LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
Faculty of Technology
Degree Program in Technomathematics and Technical Physics

Anjum Muhammad

SIMULATING THE DYNAMICS OF SILVER MARKET USING COMPUTATIONAL MARKET DYNAMICS

Examiners: Docent Ph.D. Tuomo Kauranne
D.Sc. (Tech.) Matylda Jabłońska-Sabuka

ABSTRACT

Lappeenranta University of Technology

Faculty of Technology

Degree Program in Technomathematics and Technical Physics

Anjum Muhammad

Simulating the dynamics of silver markets
using computational market dynamics

Master's thesis

2015

36 pages, 13 figures, 2 tables

Examiners: docent Ph.D. Tuomo Kauranne
D.Sc. (Tech.) Matylda Jabłońska-Sabuka

Keywords: *Animal spirits*, *Silver prices*, *Market momentum*, *Ensemble modeling*, *Silver markets modeling*, *Population dynamics*.

Traditional econometric approaches in modeling the dynamics of equity and commodity markets, have, made great progress in the past decades. However, they assume rationality among the economic agents and and do not capture the dynamics that produce extreme events (black swans), due to deviation from the rationality assumption. The purpose of this study is to simulate the dynamics of *silver* markets by using the novel computational market dynamics approach. To this end, the daily data from the period of 1^{st} March 2000 to 1^{st} March 2013 of closing prices of spot silver prices has been simulated with the Jabłońska-Capasso-Morale(JCM) model. The Maximum Likelihood approach has been employed to calibrate the acquired data with JCM. Statistical analysis of the simulated series with respect to the actual one has been conducted to evaluate model performance. The model captures the *animal spirits* dynamics present in the data under evaluation well.

Acknowledgements

First of all many sincere gratitude to Almighty God. Indeed my entire life have been granted more than my struggle and efforts from the provisions and blessing of Almighty God. And the accomplishment of this thesis is a merely one more blessing out of many that I have already been bestowed upon.

Secondly my utmost thanks to my supervisor, **Prof. Tuomo Kauranne** for all his guidance, advice and support through out this thesis for making it a success. Also my sincere thanks to my second supervisor **D.Sc. Matylda Jabłońska-Sabuka** for her advice, comments, corrections, and reviews which lead to the successful accomplishment of this work.

Lastly I would like to thank my parents and family members, especially my late mother for igniting a passionate spark within me to acquire knowledge and education. Many sincere thanks to my father for my disciplined upbringing and teaching me to thrive against all odds. Also I would like to mention my litter brother and sister theirs innocent humors made my days cheerful in Finland.

In the end I would like to thank all friends and colleagues here in Finland for their kind support and friendship.

Lappeenranta, January, 2015.

Anjum Muhammad

Contents

List of Symbols and Abbreviations

IMF	International Monetary Fund
ETF	Exchange Traded Funds
JCM	Jabłońska-Capasso-Morale
OZ	Ounce
COMEX	Commodity Exchange Inc. And New York Mercantile Exchange
ECM	Error Correction Model
AR	Auto Regressive
GARCH	General Auto Regressive Conditional Hetroskedascity
GARCH-M	General Auto Regressive Conditional Hetroskedascity In Mean
E-GARCH	Exponential General Auto Regressive Conditional Hetroskedascity In Mean
VAR	Vector Auto Regressive
ARFIMA	Auto Regressive Fractionally Integrated Moving Average
FIGARCH	Fractionally Integrated General Auto Regressive Conditional Hetroskedascity
LBMA	London Bullion Market Association
MLP	Multi Layer Perception
HONN	Higher Order Neural Networks
LTL	Large Type Limit
RE	Real Expectation
DSGE	Dynamic Stochastic General Equilibrium
ADF	Augmented Dicky Fuller
PDF	Probability Density Function
ARIMA	Auto Regressive Integrated Moving Average
PACF	Partial Autocorelation
ACF	Autocorelation
SDE	Stochastic Differential Equation
USD	United States Dollar

1 Introduction

Silver and gold have been used for centuries as monetary asset and a store of value. The modern literature refers to them as commodities. Both of them have been considered as a mean of preserving wealth for ages. In some recent decades it has been established that during political and economic turmoil institutions and individuals invest their capital in these potential asset classes to protect their savings and livelihoods. Researchers in the present decade, while examining the volatility sensitivity of precious metals to the exchange rate volatility in the presence of monetary policy, came to the conclusion that silver tends to show greater sensitivity against exchange rate volatility. Also now there is some empirical evidence that gold is the safest heaven in flight from dollar to the safety of precious metals [1]. Reviewing the past decade it is clearly evident that commodities have received an unprecedented rise in prices out of many reasons, such as excessive demand of commodities and depreciating US dollar and high financial activity by institutional investors in the form of hedge funds, exchange traded funds, (ETFs). Regarding commodities, security markets appear to be the main reason for this price rise.

Recently, equity and commodity markets have been extensively volatile. Volatility simultaneously induces risk and opportunities which are examined by investors over time, while taking investments into considerations. Several reasons can be accounted for the price volatility in precious metals market segment, such as political unrest, production hindrances, supply disruptions, all of which are reported noticeably. Moreover, with the induction of modern financial innovations and selling and buying precious metals such as gold by international monetary fund (IMF), and central banks holdings of precious metals in different countries, and variation in demand and supply due to their industrial and electrical uses, all contribute to the price fluctuations in precious metals markets. Traders participate with varying expectations in order to maximize profits. Participants perform cross-market hedging across different asset classes, by evaluating information at different levels and speeds, and processing different inventory levels. These factors contribute to the high volatility and spiky behavior of commodities over time and across the markets [2].

Among precious metals silver has been historically used as money. The evidence of silver being used as medium of exchange relates back to twelfth century. During every financial crisis silver has held its position strong among commodities and other equities and has retained its value including the most recent global financial crisis of 2008. With global economies and currency markets still struggling to regain their pre-crisis value, and global debt accumulating year by year, investors and people are looking for safest alternatives for their store of value. Therefore, modeling and predicting the dynamics of silver market is quite interesting and challenging. The behavior of silver prices is not merely dependent on basic demand and supply, but its price heavily relies on psychological dynamics of the traders and their reaction to other cross-effects prevailing over time, such as global currencies inflation monetary policies are few noticeable ones.

Many efforts have been made by previous researchers to calibrate price dynamics of silver with different mathematical and econometric models. However, they tend not to capture the accurate price dynamics especially when markets are extremely volatile, which has been the case for all commodities in general and silver in particular in the recent decade. Present models fail to predict the emergence and scale of financial crises. One reason for these failures is that econometric models do not take into account incoherent or inconsistent behavior exhibited by the market participants, usually referred to as human psychology in literature. However, the psychology of traders can be best explained by taking into account animal spirits introduced by Keynes in 1936 [3]. In addition to these in the case of extreme events, the market price deviates extremely from its global expectation by the market momentum[4]. Our aim in this study is to simulate the price dynamics of internationally traded spot silver markets by applying Jabłońska-Capasso-Morale (JCM) model introduced by Jabłońska [4].

The framework of this study is divided in *7* sections. Section *2* describes silver market, its history and description, its functioning and literature review on previous price modelling attempts. In section *3* the data set used in this study is presented. Section *4* presents model description of JCM with an introduction to stochastic differential equations.

2 Silver markets

This section provides a brief background of silver markers. The section also describes literature review, of previous silver market modelling efforts, silver as a metal itself, its applications and its tradable forms.

2.1 History & description of silver markets

Silver is a soft white lustrous transition metal. It is famous for its high levels of thermal and electrical conductivity. Silver sensitivity to reflector of light, its ability to withstand extreme temperatures, its malleability and ductility established it as a metal without replacement in many cases. The metal also occurs naturally in its pure free form (native silver), also as an alloy with gold and other minerals. Most of the silver is obtained as a byproduct while refining other metals as copper, gold and zinc.

The mining of silver began some five thousand years ago. Silver was first mined in 3000 B.C. in Anatolia, now known as Turkey. These early loads flourished the civilizations of Near East, Crete and Greece throughout antiquity. In 1870 the silver production was reported to be eighty million troy ounce annually. The time from 1876 to 1920 represented the era of technological advancement and the production of this lustrous metal quadrupled over the last quarter of 19th century and was reported to be 120 million troy ounces. At present the global mined production of *silver* is estimated to be 671 million troy ounces [5].

In 2013, the global demand for physical silver stood high to a record of 1,081 Million ounces (Moz). The largest contributor for this physical demand was due to its industrial applications. Reported industrial demand of silver was found to be 586.6 Moz in 2013, which accounts for 54 percent of total physical demand. Asia experienced a rise of demand for silver as China consumed much of the silver to its growing electronic industry. 2013 also witnessed an increase in demand in jewelry and silverware fabrication. Supply for above ground stocks for silver was dropped to 199.7 Moz with respect to 2012. Scrap supply to the market in 2013 experienced largest year on year reduction since 1980. However, supply gap was covered by substantial increase in mining sector and government sales. Mexico is the world's largest producer of silver followed by Peru, China, Australia and Russia[6].

Silver due to its attribute of being a store of value is used in currency coins historically. It is also used in making ornaments, jewelry, high-value tableware and utensils. As an investment, it is used in the forms of coins and bullion. Millions of Canadian Silver Maple Leaf coins and American Silver Eagle are purchased and invested each year. The Silver Maple Leaf is a legal tender at $5 per ounce and there are many other silver coins with higher legal tender values, including $20 Canadian silver coins. As uncertainty in financial bonds and derivatives increases, investors seek protection in commodities markets to secure their investments. Precious metal market provides great investment opportunities during unstable financial and political situations. Silver belongs to one of such asset class and can provide long and short hedging positions. Silver is a legal tender in Utah, and can be used to pay all debts [7]. In 500 BC the first silver coins were developed by Lydia, which at present is a part of Turkey; their technique of coin making was later replicated and refined by Greek, Persian, Macedonian, and later the Roman empires [8].

2.2 Functioning of silver market

As other precious metals, silver can be regarded as a form of investment and a monetary asset. Silver has been historically used as money. Nevertheless silver has lost its status of being a legal tender in many countries including the United States, China, and India many decades ago as the silver standard of money was ended.

The price of the silver is driven by elementary principles of demand and supply and also speculation. Contrary to gold the market of silver is comparatively volatile this may be because of lower market liquidity and fluctuating demand due to store of value and industrial implications. However the silver market is much smaller than the gold market in terms of value [9]. Historically the gold to silver ratio was 12:1 as set by ancient roman civilization. In 1972 the gold to silver ratio was fixed by law to 15:1 in the United States [10]. However during the entire 20th century the silver to gold ratio remained at 47:1 [11].

The functioning of silver market is somehow similar to the gold market. There are over the counter options available in which the deal is delivery based. Exchange based markets also exist. Like other equities, silver is traded on both spot and derivative segments.

Traditional ways of investing in silver involve physical ownership of the metal. And to the present era this form of trade exists. Many banks in Switzerland and Liechtenstein offer OTC trade facilitation in the form of silver bullion and coins.

Various silver bars are available for the purpose of investment. Among them the 1000 oz troy bars. These bars, 999 fine, weigh about 68.6 pounds avoirdupois (31 kg) and vary about 10% as to weight, as bars range from 900 ozt to about 1,100 ozt (28 to 34 kg). These are COMEX and LBMA good delivery bars. Other small size bars are also available for investors.

Other than bars, numerous silver coins are available to be used as an investment in terms of physical holdings. Among them Canadian Silver Maple Leaf, American Silver Eagle, Australian Silver Kookaburra, with 99.99% silver content are popular.

Silver like other stocks of equity markets is traded as an exchange traded commodity. It is an ideal way to invest in silver without having the physical ownership of the metal. The emergence of the ETFs in 2006, have absorbed much of the silver as the ETFs hold around 18,615 tonnes of metal. COMEX stocks have also risen they stood at 142.7 Moz in early November, up from 117.9 Moz at the start of the year and 106.7 Moz in November 2011 [12].

There is strong evidence that gold and silver are positively correlated with expected inflation. And these asset classes can be used as a reliable hedge against inflation in short and long run [13]. This appears to be the reason of greatest economic activity related to silver market and innovations in spot and derivative segments.

2.3 Review of previous research and previous modeling efforts

Studying the silver market has been the center of interest and fascination for many researchers. In this section general econometric models present in the previous literature have been analyzed. The latter part of this section presents recent developments and implications of behavioral economics related to animals spirits modeling have also been analyzed and discussed.

Aggarwal et al. (1987) used the silver successive price change by analyzing future contracts traded on CBOT and COMEX, to evaluate whether the price changes are stochastically independent. Furthermore, various trading rules were implemented to determine as if extraordinary profits can be yielded in case of silver futures. For this purpose they used the daily data of opening and closing prices high and low, respectively for periods of 1980-1984. It was quickly evaluated by the analysis of movements that the period of 1980 is significantly different from the rest of the period under evaluation. Nevertheless a Markov chain model was constructed to evaluate whether the process governing the price change is significantly dissimilar in 1980 as compared to other years. Also, Garman and Klass (1980) volatility estimator were implemented. Statistical inference for both methods clearly indicated that that the period of 1980 is statistically different than the other years under evaluation.

However, the year 1980 was excluded from the study and further analysis was done to remaining part of the data. The dependence for successive price change was tested by using serial correlation techniques, Markov chain models, and trend and cycle analysis. The results yielded some serial dependence in prices and strong cycles in price movements. Lastly, filter tests were conducted to evaluate the possibility of excessive returns in silver future markets. Finally, the study clearly showed weak form of efficiency in silver markets for the period under evaluation and by opting for suitable designed and disciplined trading strategies participants can make positive returns [14].

In another study Lashgari, M. K. (1992) devised an information-based theoretical design to measure the serial dependence between successive changes in daily, weekly and monthly data, respectively, from the period of January 1970 to December 1989. An index was constructed to express the level of multifariousness between the actual and the predicted price changes to obtain optimum forecast for price change of gold and silver. Optimal forecasts of respective metal price change was acquired from previously observed values by minimizing the information inaccuracy resulting in a the degree of divergence between the actual and forecasted price changes. The degree of dependence for successive price change was computed by exponential smoothing time series models. The degree of dependence on the immediate past is generally greater for gold than that for silver and some inefficiencies appear to exist for both gold and silver, as there is lack of consistent patterns in both precious metal prices under consideration [15].

Pipattadanukul et al. (2012) investigated the long and short term relationship between gold and silver futures using daily data from May 1991 to May 2001 traded on COMEX. Unit root test, co-integration tests and error correction models were applied to the data under evaluation. The results showed strong evidence of the existence of cointegration in gold and silver futures. The price dynamics of gold and silver futures show long term positive relationship. However, the error correction model ECM revealed that the changing price of silver and gold futures have short term relationship. Therefore, it can be concluded that both of these markets can be considered as substitutes against similar type of risks [16].

Wahab, M. (1995) used daily data from January 1982 and December 1991 of settlement prices of nearby-expiration gold and silver futures contracts traded on the COMEX to investigate the joint dynamics of the aforesaid markets. The study also investigated the possibility of profitable trading of precious metals futures spread. It was concluded that price change volatility exhibit statistically significant serial and cross serial dependence at least for the first lag. The unconditional distributions were non- normal. Nevertheless, a bi-variate AR(1)-GARGH(1,1)-M model provides a good description of joint process of price generating change. This yields empirical conditional distributions, which are fairly well in line with the theoretical unit normal distribution. The estimated bi-variate model was then simulated forward on a weekly basis to generate predicated changes in gold and silver futures and traced the time path for conditional co-variance matrix. Predicted changes in the mean and variances were used for calculating the conditional optimal spread ratios, which in turn were used in making optimal spread positions, based on changes in the spread. However the study found that the precious metals future markets are informationally inefficient [17].

Harper et al. (2013) analyzed the price volatility in silver spot segment by using daily data from January 2, 2008 to December 30, 2011. Family of GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models to evaluate the volatility dynamics. Both GARCH (1,1) and EGARCH (1,1) models were significant in terms of explaining silver price volatility. It was concluded that silver market is sensitive to both positive and negative news [18].

Sari et al. (2010) utilized daily time series data for four precious metal markets namely gold, silver platinum, and palladium, traded on COMEX, USD/euro exchange rate and oil. All these variables were calibrated with a Vector Auto Regressive (VAR) Model. The purpose was to evaluate the relationship between spot prices of precious metals and USD/euro exchange rate and oil. The research did not find sufficient evidence for long run equilibrium between spot prices return and exchange rate. This is probably due to the increase in disparity in economic, and monetary value and hedging between precious metals and exchange rate. The study also found evidence that the precious metals markets and exchange rate are closely related in short run after shocks. However, the speculative window is short-lived and countered by overreactions and late adjustments, which supports the fact that the "herding behavior" assumptions in commodity markets is short-lived [19].

Mutafoglu et al. (1995) analyzed the trader positions in terms of predicting the direction of gold, platinum, and silver spot price movements. For this purpose Commodity Futures Trading Commission's Commitment of Traders report for platinum, silver and gold prices using trader positions is investigated in a VAR framework. Granger causality tests are conducted to access a relationship between trader positions and market prices existence. An examination of the extreme trader positions on price movements was also deducted. The results showed that market return is a significant parameter in explaining trader's positions for all trader types in each of the precious metal markets under evaluation [20].

Arouri et al. (2012) analyzed long memory and structural breaks, in modeling the return and volatility dynamics of four precious metals, namely gold, silver, platinum and palladium using both spot and future prices into consideration from the period of January 4, 1999 to March 31, 2011. It was concluded that the dual long memory was found to be efficiently captured by ARFIMA-FIGARCH models, which adequately provides better out-of-sample forecast accuracy than several volatility models. The study provides adequate evidence that conditional volatility of precious metals is better explained by long memory rather than the structural breaks [21].

Dunis et al.(2007) used the daily data from 04/01/2000 to 31/05/2006 for spot prices of gold and silver as fixed by London Bullion Market Association (LBMA) to forecast the daily returns of gold and silver using linear and nonlinear models. ARMA was imposed as a strict criterion for comparison purposes with selected nonlinear models as Nearest Neighbors, Multi Layer Perception (MLP) and Higher Order Neural Networks (HONN). The models were assessed by statistical criteria of correct directional change and financial criteria of risk-adjusted return. The study in the overall sense accomplished that MLP and HONN models are more reliable than the benchmark with robust profits both in- and out-of-sample, MLPs being marginally better [22].

Indeed, classical econometric modeling techniques have made great progress in the past but they have assumed rationality among the economic agents. Witnessing the recent financial crises of 2008 many limitations of traditional financial modeling have arisen. These gaps seems to be fulfilled by the notation of animal spirits introduced by John Keynes in 1936. Below is a literature review with reference to animal spirits modeling.

Chang (2007) investigated a simple present value asset pricing model with social interactions and heterogeneous beliefs. The beliefs of investors were revised in each period by taking into consideration two features "The Fitness Measure" which is a measure of realization of the profits made in the past and the exogenous "Social Interactions Measure" which is a measure of the interaction among the traders exceptions from mean choice level in economy. Endogenous type of social interactions ultimately arises when there is a presence of both exogenous social interactions and heterogeneous beliefs among the traders. It was shown that the characteristics of steady state and the dynamic behavior is mainly characterized by the strength of endogenous type of social interactions [23].

De Grauwe et al. (2012) devised a heterogeneous agent model for the foreign exchange market. The main characteristic feature of the model were the traders which were unaware of underlying value of the fundamental and formed beliefs about its value. In simplest sense, the model assumed two different types of psychological beliefs namely optimistic and pessimistic, respectively. Traders were then allowed to switch beliefs depending upon the condition of how well the beliefs perform in forecasting the exchange rate. The model included simple heuristics with an imprinting from trial and error. The model generated cyclical movements of optimism and pessimism even when the fundamentals were stationary. The endogenous of either of two beliefs can be accounted as a mechanism of ascertaining the true fundamental. The model was further modified by introducing traders having an unbiased view about the fundamental exchange rate and chartists, respectively [24].

De Grauwe (2011) introduced a model of macroeconomic scale by providing specific importance to the cogitative limitations of the agents. The agents used simple and biased rules to predict output and inflation. Learning mechanism was adopted that allowed for the selection of more profitable options. This mechanism ensured that the agents used the biased rules and market forecasts are unbiased. The model generated optimism and pessimism, which further generated endogenous cycles akin to "animal spirits". The animal spirits were found to be effective when agents were willing to learn from their mistakes produced by their biased beliefs. However, some degree of forgetfullness, was associated with the errors made along, for animal spirits to emerge and play an influential role in business cycles. Model being non linear produced a degree of uncertainty while transmitting monetary policy shocks. This model takes into account an auxiliary dimension of uncertainty, which in other words means that shocks can have a different impact depending upon the condition of economy, while referring to the degree of optimism and pessimism that the agents have about the future prevailing at that time. The model established to reveal that a regime in which inflation targeting is a credible output and inflation was reduced. The reason for this appears to be that the credibility helps to reduce belief correlation and ensure self fulfilling pessimism and optimism waves. Nevertheless, the model showed that the strict inflation targeting does not yield optimal policy [25].

Brock et al. (2005) utilized Large Type Limit (LTL) approach that approximated the evolutionary dynamics of markets with various trader types in a sense that collective and relentless characteristics of an evolutionary adoptive system are saved in LTL. The LTL approach was further used to investigate the suitability of real expectation fundamental steady state as well as divergence from it. Bifurcation routes towards disorder and complex periodic or even chaotic dynamics in adaptive evolutionary systems with many trader types were analyzed. The model simulations in the end revealed that diversification of beliefs causes a divergence from unstable RE fundamental and excess volatility. Thus the model concluded that a large evolutionary system is not stable and generates complex dynamics when agents become susceptible to small differences in fitness [26].

Bofinger et al. (2013) incorporated the heuristics into a standard dynamic stochastic general equilibrium (DSGE) model that captures the significant characteristics of housing prices and provides the qualitative basis in incorporation of monetary policy effects in housing markets and in turn the overall economy when behavioral mechanisms are set to play a role. The model succeeded in implementing the notions of nonlinearity and pronounced boom-bust cycles generating waves of optimism and pessimism "animal spirits". The results were compared to a standard model having rational expectations by mean pulse response. Upon analyzing the results the research concluded that the standard Taylor rule is not suitable enough to attain macroeconomic stability. It was also concluded that monetary policy triggers endogenous and self-fulfilling "animal spirits" dynamics that drives house prices and economy in a broader scene [27].

3 Silver prices

The data used in this study is daily data from 1st March 2000 to 1st March 2013 of closing prices of spot silver in US dollars. The data has been downloaded from (http://www.livecharts.co.uk/). The weekend prices are also included in the data set under evaluation. However, they are same as the closing prices on Friday. The set of daily observations thus constitute the time series. Classical approaches of time series evaluation has been applied to the data under evaluation. This section presents the graphical representations of the data such as time line plots, the autocorrelation function, the partial autocorrelation function, and histograms plots. Also, numerical values of basic statistical moments such as mean, standard deviation, skewness and kurtosis have been analyzed to perform explanatory analysis. Figure 1 presents the time line plot of historical silver prices.

The primary analysis of the figure 1 clearly reveals that the series under consider-

Figure 1: Time line plot of the original closing price.

ation is not stationary as it is not mean reverting. The series appears to have an increasing trend and is also non stationary with respect to variance as well. The Heteroscedasticity of the closing prices of silver is presented in Figure 2. Nevertheless, an Augmented Dickey-Fuller test was conducted which also validated that the series is not stationary. In the period under consideration silver reached its high of 48.5240 an ounce on 27^{th} of April 2011 and the minimum closing price was of 4.05 on 21^{st} of November 2011. Figure 3 presents the histogram of the closing price of silver with reference to theoretical probability density function (PDF) of Gaussian distribution. It can be observed that the features of the series are not being captured by the PDF of normal distribution, hence the series in not stationary.

While considering traditional econometric approaches, the normality assumption is required to be satisfied. Hence it is preferred to use stationary data, which is usually archived by applying a suitable transformation to the time series. The reason being that the conventional approaches of ARIMA or Ornstein-Uhlenbeck takes white noise as input and are only able to produce Gaussian processes. Nevertheless, the models applied in this study to calibrate the real data are non linear in nature, therefore, even if they use only white noise, they are capable of producing leptokurtic

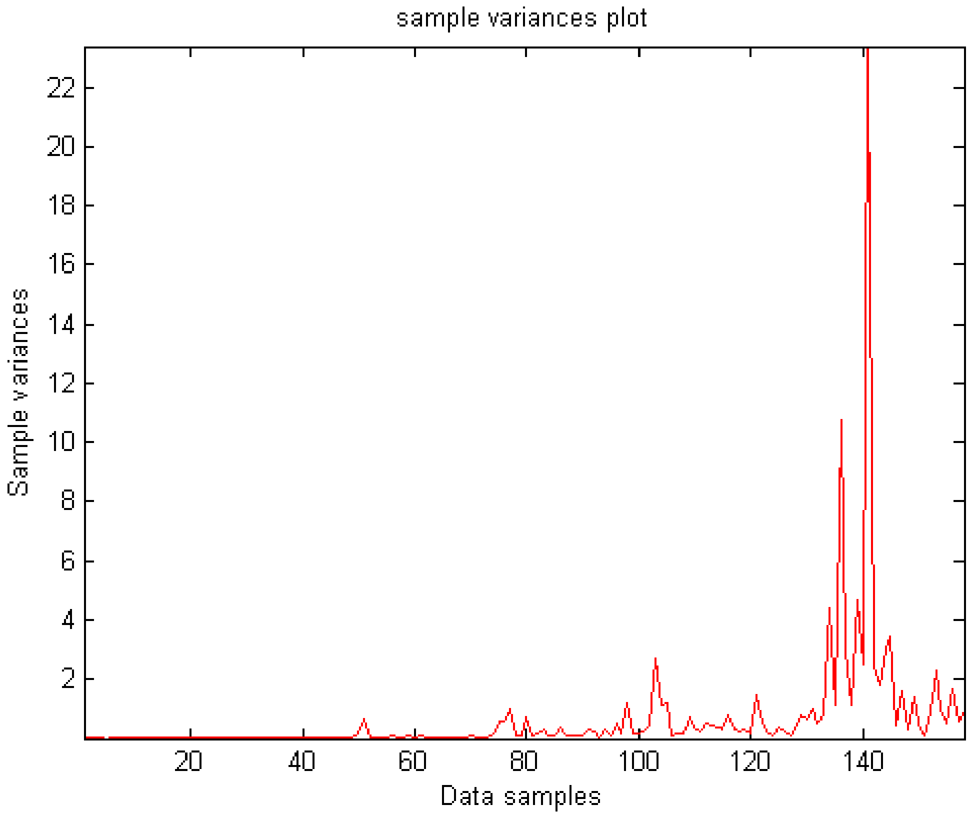

Figure 2: Sample variance plot of actual closing price.

distributions. The autocorrelation function and the partial autocorrelation function plots have been presented in Figure 4.

The slowly dying ACF validates that the series is not stationary. Also, the PACF function plot reveals that the correlation coefficient values approximately lie within the confidence limits except for lag 1. As with any other classical time series analysis the next step here is to apply some suitable transformation to the data so as to make it stationary. This has been done by taking the first difference and logarithmic returns.

Figure 5 presents the time line plots of both transformed silver prices. By taking the transform the series appears to cluster around its mean in both of the transformations. However, they have significant spikes that specifically point out that the series has not acquired constant variance over time after transformation. Figures 6 and 7, show the ACF and PACF plots of silver prices difference and logarithmic returns. It is clearly evident from the ACF and PACF plots that the series has become less auto-correlated.

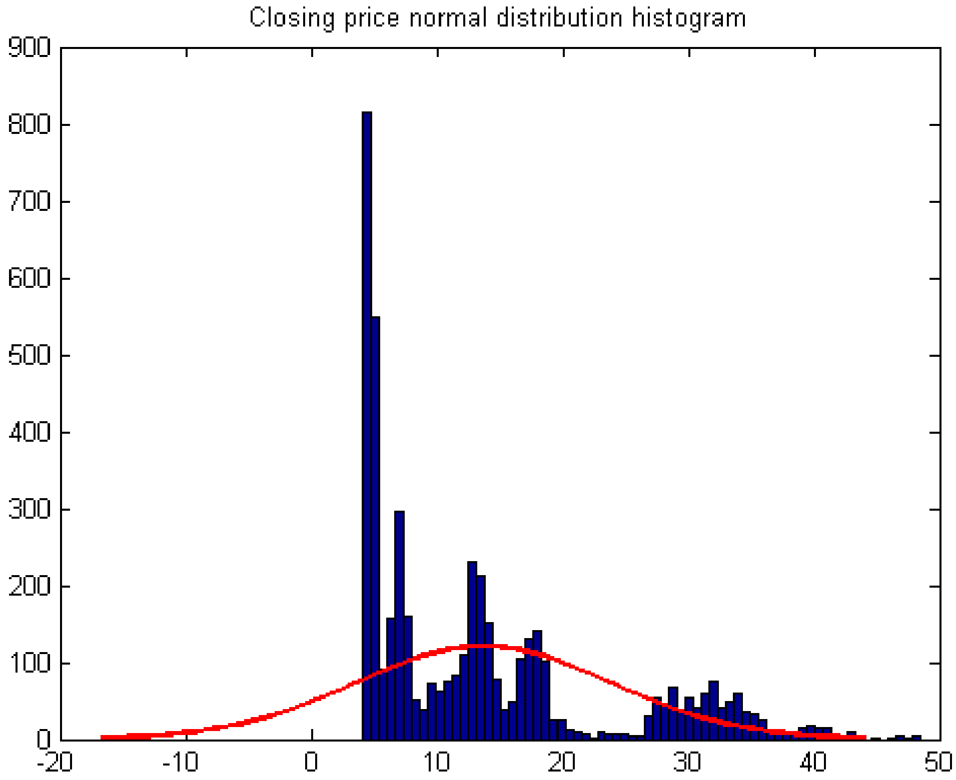

Figure 3: Histogram of the actual closing price.

Figures 8 and 9 present the histograms of the transformed data. The data transformation yields distributions that tend to follow the features of normal distribution. However, they are slightly peaked and also have fatter tails. The primary statistical values of mean, standard deviation, kurtosis and skewness have been summarized in Table 1

Evaluating the statistics it is clear that the mean value of the transformed series has significantly reduced from those of the actual series. Also, the skewness value shifted from a positive value in actual silver price series to a negative one. Hence

Table 1: Basic Statistics of silver prices.

	Mean	Std	Skewness	Kurtosis
Silver Prices	13.7067	10.1346	1.1263	3.2251
Silver Price Returns	0.0050	0.3325	-2.6565	43.221
Silver Logarithmic Returns	$3.6427\text{x}10^{-4}$	0.168	-1.398	17.5824

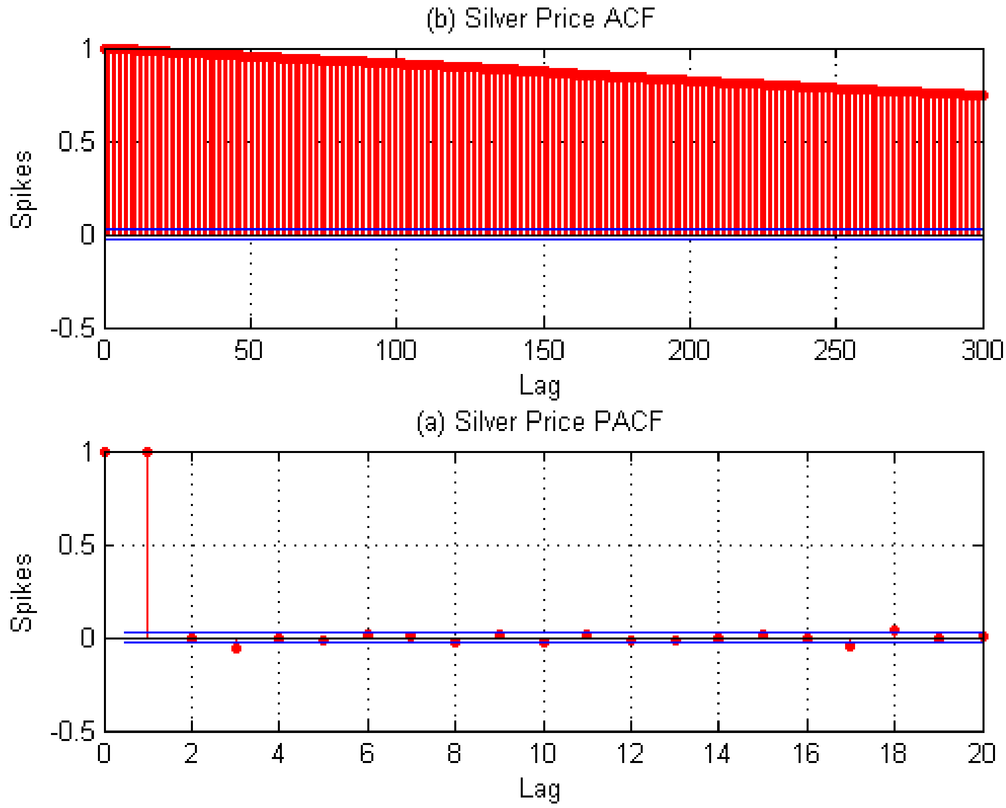

Figure 4: (a) ACF of silver price (b) PACF of silver price.

both the differencing and logarithmic data transformations have lead to negatively skewed distributions. Data transformation has lead to the generation of relatively more peaked distributions than the actual silver price series. The positive mean and standard deviation values indicate the rising trend over time. Briefly, the actual silver prices and their transformations do not tend to follow the characteristics of a Gaussian distribution.

4 Model Background

4.1 Introduction to stochastic differential equations (SDEs) and stochastic modeling

The modern world has evolved in many sectors such as telecommunications, banking, financial innovations and the list may go on. This development could be seemingly

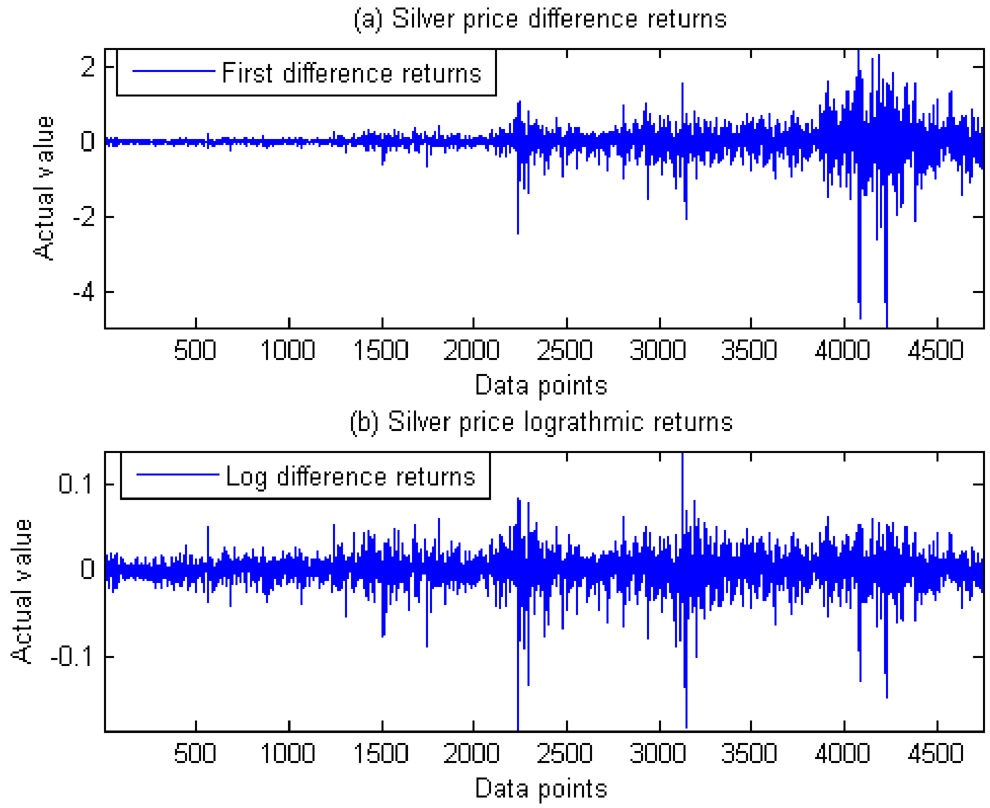

Figure 5: (a) Silver price difference returns (b) Silver price logarithmic returns.

impossible to explain by mathematical modeling. The question faced by many of us is how to model the uncertainty prevailing in science and engineering problems in general and econometric problems in particular. The stochastic differential equations are a natural tool for modeling such uncertainties. Consider the following equation.

$$dX = b(t, X)dt + \sigma(t, X)dW(t) \tag{1}$$

In equation 1, b and σ are regular functions. We say that a stochastic process $X(t, \omega)$ is a solution for the above differential equation with the initial condition

$$X(0, \omega) = X_0(\omega) \text{ if}$$

$$X(t, \omega) = X_0(\omega) + \int_0^t b(s, X(s, \omega))ds + \int_0^t \sigma(s, X(s, \omega))dW(s, \omega) \tag{2}$$

for some "suitable" $t \geq 0$. Under fairly natural conditions, such equations always have unique solution. Now again consider equation 1 with

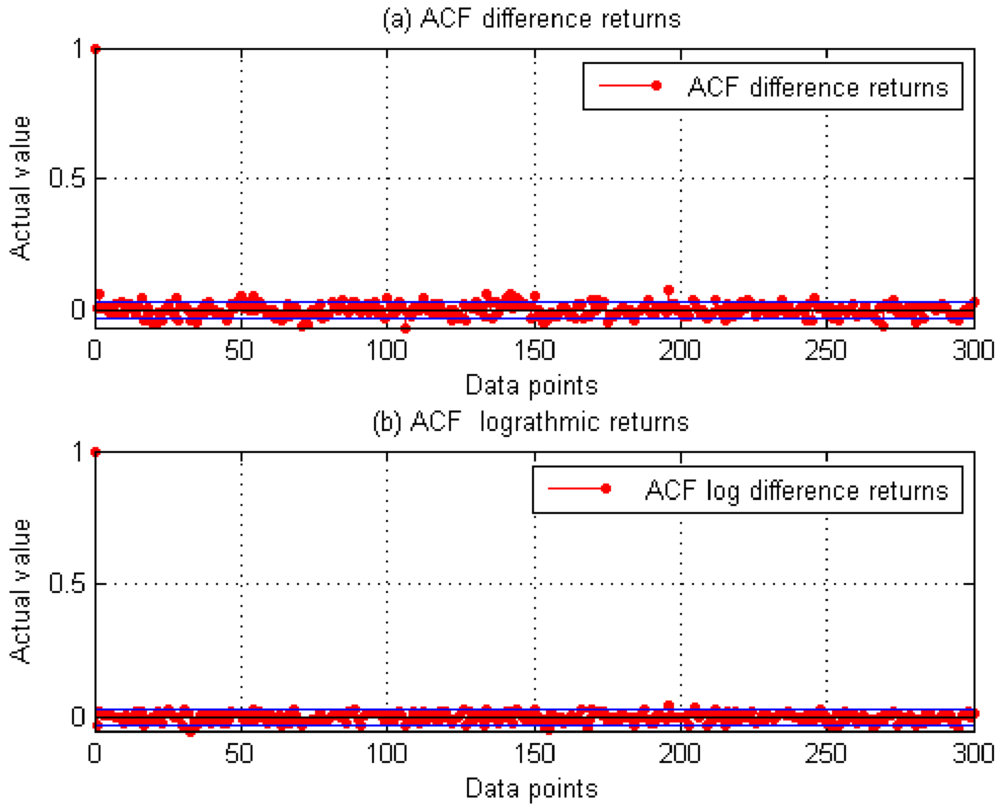

Figure 6: (a) ACF difference returns (b) ACF logarithmic returns.

$$| b(t,x) | + | \sigma(t,x) | \leq C(1+ | x |),$$
$$| b(t,x) - b(t,y) | + | \sigma(t,x) - \sigma(t,y) | \leq D | x - y |,$$

for all $t \in [0,T]$ and $x, y \in \mathbb{R}$. Furthermore, let $X_0(\omega)$ denote a square-integrable random variable which is independent of $W(t)$ for all $t \geq 0$. Then the above equation has a unique t-continuous solution $X(t,\omega)$ for $t \in [0,T]$. The solution is $\mathbf{F}_t$ adopted and

$$\mathbf{E}\left(\int_o^t | X(t,\omega) |^2) dt\right) < \infty \tag{3}$$

The presence of a unique solution can be proved with the help of Picard iteration, similarly to a deterministic solution. The necessary solution implies the use of martingale inequalities (3) mentioned as one of the stochastic integral. An analogous existence and uniqueness theorem holds in a vector valued case $X(t,X) \in \mathbb{R}^n$ for equations of the form

$$dX = b(t,X)dt + \sum_{k=1}^{m} \sigma_k(t,X)dW_k(t) \tag{4}$$

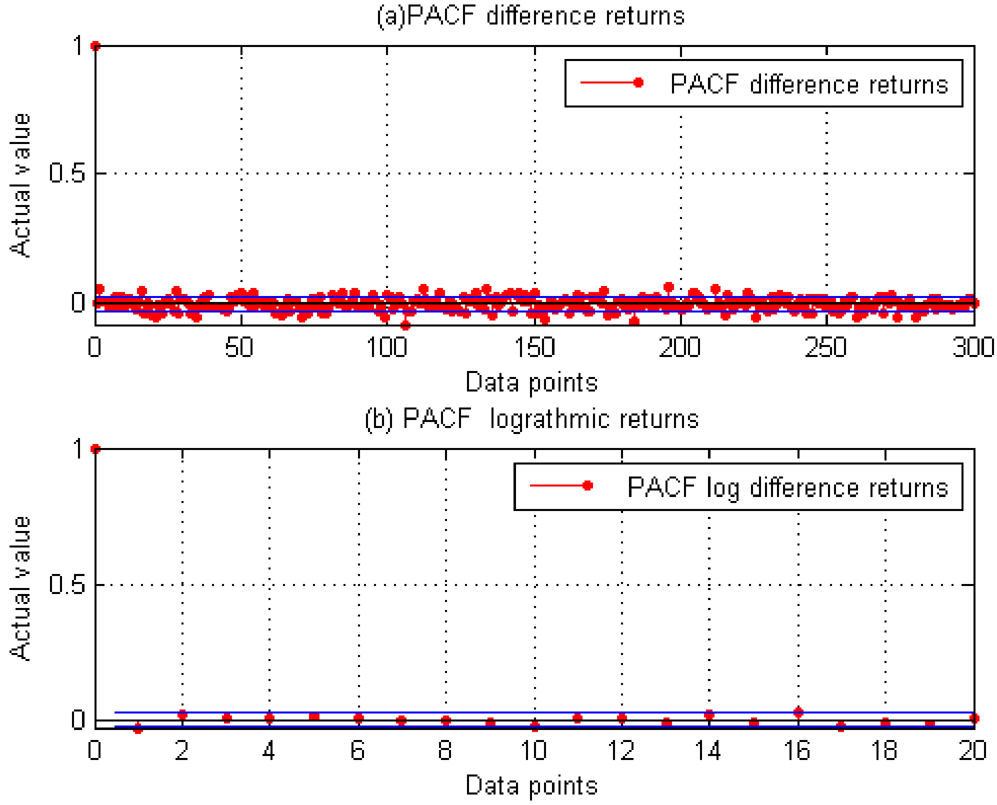

Figure 7: (a) PACF difference returns (b) PACF lograthmic returns.

Where the Wiener processes $W_1, W_2, ..., W_m$ are independent. One can simply think of equation 1 as a stochastic differential equation that is a permutation of the deterministic equation $\acute{X} = b(t, X)$. The equation is perturbed by *additive noise* if σ does not depend upon X, and it is perturbed by *multiplicative noise* if σ depends explicitly on X.

4.2 Jabłońska-Capasso-Morale (JCM) model

The year 2008 was the year when financial markets faced a global meltdown followed by a recession and declining growth rates and unemployment in developed economies. Indeed, there were many significant contributes over to the negative impacts of this recession. The most obvious one is the over-reliance on the self correcting nature of free markets. In fact, markets appeared to be less then perfect and, sadly, are not self-correcting as had been assumed in the past. The "Rational Trader" the key foundation of traditional finance, is widely prevalent in traditional econometric analysis and there was little room for "animal spirits" that were introduced early by

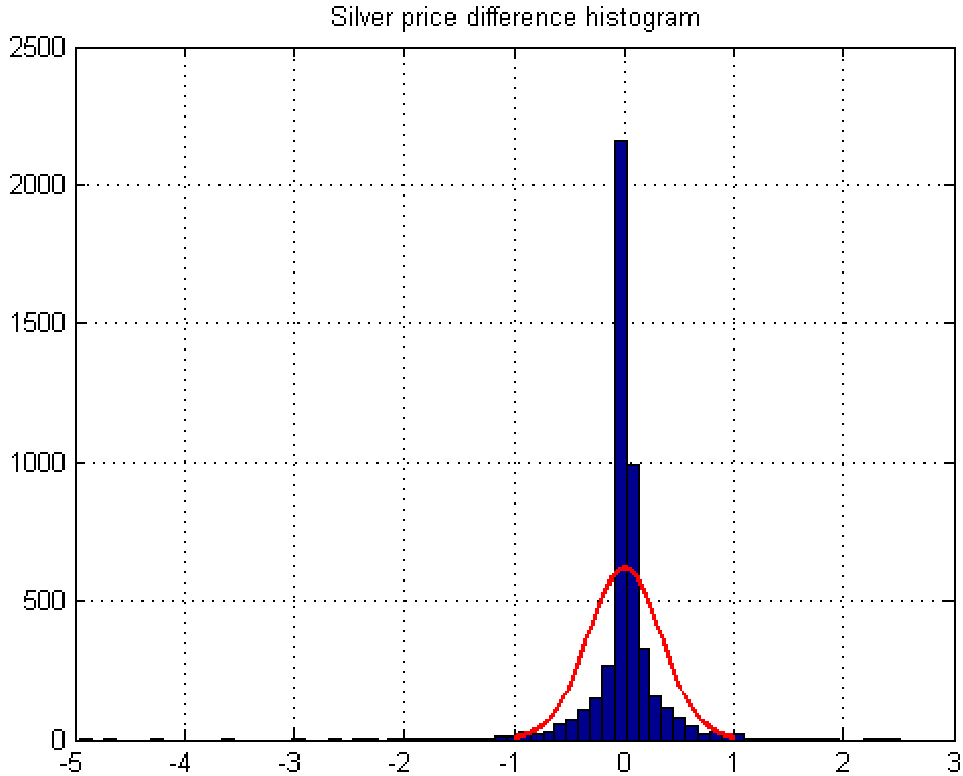

Figure 8: Silver price difference histogram

Keynes in 1936 [3, 28]. Briefly, traditional econometric approaches do not take in to account the psychological behavior of the traders, that is a key factors that accounts for market dynamics. When traders evaluate risks and uncertainties related to the future outcomes, this generates certain levels of trust and confidence [28].

Considering any stock options the price generation mechanism has always involved a group of traders and are not limited to a single trader. The movements of such groups of traders account for market shocks. Significant positive correlation has been observed in business and macroeconomic variables. There is now established evidence of herding behavior among the traders which can create positive and negative feed back to trading, when rising and falling prices collaborate for future price rise and fall respectively [4, 28, 29]. The herding behavior can be thought of as a similar process to a flock of birds flying in the sky under small groups leadership. Sufficient numerical and empirical evidence is available to support that it requires 5% of the of the population heading in a certain direction to drive the whole group to follow them. Traders bid as a reaction to the actions of theirs neighboring participants and, when a sufficiently big group bids far enough, the rest starts following that group [30]. Different mathematical biology has been used to model trader's

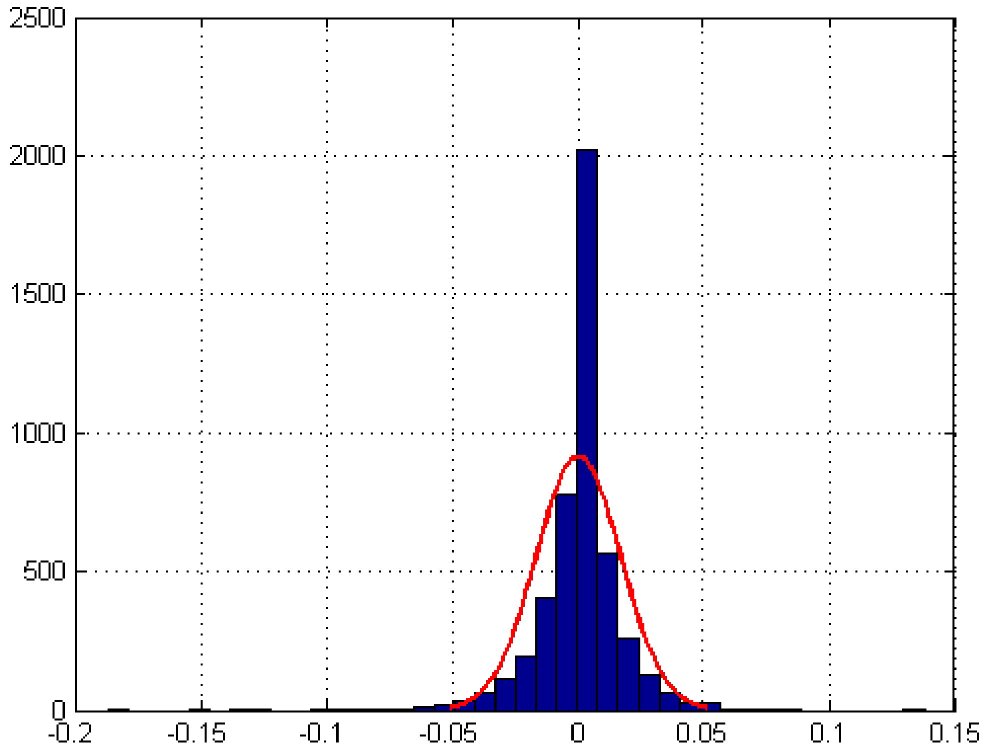

Figure 9: Silver prices log histogram

psychology in financial time series, widely used is the Capasso-Bianchi System of stochastic differential equations [31, 32, 33].

$$dX_N^k = [f(X_t^k) + h(k, X_t]dt + \sigma dW^k(t), \tag{5}$$

Equation 5 represents the Capasso-Bianchi system of differential equations in its general form. Where $f(X_t^k)$ is the representation of the forces impacting the entire population, which here is a group of traders, and a measure of their distance is the price.The term $h(k, X_t)$ is the interaction term with the closest neighbors of the individuals in the population of N members. The term dW^k accounts for the stochastic nature of the system. The animal dynamic modeling can be extended to mean reverting jump diffusion model with the system terms of 5 [34]. Therefore the model now takes the form

$$dX_t^k = \gamma_t[(X_t^* - X_t^k) + (f(k, \mathbf{X}_t) - X_t^k)dt] + \sigma_t dW_t^k +^+ j_t^k dN_t +^- j_t^k dN_t \tag{6}$$

for $k = 1, 2, ..., N$, where

- X_t^k is the price of trader k at time t,
- X_t^* is the global price mean reversion level at time t,
- γ_t is the mean reversion rate at time t,
- X_t is the vector of all traders prices' at time t,
- $f(k, t)$ is a function explaining local interaction of the trader k with his neighbors (Small range of individuals from vector) X_t
- σ_t is the standard deviation for Wiener increment at time t,
- ${}^{+}j_t^k$ is the positive jump for trader k at time t,
- ${}^{-}j_t^k$ is the negative jump for trader k at time t

Positive feedback and herding effects are apparent in a conventionalized manner in stock and commodity markets when there is a positive correlation or "momentum" at short horizons that seems to be corrected by negative correlations or mean reversions at longer horizons [28]. As there may be many anomalies in markets, there seems to be little concurrence in this value aggregation. The momentum effect prevalent in markets can be described as a fluid. The stock prices can be thought of as one dimensional measurement of fluid pressure along periodic domain. The momentum phenomena in financial markets can be modeled by taking into account Burgers' equation (7), which is a one dimensional form of Navier-Stokes equation excluding pressure and volume terms [4].

$$\mu_t + \alpha\mu\mu_x + \beta\mu_{xx} = f(x, t) \tag{7}$$

- μ stands for the price and can be perceived as unidirectional pressure measurement of fluid in a periodic domain.
- $f(x, t)$ accounts for the fundamentals of periodic character.
- $\alpha\mu_{xx}$ is the diffusion term that is related to the fact that spot markets converge to an equilibrium price.
- μ_x explains the difference between the mean and the mode of traders price on a particular day.
- $\mu\mu_x$ is the momentum term that expresses the traders' movements towards the most common price. This effect is amplified at the highest prices.

The Jabłońska-Capasso-Morale (JCM) model is an extension of the Capasso-Bianchi model with the addition of a momentum term that accounts for price spikes. However, the jump components present in the previous version of the coupled model have been removed. Now the model (6) takes the form

$$dX_t^k = [\gamma_t(X_t^* - X_t^k) + \theta_t(h(k, X_t) - X_t^k) + \xi_t(g(k, X_t) - X_t^k)]dt + \sigma_t dW_t^k, \quad (8)$$

The mean based local interaction term $f(k, X_t)$ in case of (6) has been replaced by global interaction as a Burgers type momentum component $h(k, X_t)$.

$$h(k, \mathbf{X}_t) = M(\mathbf{X}_t).[E(\mathbf{X}_t - M(\mathbf{X}_t)], \quad (9)$$

where $M(X)$ is the mode of the variable X and $E(X)$ is the expected value. And $g(k, X_t)$ is the local spread and is the representation of the maximally distant member of k-th trader's neighborhood, formed by closest $p\%$ of the population. Mathematically

$$g(k, \mathbf{X}_t) = \max_{k \in I}\{\mathbf{X}_t^k - \mathbf{X}_t\}, \text{where } I = \{k \mid x^k \in N_{p\%}^k\} \quad (10)$$

5 Silver price modeling results

This section provides the results of model calibration of silver prices. The entire data of silver prices has been analyzed on the basis of actual or original prices vs the simulated prices. Different graphical illustrations have been described in this section to evaluate the performance of the model with real prices. Also, analysis of various numerical values of moments of actual price series and model simulated series has been done.

5.1 JCM model's results

Simulation results of the JCM model after calibration is firstly presented in the form of a time line plot to check the simulated price behavior that is generated by JCM. In Figure 10 the actual silver prices have been plotted in blue colour and the simulated silver price is in red colour.

It can be observed that the JCM simulated series tends to follow the actual prices fairly well in overall sense. However, JCM produces some chaotic spikes especially at the end of the the data where prices have risen drastically and actual series is

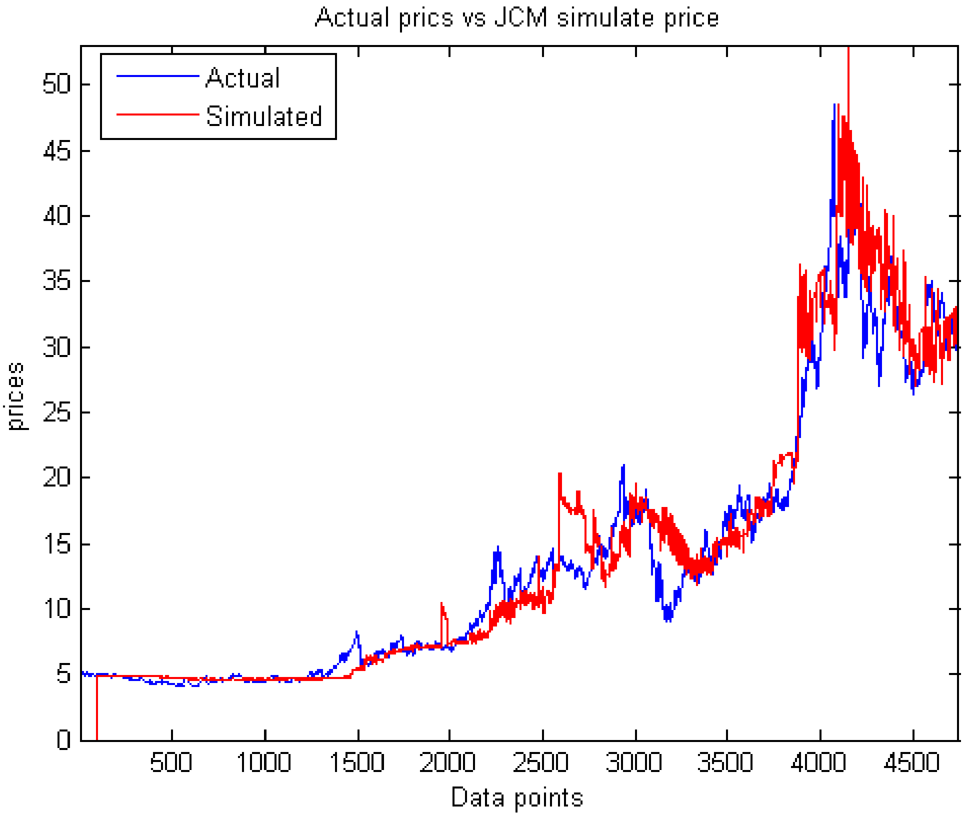

Figure 10: Actual prics vs JCM simulate price.

also somehow chaotic at these points. Nevertheless, it can be concluded that JCM tends to capture the volatility behavior in a good way. Figure 11 represents the histograms of silver prices and those simulated by JCM.

In Figure 10 the original prices have been plotted in blue while the JCM simulated prices have been plotted in red. The simulated series tend to follow the same leptokurtic distribution, as of actual silver price distribution. Also the simulated JCM tends to be positively skewed as of actual series under evaluation. More distinct evaluations to justify the distributional characteristics can be done by the analysis of statistics presented in Table 2. Figure 12 and Figure 13 the ACFs and PACFs functions plots of actual vs simulated have been illustrated.

The ACF plot of simulated JCM prices follow the same characteristics of actual silver prices. It dies away in a similar manner as of actual silver prices under evaluation. In case of PACFs the simulated PACF at lag $1, 2, 3, 4, 5$, and lag 6 are significantly spiked from the confidence limits with some, nonsignificant values below the limits with reference to actual silver price PACF. Table 2 presents the statistics of actual silver prices with simulated JCM prices.

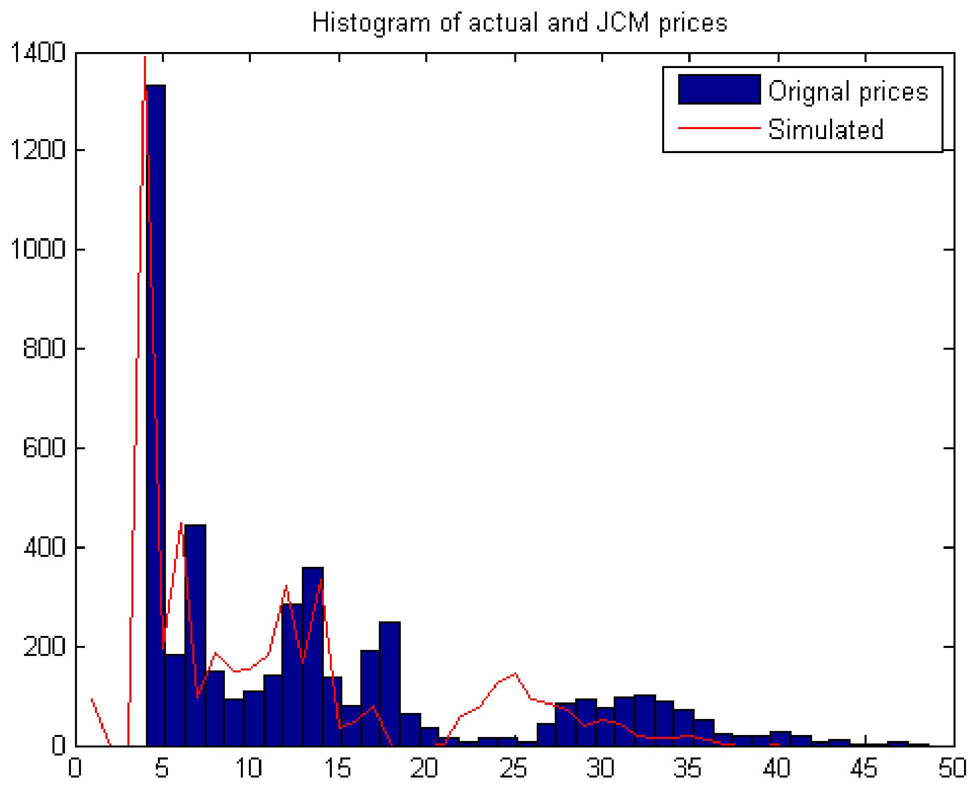

Figure 11: Histogram of actual and JCM prices

Clearly, all the statistical values of the simulated JCM are almost the same as for actual silver prices. It can be concluded with sufficient confidence that the JCM simulations tend to capture the dynamics of silver price with high performance.

6 Discussion and summary

Results obtained from the model simulations are summarized and discussed in this chapter. The model simulations were carried out using Matlab. The results ob-

Table 2: Table of statistics Actual silver prices vs Simulated JCM

Statistics	Mean	Std	Skewness	Kurtosis	Median
Actual Silver Prices	13.7067	10.1346	1.1263	3.2251	11.7800
Simulated JCM prices	14.0921	11.0424	1.0974	3.133	10.4864

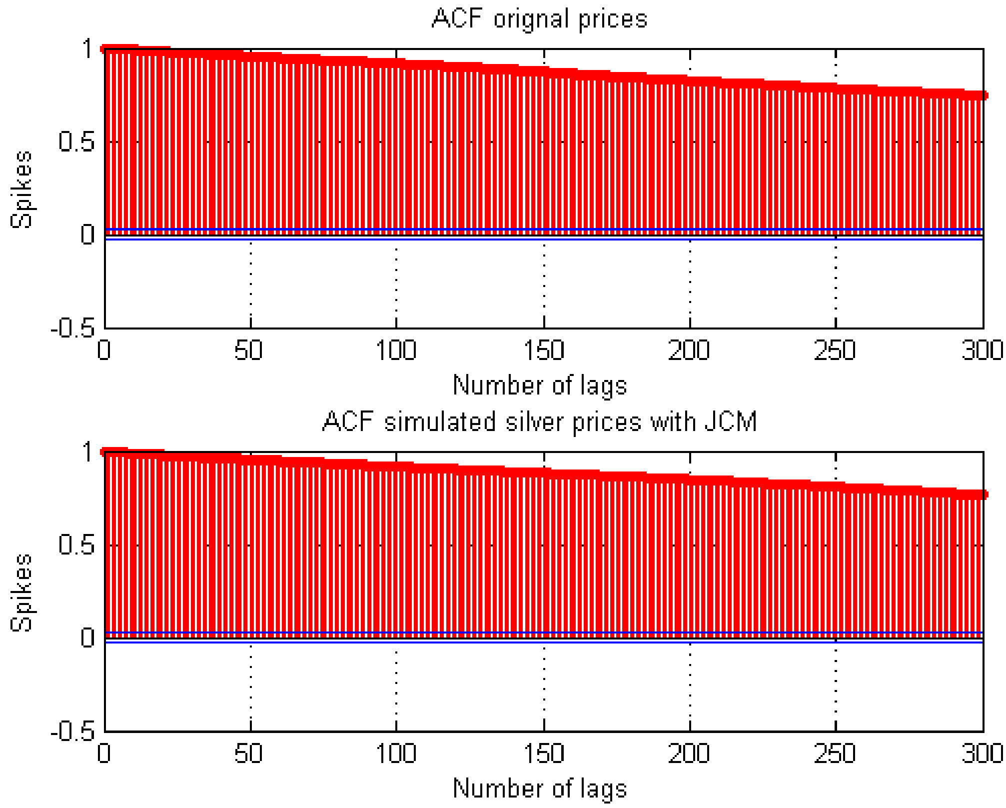

Figure 12: ACF simulated silver prices with JCM

tained from matlab simulations of the model were analyzed in the form of graphical illustrations and numerical values, such as time line plots, ACFs, PACFs, histograms and basic statistical values.

The daily real data was subjected to the JCM model, and the model parameters were tuned in by hand. The closest results to the real data that were produced were achieved by selecting an ensemble size of 100, and a time window of 91 days. However increasing the sample size did not have any impact on the improvement of the model. It is worth mentioning here that the degree of randomness in global mean reversion component mentioned in equation (8)of the model was set to zero. And the closest $P\%$ mentioned in equation (9) was set to (0.05%). Maximum likelihood approach was adopted to calibrate parameters of the model to the real data. Graphical illustrations of the simulated results of JCM were presented in figures 10 to 13 and Table 2 for statistics of actual and simulated JCM. Both the graphical illustrations and numerical values clearly reval that the JCM tends to follow the price dynamics of silver markets for the period under evaluation with high performance.

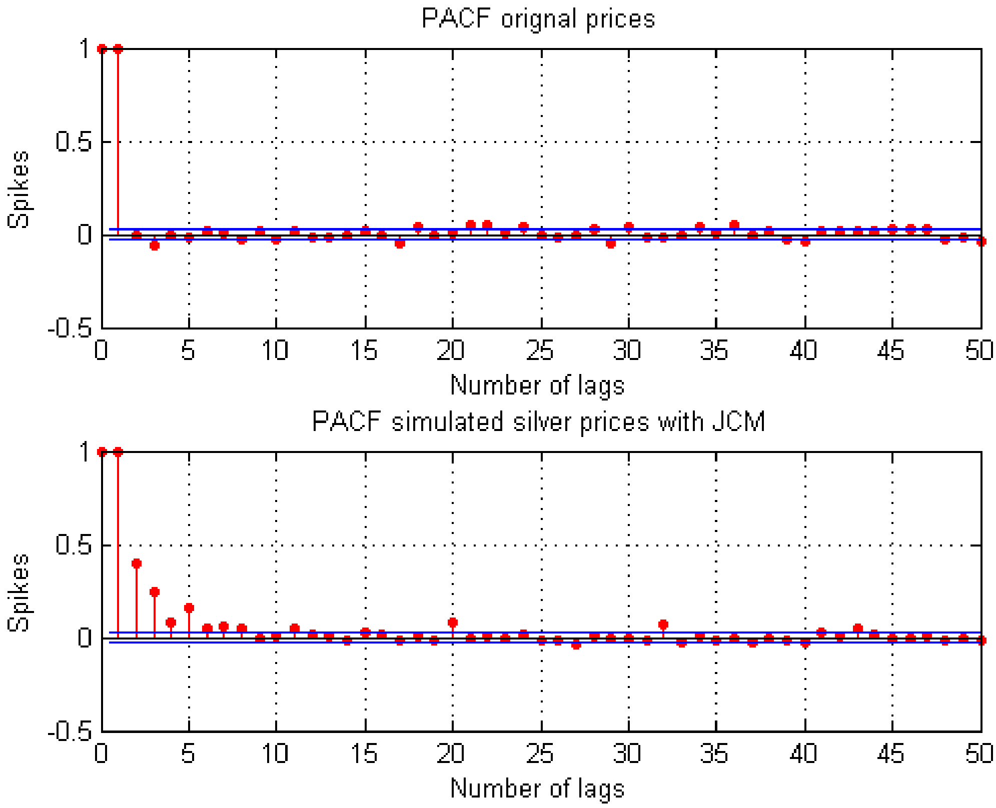

Figure 13: PACF simulated silver prices with JCM

7 Conclusions

The purpose of this study was to simulate the dynamics of silver markets with the Jabłońska-Capasso-Morale model (JCM). The data set used in this research comprised daily prices of silver in US dollars from a period of 1^{st} March 2000 to 1^{st} March 2013. Firstly the reader has been briefly introduced about the history, functioning of silver markets and recent financial innovations prevalent in this decade. Silver prices of the aforesaid period were subjected to classical econometric analysis, which revealed that the period under evaluation was non-stationary. To induce stationarity mathematical transformations were applied to the series. The transformations however failed to produce Gaussian realizations. Nevertheless the model JCM calibrated to the real series is capable of producing leptokurtic processes. Therefore the normality condition required in classic econometric approaches can be neglected here. After presenting the mathematical background of the JCM. The simulated results were analyzed using a comparison between the actual and the simulated series. Graphical realizations and analysis of numerical moments were opted as a measure of model performance.The JCM tends to capture the dynamics of actual silver prices, including pre and post recession periods, with high performance. In a simplest sense it can be said with sufficient confidence that JCM simulations tend to produce similar psychological "animal spirits" features present in the silver market for the period under evaluation.

References

[1] Hammoudeh, S. M., Yuan, Y., McAleer, M., & Thompson, M. A. (2010). *Precious metals exchange rate volatility transmissions and hedging strategies*. International Review of Economics & Finance, 19(4), 633-647.

[2] Hammoudeh, S., Malik, F., & McAleer, M. (2011). *Risk management of precious metals*. The Quarterly Review of Economics and Finance, 51(4), 435-441.

[3] Keynes, J.M.: (1936), *The General Theory of employment, Interest and Money*. London: Macmillan.

[4] Jabłońska, M.: (2011), *From fluid dynamics to human psychology. What drives financial markets towards extreme events*, Doctoral Dissertation, Lappeenranta University of Technology. Lappeenranta, Finland.

[5] "Silver In History".The Silver Institute .[Accessed 06 June 2014]. Available at https://www.silverinstitute.org/site/silver-essentials/silver-in-history/

[6] "Demand And Supply".The Silver Institute .[Accessed 06 June 2014]. Available at https://www.silverinstitute.org/site/supply-demand/

[7] Yardley,W.May 29, 2011."Utah Law Makes Coins Worth Their Weight in Gold (or Silver)". New York Times.[Accessed 06 March 2014]. Available at http://www.nytimes.com/2011/05/30/us/30gold.html?pagewanted=all&_r=0

[8] Davies,G "The history of money from Ancient Times to the Present" on line resource [Accessed 06 March 2014].Available at http://library.thinkquest.org/28718/history.html

[9] Ash,A. March 25,2010 "The Case for Silver" on line resource [Accessed 06 March 2014].Available at http://goldnews.bullionvault.com/silver_case_032520106

[10] MONEY LAW "The Coinage Act of April 2, 1792" Chapter XVI.–An Act establishing a Mint,and regulating the coins of the United States

[11] Chaize ,T (2005) Evolution of the ratio production and the price of the gold and the silver since 1900

[12] ScotiaMocatta.November 2012."Precious Metals 2013 Forecast"Silver

[13] Adrangi, B., Chatrath, A., & Raffiee, K. (2003). *Economic activity, inflation, and hedging*: the case of gold and silver investments. The Journal of Wealth Management, 6(2), 60-77.

[14] Aggarwal,R.,& Sundararaghavan, P. S. (1987). *Efficiency of the silver futures market: An empirical study using daily data*. Journal of banking & finance, 11(1), 49-64.

[15] Lashgari, M. K. (1992). *Information from successive changes in gold and silver prices during phases of the business cycles*. Journal of Economics and Finance, 16(2), 13-30.

[16] Pipattadanukul, W., & Chintrakarn, P. (2012). *Analyzing Long and Short-run Relationships Between Comex Gold and Silver Futures*. Journal of Applied Sciences, 12(7), 668-674.

[17] Wahab, M. (1995). *Conditional dynamics and optimal spreading in the precious metals futures markets*. Journal of Futures Markets, 15(2), 131-166.

[18] Harper, A., Jin, Z., Sokunle, R., Bank, U. I., & Wadhwa, M. (2013). *Price volatility in the silver spot market*: an empirical study using Garch applications.Journal of Finance & Accountancy, 13.

[19] Sari, R., Hammoudeh, S., & Soytas, U. (2010). *Dynamics of oil price, precious metal prices, and exchange rate*. Energy Economics, 32(2), 351-362.

[20] Mutafoglu, T. H., Tokat, E., & Tokat, H. A. (2012). *Forecasting precious metal price movements using trader positions*. Resources Policy, 37(3), 273-280.

[21] Arouri, M. E. H., Hammoudeh, S., Lahiani, A., & Nguyen, D. K. (2012). *Long memory and structural breaks in modeling the return and volatility dynamics of precious metals*. The Quarterly Review of Economics and Finance, 52(2), 207-218.

[22] Dunis, C. L., & Nathani, A. (2007). *Quantitative trading of gold and silver using nonlinear models*. Neural Network World, 17(2), 93

[23] Chang, S. K. (2007). *A simple asset pricing model with social interactions and heterogeneous beliefs*. Journal of Economic Dynamics and Control, 31(4), 1300-1325

[24] De Grauwe, P., & Rovira Kaltwasser, P. (2012). *Animal spirits in the foreign exchange market*. Journal of Economic Dynamics and Control, 36(8), 1176-1192.

[25] De Grauwe, P. (2011). *Animal spirits and monetary policy.Economic Theory*, 47(2-3), 423-457.

[26] Brock, W. A., Hommes, C. H., & Wagener, F. O. (2005). *Evolutionary dynamics in markets with many trader types.* Journal of Mathematical Economics, 41(1), 7-42.

[27] Bofinger, P., Debes, S., Gareis, J., & Mayer, E. (2013). *Monetary policy transmission in a model with animal spirits and house price booms and busts.* Journal of Economic Dynamics and Control, 37(12), 2862-2881.

[28] Aggarwal, R. (2014). *Animal spirits in financial economics: A review of deviations from economic rationality.* International Review of Financial Analysis, 32, 179-187.

[29] Hirshleifer, D., & Hong Teoh, S. (2003). *Herd behaviour and cascading in capital markets*: A review and synthesis. European Financial Management, 9(1), 25-66.

[30] Jabłonska, M., & Kauranne, T. (2012). *Animal spirits in population spatial dynamics.* In Parallel Problem Solving from Nature-PPSN XII (pp. 205-214). Springer Berlin Heidelberg.

[31] Bianchi, A., Capasso, V., Morale, D., & Sioli, F. (2003). *A Mathematical Model for Price Herding. Application to Prediction of Multivariate Time Series.* MIRIAM report (confidential), Milan.

[32] Capasso, V., Morale, D., & Sioli, F. (2003). *An agent-based model for "price herding", applied to the automobile market.* MIRIAM reports, Milan.

[33] Capasso, V., Bianchi, A., & Morale, D. (2005). *Estimation and prediction of a nonlinear model for price herding.* Complex Models and Intensive Computational Methods for Estimation and Prediction, Padova.

[34] Jabłońska, M., Nampala, H., and Kauranne, T. (2011), *Multiple mean reversion jump diffusion model for Nordic electricity spot prices*. The Journal of Energy Markets., 4(2) Summer 2011.

List of Tables

List of Figures