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Fusion of Data from Quadcopter's Inertial Measurement Unit Using Complementary Filter

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Bachelor's thesis 5.10.2015
LUT Institute of Energy Technology
Electrical Engineering

ABSTRACT

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A quadcopter is a helicopter with four rotors, which is mechanically simple device, but requires complex electrical control for each motor. Control system needs accurate information about quadcopter's attitude in order to achieve stable flight. The goal of this bachelor's thesis was to research how this information could be obtained.

Literature review revealed that most of the quadcopters, whose source-code is available, use a complementary filter or some derivative of it to fuse data from a gyroscope, an accelerometer and often also a magnetometer. These sensors combined are called an Inertial Measurement Unit. This thesis focuses on calculating angles from each sensor's data and fusing these with a complementary filter.

On the basis of literature review and measurements using a quadcopter, the proposed filter provides sufficiently accurate attitude data for flight control system. However, a simple complementary filter has one significant drawback – it works reliably only when the quadcopter is hovering or moving at a constant speed. The reason is that an accelerometer can't be used to measure angles accurately if linear acceleration is present. This problem can be fixed using some derivative of a complementary filter like an adaptive complementary filter or a Kalman filter, which are not covered in this thesis.

TIIVISTELMÄ

Lappeenrannan teknillinen yliopisto
LUT School of Energy Systems
Sähkötekniikka

Esa Malinen
Nelikopterin inertiaalisen mittausyksikön anturien datan yhdistäminen käyttämällä komplementtisuodinta

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Kandidaatintyö.
22 s.

Tarkastaja: TkT Joonas Talvitie

Nelikopteri on neliroottorinen helikopteri, joka on mekaanisesti yksinkertainen laite, mutta vaatii monimutkaisen sähköisen säädön jokaiselle moottorille. Stabiilin lennon aikaansaamiseksi säätöjärjestelmä tarvitsee tarkan tiedon nelikopterin asennosta. Tämän kandidaatintyön tavoite oli tutkia kuinka tämä tieto voidaan tuottaa.

Kirjallisuustutkimuksen perusteella selvisi, että suurin osa nelikoptereista, joiden lähdekoodi on saatavilla, käyttää komplementtisuodinta tai jotain siitä johdettua suodinta yhdistämään gyroskoopin, kiihtyvyysanturin ja magnetometrin tuottamat datat. Näiden sensorien yhdistelmää kutsutaan inertiaaliseksi mittausyksiköksi. Tämä tutkielma keskittyy kulmien laskemiseen yksittäisten anturien datasta ja näiden kulmien yhdistämiseen komplementtisuotimella.

Kirjallisuustutkimuksen ja nelikopterilla tehtyjen mittausten perusteella ehdotettu suodin vaikuttaa tuottavan riittävän tarkkaa tietoa nelikopterin asennosta säätöjärjestelmälle. Yksinkertaisessa komplementtisuotimessa on kuitenkin yksi merkittävä heikkous – se toimii luotettavasti vain silloin kun nelikopteri leijuu paikallaan tai liikkuu tasaisella nopeudella. Tämä johtuu siitä, että kiihtyvyysanturin avulla ei pystytä mittaamaan kulmia tarkasti, jos siihen kohdistuu lineaarista kiihtyvyyttä. Ongelma voidaan korjata käyttämällä jotain komplementtisuotimesta johdettua suodinta, kuten mukautuvaa komplementtisuodinta tai Kalman-suodinta, joita ei ole käsitelty tässä tutkielmassa.

CONTENTS

LIST OF SYMBOLS AND ABBREVIATIONS

a	Filter parameter, acceleration
g	Gravity
H	Magnetic field strength
t,T	Time
α	Magnetic inclination angle
Θ	Pitch
Φ	Roll
Ψ	Yaw
ω	Frequency
δ	Angular position
$\dot{\delta}$	Angular velocity
3-D	Three-dimensional
CCW	Counter-clockwise
CW	Clockwise
ESC	Electronic Speed Controller
EWMA	Exponentially weighted moving average
FCU	Flight Control Unit
HPF	High-pass filter
IC	Integrated circuit
IMU	Inertial Measurement Unit
LPF	Low-pass filter
MEMS	Microelectromechanical systems
PCB	Printed Circuit Board
PDB	Power Distribution Board
RF	Radio Frequency

Subscripts

a	Accelerometer
c	Cut-off
g	Gyroscope
HP	High-pass
LP	Low-pass
m	Magnetometer
n	Nominal
s	Sample
tc	Tilt compensated
x	X-axis
y	Y-axis
z	Z-axis

1. INTRODUCTION

A quadcopter is an unmanned multi-rotor helicopter with four arms (figure 1.1), each of which have a motor and a propeller at their ends (QuadcopterHQ 2013). It is controlled by changing speed of the individual motors rather than changing pitch of the blades like in traditional helicopter. This difference makes a quadcopter mechanically simple device, but instead it needs complex electrical control for each motor.

Figure 1.1. A quadcopter done as a coursework at Lappeenranta University of Technology. (Kärkkäinen 2015).

Multirotor aircrafts, such as the quadcopter, are rapidly growing in popularity. In fact, quadcopters have become a standard platform for robotics research worldwide. They are highly maneuverable and enable safe and low-cost experimentation in mapping, navigation and control strategies for robots that move in three-dimensional space. Small quadcopters have been demonstrated for exploring and mapping 3-D environments; transporting, manipulating and assembling objects; and acrobatic tricks such as juggling, balancing, and flips. (Mahony et al. 2012)

Quadcopter's ability to fly is based entirely on information about its attitude (angular position in three dimensions). Such information is normally obtained by using two or three sensors – a gyroscope, an accelerometer and optionally a magnetometer. The goal of this thesis is to provide a fundamental solution to fuse the data from these sensors in order to obtain accurate information about attitude for quadcopter's control system. Research is mainly done by reviewing current literature related to the subject. The quadcopter presented in figure 1.1 is used to confirm functionality of the provided solution in a limited extent.

The quadcopter is built as a student project which is a part of two courses: “Electronics, Laboratory Course 1” and “Electronics, Laboratory Course 2”, which are organized by Lappeenranta University of Technology. The purpose of these laboratory courses is to design and build a prototype of an electrical device. This thesis is a byproduct of the quadcopter prototype’s design process.

2. QUADCOPTER

As illustrated in figure 2.1, quadcopter’s four motors are driven by Electronic Speed Controllers (ESC), which generate three phase AC current from battery’s DC current. Speed of the motors is adjusted by changing frequency of this AC current. Frequency generated by individual ESC is controlled by Flight Control Unit (FCU). FCU receives commands wirelessly from the pilot flying the copter via radio frequency (RF). This RF system consists of RF transmitter (pilot’s controller) and RF receiver, which is connected to FCU.

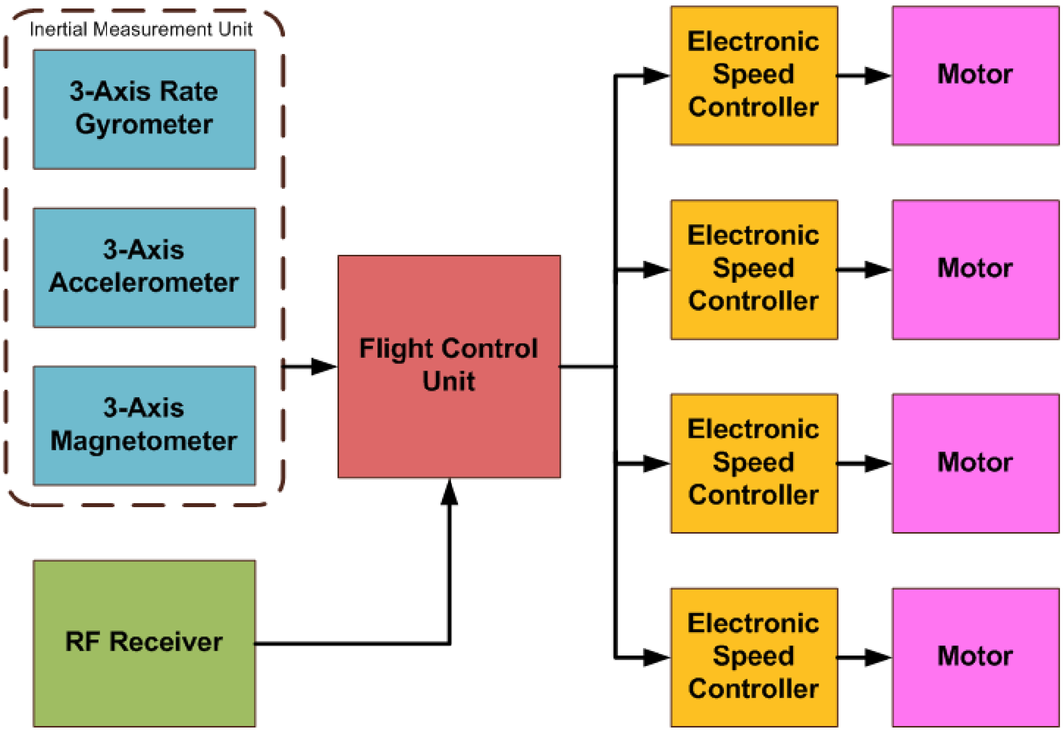

Figure 2.1 Necessary parts for controlling a quadcopter. Supply power to these parts comes from a Power Distribution Board, which is connected to a battery.

Quadcopters are aerodynamically unstable, which is why they can’t be controlled just by changing speed of the motors linearly with respect to the control sticks in the RF controller. This is why an Inertial Measurement Unit (IMU) has to be used to create a feedback loop for removing error between quadcopter’s real orientation and reference orientation from pilot’s controller.

IMU used in quadcopters is small IC (integrated circuit) sensor combination, which usually consist of a three-axis gyroscope, a three-axis accelerometer and a three-axis magnetometer.

A gyroscope measures angular speed relative to its center of mass. An accelerometer measures acceleration, including gravitational acceleration, which is used to determine orientation. A magnetometer measures strength of the magnetic field passing through the sensor, including earth's magnetic field, which is used to determine heading of the quadcopter.

3. FLIGHT CONTROL THEORY

Controlling a quadcopter is mechanically simple – everything is done by changing thrust and torque generated by four propellers. The challenging part of controlling, according to Gibiansky (2012), is that with six degrees of freedom (three translational and three rotational) and only four independent actuators (propellers), quadcopters are trivially underactuated. In order to achieve six degrees of freedom, rotational and translational motion has to be coupled. The resulting dynamics are highly nonlinear, especially after counting in quadcopter's complicated aerodynamic properties. (Gibiansky 2012)

Quadcopter's three rotational directions of movement are called roll, pitch and yaw. If quadcopter presented in figure 3.1 is flying forwards and backwards going along the X-axis, then rotation around this axis is called roll. Rotation around Y-axis is then defined as pitch and rotation around Z-axis is yaw.

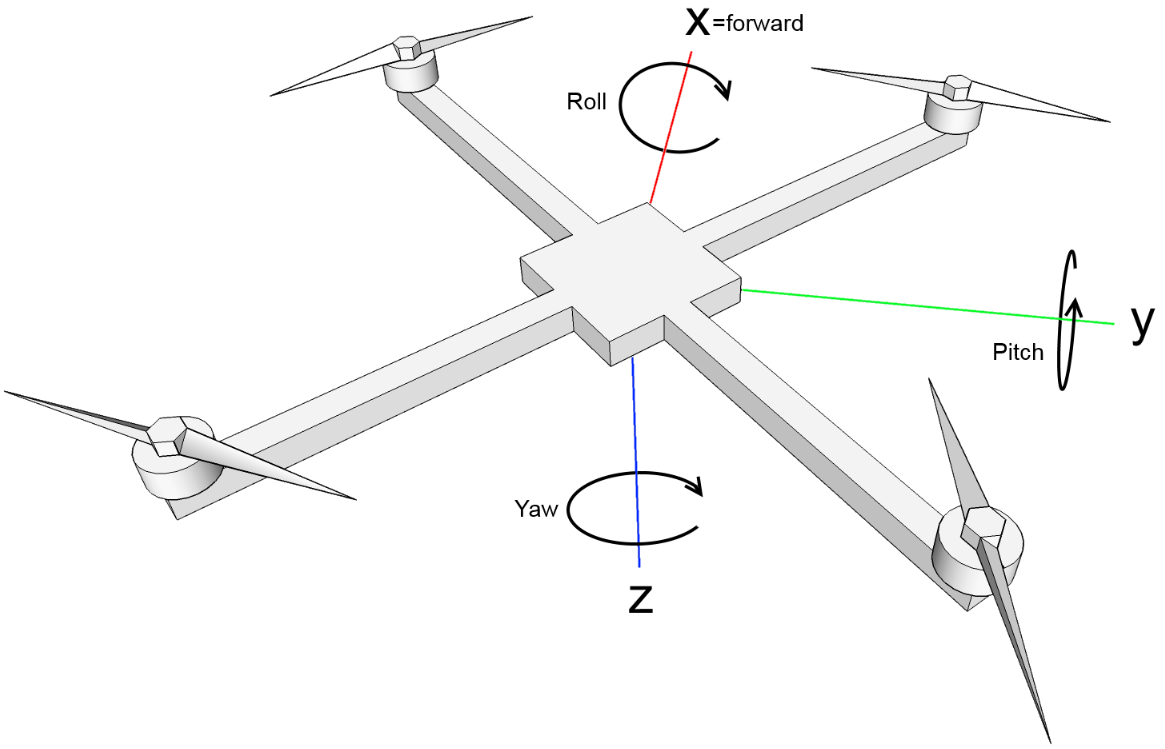

Figure 3.1. If the direction of X-axis is defined as forward, rotation around this axis is called roll. Rotation around Y-axis is called pitch and rotation around Z-axis is called yaw.

A quadcopter can fly in two configurations: "plus" configuration and "X" configuration. The quadcopter in figure 3.1 uses X-configuration, which means that front of the quad is between two motors. The "plus" configuration means that one of the motors is pointing to the front of the quadcopter. There is no meaningful difference between these configurations in terms of flying, but because quadcopters are often used in aerial photography and videography, X-configuration is often preferred for its cleaner field of view. These configurations are illustrated in figure 3.2.

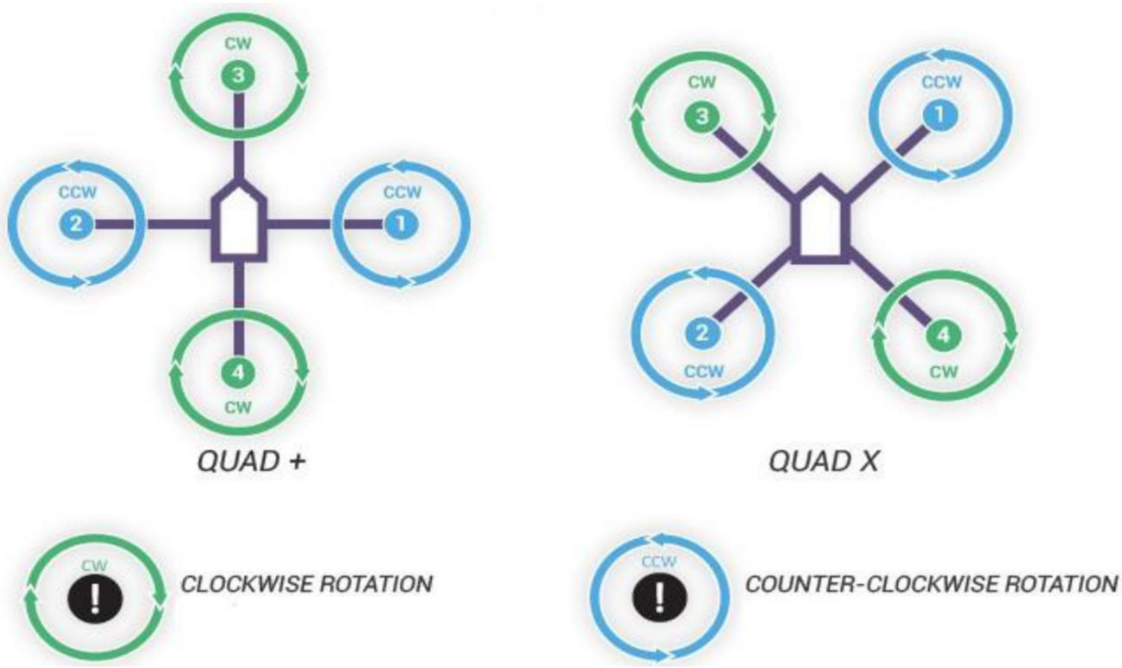

Figure 3.2. “Plus” and “X” configurations. (Tarek et al. 2014)

All four propellers don’t rotate in the same direction as shown in figure 3.2. Two of them rotates clockwise (CW) and another two rotates counter-clockwise (CCW), because torque generated by motors and propellers affecting the body of the copter has to be cancelled out. Otherwise the body of the copter would spin around yaw-axis, which would make controlling very difficult. Torque depends on the angular velocity of the propellers, which means torques cancel each other out only when all propellers are rotating at the same angular velocity. This effect can be used to change the yaw of the copter by changing angular velocity ratio between CW and CCW propellers.

Quadcopter’s movements in “plus” configuration are summarized in figure 3.3. By generating opposite variations in the angular velocities of the propellers 1 and 3, the pitch angle is changed, which results in the forward or backward movements (figure 3.3a and figure 3.3b). Similarly, angular velocity variations between the propellers 2 and 4 produce changes in the roll angle and thereby create left and right movements (figure 3.3c and figure 3.3d). To provide lift to the quadcopter, all four propellers’ angular velocities are increased and vice versa, their velocities are decreased to lower down the quadcopter e.g. in case of landing (figure 3.3e and figure 3.3f). Yaw rotation is presented in figure 3.3g and figure 3.3h. (Pham & Chew 2014)

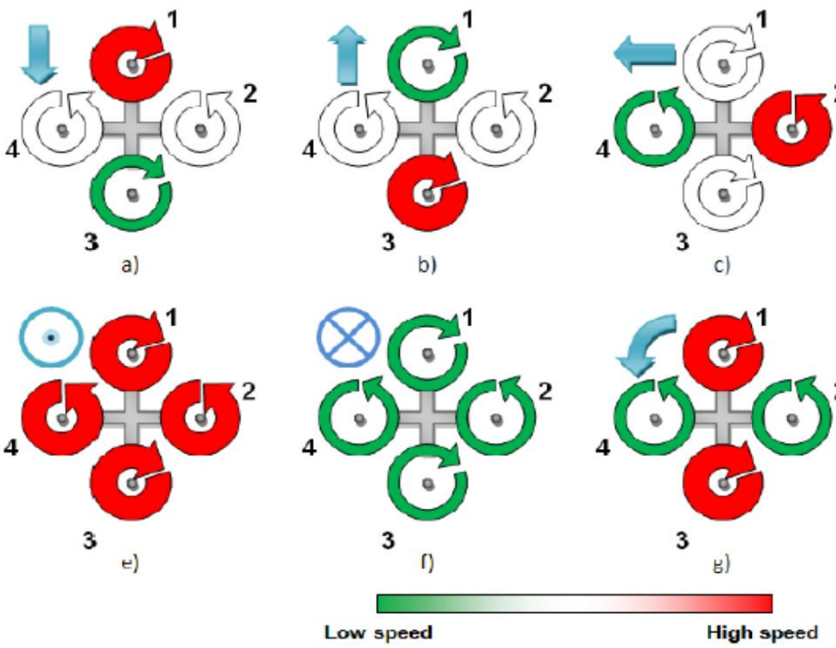

Figure 3.3. Various quadcopter's maneuvering options. (Tarek et al. 2014)

4. ORIENTATION SENSING

Sensors used in an IMU do not output angle data directly, which is why it has to be modified. A FCU is used to calculate roll (ϕ), pitch (θ) and yaw (ψ) angles from gyroscope data, roll and pitch angles from accelerometer data and yaw angle from magnetometer data. These angles are then combined and used to control individual motors in such way that differences between reference angles provided by the pilot and measured angles are minimized.

4.1 Gyroscope

Gyroscope is a device, which measures the rate of change of the angular position over time (angular velocity). This can be written as derivative of the angular position over time:

$$\dot{\delta} = \frac{d\delta}{dt}, \tag{4.1}$$

Where $\dot{\delta}$ is the angular velocity [deg/s], δ is the angular position [deg], and t is time [s].

Angular position is used for controlling a quadcopter, which can be obtained by integrating the angular velocity:

$$\delta(t) = \int_0^t \dot{\delta}(t)dt. \tag{4.2}$$

Integration is done by a microcontroller (FCU) in a quadcopter, which can't calculate a continuous integral function. However, a microcontroller can approximate this integration numerically for each three axis by calculating angular distance travelled in sample time T_s

and adding it to a previous known angular position to get a new position. This is presented for ϕ-angle in equation 4.3.

$$\phi_g(t) \approx \sum_{0}^{t} \dot{\phi}_g T_s. \tag{4.3}$$

This method makes an approximation of the travelled angular distance based on velocity at the moment when sample is taken and expects it to stay the same until next sample is taken. In reality, this is not possible and it will cause a small error between the real travelled distance and the approximated distance. Because these approximated distances are added together to calculate position, angular position error will increase over time and this is called an integration drift. Reducing sample time T_s decreases approximation error.

This error can be compensated by making a quadcopter aware of its angular position relative to the earth by using an accelerometer.

4.2 Accelerometer

An accelerometer is a device that measures the force to accelerate a proof mass; thus, it measures the acceleration of the object containing the accelerometer (Kayton & Fried 1997). An accelerometer also measures Earth's gravitational acceleration that is always pointing at known direction. This information can be used to obtain quadcopter's angular position relative to the earth, but it is accurate only when magnitude of a 3-dimensional force vector is the same as earth's gravity $g_n \approx 9.81$ m/s^2 and points to the same direction. A magnitude of the force vector can be calculated with following equation:

$$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}. \tag{4.4}$$

According to Pham & Chew (2014), pitch and yaw can be calculated using following equations:

$$\phi_a = \tan^{-1}\left(\frac{a_y}{\sqrt{a_x^2 + a_z^2}}\right). \tag{4.5}$$

$$\theta_a = \tan^{-1}\left(\frac{a_x}{\sqrt{a_y^2 + a_z^2}}\right). \tag{4.6}$$

A quadcopter is challenging environment for an accelerometer, because in addition to linear acceleration, there is always strong vibrations present caused by propellers. These vibrations can be usually filtered out using dampening material between the sensor and frame of the quadcopter and using a low-pass filter.

Trigonometric functions are computationally heavy tasks, which is why these angles are often approximated by using linearization called small angle approximation. This is done by simply replacing trigonometric function with its input. Quadcopter's tilt angle is usually low enough for this approximation to be sufficiently accurate. Approximation for equation 4.5 is:

$$\phi_a = \tan^{-1}\left(\frac{a_y}{\sqrt{a_x^2 + a_z^2}}\right) \approx \frac{a_y}{\sqrt{a_x^2 + a_z^2}}. \tag{4.7}$$

This is illustrated in figure 4.1.

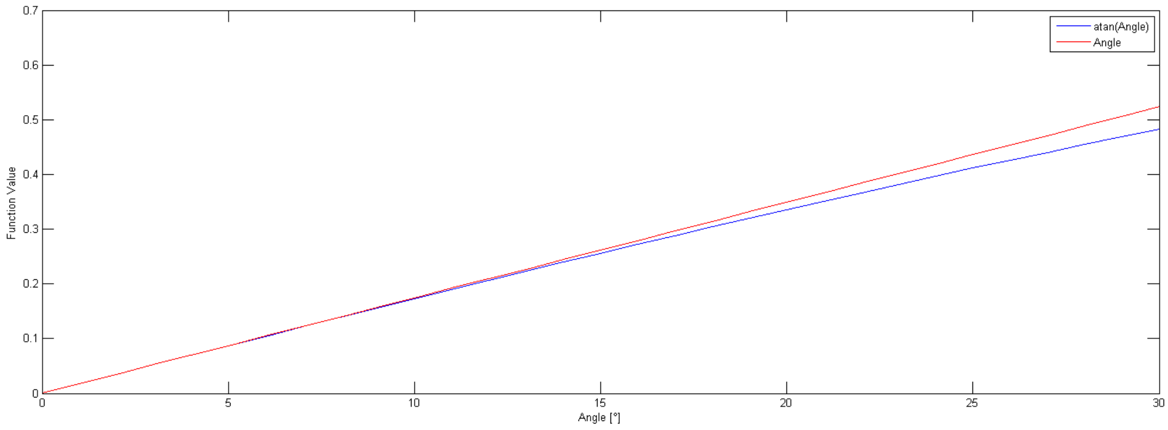

Figure 4.1. Small angle approximation for atan() -function. Unit conversion from degrees to radians is made for approximated values, because angles in degrees are usually easier to imagine.

Yaw is not calculated with an accelerometer, because it is not possible when vertical axis of the accelerometer is parallel to earth's gravity i.e. when a quadcopter is hovering in upright position. This deficiency can be corrected by using a magnetometer to measure yaw.

4.3 Magnetometer

A three-axis magnetometer measures strength of the magnetic field passing through the sensor. Quadcopter's heading relative to earth's magnetic field can be calculated from these values. If magnetometer is parallel to earth's magnetic field, heading can be calculated accurately using just magnetic field strength values H_X and H_Y (figure 4.2) with following equation

$$\psi_m = \tan^{-1}\left(\frac{H_Y}{H_X}\right). \tag{4.8}$$

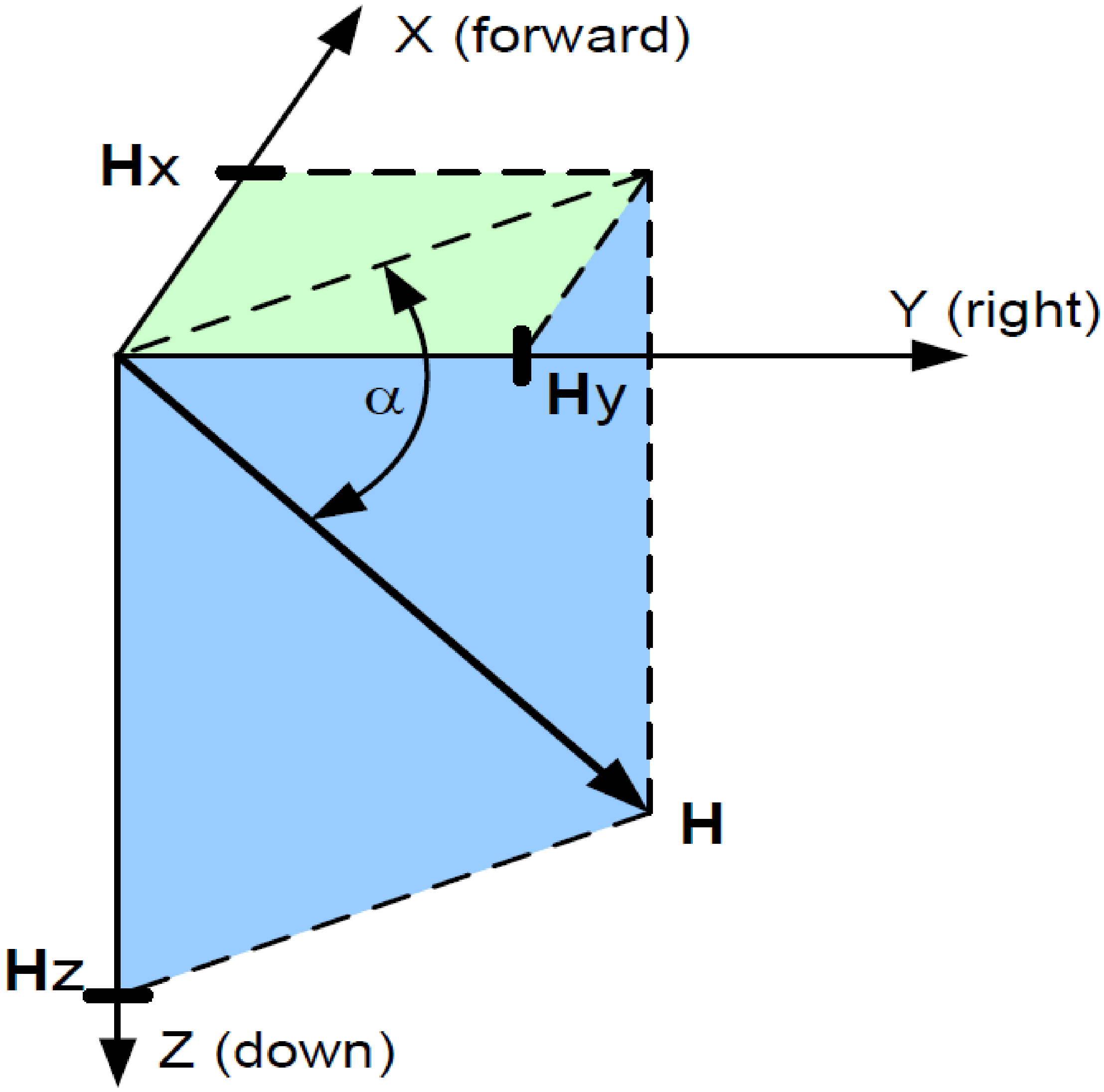

Figure 4.2. Magnetic field inclination angle (Cypress 2014)

According to Cypress (2014), if any tilt such as magnetic inclination (angle α in figure 4.2) is present it causes error and should be compensated using additional tilt sensor – an accelerometer. This compensation can be done for H_X and H_Y using following equations:

$$H_{x,tc} = H_x(1 - a_x^2) - H_y a_x a_y - H_z a_x \sqrt{1 - a_x^2 - a_y^2}, \qquad (4.9)$$

$$H_{y,tc} = H_y \sqrt{1 - a_x^2 - a_y^2} - H_z a_y, \qquad (4.10)$$

where $H_{x,tc}$ and $H_{y,tc}$ are tilt compensated values for magnetic field strengths in X and Y directions. H_x, H_y and H_z are raw measured values from magnetometer and a_x and a_y are normalized accelerometer outputs. Heading can now be calculated by replacing raw input values in equation 4.8 with tilt compensated values.

To calculate the geographic azimuth as a clockwise angle between the North Magnetic Pole and X-axis, the signs of *x* and *y* must be taken into account:

$$\text{Azimuth angle} = \begin{cases} 180 - \tan^{-1}\left(\dfrac{H_{y.tc}}{H_{x.tc}}\right) & \left(H_{x,tc} < 0\right) \\ -\tan^{-1}(\dfrac{H_{y.tc}}{H_{x.tc}}) & \left(H_{x.tc} > 0, H_{y.tc} < 0\right) \\ 360 - \tan^{-1}\left(\dfrac{H_{y.tc}}{H_{x.tc}}\right) & \left(H_{x.tc} > 0, H_{y.tc} > 0\right) \\ 90 & (H_{x.tc} = 0, H_{y.tc} < 0) \\ 270 & (H_{x.tc} = 0, H_{y.tc} > 0) \end{cases} \tag{4.11}$$

(Cypress 2014)

Magnetometers are often surrounded by disturbing electromagnetic sources and materials that can change the measured magnitude of the earth’s magnetic field, which can cause significant error in heading calculation. This is why soft-iron and hard-iron calibration has to be done for the sensor by user.

According to Gebre-Egziabher et al. (2001), soft irons are materials that generate their own magnetic field in response to an external magnetic field, which is the earth’s magnetic field in this case. If such materials are present, they will generate a magnetic field that will be superimposed on the magnetometer output. Since the orientation of the earth’s magnetic field vector relative to the soft iron materials fixed inside the aircraft changes with aircraft attitude, this gives rise to a varying bias on the magnetometer output. (Gebre-Egziabher et al. 2001)

Gebre-Egziabher et al. defines hard irons as electromagnetic sources, like ferromagnetic materials, that causes time invariant distortion on the magnetometer output. If the strength and direction of these unwanted magnetic fields is known, then their effect can be removed to unbias the magnetometer readings. (Gebre-Egziabher et al. 2001)

Items inside the aircraft can also generate unwanted magnetic fields that are time varying. For example, such an item would be a current carrying wire. If the current through the wire is time varying, the resulting magnetometer bias will also be time varying and difficult to calibrate. (Gebre-Egziabher et al. 2001) These errors can be taken into account during installation of the magnetometers by not installing it close to electromagnetic sources if possible.

5. SENSOR FUSION

A gyroscope, an accelerometer and a magnetometer have some drawbacks, but these sensors have characteristics, which can complement each other in order to create more accurate data. This can be achieved by using a complementary filter. Literature review revealed that most of the quadcopters, whose source-code is available, use this filter or some derivative of it to fuse data from these sensors. Common derivatives are adaptive complementary filter and Kalman filter. Implementation of a complementary filter for pitch angle is only presented. Roll angle can be calculated by changing gyroscope's and accelerometer's pitch angles to roll angles. Yaw angle can be calculated by replacing an accelerometer with a magnetometer.

Gyroscope's drift can be considered as low-frequency component of the signal, which can be filtered out using high-pass filter (HPF). An accelerometer is sensitive to vibrations from e.g. quadcopter's motors and a magnetometer is sensitive to interference from surrounding electronics, which can be filtered out using low-pass filter (LPF). When both filters have the same 3 dB cut-off frequency ω_c, their transfer functions add up to unity. This is illustrated in figure 5.1.

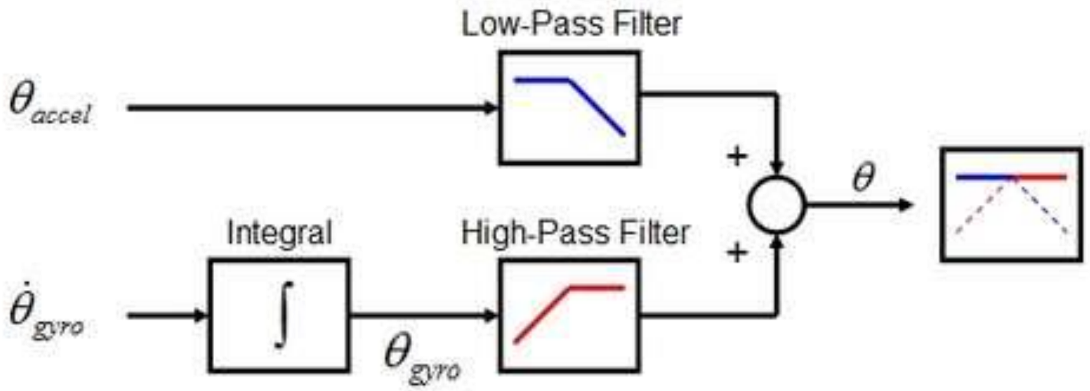

Figure 5.1. A complementary filter for fusing pitch (*θ)* data from a gyroscope and an accelerometer. (Marion 2011)

According to Colton (2007) digital implementation of the complementary filter can be presented as following exponentially weighted moving average (EWMA) equation:

$$\theta(\mathrm{n}) = a \cdot (\theta(\mathrm{n}-1) + \dot{\theta}_{gyro}(n) \cdot T_s) + (1-a) \cdot \theta_{acc}(n), \tag{5.1}$$

where *n* is the amount of samples taken and *a* is a filter parameter, which is defined as

$$a = \frac{1}{1 + \omega_c T_s}. \tag{5.2}$$

Because sample time T_s is constant and depends mostly on speed of the used FCU, boundary between trusting the gyroscope and trusting the accelerometer or magnetometer is defined by cut-off frequency ω_c. (Colton 2007). The best value of parameter *a* can be determined by analytical methods like least square method proposed by Min, H. G. & Jeung, E. T (no date) or using the most common way, which is trial-and-error method.

Figures 5.2 and 5.3 shows the roll angle calculated from gyroscope data, accelerometer data and using complementary filter with two different *a* values. The quadcopter, which was used to gather this data, was on the ground at 0° during the measurements. Figures show clearly drifting in gyroscope data and noise from vibrations in accelerometer data. Using value 0.1 for parameter *a*, filtered angle does not drift but contains some noise from accelerometer data. With value *a*=0.01 in figure 5.3, filtered angle has some drifting but does not have noticeable noise. Best value would be somewhere between 0.1 and 0.01.

The accelerometer was subjected to strong vibrations, because propellers were unbalanced and the quadcopter was on the ground. Vibrations should not be as strong in a quadcopter, which can fly.

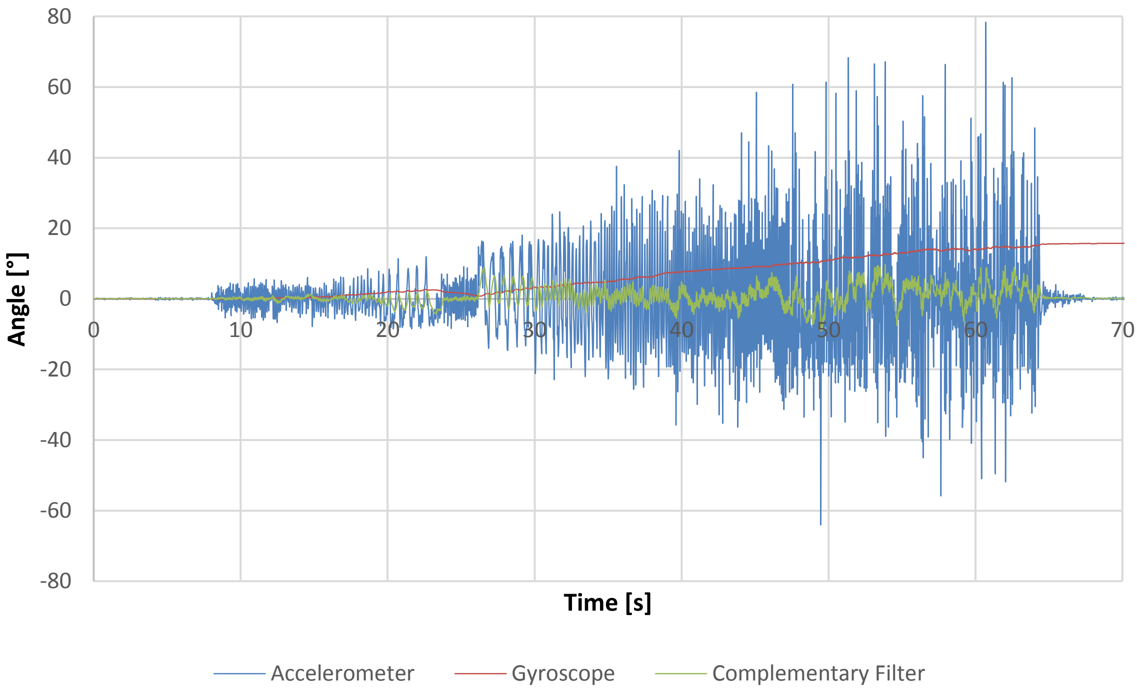

Figure 5.2. Complementary filter test using parameter $\alpha = 0.1$. Speed of the motors is increased from around 10 seconds to 65 seconds.

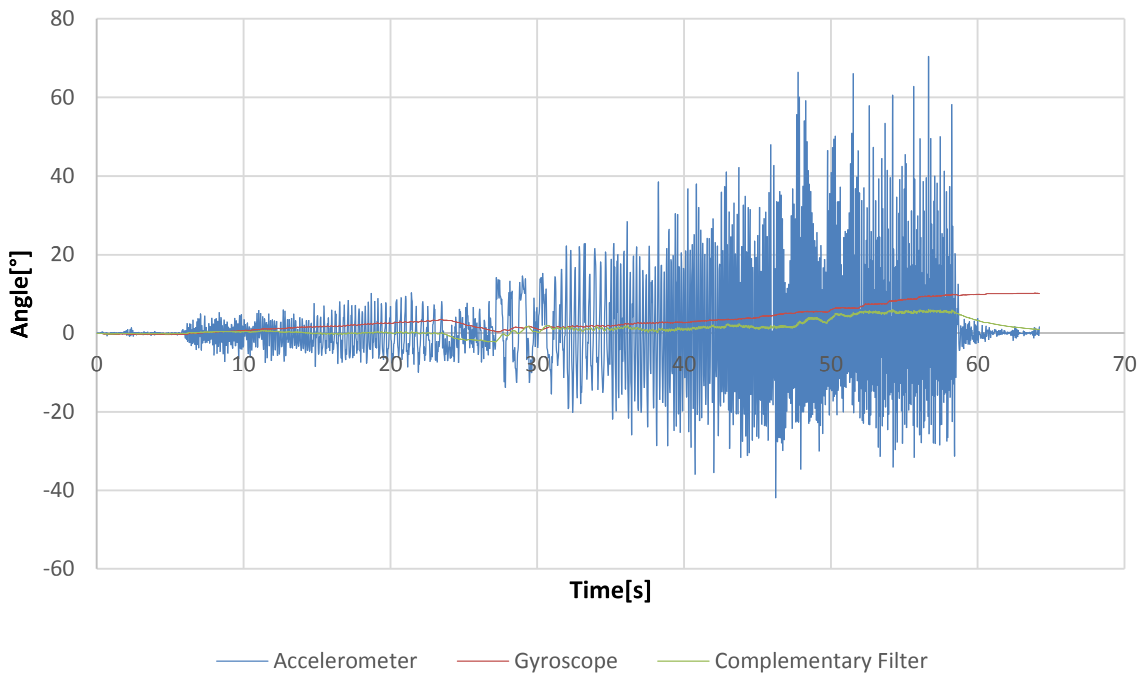

Figure 5.3. Complementary filter test using parameter α = 0.01. Speed of the motors is increased from around 5 seconds to 60 seconds.

Min, H. G. & Jeung, E. T (no date) tested complementary filter's angle tracking capabilities by comparing it to a rotational encoder. A gyroscope, an accelerometer and the encoder were attached to a pendulum. These measurements are presented in figure 5.4.

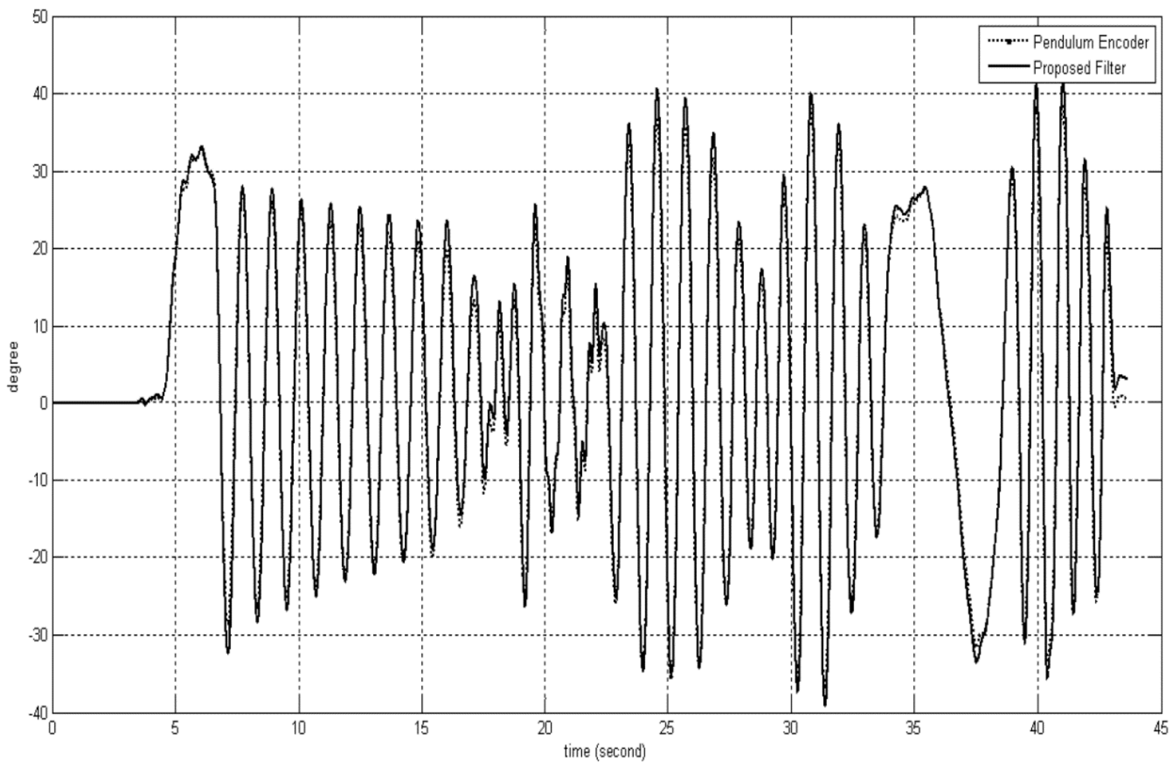

Figure 5.4. Complementary filter angle compared to angle measured by rotational encoder. Dotted line is the reference angle from encoder and solid line is filtered angle. (Min, H. G. & Jeung, E. T.)

Complementary filter seems to follow the reference angle almost perfectly. However, this test does not have interfering vibration like in a quadcopter, which does slightly hinder complementary filter's performance. Most realistic measurement found by literature review is presented in figure 5.5.

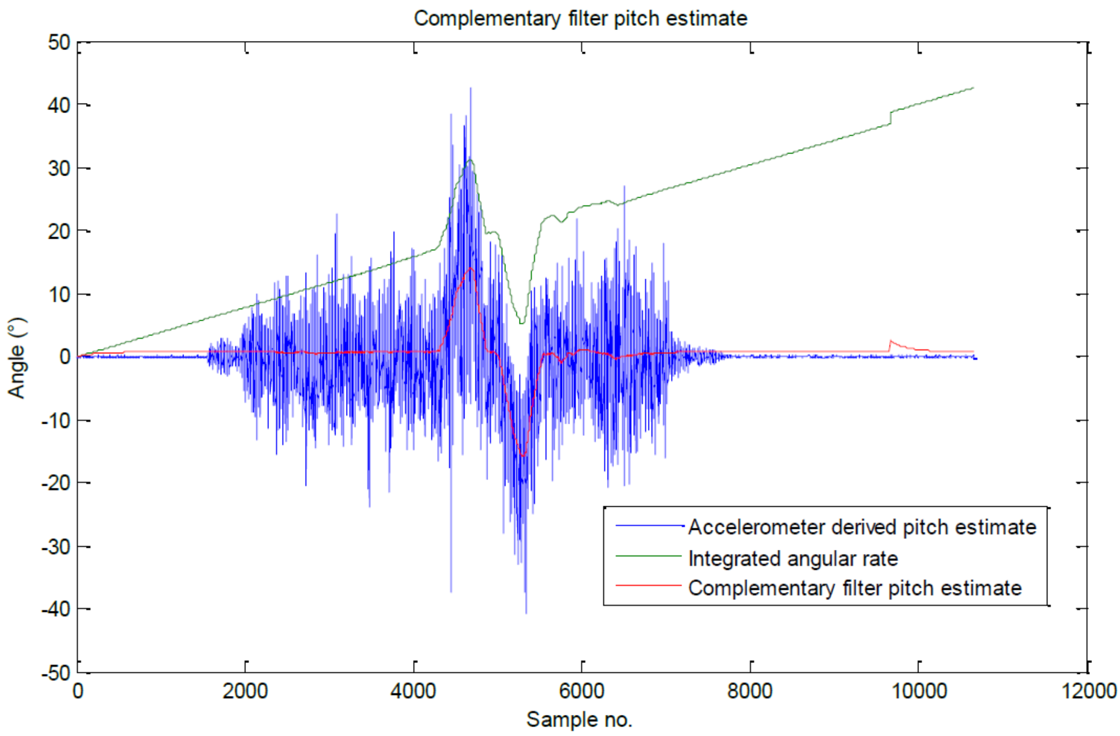

Figure 5.5. Complementary filter test, where a quadcopter's propellers are rotating at estimated hovering speed and it is tilted 15° to positive and negative pitch angle. (Watson 2013)

This measurement shows that complementary filter seems to estimate angle accurately even with strong vibration noise in accelerometer data and significant drift in gyroscope data. Watson (2013) does not give any information about how the 15° reference angle was measured. On the basis of visual reference angle estimation, complementary filter used in the quadcopter in figure 1.1 did also track angles accurately in every orientation. These measurements are presented in figure 5.6.

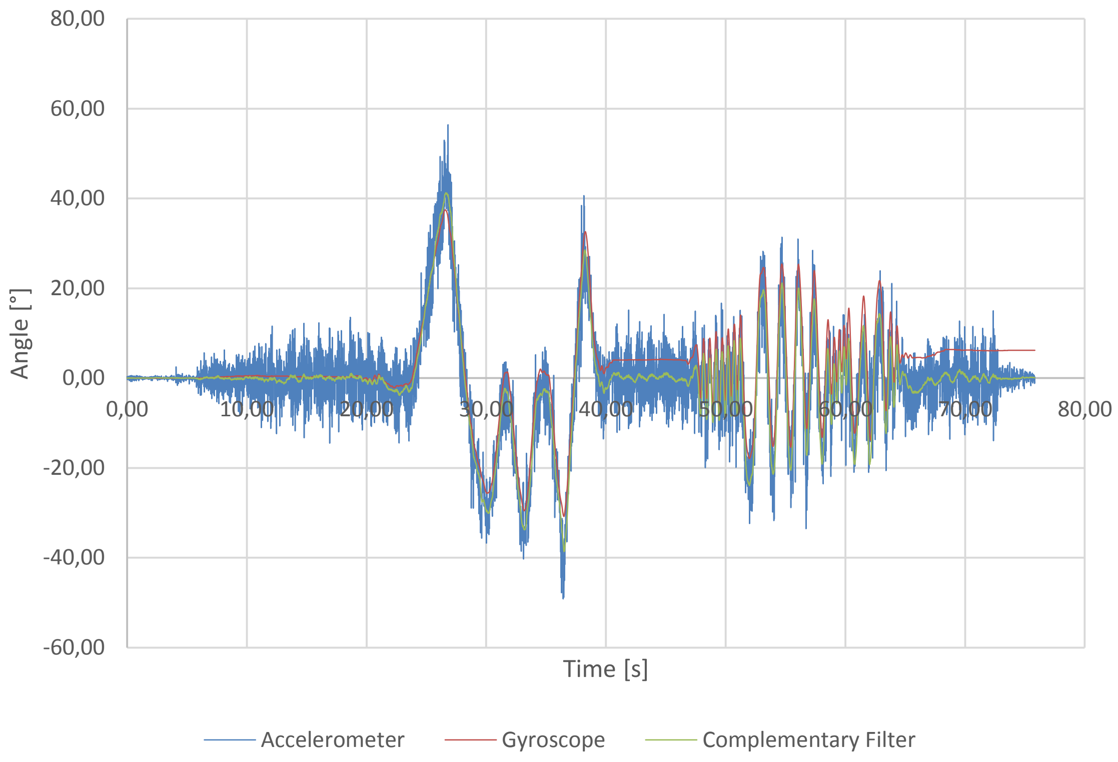

Figure 5.6. Angle estimation test with different kinds of tilting movements. A complementary filter angle seems to be accurate based on visual approximation and it responds quickly to rapid changes.

A few degrees of error in angle estimation does not have negative effect on quadcopter's flight, because the pilot usually gets visual feedback of the linear speed of the quadcopter, which he or she wants to control. Autonomous quadcopters have additional sensor for measuring linear movement accurately, which can be used to do the same compensation. Slow respond time, significant noise and increasing error in angle measurement can make a quadcopter hard to fly.

There is one major drawback in this simple complementary filter – these presented results are valid only when there is no linear acceleration force present. Acceleration on one direction makes angle estimations calculated by accelerometer unreliable, because these calculations are based on earth's gravitational acceleration as explained in chapter 4.2. Vibration from motors does not significantly affect filter performance, because it has zero average. This problem can be fixed using some derivative of a complementary filter like an adaptive complementary filter or a Kalman filter, which are not covered in this thesis.

6. CONCLUSION

In this thesis, a fundamental solution for fusing data from an accelerometer, a gyroscope and a magnetometer is proposed based on literature review and some measurements using a quadcopter. This solution is called a complementary filter, which combines best characteristics from sensors in order to achieve more accurate angle measurement compared to single sensor measurement. Literature review revealed that most of the quadcopters, whose source-code is available, uses this approach or some derivative of it for angle measurement.

On the basis of literature review and measurements using a quadcopter, the proposed filter provides sufficiently accurate attitude data. However, a complementary filter has one significant drawback – it works reliably only when the quadcopter is hovering or moving at a constant speed. The reason is that an accelerometer can’t be used to measure angles accurately if linear acceleration is present. This problem can be fixed using some derivative of a complementary filter like an adaptive complementary filter or a Kalman filter, which are not covered in this thesis and are therefore good subjects of future study.

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