

Lappeenranta University of Technology

School of Engineering Science

Technical Physics Major

Anna Savitskaya

PARAMETERS AND CHARACTERISTICS OF LIGHT SOURCES BASED ON LIGHT-EMITTING DIODES

Examiners: Erkki Lähderanta, Nikolay Yeliseev

ABSTRACT

Lappeenranta University of Technology

School of Engineering Science

Technical Physics

Anna Savitskaya

Parameters and characteristics of light sources based on light-emitting diodes

Master's Thesis

2017

69 pages, 59 figures, 4 tables, 3 appendices

Keywords: LED, correlated color temperature, color rendering, dynamic lighting

This study is devoted to experiments, simulation and analysis of color characteristics of LEDs with variable correlated color temperature (CCT). It also contains an overview of the stages of LED development, basic concepts of lighting engineering and colorimetry. Since the light sources are designed to illuminate places with a lack of natural light, the main factor for determining their properties is human perception. For this reason, it is important to consider the nonlinear response of the eye to radiation with different wavelengths. All light sources have different spectral composition and are perceived in different ways. Thus, the previously developed methods of color rendering index (CRI) and color quality scale (CQS) make it possible to evaluate the quality of the radiation. For this purpose, spectral measurements of color and white LEDs were carried out. Also a computer program in MATLAB environment was developed for spectral simulation of LEDs and to calculate their color characteristics.

LIST OF SYMBOLS AND ABBREVIATIONS

CCT correlated color temperature

CIE International Commission on Illumination (Commission Internationale de L'éclairage, French)

CRI method of color rendering index

CQS method of color quality scale

LED light-emitting diode

OS optical system

PWM pulse width modulation

Qa color quality scale

Ra general color rendering index

Ri special color rendering index

RGB LED red-green-blue light-emitting diode

RGBW LED red-green-blue-white light-emitting diode

TABLE OF CONTENTS

1. Introduction	6
1.1. Background	6
1.2. Objectives and delimitations	7
1.3. Structure of the thesis	7
2. About LEDs	8
2.1. The history of development of LEDs	8
2.2. Principle of operation	9
2.3. Adjusting luminous flux and chromaticity of LED	13
2.4. Quantitative and qualitative characteristics of LEDs	15
3. Basics of lighting engineering	17
3.1. Photometry quantities and features of human vision	17
3.2. Basic concepts of colorimetry	20
3.3. Correlated color temperature	23
3.4. Color characteristics of white LEDs and factors that affect them	25
4. Proposed methods	27
4.1. CRI	27
4.2. CQS	31
4.3. Spectra simulation	34
4.4. Calculating the proportion	35
5. Measurements	39
5.1. Equipment and measurement scheme	39
5.2. Brief description of measurement objects	41
5.3. Spectra of LEDs	44
5.3.1. LED №1	44
5.3.2. LEDs №2-4	44
5.3.3. LED №5	46
5.3.4. LED №6	46
5.4. Luminous characteristics	47
5.5. Angular characteristic	48
6. Calculations and discussion	49
6.1. Program	49

6.1.1.	The program for two-crystal LED	50
6.1.2.	The program for four-crystal RGBW LED	51
6.2.	Coefficients	52
6.3.	Spectra for the required range of CCTs	55
6.4.	CRI and CQS	58
7.	Conclusion	61
	References	62
	Appendices	

1. INTRODUCTION

1.1. BACKGROUND

On average, lighting needs up to 20% of the total electricity consumption [1]. However, it should be taken into account that in reality the norms of illumination are not always obeyed; sometimes they are two times less than normalized. Also the regulated levels of illumination are far from optimal values. If no actions are taken, then the need for electricity for lighting will subsequently grow much faster than was expected. It explains the increase in demand for electricity.

Speaking about expenses especially for lighting purposes, it is necessary not to reduce the level of illumination, but to use electricity more efficiently and rationally. To reduce energy costs for lighting, one need to look for ways to save electricity consumed by lighting installations. There are two possible ways: using energy-saving light sources and optimizing the operating mode of the lighting system. These are also called extensive and intensive methods.

The second method makes increases the efficiency of the lighting system by optimizing its operation modes using appropriate equipment. One of the ways of optimization is the possibility to adjust the luminous flux of the lighting installation as a function of natural light. In addition to the luminous flux control function, the lighting control systems may be capable to change the correlated color temperature of the radiation. In this case they are called “systems of dynamic lighting”. Thus, such systems of lighting not only save energy, but have a beneficial effect on human health [2].

On the basis of the foregoing, following requirements for light sources can be formulated:

- Energy efficiency;
- Luminous flux adjustment;
- Adjustment of correlated color temperature (CCT).

Such requirements are best suited to LEDs. LEDs are a promising and energy-efficient light source with a number of advantages over traditional light sources: long service life, high light output, high color rendering, aesthetics, environmental friendliness, reliability, high durability.

Using LEDs in systems of dynamic lighting it is necessary to take into account their quality characteristics and parameters, such as correlated color temperature (CCT) and color rendering. It is important to understand that the color rendering index also changes when the correlated color temperature changes. These and many other features of LEDs have not yet been studied. Therefore, it is important to investigate the color characteristics of light sources based on light-emitting diodes and to formulate criteria for selecting the types of LEDs that can be used in systems of dynamic lighting.

1.2. OBJECTIVES AND DELIMITATIONS

The purpose of this work is to formulate requirements for the characteristics and types of white LEDs for their application in systems of dynamic lighting.

The task of the work can be defined as the investigation of the influence of the type of LED on the spectral, as well as the color characteristics of white LEDs of various types:

- two-crystal LEDs with two luminophore-based light-emitting diodes;
- four-crystal LEDs based on red, green, blue and white light-emitting diodes (RGBW);

To solve this problem, it is necessary carry out measurements of electrical, spectral and angular characteristics. It is also required to develop a program for computation of the spectral and color parameters (CCT, Ra, Ri, Qa), to make calculations and to analyze the results.

1.3. STRUCTURE OF THE THESIS

Chapter 2 contains detailed information on the development of the LED industry, the main principles of their operation and their properties. Chapter 3 presents an introduction to lighting engineering, the main terms and quantities that are important for describing the LED operation of LEDs in terms of human perception. In this chapter, some concepts of colorimetry are also highlighted. The next chapter consists of a description of the calculation sequence of color rendering for two main methods. Chapter 4 contains also the way of calculation the spectral proportion for different kinds of LEDs. Chapter 5 is devoted to measurements. Chapter 6 consists of a brief description of interface of developed program, calculations and their analysis.

2. ABOUT LEDS

2.1. THE HISTORY OF DEVELOPMENT OF LEDS

Despite the fact that nowadays the LED is often called a modern, innovative and fundamentally new light source, its history dates back to the twentieth century.

The first who discovered the phenomenon of photoluminescence was the British scientist Henry Round, who in 1907 worked with SiC crystals. His scientific work was about how visible radiation was generated from SiC under the influence of voltage. Round noticed that under the influence of low voltage the light was yellowish, while with increasing voltage, the color changed to blue [3]. However, in those days there were no methods to determine the exact properties of materials, which did not allow the physicist to explain this phenomenon.

Much later, in 1928, O.V. Losev published the results of his research on the phenomenon of luminescence. He found that the appearance and disappearance of luminescence in SiC diodes occurred so quickly that it made possible the production of "light relays" on their basis.

Silicon carbide diodes were the progenitors of modern light-emitting diodes. Their conversion efficiency of electric energy into light was 0.005%. Further studies did not significantly improve the light output of light-emitting diodes from silicon carbide. This connection refers to indirect conductors, in which the probability of interband optical transitions is very low. Therefore, by the beginning of the 90s, SiC could no longer compete with compounds of the A^{III}B^V type.

In the 1960s, Nick Holonyak from General Electric experimented with gallium, arsenic and phosphides. Together with Robert Hall, scientists created a laser with visible radiation. After a while, LEDs with red light began to be used in commercial structures.

A student of Holonyak, George Creedord in 1972 invented a yellow LED and crystals with a higher brightness of red and red-orange LEDs [4].

A big step in the history of the development of LEDs is the use of heterostructures. Zhores Alferov and his co-workers developed double heterostructures in the 1970s. It allowed to significantly increasing the external quantum efficiency. The use of heterostructures based on gallium-aluminum arsenides made it possible to achieve an external quantum efficiency up to 15% for the red part of the spectrum (light output to 10 lm / W) and more than 30% for infrared [5]. In 2000, Zhores Ivanovich Alferov won the Nobel Prize in Physics for the development of semiconductor heterostructures and the creation of fast opto- and microelectronic components.

In 1987, HP was able to improve the technology of GaAlAs, and by 1990, the red, yellow and green LEDs had achieved a luminous flux of 1 lumen. Thus, LEDs could already be used as separate light elements, such as lamps in cars.

In the beginning of 1990, a more efficient AlGaInP based semiconductor (aluminum-gallium-indium phosphide) began to be used. This semiconductor made it possible to reduce the degradation of LEDs significantly and expand the range of colors. Since then, the LEDs of almost all colors began to be produced.

LEDs of all the basic colors were already known to mankind. The only problem was to find how to get the blue LED. In the late 1980s, Japanese scientists Isamu Akasaki, Hiroshi Amano and Shuji Nakamura solved the important issue of growing epitaxial gallium nitride structures. By 1993, engineers had introduced a new blue LED to the world. In 2014 this group of Japanese scientists was awarded the Nobel Prize in Physics " for the invention of efficient blue light-emitting diodes which has enabled bright and energy-saving white light sources " [6].

In the years 2000-2005, luminous flux of the LEDs has already reached the value of 100 lm and even higher. By that time, white LEDs of warm and cool colors, similar to the color of incandescent lamps, fluorescent lamps and to natural light, had already appeared. Gradually, LEDs competed with traditional light sources and began to be used in theatrical and stage lighting.

Currently, LEDs are widely used in various general lighting systems. Every year they are more and more popular, displacing other sources of light.

2.2. PRINSIPLE OF OPERATION

The LED's work is based on the p-n junction and the processes occurring in it. This pn-junction represents the following compound: by doping, the n-type material is doped with donors (elements from the fifth group, such as phosphorus or arsenide, called donor impurities), and the p-type material is doped with acceptors (elements from the third group, such as boron, called acceptor impurities) Atoms in the n-type material give additional electrons, and atoms in the p-type material give holes which can be defined as places in the outer electron orbits of atoms in which there are no electrons [7]. If a contact is established between two such semiconductors, there will appear diffusion current. Diffusion current is the charge carriers, chaotically moving, flow from the region where there excess of them to the region with shortage of them. During diffusion, electrons and holes carry a charge. As a consequence, the region at the boundary becomes charged, and the region in the p-type semiconductor that adjoins the boundary will receive an additional negative

charge brought by the electrons, and the boundary region in the n-type semiconductor will receive a positive charge brought by the holes.

The electric field produced by the formation of space charge region causes a drift current in the direction opposite to the diffusion current. In the end, dynamic equilibrium is established between the diffusion and drift currents, and the overflow of charges ceases. The scheme of directions of diffusion and drift currents can be seen in Fig. 1.

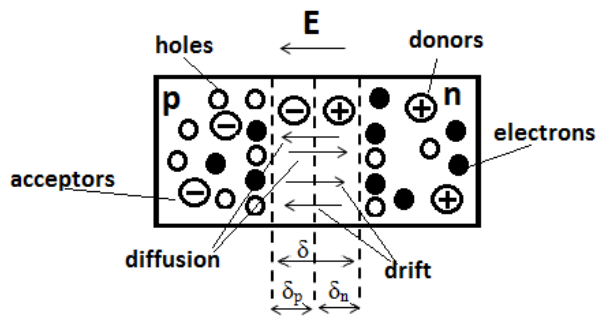


Fig. 1. Diffusion and drift currents in the pn junction.

With forward bias, the diffusion current predominates over the drift current, since the electric field created by the applied voltage is directed opposite to the direction of the electric field between the space charge regions, and the dynamic equilibrium is violated. As a result, the potential barrier between p and n regions will decrease.

If the external voltage is applied so that the field created by it is of the same direction as the field between space charge region, this will lead to increasing the space charge region, and the current does not pass through the p-n junction. This connection of voltage to the p-n junction is called reverse bias (Fig. 2).

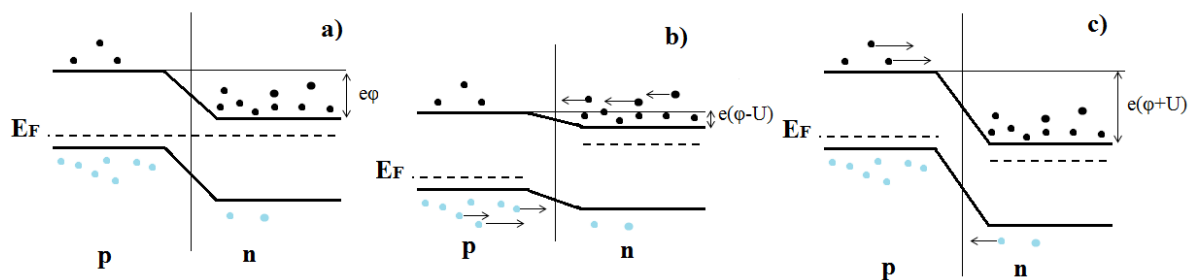


Fig. 2. Energy diagram of the pn junction. a) Equilibrium condition b) With applied forward voltage c) With applied reverse voltage.

All diodes work on this principle. As for the LEDs, principle is the same: the current flows from the p-zone, or anode, to the n-zone, or cathode, but not in the opposite direction. When an electron meets a hole, it falls into a lower energy level and releases energy in the form of a photon.

The wavelength of the emitted light and, thus, its color, depends on the width of energy gap of the materials forming the pn-junction. In silicon or germanium diodes, electrons and holes recombine with a non-radiative transition, which does not produce optical radiation.

All modern LEDs are developed on the basis of heterojunctions. The use of heterostructures makes it possible to improve the efficiency of LEDs. This is due to the limitation of carriers in the active region, which eliminates the diffusion of minority carriers over long distances and increases the probability of recombination [3]. At the border section of semiconductor usually varies the width of forbidden band, the carrier mobility, their effective mass and other characteristics.

Because of the different widths of the forbidden band in different materials, there is a break in the bottom of the conduction band and in the top of the valence band. (Fig. 3).

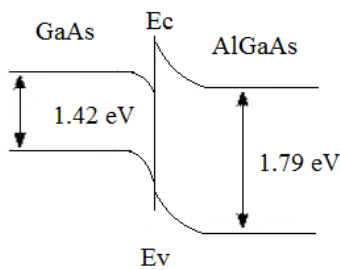


Fig. 3. Heterojunction of GaAs and AlGaAs.

As a result, the height of the potential barrier for electrons and holes is different (Fig. 4). This is the main difference of the heterojunction from the pn-junction.

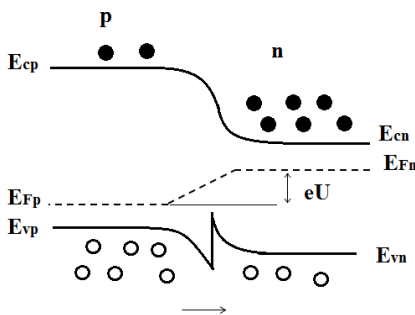


Fig. 4. Energy diagram of a straight-shifted pn heterojunction.

Ways to create white light emitting diodes

There are several ways to obtain white light with LEDs:

White color can be obtained by mixing red, green and blue colors in a certain proportion. In the case of LEDs white light is obtained by combining red, green and blue LEDs (Fig. 5).

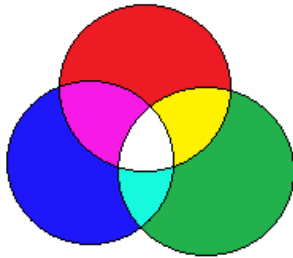


Fig. 5. Additive model of color (RGB LED).

Luminophore technologies for obtaining white light involve the use of one short-wave radiation LED, for example, blue, in combination with a yellow phosphor coating. Photons of blue radiation generated by the LED, either pass through the phosphor layer unchanged, or are converted into photons of yellow light. The combination of photons of blue and yellow creates white light. The example of spectrum of phosphor LED is presented in Fig. 6.

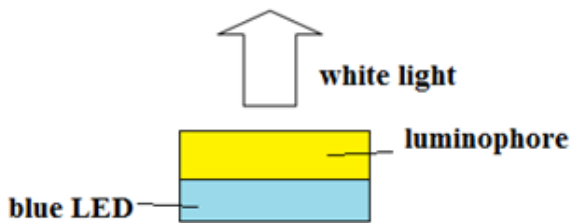


Fig. 6. Phosphor LED based on blue radiation.

The third method of obtaining white light by means of LED is the conversion of ultraviolet radiation with the help of three phosphors: red, green and blue.

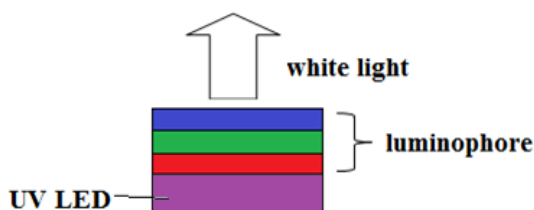


Fig. 7. Spectrum of phosphor LED based on UV radiation.

In comparing these three types of LEDs each has advantages and drawbacks.

RGB systems have their advantage due to the possibility to obtain not only white light but also moving along the color diagram, by controlling the current on each LED. Also a large number of LEDs in the matrix allows obtaining a high value of the luminous flux. The disadvantages of such LEDs are higher cost and the presence of aberrations of the optical system, because of which the light spot has different color in the center and at the edges. This phenomenon is quite difficult and expensive to compensate.

The advantage of luminophore-based light-emitting diodes is relatively low price. Among drawbacks are aging of the phosphor, which occurs much earlier than the LED itself, lower light output compared to multi-chip LEDs, and also difficulty to control the applying the phosphor coating. The result can be deviation from the required color temperature [8].

2.3. ADJUSTING LUMINOUS FLUX AND CHROMATICITY OF LED

The brightness and color of multi-chip LEDs can be changed by operating in pulse width modulation (PWM) mode. PWM control consists of controlling the on and off times of the current through the LED, repeated with a sufficiently high frequency, which, given the physiology of the human eye, should not be less than 200 Hz. Otherwise, there can appear the flicker effect [9]. The average current through the LED becomes proportional to the fill factor and is expressed by the formula:

$$I_{av} = D \times I_{max} \quad (1)$$

where I_{av} is average current,

D is the fill factor,

I_{max} is maximum current.

Changing the fill factor of the signal applied to each crystal changes the average current (Fig. 8).

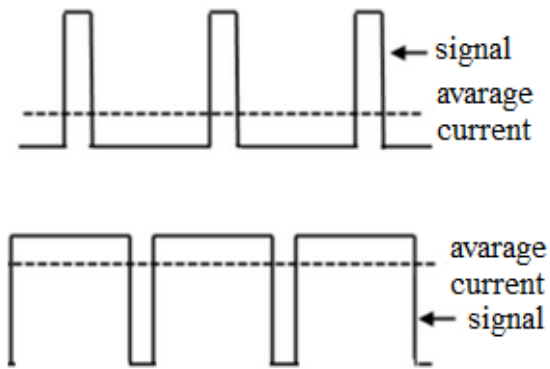


Fig. 8. Principle of PWM.

When the operating current varies through each LED, the spectral characteristics of the radiation of each crystal will change too. The results of investigation carried out by Nikiforov [10] are shown in Fig. 9.

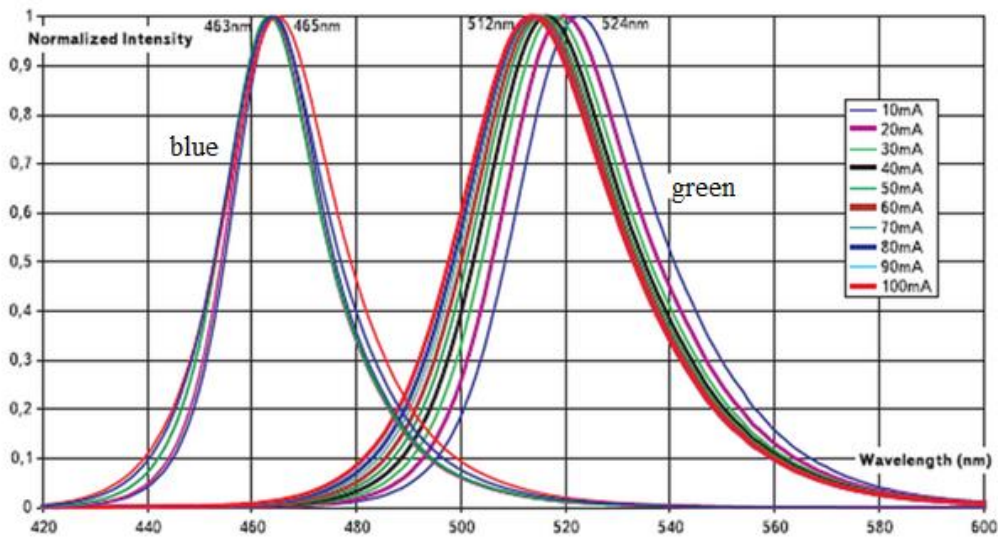


Fig. 9. The relative spectral characteristics of the emission of blue and green LEDs as a function of the current. [10]

All changes in the radiation of light-emitting diodes are caused by physical processes occurring in the radiating region of the crystal. The change in the current density through the p-n junction is associated with a change in the external electric field (voltage) and represents the shape of the current-voltage characteristic. As the voltage increases, the energy of the charge carriers will increase, and they will be able to overcome the band gap with higher energy. Proportionally there will be increase in their number and the current density, and as a result increase in the intensity of

radiation. It is obvious that at low current values the radiation will have a spectrum whose composition will correspond to the longest wave characteristics of a given type of radiating structures.

However, an increase in the current density will cause not only an increase in the radiation in the blue region of the spectrum, but also a simultaneous increase in the radiation intensity in regions with lower energy, which is disproportionate to this growth. This explains the presence in the emission spectrum of a more smooth slope of the characteristic from the long-wave radiation side. Nevertheless, this does not mean that there would be a shift of the maximum of the spectrum into the short-wave region due to disproportion [10].

2.4. QUANTITATIVE AND QUALITATIVE CHARACTERISTICS OF LEDS

Due to the fact that the LED is a new light source, it is necessary to introduce its own system of characteristics, different from other light sources. However, this issue has not yet been solved. In order to completely characterize the LED, it is necessary to describe it from the point of view of lighting engineering, physics and to consider its operation as an element of the electrical circuit. For this, the following technical and operational parameters are suggested:

- Peak wavelength of radiation , λ_p ;
- Full width at half maximum, $\Delta\lambda_{0.5}$;
- The relative distribution of the spectral density of the radiation flux;
- Luminous flux, Φ_v , or axial luminous intensity, I_0 ;
- Distribution curve of luminous intensity, photometric body (spatial distribution of luminous intensity);
- Light output and $h\nu$ of crystals;
- Voltage U and current I of the LED;
- The current-voltage characteristic of the LED;
- Lifetime τ of the LEDs;
- Temperature of p-n junction;
- Internal (η_{in}) quantum yield of crystals;
- External (η_{ex}) quantum yield of crystals;
- The efficiency of the optical system (η_{OS}).

The internal quantum yield is defined as the ratio of the number of photons emitted by the pn junction to the number of injected electrons. It depends on the quality of the material, the presence of impurities and defects in it, as well as the structure and composition of the epitaxial

layer. However, not all of the radiation “leaves” the crystal, while some of it is absorbed in the crystal.

The external quantum yield is the ratio of the number of photons emitted from the crystal to the number of injected photons. In its turn, the output of the optical system (OS) is the fraction of light emerging from the crystal and its surrounding optical system. It depends on the refractive index, internal absorption and the geometry of the lens. The overall efficiency of the LED is the product of the external quantum yield with the efficiency of the output of the OS. All these values are measured in percent.

$$\eta = \eta_q \cdot \eta_{OS} \quad (2)$$

3. BASICS OF LIGHTING ENGINEERING

3.1. PHOTOMETRY QUANTITIES AND FEATURES OF HUMAN VISION

It is known that the human eye can distinguish only radiation of certain wavelengths. This range is called the visible range and looks like a rainbow (Fig. 10).

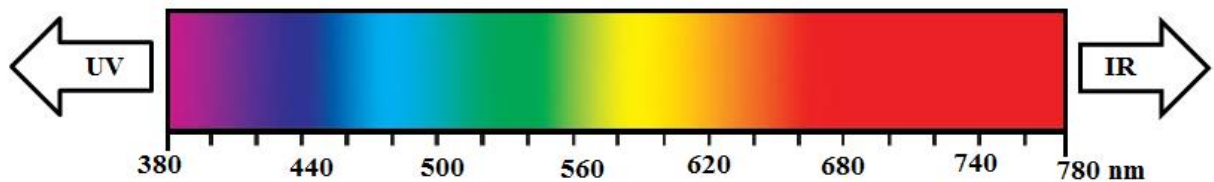


Fig. 10. Visible region of EM spectrum.

This visible light region is located almost in the middle of so-called optical region. Optical region includes ultraviolet, visible and infrared regions. People see radiation with wavelength between 380 nm and 780 nm. According to International Commission on Illumination (CIE), only radiation of these wavelengths can be called “light”. To describe infrared and ultraviolet should be used the term “radiation”. This difference can be explained from the point of human eye. The eye is a non-linear receiver. It has different sensitivity on different wavelength. Many investigations were re carried out to obtain values of this dependence. The sensitivity curve of human visual perception is on Fig. 11:

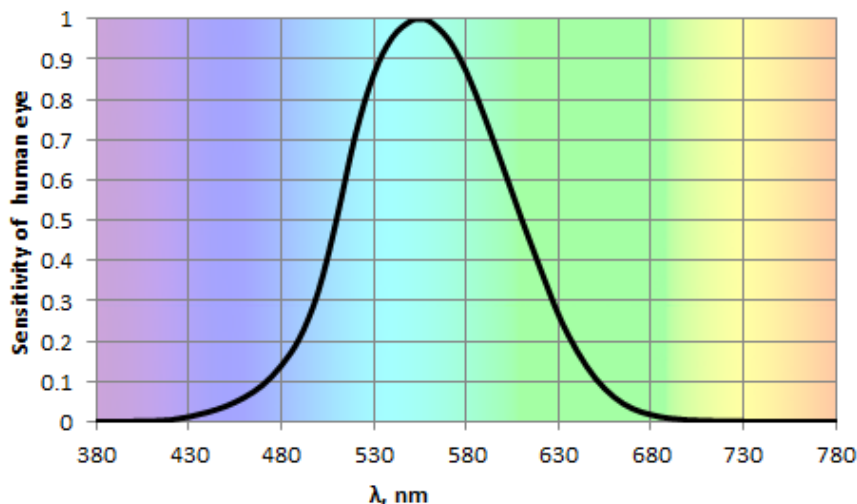


Fig. 11. Relative spectral sensitivity of human eye.

The maximum of sensitivity is located on the wavelength equal to 555 nm. The curve itself has Gaussian shape. Speaking about meaning and importance of this dependence, it is easy to conclude

that the same power of two sources with different wavelength is perceived by the eye in different ways. In other words, one watt of green light seems much brighter than one watt of red or blue light.

That is why there is difference between light and radiation not only in words that are used to describe them. Usually, power of radiation is measured in Watts, while power of light has different units. According to lighting engineering, the whole amount of light that light source emits is "luminous flux". Usually there is also radiant flux which can be measured in watts.

In other words, "the luminous flux is a measurand which is proportional to the radiation flux and is estimated in accordance with the relative spectral sensitivity of the average human eye" [11]. In turn, the "radiation flux" is defined as the power transferred by radiation through a surface.

More formally, the luminous flux can be defined as a light quantity that estimates the radiation flux by its impact on a selective light receiver whose spectral sensitivity is determined by the function of the relative spectral luminous efficiency of the radiation.

A symbol for luminous flux is Φ_v . Usually index "v" is used to denote a photometric system and "e" for radiometry system.

Unit of measurement of luminous flux in the International System of Units (SI) is lumen (lm). To recalculate flux from radiometry system into photometric it is necessary to know not only value of power or flux but also spectral distribution:

$$\Phi_v = K_m \int_{380nm}^{780nm} \Phi_e(\lambda) V(\lambda) d\lambda \quad (3)$$

Where K_m is the maximum value of the spectral luminous efficacy of monochromatic radiation (photometric radiation equivalent) equal to 683 lm / W [11, 12], $\Phi_e(\lambda)$ is the spectral density of the radiant flux and $V(\lambda)$ is relative spectral sensitivity of human eye. This is true for transfer of each radiometric to photometry quantity.

Photometry also includes the concept of solid angle. It is a region of space bounded by a conical surface. In contrast to the usual angle, the solid angle is three-dimensional. Units of solid angle is "steradian". The whole space is 4π steradians. There is the illustration of solid angle in Fig. 12:

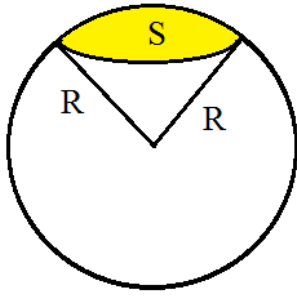


Fig. 12. Solid angle.

Thus, solid angle can be calculated as the ratio of the surface area (S) of the sphere to the radius (R) in the square:

$$\Omega = \frac{S}{R^2} \quad (4)$$

Where S is area of space bounded by a conical surface and R is the radius of the sphere.

This concept is important for understanding the link between luminous flux and luminous intensity. There is a whole list of measurands in a system of SI photometry quantities (Table 1):

Table 1. Values and definitions of SI photometry quantities

Name	Sym.	Unit	Formula	Comments
Luminous flux	Φ_v	lumen (lm)		
Luminous intensity	I_v	candela (cd)	$=d\Phi/d\Omega$	
Luminance	L_v	cd/m ²	$=dI/dA$	A is the area of the projection of the radiating body on a plane perpendicular to the chosen direction
Illuminance	E_v	lux (lx)	$=d\Phi/dA$	Where Φ is incident flux on a surface (A)
Luminous emittance	M_v	lm/m ² (but not lux!)	$=d\Phi/dA$	Where Φ is emitted from a surface (A)
Luminous exposure	H_v	lx·s	$E \cdot t$	
Luminous efficacy	η_v	lumen per watt (lm/W)	Φ/P	P can be either power or radiant flux

It is important to take into account limits of luminous efficacy. The maximum luminous flux that can be obtained in the result of transformation of radiant energy with flux equal to 1 W is 683 lm. Hence, the maximum value of luminous efficacy is 683 lm/W. However, this limit is complicated to reach. First of all, there are some losses in transformation of consumptive energy into radiation. The second reason is that not all of radiation belongs to visible region. And the last reason is sensitivity of human eye. Even if first two conditions are fulfilled perfectly, the luminous efficacy will be 683 lm/W only for green light with the wavelength 555 nm. Thus, there is a limit of light efficacy of LEDs that is necessary to achieve but impossible to break this limit [13].

3.2. BASIC CONCEPTS OF COLORIMETRY

Colorimetry is the science of color and color measurement. It is a science that arose in the 19th century and it studies the methods of measurement, the expression of the quantity of color and the differences in colors. The scientific basis of colorimetry as a combination of several primary colors was laid by Isaac Newton. Since then there have been developed a great many of laws, rules and principles concerning color itself and the perception of color by human.

To determine the color properties, so-called color systems were created. The main principle of each system is color-matching functions (curves of efficacy for the color versus wavelength). Based on these curves, a color space is created. The first standard colorimetric system accepted by the CIE in 1931 was the RGB system, in which, red, green and blue were used for the primary colors. In the same year, the CIE accepted one more colorimetric system which is called XYZ. This system is obtained artificially by recounting from the RGB the color coordinates so that there were no negative coordinates in the XYZ system. This system is the main colorimetric system used today.

The colorimetric system XYZ is a mathematical model that can be used to represent color in the form of base color coefficients, and also store information about color and color processing in a discrete form [14].

In this system, an addition function $\bar{y}(\lambda)$ coincides with the function of the sensitivity of human vision perception and the Y coordinate corresponds to the brightness of the color. The chromaticity coordinates are determined by the formulas:

$$x = \frac{X}{X+Y+Z}; y = \frac{Y}{X+Y+Z}; z = \frac{Z}{X+Y+Z} \quad (5)$$

Dependences of CIE-matching functions [15] vs. wavelength can be seen in Fig. 13:

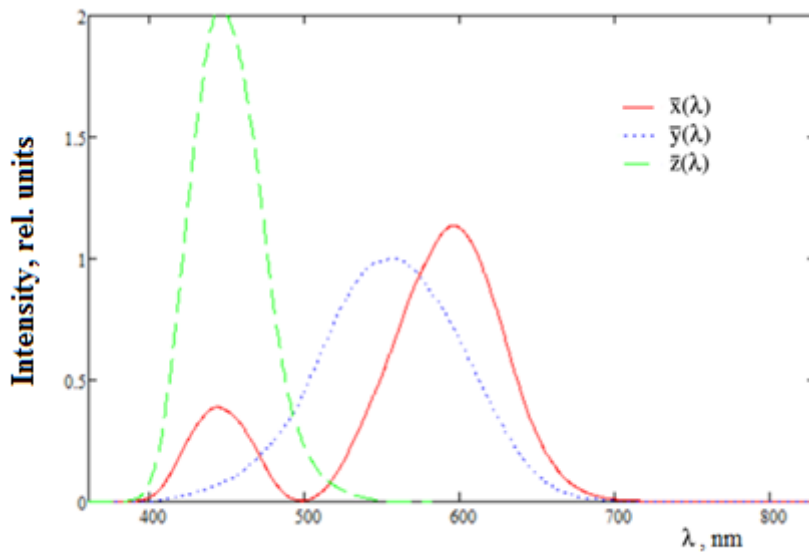


Fig. 13. Spectral distribution of CIE-matching functions.

The chromaticity coordinates do not have any quantitative meaning; they only indicate the position of the point on the model color chart.

The diagram XY is the projection of the system with the basic points of XYZ onto the unit plane. This diagram allows illustrating the chromaticity of different radiations in a convenient form, for example, the color coverage of various devices [14]. The diagram has one useful property: the chromaticity coordinates of the mixture of the two emissions will be strictly on the line that connects the points of these two radiations on the diagram. Therefore, the color coverage of the monitor, for example, on such a diagram will be a triangle.

The XY diagram also has one drawback, which should be taken into account: equal segments on different sections of the chart do not mean the same perceived difference in color. This disadvantage is the reason to create another one color system that will be described later. In Fig. 14 is the image of XY diagram.

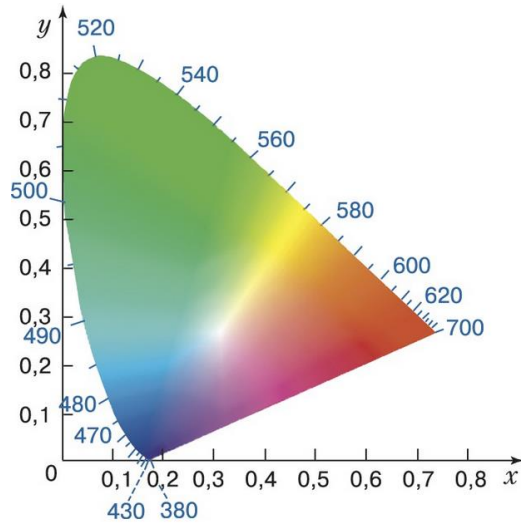


Fig. 14. XY diagram. [16]

Generally, XYZ space is three-dimensional, while XY diagram is two-dimensional. This diagram is a unit plane, where each point satisfies condition: $x+y+z=1$. Thus, the value of z can be calculated for each point in the diagram as $z=1-x-y$. The non-closed curve represents the all monochromatic colors of the spectrum, where wavelengths in nm are signed in blue numbers. This curve is called “locus”. The line in the bottom is called “line of purples”. In the result, all existing colors are inside of the diagram.

There is also another color space which was developed by CIE. This color space is called CIE LAB. Illustration of this space is shown in Fig. 15.

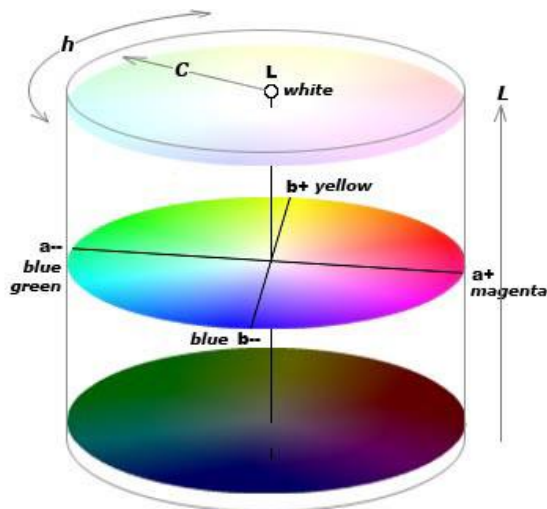


Fig. 15. Schematic representation of the space Lab. [17]

It is considered that LAB space is more understandable for human. L defines lightness, while a and b refers to colors themselves. All colors between blue green and magenta is ‘ a ’ and all colors from blue to yellow is ‘ b ’ [18].

3.3. CORRELATED COLOR TEMPERATURE

Color temperature (spectrophotometric or colorimetric temperature, measured in Kelvins) is a characteristic of the the emission intensity of a light source as a function of the wavelength in the optical range. According to Planck's formula, the color temperature is defined as the temperature of blackbody, at which it emits radiation of the same chromaticity as the radiation under consideration [19]. It characterizes the relative contribution of the radiation of each color to the source radiation. It is used in colorimetry and astrophysics (in studying the energy distribution in the spectra of stars). It is measured in Kelvins and mirades.

The correlated color temperature CCT is defined as the temperature of a blackbody at which the chromaticity coordinates of its radiation are close within a given tolerance to the chromaticity coordinates of the radiation under consideration of the XY diagram [14]. In Fig. 16 is shown XY diagram with line of chromaticities of black body radiation (Planckian locus) with lines of constant CCT.

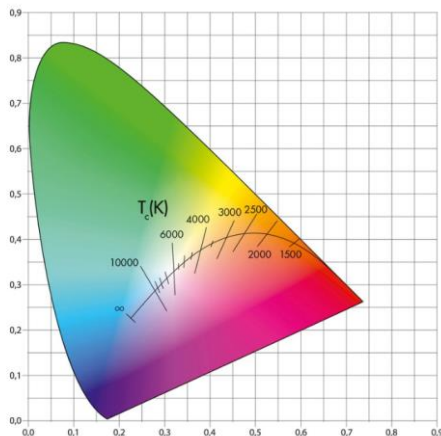


Fig. 16. XY diagram with Planckian locus. [20]

Usually all light sources are divided by color into three groups:

- Warm white (2700-3500 K)
- Neutral white or daylight (3500-5000 K)
- Cool white (CCT above 5000 K)

The color temperature of the usual incandescent lamp is about 2800 K, so the warm white light of the LED lamps is most familiar to the eye (from 2700 to 3500K). Other widespread sources of certain CCTs are in Fig. 17:

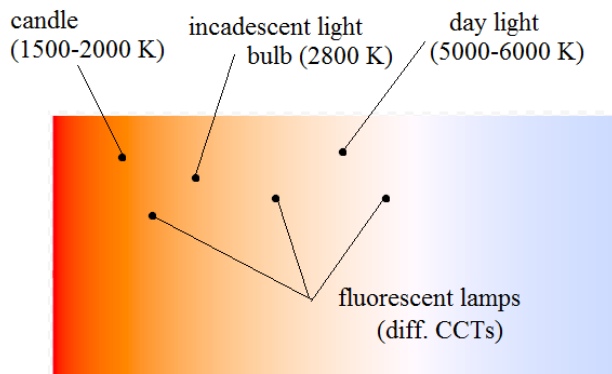


Fig. 17. Scale of CCTs.

If the chromaticity coordinates of all CCTs are on the line of blackbody radiation, then it can be concluded that color quality of light is perfect. However, in reality, the chromaticity coordinates of light source are close to this line but don't lie on it. In this case CCT can be defined as a point of intersection of the line of the Planckian locus and the perpendicular dropped from it from the point of the radiant body. For LEDs there were introduced such a concept as "binning" (Fig. 18). It looks like a line of blackbody radiation which is surrounded by different rectangles. Being inside of each rectangle provides certain quality of white light.

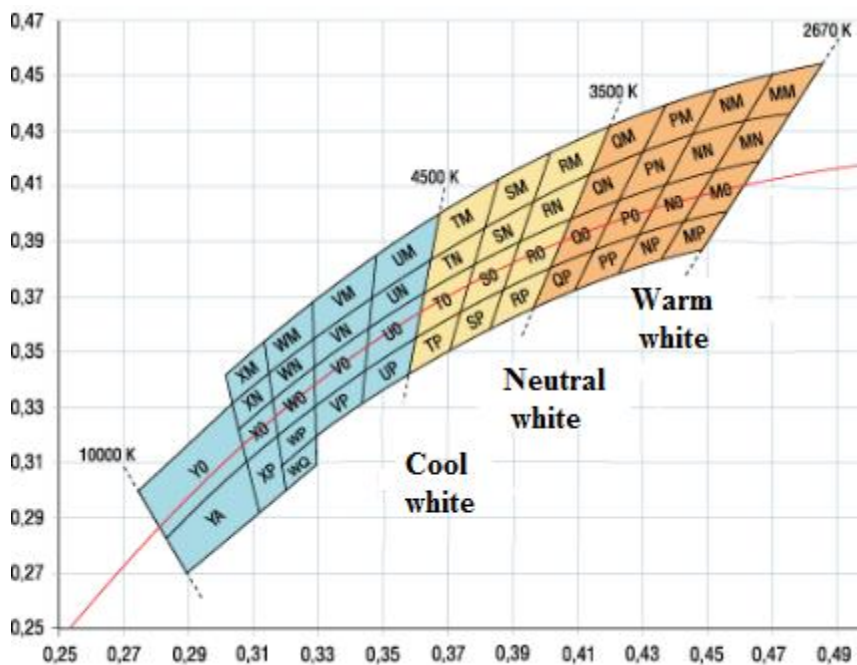


Fig. 18. Binning of LEDs [21]

Thus, the closer the point of the chromaticity coordinates of light source is to the line, the better or “more white” is the radiation.

3.4. COLOR CHARACTERISTICS OF WHITE LEDS AND FACTORS THAT AFFECT THEM

One of the most important quality characteristics of radiation is color rendering. It describes the spectrum of radiation from the point of view of its perception of human by means of the possibility of distinguishing colors. **Ошибка! Источник ссылки не найден.** can help to understand difference between poor and good color rendering of light source.



Fig. 19. Comparison of poor and perfect color rendering. [22]

It can be seen that the object with the lowest CRI looks dim and dull and it is almost impossible to distinguish colors. With increasing CRI the object looks brighter and colorful, so colors become closer to reality.

Nowadays there are two methods to determine the quantitative value of color reproduction. Both methods are based on comparison (test-sample method) of the investigated spectrum with standard radiation, which is selected according to the correlated color temperature of the radiation being studied. The calculation using both methods is based on the use of control colors. Also both methods are theoretical and do not require any measurements except for the investigated spectrum.

The first way to describe the color rendering is called CRI (Color Rendering Index) or the color rendering index [22]. The calculations are carried out using the method of measuring and specifying the color transmittance of light sources in the CIE of 1974 (the last amendment was in 1994), in which the general color rendering index R_a is calculated, as well as special (R1-8) and additional (R9-14) color rendering indices .

However, the applicability of this method to LEDs is a matter of dispute, since the spectra of semiconductor light sources are predominantly monochromatic. In the technical report of the CIE devoted to this method, it is written that this method the color rendering index, developed by the commission, is usually not applicable for predicting the color rendering parameters of a set of light sources if this set includes white LEDs [24].

Also, in order for the calculation results to be reliable, there is a condition that $\Delta C \leq 5.4 \cdot 10^{-3}$. For many LEDs and their reference sources, this condition is not met.

It is necessary to take into account complicated choice of the standard radiation source, since a sharp limit of 5000 K can lead to incorrect results.

Even with a high value of Ra, it may turn out that the investigated source transmits the saturated colors poorly. This is due to the limited number of samples and due to the fact that they all have too low saturation [25].

In the CRI method general color rendering index is defined as the arithmetic average. This means that even with a low value for one or more private color rendering indexes, the overall index may still be quite high. This is typical for light source with a narrow spectrum.

Another way is Color Quality Scale (CQS). CQS method uses a wider and more diverse set of control colors with different brightness and saturation [26]. This allows us to obtain a more reliable calculation result.

Also in this method, the result of the index of the color scale is proportional to the geometric average, which will not give high final result with a small value of the special indices. Thus, the CQS method is designed taking into account the disadvantages of the CRI method and allows to obtain estimation of the color rendering of LEDs with greater accuracy.

4. PROPOSED METHODS

4.1. CRI

Sequence of calculation:

First of all it is necessary to choose the reference illuminant with a known spectrum of radiation $\Phi_{\text{ref}}(\lambda)$. The choice is made based on the correlated color temperature of the investigated source with spectral distribution $\Phi_{\text{inv}}(\lambda)$. In other words, one must choose a spectrum which is the closest in chromaticity to the investigated and chromaticity coordinates $(x_{\text{ref}}, y_{\text{ref}})$. If CCT is below 5000 K, then for a standard source can be taken a black body radiation whose spectrum is described by the Planck formula. If the color temperature of the investigated radiation is higher than 5000 K, then as a standard source is taken D illuminant, which reproduces the phases of daylight. An example of two kinds of reference illuminant is in Fig. 20:

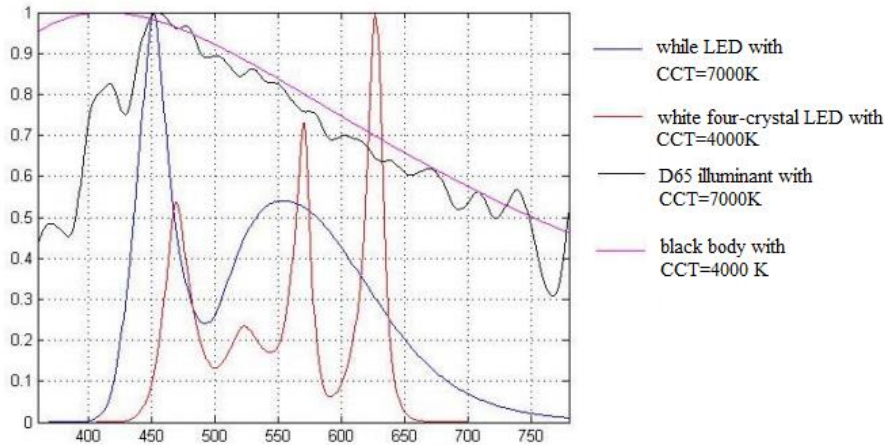


Fig. 20. Spectra of white LEDs with their reference illuminants.

It is necessary to carry out the normalization of the spectrum, that is, to determine the k coefficient. The spectra $\Phi_{\text{inv}}(\lambda)$ and $\Phi_{\text{ref}}(\lambda)$ are normalized with the help of these coefficients so that the following conditions are satisfied: $Y_{\text{inv}} = Y_{\text{ref}} = 100$.

$$k = \frac{100}{\int_{380}^{780} \Phi_{e\lambda}(\lambda) \cdot \bar{y}(\lambda) d\lambda} \quad (6)$$

The next step is computation of chromaticity coordinates $(x_{\text{inv}}, y_{\text{inv}})$ and $(x_{\text{ref}}, y_{\text{ref}})$ Tristimulus values (of a color stimulus):

$$\begin{aligned}
X &= \int_{380}^{780} \Phi_{e,inv}(\lambda) \cdot \bar{x}(\lambda) d\lambda \\
Y &= \int_{380}^{780} \Phi_{e,inv}(\lambda) \cdot \bar{y}(\lambda) d\lambda \\
Z &= \int_{380}^{780} \Phi_{e,inv}(\lambda) \cdot \bar{z}(\lambda) d\lambda
\end{aligned} \tag{7}$$

Chromaticity coordinates:

$$\begin{aligned}
x &= \frac{X}{X + Y + Z} \\
y &= \frac{Y}{X + Y + Z} \\
z &= \frac{Z}{X + Y + Z}
\end{aligned} \tag{8}$$

All colorimetric data must be converted from a standard colorimetric system into the coordinates of a uniform-chromaticity-scale diagram using the formulas:

$$\begin{aligned}
u &= \frac{4x}{-2x + 12y + 3} \\
v &= \frac{6y}{-2x + 12y + 3}
\end{aligned} \tag{9}$$

The difference in chromaticity ΔC between investigated spectrum (u_{inv}, v_{inv}) and reference spectrum (u_{ref}, v_{ref}) should be less than $5,4 \cdot 10^{-3}$ and can be calculated by the formula:

$$\Delta C = \left[(u_{inv} - u_{ref})^2 + (v_{inv} - v_{ref})^2 \right]^{1/2} \tag{10}$$

Spectral distributions of reflectance of samples are in Fig. 21 [22].

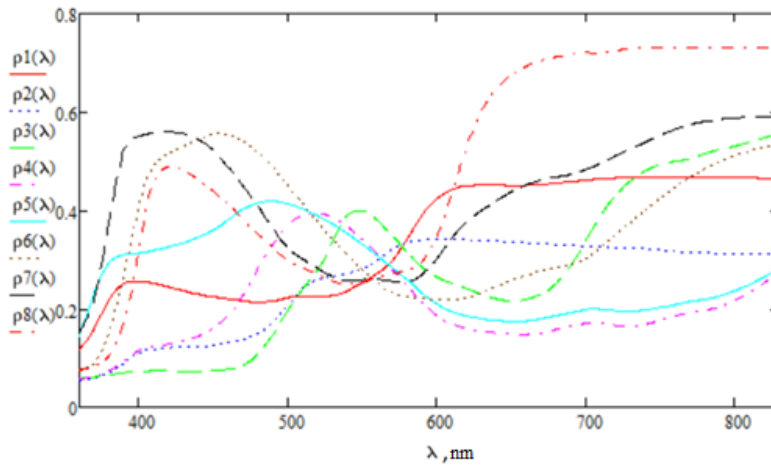


Fig. 21. Spectral distributions of reflectance of samples №1-8.

Fig. 22 shows image of the set of main samples.

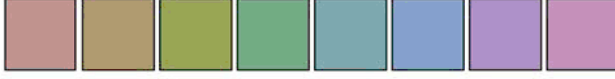


Fig. 22. Main samples. [26]

Then it is necessary to calculate the tristimulus values and chromaticity coordinates of the reflected light in the result of illumination of samples by the investigated and reference sources:

$$\begin{aligned} X_{inv,i} &= \int_{380}^{780} \Phi_{e\lambda inv}(\lambda) \cdot \bar{x}(\lambda) \cdot \rho_i(\lambda) d\lambda \\ Y_{inv,i} &= \int_{380}^{780} \Phi_{e\lambda inv}(\lambda) \cdot \bar{y}(\lambda) \cdot \rho_i(\lambda) d\lambda \\ Z_{inv,i} &= \int_{380}^{780} \Phi_{e\lambda inv}(\lambda) \cdot \bar{z}(\lambda) \cdot \rho_i(\lambda) d\lambda \end{aligned} \quad (11)$$

$$\begin{aligned} X_{ref,i} &= \int_{380}^{780} \Phi_{e\lambda ref}(\lambda) \cdot \bar{x}(\lambda) \cdot \rho_i(\lambda) d\lambda \\ Y_{ref,i} &= \int_{380}^{780} \Phi_{e\lambda ref}(\lambda) \cdot \bar{y}(\lambda) \cdot \rho_i(\lambda) d\lambda \\ Z_{ref,i} &= \int_{380}^{780} \Phi_{e\lambda ref}(\lambda) \cdot \bar{z}(\lambda) \cdot \rho_i(\lambda) d\lambda \end{aligned} \quad (12)$$

, where i is the number of the sample

After that should be calculated tristimulus values and transformed into coordinates of a uniform-chromaticity-scale diagram UV [27].

Further it is necessary to find the coordinates of chromaticity of the control samples ($u_{inv,i}^{\wedge}, v_{inv,i}^{\wedge}$), after taking into account the adaptive shift obtained by moving the investigated source to the reference source:

$$\begin{aligned} u_{inv,i}^{\wedge} &= \frac{10,872 + 0,404 \cdot \frac{c_{inv,i}}{c_{ref}} \cdot c_{ref} - 4 \cdot \frac{d_{inv,i}}{d_{inv}} \cdot d_{ref}}{16,518 + 1,481 \cdot \frac{c_{inv,i}}{c_{inv}} \cdot c_{ref} - \frac{d_{inv,i}}{d_{inv}} \cdot d_{ref}} \\ v_{inv,i}^{\wedge} &= \frac{5,520}{16,518 + 1,481 \cdot \frac{c_{inv,i}}{c_{inv}} \cdot c_{ref} - \frac{d_{inv,i}}{d_{inv}} \cdot d_{ref}} \end{aligned} \quad (13)$$

Functions c and d are defined as follows:

$$c = \frac{1}{v}(4 - u - 10v)$$

$$d = \frac{1}{v}(1,708v + 0,404 - 1,481u) \quad (14)$$

Then the chromaticity coordinates should be transformed into the coordinates of CIE 1964 uniform color space [28] using the following formulas:

$$W_{inv,i}^* = 25(\bar{Y}_{inv,i})^{1/3} - 17;$$

$$W_{ref,i}^* = 25(\bar{Y}_{ref,i})^{1/3} - 17; \quad (15)$$

$$\bar{Y}_{ref,i} = \frac{Y_{ref,i}}{Y_{ref}}; \quad \bar{Y}_{inv,i} = \frac{Y_{inv,i}}{Y_{ref}}$$

$$U_{inv,i}^* = 13 \cdot W_{inv,i}^* (u_{inv,i} - u_{inv});$$

$$U_{ref,i}^* = 13 \cdot W_{ref,i}^* (u_{ref,i} - u_{ref});$$

$$V_{inv,i}^* = 13 \cdot W_{inv,i}^* (v_{inv,i} - v_{inv}); \quad (16)$$

$$V_{ref,i}^* = 13 \cdot W_{ref,i}^* (v_{ref,i} - v_{ref});$$

Values $u_{inv} = u_{ref}$, $v_{inv} = v_{ref}$ are chromaticity coordinates of investigated source after taking into account adaptive color shift.

To calculate the difference between the perceived color of the control sample illuminated by the investigated light source and the color of the same sample illuminated by the standard source, the color difference formula CIE 1964 is used [28]:

$$\Delta E_i = \left[(U_{ref,i}^* - U_{inv,i}^*)^2 + (V_{ref,i}^* - V_{inv,i}^*)^2 + (W_{ref,i}^* - W_{inv,i}^*)^2 \right]^{1/2} \quad (17)$$

For each sample, a special color rendering index is:

$$R_i = 100 - 4,6\Delta E_i \quad (18)$$

General color rendering index is calculated as the arithmetic average of the partial color rendering indices:

$$Ra = \frac{1}{8} \sum_{i=1}^8 R_i \quad (19)$$

For completeness of color rendering estimation, there are defined additional color rendering indices, which refer to six standard additional samples № 9-14 (Fig. 23). The colors of these samples are defined as red, yellow, blue, pinkish (the color of the skin of a European man), olive green (foliage of a tree).

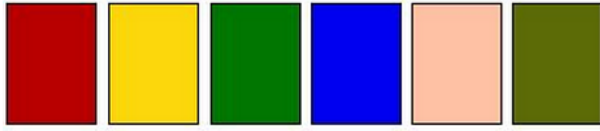


Fig. 23. Additional samples. [26]

4.2. CQS

Sequence of calculation:

Preparation for the calculations is carried out in a similar manner to the CRI method. Further, all the colorimetric data are recalculated from the standard colorimetric system of the CIE (1931 Publication) into the coordinates of the uniform color space CIE LAB by the following formulas:

$$\begin{aligned}
 L^*_{inv} &= 116 \cdot \left(\frac{Y_{inv}}{Y_{ref}} \right)^{1/3} - 16 \\
 a^*_{inv} &= 500 \cdot \left[\left(\frac{X_{inv}}{X_{ref}} \right)^{1/3} - \left(\frac{Y_{inv}}{Y_{ref}} \right)^{1/3} \right] \\
 b^*_{inv} &= 200 \cdot \left[\left(\frac{Y_{inv}}{Y_{ref}} \right)^{1/3} - \left(\frac{Z_{inv}}{Z_{ref}} \right)^{1/3} \right]
 \end{aligned} \tag{20}$$

The coordinates for a standard source are calculated in a similar way.

The reflectances of the samples are shown in Fig. 24:

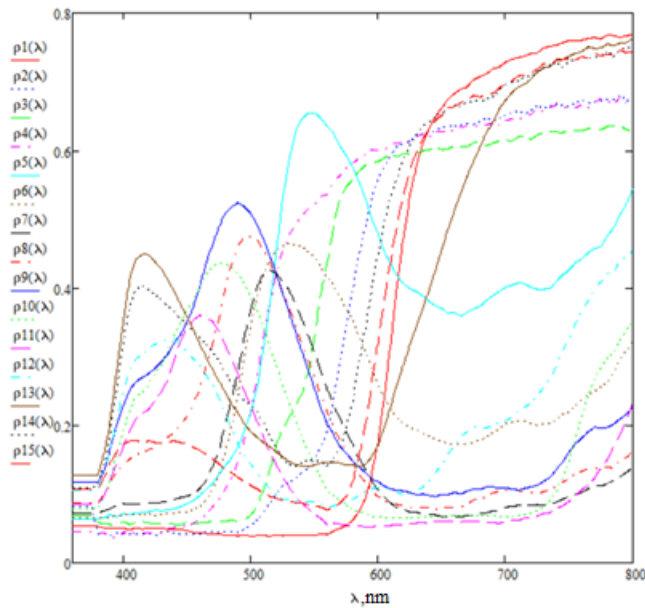


Fig. 24. Spectral distribution of reflectance of samples №1-15.

Images of samples №№1-15 are presented respectively in Fig. 25 in the appropriate order:



Fig. 25. Samples for CQS. [26]

Then it is necessary to calculate the tristimulus values and chromaticity coordinates of the reflected light in the result of illumination of samples by the investigated and reference source:

$$\begin{aligned}
 X_{inv,i} &= \int_{380}^{780} \Phi_{e\lambda inv}(\lambda) \cdot \bar{x}(\lambda) \cdot \rho_i(\lambda) d\lambda \\
 Y_{inv,i} &= \int_{380}^{780} \Phi_{e\lambda inv}(\lambda) \cdot \bar{y}(\lambda) \cdot \rho_i(\lambda) d\lambda \\
 Z_{inv,i} &= \int_{380}^{780} \Phi_{e\lambda inv}(\lambda) \cdot \bar{z}(\lambda) \cdot \rho_i(\lambda) d\lambda
 \end{aligned} \tag{21}$$

$$\begin{aligned}
 X_{ref,i} &= \int_{380}^{780} \Phi_{e\lambda ref}(\lambda) \cdot \bar{x}(\lambda) \cdot \rho_i(\lambda) d\lambda \\
 Y_{ref,i} &= \int_{380}^{780} \Phi_{e\lambda ref}(\lambda) \cdot \bar{y}(\lambda) \cdot \rho_i(\lambda) d\lambda \\
 Z_{ref,i} &= \int_{380}^{780} \Phi_{e\lambda ref}(\lambda) \cdot \bar{z}(\lambda) \cdot \rho_i(\lambda) d\lambda
 \end{aligned} \tag{22},$$

where i is the number of the sample.

In this case, formulas (20) take the following form:

$$\begin{aligned}
 L^*_{inv,i} &= 116 \cdot \left(\frac{Y_{inv,i}}{Y_{ref}} \right)^{1/3} - 16 \\
 a^*_{inv,i} &= 500 \cdot \left[\left(\frac{X_{inv,i}}{X_{ref}} \right)^{1/3} - \left(\frac{Y_{inv,i}}{Y_{ref}} \right)^{1/3} \right] \\
 b^*_{inv,i} &= 200 \cdot \left[\left(\frac{Y_{inv,i}}{Y_{ref}} \right)^{1/3} - \left(\frac{Z_{inv,i}}{Z_{ref}} \right)^{1/3} \right]
 \end{aligned} \tag{23}$$

$$\begin{aligned}
L_{ref,i}^* &= 116 \cdot \left(\frac{Y_{ref,i}}{Y_{ref}} \right)^{1/3} - 16 \\
a_{ref,i}^* &= 500 \cdot \left[\left(\frac{X_{ref,i}}{X_{ref}} \right)^{1/3} - \left(\frac{Y_{ref,i}}{Y_{ref}} \right)^{1/3} \right] \\
b_{ref,i}^* &= 200 \cdot \left[\left(\frac{Y_{ref,i}}{Y_{ref}} \right)^{1/3} - \left(\frac{Z_{ref,i}}{Z_{ref}} \right)^{1/3} \right]
\end{aligned} \tag{24}$$

For each i-th sample, the color difference is determined:

$$\Delta E_i = \sqrt{(\Delta L_i^*)^2 + (\Delta a_i^*)^2 + (\Delta b_i^*)^2} \tag{25}$$

where

$$\begin{aligned}
\Delta L_i^* &= L_{ref,i}^* - L_{inv,i}^* \\
\Delta a_i^* &= a_{ref,i}^* - a_{inv,i}^* \\
\Delta b_i^* &= b_{ref,i}^* - b_{inv,i}^*
\end{aligned} \tag{26}$$

Now it is necessary to determine the geometric mean of the color difference:

$$\Delta E_{RMS} = \sqrt{\frac{1}{15} \cdot \sum_{i=1}^{15} \Delta E_i^2} \tag{27}$$

After this, the color quality scale should be determined by the formula:

$$Q_{a,RMS} = 100 - 2.81 \cdot \Delta E_{RMS} \tag{28}$$

The coefficient 2.81, according to the method, was chosen so that the average value of the index of the color scale for the standard fluorescent lamp of the CIE was equal to the average value of the color rendering index for such a lamp meaning 75.1.

To exclude negative values of the color quality scale and to bring it to a scale of 0-100, the following transformation is necessary:

$$Q_{a,0-100} = 10 \cdot \ln\left(\exp\left(\frac{Q_{a,RMS}}{10}\right) + 1\right) \tag{29}$$

Further, it is necessary to take into account the coefficient M_{cct} , which depends on the color temperature of the investigated radiation.

$$Q_a = M_{cct} \cdot Q_{a,0-100} \quad (30)$$

If $CCT > 3500$ K, then the coefficient is 1. Then:

$$Q_a = Q_{a,0-100} \quad (31)$$

4.3. SPECTRA SIMULATION

One of the stages of modeling the color characteristics of white LEDs based on multichip LEDs is the choice of an approximation that satisfies the minimum error in the difference in spectra for the subsequent color calculations.

Modeling of spectra can be carried out by one of two methods:

1) Approximation on the recommendation of the CIE 107 [29]:

$$\Phi_{e,\lambda}(\lambda, \lambda_0, \Delta\lambda_{0.5}) = \frac{g(\lambda, \lambda_0, \Delta\lambda_{0.5}) + 2 \cdot g^5(\lambda, \lambda_0, \Delta\lambda_{0.5})}{3}$$

$$g(\lambda, \lambda_0, \Delta\lambda_{0.5}) = \exp \left[- \left(\frac{\lambda - \lambda_0}{\Delta\lambda_{0.5}} \right)^2 \right] \quad (32)$$

where λ_0 is wavelength corresponding to the peak wavelength,

λ is current wavelength,

$\Delta\lambda_{0.5}$ is width of the emission spectrum at half height from the spectral maximum of the radiation.

2) Approximation by normal distribution:

$$\Phi_{e,\lambda}(\lambda, \lambda_0, \sigma_1, \sigma_2) = \frac{1}{\frac{\exp\left(\frac{\lambda - \lambda_0}{2\sigma_1^2}\right)}{\sigma_1\sqrt{2\pi}} + \frac{\exp\left(\frac{\lambda_0 - \lambda}{2\sigma_2^2}\right)}{\sigma_2\sqrt{2\pi}}} \quad (33)$$

where λ_0 is wavelength corresponding to the peak wavelength,

λ is current wavelength,

σ_1, σ_2 correspond to the width of the emission spectrum (in the case of a symmetric function they are equal)

When comparing two methods, one can conclude that the second one has more advantages. Normal distribution takes into account the asymmetry of the emission spectrum of the LED. However, in this investigation color spectrum are not quite asymmetric and CIE approximation let to obtain precise results. That is why calculation in computer program is based on Approximation on the recommendation of the CIE 107.

4.4. CALCULATING THE PROPORTION

Two-crystal LED

Fig. 26 shows that all colors that can be obtained by mixing two colors with chromaticity coordinates (x_1, y_1) and (x_2, y_2) lie on the line connecting these points. CCT of the light source is defined as the nearest point on the line of the black body radiation to the point of the chromaticity of the investigated source.

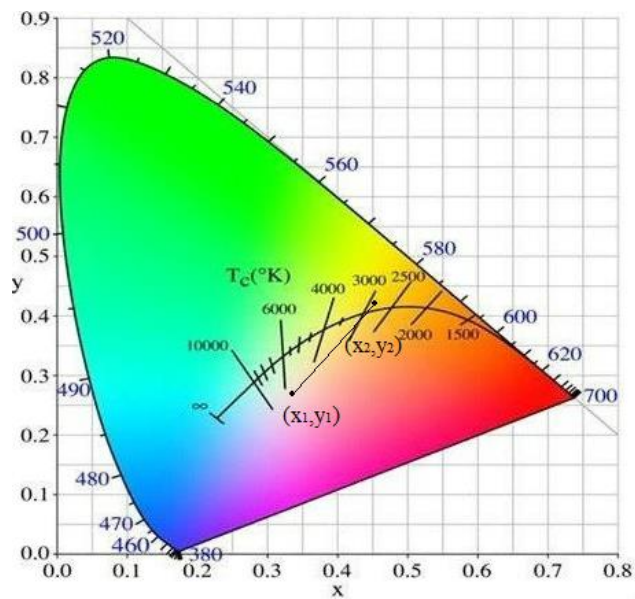


Fig. 26. XY diagram with Planckian locus and the line of CCT of two-crystal LED.

Chromaticity coordinates of the combined color:

$$x = \frac{x_1 \cdot L_1 + x_2 \cdot L_2}{L_1 + L_2} \quad (34)$$

$$y = \frac{y_1 \cdot L_1 + y_2 \cdot L_2}{L_1 + L_2} \quad (35)$$

The total spectrum :

$$\Phi_{e,\lambda_tot}(\lambda) = \Phi_{e,\lambda1}(\lambda) \cdot b1 + \Phi_{e,\lambda2}(\lambda) \cdot b2 \quad (36),$$

where the intensities b_1 and b_2 vary from 0 to 1 so that at the point (x_1, y_1) $b_1 = 1$ and $b_2 = 0$, and at the point (x_2, y_2) $b_1 = 0$ and $b_2 = 1$.

Three-crystal LED

The calculation is carried out according to the following algorithm:

- 1) Spectra of the three crystals are taken (in relative units);
- 2) The spectrum of a black body at a given temperature is found by Planck's formula. Its tristimulus values are calculated X_{bb} , Y_{bb} , Z_{bb} ;
- 3) Calculation of tristimulus values of the crystals is carried out;
- 4) A matrix is formed from the tristimulus values of the radiation of the LED:

$$M1 = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ Z1 & Z2 & Z3 \end{pmatrix} \quad (37)$$

$m1 = M1^{-1}$ is inverse matrix

$$Mst = \begin{pmatrix} X_{bb} \\ Y_{bb} \\ Z_{bb} \end{pmatrix} \quad (38)$$

where M_{st} is a matrix of the coordinates of the color of the black body (for a given T)

Equation:

$$abc = m1 \times Mst \quad (39)$$

where abc is a matrix of three values.

$abc = \begin{pmatrix} b1 \\ b2 \\ b3 \end{pmatrix}$ is matrix of intensities of each LED, which it should have so that the total radiation

had the same chromaticity coordinates as the blackbody at a given temperature.

$$\Phi_{e,\lambda_tot}(\lambda) = \Phi_{e,\lambda1}(\lambda) \cdot b1 + \Phi_{e,\lambda2}(\lambda) \cdot b2 + \Phi_{e,\lambda3}(\lambda) \cdot b3 \quad (40)$$

Taking the intensity of the third LED as 1, the total spectrum can be written as:

$$\Phi_{e,\lambda_tot}(\lambda) = \Phi_{e,\lambda1}(\lambda) \cdot \frac{b1}{b3} + \Phi_{e,\lambda2}(\lambda) \cdot \frac{b2}{b3} + \Phi_{e,\lambda3}(\lambda) \quad (41)$$

Four-crystal LED

In the case of a four-crystal LED there exist not only one solution. Hence, various combinations will be possible in a result of the calculations because "the spectral composition of the radiation uniquely determines its chromaticity, while a number of different spectral compositions may correspond to a given chromaticity" [14]. This is because the color perception of human is determined by the ratio of the excitation levels of the three kinds of receptors, respectively sensitive in the red, green and blue areas of the visible optical range. Thus, the same ratio of the excitation levels of the color receptors can be provided by different spectral compositions of the radiation.

Various proportions providing the required chromaticity of white radiation can be defined as follows:

Beginning of calculations (points 1-3) is performed similarly to the case with a three-crystal white LED.

4) The equation is as follows:

$$M1 = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ Z1 & Z2 & Z3 \end{pmatrix} \quad (42)$$

matrix of tristimulus values of the radiation of the LED

$m1 = M1^{-1}$ is inverse matrix

$$Mst = \begin{pmatrix} X_{bb} - X_4 \cdot a \\ Y_{bb} - Y_4 \cdot a \\ Z_{bb} - Z_4 \cdot a \end{pmatrix} \quad (43)$$

Equation:

$$abc = m1 \times Mst \quad (44)$$

where abc is a matrix of three values.

$abc = \begin{pmatrix} b1 \\ b2 \\ b3 \end{pmatrix}$ is matrix of intensities of each LED, which they should have to make total radiation

with the same color coordinates as the blackbody at a given temperature. And for the fourth crystal this factor will be "a". Total spectrum:

$$\Phi_{e,\lambda_tot}(\lambda) = \Phi_{e,\lambda1}(\lambda) \cdot b1 + \Phi_{e,\lambda2}(\lambda) \cdot b2 + \Phi_{e,\lambda3}(\lambda) \cdot b3 + \Phi_{e,\lambda4}(\lambda) \cdot a \quad (45)$$

The result is a set of proportions colored LED radiation (for different values of "a"), which create white light with the same hue, but with different characteristics, i.e. with different color rendering, with different luminous efficacy.

From this set of different proportions it is necessary to choose one. The program offers a choice of such a proportion, which provides the best color rendering of white LEDs.

5. MEASUREMENTS

5.1. EQUIPMENT AND MEASUREMENT SCHEME

Measurement of the spectral distribution of irradiance was carried out with the aid of a spectrometer Instrument Systems CAS-140. Its image is shown in Fig. 27.



Fig. 27. Spectrometer CAS-140. [30]

The working principle is based on Czerny-Turner optical scheme. The cross-correlation scheme consists of two concave mirrors and one diffraction grating, as presented in Fig. 28.

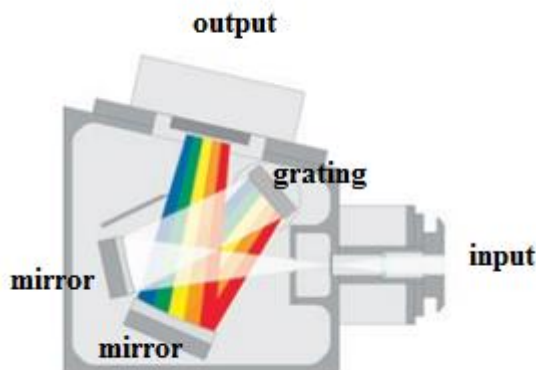


Fig. 28. Czerny-Turner optical scheme [30]

The focal length of mirror 1 is chosen in such a way that it collimates the light beam from the input slit and directs it to the diffraction grating. After the light is decomposed into separate components, the mirror 2 focuses the scattered light with a diffraction grating into the plane of the detector.

This model is convenient for a compact spectrograph. For a diffraction grating with an angular dispersion value, the focal length of the two mirrors can be varied to obtain different values of the

linear dispersion. This determines the spectral range, sensitivity and resolution of the system. The optimal geometry of the cross-correlation scheme of the spectrograph can create a scattered spectral field and good measurement accuracy.

In addition to spectrometer there were necessary to choose optical probes or so-called photometric head that receive data of light such as irradiance or illuminance (Fig. 29). For the conducted experiments, there was used the probe №1.



Fig. 29. Optical probes. [31]

Measurements of photometric values should have been carried out due to CIE Standard S025 [32]. Thus, the scheme of measurements was the same as in Fig. 30.

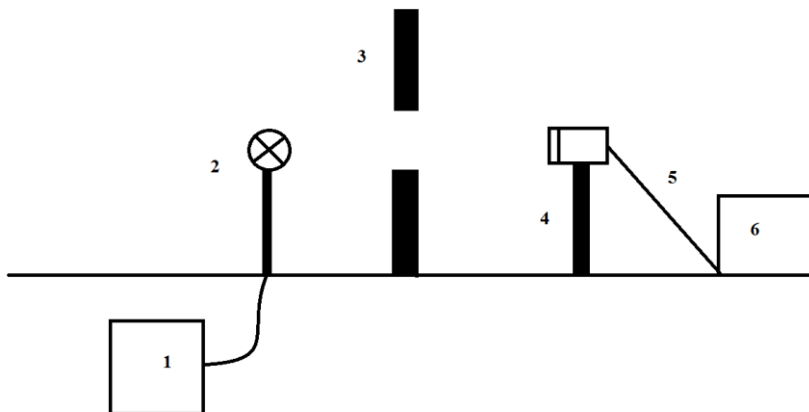


Fig. 30. Scheme of measurements.

Parts of the scheme:

1 is power supply

2 is light source (in this work it is LED line)

3 is black screen that helps to avoid scattered light and light from other sources

4 is optical probe

5 is optical fiber

6 is spectrometer

For different LEDs was used various types of power supply. For some of them there were special drivers that were connected directly to the socket. Another group of LEDs were powered by stabilized current source. Thus, all investigated LEDs were in their nominal mode without deviations of electrical parameters.

All measurements were conducted without surrounding light in the room. Also one of the main conditions of such measurements is specific room requirements. During photometric works it is necessary to make measurements in so-called black room. Such rooms have black walls, floor and ceiling painted with special dye to eliminate specular reflection. This condition is needed to avoid inaccuracies in obtained results.

Sequence of measurements:

1. Switching on the spectrometer;
2. Fixing the sample on the stand in the off mode;
3. Switching on the sample;
4. Adjustment of optical line (main direction of intensity of the sample should coincide with a center of optical probe);
5. Checking for the absence of shadows on the probe that can appear because of screen;
6. Turning off the ambient lighting (or closing curtains);
7. Making a measurement.

To obtain more accurate results, it is recommended to carry out the measurement several times for each sample. In this study was made three measurements for each LED.

5.2. BRIEF DESCRIPTION OF MEASUREMENT OBJECTS

All measured samples are summarized in Table 2. There is a number of the object, its type (two- or four-crystal), properties and manufacturer. There were only two types of LEDs:

- Four-crystal with three color and one white luminophore-based LEDs (RGBW);
- Two-crystal with two white luminophore-based LEDs (2LEDs).

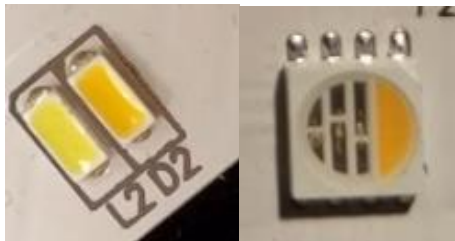
Regarding column “properties”, it is useful to note that there were four different RGBW LEDs. Three of them contained the same set of RGB crystals but different white crystals. However, in LED №1 were both white and colorful crystals.

Table 2. Measured LEDs.

Number of LED	Type	Properties	Manufacturer
	RGBW	$R_1 G_1 B_1 W_1$ (2700 K)	Cree
	RGBW	R	Edison
	RGBW	R	Edison
	RGBW	R	Edison
	2LEDs	$W_5 + W_6$ (3000 K +6500 K)	Edison
	2LEDs	$W_7 + W_8$ (3000 K +5000 K)	Galad

Information on brackets for LEDs №1-4 is CCT of white crystal. LEDs №5 and №6 are two-crystal and the column “properties” consists of information about CCTs of both crystals.

Photos of two kind of the investigated LEDs are provided in Fig. 31.



(a) (b)

Fig. 31. Photos of multicrystal LEDs: a) 2-crystal LED where both are based on luminophore; b) 4-crystal RGBW LED

Table 3 contains the set of CCTs that can be obtained using each LED. As it was mentioned before, such LEDs as RGBW allow to obtain any CCT, while two-crystal LEDs are limited by their own CCT. That is why it is impossible to get CCT below 3000 K for LED №5 and 6 or CCT higher 5000 K for LED №6.

Table 3. Set of measurements.

№ of LED CCT, K						

At first glance, the scale of CCTs may seem illogical. However, the choice of these values can be explained from the point of required values of CCTs in lighting engineering. Although human eye can distinguish CCTs with difference about 100 K, such accuracy is not necessary. Thus, the scale that is used in the table above is the most widespread.

One more important detail is the way of obtaining results. While spectra of LEDs №5 and 6 were measured directly, spectra of LEDs №1-4 were created artificially. In other words, spectrum of each crystal was measured separately and the next step was to model the final spectrum for each CCT with the aid of the developed program.

In measurements was investigated also following:

- irradiance vs. current;
- CCT vs. angle of observation.

In this part there were measured only crystals R₂, G₂, B₂, W₂, W₃ and W₄ separately.

5.3. SPECTRA OF LEDS

5.3.1. LED №1

Spectral distribution of emission of each crystal is shown in Fig. 32:

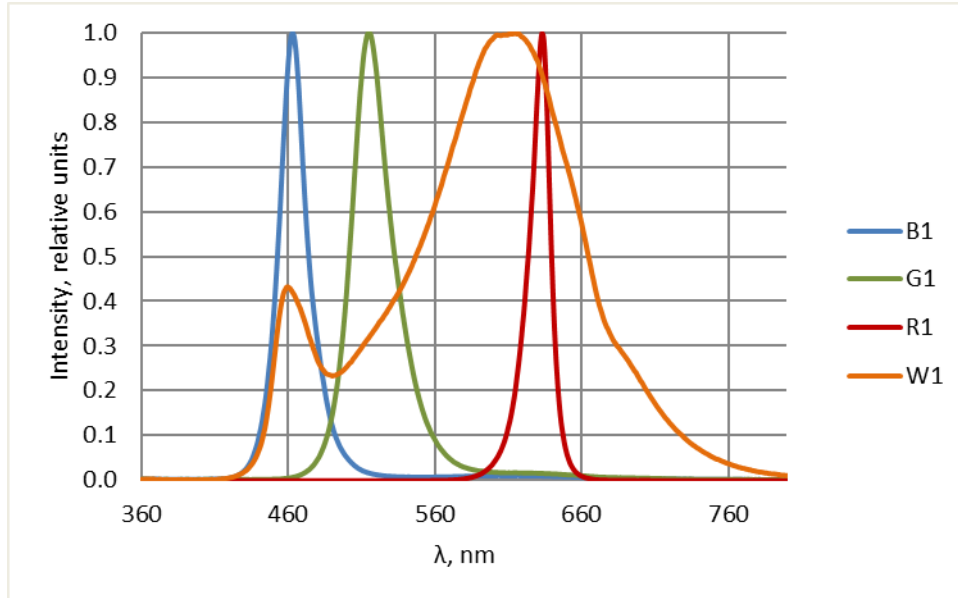


Fig. 32. Spectra of LED №1 crystals.

In LED №1 is three colorful crystals and one phosphor. Blue crystal (B_1) has peak wavelength (λ_P) equal to 463 nm and full width at half maximum ($\Delta\lambda_{0.5}$) is 21 nm. Green crystal (G_1) has $\lambda_P=515$ nm and $\Delta\lambda_{0.5}=31$ nm. These values for red crystal (R_1) are 633 nm and 16 nm respectively. White crystal (W_1) can be characterized by CCT. It was written before that CCT=2700 K but it is widespread situation when there is difference in theoretic value and results of measurements. Thus, CCT=2627 K. Measured CCT is equal to theoretical with good accuracy.

5.3.2. LEDS №2-4

Spectral distributions of LED №2-4 are presented on two graphs. In the Fig. 33 there are three spectra of white crystals. Each of them relates to a certain LED. Thus, W_2 is for LED №2, W_3 is for LED №3 and W_4 is for LED №4.

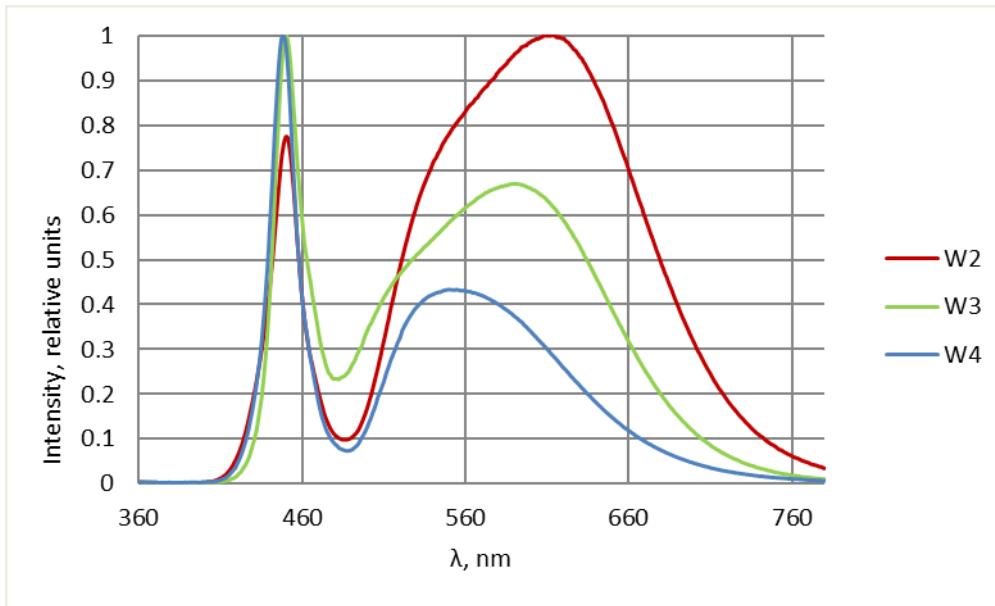


Fig. 33. Spectra of crystals W₂, W₃ and W₄.

Measured values of CCT are 3079 K (W₂), 4450 K (W₃) and 6597 K (W₄).

Fig. 34 contains three spectra of colorful crystals. These spectra are used in all of three LEDs №2-4.

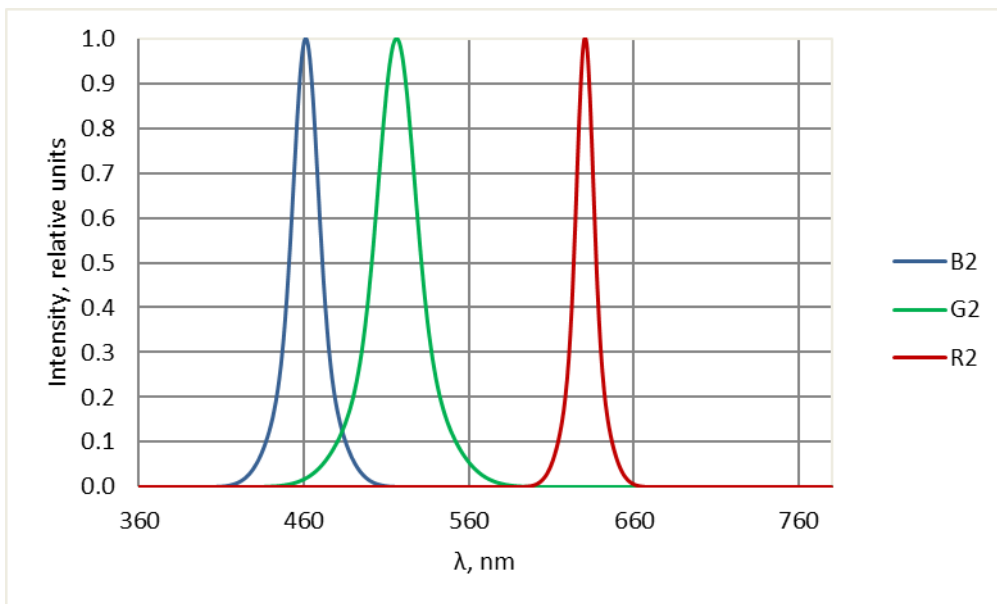


Fig. 34. Spectra of crystals B₂, G₂ and R₂.

Characteristics of color crystals are following: $\lambda_P=461$ nm and $\Delta\lambda_{0.5}=22$ nm for blue crystal (B₂), $\lambda_P=517$ nm and $\Delta\lambda_{0.5}=33$ nm for green crystal (G₂), $\lambda_P=632$ nm and $\Delta\lambda_{0.5}=16$ nm for red crystal (R₂).

5.3.3. LED №5

As it was mentioned before, this type of LED consists of two phosphor crystals with different CCTs. Spectral distributions of crystals composing LED №5 is shown in the Fig. 35. CCTs of these crystals are 3088 K (W_5) and 6689 K (W_6).

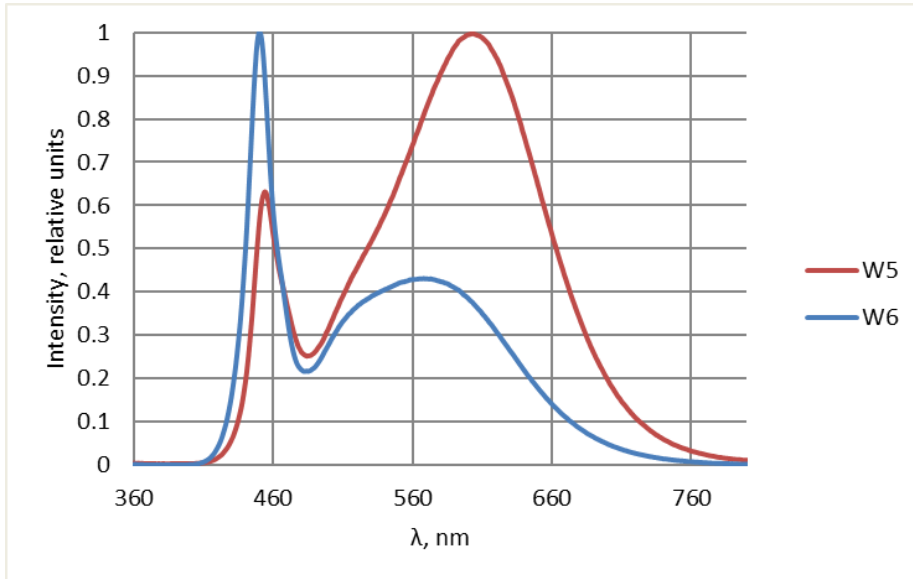


Fig. 35. Spectra of crystals W_5 and W_6

5.3.4. LED №6

Spectral distribution of radiant intensity in relative units of crystals for LED №6 are shown on the Fig. 36. These crystals have CCT= 3160 for W_7 4911 K for W_8 .

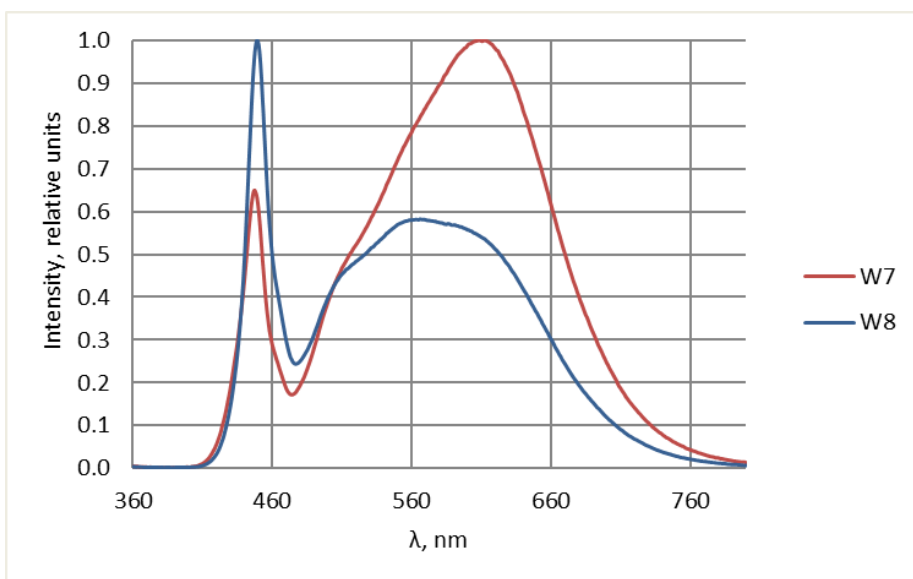


Fig. 36. Spectra of crystals W_7 and W_8

5.4. LUMINOUS CHARACTERISTICS

All main measurements were carried out in the nominal mode of the operating current. However, in fact, for this type of LEDs, it is necessary to change the intensity of each crystal. To determine how the intensity depends on the current, so-called luminous characteristics were measured. There are luminous characteristics for crystals W_2 , W_3 , W_4 in Fig. 37 and for crystals R_2 , G_2 and B_2 in Fig. 38.

Both dependences allow to conclude that intensity of light and, as a consequence, luminous flux and illuminance is non-linear function of current. This condition should be taken into account during designing the lighting systems and writing programs of scenario. In this study were calculated spectra and coefficients themselves but they should be recalculated for implementation with real light sources.

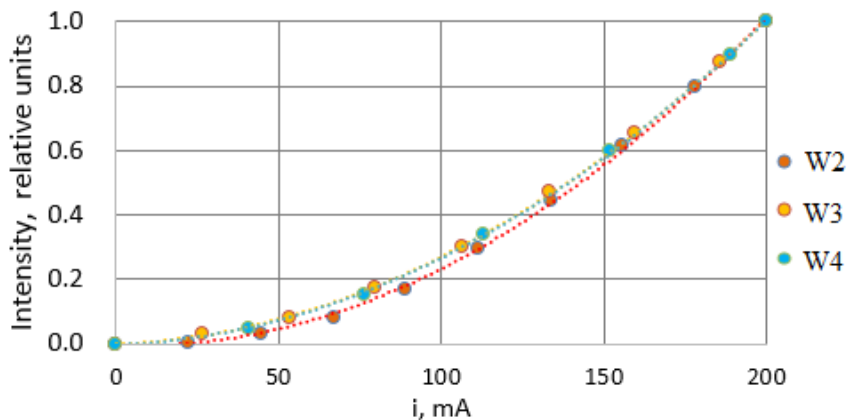


Fig. 37. Luminous characteristic for crystals W_2 , W_3 and W_4 .

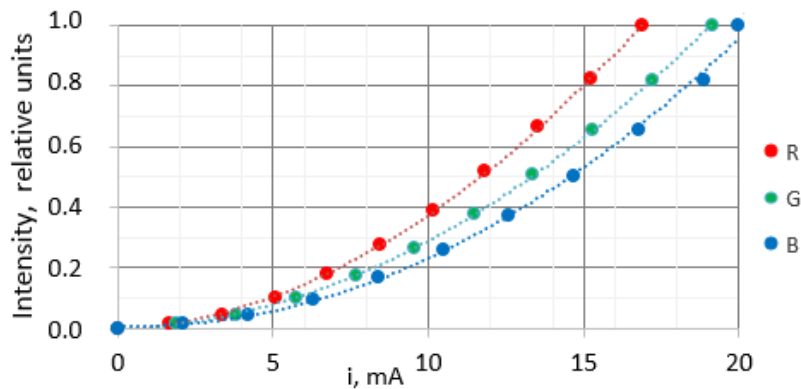


Fig. 38. Luminous characteristic for crystals R_2 , G_2 and B_2 .

5.5. ANGULAR CHARACTERISTIC

One more kind of measurements is getting information about spatial distribution of white LEDs. It is known that CCT of luminophore LED varies depending on angle because of the different optical path length. There were measured values of CCT from 0° to 65° (Table 4). Such limit of the angle is explained by absence of radiation for larger values of the angle.

Table 4. Angular characteristics for crystals W₂, W₃ and W₄

$\alpha, ^\circ$	CCT, K		
	W ₂	W ₃	W ₄
0	3079	4453	6605
5	3072	4446	6585
20	3068	4427	6489
35	3048	4360	6351
50	3009	4248	6160
65	2955	4107	5886

Fig. 39 shows relative deviation of CCT of each LED from the nominal value.

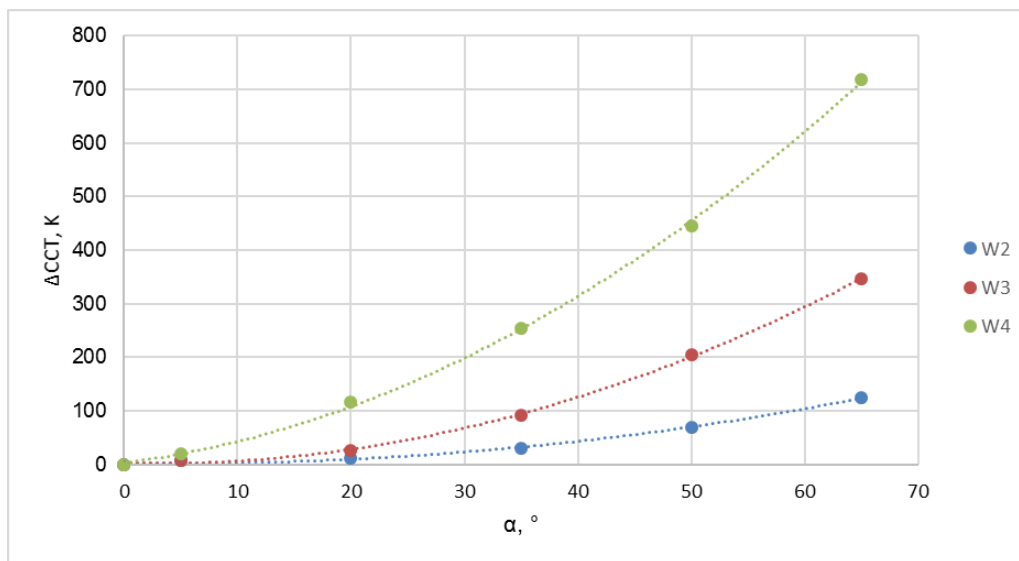


Fig. 39. Angular characteristics for crystals W₂, W₃ and W₄

It can be seen a strong dependence of value of CCT on angle. Also it can be noticed that difference in CCT is much higher for W₄ which has the highest CCT and refers to cool white LEDs. Light sources usually are not applied themselves. That is why for a purpose of lighting are created lighting devices with diffusers and others components. As a result, output light can have other characteristics because this light is a mixture of light from all directions of radiation. Because of this reason, it is important to take into account the angle characteristics in a process of creating the lighting device.

6. CALCULATIONS AND DISCUSSION

All measured spectra were processed in a program. The description is given below.

6.1. PROGRAM

In order to optimize calculations, to reduce the consumed time and to simplify the process, a program in MATLAB was written. It allows getting the necessary calculations quickly and conveniently and obtaining results in a visual form.

For each type of LED was created its own program, taking into account the characteristics of the type of input data for modeling the spectrum.

Input data for each program:

- Spectra of each crystal (LED);
- Required CCT, K.

The output of each program is as follows:

- R1-R8 are special color rendering indices;
- R9-R14 are additional color rendering indices;
- DE1-DE15 are special indices calculated by the CQS method;
- Ra is general color rendering index by the MCO method;
- Qa is color quality scale by the CQS method;

6.1.1. THE PROGRAM FOR TWO-CRYSTAL LED

Screenshot of the program window is shown in Fig. 40.

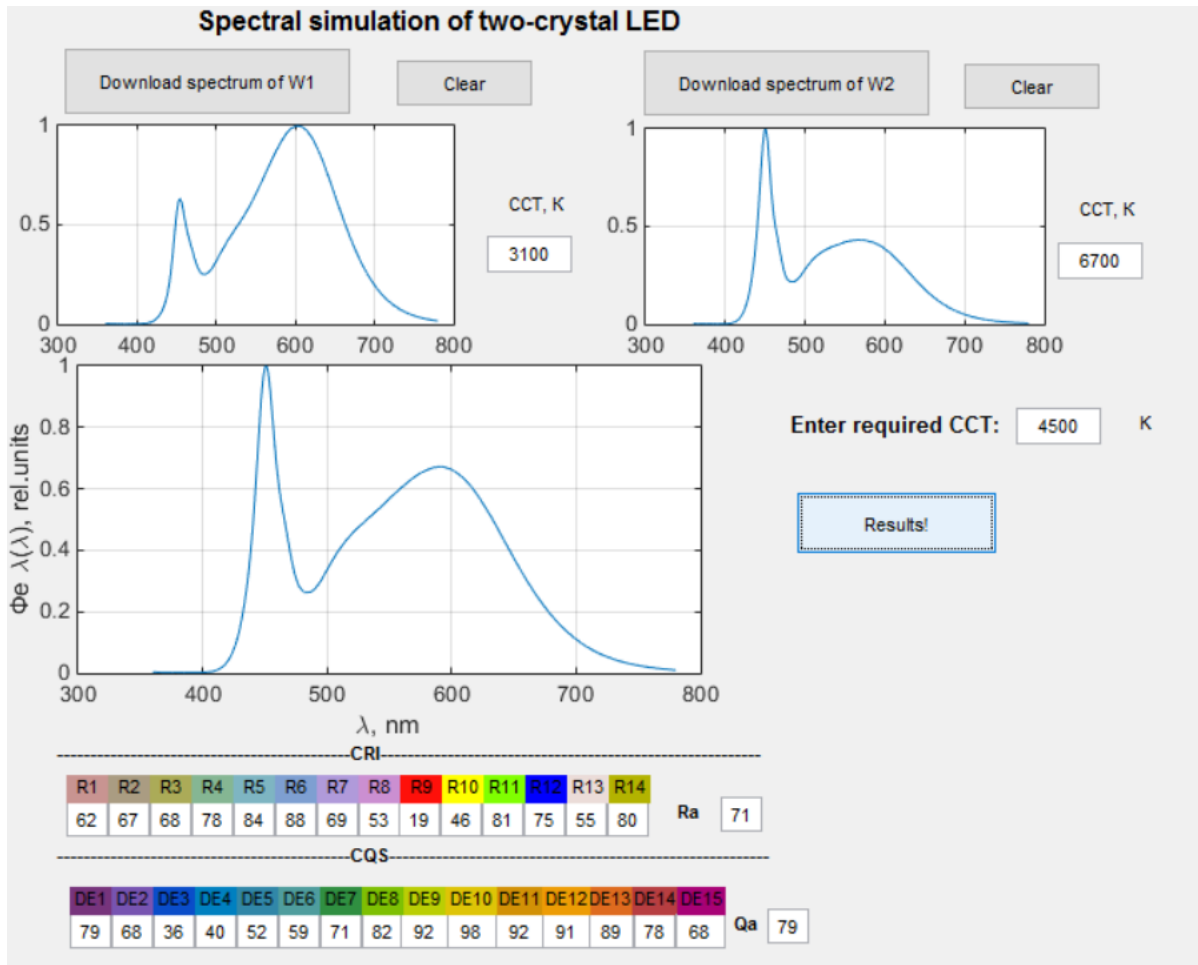


Fig. 40. Interface of the program for two-crystal LED.

The program allows:

- To import the spectra of two white LEDs;
- To calculate the radiation proportion to obtain the spectrum with required CCT;
- To calculate the color characteristics of the white light source (chromaticity coordinates, CCT), as well as to evaluate the color rendering using the CRI method (special and general color rendering indices) and the CQS method (color differences and color quality scale).

6.1.2. THE PROGRAM FOR FOUR-CRYSTAL RGBW LED

Screenshot of the program window is shown in Fig. 41.

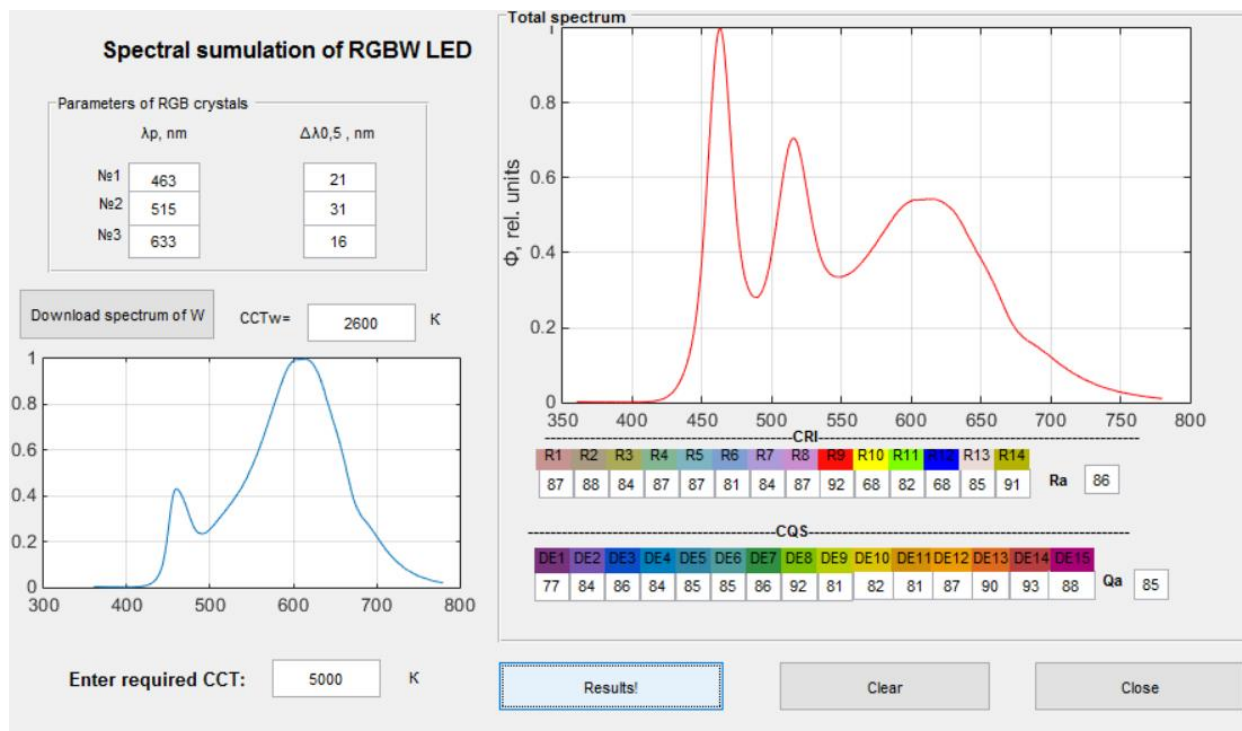


Fig. 41. Interface of the program for four-crystal LED (RGBW).

The program allows:

- To import the spectra of two white LEDs;
- To determine parameters of the spectra of color LEDs and white LED, in which the total LED will have the best color rendering;
- To calculate the color characteristics of the white light source (chromaticity coordinates, CCT), as well as to evaluate the color rendering using the CRI method (special and general color rendering indices) and the CQS method (color differences and color quality scale).
- To model the spectra of color LEDs using the approximation by the recommendation of the CIE (input data are λ_p , $\Delta\lambda_{0.5}$).

6.2. COEFFICIENTS

After calculations of the measured spectra of the samples, results were obtained in the form of spectra, R_i , R_a , Q_i and Q_a for each combination of LEDs. Fig. 42-Fig. 47 show values of coefficients of four crystals for every required CCT. Numerical values for all LEDs are in Appendix 1. Lines are guide for the eye.

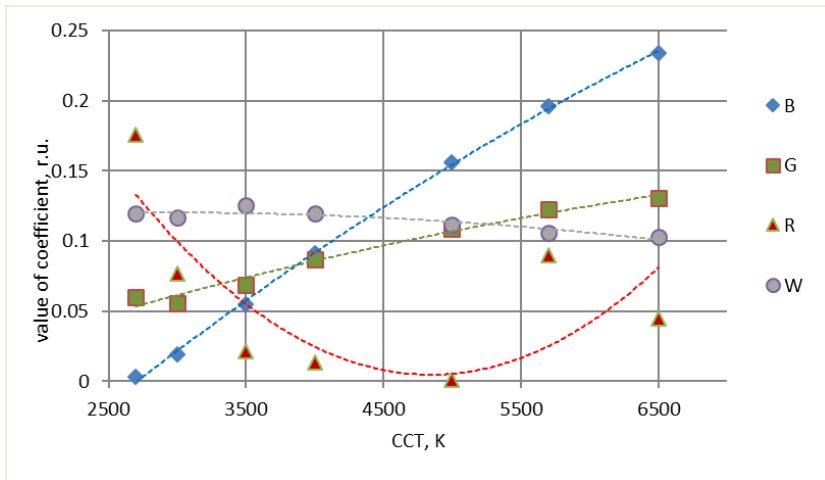


Fig. 42. Coefficients of each crystal vs. CCT (LED №1).

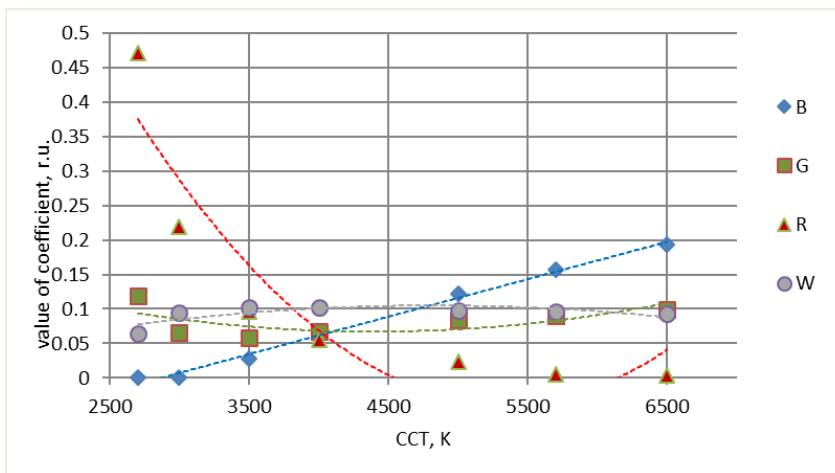


Fig. 43. Coefficients of each crystal vs. CCT (LED №2).

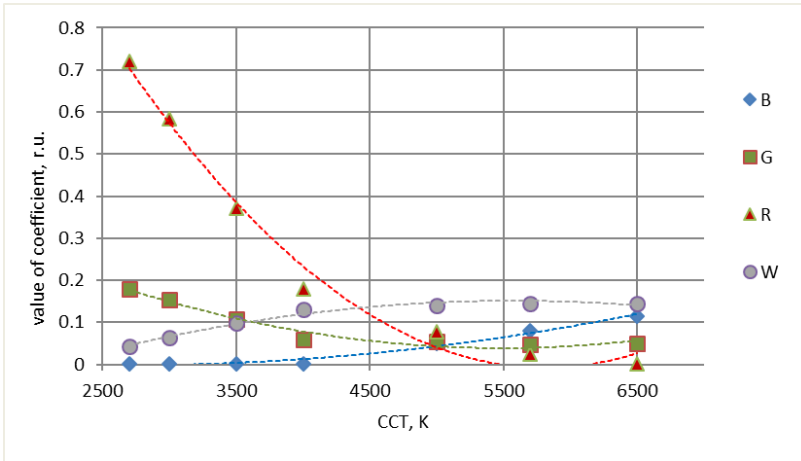


Fig. 44. Coefficients of each crystal vs. CCT (LED №3).

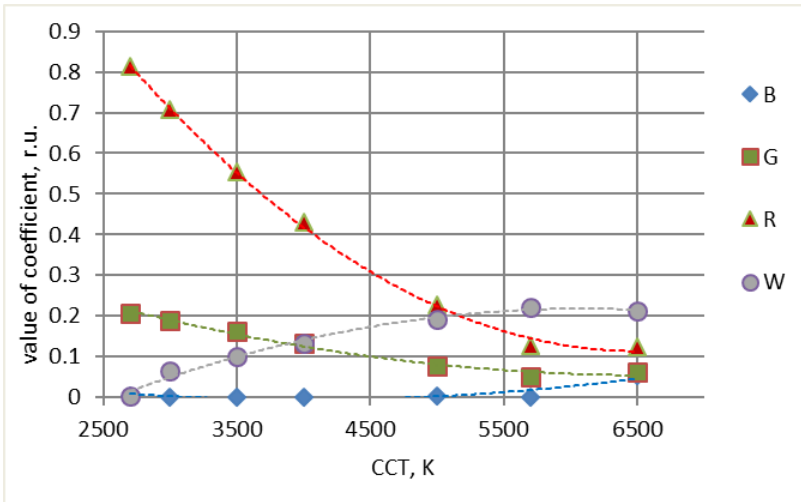


Fig. 45. Coefficients of each crystal vs. CCT (LED №4).

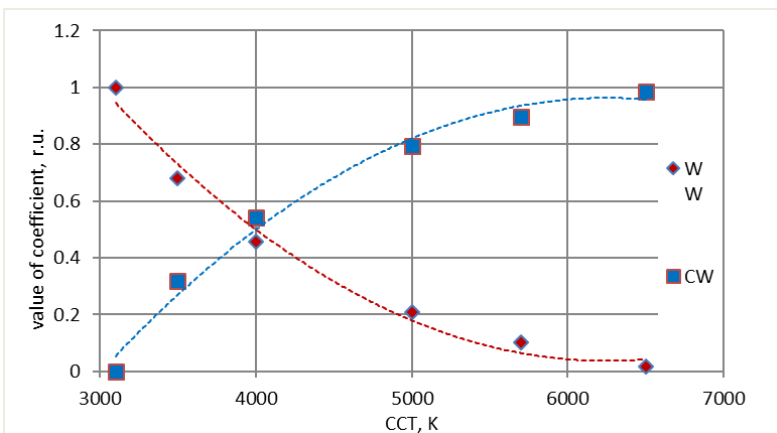


Fig. 46. Coefficients of each crystal vs. CCT (LED №5).

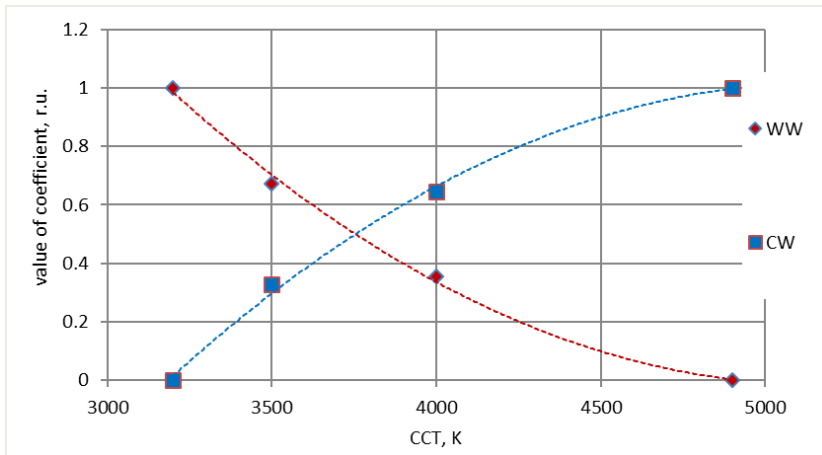


Fig. 47. Coefficients of each crystal vs. CCT (LED №6).

Discussion: in Fig. 42 it can be seen that coefficients for blue, green and white components grow with increasing CCT, while red crystal has different behavior. Value of red is relatively high for low CCTs, then it rapidly decrease and in the end there is slight increase. However, the value for red coefficient is much smaller than coefficients of others crystals for CCT=5000 K. The same behavior of red crystal can be noticed in Fig. 43 (LED №2). This LED has different color LEDs but their spectra are similar to color spectra of LED №1. The main distinguish feature is CCT of white crystal (**further - CCT_w , do not confuse with CCT of the total spectrum**). LED №2 has white crystal with higher CCT_w in comparison to LED №1. Although both of them have warm white light (2700 K and 3100 K) there is difference in behavior of red crystals. The rest of crystals coefficients (G, B and W) have the same behavior, but coefficient of red crystal of LED №2 drop more rapidly and its value is much higher for low CCTs. Fig. 44 and Fig. 45 prove that such behavior of red coefficients is not a coincidence. There can be formulated rules for RBGW LEDs:

- *The higher is CCT_w of white crystal, the higher the coefficient of the red crystal is for low CCTs;*
- *The higher is CCT_w of white crystal, the smaller the contribution is made by the red crystal at high color temperatures.*

Last two investigated LEDs are two-crystal and behavior seems logical, because there is only shift from CCT of one crystal to CCT of another. In Fig. 46 and Fig. 47 is gradual decreasing of contribution of crystal with low CCT and increasing of coefficient of cool white crystal.

6.3. SPECTRA FOR THE REQUIRED RANGE OF CCTS

According to formula (45) total spectrum for each CCT from the required range was calculated. First, the spectrum was found in the program. However, for clarity is was recalculated using proportions and original spectra of each crystal. Spectral distributions of irradiance for all investigated LEDs are given in Fig. 48-Fig. 53:

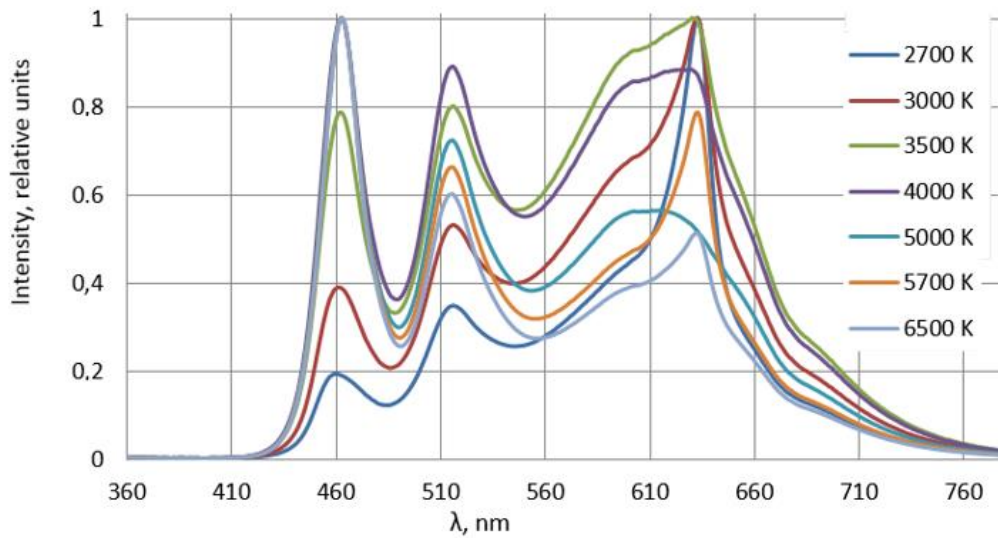


Fig. 48. Spectra of LED №1 for each CCT.

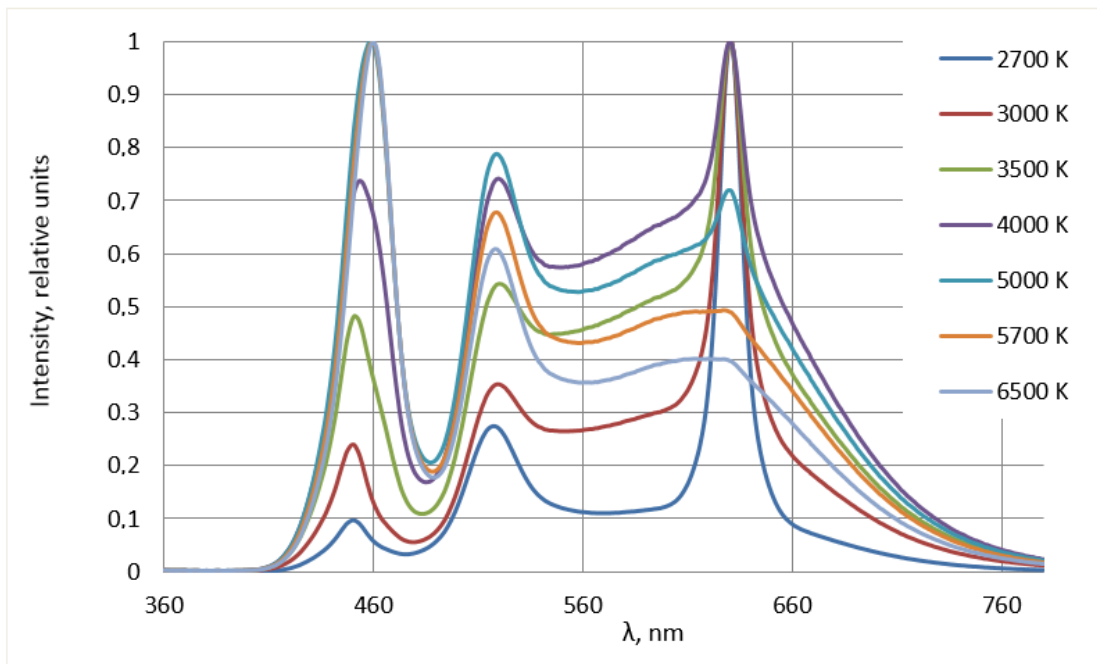


Fig. 49. Spectra of LED №2 for each CCT.

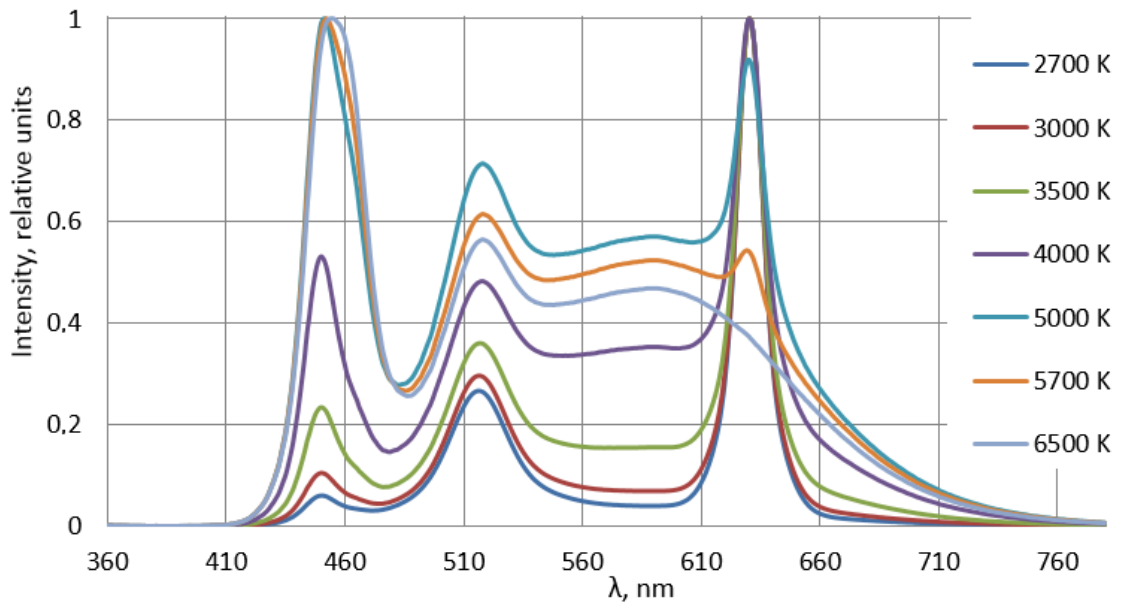


Fig. 50. Spectra of LED №3 for each CCT.

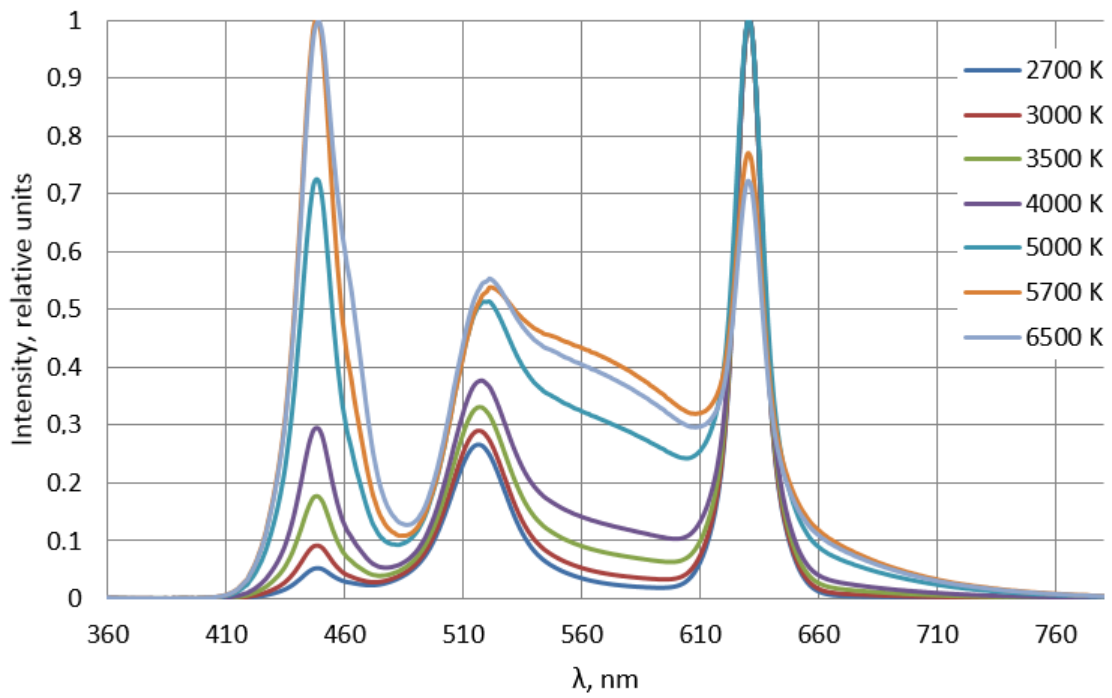


Fig. 51. Spectra of LED №4 for each CCT.

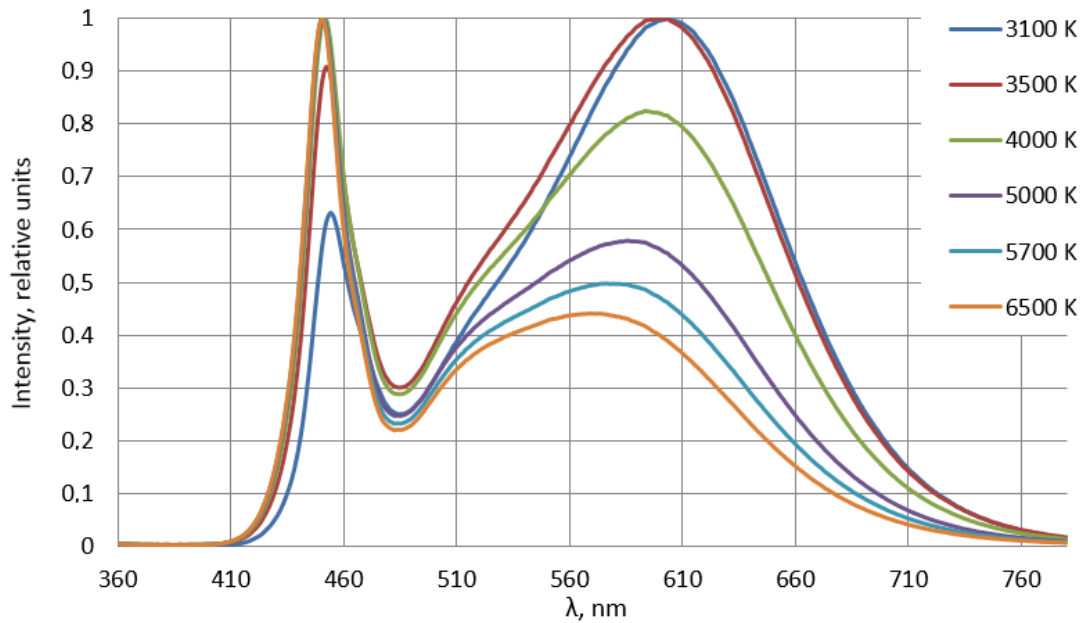


Fig. 52. Spectra of LED №5 for each CCT.

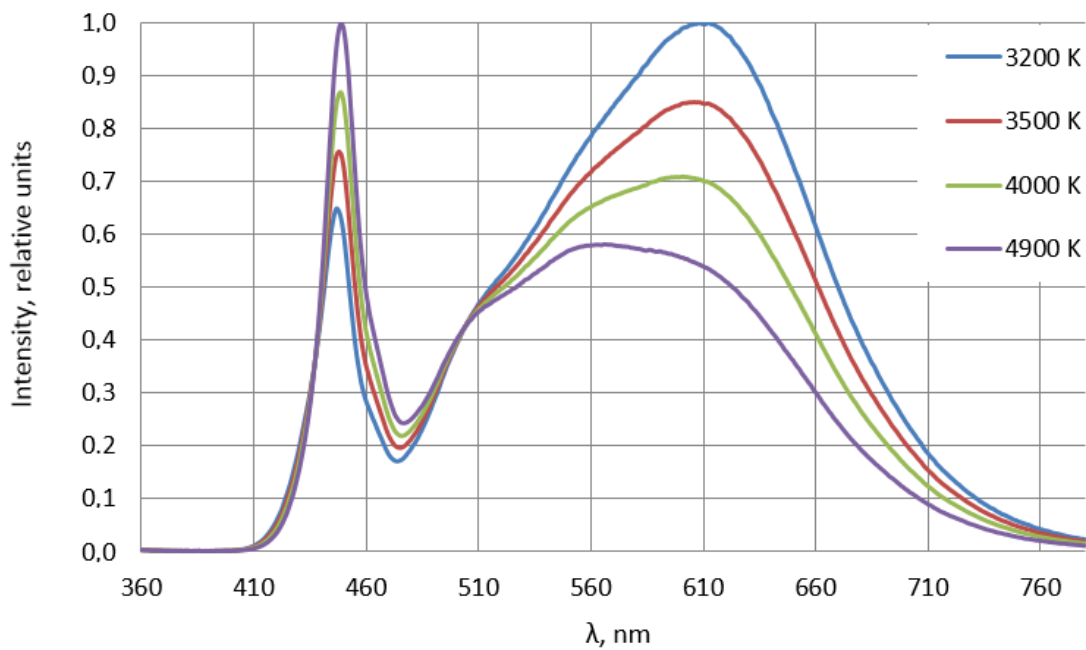


Fig. 53. Spectra of LED №6 for each CCT.

Discussion: For LEDs №1-4 it can be noticed difference in the shapes of spectral distributions. While LED №1 and LED №2 have relatively smooth spectra (Fig. 48 and Fig. 49), there are gaps in blue and green parts of the spectrum. It is important to notice that total spectrum of RGBW LED has unique shape with three peaks. There are no four peaks because of mixing two peaks: the peak of blue crystal and peak of white crystal are in blue part of spectrum. Thus, one can observe

elimination of one thin peak in spectrum of usual LED. Spectra of two-crystal LEDs (Fig. 52 and Fig. 53) represent relatively gradual changing from warm to cool white LED. Each spectra looks like classic spectral distribution of LED based of blue radiation.

6.4. CRI AND CQS

The exact calculation results for methods CRI (special additional and general indices) and CQS (color differences and color quality scale) are presented in Appendices 2 and 3. On the basis of these tables there were drawn graphs of Ra and Qa as a function of CCT (Fig. 54-Fig. 59). Lines are guide for the eye.

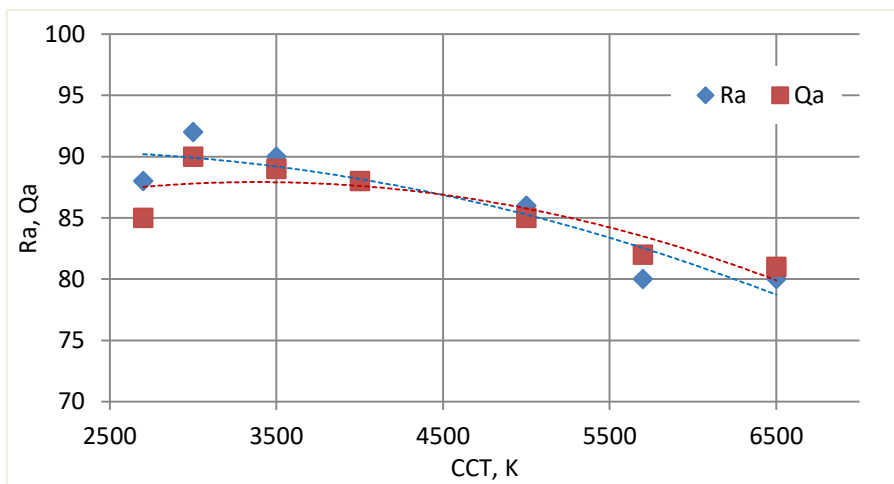


Fig. 54. Color rendering of LED №1 vs. CCT.

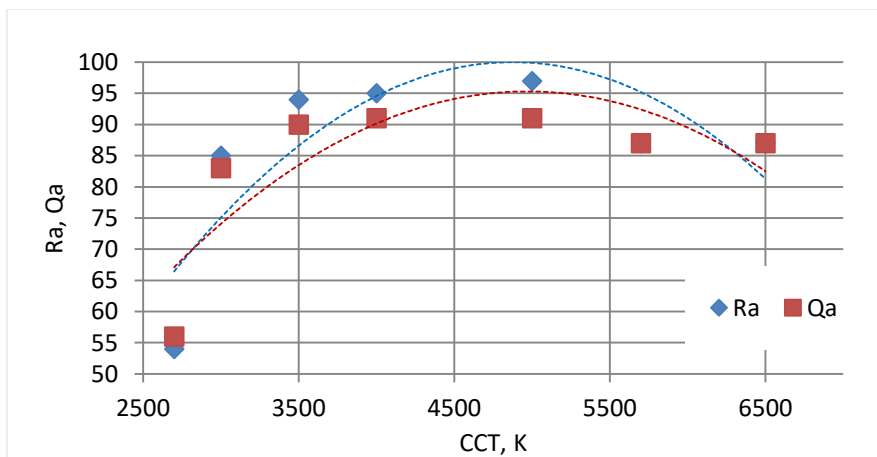


Fig. 55. Color rendering of LED №2 vs. CCT.

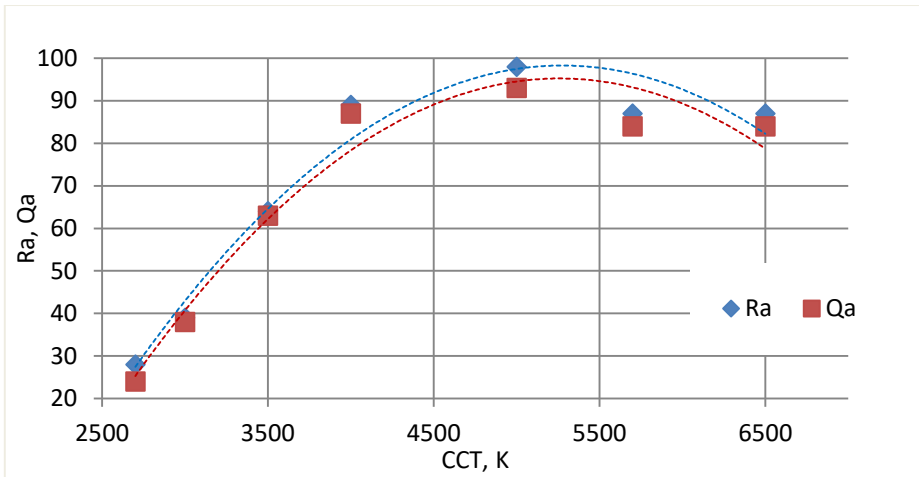


Fig. 56. Color rendering of LED №3 vs. CCT.

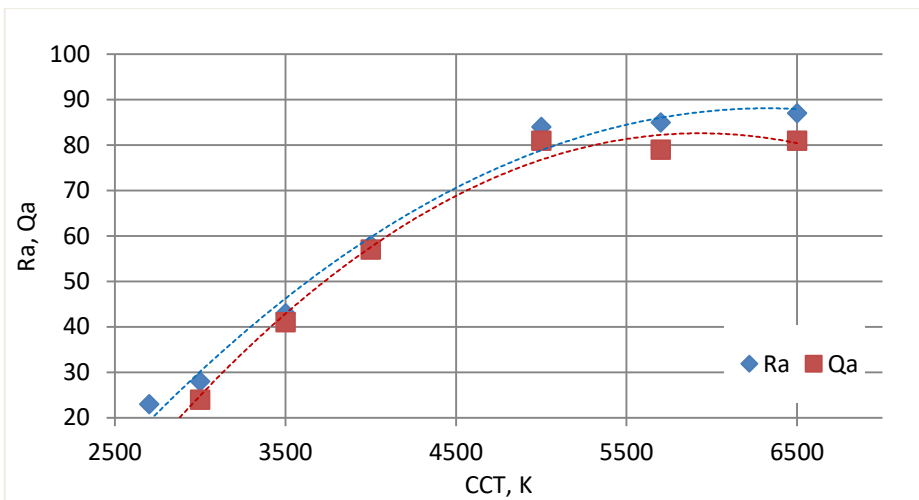


Fig. 57. Color rendering of LED №4 vs. CCT.

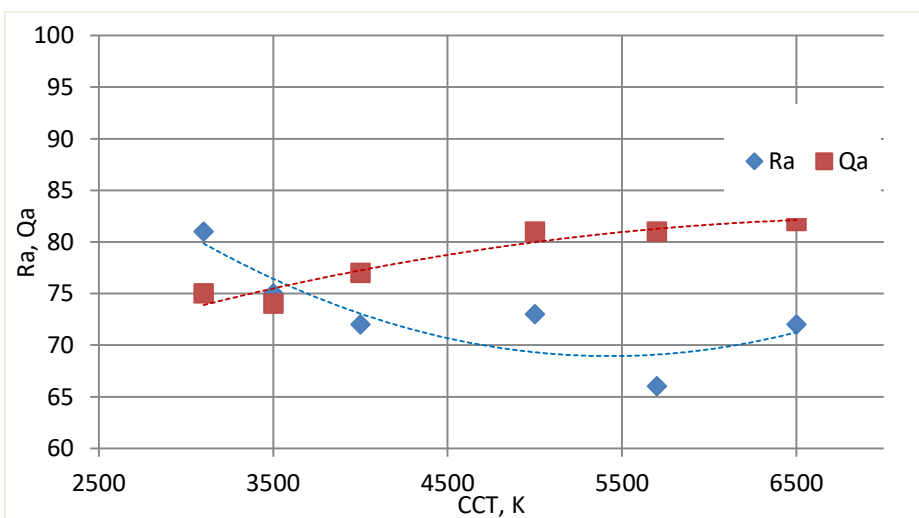


Fig. 58. Color rendering of LED №5 vs. CCT.

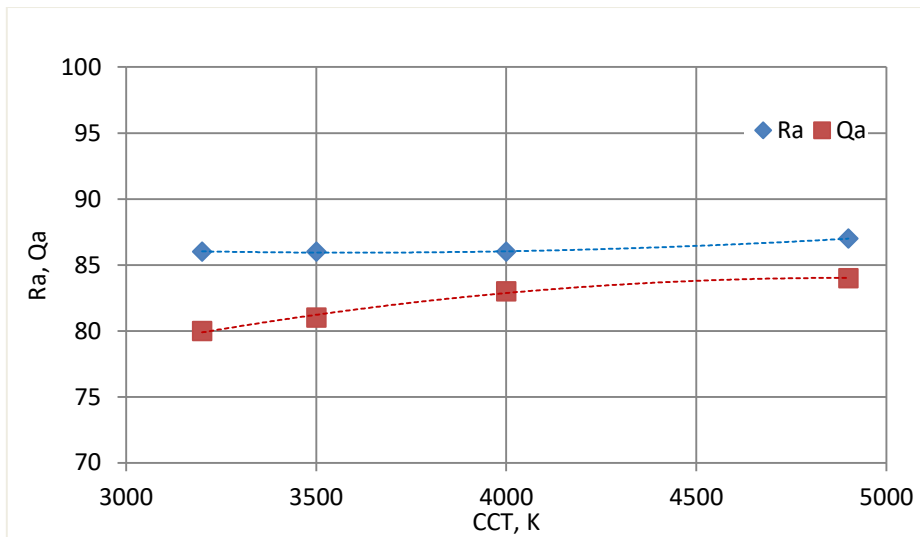


Fig. 59. Color rendering of LED №6 vs. CCT.

Discussion: Results of color rendering are quite different. LED №1 has decreasing Ra and Qa, although these values are in the range from 80 to 92 for Ra and from 81 to 90 for Qa. The maximum values of Ra and Qa were obtained for those CCTs which were the closest to CCT of white crystal. This observation is true for LEDs №1-4. This can be seen in corresponding graphs for these LEDs. *If CCTs are lower than CCT_w , Ra and Qa is low.* Furthermore, it is necessary to take into account not only general indices for both methods but also special, additional rendering indices and color differences in CQS. It is noteworthy that even with moderately Ra and Qa quite low special indices were obtained. Performed calculations showed and approved advantages of CQS in comparison to CRI. On average, Qa is lower than Ra. This happened because of influence of the method of calculation. Qa is the result of geometrical average, while CQS is the result of arithmetical average. This is why even with low Ri high values of Ra were obtained. As it was mentioned before, CQS does not allow to get high Qa with low color differences. Regarding LED №5 and LED №6 in Fig. 58 and Fig. 59 is shown that color rendering changes within values of Ra and Qa of each crystal.

7. CONCLUSION

1. Non-linear dependences of intensity on forward current for both luminophore and color LEDs were obtained. There were observed considerable deviations of CCT from the nominal value depending on the angle of observation. These factors should be taken into account for implementation of systems of dynamic lighting.
2. Calculations confirmed that to evaluate the color rendering of light sources based of LEDs it is more appropriate to use the color scale method (CQS), since it takes into account the features of such as light sources much better than the method of color rendering indexes (CRI).
3. The color rendering for RGBW LEDs is determined by the CCT_w of white crystal: for high values of color rendering it is necessary to select the LED with a low color temperatures (2300-4000 K). Such combination there would give high color rendering for the whole range of required CCTs.
4. The developed program is a convenient and intuitive tool for simulation of spectral and color characteristics of multicrystal LED, such as two-crystal and four-crystal ones. These two interfaces of the program can be used by developers of systems of dynamic lighting to select the intensity ratio of each crystal and to determine the color rendering.

REFERENCES

1. Shponkina, Yu., Energy saving in the electric power industry, *Electrotechnical market*, No 3(57), 2014. Web site: <http://market.elec.ru/nomer/52/energoberezhenie-v-elektroenergetike/> RUS
2. Henri Juslén, Dynamic Lighting and human performance. Web site: https://static.aminer.org/pdf/PDF/000/240/462/effects_of_lighting_on_human_performance_in_training.pdf
3. Schubert E. F., Light Emitting Diodes, Second edition, Cambridge University Press, 2006
4. History of LEDs. Web site: <http://leds-magazine.ru/istoriya-svetodiodov.html> RUS
5. LEDs. Web site: <http://sveti.ru/handbook/1212827173.html> RUS
6. The Nobel Prize in Physics 2014, Web site: http://www.nobelprize.org/nobel_prizes/physics/laureates/2014/
7. P-N Junction. Web site: <http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/pnjun.html>
8. Huai Zheng, Yiman Wang, Lan Li, Xing Fu, Yong Zou and Xiaobing Luo, Dip-transfer phosphor coating on designed substrate structure for high angular color uniformity of white light emitting diodes with conventional chips, *Journal of The Optical Society of America (OSA)*, 4 November 2013, Vol. 21, No. S6
9. Rich Rosen, Dimming Techniques for switched-mode LED drivers, *RadioLotsman*, November 2011, Web site: <http://www.radiolocman.com/review/article.html?di=124982> RUS
10. Nikiforov, S. Now the electrons can be seen: the LEDs make the electric current very noticeable, *Components and Technologies*, 2006, № 3. Web site: http://www.kite.ru/articles/led/2006_3_20.php RUS
11. Meshkov V.V., Matveev A.B., Basics of lighting engineering, Part 1, Moscow, Energiya, 1979 RUS
12. Aizenberg J., Lighting Handbook, Moscow, Znak, 2007 RUS
13. Yeliseev N., Reshenov, On limit light and colour characteristics of white light emitting diodes, *Light & Engineering*, 2013, №3 RUS
14. Meshkov V.V., Matveev A.B., Basics of lighting engineering, Part 2 Physiological optics and colorimetry, Moscow, Energiya, 1979 RUS
15. Colorimetry - Part 3: CIE Tristimulus Values, Joint ISO/CIE Standard ISO 11664-3:2012(E)/CIE S 014-3/E:2011
16. Color space XYZ, Web site: <http://bigenc.ru/physics/text/2082110>
17. L*a*b color space, Web site: <https://www.handprint.com/HP/WCL/color7.html>

18. CIE Colorimetry - Part 4: 1976 L*a*b* Colour Space, Joint ISO/CIE Standard, ISO 11664-4:2008(E)/CIE S 014-4/E:2007
19. Javier Herna'ndez-Andre's, Raymond L. Lee, Jr., and Javier Romero, Calculating correlated color temperatures across the entire gamut of daylight and skylight chromaticities, *Journal of The Optical Society of America (OSA)*, 20 September 1999, Vol.38, No. 27
20. XY diagram with Planckian locus, Web site: <https://www.led-professional.com/resources-1/articles/guidance-on-specifying-solid-state-lighting-luminaires>
21. Binning of LEDs, Web site: http://www.svetozone.ru/press/news/2015/02/27/news_8481.html RUS
22. Color rendering index, Web site: <http://www.beamled.com/info/blog/your-guide-to-cri/>
23. CIE (Commission Internationale de l'Eclairage), Method of Measuring and Specifying Colour Rendering Properties of Light Sources, Publication 13.3, 1995
24. Technical report «Measurement of LED's», CIE127-1997
25. Yoshi Ohno, «Color rendering of Light Sources», Web site: <https://www.nist.gov/pml/sensor-science/optical-radiation/color-rendering-light-sources>
26. Wendy Davis, Yoshi Ohno. Color quality scale, *Optical Engineering*, No. 49(3), March 2010
27. CIE (Commission Internationale de l'Eclairage), Technical Note: Brussels Session of the International Commission on Illumination, *Journal of The Optical Society of America (OSA)*, Vol.50, 1960
28. Günter Wyszecki, Proposal for a New Color-Difference Formula, *Journal of the Optical Society of America*, Vol. 53, Issue 11, pp. 1318-1319, 1963
29. CIE(Commission Internationale de l'Eclairage), Review of the official recommendations of the CIE for the colours of signal lights, Publication 107, 1997
30. Instrument systems CAS-140, Web site: <http://www.instrumentsystems.com/produkte/spektrometer/cas-140ct/>
31. Optical probes for CAS-140, Web site: <http://www.instrumentsystems.com/products/general-accessories/eop-optical-probes/>
32. Test Method for LED Lamps, LED Luminaires and LED Modules, CIE International Standard S 025/E:2015

APPENDIX 1. Proportions for investigated LEDs

LED №1

CCT, K	B	G	R	W
2700	0,003	0,0603	0,176	0,1195
3000	0,0189	0,0559	0,077	0,1169
3500	0,055	0,069	0,0215	0,1255
4000	0,0915	0,0869	0,0135	0,1195
5000	0,1563	0,1091	0,001	0,1117
5700	0,1958	0,1226	0,09	0,106
6500	0,2339	0,1304	0,045	0,1029

LED №2

CCT, K	B	G	R	W
2700	0,0003	0,1184	0,471	0,0647
3000	0,0005	0,0661	0,219	0,0943
3500	0,028	0,0584	0,0965	0,103
4000	0,0606	0,0668	0,0555	0,1022
5000	0,1214	0,0842	0,024	0,0976
5700	0,1572	0,0905	0,005	0,0961
6500	0,1939	0,0999	0,004	0,0926

LED №3

CCT, K	B	G	R	W
2700	0,0007	0,1789	0,721	0,0437
3000	0,0014	0,1548	0,5845	0,0635
3500	0,0014	0,1081	0,3705	0,0979
4000	0,0015	0,0602	0,179	0,1311
5000	0,0506	0,0534	0,077	0,1401
5700	0,0802	0,0477	0,024	0,1455
6500	0,115	0,0506	0,001	0,1451

LED №4

CCT, K	B	G	R	W
2700	0,0044	0,2073	0,814	0,00409
3000	0,0017	0,19	0,7065	0,0648
3500	0,0005	0,1611	0,555	0,1014
4000	0,0017	0,1327	0,4295	0,1342
5000	0,0046	0,0784	0,2265	0,1926
5700	0,00116	0,0499	0,126	0,2221
6500	0,0546	0,0619	0,123	0,2131

APPENDIX 1. (continues)

LED №5

CCT, K	WW	CW
3100	1	0
3500	0,682	0,318
4000	0,457	0,543
5000	0,207	0,793
5700	0,102	0,898
6500	0,016	0,984

LED №6

CCT, K	WW	CW
3200	1	0
3500	0,672	0,328
4000	0,354	0,646
4900	0	1

APPENDIX 2. Results of the CRI calculation

LED №1

CCT, K	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	Ra
2700	87	92	93	82	82	78	94	92	83	77	70	72	87	96	88
3000	95	94	90	94	93	86	90	90	85	85	86	79	94	95	92
3500	95	94	88	93	93	86	88	86	77	83	88	77	94	93	90
4000	91	91	86	90	90	83	87	88	89	77	84	73	90	92	88
5000	87	88	84	87	87	81	84	87	92	68	82	68	85	91	86
5700	85	93	93	79	75	67	73	78	77	75	67	70	89	96	80
6500	85	93	92	78	73	66	72	77	69	72	67	68	89	95	80

LED №2

CCT,K	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	Ra
2700	46	79	83	30	43	59	71	24	0	51	6	47	53	88	54
3000	83	97	86	74	84	93	87	73	50	95	64	98	86	90	85
3500	96	96	88	89	97	96	93	93	91	91	84	78	98	92	94
4000	97	96	90	92	100	95	96	97	98	91	87	73	99	94	95
5000	96	98	94	95	98	96	99	97	96	95	88	68	97	96	97
5700	89	87	83	83	88	90	88	88	91	81	77	76	87	89	87
6500	90	89	84	83	86	87	87	88	89	86	76	73	90	89	87

LED №3

CCT,K	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	Ra
2700	6	56	81	0	0	23	52	0	0	0	0	0	15	86	28
3000	23	67	83	10	23	43	59	0	0	20	0	25	32	87	39
3500	56	83	87	48	57	71	74	33	0	59	30	69	62	90	64
4000	86	97	92	82	88	93	92	78	53	92	73	85	89	94	89
5000	97	99	96	97	98	96	99	98	97	98	90	70	98	97	98
5700	83	86	86	93	95	92	82	75	57	77	89	70	82	91	87
6500	83	86	86	93	95	92	82	75	57	77	89	70	82	91	87

LED №4

CCT,K	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	Ra
2700	0	52	77	0	0	14	43	0	0	0	0	0	2	83	23
3000	5	61	76	0	5	33	46	0	0	5	0	10	17	83	28
3500	29	75	75	16	32	58	55	0	0	39	0	50	40	83	43
4000	50	86	76	39	55	76	64	14	0	66	22	80	60	84	58
5000	87	96	77	75	90	95	82	66	27	89	68	74	92	86	84
5700	85	76	65	74	89	94	95	98	95	53	74	66	78	79	85
6500	88	79	67	75	90	97	97	99	88	59	73	68	82	80	87

APPENDIX 2. (continues)

LED №5

CCT,K	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	Ra
3100	76	85	89	81	85	98	79	59	16	73	78	72	76	93	81
3500	68	74	77	80	85	91	73	55	17	59	80	77	64	86	75
4000	63	68	71	78	83	87	69	53	19	50	80	77	57	81	72
5000	63	68	69	81	86	90	71	55	19	47	83	72	57	80	73
5700	56	59	59	73	80	84	65	49	13	34	79	77	47	74	66
6500	61	65	65	80	88	91	72	55	12	39	84	71	54	78	72

LED №6

CCT,K	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	Ra
3200	84	89	93	85	85	88	89	71	32	76	85	73	84	96	86
3500	83	87	90	86	87	91	89	72	36	73	86	73	82	94	86
4000	83	86	88	88	89	91	90	75	39	72	87	71	82	93	86
4900	88	92	95	86	84	83	91	80	40	79	85	61	89	97	87

APPENDIX 3. Results of the CQS calculation

LED №1

CCT, K	DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8	DE9	DE10	DE11	DE12	DE13	DE14	DE15	Qa
2700	85	88	85	80	82	81	82	88	85	85	84	88	902	91	90	85
3000	85	90	89	88	89	89	91	93	88	90	91	95	97	95	91	90
3500	82	89	88	88	89	89	92	94	87	89	91	96	97	93	89	89
4000	80	8	87	86	87	86	89	93	84	86	86	92	94	94	90	88
5000	77	84	86	84	85	85	86	92	81	82	81	87	90	93	88	85
5700	85	92	85	84	84	83	86	87	73	74	72	79	83	90	90	82
6500	82	89	85	85	84	83	86	87	72	72	69	77	81	87	88	81

LED №2

CCT,K	DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8	DE9	DE10	DE11	DE12	DE13	DE14	DE15	Qa
2700	72	95	67	42	41	35	32	64	81	75	57	57	57	56	63	56
3000	91	90	97	86	79	71	68	79	93	98	89	87	85	84	88	83
3500	97	90	83	95	94	86	79	82	91	93	98	97	95	97	98	90
4000	99	92	79	91	97	92	83	84	92	94	99	96	97	99	99	91
5000	93	96	77	88	96	98	86	88	95	97	98	97	96	97	97	91
5700	99	87	66	76	84	91	95	94	97	97	93	94	93	96	98	87
6500	96	90	68	77	84	91	95	96	95	95	91	91	91	95	95	87

LED №3

CCT,K	DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8	DE9	DE10	DE11	DE12	DE13	DE14	DE15	Qa
2700	47	81	31	5	7	4	2	35	57	45	22	24	27	28	36	24
3000	56	88	49	20	21	15	11	48	69	59	36	37	38	36	45	38
3500	73	95	78	57	54	48	43	69	85	79	63	62	62	57	64	63
4000	92	96	91	92	87	81	76	85	96	97	89	88	87	85	88	87
5000	93	96	80	89	97	96	98	90	91	98	98	98	97	98	97	93
5700	92	90	62	68	76	81	90	91	95	98	98	98	97	90	91	84
6500	92	90	62	68	76	81	90	91	95	98	98	98	97	90	91	84

LED №4

CCT,K	DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8	DE9	DE10	DE11	DE12	DE13	DE14	DE15	Qa
2700	41	81	22	2	2	1	0	26	52	39	12	14	17	19	28	14
3000	47	90	38	8	7	3	2	35	64	53	23	23	24	24	34	24
3500	57	97	63	31	27	18	12	48	76	70	42	40	38	33	43	41
4000	66	94	85	57	49	38	30	58	85	85	61	57	53	46	55	57
5000	86	85	78	96	87	76	65	71	84	92	94	89	84	77	82	81
5700	84	70	50	65	81	85	83	80	85	85	92	96	95	95	91	79
6500	88	75	53	67	82	88	85	83	88	89	95	98	955	93	92	81

APPENDIX 3. (continues)

LED №5

CCT,K	DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8	DE9	DE10	DE11	DE12	DE13	DE14	DE15	Qa
3100	77	80	57	58	67	71	78	87	97	94	89	87	85	74	76	75
3500	74	68	42	45	56	63	73	85	93	97	91	88	86	75	69	74
4000	75	66	37	40	52	60	71	83	92	98	92	90	88	76	67	77
5000	83	71	38	41	51	61	71	82	93	96	92	91	90	79	69	81
5700	80	64	32	36	49	56	67	78	90	95	93	92	91	79	66	81
6500	85	70	37	41	54	61	70	80	91	93	89	90	89	78	69	82

LED №6

CCT,K	DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8	DE9	DE10	DE11	DE12	DE13	DE14	DE15	Qa
3200	80	85	75	78	84	83	84	85	89	86	87	88	88	80	83	80
3500	82	82	69	73	82	84	86	87	92	89	89	89	89	81	82	81
4000	85	84	66	71	80	83	86	88	92	89	89	89	89	82	83	83
4900	87	93	71	76	85	86	87	87	88	86	85	86	87	82	84	84