hunt-Crossley model remains stable at 0.0225 s time-step while the Flores model is stable at 0.0165 s. The simple test reveals the link between the system stability and the system stiffness.

One consideration in the energy conservation is to note what kind of an integrator

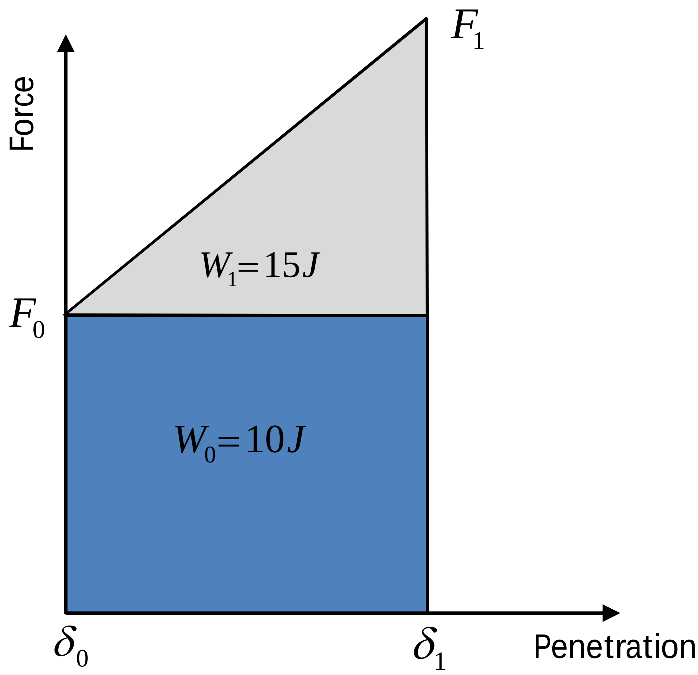


Figure 4.5. Discretization error in penalty methods using work during one time-step.

is being used. With multistep integrators the integrable values are updated within the integration loop during the integration (for example Runge-Kutta4). It is not practically possible in a real-time application to call the collision detection at each update within the integration but it possible to update the individual collision elements thus relieving the problem. Depending on the software environment it is possible to do this and call the collision detection only before the integration loop. The integrator in general plays an important role in the stability of the collision response model. Some integration methods suit stiff systems very well such as the one presented by Negrut et al.[54] and are therefore better for simulating collisions than others.

There are different methods for determining the stability limits of stiff systems. Shinya et al.[67] introduces a method for determining the allowable parameters in order for the simulation to remain stable. For example one can calculate the maximum allowable time-step. Such calculations make the stability of the system more determinable and reduce the need for guesswork and they are highly recommended.

With all the problems in the energy balance it should be noted that in most practical cases the energy conservation is not a problem. This can be seen in Fig. 6.2 in which, a fully elastic sphere is bouncing against another fully elastic sphere, consistently at the same height, showing energy conservation. Although it is a major problem it can be avoided by choosing the right parameters. It does mean though that in practical applications the realistic material values need to be modified. In a non-realtime the time-step size can be reduced in order to allow for the use of realistic material parameters but such is not the case with realtime simulations.

The stabilization methods for stiff systems are not covered in this thesis as it is a very difficult issue and worth another thesis in its own right.

4.2.5 Geomertic dependency

The geometry based penalty methods are in some respect dependent on the object geometries. For example the PCM [35] method relies on the geometry mesh in order to form the colliding surface. The collision detection provides a mesh of vertices to the collision response that determines the penetrated area. Using a coarse mesh in comparison to using a dense mesh provides less accurate results in terms of collision response. This is not a desired feature in a collision model. Some models also place temporal springs directly on the geometry vertices. This creates problems in the sense that the collision stiffness varies as a function of the number of penetrated vertices unless it is taken into account otherwise. A densely meshed sphere would be stiffer than a similar sphere but with a coarse mesh.

The model presented by Tang et al. [68] is not dependent on the number of penetrated vertices but forms collisions for vertice-face and edge-edge collision pairs. Therefore the collision depends on the object mesh also.

There are three aspects to geometric dependency, force related, efficiency related and method related. The force related geometric dependency means that the collision force is dependent on the geometry mesh. For example the PCM model presents different results for the same geometry using different mesh densities. The same applies to models where the amount of collision points effects the stiffness. The efficiency related geometric dependency is connected to the model geometry requirements. If a coarse mesh can be used to represent a geometry it is always advantageous in terms of calculation efficiency which is imperative to real-time solutions. Also the geometry can complicate the calculations for example if an inside/outside point is required. The calculations can be slow and could result in false results. The method related geometric dependency is related to the model failing due to certain types of gemetries being used. For example the box stacking experiment shown in section 6.6 results in failure with some models. This is due to either the collision force placement, normal direction definition or normal force magnitude definitions failing.

4.2.6 Normal and penetration definitions

In a penalty method the force on the two colliding bodies is always applied through the use of temporal spring-dampers. The use of these spring-dampers always requires some information to solve for the position, magnitude and direction of the normal force. As the collision detection routine can only provide information on the surfaces of the two bodies the only information available on the collision is the intersection circumference and the penetrated surfaces. The circumference can be solved using triangle-triangle intersection tests and the penetrated triangles can be solved calculating outwards from the triangle-triangle intersections. If a method is using the penetrated geometry features to determine the collision points a problem arises when determining the penetration or magnitude of the normal force. The normal direction can be defined as the surface normal but the penetration is more complicated. If the penetration depth is calculated to the nearest surface in the normal direction serious problems can occur. For example in a case such as presented in Fig 4.6 the penetration would be calculated from the opposite side of the cube and the normal force would be enormous.

Another way is to find a closest point of exit for the collision points. This means that the normal direction is no longer along the geometry normal but directed to the closest point of exit. This also creates problems in determining the normal direction as well as the normal force magnitude. For example a body interpenetrating another close to the edge shown in Fig. 4.7 can cause multiple solutions on the collision normal direction. In Fig. 4.7 the solid line rectangle (moving at velocity v in the direction of the red line) initially penetrates the filled rectangle and it can clearly be seen that the normal direction on the corner vertice should repel the body upwards. But if the colliding rectangle is moved so that the side of the filled rectangle shows the closest exit point it is no longer clear where the normal force should point. This creates a discontinuity in the model.

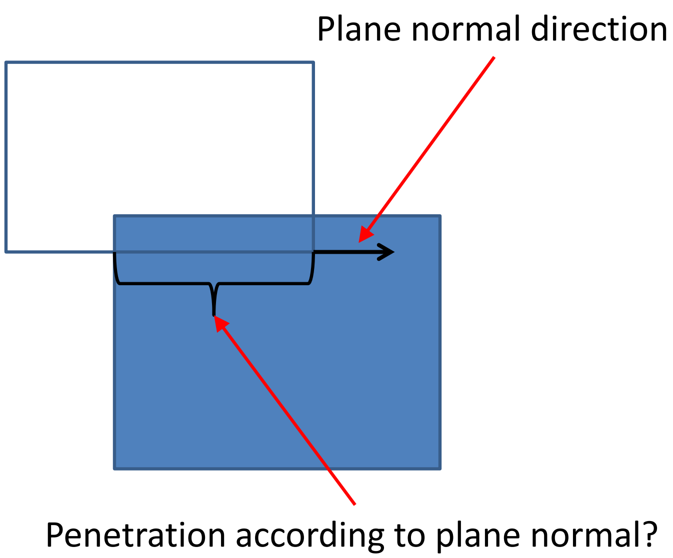


Figure 4.6. Penetration depth problem in determining the normal force magnitude.

Without the knowledge of where the colliding point came from there can be no certainties as to the normal direction.

Some methods such as Heidelberger [32] modify the normal in order to avoid inconsistencies. This can lead to problems in the behavior of the collision regarding sliding. The same problems can arise when regarding the closest point of exit models. When two objects penetrate as in Fig. 4.8 the closest point of exit determines the normal force direction as pushing the penetrating cube upwards but it would not stop the cube from sliding on the surface even though it has penetrated. In reality the penetrated cube should snag on the opposing cube and it should stop. In these methods the only force that will stop the objects from sliding is friction. This problem of penetrated objects being allowed to slide occurs actually in quite a few collision methods and it leads to excessive friction parameters being used in order to avoid sliding.

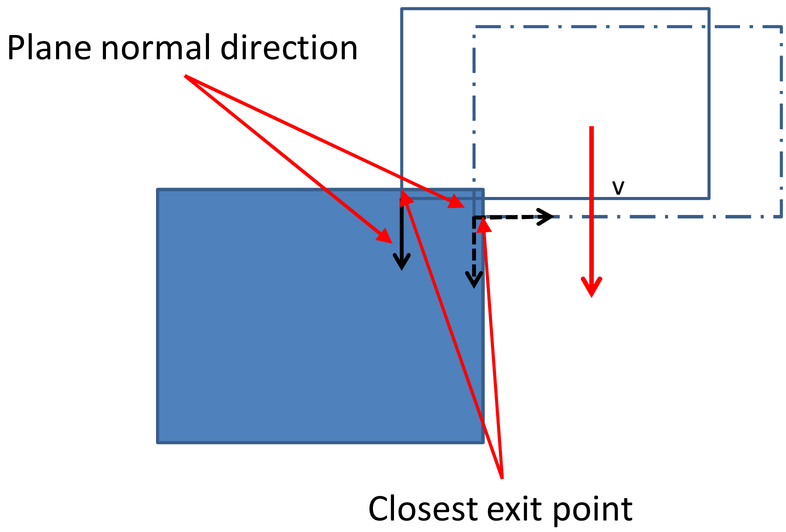


Figure 4.7. Closest face problem in determining the normal direction.

4.2.7 Volumetric models

Some collision response methods use the intersecting volume to determine the collision force as well as the collision normal direction. The major problems in volumetric contact responses are the normal direction and the added calculations for determining the penetrated volume. When considering the intersecting volume between the two colliding bodies it is not easy to determine the correct normal direction for the colliding force from just a volume. Some additional assumptions are needed which in turn lead to cases where the model fails as the assumption is broken. Models such as the one presented by Hasegawa et al. [30] rely on the assumption that the colliding objects are convex or that they are closed forms as in the model by Heidelberg et al. [32]. This limits the modeling quite a bit. In the case of non-convex objects the geometries can be divided into convex sections but if the method also requires closed forms in order to formulate volumes it will complicate the calculations considerably. For example the closed form requirement complicates the division of the colliding geometry for the purposes of the collision detection. Also the Hasegawa method requires that an inside point of the collision volume is solved which can be quite

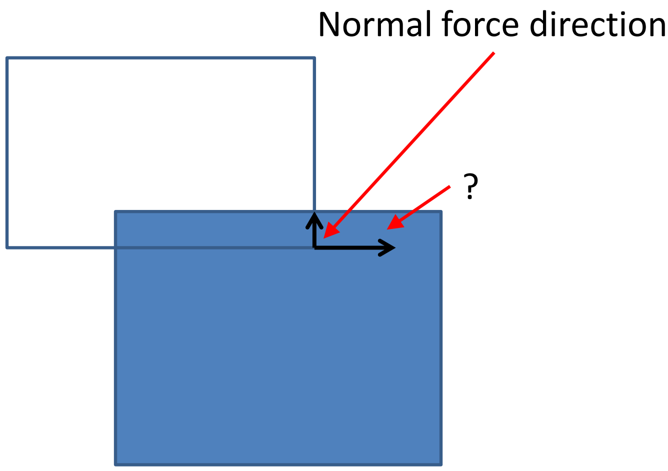


Figure 4.8. Sliding problem in normal direction determination. Normal force and missing opposing force.

troublesome.

CHAPTER 5

The proposed method

The proposed method addresses the two areas in a collision algorithm: Collision detection and collision response. The collision detection in itself is very briefly covered as it is not a key area in this dissertation. The focus in terms of collision detection lies in providing the collision response model with the required information on the collision. In addition to solving the intersections of objects the proposed collision detection method addresses the problem of providing temporal information to the collision response which is one of the key areas in this dissertation.

Solving single colliding instances without knowledge of what happened before, requires a lot of assumptions on the collision modeling. By introducing temporal knowledge to the system the guesswork can be reduced to a minimum removing sources for error. The temporal knowledge allows the use of integration of parameters as well as providing more information as to how the collision normal force should act.

The proposed collision response method is based on the idea that the only correct information on the collision is located at the edges of the collision surface. This is due to the fact that the rigid bodies do not deform and if a point in a body has penetrated another body it is considered inaccurate. The assumption holds true to small local deformations such as should be the case in soft contacts. The method will work with greater penetrations as well but the chance for error will increase as the deformation would no longer be local. Hertz stated that the deformations during a collision are very local in comparison to the deformations of the entire body. Therefore the assumption that the collision circumference is undeformed can be made with certain precision. In figure 5.1 two colliding spheres are shown.

On the left hand side the the spheres are intersecting and on the right hand side the colliding instance is separated to show the areas of interest. In the image are shown the deformed area, the line between the deformed and undeformed areas and the undeformed area. The deformed area is considered incorrect and is discarded from the calculations, the collision line is used in the calculations and the undeformed area does not affect the calculations. This assumption of incorrectness of the penetrated volume leads to a situation where almost all the errors are the result of the initial assumption of rigid-bodies penetrating each other. No assumptions on the modeled system or geometries have to be made. The collision detection calculations remain simple as only the intersection lines are needed.

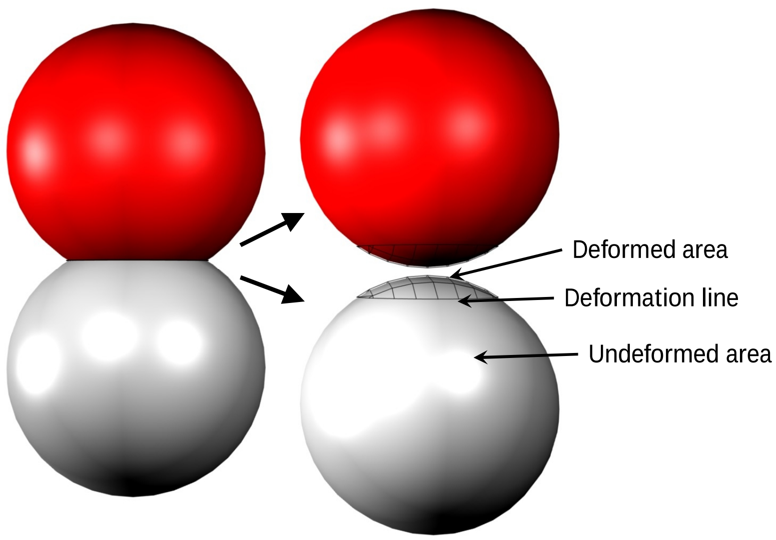


Figure 5.1. Intersecting volume between two colliding spheres

The following sections cover the model composition. Firstly the collision detection and temporal history methods are covered in section 5.1. The collision response section 5.2 covers the collision normal force formulation, friction force formulation as well as the damping behavior.

5.1 Collision detection

The main aspect of this study is not the collision detection but rather the collision response and the problems related to it. It is important to cover the collision detection to some extent as well because the collision response is dependent on the information the collision detection can provide. When creating or choosing from available collision detection methods it is crucial to select one which can provide the information necessary for the collision response. A closest point collision detection can provide only one collision point and no collision surface. Therefore it is necessary to take into consideration the collision detection and response chain as a whole.

The proposed model uses a tree hierarchy consisting of oriented bounding boxes (OBB). The trunk and branches are made up of OBB and the leaf level consists of triangles. Triangle-triangle collision tests, calculated using an algorithm by Devillers et al. [14], are used in order to determine the collision surface circumference. The circumference consists of triangle-triangle intersection lines forming the perimeter of the collision surface.

One of the advantages of the proposed method is that it does not require any assumptions on the geometry. It will function exactly the same regardless of the shape of the object. For example it does not make any assumptions that the object should be convex or that it should be a closed form. It even works for polygon soups. Also there is no requirement on the mesh density. This enables the optimization of the collision detection algorithm quite freely and there is no need for any extra inside/outside calculations. The collision detection simply detects the triangle-triangle intersection lines and provides the collision response with the outline of the colliding surface.

The collision response requires the triangle normal directions and the intersection lines for use in the normal force calculation. The proposed model also uses integration over time so the collision response method has to be provided with a temporal memory of the previous collision states.

5.1.1 Temporal collision memory

In order to use parameter integration in the collision response model temporal memory has to be provided by the collision detection model.

By using temporal memory a history of the previous collision states is made available to the collision response model and linking the current state to the previous state is therefore possible. For example the use of a dynamic friction model requires temporal memory for the model in order to integrate for the sliding motion.

The collision detection is usually performed at a certain configuration only, without any regard of the previous states, which causes problems in determining the normal directions as well as the normal force magnitudes. The main problem in creating the temporal history is linking the current state with the previous state. Usually when solving for the collision points it is not possible to directly link them to a collision point in a previous state. Meaning that a number of points solved at two different states and would need to connected, which ones are the same points. Some sort of a link in between these collision points is needed. As the proposed method relies on triangle-triangle intersection tests, these triangles can be used as that link. This is done by creating a unique id number for each individual triangle in the model. These id numbers are then used in order to track for previous collision states between two colliding triangles. A collision point is assigned the ids of the two colliding triangles defining the collision point. This information can then be used in order to link the current and previous states.

As the identification as well as collision point formulations are based on triangles it is necessary to make sure the collision parameters continue over triangle limits. This means for example that when a triangle collision moves to intersect a new triangle the collision parameters need to be combined with the previous existing collision. This data interchange is shown in Figure 5.2. In Fig 5.2(a) the intersecting triangle collides only with one of the opposite triangles. In Fig. 5.2(b) the colliding triangle has been sliding sideways to intersect with a new triangle. In this case the collision parameters of the old and new collision need to be combined. When a collision between two triangles is detected a list of previously colliding triangle pairs is sought for an identical pair. If the pair already exists the collision response parameters are updated and integrated. If the collision does not exist before, both colliding triangles are checked if they share a colliding triangle with the other collision pairs. If so the common triangle parameters (penetration and initial velocity) are combined and the adjoining triangles collision parameters are calculated using scalar projections. This procedure will ensure the integration over triangle limits. The algorithms for combining the collision parameters are presented in Algorithms 2 and 3.

The procedure will ensure that even using a tighter mesh the collision will tighten even when passing over triangle limits. Without this combination method the model would lose for example the integrated collision penetration information when passing from one triangle to another.

This proposed collision tracking method provides the needed temporal history

Algorithm 2: Searching for common triangles

Data: All detected Triangle-Triangle collisions
Result: A new triangle-triangle collision pair sharing a common triangle with an existing one

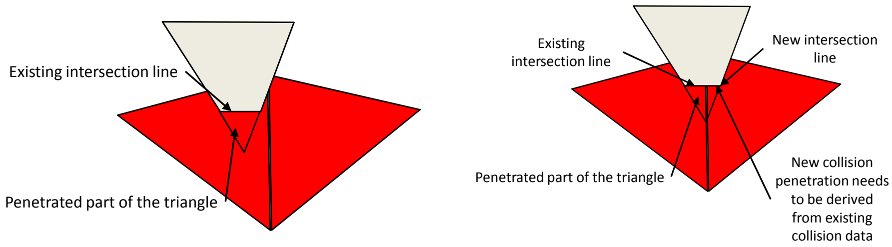
```
for All detected triangle-triangle collisions do
    Search list of previous collision triangle-triangle collisions for
    identical ids;
    if Found existing identical id pair then
        Existing collision pair:Update and integrate collision parameters;
    else
        if Found single existing identical id then
            Found new collision with a shared triangle to another collision
            pair -> go to algorithm 3
        else
            New triangle-triangle collision pair: Integrate collision
            parameters;
        end
    end
end
```

Algorithm 3: Combining collision data over triangle limits

Data: A new triangle-triangle collision pair sharing a common triangle with an existing one
Result: Combined collision data over triangle limits

```
for Common triangle do
    Set penetration depth on the new contact as equal to the old contact;
    Set initial velocity on the new contact to equal to the old contact;
end
for Adjoining triangles do
    Take vector scalar projection of old collision triangle initial velocity to
    new triangle normal direction and set as initial velocity;
    Take vector scalar projection of old collision triangle penetration to
    new triangle normal direction and set as initial velocity;
end
```

for the collision response model. By enabling the linking of the current collision state to the previous state many different possibilities are made available.



(a) Triangle intersecting initial triangle (b) Triangle moved to intersect a new triangle

Figure 5.2. Example of a triangle passing over a triangle edge

Integration of parameters enables the dynamic friction models as well as the temporal history can make the collision normal force calculations more accurate. There is no more need to guess the normal direction as the integrated parameters remember where the colliding object came from and thus where it should be returning to.

5.1.2 Collision tracking limitations

The tracking method has some limitations. The penetration depth can be carried over the triangle boundaries by combining the parameters but the initial velocity of the collision presents a problem. In combining the collision data between bondary triangles the initial velocity is calculated as a scalar projection to the surface normal based on the previously existing collision initial velocity. This means that in a case where the colliding triangle travels over several opposing triangles the initial velocity of the collision is reduced. This is presented in figure 5.3. If the grey triangle collides initially with the blue triangle and then moves along the dashed line to intersect the red and green triangles the parameters are a combination of the route. First the parameters are combined from the grey-blue to the grey-red collision. Then the parameters from the grey-red collision are combined to the grey-green collision. At the grey-green collision the initial velocity will be from the grey-red collision instead of the grey-blue collision. This means that the damping in such cases is increased as the initial velocity is smaller than the original velocity. The damping term $\frac{\dot{\delta}}{\dot{\delta}^{-}}$ becomes greater thus increasing the damping. Combining the initial velocity from the original initial velocity would be possible but it would endanger collisions with a longer duration as the initial velocity could linger in the contact for unrealistic periods. For example in a fully plastic collision the initial collision velocity could linger in the static collision state.

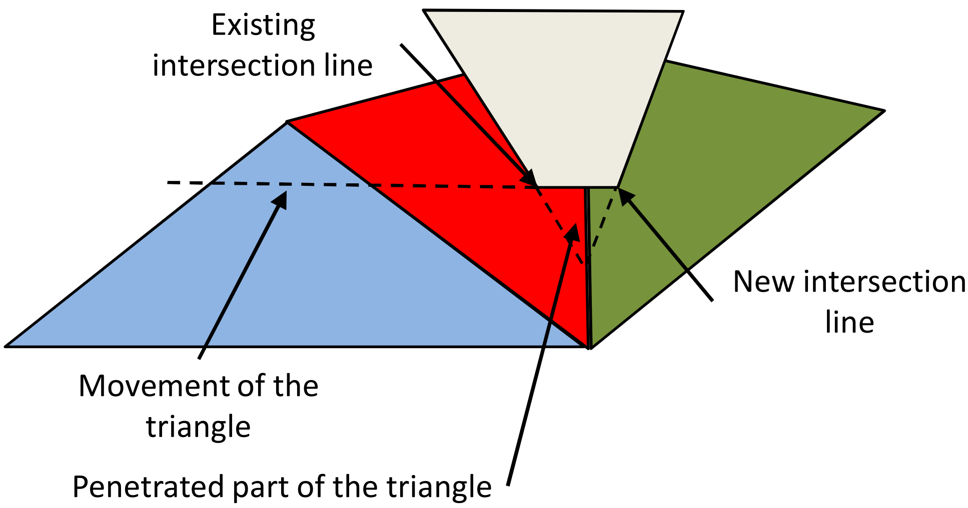


Figure 5.3. Initial velocity continuity problem.

5.2 Collision response

The proposed method consists of using triangularized geometries in order to calculate soft contacts between bodies. The main problem areas in a penalty based collision response method are determining the collision point locations, collision force normal directions and collision force magnitudes. The proposed method places the collision points to the collision area circumference and solves the normal directions using the surface normals. The problem of collision force magnitude is solved by then integrating the penetration at the collision points.

A key aspect of a collision response model is the friction modeling. All of the existing triangle-triangle collision response methods employ a static friction model. These models suffer from some problems which can be avoided by using a dynamic friction model. The use of a dynamic friction model requires temporal memory and it is not possible to utilize them in a collision system where no temporal history exists. Therefore the proposed method introduces an approach for providing the temporal history and implements a LuGre [8] based friction model to the collision response. This enables for example grasping situations where perfectly flat surfaces are grasped using friction alone without the object slipping away over time.

The solution proposals to the common problems are further explained in the following sections.

Section 5.2.1 introduces basic contact kinematics.

Section 5.2.2 proposes a solution for placing the collision points.

Section 5.2.3 proposes a solution for solving for the normal directions.

Section 5.2.4 proposes a solution for solving for the penetrations at the collision points.

Section 5.2.5 proposes a solution for solving the normal force magnitude.

Section 5.2.6 proposes a solution for determining the frictional force.

5.2.1 Rigid body contact kinematics

When a collision occurs the contact information (position, relative velocity, penetration, etc.) is used to calculate the force. The kinematics of the contact points between two bodies i and j can be described using knowledge of the geometries and states of the bodies (see Fig. 5.4).

The distance between contact points P_i and P_j can be written as follows:

$$\boldsymbol{s} = \boldsymbol{r}^{P_j} - \boldsymbol{r}^{P_i} \tag{5.1}$$

where $\boldsymbol{r}^{P_i}$ and $\boldsymbol{r}^{P_j}$ are the position vectors of each contact point in the global reference frame. If $\boldsymbol{R}^i$ and $\boldsymbol{R}^j$ are defined as the center position vector of each body, the distance can be written as:

$$\boldsymbol{s} = \boldsymbol{R}^j + \boldsymbol{A}^j \bar{\boldsymbol{u}}^{P_j} - \boldsymbol{R}^i - \boldsymbol{A}^i \bar{\boldsymbol{u}}^{P_i} \tag{5.2}$$

where $\boldsymbol{A}^i$ and $\boldsymbol{A}^j$ are rotation matrices from the body reference frame to the global reference frame and $\bar{\boldsymbol{u}}^{P_i}$ and $\bar{\boldsymbol{u}}^{P_j}$ are the position vectors of the contact points within the body reference frames.

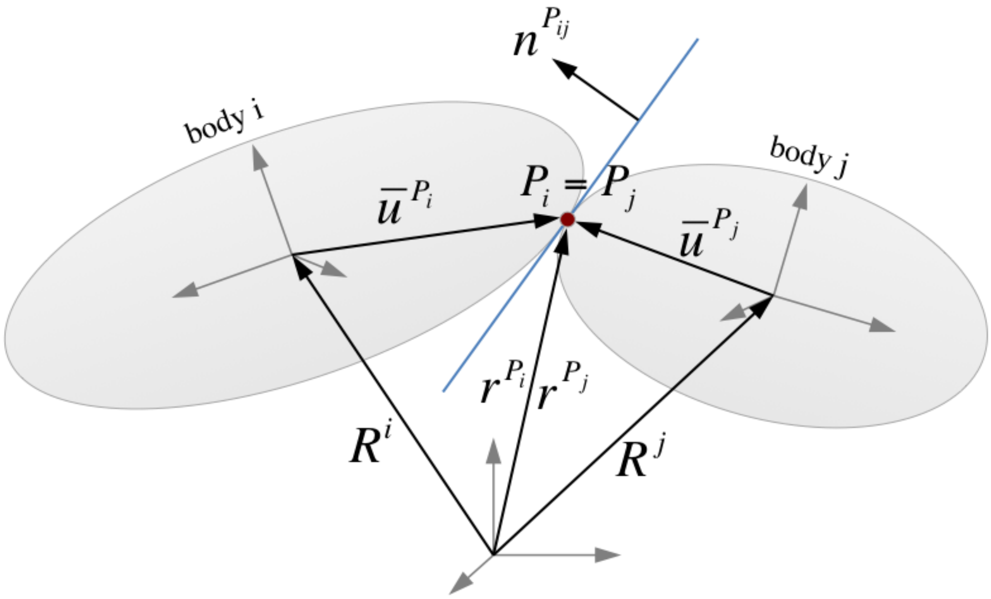


Figure 5.4. Contact between bodies i and j.

In this case, P_i is coincident to P_j and the distance is therefore zero, but it can be used to calculate the relative velocity between the contact points:

$$\dot{\boldsymbol{s}} = \dot{\boldsymbol{R}}^j + \tilde{\boldsymbol{\omega}}^j \boldsymbol{A}^j \bar{\boldsymbol{u}}^{P_j} - \dot{\boldsymbol{R}}^i - \tilde{\boldsymbol{\omega}}^i \boldsymbol{A}^i \bar{\boldsymbol{u}}^{P_i} \tag{5.3}$$

where $\dot{\boldsymbol{R}}^i$ and $\dot{\boldsymbol{R}}^j$ are the velocity vectors of bodies i and j, and $\tilde{\boldsymbol{\omega}}^i$ and $\tilde{\boldsymbol{\omega}}^j$ are skew-symmetric matrices of the angular velocities.

The distance between contact points in this normal direction can be written as:

$$d = \boldsymbol{s}^T \boldsymbol{n}^{P_{ij}} \tag{5.4}$$

where $\boldsymbol{s}^T$ is the transpose vector of $\boldsymbol{s}$ and $\boldsymbol{n}^{P_{ij}}$ is the normal vector of the contact plane. Accordingly, the velocity in the direction of the normal of the contact plane can be written as:

$$\dot{d} = \dot{\boldsymbol{s}}^T \boldsymbol{n}^{P_{ij}} \tag{5.5}$$

The relative velocity in the tangential direction of the contact plane can be obtained as follows:

$$\dot{\boldsymbol{s}}_{\boldsymbol{t}} = \dot{\boldsymbol{s}} - \dot{d}\boldsymbol{n}^{P_{ij}} \tag{5.6}$$

On each contact point, the contact force $\boldsymbol{F_C}$ can be written as:

$$\boldsymbol{F_C} = \boldsymbol{F_n} + \boldsymbol{F_t} \tag{5.7}$$

where $\boldsymbol{F_n}$ is the normal force produced by the contact and $\boldsymbol{F_t}$ is the tangential friction force.

The contact force in the normal direction of the contact ($\boldsymbol{F_n}$) can be written as:

$$\boldsymbol{F_n} = f_n \boldsymbol{n} \tag{5.8}$$

where f_n is the magnitude of the normal force component and $\boldsymbol{n}$ is the normal vector.

For bodies i and j, the resulting contact force can be applied as follows:

$$\boldsymbol{F_C}^i = \boldsymbol{F_C} \tag{5.9}$$

$$\boldsymbol{F_C}^j = -\boldsymbol{F_C} \tag{5.10}$$

Accordingly, the resulting torque of contact can be written as follows:

$$\boldsymbol{T_{F_C}}^i = \widetilde{\boldsymbol{A}^i \bar{\boldsymbol{u}}^{P_i}} \boldsymbol{F_C}^i \tag{5.11}$$

$$\boldsymbol{T_{F_C}}^j = \widetilde{\boldsymbol{A}^j \bar{\boldsymbol{u}}^{P_j}} \boldsymbol{F_C}^j \tag{5.12}$$

where $\widetilde{\boldsymbol{A}^i \bar{\boldsymbol{u}}^{P_i}}$ and $\widetilde{\boldsymbol{A}^j \bar{\boldsymbol{u}}^{P_j}}$ are skew-symmetric matrices. The torque constitutes the transferral of the contact force to the body frame of reference coordinates.

5.2.2 Collision force placement

The collision detection provides the collision surface circumference as a result of triangle-triangle intersections. Each intersection line can be assigned a temporal spring damper element at both ends of the line, thus resolving the collision force

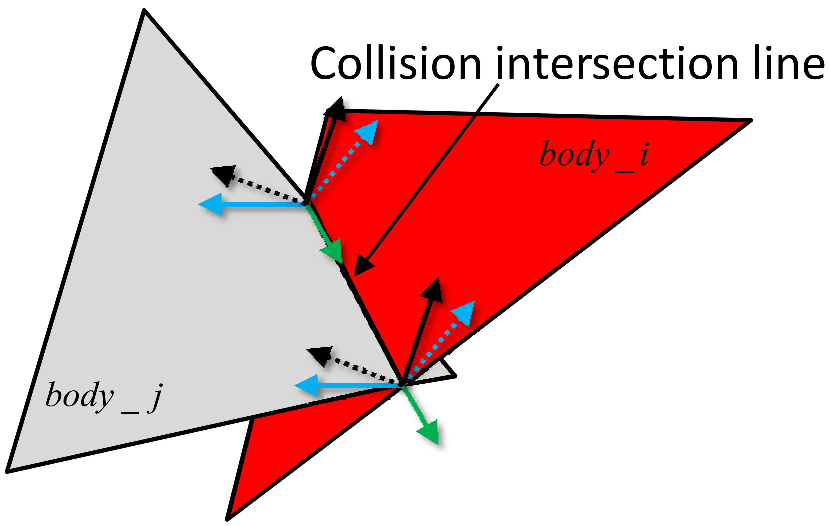


Figure 5.5. The normal and tangential directions for a single contact point.

placement problem (see figure 5.5). In the figure both ends of the collision intersection line are assigned a collision point.

By placing the collision points at the edges of the collision circumference and making the collision normal force a function of the circumference, the problem with the number of collision points affecting the stiffness of the collision can be avoided. Also the placement is relatively simple and requires no additional calculations beyond the triangle-triangle test in the collision detection section. The collision points will also be always valid and consistent. Some methods have to eliminate or sort the detected collision points for example by removing double points in order to function properly. This can lead to errors in the sorting which is not necessary in placing the points in the proposed manner.

The problems arising from placing the collision points at the collision surface circumference are where to point the force normal and how to solve the penetration.

5.2.3 Solving the normal directions

In the proposed method neither of the bodies is considered to be deformed. Since the penetrated sections of the colliding bodies are considered invalid, the edges of the collision surface (see Fig. 5.6) should repel the opposing objects in the normal direction. In the figure 5.6 the black lines represent the normal directions of both objects and the red and gray arrows represent the direction of the resulting force. In the proposed solution both of the normal directions are used to produce a correct resultant force.

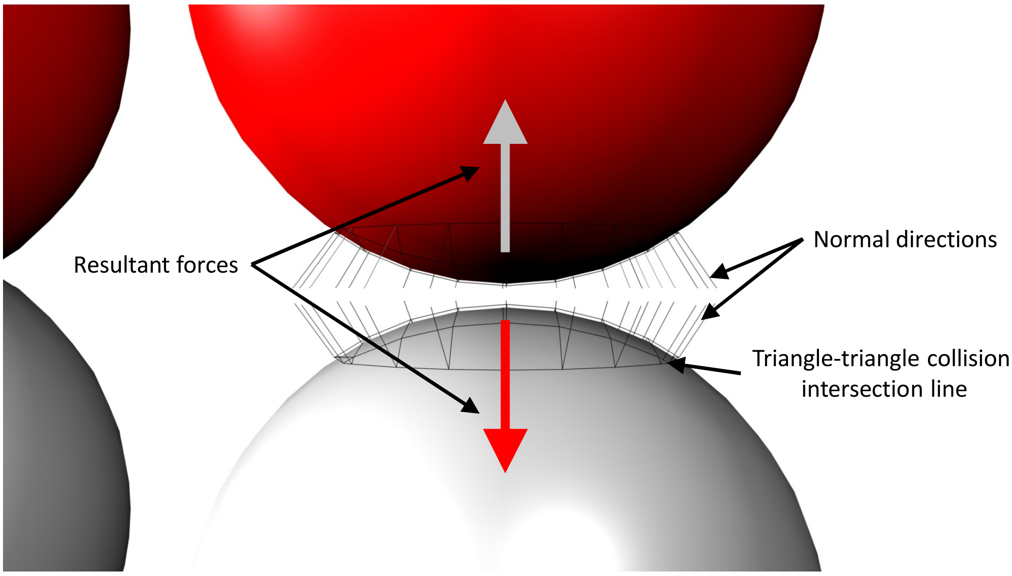


Figure 5.6. Determining the normal directions using the rigid-body assumption.

A detailed drawing of the normal directions in the case of a single triangle-triangle collision can be seen in figure 5.5. In the image the dashed black arrow and the solid black arrow represent the normal directions in the respective bodies i and j in both collision points of a single collision intersection line.

The proposed method seems odd at first but provides very reliable results. The results provided by the method are consistent completely regardless of the used geometry. No assumption on the penetrated volume is made but only the exact information of the circumference is used. Also the normal direction problem does not exist in the sense that it is always a known surface normal and never an assumed direction.

This leaves the method with only the problem of solving the penetration which is presented in the following section.

5.2.4 Solving the penetration at the collision point

The proposed method solves the problem of penetration at the collision points by integrating the relative velocity along the normal direction. In Fig 5.5 the penetration of body *i* for example would be calculated by integrating the relative velocity in the direction of the black dashed arrow. This in addition to placing the collision points at the collision surface circumference makes sure that the collision forces do not lose the temporal history and cause problems as the bodies interpenetrate. The solution seems very simple but it has to be noted that it would not work without the addition of the temporal history through the collision detection and triangle identification system.

5.2.5 Solving the normal force magnitude

The existing normal force methods cannot be directly applied for solving the normal force magnitude as they require some information on the colliding geometry. For example in case of two colliding spheres the radii of the spheres is needed. This information is not available in a general case and therefore a new collision normal force method has to be created. The proposed method utilizes the correlation between the collision surface area and the collision area circumference. The equations can be derived using a case where a sphere collides with a plane (see Fig 5.7).

If an assumption is made where neither body deforms the contact area radius can be defined as follows

$$a = \sqrt{2R_1\delta} \tag{5.13}$$

and the contact area will be

$$A_{stiff} = 2\pi R_1 \delta \tag{5.14}$$

According to Popov [59] the normal force in such a case can be calculated as

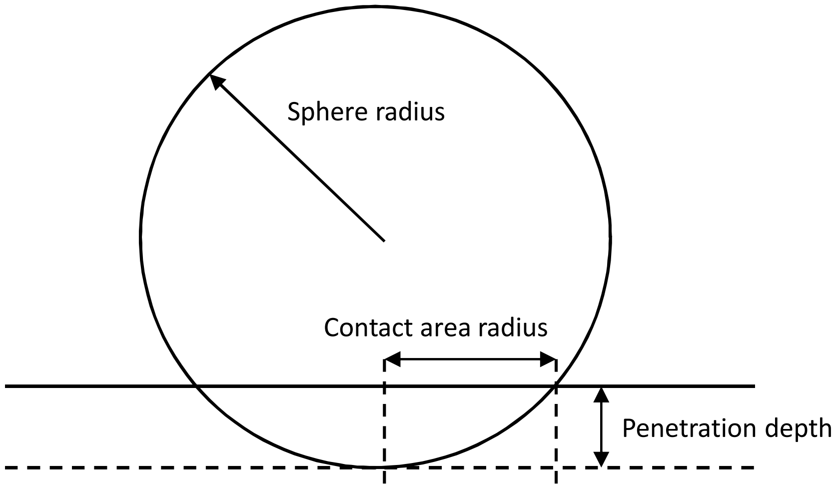


Figure 5.7. Non-deforming Hertzian collision between sphere and plane

$$F_{popov} = \frac{K_p \delta}{2a} \pi a^2 = \frac{K_p \delta}{2} \pi \sqrt{2R\delta} \tag{5.15}$$

in which the stiffness K_p is

$$K_p = \frac{1}{(\sigma_i + \sigma_j)} \tag{5.16}$$

in which σ_i and σ_j are presented in equation 3.3. Popov [59] states that the relation between the undeformable model and the deformable Hertzian model is approximately 1.5.

In the case of a sphere the collision area radius a is in direct relation to the collision area circumference L_c

$$a = \frac{L_c}{2\pi} \tag{5.17}$$

By substituting the relation between equations 5.13 and 5.17 to the equation 5.15 a circumference based force equation can be formed that is dependent only on the circumference and the penetration. This same model can be used as an approximation of different shapes as well, assuming that the behavior between the collision area vs. circumference behaves similarly to a sphere.

$$F_{mod} = \frac{K_p \delta L_c}{4} \tag{5.18}$$

In Fig. 5.8 is presented a comparison of the original Hertz model and the modified model where a sphere with radius R_1= 0.2 m, mass 1 kg, Poisson's ratio ν=0.3, Young's modulus E=210000 N/m^2 collides with a plane with the same material properties. Integration is done using Runge-Kutta4 with a fixed time-step of 0.001 s. From the figure it can be seen that the model behaves very similar to the original model. The figure shows a comparison of the sphere positions over time, force vs. penetration curves as well as the differences in peak positions (sphere reaches highest position) in relation to the theoretically correct value (1 m). Also the time difference in the peak times is shown to illustrate the phase shift caused by the differing stiffness in the models. It can be seen from the force vs penetration curve that the proposed model is stiffer than the original model. This also causes a phase shift in the models making the modified model bounce faster than the original Hertz model.

If a factor L_r is introduced in order to compensate for the difference in the deformation between the stiff and deformable model the results will be more accurate. This can be done by adding a stiffness factor L_r to the equation 5.18

$$F_{modFr} = \frac{K_p \delta L_c}{4} L_r \tag{5.19}$$

For example using a L_r = 1/2 to compensate for the factor 1.5 stated by Popov the results are better in comparison to the original model as can be seen in Fig 5.9. The modification is analogous to the modification of the power n in the Hertz model. In comparing the figures 5.8 and 5.9 it can be seen that the model with L_r = 1/2 is slightly softer than the Hertz model and therefore has a negative phase shift. It can also be seen that the peak heights vary irregularly showing that the time-step is not small enough to show consistent performance.

By substituting the eqn. 5.18 to the Flores model eqn. 3.10 a force model based on the circumference can be obtained.

$$F_{Floresmod} = \frac{K_p \delta L_c}{4} L_r [1 + \frac{8(1-c_r)}{5c_r} \frac{\dot{\delta}}{\dot{\delta}^-}] \tag{5.20}$$

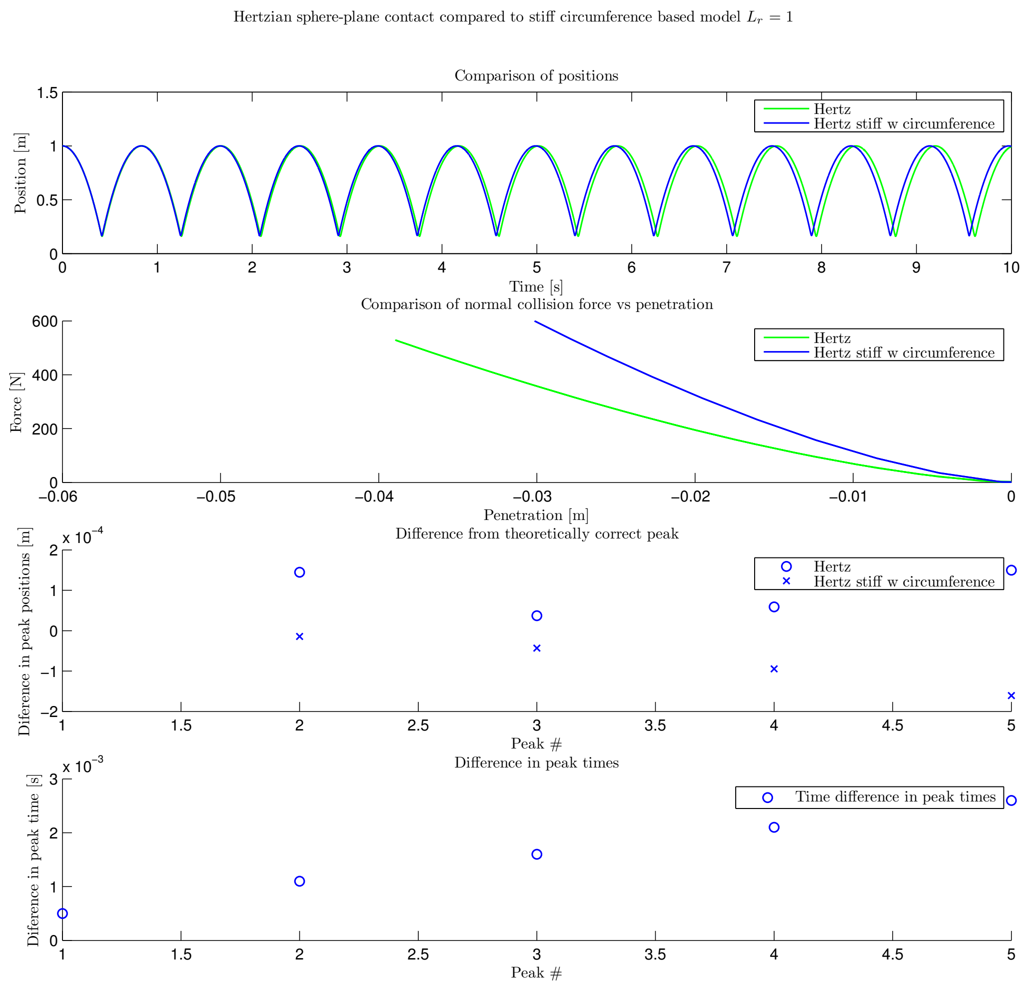


Figure 5.8. Collision model comparison using the Hertz model and the modified model

Fig 5.10 shows a comparison of the original Flores model versus the modified model using a $L_r = \frac{1}{2}$. In the comparison a time-step size of 0.0001 s was used to reduce the error caused by the penalty method stiffness. This way the differences in the peak heights will be consistent enough to make conclusions about the model. From the figure it can be seen that the proposed model is slightly softer than the original Flores model making it fall behind the original model. It can also be seen from the peak heights that the original Flores model is gaining in

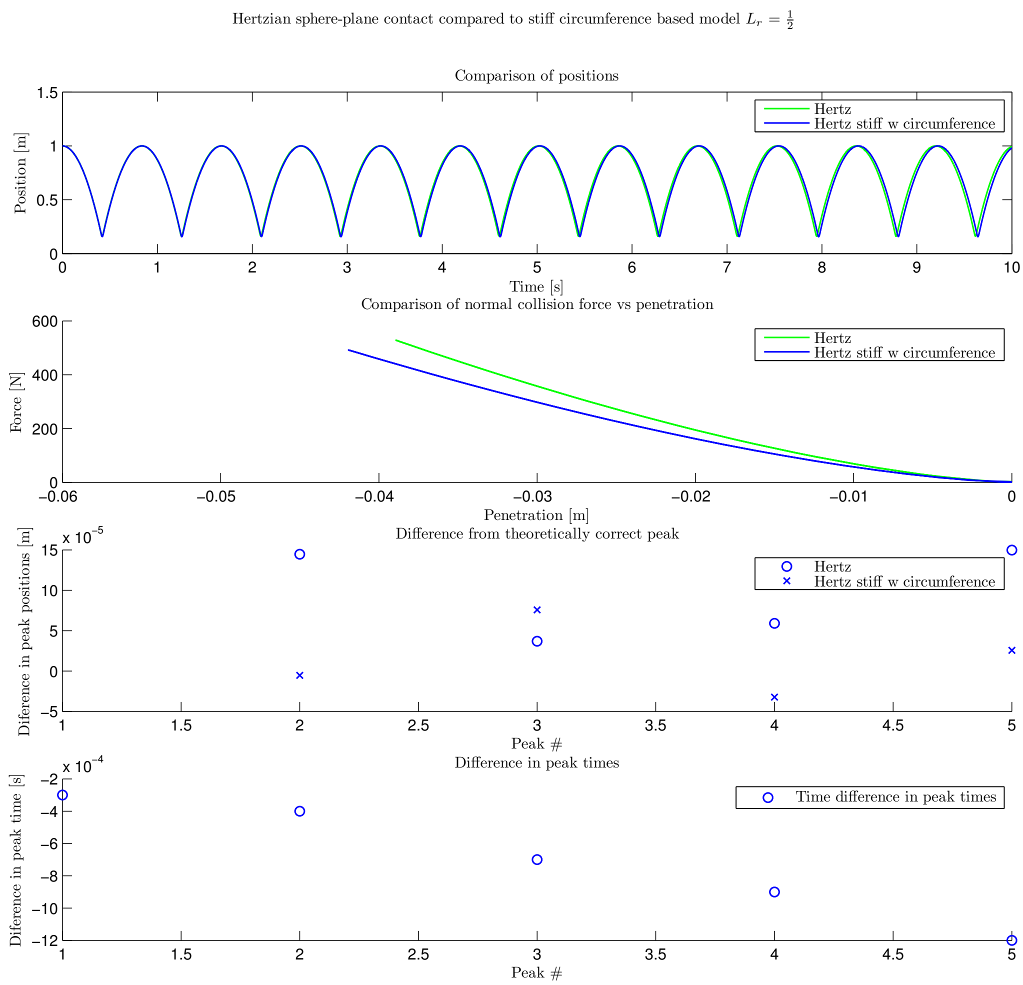


Figure 5.9. Collision model comparison using the Hertz model and the modified model with $L_r = \frac{1}{2}$

height whereas the proposed model is losing height.

The proposed model can be used to remove the need to know anything about the colliding object shape. The link between the collision geometrical stiffness to the normal force magnitude is represented through the circumference of the colliding surface. The model is an approximation and holds true to collision area shapes that behave similar to spherical contact areas. In most cases this is a valid assumption and in general it is a good general approximation.

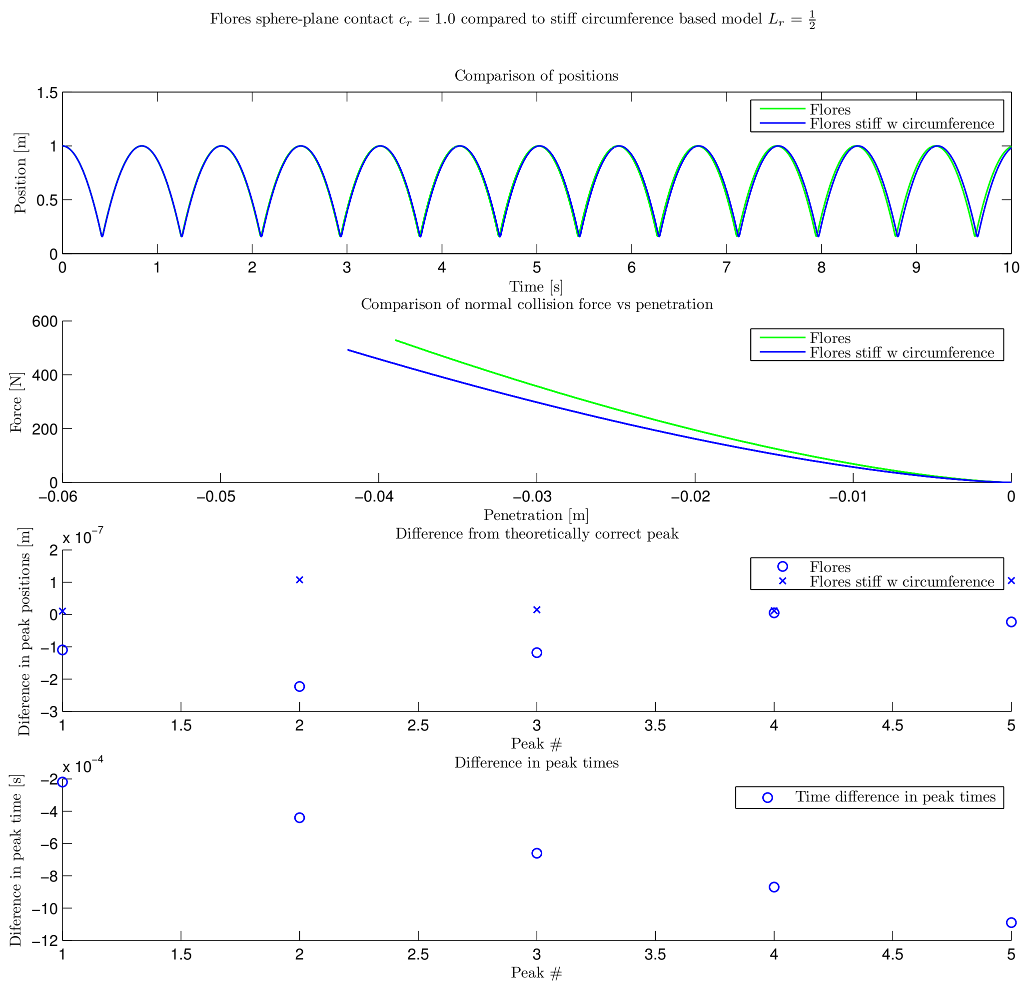


Figure 5.10. Collision model comparison using the Flores model and the modified model with softer L_r and $c_r = 1.0$

The Flores model was chosen as the basis for the model as it is able to reproduce the low c_r values as well. The Flores model behaves correctly in low and high restitution values but is over damped in mid range values. In comparison to for example the Lankarani-Nikravesh model the Flores model behaves better overall. One potential problem with the Flores model is the high stiffness related to the damping in the model. In a low restitution collision the collision force peaks suddenly (see Fig. 6.3)causing instability with longer time-step values.

In the proposed model also the friction needs to be accounted for as a damping element. Implementing the modification to the Flores model and adding the friction, can result in an over damped model. It is later shown in section 6.3 how the friction effects the damping of the system.

Using the force placement data from section 5.2.2 normal direction information from section 5.2.3 and the penetration from section 5.2.4 the normal force in a triangularized case can be solved by adapting the circumference based normal force model.

In order to account for the multiple normal directions and sectioned circumference in the proposed model, the presented model has to be modified. In the triangularized model the different sections of the collision area circumference are calculated separately. This means that the total force acting on the bodies is a sum of all the circumference sections. The normal force $F_{nTriangularized}$ equation can be written as follows:

$$F_{nTriangularized} = \sum \frac{K_p \delta \frac{L_{sec}}{2}}{4} L_r [1 + \frac{8(1-c_r)}{5c_r} \frac{\dot{\delta}}{\dot{\delta}^-}] \tag{5.21}$$

In which L_{sec} is the intersection circumference section length. There are two collision points to each intersection line and therfore the section length is divided by two. In this way the total force can be calculated as a sum of the separate triangle-triangle intersection lines.

5.2.6 Determining the friction force

In determining the tangential directions for the contact the non-deformation assumption is used as the basis. In having two normal directions for each collision point we also need to assume two tangential planes for the force. The two intersecting triangles form an intersection line (Fig. 5.5). This line direction can be used as the common friction surface direction for both normal directions Fig 5.5 green arrow. To complete a friction surface for both normal directions a second axis for both normal directions is calculated as the cross product of the normal direction and the common friction axis (Fig. 5.5 blue solid and dashed arrows representing bodies *i* and *j*). This leads to two friction components representing the friction plane perpendicular to the normal direction for each of the two collision points at the ends of the triangle-triangle intersection line.

The tangential friction forces can be evaluated using the LuGre friction model [8] discussed in section 3.4. The tangential forces can then be applied directly to the body as external forces in the same manner as the normal forces.

The LuGre friction model has been successfully verified for example in [41]. The friction model was not verified in this paper since the only decisive factor in the application is the determination of the friction surface. As the friction surface in itself is tied to the normal directions the verification of the normal forces account for the behavior of the friction force as well.

5.2.7 Damping behavior

The damping behavior of the proposed model consists of two different parts; the damping in the normal force model and the damping introduced by the friction in the system. The normal force models presented in section 3.3 are all presented for cases with no tangential friction. Gilardi [26] presents that the dissipated energy in the normal directional collision can be separated into wave propagation, plasticity, material damping and others(sound, heat). In this separation the others section can be considered to include partially the effects of friction turning kinetic energy into heat. In the proposed model the friction is also calculated separately adding damping into the system. This means that even with a fully elastic $c_r = 1.0$ collision case with friction the system will be damped. Some experiments on the frictional damping behavior of the proposed model are presented in sections 6.3 and 6.4. In the normal force model the damping is presented using the coefficient of restitution. The coefficient of restitution is dependent on many elements such as geometry of the colliding bodies, approach velocity, material, duration of the contact and possibly friction [27]. The effect of friction on the coefficient of restitution is a subject of debate. It may effect and it may not, there is no certainty on it. The proposed method is influenced by all of these aspects in some way. The body geometries effect the location of the collision points as well as the normal force through the length of the collision surface circumference. Approach velocity and material affect the normal force model. The duration could be included in the normal force model in the way Lankarani and Nikravesh did in [46] but it is always based on an estimation at the beginning of the contact and therefore not necessarily applicable to a general case.

In the proposed method the frictional damping is separated from the normal direction damping so the effect can be seen separately. This gives the possibility to study the effect of friction on the collision separately and it may even offer insights on how friction effects real collisions. It should also be noted that such inclusion of frictional damping is not possible in the non-penetrating contact models as there is no tangential displacement.

CHAPTER 6
Numeric validation

In order to validate the proposed model, different experiments were conducted.

One of the characteristics of collision response modeling is the difficulty to verify the models against measurements from real systems. The real collisions depend on so many different parameters ranging from temperature and air humidity to geometry surface roughness whereas the simulation models are affected by a limited number of parameters. This makes conducting exact measurements extremely challenging and usually the results can not be verified separately but as a whole. For example measuring the effect of friction to a collision is next to impossible but a collision occurrence including friction can be verified as a whole to a certain accuracy. Therefore the validation of the model is done by comparing it to existing Hertzian models.

Colliding frictionless spheres have been a favorite of researchers since Heinirich Hertz developed his initial contact theory in the 1880's [33, 40] and therefore the natural selection for validating the model is through using two colliding spheres. The initial validation is done through a series of tests:

- In section 6.1 a theoretical 1D mass model is created using Matlab/Simulink in order to compare the developed mathematical normal force model to the currently existing models.

- In section 6.2 a test is conducted using the MeVEA [49] simulation software. The complete geometric collision model is implemented into the MeVEA software and a model with two identical spheres similar to the mathematical model presented in 6.1 is created using triangularized geometries instead of mathematical representations of spheres. The triangularized model is compared to the simulated results from section 6.1

- In section 6.3 the effect of friction on the damping of the system is shown using a perfectly elastic case damped only with friction.

In order to validate the model behavior in general cases, further experiments are conducted in sections 6.4, 6.5 and 6.6.

- In section 6.4 the method is demonstrated on a pair of cubes in order to demonstrate the working principle on different shapes other then spheres.
- In section 6.5 is presented a simple case where multiple differently shaped objects are dropped into a box. This test was conducted to show that the proposed method works well regardless of the used geometries.
- In section 6.6 is presented another typical collision test secenario, the box stacking.

As the model is designed for use in real-time simulators some guideline timings on the proposed method are presented in section 6.7.

6.1 1D mathematical comparison of normal force models

A 1D mass model was constructed in Matlab/Simulink in order to compare the different models. The model consists of two spheres with radius R=0.2 m, mass 1 kg, Poisson's ratio ν=0.3, Young's modulus E=2100000 N/m^2 and coefficient of restitution c_r=0.5. One of the spheres is stationary and the other is dropped on it from a height of 1 m. The simulation was done using the Runge-Kutta 4 integrator and a time-step of 0.0001 s.

The results can be seen in Fig. 6.1. It can be seen that the proposed model behaves as the Flores model does. The differences in the model behavior between the proposed and the Flores model are insignificant.

The different models behave very differently with different coefficients of restitution. The Hunt-Crossley and Lankarani-Nikravesh models were initially made for stiff materials such as steel (coefficient of restitution above at least 0.4) which can be seen in Fig 6.2 and 6.1. The Flores model behaves well in high

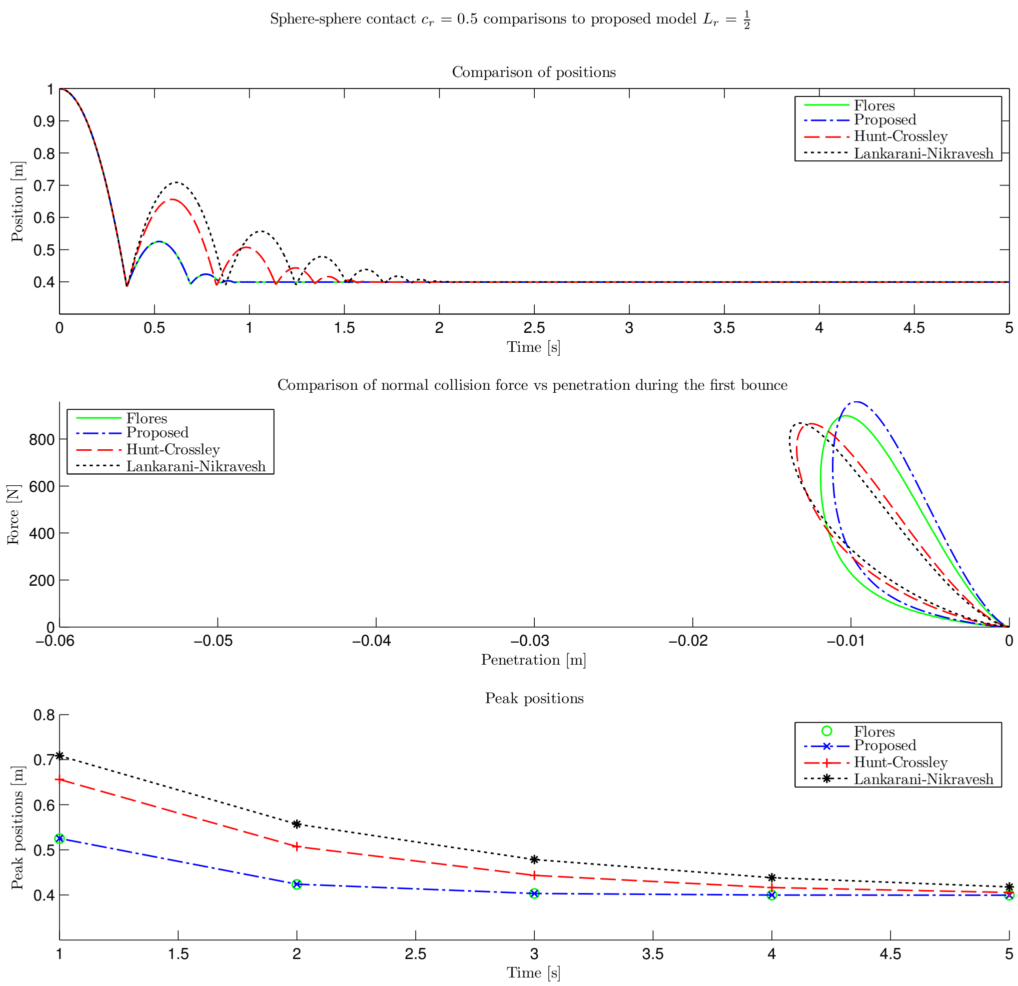


Figure 6.1. Collision model comparison using coefficient of restitution 0.5. Two Spheres R = 0.2 m

restitution collisions as well but is over damped in midrange restitutions. It can be calculated that with two 0.2 radius spheres, c_r=0.5 and a drop height of 1.0 m the initial bounce peak position should be 0.7 m. The Flores model only reaches the height of approximately 0.5 which is considerably less than the calculated value. The Lankarani-Nikravesh model bounces to 0.7 and the Hunt-Crossley to 0.65 which can be considered to be correct results. The use of a lower coefficient of restitution on the other hand reverses the situation. When using a coefficient of restitution of 0.05 (as the Flores model cannot represent a fully plastic contact of $c_r = 0$) it can be seen (see 6.3) that the Hunt-Crossley and Lankarani-Nikravesh models still exhibit similar behaviour to $c_r = 0.5$. When the first bounce should be 0.43 m it is 0.63 with the Hunt-Crossley model and 0.6 with the Lanakrani-Nikravesh contact. The Flores model bounces to a height of 0.4 m behaving the best of the original models. The comparisons show a general trend. Hunt-Crossley original model and Lankarani-Nikravesh tend to under dampen the systems with a low c_r whereas the Flores model tends to over dampen all medium restitution systems. It has to be pointed out that the results are all relative to the time-step used as well as the ν and E. Different time-steps produce different ratios in the different methods.

From Figures 6.1, 6.2 and 6.3 It can be seen that the modified Flores model works very similar to the original Flores model. The proposed model is somewhat harder than the original Flores model but the difference is not significant. The added stiffness also increases the amount of damping in the model.

6.2 Geometry based spheres compared to mathematical spheres

The proposed model has been implemented in the MeVEA ltd. [49] simulation environment. Using the MeVEA simulation environment a test model of two frictionless triangularized spheres was created in similar fashion to the example in section 6.1. The force model is implemented outside of the dynamics integration loop (runge-Kutta4) in the simulation environment, so the resulting force is a mix of integration methods. The force related integrable variables are integrated outside of the dynamics Runge-Kutta4 integrator using an Euler integrator. The 1D comparison models were all integrated using the Runge-Kutta4. The original Flores model was compared to the modified Flores model as well as the geometric proposed model. The Fig. 6.4 shows that both the proposed theoretical and geometric models behave very similar to the Flores model. In the figure the force showed as the proposed geometric force is the total force in vertical direction acting on the bouncing sphere which is comparable to the vertical force produced by the 1D models. The geometric model has non-vertical components as well but they are not shown in the comparison as they do not affect the vertical motion.

It can clearly be seen in figures 6.4, 6.5 and 6.6 that the proposed method is harder than the Flores model. The blue dashed line represents the modified 1D Flores model presented in section 6.1, the red dashed line is the triangularized geometry based model and the green line is the original 1D Flores model. The

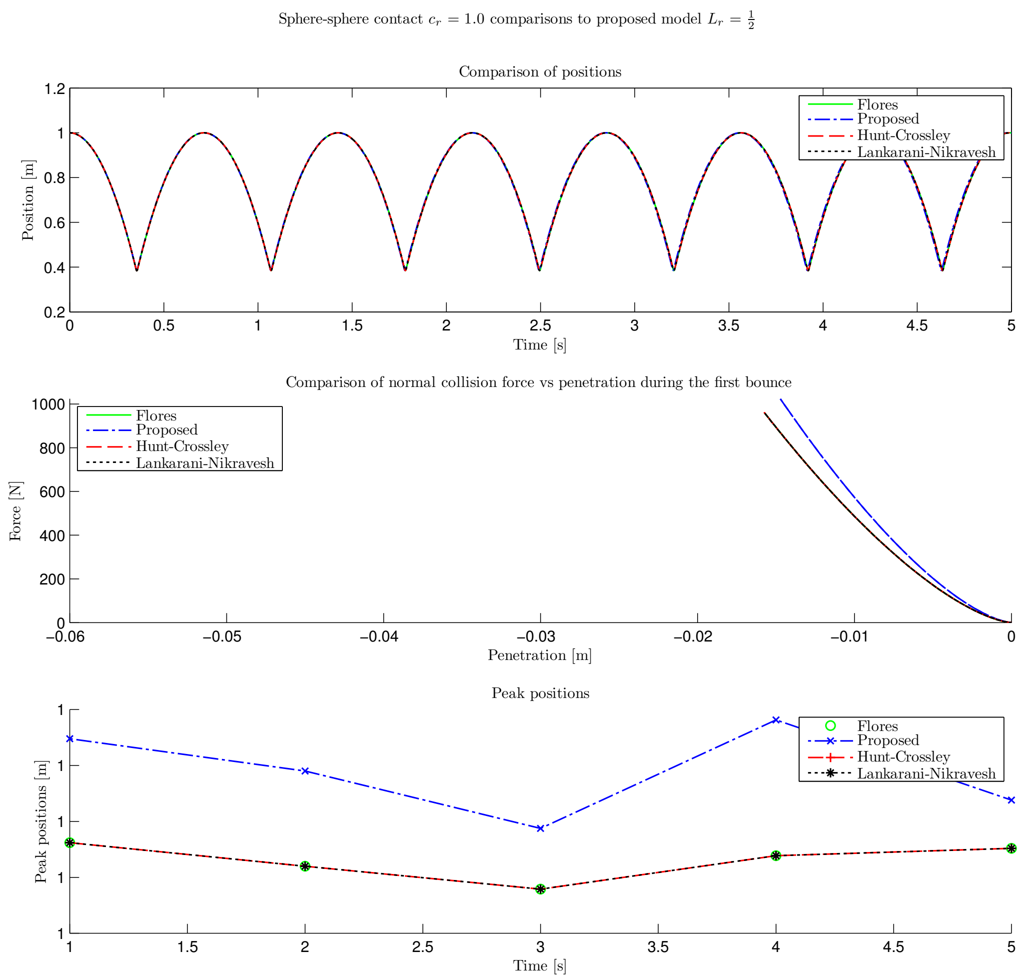


Figure 6.2. Collision model comparison using coefficient of restitution 1.0. Two Spheres R = 0.2 m

difference in the stiffness of the systems can be noted in the penetration vs. force curves and the damping in the time vs. position curves.

The slight differences in the force rise times and resting positions are caused by three things. First of all the entry level error presented in section 4.2. As the integration for the penetration is initiated only when the initial contact is detected, instead of the exact moment of collision, there is a level error in the surface. This can be seen as a later force build up point as well as a lower resting position. Secondly in the MeVEA implementation the integration is

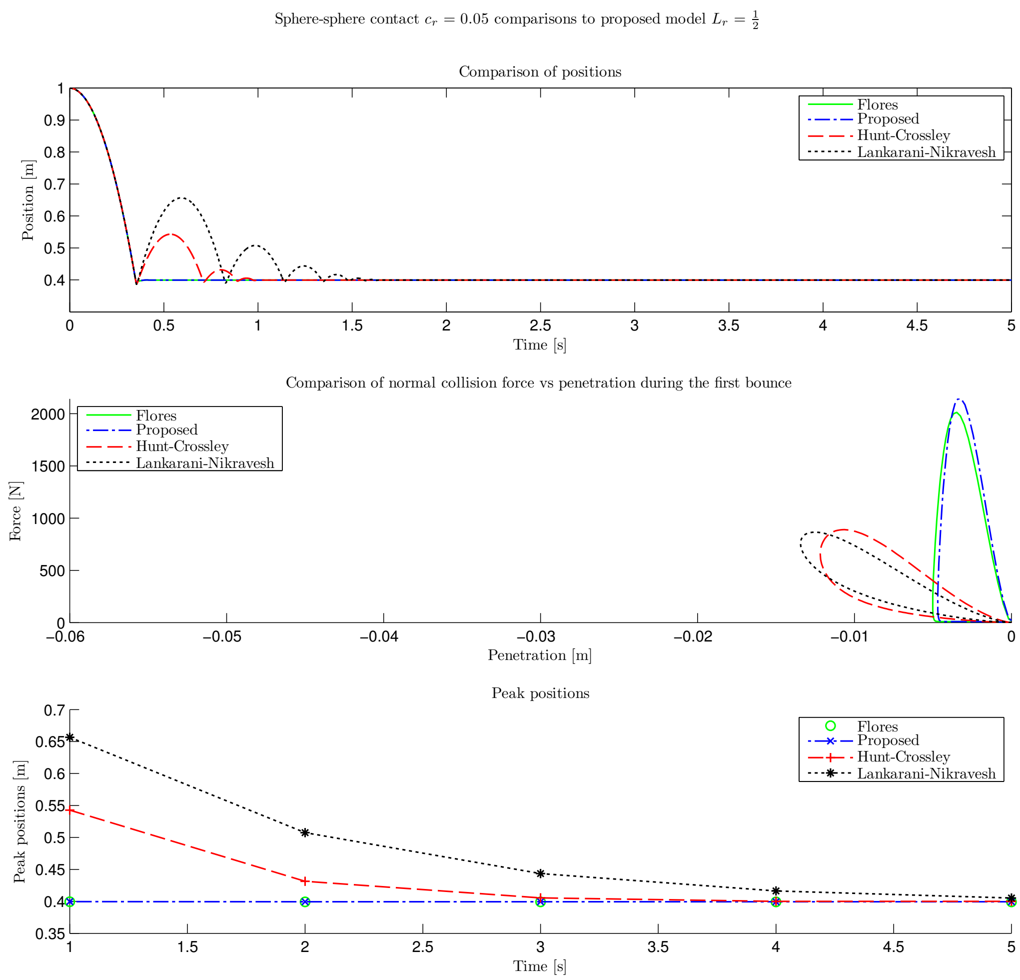


Figure 6.3. Collision model comparison using coefficient of restitution 0.05. Two Spheres R = 0.2 m

done based on the velocities from the previous calculation cycle and therefore there is a phase shift in the forces in comparison to the positions. This also causes a delayed force build up. Also as the triangularized spheres are only approximations of a sphere there is always a difference in the radii of the spheres at any given point. This also causes a small difference in the resting position.

From the results presented in figure Fig 6.5 it can be seen that both modified models perform well in a frictionless highly damped case.

In the fully elastic case shown in 6.6 the proposed geometry based model is

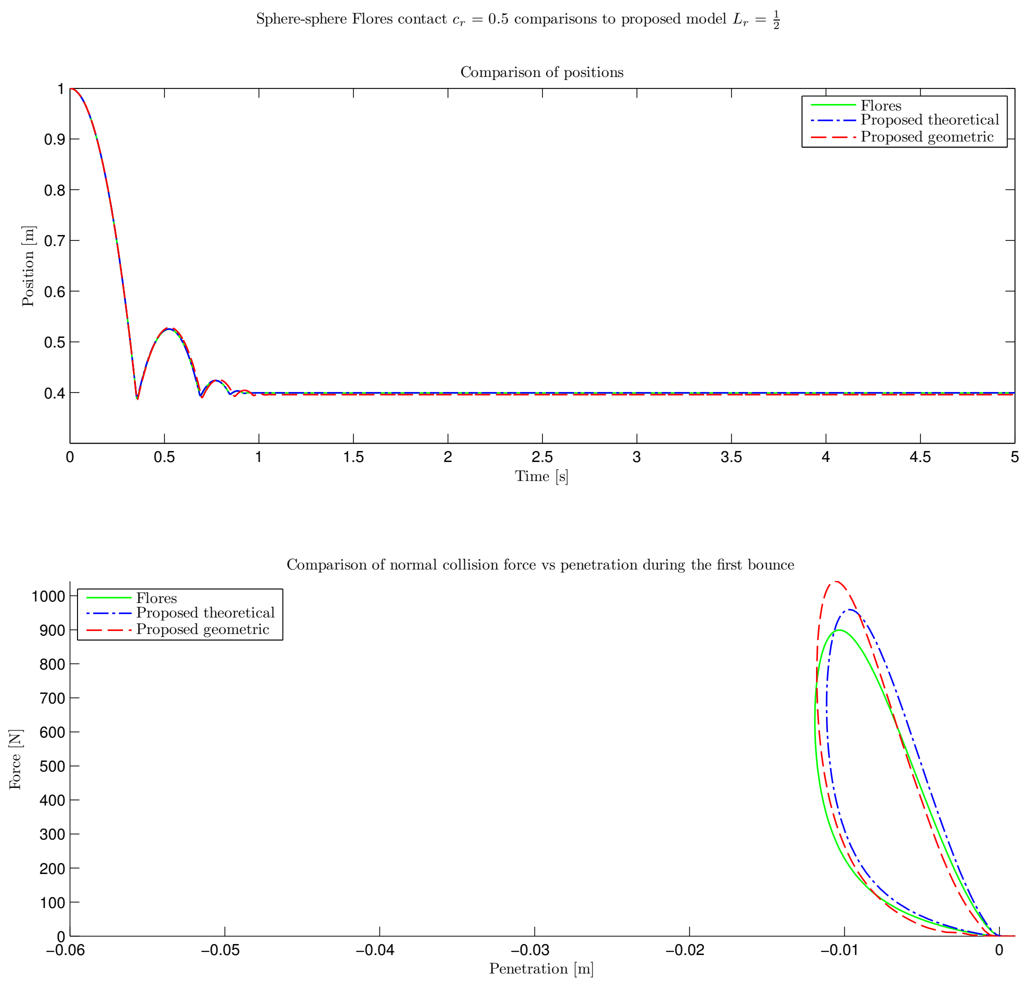


Figure 6.4. Triangularized collision model comparison using coefficient of restitution 0.5. Two Spheres R = 0.2 m

damped when it should not be. This is due to the initial velocity problem described in section 5.1.2. This can be seen as hysteresis in the force vs. penetration curve of the proposed geometry based model. It can also be seen that once the penetration drops to a level where no more successive boundary crossings occur (after 2.5 s) the collision becomes fully elastic again. It should be noted that even though the force model itself is continuous the triangularized model of a geometry is not continuous. Any triangularized geometry is just an approxima-

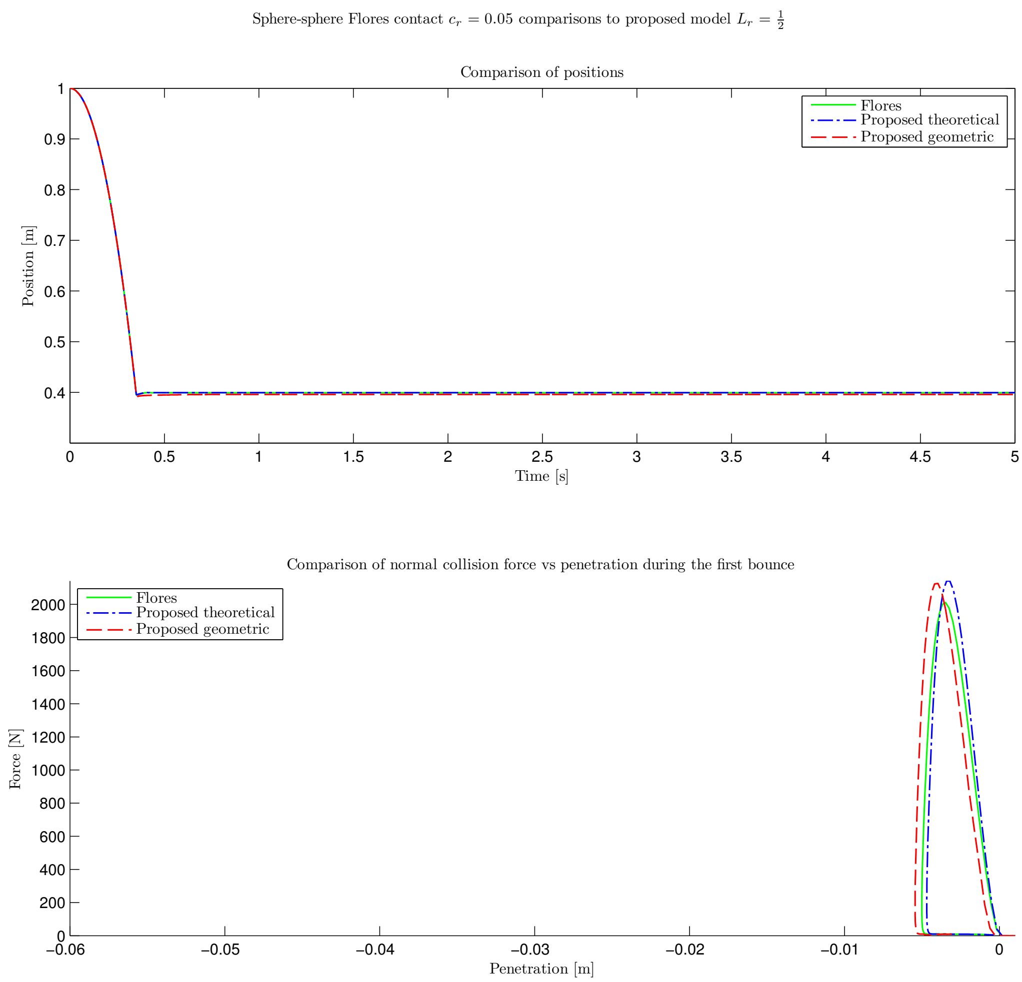


Figure 6.5. Triangularized collision model comparison using coefficient of restitution 0.05. Two Spheres R = 0.2 m

tion of the original form and therefore there will always be fluctuations because of the geometry as the surface normal directions change abruptly. This can be seen in the proposed geometry based model force vs. penetration curve as drops in the force amount. The penetration is carried over the triangle bound crossings as described in section 5.1.2 but as the triangle normal direction changes the vertical force also changes.

From the results it can be seen that the model behaves similarly to the Flores model. The proposed model can be made to show low restitution despite the c_r

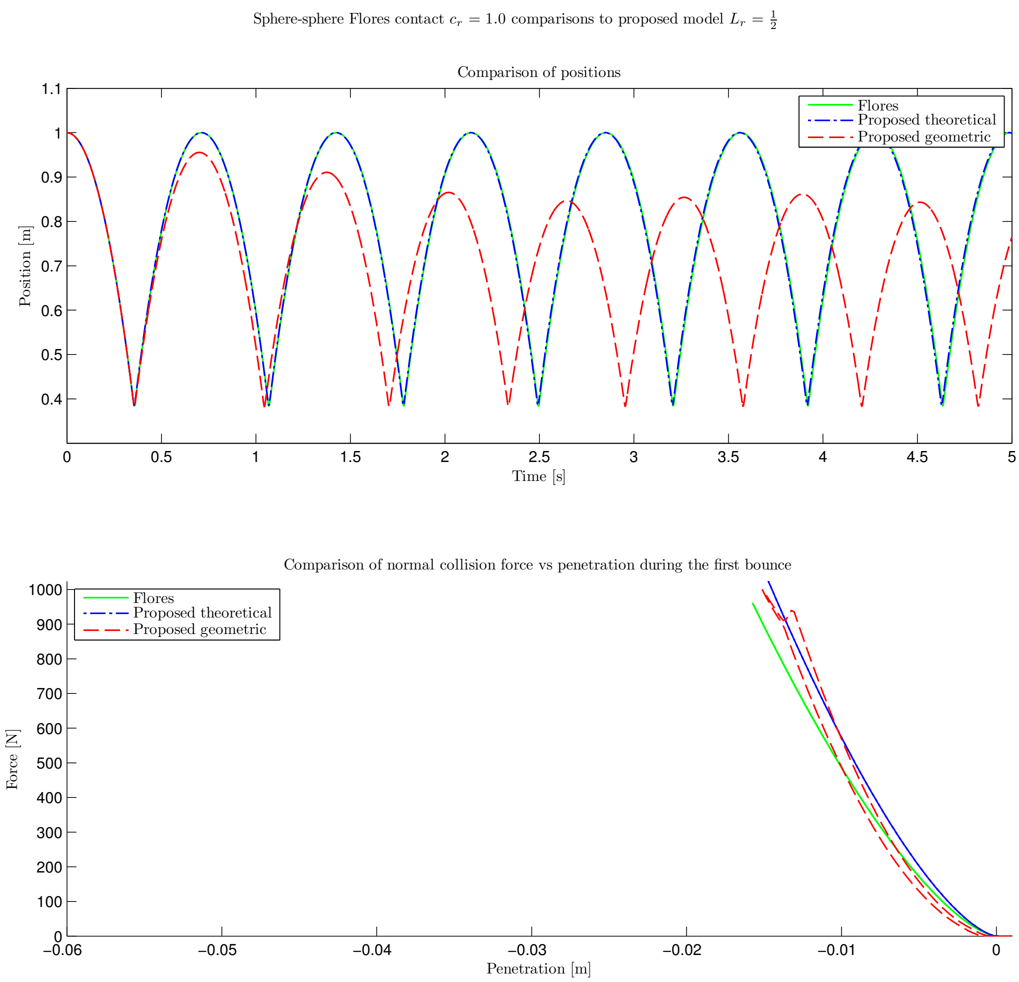


Figure 6.6. Triangularized collision model comparison using coefficient of restitution 1.0. Two spheres R = 0.2 m

used by adding some friction to the model. The tangential friction term adds damping to the system allowing for higher damping to be displayed. More on the frictional damping behaviour of the test model is presented in section 6.3. The biggest problem in the model regarding the accuracy is the multiple triangle bound crossing that causes additional damping in the system. In cases where there are few or no triangle boundary crossings these effects do not show (see Fig. 6.5). It should be noted that in cases where multiple triangle boundary crossings occur, the initial assumption of local deformations is already broken.

6.3 Sphere collisions with friction

The normal force models presented in section 3.3 all address frictionless spheres. In trying to include the energy dissipation to heat caused partially by friction, some frictional damping has been included in the model damping terms. The proposed model includes the tangential friction term as well which causes additional damping to the system in addition to the normal force damping. In order to show the damping behavior of the friction term a test with the spheres described in section 6.2 was conducted. The spheres were set as fully elastic (c_r = 1.0) but the friction parameters were set very high (σ_0 = 200, σ_1 = 1, $\dot{x_0}$ = 0.001, μ_s = 0.3, μ_d = 0.1 and c_t = 0.1).

It can be seen in Fig 6.7 that even fully elastic spheres can be made to behave damped by just adding a tangential friction term to the model. In the figure the green line is the original 1D Flores model, blue dashed line is the 1D modified Flores model without friction and the red dashed line is the proposed geometric model with friction. The friction in the example is exaggerated in order to show the effect of friction on the damping behaviour of the model. The exaggerated friction also brings out the triangle limits in the plot. In the friction model the collision parameters cannot be combined over triangle limits in the same manner as in the normal force parameters (see 5.1.1). The friction force is relative to the normal force in the collision point. The normal force is carried over the triangle limit but the sliding state of the collision point can not be carried over as it would cause multiplication of the collision force. This problem causes some fluctuations in the colliding force with high friction values. In comparison to the friction force the normal force does not change on triangle boundaries unless the normal direction changes whereas the friction does not transfer over the triangle limits. Therefore the friction is changed every time a triangle boundary crossing occurs.

6.4 Colliding cubes

In similar fashion to section 6.2 a model with two cubes was tested (see Fig. 6.8). The model consists of two 1 kg cubes, with the same material properties as the spheres in section 6.2, being dropped on each other. The simulation settings are the same as in the example with spheres. The lower cube is fixed on the ground and the higher cube is fixed with a translational joint in order to show the linear collision performance. The purpose of this experiment is to show the functionality of the normal force model in a general case.

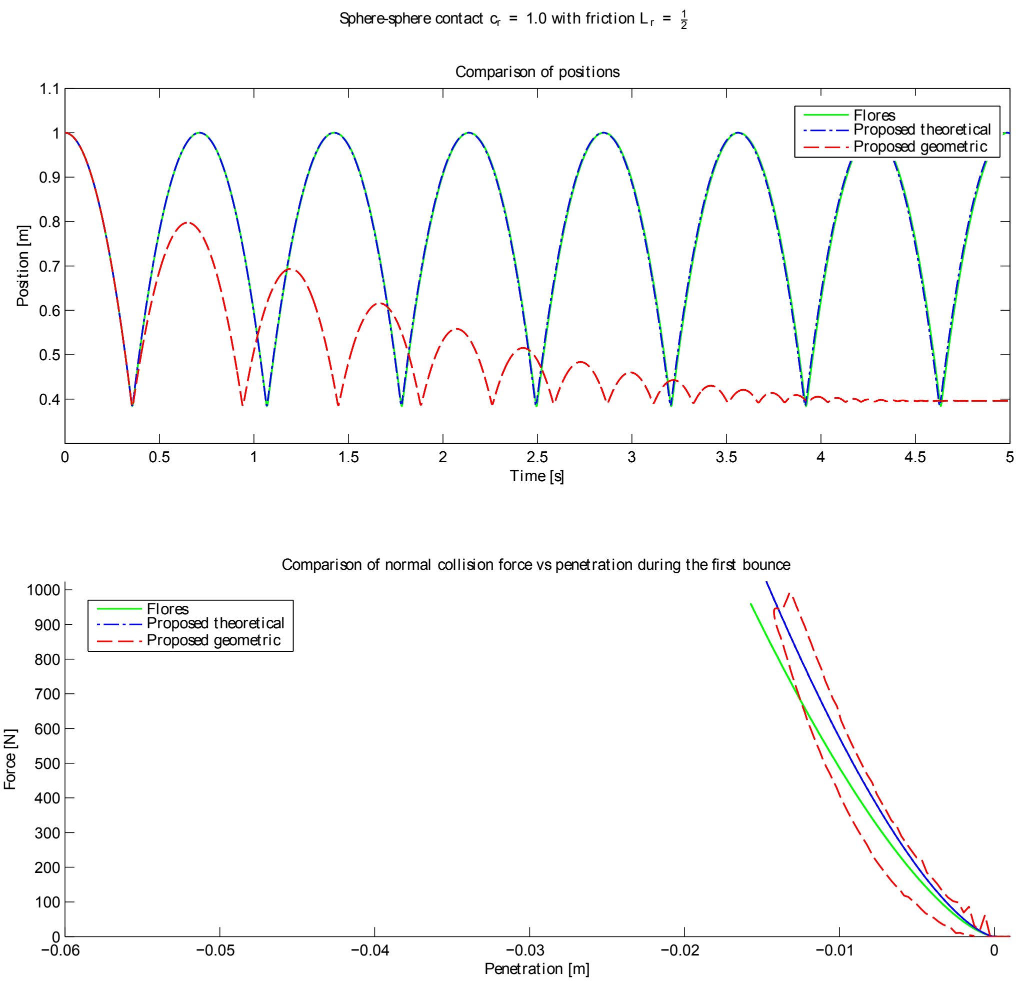


Figure 6.7. Triangularized collision model comparison using coefficient of restitution 1.0 with friction. Two Spheres R = 0.2 m

In figure Fig. 6.9 is presented the position of the bouncing cube in a frictionless case using different coefficients of restitution. As can be seen in the figure the cube behaves consistently with the results in the earlier experiments with spheres. It can also be seen that since in the cube case there are no multiple triangle boundary crossings there is no additional damping (c_r = 1.0 does not dampen out).

In addition the cube was tested using high friction in order to show the effect of

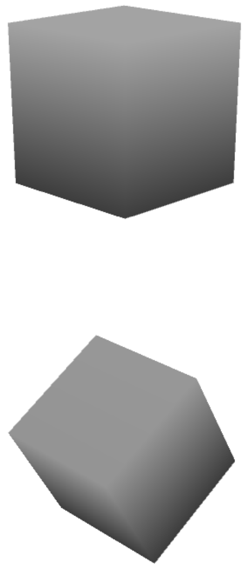


Figure 6.8. The colliding cubes.

the frictional damping in the case of cubes. In Fig. 6.10 is presented the position of the cube in the frictional case compared to the frictionless case. As can be seen the system becomes damped with the addition of friction.

These experiments show that the proposed model works well with different geometries other than spheres as well. These experiments also show how the circumference based system enables the collision stiffening based not only on the penetration depth but also the colliding object shapes.

6.5 Multiple colliding objects

A test was conducted where differently shaped objects were dropped into a box. The test model consists of a sphere, box and a monkey head (a standard shape from Blender 3D content creation program [5]) being dropped into a box. The initial positions of the objects are shown in Fig. 6.11(a).

The Fig. 6.11(b) shows the final resting positions of the objects. The simulation is run at a time-step of 1.5 ms which is acceptable for soft dynamic systems. Usually stiff simulation models containing for example hydraulics should not

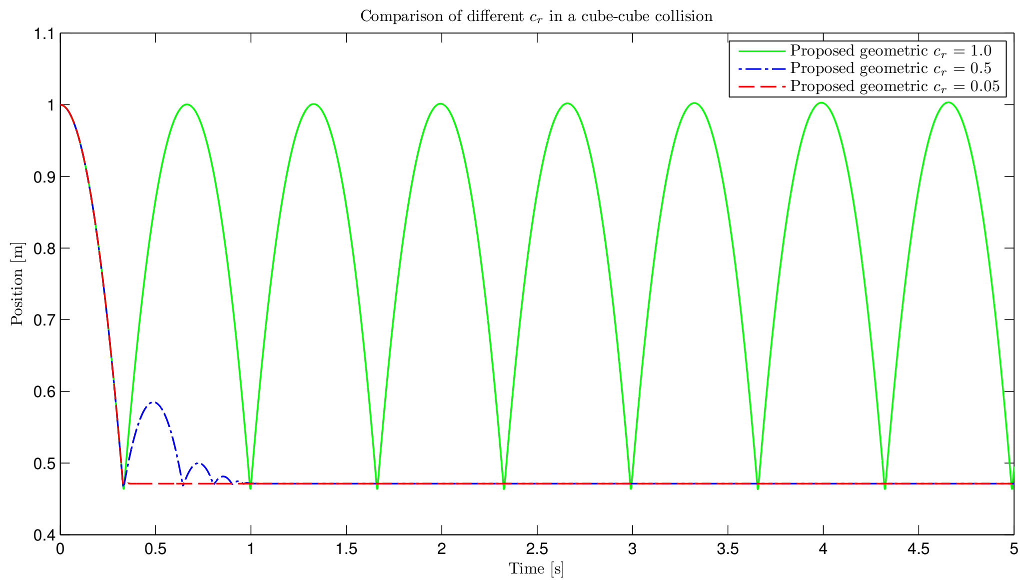


Figure 6.9. Triangularized cube collision model using different coefficient of restitution. Two cubes

Object	Box	Sphere	Monkey
Number of vertices	8	137	507
Number of triangles	12	270	968

Table 6.1. Table of the number of vertices and triangles

be run using time-steps over 1.5 ms depending on the used integrator. The monkey head is quite a complex geometry which makes the collision detection computationally expensive. The table 6.1 shows the number of triangles and vertices in each object.

The objects were dropped from different configurations and no configuration presented any problems. The same experiment was also conducted using different shapes as well as mesh densities. The proposed method performed consistently with each different shape combination. These experiments show that the proposed method performs consistently also regardless of the colliding shapes.

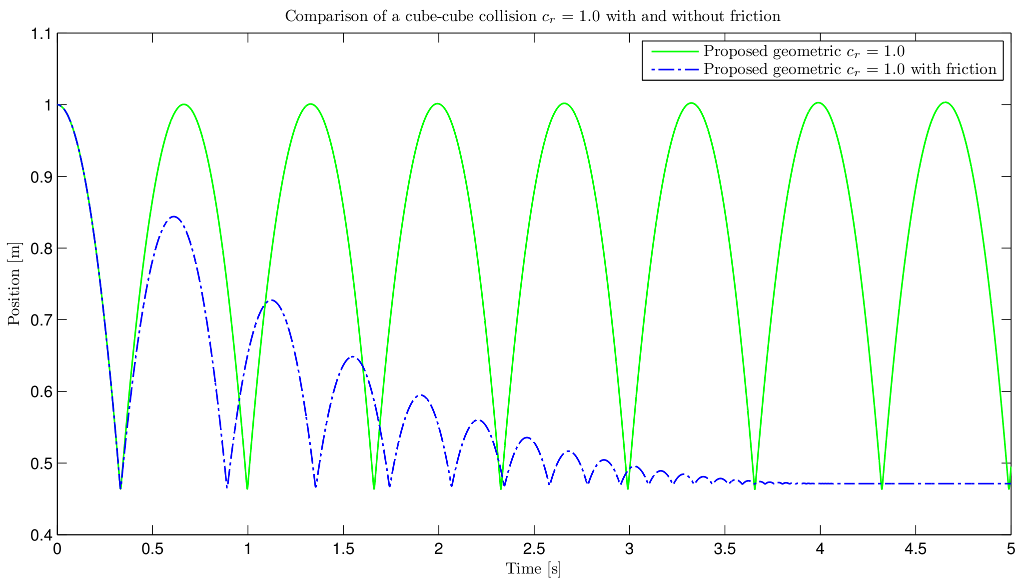
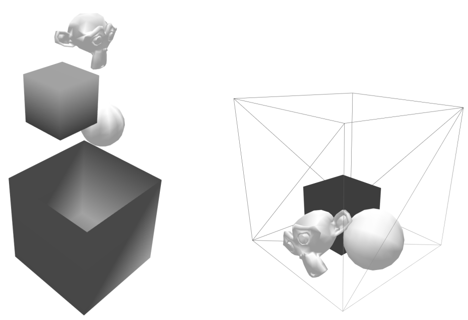


Figure 6.10. Triangularized collision model comparison using coefficient of restitution 1.0 with and without friction. Two cubes



(a) The initial positions of the objects (b) Resting positions of the objects

Figure 6.11. Example of multiple colliding objects

6.6 Box stacking

One of the established collision modeling performance tests is the box stacking experiment. The box stacking is used in order to show the behavior of the modeling method in different stack configurations. Originally the overhanging box stack test was performed by Baraff in [3]. In this experiment the boxes are 10 kg cubes with inertias $I_{xx} = 0.0666$, $I_{yy} = 0.0666$, $I_{zz} = 0.0666$. The cubes are 0.2 m in each direction. Three different stack configurations are presented straight identical tower, shifted tower and a balanced configuration.

The first case is a stack of identical cubes placed exactly on top of each other shown in Fig 6.12. Some modeling methods such as methods where the collision points are placed on the geometry vertices will fail in such cases as the penetration formation will fail due to the vertices sliding exactly along the opposing surface. Another cause for failure can be the formation of the normal directions. If the normal direction is calculated to the opposing surface or towards the nearest point of exit the method will potentially fail. The proposed method is able to solve the configuration without any problems the figure 6.12 shows the cubes after 10 minutes of simulation time. The cubes stay aligned and the collision behaves correctly.

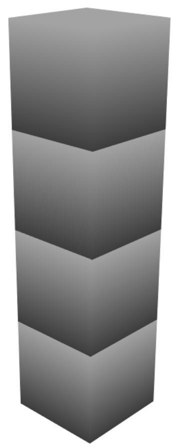


Figure 6.12. Identical cubes stacked exactly on top of each other.

In the second case the box stack is shifted so that the tower over balances causing it to fall over as shown in Fig. 6.13. This test will show the method capability to carry a load over the cube corner. Some modeling methods have trouble forming the load carrying collision points at the edges of two colliding cubes. A common problem is ending up in a situation where the cubes get pushed away from each other when the normal direction shifts from vertical to horizontal. This can be seen in that the stack of cubes does not remain in the same position in relation to each other with the exception of the lowest cube where the toppling over occurs. As can be seen in the Fig. 6.13 the proposed model behaves correctly in the toppling over.

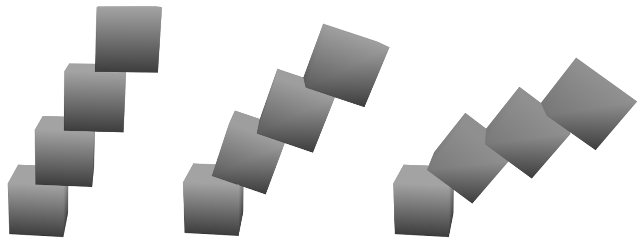


Figure 6.13. Identical cubes stacked with an offset to form a tilted stack.

In the third case the the cubes form a balanced stack as shown in Fig. 6.14. The method allows the balanced stack to remain in place for extended periods without any problems. Some methods suffer from drifting of the stack due to the stack tilting slightly. This is caused by the cubes penetrating each other very slightly and causing the tower to tilt. The slight tilt in turn causes models with weaker friction models to creep eventually out of position. The proposed method does not suffer from such problems.

As a conclusion of these and other similar experiments the cubes can be stacked in any configuration without any problems. The method performs consistently regardless.

6.7 Method performance timing

A test for timing the collision detection was not conducted as a separate test as the timing is completely case specific. Some benchmark cases have been presented for example by [69] but the proposed cases are not necessarily suitable for real-time simulations nor are they conclusive in proving the performance of the system. Separate testing benchmarks should be suggested in order to test separately the collision detection and collision response methods. As a combination the results are not conclusive. Some general timings of the different test cases were made though. For example the model in section 6.2 can be run at 0.05 ms using the same sphere as in this example with a quadcore 2.3 MHz PC. This however does not give a conclusive information on the effectiveness of the calculation method.

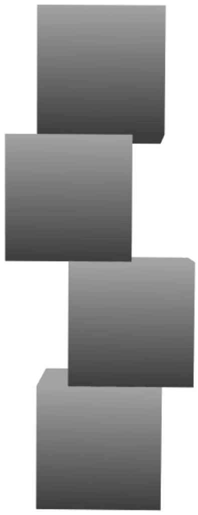


Figure 6.14. Identical cubes stacked with an offset to form a balanced stack.

The collision force calculations were also timed in comparison to a single triangle-triangle intersection test. The triangle-triangle intersection test was performed using the method proposed by Devillers [14]. Adding the force calculations to the simulation model approximately doubles the processing time of a single triangle-triangle collision in comparison to just doing the collision detection. This doubling affects only the colliding triangles, which usually is a fairly low number. This means that the decisive factor in the proposed method in terms of calculation efficiency is the collision detection phase and in particular how the triangles are eliminated from the collision.

In the developed model the collision detection phase can be further improved by adding optimizations. This study focuses on the creation of a collision response method and therefore all possible optimizations on the part of the collision detection have not been utilized yet.

6.8 Advantages of the proposed method

The proposed model solves some of the key issues in the geometric penalty based collision models dealing with collision point placement, collision normal direction and collision force magnitude.

The assumption that the deformation in the collision area is relatively small and that the penetrated volume is inaccurate leads to a simplified model of the collision. The accuracy of the model is not compromised as long as the assumption of relatively small deformations in the collision is not broken. Using the collision circumference as the source of accurate information on the colliding objects eliminates the need for inside/outside calculations or other calculations required to determine the collision surface/points.

By placing the collision points at the ends of the triangle-triangle intersection lines further calculations on placing the collision points can be avoided. There is no need to calculate for points on the penetrated volume because it is considered inaccurate information. Also the intersection lines capture all the relevant surface features related to the colliding objects and therefore no geometry feature is neglected.

Using the collision area circumference to place the collision points also simplifies greatly the determination of the normal directions for the forces. The triangle normal directions can be applied directly as the normal directions and therefore discontinuities in the normal directions can be completely avoided. Also the collision force will always point consistently with the colliding surfaces avoiding problems with normal force pointing in an incorrect direction as a result of an incorrect assumption in calculating the normal direction. In terms of efficiency the solving of the normals directly from the colliding surfaces saves a number of calculations as opposed to solving for the normal direction for example through the nearest point of exit search.

In order to use the collision area circumference as the source of accurate information a key issue is providing temporal information in order to utilize integration. Without the possibility to integrate variables the collision surface edges could not be used to determine the collision. Using the possibility to integrate variables the normal force magnitude at the collision points can be solved by integrating the penetration at the collision points. This also leads to a consistent solution without any discontinuities due to normal direction changes or geometry shape.

The dynamic friction model utilized in the proposed model also alleviates some of the problems occurring for example in robotic grasping by eliminating the near zero velocity creep, by allowing a friction force at zero velocities. With the improved friction models more different scenarios can be modeled more accurately. The use of static friction models can cause numeric instability if the velocity oscillates around zero causing discontinuities in the system. This can be avoided by using a dynamic friction model.

The behavior of the proposed model is exactly according to the rigid body soft contact assumption. A key feature of the proposed method is that it is completely independent of the geometry used. The collision response is of course affected by the geometry as it should but there are no negative or undesired effects of the geometry. Different geometries pose no discontinuities or problem instances. This allows the model to be used as a general purpose collision model in a wide variety of cases.

Assuming local deformations according to the soft contact theory the assumption of the correctness of the collision surface circumference is valid. Also the tests show that the model behavior is in accordance with the Hertzian contact models.

6.9 Weaknesses of the proposed method

As all models, the proposed model has some weaknesses. No model is perfect and it is important to realize and account for the weaknesses in dealing with any method. The model has some weaknesses due to the discrete nature of the real-time simulations as well as the original assumption that rigid bodies can penetrate each other.

Almost all of the problems related to the actual collision response method can be derived from the rigid-body assumption. Some other methods add their own assumptions on top of that creating even more problems or modifying the existing errors to take place in other parts of the collision. The proposed method does not make any extra assumptions on the deformation during collision. Therefore all of the geometry related problems can be traced to the initial assumption.

One of the problems with the developed method is the snagging problem. This occurs when for example a cube is sliding on a flat surface. It is not possible to use high enough spring constants to keep the cube from penetrating the plane because the discrete simulation becomes unstable. The stability could be improved by reducing the time-step but that will compromise the real-time requirement. Therefore the cube penetrates the flat surface and the leading edge of the cube snags on the surface. This is in accordance with the rigid-body assumption so the model itself behaves correctly. It is possible to avoid this by neglecting the other normal direction in the force creation but this will add another assumption and therefore increases the risk of further errors. This error appears to be of very little consequence in practical applications and with the correct parameters it can be minimized. The behavior is exactly what you would get by letting a real box penetrate the surface and therefore in itself cannot be considered as an error. It is just a feature that requires some relaxing in light of practical applications.

Another problem that occurs is the sticking problem. If the said cube is pushed through the flat surface it can become stuck there. The friction caused by the pressure on the cube side walls makes the cube stick on the flat surface. This is in accordance with the rigid-body assumption as well. If for example a steel dart is pushed inside a wooden table it becomes stuck there. Again as with the snagging problem the behavior is physically correct but in some occasions it may become undesirable. For example in cases where it is not possible to use stiff enough material parameters objects may penetrate slightly causing objects to stick. In such cases it is also possible to neglect the other normal direction and thus removing the compressing force. Although, again the further assumption creates another error.

The definition of damping can also be problematic in the developed model. The model has two sources of damping; the normal force model as well as the friction model. This means that in order to determine the desired damping two sets of parameters need to be addressed. In practical cases (metal-metal etc...) the friction is usually fairly small making the damping from the friction less significant and thus simplifying the selection of parameters. The parameter selection difficulty is in direct relation to the required accuracy. For example in the case of the colliding spheres a coefficient of restitution can be selected slightly lower than the material. As the normal force model is then less damped the friction parameters can then be selected to give proper results in comparison to the other methods. In such manner some general parameters can be obtained. In cases where the model is used to model some general structure such as a shipping container the parameters need to be practically selected based on a good estimation. Measuring the restitution of a shipping container would be very inaccurate and difficult.

The boundary crossing problem described in section 5.1.2 is a weakness of the model. With greater penetrations or extremely dense meshes the model will suffer from excessive damping. Extremely dense meshes are usually not suited for real-time simulations in general, so the problem usually does not present itself. Also the large penetrations causing multiple triangle boundary crossings usually violate the small local deformations assumption.

CHAPTER 7
Conclusions

A new penalty based collision model was proposed with the aim of creating a general collision model to be used in large varying simulation environments. The creation of the model is presented along with some tests showing the functionality of the proposed model.

New methods for the common problems of geometry based collision modeling were presented. Methods for collision point placement, normal direction handling as well as normal force magnitude were presented. The proposed methods enable continuous contact forces regardless of the collision case. Common problems of certain geometries or collision configurations presenting problems can be avoided using the proposed method. This feature enables the use of the proposed method as a general collision modeling method in varied simulation environments.

A new method for providing temporal history to the collision response model was presented. The method relies on the triangularized data to link the previous collision states to the current states making possible integration of parameters over time. This possibility of integration enables the use of new methods that require integration such as dynamic friction models. The temporal history also enables the collision response model to use integration in determining the collision point penetration. This removes the need to make assumptions on the penetration depth and provides a continuous collision response regardless of the case.

The proposed normal force model removes the requirement to need to know something about the colliding geometries for use with the Hertzian collision models by linking the geometric stiffness to the collision area circumference. Typical cases for Hertzian contacts are spheres and rods where the geometry shape is known. In a case where the geometry shape is irregular or unknown the Hertzian forces can be estimated using the collision area circumference. With the previous models one would need some information or assumption on the colliding object such as radius in case of spheres.

7.1 Future prospects and development

As any stiff penalty based method the proposed model suffers from instability at higher time-steps as well as problems with the energy conservation. These problems should be solved in order for the penalty methods to advance further. The proposed method solves the common problems regarding geometry based collisions so the next focus should be on the stabilization and energy conservation of the system. The proposed temporal history method could possibly be used in order to integrate the energy created in the collision and be used as a constraining feature.

The proposed method is suited for use in general environments as it is able to produce reliable results in a wide variety of cases. The method has been implemented in MeVEA inc. modeling software and is currently used in creating simulations for product development simulators as well as training simulators.

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