LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
School of Engineering Science
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KALMAN FILTER FOR COMPUTATIONAL MARKET DYNAMICS

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D.Sc. (Tech.) Matylda Jablonska-Sabuka

ABSTRACT

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Kalman Filter for Computational Market Dynamics

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Kalman filter is a recursive mathematical power tool that plays an increasingly vital role in innumerable fields of study. The filter has been put to service in a multitude of studies involving both time series modelling and financial time series modelling. Modelling time series data in Computational Market Dynamics (CMD) can be accomplished using the Jablonska-Capasso-Morale (JCM) model. Maximum likelihood approach has always been utilised to estimate the parameters of the JCM model. The purpose of this study is to discover if the Kalman filter can be effectively utilized in CMD. Ensemble Kalman filter (EnKF), with 50 ensemble members, applied to US sugar prices spanning the period of January, 1960 to February, 2012 was employed for this work. The real data and Kalman filter trajectories showed no significant discrepancies, hence indicating satisfactory performance of the technique. Since only US sugar prices were utilized, it would be interesting to discover the nature of results if other data sets are employed.

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List of Symbols and Abbreviations

CMD	Computational Market Dynamics
JCM	Jablonska-Capasso-Morale
KF	Kalman Filter
MCMC	Markov Chain Monte Carlo
GARCH	Generalized AutoRegressive Conditional Heteroskedasticity
GBM	geometric Brownian motion
ARCH	AutoRegressive Conditional Heteroskedasticity
DTARCH	Double-Threshold AutoRegressive Conditional Heteroskedasticity
HMM	Hidden Markov model
PDF	Probability density function
CDF	Cumulative distribution function
VaR	Value at risk
AVaR	Average value at risk
TS	Tempered Stable (distribution)
TID	Tempered Infinitely Divisible (distribution)
STAR	Smooth Transition Autoregressive (models)
BEKK	Baba, Engle, Kraft and Kroner (model)

1 INTRODUCTION

Kalman filter is an optimal estimator in the sense that parameters of interest are infered from indirect, inaccurate and uncertain observations. Its recursive nature allows new measurements to be processed as they arrive (Kleeman, 1996). The filter is so popular because of numerous reasons. Not only does it give good results in practice (due to optimality and structure), but also is a convenient form for online real time processing. Furthermore the filter is easy to formulate and implement given a basic understanding, and moreover measurement equations need not be inverted. The term filter is employed owing to the fact that the process of finding the "best estimate" from noisy data amounts to "filtering out" the noise. However, Kalman filter also projects data measurements onto the state estimate besides cleaning up these measurements.

This mathematical power tool is playing an increasingly vital role in innumerable fields of study. Its usage in computer graphics spans from simulating musical instruments in virtual reality (VR), to head tracking, to extracting lip motion from video sequences of speakers, to fitting spline surfaces over collections of points (Welch, 2001). Adding to the vast applications of this filter, we include determination of planet orbit parameters from limited earth observations, tracking targets such as aircraft, missiles using RADAR, robot Localisation and Map building from range sensors (Kleeman, 1996).

Kalman filter has been put into service in a multitude of studies, from time series at large to financial time series in particular, such as in the works of Srinivasan and Mitra (2012), Valadkhani and Araee (2013), Bit-Kun et al. (2010), and Dacass (2012). Srinivasan and Mitra (2012) apply the Kalman filter to estimate the natural rate of unemployment for Germany and France while Valadkhani and Araee (2013) employ this approach together with annual time series data spanning from 1959 to 2008, to present two estimates of the non-accelerating inflation rate of unemployment (NAIRU) for Iran. Additionally, Dacass (2012) simultaneosly estimates the natural rate interest and pontential output while wielding the Kalman filter technique. Lastly, Bit-Kun et al. (2010) examine the time-varying world integration of the Malaysian Stock Market as well as if the paths of the time-varying integration match the economic events of the country.

Various models are utilized in the representation of financial time series. By way of illustration, stochastic time-serie models, threshold time-series models, the Geometric Brownian motion, state-space models, and autoregressive conditional heteroscedastic models. Stochastic time-serie models of conditional heteroscedasticity, have been applied to explore the relationship between return volatility and trading volumes, such as in Bohl and Henke (2003), Darrat et al. (2003) and Omran and McKenzie (2000). A family of nonlinear threshold time-series models (which incorporate changes in logarithm of volumes), together with an implementation of MCMC procedure to obtain estimates of unknown parameters, was used by So et al. (2007). A new class of tree structured multivariate GARCH models was proposed for the the analysis of volatility and co-volatility asymmetries which appeared in financial time series (Dellaportas and Vrontos, 2007). The geometric Brownian motion (GBM) is a standard method for modelling financial time series whose parameters are assumed to be constant (Erlwein et al., 2012). Campagnoli et al. (2001) consider a class of conditionally Gaussian state-space models. Hasanov and Omay (2008) employ STAR family models for modelling monthly returns on stock exchange indices of the Athens Stock Exchange and Istanbul Stock Exchange since they allow to model nonlinearities in the conditional mean. Also, Autoregressive conditional heteroscedastic(ARCH) models together with its extensions are extensively employed in modelling volatility in financial time series. Therefore, to model the conditional variance and conditional mean that are piecewise linear, Van Hui and Jiancheng (2005) proposed the double-threshold autoregressive conditional heteroscedastic (DTARCH) model (one of the variants of ARCH models).

In computational market dynamics, one of the models utilized to model time series data is, locally, refered to as the JCM model (see 4 in section 4). This model preserves the most important trading biases and has proven to be efficient in reproducing statistical features of comodity markets' price dynamics as evidenced in the works of Jabłońska (2011), Gasana (2013) and Uwamariya (2012). However, estimates for the parameters of this model have always been obtained by the maximun likelihood method. Motivated by the efficacy of the JCM model and the popularity of Kalman filter (in parameter estimation), in this study, we impliment Kalman filter, on the JCM model to discover whether or not it can also be put to effective utilization in Computational Market Dynamics.

This document comprises of seven distinct sections. Section 2 provides a review of part of the work that has been done in the field of time series. Section 3 discusses the psychology behind price formation. Section 4 provides an overview of some existing Kalman filtering methods that are of much relevance to our study. We begin by presenting how the basic Kalman filter and extended Kalman filter work and proceed to ensemble filtering methods. Section 5 shall discuss the data employed in our study as well as present the results obtained after the implementation of the Kalman filter technique. The penultimate section (section 6) summarises and discusses the results obtained in section 5. Finally, section 7 terminates the whole study, gives final remarks and certain recommendations for future works.

2 LITERATURE REVIEW

Financial time series are continually brought to our attention. Daily news reports in newspapers, on television and radio inform us for instance of the latest stock market index values, currency exchange rates, electricity prices, and interest rates. It is often desireable to monitor price behavior frequently and try to understand the probable development of the prices in the future. Private and corporate investors, businessmen, anyone involved in international trade, the brokers and analysts who advice these people can all benefit from a deeper understanding of price behaviour. All traders deal with the risks associated with changes in prices. These risks can frequently be summarised by the variances of future returns, directly or by their relationship with relavant covariances in a portfolio context. Forecasts of future standard deviations can provide up-to-date indications of risk, which might be used to avoid unacceptable risks perhaps by hedging (Aas Kjersti, 2004).

There are two main objectives of investigating financial time series. First, it is important to understand how prices behave. The variance of the time series is particularly relevant. Tomorrow's price is uncertain and it must therefore be described by a probability distribution. This means that statistical methods are the natural way to investigate prices. Usually, one builds a model, which is a detailed description of how successive prices are determined. The second objective is to use our knowledge of price behaviour to reduce risk and take better decisions. Time series models may for instance be used for forecasting, option pricing and risk management (Aas Kjersti, 2004).

In this section, we provide a review of part of the work that has been done in the field of time series. We commence with work in the field of modelling financial time series, then we address the use of filtering methods for time series in general and lastly consider the utilization of filtering methods for financial time series in particular.

2.1 Modelling financial time series

Stochastic time-series models of conditional heteroscedasticity have been applied to explore the relationship between return volatility and trading volumes, such as in Bohl and Henke (2003), Darrat et al. (2003) and Omran and McKenzie (2000). A drawback of standard autoregressive conditional heteroscedasticity-type models is that the measure of volatility is assumed to be symmetric and thereby fails to take into account the asymmetric effects of positive and negative shocks to stock returns.

So et al. (2007) use an implementation of MCMC procedure to obtain estimates of unknown parameters of a family of nonlinear threshold time-series models. These models incorporate changes in logarithm of volumes. After analyzing five markets, three major observations were made. Firstly, an optimum selection can be achieved if differences of log-volumes are involved in the system of log-return and volatility models. Secondly, it is also noted that volumes play an important role in governing the regime changes. Finally, both trading volume and conditional variance are important when describing stock market risk.

There exist numerous data mining techniques. However, to forecast the foreign exchange time series process, Vojinovic et al. (2001) employ one of the relatively new data mining techniques. This is aimed at contributing to the development and application of such techniques by exposing them to difficult real-world (non-toy) data sets. A Radial Basis function Neural Netivork model and traditional linear auto regressive model are applied. It is revealed that the prediction of the former, for forecasting the daily $US/$NZ closing exchange rates, is significantly better than that of the latter. Furthermore, the prediction of the RBF NN dominates in both forecast of the exchange rates and directional change.

To rigorously analyze the convergence of a recursive estimation method for GARCH processes with restricted stability margin under reasonable technical conditions, Gerencsér and Orlovits (2012) use an appropriate modification of the theory of recursive estimation within a Markovian framework. It is noted that the best alternative to this method is the two pass recursive estimation method. The method exploits relatively simple structure of the dynamics of the model and this allows its inversion. Additionally, in more complex stochastic volatility models it may be obligatory to compute conditional expectations in order to compute the likelihood of a specific observation.

In order to understand the dynamics of the Indian and Saudi stock markets, Manchanda et al. (2007) conducted an analysis. Manchanda et al. (2007) looked for similarities, point of abrupt changes, normalized data, return, volatility, graph, pure and noise part, correlation lengths, and signal-to-noise ratio. To accomplish this, trends in the $S\&P500$ (The Standard and Poor's 500 Index) for various time periods using wavelet tools, are studied. After the analysis, it was found that highly volatile stocks should have lower correlation lengths (since they have more random elements in them) than that for more stable stocks. However, it was not observed in the case of $S\&P500$ which was expected. Lastly, it was recommended to investigate further the factors influencing the value of correlation length.

For the analysis of volatility and co-volatility asymmetries which appeared in financial time series, Dellaportas and Vrontos (2007) propose a new class of tree structured multivariate GARCH models. The model is based on a binary decision tree, according to some splitting rule on the time series, where every terminal node parametrizes a multivariate GARCH model. With the help of a sequence of binary decisions, the space of time series is partitioned recursively while dealing with these models. Different sections of the partition are represented by the terminal nodes of the tree, and splitting rules associated with the time series at each of the internal nodes dictate the partition. As a way of analyzing this proposed model, a Bayesian stochastic method is developed. It includes parameter estimation, model selection and volatility prediction. The method is then referred to as a Markov chain Monte Carlo stochastic search algorithm. After performing simulation experiments to assess the performance of this method, it results that the method is seemingly flexible and efficient. The reason for this being the fact that it reached the correct tree structures after at most 1000 iterations.

Another common concern in financial modeling is whether returns follow a normal distribution. To answer this query, Melas (2009) used the fact that if returns followed a normal distribution, then volatility would be a complete measure of risk. Examples of asset, factor and index returns where the assumption of normality does not capture the empirical properties of returns and volatility alone is not a reliable measure of portfolio risk were used. For empirical envidence, a price series of a large pharmaceutical company was employed. The comparison of the simulated series showed how extreme value theory can help to more accurately model the tails of the return distribution. The implication here is that large positive and negative returns have a higher empirical likelihood of appearing than the normal distribution would demand.

The Geometric Brownian motion (GMB) is a standard method for modelling financial time series whose parameters are assumed to be constant. Erlwein et al. (2012) suggested an approach where the parameters of the GMB are able to switch between regimes, more precisely, they are governed by a hidden Markov chain. According to Erlwein et al. (2012), a hidden Markov model (HMM) is a particular regime-switching model, in which there are two stochastic processes involved: apart from the one of interest, that is observable (e.g asset prices), there is an underlying stochastic process describing the system's state over time, that is not observable, that is, "hidden". Moreover, once optimal parameters are estimated using historical information of the prices, scenario paths can be generated according to the parameter estimates and the filtered transition probability of the underlying Markov chain estimation of parameters. These parameters are estimated using a parameter estimation filtering approach. As a result, the scenario generator based on independent observations gives a good in-sample stability. HMM (3 states) gives a better representation of the data, especially with respect to unfavourable events.

In their study, Folpmers (2009) show that a straightforward model to identify local extremes in a financial index can be applied to a subsequent period as a trading algorithm. Even more, it is showed how a straightforward algorithm can detect regularities in the timing of upswings and downswings and how a trading routine can capture the benefits of these moments of exit and entry. Theoretically, the algorithm is vindicated by the short-term behaviour of the index as an oscillation with predictable frequency. The interpretation, economically, of this behaviour relates to the overshooting and mean reversion properties of the financial time series. The modelling of a financial time series as a short-term trendless oscillation with predictable frequency is the fundamental mechanism of the algorithm. The economic rationale of the algorithm can be explained by referring to the well-known overshooting and mean reversion properties of financial time series, which we apply here to index prices. The two main Euronext indicies, the AEX index and CAC40 index, were used for testing the algorithm. Consequently, it was noted that the algorithm performs better than the buy and hold investment strategy noticeably in both cases.

Campagnoli et al. (2001) consider a class of conditionally Gaussian state-space models. Even more, a discussion of how they can provide a flexible and fairly simple tool for modelling financial time series, even in the presence of different components in the series, or of stochastic volatility. Furthermore, discussion is also done on how some models traditionally employed for analysing financial time series can be regarded in the state-space framework. In this study, it was also mentioned that estimation can be computed by recursive equations. These provide the optimal solution under rather mild assumptions but in more general models, the filter equations can still provide approximate solutions.

Since nonlinearities in financial variables in developing economies had not been completely explored yet, Hasanov and Omay (2008) dedicated their effort towards the investigation of potential nonlinearities in conditional mean of many economic and financial variables, principally concentrating in developed economies. This investigation was conducted on stock returns in Europe's two largest emerging stock markets, chiefly in the Greek and Turkish stock markets. In the investigation, STAR family models were employed for modelling monthly returns on stock exchange indices of the Athens Stock Exchange and Istanbul Stock Exchange since they allow to model nonlinearities in the conditional mean. Consequently, strong evidence in favour of nonlinear adjustment of stock returns is found but no nonlinearity in conditional variance. Besides, discovery is made that for a superior model and good out-of-sample forecasts, allowing for nonlinearity in conditional mean is vital (a contradtion to efficient market hypothesis). Also, it was found that the linear models for both the Turkish and Greek stock markets exhibit nonlinearities in the conditional mean but only the Greek stock market displays nonlinearities in the conditional variance. Additionally, the Greek stock market exhibits multiple nonlinearity in the mean. Nonetheless, once the nonlinearity in the mean is taken into account, no nonlinearity is found for the variance (Hasanov and Omay, 2008).

Particularly, Chavez-Demoulin et al. (2005) studied estimation of value-at-risk for return series as a measure of market risk, and tail estimation for financial time series in general. An approach that models within cluster behaviour is suggested which is based on an extension of the classical POT model involving a self-exciting process for the exceedance times. This form of the self-exciting process allows realistic models in which distant events have less effect on the current intensity than recent ones, and which allows reliance of the intensity on the event size. After application of the test to several real financial time series of different types (index data, currency exchange data, share prices data), it was revealed that "the method correctly estimates the conditional quantiles" hypothesis was never rejected on any occassion. Hence it seems that our market point process approach for the excess over high threshold provides a reasonable model for the extremal behavior of the negative log returns of the types studied here.

The considerable empirical evidence that financial returns exhibit leptokurtosis and nonzero skewness has made it possible to suggest alternative distributions for modelling a time series of the financial returns. The tempered stable and tempered infinitely divisible distributions forms a class of distributions that has shown considerable promise for modelling financial returns but two representative distributions are the classical tempered stable and the Rapidly Decreasing Tempered Stable (RDTS) (Scherer et al., 2012). Scherer et al. (2012) explain the pragmatic implementation of these two distributions. This is done by illustrating (firstly) how the density functions can be computed efficiently by employing the Fast Fourier Transform (FFT) and (secondly) how efficiency and effectiveness of maximum likelihood inference is aided by standardization. Additionally, an efficient approximation for the PDF, CDF,VaR and AVaR of TS and TID distributions is also outlined. Furthermore, to compute density and distribution functions, the FFT method is employed with basis on knowledge of the characteristic function since this procedure is arguably computationally efficient. Also, it is explained why standardization is vital for the parameter selection of the FFT and presented the way the standardized CTS and RDTS can be employed to attain PDF values for any parameterization. A two-step for a near optimal FFT parameter selection is then suggested for the PDF of CTS distribution where in each step a regression model is used to determine one optimal parameter. Consequently, empirical results using $S\&P$ 500 return data supported the two main theoretical results for the CTS case. These two are (first) applying the FFT method to the CTS distribution delivers good approximation quality and (second) using standardization improves the effectiveness of the MLE.

Jiancheng et al. (2001) studies ARCH modelling for the conditional scale. L_1-estimation of ARCH models is examined and the limiting distributions of the estimators derived. It also proposes a robust absolute residual autocorrelation based on least absolute deviation estimation. As a way of testing the adequacy of the model, markedly the specification of the conditional scale, a robust portmanteau statistic is constructed. Also their asymptotic distributions under mild conditions are obtained. Both suggested L_1-norm estimators and the goodness-of-fit are robust against error distributions as well as correct for samples of moderate size. In addition, Jiancheng et al. (2001) renders a beneficial tool in modelling conditional heteroscedastic time series data.

Autoregressive conditional heteroscedastic(ARCH) models together with its extensions are extensively employed in modelling volatility in financial time series. To model the conditional variance and conditional mean that are piecewise linear, Van Hui and Jiancheng (2005) proposed the double-threshold autoregressive conditional heteroscedastic (DTARCH) model (one of the variants of ARCH models). Another use of the DTARCH model, among many others, is to model conditional heteroscedasticity having nonlinear structures; for instance, asymmetric cycles, jump resonance and amplitude-frequence dependence. Furthermore, Van Hui and Jiancheng (2005) asserts that without supposing specific distribution, it is worth studying robust DTARCH modelling given the fact that asset returns frequently display heavy tails and outliers. In Van Hui and Jiancheng (2005), DTARCH structures for conditional scale is studied in lieu of conditional variance. In order to accomplish the above examination of L_1-estimation of the DTARCH model is conducted together with derivation of limiting distributions for the suggested estimators. Moreover, in oder to test the model competence, a robust portmanteau statistic is constructed. In addtion, the approach which is based on the L_1-norm fit captures various nonlinear phenomena and stylized facts having desirable robustness. After simulations, it is evident that the L_1-estimators are robust against innovation distributions and accurate for a moderate sample size. It is further noted that the suggested test in addtion to being robust against innovation distributions is also highly effective in discriminating the delay parameters and ARCH models. Finally, it was also observed that the quasi-likelihood modelling approach used in ARCH models is inappropriate to DTARCH models in the presence of outliers and heavy tail innovations.

Kasch-Haroutounian and Price (2001) models returns from four emerging equity markets of central europe (Poland, Hungary, Slovakia and Czech Republic) econometrically. Before turning to univariate as well as multivariate GARCH models of volatility, which have proven to be specifically suited for modelling the behaviour of financial time series, the authors first gathered statistical properties of returns. Equally important is that they are capable of capturing the three most common empirical observations in daily return data, namely leptokurtosis, skewness, and volatility clustering. Resultingly strong GARCH effects are visible in all series examined, estimates of asymmetric models of conditional volatility show rather weak envidence of asymmetries in the markets, the outcomes of the multivariate specifications of volatility possess implications for comprehending the parttern of information flow between the markets. Also it was observed that there was signifcant conditional correlations between two pairs of countries: Hungary and Poland, and Hungary and Czech Republic. Additionally, though the BEKK model of multivariate volatility indicated proof of return volatility spillovers from Hungary to Poland, there existed no evidence of volatility spillover effects in the counter direction (Kasch-Haroutounian and Price, 2001).

2.2 Use of filtering methods for time series

Srinivasan and Mitra (2012) apply the Kalman filter to estimate the natural rate of unemployment for Germany and France. The annual data employed for Germany and France spans the period 1955-2010 and is the survey based measure as reported in OECD's Main Economic Indicators. When the hysteresis theory (equilibrium unemployment rate is dependent on the history of actual unemployment rate) is tested against the alternative of a moving natural rate model, the hysteresis model is resoundingly rejected. Hence it appears that possibly a large part of the rise in unemployment in Germany and France can be attributed to a rise in the natural rate.

There already exist estimates of Iran's time varying non-accelerating inflation rate of unemployment (NAIRU) in literature but Valadkhani and Araee (2013) purposely provides more accurate estimates. To be more specific, Valadkhani and Araee (2013) employs the the Kalman filter approach together with annual time series data spanning from 1959 to 2008, to present two estimates of the NAIRU for Iran. As a result, the estimated two measures appear robust and consistent as regards their magnitude and pattern, having a more logical upper limit of 11.1 percent. Furthermore, the results clearly show that overall Iran's NAIRU has been on the rise since the 1960s regardless of which of the two models is considered. In addition whenever unemployment rate lies below the NAIRU, the rate of inflation has manifested an explosive bahaviour which was noticed in both 1995-1996 and the post 2006 era. However, the NAIRU's upper limit ranges from 14 to 20.7 per cent according to previous studies, which happens to be an over-estimate. Finally, Valadkhani and Araee (2013) concludes that such incredible high rates are due to the over-estimation affiliated with misspecification erros in their model.

Patuelli et al. (2012) present a novel econometric method to the study of regional unemployment persistence, for the sake of accounting for spatial heterogeneity and/or spatial autocorrelation in both the dynamics and the levels of unemployment. First, Patuelli et al. (2012) suggests an econometric approach proposing the employment of spatial filtering techniques as a replacement for fixed effects in a panel estimation framework. Not to mention the spatial filter computed, there is proxy for spatially distributed region-specific information which is usually embedded in the fixed effects coefficients. The procedure was chosen inasmuch as spatial filter, by incorporating region-specific information that generates spatial autocorrelation, frees up degrees of freedom, simultaneously corrects for time-stable spatial autocorrelation in the residuals, and provides insights about the spatial patterns in regional adjustment processes. For the purpose of investigating the spatial pattern of the heterogeneous autoregressive coefficients (estimated for unemployment data for German NUTS-3 regions used), numerous experiments were presented. Consequently, widely heterogeneous but generally high persistence was noticed in regional unemployment rates.

2.3 Use of filtering methods for financial time series

Lopes and Tsay (2011) review sequential Monte Carlo methods (SMC), or particle filters (PF), putting special accentuation on its application in financial time series analysis and econometrics. After the review, Lopes and Tsay (2011) also finally argued that, after almost two decades, SMC methods now belong in the toolbox of researchers and practitioners in many areas of modern science, ranging from signal processing and target tracking to robotics, bioinformatics and financial econometrics.

Bit-Kun et al. (2010) examine the time-varying world integration of the Malaysian Stock Market as well as if the paths of the time-varying integration match the economic events of the country. Bit-Kun et al. (2010) used the weekly time series data for the period between February 1988 and September 2009 which corresponds exactly with the liberalisation of the Malaysian market since the late 1980s. Then the Kalman filter technique, which produces time-varying coefficients in estimating international Capital Asset Pricing Model (ICAPM), is utilized to capture the time-varying degree of market integration. After all, it is found that the capital reform and capital control measures that were imposed by the Malaysian government had an effect on the levels of integration of the Malaysian stock market with the global market. Finally, results also show that the Malaysian stock market was dismembered from the world market during the 1997 to 1998 Asian financial crisis yet is now closely integrated with the world market during a time of global crisis.

Through the exponential smoothing filter, Guerrero and Galicia-Vázquez (2010) seeks to decompose a financial time series into trend plus noise. It is also noted that the estimates of the trend generated by this filter are statistically efficient and can be computed by a straightforward utilization of the Kalman filter. Hence, Kalman filter was applied to execute the needed calculations for the trend estimation thereby avoiding probable numerical difficulties due to inverting large dimensional matrices.

Motivated by the proposition that the natural rate of interest should perform a significant role in the conduct of monetary policy (Economic theory), Dacass (2012) tries to identify the probable significance of the natural rate of interest for monentary policy in Jamaica. With this in mind, Dacass (2012) simultaneosly estimates the natural rate interest and pontential output and employs the Kalman filter technique. To explain, the Kalman filter technique was utilized to a state-space model of the economy to approximate these unobserved variables for the period 1990 to 2011. All in all, it was established that not only is there anticorrelation between the interest rate gap and inflation but also the impact of the interest rate gap on inflation was found to be weak and the interest rate gap affects inflation with a lag.

3 COMPUTATIONAL MARKET DYNAMICS

This section is dedicated to discussing the psychology behind price formation. First, we present a model that is used for modelling animal populations (price herding). Secondly, we discuss the transformations that have been made to this model, the Capasso-Bianchi system of stochastic differential equations. Finally, the outcome of all the discussed transformations, locally refered to as the JCM model (a model that preserves the most important trading biases), is presented.

3.1 Background and motivation

Since price formation for any commodity is always a process involving a group of traders, an intuitive analogy would be to treat traders as individuals of a bigger population. For one thing, the price can be treated as the measure of their distance (Jabłońska and Kauranne, 2012; Jabłońska, 2011).

A Capasso-Bianchi system of stochastic differential equations, used for modeling animal populations (price herding), in which the movement of each particle k in the population of N individuals is based on the location of each individual with respect to the whole population $f(X_t^k)$, as well as its local interaction with the nearest neighbours $h(k, X_t)$, is in its general form (Jabłońska and Kauranne, 2012; Bianchi et al., 2005; Morale et al., 2005):

$$dX_N^k(t) = [f(X_t^k) + h(k, X_t)]dt + \sigma * dW^k(t), \quad (1)$$

for $k = 1 \dots N$.

A connection of mathematical biology with financial time series modeling has been proposed, by studying a phenomenon called *price herding*. The proposed model was not able to generate jumps which are an important characteristic of the real prices. As a result, spikes were generated through separate jump processes dependent on the price level (Jabłońska, 2011).

Alternatively, researchers have continuously been facing Efficient Market Hypothesis failure as a result of the presence of the *momentum* in the financial markets. The momentum effect is consistently too strong to be a simple market anomaly. Chiefly, clients are attracted to invest more money where managers hold the most well known, liked and valuable stocks. To put it differently, investors simply buy stocks just because their price has risen. So then, the money again goes into the same investments and additionally boosts shares that are performing well already. In brief, the momentum effect consists of the fact that the individual prices deviate from the global tendency due to the traders' pursuit of an extra profit (Jabłońska, 2011).

Furthermore, physical analogy to the momentum phenomenon that can be found in the fluid dynamics; the Burgers equation (2), is thereupon applied (Jabłońska, 2011).

$$u_t + \alpha u u_x + \alpha u_{xx} = f(x,t) \tag{2}$$

The following analogies surfaced as regards to the market dynamics (Jabłońska, 2011):

1. u specifies the price,
2. $f(x,t)$ denotes the fundamentals (of a periodic character),
3. αu_{xx} is the diffusion term related to the fact that the market tends to reach an equilibrium price,
4. uu_x is the momentum term expressing traders' movement towards higher prices.

Likewise, the jump processes have been eliminated and the global interaction as Burger's type momentum component $h(k, X_t)$ was introduced in the model and thus, the model became Jabłońska (2011);

$$dX_t^k = [\gamma_t(X_t^* - X_t^k) + \theta_t(h(k, X_t) - X_t^k)]dt + \sigma_t dW_t^k \tag{3}$$

Where $h(k, X_t) = M(X_t).[E(X_t) - M(X_t)]$ and $M(X)$ connotes the mode of a random variable X. And θ_t represents the strength of that global interaction at time t and is allowed to be different from the mean field force γ. And,

1. X_t^k is the price of the trader k at time t,
2. X_t^* is the global price reversion level at time t,
3. γ_t is the mean reversion rate at time t,
4. X_t is the vector of all traders' prices at time t,
5. W_t^k is the Wiener process value for trader k at time t,
6. σ_t is the standard deviation for Wiener increment at time t.

By and large, the model was extended, with the goal of accounting for the main components of the Capasso-Bianchi population dynamics model, into a model that preserves the major trading biases.

3.2 Jablonska-Capasso-Morale (JCM) model

For decades researchers have been trying to use econometric models to predict the behaviour of financial markets. These could work very well for “perfect markets”, that is such ones that have clear periodicities and never collapse. However, reality is far from this. Markets often change local trends unexpectedly, and that is what makes classical econometric models of not much use (Gasana, 2013; Jabłońska, 2011; Jablonska and Kauranne, 2011).

The major source for those failures was determined to be *animal spirits*, a term suggested by John Maynard Keynes in 1936. These animal spirits in financial markets, also called trading biases, refer in principle to human psychological factors influencing traders’ behaviour in the markets. Fear and greed were identified as the most general and at the same time the most important ones (Keynes, 2006).

With focus on the above stated observations, extensive research has been conducted and a model that preserves some of the trading biases was developed. This model, commonly referred to as the Jablonska-Capasso-Morale (JCM) model, accounts for the main components of the Capasso-Bianchi population dynamics model. Mathematically, the model is written as (Jabłońska, 2011; Jabłońska and Kauranne, 2012);

$$dX_t^k = [\gamma_t(X_t^* - X_t^k) + \theta_t(h(k, X_t) - X_t^k) + \xi_t(g(k, X_t) - X_t^k)]dt + \sigma_t dW_t^k \quad (4)$$

where

- X_t^k is the price of the trader k at time t,
- X_t^* is the global price reversion level at time t,
- γ_t is the mean reversion rate at time t,
- X_t is the vector of all traders’ prices at time t,
- W_t^k is the Wiener process value for trader k at time t,

- the new component $g(k, X_t)$ represents the maximally distant member of k^{th} trader's neighborhood, formed by the closest $p\%$ of the population.
- $h(k, X_t) = M(X_t) \cdot [E(X_t) - M(X_t)]$ and $M(X)$ stands for the mode of a random variable X.
- θ_t represents the strength of the global interaction at time t and is permited to be different from the mean field force γ.
- σ_t is the standard deviation for Wiener increment at time t.

In a nutshell, the above model is comprised of four main components, namely;

- **Global mean** which connotes the herding phenomenon and is related to the aggregation forces suggested by Morale et al. (2005)
- **Momentum**. In the work of Couzin et al. (2005) it has been noticed that when a sufficiently large subgroup of the whole population exhibits significantly deviant behaviour the momentum effect is expected to occur.
- **Local interaction** which denotes that each individual will follow the furthest neighbour within a range that caters for the closest $p\%$ of the whole population.
- **Randomness**. A Wiener increment is then included in each and every individual move.

4 Filtering techniques

In this section, we provide an overview of some existing Kalman filtering methods that are of much relevance to our study. We begin by presenting how the basic Kalman filter and extended Kalman filter work and proceed to ensemble filtering methods.

4.1 Kalman filter

The Kalman filter (see, Kalman, 1960) can be used to estimate the state x_k at discrete times k from observations y_k, when the model and observation equations are linear:

$$\mathbf{x}_k = \mathbf{M}_k x_{k-1} + \epsilon_k^p \quad (5)$$

$$\mathbf{y}_k = \mathbf{K}_k x_k + \epsilon_k^o. \quad (6)$$

In the above system, M_k is the $d \times d$ evolution model and K_k is the $m \times d$ observation operator. The $d \times 1$ vector x_k represents the model state, and the observed data are denoted by the $m \times 1$ vector y_k. The model error ϵ_k^p and the observation error ϵ_k^o are assumed to be normally distributed zero mean random variables with covariance matrices $C_{\epsilon_k^p}$ and $C_{\epsilon_k^o}$, respectively. The Kalman filter algorithm for estimating states and their error covariances is presented in Algorithm 1 (Solonen, 2011; Solonen et al., 2012; Särkkä, 2013);

Algorithm 1 Kalman filter

- Initialize $\mathbf{x}_0^{\text{est}}$ and $\mathbf{C}_\mathbf{0}^{\text{est}}$
- For $k = 1, 2, \cdots$
  - Move the state estimate and covariance in time:
    - Compute $x_k^p = \mathbf{M}_k x_{k-1}^{est}$.
    - Compute $\mathbf{C}_k^p = \mathbf{M}_k \mathbf{C}_{k-1}^{est} \mathbf{M}_k^T + \mathbf{C}_{\epsilon_k^p}$.
  - Combine the prior with observations:
    - Compute the Kalman gain $\mathbf{G}_k = \mathbf{C}_k^p \mathbf{K}_k^T (\mathbf{K}_k \mathbf{C}_k^p \mathbf{K}_k^T + \mathbf{C}_{\epsilon_k^o})^{-1}$.
    - Compute the state estimate $x_k^{est} = x_k^p + \mathbf{G}_k(\mathbf{y}_k - \mathbf{K}_k x_k^p)$.
    - Compute the covariance estimate $\mathbf{C}_k^{est} = \mathbf{C}_k^p - \mathbf{G}_k \mathbf{K}_k \mathbf{C}_k^p$.
  - k ← k+1

4.2 Extended Kalman Filter

The extended Kalman filter (EKF, Särkkä, 2013) is the extension of KF to nonlinear optimal filtering problems by forming a Gaussian approximation to distribution of states and measurements using a Taylor series expansion. Algorithm 2 represents the extended Kalman filter algorithm.

The extended Kalman filter directly uses the Kalman filter formulas in the nonlinear case by replacing the nonlinear model and observation operators with appropriate linearizations: $\mathbf{M}_k = \frac{\partial}{\partial x}\mathbf{M}(x_{k-1}^{est})$ and $\mathbf{K}_k = \frac{\partial}{\partial x}\mathbf{K}(x_k^p)$.

Algorithm 2 Extended Kalman filter

The prediction and update step for EKF with additive noise are

- Model equation

$$\mathbf{x}_k = \mathbf{M}_k x_{k-1} + \epsilon_k^p \tag{7}$$

$$\mathbf{y}_k = \mathbf{K}_k x_k + \epsilon_k^o. \tag{8}$$

- Prediction step

$$\begin{aligned} \mathbf{x}_k^- &= \mathbf{f}(\mathbf{x}_{k-1}) \\ \mathbf{P}_k^- &= \mathbf{F}_x(\mathbf{x}_{k-1})\mathbf{P}_{k-1}\mathbf{F}_x^{\mathrm{T}}(\mathbf{x}_{k-1}) + \mathbf{Q}_{k-1} \end{aligned} \tag{9}$$

- Update step:

$$\begin{aligned} \mathbf{S}_k &= \mathbf{H}_x(\mathbf{x}_k^-)\mathbf{P}_k^-\mathbf{H}_x^{\mathrm{T}}(\mathbf{x}_k^-) + \mathbf{R}_k \\ \mathbf{K}_k &= \mathbf{P}_k^-\mathbf{H}_x^{\mathrm{T}}(\mathbf{x}_k^-, t)\mathbf{S}_k^{-1} \\ \mathbf{x}_k &= \mathbf{x}_k^- + \mathbf{K}_k\left(\mathbf{y}_k - \mathbf{h}(\mathbf{x}_k^-)\right) \\ \mathbf{P}_k &= \mathbf{P}_k^- - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^{\mathrm{T}}, \end{aligned} \tag{10}$$

where $\mathbf{F}_x(\mathbf{x}, t)$ is the Jacobian matrix of $\mathbf{f}(\mathbf{x}, t)$, $\mathbf{H}_x(\mathbf{x})$ is the Jacobian matrix of $\mathbf{h}(\mathbf{x})$.

4.3 Ensemble Kalman Filter

The ensemble Kalman filter (EnKF) which was first proposed by Evensen (2003, 2009) is a stochastic or Monte Carlo alternative to the EKF. It solves the problems of dimensionality and nonlinearity suffered by EKF. Like KF, there are two steps in EnKF: prediction step (forecast step) and update step (analysis step). In the prediction step ensemble of forecast states are computed, and are used to compute the sample mean and error covariances. The Kalman gain is computed from these sample mean and error covariances and it is used to assimilate the measurements to produce the analysis ensemble states. For a linear model the EnKF converges exactly to the KF with increasing ensemble size.

There are various versions of EnKF that differ in the computation of update ensemble (Tippett et al., 2003; Wang et al., 2007). The EnKF can be stochastic filter or deterministic filter, depending to the added vectors. In stochastic case, the EnKF uses Kalman gain together with random perturbations while in the deterministic case the EnKF uses a non-random transformation on the forecast ensemble. The *perturbed observation filter* is the EnKF where the measurement ensemble is created by adding random vectors to actual measurements. In the update step, the intuition is to use the Kalman gain to combine the forecast ensembles, measurement and measurement noise.

Now consider a bunch of N-dimensional random vectors $s_{k,i} \sim \mathcal{N}(x_k^{est}, C_k^{est})$, where $k \in \mathbb{N}$, $i = 1, \ldots, S$, and S is the ensemble cardinality. Consider an N-by-S matrix $\mathbf{X}_k$ depending on $s_{k,i}$, which is defined by the following:

$$\mathbf{X}_k = \left((s_{k,1} - \bar{s}_k), \ldots, (s_{k,S} - \bar{s}_k) \right) / \sqrt{S-1}. \tag{11}$$

Here $\bar{\mathbf{s}}_k = \frac{1}{S} \sum_{i=1}^{S} \mathbf{s}_{k,i}^p$ denotes the mean of ensemble $s_{k,i}$. The perturbed ensemble Kalman filter algorithm can be written as algorithm 3 (Solonen, 2011);

Algorithm 3 Perturbed ensemble Kalman filter

- Prediction step
  1. Propagate each ensemble member forward using a stochastic model

$$\mathbf{s}^{p}_{k,i} = \mathbf{f}(\mathbf{s}^{\text{est}}_{(k-1),i}) + \mathbf{q}_{k,i}$$

  2. Compute sample mean and sample covariance

$$\bar{\mathbf{s}}_k = \frac{1}{N}\sum_{i=1}^{N}\mathbf{s}^{p}_{k,i}$$
$$\mathbf{C}^{p}_{k} = \mathbf{X}_k\mathbf{X}_k^{\mathrm{T}},$$

     where

$$\mathbf{X}_k = \frac{1}{\sqrt{N-1}}\left[(\mathbf{s}_{k,1}-\bar{\mathbf{s}}_k),(\mathbf{s}_{k,2}-\bar{\mathbf{s}}_k),\ldots,(\mathbf{s}_{k,N}-\bar{\mathbf{s}}_k)\right]$$

- Update step
  1. Compute the Kalman gain

$$\mathbf{K}_k = \mathbf{C}^{p}_{k}\mathbf{H}^{\mathrm{T}}_{x}(\mathbf{x}_k)\left(\mathbf{H}_x(\mathbf{x}_k)\mathbf{C}^{p}_{k}\mathbf{H}^{\mathrm{T}}_{x}(\mathbf{x}_k)+\mathbf{R}_k\right)^{-1}$$

  2. Update ensembles

$$\mathbf{s}^{\text{est}}_{k,i} = \mathbf{s}^{p}_{k,i} + \mathbf{K}_k\left(\mathbf{y}_k - \mathbf{H}_x(\mathbf{x}_k)\mathbf{s}^{p}_{k,i} + \mathbf{r}_k\right)$$

  3. State estimate

$$\mathbf{x}_k = \frac{1}{N}\sum_{i=1}^{N}\mathbf{s}^{\text{est}}_{k,i}$$

5 SIMULATION RESULTS

In this section we shall discuss the data employed in our study as well as present the results obtained after the implementation of the Kalman filter technique. We will commence with the case study data discussion and then present the

JCM model together with the KF simulation results.

5.1 Case study data

The data utilized in this study is a representative of a set of monthly US sugar prices realised from January 1960 to February 2012 for 626 observations. This data was extracted from the World Bank Comodity Price Data (Pink Sheet), in cents (US dollars)/kg. However, this data can easily be obtained from the World Bank website.

Figure 1 gives a visual of nearly 700 (626 observations) consecutive monthly US sugar prices, covering 52 years from January, 1960 to February, 2012. We can observe peaks and troughs which do occur at irregular time intervals, to be more specific they are random. From the figure we can note that there is a general slight positive secular trend with random variation. The volatility seems to be larger in the latter years of the data. The highest peak is realised towards the 200th observation.

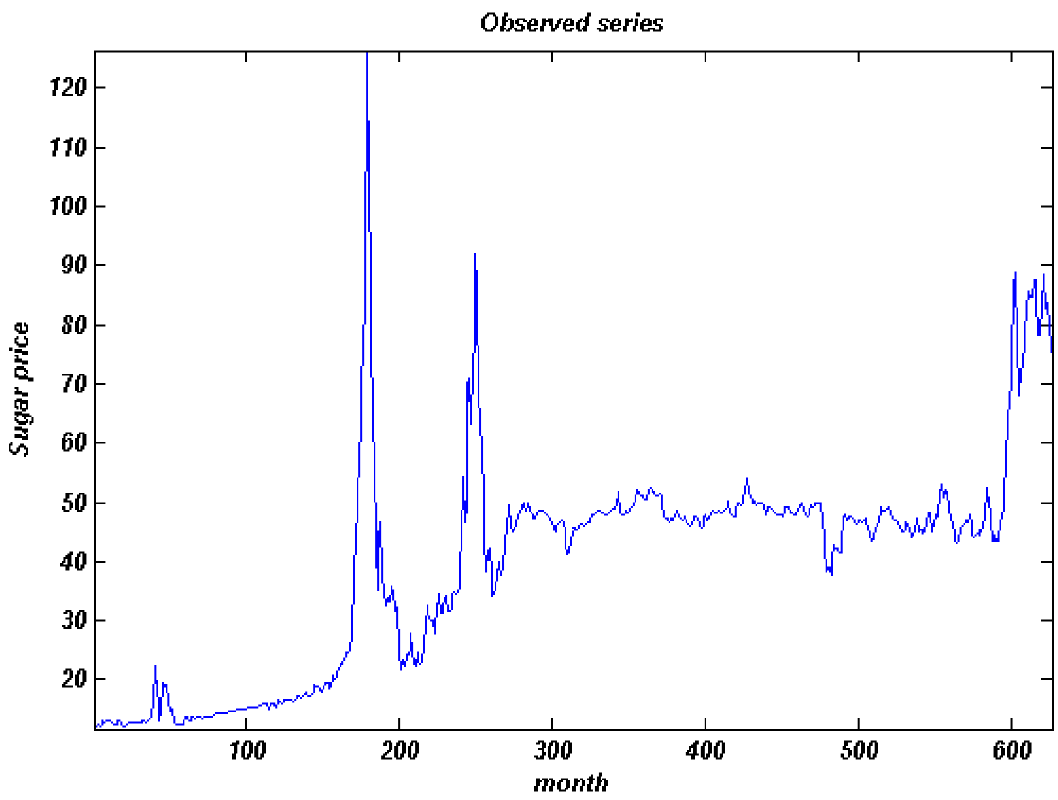


Figure 1: Time plot of monthly US sugar prices from from January, 1960 to February, 2012

To study the distribution of the data in question, we plotted a histogram (Figure 2). The data shows no tendency of being around a central value with no bias left or right, and hence is far from a "Gaussian Distribution".

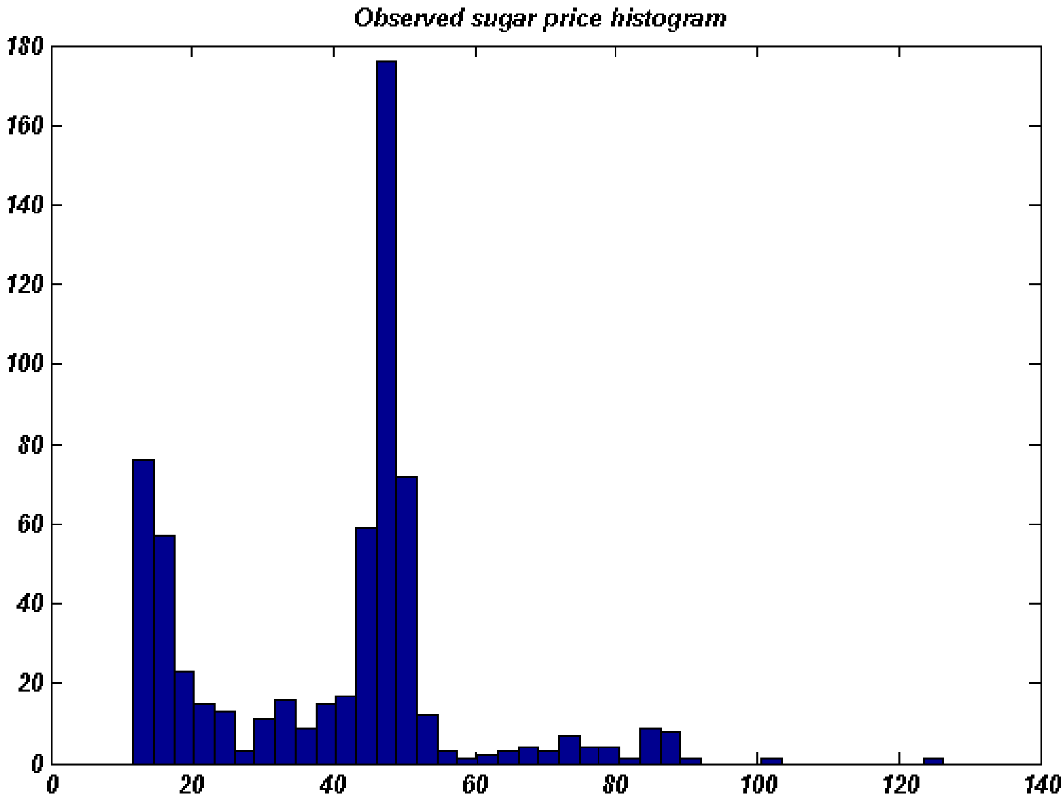


Figure 2: A plot of the observed sugar price histogram

Basic statistics of the data is summarized in the Table 1. From Table 1 it can be seen that the mean values for both the price returns and price logarithmic returns are zero, to the nearest whole number. Also, the standard deviation values suggest that the series in the logarithmic returns are less deviated. Additionally, we can argue that the original sugar prices illustrate less flat tails and shorter peaks on basis of the kurtosis and skewness values. However, the price returns present worse distribution than the sugar price log returns due to the fact that the skewness value of the price returns is further from 0 (which also means further from symmetry) as well as it is kurtosis value further from 3 (which would also mean more extreme observations). Finally, it is indicated that all the distributions are asymmetric and right skewed since the skewness values are all nonnegative and bigger than zero.

Table 1: Basic statistics of sugar prices

	Mean	Std	Skewness	Kurtosis
Sugar price	39.7225	18.3770	0.3730	3.6615
Price returns	0.0999	3.3277	2.156	46.6240
Price logarithmic returns	0.00123	0.0259	0.5072	10.7330

5.2 Kalman results

Figure 3 shows the simulated JCM series of monthly US sugar prices for 626 time points. We observe peaks and troughs which do occur at irregular time intervals. We can note that there is a general slight positive secular trend with random variation. The volatility, as well, seems to be larger in the latter years of the data. The highest peak is realised towards the 200th observation.

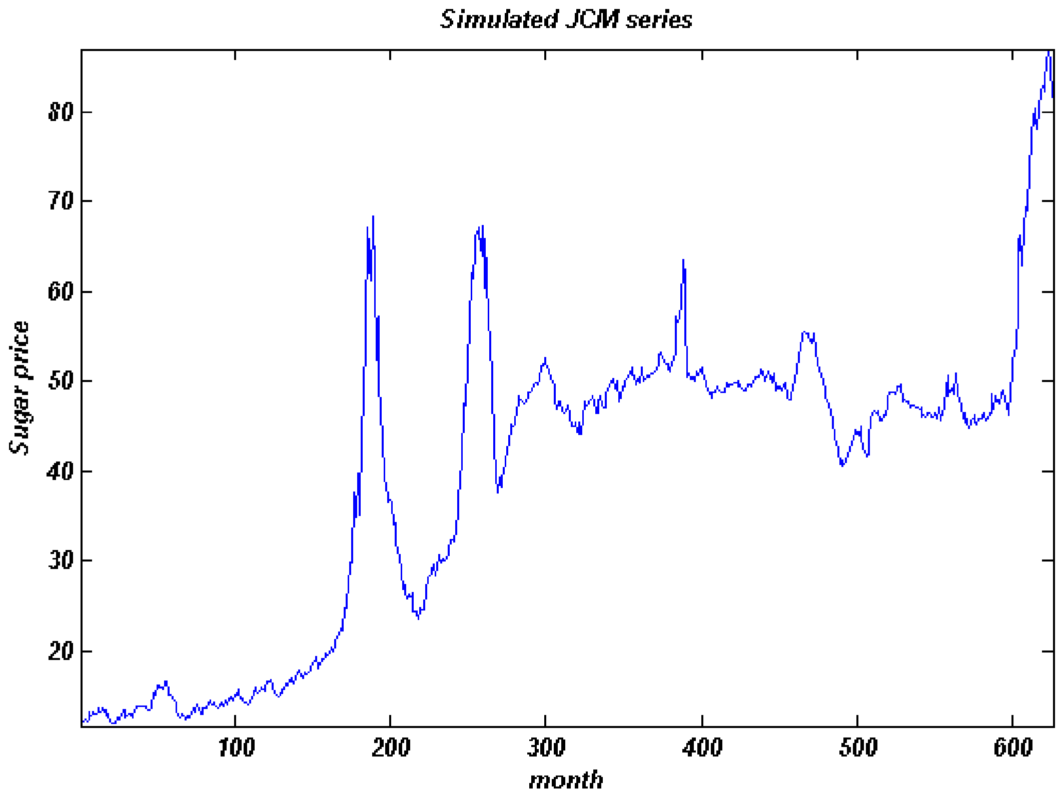


Figure 3: A plot of the simulated JCM series

The Figure 4 shows the simulated KF ensemble. This figure is synonymous to Figures 3 and 1 in the sense that ensemble members follow each other quite closely. The members realize peaks almost simultaneously with the highest value of about 110. In general the ensemble members exihibit an upward trend.

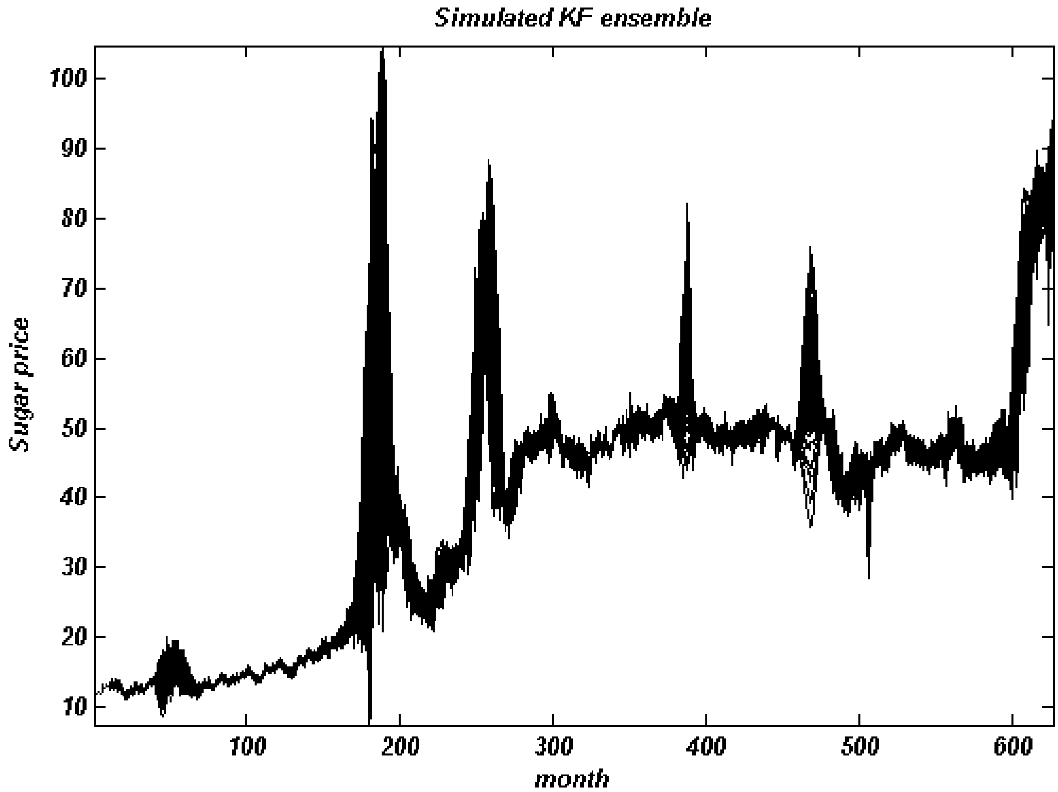


Figure 4: A plot of the simulated KF ensemble

Figure 5 illustrates that the simulated JCM series follows the simulated KF series quite closely. This is envidenced by the fact that it almost appears as the mean of the KF simulated ensemble. The highest peak of the ensemble is significanly higher than that of the JCM estimated series and two peaks occur at different observations.

Figure 6 represents the ensemble spread as given by the JCM simulated series. As seen from Figure 6, there is no indication of whether the spread generally decreases with time or not. What is clearly observed is that most of the spread values fall below 10. From the figure one would conclusively argue that the spread values are random and the spikes or peaks can not be predicted. We can not easily tell when the next peak will happen. The peaks are realised around the 40th, 100th, 110th, 240th, 320th, 460th observations with highest peak, which occurs about the 40th observation, having a value of slightly over 80.

Figure 7 shows the behaviour of the ensemble spread computed from the Kalman filter results. The spread values are the difference between the highest

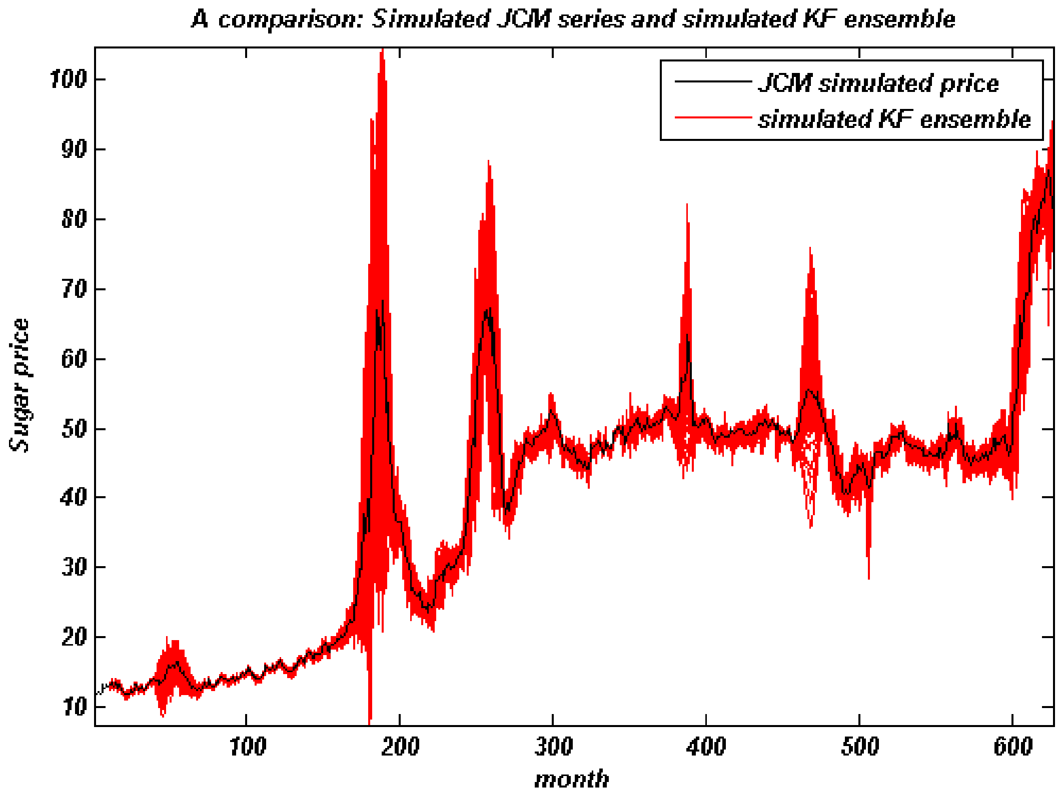


Figure 5: A comparison: Simulated JCM series and simulated KF series

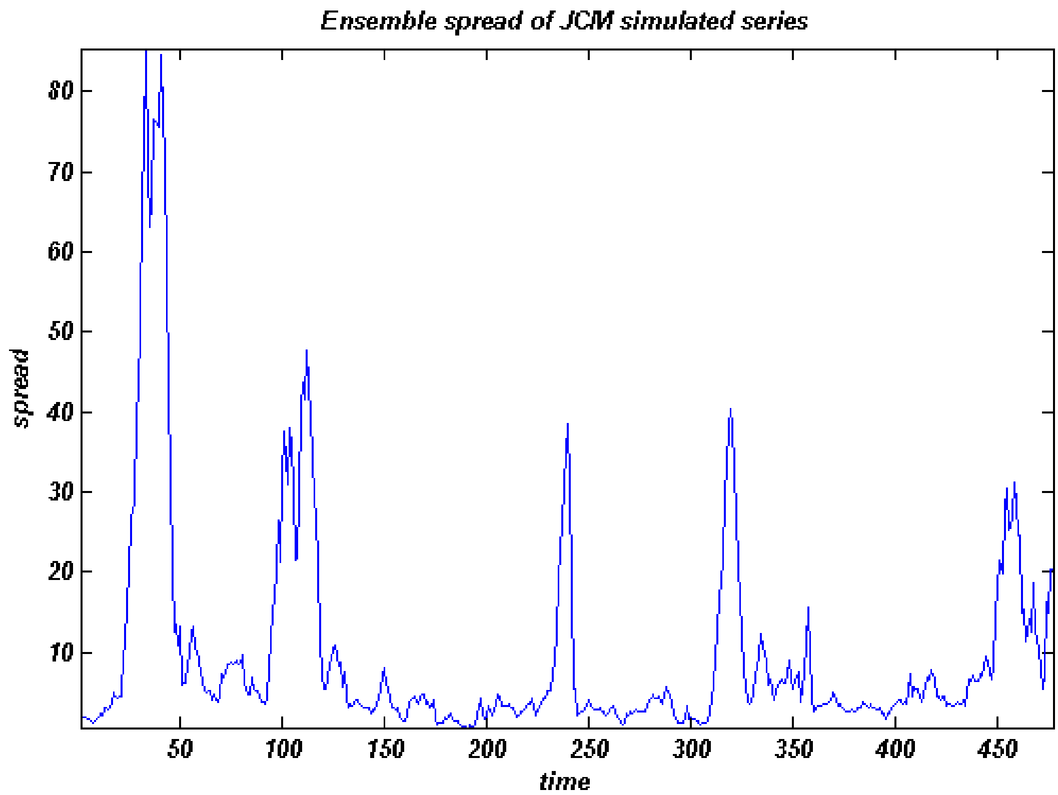


Figure 6: Ensemble spread for the JCM simulated series

and the lowest member values of the ensemble at each time point. Most of the spread values lie below 10 as expected. About 8 spikes are realised but unpredictable as well (highly random). The highest peak value for the spread is about 95.

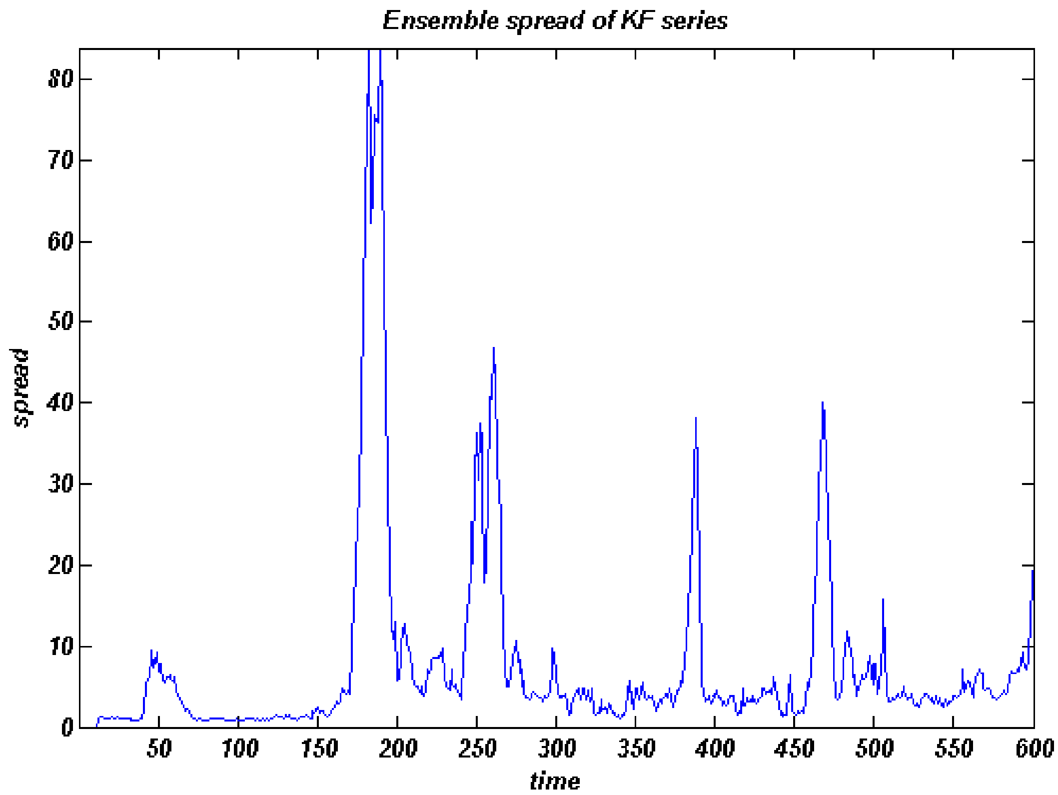


Figure 7: Ensemble spread for the KF simulated series

Figure 8 is a visualisation for the comparison between the oberved price (blue line), the simulated price as given by the JCM model (the black line) as well as the JCM model ensemble (green shade). Generally, the JCM simulated price follows the original price quite closely though the peaks happen at slightly different times. The highest peak, as well as the second highest, for the original price occurs a little before that for the JCM simulated price together with the ensemble.

Figure 9 shows a simultaneous plot of the original price (blue line), KF simulated price (black line) and KF model ensemble (red shade). From the plot it can be seen that the three move relatively in unison but the original price appears to be more volatile.

Figure 10 indicates the time plots of the two ensemble spreads with the green line denoting the JCM ensemble spread and the red line the KF ensemble

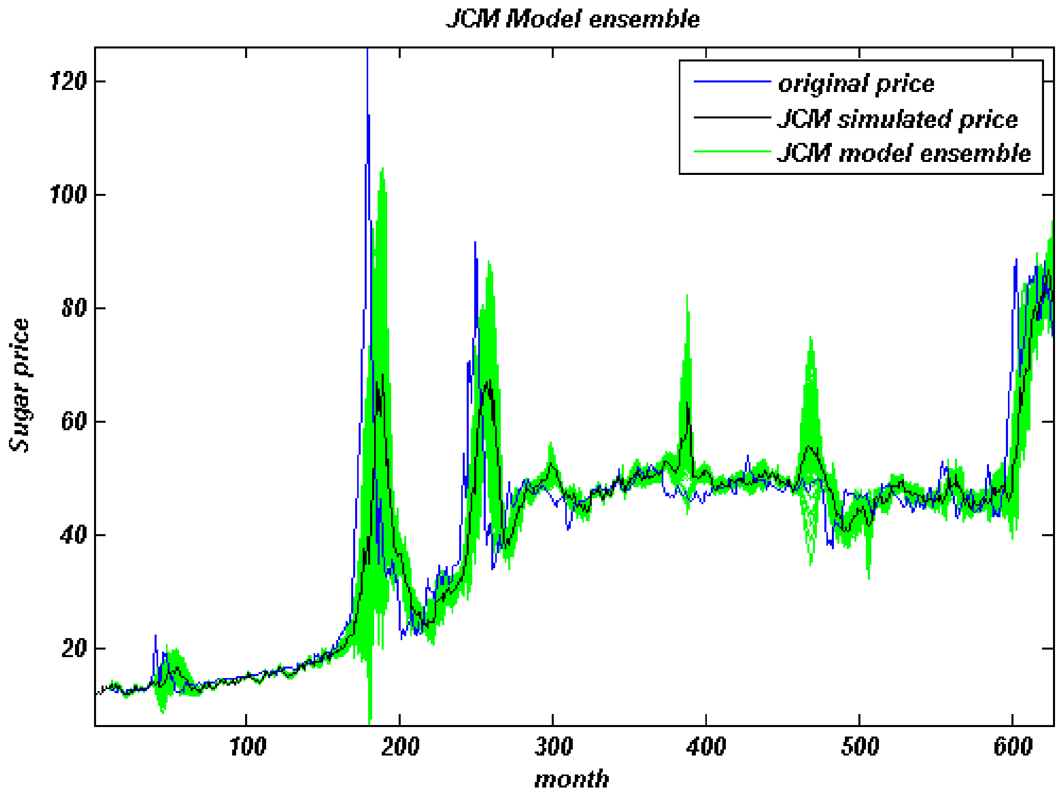


Figure 8: Observed price, JCM simulated price and JCM ensemble

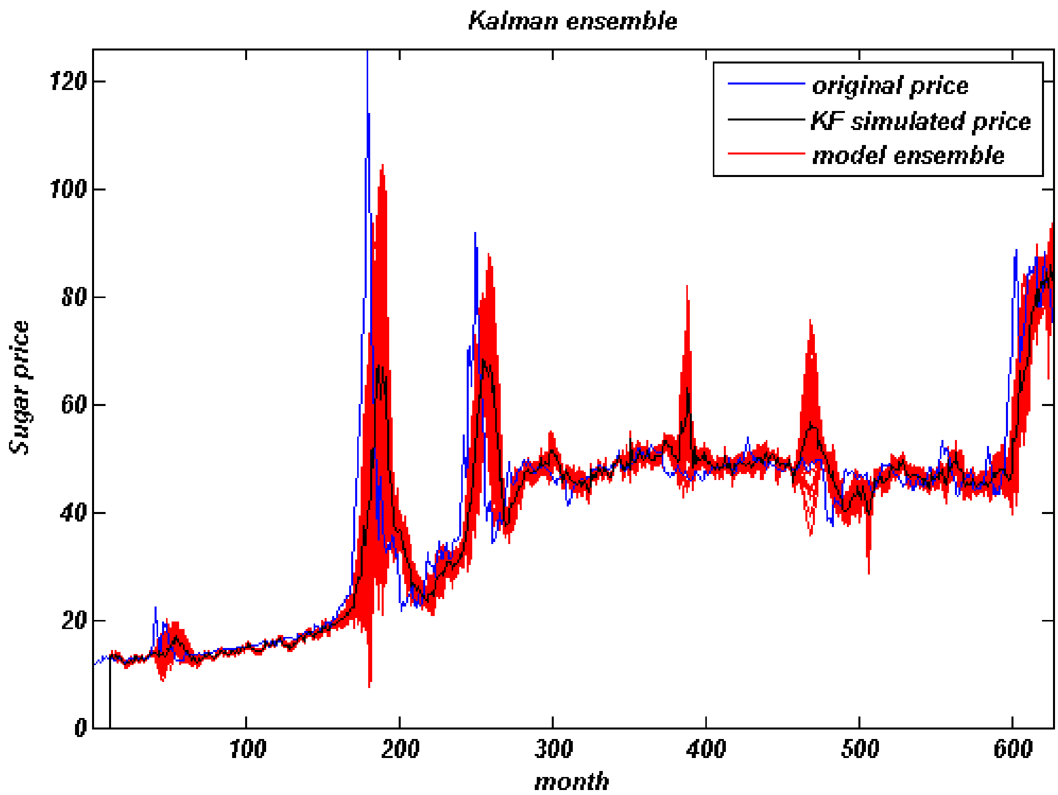


Figure 9: Realised price, KF simulated series and KF ensemble

spread. As can be seen the two ensemble spreads exhibit similar patterns and are in the same magnitude.

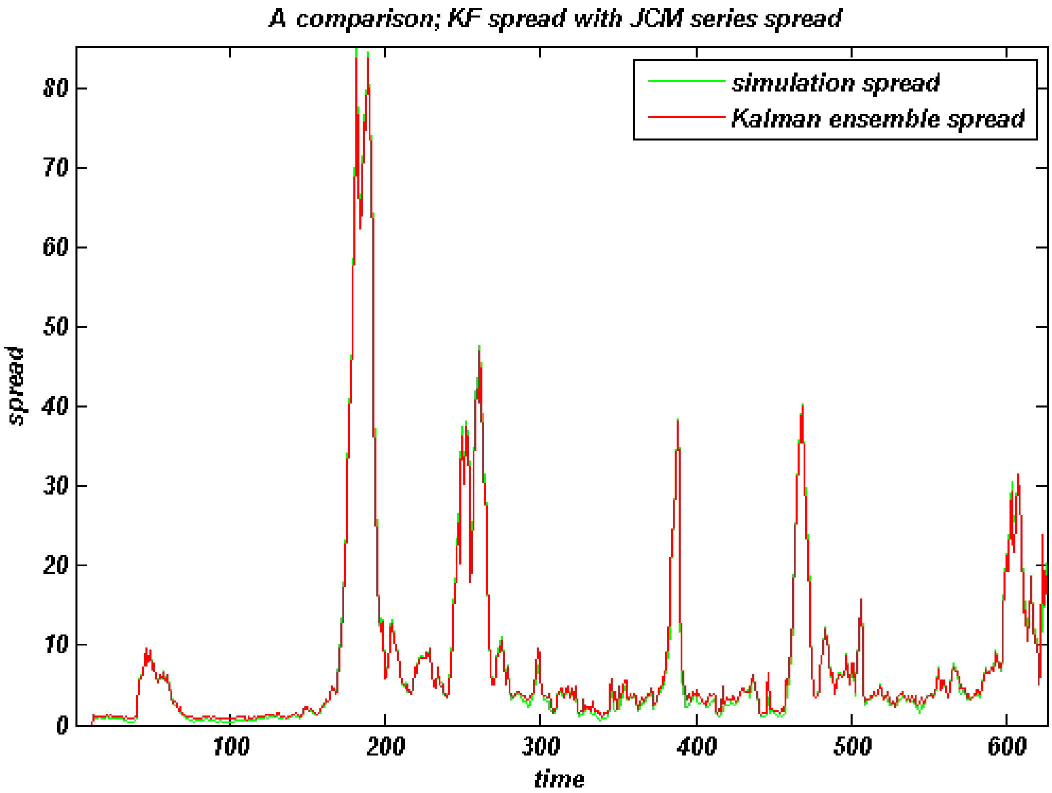


Figure 10: KF ensemble spread and JCM simulated ensemble spread

Figure 11 represents a whole ensemble of histograms (red shade) together with the realised price histogram (blue bars), as regards the JCM model simulation. In the green color, is a curve that illustrates the distribution of the JCM model price estimates or simply the ensemble mode histogram. As can be viewed, the ensemble envelope does almost cover the original price histogram satisfactorily. Hence the discrepancy between the shapes of the histograms of true data and the ensemble is negligible.

Just like figure 11, figure 12 again presents a full ensemble of histograms (red shade) as well as the true price histogram (blue bars), but with correspondence to the KF simulation results. The green curve indicates the ensemble (KF ensemble) mode histogram. It is also conveyed by figure 12 that the inconsistency between the shapes of the histograms of original data and the ensemble is insignificant since the ensemble envelope quite covers the real price histogram suitably.

A simultaneous comparison of the series (original sugar price, JCM final price

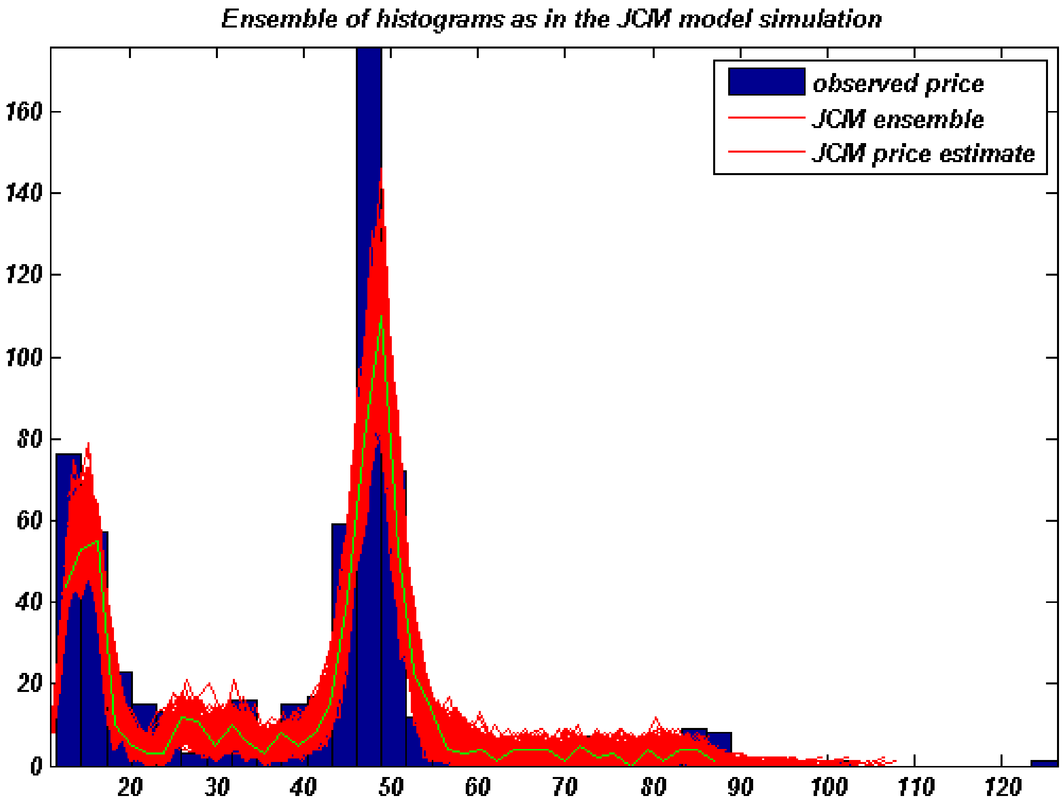


Figure 11: Observed price histogram and the JCM model ensemble histograms

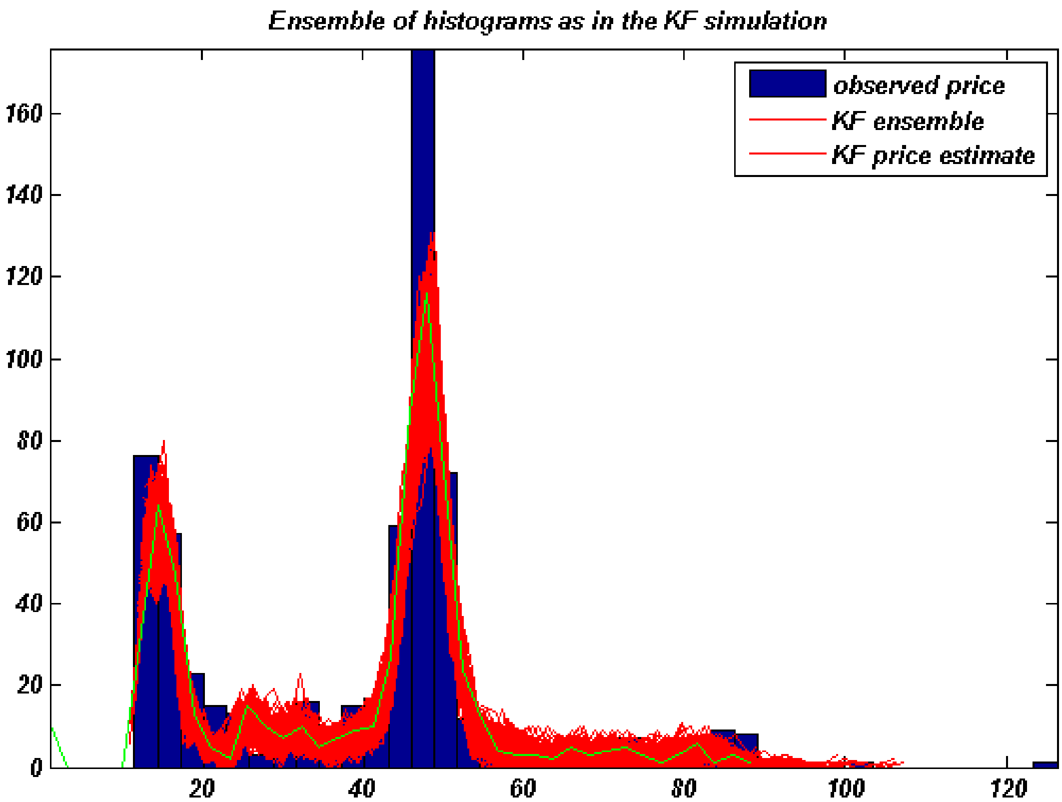


Figure 12: Observed price histogram and the KF ensemble histograms

Table 2: A statistical comparison

	Mean	Std	Skewness	Kurtosis
Original Sugar price	39.7225	18.3770	0.3730	3.6615
Final JCM price	39.1360	17.2380	-0.1183	2.3568
Final KF price	38.6249	17.5636	-0.1750	2.5116

and KF final price) yielded the following results. The values of the basic statistics (mean, standard deviation, skewness and kurtosis) for the original sugar price, final JCM price and final KF price are almost indistinguishable (table 2). Also, the trajectories for the three price series appear to be almost in unison (see figure 13). Furthermore, from the autocorrelograms of these three price series (figure 14), we observe that there exists both positive and negative ACFs with the transition from positive to negative being realised slightly after the 200^{th} lag. Moreover, only one highly significant spike is noticed from the partial correlograms in all the three cases (figure 15), together with a few other spikes of relatively very low significance.

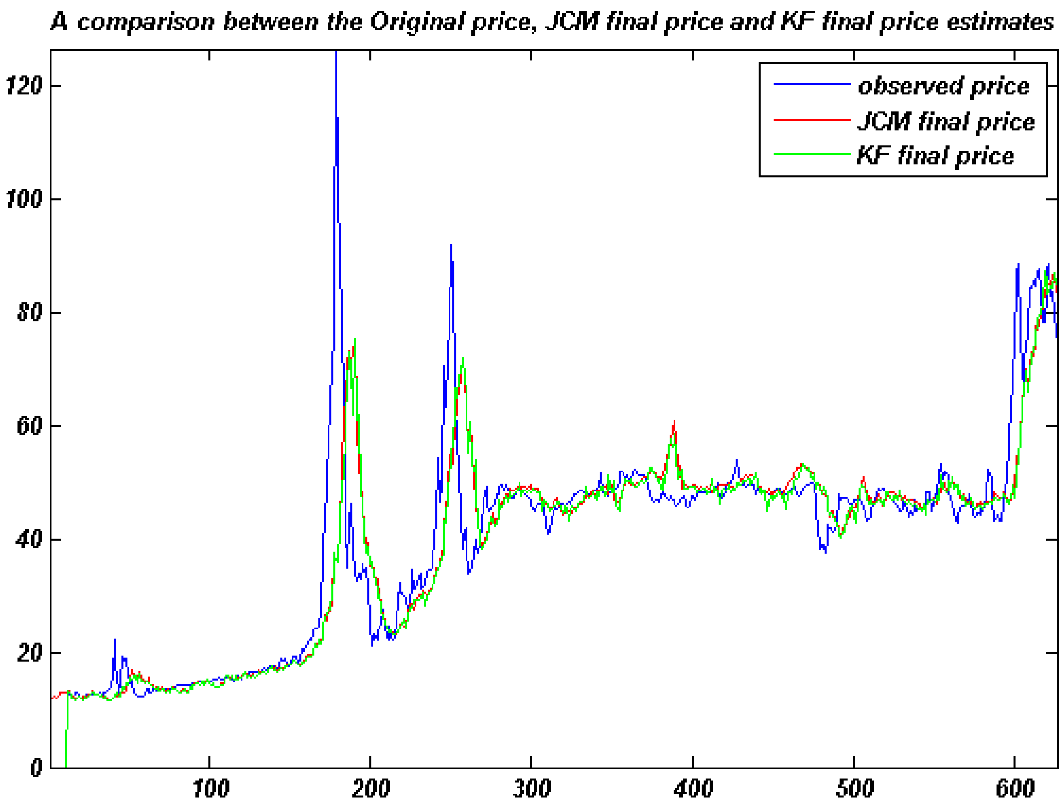
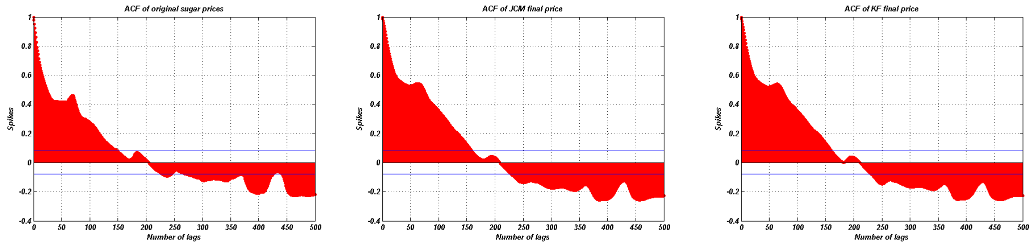


Figure 13: Comparison; Original sugar price, JCM final price and KF final price

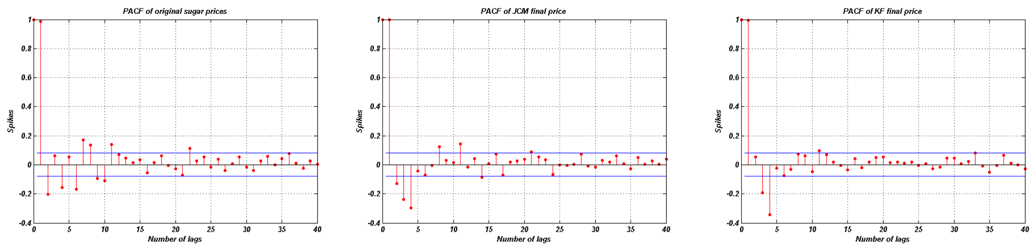


(a) ACF of original sugar price

(b) ACF of JCM final price

(c) ACF of Kalman Filter final price

Figure 14: ACF comparison



(a) PACF of original sugar price

(b) PACF of JCM final price

(c) PACF of Kalman Filter final price

Figure 15: PACF comparison

6 RESULTS SUMMARY AND DISCUSSION

This penultimate section summarises and the discusses the results obtained in section 5.

The JCM model, locally named so, was used. The maximum likelihood was used to obtain the model parameter values. These parameter estimates were used as initialisations for the KF technique. We then implimented the KF approach, ensemble Kalman filter, with 50 ensemble members. All the required simulations and visualisations were acquired by use of the MATLAB computer language.

We used a set of monthly US sugar prices. To visualise this set of data we presented a time plot (figure 1). From the figure we observed a general positive secular trend. Also, we noted that the data was evidently non-Gaussian (figure 2). From the basic statistics (table 1), we discovered three most important revelations. Firstly, it was revealed that the series in the logarithmic returns are less deviated. Secondly, it was brought to attention that price returns presented worse distributions than the sugar log-returns. Thirdly, it was apparent that all the distributions were asymmetric and right skewed.

From a visual study of the KF ensemble (figure 4), it is clear that ensemble members follow each other quite closely. A visual inspection of the ensemble spread shows that these are random and most of the values fall below 10. The plot of the original price and estimated price from KF, shows that these two move relatively in unison. Hence the KF performs satisfactorily. Also, the discrepancies betweeen the shapes of the histogram of true data and the ensembles is negligible. Hence good performance of the KF method.

7 CONCLUSIONS

The purpose of this study was to apply the Kalman filter technique (ensemble Kalman filter) to CMD, particularly on the JCM model. The major aim was to discover how Kalman filter performs on the JCM model. For this discovery, we used ensemble Kalman filter with 50 ensemble members. This was applied to US sugar prices spanning the period of January, 1960 to February, 2012. The results, that is to say figures and tables, were obtained with the help of the MATLAB computer programm (matlab codes of the JCM model and KF).

Consequently, it was found that the technique performs just well since there were no significant differences between the real data and the Kalman filter trajectories. This result was in line with other Kalman filter studies, for instance Srinivasan and Mitra (2012), Valadkhani and Araee (2013), and Guerrero and Galicia-Vázquez (2010), where Kalman filter works satisfactorily. However, the results obtained in this study are limited to only the US sugar prices. Therefore, we would recommend the employment of Kalman filter on the JCM model with other data sets such as electricity prices, gold prices and many more.

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