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LIQUIDITY AND ASSET PRICING IN THE FRENCH STOCK MARKET

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ABSTRACT

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This thesis investigates pricing of liquidity in the French stock market. The study covers 835 ordinary shares traded in the period of 1996-2014 on Paris Euronext. The author utilizes the Liquidity-Adjusted Capital Asset Pricing Model (LCAPM) recently developed by Acharya and Pedersen (2005) to test whether liquidity level and risks significantly affect stock returns. Three different liquidity measures – *Amihud*, *FHT*, and *PQS* – are incorporated into the model to find any difference between the results they could provide. It appears that the findings largely depend on the liquidity measure used. In general the results exhibit more evidence for insignificant influence of liquidity level and risks as well as market risk on stock returns. The similar conclusion was reported earlier by Lee (2011) for several regions, including France. This finding of the thesis, however, is not consistent across all the liquidity measures. Nevertheless, the difference in the results between these measures provides new insight to the existing literature on this topic. The *Amihud*-based findings might indicate that market resiliency is not priced in the French stock market. At the same time the contradicting results from *FHT* and *PQS* provide some foundation for the hypothesis that one of two leftover liquidity dimensions – market depth or breadth – could significantly affect stock returns. Therefore, the thesis' findings suggest a conjecture that different liquidity dimensions have different impacts on stock returns.

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Arseny Gorbenko
Lappeenranta, October 2014

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LIST OF ABBREVIATIONS

ASX - Australian Stock Exchange

AMEX – American Stock Exchange (from 2012 it is called NYSE MKT LLC)

CAPM – (Traditional) Capital Asset Pricing Model

ECB - European Central Bank

FHT – (Illiquidity measure of) Fong, Holden, and Trzcinka (2014)

GMM - Generalized Method of Moments

LCAPM – Liquidity-Adjusted Capital Asset Pricing Model

LD-CAPM - Liquidity-Adjusted Downside Capital Asset Pricing Model

LDV – Limited Dependent Variable

NASDAQ – National Association of Securities Dealers Automated Quotation

NYSE – New York Stock Exchange

OLS - Ordinary Least Squares

OTC - Over-the-Counter (Market)

PES – Percent Effective Spread

PQS – (Closing) Percent Quoted Spread

RMSE – (Average) Root Mean Squared Error

RSS - Residual Sum of Squares

SEC – Securities and Exchange Commission

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1 INTRODUCTION

Liquidity is one of the major concerns for market participants. Traders tend to make transactions with more liquid stocks, exchanges try to increase and support liquidity on their markets to attract more participants, and regulators care about sudden liquidity leakages that may force further panic in the market. Consequently, any risks associated with liquidity changes over time are important as well. Therefore, it is broadly believed that liquidity level itself and its respective risks affect required returns of different securities.

The influence of liquidity on assets returns has been extensively studied recently (see Section 2.4 for details). It was generally found that the higher is illiquidity level the higher is the required stock return (e.g., Brennan and Subrahmanyam, 1996; Amihud, 2002; Chan and Faff, 2005), although this result was not always confirmed (Dalgaard, 2009; Lam and Tam, 2011). There were also studies considering the risk associated with commonality in liquidity (e.g., Huberman and Halka, 2001; Karolyi, Lee, and van Dijk, 2012) and the systematic liquidity risk (e.g., Pástor and Strambaugh, 2003; Bekaert, Harvey, and Lundblad, 2007; Liang and Wei, 2012) and their impacts on securities returns. However, there are few papers that consider all these aspects together.

Acharya and Pedersen (2005) recently developed the Liquidity-Adjusted Capital Asset Pricing Model (LCAPM) that is able to consider both liquidity level and risks in a single framework. The model was tested on the US (Acharya and Pedersen, 2005; Kim and Lee, 2014) and Australian (Vu, Chai, and Do, 2014) markets as well as on the global level (Lee, 2011). It was found that the aggregate liquidity risk could affect or be insignificant with respect to stock returns in different regions. The findings were also sensitive to the liquidity measures incorporated in the LCAPM.

This study tests the LCAPM on the French equity market employing the data from December 1995 to March 2014. The author utilizes two recently developed liquidity measures – *FHT* that was introduced by Fong, Holden, and Trzcinka (2014) and *Closing Percent Quoted Spread* (*PQS*) created by Chung and Zhang (2014) – together with the most widely used proxy in the financial literature, namely *Amihud* (Amihud, 2002), to check whether the is a difference in findings based on various liquidity measures. The measures are picked up so as to represent several economic viewpoints on liquidity and also on the basis of correlation with the more precise high-frequency liquidity proxies. In particular, Amihud, FHT, and PQS concentrate on different dimensions of liquidity. In addition, all the measures represent different sizes of transactions, i.e. they are designed for different types of investors. By using economically different liquidity proxies this study aims at building a more comprehensive view on the problem of liquidity pricing.

This study finds more evidence for insignificant influence of both liquidity level and risks on stock returns. The market risk also appears to be unimportant in this respect. However, the findings and their robustness largely depend on illiquidity measure used. This fact provides a new insight to the existing literature on liquidity. The *Amihud*-based analysis revealed some evidence for insignificant influence of one of the liquidity dimensions – market resiliency – on stock returns. At the same time the contradicting findings from *FHT* and *PQS* may indicate that one of the rest two liquidity dimensions – market depth or breadth – is priced in the French equity market. However, from the conducted study it was not possible to distinguish to which dimension this impact could belong. Nevertheless, the findings suggest a new interesting conjecture: different liquidity dimensions could affect stock returns in different ways.

Therefore, this thesis contributes to the existing literature in several ways. First, it tests the LCAPM on the French equity market which was not done before. Second, it is the first study that incorporates two recent liquidity measures, namely *FHT* and *PQS*, into the LCAPM framework. Moreover, the focus on economically different liquidity proxies was not practiced before. It helped to bring the new insight to the existing literature about pricing of liquidity dimensions rather than liquidity as a whole.

The rest of the paper is organized as follows. Section 2 describes the concept of liquidity and how it could be measured, then it presents the LCAPM and discusses the previous literature related to the topic of the thesis. Section 3 describes in detail the methodology that is employed in order to test the LCAPM on the French stock market. Further Section 4 characterizes the data that is used in this study. Then Section 5 presents the empirical results of the analysis and tests them for robustness. Section 6 provides the economic interpretation of the obtained findings. Finally, Section 7 completes the thesis with the summary of the results and conclusions.

2 THEORETICAL FRAMEWORK

2.1 Liquidity and Liquidity Risk

Liquidity is a complex phenomenon that does not have a commonly acceptable definition. Usually this term is described according to the contexts of particular models that utilize it. Nonetheless, liquidity is an important factor in asset pricing and is a key concern for market participants.

Asset managers and ordinary investors care about liquidity as it affects returns on their investments, simply because illiquid securities cost more to buy, and sell for less (Foucault, Pagano, and Röell, 2013, p. 4). Consequently, exchanges want liquidity to attract these investors. Finally, regulators like liquidity because liquid markets are often less volatile than illiquid markets (Harris, 2003, p. 394).

Roughly speaking liquidity is the ease of converting of an asset into cash. It is sometimes described as ability to trade a significant quantity of a security at a low cost in a short time (Holden, Jacobsen, and Subrahmanyam, 2014, p. 1). However, it is quite difficult to estimate “ease” or “ability” that makes these definitions too abstract.

Therefore, it is more common to talk about liquidity in terms of its three inherent attributes – depth, breadth and resiliency of a market. In a deep market if we look a little above or below the current price, there is a large incremental quantity available for sale and buy. A broad market has many participants and none of them possess significant market power. Resilient market means that temporary price changes associated with the trading process (as opposed to the fundamental valuations) are small and fade away quickly (Hasbrouck, 2007, p. 4-5). Thus, liquidity is a multi-dimensional phenomenon by nature.

The above mentioned liquidity attributes are translated in two practically important liquidity indicators: bid-ask spread and market impact. The bid-ask spread is the execution cost of a round-trip (buying at the offer and subsequently selling at the bid or vice versa), whereas market impact refers to the additional cost above the spread that a trader may incur to have a large order execute quickly (Schwartz and Francioni, 2004, p. 66). It is worth to notice that the simple difference between the lowest quoted ask and the highest quoted bid prices is not the same as actual buying at the best offer price followed by a sale. The latter figure depends on time between transactions. Furthermore, both indicators would vary depending on sizes of orders. Bid-ask spread and price impact would be described in more detail in Section 2.2.1 of this chapter.

Liquidity of a security or overall market depends on different factors. In general Amihud, Mendelson, and Pedersen (2005) distinguish four potential sources of illiquidity. First, exogenous transaction costs influence on liquidity in the form of brokerage fees, transaction taxes, and order-processing costs. Every time a market participant is involved in a trade she incurs these costs. Second source is demand pressure and inventory risk. Demand pressure arises when you need to sell an asset quickly, but there are no natural buyers available in the market at that moment. As a result you make a deal with a market maker who anticipates to liquidate her position in future, but has to hold a security in inventory by then and thus asks a compensation for the risk of price changes for this period. Moreover, informed trading put additional cost on trading. Buyer and seller of an asset often worry whether their counterparties have some private information about a company's performance that forces them to make a transaction. This cost may also be associated with private information about order flow when, for example, a counterparty knows in advance about a large stock repurchase that would affect the current price, and thus she can buy an asset now at a lower price to sell it later at a higher price. In these cases trading with an informed counterparty would lead to a loss. Finally, illiquidity might appear due to difficulties to find a counterparty who is eager to trade a particular asset or a large amount of a given security. This search friction is particularly relevant in Over-the-Counter (OTC) markets in which there is no central marketplace (Amihud, Mendelson, and Pedersen, 2005, p. 271).

All these sources of illiquidity are also called trading costs or illiquidity cost. Investors require compensation for bearing these costs that is reflected on securities' prices. Moreover, because liquidity may change from time to time, a compensation is required for being exposed to liquidity risk as well (Amihud, Mendelson, and Pedersen, 2005, p. 271). Consequently, less liquid assets are expected to produce higher returns. This does not necessarily mean that investors are better off holding securities with low liquidity, because higher transaction costs can eat up return gains (Amihud and Mendelson, 1991, p. 56). It basically means that liquid assets could be a better option than illiquid for a short investment horizon.

However, it is important to distinguish between liquidity risk and liquidity level itself. Illiquid security does not necessarily imply high liquidity risk. More precisely the distinction between these two terms is provided in Section 2.3 of this chapter.

Liquidity is not observed directly, but rather has different dimensions. Because of that it is very hard to capture all the aspects of this phenomenon in a single measure. In the next Section the author describes the existing measures and their interpretation with regard to liquidity.

2.2 Liquidity Measures

Liquidity measures can be computed using data with different frequency. High-frequency data implies using intraday observations of each and every trade, while low-frequency data often assumes the use of end of the day data (Bundgaard and Ahm, 2012, p. 18). Consequently, low-frequency data is less precise, but easier to calculate. Moreover, high-frequency data is not always available for large time horizons: for the US – not available before 1983 (Hasbrouck, 2009, p. 1445), globally – before 1996 (Thomson Reuters, 2012). Finally, this data is quite costly and thus is not available for each and every researcher.

There are a number of studies questioning the ability of low-frequency liquidity measures to capture the same information as their high-frequency counterparts (see for example, Hasbrouck, 2009; Goyenko, Holden, and Trzcinka, 2009; Holden, 2009; Chung and Zhang, 2014; Fong, Holden, and Trzcinka 2014). The results vary across different liquidity measures. Thus, it is important to begin with a description of high-frequency liquidity benchmarks that are usually used for comparison.

2.2.1 High-Frequency Liquidity Benchmarks

High-frequency benchmarks are usually divided on percent-cost and cost-per-volume proxies. The first ones capture trading cost as a percentage of the price or a percent bid-ask spread. The latter ones calculate the concession of the price per a quantity unit traded (in this study it is €1,000) or a price impact. This study utilizes *Percent Effective Spread* (*PES*) and *Lambda* as the proxies for the percent bid-ask spread and the price impact.

As was already mentioned above simple quoted spread is not always the same as what a trader would pay for the execution of buy and sell orders. Consequently, market participants are more concerned of the so called effective spread. Effective spread is the signed difference between the trade price and the bid-ask midpoint prevailing at the time of order submission (Madhavan et al., 2002, p. 2). Therefore, effective spread measures that difference between the real spread that a trader experience and a prevailing hypothetical quoted spread which she could see. *PES* is conceptually the same, but expressed in relative terms.

PES on the kth trade of a given stock:

$$PES_k = 2D_k(\ln(P_k) - \ln(M_k)), \tag{1}$$

where D_k is an indicator variable that equals *+1* if the k^{th} trade is a buy and *-1* for a sell trade, P_k is the price of the k^{th} trade, and M_k is the consolidated best bid or ask price prevailing immediately prior to the time of the k^{th} trade (Fong, Holden, and Trzcinka, 2014, p. 5). To aggregate this indicator over a month a euro-volume-weighted average across all trades in a month is calculated.

The price impact benchmark is calculated over a five-minute period. It is calculated as the slope coefficient, λ_i, of the following regression as in Hasbrouck (2009) and Goyenko, Holden, and Trzcinka (2009):

$$r_{ni} = \lambda_i \cdot S_{ni} + u_{ni}, \tag{2}$$

where r_{ni} is the stock *i*'s return for the n^{th} five-minute period, $S_{ni} = \sum_k Sign(v_{kni})\sqrt{|v_{kni}|}$, v_{kni} is the signed euro volume of the k^{th} trade for the stock *i* in the n^{th} five-minute period, and u_{ni} is the error term of the stock *i* for the n^{th} five-minute period. For each month separate estimates are calculated.

These two measures are used further in the next Section to select their best low-frequency counterparts for the French equity market. Obviously there are other liquidity benchmarks that are sometimes used, but *PES* and *Lambda* to the knowledge of the author are the most widely used for the same purpose. The possible reason is that both measures implicitly include both temporary and permanent price impacts, whereas other measures often concentrate only on one of these terms (e.g., *Percent Realized Spread*, *Percent Price Impact* or *Permanent Price Impact*) or do not take time into consideration (*Static Price Impact*).

2.2.2 Low-Frequency Liquidity Measures

One could say that although easier to calculate low-frequency measures should be significantly less precise than their high-frequency counterparts. However, empirical studies disapprove this suspicion. For example, Hasbrouck (2009) found out that his daily measure of liquidity (*Gibbs*) achieves a correlation of 0.965 with the high-frequency *PES* for a sample of stocks in the New York Stock Exchange (NYSE) for a period 1993-2005. For the same market and dates Goyenko, Holden, and Trzcinka (2009) measured performance of liquidity proxies monthly and annually. The concluded that the effort of using high-frequency measures is not worth the cost. More precisely, they found three dominating measures in estimating monthly effective spread – *Effective Tick* (Goyenko, Holden, and Trzcinka, 2009), *Holden* (Holden, 2009) and *LOT Y-split* (Goyenko, Holden, and Trzcinka, 2009), and six measures for annual effective spread – *Roll* (Roll, 1984), *Effective Tick*, *Effective Tick2* (Goyenko, Holden, and Trzcinka, 2009), *Holden*, *Gibbs*, and *LOT Y-split*. However, they documented the failure of low-frequency measures in estimating the magnitude of *Lambda*. Finally, the study of Holden (2009) on the same market argues that combining several measures into one can significantly increase the performance – his *Multi-Factor2* measure was significantly better than others on all three performance dimensions for high-frequency *PES*.

Nevertheless, there is not so many literature of the same type dedicated to other markets than the US. Probably the only two papers comparing low- and high-frequency measures in other markets are Kang and Zhang (2013) and Fong, Holden, and Trzcinka (2014). Kang and Zhang (2013) conducted the research on 20 emerging stock markets[1] for the period 1996-2007. They found out that their new measure *AdjILLIQ*, combining *Amihud* and *ZeroVol* (Kang and Zhang, 2013) measures, outperformed others for all markets both for *PES* and *Lambda*. The authors highlight that emerging markets need special liquidity measures adjusted for less actively-traded markets. Fong, Holden, and Trzcinka (2014) on the other side made an investigation of low-frequency liquidity proxies globally (43 stock exchanges[2]) for the period 1996-2007. They found that *PQS* measure was the best

[1] Argentina, Brazil, Chile, China, Greece, India, Indonesia, Israel, Korea, Malaysia, Mexico, Philippines, Poland, Portugal, Russia, Singapore, South Africa, Taiwan, Thailand, Turkey

[2] With the total number of 38 countries. Three exchanges in US (NYSE, AMEX, NASDAQ), three in China (Hong Kong, Shanghai, Shenzhen), two in Japan (Tokyo, Osaka) and one main exchange per each other country: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Denmark, France, Finland, Germany, Greece, India (Bombay), Indonesia, Ireland, Israel, Italy, Malaysia, Mexico, Nethelands, New Zealand, Norway, Philippines, Poland, Portugal, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, UK

globally both for daily and monthly values with the highest correlation with *PES*. There were six nearly equivalent measures when comparing with monthly *Lambda*: *PQS*, *LOT Mixed* (Lesmond, Ogden, and Trzcinka, 1999), *High-Low* (Corwin and Schultz, 2012), *FHT*, *Extended Roll* (Holden, 2009) and *Amihud*. The best daily proxy globally for *Lambda* was *Amihud*. However, the authors also noted that none of the measures captured the level of *Lambda* at any frequency.

This study utilizes the results of the last mentioned paper for choosing the best monthly low-frequency measures for the French stock market. The author uses the same comparison criteria with *PES* and *Lambda* for selection as in the original paper of Fong, Holden, and Trzcinka (2014) – average cross-sectional and portfolio correlations with the high-frequency benchmarks, and prediction accuracy in the form of average Root Mean Squared Error (*RMSE*).

Because one of the aims of this study is to try different types of measures in the LCAPM, the author has chosen two best liquidity proxies in their respective classes – *PQS* and *FHT*. Table 1 below provides the top 10 liquidity estimates for the French equity market. *PQS* and *High-Low* are pure spread measures that are estimated based on closing, highest, and lowest bid and offer prices. *FHT*, *LOT Mixed*, *LOT Y-split*, *Zeros* (Lesmond, Ogden, and Trzcinka, 1999), and *Zeros2* (Lesmond, Ogden, and Trzcinka, 1999) are all based on amount of zero-return observations of stocks. You can see from Table 1 that *FHT* and *PQS* almost exclusively occupied the top positions in all the comparison criteria. In the cases where they do not represent the best estimates overall both measures seem not to be significantly numerically different from the better proxies of their respective classes.

Concerning other liquidity proxies classes, *Roll* and *Extended Roll* utilize the price changes for calculation, while *Effective Tick* use an idea of price clustering for finding the effective spread. *Amihud* is based on stock returns and absolute trading volumes. It should be also mentioned that all *Lambda* low-frequency estimates, except for *Amihud*, are obtained via simple division of the original measures by trading volumes. Nevertheless, this study uses the basic *PQS* and *FHT* as they are initially designed in a more comprehensive way with regard to liquidity, not only for capturing *Lambda*, and thus should be more efficient.

Table 1. Top 10 Low-Frequency Liquidity Proxies for Percent Effective Spread and Lambda in the French Stock Market by Different Criteria, 1996 - 2007

No.	PES						Lambda	
	Average Cross-Sectional Correlation		Portfolio Time-Series Correlation		RMSE		Average Cross-Sectional Correlation	
1	PQS	0.753	PQS	0.980	PQS	0.0133	High-Low	0.611
2	FHT	0.562	Ext.Roll	0.922	FHT	0.0176	LOT M	0.574
3	LOT M	0.560	High-Low	0.920	LOT Y	0.0178	PQS	0.569
4	LOT Y	0.549	LOT Y	0.919	High-Low	0.0182	Amihud	0.543
5	High-Low	0.497	FHT	0.916	Ext.Roll	0.0196	Ext.Roll	0.517
6	Zeros	0.433	LOT M	0.887	Eff.Tick	0.0229	FHT	0.500
7	Ext.Roll	0.341	Roll	0.624	LOT M	0.0232	LOT Y	0.492
8	Eff.Tick	0.273	Zeros	0.573	Roll	0.0235	Zeros	0.480
9	Zeros2	0.230	Eff.Tick	0.311	Zeros2	0.1170	Zeros2	0.440
10	Roll	0.213	Zeros2	0.259	Zeros	0.1522	Eff.Tick	0.378

This Table is based on Fong, Holden, and Trzcinka (2014)

The table presents the correlation coefficients between low-frequency and high frequency liquidity measures. The first column indicates the place of a low-frequency measure according to four criteria indicated in the following columns. The second column reports the average cross-sectional correlation of monthly percent-cost illiquidity measures with PES. The third column shows the portfolio time-series correlation of monthly percent-cost illiquidity proxies with PES. The fourth column presents RMSE of monthly percent-cost proxies with PES. The last fifth column reports the average cross-sectional correlation of monthly cost-per-volume measures with Lambda

Abbreviations: LOT M – LOT Mixed, LOT Y – LOT Y-split, Ext.Roll – Extended Roll, Eff.Tick – Effective Tick

Therefore, this study employs three low-frequency liquidity measures – *Amihud* that was initially used by the LCAPM creators (Acharya and Pedersen, 2005) and *FHT* together with *PQS* that were selected as the best proxies of different types for high-frequency benchmarks. Further each of these measures is described in detail.

Amihud

Amihud illiquidity measure is perhaps the most widely used liquidity proxy with over 100 papers utilizing it for empirical analyses in The Journal of Finance, Journal of Financial Economics, and The Review of Financial Studies during the period of 2009-2013 (Lou and Shu, 2014, p. 1). The main advantages of this measure are ease of calculation and interpretation.

Amihud (2002) defines the illiquidity of stock *i* in month *t* as follows:

$$Amihud_t^i = \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{\left|R_{td}^i\right|}{V_{td}^i}, \tag{3}$$

where R_{td}^i is the return on day *d* in month *t*, V_{td}^i is the euro trading volume (in thousands) on day *d* in month *t*, $Days_t^i$ is the number of valid observation days in month *t*. *Amihud* estimates a monthly price response associated with one thousand euros of trading volume. In addition Acharya and Pedersen (2005, p. 386) argue that it is an instrument of the cost of selling, although it does not directly measure the cost of a trade.

However, *Amihud* is not the best liquidity measure for France when it is compared with high-frequency benchmarks as can be seen from Table 1 above. Moreover, return premium often associated with this measure (*Amihud* was used in many studies on this topic) is driven by its association with trading volume, but not by its construct of return-to-volume ratio that captures price impact (Lou and Shu, 2014, p. 24). But trading volume itself seems to be a noisy estimate of liquidity. In fact, a large number of studies explain the pricing of this factor by other reasons. For example, Blume, Easley, and O'Hara (1994) relate it with investor disagreement, Baker and Wurgler (2006) – with investor sentiment, and Barinov (2014) - with information uncertainty.

FHT

FHT was derived by Fong, Holden, and Trzcinka (2014) to simplify computationally intensive *LOT Mixed* model of Lesmond, Ogden, and Trzcinka (1999). Despite simplification the measure showed better performance overall than any other zero-return based proxies.

The model originates from the Limited Dependent Variable (LDV) model of Tobin (1958) and Rosett (1959). The idea is that the true market model of security returns is suppressed by the effects of transaction costs. In this model the marginal informed investor will make a deal only if the expected gains from information exceed transaction costs.

Figure 1 below illustrates the LDV concept. The bold red line represents the observed return and the thin blue line – the return expectation. The red line remains at zero level between two dashed lines – the area where transaction costs exceed the expected gains of a trade for the marginal investor. Consequently, the true return is not revealed by the measured return there. Therefore, the size of the zero-return area might serve as an indicator of liquidity for a given asset.

Lesmond, Ogden, and Trzcinka (1999) develop the model based on the described concept. Their *LOT Mixed* model relates measured returns, R_{it}, and true returns, R_{it}^*, of a stock *i* in month *t*:

$$R_{it}^* = \beta_i R_{mt} + u_{it}, \tag{4}$$

where β_i is the sensitivity of stock *i* to the market return R_{mt} in month *t*, u_{it} is the error term of stock *i* in month *t*, and $u_{it} \sim N(0, \sigma^2)$. It is assumed that:

$$\begin{aligned} R_{it} &= R_{it}^* - \alpha_{1i} && \text{if} && R_{it}^* < \alpha_{1i} \\ R_{it} &= 0 && \text{if} && \alpha_{1i} < R_{it}^* < \alpha_{2i} \\ R_{it} &= R_{it}^* - \alpha_{2i} && \text{if} && R_{it}^* > \alpha_{2i}. \end{aligned} \tag{5}$$

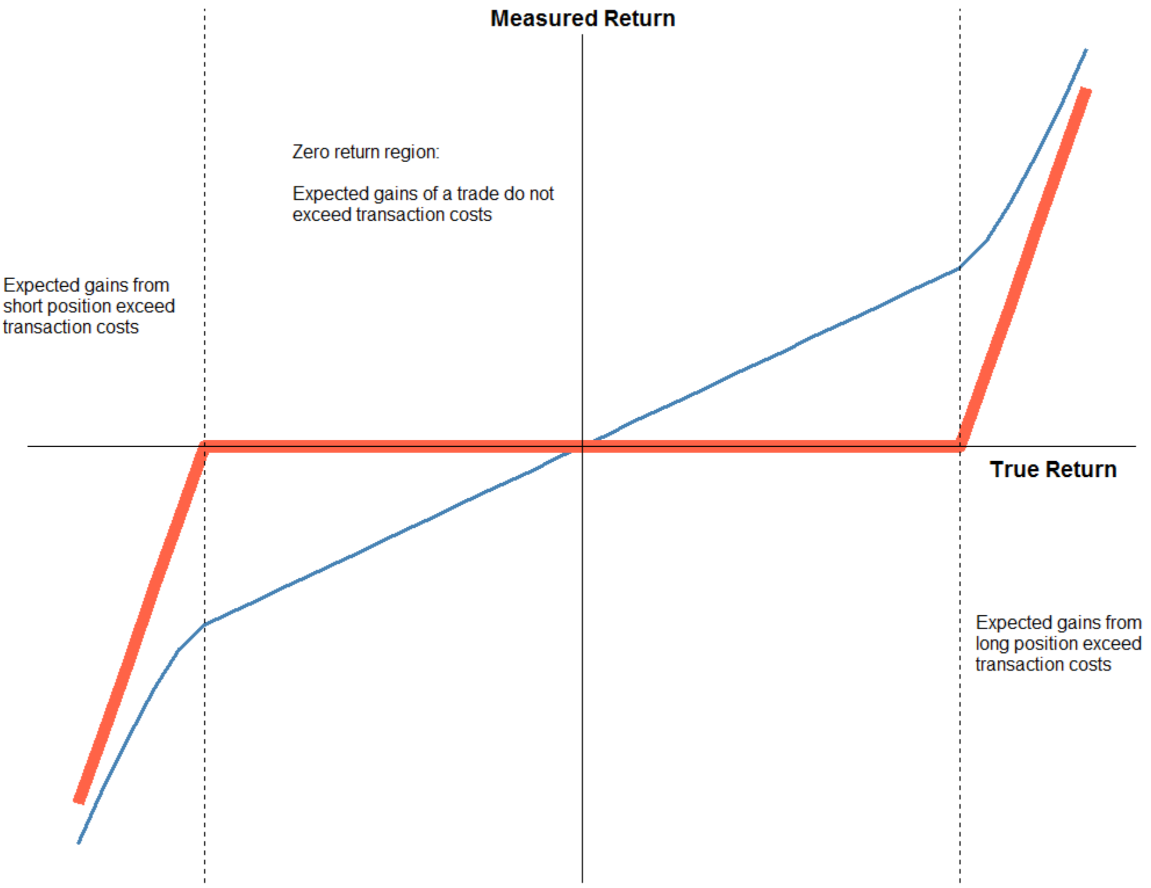

Figure 1. LDV Model Illustration
Based on Lesmod, Ogden, and Trzcinka (1999)

Here $\alpha_{1i} \leq 0$ and $\alpha_{2i} \geq 0$ are the percent transaction costs of selling and buying stock *i* respectively. However, Fong, Holden, and Trzcinka (2014) simplify equations (5) by assuming that selling and buying costs are equal, i.e. $\alpha_{1i} = -\frac{S}{2}$ and $\alpha_{2i} = \frac{S}{2}$, where *S* is the round-trip, percent transaction cost. After required substitutions and suppressing the transcripts we get:

$$
\begin{aligned}
R &= R^* + \frac{S}{2} && \text{if} \quad R^* < -\frac{S}{2} \\
R &= 0 && \text{if} \quad -\frac{S}{2} < R^* < \frac{S}{2} \\
R &= R^* - \frac{S}{2} && \text{if} \quad R^* > \frac{S}{2}.
\end{aligned} \tag{6}
$$

Concerning the true return distribution of a stock, the authors imply that $R^* \sim N(0, \sigma^2)$. It allows calculating the probability of being in the zero-return region:

$$N\left(\frac{S}{2\sigma}\right) - \mathrm{N}\left(-\frac{S}{2\sigma}\right). \tag{7}$$

Then the authors compute the empirically observed frequency of zero return in a month *t*:

$$z \equiv Zeros = \frac{ZRD}{TD + NTD}, \tag{8}$$

where *ZRD* is the total number of zero-return days in month *t*, *TD* is the number of trading days, and *NTD* is the number of non-trading days. Equating theoretical probability (7) and empirical frequency (8) of a zero return gives the following:

$$N\left(\frac{S}{2\sigma}\right) - N\left(-\frac{S}{2\sigma}\right) = z. \tag{9}$$

Formula (9) can be rewritten by utilizing the symmetry of cumulative normal distribution:

$$N\left(\frac{S}{2\sigma}\right) - \left[1 - N\left(\frac{S}{2\sigma}\right)\right] = z. \tag{10}$$

FHT is obtained by solving equation (10) for S:

$$FHT \equiv S = 2\sigma N^{-1}\left(\frac{1+z}{2}\right), \tag{11}$$

where $N^{-1}()$ is the inverse (quantile) function of the cumulative normal distribution. Thus, *FHT* rises with the increased frequency of zero returns and volatility of the return distribution.

Although the measure appears to be the most efficient in its class and pretty simple, the author would like to make some critical remarks on it. First of all, the assumption of normal distribution of returns is arguable (see e.g., Mandelbrot, 1963; Fama, 1965; Clark, 1973; Cont, 2001). However, the authors relate Gaussian distribution to the true theoretical returns, where this assumption is more reasonable compared to empirical returns. In addition, although original measure that is going to be used in this study does estimate the cost of a trade, it is not originally designed for capturing the price impact of a trade. Moreover, the model equates cost of buying and selling a stock which might be true for a stable quiet market, but may become wrong in a crisis period when everybody wants to get rid of long positions. Finally, *FHT* is quite a new measure and has not been substantially studied. Fong, Holden, and Trzcinka (2014) and Bundgaard and Ahm (2012) are among the rare papers that utilized this liquidity measure for their empirical analyses.

PQS

Chung and Zhang (2014) recently have proposed a simple measure for the bid-ask spread – *PQS*. Despite its goal *PQS* has the top results for estimating both high-frequency benchmark spreads, including *PES*, and *Lambda*.

PQS of stock *i* in month *t* is defined as follows:

$$PQS_t^i = \frac{1}{Days_i^t} \sum_{d=1}^{Days_t^i} \frac{2(Ask_{td}^i - Bid_{td}^i)}{Ask_{td}^i + Bid_{td}^i}, \qquad (12)$$

where Ask_{td}^i is the closing offer price of stock *i* on day *d* in month *t*, Bid_{td}^i is the closing bid price of stock *i* on day *d* in month *t*, and $Days_t^i$ is the number of valid observation days in month *t*. In other words *PQS* is the bid-ask spread divided by the quote midpoint.

Although *PQS* has proven to be an effective low-frequency measure for the high frequency benchmarks it was developed as a spread measure. Thus, the original measure does not evaluate the price impact, but rather cost of a single trade. Compared to *FHT*, *PQS* is much more simple and straightforward in terms of calculation. However, it lacks some inherent analytical sense and possesses less explanatory power. Finally, *PQS* is a relatively new liquidity measure that was not widely studied before. To the knowledge of the author Chung and Zhang (2014) and Fong, Holden, and Trzcinka (2014) are the only papers that empirically tested this measure.

Comparison between Liquidity Measures

The brief comparison between *Amihud*, *FHT*, and *PQS* is provided in Table 2. First, it could be noticed that *Amihud* is the price impact measure while *FHT* and *PQS* represent trading cost as percent of the stock price. The LCAPM framework treats illiquidity as the cost of selling (see Section 2.3 for details). Thus, *FHT* and *PQS* seem more preferable in this respect, although *Amihud* could be considered as an indirect proxy for the selling cost as well. At the same time, it is possible to say that because *Amihud* captures a price impact it is more important for large trades and, consequently, for large institutional investors who are involved in such kinds of transactions. In contrary, *PQS* is derived from the bid-ask spreads and does not consider the size of trades at all. Hence, it is more appropriate for the small trades and small private investors whose transactions could not influence the price of a stock. *FHT* in this respect appears something in between *Amihud* and *PQS* because its analytical explanation described above implies some sort of price impact, but does not concentrate on it. Therefore, *FHT* could be seen as a representative of medium trades that are of the main interest of medium-sized investors. Finally, different liquidity measures capture various liquidity dimensions differently. *Amihud* gives the highest weight to the market resiliency, leaving the rest of the dimensions relatively unimportant. *FHT* seems to be a more balanced measure that treats various liquidity dimensions relatively equally. *PQS* from its side is a bid-ask spread measure that ignores the trade size, i.e. it considers market resiliency as a relatively unimportant dimension. For the leftover dimensions the author assigned the medium priority as it was difficult to

Table 2. Qualitative Comparison between Liquidity Measures

	Amihud	**FHT**	**PQS**
What it measures?	Price impact per €1,000 traded	Trading cost as a percentage of the price	Trading cost as a percentage of the price
The focus of the measure	Large trades → large institutional investors	Medium trades → medium-sized investors	Small trades → small private investors
Priorities for liquidity dimensions	Depth – low, Breadth – low, Resiliency - high	Depth – medium, Breadth – medium, Resiliency - medium	Depth – medium Breadth – medium, Resiliency - low

Liquidity dimensions are market depth, breadth, and resiliency (see Section 2.1 for details).
The last row in the table ("Priorities for liquidity dimension") shows the relative weights of liquidity dimensions to each other (assigned by the author) that are captured by different liquidity proxies

identify which dimension is more important here. Due to this fact *PQS* seems to possess less analytic power compared to *Amihud* and *FHT*. Therefore, different liquidity measures that are used in this study represent quite different economic viewpoints on liquidity.

In conclusion, this study uses three liquidity measures – *Amihud*, *FHT*, and *PQS* – that are quite different in their sense. It helps to view liquidity from different angles that could make the economic interpretation of the further results more comprehensive. The next Section will introduce the pricing model that can utilize these measures for calculation of liquidity risk.

2.3 LCAPM

There are number of studies investigating asset pricing with liquidity. The majority of the papers in this field so far have focused either on liquidity level (e.g., Amihud and Mendelson, 1986; Brennan and Subrahmanyam, 1996; Amihud, 2002) or a single liquidity risk factor (e.g., Pástor and Strambaugh, 2003; Korajczyk and Sadka, 2008). This study is based on the LCAPM framework developed by Acharya and Pedersen (2005), which combines both illiquidity level and three different types of liquidity risks. Therefore, the LCAPM appears to be a comprehensive tool in testing whether liquidity influences on stock returns.

Acharya and Pedersen (2005) start their model by the common assumption of risk-averse investors maximizing their expected utility by choosing consumption and portfolios under a wealth constraint. They derive the LCAPM from the traditional Capital Asset Pricing Model (CAPM) by expanding the frictionless economy to an economy with illiquidity costs:

$$E_t(r_{t+1}^i - c_{t+1}^i) = r_f + \lambda_t \frac{\operatorname{cov}_t(r_{t+1}^i - c_{t+1}^i, r_{t+1}^M - c_{t+1}^M)}{\operatorname{var}_t(r_{t+1}^M - c_{t+1}^M)}, \tag{13}$$

where $E_t(r_{t+1}^i - c_{t+1}^i)$ is the conditional expected net return of stock *i*, $\lambda_t = E_t(r_{t+1}^M - c_{t+1}^M - r_f)$ is the risk premium, r_{t+1}^i is the gross return of stock *i*, r_f is the gross risk-free rate, and c_{t+1}^i is the relative illiquidity (trading) cost. It is easy to notice that if trading cost, c_{t+1}^i, is neglected equation (13) transforms into a common CAPM model.

Equivalently, the conditional expected gross return is:

$$E_t(r_{t+1}^i) = r_f + E_t(c_{t+1}^i) + \lambda_t \frac{\mathrm{cov}_t(r_{t+1}^i, r_{t+1}^M)}{\mathrm{var}_t(r_{t+1}^M - c_{t+1}^M)} + \lambda_t \frac{\mathrm{cov}_t(c_{t+1}^i, c_{t+1}^M)}{\mathrm{var}_t(r_{t+1}^M - c_{t+1}^M)} - \lambda_t \frac{\mathrm{cov}_t(r_{t+1}^i, c_{t+1}^M)}{\mathrm{var}_t(r_{t+1}^M - c_{t+1}^M)} - \lambda_t \frac{\mathrm{cov}_t(c_{t+1}^i, r_{t+1}^M)}{\mathrm{var}_t(r_{t+1}^M - c_{t+1}^M)}, \tag{14}$$

where $E_t(c_{t+1}^i)$ is the expected relative illiquidity cost/level and four covariances are betas depending on the stock's payoff and liquidity risks (Acharya and Pedersen, 2005, p. 381). In order to make the equation (14) unconditional on the information set available up to time *t*, the authors assume constant conditional variances (or constant risk premiums *λ*). It leads to the result:

$$E(r_t^i) = E(c_t^i) + \lambda\beta^{1i} + \lambda\beta^{2i} - \lambda\beta^{3i} - \lambda\beta^{4i}, \tag{15}$$

where $\lambda = E(r_t^M - c_t^M)$ and

$$\beta^{1i} = \frac{\mathrm{cov}(r_t^i, r_t^M)}{\mathrm{var}(r_t^M - [c_t^M - E_{t-1}(c_t^M)])}, \tag{16}$$

$$\beta^{2i} = \frac{\mathrm{cov}(c_t^i - E_{t-1}(c_t^i), c_t^M - E_{t-1}(c_t^M))}{\mathrm{var}(r_t^M - [c_t^M - E_{t-1}(c_t^M)])}, \tag{17}$$

$$\beta^{3i} = \frac{\mathrm{cov}(r_t^i, c_t^M - E_{t-1}(c_t^M))}{\mathrm{var}(r_t^M - [c_t^M - E_{t-1}(c_t^M)])}, \tag{18}$$

$$\beta^{4i} = \frac{\mathrm{cov}(c_t^i - E_{t-1}(c_t^i), r_t^M)}{\mathrm{var}(r_t^M - [c_t^M - E_{t-1}(c_t^M)])}. \tag{19}$$

Due to persistence of illiquidity (Amihud, 2002; Pástor and Strambaugh, 2003; Korajczyk and Sadka, 2008) the unconditional model focuses on its innovations, $c_t - E_{t-1}(c_t)$. Note that risk premiums of different betas, *λ*, do not have any subscripts, because the authors of the LCAPM impose the restriction that $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$. They also do not allow short-selling implying that an investor could buy stock *i* at price p_t^i in time period *t*, but must sell it at $p_t^i - c_t^i$.

In addition, this study omits using the risk-free rate, r_f, in calculations because of several reasons. First, the risk-free rate for France before introduction of euro in 1999 was tied to French frank that hinders r_f usage. Furthermore, measuring illiquidity premiums for returns net of trading costs itself makes sense. Although it has not been studied so far the risk-free rate that is usually tied to the local government debt rate might also incorporate some liquidity premium. For example, even the US T-bills, which are often perceived as the best risk-free instruments globally, cannot be more liquid than cash. The debt rates of the European Central Bank (ECB) and the French government may incorporate some more liquidity premiums compared to the higher ranked US debt instruments. Therefore, omitting the usage of the risk-free rate can help in capturing all the liquidity risk.

As could be seen from equations (15)-(19) the unconditional LCAPM have four different betas. First, β^{1i} is the common CAPM market beta with the only difference in denominator where market return is adjusted for innovations in illiquidity costs. The required return of stock *i* increases linearly with the market beta that binds the stock's return with the market return. The other three betas are interpreted as different forms of liquidity risks.

The second beta, β^{2i}, represents the effect associated with the covariance between the stock and market liquidities. It is sometimes referred to a phenomenon of commonality in illiquidity, meaning that a stock becomes illiquid when the market becomes illiquid. This situation is carefully explained in Acharya and Pedersen (2005, p. 382). An investor who holds a stock that has become illiquid together with the market can choose not to trade this asset. Instead she can trade another similar stock at lower cost if the liquidity of this stock does not co-move with the market liquidity. Therefore, investors require higher returns for securities with positive covariance between individual and market illiquidity.

The third beta, β^{3i}, captures the covariance between the stock return and market illiquidity. It is sometimes called systematic liquidity risk. The co-movement of these two elements has a negative impact on the required return of an asset. When illiquidity costs are rising for the whole market investors want stocks to provide additional returns to compensate the increasing expenses. If a security cannot offer additional yield in this situation an investor would initially require a higher return for this asset and vice versa. Hence, the required return decreases with positive covariation between the stock return and market illiquidity.

The fourth beta, β^{4i}, links the covariance between the stock illiquidity and market return with the required return of an asset. The effect associated with it in fact was firstly introduces and tested by Acharya and Pedersen (2005). They explain that when the marketwide returns are low investors are poor and are willing to sell their assets. Consequently, they appreciate if a security has lower illiquidity cost in a down market.

Therefore, investors would accept a lower required return on stocks with positive covariance between individual illiquidity and market return.

In summary, the LCAPM appears to be a comprehensive model that combines illiquidity cost, market beta and three different liquidity betas to explain the required return of an asset. Whereas illiquidity cost/level together with the first and the second betas increase the required return, the third and the fourth betas have a negative impact on it.

There are number of empirical studies testing all these factors in different markets and on various assets. The next Section would describe the previous findings with the focus on stock markets and empirical results for the LCAPM.

2.4 Literature Review

The empirical studies in this field have mainly focused on investigation of influence of a single liquidity factor on return of an asset. The vast majority of researches are conducted on the US stock market, but there are some findings with regard to other parts of the world. First findings in this area are related to liquidity level, whereas the latter papers are more concentrated on liquidity risks.

One of the first evidences that liquidity level is priced was documented by Amihud and Mendelson (1986) who studied this phenomenon on NYSE stocks in the period of 1961-1980. Eleswarapu (1997) confirmed this result on stocks traded on National Association of Securities Dealers Automated Quotation (NASDAQ) for the period 1973-1990 using the same measure of liquidity – quoted bid-ask spread. Later Brennan and Subrahmanyam (1996) argued against using the bid-ask spread as a liquidity proxy. They used price impact measures instead and found that liquidity level positively affected stock returns on NYSE and American Stock Exchange (AMEX) in 1984-1988 as well. Latter Amihud (2002) proved their results for NYSE stocks for the period 1964-1997 using his own price impact measure and also discovered that market expected illiquidity could predict stock excess returns.

Concerning other markets, Chan and Faff (2005) and Chang, Faff, and Hwang (2010) confirmed the positive relationship between illiquidity level and stock returns on the Australian Stock Exchange (ASX) (1990-1998) and Tokyo Stock Exchange (1975-2004) respectively. On the other side, Dalgaard (2009) found that liquidity level is not an important factor for the Danish equity market (1997-2008), but his result was not robust. Furthermore, the results of Lam and Tam (2011) with regard to Hong-Kong stock market (1981-2004) showed significant influence of liquidity only for some of the used liquidity measures. Therefore, the results from less liquid stock markets than the US often diverge.

The majority of studies dedicated solely to commonality in liquidity do not consider the influence of this factor on stock returns. Nevertheless, Huberman, and Halka (2001) who used high-frequency liquidity proxies for 60 NYSE stocks in 1996 concluded that commonality in liquidity was not reflected in stock returns. They also mentioned that decreases in common component of liquidity were followed by a negative return, while increases – were not. Other papers produced results which could be influential with regard to stock returns. Brockman, Chung, and Perignon (2009) studied intraday data for the period 2002-2004 and reported that for the most markets out of 47 stock exchanges across 38 countries commonality in liquidity is significant. However, a closer look at their results reveals that for the majority of individual stocks worldwide this factor does not matter. In particular, more than 90% of firms traded on Euronext Paris exhibit insignificant commonality in liquidity at 95% confidence level. Besides, Karolyi, Lee, and van Dijk (2012) investigated the daily data of 27,447 individual stocks of 40 different countries for the period1995-2009 and found that developed markets exhibit lower commonality in liquidity compared to emerging markets. Therefore, it is expected that for the developed French market the second beta of the LCAPM would insignificantly influence on stock returns.

The third beta, which states for covariance between asset returns and market illiquidity, has been studied quite extensively together with its impact on stock returns. Pástor and Strambaugh (2003) explored all common shares traded on NYSE, AMEX, and NASDAQ for the period of 1966-1999 and found the cross-sectional evidence that the expected stock return was positively related to the sensitivity of stock return to innovations in market liquidity[1]. Latter Korajczyk and Sadka (2008) confirmed previous findings on the intraday data for NYSE stocks in the period of 1983-2000 using eight different liquidity measures. Liu (2009) also proved that investors require higher return for stock sensitive to market liquidity fluctuations on the extended period of 1926-2005 using ordinary common stocks

[1] Because the authors used their own liquidity measure that was typically negative they assumed a negative relation between stock returns and innovations in market liquidity. One should bear it in mind when interpreting 'sensitivity' term here. Putting the authors' conclusions in other words, required return of a stock increases with the absolute increase of the negative covariation between the stock return and market illiquidity

from NYSE, AMEX, and NASDAQ. Concerning other markets, Bekaert, Harvey, and Lundblad (2007) found that the local systematic liquidity risk significantly affected stock returns on 19 emerging markets[1] for the period 1987-2003, even more than local market risk. More importantly for this study, Liang and Wei (2012) investigated the significance of the liquidity risk factor on 21 developed markets[2] for different periods depending on data availability (1989-2005 is the most common analysis period, including France). They used two different liquidity measures (including *Amihud*) and discovered that systematic liquidity risk is priced based on both measures only for three markets – France, Ireland, and Japan[3]. Hence, it is expected that the LCAPM model would exhibit similar results in this study. It is also worth to notice that this result contradicts the previous findings of the significance of this liquidity risk for the US market.

The fourth beta is not usually studied solely, but rather in the context of the LCAPM. Thus the results for this liquidity risk factor would be mentioned while talking about empirical test of the LCAPM that are found in academic papers. The model of Acharya and Pedersen (2005) has not been studied that extensively, but there were several papers using this model in different contexts.

To begin with, Acharya and Pedersen (2005) not only developed the LCAPM, but also tested its unconditional version empirically on the cross-section of NYSE and AMEX stocks in the period of 1962-1999. The authors used *Amihud* measure as a liquidity proxy and tested the model on 25 portfolios using the Generalized Method of Moments (GMM) framework that takes into account the pre-estimation of the betas (as in Cochrane, 2005). They found that liquidity level and all three liquidity risks significantly affect stock returns, with the fourth beta being the most influential and the second beta being the least influential. However, the multicollinearity problem, which arose when all the betas were used simultaneously, forced them to make an assumption of equal liquidity premiums, λ. Nevertheless, it helped them to report higher R^2 for the LCAPM compared to the CAPM (73.2% and 65.3%) without increasing degrees of freedom.

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[1] Argentina, Brazil, Chile, Colombia, Greece, India, Indonesia, Korea, Malaysia, Mexico, Pakistan, Philippines, Portugal, Taiwan, Thailand, Turkey, Venezuela, Zimbabwe

[2] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Italy, Ireland, Japan, the Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States

[3] However, there were 11 countries with significant influence of systematic liquidity risk on stock returns based on at least one liquidity measure: Australia, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Japan, New Zealand, Norway, and Sweden. This result remains the same after controlling for the local market value and size risk factors

Besides, there is a huge study conducted by Lee (2011) who tested the LCAPM on around 30 thousand stocks from 50 countries[1] for the period of 1988-2007. Compared to the original model, the author employed another illiquidity measure – *Zeros* - and tested the model on individual stocks, rather than portfolios, generally following the Fama-MacBeth procedure (Fama and MacBeth, 1973). The paper focused on global and regional level tests of the LCAPM, rather than on country-by-country analysis (except the US). The findings showed that the local aggregated liquidity risk is priced in the US and emerging markets, but not in the developed and overall world markets. The authors also discovered that the global aggregated liquidity is priced worldwide, in developed and emerging markets, but not in the US. It should be noticed that the result for the developed markets is driven solely by the significance of the fourth beta, leaving other liquidity risks insignificant. The authors also found that the largest market in the world – the US – significantly affects other markets returns. They reported that the US aggregated liquidity risk is priced generally in the world and in the developed countries, but not in the emerging markets. The result for the developed markets is again driven solely by the significance of the fourth beta. Concerning smaller geographical regions, the authors reported the same result for developed Europe – not significant local aggregated liquidity risk, but significant US aggregated liquidity risk. However, the author found that neither of risks, including separate and aggregated, local and the US liquidity risks, affect returns in the French equity market[2]. All the regressions run in this study revealed that liquidity level and market beta do not significantly influence stock returns. In addition, the paper provided some results for portfolios based tests that do not appear that robust. Illiquidity sorted portfolios for developed countries showed statistically significant, but negative aggregated liquidity risk, the first and the second betas coefficients that does not make sense economically. At the same time size-based portfolios showed that aggregated liquidity risk is not priced, whereas the first beta was statistically and economically significant for stock returns.

Another interesting study was done by Kim and Lee (2014) who tested the LCAPM on NYSE and AMEX common shares of non-financial firms for the period 1962-2011. The

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[1] Developed countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, UK, US.
Emerging markets: Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Israel, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Portugal, Russia, South Africa, South Korea, Sri Lanka, Taiwan, Thailand, Turkey, Venezuela.

[2] I thank the author of the paper, Kuan-Hui Lee, for kindly providing me with the country-specific results for France.

paper utilized eight different liquidity measures for the model's factors calculation, but used only Amihud from the liquidity proxies we are interested in. Although all the regressions revealed that liquidity level, market beta and second beta did not affect stock returns, the findings showed that different liquidity measures usage could lead to different results. The half of the measures exhibited statistically significant, but negative coefficient for the aggregated liquidity risk, whereas other liquidity proxies did not found this factor significant. Besides, two out of eight measures led to the conclusion that the systematic liquidity risk, β^3, significantly influenced on the stock returns. Finally, only Amihud measure usage brought statistically and economically significant coefficient for β^4.

The similar study was done recently by Vu, Chai and Do (2014) on the ASX stocks for the period of 1995-2010. They employed the fixed effects panel regressions instead of the GMM or the Fama-MacBeth procedure. They generally found that the aggregated liquidity risk is priced, however the result largely depends on the measure used. Although statistically significant, two out of five measures reported positive sign for the aggregate liquidity risk, while the rest of the proxies – negative. Similarly β^2 was found to be significant in all cases, but turned to be positive for two illiquidity measures and negative – for others three. β^3 was insignificant in one case, significant and negative in another case, and significant and positive for the rest three liquidity proxies. Finally, β^4 was significant and negative once, and turned out to be significant and positive for the other four cases. The similar situation appeared for liquidity cost/level: one measure exhibited insignificant results, two – significant and positive, and the remaining two – significant and negative[1].

Unfortunately, to the knowledge of the author there are no similar studies dedicated solely to one market other than the US or Australia. Although Lee (2011) performed the global empirical test of the LCAPM a single market exploration might help to understand whether the model works for a particular stock exchange, rather than for a notional region. Moreover, there have not been any researches testing how different liquidity measures usage could influence the results of the LCAPM in the non-US markets. Furthermore, the focus of this study on using the most effective low-frequency liquidity proxies representing different economic viewpoints on liquidity may help to understand liquidity pricing more comprehensively. Finally, it is interesting to see the performance of *FHT* and *PQS* in the LCAPM which have not been used before.

[1] I thank the corresponding author of the paper, Van Vu, for kindly providing me with the more detailed results for alternative liquidity measures utilized in her article

Previous researches used different methodologies for empirical test of the model. The next Section will describe the framework we are going to use for this study.

3 METHODOLOGY

In order to estimate the LCAPM on empirical data this study uses Fama-MacBeth procedure (Fama and MacBeth, 1973). While original paper of Acharya and Pedersen (2005) used the GMM, Fama-MacBeth is also spread in similar papers (Lee, 2011; Kim and Lee, 2014) and seems to be a simpler and more widely used technique in finance literature.

There are different modifications of this procedure. This study utilizes the one described in Cochrane (2005). However, Lee (2011) and Kim and Lee (2014) employ the modification of Fama and French (1992). This way takes longer time period of data for pre-estimation and leaves less for the final analysis (see Section 3.1.4 for details). Thus, due to a rather short time interval available for analysis (December 1995 – March 2014), the author decided to use the version of Fama-MacBeth that is described further.

In brief, the procedure looks as follows. First, stocks are united into portfolios in order to eliminate the noise attributable to individual stocks. Second, the betas are computed according to equations (16)-(19) using the full data sample. Finally, different specifications of the LCAPM are estimated by cross-sectional time-series regressions following Cochrane (2005). Section 3.1 deals with the first two steps, while Section 3.2 describes the final empirical estimation procedure.

3.1 Preparatory Processing

3.1.1 Data Preprocessing and Filtration

Before forming portfolios all the illiquidity estimates for stocks are adjusted by market index as in Acharya and Pedersen (2005):

$$c_t^i = illiq_t^i P_{t-1}^M, \tag{20}$$

where c_t^i is the illiquidity of stock *i* in month *t*, $illiq_t^i$ is illiquidity of stock *i* in month *t* measured by one of the three proxies – *Amihud, FHT or PQS*, and $P_{t-1}^M = \frac{p_{t-1}^M}{p_{Dec1995}^M}$ with p_{t-1}^M representing the overall market capitalization at the end of month *t-1* and $p_{Dec1995}^M$ -

the overall market capitalization at the end of December 1995. This adjustment helps to make all the illiquidity measures relatively stationary.

Besides, all the penny stocks[1] are filtered out because returns on these stocks might be affected by the minimum tick, thus adding noise to the further estimations. Moreover, stock-day observations of prices (including closing, bid, and ask prices) with missing volume figures are also treated as missing. The latter filtration is important because Datastream, which is used for downloading of all the required data, might report an outdated price if the today's value is not available. For example, if the stock data quality is suspected, trading of a security may be suspended until a company makes all the required actions. One can still observe the last available quoted price after some days, but the volume data would be missing. Hence, adjustment for missing volume allows to eliminate junk stock-day observation of prices.

3.1.2 Portfolios formation

Then rest of the shares are sorted into 25 portfolios for each year *y* during the period of 1997-2014 based on their illiquidity in year *y-1*, from the most to the least liquid. The annual illiquidity for the year *y-1* for each eligible stock is calculated as the average of its daily illiquidities for *Amihud* and *PQS* measures if there are at least 100 stock-day observations (as in Acharya and Pedersen, 2005), and for *FHT* – as the average of the stock's monthly figures if there are at least five stock-month observations. All the stocks included into these portfolios form one market portfolio. Finally, we get 25 illiquidity portfolios sorted by three different measures and the market portfolio that includes all the eligible shares.

Analogously, 25 portfolios are formed on the basis of illiquidity variation in year *y-1*, from the lowest to the highest variation. It results into 25 illiquidity-variation portfolios (and one market portfolio) sorted by three measures discussed above. Furthermore, we make 25 size portfolios by ranking eligible stocks by their market capitalization (from largest to smallest) at the beginning of the year *y*. Illiquidity-variation and size portfolios are employed for the robustness check of the results acquired on illiquidity portfolios. Although

[1] Basically penny stocks mean that shares are traded for pennies, i.e. the price should be less than €1. While Paris Euronext does not have the official definition of this term, the US Securities and Exchange Commision (SEC) defines penny stocks as those traded at less than $5 per share (SEC, 2014)

Acharya and Pedersen (2005) also utilized portfolios sorted together by book-to-market ratio and size, this study omits this procedure.

The number of stock in each portfolio is computed as in Fama and MacBeth (1973). If *N* is the total number of eligible stocks to be allocated to portfolios in year *y*, the integral part of the number *N/25, int(N/25),* is the amount of shares in each of the middle 23 portfolios. Two leftover extreme portfolios get the half of the *N* – *23·int(N/25)* if this number is even. If it is odd the first portfolio (most liquid, with least illiquidity variation or with the largest market capitalization) gets one more share than the last portfolio.

As the empirical estimation of the LCAPM is based on monthly figures the next task is to compute monthly returns and illiquidities for these portfolios. For each portfolio *p* its return in month *t* is calculated as follows:

$$r_t^p = \frac{1}{n}\sum_{i=1}^{n} r_t^i \ , \tag{21}$$

where r_t^i is the return of stock *i* from portfolio *p* in month *t* and *n* is the number of stocks in portfolio *p*. Similarly, the adjusted illiquidity of a portfolio *p* in month *t* is:

$$c_t^p = \frac{1}{n}\sum_{i=1}^{n} c_t^i \ , \tag{22}$$

where c_t^i is the adjusted illiquidity of stock *i* from portfolio *p* in month *t*. Following Acharya and Pedersen (2005) and Kim and Lee (2014) the monthly illiquidities are estimated only if there are at least 15 required observations per each input variable required for a particular liquidity proxy computation in that month. Therefore, *Amihud* measure requires 15 days of return and volume data available, *FHT* – only return data, and *PQS* – bid and ask prices data.

By the end of this section we have formed all the portfolios and calculated their returns and illiquidities. To estimate the LCAPM as in equation (15) one has to compute all four betas used in this formula as well. However, betas involve innovations in illiquidity that are not calculated yet. The next Section solves this task.

3.1.3 Illiquidity innovations

As was mentioned in Section 2.3 the persistence of illiquidity pushes to focus on its innovations. The persistence results from the notion that high illiquidity today means high expected illiquidity tomorrow. This view has an empirical evidence (Amihud, 2002; Pástor and Strambaugh, 2003; Korajczyk and Sadka, 2008). Therefore, this study concentrates on the innovations in illiquidity on a portfolios level, $c_t^p - E_{t-1}(c_t^p)$. The computation process is in line with Acharya and Pedersen (2005).

First, the portfolios' illiquidities are un-normailized by market index, P_{t-1}^M, the operation reciprocal to (20):

$$illiq_t^p = \frac{c_t^p}{P_{t-1}^M} \, , \tag{23}$$

where $illiq_t^p$ is the un-normalized illiquidity of portfolio *p* in month *t*, c_t^p is the adjusted illiquidity as in (22), and P_{t-1}^M is the market index as in (20).

To calculate innovations, the autoregressive model with 2 lags, *AR(2)* is employed:

$$illiq_t^p P_{t-1}^M = \alpha_0 + \alpha_1 illiq_{t-1}^p P_{t-1}^M + \alpha_2 illiq_{t-2}^p P_{t-1}^M + u_t \, , \tag{24}$$

where u_t, residual of the regression, is interpreted as illiquidity innovation, $c_t^p - E_{t-1}(c_t^p)$ attributable to portfolio *p* (including market portfolio) in month *t*. Note that market index, P_{t-1}^M, is used with the same subscript, *t-1*, with all three un-normalized illiquidity terms. It guarantees that the measured innovations are related only to liquidity and not to changes in market capitalization of portfolios, P^M.

Therefore by the end of this Section we have obtained all the variables required for calculation of betas.

3.1.4 Betas Calculation

The betas computation is done similar to Acharya and Pedersen (2005) who utilized the GMM, but different to Lee (2011) and Kim and Lee (2014) who used the Fama-MacBeth procedure. The latter approach implies two-step procedure assessing pre- and post-ranking betas. This procedure helps to minimize the loss of information attributable to individual stocks when forming portfolios. However, in order to obtain the post-ranking betas, which are used in the final regression, a researcher has to spend the first five years of his analysis period to get pre-ranking betas. At the same time decreasing the period of pre-estimation might harm the validity of results. Thus, this study uses the full-sample betas for running the Fama-MacBeth procedure as described in Cochrane (2005).

Therefore, the author utilizes the full sample of required variables from 1997 to 2014. It allows to make a valid estimation of betas without sacrificing a part from the final analysis period. The drawback is the loss of information mentioned in the previous paragraph. However, shortening the final analysis period would be inappropriate for solving this problem.

The computation of betas is done as in equations (16)-(19) using portfolios instead of stocks. This study utilizes monthly portfolios' and market's returns as well as portfolios' and market's innovations in illiquidity for calculations.

3.2 Final Analysis

The final part of the analysis is done as described in Cochrane (2005). The general idea is to use the full-sample betas and run cross-sectional regression at each time period. However, at first it is important to specify the models that this study is going to use.

First, following Acharya and Pedersen (2005) the author defines "net beta" in order to impose the LCAPM restriction that $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$:

$$\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}. \tag{25}$$

The idea behind the net beta is to eliminate the potential collinearity between different betas. Collinearity is a problem because adding or removing one beta from a regression may cause changes in values of the coefficients of the other betas (Brooks, 2008, p. 170).

Second, following Lee (2011) the aggregated liquidity risk or illiquidity beta is defined as the combination of all the liquidity risks:

$$\beta^{illiq,p} = \beta^{2p} - \beta^{3p} - \beta^{4p}. \tag{26}$$

It helps to separate the market risk from the liquidity risk for further analysis.

This study employs four types of regressions:

$$E(r_t^p) = \alpha + kE(c_t^p) + \lambda_t^{net} \beta^{net,p} + u_t^p, \tag{27}$$

$$E(r_t^p) = \alpha + \lambda_t^1 \beta^{1p} + u_t^p, \tag{28}$$

$$E(r_t^p) = \alpha + kE(c_t^p) + \lambda_t^1 \beta^{1p} + \lambda_t^{illiq} \beta^{illiq,p} + u_t^p \text{, and} \quad (29)$$

$$E(r_t^p) = \alpha + kE(c_t^p) + \lambda_t^1 \beta^{1p} + \lambda_t^2 \beta^{2p} - \lambda_t^3 \beta^{3p} - \lambda_t^4 \beta^{4p} + u_t^p, \quad (30)$$

where *α* is the intercept of the regression, *k* is a constant representing a typical investor's holding period, and u_t^p is a pricing error. Line (28) aims at representing the traditional CAPM model, although the market beta here is adjusted for innovations in illiquidity costs as seen from equation (16). In the last regression (30) the model implied restriction that $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$ is eliminated.

Although the LCAPM does not imply the intercept, *α*, it is allowed in the regressions but expected to be insignificant. The constant, *k*, is assumed to equal *1* in the original model because investors incur the illiquidity cost exactly once each model period (Acharya and Pedersen, 2005, p. 393). However, monthly estimation period is likely to be different from the typical investor's holding period that was implicitly imposed in the LCAPM. At the same time a portfolio return and all the betas are nearly linearly scalable because returns and innovations should not be significantly correlated across time and due to the fact that a sum of *k* one-period returns (or illiquidity innovations) is approximately *k*-period return (or illiquidity innovation) (Acharya and Pedersen, 2005, p. 393). Consequently, $E(r_t^p)$ and β^p are approximately *k* times the holding period return or beta. However, $E(c_t^p)$ is not linearly scalable because illiquidity for a longer period is basically an average of illiquidities for shorter periods (as it was calculated in Section 3.1.2, for example). Consequently, *k* is treated either as a free parameter or as a constant. When *k* is fixed the term $E(r_t^p) - kE(c_t^p)$ is used as the dependent variable following Acharya and Pedersen (2005). The constant for *k* in this study corresponds to a holding period of 17 months based on Della Croce, Stewart, and Yermo (2011). They provided information on investment horizons for different exchanges, and it appeared that the average holding period for Euronext from 1995 to 2010 was approximately 1.5 years or 17 months. Thus, $k = 1/17 \approx 0.059$. This estimate should be credible enough for France taking into account that it has been always being the biggest market of Euronext Europe.

Because this study employs several types of regressions and also uses different *k*'s there, the demonstration of the Fama-MacBeth procedure is done on the basis of equation (27). If *k* is constant, the regression looks as follows:

$$[E(r_t^p) - 0.059E(c_t^p)] = y_t^p = \alpha + \lambda_t^{net} \beta^{net,p} + u_t^p, \quad (31)$$

where *p = 1, 2, …, 25* for each *t*, and $E(r_t^p) - 0.059E(c_t^p)$ is treated as the dependent variable.

First, the cross-sectional regressions at each moment *t* are run to get the estimates $\tilde{\lambda}_t^{net}$ for each period. They are obtained by using Ordinary Least Squares (OLS) approach. It basically minimizes the sum of squared distances between the factual points and the estimated regression line across all the portfolios at a given time period *t*, $\sum_{p=1}^{N}(y_t^p - \tilde{y}_t^p)^2$, where *N* is the number of portfolios, and $\tilde{y}_t^p$ is the fitted value from the regression line. It is equivalent to minimizing the *Residual Sum of Squares* (*RSS*) or the sum of the squared pricing errors, $RSS_t = \sum_{p=1}^{N} \tilde{u}_t^{p\,2}$, where $\tilde{u}_t^p = y_t^p - \tilde{y}_t^p$ (Brooks, 2008, p. 33). Consequently, estimates for *α* and λ_t^{net} at time *t* are selected so as to minimize RSS_t in the same time period.

Then the cross-sectional estimates are averaged in order to get estimates $\tilde{\lambda}^{net}$ and $\tilde{u}^p$:

$$\tilde{\lambda}^{net} = \frac{1}{T}\sum_{t=1}^{T}\tilde{\lambda}_t^{net} \qquad \tilde{u}^p = \frac{1}{T}\sum_{t=1}^{T}\tilde{u}_t^p , \tag{32}$$

while the sampling errors of these estimates are computed by employing their standard deviations (Cochrane, 2005, p. 246):

$$\sigma^2(\tilde{\lambda}^{net}) = \frac{1}{T^2}\sum_{t=1}^{T}(\tilde{\lambda}_t^{net} - \tilde{\lambda}^{net}) \qquad \sigma^2(\tilde{u}^p) = \frac{1}{T^2}\sum_{t=1}^{T}(\tilde{u}_t^p - \tilde{u}^p). \tag{33}$$

The test whether all the residuals are jointly zero is conducted as before.

Cochrane (2005) notes that Fama-MacBeth major drawback is the assumption that the time series is not autocorrelated. He adds that the implementation of this procedure on corporate finance data might be not that efficient due to this problem. However, he points out that asset pricing applications should be reasonable because returns are approximately independent.

All the calculations are implemented in RStudio – integrated development environment for the R programming language. The details of the program, the required packages for the analysis, and parameters of the computer used in this study are described in Appendix 1.

4 DATA

The initial dataset consist of all common shares traded on Paris Euronext in the period from December 01, 1995 to March 31, 2014. There is a total number of 835 stocks included in the analysis. The study uses daily values of closing prices, closing bid and ask prices, absolute trading volumes and market capitalization that are downloaded from Datastream. All the stock prices are adjusted for subsequent capital actions. The volumes used in the analysis are expressed in thousands of euros, while market capitalization - in millions of euros. The daily observations are filtered by penny stocks and missing volume data as described in Section 3.1.1. Totally there are 3,992,970 stock-day observations of different variables including missing values (*4,782 trading days x 835 stocks*) that are used for initial computations.

Nevertheless, as this study concentrates on monthly figures it employs 172,845 stock-month observations including missing values (*207 months x 835 stocks*) that are left after portfolios formation (only for illiquidity and illiquidity-variation portfolios) [1]. Therefore, the final analysis period is from January 01, 1997 to March 31, 2014. Table 12 of Appendix 2 provides descriptive statistics on individual stock level for all liquidity measures and returns – variables that are directly used for the final analysis. Although different illiquidity proxies show different amounts of valid observations – from 82,657 for *PQS* to 87,458 for *FHT* - the total number of observations for each measure is still enough for statistical analysis. *FHT* has a big number of zero values that decreases its variability compared to other measures. It can also be noticed that *FHT* and *PQS* have larger ranges than *Amihud* because the later proxy is divided by trading volume. Concerning stock returns for the analyzed period they vary from approximately -86% to +378%, whereas the average monthly figure for all stocks is about +5%.

It is worth to mention the portfolios characteristics depending on liquidity measure used for their formation before moving forward to portfolios analysis. Table 3 shows the amount of stocks per first, middle, and last portfolios depending on different formation criteria. The last column is not divided by different liquidity measures because the size portfolios are the same for each measure. The figures represent the minimum and maximum numbers of shares one can observe in different years for the analyzed period. It can be noticed that

[1] For portfolios formed by firm size there are additional 12 months of observations available because they are based on the beginning of the year figures while illiquidity and illiquidity-variation portfolios are based on the previous year figures (check Section 2.1.2 for details). So, there are totally 182,865 stock-month observations used for size portfolios.

Table 3. Variability of Number of Stocks per Portfolio Depending on Formation Criteria and Illiquidity Measure Used, 1996-2014

Portfolios Formed by:	Illiquidity and Illiquidity-Variation (Increasing)			Size (Decreasing)
	Amihud	FHT	PQS	
Portfolio No.1	12-32	9-33	17-32	12-33
Portfolios No. 2-24	8-23	7-22	6-23	7-21
Portfolio No. 25	11-32	9-33	16-32	11-32
Total	219-591	196-566	173-586	184-548

Portfolios formed by size are the same for each liquidity measure. Thus, the last column shows the amount of stocks per first, middle, and last size portfolios for *Amihud*, *FHT*, and *PQS*

the amount of stocks per portfolios significantly vary from year to year. The variability of total number of shares eligible for analysis is most wide for *PQS*, whereas *Amihud* and *FHT* exhibit more modest and approximately the same variation of stocks per portfolio.

All the portfolios have 207 monthly observations of illiquidity and return each. Both illiquidities and returns time series appear to be stationary according to the *Augmented Dickey-Fuller Test* (at 95% confidence level) disregard to liquidity measure used[1]. Therefore, the data is valid for further usage in computation of illiquidity innovations and betas as well as in the cross-sectional time series regression.

Further Table 4 shows the mean illiquidity, c^p, return, r^p, and their standard deviations, $\sigma(c^p)$ and $\sigma(r^p)$, for different portfolios. The statistics is provided for illiquidity-sorted portfolios as they represent the key dataset for analysis. In line with the results for the US market (Acharya and Pedersen, 2005) both c^p and $\sigma(c^p)$ are increasing from the first to the last portfolios in all three panels. It means that the more liquid is the stock the less liquidity variation it has. Hence, portfolios based on illiquidity and illiquidity-variation should be characterized by similar sorting. At the same time Panel A shows that mean return is generally increasing with illiquidity only for portfolios based on *Amihud* measure. On the other hand, Panel B and Panel C representing *FHT*- and *PQS*-based portfolios accordingly do not exhibit the same upward trend for returns. In contrast with the US market characteristics where returns variation increases with illiquidity (Acharya and Pedersen, 2005), $\sigma(r^p)$ in the French stock market does not seem to have any trend disregard to liquidity proxy used.

[1] Although the topic of stationarity was not discussed before for the sake of brevity, the author would like to notice that using non-stationary data can lead to spurious regressions where standard assumptions for asymptotic analysis are not valid (Brooks, 2008, pp. 319-320).

Table 4. Characteristics of Illiquidity Portfolios Based on Different Liquidity Measures

Portfolio No.	Mean Illiq-ty, c^p	Mean Illiq-ty StD, $\sigma(c^p)$	Mean Return, r^p	Mean Return StD, $\sigma(r^p)$
Panel A: Amihud				
1	0.0000	0.0000	0.0062	0.0694
5	0.0001	0.0001	0.0080	0.0737
10	0.0028	0.0035	0.0080	0.0639
15	0.0126	0.0120	0.0080	0.0694
20	0.0346	0.0234	0.0094	0.0547
25	0.1284	0.0929	0.0198	0.0843
Panel B: FHT				
1	0.0137	0.0095	0.0069	0.0685
5	0.0349	0.0195	0.0102	0.0686
10	0.0884	0.0374	0.0089	0.0643
15	0.1494	0.0613	0.0066	0.0565
20	0.2285	0.0925	0.0082	0.0657
25	0.6076	0.3299	0.0141	0.0601
Panel C: PQS				
1	0.0061	0.0037	0.0070	0.0687
5	0.0301	0.0137	0.0043	0.0665
10	0.0657	0.0266	0.0123	0.0714
15	0.0996	0.0393	0.0072	0.0670
20	0.1546	0.0632	0.0116	0.0648
25	1.1495	1.0239	0.0073	0.0605

Abbreviations: Illiq-ty – Illiquidity, StD – Standard Deviation
All the mean figures are average monthly means across the period of January 1997 – March 2014

It is also interesting to see whether illiquidity portfolios differ depending on liquidity proxy used or in other words how similarly different liquidity measures sort the shares. Figure 2 shows how many times the same stocks appear in *Amihud*-, *FHT*-, and *PQS*-based illiquidity portfolios. As could be seen from the figure only the most and the least liquid portfolios are compared. In particular, *Matches* indicate how many times a certain stock occurs in all three portfolios for a given year, and *Number of Unique Values* represents the amount of unique stocks bounded in these portfolios. Hence, the blue part of the bars indicates the amount of unique stocks included in all three portfolios, the green part – the amount of unique stocks included in only two out of three portfolios, and the red part – the amount of unique stocks that one can observe only once in portfolios formed by different

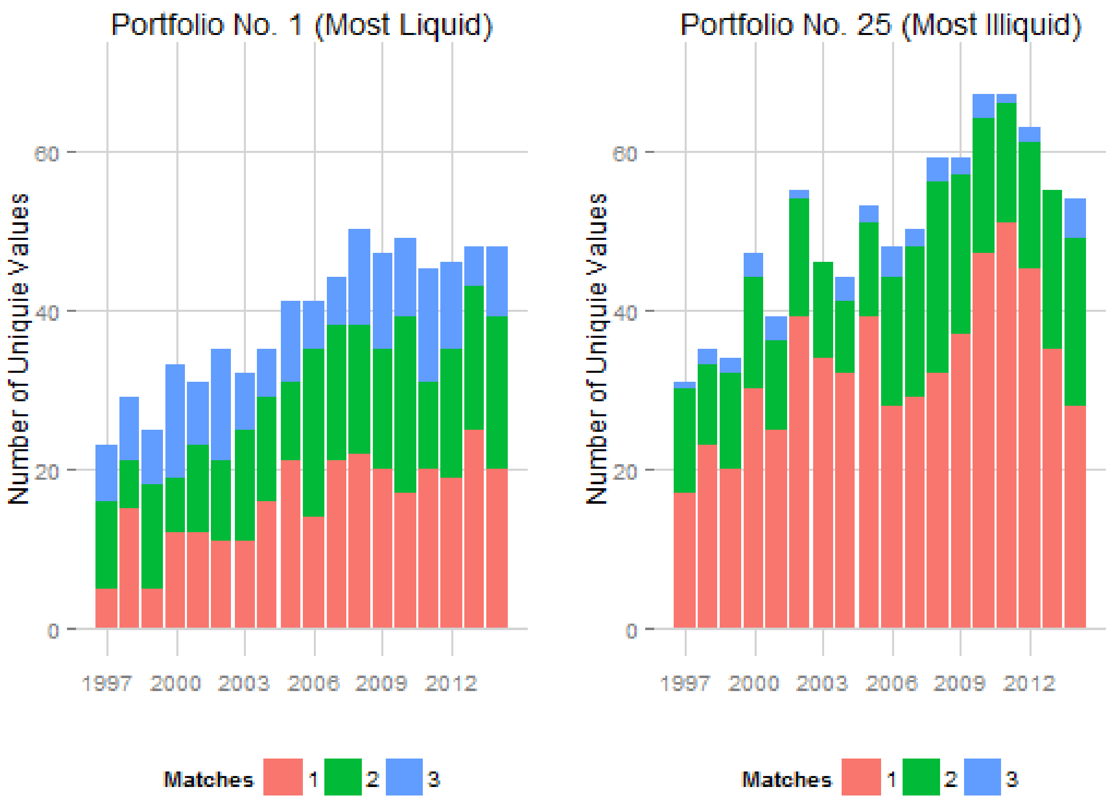

Figure 2. Yearly Matches of Stocks Included in Illiquidity Portfolios Between Different Liquidity Measures

Matches is the number of times a particular stock occurs in *Amihud*-, *FHT*-, and *PQS*-based portfolios together. So, *Matches = 1* means that a stock is included only in one portfolio and does not appear in others, *Matches = 2* – a stock occurs in two portfolios, but is not included in the leftover portfolio, *Matches = 3* – a stock can be found in every portfolio

Number of Unique Values is the number of unique stocks in *Amihud*-, *FHT*-, and *PQS*-based portfolios altogether.

Portfolio No. 1 is the most liquid portfolio, *Portfolio No. 25* – the most illiquid

liquidity measures. It can be noticed that stocks are jointly recognized as the most liquid (Portfolio No. 1) by several liquidity measures much more often compared to the most illiquid shares (Portfolio No. 25). Comparing the left and the right plots one can notice from the blue parts of the bars that *Amihud*, *FHT*, and *PQS* rarely put the same shares in the last portfolio together. For example, in 2003 and 2013 there were no stocks identified as the most illiquid mutually by all three liquidity measures. Therefore, different liquidity measures seem to agree more often about which stocks are the most liquid. However, they provide much more diverse results with regard to the most illiquid shares.

To conclude, different liquidity measures could cause diverse results. *Amihud*, *FHT*, and *PQS* often identify different stock to be the most and the least liquid. Thus, the author expects that the choice of illiquidity measure should significantly affect the output of the LCAPM. The next Section provides the results of testing of the different LCAPM versions.

5 RESULTS

This section presents the empirical analysis implemented via RStudio according to the methodology described in Section 3. Generally all the computational processes could be divided into four parts: data pre-processing and portfolios formation, calculation of illiquidity innovations, estimation of betas, and running Fama-MacBeth regressions. The previous section has introduced the data both from stock and portfolio level perspectives, thus, covering the first step of the analysis. This chapter is going to discuss the innovations in illiquidity at first. Further, the estimates of betas are shown and analyzed. Finally, different specifications of Fama-MacBeth regressions are described. Note that all these results are based on illiquidity portfolios.

After that the obtained results are checked for robustness. For this reason illiquidity-variation and size portfolios are utilized as in Acharya and Pedersen (2005). Then the controlling of the obtained results for a size effect is done. Finally, the study also presents the specification test for the LCAPM variations used and summarizes the main conclusions from the results.

5.1 Innovations in Illiquidity

Illiquidity innovations are estimated on a portfolio level according to equation (24). Because they are computed with two lags the final dataset is reduced by two months to 17 years and 1 month, from March 1997 to March 2014. The average innovations in illiquidity – those attributed to market portfolio – are estimated with the R^2 of 64%, 69%, and 85% for *Amihud*, *FHT*, and *PQS* accordingly. Figure 3 visualizes the dynamics of these market illiquidity innovations for the analyzed period.

The upper spikes of Figure 3 indicate problems with liquidity. It could be noticed that innovations in illiquidity computed with *FHT* and *PQS* share the similar behavior, while *Amihud*'s innovations look quite more different. The upper peaks of the *FHT* and *PQS* graphs coincide with the important events associated with liquidity leakage. For example, three spikes in the left part are matched with such events in the end of summer 2000 (acceleration of dot-com bubble started in March), September 2001 (9/11 terrorists' attack in the US), and summer 2002 (last big collapses of dot-com bubble, including WorldCom). In general the time lapse of these three peaks – 2000-2002 – coincides with the dot-com

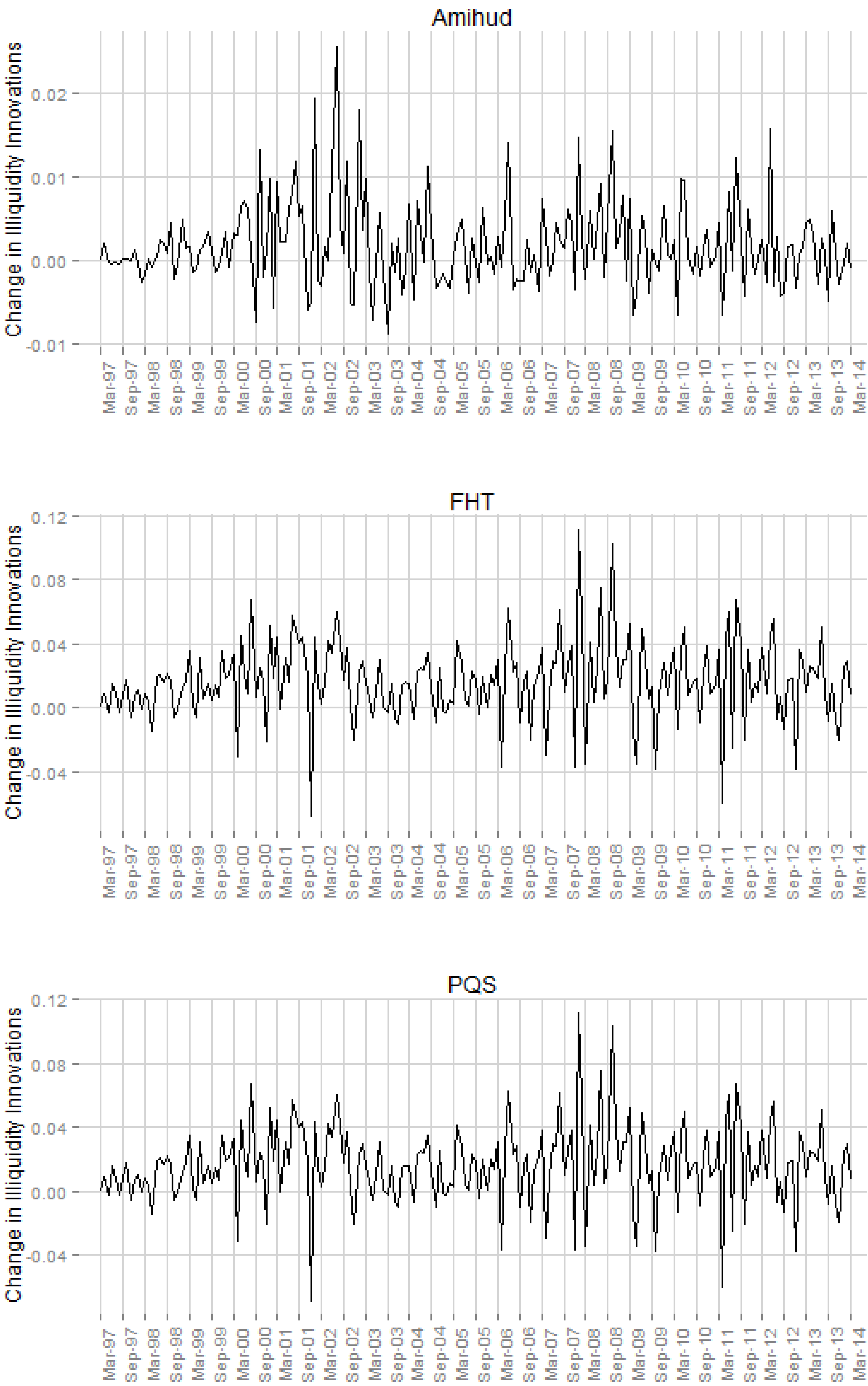

Figure 3. Dynamics of Market Illiquidity Innovations, 1997-2014

All the changes are computed with regard to previous values, the first value is taken as zero bubble. Further in the picture the highest peaks in the end of 2007 - beginning of 2008 and in September 2008 are associated with rapid spread of global financial crisis and bankruptcy of Lehman Brothers accordingly. Finally, a spike in August 2011 may be noticed when Standard & Poor's downgraded the US credit rating from *AAA* to *AA+*. There were also rising concerns about downgrading of France at that time (see e.g., Bremer and Dmitracova, 2011; Alderman, 2011).

However, the results for *Amihud* are not consistent with other illiquidity measures. For instance, although its innovations also exhibit spikes in the period of the global financial crisis they are much lower than in 2002. It seems strange bearing in mind that dot-com bubble of 2002 had much lower scope and power compared to the crisis of 2007 - 2008. Nevertheless, the measure's spikes generally occur in the same places but with different magnitudes.

Therefore, it appears that events associated with liquidity leakage coincide with the spikes in illiquidity innovations. The same relationship was documented for the US stock market by Acharya and Pedersen (2005). More reasonable results of *FHT* and *PQS* may indicate that these measures are more adequate for liquidity estimation.

5.2 Betas

The betas are computed using equations (16)-(19). β^1 and β^2 appear to be positive values, while β^3 and β^4 – negative. This result is in line with Acharya and Pedersen (2005) and Lee (2011). Figure 4 further illustrates how the betas are changing when moving from the most to the least liquid portfolio. It could be noted that each beta shares the same dynamics disregard to illiquidity measure used. At the same time it can be noticed that illiquidity betas estimated with *Amihud* exhibit smoother trends. In addition *PQS*' β^2 and β^4 have very large jumps of values for the last portfolio.

Dynamics of β^1 generally shows a downward trend. Although it is not that explicit it is telling that the most illiquid stocks' returns are less related to overall market return movements than the returns of the most liquid stocks. The possible explanation has the following logic. If a stock is illiquid it is not frequently traded (according to *Amihud* and *FHT* formulas) and has a large bid-ask spread (according to *PQS* definition). Because potential investors trade only when potential gains exceed trading costs (see Section 2.2.2), they would probably trade based on crucial company or industry specific news

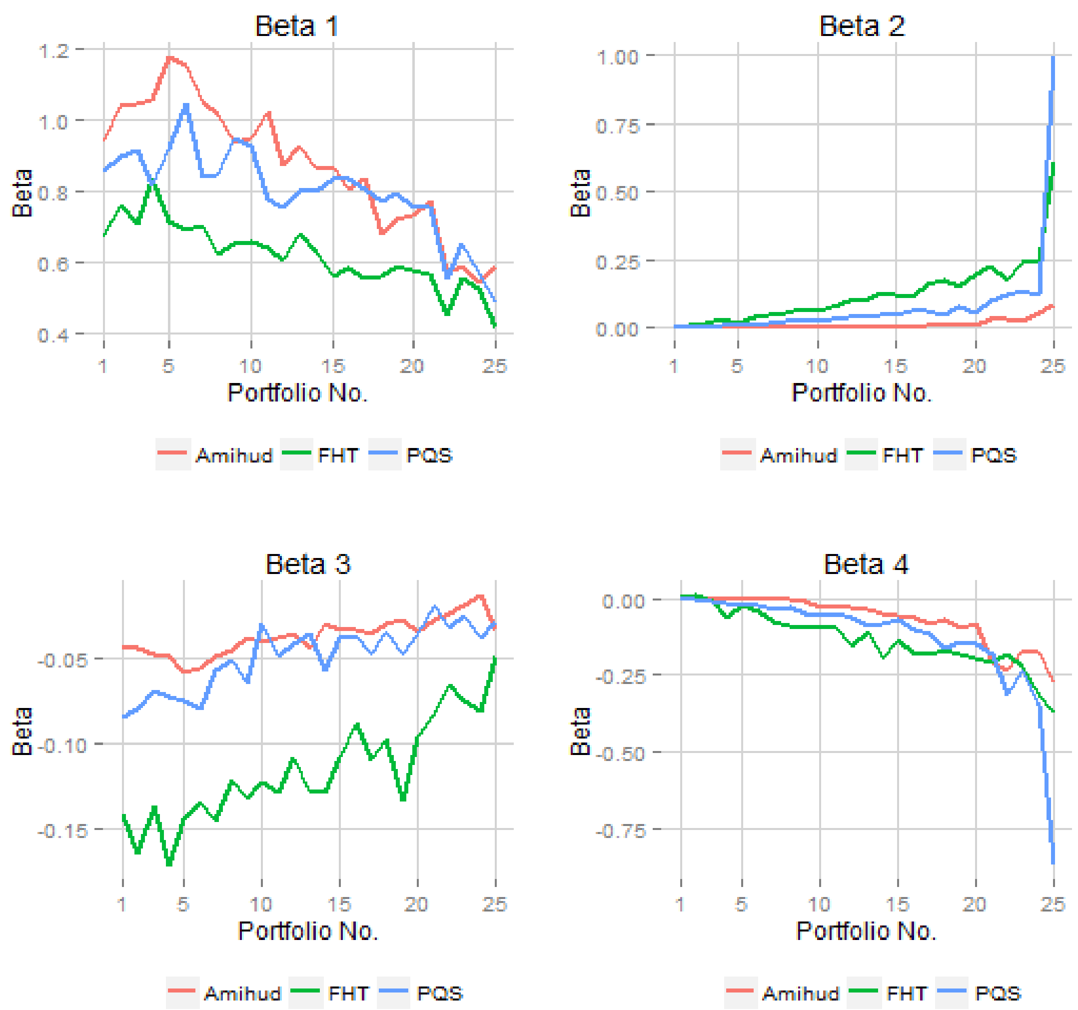

Figure 4. Dynamics of the Betas across Portfolios for Different Liquidity Measures

while overall market movements should not generally be that groundbreaking to cause potential profits to cross the high return threshold taking into account very large costs associated with very illiquid stocks. However, this result is in contrast with Acharya and Pedersen (2005) and Lee (2011) who documented also not that clear but generally upward trend of β^1 for the US and global stock markets.

Second beta, in contrast, has an upward trend that is in line with both Acharya and Pedersen (2005) and Lee (2011). The reason for such a behavior could lie in a response of market participants to increased market illiquidity. When this scenario is realized investors who were involved into transactions with illiquid stocks before would likely switch to more liquid shares pursuing to trade at the same liquidity level and trying to avoid paying more trading costs for previously held securities. Therefore, more liquid stocks receive new traders whereas the most illiquid assets become extremely illiquid. As an evidence Figure 4 shows the jumps at the end of the top right graph representing behavior of β^2 for the most illiquid shares (for *FHT* and *PQS*).

Third beta shows an upward direction with *FHT* having the largest range of the observed values. Such a trend is in line with Lee (2011), but in contrast with Acharya and Pedersen (2005). Most liquid stocks' returns could be more sensitive to market liquidity movements because of the side effect associated with β^2. When market liquidity decreases investors switch to more liquid stocks, thus pushing the total demand for shares to shift to more liquid assets. It might cause lowering of returns of more favorable securities due to the changed price equilibrium. In contrary the most illiquid stocks should provide higher returns in this situation. In fact they provide slightly less returns (as seen from a negative sign of β^3), but their returns have a relatively small negative change compared to more liquid shares.

Finally, the fourth beta has a downward trend which is in line with Acharya and Pedersen (2005), but in contrast with Lee (2011). To explain this result the logic of the LDV model could be applied (see Section 2.2.2 for details). As was discussed in Section 2.3 investors are poor in a down market and especially appreciate liquidity in such a situation. When returns are decreased market participants' expected gains of a trade cannot exceed trading costs anymore. Consequently, for most illiquid stocks that were not traded frequently before the situation should become extremely bad taking into account that investors themselves start to worry more about liquidity. Hence, the more illiquid is an asset the more sensitive it should be to market return movements.

Concerning co-movements between betas it was expected that collinearity there should be strong that could cause problems with running the full regression as in equation (30) (see Section 3.1.4 for details). Empirically this expectation is confirmed for all the liquidity measures. From Table 5 it is seen that all the betas are strongly correlated between each other, except $cor(\beta^2, \beta^3)$ for *Amihud* and $cor(\beta^2, \beta^3)$ and $cor(\beta^3, \beta^4)$ for *PQS* which are moderately correlated. Therefore, the use of the net beta in the final regression is empirically justified.

5.3 Regression Analysis

Table 6 provides the results for Fama-MacBeth regressions run for illiquidity portfolios. The full results with *t-statistics* are provided in Table 14, Appendix 3. Different panels are

Table 5. Betas Correlations for Different Liquidity Measurements

	β^1	β^2	β^3	β^4
Panel A: Amihud				
β^1	1			
β^2	-0.7508	1		
β^3	-0.9209	0.5869	1	
β^4	0.8632	-0.9014	-0.7330	1
Panel B: FHT				
β^1	1			
β^2	-0.7998	1		
β^3	-0.9376	0.8162	1	
β^4	0.8362	-0.9043	-0.8467	1
Panel C: PQS				
β^1	1			
β^2	-0.6421	1		
β^3	-0.6474	0.3716	1	
β^4	0.8200	-0.9365	-0.5386	1

dedicated to different liquidity measures. Each section 1 – 4 (column 1) in these panels represents regression equations (27) - (30) accordingly. Because *k* was assumed to be a constant value (see Section 3.2) this study also employed regressions where *k* is a free unfixed parameter. Nevertheless, the resulted *k* in this situation should be positive due to initial short-sale constrain imposed by the LCAPM.

The results from the Panel A, Table 6 indicate that when *Amihud* illiquidity measure is used the net beta is not reflected in the French stock returns. The second section also says that market beta does not significantly influence on stock returns. From the third section it can be seen that the aggregate illiquidity risk does not affect securities returns when *k* is fixed, while it has a significant impact when *k* is treated as a non-constant parameter. However, in the latter case the assumption of positive *k* is violated (see Section 3.2 for details), that is why in the last row of the third section the same regression is run with $k = 0$. It helps to avoid the imposed restriction by artificially implying that the illiquidity level, $E(c)$, does not influence on stock returns. It could be noted that when the assumption of positive *k* is not violated the aggregated liquidity risk does not affect securities returns. The similar situation is observed in the third section when all the liquidity risks are used separately in the model: β^2 influences on French stock returns only when the assumption of positive *k* is violated. In summary, the *Amihud*-based results

Table 6. Fama-MacBeth Regression Results for Illiquidity Portfolios

	α	*E(c)*	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	R^2
Panel A: Amihud									
1	0.003						0.003		66.9%
	0.015*	-0.041					-0.007		73.7%
2	0.015***		-0.008						69.0%
3	0.006		0.001					-0.005	73.9%
	0.010	-0.201	-0.006					0.047**	78.1%
	0.006		-0.001					0.019	73.8%
4	0.004		0.010	0.038	0.142	0.011			80.0%
	0.011*	-0.263*	0.003	0.377*	0.166	-0.003			82.9%
	0.006		0.006	0.144	0.128	0.011			80.0%
Panel B: FHT									
1	0.039***						-0.041***		73.5%
	-0.004	-0.015					0.012*		76.8%
	-0.003						0.010*		73.3%
2	0.011*		-0.006						76.2%
3	-0.012		0.031**					-0.023***	78.1%
	0.003	-0.024	0.001					0.019*	79.4%
	-0.004		0.012					0.011**	77.7%
4	-0.015*		0.054***	-0.022	0.115**	0.030			81.7%
	0.002	-0.041**	0.031	0.054***	0.111*	0.005			82.8%
	-0.006		0.042**	0.023*	0.116**	0.015			81.4%
Panel C: PQS									
1	0.048***						-0.046***		71.7%
	0.015**	-0.013					-0.008		72.8%
2	0.009*		-0.002						71.5%
3	0.012		-0.004					-0.038***	75.5%
	0.011	-0.019	-0.003					0.001	75.5%
4	0.010		-0.003	-0.042***	0.022	0.033*			80.3%
	0.007	-0.045	0.003	0.013	0.033	-0.003			80.6%

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, and ***** indicates significance of a parameter at 10% level

Note also that when *E(c)* is not specified in the table it is not treated as an independent variable (see Section 3.2 for details), thus implying that illiquidity level, *E(c)*, does affect stock returns. However, in the second section of the table variable *E(c)* is not used at all

indicate that neither illiquidity level nor liquidity risks significantly affect stock returns. Market beta also appeared to be unimportant with regard to returns.

Panel B of Table 6 shows the results for *FHT* illiquidity measure. The test of the regression equation (27) shows that the net beta significantly affects French stock returns.

However, when *k* is fixed the negative sign of β^{net} is not economically valid, whereas non-constant *k* leads to a negative, although insignificant *k*. Nevertheless, when liquidity level is not taken into account ($k = 0$) the net beta is both statistically and economically significant. The second section of Panel B does not confirm the validity of the standard CAPM as in the case of *Amihud*. The results of the third section provide additional insight to the conclusions from the first section. In particular, according to significant β^{illiq} and insignificant β^{1} of the second and third rows aggregated illiquidity risk appeared to add the most to significance of β^{net}. Hence, liquidity risks influence on stock returns, whereas market beta does not when *E(c)* is not taken into account. On the other hand, the fixed *k* used in the same regression equation (29) results in significant and positive β^{1} and significant, but having a wrong sign β^{illiq}. The findings from the fourth section provide some confirmation of the relative importance of β^{2} compared to other liquidity betas (as β^{3} has a wrong sign) and of β^{1}, but this result is not that robust because of the multicollinearity problem (see Section 3.2 for details). In general, *FHT*-based Fama-MacBeth regressions show that liquidity risks significantly affect stock returns, while liquidity level together with market beta are not that important in this respect.

Concerning the *PQS*-based regression analysis, Panel C of Table 6 generally reports statistically but not economically significant results about liquidity risks' relation to stock returns. The first section says that when *k* is fixed the net beta significantly affects stock returns, but the wrong sign eliminates the economic validity of this result. In addition, non-constant *k* leads to insignificant impact of the net beta on stock return. The standard CAMP is again not confirmed as can be seen from the second section. Similarly to the previous results the third section reports significance of illiquidity beta with the wrong sign for the fixed *k*, whereas for non-constant *k* the aggregate liquidity risk appeared not influential with respect to the stock returns. Finally, the fourth section found β^{2} and β^{4} statistically but not economically significant. In summary, *PQS*-based regressions did not show economically valid results, except for inconsistence of CAPM and cases where *k* is treated as a free parameter.

In conclusion, the Fama-MacBeth regressions for illiquidity portfolios reveal the following results. First of all, disregard to illiquidity measure used the standard CAPM model appears not to be working for the French stock market. Moreover, liquidity level generally appears unimportant with regard to securities returns. It is worth to mention also that fixed *k* often leads to invalid economic results.

However, all the liquidity measures used in this study bring different conclusions. *Amihud*-based analysis shows that neither liquidity level nor liquidity risks affect stock returns. On the other hand, *FHT*-based results indicate that the aggregate liquidity risk significantly influence on equity return, while liquidity level does not appear that important in this respect. Finally, *PQS* regressions produce inconsistent results which partly indicate insignificant impact of aggregate liquidity risk and partly provide economically invalid output. Therefore, the choice of liquidity measure significantly affects the results of the LCAPM.

5.4 Robustness Check

The robustness check consists of the three parts. First, the study follows Acharya and Pedersen (2005) and considers different portfolios. If illiquidity-variation and size portfolios provide similar outputs the conclusions derived from the main analysis are strong. Then the obtained results are controlled for the potential size effect. Finally, the study presents the specification tests for the used LCAPM variations on the basis of illiquidity portfolios.

5.4.1 Considering Other Portfolios

Table 7 provides the results of Fama-MacBeth regressions run for illiquidity-variation portfolios. The full table with according *t-statistics* can be found in Table 15 of Appendix 3. The organization of the table is the same as in Table 6.

Panel A of Table 7 provides the results for *Amihud*-based Fama-MacBeth regressions. It could be noticed that the first, the second, and the fourth sections confirm the previous results that the standard CAPM does not work in the French equity market and that liquidity level and risks do not significantly affect stock returns. On the other hand, the third section contradicts the findings based on illiquidity portfolios. It is found that the aggregate liquidity risk does influence on stock returns with the 10% confidence level when *k* is treated as a free parameter. Nevertheless, except the later result illiquidity-variation portfolios analysis is consistent with the previous output for *Amihud* measure: illiquidity level and liquidity risks do not significantly affect French stock returns.

Panel B shows the results for the *FHT*-based regression analysis. Illiquidity-variation portfolios generally provide the output inconsistent with the previous conclusions. Only the second section confirms that the CAPM is inappropriate for the French equity market.

Table 7. Fama-MacBeth Regression Results for Illiquidity-Variation Portfolios

	α	*E(c)*	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	R^2
Panel A: Amihud									
1	0.001						0.005		70.1%
	0.005	0.032					0.002		74.2%
2	0.014***		-0.007						70.8%
3	-0.002		0.008					0.015	73.1%
	-0.001	0.001	0.005					0.037*	77.0%
4	-0.002		0.009	0.043	0.008	-0.010			79.2%
	0.001	-0.003	0.007	0.051	0.029	-0.046			81.6%
Panel B: FHT									
1	0.035***						-0.036***		77.6%
	0.004	-0.003					0.003		79.1%
2	0.012*		-0.007						75.2%
3	0.004		0.010					-0.033***	79.4%
	0.004	0.001	0.003					0.003	80.7%
4	-0.001		0.010	-0.025**	-0.005	0.037**			82.7%
	0.002	-0.009	0.012	0.028*	0.022	0.017			83.7%
	0.002		0.013	0.023**	0.026	0.022			77.4%
Panel C: PQS									
1	-0.014**						0.013**		69.3%
	-0.005	-0.024					0.014**		73.5%
	-0.005						0.013**		69.9%
2	0.008		0.000						70.8%
3	-0.054***		0.065***					0.007	74.1%
	0.000	-0.022	0.006					0.015**	76.7%
	-0.003		0.010					0.013**	74.3%
4	-0.017**		0.015	-0.052***	-0.126*	-0.029***			78.3%
	0.007	-0.036	0.001	0.020	0.004	-0.014*			80.3%
	-0.001		0.006	0.011	-0.030	-0.014*			78.3%

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, and ***** indicates significance of a parameter at 10% level

Note also that when *E(c)* is not specified in the table it is not treated as an independent variable (see Section 3.2 for details), thus implying that illiquidity level, *E(c)*, does affect stock returns. However, in the second section of the table variable *E(c)* is not used at all

From the first and the third section it could be noted that the net beta and the illiquidity beta do not significantly affect stock returns when k is non-constant. However, when k is fixed both betas are statistically significant with the wrong negative sign. Finally, the fourth section shows different results to what was discovered for illiquidity portfolios. When k is fixed β^4 appears statistically and economically significant, whereas non-constant k leads to significance of only β^2 which is different from the previous findings. Therefore, analysis based on illiquidity-variation portfolios does not confirm the previously derived conclusions of importance of liquidity risks for stock returns.

Lastly, Panel C presents the results with regard to *PQS* illiquidity measure. Here only the invalidity of CAPM for the French stock market appears to be consistent with the previous results. On the other hand, the first section indicates that the net beta is priced both when k is fixed and non-constant. However, the third section reports significant β^1 and insignificant β^{illiq} for fixed k and the opposite result for non-constant k. In other words, fixed k explains the pricing of β^{net} by the market risk, while non-constant k – by the aggregate liquidity risk. The fourth section reports statistically and economically significant β^3 and β^4 for $k = 0.059$ and only significant β^4 when k is treated as a free parameter. However, multicollinearity problem is again should be born in mind. Therefore, illiquidity-variation portfolios provide completely different results compared to illiquidity portfolios for *PQS* measure.

In summary, results derived from analyzing illiquidity-variation portfolios differ from those of illiquidity portfolios. Only for *Amihud* measure the conclusions were generally consistent. For *PQS* the results were completely opposite, whereas for *FHT* the important part dedicated to illiquidity risks pricing for non-constant k contradicted the previous findings. The only finding that was consistent across all the measures is that the standard CAPM does not work in the French equity market. Therefore, the initial results obtained for illiquidity portfolios appear to be weak for *FHT* and *PQS*, but are generally consistent for *Amihud*.

To further check the robustness of the initial findings this study analyzes size portfolios. Table 8 provides the according results for them (more details including *t-statistics* are shown in Table 16 of Appendix 3).

Panel A presents the results for *Amihud* illiquidity measure. The figures from the sections 1 and 3 are consistent with the previous findings and suggest that both the net and illiquidity betas are not priced in the French equity market. In line with illiquidity and illiquidity-variation portfolios size portfolios lead to unimportance of liquidity level with respect to stock returns and invalidity of the standard CAPM. Although the fourth section provides some statistically and economically significant results for the relative importance of β^1 and β^2, these findings are not robust in the presence of the multicollinearity problem.

Table 8. Fama-MacBeth Regression Results for Size Portfolios

	α	$E(c)$	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	R^2
Panel A: Amihud									
1	0.000						0.007		73.3%
	-0.002	-0.044					0.010		77.0%
2	0.008		0.001						74.8%
3	0.000		0.009					-0.008	77.5%
	0.000	-0.165*	0.007					0.031	79.1%
4	-0.006		0.035***	0.502***	0.549***	0.084**			80.5%
	-0.007	-0.270***	0.037***	0.845***	0.604***	0.060			82.0%
	-0.005		0.035***	0.636***	0.584***	0.081**			80.4%
Panel B: FHT									
1	0.032***						-0.032***		75.4%
	0.002	-0.039*					0.008		78.0%
2	0.008		0.001						74.8%
3	-0.006		0.023**					-0.029***	78.2%
	0.007	-0.036*	0.000					0.012	79.6%
4	-0.005		0.020	-0.045***	0.021	0.010			81.9%
	0.009	-0.036	0.005	0.008	0.043	-0.027			83.2%
Panel C: PQS									
1	-0.004						0.005		73.3%
	-0.003	-0.029					0.012**		77.4%
	-0.004						0.012**		74.1%
2	0.008		0.001						74.8%
3	-0.041***		0.050***					0.005	76.6%
	0.003	-0.021	0.006					0.013**	79.7%
	-0.004		0.012					0.012**	77.3%
4	-0.021***		0.026**	-0.039***	-0.048	-0.029***			80.8%
	-0.002	-0.026	0.011	0.032**	-0.018	-0.001			83.3%
	-0.010		0.017*	0.025*	-0.043	-0.007			81.2%

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, and ***** indicates significance of a parameter at 10% level

Note also that when *E(c)* is not specified in the table it is not treated as an independent variable (see Section 3.2 for details), thus implying that illiquidity level, *E(c)*, does affect stock returns. However, in the second section of the table variable *E(c)* is not used at all

Therefore, disregard to portfolio sorting criteria used *Amihud* measure provides consistent result: illiquidity level and aggregate liquidity risk do not affect stock returns.

Panel B contains the results for the *FHT*-based regression analysis. Similar to illiquidity-variation portfolios size portfolios report statistical significance (although not economical) of the net beta and the aggregated liquidity risk when *k = 0.059*. On the other hand, it is found that when *k* is treated as a free parameter both β^{net} and β^{illiq} appear unimportant with respect to stock returns. The fourth section does not provide any relevant economic results. Finally, CAPM is again not working for the French stock market. Thus, the initial result reporting significant influence of liquidity risks on stock return appears to be weak taking into account the contradicting findings of illiquidity-variation and size portfolios.

Lastly, Panel C shows the results for the *PQS* regressions. The size portfolios lead to output different from the initial findings. The only coincidence is the insignificant standard CAPM model. On the other hand, sections one and three report the significant net and illiquidity betas when *k* is treated as a free parameter. At the same time when *k* is fixed the aggregate liquidity risk is not priced while the market beta is. The fourth table provides some evidence of the relative importance of β^1 and β^2 with regard to other betas when *k* is non-constant. However, when *k* is fixed β^1 and β^4 appear both statistically and economically significant whereas β^2 changes its sign. Therefore, the results are similar to illiquidity-variation portfolios, but not to illiquidity portfolios. Thus, if originally there were no economically valid findings illiquidity-variation and size portfolios provide some evidence for significant influence of liquidity risks on stock returns.

In conclusion, the robustness check shows the weakness of the previously obtained results. The only finding that remains stable among all the illiquidity measures is that the standard CAPM does not work in the French equity market. The main result of *Amihud*-based regression analysis – that neither illiquidity level nor liquidity risks significantly affect stock returns – is generally robust disregard to portfolio sorting criteria. In contrary, the findings based on *FHT* measure – significance of liquidity risks with regard to stock returns – are not confirmed by illiquidity-variation and size portfolios. At the same time, the new results for *PQS* lead to the conclusion that liquidity risks affect stock returns.

Therefore, only *Amihud* illiquidity measure performs stably across different types of portfolios. Other measures show conflicting behavior depending both on portfolio sorting criteria and on *k* parameter used (fixed or non-constant). Nevertheless, the results still have to be controlled for potential size effect.

5.4.2 Controlling for Size

The controlling for size and book-to-market (B/M) effects is a standard procedure in financial literature to test the robustness of results. However, B/M ratios at a company level are not generally available in Datastream. The database provides the ratios at a security level which is not valid for the according robustness check. Moreover, Datastream sometimes reports values that are negative which should not be feasible. For example, for our sample of the French stocks 113,621 values out of 2,819,099 non-missing observations are negative. Therefore, the author decided to use only size for the further robustness check.

The main goal of controlling for a size effect is to check whether earlier findings remain the same and whether stock returns contain a size premium. To test it the additional size variable, $\ln(MC_t^p)$, is included in regression equations (27) - (30). It is the natural log of market capitalization at the beginning of the month *t* of a portfolio *p*. A portfolio's market capitalization is calculated as the average of the respective figures of stocks included there. This study employs an average instead of a sum of market capitalizations (as was done in Acharya and Pedersen, 2005) in order to diminish the potential problem that could arise from difference between the number of securities of the extreme and middle portfolios. It should be noticed that the difference in exact coefficients attributed to previously studied variables is partly caused by the inclusion of the additional size parameter in regression models. Therefore, the author is focused on whether the findings of the new regressions remain generally the same. The results of the fourth section of each table are not considered that strictly because of the multicollinearity problem comprised in the regression specification (30) that is represented by this section.

Table 9 shows the findings for the illiquidity portfolios. The full table with according R^2 and *t-statistics* can be found in Table 17 of Appendix 4. The organization of the table is the same as in Table 6.

From the Panel A it can be noticed that the findings for the *Amihud*-based regression analysis remain robust to inclusion of the additional size variable. However, Panel B only partly confirms the result of a significant influence of the aggregated liquidity risk on stock returns for *FHT*. From the first section it is seen that the net betas is priced whereas the third section does not reveal any significant influence of both market and aggregate liquidity risks on stock returns. Finally, Panel C reports the findings for the *PQS* illiquidity

Table 9. Fama-MacBeth Regression Results for Illiquidity Portfolios Controlled for the Size Effect

	α	*E(c)*	*β¹*	*β²*	*β³*	*β⁴*	*β^net*	*β^illiq*	*ln(MC)*
Panel A: Amihud									
1	0.005						-0.001		0.000
	0.014*	-0.056					-0.004		0.000
2	0.014***		-0.009						0.000
3	0.006		0.000					-0.007	0.000
	0.011	-0.194	-0.007					0.042*	0.000
	0.006		-0.002					0.017	0.000
4	0.004		0.007	0.015	0.116	0.009			0.000
	0.012*	-0.265*	0.000	0.364*	0.126	0.000			0.000
	0.007		0.004	0.122	0.102	0.010			0.000
Panel B: FHT									
1	-0.001						0.002		0.001
	-0.013	0.030					0.014**		0.001
2	0.012*		-0.011						0.000
3	-0.006		0.012					0.002	0.001
	-0.008	0.039	0.007					0.012	0.001
4	-0.007		0.038**	0.007	0.095*	0.015			0.000
	-0.012	-0.051	0.037**	0.024	0.109*	0.000			0.001
	-0.009		0.040**	0.019	0.122**	0.005			0.001
Panel C: PQS									
1	0.009						-0.006**		0.001
	0.002	-0.010					0.003		0.000
2	0.007		-0.001						0.000
3	0.002		0.004					-0.005**	0.000
	0.009	-0.050	-0.004					-0.004	0.000
4	0.000		0.007	-0.005	0.102*	0.006			0.001
	0.000	-0.115	0.006	-0.009	0.067	-0.009			0.001

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly, where the additional variable, $ln(MC)^p$, is added. It is calculated as the natural log of the average market capitalization (MC) across all the stocks included in portfolio *p* at the beginning of the month

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, and ***** indicates significance of a parameter at 10% level

Note also that when *E(c)* is not specified in the table it is not treated as an independent variable (see Section 3.2 for details), thus implying that illiquidity level, *E(c)*, does affect stock returns. However, in the second section of the table variable *E(c)* is not used at all

measure that are similar to those obtained without controlling for a size effect. It is worth to mention that the size variable, $\ln(MC_t^p)$, itself is not priced in any of the regression model specifications. Therefore, French stock returns appear not to have a size premium.

Table 10 further reports the results for the illiquidity-variation portfolios (see the full table with R^2 and *t-statistics* in Table 18 of Appendix 4). The findings are generally similar to those obtained from the original illiquidity-variation portfolios. Panel A shows the identical results for the *Amihud*-based regression. Panel B instead indicates different economic

Table 10. Fama-MacBeth Regression Results for Illiquidity-Variation Portfolios Controlled for the Size Effect

	α	$E(c)$	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	$ln(MC)$
Panel A: Amihud									
1	0.001						0.005		0.000
	0.005	0.035					0.003		0.000
2	0.013**		-0.006						0.000
3	-0.003		0.006					0.015	0.000
	-0.001	0.003	0.004					0.040*	0.000
4	-0.003		0.008	0.006	0.027	-0.018			0.000
	-0.001	0.018	0.007	0.025	0.028	-0.056			0.000
Panel B: FHT									
1	-0.008						0.005		0.001**
	-0.014	0.020					0.012*		0.001**
2	0.013*		-0.011						0.000
3	-0.009		0.008					0.004	0.001**
	-0.012	0.016	0.010					0.010	0.001**
4	-0.008		0.009	0.002	0.000	-0.005			0.001*
	-0.007	0.024	0.014	0.006	0.018	-0.003			0.001
Panel C: PQS									
1	-0.016**						0.016***		0.001*
	-0.009	-0.026					0.014**		0.000
	-0.008						0.014**		0.000
2	0.006		0.002						0.000
3	-0.021***		0.024**					0.014**	0.001
	-0.003	-0.011	0.003					0.017**	0.001
	-0.005		0.007					0.015**	0.015
4	-0.013		0.017	-0.003	0.021	-0.019**			0.001
	-0.002	-0.036	0.004	0.028*	-0.006	-0.015*			0.001
	-0.006		0.009	0.015	0.023	-0.016*			0.001

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly, where the additional variable, $ln(MC)^p$, is added. It is calculated as the natural log of the average market capitalization (MC) across all the stocks included in portfolio *p* at the beginning of the month

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, and ***** indicates significance of a parameter at 10% level

Note also that when *E(c)* is not specified in the table it is not treated as an independent variable (see Section 3.2 for details), thus implying that illiquidity level, *E(c)*, does affect stock returns. However, in the second section of the table variable *E(c)* is not used at all

Table 11. Fama-MacBeth Regression Results for Size Portfolios Controlled for the Size Effect

	α	*E(c)*	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	*ln(MC)*
Panel A: Amihud									
1	-0.017**						0.006		0.002***
	0.005	-0.001					0.004		0.000
2	0.008		-0.001						0.000
3	-0.010		0.010					-0.038**	0.001**
	0.003	-0.017	0.001					0.044**	0.001
	-0.004		0.005					0.033*	0.001
4	-0.017**		0.039***	0.221	0.633***	0.069*			0.001**
	-0.001	-0.021	0.032**	0.727***	0.589***	0.062			0.000
	-0.009		0.036***	0.558***	0.605***	0.050			0.001
Panel B: FHT									
1	-0.008						-0.002		0.002***
	-0.002	-0.007					0.009		0.000
2	0.008		-0.001						0.000
3	-0.014*		0.022*					-0.012	0.001*
	0.006	-0.018	0.000					0.018**	0.001
	-0.001		0.000					0.012*	0.001
4	-0.018**		0.020	-0.039***	0.066	-0.049***			0.002***
	0.002	-0.006	-0.002	0.027**	0.059	-0.035**			0.002**
	-0.004		0.006	0.002	0.072	-0.047***			0.002***
Panel C: PQS									
1	-0.032***						0.019***		0.003***
	-0.005	-0.002					0.011*		0.000
	-0.007						0.013**		0.000
2	0.008		-0.001						0.000
3	-0.038***		0.029***					0.018***	0.002***
	0.001	0.007	0.003					0.013**	0.000
4	-0.028***		0.024**	-0.006	0.075	-0.022***			0.002***
	-0.006	0.006	0.009	0.035**	-0.002	0.001			0.001

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly, where the additional variable, $ln(MC)^p$, is added. It is calculated as the natural log of the average market capitalization (MC) across all the stocks included in portfolio *p* at the beginning of the month

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, and ***** indicates significance of a parameter at 10% level

Note also that when *E(c)* is not specified in the table it is not treated as an independent variable (see Section 3.2 for details), thus implying that illiquidity level, *E(c)*, does affect stock returns. However, in the second section of the table variable *E(c)* is not used at all

results for *PQS*-based analysis. The first section shows both statistically and economically significant impact of the net beta on stock returns. At the same time *ln(MC)* appears significant in the most of regression specifications indicating that securities returns comprise a size premium. Panel C generally confirms the previous results for the same type of portfolios, although one regression specification also indicates the existence of a size premium in the French stock market.

Finally Table 11 presents the output for the size portfolios (see the full table with R^2 and *t-statistics* in Table 19 of Appendix 4). Here the results generally contradict the previous findings. At the same time *ln(MC)* is priced in half of the cases thus undermining the earlier results. From the third section of Panel A it can be seen that the aggregate liquidity risk becomes priced for the *Amihud*-based regressions. Section 3 of Panel B reaches the same conclusion for *FHT*-based analysis. Only the findings for *PQS* illiquidity measure reported in Panel C remains generally the same although indicating that the size effect also significantly affects stock returns. Therefore the last results provide some evidence for existence of the size premium in the French stock market.

In summary, the size effect found no confirmation for illiquidity portfolios, and was confirmed only for *FHT* measure for illiquidity-variation portfolios. However, the size portfolios reported significant size premium in approximately half of the regression specifications used. At the same time the previous findings for *PQS*-based analysis remained generally the same after controlling for the size effect. For *FHT* results were mostly contradicting to the earlier findings, whereas for *Amihud* they remain the same for two portfolios out of three that may be considered as an indication of the robustness with some reservations.

5.4.3 Specification Test

This study generally uses four different regression specifications according to equations (27) - (30). However, other variations of regressions also arise after treating investor average holding period, *k*, as a fixed, non-constant or zero variable. Thus, one has to test each specification for consistency.

In order to check the regression specifications for consistency this study applies the Hausman test. Hausman test allows to identify which type of regression model is preferred: fixed effects or random effects. The crucial difference between these models is that fixed affects approach assumes that unobserved individual effects of a panel regression embody elements that are correlated with the regressors (Greene, 2003, p. 285). Fixed effects model is more appropriate when the constituents of the sample represent the entire population as in our case when the dataset consists of all the stocks from Paris Euronext (Brooks, 2008, p. 500). More importantly the parameters of a random effects model cannot be efficiently estimated with OLS that is employed for running Fama-MacBeth regressions. Therefore, only those LCAPM specifications implying fixed effects model application should be further economically interpreted.

Table 12 provides the results of the Hausman test for according regression specification. The findings are the same for all three illiquidity measures. Hausman test is implemented only on illiquidity portfolios, but the results for other portfolios should remain the same. It can be seen that only the regression models that treat k as a free parameter are robust enough to be economically interpreted. Therefore, the findings obtained from using other variations of the LCAPM should be neglected because the estimation of the coefficients is not efficient. Consequently, the economic interpretation of the results provided in Section 6 focuses on conclusions drawn from regression models where k is non-constant.

Moreover, the regressions run by the author imply existence of an intercept, α, while the original LCAPM assumed no intercept. However, from Tables 6-11 it can be seen that α is sometimes significant. For the main findings based on illiquidity portfolios roughly one third of the regressions have a significant intercept. For the LCAPMs of the robustness check section 19% of the models appear to have significant α. When only cases that passed the Hausman test are considered, only 9% of them overall violated the LCAPM assumption. Nevertheless, the assumption of insignificant intercept should not be considered very strict.

In conclusion, after all the robustness checks the initial results obtained from the illiquidity portfolios remain strong only for the LCAPMs using investor average holding period, k,

Table 12. Results of the Hausman Test for Different Regression Specifications

Regression Model	***k* = 0.059**	**non-constant *k***	***k* = 0**
$E(r) = \alpha + kE(c) + \lambda^{net}\beta^{net}$	RE	FE	RE
$E(r) = \alpha + \lambda^{1}\beta^{1}$	-	-	RE*
$E(r) = \alpha + kE(c) + \lambda^{1}\beta^{1} + \lambda^{illiq}\beta^{illiq}$	RE	FE	RE
$E(r) = \alpha + kE(c) + \lambda^{1}\beta^{1} + \lambda^{2}\beta^{2} + \lambda^{3}\beta^{3} + \lambda^{4}\beta^{4}$	RE	FE	RE

* For this model (standard CAPM) there is actually no term $kE(c)$ at all

RE stands for random effects model, whereas FE – for fixed effects model

Note also that when k is fixed and equals *0.059*, $E(c)$ is not treated as an independent variable (see Section 3.2 for details), thus implying that illiquidity level, $E(c)$, does affect stock returns unfixed. The results for *Amihud* measure remain quite strong and indicate that illiquidity level and risk are not priced in the French stock market after considering different portfolio sortings and controlling for the size effect. On the other hand, *FHT*-based analysis shows that liquidity level does not affect stock returns, while the aggregate liquidity risk does. However, this result is not robust after considering different portfolios and including a size variable into consideration. At the same time, *PQS*-based regressions indicate that both illiquidity level and risks do not matter for stock returns. But this finding remains robust only with regard to liquidity level after considering other portfolios and controlling for size. Finally, we have found little evidence of pricing of the market risk and existence of the size premium in the French equity market.

6 RESULTS DISCUSSION

This section describes the economic interpretation of the results. It also discusses why the obtained findings differ across liquidity measures. Finally, it emphasizes the limitations of the LCAPM that could have driven such results.

To begin with, the contradicting findings based on different liquidity measures do not allow us to make a unified conclusion about the French equity market. However, it could be stated that according to the obtained results there is more evidence that liquidity level and risks are not priced on Paris Euronext. Market premium was not found to be significant as well. Nevertheless, it is possible to consider conclusions from different liquidity proxies separately because they could provide different insights to the problem.

According to the *Amihud*-based analysis both liquidity level and aggregate liquidity risk do not affect stock returns. It should be remembered that *Amihud* is a cost-per-volume proxy that captures the price response to €1,000 trading volume. So, this measure is more concerned about market resiliency compared to other two liquidity dimensions (see Section 2.1 for details). Hence, if we look at liquidity from the perspective of *Amihud* we can say that the stock returns are not significantly affected by the risks associated with price response changes. This conclusion should be more of an interest of large institutional investors who execute large transactions and would rather use cost-per-volume measures to estimate liquidity. It should also be mentioned that the conclusions for *Amihud* are robust both for different portfolio sortings and size effect.

When small private investors are considered they should prefer to use spread measures because their transactions do not impact the price of a stock. In this case they would rather rely on results of *PQS* because it is not only a proxy for *PES*, but is also based on closing bid-ask spreads. Therefore, this measure is associated more with the liquidity of small transactions, i.e. giving a low weight to market resiliency if we talk about different liquidity dimensions. The findings from *PQS*-based analysis indicate that both liquidity level and risks do not influence on stock returns. However, this result is robust only with regard to liquidity level when other portfolios and size effect are taken into consideration.

Finally, *FHT* could be seen as a measure treating all three liquidity dimension equally, and thus representing medium-sized transactions. The logic behind this proxy indicates that investors would not trade until anticipated returns of a stock exceed transaction costs. The larger the transaction the less expenses per share traded an investor has. Thus, it is more likely that a larger investor would execute a deal compared to a small private investor. At the same time, the largest investors also care about price impact of their trade that could prevent them to get involved into a transaction, leaving the way to the medium-sized investors. The findings of *FHT*-based analysis show that the aggregate liquidity risk significantly affects stock return, while the results for liquidity level do not always remain economically valid. The author computes the liquidity premium of different types of liquidity risks to give a perspective on the magnitude of the effect. The premiums are computed according to the results employing β^{net} in order to avoid the multicollinearity problem. The difference in annualized expected returns between the most and the least liquid portfolios attributable to commonality in liquidity (covariance between individual portfolio's and marketwide illiquidities) or β^2 is:

$$\lambda(\beta^{2,p25} - \beta^{2,p1})12 = 0.012(0.176 - 0.001)12 \approx 0.025\%. \quad (34)$$

Hence, holding the most illiquid portfolio of French stocks gives additional 0.025% return compared to the most liquid portfolio. On the other hand, annualized return difference between the same portfolios due to systematic liquidity risk (covariance between individual portfolio's return and market liquidity) or β^3 is:

$$-\lambda(\beta^{3,p25} – \beta^{3,p1})12 = -0.012(-0.041 - 0.014)12 \approx -0.004\%. \quad (35)$$

Finally, there is also a difference in annualized expected returns attributable to β^4 (covariance between individual portfolio's illiquidity and market return) between the most and the least liquid portfolios:

$$-\lambda(\beta^{3,p25} – \beta^{3,p1})12 = -0.012(-0.003 - 0.168)12 \approx 0.024\%. \quad (36)$$

So, the total annual liquidity risk premium for holding the most illiquid portfolio is 0.045%. It is more than 20 times lower compared to 1.1% premium discovered for the US stock market (Acharya and Pedersen, 2005) and more than 30 times lower compared to the 1.53% worldwide (Lee, 2011). However, the effect of liquidity risk on stock returns for *FHT* is not confirmed after analyzing other portfolio sortings and controlling for the potential size effect. Thus, this conclusion is weak and should be considered with caution.

Because different liquidity measures produce different conclusions that are often not quite robust the unified conclusion with regard to the French equity market could not be drawn. At the same time different proxies used capture different aspects of liquidity. It means that stock returns could be sensitive or indifferent to various liquidity dimensions, namely market depth, breadth, and resiliency. The latter term seems more measurable compared to others two through cost-per-volume price impact proxies such as *Amihud*. Therefore, the *Amihud*-based robust findings may indicate that the market resiliency and its changes are relatively unimportant and do not significantly influence on securities returns. In contrast *FHT* and *PQS* give a lower weight to resiliency. Thus, their contradicting results could provide some foundation for the hypothesis that one of the two leftover liquidity dimensions – market depth and breadth – significantly affects stock returns. Perhaps, if one other liquidity dimension could be eliminated (or treated with the less weight) inside a liquidity measure it would be possible to identify which dimension is priced. The problem is that while one can give resiliency less importance by employing percent-cost measures such as *FHT* or *PQS* it is quite problematic to configure the weight of market depth and breadth inside a liquidity measure. The author supposes that one of these two liquidity aspects could be priced in the French stock market. Because *Amihud* gives resiliency more weight the effects of other liquidity dimensions on stock returns are ignored, and consequently, we obtain no evidence of pricing of aggregate liquidity risk. Nevertheless, it would be interesting to differentiate the effect on stock returns attributable to the market depth or breadth. However, to the knowledge of the author there are no measures that aim at estimating only one dimension of liquidity. Thus, the future research may concentrate on this issue to distinguish between the impacts of different liquidity aspects on stock returns.

To develop this discussion the author would like to share his view on this problem. The initial proposition is that three liquidity dimensions affect stock returns differently. If all three dimensions are separated then which of them could be more important for an investor? Perhaps, one would first look at the order flow of a particular share: big quantities for sale and to buy indicate that there is an interest from market participants to this share and thus it should be liquid. If an investor is willing to engage in trading with this stock then she would start to worry whether there are some market makers that have a lot of power to influence on the stock price or whether her trade is large enough to affect the price. Therefore, an investor’s first checkpoint to be engaged in transactions with a particular stock is the amount of order flow that is captured by the market depth. If it is deep enough then she starts to consider other issues that are more related to a particular trade she is going to implement, namely whether her trade volume is so big that it could affect the stock price (market resiliency) and whether there are several market participants that could manipulate the price against her trade (market breadth). Hence, the market depth should be of the first importance for potential investors. Consequently the author supposes that this liquidity dimension should be more influential with regard to stock returns compared to other dimensions. It would be interesting to empirically test this hypothesis on different markets. However, the liquidity measures should be picked up in such a way that each one is representing only one dimension of liquidity which seems to be a challenging task.

Except the explanation of the results that was discussed above, it is worth to notice the limitations of the LCAPM. First, it puts a selling constraint and considers illiquidity parameter of the model as a cost of selling. In this respect, *FHT* and *PQS* should better fit the model because they are percent-cost measures that estimate a trading cost as a percent of the stock price, while *Amihud* captures a price impact associated with a certain volume of a transaction. It is also worth to mention here that illiquidity innovations produced from *FHT* and *PQS* are more consistent with the liquidity leakage events compared to *Amihud*. It might indicate that the later measure is worse than the other two. Moreover, the full LCAPM as in equation (30) is difficult to estimate effectively because of the collinearity between the betas used in the model. Thus, although different liquidity risks are separated from each other the multicollinearity problem forces to employ a restriction of equal premiums, $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$, when running regressions. Finally, this study uses the unconditional LCAPM version that assumes constant premiums that might differ from the reality.

Furthermore, there could be a difference between the downside and upside liquidity risks. Investors in general view positive and negative shocks differently (e.g. Kahneman and Tversky, 1979). In particular the risk is often perceived as the probability of loss, while the gain side of the risk is not taken into account. Moreover, Pástor and Strambaugh (2003) found asymmetry in co-movements of the asset liquidity and returns: they reported 0.5 correlation between stock liquidity and market return in negative-return months and near zero correlation in positive-return months. In addition there is a recent study by Anthonisz and Putniņš (2014) that found that the return premium associated with downside liquidity risk is six to eight times larger than suggested by symmetric liquidity models. Therefore, the contradicting results of this study may also be driven by this problem. It would be interesting to test the Liquidity-Adjusted Downside Capital Asset Pricing Model (LD-CAPM) proposed by Anthonisz and Putniņš (2014) on other less liquid markets than the US such as the French equity market.

7 CONCLUSIONS

This study utilizes the LCAPM developed by Acharya and Pedersen (2005) to test whether liquidity level and risks are priced in the French equity market. Three different liquidity measures were incorporated one by one into the model in order to see whether they could provide significantly different results. Two of these measures – *FHT* and *PQS* – were employed within the LCAPM for the first time.

This paper analyzed 835 French common stocks from December 1995 to March 2014 and found little evidence of significant influence of both liquidity level and risks on stock returns. The market risk was also generally found to be insignificant in this respect. In addition, the size effect was mostly found to be unimportant with regard to stock returns. However, the findings were often not robust after considering different portfolio sortings, controlling for size, and specification tests and thus should be used with caution.

It was found that results of the analysis largely depend on liquidity measure incorporated in the LCAPM. The Amihud-based findings appeared quite robust and showed that neither liquidity level nor liquidity risks significantly affect stock returns. On the other hand, *FHT*-based results indicate that the aggregate liquidity risk influence on securities returns, although these findings were not robust. Finally, *PQS* presented economically contradicting results in the main analysis that was found to be weak after running several robustness tests.

Nevertheless, the variability in results from different liquidity measures is able to provide a new insight to the existing literature dedicated to asset pricing with liquidity. The robust *Amihud*-based findings may indicate that one of liquidity dimensions – market resilience – is not priced in the French equity market. Moreover, the contradicting results of *FHT* and *PQS* provide some foundation for the hypothesis that one of the two leftover liquidity dimensions – market depth and breadth – significantly affects stock returns. However, unavailability of liquidity measure capturing only one of these two liquidity aspects makes it difficult to test this hypothesis. Nonetheless, the findings suggest a new interesting conjecture that different liquidity dimensions have different impacts on stock returns.

The findings of the study could be also affected by the LCAPM limitations. First, the short selling constraint is not realistic. Moreover, liquidity premium, λ, is treated as constant whereas it may vary over time. Finally, the LCAPM implies symmetry of liquidity in bad and good times. This assumption might cause underestimation of the impact of liquidity risks on stock returns as was recently documented by Anthonisz and Putniņš (2014).

This study calls for future research to create the new liquidity measures that would be able to represent each of the liquidity dimensions separately. It can help to understand the difference between impacts of different liquidity dimensions on securities returns and test.

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Appendix 1. Software and Computer Characteristics

This appendix provides information on software that was used for implementation of the study and parameters of the computer where this software was run. The software is also available for Mac and Linux OS. Note that although the programming scripts that were used in this study do no require better hardware characteristics to be implemented, the worse hardware parameters might be problematic for loading all the data into R.

Hardware:	CPU type: *Intel Core i3* CPU speed: *3120M (2.5GHz)* RAM size: *4GB* RAM speed: *DDR3 1600*
Operating System:	*Windows 8.1 64-Bit*
Software:	*RStudio Desktop version 0.98.953* *R version 3.1.0 (2014-04-10)*
Required R Packages*:	General usage: *'xlsx', 'zoo', 'reshape2', 'ggplot2'* Portfolios formation: *'plyr'* Fama-MacBeth regression + Specification test: *'plm', 'lmtest'* Descriptive statistics: *'pastecs'* Stationarity test: *'tseries'*

* Other R packages used in this study are the base packages built in R version 3.1.0
More information about RStudio is available on the software's official website www.rstudio.com

Appendix 2. Descriptive Statistics on Individual Stock Level

This appendix provides descriptive statistics on all the stocks' variables – Amihud, FHT, and PQS illiquidities (adjusted for market capitalization) as well as returns. The variables represent stock-month observations from January 01, 1997 to March 31, 2014. All the values are provided in absolute terms

Table 13. Descriptive Statistics of Monthly Stock Observations, January 01, 1997 – March 31, 2014

	Amihud	FHT	PQS	Returns
Number of Valid Observations	85,231	87,458	82,657	83,338
Number of Null Observations	112	22,063	8	1,556
Number of Missing Observations	87,614	85,387	90,188	89,507
Minimum Value	0	0	0	-0.859
Maximum Value	4.172	20.909	16.828	3.781
Range	4.172	20.909	16.828	4.640
Median	0.007	0.130	0.093	0
Mean	0.046	0.218	0.210	0.005
Standard Error of the Mean	0.011	0.028	0.024	0.017
Mean Variance	0.010	0.100	0.068	0.018
Minimum Variance	0	0	0	0
Maximum Variance	0.743	40.317	7.742	0.286
Mean Standard Deviation	0.056	0.180	0.121	0.122
Minimum Standard Deviation	0	0	0.001	0
Maximum Standard Deviation	0.862	6.350	2.782	0.535
Mean Coefficient of Variation	1.504	1.036	0.644	4.365
Minimum Coefficient of Variation	0.075	0.082	0.014	-1,362.649
Maximum Coefficient of Variation	9.610	3.465	3.463	846.428

Appendix 3. Main Results of Fama-MacBeth Regressions

Table 14. Detailed Results of Fama-MacBeth Regressions for Illiquidity Portfolios

	α	*E(c)*	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	R^2
Panel A: Amihud									
1	0.003	*0.059*					0.003		66.9%
	(-0.50)	*(-)*					*(-0.34)*		
	0.015*	-0.041					-0.007		73.7%
	(-1.92)	*(-0.81)*					*(-0.77)*		
2	0.015***		-0.008						69.0%
	(-3.04)		*(-1.21)*						
3	0.006	*0.059*	0.001					-0.005	73.9%
	(-0.73)	*(-)*	*(-0.11)*					*(-0.22)*	
	0.010	-0.201	-0.006					0.047**	78.1%
	(-1.45)	*(-1.37)*	*(-0.71)*					*(-2.12)*	
	0.006		-0.001					0.019	73.8%
	(-0.77)		*(-0.07)*					*(-0.95)*	
4	0.004	*0.059*	0.010	0.038	0.142	0.011			80.0%
	(-0.57)	*(-)*	*(-0.74)*	*(-0.33)*	*(-0.69)*	*(-0.42)*			
	0.011*	-0.263*	0.003	0.377*	0.166	-0.003			82.9%
	(-1.71)	*(-1.84)*	*(-0.28)*	*(-1.94)*	*(-0.91)*	*(-0.08)*			
	0.006		0.006	0.144	0.128	0.011			80.0%
	(-0.98)		*(-0.50)*	*(-1.28)*	*(-0.63)*	*(-0.43)*			
Panel B: FHT									
1	0.039***	*0.059*					-0.041***		73.5%
	(-4.76)	*(-)*					*(-6.54)*		
	-0.004	-0.015					0.012*		76.8%
	(-0.57)	*(-0.90)*					*(1.90)*		
	-0.003						0.010*		73.3%
	(-0.38)						*(1.73)*		
2	0.011*		-0.006						76.2%
	(-1.65)		*(-0.46)*						
3	-0.012	*0.059*	0.031**					-0.023***	78.1%
	(-1.41)	*(-)*	*(-2.30)*					*(-4.42)*	
	0.003	-0.024	0.001					0.019*	79.4%
	(-0.38)	*(-1.46)*	*(-0.05)*					*(-2.48)*	
	-0.004		0.012					0.011**	77.7%
	(-0.49)		*(-0.90)*					*(-2.17)*	
4	-0.015*	*0.059*	0.054***	-0.022	0.115**	0.030			81.7%
	(-1.83)	*(-)*	*(-2.97)*	*(-1.63)*	*(-2.06)*	*(-1.60)*			
	0.002	-0.041**	0.031	0.054***	0.111*	0.005			82.8%
	(-0.21)	*(-2.52)*	*(-1.63)*	*(-3.17)*	*(-1.95)*	*(-0.23)*			
	-0.006		0.042**	0.023*	0.116**	0.015			81.4%
	(-0.73)		*(-2.31)*	*(-1.75)*	*(-2.11)*	*(-0.83)*			
Panel C: PQS									
1	0.048***	*0.059*					-0.046***		71.7%
	(-8.17)	*(-)*					*(-12.35)*		
	0.015**	-0.013					-0.008		72.8%
	(-2.32)	*(-0.60)*					*(-1.21)*		
2	0.009*		-0.002						71.5%
	(-1.67)		*(-0.26)*						
3	0.012	*0.059*	-0.004					-0.038***	75.5%
	(-1.60)	*(-)*	*(-0.39)*					*(-10.54)*	
	0.011	-0.019	-0.003					0.001	75.5%
	(-1.38)	*(-0.70)*	*(-0.34)*					*(-0.07)*	
4	0.010	*0.059*	-0.003	-0.042***	0.022	0.033*			80.3%
	(-1.48)	*(-)*	*(-0.31)*	*(-2.64)*	*(-0.37)*	*(-1.71)*			
	0.007	-0.045	0.003	0.013	0.033	-0.003			80.6%
	(-0.96)	*(-1.29)*	*(-0.36)*	*(-0.40)*	*(-0.52)*	*(-0.13)*			

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, ***** indicates significance of a parameter at 10% level

Values in brackets indicate *t-statistics* of the abovementioned figures

Note also that when *k = 0.059*, *E(c)* is not treated as an independent variable, thus it does not have a *t-statistic* in brackets below

Table 15. Detailed Results of Fama-MacBeth Regressions for Illiquidity-Variation Portfolios

	α	E(c)	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	R^2
Panel A: Amihud									
1	0.001	*0.059*					0.005		70.1%
	(-0.20)	*(-)*					*(-0.57)*		
	0.005	0.032					0.002		74.2%
	(-0.76)	*(-0.71)*					*(-0.20)*		
2	0.014***		-0.007						70.8%
	(-2.92)		*(-1.05)*						
3	-0.002	*0.059*	0.008					0.015	73.1%
	(-0.25)	*(-)*	*(-0.94)*					*(-0.92)*	
	-0.001	0.001	0.005					0.037*	77.0%
	(-0.12)	*(-0.02)*	*(-0.70)*					*(-1.70)*	
4	-0.002	*0.059*	0.009	0.043	0.008	-0.010			79.2%
	(-0.33)	*(-)*	*(-0.84)*	*(-0.33)*	*(-0.05)*	*(-0.32)*			
	0.001	-0.003	0.007	0.051	0.029	-0.046			81.6%
	(-0.14)	*(-0.03)*	*(-0.66)*	*(-0.17)*	*(-0.20)*	*(-1.05)*			
Panel B: FHT									
1	0.035***	*0.059*					-0.036***		77.6%
	(-4.67)	*(-)*					*(-5.82)*		
	0.004	-0.003					0.003		79.1%
	(-0.54)	*(-0.20)*					*(-0.44)*		
2	0.012*		-0.007						75.2%
	(-1.79)		*(-0.60)*						
3	0.004	*0.059*	0.010					-0.033***	79.4%
	(-0.47)	*(-)*	*(-0.75)*					*(-5.36)*	
	0.004	0.001	0.003					0.003	80.7%
	(-0.45)	*(-0.09)*	*(-0.23)*					*(-0.33)*	
4	-0.001	*0.059*	0.010	-0.025**	-0.005	0.037**			82.7%
	(-0.08)	*(-)*	*(-0.72)*	*(-2.35)*	*(-0.11)*	*(-2.38)*			
	0.002	-0.009	0.012	0.028*	0.022	0.017			83.7%
	(-0.30)	*(-0.59)*	*(-0.85)*	*(-1.73)*	*(-0.56)*	*(-1.08)*			
	0.002		0.013	0.023**	0.026	0.022			77.4%
	(-0.31)		*(-0.90)*	*(-2.21)*	*(-0.64)*	*(-1.41)*			
Panel C: PQS									
1	-0.014**	*0.059*					0.013**		69.3%
	(-2.11)	*(-)*					*(-2.30)*		
	-0.005	-0.024					0.014**		73.5%
	(-0.78)	*(-1.34)*					*(-2.37)*		
	-0.005						0.013**		69.9%
	(-0.80)						*(-2.24)*		
2	0.008		0.000						70.8%
	(-1.14)		*(-0.01)*						
3	-0.054***	*0.059*	0.065***					0.007	74.1%
	(-7.97)	*(-)*	*(-7.21)*					*(-1.06)*	
	0.000	-0.022	0.006					0.015**	76.7%
	(-0.03)	*(-0.97)*	*(-0.54)*					*(-2.37)*	
	-0.003		0.010					0.013**	74.3%
	(-0.44)		*(-1.07)*					*(-2.14)*	
4	-0.017**	*0.059*	0.015	-0.052***	-0.126*	-0.029***			78.3%
	(-2.21)	*(-)*	*(-1.28)*	*(-4.36)*	*(-1.87)*	*(-3.69)*			
	0.007	-0.036	0.001	0.020	0.004	-0.014*			80.3%
	(-0.77)	*(-1.29)*	*(-0.04)*	*(-1.32)*	*(-0.06)*	*(-1.84)*			
	-0.001		0.006	0.011	-0.030	-0.014*			78.3%
	(-0.14)		*(-0.53)*	*(-0.96)*	*(-0.44)*	*(-1.84)*			

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, ***** indicates significance of a parameter at 10% level

Values in brackets indicate *t-statistics* of the abovementioned figures

Note also that when *k = 0.059*, *E(c)* is not treated as an independent variable, thus it does not have a *t-statistic* in brackets below

Table 16. Detailed Results of Fama-MacBeth Regressions for Size Portfolios

	α	$E(c)$	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	R^2
Panel A: Amihud									
1	0.000	*0.059*					0.007		73.3%
	(-0.02)	*(-)*					(-0.84)		
	-0.002	-0.044					0.010		77.0%
	(-0.25)	*(-0.77)*					*(-1.19)*		
2	0.008		0.001						74.8%
	(-1.01)		*(-0.07)*						
3	0.000	*0.059*	0.009					-0.008	77.5%
	(-0.02)	*(-)*	(-1.02)					(-0.51)	
	0.000	-0.165*	0.007					0.031	79.1%
	(-0.00)	*(-1.85)*	*(-0.79)*					*(-1.43)*	
4	-0.006	*0.059*	0.035***	0.502***	0.549***	0.084**			80.5%
	(-0.77)	*(-)*	(-2.77)	(-2.58)	(-2.77)	(-2.24)			
	-0.007	-0.270***	0.037***	0.845***	0.604***	0.060			82.0%
	(-0.88)	(-2.60)	(-3.00)	(-3.94)	(-3.14)	(-1.55)			
	-0.005		0.035***	0.636***	0.584***	0.081**			80.4%
	(-0.70)		(-2.83)	(-3.26)	(-2.96)	(-2.17)			
Panel B: FHT									
1	0.032***	*0.059*					-0.032***		75.4%
	(-4.52)	*(-)*					*(-5.48)*		
	0.002	-0.039*					0.008		78.0%
	(-0.35)	*(-1.83)*					*(-1.19)*		
2	0.008		0.001						74.8%
	(-1.01)		*(-0.07)*						
3	-0.006	*0.059*	0.023**					-0.029***	78.2%
	(-0.80)	*(-)*	*(-2.03)*					*(-5.15)*	
	0.007	-0.036*	0.000					0.012	79.6%
	(-1.00)	*(-1.93)*	*(-0.01)*					*(-1.55)*	
4	-0.005	*0.059*	0.020	-0.045***	0.021	0.010			81.9%
	(-0.64)	*(-)*	*(-1.55)*	*(-4.20)*	*(-0.46)*	*(-0.76)*			
	0.009	-0.036	0.005	0.008	0.043	-0.027			83.2%
	(-1.21)	*(-1.60)*	*(-0.42)*	*(-0.68)*	*(-0.92)*	*(-1.59)*			
Panel C: PQS									
1	-0.004	*0.059*					0.005		73.3%
	(-0.54)	*(-)*					*(-0.83)*		
	-0.003	-0.029					0.012**		77.4%
	(-0.44)	*(-1.31)*					*(-2.10)*		
	-0.004						0.012**		74.1%
	(-0.53)						*(-2.03)*		
2	0.008		0.001						74.8%
	(-1.01)		*(-0.07)*						
3	-0.041***	*0.059*	0.050***					0.005	76.6%
	(-5.15)	*(-)*	*(-5.01)*					*(-0.76)*	
	0.003	-0.021	0.006					0.013**	79.7%
	(-0.37)	*(-0.87)*	*(-0.58)*					*(-2.16)*	
	-0.004		0.012					0.012**	77.3%
	(-0.53)		*(-1.26)*					*(-2.02)*	
4	-0.021***	*0.059*	0.026**	-0.039***	-0.048	-0.029***			80.8%
	(-2.61)	*(-)*	*(-2.55)*	*(-2.98)*	*(-0.83)*	*(-3.68)*			
	-0.002	-0.026	0.011	0.032**	-0.018	-0.001			83.3%
	(-0.23)	*(-0.81)*	*(-1.03)*	*(-2.00)*	*(-0.30)*	*(-0.15)*			
	-0.010		0.017*	0.025*	-0.043	-0.007			81.2%
	(-1.26)		*(-1.69)*	*(-1.94)*	*(-0.74)*	*(-1.06)*			

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, ***** indicates significance of a parameter at 10% level

Values in brackets indicate *t-statistics* of the abovementioned figures

Note also that when $k = 0.059$, $E(c)$ is not treated as an independent variable, thus it does not have a *t-statistic* in brackets below

Appendix 4. Results of Fama-MacBeth Regressions Controlled for the Size Effect

Table 17. Detailed Results of Fama-MacBeth Regressions for Illiquidity Portfolios Controlled for the Size Effect

	α	$E(c)$	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	$ln(MC)$	R^2
Panel A: Amihud										
1	0.005	0.059					-0.001		0.000	71.3%
	(0.76)	(-)					(-0.10)		(0.67)	
	0.014*	-0.056					-0.004		0.000	77.6%
	(1.82)	(-0.97)					(-0.45)		(-0.47)	
2	0.014***		-0.009						0.000	71.7%
	(2.99)		(-1.27)						(0.34)	
3	0.006	0.059	0.000					-0.007	0.000	76.5%
	(0.74)	(-)	(-0.04)					(-0.35)	(0.32)	
	0.011	-0.194	-0.007					0.042*	0.000	80.6%
	(1.46)	(-1.36)	(-0.91)					(1.91)	(0.38)	
	0.006		-0.002					0.017	0.000	76.5%
	(0.77)		(-0.29)					(0.80)	(0.45)	
4	0.004	0.059	0.007	0.015	0.116	0.009			0.000	82.3%
	(0.63)	(-)	(0.50)	(0.14)	(0.57)	(0.37)			(0.31)	
	0.012*	-0.265*	0.000	0.364*	0.126	0.000			0.000	85.2%
	(1.65)	(-1.76)	(0.02)	(1.85)	(0.70)	(-0.01)			(0.20)	
	0.007		0.004	0.122	0.102	0.010			0.000	82.2%
	(1.02)		(0.28)	(1.10)	(0.50)	(0.38)			(0.27)	
Panel B: FHT										
1	-0.001	0.059					0.002		0.001	78.0%
	(-0.14)	(-)					(0.28)		(1.39)	
	-0.013	0.030					0.014**		0.001	79.5%
	(-1.58)	(0.98)					(2.32)		(1.47)	
2	0.012*		-0.011						0.000	78.2%
	(1.95)		(-0.93)						(0.75)	
3	-0.006	0.059	0.012					0.002	0.001	80.2%
	(-0.64)	(-)	(0.93)					(0.29)	(0.76)	
	-0.008	0.039	0.007					0.012	0.001	81.5%
	(-0.93)	(0.83)	(0.49)					(1.43)	(1.30)	
4	-0.007	0.059	0.038**	0.007	0.095*	0.015			0.000	83.1%
	(-0.74)	(-)	(2.08)	(0.52)	(1.70)	(0.77)			(0.29)	
	-0.012	-0.051	0.037**	0.024	0.109*	0.000			0.001	84.1%
	(-1.27)	(-0.79)	(1.99)	(1.47)	(1.92)	(0.02)			(1.26)	
	-0.009		0.040**	0.019	0.122**	0.005			0.001	83.0%
	(-0.99)		(2.20)	(1.53)	(2.19)	(0.26)			(0.94)	
Panel C: PQS										
1	0.009	0.059					-0.006**		0.001	74.4%
	(1.36)	(-)					(-2.45)		(1.18)	
	0.002	-0.010					0.003		0.000	76.8%
	(0.29)	(-0.19)					(0.66)		(0.59)	
2	0.007		-0.001						0.000	0.7499
	(1.36)		(-0.13)						(0.30)	
3	0.002	0.059	0.004					-0.005**	0.000	77.0%
	(0.30)	(-)	(0.39)					(-2.10)	(0.57)	
	0.009	-0.050	-0.004					-0.004	0.000	78.5%
	(1.14)	(-0.48)	(-0.39)					(-0.62)	(0.42)	
4	0.000	0.059	0.007	-0.005	0.102*	0.006			0.001	80.5%
	(-0.03)	(-)	(0.75)	(-0.33)	(1.79)	(0.33)			(1.55)	
	0.000	-0.115	0.006	-0.009	0.067	-0.009			0.001	82.1%
	(-0.02)	(-1.11)	(0.64)	(-0.47)	(1.17)	(-0.32)			(1.24)	

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly, where the additional variable, $ln(MC)^p$, is added. It is calculated as the natural log of the average market capitalization (MC) across all the stocks included in portfolio *p* at the beginning of the month

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, ***** indicates significance of a parameter at 10% level

Values in brackets indicate *t-statistics* of the abovementioned figures

Note also that when *k = 0.059*, *E(c)* is not treated as an independent variable, thus it does not have a *t-statistic* in brackets below

Table 18. Detailed Results of Fama-MacBeth Regressions for Illiquidity-Variation Portfolios Controlled for the Size Effect

	α	E(c)	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	ln(MC)	R^2
Panel A: Amihud										
1	0.001	0.059					0.005		0.000	73.0%
	(0.12)	*(-)*					*(0.60)*		*(0.26)*	
	0.005	0.035					0.003		0.000	77.1%
	(0.69)	*(0.76)*					*(0.33)*		*(-0.11)*	
2	0.013**		-0.006						0.000	73.3%
	(2.74)		*(-0.91)*						*(0.03)*	
3	-0.003	0.059	0.006					0.015	0.000	75.5%
	(-0.32)	*(-)*	*(0.83)*					*(0.87)*	*(0.45)*	
	-0.001	0.003	0.004					0.040*	0.000	79.5%
	(-0.17)	*(0.05)*	*(0.55)*					*(1.82)*	*(0.40)*	
4	-0.003	0.059	0.008	0.006	0.027	-0.018			0.000	81.3%
	(-0.39)	*(-)*	*(0.75)*	*(0.04)*	*(0.18)*	*(-0.55)*			*(0.65)*	
	-0.001	0.018	0.007	0.025	0.028	-0.056			0.000	83.7%
	(-0.12)	*(0.15)*	*(0.67)*	*(0.08)*	*(0.19)*	*(-1.24)*			*(0.38)*	
Panel B: FHT										
1	-0.008	0.059					0.005		0.001**	79.4%
	(-0.84)	*(-)*					*(0.75)*		*(2.40)*	
	-0.014	0.020					0.012*		0.001**	81.0%
	(-1.55)	*(0.58)*					*(1.76)*		*(2.35)*	
2	0.013*		-0.011						0.000	79.3%
	(1.93)		*(-0.93)*						*(0.56)*	
3	-0.009	0.059	0.008					0.004	0.001**	81.1%
	(-0.90)	*(-)*	*(0.63)*					*(0.59)*	*(2.11)*	
	-0.012	0.016	0.010					0.010	0.001**	82.7%
	(-1.10)	*(0.37)*	*(0.68)*					*(1.32)*	*(2.13)*	
4	-0.008	0.590	0.009	0.002	0.000	-0.005			0.001*	83.9%
	(-0.89)	*(-)*	*(0.61)*	*(0.19)*	*(0.00)*	*(-0.27)*			*(1.83)*	
	-0.007	0.024	0.014	0.006	0.018	-0.003			0.001	85.2%
	(-0.74)	*(0.32)*	*(0.94)*	*(0.35)*	*(0.47)*	*(-0.15)*			*(1.39)*	
Panel C: PQS										
1	-0.016**	0.059					0.016***		0.001*	74.5%
	(-2.26)	*(-)*					*(2.82)*		*(1.70)*	
	-0.009	-0.026					0.014**		0.000	77.1%
	(-1.11)	*(-0.75)*					*(2.38)*		*(0.62)*	
	-0.008						0.014**		0.000	74.3%
	(-1.09)						*(2.44)*		*(0.39)*	
2	0.006		0.002						0.000	74.2%
	(0.96)		*(0.23)*						*(-0.11)*	
3	-0.021***	0.059	0.024**					0.014**	0.001	78.0%
	(-2.94)	*(-)*	*(2.54)*					*(2.01)*	*(0.97)*	
	-0.003	-0.011	0.003					0.017**	0.001	79.6%
	(-0.30)	*(-0.17)*	*(0.30)*					*(2.43)*	*(0.97)*	
	-0.005		0.007					0.015**	0.015	77.9%
	(-0.74)		*(0.77)*					*(2.23)*	*(0.81)*	
4	-0.013	0.059	0.017	-0.003	0.021	-0.019**			0.001	80.7%
	(-1.55)	*(-)*	*(1.38)*	*(-0.23)*	*(0.33)*	*(-2.36)*			*(1.20)*	
	-0.002	-0.036	0.004	0.028*	-0.006	-0.015*			0.001	82.1%
	(-0.26)	*(-0.49)*	*(0.29)*	*(1.78)*	*(-0.09)*	*(-1.86)*			*(0.91)*	
	-0.006		0.009	0.015	0.023	-0.016*			0.001	80.6%
	(-0.73)		*(0.76)*	*(1.30)*	*(0.36)*	*(-1.93)*			*(1.10)*	

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly, where the additional variable, *ln(MC)*^{p}, is added. It is calculated as the natural log of the average market capitalization (MC) across all the stocks included in portfolio *p* at the beginning of the month

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, ***** indicates significance of a parameter at 10% level

Values in brackets indicate *t-statistics* of the abovementioned figures

Note also that when *k = 0.059*, *E(c)* is not treated as an independent variable, thus it does not have a *t-statistic* in brackets below

Table 19. Detailed Results of Fama-MacBeth Regressions for Size Portfolios Controlled for the Size Effect

	α	E(c)	β^1	β^2	β^3	β^4	β^{net}	β^{illiq}	ln(MC)	R^2
Panel A: Amihud										
1	-0.017**	0.059					0.006		0.002***	77%
	(-2.12)	*(-)*					(0.77)		(4.86)	
	0.005	-0.001					0.004		0.000	79%
	(0.68)	*(-0.03)*					*(0.52)*		*(-0.29)*	
2	0.008		-0.001						0.000	78%
	(1.24)		*(-0.07)*						*(0.25)*	
3	-0.010	0.059	0.010					-0.038**	0.001**	80%
	(-1.27)	*(-)*	(1.15)					(-2.08)	(1.98)	
	0.003	-0.017	0.001					0.044**	0.001	81%
	(0.31)	*(-0.57)*	*(0.07)*					*(2.22)*	*(0.99)*	
	-0.004		0.005					0.033*	0.001	80%
	(-0.47)		*(0.57)*					*(1.94)*	*(1.41)*	
4	-0.017**	0.059	0.039***	0.221	0.633***	0.069*			0.001**	82%
	(-2.02)	*(-)*	(2.98)	(1.16)	(3.13)	(1.92)			(1.98)	
	-0.001	-0.021	0.032**	0.727***	0.589***	0.062			0.000	84%
	(-0.17)	(-0.60)	(2.41)	(3.50)	(2.83)	(1.62)			(0.32)	
	-0.009		0.036***	0.558***	0.605***	0.050			0.001	83%
	(-1.16)		(2.83)	(3.04)	(3.05)	(1.43)			(1.12)	
Panel B: FHT										
1	-0.008	0.059					-0.002		0.002***	78%
	(-0.92)	*(-)*					*(-0.39)*		*(3.88)*	
	-0.002	-0.007					0.009		0.000	79%
	(-0.19)	*(-0.25)*					*(1.48)*		*(0.42)*	
2	0.008		-0.001						0.000	78%
	(1.24)		*(-0.07)*						*(0.25)*	
3	-0.014*	0.059	0.022*					-0.012	0.001*	80%
	(-1.73)	*(-)*	*(1.82)*					*(-1.61)*	*(1.68)*	
	0.006	-0.018	0.000					0.018**	0.001	81%
	(0.73)	*(-0.58)*	*(-0.78)*					*(2.28)*	*(1.21)*	
	-0.001		0.000					0.012*	0.001	80%
	(-0.16)		*(0.03)*					*(1.69)*	*(1.51)*	
4	-0.018**	0.059	0.020	-0.039***	0.066	-0.049***			0.002***	83%
	(-2.12)	*(-)*	*(1.51)*	*(-3.24)*	*(1.38)*	*(-2.83)*			*(3.11)*	
	0.002	-0.006	-0.002	0.027**	0.059	-0.035**			0.002**	84%
	(0.24)	*(-0.11)*	*(-0.13)*	*(2.17)*	*(1.21)*	*(-2.01)*			*(2.24)*	
	-0.004		0.006	0.002	0.072	-0.047***			0.002***	83%
	(-0.50)		*(0.48)*	*(0.18)*	*(1.51)*	*(-2.77)*			*(2.61)*	
Panel C: PQS										
1	-0.032***	0.059					0.019***		0.003***	78%
	(-4.23)	*(-)*					*(3.16)*		*(5.43)*	
	-0.005	-0.002					0.011*		0.000	80%
	(-0.58)	*(-0.06)*					*(1.89)*		*(0.44)*	
	-0.007						0.013**		0.000	79%
	(-1.03)						*(2.38)*		*(0.70)*	
2	0.008		-0.001						0.000	75%
	(1.24)		*(-0.07)*						*(0.25)*	
3	-0.038***	0.059	0.029***					0.018***	0.002***	80%
	(-4.87)	*(-)*	*(2.99)*					*(2.70)*	*(4.47)*	
	0.001	0.007	0.003					0.013**	0.000	82%
	(0.12)	*(0.24)*	*(0.34)*					*(1.97)*	*(0.70)*	
4	-0.028***	0.059	0.024**	-0.006	0.075	-0.022***			0.002***	84%
	(-3.25)	*(-)*	*(2.36)*	*(-0.39)*	*(1.28)*	*(-2.86)*			*(3.72)*	
	-0.006	0.006	0.009	0.035**	-0.002	0.001			0.001	85%
	(-0.66)	*(0.16)*	*(0.90)*	*(2.13)*	*(-0.03)*	*(0.11)*			*(1.07)*	

The first column stands for the section number (1 - 4) corresponding to regression equations (27) - (30) accordingly, where the additional variable, $ln(MC)^p$, is added. It is calculated as the natural log of the average market capitalization (MC) across all the stocks included in portfolio *p* at the beginning of the month

******* indicates significance of a parameter at 1% level, ****** indicates significance of a parameter at 5% level, ***** indicates significance of a parameter at 10% level

Values in brackets indicate *t-statistics* of the abovementioned figures

Note also that when *k = 0.059*, *E(c)* is not treated as an independent variable, thus it does not have a *t-statistic* in brackets below