LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
LUT School of Energy Systems
LUT Mechanical Engineering

Seppo Uimonen

STABILITY AND DURABILITY OF ALUMINIUM FRAME STRUCTURES

Examiners: Professor Timo Björk
Ilkka Pöllänen

TIIVISTELMÄ

Lappeenrannan teknillinen yliopisto
LUT School of Energy Systems
LUT Kone

Seppo Uimonen

Stability and durability of aluminium frame structures

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Ilkka Pöllänen

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Äärellisten elementtien menetelmällä on suuri merkitys nykyajan lujuuslaskennassa. Äärellisten elementtien menetelmällä ja kehittyneen tietokoneavusteisen laskennan avulla on mahdollista ratkaista jännityksiä ja tukireaktioita monimutkaisissa kolmiulotteisissa rakenteissa. Tutkimuksen taustalla oli halu parantaa lujuuslaskennan tehokkuutta ja tarkkuutta, korvaamalla yksinkertaistetut analyyttiset kaavat äärellisten elementtien menetelmällä alumiinikehärakenteissa.

Tutkimuksen tavoitteena oli luoda laskentamalli joka antaisi tiedon stabiliteetista, kuormituskestävyydestä ja tukireaktioista alumiinikehärakenteissa, kun lähtötietoina annetaan rakenteen mittasuhteet ja siihen kohdistuvat voimat. Tutkimus antoi tietoa tukien optimaalisesta määrästä, kulmista, sijainnista ja jännitysjakaumasta rakenteessa. Tutkimusta voidaan käyttää taustana varmistaessa rakenteen stabiliteetti ja kestävyys sekä hienosäätäessä omamassaa sekä ulkonäköseikkoja.

ABSTRACT

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Seppo Uimonen

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Finite element analysis performs an important role in material strength analysis of today. By finite element analysis and aid of developed computational methods it is possible to solve stresses and reactions for complex structures in three dimensions. Background of the study was to improve efficiency and accuracy of material strength analysis by replacing simplified analytical formulae and employ the use of finite element analysis in aluminum frame structures.

The objective of the research was to create a computational model that would be give stability, design resistance and support reactions of defined aluminum frame structures when inputting environmental loads and dimensions of the structure. Research gave out information about the optimal number, angle and location of supports and stress distribution of structure. The research can be used as basis for ensuring stability, durability, and refining self-weight and visual aspects.

TABLE OF CONTENTS

LIST OF SYMBOLS AND ABBREVIATIONS

a	Lower limit of the interval
A_g	Either the cross-section or reduced cross-section that regards HAZ softening in longitudinal welds
A_e	Cross-sectional area with no welds
A_{eff}	The effective area of a cross-section
A_{net}	Net section area with deduction for holes
A_{ref}	Reference area of the structure
A_v	Shear area
b	Step size of the interval
b_{haz}	Total height of the HAZ material between flanges
c	Upper limit of the interval
c_o	Orography factor
C_1	Factor depending on restrain conditions
C_2	Factor depending on restrain conditions
C_3	Factor depending on restrain conditions
C_e	Exposure coefficient
c_{dir}	Wind direction coefficient
c_f	Force coefficient
c_{pe}	External pressure coefficient
c_{pi}	Internal pressure coefficient
$c_s c_d$	Structural factor
c_{season}	Seasonal factor
C_t	Thermal coefficient
d	Diameter of holes along the shear plane
E	Modulus of elasticity
f_{oc}	Yield strength of cast material
f_0	Yield strength of the material
$f_{0,V}$	Reduced strength in combined bending and shear forces
f_u	Ultimate strength of the material
f_{uc}	Ultimate strength of cast material
F_w	Wind force
G	Glide modulus

h_w	Height of the web between flanges
i	Radius of gyration
i_s	Radius of gyration
I_p	Polar moment of inertia
I_t	Torsional second moment of inertia
I_v	Turbulence intensity
I_x	Second moment of inertia around x-axis
I_y	Second moment of inertia around stronger axis
I_w	Warping second moment of inertia
I_z	Second moment of inertia around weaker axis
k	Buckling length factor
k_l	Turbulence factor
k_r	Terrain factor
k_x	Buckling length factor around x-axis
k_y	Buckling length factor around y-axis
k_z	Restrain factor
k_w	Restrain factor
L	Length
L_{cr}	Critical length
Mcr	Elastic critical moment
M_{Ed}	The design bending moment
M_{equ}	Equivalent system of horizontal forces
M_{Rd}	The design bending moment resistance
$M_{u,Rd}$	Bending moment resistance in net cross-section
$M_{o,Rd}$	Bending moment resistance in all cross-sections
$M_{y,Ed}$	Design bending moment around y-axis
$M_{z,Ed}$	Design bending moment around z-axis
$N_{b,Rd}$	Buckling resistance of a compressed part
N_{cr}	Critical load
$N_{Cr,x}$	Critical buckling load around x-axis
$N_{Cr,y}$	Critical buckling load around y-axis
$N_{c,Rd}$	The design resistance in compression
$N_{Cr,T}$	Critical normal force
N_{Ed}	The design value of normal force
$N_{t,Rd}$	Tensile design resistance

$N_{o,Rd}$	General yielding along the member
$N_{Rd,I}$	Buckling resistance according to I. order analysis
$N_{Rd,II}$	Buckling resistance according to II. order analysis
$N_{u,Rd}$	Local failure at section with holes
n_v	Shape coefficient
q_p	Peak velocity pressure
s	Snow load
S_k	Characteristic value of snow load on the ground
t_w	Thickness of the web
T_{Ed}	Torsion design value
T_{Rd}	Design torsion moment resistance
$T_{t,Ed}$	The internal St. Venants torsion moment
$T_{w,Rd}$	The internal warping torsion moment
V_{Ed}	The design shear force
V_{Rd}	The design shear resistance
V_T	Combined shear force and torsional moment resistance in hollow sections
$V_{T,Rd}$	Combined shear force and torsional moment resistance
v_b	Basic wind velocity
$v_{b,0}$	Basic wind velocity, initial value
v_m	Wind speed profile
$W_{el,y}$	Elastic bending resistance of the cross-section
w_e	Wind pressure acting on external surfaces
w_i	Wind pressure acting on internal surfaces
W_{net}	Elastic bending resistance of a net cross-section
$W_{T,pl}$	Torsion modulus according to plastic theory
x	Variable
X	Meshed variable
Y	Meshed variable
y_s	Shear center coordinate
z	Variable, in chapter Post-processing of results
z_0	Roughness length
$z_{0,II}$	Reference terrain class II
z_e	Reference height
z_g	Coordinate of the point load application
z_j	Factor related to load application

z_{max}	Maximum height
z_{min}	Minimum height
z_s	Coordinate of the shear center related to centroid
α_{h}	Reduction factor for the height of columns
α_{LT}	Imperfection factor
α_{m}	Reduction factor for the number of columns
α_y	Combination coefficient
$\alpha_{yw}(k_y, k_w)$	Boundary condition factor
α_z	Combination coefficient
$\alpha_{zw}(k_y, k_w)$	Boundary condition factor
γ_0	Combination coefficient
$\gamma_{\mathrm{Mo,c}}$	Partial factor related to yield limit of cast material
γ_{M1}	Partial safety factor related to yield limit
γ_{M2}	Partial safety factor related to ultimate limit
$\gamma_{\mathrm{Mu,c}}$	Partial factor related to ultimate limit of cast material
ε	Slenderness limit, in chapter General rules
ζ_{g}	Relative non-dimensional coordinate of the point load position
ζ_{j}	Relative non-dimensional mono-symmetry parameter
κ	Impact of welds
κ_{wt}	Non-dimensional torsion parameter
λ	Relative slenderness
$\lambda_{\mathrm{0,LT}}$	Limit of the horizontal plateau
λ_{LT}	Relative slenderness
λ_{T}	Slenderness
η_0	Combination coefficient
μ_1	Coefficient for monopitch roofs
μ_{cr}	Relative non-dimensional critical moment
μ_i	Snow load shape coefficient
ν	Poisson’s ratio
ξ_0	Combination coefficient
ρ	Density of air
$\rho_{\mathrm{0,haz}}$	Heat affected zone yield limit
$\rho_{\mathrm{u,haz}}$	Heat affected zone ultimate limit
$\sigma_{\mathrm{eq,Ed}}$	Equivalent design load
σ_{v}	Standard deviation of turbulence

$\sigma_{x,Ed}$	Longitudinal local stress design value
$\sigma_{y,Ed}$	Transverse local stress design value
σ_{Rd}	Design resistance value
$\tau_{t,Ed}$	Design value of shear stress in torsion
τ_{w}	Shear stress in web
$\tau_{xy,Ed}$	Local shear stress design value
Φ_{0}	Reduction factor
ϕ_{LT}	Lateral torsional buckling coefficient
χ	Reduction factor
χ_{LT}	Reduction factor to lateral torsional buckling resistance
ψ	Combination coefficient
ω_{0}	Combination coefficient
CBAC	Combined bending, axial and shear force check
Deflection	Maximum deflection of the horizontal beam normal to length in xy-plane
EQU	Loss of equilibrium of the structure or any part of it considered as rigid body
FAT	Fatigue failure of the structure or structural members
FB	Flexural buckling
GEO	Failure or excessive deformation of the ground
HAZ	Heat affected zone
Hstress	Maximum combined stress within a horizontal member
HRx	Reaction force in horizontal beam parallel to global x-axis
HRy	Reaction force in horizontal beam parallel to global y-axis
HYD	Hydraulic heave, internal erosion and piping in the ground caused by hydraulic gradients
Length	Length of the horizontal beam or frame parallel to global x-axis
Line load	Line load to the side of a horizontal member
LTB	Lateral torsional buckling
meshgrid	Command for meshing a set of data
plot	Command for plotting one variable
plot3	Command for plotting two variables
Stability	Stability of the frame
STR	Internal failure or excessive deformation of the structure or structural members
surf	Creates a surface
TFB	Torsional flexural buckling

UPL	Loss of equilibrium of the structure or ground due to uplift by water pressure
VRy	Reaction force in vertical column parallel to global y-axis
Vstress	Maximum combined stress within a vertical member
Width	Length of the horizontal beam or frame parallel to global y-axis

The following symbols have different meanings according to their context. The different meanings of these symbols have been presented in following table 1.1.

Table 1.1. Different meanings of following symbols according to their context.

A	The site altitude above sea level in meters, in chapter Snow loads
A	Is the area of cross-section, in chapter General rules
n	Value for plastic analysis, in chapter Modeling of materials in FEA-software
n	The number of webs, in chapter General rules
t	Thickness of the plate, in chapter General rules
t	Variable, in chapter Post-processing of results
y	Variable, in chapter Post-processing of results
y	Coordinate, in chapter Stability
Z	Zone number, in chapter Snow loads
Z	Meshed variable, in chapter Post-processing of results
z	Height of the building, in chapter Wind loads
z	Coordinate, in chapter Stability
α	Angle of roof, in chapter Snow loads
α	Shape coefficient, in chapter General rules
α	Shape factor, in chapter Stability
α	Thermal expansion coefficient, in chapter Modeling of materials in FEA-software
Φ	Global initial sway factor, in chapter Case study
Φ	Reduction factor, in chapter Flexural buckling

1 INTRODUCTION

The following chapter presents background, objective and the scope of the study.

1.1 Background

Finite element analysis performs an important role in material strength analysis of today. By finite element analysis and aid of developed computational methods it is possible to solve stresses and reactions for complex structures in three dimensions. Background of the study was to improve efficiency and accuracy of material strength analysis by replacing simplified analytical formulae and employ the use of finite element analysis in aluminum frame structures.

1.2 Objective

The objective of the research was to create a computational model that would be give stability, design resistance and support reactions of defined aluminum frame structures when inputting environmental loads and dimensions of the structure. Research should give out information about the optimal number, angle and location of supports and stress distribution of structure. Aim is that this research could be used as basis of ensuring stability, durability, and refining self-weight and visual aspects.

1.3 Scope

The stability and durability of the structure will be verified according to standards EN 1990 (2002), SFS-EN 1991-1-1 (2002), SFS-EN 1991-1-3 (2015), SFS-EN 1991-1-4 (2011) and SFS-EN 1999-1-1 (2009). Stress at all members should not exceed yield limits of corresponding materials. Support reactions due to loading of a structure should not exceed the capacity of anchor bolts. In general stress distribution should be evenly divided between members.

2 METHODS

Methods used in the study are divided between structural design according to Eurocodes EN 1990 (2002), SFS-EN 1991-1-1 (2002), SFS-EN 1991-1-3 (2015), SFS-EN 1991-1-4 (2011) and SFS-EN 1999-1-1 (2009) and structural design by finite element analysis.

2.1 Structural design by Eurocodes EN 1990 (2002), SFS-EN 1991-1-1 (2002), SFS-EN 1991-1-3 (2015), SFS-EN 1991-1-4 (2011) and SFS-EN 1999-1-1 (2009).

The following chapter deals with design rules associated with tension, compression, bending moment, shear, torsion, wind loads, snow loads and design in ultimate and serviceability limit states.

2.1.1 General rules

The design load in each cross-section cannot exceed the corresponding design resistance. When several loads act simultaneously, they cannot exceed the resistance value to that combination.

Tension

According to SFS-EN 1999-1-1 (2009, p. 72) the design value of normal force N_{Ed} in a member cannot exceed the tensile design resistance $N_{t,Rd}$.

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1{,}0 \tag{2.1}$$

Tensile design resistance $N_{t,Rd}$ is the smallest value of following cases:

a) General yielding $N_{o,Rd}$ along the member

$$N_{o,Rd} = \frac{A_g f_0}{\gamma_{M1}} \tag{2.2}$$

A_g is either the cross-section or reduced cross-section that regards HAZ, heat affect zone softening in longitudinal welds. The latter case A_g is calculated using the area of cross-section A multiplied by HAZ yield limit $\rho_{0,haz}$. f_0 is the yield strength of the material and γ_{M1} is the partial safety factor related to yield limit.

b) Local failure at section with holes $N_{u,Rd}$

$$N_{u,Rd} = \frac{0.9\,A_{net}\,f_u}{\gamma_{M1}} \tag{2.3}$$

A_{net} is the net section area with deduction for holes and if necessary deduction for HAZ softening in the net section through the hole. The latter is based on reduced effective thickness $\rho_{u,haz}\,t$, where $\rho_{u,haz}$ HAZ ultimate limit and t is thickness of the plate. f_u is the ultimate strength limit of the material.

c) Local failure at heat affected zone

$$N_{u,Rd} = \frac{A_{eff} f_u}{\gamma_{M2}} \tag{2.4}$$

A_{eff} is the effective area of a cross-section that is based on reduced thickness $\rho_{u,haz}\,t$. γ_{M2} is the partial safety factor related to ultimate strength of the material.

Compression

According to SFS-EN 1999-1-1 (2009, p. 72) the design of value of normal force N_{Ed} should satisfy

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1{,}0 \tag{2.5}$$

The design resistance value $N_{c,Rd}$ in uniform compression should be selected smallest of following two equations.

If cross-section has holes

$$N_{u,Rd} = \frac{A_{net}\,f_u}{\gamma_{M2}} \tag{2.6}$$

With other cross-sections

$$N_{c,Rd} = \frac{A_{eff}\,f_0}{\gamma_{M1}} \tag{2.7}$$

Bending moment

According to SFS-EN 1999-1-1 (2009, p. 73) bending moment M_{Ed} at all cross-sections should satisfy

$$\frac{M_{Ed}}{M_{Rd}} \leq 1.0 \quad (2.8)$$

Where the design bending moment resistance M_{Rd} is the least of following two values bending moment resistance in net cross-section $M_{u,Rd}$ and bending moment resistance in all cross-sections $M_{o,Rd}$.

$$M_{u,Rd} = \frac{W_{net}\, f_u}{\gamma_{M2}} \quad (2.9)$$

$M_{u,Rd}$ in net cross-section and

$$M_{o,Rd} = \frac{\alpha\, W_{el,y}\, f_0}{\gamma_{M1}} \quad (2.10)$$

$M_{o,Rd}$ in all cross-sections. Where α is the shape coefficient. $W_{el,y}$ is the elastic bending resistance of the cross-section. W_{net} is the elastic bending resistance of a net cross-section, that regards holes and heat affected zone effects. The latters is based on reduced thickness $\rho_{u,haz}\, t$.

Shear

The design value of shear force V_{Ed} at every cross-section should satisfy (SFS-EN 1999-1-1 2009, p. 76)

$$\frac{V_{Ed}}{V_{Rd}} \leq 1.00 \quad (2.11)$$

Where V_{Rd} is the design shear resistance value of a cross-section.

Parts that are not slender, $\frac{h_w}{t_w} \leq 39\varepsilon$, (SFS-EN 1999-1-1 2009, p. 76)

$$V_{Rd} = A_v \frac{f_0}{\sqrt{3}\, \gamma_{M1}} \quad (2.12)$$

Where A_v is the shear area and ε is the slenderness limit. h_w is the height of the web and t_w is the thickness of the web. For webs (SFS-EN 1999-1-1 2009, p. 76)

$$A_v = \sum_{i=1}^{n}\left[(h_w - \sum d)(t_w)_i - \left(1 - \rho_{0,\mathrm{haz}}\right)b_{haz}\left(t_w\right)_i\right] \qquad (2.13)$$

In equation 2.13 the symbols are:

- h_w is the height of the web between flanges
- b_{haz} is the total height of the HAZ material between flanges. If there are no welds in cross-section, $\rho_{0,\mathrm{haz}} = 1$. If heat affected zone is height of the web, then $b_{haz} = h_w - \sum d$.
- t_w is the thickness of the web.
- d is the diameter of holes along the shear plane.
- n is the number of webs.

For solid bars and round tubes (SFS-EN 1999-1-1 2009, p. 76)

$$A_v = n_v\, A_e \qquad (2.14)$$

In the equation 2.14 the symbols are:
$n_v = 0.8$ for solid bars
$n_v = 0.6$ for round tubes
A_e is cross-sectional area with no welds and effective area can be calculated using reduced thickness $\rho_{\mathrm{u,haz}}\, t$.

Torsion without warping
Torsion with without warping and distortional deformations should satisfy (SFS-EN 1999-1-1 2009, p. 77):

$$\frac{T_{Ed}}{T_{Rd}} \leq 1.0 \qquad (2.15)$$

$T_{Ed}$is the torsion design value. T_{Rd} is the design St. Venants torsion moment resistance of the cross-section, defined by (SFS-EN 1999-1-1 2009, p. 77):

$$T_{Rd} = \frac{W_{T,pl}\, f_0}{\sqrt{3}\,\gamma_{\mathrm{M1}}} \qquad (2.16)$$

Where $W_{T,pl}$ is the torsion modulus according to plastic theory.

Torsion with warping

Members that are subjected to warping but distortional deformations can be disregarded should satisfy (SFS-EN 1999-1-1 2009, p. 78)

$$T_{Ed} = T_{t,Ed} + T_{w,Rd} \tag{2.17}$$

$T_{t,Ed}$ is the internal St. Venants torsion moment

$T_{w,Rd}$ is the internal warping torsion moment.

Combined shear force and torsional moment

In the influence of simultaneous shear force and torsional moment, shear resistance is reduced from V_{Ed} to $V_{T,Rd}$ and design shear force should satisfy (SFS-EN 1999-1-1 2009, p. 78)

$$\frac{V_{Ed}}{V_{T,Rd}} \leq 1.0 \tag{2.18}$$

Where combined shear force and torsional moment resistance $V_{T,Rd}$ depends on the cross-sections. For I and H cross-sections can be used equation (SFS-EN 1999-1-1 2009, p. 78).

$$V_{T,Rd} = \sqrt{1 - \frac{\tau_{t,Ed}\sqrt{3}}{1.25\frac{f_0}{\gamma_{M1}}}}\, V_{Rd} \tag{2.19}$$

For a channel section according to SFS-EN 1999-1-1 (2009, p. 78)

$$V_{T,Rd} = [\sqrt{1 - \frac{\tau_{t,Ed}\sqrt{3}}{1.25\frac{f_0}{\gamma_{M1}}}} - \frac{\tau_w\sqrt{3}}{\frac{f_0}{\gamma_{M1}}}]\, V_{Rd} \tag{2.20}$$

And combined shear force and torsional moment resistance in hollow sections according to SFS-EN 1999-1-1 (2009, p. 78)

$$V_T = \left[1 - \frac{\tau_{t,Ed}\sqrt{3}}{\frac{f_0}{\gamma_{M1}}}\right] V_{Rd} \tag{2.21}$$

Combined bending and shear force

According to SFS-EN 1991-1-1 (2009, p. 78), when a section is loaded by both bending and shear force, shear force is taken in account. If V_{Ed} is smaller than half of the shear resistance V_{Rd} and shear buckling does not reduce section resistance, the influence to bending moment resistance can be ignored.

In other cases bending moment resistance is reduced by using cross-sectional resistance value that has been calculated using reduced strength in combined bending and shear forces (SFS-EN 1999-1-1 2009, p. 78)

$$f_{0,V} = f_0 \left(1 - \left(\frac{2V_{Ed}}{V_{Rd}} - 1\right)^2\right) \tag{2.22}$$

Where V_{Rd} is calculated according to instructions given in chapter Shear.

Combined bending and axial force

Double symmetrical open cross-sections should satisfy two following conditions (SFS-EN 1999-1-1 2009, p. 79).

$$\left(\frac{N_{Ed}}{\omega_0\, N_{Rd}}\right)^{\xi_0} + \frac{M_{y,Ed}}{\omega_0\, M_{y,Rd}} \le 1.00 \tag{2.23}$$

$$\left(\frac{N_{Ed}}{\omega_0\, N_{Rd}}\right)^{\eta_0} + \left(\frac{M_{y,Ed}}{\omega_0\, M_{y,Rd}}\right)^{\gamma_0} + \left(\frac{M_{z,Ed}}{\omega_0\, M_{z,Rd}}\right)^{\xi_0} \le 1.00 \tag{2.24}$$

Where in equations 2.23 and 2.23 the symbols are:

η_0 = 1.0 or between 1 and 2 when calculated from $\alpha_z^2\alpha_y^2$

γ_0 = 1.0 or between 1 and 1.56 when calculated from α_z^2

ξ_0 = 1.0 or between 1 and 1.56 when calculated from α_y^2

N_{Ed} is the design value of axial compression or tensile force.

$M_{y,Ed}$ and $M_{z,Ed}$ are bending moments around y-y and z-z axis.

ω_0 is combination coefficient.

$N_{Rd} = A_{eff}\, f_0 \,/\, \gamma_{M1}$

$M_{y,Rd} = \alpha_y W_{y,el} f_0 / \gamma_{M1}$

$M_{z,Rd} = \alpha_z W_{z,el} f_0 / \gamma_{M1}$

Double symmetrical hollow and solid cross-sections should satisfy (SFS-EN 1999-1-1 2009, p. 80)

$$\left(\frac{N_{Ed}}{\omega_0 N_{Rd}}\right)^{\psi} + \left[\left(\frac{M_{y,Ed}}{\omega_0 M_{y,Rd}}\right)^{1.7} + \left(\frac{M_{z,Ed}}{\omega_0 M_{z,Rd}}\right)^{1.7}\right]^{0.6} \leq 1.00 \qquad (2.25)$$

Where in the equation 2.25 symbols are combination coefficient ψ, for hollow sections $\psi = 1.3$ and for solid sections $\psi = 2$. Alternatively ψ can be calculated $\alpha_z^2 \alpha_y^2$, but then $1 \leq \psi \leq 1.3$ for hollow and $1 \leq \psi \leq 2$ for solid cross-sections.

2.1.2 Design loads of structures

The following chapter deals with design loads of structures such as wind loads, snow loads.

Wind loads

According to SFS-EN 1991-1-4 (2011, p. 36) terrain classes are divided into five categories as in table 2.1.

Table 2.1. Terrain categories according to SFS-EN 1991-1-4 (2011, p. 36)

Class	Terrain description	z_0 [m]	z_{min} [m]
0	Open sea or exposed coastal area	0.003	1
I	An area close to lakes or an area where there are no wind barriers or notable vegetation	0.01	1
II	An area where there are low vegetation such as hey and separate obstacles such as trees or buildings whose distance to each other is at least 20 times the height of the obstacles.	0.05	2
III	Areas that have regular vegetation, buildings or separate wind barriers whose distance to each other is at most 20 times the height of the obstacles. Example villages, suburban areas, permanent forest.	0.3	5
IV	Areas whose surface is covered at least 15 % by buildings and their average height exceeds 15 meters.	1.0	10

Where z_0 is roughness length and z_{min} is a minimum height.

When changing from exposed to more protected terrain class the transition zones between zones are

- 2 km when shifting from class 0 to class I
- 1 km when shifting from I to II or from/to any other class

Peak velocity pressure $q_p(z)$ is defined by SFS-EN 1991-1-4 (2011, p. 40)

$$q_p(z) = [1 + 7\, I_v(z)]\,\frac{1}{2}\,\rho\, \mathrm{v}_\mathrm{m}^2(z) \tag{2.26}$$

where $I_v(z)$ is the turbulence intensity, $v_m(z)$ is the wind speed profile, ρ is the density of air, which depends on the altitude, temperature and air pressure during storms. The recommended value is $\rho = 1.25\ \mathrm{kg/m^3}$.

The turbulence intensity at height z defined as the standard deviation of the turbulence divided by the mean wind velocity and can be defined from formula SFS-EN 1991-1-4 (2011, p. 38)

$$I_v(z) = \frac{\sigma_v}{v_m(z)} = \frac{k_l}{c_o(z) * \ln\left(\frac{z}{z_0}\right)} \qquad z_{min} \leq z \leq z_{max} \tag{2.27}$$

k_l is the turbulence factor, $c_o(z)$ is the orography factor. The standard deviation of turbulence σ_v may defined as follows

$$\sigma_v = k_r * v_b * k_l \tag{2.28}$$

where k_r is the terrain factor, and v_b is the basic wind velocity. Turbulence factor k_l is presented in national annexes, however it has recommended value of 1.0.
Terrain factor k_r can be calculated from SFS-EN 1991-1-4 (2011, p. 34)

$$k_r = 0.19 * \left(\frac{z_0}{z_{0,II}}\right)^{0.07} \tag{2.29}$$

The basic wind velocity is defined as function of wind direction and season of the year at 10 meters above ground and can be evaluated from SFS-EN 1991-1-4 (2011, p. 32)

$$v_b = c_{dir} * c_{season} * v_{b,0} \tag{2.30}$$

Recommended values for wind direction coefficient c_{dir} and seasonal factor c_{season} are 1.0. $v_m\,(z)$ is the wind speed profile at height above terrain depending on terrain roughness, orography and basic wind velocity defined by SFS-EN 1991-1-4 (2011, p. 34).

$$v_m\,(z) = c_r(z)\; c_0(z)\; v_b \tag{2.31}$$

Where $c_r(z)$ is roughness factor and v_b is the basic wind velocity.

If orography such as hills or cliffs increases wind velocity more than 5 %, the effects accounted by using orography factor, $c_0(z)$.

Roughness factor can be calculated from SFS-EN 1991-1-4 (2011, p. 34)

$$c_r(z) = k_r * \ln\left(\frac{z}{z_0}\right) \qquad z_{min} \leq z \leq z_{max} \tag{2.32}$$

According to SFS-EN 1991-1-4 (2011, p. 46) Wind actions should be determined using sum of both external and internal wind pressures. According to SFS-EN 1991-1-4 (2011, p. 42) calculation procedure for wind actions is as follows in table 2.2.

Table 2.2. Wind action table according to SFS-EN 1991-1-4 (2011)

Peak velocity pressure q_p Basic wind velocity v_b Reference height z_e Terrain category Characteristic peak velocity pressure q_p Turbulence intensity I_v Mean wind velocity v_m Orography coefficient $c_0(z)$ Roughness coefficient $c_r(z)$
Wind pressures for claddings, fixings and structural parts External pressure coefficient c_{pe} Internal pressure coefficient c_{pi}
Wind forces on structures, overall wind effects Structural factor $c_s c_d$ Wind force F_w calculated from force or pressure coefficients

The wind pressure acting on the external surfaces w_e can be calculated from formula SFS-EN 1991-1-4 (2011, p. 42)

$$w_e = q_p(z)\, c_{pe} \tag{2.33}$$

The wind pressure to the internal surfaces w_i of a structure can be calculated from formula (SFS-EN 1991-1-4 2011, p. 44)

$$w_i = q_p(z)\, c_{pi} \tag{2.34}$$

And finally the wind force acting on the structure can be calculated from SFS-EN 1991-1-4 (2011, p. 44)

$$F_w = c_s c_d \, c_f \, q_p(z) \, A_{ref} \tag{2.35}$$

Where $c_s c_d$ is a structural factor, c_f is the force coefficient for the structure or structural element and A_{ref} is the reference area of the structure or structural element.

Case study

Let’s take a look how the height and the terrain class affect on peak wind velocity pressure.

Let’s define following initial values:

Turbulence factor $k_l = 1.0$

Orography coefficient in open areas $c_o = 1.0$

Density of air $\rho = 1.25 \text{ kg/m}^3$

Roughness length $z_0 = [0.003 \; 0.01 \; 0.05 \; 0.3 \; 1.0]^T$

Minimum height $z_{min} = [1 \; 1 \; 2 \; 5 \; 10]^T$

Maximum height $z_{max} = 200$ meters.

Height of the building $z = [z_{min}, \; z_{max}]$, z € $\mathbb{N}$

Basic wind velocity $v_b = 21 \text{ m}/s$

Results are shown in figure 2.1. Highest curve in belongs to terrain class 0 and lowest curve belongs to terrain class IV. The logarithmic behavior of phenomenon is clearly seen from graph: after rapid growth between 10 and 50 meters, the growth of peak wind velocity starts to slow down.

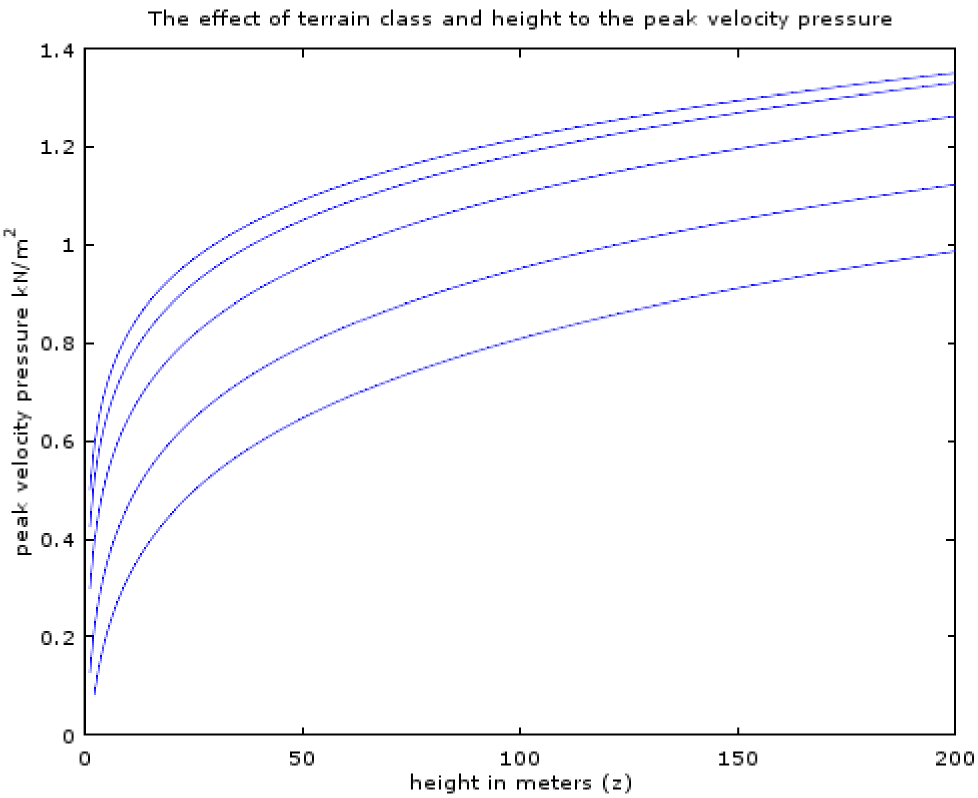


Figure 2.1. The effect of terrain class and height of the building to the peak wind velocity pressure.

Snow loads

Snow loads of roofs are defined in normal situations SFS-EN 1991-1-3 (2015, p. 28)

$$s = \mu_i \, C_e \, C_t \, S_k \tag{2.36}$$

μ_i is the snow load shape coefficient

S_k is the characteristic value of snow load on the ground

C_e is the exposure coefficient

C_t is the thermal coefficient

Snow load shape coefficient μ_i is defined using SFS-EN 1991-1-3 (2015, p. 32). For monopitch roofs $\mu_1(\alpha)$:

$$0° \leq \alpha \leq 30°: \mu_1 = \mu_1(0°) \geq 0.8 \tag{2.37}$$

$$30° \leq \alpha \leq 60°: \mu_1 = \frac{\mu_1(0°)(60°-\alpha)}{30°} \tag{2.38}$$

$$\alpha \geq 60°: \mu_1 = 0 \tag{2.39}$$

Recommended values of exposure coefficient C_e according to SFS-EN 1991-1-3 (2015, p. 30). Windswept: $C_e = 0.8$. Normal: $C_e = 1.0$. Sheltered: $C_e = 1.2$.

Thermal coefficient C_t takes in count situations where high thermal conductivity of a roof causes snow to melt and therefore the amount of snow load can be reduced according to National Annex. High thermal conductivity is defined more than $1\ W/m^2K$, and in all other cases:

$$C_t = 1.0 \tag{2.40}$$

The characteristic value of snow load on the ground S_k is defined according to SFS-EN 1991-1-3 (2015, p. 64) by climate region. For example for Finland and Sweden:

$$S_k = (0.790\ Z + 0.375) + \frac{A}{336} \tag{2.41}$$

A is the site altitude above sea level in meters.
Z is the zone number referring to snow load within the climate region. And can be defined for example from 2.0 to 2.7 kN/m for most of the Finland.

Case study
For example for a site located at 100 meters above sea level in climate zone of 2.0 kN/m snow load, at normal thermal conductivity, the effect of roof angle between 30 and 60 degrees and the exposure coefficient between windswept and sheltered on snow load would be as in figure 2.2.

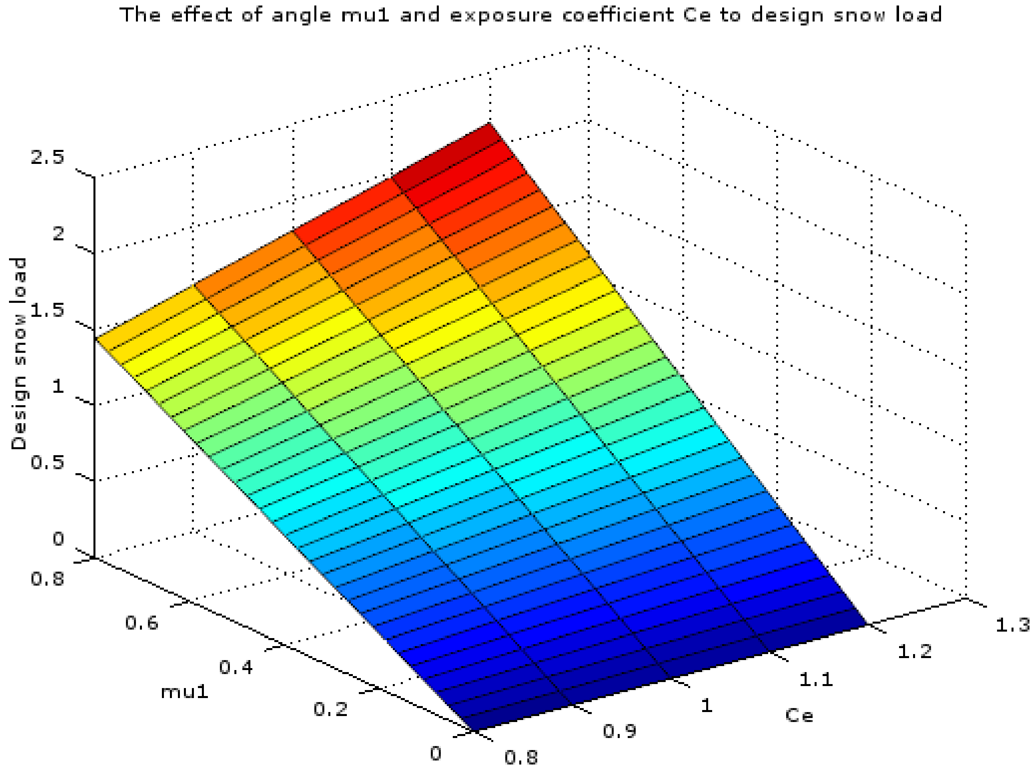


Figure 2.2. The effect of roof angle and exposure coefficient to design snow load kN/m^2.

As seen from figure 2.2, the snow load may be ignored when the angle of the roof exceeds $60 \text{ degrees } (\text{mu1} = 0)$. This is due the low friction between snow and roof material, and in this case gravity takes care of winter maintenance. The difference between a windswept and sheltered area is $40\ \%\ (1.2 - 0.8)$ due to linear nature of the snow load formula *s*.

Imposed loads

Imposed loads of a building should be classified as variable free actions as specified in EN 1990 (2002, p. 33). Imposed loads are classified as quasi-static loads. If there is a possibility of resonance, significant acceleration or other dynamic response, then load models should take in account dynamic effects. According to SFS-EN 1991-1-1 (2002, p. 20) when imposed loads act simultaneously other variable actions such as wind, snow, cranes or loads generated by machinery, imposed loads should be considered as a single action.

Self-weight

The self-weight of a building is defined as permanent load according to EN 1990 (2002, p. 33). The self-weight of structural and non-structural members should be considered single action in load combinations. According to SFS-EN 1991-1-1 (2002, p. 20) when designing areas where there is intended to add or remove structural or non-structural parts after completion, the critical load case should be considered.

2.1.3 Stability

Classification of cross-sections

According to SFS-EN 1999-1-1 (2009, p. 59), cross-sections can be classified to four different classes. These are:

1) Cross-section can develop a plastic hinge, and therefore can be loaded and calculated by plastic theory under static loading. The mechanism is called plastic – plastic.
2) Cross-section can develop a plastic hinge, but local buckling limits its rotation capacity. The mechanism is therefore plastic – elastic.
3) Cross-section cannot develop a plastic hinge, local buckling develops as cross-section achieves yield stress in its outernmost point.
4) Local bucking develops before cross-section achieves yield stress in its outernmost point. These cross-sections are slender.

Buckling modes

Lateral torsional buckling

Lateral torsional buckling occurs when a member is subjected to a critical load from its stronger inertia axis and part under critical stress rotates and buckles by weaker inertia axis.

By SFS-EN 1999-1-1 (2009, p. 81) lateral torsional buckling is evaluated by following steps: Calculation of a elastic critical moment Mcr for lateral torsion (SFS-EN 1999-1-1 2009, p. 196)

$$Mcr = \frac{\mu_{cr}\pi\sqrt{EI_zGI_t}}{L} \tag{2.42}$$

Where E is the modulus of elasticity, I_z is second moment of inertia around weaker axis, G is the glide modulus, I_t torsional second moment of inertia, L is length and relative non-dimensional critical moment μ_{cr} is by SFS-EN 1999-1-1 (2009, p. 196)

$$\mu_{cr} = \frac{C_1}{k_z}[\sqrt{1+\kappa_{wt}^2+\left(C_2\zeta_g - C_3\zeta_j\right)^2} - \left(C_2\zeta_g - C_3\zeta_j\right)] \tag{2.43}$$

Where C_1, C_2 and C_3 are factors depending on restrain conditions, ζ_g is relative non-dimensional coordinate of the point load position, ζ_j is relative non-dimensional mono-symmetry parameter, κ_{wt} is non-dimensional torsion parameter and k_z is restrain factor.

Non-dimensional torsion parameter κ_{wt} is

$$\kappa_{wt} = \frac{\pi}{k_w L}\sqrt{\frac{EI_w}{GI_t}} \quad (2.44)$$

Where k_w is restrain factor and I_w is warping second moment of inertia. Elastic critical moment is scaled by influence of load position. Load position is defined by influence coordinates. Relative non-dimensional coordinate of the point of load position related to shear center.

$$\zeta_g = \frac{\pi z_g}{k_z L}\sqrt{\frac{EI_z}{GI_t}} \quad (2.45)$$

Where z_g is coordinate of the point load application. Relative non-dimensional cross-section mono-symmetry parameter $\zeta_j$is defined by

$$\zeta_j = \frac{\pi z_j}{k_z L}\sqrt{\frac{EI_z}{GI_t}} \quad (2.46)$$

Where z_j is factor related to load application. Coordinate of the point of load application related to shear center.

$$z_g = z_a - z_s \quad (2.47)$$

$$z_j = z_s - \frac{0.5}{I_y}\int_A (y^2 + z^2)z\, dA \quad (2.48)$$

z_a is the coordinate of the point of load application related to centroid.
z_s is the coordinate of the shear center related to centroid.
z_g is the coordinate of the point of load application related to shear center.

Where C1, C2 and C3 are factors depending mainly on loading and end restrain conditions (SFS-EN 1999-1-1 2009, p. 197).

k_z and k_w are restrain factors so that:

$k_z = 1$ restrained against lateral movement, free to rotate on both ends

$k_w = 1$ restrained against rotation about the longitudinal axis, free to warp on both ends

$k_z = 0.7$ and $k_w = 0.7$ is situation where first end is fixed and second end is pinned.

$k_z = 0.5$ and $k_w = 0.5$ are used when both ends are fixed.

The design buckling resistance moment Mb, Rd against lateral torsional buckling is defined by SFS-EN 1999-1-1 (2009, p. 87)

$$Mb, Rd = \frac{\chi_{LT} \alpha \, \mathrm{W}_{\mathrm{el,y}} f_o}{\gamma_{\mathrm{M1}}} \tag{2.49}$$

$\mathrm{W}_{\mathrm{el,y}}$ is the elastic bending resistance of a cross-section and χ_{LT} is reduction factor to lateral torsional buckling resistance. α is shape factor and taken from SFS-EN 1999-1-1 (2009, p. 74) regarding

$$\alpha \leq \frac{\mathrm{W}_{\mathrm{pl,y}}}{W_{el,y}} \tag{2.50}$$

Reduction factor to lateral torsional buckling resistance is defined by SFS-EN 1999-1-1 (2009, p. 87)

$$\chi_{LT} = \frac{1}{\phi_{\mathrm{LT}} + \sqrt{\phi_{\mathrm{LT}}^2 - \lambda_{\mathrm{LT}}^2}} \leq 1 \tag{2.51}$$

where λ_{LT} is the relative slenderness and ϕ_{LT} is defined by SFS-EN 1999-1-1 (2009, p. 87)

$$\phi_{\mathrm{LT}} = 0.5\,[1 + \alpha_{\mathrm{LT}}(\lambda_{\mathrm{LT}} - \lambda_{0,\mathrm{LT}}) + \lambda_{\mathrm{LT}}^2] \tag{2.52}$$

α_{LT} is an imperfection factor and $\lambda_{0,\mathrm{LT}}$ is the limit of the horizontal plateau. The relative slenderness is determined from SFS-EN 1999-1-1 (2009, p. 88)

$$\lambda_{\mathrm{LT}} = \sqrt{\frac{\alpha \, \mathrm{W}_{\mathrm{el,y}} f_0}{M_{cr}}} \tag{2.53}$$

For cross-section classes 1 and 2 $\alpha_{\mathrm{LT}} = 0.10$ and $\lambda_{0,\mathrm{LT}} = 0.6$
For cross-section classes 3 and 4 $\alpha_{\mathrm{LT}} = 0.20$ and $\lambda_{0,\mathrm{LT}} = 0.4$

Finally structure is checked against lateral torsional buckling by comparing design load to buckling resistance (SFS-EN 1999-1-1 2009, p. 86)

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0 \qquad (2.54)$$

Case study
For example let's take a look how the length of a beam affects to lateral torsional buckling resistance. Let's define some constants for the evaluation:
Modulus of elasticity $E = 70\,000\ \mathrm{MPa}$
Glide modulus $G = 27\,000\ \mathrm{MPa}$
0.2 % limit $f_0 = 120\ \mathrm{MPa}$
Partial safety factor against buckling $\gamma_{\mathrm{M1}} = 1.1$
Elastic bending resistance $W_{el,y} = 38748\ \mathrm{mm}^3$
Moment of inertia with respect to weaker axis $Iz = 26093\ \mathrm{mm}^4$
Torsional inertia of a beam $It = 40967\ \mathrm{mm}^4$
Warping inertia of a beam $Iw = 75405000\ \mathrm{mm}^6$
Length of a beam $L = [0, 10, 20, \ldots, 3000]\ \mathrm{mm}$
Boundary conditions of supports $k_z = 0.7$, $k_w = 0.7$
And the position of force equals to shear center.

The results can be now plotted to figure 2.3. As we see from the plot and previous equations, the maximum capacity with corresponding elastic resistance and yield strength is achieved when reduction factor $\chi_{LT} = 1$, meaning simply that the shape or length of the beam is so, that lateral torsional buckling cannot happen and capacity of the beam is defined by bending moment resistance of the cross-section.

Figure 2.3. The effect of length of a beam to lateral torsional buckling resistance.

Flexural buckling

Flexural buckling occurs when a member is subjected to compression and buckles by its weaker inertia axis. Relative slenderness in flexural buckling according to SFS-EN 1999-1-1 (2009, p. 85)

$$\lambda = \sqrt{\frac{A_{eff} f_0}{N_{cr}}} = \frac{L_{cr}}{i\pi} \sqrt{\frac{A_{eff} f_0}{AE}} \tag{2.55}$$

Where effective area A_{eff} equals area A in cross-sections 1, 2 and 3. L_{cr} is buckling length, A is the area of cross-section and N_{cr} refers to critical load according to elastic theory in double symmetrical cases.

$$N_{cr} = \left(\frac{i\pi}{L_{cr}}\right)^2 AE \tag{2.56}$$

Buckling length is calculated according to

$$L_{cr} = kL \tag{2.57}$$

And i is the radius of gyration about the relevant axis, determined from the properties of cross-section.

$$i = \sqrt{\frac{I}{A}} \tag{2.58}$$

Buckling length factors are compiled to table 2.3.

Table 2.3. Buckling length factor k according to SFS-EN 1999-1-1 (2009, p. 86)

Method of support	k factor
Fixed support on both ends	0.7
First end fixed, second end pinned	0.85
Both ends pinned	1.0
Both ends supported against torsion, first end laterally supported, second end no lateral support	1.25
First end fixed, second end torsion partly supported, but laterally no support.	1.5
First end fixed, second end no support	2.1

Buckling resistance of a compressed part $N_{b,Rd}$ can be calculated from SFS-EN 1999-1-1 (2009, p. 82)

$$N_{b,Rd} = \frac{\kappa\, \chi\, A_{eff}\, f_0}{\gamma_{M1}} \tag{2.59}$$

κ is a factor that considers weakening impact of welds. Factor values for a member that includes longitudinal welds can be seen from SFS-EN 1999-1-1 (2009, p. 83). If a member contains no welds, then κ is given value one. Where reduction factor χ is evaluated using SFS-EN 1999-1-1 (2009, p. 82)

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} \tag{2.60}$$

Reduction factor Φ can be calculated from

$$\Phi = 0.5\,(1 + \alpha(\lambda - \lambda_0) + \lambda^2) \tag{2.61}$$

Where α is an imperfection factor and λ_0 is the limit of horizontal plateau.
Finally compressed part should satisfy stability criteria (SFS-EN 1999-1-1 2009, p. 81)

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1.0 \tag{2.62}$$

Case study
Let's take a look to a following example. First some constants are defined:
Modulus of elasticity $E = 70\,000$ MPa
Area of the cross section $A = 1200\ \mathrm{mm}^2$
Effective area of the cross-section $Aeff = 1165\ \mathrm{mm}^2$
0.2 % yield limit $f_o = 140$ MPa
The radius of gyration to the weaker axis $i = 13$ mm
Length on an interval between $L = [1000, 5000]$
Partial factor with regard to instability $\gamma_{M1} = 1.1$
Both ends are pinned, so $k = 1.0$
The results have been plotted to figure 2.4. As seen from figure, first order theory gives remarkably greater values than the threshold to use second order analysis. The second order analysis limit α_{cr} is defined as $N_{cr}/10$ by SFS-EN 1999-1-1 (2009, p. 49).

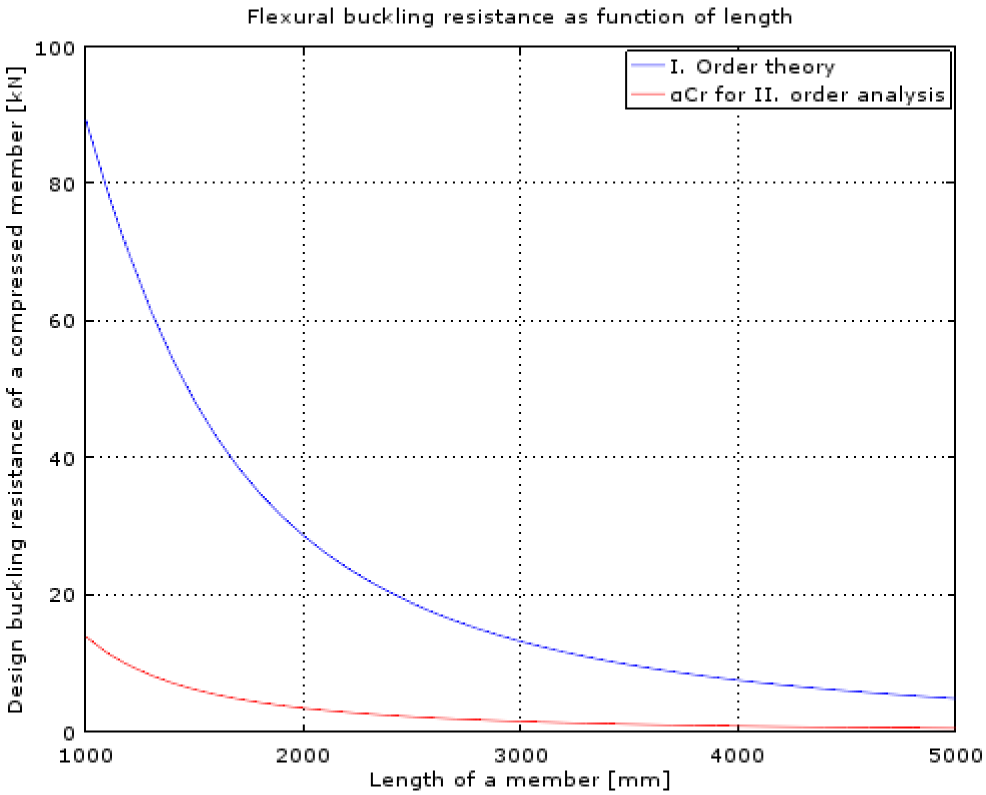


Figure 2.4. Flexural buckling resistance as a function of length.

The limit for II. order analysis is so low, so further analysis is required. While formulae for N_{Rd} include the effect of local bow imperfections, effects of initial sway imperfections due manufacturing of frame are not included. The method used to include initial sway imperfections is use of equivalent horizontal forces. For that it is necessary to calculate global initial sway imperfection factor

$$\Phi = \Phi_0 \, \alpha_h \, \alpha_m \tag{2.63}$$

Where $\Phi_0 = 1/200$, α_h is reduction factor for columns and α_m is reduction factor that regards the number of columns in a row.

$$\alpha_h = \frac{2}{3} \leq \frac{2}{\sqrt{h}} \leq 1.0 \tag{2.64}$$

$$\alpha_m = \sqrt{0.5(1 + \frac{1}{m})} \tag{2.65}$$

So the equivalent system of horizontal forces is $M_{equ} = \Phi \, \mathrm{N}_{Ed} L$. So let's take these into consideration, and the capacity is checked against combined moment and axial force. In combined moment and axial force check, double symmetrical hollow and solid cross-sections should satisfy SFS-EN 1999-1-1 (2009, p. 80)

$$\left(\frac{N_{Ed}}{\omega_0 \, N_{Rd}}\right)^{\psi} + \left[\left(\frac{M_{y,Ed}}{\omega_0 \, M_{y,Rd}}\right)^{1.7} + \left(\frac{M_{z,Ed}}{\omega_0 \, M_{z,Rd}}\right)^{1.7}\right]^{0.6} \leq 1.00 \tag{2.66}$$

Where $\psi = 2$ for solid open profiles, $\omega_0 = 1$ for no welds. The equivalent force is has to be inputting to one direction only, so check according to weaker inertia is required. Therefore check equation can be reduced to

$$\left(\frac{N_{Ed}}{N_{Rd}}\right)^{2} + \left(\frac{M_{y,Ed}}{M_{y,Rd}}\right)^{1.37} \leq 1.00 \tag{2.67}$$

And by substituting $M_{y,ed} = M_{equ}$, and solving the equation for N_{Ed} one can get to

$$N_{Ed} \leq N_{Rd} \sqrt{1.00 - \left(\frac{M_{equ}}{M_{y,Rd}}\right)^{1.37}} \tag{2.68}$$

Now it's possible to take a look at the results at figure 2.5 and notice that N_{Rd} is scaled by a root function that depends on the relationship between II. order analysis equivalent moment and moment resistance to the weaker axis of inertia.

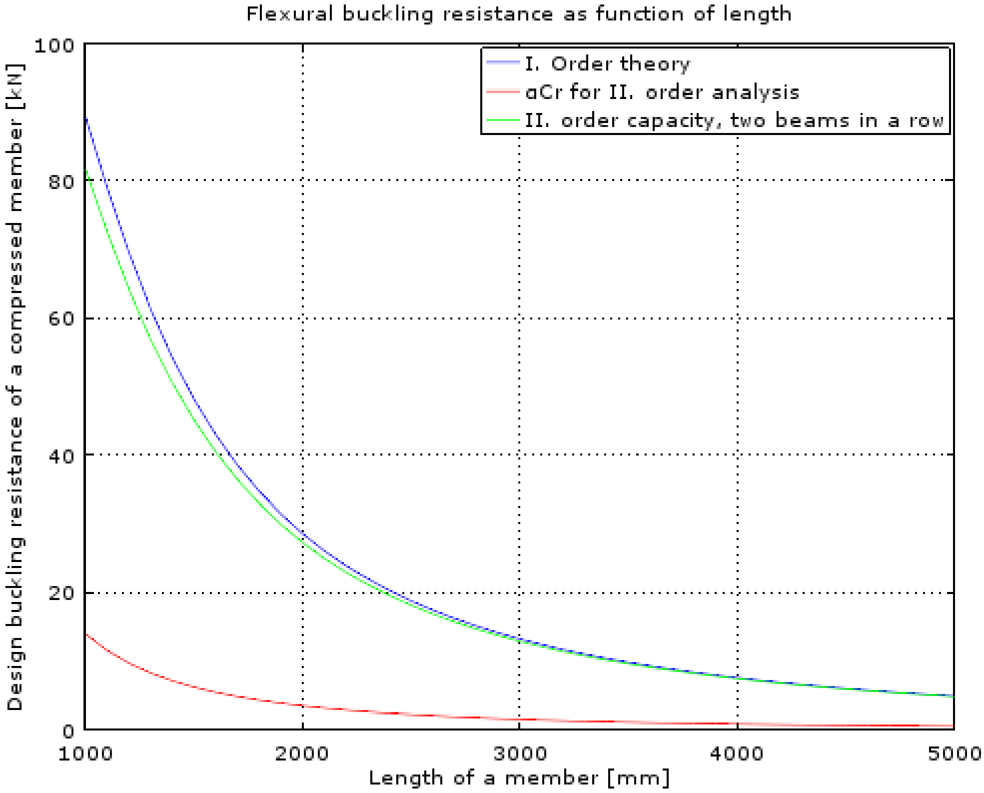


Figure 2.5. The effect of II. order analysis to the capacity of a compressed member

From figure 2.5 it can be seen that equivalent moment method has notable effect to results for member lengths between one and two meters and some effect between two and three meters. For beam lengths longer than three meters, in this case II. order effect is negligible. The percentage difference between results gained by I. and II. order theory analysis can be evaluated using equation (2.69) and can be plotted to figure 2.6.

$$\frac{N_{Rd,I}-N_{Rd,II}}{N_{Rd,I}} \tag{2.69}$$

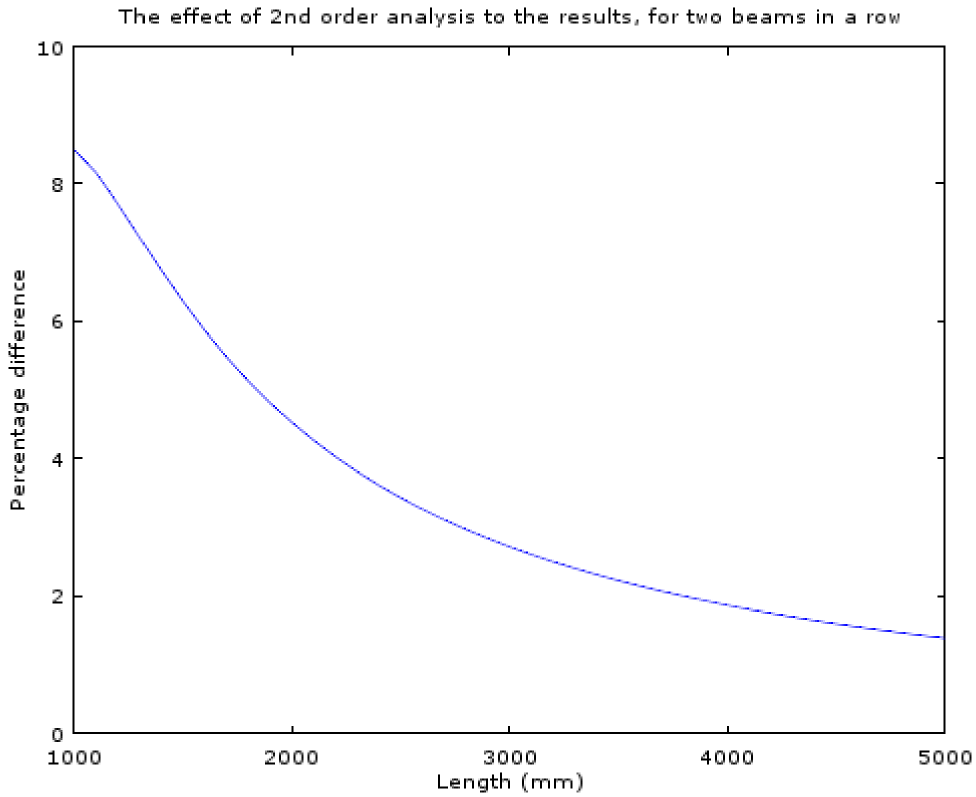


Figure 2.6. Percentage difference between I. and II. order analyses

Torsional buckling

Torsional buckling occurs when a member is subjected to compression and buckles by rotating around its length. So slenderness λ_T for torsional and torsional flexural buckling is calculated using SFS-EN 1999-1-1 (2009, p. 85).

$$\lambda_T = \sqrt{\frac{A_{eff} f_0}{N_{cr,T}}} \tag{2.70}$$

And critical normal force $N_{Cr,T}$ is calculated by

$$N_{Cr,T} = \frac{A}{I_p}[GI_t + \frac{\pi^2 EI_w}{L^2}] \tag{2.71}$$

Torsional flexural buckling

Torsional flexural buckling occurs when a member is subjected to a compression stress and opens, rotates and then buckles to its weaker inertia axis.

Slenderness λ_T for torsional and torsional flexural buckling

$$\lambda_T = \sqrt{\frac{A_{eff} f_0}{N_{cr}}} \tag{2.72}$$

$$N_{Cr,x} = \frac{\pi^2 E I_x}{(k_x L)^2} \tag{2.73}$$

$$N_{Cr,y} = \frac{\pi^2 E I_y}{\left(k_y L\right)^2} \tag{2.74}$$

Where $N_{Cr,x}$ is critical buckling load around x-axis, $N_{Cr,y}$ is critical buckling load around y-axis, I_x is second moment of inertia around x-axis. $N_{cr,T}$ regards critical torsional buckling load combined with flexural buckling effect according to SFS-EN 1999-1-1 (2009, p. 208).

$$N_{Cr,T} = \frac{A}{I_p}\left[G I_t + \frac{\pi^2 E I_w}{L^2}\right] \tag{2.75}$$

Where I_p is polar moment of inertia. Constant cross-sectional beam with variable boundary conditions at the ends and evenly distributed normal force at the shear center, the critical normal force due at torsion and lateral torsional buckling according to elastic theory is calculated from SFS-EN 1999-1-1 (2009, p. 208)

$$\left(N_{cr,y} - N_{cr}\right)\left(N_{cr,z} - N_{cr}\right)\left(N_{cr,T} - N_{cr}\right) i_s^2 - \alpha_{zw} z_s^2 N_{cr}^2 \left(N_{cr,y} - N_{cr}\right) - \alpha_{yw} y_s^2 N_{cr}^2 \left(N_{cr,z} - N_{cr}\right) = 0 \tag{2.76}$$

Where boundary condition factors $\alpha_{yw}(k_y, k_w)$ and $\alpha_{zw}(k_y, k_w)$ depend on combined bending and torsion boundary conditions. i_s^2 is radius of gyration defined by

$$i_s^2 = \frac{I_y + I_z}{A} + y_s^2 + z_s^2 \tag{2.77}$$

Where y_s and z_s are shear center coordinates related to centroid

2.1.4 Design loads of materials

Extruded profiles

SFS-EN 1999-1-1 (2009) standard covers a range of structural aluminium alloys listed in article 3.2.1. Wrought aluminium alloys for structures come in the form of sheet (SH), strip (ST), plate (PL), extruded tube (ET) , extruded profiles (EP), extruded rod and bar (ER/B), drawn tube (DT) and forgings (FO).

SFS-EN 1999-1-1 (2009) standard gives material strength values for wrought aluminium alloys in SFS-EN 1999-1-1 (2009, p. 44). The design resistance values are calculated normally according to the standard.

Cast parts

SFS-EN 1999-1-1 (2009, p. 170) gives design rules for heat treated cast alloys EN AC-42100, EN AC-42200, EN AC-43000, EN AC-43300 and non-heat treatable alloys EN AC-44200 and EN AC-51300.

Special rules are applied to cast structures that have the kind of shape and load that buckling does not occur. Cast structures cannot be formed, welded or machined so, that there will be sharp internal edges.

Load carrying cast parts should be designed so that equivalent design load $\sigma_{\mathrm{eq,Ed}}$ according to elastic theory SFS-EN 1999-1-1 (2009, p. 170)

$$\sigma_{\mathrm{eq,Ed}} = \sqrt{\sigma_{\mathrm{x.Ed}}^2 + \sigma_{\mathrm{y,Ed}}^2 - \sigma_{\mathrm{x,Ed}}\sigma_{\mathrm{y,Ed}} + 3\tau_{\mathrm{xy,Ed}}^2} \tag{2.78}$$

$\sigma_{\mathrm{x,Ed}}$ is longitudinal local stress design value, $\sigma_{\mathrm{y,Ed}}$ is transverse local stress design value and $\tau_{\mathrm{xy,Ed}}$ local shear stress design value. $\sigma_{\mathrm{eq,Ed}}$ should be compared to design resistance value σ_{Rd} which is smaller of following values

$$\sigma_{\mathrm{Rd}} = \frac{f_{oc}}{\gamma_{\mathrm{Mo,c}}} \tag{2.79}$$

$$\sigma_{\mathrm{Rd}} = \frac{f_{uc}}{\gamma_{\mathrm{Mu,c}}} \tag{2.80}$$

Where recommended partial factors are $\gamma_{\mathrm{Mo,c}} = 1.1$ and $\gamma_{\mathrm{Mu,c}} = 2.0$.

2.1.5 Ultimate limit states

According to EN 1990 (2002, p. 45) following ultimate limit states are verified

a) "EQU : Loss of equilibrium of the structure or any part of it considered as rigid body where:
   - minor variations in the value or the spatial distribution of permanent actions from a single source are significant
   - the strengths of the construction materials or ground are generally not governing ;
b) STR : Internal failure or excessive deformation of the structure or structural members, including footings, piles, basement walls, etc., where the strength of construction materials governs ;
c) GEO : Failure or excessive deformation of the ground where the strengths of soil or rock are significant in proving resistance ;
d) FAT : Fatigue failure of the structure or structural members.
e) UPL : Loss of equilibrium of the structure or ground due to uplift by water pressure (buoyancy) or other vertical actions ;
f) HYD : Hydraulic heave, internal erosion and piping in the ground caused by hydraulic gradients."

In generally design due to ultimate limit states require use of partial factor against failure with value 1.1 for material and stability and with 1.5 for variable loads.

2.1.6 Serviceability limit states

In the serviceability states the equilibrium of the structure is not compromised under loading, but the deformation of the structure is limiting functionality. This limitation comes usually in the form of a deflection that is considered to have significant value. In the serviceability limit states the use of partial factors γ_M can be taken as 1.0 unless specified differently.

2.2 Structural design by finite element method

The following chapter deals with the design of structures by finite element method. The chapter consists of modeling of materials, modeling of geometry and analysis settings in FEA-software.

2.2.1 Modeling of materials in FEA-software

General rules

In generally modeling of materials requires certain material properties defined by material testing. These are:

- The relation of stress and strain, modulus of elasticity E
- The relation of torsion and glide, modulus of glide G
- Yield strength of a corresponding material f_0
- Ultimate strength of a corresponding material f_u

All aluminium alloys covered in SFS-EN 1999-1-1 (2009) can be modeled using $E = 70\ \mathrm{GPa}$, $G = 27\ \mathrm{GPa}$, $\nu = 0.3$ for Poisson ratio, $\alpha = 23 * 10^{-6}\ 1/\mathrm{K}$ for thermal expansion and $\rho = 2\ 700\ \mathrm{kg/m^3}$ for density.

Extruded aluminium profiles

As material sense extruded aluminium profiles were modeled using following properties for EN AW-6060 T6 alloy with shape EP as in table 2.4.

Table 2.4. Extruded profile properties according to SFS-EN 1999-1-1 (2009).

Property	Value
Yield strength f_0	140 MPa
Ultimate strength f_u	170 MPa
n value for plastic analysis	24
Buckling class	A

Pressure cast aluminium parts

Pressure cast aluminium parts were modeled using following properties given in material certificate by manufacturer for AlSi10Mg alloy as in table 2.5.

Table 2.5. Cast part properties according to SFS-EN 1999-1-1 (2009).

Yield strength	120 MPa
Ultimate strength	180 MPa
Buckling class	B
Minimum elongation	1 %

2.2.2 Modeling of geometry in FEA software

The cross-sections were modeled as beam elements with following principal cases

1) One-sided frame with horizontal beam and one or two vertical columns. (figure 2.7)

2) Two-sided frame with two horizontal beams, without and with one or two vertical columns. (figure 2.8)
3) Three-sided frame with three horizontal beams, without and with one (or two) vertical columns. (figure 2.9)

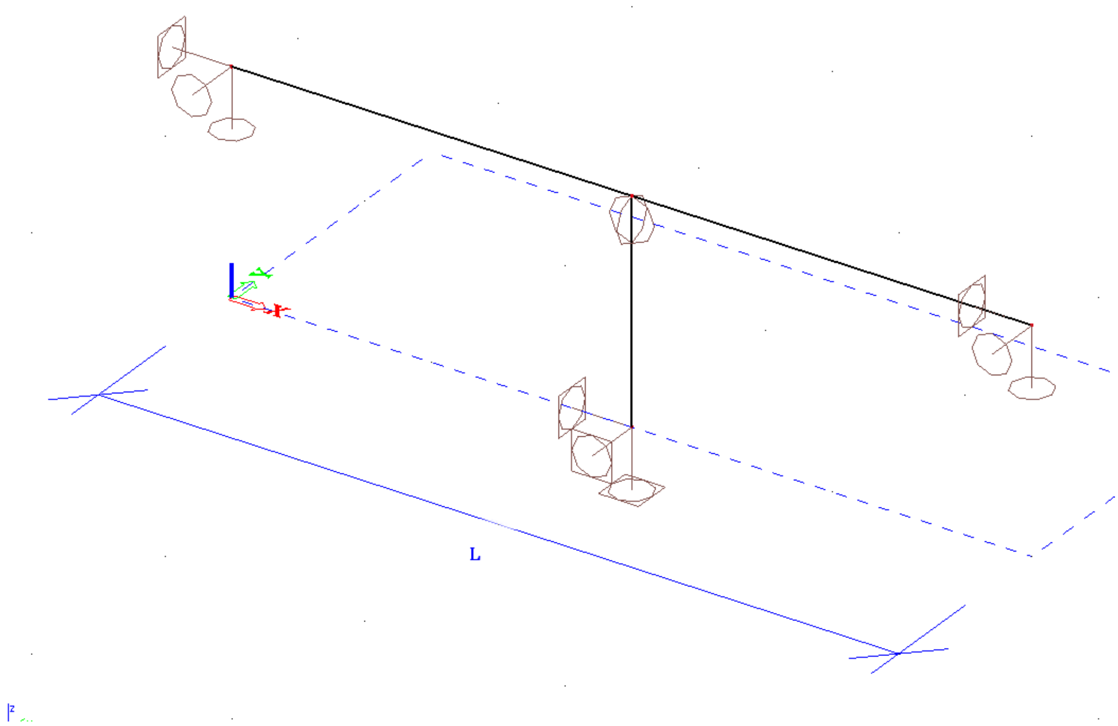


Figure 2.7. One-sided frame with horizontal beam and one vertical column.

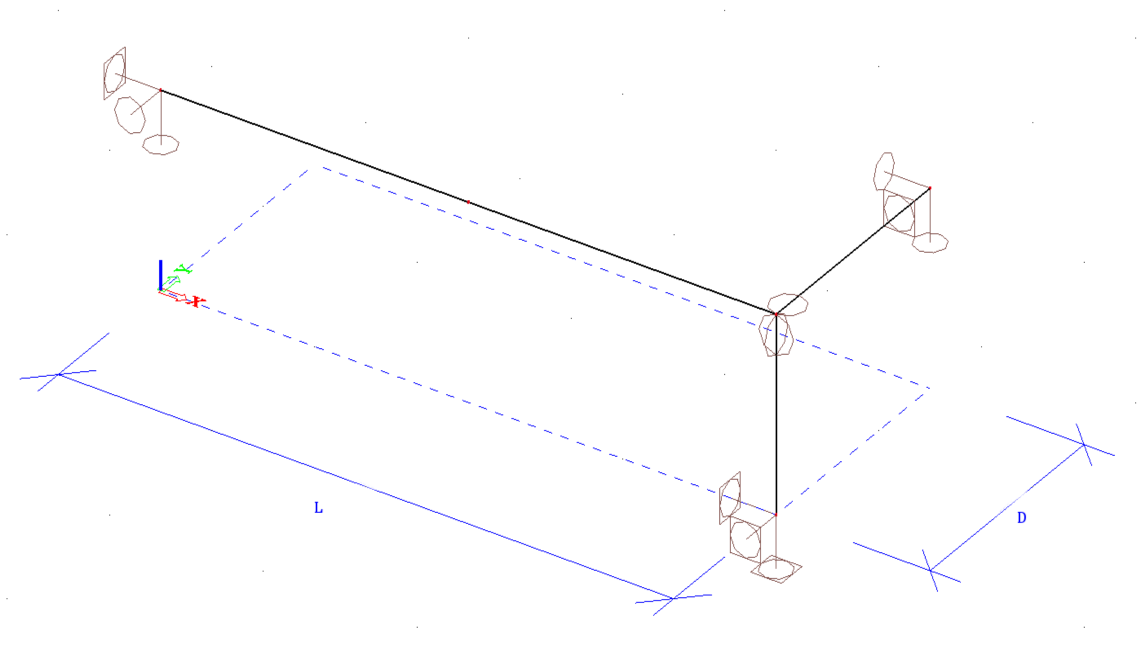


Figure 2.8. Two-sided frame with two horizontal beams and one vertical column.

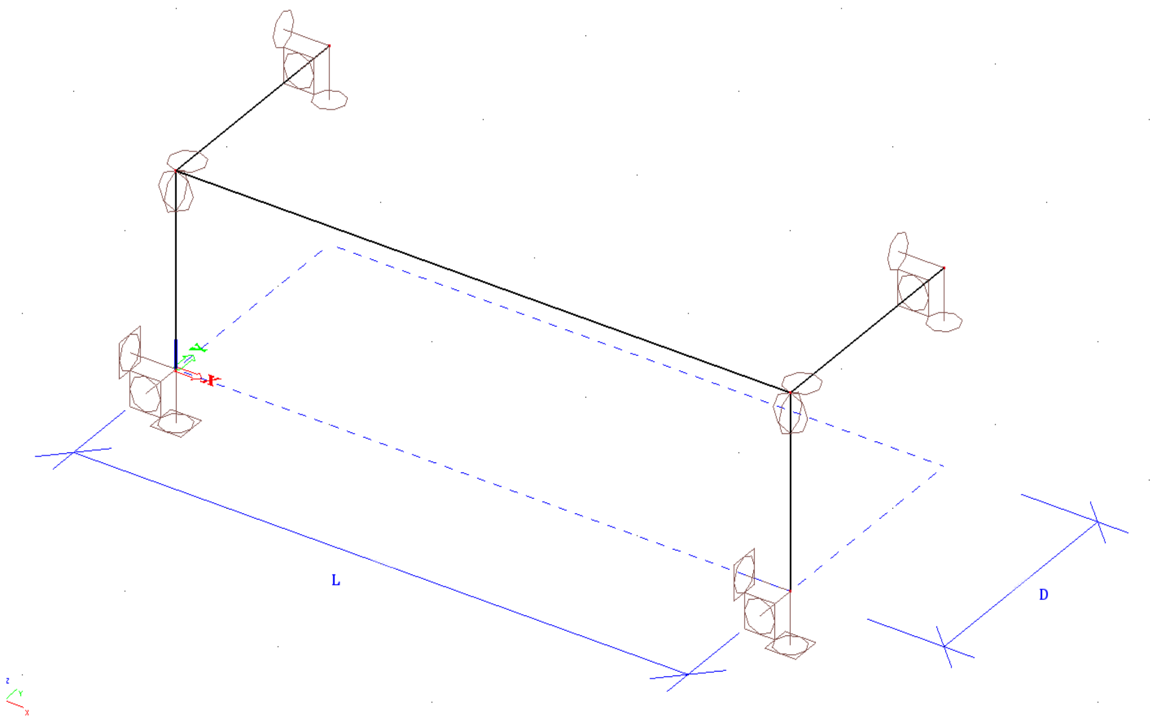


Figure 2.9. Three-sided frame with three horizontal beams and two vertical columns.

Modeling of cross-sections

Horizontal beams were modeled importing cross-sections into FEA software SCIA Engineer. Modeling of cross-sections consisted of following phases:

1) Importing cross-section as polygon containing no information valuable to FE analysis. (figure 2.10)
2) Creating a thick-walled presentation from a polygonal model for calculating cross-sectional properties enabling use up to cross-section class 3. (figure 2.11)
3) Creating a thin-walled presentation from a thick-walled model for calculating effective cross-sectional properties according to cross-section class 4. (figure 2.12)
4) Setting up type of reinforcement in fibers and setting up HAZ data for cross–section. (figure 2.13)

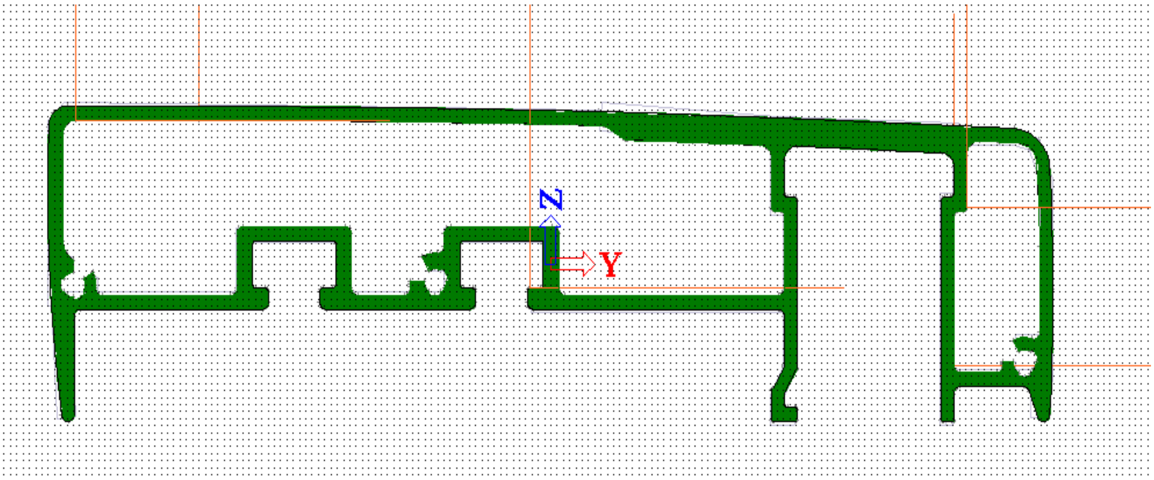


Figure 2.10. Importing geometrical shape into polygonal presentation.

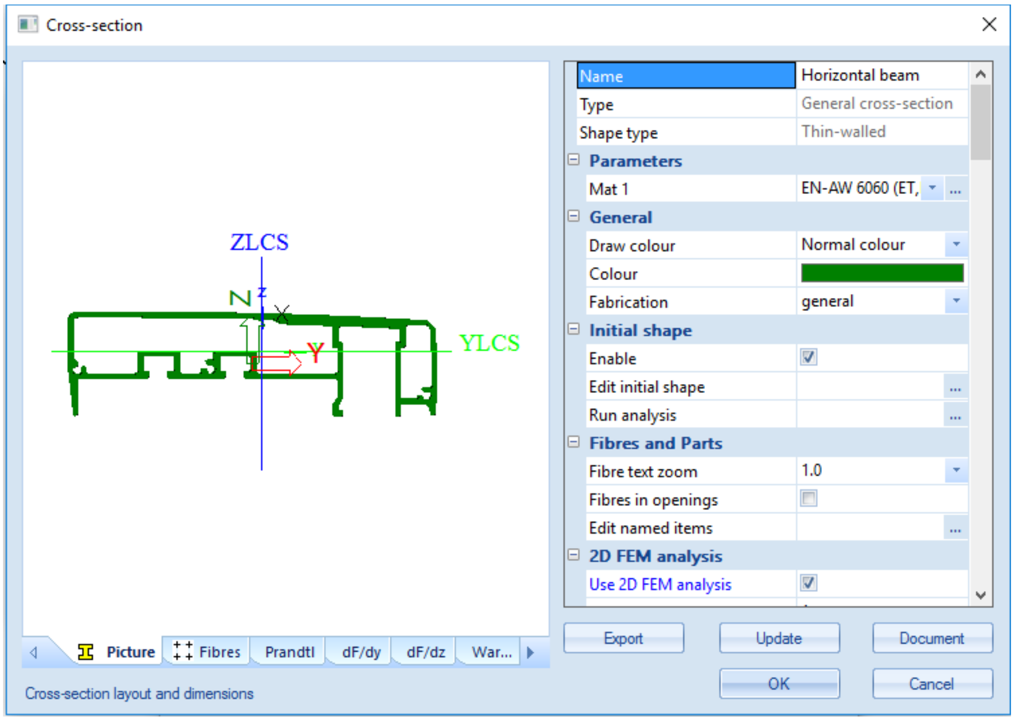


Figure 2.11. Thick-walled presentation of cross-section of the horizontal beam.

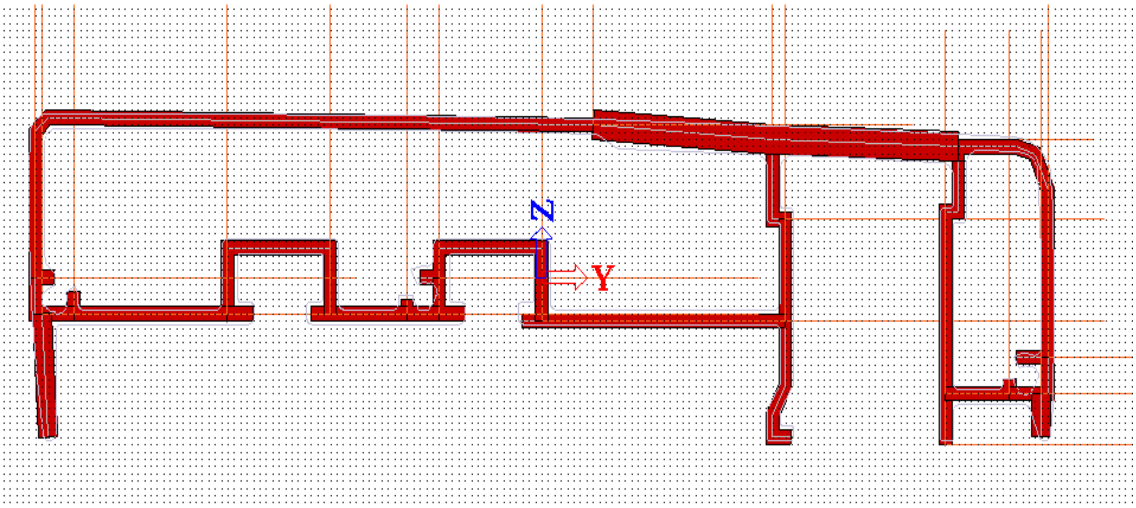
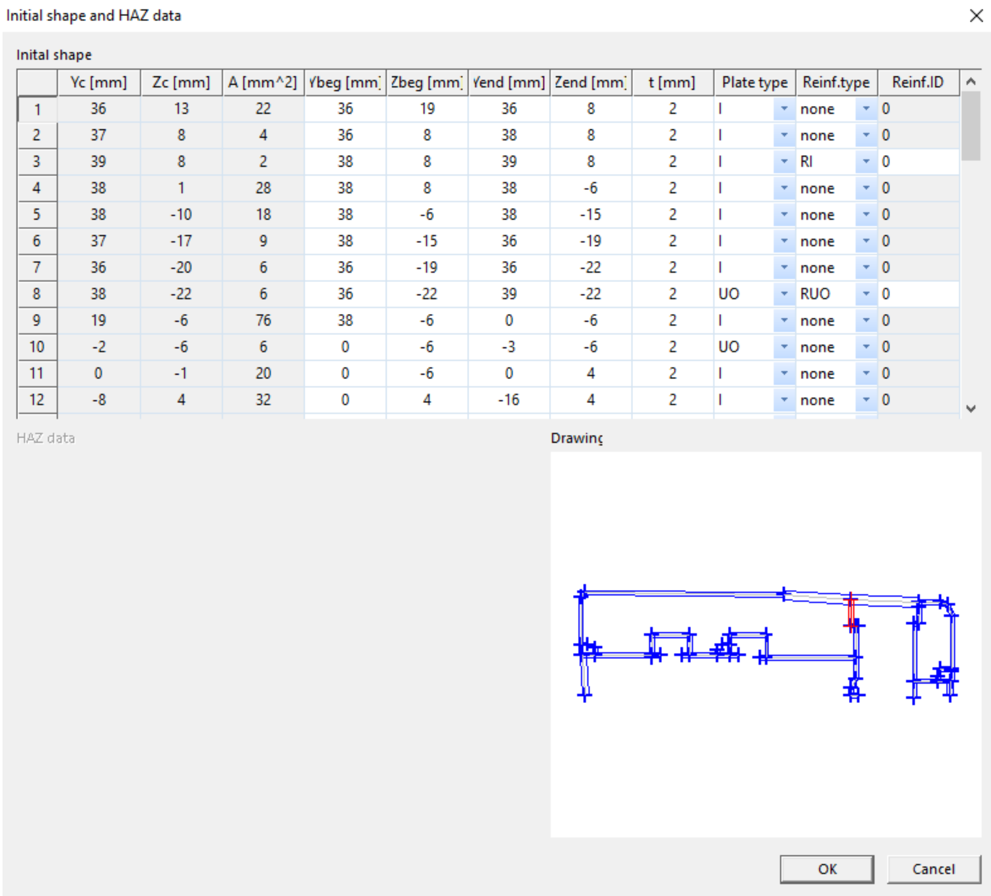


Figure 2.12. Thin-walled presentation. The cross-section is divided into fibers with individual thicknesses.



	Yc [mm]	Zc [mm]	A [mm^2]	Ybeg [mm	Zbeg [mm	Yend [mm]	Zend [mm	t [mm]	Plate type	Reinf.type	Reinf.ID
1	36	13	22	36	19	36	8	2	I	none	0
2	37	8	4	36	8	38	8	2	I	none	0
3	39	8	2	38	8	39	8	2	I	RI	0
4	38	1	28	38	8	38	-6	2	I	none	0
5	38	-10	18	38	-6	38	-15	2	I	none	0
6	37	-17	9	38	-15	36	-19	2	I	none	0
7	36	-20	6	36	-19	36	-22	2	I	none	0
8	38	-22	6	36	-22	39	-22	2	UO	RUO	0
9	19	-6	76	38	-6	0	-6	2	I	none	0
10	-2	-6	6	0	-6	-3	-6	2	UO	none	0
11	0	-1	20	0	-6	0	4	2	I	none	0
12	-8	4	32	0	4	-16	4	2	I	none	0

Figure 2.13. Setting initial shape, reinforcements and HAZ data for the cross-section.

Vertical columns were modeled using existing geometrical shape templates from SCIA Engineer's profile library and then adjusting measurements according to technical drawings. SCIA Engineer then calculated cross-sectional properties used in analysis (figure 2.14).

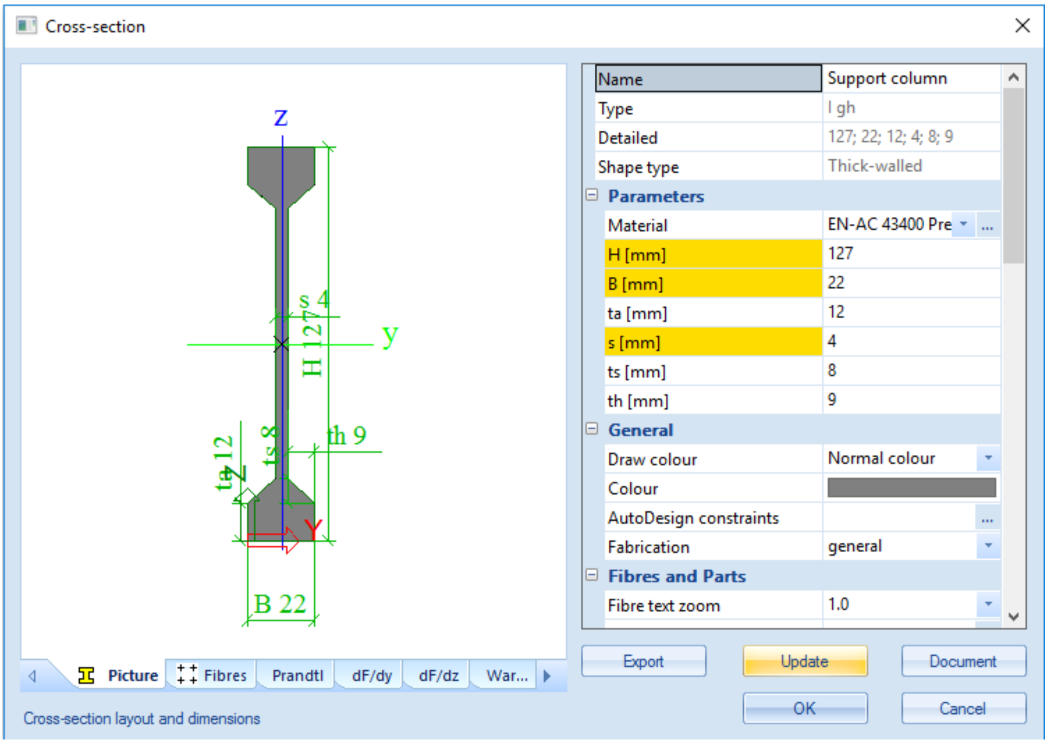


Figure 2.14. Cross-section of the support column.

Modeling of structures

Modeling of structures consisted of

1) Setting up lengths of columns
2) Setting locations of beginning nodes
3) Setting orientations of the columns
4) Setting up members between the columns and supports
5) Checking the orientations of the members.

Modeling of joints

Joints in the structure have limited capacity due the use of frictional bolts joints. Frictional bolt joints have been tested fail-safe in ultimate limit state. Tests to define capacity in serviceability limit state have proven to be difficult. Until tests have been made to define spring stiffness and capacity in serviceability limit state, the joints in the structure are simplified to the safe side and modeled with no rotational resistance, bearing translational forces only.

Modeling of supports

Horizontal supports of the frame were modeled as hinged (fixed translation on all axes, free rotation on all axes). However there were cases where modeling of the horizontal supports would have led to matrix singularities (free rotation), in this case rotation was set fixed around beam's local x-axis. Vertical supports of the frame were modeled as fixed.

Modeling of load cases

Self-weight was modeled towards global –Z axis. Wind loads and imposed loads were modeled as line loads per length (kN/m). The benefit of this is that the computational model can be built directly as a function of line load and can be used with different heights of wind areas and combined load coefficients. Loads however were divided into four cases: wind from positive X, wind from negative X, wind from positive Y and wind from negative Y.

Load groups

Load groups were modeled the following way

1. Self-weight
2. Imposed loads
3. Wind (exclusive)

Exclusive means that the only one load case within a load group is taken into creation of combinations at once. For example in this case the least preferable combination as exclusive would be $c_1 * \mathrm{self-weight} + c_2 * \mathrm{imposed\ load} + c_3 * \mathrm{wind1, 2, 3\ or\ 4}$ instead of $c_1 * \mathrm{self-weight} + c_2 * \mathrm{imposed\ load} + c_3 * \mathrm{wind1} + c_4 * \mathrm{wind2} + c_5 * \mathrm{wind3} + c_6 * \mathrm{wind4}$. Where c_i are combination coefficients according to EN 1990 (2002, p. 47). This is for a physical reason, that wind loads cannot happen simultaneously from different directions.

Modeling of load combinations

Load combinations were modeled using the following load cases

1) Self-weight (direction global –Z)
2) Imposed loads
3) Wind 1 (direction global X)
4) Wind 2 (direction global –X)
5) Wind 3 (direction global Y)
6) Wind 4 (direction global –Y)

So given in according to combinatorics, the exclusive combination would result in *k* possible choices out of *n* alternatives

$$\binom{n}{k} = \binom{6}{3} = \frac{6!}{3!(6-3)!} = 20 \tag{2.80}$$

Whereas standard combination would give a result based on the sum of combinations depending on the *k* number of choices, to select from constant amount of *n* alternatives

$$\sum_{i=1}^{n}\binom{n}{i} = \sum_{i=1}^{6}\binom{6}{i} = 63 \tag{2.81}$$

2.2.3 Analysis settings in FEA software

Element mesh

The element type used for analysis was 1D beam element. Element mesh was generated as an average of 10 elements for a beam. The minimal distance between two points (nodes) was set to 1 mm. Nodes were generated in connections of beam elements. Nodes were generated under concentrated loads on beam elements.

Settings of FEA solver

Shear force deformation was taken in count (Ay, Az >> A) in computation. Analysis type was linear static. Number of sections per average member was 10. Considering non-linearity, the method to calculate 2. and 3. order effects was according to Timoshenko beam theory with maximum of 50 iterations.

3 RESULTS

This chapter presents results for different geometry and load cases as well as post-processing of results, regarding one, two and multivariable models.

3.1 Results for different geometry and load cases

After each analysis the following data (table 3.1 and 3.2) was read from SCIA Engineer and then compiled to the result tables below (tables 3.3 - 3.11).

Table 3.1. Result parameters

Parameter	Description
Line load	Line load to the side of a horizontal member
Deflection	Maximum deflection of the horizontal beam normal to length in xy-plane
Hstress	Maximum combined stress within a horizontal member
Vstress	Maximum combined stress within a vertical member
Stability	Is the frame stable? If not, which buckling mode activates? See stability table.
My	Reaction moment around support column around global y-axis
Mz	Reaction moment around support column around global z-axis
VRy	Reaction force in vertical column parallel to global y-axis
HRy	Reaction force in horizontal beam parallel to global y-axis
HRx	Reaction force in horizontal beam parallel to global x-axis
Length (L)	Length of the horizontal beam or frame parallel to global x-axis
Width (D)	Length of the horizontal beam or frame parallel to global y-axis
-	Value is either non-existent or irrelevant by the scale of magnitudes

Table 3.2. Stability parameters

Parameter	Description
LTB	Lateral torsional buckling
TFB	Torsional flexural buckling
FB	Flexural buckling
CBAC	Combined bending, axial and shear force check

3.1.1 Results for one sided frame

Results for one sided frame were read from SCIA Engineer and compiled to the following tables below (tables 3.3 – 3.5). Results for lateral torsional stability were plotted to figure 3.1.

Table 3.3. Length = 2000, one support column at L/2.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y, LTB 0.12	Y, LTB 0.26	Y, LTB 0.34	Y, LTB 0.42	Y, LTB 0.51
Deflection [mm]	1.2	1.8	2.4	3.1	3.7
HStress [MPa]	-	-	-	-	-
VStress [MPa]	13.4	20.1	26.8	33.5	40.2
My [kNm]	-	-	-	-	-
Mx [kNm]	0.47	0.70	0.93	1.17	1.40
VRy [kN]	0.42	0.64	0.85	1.06	1.27
HRy [kN]	1.29	1.93	2.58	3.22	3.86
HRx [kN]	-	-	-	-	-

Table 3.4. Length = 3000, one support column at L/2.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y, LTB 0.56	Y, LTB 0.83	N, LTB 1.10	N, LTB 1.38	N, LTB 1.65
Deflection [mm]	4.0	6.0	8.0	10.0	12.0
HStress [MPa]	-	-	-	-	-
VStress [MPa]	39.4	59.1	78.8	98.5	118.1
My [kNm]	-	-	-	-	-
Mx [kNm]	1.52	2.29	3.05	3.81	4.57
VRy [kN]	1.39	2.08	2.77	3.47	4.16
HRy [kN]	1.56	2.34	3.11	3.89	4.67
HRx [kN]	-	-	-	-	-

Table 3.5. Length = 4000, one support column at L/2.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	N, 1.04	N, 1.56	N, 2.07	N, 2.59	N, LTB 3.1
Deflection [mm]	6.9	10.4	13.9	17.3	20.8
HStress [MPa]	-	-	-	-	-
VStress [MPa]	74.3	111.3	148.4	185.4	222.6
My [kNm]	-	-	-	-	-
Mx [kNm]	2.87	4.31	5.75	7.18	8.62
VRy [kN]	2.61	3.92	5.22	6.53	7.83
HRy [kN]	1.69	2.54	3.39	4.24	5.08
HRx [kN]	-	-	-	-	-

Results for lateral torsional stability for length and line load were plotted to figure 3.1.

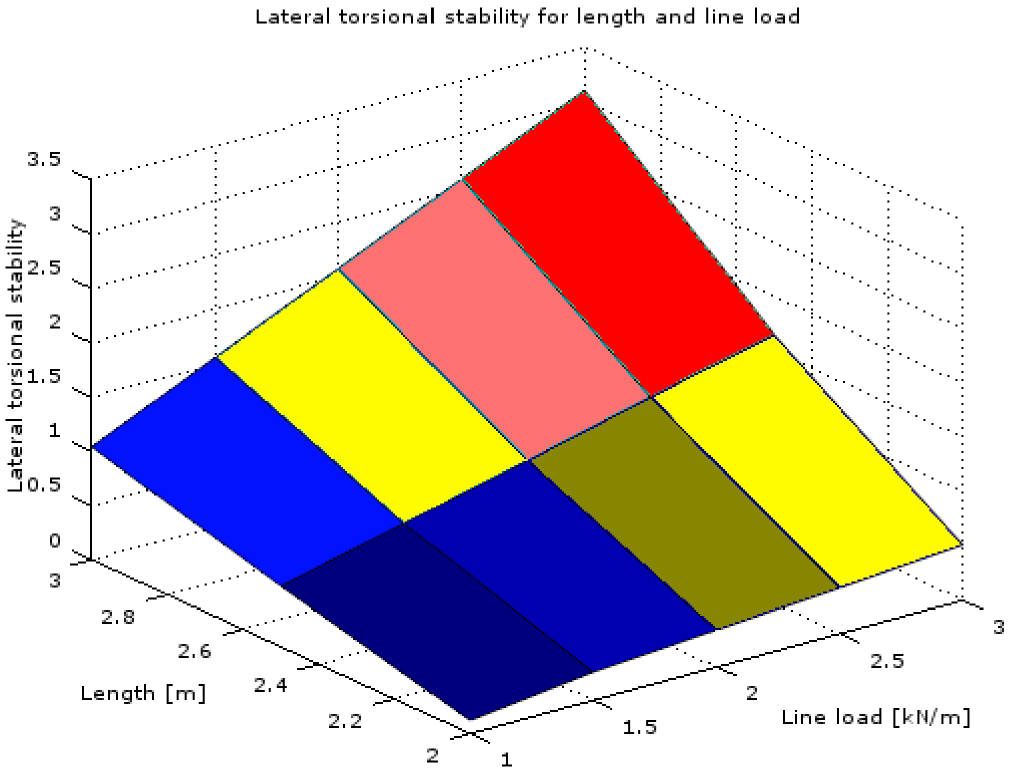


Figure 3.1. Lateral torsional stability for length and line load.

3.1.2 Results for two sided frame

The results for two-sided frame were been inputted to tables 3.6 - 3.8. Line marker means that those values are not remarkable. Stress in the horizontal member was plotted to figure 3.2.

Table 3.6. Length = 2000, width = 1500, one support column at edge, two horizontal beams with one hinged joint.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y	Y	Y	Y	N, 1.07
Deflection [mm]	1	1.5	2	2.5	3
HStress [MPa]	-15.9	-23.3	-30.6	-38.0	-45.3
VStress [MPa]	-	-	-	-	-
Mx [kNm]	-	-	-	-	-
VRy [kN]	-	-	-	-	-
HRy [kN]	1.72	2.56	3.42	4.28	5.13
HRx [kN]	-	-	-	-	-

Table 3.7. Length = 3000, width = 1500, one support column at edge, two horizontal beams with one hinged joint.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y	Y	Y	Y	N, CBAC 1.04
Deflection [mm]	7.4	11.0	14.6	18.2	21.9
HStress [MPa]	-46.5	-68.6	-90.7	-112.8	-135.3
VStress [MPa]	-	-	-	-	-
Mx [kNm]	-	-	-	-	-
VRy [kN]	-	-	-	-	-
HRy [kN]	2.25	3.38	4.50	5.63	6.76
HRx [kN]	-	-	-	-	-

Table 3.8. Length = 4000, width = 1500, one support column at edge, two horizontal beams with one hinged joint.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y, LTB 0.11	Y, LTB 0.16	Y, LTB 0.22	Y, LTB 0.27	N, LTB 0.33
Deflection [mm]	22.9	34.3	45.5	57.1	68.6
HStress [MPa]	-82.5	-121.8	-161.1	-200.4	-240.2
VStress [MPa]	-	-	-	-	-
Mx [kNm]	0.02	0.03	0.04	0.05	0.05
VRy [kN]	-	-	-	-	-
HRy [kN]	3	4.51	6.01	7.51	9.01
HRx [kN]	-	-	-	-	-

In this case the limiting factor was stress in horizontal member, stresses were plotted to figure 3.2 as a function of length and line load.

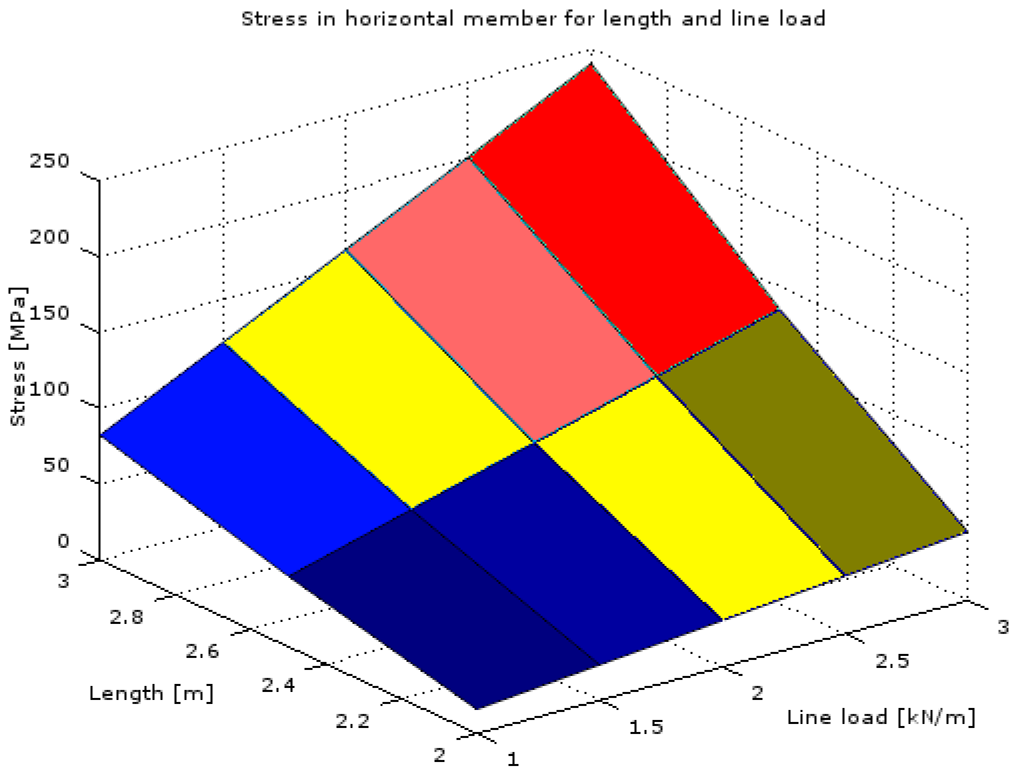


Figure 3.2. Stress in a horizontal member as a function of length and line load.

3.1.3 Results for three sided frame

Following results such as stability, deflection, stresses and support reactions were read from SCIA Engineer and compiled to tables 3.9 – 3.11.

Table 3.9. Length = 2000, width = 1500, two support columns at corners, three horizontal beams with two hinged joints.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y	Y	Y	Y	Y
Deflection [mm]	1.5	2.2	3.0	3.7	4.4
HStress [MPa]	-20.8	-30.6	-40.5	-50.3	-60.1
VStress [MPa]	-	-	-	-	-
Mx [kNm]	-	-	-	-	-
VRy [kN]	-	-	-	-	-
HRy [kN]	1.5	2.25	3.00	3.75	4.50
HRx [kN]	-	-	-	-	-

Table 3.10. Length = 3000, width = 1500. Two support columns at corners, three horizontal beams with two hinged joints.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y	Y	Y	Y	N, CBAC 1.04
Deflection [mm]	7.3	10.9	14.6	18.2	21.9
HStress [MPa]	-46.3	-68.4	-90.5	-112.6	-134.9
VStress [MPa]	-	-	-	-	-
Mx [kNm]	-	-	-	-	-
VRy [kN]	-	-	-	-	-
HRy [kN]	2.25	3.37	4.5	5.62	6.75
HRx [kN]	-	-	-	-	-

Table 3.11. Length = 4000, width = 1500. Two support columns at corners, three horizontal beams with two hinged joints.

Line load [kN/m]	1	1.5	2	2.5	3
Stability [Y/N, reason]	Y	Y	N, CBAB 1.31	N, CBAB 1.62	N, CBAC 1.93
Deflection [mm]	22.9	34.3	45.7	57.1	68.5
HStress [MPa]	89.3	129.6	169.8	210.0	250.2
VStress [MPa]	-	-	-	-	-
Mx [kNm]	-	-	-	-	-
VRy [kN]	-	-	-	-	-
HRy [kN]	3.0	4.5	6.0	7.5	9.0
HRx [kN]	-	-	-	-	-

In three sided frame the most limiting factor was stress in a horizontal member, that was plotted into figure 3.3.

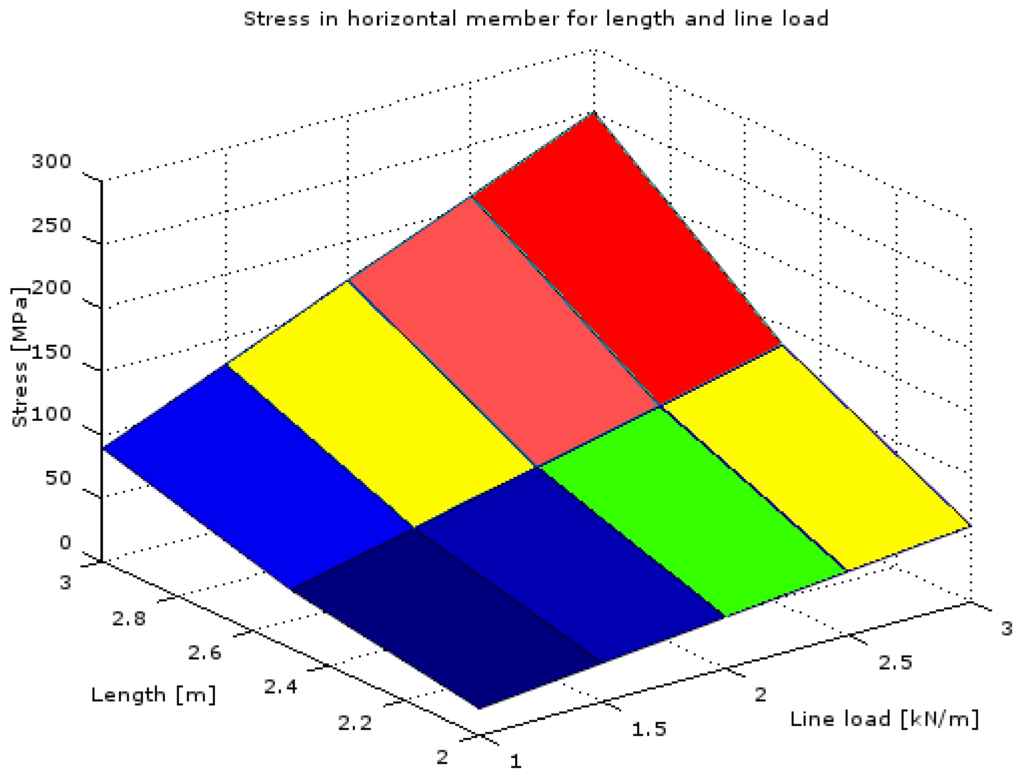


Figure 3.3. Stress in a horizontal member as function of length and line load.

3.2 Post-processing of results

Since FE-analysis is not stochastic method, post-processing of results does not require use of regression models such as least squares method or equivalent. Results are pretty much same and accurate to same initial conditions and solver settings.

Inaccuracy of FEM-models comes mostly from user definitions such as rigidity of joints, initial deflections, simplifications of geometries, deviations in manufacturing process et cetera. Not from the computation process itself, at least not in the case of beam elements that are based on analytical formulae.

The results were imported into technical computation program called Octave. Octave is matrix based programmable calculator that also handles differential equations and statistic operations. The syntax used below is mostly equivalent to syntax used in commercial software such as Matlab.

3.2.1 One variable models

A function of one variable is in the form of $(y = f(x))$. A simple way to plot those results is to create a vector space of input values $x = [a: b: c]$ where *a* and *c* are limits and *b* is corresponding step size of the interval. Next step would be calculating the output values, example $y = @(x)\ x^2 - 2x + 3$ and finally plot the result in the form of *plot(first variable x, corresponding value y, color and style of the plot).*

3.2.2 Two variable models

A function of two variables is in the form of $(z = f(x, y))$. Octave enables two different possibilities of plotting a two variable function. First method is the use of *plot3* command which enables plotting a single graph in three dimensions. Second method is the surface, command *meshgrid* enables user to mesh a large set of data and command *surf* allows user to visualize it.

Creating a surface consists of

1. defining a data set *x, y* and *z = f(x,y)*
2. meshing the data set, *[X,Y] = meshgrid(x,y)*
3. meshing the function *Y = f(X,Y)*
4. plotting a surface from the function: *surf(X,Y,Z)*

3.2.3 Multivariable models

There is no general method to present multivariable models in visual form. However 4 parameter models could be presented as in animation, so that resulting parameter could be a surface function of time *z = f(x,y,t)*. Otherwise user has to deal with separate graphs while keeping remaining variables as constants.

4 DISCUSSION

Results gained by finite element analysis can be recognized as reliable. Computation process can be assumed free of human errors, while the modeling stage cannot. There is always some error what comes to modeling of joints and supports. As written before, ideal supports such as fixed or hinged rarely exist in real world, so some assumptions and simplifications have to be made.

Due the geometry of support column, in many cases the phenomenon limiting the capacity of the structure was lateral torsional buckling of the vertical column. So therefore it can been seen that further development concerns the vertical column. Mainly by increasing the reduction coefficient χ_{LT} to value 1.00, meaning that lateral torsional buckling is not possible. This way it is possible to increase the capacity up to bending moment resistance given in SFS-EN 1999-1-1 (2009, p. 73).

Another method to increase the capacity of the vertical column would be increase the yield limit of the material. Current 120 MPa yield limit is no way good or advanced in the world of modern material technology. Standards come slow behind the latest methods and research, but already SFS-EN 1999-1-1 (2009) allows switching current cast material to another with higher yield limit. For example switching current cast alloy to a best alloy in the SFS-EN 1999-1-1 (2009, p. 43) EN AC-42200 T6, would make 40 % increase to the capacity of the vertical column possible this way.

If these modifications are not enough, then the next step would be increasing the elastic bending resistance of the vertical column. This means putting more material to the cross-section and keeping the relationships so that reduction factor $\chi_{LT} = 1.00$ at all times. There is no upper limit with this procedure. Permanent deformations are not allowed, so design according to plastic theory and cross-sectional classes 1 and 2 is not possible.

It can be seen reasonable to increase the capacity of the vertical column so that the stress levels between all load carrying members would be evenly distributed. It also can be seen as reasonable to increase the capacity of the vertical column to the point that the capacity is not greater than the capacity of the connection to the ground. The ample capacity of the future vertical column cannot be used when the connection to the ground is the limiting factor.

When comparing results to the goals set in introduction the research was successful by answering questions such as the optimal amount of supports, angle of supports, location of supports and stress distribution among members. The research also serves as basis for future development concerning cross-sectional optimization, refining self-weight and visual aspects. By above-mentioned calculations, the aluminum frames have been verified to satisfy standards EN 1990 (2002), SFS-EN 1991-1-1 (2002), SFS-EN 1991-1-3 (2015), SFS-EN 1991-1-4 (2011) and SFS-EN 1999-1-1 (2009) and therefore can be seen as structurally fail-safe.

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