LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
School of Engineering Science
Master's Programme in Computational Engineering and Technical Physics

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COMPUTATIONAL MARKET DYNAMICS OF VIRTUAL ECONOMIES

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D.Sc. (Tech.) Matylda Jabłońska-Sabuka

ABSTRACT

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The last two decades have provided a vast opportunity to live and explore the compulsive imaginary world or virtual world through massively multiplayer online role-playing games (MMORPGs). MMORPG gives a wide range of opportunities to its users to participate with multi-players on the same platform, to communicate and to do real time actions. There is a virtual economy in these games which is largely player-driven. In-game currency provides its users to build up their Avatars, to buy or sell the necessary goods to play, survive in the games and so on. As a part of virtual economies generated through EVE Online, this thesis mainly focuses on how the prices of the minerals in EVE Online behave by applying the Jabłońska-Capasso-Morale (JCM) mathematical simulation model. It is to verify up to what degree the model can reproduce the virtual economy behavior. The model is applied to buy and sell prices of two minerals namely, isogen and morphite. The simulation results demonstrate that JCM model fits reasonably well to the mineral prices, which lets us conclude that virtual economies behave similarly to the real ones.

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Lappeenranta, January 21, 2016.

Rajshree Patel

Contents

List of Symbols and Abbreviations

ABM	Agent Based Model
ACF	Autocorrelation Function
AR	Autoregressive Process
ARPU	Average Revenue Per User
bot	Artificial Player
EMH	Efficient Market Hypothesis
EVE	EVE Online
GMMS	Generalized Market Microstructure Model
ISK	Icelandic Króna
JCM	Jabłońska-Capasso-Morale Model
MMOG	Massively Multiplayer Online Games
MMORPG	Massively Multiplayer Online Role-playing Games
NPC	Non-playing Character
ODE	Ordinary Differential Equation
PACF	Partial Autocorrelation Function
PDF	Probability Density Function
PLEX	(Concord) Pilot Licence Extension
SA	Sentimental Analysis
SNA	Social Language Analysis
SDE	Stochastic Differential Equation
USD	United States Dollar
WoW	World of Warcraft

1 INTRODUCTION

Virtual reality is assumed to have begun during 1950s, but the traces of its existence can be found around 1860s before the innovations in digital technology world. At first virtual reality was in the form of 360-degree panaromic paintings. Later artists like Baldassare Peruzzi and Antonin Artaud combined illusion and reality to be the same for the audiences experience in the theatre to have the real feeling of the performance[1]. Edwin Link developed the first flight simulator during 1920s which was designed to train pilots virtually. In 1962, it took one step further in the innovation of virtual worlds, when Morton Heilig created a device known as 'Sensorama' [Mazuryk and Gervautz, 1996]. The device was designed to create an interactive theatre experience. The primitive research in virtual worlds was implemented through computers via virtual reality simulators during 1990s [Bartle, 2003].

At present, virtual reality plays a huge role via its applications in education, business, entertainment, scientific visualisation etc. But what makes the evolution of virtual reality worthwhile? After all, it is mostly noticeable through games and their virtual worlds which is almost a multi billion dollar business [Hamari and Lehdonvirta, 2010]. In reality, if some tasks are too dangerous, or very expensive or impractical, virtual reality provides the platform to do the impossible. It allows to take risks virtually in order to gain real world experience, with the only tangible risk factor being the money spent on it.

1.1 Virtual world and virtual economy

Richard Bartle, during 1970s and 1980s created text-based virtual worlds. He defined the virtual worlds where the imaginary meets the real [Bartle, 2003]. Castronova described interactivity, physicality and persistence as three important features of virtual worlds [Castronova, 2004]. Although, there is no general definition of virtual worlds, one thing they do require is the world to be persistent. In 2008, a new definition of virtual worlds by Bell [2008] was proposed which says that it is "a synchronous, persistent network of people, represented as avatars, facilitated by networked computers". In other words, virtual world keeps on going and changes

[1]Source:`http://www.vrs.org.uk/virtual-reality/beginning.html`, accessed 1st April, 2013

made by the user should be retained after each exit. It requires real time interaction among the users via computer based simulation environment.

Virtual worlds became much more complex since massively multiplayer online games (MMOGs) were introduced in 1996. Due to the increasing popularity of MMOGs, the developers decided to take them to the next level by upgraded version of games as massively multiplayer online role-playing games (MMORPGs). They also implemented the possibility for the players to trade real money for virtual goods. Therefore, a unique economy is generated by a large number of players buying and selling things in virtual worlds [Chambers, 2011]. These virtual economies have their own markets and currency. It is defined as an emergent economy which exists in a virtual world [Thorpe et al., 2007]. The largest virtual economies exist through MMORPGs.

According to the online data statistics of virtual economy, users are willing to pay thousands of dollars for certain goods via real-money trade. The real-money trading market forms a connection between virtual property inside the game service and the real-world economy outside the service [Heeks, 2010]. The global markets of real-money trade estimated USD 2.1 billion of real money spent on virtual items in year 2007 only [Lehtiniemi and Lehdonvirta, 2007].

1.2 Aim of the thesis

The aim of this research is to study the mineral prices taken from an MMORPG, EVE Online by simulating their market dynamics. To understand the market dynamics, non-linear dynamic equations have been applied. In this particular study, Jabłońska-Capasso-Morale (JCM) model has been used to capture the dynamics of the mineral prices and to verify up to what degree the model can reproduce the virtual economy behavior.

1.3 Structure of the thesis

This thesis explores the market dynamics of virtual economy by targeting on an MMORPG, EVE Online. Section 2 contains the literature review in three parts. First, it gives a brief history of MMOGs and MMORPGs. Second, it presents the virtual economy generated through MMORPGs. Finally, it introduces econophysics with agent based modelling in virtual economies. Section 3 focuses on a brief introduction to EVE Online with how trading and mining takes place in the game. Further, it includes a careful statistical analysis of the buying and selling prices of the minerals, isogen and morphite. This very chapter builds readers interest. Section 4 introduces the spatial population dynamics behind the Jabłońska-Capasso-Morale (JCM) model which applied to the mineral prices of EVE Online in the next chapter. It also provides a short description on stochastic differential equations (SDE) used by JCM model, as well as the behavior and decision making of investors in financial markets. Section 5 includes the main contribution to this thesis. The simulations are done using MATLAB software to check the performance of the JCM model. It provides the modelling results. Section 6 reflects a brief summary over the results and the proposal of further research. Lastly, Section 7 provides the conclusion of this thesis.

2 LITERATURE REVIEW

The success of video games and increasing use of internet in early 1980s led to prototype of games but on a larger scale, known as multi-user dungeons (MUDs) were text based computer games [Bartle, 2010]. The increasing demand and innovations in computer games gave birth to a very popular genre in gaming industry, namely massively multiplayer online games (MMOGs) and massively multiplayer online role-playing games (MMORPGs). In MMOGs and MMORPGs, thousands of players interact with each other via created avatars (visual image of a person in a digital environment) in real time through the graphical virtual environment simultaneously connected to the same server. This world continues to function even in the absence of a player's participation. Game developers see MMORPGs as a potentially profitable business, but in reality only few became a success [Achterbosch et al., 2008]. Table 1 shows the brief history of MMOGs and MMORPGs since 1970s with different genre and modes of the games.

The profit or loss of a company developing an MMORPG is proportional to the subscription of users. One of the main key factors to understand the popularity of an MMORPG, is through instant number of online avatars. The number of users are increasing at an exponential rate [Papagiannidis et al., 2008]. Jiang et al. [2010] discovered that the variations in the number of avatars exhibit an intra-day pattern and has a leptokurtic non-Gaussian shape. On the other hand, Gubiec and Wiliński [2015] suggested that intra-day pattern of volatility in stock market has comparably

Table 1: History of MMOGs and MMORPGs

Year	Name	Genre and mode
1974	Mazewar	First person shooter, single/multi-player
1978	MUD1	First MUD, multi-player
1986	Habitat	First graphical MUD, multi-player
1995	Meridian 59	First 3D graphical MMOG, multi-player
1997	Ultima online	First popular game, multi-player
2003	EVE Online	Space simulation MMORPG, multi-player
2004	World of warcraft	Most subscribed game, multi-player

less significance than the seasonality of inter-transaction times.

Furthermore, virtual goods attract players in MMORPGs to gather, move forward, and distribute different items for real money. Keegan et al. [2010] analyzed players in MMORPGs complex networks and discovered that they follow distinctive behavioral attitude in the games such as connectivity, assortivity, and attack tolerance. Their research is about comparison of MMORPGs and the real world which has similarities in both networks which reflects comparable effects of secrecy, resilience and efficiency. To understand the economic behaviors of players in the evolution of wealth distribution, an MMORPGs acts as a place for real time social activities, more opportunities to make real money [Jiang et al., 2010].

Apart from the popularity of MMORPGs, their market has dramatically changed in terms of revenue over the last few years. MMORPGs are approximated to generate USD 11 billion revenue by the end of the year 2015. The global market revenue of MMORPGs is estimated to exceed USD 13 billion by the end of the year 2017[2]. Virtual worlds are not the only ones to sell virtual goods for real money. Social networking sites and other online media also practise to increase the economy through these platforms. What encourages users to purchase a virtual item is definitely a keen point to think about. Although, virtual items are valued as more tangible commodities [Lehdonvirta, 2009].

Moreover, Wang et al. [2013] considered monetary value of virtual goods in twenty-four popular MMORPGs. By combining experimentation and cross-sectional time

[2]Source:`https://www.superdataresearch.com/market-data/mmo-market/`, accessed 15th March, 2015

series data analysis, they determined that social networking and structure of the MMORPGs can be two effective points for the game developers to consider and address the issue of real-money trading of virtual goods. From the study on market efficiency, Kim et al. [2002] concluded that possible reactions in the competitive virtual marketplace are close to the efficient market equilibrium. The comparison of both market mechanisms like categorization process and buyer-seller survey, they found that customers absorb higher inflow of money from trading virtual goods through auctions then posted price of them in markets. They also showed that third party vendors improve the effectiveness of market place for virtual goods.

2.1 Virtual economy through MMORPGs

The monetary development of large-scale, multi-user virtual worlds as MMORPGs are constructed to earn profit. In these worlds, game developers designed universal scarcity of time [Chambers, 2011]. Soon after, MMORPGs came into existence, players began to spend and exchange in-game currency for goods for real money [Castronova, 2002]. Players were and are willing to spend real money to build more powerful in-game characters and an effective virtual infrastructure. Thereafter, virtual goods had real-world value.

Virtual economy is related to many concepts such as virtual world [Bartle, 2003], virtual goods [Hamari and Lehdonvirta, 2010], virtual items [Lehdonvirta, 2009] and virtual money [Lehtiniemi and Lehdonvirta, 2007]. It certainly plays an important role in real world economies, with their own markets and currency. Although, they are not as virtual as they appear to be. The technology in these virtual worlds promotes open standards. An open standard does not require any legal formalities. It allows the movements of virtual goods and trade within worlds. So, even though the economies are thriving in each individual world, they are also stuck within those worlds [Chambers, 2011].

Many research scholars are interested in developing different approaches and methodologies in the analysis of virtual economies. A latest approach, a virtual field experiment suggested by Kim et al. [2002], makes it more adequate and advantageous to test research hypothesis and to explore more effective business strategies in a real world. Virtual economies tend to behave like any other economy and have their predictability with the so-called real world economy [Castronova et al., 2015].

To understand this briefly, consider an MMORPG, World of Warcraft (WoW) released in 2004, which is a fantasy role-playing game developed by Blizzard Entertainment. It had the largest over 9.6 million active subscribers worldwide in the last quarter of 2012[3]. The popularity of the WoW has decreased over the time. In September 2015, WoW game developers have announced 5.5 million subscribers[4]. WoW players can explore a virtual fantasy world, complete quests, kill monsters, interact with other players or fight in battlegrounds to earn the reward in terms of the in-game currency, gold coins.

It is important to see virtual goods sales as an economic model. Around 1999, trading of real money for virtual goods begun amongst players. MMORPGs require significant time commitment from players to establish or make virtual goods, players who do not have time but if they do have money in a real world they will go for a shortcut to buy those from virtual world platforms [Ondrejka, 2004]. Users used eBay as a platform to sell their hard earning game possessions and others to let bid for those [Lehdonvirta, 2008].

Likewise, players encouraged themselves to go on third-party vendors to buy the in-game assets. Particularly after these events, WoW took control over its economy by putting a ban on the resale of in-game assets and removing accounts of almost over one million USD worth of gold coins from players who exchange gold coins or use third-party vendors to buy gold coins [Thorpe et al., 2007]. Undoubtedly, WoW became one of the most profitable computer games in history by generating the revenue close to one billion USD a year through monthly subscriptions and other in-game assets[5].

As an economic oriented attitude from game developers, apart from subscribed users, they also allowed users to enter the games and play without paying fees assuming that curious users might spend real-money on virtual goods [Nojima, 2007]. For this reason, such games like Korean-based 'Maplestory' are considered as 'free-to-play' games. Many successful MMORPGs charge USD 10 - USD 20 per month as a monthly subscription fees[6]. On the other side, game operators who offer 'free-to-play' games earn approximately USD 1 - USD 2 in a month considering average

[3]Source:`http://www.statista.com/statistics/276601/number-of-world-of-warcraft-subscribers-by-quarter/`, accessed 1st February, 2015

[4]Source:`http://www.gamespot.com/articles/blizzard-will-no-longer-report-world-of-warcraft-s/1100-6431943/`, accessed 1st February, 2015

[5]Source:`http://www.nytimes.com/2007/06/17/magazine/17lootfarmers-t.html`, accessed 18th May, 2015

[6]Source: `https://lsvp.wordpress.com/2008/05/27/`, accessed 15th April, 2015

revenue per user (ARPU). The subscription model in MMORPGs is often the more attractive option to generate virtual economy. Although it will lead to more economic benefit if the game developers also consider other aspects such as number of subscribed/non-subscribed users, how active the users are and the rate at which virtual goods trade for real-money [Hamari and Lehdonvirta, 2010].

Previous studies followed how an individual user focuses on the motivations and decision making process in purchase of virtual goods. Hamari and Lehdonvirta [2010] raised the question of how MMORPGs rules and mechanics are made in such a way to encourage their users to purchase virtual goods. They proposed that the mechanism and design patterns of the game should be observed as a set of marketing tactics to increase the sales of virtual goods. Moreover, they also suggested entirely new perspective as "viewing marketing as a form of game design".

2.2 Econophysics and virtual economies

Researchers have been motivated to accomplish more persistent understanding of the formation of economy bubbles, looking at the financial crises that occurred till now. When stock markets around the world crashed on Monday, 19th October, 1987, it created a catastrophic fall in a very short time and gave finance a new term called "Black Monday" [Silva et al., 2005]. However, in virtual sense, biased random walk model denies the possibility of Black Monday. Somehow, the economy is still suffering the consequences of the latest international financial crisis, originally focused on credit defaults in the U.S. housing market in 2007 [Kotios and Galanos, 2012]. It spread out through liquidity in lending from banks and thus affecting the financial market. All the development of financial crisis coincide with reactions from stock markets.

A probability density function of biased random walk model is Gaussian in nature proposed by Bachelier [1900], meaning that evolution of prices is completely random. The fluctuations in the model are normalized to 1 standard deviation, but the possibility for the model to have more than 5 standard deviations is absolutely zero. However, standard deviations for Black Monday are as high as 34 [Stanley et al., 2001]. Economists elaborate these crisis events in the category of outliers [Preis, 2011]. Practically, these financial crises have socially severe extreme impacts. Economists admitted the failure in a journal called *The Economist*. This gave a strong motivation to physicists to think differently than economists.

From a physicist perspective, economy is a complex collection of collaborating things, where everything depends on everything else. But the curiosity formed when physicists gave a critical thinking about, how everything depends on everything else [Stanley et al., 2001]? To build the models, they have decided to investigate economic and financial facts. And a new interdisciplinary research field came into existence known as *econophysics*. The term econophysics first introduced by H. Eugene Stanley in 1995, in a conference on statistical physics in Kolkata[7].

Econophysics is based on composed human experiments mainly developed by physicists to have insight into problems related to economics and finance [Huang, 2015]. It is concerned with the statistical properties of human experiments in the laboratory along with agent-based modeling (analytically or simulationwise) to explain cause-effect relationship between precise conditions and emergent properties of financial markets. Empirical research in the field of econophysics aims at demonstrating devastating events such as Black Monday, should not be treated as deviations but they should be treated as a part of the whole scenario [Silva et al., 2005].

Any marketplace is a place where buyers and sellers participate in the trade of assets. Similarly, financial market can be defined as a place in which people trade equities, bonds, currencies, derivatives and commodities[8]. Conventionally, financial markets are a powerful combination of robust nonlinearity, randomness, jumps and stochastic volatility. For theoretical research and for practical purpose, it is very essential to observe and identify the jump components in financial market by using mathematical models and estimation methods.

However, to characterize the dynamics of financial markets, jump diffusion model, stochastic volatility model and many other mathematical models have been proposed. To describe the dynamic behavior of measurable market price, demand and liquidity along with the interactions between them, Peng et al. [2015] suggested a continuous-time generalized market microstructure (GMMS) model. The model includes a jump component for capturing the abnormality of financial assets. They proposed a jump detection algorithm on the basis of the discrete-time GMMS model.

Many academic researchers are working on understanding how simulation, prediction and hedging of financial markets can be done. A simple approach was given by Jefferies et al. [2001] in the form of agent-based market simulations, where they investigated the effect of different market mechanisms. They combined the formalism

[7]Source:`https://en.wikipedia.org/wiki/Econophysics`, accessed 11th June, 2015

[8]Source:`https://en.wikipedia.org/wiki/Financial\_market`, accessed 14th June, 2015

of Cont and Bouchaud [2000] with agent-based models (ABMs) to show that trading with these models, risk cannot be totally eliminated. However, it can be reduced through the use of time-dependent volatility structure of delta-hedging strategy. Agent-based models exhibit some of the statistical properties which can be seen in real-world financial markets such as fat tailed distribution of returns, fluctuations in financial time series, and so on. The key aspects of these models are feedback, frustration, adaptability and evolution.

One might think, why use agent-based models? Simply, because they simulate operations and interactions of multiple agents simultaneously in complex phenomena's prediction and recreation[9]. ABMs are a system of individual agents that work together on the same goal. Tucnik [2015] emphasized on agent-based economic systems. These individual agents can be either manufacturers or traders or consumers of any virtual economy generated through virtual worlds. The systems are different from the most economic models as it is a class of computer simulation models and it concentrates on a singular economic problem.

Furthermore, some important questions arise in MMORPGs. How do players implement their tasks in order to fulfill the game objectives? How it affects the virtual economy? Schatten et al. [2015] provided some insights on how to organize and analyze, model behavioral and social interaction player data using social language analysis (SNA), natural language processing (NLP), sentiment analysis (SA) and ABM techniques. Their work proposed a possible agent-based approach to modelling MMORPG players including real players, artificial players (bots) and non-playing characters (NPCs), to model the social interaction and behavior of players.

3 EVE Online (EVE)

EVE Online (EVE) was published by CCP games in 2003. It is a persistent player-driven science fiction with a space setting MMORPG having almost 7,800 star systems in the galaxy. Star systems are associated to each other via stargates and players pilot customizable ships to fly around through star systems, which consists of asteroid belts, possible wormholes, space stations, variety of moons and planets to engage in a combat[10]. EVE subscribers' number reached to 500,000 in February 2013. But for the last two years, CCP has not launched any official numbers of

[9]Source:`https://en.wikipedia.org/wiki/Agent-based\_model`, accessed 1st July, 2015
[10]Source:`https://en.wikipedia.org/wiki/Eve\_Online`, accessed 15th March, 2013

subscribers[11].

The players' goal in EVE is about acquiring power, which can be over the market, financially, politically or military dominance. A player creates a new avatar by preference amongst the four empires named Amarr, Gallente, Minmatar or Caldari. The players as spaceship pilots, travel through these galaxies and interact with each other in mining, trading, piracy, fighting in combats and producing goods. Recently in 2014 biggest epic fight happened in a virtual world of EVE where 7,548 players who fought and lost virtual ships worth more than USD 300,000 in real-world money[12].

3.1 Trading in EVE markets

All EVE's subscribers are competing agents in the same economy. The economic model of EVE is based on subscriptions. Normally, subscribers pay USD 15 fee per month to access and play the game[13]. The developers of EVE do not sell virtual property directly, but there exists a secondary market, where players can buy, sell or bid for virtual items related to the game. There are more than 10,000 items in the database of EVE[14].

As EVE is set in the distant future, the form of virtual items include building materials for spaceships, exotic minerals, metals and virtual currency Icelandic Króna (ISK)[15]. According to concord pilot license extension (PLEX), which is a 30-days license for USD 19.99, a player accesses and plays the game, gets 600 million ISK. As per weekly statistics, trading in minerals alone was about 1.6 trillion ISK (1600 billion ISK) at the end of June 2007[16].

[11]Source:http://massivelyop.com/2015/04/19/eve-evolved-how-many-subscriptions-does-eve-have/, accessed 31st March, 2015

[12]Source:http://www.wired.co.uk/news/archive/2014-02/10/eve-online-battle, accessed 25th March, 2015

[13]Source:https://support.eveonline.com/hc/en-us/articles/203599091-Subscription-Costs, accessed 31st March, 2015

[14]Source:https://www.reddit.com/r/Eve/comments/1z15t0/how_many_different_items_are_there_in_eve/, accessed 10th March, 2015

[15]Source:https://wiki.eveonline.com/en/wiki/About_EVE_Online, accessed 10th March, 2013

[16]Source:http://community.eveonline.com/news/dev-blogs/econ-dev-blog-market-overview-for-mineral-markets/, accessed 20th May, 2015

However, a bigger part of trading in EVE happens using a built-in market which is divided into regional sub-markets. The built-in market is actually an exchange platform for players to list their buy and sell offers. It has two positions, ask and bid. When a player offers money and waits for other players to sell the virtual item, he falls into position 'ask'. On the other hand, when a player offers items and waits for players to buy them from him, he is in the position 'bid'. Players can trade with their best bidder. Furthermore, 'ask' refers to buying the items and 'bid' refers to selling the items offered in EVE markets. In sub-markets, players may view a subset of total market and as they travel further in the game, they can view needed items. Some items cannot be listed on the market but are sold in a form of contracts that include a type of English auctions, loans and courier contracts.

3.2 Mining in EVE world

In EVE Online there are various ways for players to earn ISK, for instance through participating in professions and in-game activities. Mining is one of those professions in EVE listed earlier[17]. To mine, a player gets a ship with built-in mining lasers. Then he flies out that ship to an asteroid belt. The next step is to target an asteroid and, while activating mining lasers, the ore will be taken from the asteroid into the cargo hold. This ore can either be sold directly as raw material or refined into minerals which are much more valuable.

There are eight different types of minerals in EVE. Table 2 contains minerals with their short names and their base ISK values[18]. For current actual prices, the players have to visit the local EVE market. Determination of prices based on their scarcity and demand to construct ships and equipments. Tritanium is the cheapest and easily available mineral, while morphite is the rare and expensive one. The powerful miners with mature skills and better battleships can make approximately ten million ISK in an hour[19], which is USD 0.10. Although, it requires lots of gaming hours and practise and training to build up skills and better equipment to reach up to a level of money making.

[17]Source:https://wiki.eveonline.com/en/wiki/Mining_guide, accessed 15th March, 2013

[18]Source:https://wiki.eveonline.com/en/wiki/Item_Database:Manufacture_&_Research:Materials:Ore_&_Minerals_:Minerals, accessed 15th March, 2013

[19]Source:http://www.thonky.com/eve-online-guide/mining-in-eve, accessed 20th March, 2013

Table 2: Minerals in EVE Online

Mineral	Shortname	ISK value
Tritanium	Trit	2
Pyerite	Pye	8
Mexallon	Mex	32
Isogen	Iso	128
Nocxium	Nocx	512
Zydrine	Zyd	2048
Megacyte	Mega	8192
Morphite	Morph	32,768

3.3 Mineral prices - a case study data

The mineral price data used for this thesis are taken from 'third party websites' having a wide range of data collection [Wurtz, 2015]. However, these websites are mostly incomplete, only popular items and locations in demand are sampled. The basic structure of these data consists of 'ask' and 'bid' prices. These data are highly compressed because they are quite big in size, which yields a number of items into individual data vectors. The individual four data vectors named isogen and morphite, both ask and bid prices are being statistically analyzed in this section.

As discussed earlier, the term econophysics has existed for the last two decades. But physicists started to give more concentration on applying concepts and methods of statistical physics to have more broader view about economic problems, in recent years. They focused more carefully on observations of statistical properties of financial time series, simply because financial markets behave as complex interacting systems, where huge amount of data exists and reviewing these data might yield new results [Plerou et al., 2000].

However, time-series analyses in financial markets reflect market characteristics with time scale and suggest the reasons why traders mainly focus on weighted feedbacks of past prices [Sato, 2006]. Time series analysis is associated with uncertainty. It is extremely common in many areas to observe the data sequentially over a span of time, for example, in business, stock price data observed on weekly, daily, monthly and yearly basis. Likewise, time series has its application in many areas. The purpose of time series is to understand the stochastic behavior and forecast future values of a series, either alone or incorporated with other series [Cryer and Chan, 2008].

In Box-Jenkins approach to make the forecast of the formation of a stochastic process based on an observed data, stationarity is the most important assumption. Foundation of the classical time series analysis is based on its stationarity. Conceptually, stationarity is governed by probability laws which are responsible for the process to be in a static equilibrium [Cryer and Chan, 2008]. In a broader sense, the time series is said to be stationary if there is no precise change in its mean (no trend) nor variance with removal of periodic variations. Due to this property, a stationary process will not move further from its mean because of its finite variance. White noise is considered as a very important example of a stationary process, which is defined as a sequence of independently identically distributed random variables.

This section of the thesis contributes useful plots for an initial look at the time series data and to analyze its spread and shape. Figure 1 shows the time line plot of the minerals isogen and morphite with buy and sell prices of both. It is easy to understand from the time line plot that time series data are persistent, having trend and variance. Both the time series used buy and sell prices of the minerals, isogen and morphite, are non-stationary, as their mean and variance change over time and they have high peaks at several points in time.

Skewness and kurtosis are known as the third and fourth *'moment'* of a distribution. Moreover, the level of symmetry of the shape of a data (in the form of histogram) is the sample skewness. In the histogram plot, the position of mean and median describes skewness where, as mode is the peak in the histogram. Data is said to be positively skewed if mean $>$ median $>$ mode and negatively skewed if mean $<$ median $<$ mode. The mean and median are two statistical measures used as measures of location. The variability of the data can be seen through its spread. The standard deviation is also one of the measures of the spread.

Figure 2 pictures the histogram and a normal distribution PDF of buy and sell prices of isogen, which shows that the prices are not normally distributed. Also, Figure 3 has an analogical plot for buy and sell prices of morphite.

Table 3 collects the statistical properties of buy and sell prices of minerals isogen and morphite. Looking at the values of the mean and standard deviation, it displays that the data is widely spread out and is more deviated. It shows higher indication of

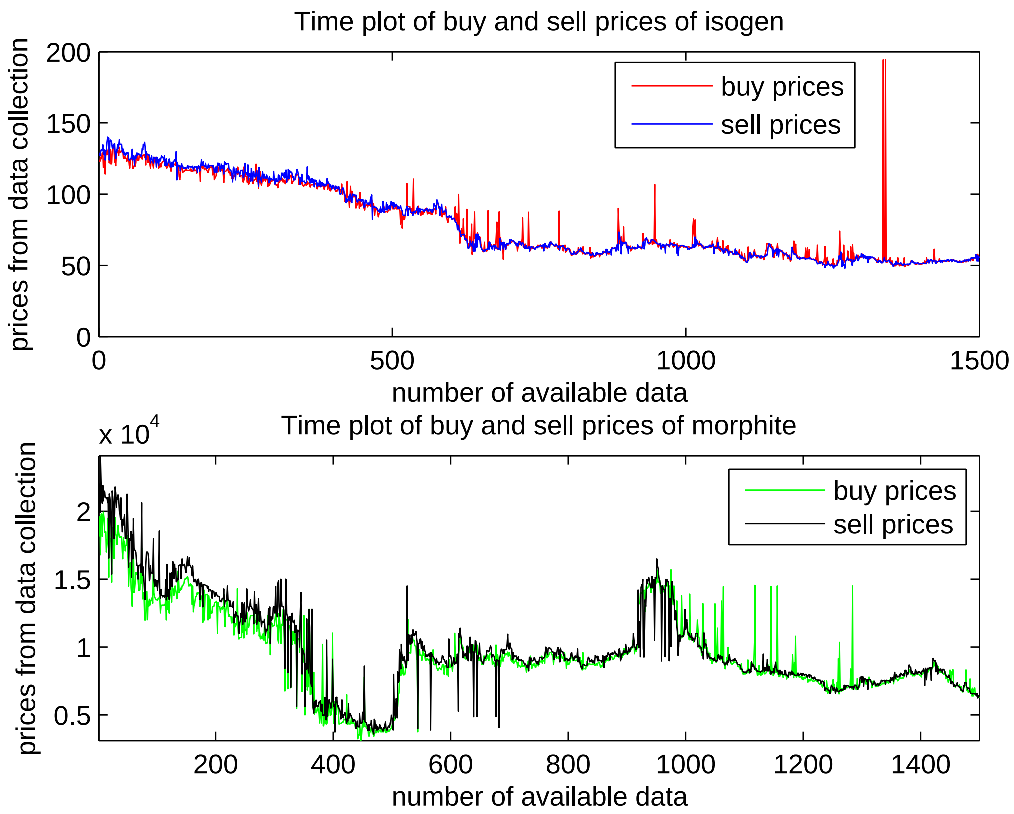

Figure 1: Time line plot of buy and sell mineral prices of isogen and morphite.

Table 3: Basic statistical properties of buy and sell prices of isogen and morphite

	Mean	Standard deviation	Skewness	Kurtosis
Isobuy prices	79.2234	25.8102	0.6727	2.2233
Isosell prices	79.4351	26.8496	0.6432	1.8537
Morphbuy prices	9602.1700	3156.3485	0.6608	3.5065
Morphsell prices	9973.5518	3556.2260	1.0037	4.0924

volatility of these commodities. Furthermore, both the prices of isogen have positive skewness, meaning they have right-skewed distributions where the right side tail of a probability density function is longer than the left side tail. Which implicates that most values are concentrated on the right side of the mean and left side with extreme values. Likewise, both the prices of morphite have positive skewness as well, which interprets frequent but small negative outcomes and extreme bad outcomes are less likely to occur.

Kurtosis is also a measure of shape of a distribution. Kurtosis of buy and sell prices

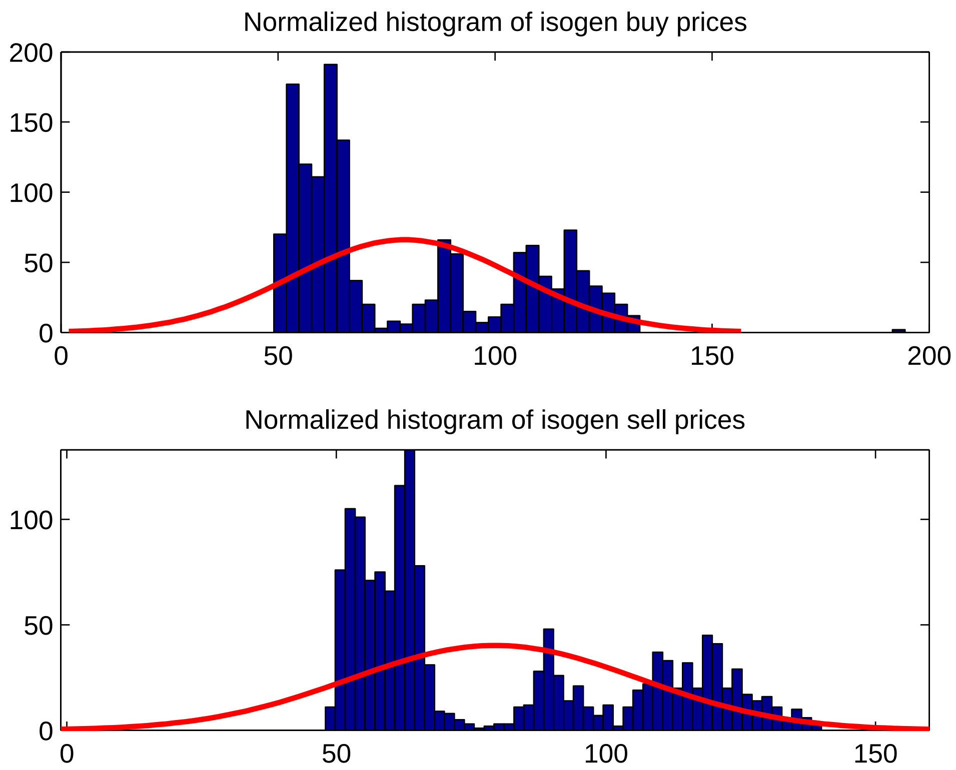

Figure 2: Normal distribution histogram of buy and sell prices of isogen.

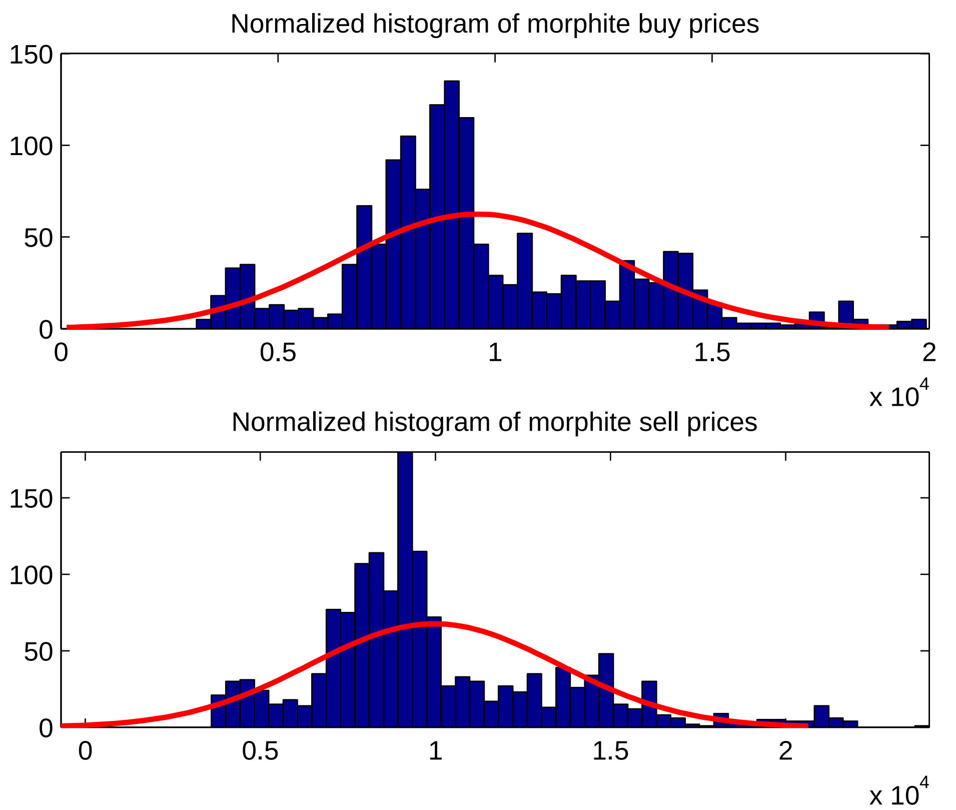

Figure 3: Normal distribution histogram of buy and sell prices of morphite.

of isogen are less than 3, which means that they have a platykurtic distribution (see Table 3). It means that it is flatter than the univariate normal distribution, with a visible wider peak. On the other hand, kurtosis in buy and sell prices of morphite are higher than 3, therefore, they follow a leptokurtic distribution which does not have positive-valued tails. Furthermore, higher kurtosis coefficients are more likely to be responsible for future returns to be larger or smaller in an extreme scenario. In finance, kurtosis is also known as 'volatility of volatility'.

Figure 4 shows the normal probability plot, curvature in the plot indicates departure from normality. Normal probability plots for both buy and sell prices of the isogen, shows that data are not sampled normally. The straight line connecting 0.25 and 0.75 quantiles of the data verifies that an assumption of normality is not reasonable, therefore, the normal distribution does not fit these data well. Similarly, Figure 5 interprets the normal probability plot for morphite's buy and sell prices. The points form a nearly linear pattern and a straight reference line indicates that the further the points are from the straight line, the higher chance of non-normality there is. Herein, straight line between 0.25 and 0.75 quantiles of the data illustrates that normal distribution is not the best choice for this data of buy and sell prices of morphite.

Intuitively, a stationary time series is characterized by its mean, fluctuations and autocorrelation function (ACF). As a result of previous statistics, it is clear that both the time series for isogen and morphite, buy and sell prices are non-stationary. For further analysis, ACF and PACF plots of all four time series are used. ACF plot is a stem plot of coefficient of correlation between time series and its own lagged version, whereas PACF is a plot of partial correlation coefficients between series and its own lagged version.

From Figure 5 it is more clear that the data of buy and sell prices of isogen are less autocorrelated. ACF plots of buy and sell prices of isogen shows that the autocorrelations are high at lag 1 but decreasing gradually. PACF plots of isogen's buy prices have a very high peak of spike at lag 1 only, meaning that all the other autocorrelations in the series can be completely explained by the lag 1 autocorrelation. But isogen sell price has significant autocorrelations at lag 1 and lag 2, then decreasing gradually with frequent spikes. Figure 6 shows ACF and PACF plots of buy and sell prices of morphite. The ACF plots of buy and sell prices indicate strong persistence throughout all the lags. It indicates significant correlation at lag 1 only, which gradually decreases. Whereas, PACF plots of buy and sell prices of morphite show very few significant spikes at low lags.

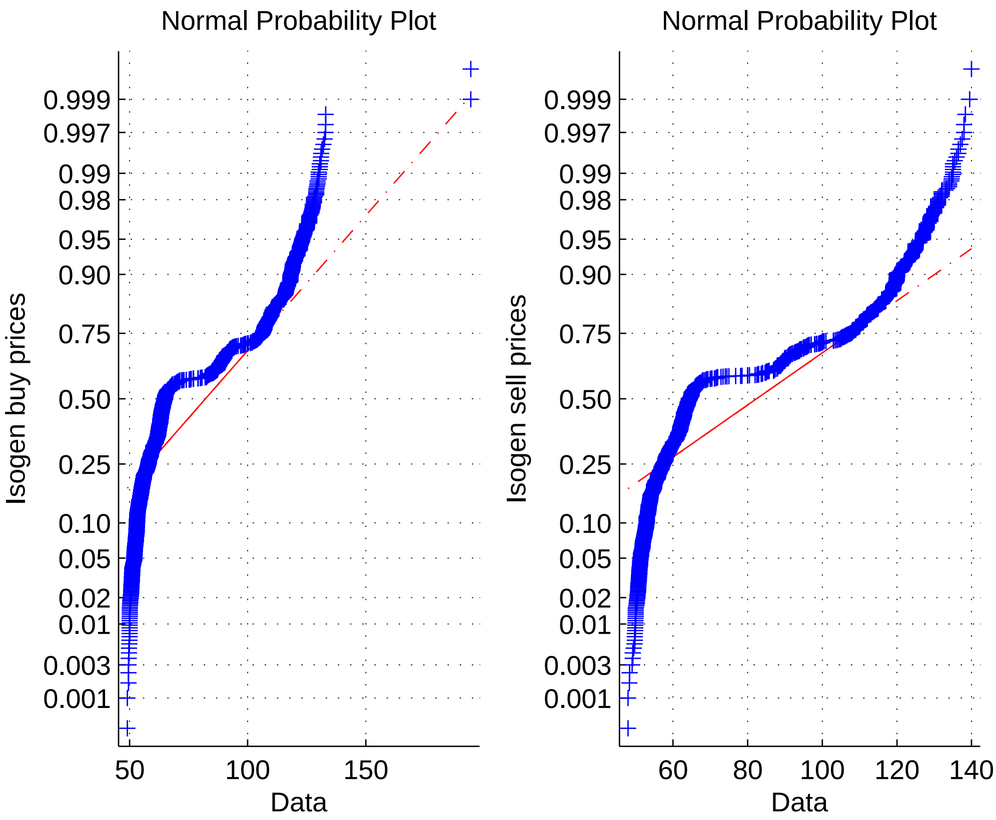

Figure 4: Normal probability plots for buy and sell prices of isogen

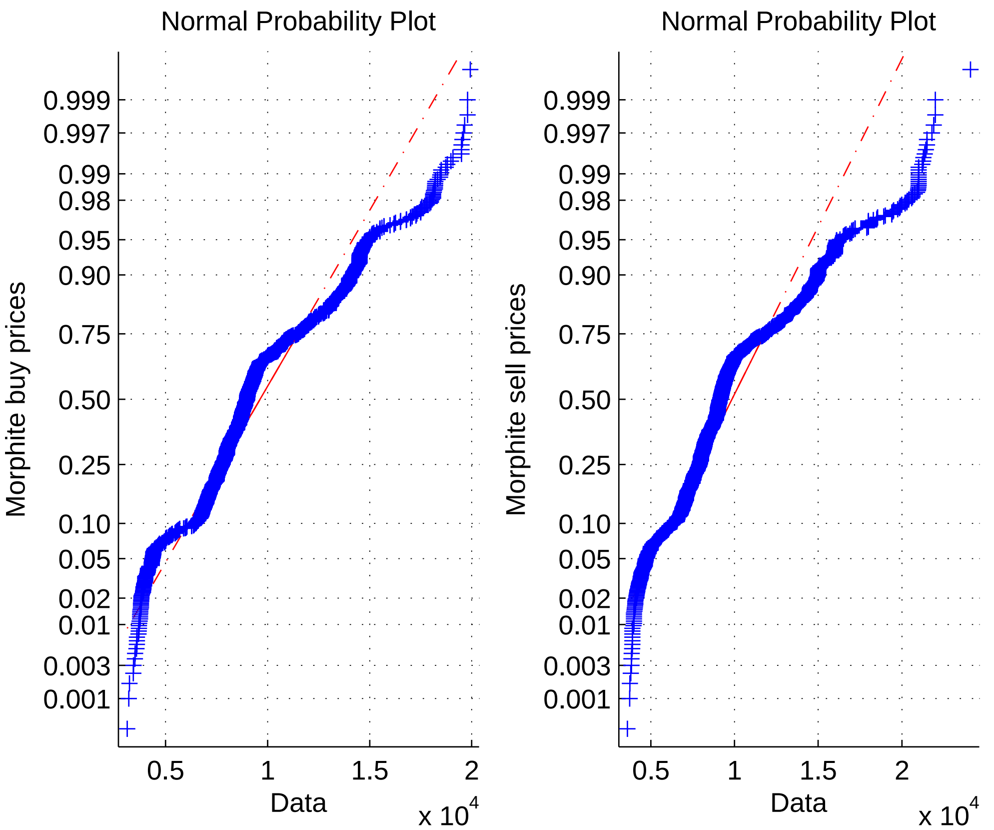

Figure 5: Normal probability plots for buy and sell prices of morphite

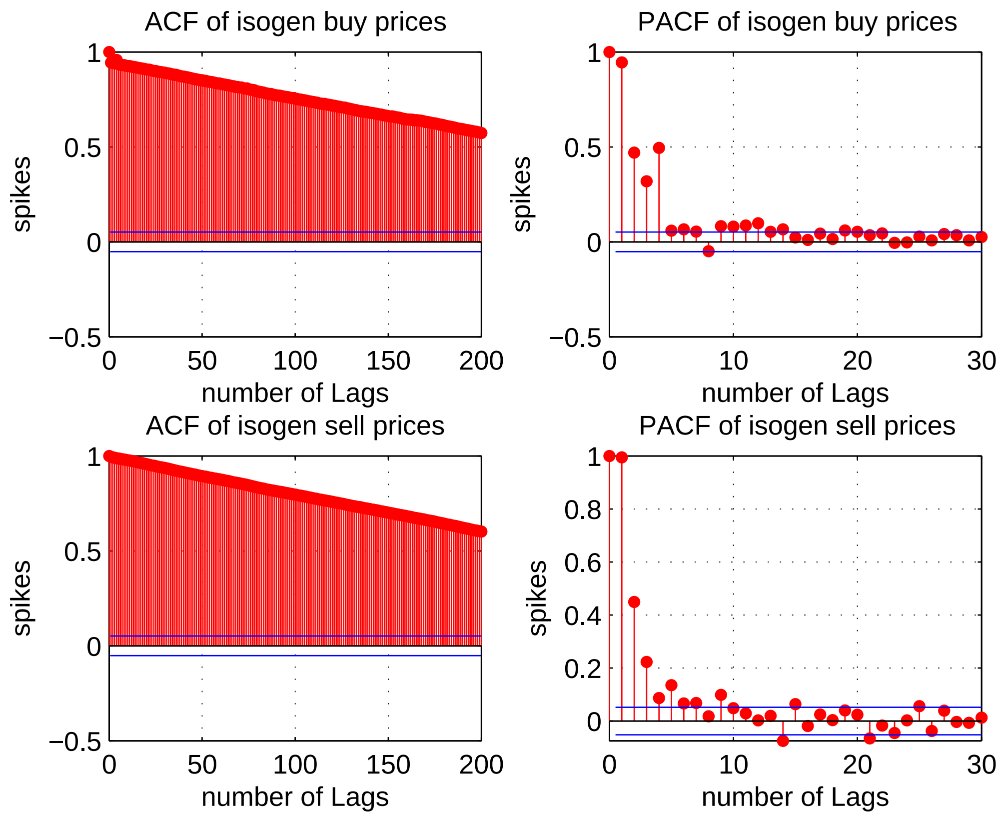

Figure 6: ACF and PACF of buy and sell prices of isogen.

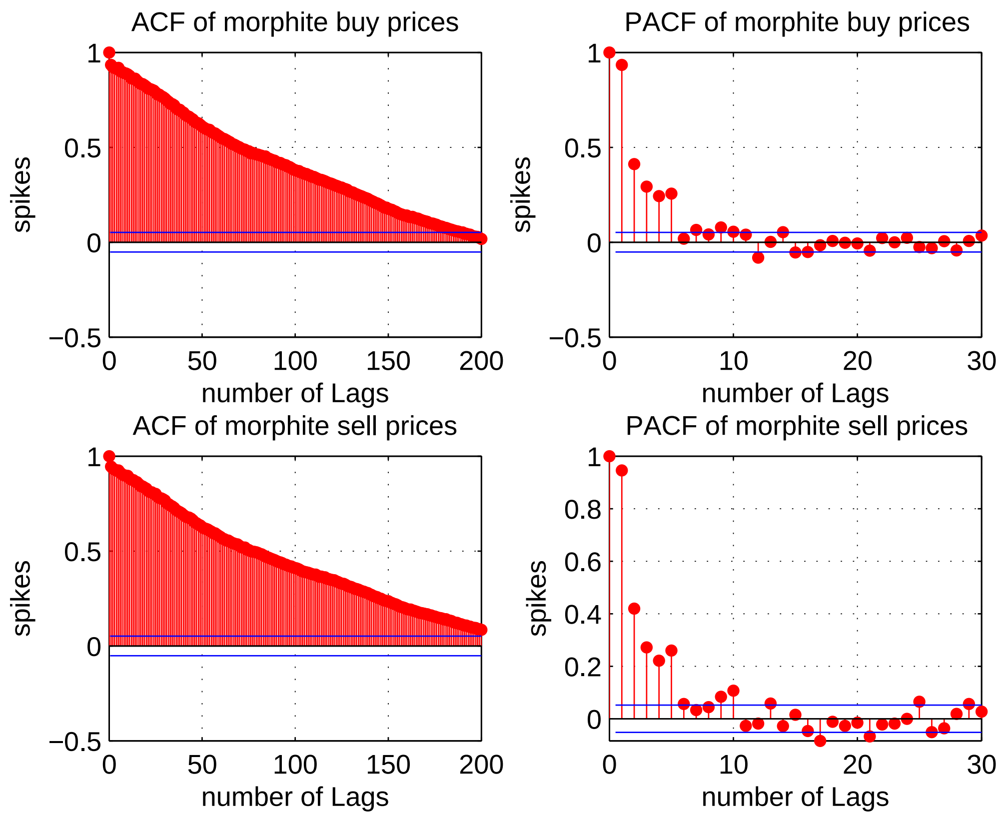

Figure 7: ACF and PACF of buy and sell prices of morphite.

4 MODELLING BACKGROUND

4.1 Introduction to stochastic differential equations and stochastic modelling

There is a wide range of processes in finance, economics, engineering, whose mathematical dynamics can be described in the form of differential equations. In many applications, trajectories measured by system of ordinary differential equations (ODEs) do not behave as the real phenomenon, which points out the possible formation of randomness in the system. Such systems can be represented by adding noise terms to ODEs which creates stochastic differential equations (SDE). In order to properly understand their origin and structure, it is necessary to recall the concept of *Brownian motion.*

Robert Brown, during his study in 1826-27, discovered an irregular or random motion in pollen grains suspended in the water which is known as *Brownian motion.* Later, this same process was mathematically described by Norbert Wiener and thus it is called *Wiener processes.*

4.1.1 Brownian motion

Definition 1 *A real-valued stochastic process $W(\cdot)$ is called a Brownian motion or Wiener process if*

1. $W(0) = 0$,
2. $W(t) - W(s) \sim \mathcal{N}\ (0, t-s)\ , \forall \ \mathrm{t} \geq \mathrm{s} \geq 0$
3. *The process W has independent increments, i.e., if* $\mathrm{r} < \mathrm{s} \leq \mathrm{t} < \mathrm{u}$ *then* $W(u) - W(t)$ *and* $W(s) - W(r)$ *are independent stochastic variables.*

Once randomness is included in a classical ODE, it results in formation of the following stochastic differential equation (1).

$$\begin{aligned} dX_t &= \mu(t, X_t)dt + \sigma(t, X_t)dW_t \\ X_0 &= x_0 \end{aligned} \tag{1}$$

where,

- Wiener process W is a d-dimensional column vector
- function $\mu : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is the drift coefficient
- function $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \to \mathcal{M}(n,d)$ is the diffusion coefficient
- $\mathcal{M}(n,d)$ - class of $n \times d$ matrices
- $x_0 \in \mathbb{R}^n$, a real column vector

In an integrated form,

$$X_t = x_0 + \int_0^t \mu(s, X_s)ds + \int_0^t \sigma(s, X_s)dW_s, \forall t \geq 0 \tag{2}$$

Equation (2) represents the behavior of the stochastic process X_t in continuous time, where the first integral is an ordinary Lebesgue integral and the second integral is an Itô integral. The possible interpretation of the SDE can be such that the stochastic process X_t changes its value by normally distributed expectation $\mu(X_t, t)\triangle$ and variance $\sigma(X_t, t)^2\triangle$ in a small length of time interval $\triangle$ and it is entirely independent of the past performance of the process.

Moreover, to find out if the solution to the SDE (1) exists and whether it is unique, some inequalities are required.

4.1.2 Existence and uniqueness of solutions of an SDE

Inequalities: Suppose that there exists a constant K such that the following condition are satisfied $\forall$ x,y and t:

$$\begin{aligned} \parallel \mu(t,x) - \mu(t,y) \parallel &\leq K \parallel x - y \parallel \\ \parallel \sigma(t,x) - \sigma(t,y) \parallel &\leq K \parallel x - y \parallel \\ \parallel \mu(t,x) \parallel + \parallel \sigma(t,x) \parallel &\leq K(1+ \parallel x \parallel) \end{aligned} \tag{3}$$

Then there exists a unique solution to the SDE (1) [Björk, 2003]. The solution has the following properties:

- X is $\mathbb{F}_t^W$ - adapted.

- X has continuous trajectories.
- X is a Markov Process.
- There exists a constant C such that,

$$\mathbb{E} = [\| X_t \|^2] \leq Ce^{Ct}(1+ \| x_0 \|^2) \tag{4}$$

The explicit solution of the SDE (1) is excessively complicated. However, in few cases the solution of the SDE can be found analytically, for example a *geometric Brownian motion* (GBM).

$$\begin{aligned} dX_t &= \alpha X_t dt + \sigma X_t dW_t \\ X_0 &= x_0 \end{aligned} \tag{5}$$

The equation can be viewed in a simplest linear ODE form as follows:

$$\dot{X}_t = (\alpha + \sigma \dot{W}_t) X_t \tag{6}$$

where W is a white noise.

4.1.3 Ornstein-Uhlenbeck process

The *Ornstein-Uhlenbeck process* is a linear stochastic differential equation. It introduced as a velocity of a particle under Brownian motion. The time derivative does not exist, as the position of the particle is following a Brownian motion. The Ornstein-Uhlenbeck process has emerged in finance as a model of the volatility of the underlying asset prices.

Suppose, the SDE [Smith, 2010],

$$ds = \lambda(\mu - S_t)dt + \sigma dW_t \tag{7}$$

where,

- $S_t, t \geq 0$ is the price of a stock
- W_t is a Brownian motion
- λ is the mean reversion rate
- μ is the long run mean
- σ is the volatility

4.2 Jabłońska-Capasso-Morale (JCM) model

The Wall street stock market is a wide collection of markets and financial institutions. It may appear at first that financial markets are not chaotic. But there is a strong reason and purpose for every financial institution and market. Therefore, it is clear that financial markets grow and run to the response of current technologies, greed and needs of the investors in the economy. Wealth of an economy is determined by the real assets. On the other hand, the financial assets purely rely on the claims of the real assets. For the growth of an economy, this plays a very important role.

Economic time series started to be analyzed through the development and application of computers in economics since 1950s. Business experts believed that tracing certain economic variables over a period of time could help them in prediction of the economy's progress through boom and bust times. Since long time, experts are trying to figure out a way to predict the stock prices over time, assuming that the stock prices reflect on firm's performance and other prospects related to the firm. Maurice Kendall [1953] examined this proposition. His research shows that there is no predictability in the pattern of the stock prices. Prices change randomly, regardless of the past performance.

Though one can possibly say that any information regarding to prediction of stock performance should be reflected in the stock prices on the availability of the information. Kendall's theory seemed to imply that "the stock market is dominated by erratic market psychology or animal spirits which explain the market irrationality" [Bodie et al., 2003].

What if the stock prices were predictable? It would be like winning a million dollar lottery. But why the investors expect the stock prices to reflect all the available information? This leads to the notion of efficient market hypothesis (EMH). There are three types of efficient market hypothesis. The weak form of EMH suggests that all available information from past stock prices is reflected in the current stock prices. The semi-strong form of EMH claims that entire publicly available information is reflected in the current stock prices. The strong form of EMH is the extreme one, states that all the available information along with insider information is also reflected in the stock prices.

4.2.1 Animal spirits and financial markets

An ecosystem runs smoothly because of the crucial role played by preys and predators. Almost all the species in the ecosystem are interdependent. Preys and predators reflect the most dramatic aspects for struggle in life. By nature, preys are weak individuals who fall victims to the predators whose main focus it to satisfy their own hunger (greed). On the other hand, preys have both fear and greed. They have to be less protective in order to get enough food, but to satisfy this greed they have to expose themselves. But they do have fear of exposing which means that they may die of hunger. There is not much difference between this scenario and financial markets.

In financial markets, the investors behavioral approach for decision making undertaking risk also reflects the real animal behavior. The term *'animal spirits'* first introduced by John M. Keynes [1936]. It means 'of the mind' or 'animating' from ancient and medieval Latin form. Keynes proposed that there are strong emotions like confidence and trust in humans which influence their actions in financial markets [King, 2010]. In Keynes' views, these animal spirits are the main cause for the fluctuations in economy [Akerlof and Shiller, 2009].

4.2.2 Spatial population dynamics

Research in spatial population dynamics shows that different patterns of spatial covariance gets generated from three synchronizing mechanisms which are diffusion, geographic climatic variable and conflict enemies amongst the population [Bjørnstad et al., 1999]. In order to model population's spatial spread one can use Skellam's models [Skellam, 1991] which are considered as persuasive tool to approach the dispersal problem of population.

Decision making in animals travelling in groups depends on social interaction among group members. Many species, like herds of sheep, have a tendency to swarm close to other members of the herd. Studies have shown that during swarming, there is always not a single individual but a group of leaders that pull others to move and follow the directions. By the use of numerical simulation, Couzin et al. [2005] showed that there has to be approximately 5% of the overall population moving in a specific direction to drag the entire population behind them. Moreover, in financial markets, like animals have animal spirits, humans have fear and greed [Jabłońska, 2011] which reflect clearly in their decision making. This forms the so called risk averse, risk neutral and risk seeker behaviors.

4.2.3 JCM model

Econophysics, the physics of finance, emerged to study the dynamic behavior of financial and economic markets. In financial markets, many managers hold the most popular and valuable stocks in order to be rewarded for good performance. When these stocks perform well, investors invest more money in the same investments which leads to a hike in the stock market. This introduces to the concept of momentum effect in the financial markets. It simply means, the rate of acceleration of a stock price or volume.

To understand the physical analogies in terms of financial market, it is relevant to mention the one dimensional Burgers' equation as follows,

$$u_t + \alpha u u_x + \alpha u_{xx} = f(x,t) \tag{8}$$

In terms of market dynamics, the following analogies are formed.

- u stands for price of the stock,
- *f(x,t)* denotes the fundamentals (of a periodic character),
- u_x is the spread between the most common bids and given day's average,
- $\alpha u u_x$ describes the momentum term expressing traders' movement towards higher prices,
- αu_{xx} specifies the diffusion term reflecting the fact that markets tend to reach an equilibrium price.

Furthermore, the jump processes have been eliminated. Also the mean-based local interaction $f(k,X_t)$ is replaced by global interactions as Burgers' type momentum component $h(k, X_t)$.

The JCM model stands on the Capasso-Bianchi system of stochastic differential equations in a general form,

$$dX_N^k(t) = [f(X_t^k) + h(k, X_t)]dt + \sigma dW^k(t) \tag{9}$$

for $k = 1,\ldots,N$, used for modelling dynamics of animal population by Morale et al. [2005] or price herding by Bianchi et al. [2003] and Capasso et al. [2005].

In other words, this equation shows that the movement of each particle k of N individuals is based on the local interaction with the nearest neighbors $h(k, X_t)$ and location of each individual with respect to the entire population $f(X_t^k)$. The randomness is presented by Wiener process increments dW^k with volatility σ in the model.

A modified version of this approach proposed by Morale et al. [2005] was applied by Jabłońska [2011] in modelling of prices in commodity markets, including the electricity spot markets. JCM is an example of such a model and it is a type of stochastic population dynamics model with Burgers'-type interaction. Its structure is mainly related to the Ornstein-Uhlenbeck mean reverting process. In this model, the position of each individual (market participant) is described via an SDE which is nonlinearly coupled with all other individuals. Consequently, each trader's location is expressed as his price bid.

The following are the proposed four components of the JCM model:

- **Global mean** stands for the willingness of each individual to stay within the whole group. The whole population is moving around its mean X_t^*.
- **Momentum** effect $h(k, X_t)$ should take place when a sufficiently large subgroup of the population has significantly different behavior in comparison to the whole population.
- **Local interaction** means that each individual follows his neighbors up to some extent. In the model, $g(k, X_t)$ followed by each individual, is the furthest neighbor within a range closest to 5% of the whole population. This component plays and additional role of avoiding overcrowding.
- **Randomness** is present in the model via Wiener increments in each individual's move.

The model is therefore constructed as follows,

$$dX_t^k = [\gamma(X_t^* - X_t^k) + \theta(h(k, X_t) - X_t^k) + \xi(g(k, X_t) - X_t^k)]dt + \sigma_t dW_t^k \tag{10}$$

where,

$$h(k, X_t) = M(X_t) \cdot [E(X_t)) - M(X_t)] \tag{11}$$

M(X) and *E(X)* represents the mode and expected value of a random variable X.

$$g(k, X_t) = \max_{k \in I}\{X_t^k - X_t\}, where, I = \{k \mid_{X^k \in N_{p\%}^k}\} \tag{12}$$

where,

- X^k are continuous stochastic processes.
- $N^k_{p\%}$ is the neighborhood of the k^{th} individual generated by the closest $p\%$ of the whole population.
- parameters $\gamma,\ \theta\ and\ \xi$ are the forces happens because of each interactions.

5 MODELLING RESULTS

In recent years, there has been quite an attempt to uncover and explain the forecasting of financial time series by analyzing their statistical properties. Agents interaction and behavior compromises financial markets price fluctuations. Thus, the empirical studies of financial markets can be done through visualization and prediction of the fluctuations.

Significant discontinuities, so-called jumps, widely occur in financial markets as a result of randomness. There are different mathematical models to characterize the randomness and describe the dynamics in financial markets by some models like jump diffusion model, stochastic volatility model and many more. Their focus is mainly on modelling dynamics of asset prices along with its conditional variance. Daily data of financial assets are typically with high-frequency and low-frequency jumps which can be identified and estimated with the use of those jump-diffusion models.

This section is mainly focusing on interpreting and explaining the statistical properties of financial time series of EVE Online's mineral prices to see their non-Gaussian behavior in the presence of jump component and simulation with the help of JCM model. Also, the results cover the presentation of autocorrelation functions of the original time series and the one simulated via JCM model.

5.1 JCM simulation results

After JCM model was fitted to the minerals isogen and morphite, buy and sell prices, the following results were obtained. Figures 8 and 9 represents the time plot of original isogen and morphite, buy and sell prices along with simulated prices after fitting the JCM model. There are more prominent spikes and steps in all the four time plots when compared with the JCM simulation. Although, even after applying JCM model, the most visible feature of these plots is, random looking irregular component which is also known as residual. This irregular component is a result from short term fluctuations which are not easy to predict and they appear in an unsystematic manner. These fluctuations dominate the movement of the all four time series. Figure 9, the buy and sell prices of morphite are highly irregular and these fluctuations can dominate movements, which covers the trend and seasonality of the series. Although, from the Figures 8 and 9, it is quite clear that future prices will also have a lot of fluctuations. To sum up, the comparison of these four time series with JCM model can be decomposed into two components, a trend with an irregular component.

To determine whether the data in these four time series are skewed or not, their histograms are used to compare the simulated prices with JCM model. Histograms show the distributions of the observed and simulated data. Certainly from Figures 10 – 13, it is clear that all four time series are non-Gaussian in nature. JCM model simulates the prices of these four time series quite well.

Furthermore, normal probability plot is used to check the normality of the simulated prices with JCM model. Normal plot represents theoretical percentiles of normal distributions verses the observed percentiles should be relatively linear. Figures 14 – 17 show that the relationship between the observed and theoretical percentiles is not linear meaning that the residuals are not normally distributed in any of the four time series.

Table 4 demonstrate the statistical properties of original prices compared with the simulated prices of buy and sell, isogen with JCM model. The values of mean and standard deviation displays widely spreaded data. Furthermore, the simulated buy and sell prices of isogen and morphite have negative skewness. The simulated prices have left-skewed distributions with the tail on the left side. Hence, from skewness of simulated prices, it concludes that the prediction and estimation of the future prices will be more concentrating on the left side of the mean and right side with extreme values. The values of kurtosis for buy and sell simulated prices of isogen are less than 3 which means that the distribution appears to be platykurtic. Whereas, the values of kurtosis for buy and sell simulated prices of morphite are higher than 3, follows a leptokurtic distribution. This higher values of kurtosis can be responsible for future predictions.

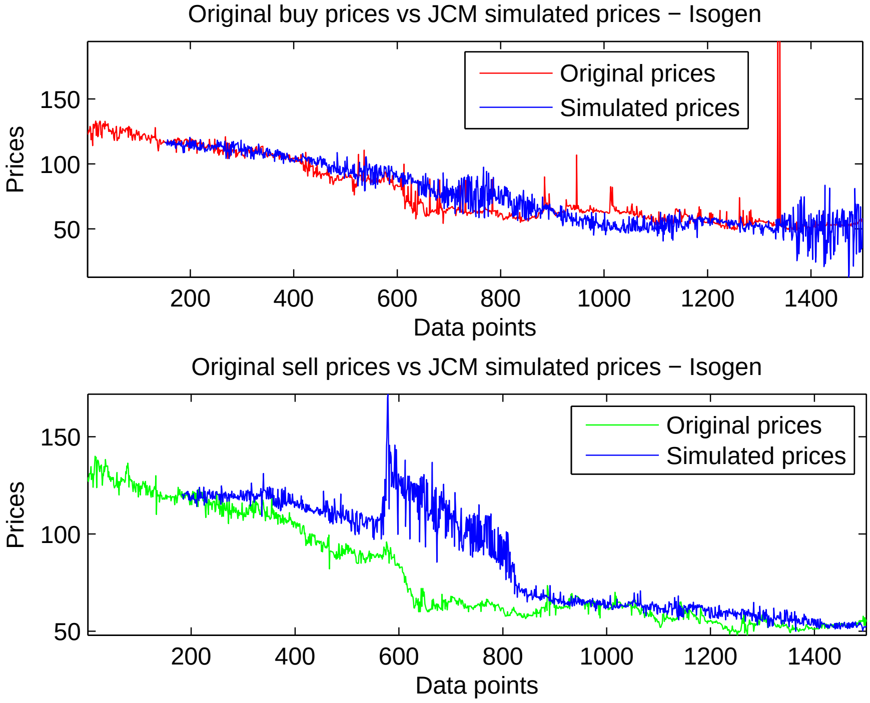

Figure 8: Time line plots of original vs simulated isogen prices.

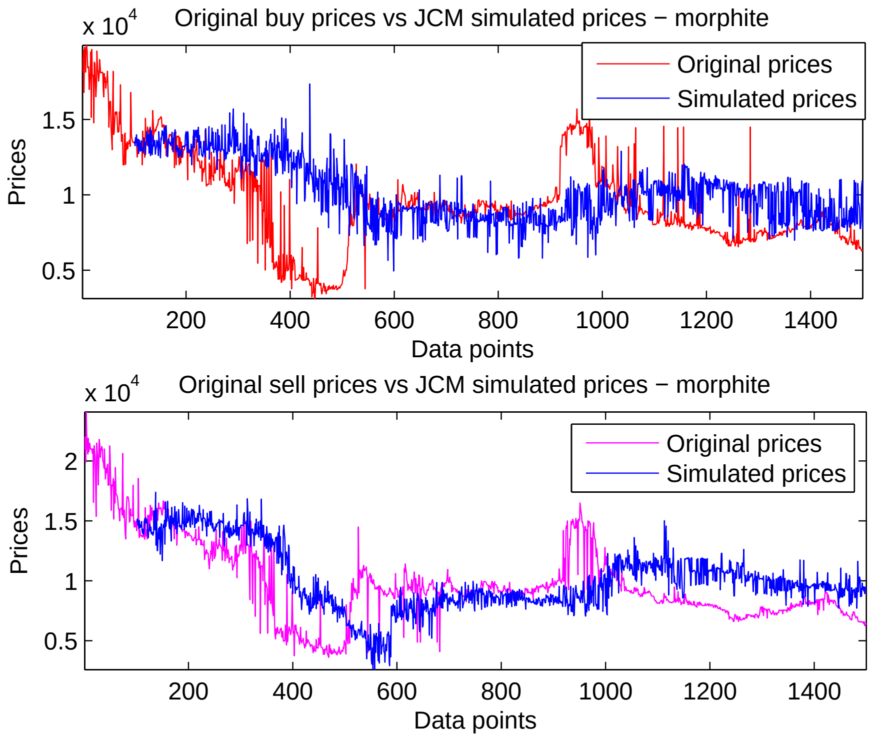

Figure 9: Time line plots of original vs simulated morphite prices.

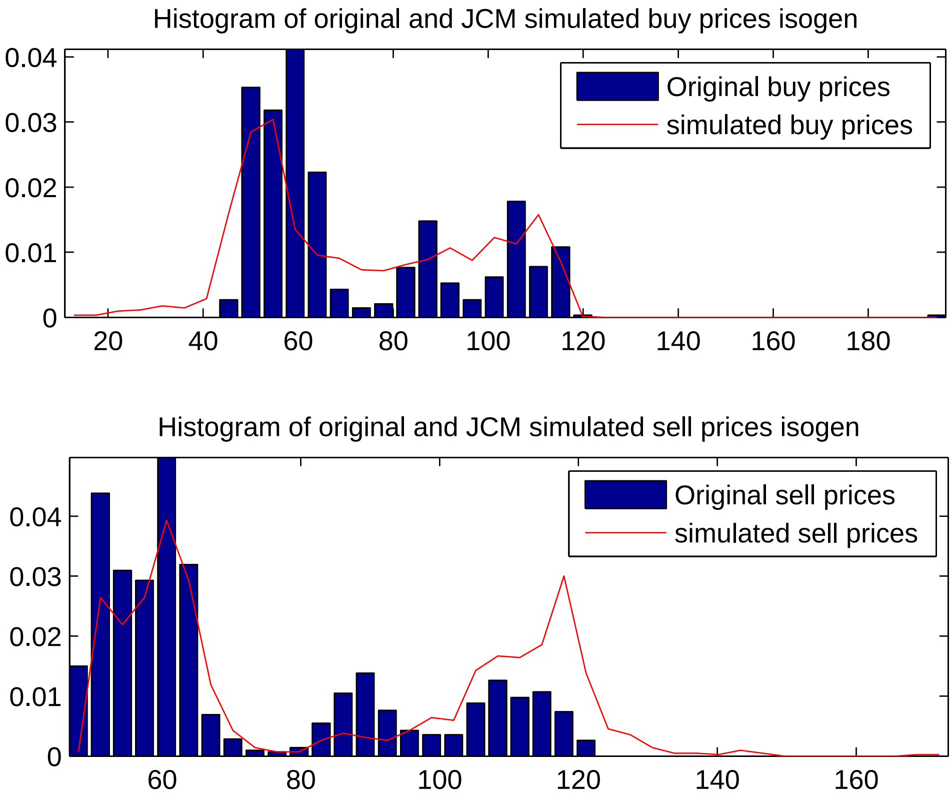

Figure 10: Histogram of original vs simulated isogen buy and sell prices.

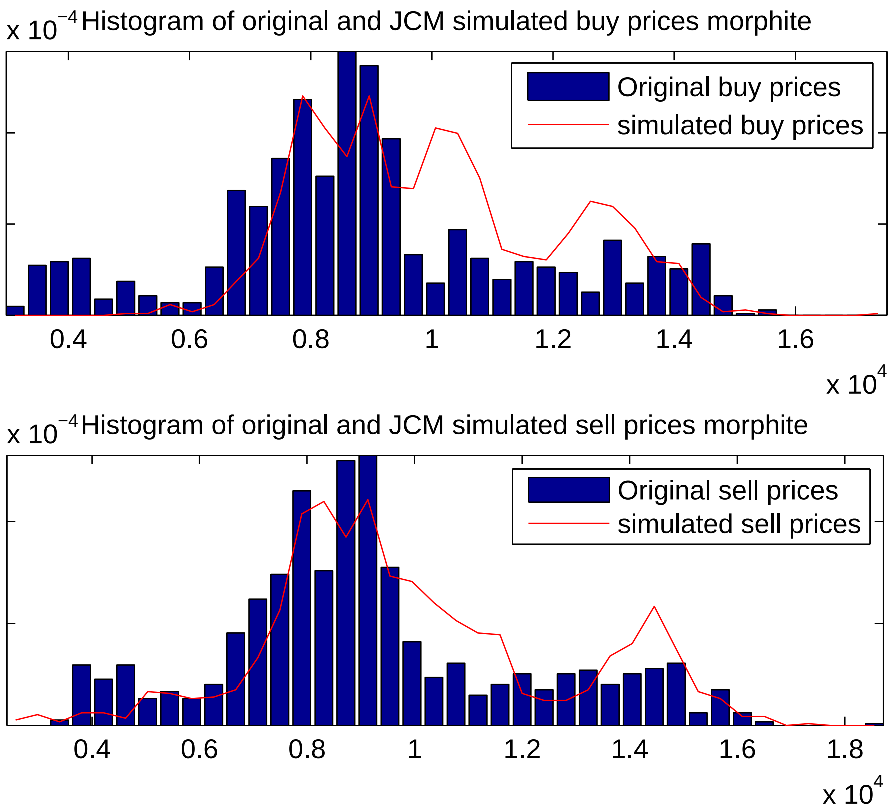

Figure 11: Histogram of original vs simulated morphite buy and sell prices.

Table 4: Statistical properties of original and simulated prices of isogen and morphite.

	Mean	Standard deviation	Skewness	Kurtosis
Original isobuy	79.2234	25.8102	0.6727	2.2233
Simulated isobuy	67.8422	32.2649	-0.4629	2.7338
Original isosell	79.4351	26.8495	0.6432	1.8536
Simulated isosell	72.4572	34.8879	-0.6319	2.8000
Original morphbuy	9602.1700	3156.3485	0.6608	3.5064
Simulated morphbuy	9525.0118	3214.3273	-1.3911	5.6805
Original morphsell	9973.5518	3556.2260	1.0037	4.0924
Simulated morphsell	9458.8985	3653.7327	-0.7275	4.0070

Figures 18 – 21 represent autocorrelation function of original versus simulated prices of isogen and morphite. The simulated prices of isogen, both buy and sell (see Figures 18 and 19) show that the autocorrelations are high at lag 1 but decreasing gradually. This slow decay of ACF points out the long-memory process of the time series. The interval of shocks are comparatively persistent and it will influence the forecasting of the future prices of the non-stationary time series. However, ACF of the morphite, buy and sell prices (see Figures 20 and 21) have highest correlation at lag 1 but it is exponentially decaying to zero as the lag increases, which can be an indication of a future stationary time series. This exponential decay of ACF points out the short-memory of the time series. These patterns of autocorrelations can be explained more precisely with the help of autoregressive (AR) terms.

The plots of PACF in Figures 22 – 25 demonstrate the isogen and morphite, buy and sell prices compared with simulated prices. The PACF of simulated prices have a cut off after lag 2 which means this follows mostly an AR(2) process. Hence, ACF and PACF plots has statistical significance to has strong evidence of forecasting the future prices.

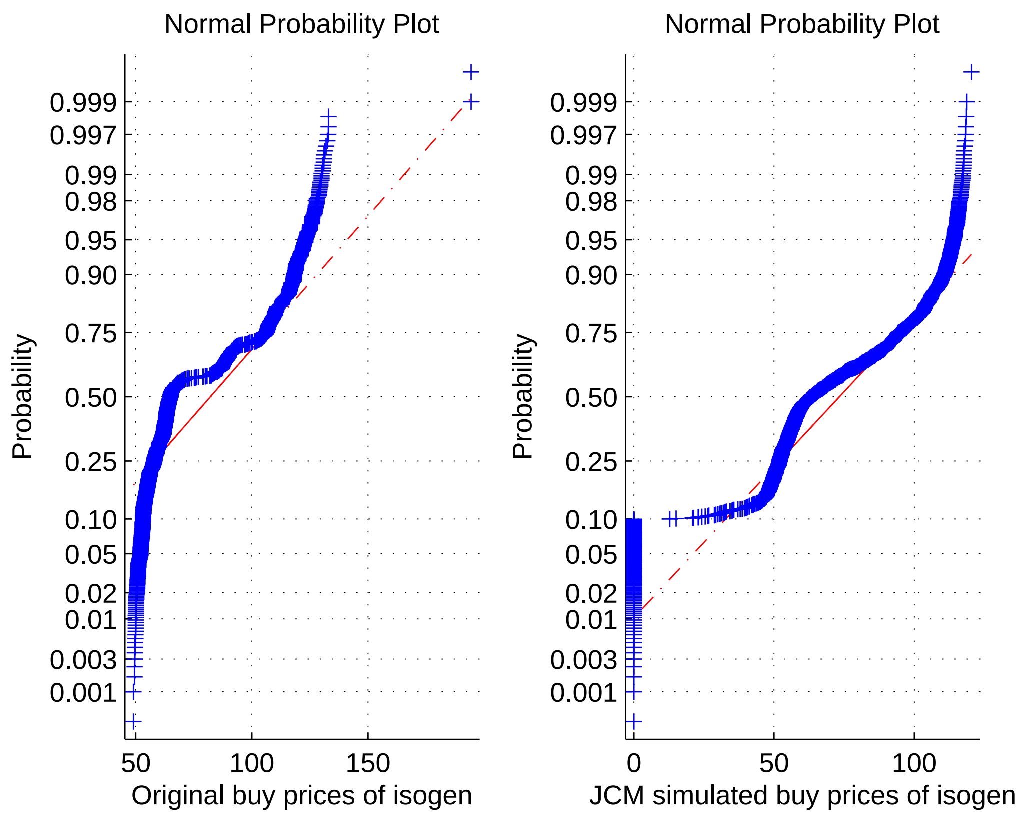

Figure 12: Normality plot of original and simulated isogen buy prices.

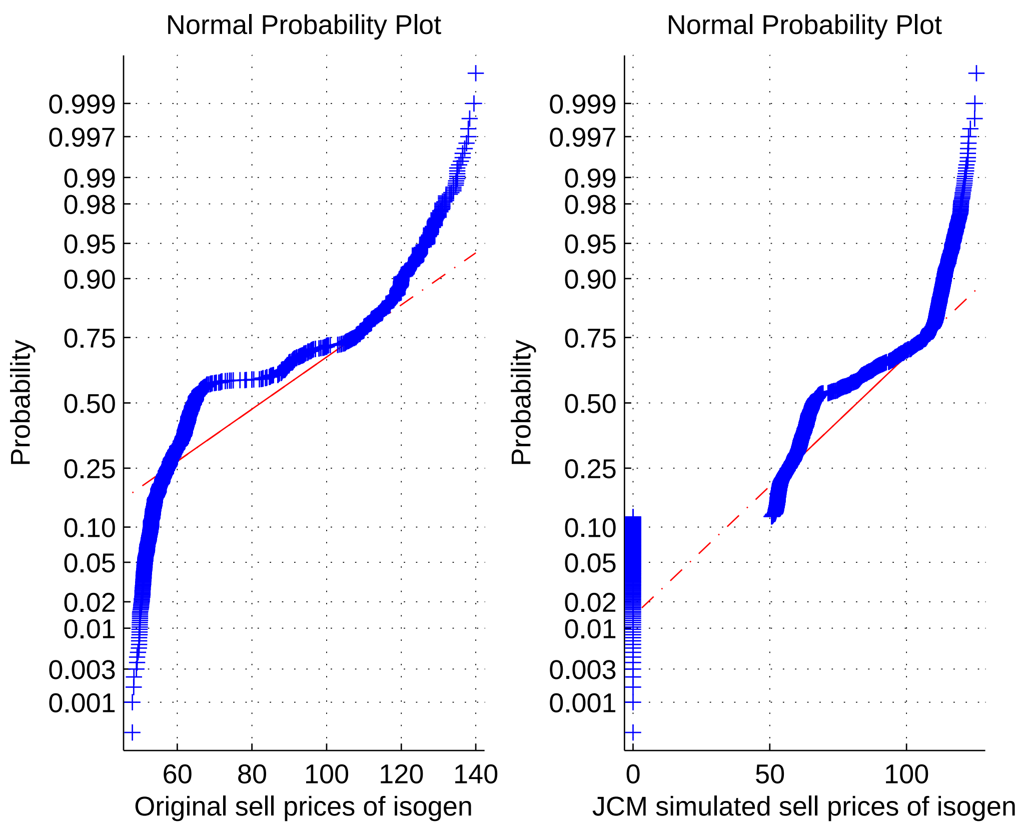

Figure 13: Normality plot of original and simulated isogen sell prices.

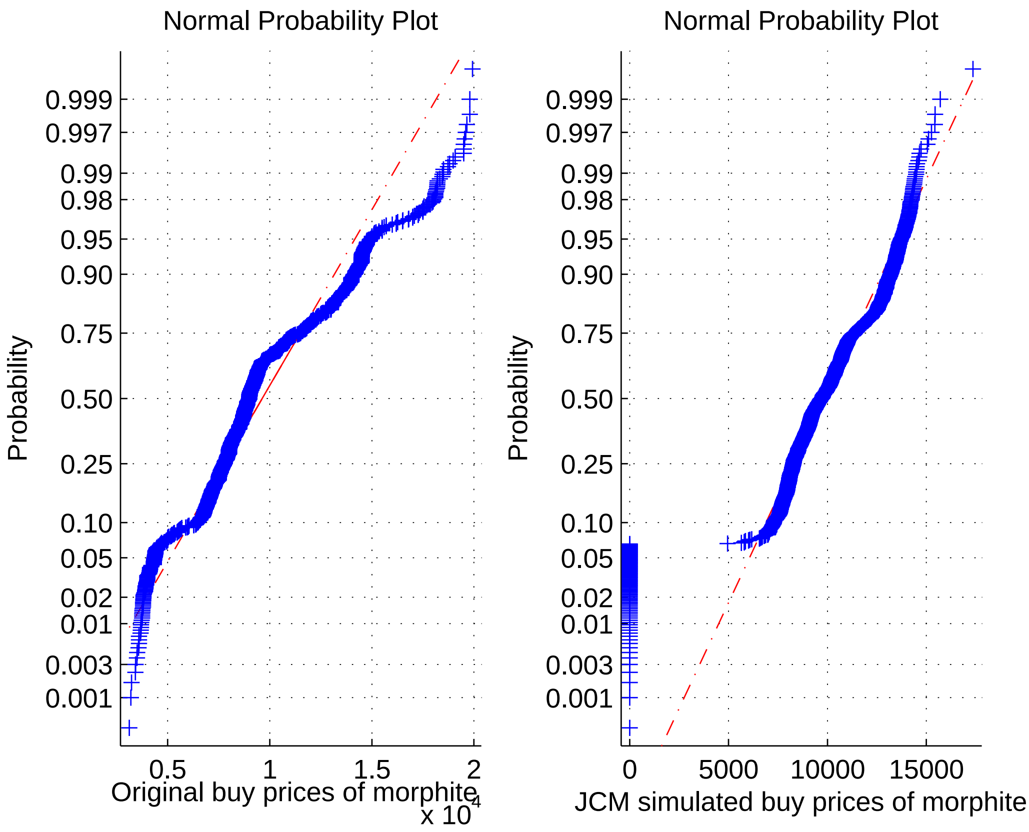

Figure 14: Normality plot of original and simulated morphite buy prices.

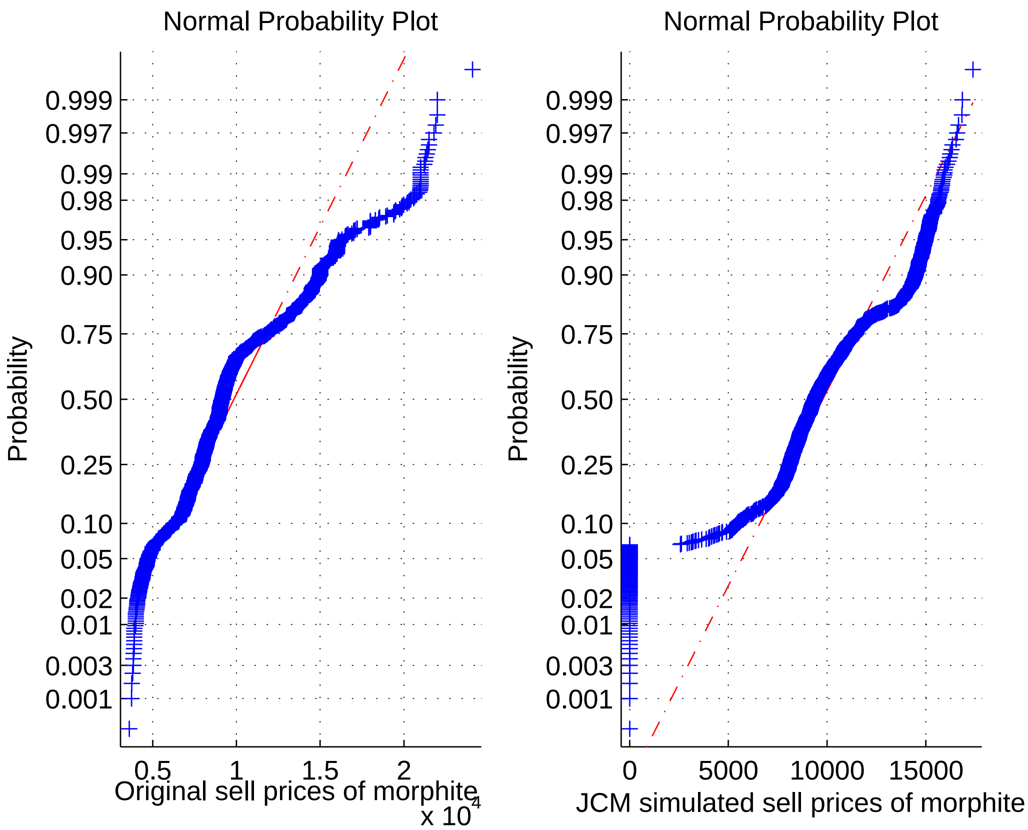

Figure 15: Normality plot of original and simulated morphite sell prices.

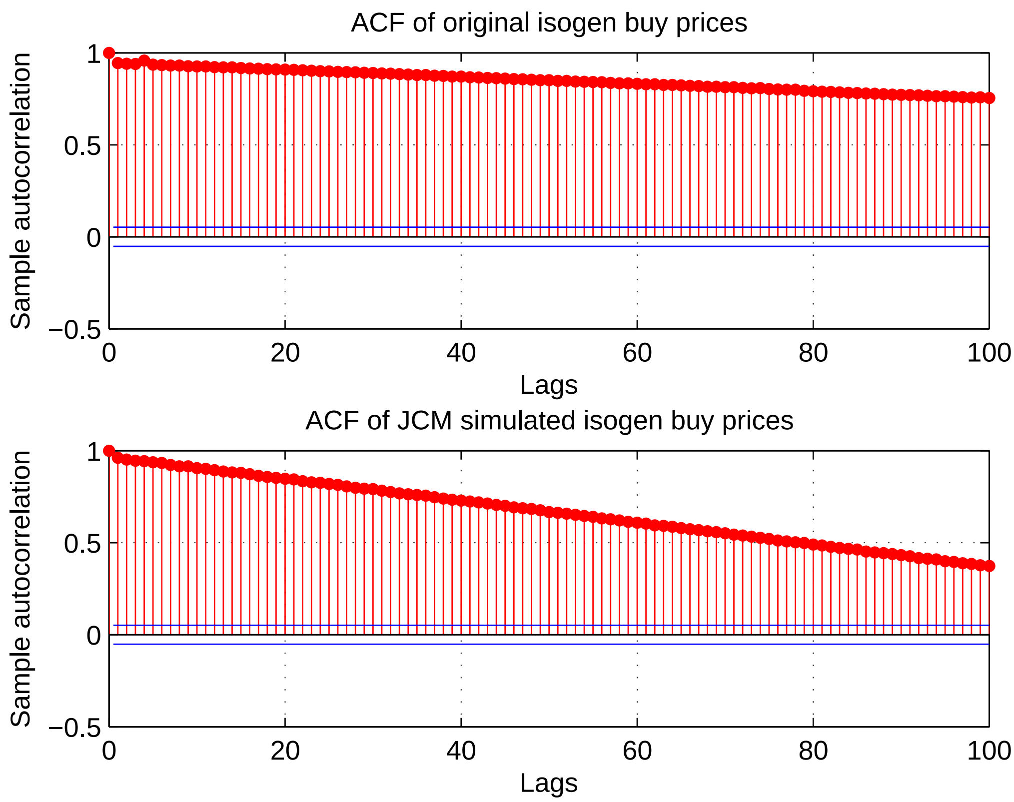

Figure 16: ACF of original vs simulated isogen buy prices.

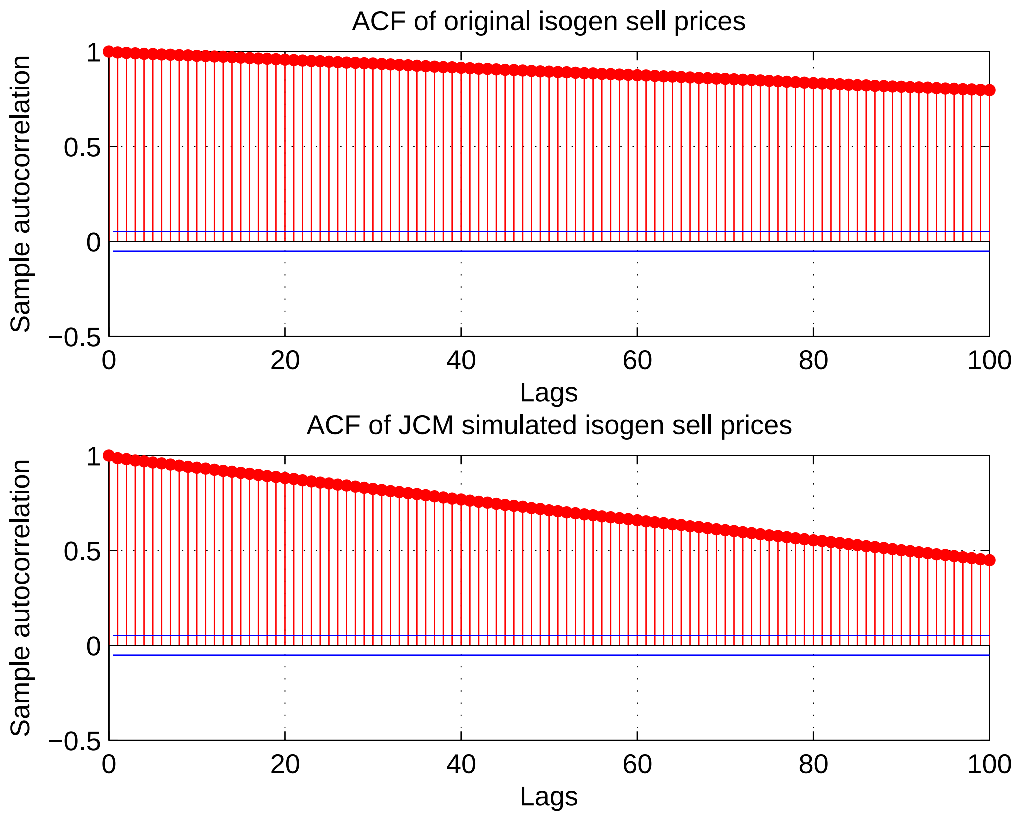

Figure 17: ACF of original vs simulated isogen sell prices.

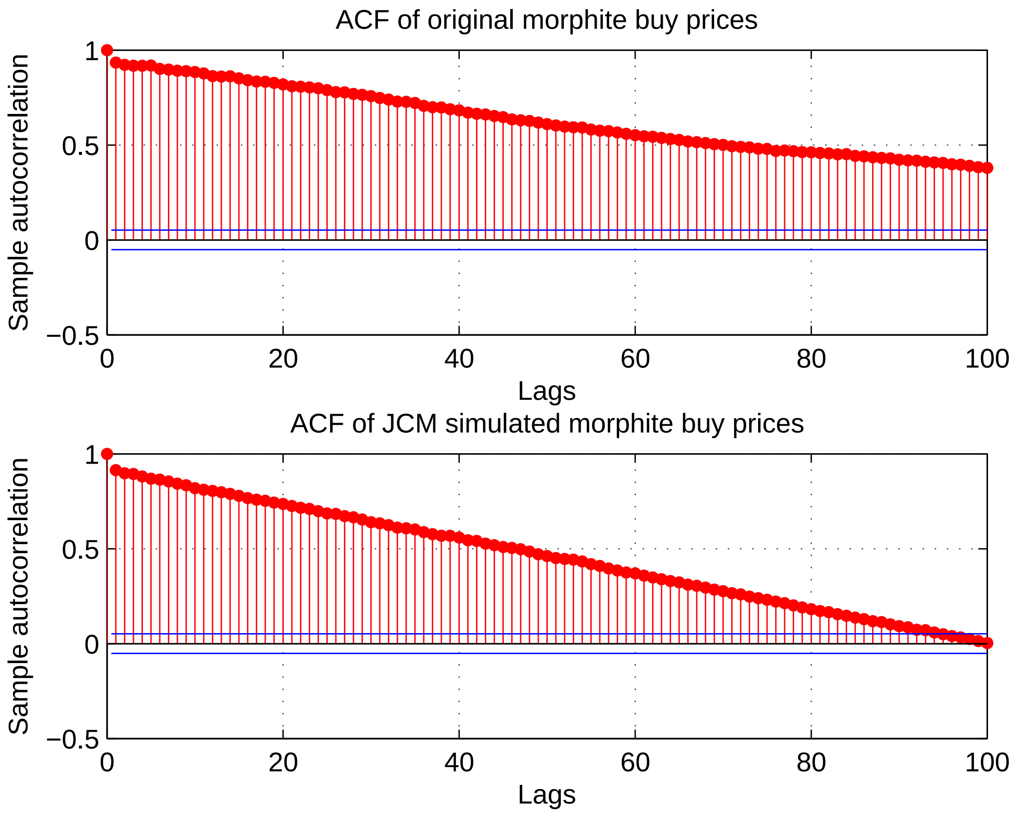

Figure 18: ACF of original vs simulated morphite buy prices.

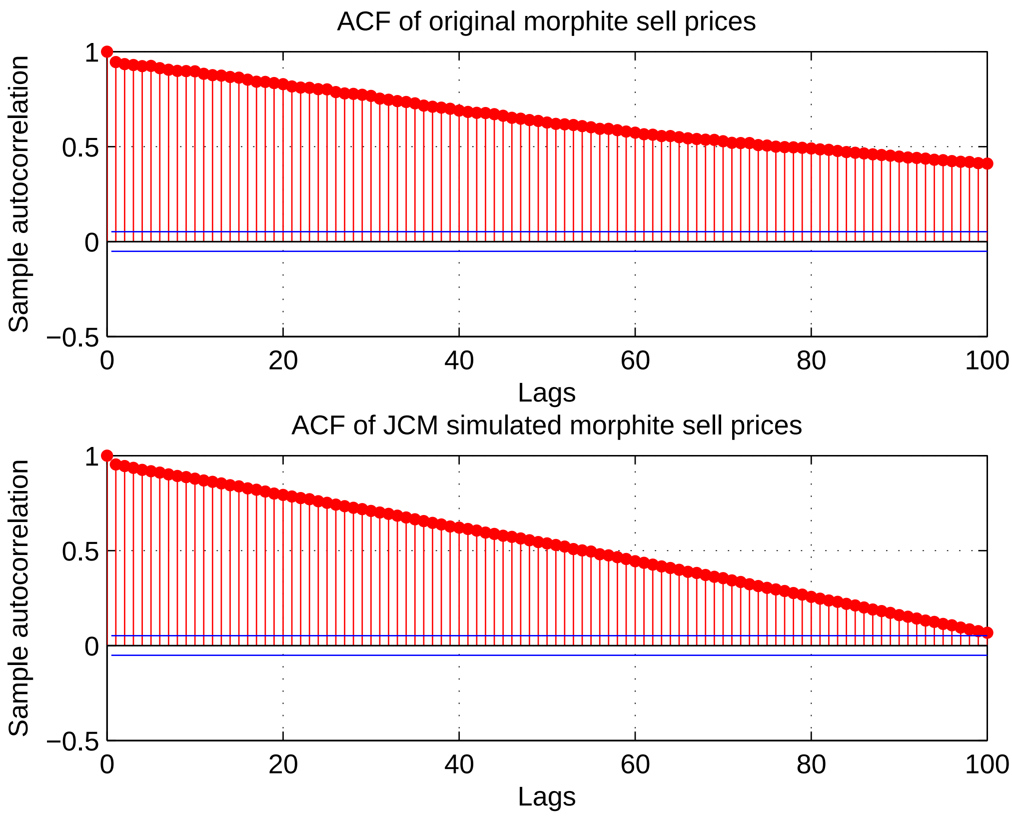

Figure 19: ACF of original vs simulated morphite sell prices.

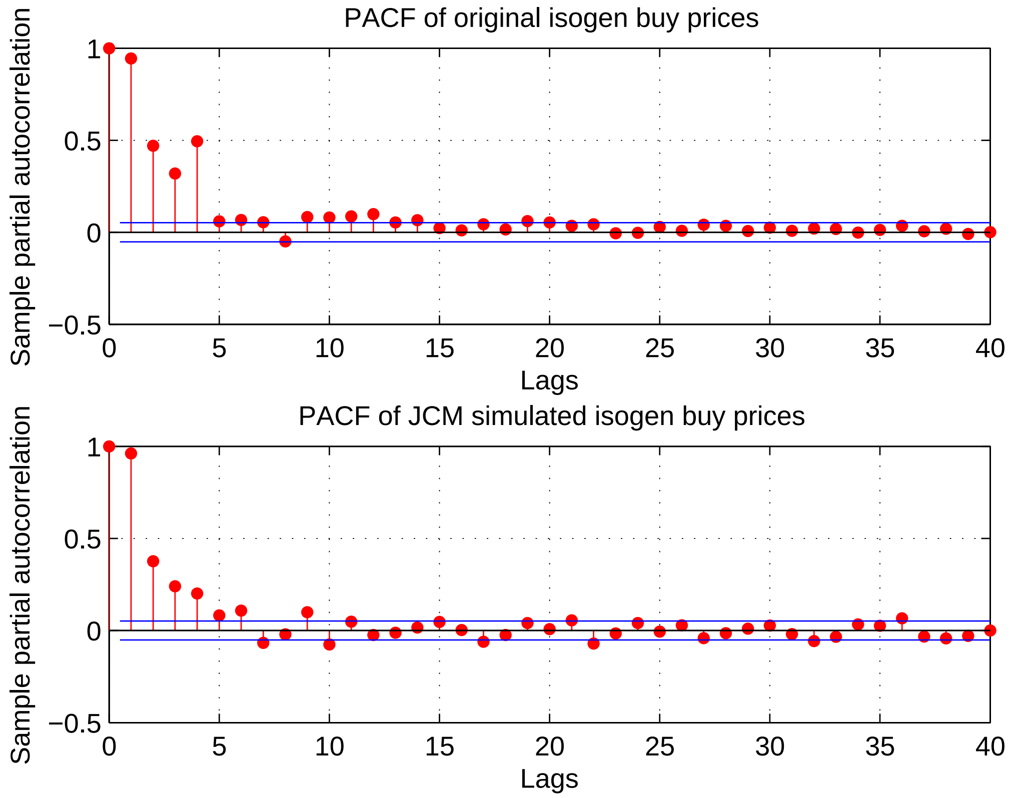

Figure 20: PACF of original vs simulated isogen buy prices.

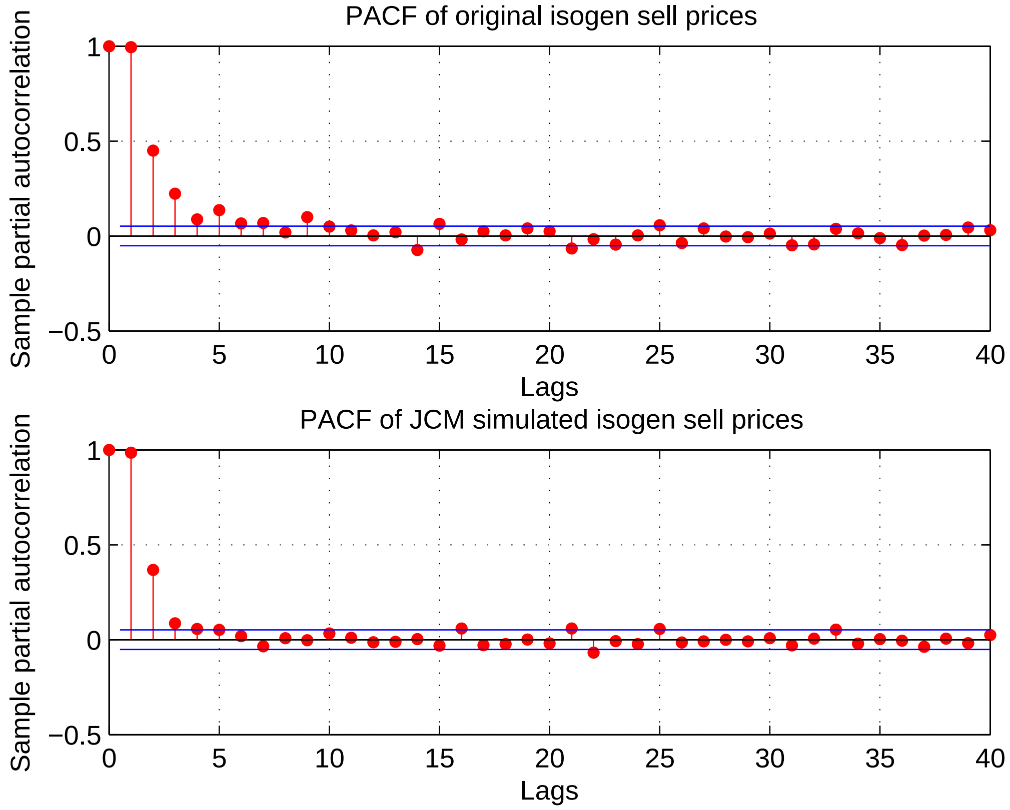

Figure 21: PACF of original vs simulated isogen sell prices.

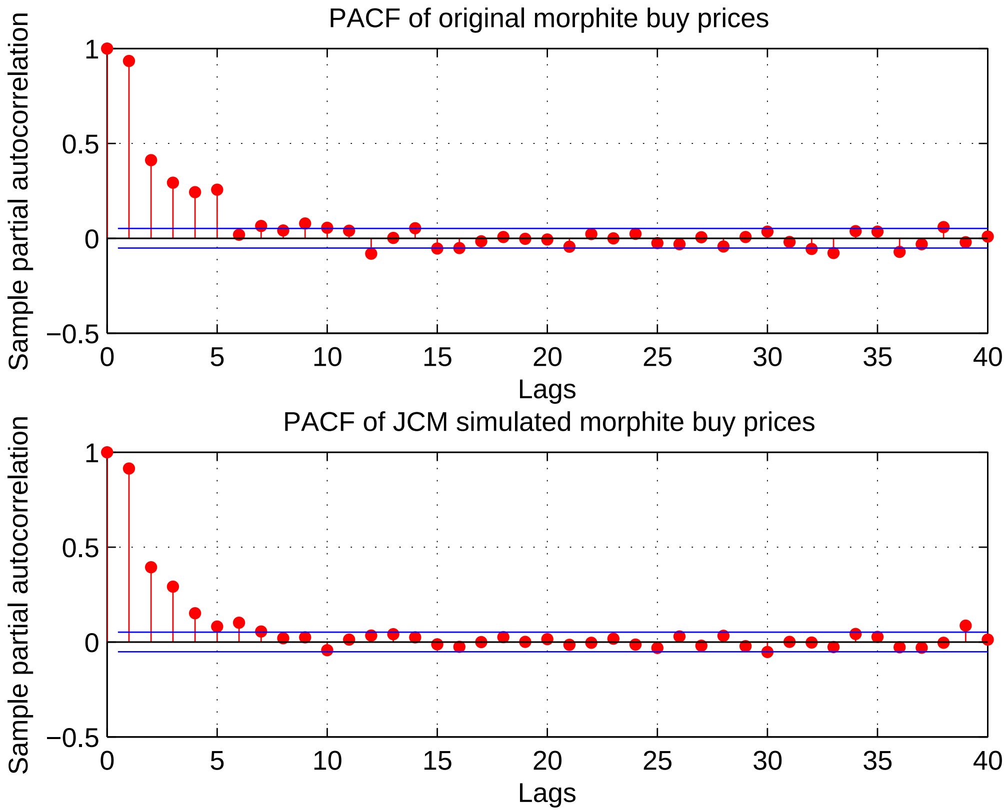

Figure 22: PACF of original vs simulated morphite buy prices.

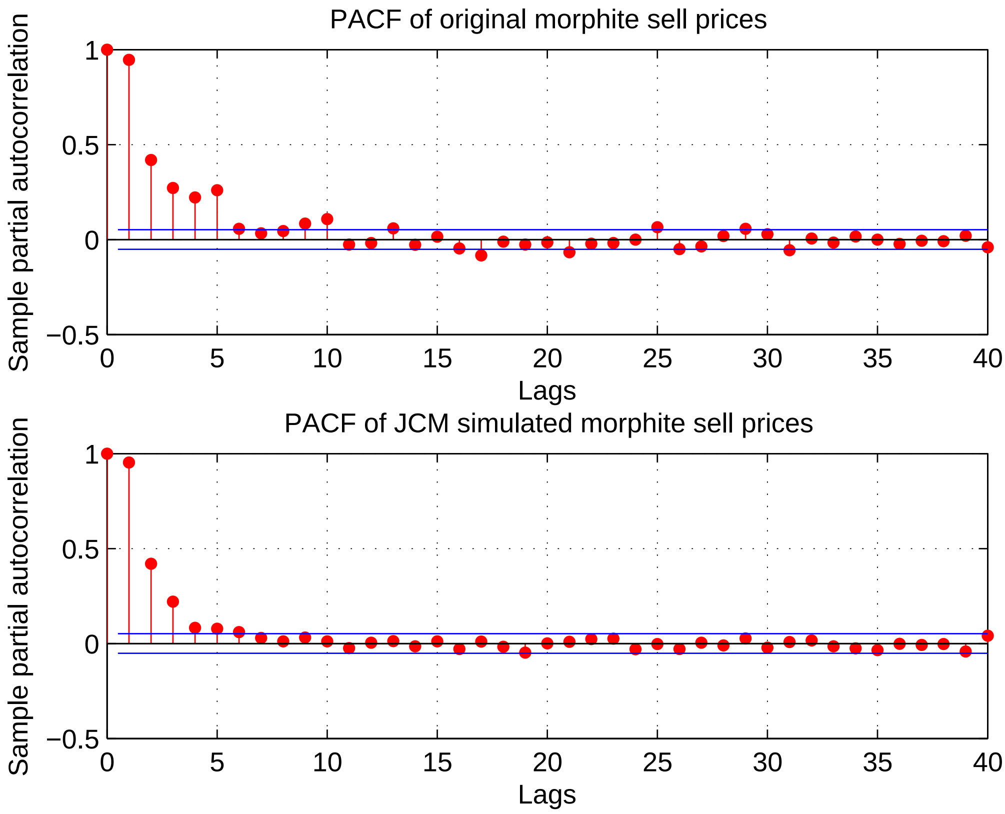

Figure 23: PACF of original vs simulated morphite sell prices

6 SUMMARY AND DISCUSSION

This obtained section discusses and summarizes brief methodology and results in Sections 4 and 5 in simulating prices by fitting Jabłońska-Capasso-Morale (JCM) model. The statistical analysis of the prices was obtained by executing the MATLAB software code of JCM model, fit the model parameters and reproduce the dynamics of the original price series.

In this thesis, the market fluctuations in mineral prices of EVE Online have been analyzed and compared with the JCM model simulation. The model parameters were set by using maximum likelihood method, to obtain the close fit to the original prices. The ensemble size was set to 100 with 5% of the population for local interaction. The JCM model assumption of local interaction from investors or traders also fulfilled as the significant fluctuations in the variability of the prices in the financial markets are result of a collective crowd effect or herding behavior.

From the results, its clear that JCM model was able to reproduce prices given in the time series. After careful statistical analysis, the distributions of these time series were asymmetric and positively skewed. Hence, the fitting of JCM model to the mineral prices of EVE Online concludes that the model was able to capture the dynamics in the behavior of virtual economy.

7 CONCLUSION

This work has begun by primary research in the virtual world and its impact on virtual economies, and to study the market dynamics in terms of behavioral human patterns. To understand virtual economy and its impact on real economy, econophysics approach was implemented. The efficient market hypothesis is a very strong assumption in econophysics. Although, Jabłońska-Capasso-Morale (JCM) model suggested another approach to study the market dynamics by including the mean-reversion and the momentum effect. Moreover, the background of the JCM model was discussed and its application to capture the characteristics and forecast of the price time series. As a result, it successfully modeled the dynamics of non-stationary time series, by use of mean reversion.

Emerging virtual economies from MMORPGs brought a new era in the virtual worlds. As it is a new and challenging field, there has been less mathematical studies done investigating agent-based modelling. Therefore, for further research it is important to understand how fluctuations in financial markets depend on the human behavioral patterns to predict the future prices and what other different kinds of agent-based mathematical models can be used to meet the requirement.

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