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Master's Thesis

Profit potential of DAX index option trading based on implied volatility surfaces

Jouko Juvakka, 2020
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ABSTRACT

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The purpose of this thesis is to investigate DAX 30 option price information and evaluate trading profits of these options based on implied volatility surfaces. The used option data consists of DAX call and put options from the beginning of 2018 until the end of 2019. The options are divided in three maturity- and five moneyness categories. Implied volatilities are extracted by using the Black-Scholes formulas. Trading simulations for evaluating possibilities for profits are built by constructing daily updated delta-hedge portfolios with two different hedge-volatilities. The results from the trading simulations are conducted from an individual investor's perspective before and after transaction costs. Also, statistical significance and robustness checks are performed for the trading simulation results.

The results for implied volatilities show that the Black-Scholes model assumption for constant implied volatilities across the volatility surface is not valid. The implied volatilities of DAX options created a shape of a smile or a smirk and put options exhibited higher implied volatilities than call options. The results for the trading simulations indicated that the profits of selling DAX options combined with delta-hedge are not zero nor constant through moneyness categories before transaction costs. Especially deep-out-of-the-money and deep-in-the-money options generated significant profits. Abnormal profits and profitability patterns across moneyness categories disappeared when transaction costs were included.

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Tämän tutkielman tarkoituksena on tutkia DAX 30 optioiden hintainformaatiota ja määrittää kaupankäyntituottoja liittyen optioiden volatiliteettipintoihin. Data koostuu DAX osto- ja myyntioptioista vuoden 2018 alusta vuoden 2019 loppuun. Optiot jaetaan kolmeen maturiteetti- ja viiteen rahallisuuskategoriaan ja implisiittiset volatiliteetit uutetaan optioista Black-Scholes mallin avulla. Kaupankäyntisimulaatiot tuottojen määrittämiseksi koostuvat portfolioista joissa optiopositio on suojattu päivittäin päivitetyllä delta-suojauksella. Kaupankäyntisimulaatiot toteutetaan yksityissijoittajan näkökulmasta sekä kaupankäyntikustannusten kanssa että ilman. Tilastollinen merkitsevyys testataan kaupankäyntituotoille ja outlierit poistetaan tulosten verifioimiseksi.

Tulokset implisiittisille volatiliteeteille osoittivat että Black-Scholes mallin mukainen oletus muuttumattomille implisiittisille volatiliteeteille on virheellinen. Implisiittiset volatiliteetit muodostivat volatiliteettihymyn, minkä lisäksi myyntioptioiden implisiittiset volatiliteetit olivat korkeammat kuin osto-optioiden. Tulokset kaupankäyntisimulaatioille osoittivat että tuotot ennen kaupankäyntikustannuksia eivät ole nolla. Kaupankäyntituotot olivat tilastollisesti merkitsevästi positiivia erityisesti vahvoille miinus- ja plusoptiolle. Epänormaalit tuotot katosivat kun kaupankäyntikustannukset otettiin huomioon.

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In Berlin, 1st of October 2020

Jouko Juvakka

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1 Introduction

Financial option is a potential asset for one market participant and a potential liability for another. Price of an option depends on contingent events which subjects options to probability theory. (Taleb, 16, 1997) At the early stages of the options market in the mid-1980's probabilities for certain events on the underlying price distribution were misjudged and well informed investors were able to gain abnormal profits on options (Hull, 435, 2014). Investigation has continued later with contradictory results regarding option markets efficiency.

Market prices for options can be evaluated by using implied volatilities as a measure for the relative price of an option. Soon after the release of Black-Scholes option pricing model in 1973 it was discovered that the implied volatility of stock options was not constant unlike assumed by Black and Scholes (Rubinstein, 1985). Market prices for stock and stock index options differed based on the strike price of an option. For example, implied volatility function based on strike prices of S&P 500 options had a shape of a smile, which is recognised in literature as volatility smile (Benzoni et al. 2011). However, before 1987 Black-Scholes option pricing model did produce reasonably accurate values for S&P 500 option prices (Rubinstein, 1985).

The stock market crash of 1987 changed the market's view concerning the risk on equity markets and implied volatilities on S&P index options became heavily negatively skewed (Bates, 2000), which led to more severe mispricing of the Black-Scholes formula (Rubinstein, 1994). In practice negatively skewed implied volatilities on index options are referred as out-of-the-money puts and in-the-money calls having relatively higher prices compared to their Black-Scholes benchmark (Christoffersen et al. 2009). Index option markets are well correlated (Äijö, 2008) and after the 1987 crash literature refers to the globally recognized shape of the implied volatility function on stock index options as volatility smirk. In this thesis the term "volatility smile" is used to describe both the smile and smirk shapes of implied volatilities across moneyness categories.

Volatility smirk is a widely accepted phenomenon within index option markets. However, it has been shown that the moneyness, the relationship of spot price and strike price of an option is not the only factor generating inconsistent implied volatilities. The time-to-maturity of an option also affects its level of implied volatility and create a level and slope for implied volatilities, known as term structure (Christoffersen et al. 2009). Unlike for implied volatilities across moneyness categories, there has been no clear recognized pattern for the term structure of index options (Mixon, 2007, Äijö, 2008). The combination of option moneyness and term structure factors describing implied volatility produces a surface, which is referred to as volatility surface in literature.

Differentiating option prices from the Black-Scholes model do not indicate that the markets are mispricing options for stock indexes. However, the nature of options as an insurance is interesting and multiple papers (Bollen & Whaley, 2004, Chan et al. 2004, Duan & Hung, 2010 and Larkin et al. 2012) have published results indicating that especially put options on stock indices in the lower strike price categories have been relatively expensive.

1.1 Motivation and Objectives

European style options on DAX 30 index started trading in August 1991 (Mittnik & Rieken, 2000). Given short history of one of the biggest index option market it is worthwhile to examine whether or not the DAX options are efficiently priced. The assumption is that inefficiencies occurred at the early stages of financial option trading would have disappeared.

Options are used for hedging, speculating and arbitraging purposes. Hedgers reduce their risks related to the movement in the underlying asset, speculators bet on the future movements of the underlying and arbitrageurs enter two or more offsetting positions to lock in a profit. (Hull, 2014, 11) In case that inefficiencies in the market and price differences across options with different features exists, each market participant can either lose or benefit from the mispricing of options.

This thesis comprises only the conventional level of European options. Previous literature on vanilla index option implied volatilities and index option trading profits has documented the relative overpricing of out-of-the-money options. Overpricing of options can be benefited by selling the options when the price is relatively high and by closing the position when the price is relatively low or close to its equilibrium. After the crash of 1987, a concept known as crash phobia has been recognized as one of the potential factors behind the smirk pattern of put index option implied volatilities (Mo et al. 2015). If hedgers are willing to pay abnormally high prices for an insurance against market crash, market participants selling the options should benefit from the relative overpricing.

The first objective of this thesis is to investigate the price information extracted from DAX 30 options against the Black-Scholes assumption of constant implied volatilities across option types, moneyness and maturity. Implied volatilities are used as a measure for relative prices of call and put options separately across multiple moneyness and maturity categories. The second objective is to evaluate the trading profits of DAX 30 options. This study aims to find out whether there are inefficiencies in the DAX 30 option market and whether abnormal profits can be generated, by using a familiar trading framework from previous studies. The role of market makers is taken into account in this study by making the calculations with and without transaction costs. The third objective of this study is to compare the predictive power of option implied volatilities. Assumption is that different option types with different moneyness provide the same forecast for future volatilities of the underlying index. If the Black-Scholes assumption of constant volatilities for the underlying is valid, all the options should provide the same forecast for future volatilities.

1.1 Research Questions and Limitations

This thesis examines the existence of the volatility smile, the term structure of implied volatilities and possible overpricing of DAX 30 index options. In addition, forecasting power of implied volatilities is evaluated on simple terms. Research questions referring to main hypotheses of constant implied volatilities and zero abnormal trading profits along with more specific sub-questions are formed as follows:

1. *Are extracted implied volatilities of DAX 30 call and put options constant through moneyness and maturity as the Black-Scholes option pricing model suggests?*
    a) *Are there differences in mean implied volatilities across call and put option types?*
    b) *Does information for future volatilities of the underlying vary systematically across option moneyness categories?*
2. *Can systematic selling of DAX 30 options together with delta-hedge generate statistically significant profits before or after transaction costs?*
    a) *Do trading profits of DAX 30 options fluctuate systematically across option types or moneyness categories before or after transaction costs?*

This thesis is limited to focus on the existence of inconsistencies in implied volatilities and in possibility to gain abnormal profits on DAX options. Therefore, this study does not determine reasons for possibly inconsistent volatilities or possibly above zero abnormal profits. This paper focuses on finding inefficiencies from DAX option market in the context of option overpricing. This is a decisive factor in deciding the trading simulation designs for the study. Also, testing of put-call parity is out of the scope of this study since the used dataset does not enable accurate pairing of call and put options. As a last limitation, the determination of risk factor related to delta hedging and risk-adjusted profits related to delta-risk are not considered.

1.2 Structure of the Thesis

This thesis consists of two main parts preceded by an introduction chapter. First main part gathers the theory on option pricing and option trading combined with literature review. In sub-sections 2.1-2.4 the key theories of option pricing are discussed, followed by the sub-section dedicated to volatilities. Sub-section 2.6 reviews option trading strategies related to the objective of the study. The theoretical part is finished by section 3 where previous literature on index option implied volatilities and trading profits are reviewed and discussed.

The second main part of this thesis consists of presenting the inputs and the outputs of the empirical work. Data and filtering of the data is presented in section 4, including the characteristics of the underlying stock index. Section 4 is followed by a methodology description where the extraction of implied volatilities, conduction of trading simulations and evaluation of trading profits are explained. The results for implied volatilities and option trading profits are presented separately in section 6. Conclusions are drawn in the last chapter with discussion on the contribution of the thesis and further research.

2 Theoretical Framework

In this chapter the main concepts and tools within option pricing will be introduced. After discussing the basic factors and glossary of option pricing, put-call parity and the two most known option pricing models are introduced together with fundamental market limitations concerning prices of options. The chapter continues by reviewing volatility, the most elemental factor in determination of option prices. Finally, the theory part of this paper is finished by briefly addressing different option trading strategies.

2.1 Introduction to Option Pricing

The total value of an option can be seen to consist of its intrinsic value and its time value. The intrinsic value, or in-the-money part of an option price is defined as the maximum of zero and the value of the option if it were exercised immediately. (Hull, 2006, 188, Taleb, 1997, 18) Since a European option can only be exercised at the expiry, traders typically express the intrinsic value as the difference between the strike price and the corresponding forward price. For a call, in-the-money part is the subtraction between the asset value and the strike price of an option, assuming that the difference is positive. For a put option, relationship is the reverse. (Taleb, 1997, 18)

The value of an option cannot be below zero (Hull, 2006, 188). Hull reminds that the longer the time to expiration is the higher the option value usually is, due to time-value part of an option. Exceptions to this are European options that pay out dividends during the life of an option. For example, if an option with maturity of two months is expected to pay out a dividend before the expiry of an option the option can be less valuable than a corresponding, non-dividend paying option with one month to expiration. (Hull, 2006, 206)

Option prices are affected by six factors (Hull, 2006, 205):

1. The current stock price, S_0
2. The strike price, K
3. The time to expiration, T
4. The volatility of the stock price, σ
5. The risk-free interest rate, r

6. The dividends d expected during the life of an option

All the factors except the volatility of the stock price can be observed from the market. Strike price K and time to expiration T are extracted from option data and stock price S_0 is matched with the options. The risk-free rate r and the expected dividends can be observed from the market. Effect of dividends on option value must be determined carefully, in option pricing they can be included as dividend yield at rate q (Hull, 2012, 350). Other option for including dividend payment for European options is to calculate the present value for the expected dividends and subtract it from the current stock price (Hull, 2012, 320).

The fifth factor on the list, the risk-free interest rate can also affect option pricing through underlying prices. In some cases, interest rate changes have had correlation on stock prices (Hamrita & Trifi, 2011). While chancing interest rates put pressure on stock prices the combined effect can be that the option prices are also affected (Hull, 2012, 217). However, in this paper the risk-free interest rates for different maturities are determined based on current market information as given and focus is kept on analysing factors that are known to have greater impact on option prices. The effects of the above listed variables on option prices are presented in Table 1. The arrows describe the effect on option price when the value of each variable increases (Hull 2012, 215).

Table 1. Effect of variables in the prices of European options

Variable	Call option	Put option
S_0	↑	↓
K	↓	↑
T	↑	↑
σ	↑	↑
r	↑	↓
d	↓	↑

For both option types the effect of volatility increase is always the same, prices increase. For a call option, an increase in the underlying price or in the risk-free interest rate increases the call price. For a put option, the effect is the opposite. For a put option, price rise is expected if either strike price or the expected paid dividends increase. For calls, the effect of strike price and dividends are vice versa. The only remaining variable is time-to-maturity, the effect of which on European option prices is uncertain due to earlier explained possible dividends. (Hull, 2012, 215) However, if dividends during the life of an option are zero, time-to-maturity has an increasing effect on the option price (Cuthbertson & Nitzsche, 2001, 192). This thesis will discuss the option pricing models that use the information listed above to price options. The most popular models in this category, binomial tree and Black-Scholes model are reviewed in detail in sub-sections 2.3 and 2.4.

2.2 Put-Call Parity

Put-call parity is one of the most plain and well-known no-arbitrage relations. Unlike many option pricing models, put-call parity does not require assumptions for the future price probability distribution of the underlying. Also, the assumption for continuous trading along with host of other assumptions are not required. (Cremers & Weinbaum, 2010) In the put-call parity framework the payoff of an asset can be synthetically replicated using a put option, a call option, and a bond (Chen et al. 2014).

Put-call parity describes the relationship between put and call options for European options. It assumes that arbitrage opportunities do not exist, and therefore the example portfolios A and B in Table 2 should have the same value. Portfolio A consists of one call option combined with a zero-coupon bond providing a payoff K at time T. Portfolio B consists of one put option combined with one share of the stock. Both options have the same strike price K and time-to-maturity T. In addition, assumption is that the stock does not pay dividends. Since options can only be exercised at the expiry, there are two possible outcomes for each portfolio A and B. If stock price S_T ends up being above K at time T, both portfolios are worth the stock price at time T. If the stock price at time T is below K, value of the portfolios is K. (Hull, 2012, 221-222)

Table 2. Values of portfolio A and portfolio B at the expiry of the options

		$S_T > K$	$S_T < K$
Portfolio A	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio B	Put option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

Since the values for both portfolios (Table 2) are the same at the expiration of the options the values must be equal also today. (Hull, 2012, 221-222) In other words, under put-call parity framework the net profits from riskless hedge is expected to be non-positive (Mittnik & Rieken, 2000). Put-call parity is written in its basic form as values of two portfolios today in Equation 1, where C is the call option price, Ke^{-rT} is the value of zero-coupon bond discounted with the risk-free rate *r* and the time-to-maturity T. P is the put option price and S_0 is the price of the stock today. (Hull, 2012, 221-222)

$$C + Ke^{-rT} = P + S_0 \tag{1}$$

The relationship of Equation 1 is used to deduce put option value from a call option value with the same exercise price and time-to-maturity and vice versa (Hull, 2012, 221-222).

Violations of put-call parity are known to exist on some degree, but most empirical studies that found violations of put-call parity concluded that these violations often do not represent tradable arbitrage opportunities. Factors making the utilization of the violations difficult are transaction costs, dividend payments, nonsynchronous trades and margin requirements. (Cremers & Weinbaum, 2010, Chen et al. 2014)

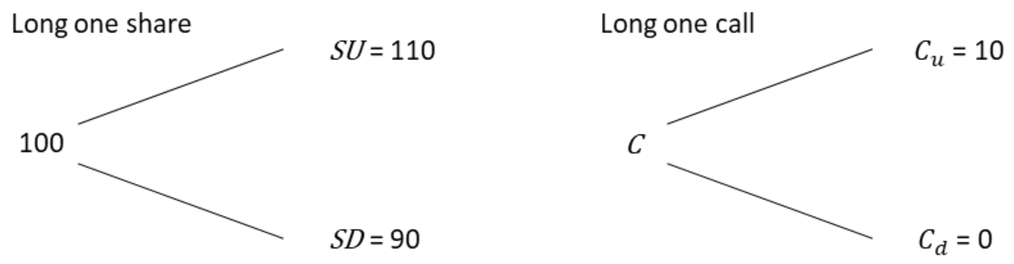
2.3 Binomial Pricing Model and Risk-Free Portfolio Framework

In 1979 John C. Cox, Stephen A. Ross and Mark Rubinstein introduced a simple discrete-time model for valuing options. Cox, Ross and Rubinstein claimed that the development of the model requires only elementary mathematics, despite that it includes a limiting case known as Black-Scholes model. This discrete-time model is therefore another application as an option for more complex Black-Scholes derivation method. (Cox et al. 1979) The binomial model becomes the same as Black-Scholes model when the amount of discrete time steps gets towards infinity (Hull, 2014, 274).

Determination of vanilla European option value with the discrete model in an environment with no dividends, zero transaction costs, no margin requirements and no taxes requires only the values for the following variables: option strike price, underlying price, the rate of interest and the range of movement in the underlying. Probability for the underlying to go up or down is not needed, and therefore all market participants, bulls and bears, must agree on the value of the option. (Cox et al. 1979)

The basic idea of binomial pricing model can be presented by considering one-period problem for a non-dividend paying stock. Suppose that the current price of an underlying is 100, and at the end of the period its price must be either 90 or 110. A call option on the underlying stock, expiring at the end of the period is available with a strike price of 100. Also, for an investor it is assumed to be possible to borrow and lend at a 5 % of interest rate. Current value of the call option can be deduced from the given information, assuming no riskless arbitrage profits exists. The payoffs from holding either a long call with strike price of 100 or one share of the underlying with current price of 100 are given in Figure 1, where SU is the higher possible outcome for the underlying at the end of the period and SD is the lower corresponding underlying value. C is the call option value at t = 0, C_u and C_d represent the higher and lower possible long call option values at t = 1. (Cox et al. 1979, Cuthbertson & Nitzsche, 2001, 197-198)

Figure 1. Stock and option prices in one-step binomial tree

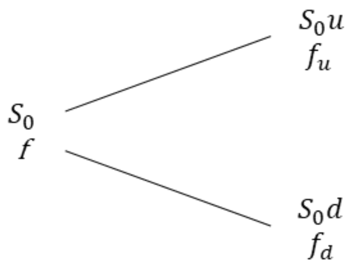


The difference between the possible payoffs is 20 for the share and +10 for the long call. Corresponding the difference for written call would be -10. Now can be written ΔS = 20 and ΔC = 10 and therefore the hedge ratio for the call is $\mathrm{h} = \Delta C/\Delta S$ = 1/2. Portfolio A, consisting of one short call and 1/2 long stocks with $SU = 110$ and $SD = 90$ can be considered to exemplify the meaning of the hedge ratio. Since 1/2 shares is bought and one call sold, the cost of the portfolio A at time zero is (1/2)S – C. The short call position is hedged with long 1/2 a share, meaning that the difference in the payoff on 1/2 a share will offset the difference in the payoff on the written call. Possible payoffs for the portfolio A at t = 1 are (1/2)*(110) – 10 = 45 or (1/2)*(90) – 0 = 45.

Above discussed risk-free portfolio with hedge ratio of -0,5 produces a payoff of 45 independent if the price of the stock at the end of the period is 90 or 110. (Cox et al. 1979, Cuthbertson & Nitzsche, 2001, 197-198) Now the call option price at t = 0 can be determined. Risk-free payoff of 45 discounted back to time zero must equal the cost of the portfolio A: 45/1.05 = (1/2)100 – C. Hence C, the call option value at t = 0 equals to 7.1428517. (Cox et al. 1979, Cuthbertson & Nitzsche, 2001, 197-198)

A simple binomial model for a non-dividend paying stock or stock index with single time step can be extracted to a generalization of the binomial model. Stock and call option prices for general one-step binomial tree are demonstrated in Figure 2: (Hull, 2012, 255)

Figure 2. General stock and option prices in one-step binomial tree



Again, like in Figure 1 only two outcomes are assumed for the underlying stock: S_0u and S_0d where S_0 is the underlying price at t = 0 multiplied by u and d. Value for the option if stock price goes up is f_u ($f_u = S_0u - K$) and correspondingly f_d ($f_d = S_0d - K$) if the stock price goes down.

Multipliers u and d represent the stocks volatility: $u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$, where σ = the observed annual standard deviation of the stock return and Δt = the time to expiration in years. These u and d values proposed by Cox, Ross and Rubinstein (1979) are based on a general theory called Girsanov's theorem. Girsanov's theorem states that even the expected returns on a stock change, for small Δt the volatility of a stock stays the same in both risk-neutral and the real world. (Hull, 2012, 267, Cuthbertson & Nitzsche, 2001, 199) Arbitrage condition is that the risk-free rate must lie between the rate of return in case the stock goes up and in case the stock goes down, so that $u > r > d$ (Cuthbertson & Nitzsche, 2001, 199).

As before, in Equation 2 a portfolio consisting of a long position in Δ shares and a short position in one option is constructed. The two possible values for the portfolio at t = 1 are equal when:

$$S_0 u\Delta - f_u = S_0 d\Delta - f_d \tag{2}$$

or

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d} \tag{3}$$

Now the hedged portfolio is riskless, and it must earn the risk-free rate, otherwise arbitrage opportunities exist. Next, general formulas for finding the present value for the option are presented. Again, risk-free rate of return is discounted with risk-free rate r and it must be equal to the cost of setting up the portfolio. (Cuthbertson & Nitzsche, 2001, 200) Therefore, from the value of the portfolio (Equation 4) and the cost of setting up the portfolio (Equation 5) can be obtained the value of an option (Equation 6):

$$(S_0 u\Delta - f_u)e^{-rT} \tag{4}$$

$$S_0\Delta - f \tag{5}$$

$$f = e^{-rT}[pf_u + (1-p)f_d] \tag{6}$$

where

$$p = \frac{e^{rT} - d}{u - d} \tag{7}$$

The weight p in Equation 7 is known as the risk neutral probability for a stock price to go up, which is simply derived under the assumption that portfolio is riskless and earns the risk-free rate. Probability p must not be confused with the actual probability of a rise in the stock price, which does not affect the option premium. (Cuthbertson & Nitzsche, 2001, 200)

2.3.1 Lower and Upper Boundaries for Option Prices

Equations 6 and 7 make it possible to price an option when possible stock price values are given by one-step binomial tree. The only assumption is that no arbitrage opportunities exists. (Hull, 2012, 256) Price for non-dividend paying call and put options must satisfy the lower option price condition during the options life, presented in Equations 8 and 9:

$$C_t \geq max\ (S_t - Ke^{-rT}, 0) \tag{8}$$

$$P_t \geq max\ (Ke^{-rT} - S_t, 0) \tag{9}$$

Where C_t = call option market price, P_t = put option market price, S_t = level of the underlying, K = strike price of the option, T = time-to-maturity of the option and r = continuously compounded and annualized risk-free rate of return. (Hull, 2014, 240)

At the maturity of an option values are the same as in Equations 8 and 9 without the discounting factor for the strike price K (Hull, 2012, 310). In case arbitrage condition for the lower limit of call option price is not fulfilled, trader can generate arbitrage profits by buying the call options, shorting the underlying index, and lending at the risk-free rate. A possibility for lower limit arbitrage profits for put options can be exploited by buying the puts and the underlying and borrowing at the risk-free rate. (Mittnik & Rieken, 2000)

European vanilla options also have an upper limit for the individual price, which is derived from the character of option contracts. Upper boundary conditions for European call and put options are written in Equations 10 and 11, where C = call option price, P = put option price, $S_0 =$ price of an underlying, r = risk-free rate, $K =$ strike price of an option and $t =$ time-to-maturity (Hull, 2014, 239):

$$C \leq S_0 \tag{10}$$

$$P \leq Ke^{-rt} \tag{11}$$

One European call option gives the buyer the right to purchase one share of an underlying for a certain price and therefore option cannot be worth more than one unit of the underlying. For European put option, upped boundary is the maximum profit from selling the stock at the maturity of an option. (Hull, 2014, 239)

2.4 Black-Scholes-Merton Option Pricing Model

The Black-Scholes-Merton model was a massive breakthrough by Fischer Black, Myron Scholes and Robert Merton in the world of finance in the early 1970s. The Black-Scholes-Merton model, or Black-Scholes model, changed the way how traders price and hedge derivatives. (Hull, 2012, 299) In this thesis focus is kept on the more general Merton's approach to deriving the Black-Scholes-Merton model.

Black and Scholes relied on the capital asset pricing model in order to determine a relationship between the required rate of return on the stock and the required rate of return on the option. Merton's approach involves constructing a riskless portfolio, consisting of the option and the underlying stock. (Hull, 2012, 299, 311) Merton argued that this approach should result in a risk-free rate of return over a short period of time. A significant factor in Merton's approach is that it does not include estimate for expected return, and therefore it follows the earlier presented risk-neutral valuation concept. According to Merton volatility of the underlying can be either estimated from the historical price data or implied from option prices. (Hull, 2012, 299, 311)

Black, Scholes and Merton developed a differential equation (Equation 12), which has many solutions for derivatives with different boundary conditions. Equation 12 corresponds to every derivative that can be defined with S as the underlying. For European call and put options the boundary conditions are corresponding to earlier presented Equations 8 to 11. This solution produces a riskless portfolio for short period of time and therefore over a short period of time the portfolio must earn the short-term risk-free rate of return. If this condition is not fulfilled, possibilities for an arbitrageur appear. Like mentioned earlier, any of the variables in Equation 12 are not affected by investors risk preferences. In Equation 12 f is the price of an option, r is the continuously compounded risk-free rate, S is the price of an underlying, t is the time to maturity and σ is the stock price volatility. (Hull, 2012, 310, Hull, 2014, 332)

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \tag{12}$$

Specific solutions to the differential equation for the prices of European call and put options are presented in Equations 13 and 14 (Hull, 2012, 313-314):

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{13}$$

$$P = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \tag{14}$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{15}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{16}$$

Where $C =$ call option price, $P =$ put option price, $S_0 =$ stock price at time zero, $K =$ strike price of an option, $N(x) =$ cumulative probability distribution function for standardized normal distribution , $r =$ continuously compounded risk-free rate, $\sigma =$ stock price volatility and $T =$ option's time-to-maturity.

Assumptions for the Black-Scholes model are (Cuthbertson & Nitzsche, 2001, 222):

1. Riskless arbitrage opportunities are eliminated
2. There are no transaction costs or taxes
3. Trading is continuous, stocks are perfectly divisible and there are no dividends during the life of the option
4. Investors can borrow and lend at the risk-free interest rate which is constant during the life of the option
5. Stock prices follow stochastic process known as geometric Brownian motion, which gives a lognormal distribution for the stock prices with a constant expected return μ and variance σ

If assumptions for frictionless market, geometric Brownian motion and constant volatility are valid, the Black-Scholes model gives the no-arbitrage price of an option (Lee, 2004). On the other hand, given the Black-Scholes assumptions, all option prices on the same underlying security with different exercise price and the same expiration date should have the same implied volatility. The volatility smile pattern suggests that the Black-Scholes model misprices in-the-money and out-of-the-money options. (Pena et al. 1999) The failure of the Black-Scholes model to characterize the structure of option prices is often thought to be a result from its constant volatility assumption (Dumas et al. 1998).

It was known by the developers of the Black-Scholes valuation formulas and by its sophisticated users that the posited market conditions of Black-Scholes model were idealizations and Black warned of this repeatedly (MacKenzie, 2006). Black and Scholes had done empirical tests with call-option data before releasing their valuation formulas in 1973, and they noticed that bid market prices for the call options were higher than the option prices implied by their model. Their tests indicated that this was due to large transactions costs. However, the markets have changed after the release of the Black-Scholes model and made the assumptions more realistic. Technological change and the growing influence of free-market economics are factors that have affected option trading. Transaction costs are lower, which means even smaller discrepancies can be exploited. MacKenzie stated already in 2006 that the assumption of zero transaction costs is now close to true for major investment banks. (MacKenzie, 2006)

It is natural to question why a failing option pricing model deserves such attention. The Black-Scholes model is still helpful as a language in which to express price of an option. Formulas give a metric by which investors can compare option prices across different underlyings, strikes, maturities and observation times. (Lee, 2004)

2.5 Volatility

In this chapter the role of volatility in option pricing and methods that are used to evaluate the future volatility of returns are discussed. Volatility and forecasted volatility have become an important subject within trading. Volatility forecasting is essential for risk management, portfolio creation and option pricing.

Volatility can be defined as the spread of all outcomes of an uncertain variable. In financial markets volatility is often referred to as equal to risk but this is not exactly the case. Risk is linked with undesired outcome, whereas high or changed volatility can be also a result from positive outcome. From stock owner perspective positive outcome would mean large increase in the stock price. (Poon, 2005, Schwert, 1990) However, it must be mentioned that the changes in the underlying price tends to be negatively correlated with volatility (Vähämaa, 2004). Factors causing volatility are not unambiguous. Potential reasons include information releases, volatility spillovers and returns (Gutierrez et al. 2009), but also the trading itself is recognized to generate price fluctuation (Ghufran, 2016, Hull, 2014, 329).

Volatility is the only variable that cannot be observed from the market when valuing derivative securities. In other words, volatility is the most important variable considering derivatives pricing. In order to price an option, the volatility of the underlying asset from now until the expiration of the option needs to be known. (Poon & Granger, 2003)

Future volatility can be estimated either by historical methods or by using implied volatilities. Models using historical data are for example equally weighted standard deviation (EWSD) model, exponentially weighted moving average (EWMA) model, autoregressive conditional heteroscedascity (ARCH) model and generalized autoregressive conditional heteroscedascity (GARCH) model. (Brooks, 2014, 421-422) In order to retain focus on the key subjects considering objectives of this paper, only the two methods used in the empirical part of this study are outlined in the next two chapters.

2.5.1 Historical Volatility

Typically, in the financial world volatility describes the standard deviation of returns. It can be determined as the standard deviation of the change in a price or a price index, calculated in terms of continuously compounded return. Volatility can be interpreted in multiple ways, one of the most common ways being annual volatility presented in percentages. If it is assumed that the price follows the geometric Brownian motion, the estimate for the standard deviation for any given period T can be derived from annualized, or for example daily volatility. Historical, realized volatility based on equally weighted standard deviation is often used as a benchmark for other volatility estimations and can be calculated based on high- or low frequency data. (Brooks, 2005, 189-190)

Standard deviation s of a set of previous returns is dependent on n trading periods, continuously compounded returns $r_t - n, \ldots, r_t - 1$ and the average return $\overline{r}$ (Equation 17): (Brooks, 2005, 189-190)

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_{t-i} - \overline{r})^2} \tag{17}$$

Uncertainty of future outcome increases the longer time scope is used, which can be demonstrated by observing the changes in standard deviation of a stock or stock index. Standard deviation increases approximately with the square root of the forward-looking time span. Hull states that typical annual volatility for a stock is between 15 % and 60 %. (Hull, 2012, 303) Annualized volatility for DAX 30 index from January 1999 to January 2009 was 20.9 % (Bentes, 2014).

2.5.2 Implied Volatility

Implied volatilities are forward looking volatilities implied by option prices. They are used to monitor the market's view about the volatility of an underlying asset. (Hull, 318-319, 2012) In other words, implied volatility is the market's expectation of underlying future volatility, time-averaged over the remaining life of the option. In practice Black-Scholes option pricing model has an important role in discussion about implied volatilities; implied volatility is the unique underlying volatility parameter for which the Black-Scholes model recovers the option price. In fact, standard in industry is to quote options prices in "vol" points. (Lee, 2004) From option pricing perspective implied volatility informs how market is currently pricing an option (Ahmad & Wilmott, 2005).

2.5.2.1 Potential Explanations for Existence of Volatility Smile

If instantaneous volatility of an underlying asset is nonconstant, then implied volatilities on that underlying will exhibit variation in respect of strike price and/or expiry. These variations are described graphically as a volatility smile and volatility term structure. (Lee, 2004)

Previous literature offers multiple explanations especially for the volatility smile without clear consensus.

Literature from 21st century set out from the assumption of a frictionless market and explains the volatility smile by market failures. Market order imbalance, non-synchronized trading, discrete trades, and transaction costs are potential components of imperfect markets. (Duan & Hung, 2010) After the market crash of 1987 common smirk pattern for index options is often referred as crash phobia. Higher demand for OTM put options are seen to make them more expensive over OTM call options. (Mo et al. 2015) Also Bollen and Whaley (2004) explained the volatility smirk of put index options by order imbalance. Net buying pressure of put options, recognized widely in the index option market in the beginning of 2100 century has been observed as factor to result implied volatilities to vary across moneyness. Also transactions costs, derived from bid-ask spreads, seem to have a key effect on the curvature of the volatility smile (Pena et al. 1999).

More traditional explanation for the volatility smile or skew is the time series characteristics of the stock and stock index prices. Brooks (2014, 415-416) reminds that financial data has a number common, important features that cannot be explained by linear models. Linear nature of a model means that it is linear in the parameters, so that one parameter is multiplied by each variable. Features that linear models are unable to explain include leptokurtosis, volatility clustering and leverage effects. (Brooks 2014, 415-416) In addition, one recognized quality of variance is that it tends to reverse to its long-term average. It means that variance has either a positive or a negative drift (Hull, 2012, 503).

Black-Scholes model expects normal distribution for returns, which is not consistent with leptokurtic distribution. Financial asset returns tend to have excess peak, fat tails and stochastic volatility. (Brooks, 2014, 415-416, Schoutens et al. 2003) These characteristics, kurtosis above or below 3 and skewness different from zero is correlated with volatilities implied from option prices (Blaskowitz et al. 2004). For example, left-skewed distribution of stock returns entails deep in-the-money call options to have higher implied volatilities compared to at-the-money or deep out-of-the-money call options (Corrado & Su, 1997). This is a consequence of left-skewed underlying distribution, where the probability for the stock to go down is larger than normal distribution expects and correspondingly the probability for up movement in the underlying price is smaller than normal distribution assumes (Hull, 2014, 437).

Correspondingly, if implicit distribution of the underlying returns is leptokurtic in both the left and the right tail, deep-in-the-money and deep-out-of-the-money options are more valuable than predicted by Black-Scholes model. (Pena et al. 1999) Since put option prices can be derived from the call option prices within put-call parity framework, effects of the underlying return distribution are the same as for call options with the same strike prices (Hull, 2014, 431).

Effects of stochastic volatility and non-normal distributed returns can be included in option pricing by using pricing models where restrictive assumptions for volatility and distribution of returns are relaxed (Schoutens et al. 2003). Therefore, the Black-Scholes model has been attempted to be modified to describe the volatility smile with a one factor stochastic model assumption (Duan & Hung 2010).

2.5.2.2 Potential Explanations for Existence of Inconsistent Term Structure

The term structure, the variation of implied volatilities across maturities is the other dimension of volatility surface (Fouque et al. 2004). Under the Black-Scholes model options term structure is a flat line (Mixon, 2007). From a trader's viewpoint, implied volatilities are allowed to be dependent on time-to-maturities, depending on the market expectations at the current moment. If the expectation is that the volatility will go up, implied volatilities tend to have an increasing function of maturity. Similarly, implied volatilities tend to have a decreasing function of maturity if the volatility of the underlying is expected to go down. (Hull, 2012, 416) Market's expectations linked to the term structure of options is referred in literature as expectations hypothesis (Mixon, 2007).

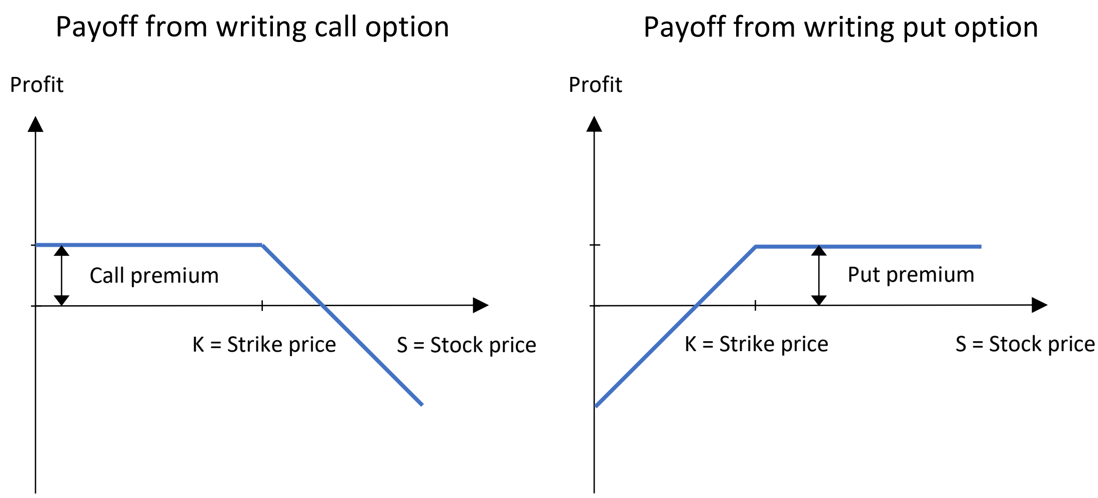
Differing implied volatilities across maturities form the implied volatility term structure where the slope of the term structure should reflect the market participant's expectations of underlying volatilities through future horizons (Xu & Taylor, 1994). In overall, it is found that term structure of index option volatilities changes its slope from increasing to decreasing from time to time without obvious patterns (Mixon, 2007, Äijö, 2008) and the forecast power of implied volatilities weakens the longer time-to-maturities are considered (Mixon, 2007).

2.6 Trading Strategies for Mispriced Options

Option mispricing for European call or put option is based on a belief or intuition that the options price implies greater or lower volatility in the underlying than is likely accurate (Barry & Taggart, 2007). Market participants value options with volatility derived from the distribution of returns of the underlying. Opportunity for abnormal profit rises if probabilities for price or value changes are estimated incorrectly by the market. (Hull, 2012, 412-413)

If the investor estimates that the volatility of the underlying will be lower than what is implied from the underlying's option price, he or she can use vanilla options with or without hedging to benefit from possible option overpricing. Simple trading strategy for overpriced call or put option is to sell the option when the price is high and buy it back when the price has fallen. This kind of naked position is not hedged at all, hence there is a risk for high losses. (Barry & Taggart, 2007) Payoffs for writing call and put vanilla options are described in Figure 3 (Cuthbertson & Nitzche, 2001, 237):

Figure 3. Payoffs from writing call and put options

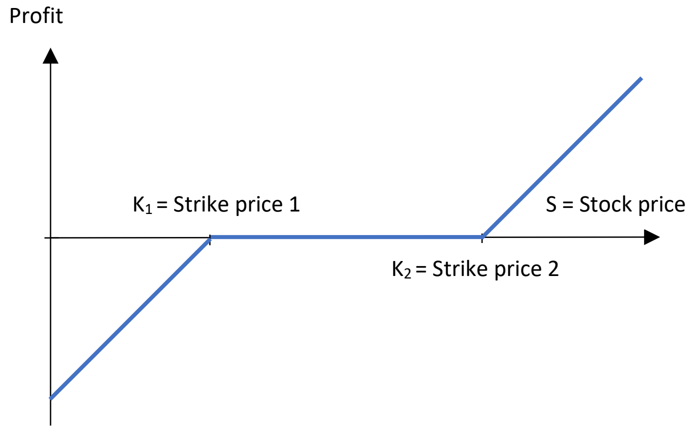


Volatility of the underlying is only one of the factors affecting the price of an option. Other factors could cause the underlying price to rise or fall while the short or long option position is in place. To prevent this, one can use the underlying stock and other options on that underlying to set up a hedged portfolio position. (Barry & Taggart, 2007)

There are multiple strategies for hedging position of potentially mispriced options, some being more common than others. As the first alternative to not hedge is to adopt a static covered position. In covered position an investor sells an option and either buys or sells shares of the underlying asset, depending of the option type. The number of shares bought or sold is equivalent to the number of shares the option gives the owner the right to buy or sell. However, a covered position can still lead to a significant loss and it does not provide consistent results. (Hull, 2012, 378)

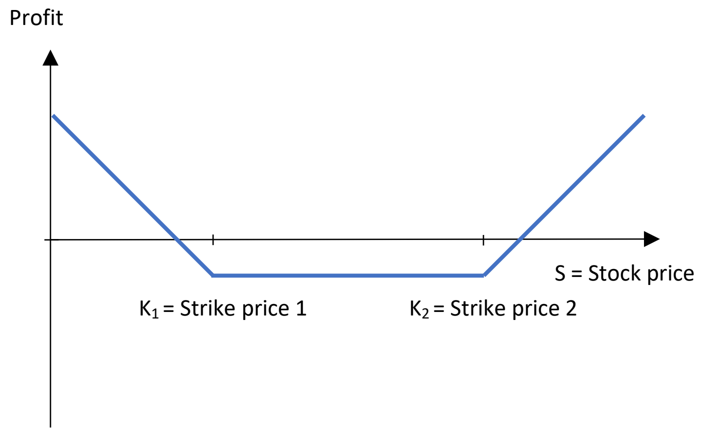
A more specific example of option strategies for exploiting asymmetric and fat-tailed properties of the underlying's price distribution is to use risk-reversals and strangles (Wilmott, 2007). If the investor considers that high prices of the underlying are more likely to occur over low prices, the investor can create a risk-reversal position consisting of long European call and short European put, both options having the same maturity date and the call strike being higher than the put strike (Blaskowitz et al. 2004). Net position of risk-reversal position in the setting up stage is depending on the option prices. If the price of the sold put option is higher than the price of the bought call price, the strategy generates an initial cash inflow and vice versa. An example of risk-reversal strategy is provided in Figure 4, where vanilla put option is sold at strike price 1 and vanilla call option is bought at strike price 2 (Kapner, 2014):

Figure 4. Payoffs form risk-reversal strategy



Similarly, an investor expecting that large moves of the underlying are likely to occur can enter a long strangle consisting of a long position in a call and a long position in a put with same maturity and lower strike price. Example of long strangle is presented in Figure 5, where put option is bought at strike price 1 and call option is bought at strike price 2: (Blaskowitz et al. 2004)

Figure 5. Payoffs from long strangle



Third popular option is to create dynamically delta-hedged portfolios. One benefit of dynamic hedging is that it unifies different option types. A dynamic hedger is not interested in whether the hedged instrument is call or put option. Delta hedging will make them identical, consequently only the strike price and the expiration of an options are the factors that matter. (Taleb, 1997, 11) Delta hedging can be implemented with multiple different setups and limitations, of which some are discussed in the next section.

2.6.1 Delta Hedging

Delta, also known as hedge ratio, is the most important hedging parameter and the easiest to adjust. Putting up and maintaining delta hedged position only requires trading of the underlying asset. (Hull & White, 2017) The reason why options are hedged with the underlying instead of underlying being hedged with the options is that dynamic trading strategies in options are expensive (Carr, 1999).

Delta hedging is based on the earlier presented riskless portfolio framework. In delta hedging a portfolio of stock shares and option contracts is constructed in a way that the loss or gain from the shares is offset by the loss or gain from the option contracts over a short period of time. (Cuthbertson & Nitzche, 2001, 237) Delta hedging protects the investor for small price changes in the underlying by including positions in the two negatively correlated, offsetting positions. Trading mispriced options by using delta hedging should be moderately effective if mispricing of the option is corrected quickly and the underlying price does not move too much. If this is not the case, risk for delta changes arises. Gamma, the rate of change of delta with respect to the underlying price is used to measure the risk for changes in option's delta. Gamma is the option's second partial derivative corresponding to the underlying price. (Barry & Taggart, 2007)

Delta is often calculated from option valuation model introduced by Black, Scholes and Merton (Hull & White, 2017). Delta based on the Black-Scholes option pricing model enables users to construct a portfolio that makes it possible to exploit a mispriced option and still hedge against changes in the underlying price (Barry & Taggart, 2007). One of the alternative methods for delta determination is to use binomial model introduced in section 2.3. However, binomial model sacrifices some of the accuracy due to the discrete payoffs in the tree model. (Chung & Shackleton, 2002) Delta calculation based on the Black-Scholes model for call and put options are presented in Equations 18 and 19:

$$\Delta(Call) = N(d_1) \tag{18}$$

$$\Delta(Put) = N(d_1) - 1 \tag{19}$$

where

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{20}$$

$N(x)$ is the cumulative distribution function of the standard normal distribution, S_0 is the stock price, K is the strike price, r is the continuously compounded annual risk-free rate, σ is the volatility of the underlying and T is the time-to-maturity of an option in years. (Hull, 382, 2012)

Since the prices of the two assets have either a positive or negative correlation based on the option type, the construction of the portfolio is the opposite for call and put options. Delta hedging with calls involve being short on one asset and long in the other (Cuthbertson & Nitzche, 2001, 237). In order to construct a fully hedged portfolio V (Equation 21) consisting of N_S stocks with price S together with N_C call options with price C hedge ratio h_C (Equation 22) is set to neutralize the relative difference in the price changes of the stock and the option (Equation 23) (Cuthbertson & Nitzche, 237-238, 2001):

$$V = N_S S + N_C C = N_C(h_C S + C) \tag{21}$$

where

$$h_C = N_S/N_C \tag{22}$$

and

$$\frac{\partial V}{\partial S} = h_C + \frac{\partial C}{\partial S} = 0 \tag{23}$$

Now the portfolio is independent of the changes in the stock. Delta of call option lies between 0 and 1, stating not only that the correlation between the underlying and call option is negative, but also that the option premium increases or decreases always less than the share price. Therefore, in delta hedging the number of shares is always less than the number of options. (Cuthbertson & Nitzche, 2001, 237-238)

A delta hedged portfolio construction for put options is the opposite from call options since delta for put options is negative and lies between 0 and -1 due to positive correlation between the underlying and the options. Long put position is hedged by buying the shares and short put position is hedged by entering a short position in the shares. The construction of fully hedged portfolio V of N_P puts and N_S shares is presented in Equations 24, 25 and 26 where S is the stock price, P is the put option premium and h_P is the hedge ratio. (Cuthbertson & Nitzche, 2001, 240)

$$V = N_S S + N_P P = N_P(h_P S + P) \tag{24}$$

$$h_P = N_S / N_P \tag{25}$$

$$\frac{\partial \mathrm{V}}{\partial \mathrm{S}} = h_P + \frac{\partial \mathrm{P}}{\partial \mathrm{S}} = 0 \tag{26}$$

Proper delta hedging requires volatility input to be correct. Dynamic hedging based on deltas involves continuous trading to maintain delta neutrality. In practice delta hedging is never perfect, maintaining delta neutrality at all times would theoretically entail an infinite number of hedging transactions on the underlying security. Transaction costs and restricted opening hours of exchanges mean that in practice the rebalancing of the hedge cannot always be done as often or at a time when it would be needed. Instead, position delta is neutralized approximately, with frequent periods when the hedging position is somewhat imprecise. The result of this kind of hedging is that the price risk is not fully eliminated. (Green & Figlewski, 1999)

Overall, determination of delta is complicated and problems especially with the non-constant risk-neutral distribution combined with inconsistent interest rates are addressed (Mykland, 2000). Risk-neutral distribution is the forward-looking view of the price distribution of the underlying asset, and therefore it is used to assess market expectations on the future changes in the underlying. The Black-Scholes option pricing model assumes risk-neutral distribution to follow the normal distribution. (Kim & Kim, 2015) Distribution of the underlying returns is connected to volatility smiles and option term structure, which need to be included in efficient delta hedging (Vähämaa, 2004).

2.6.1.1 Choosing Volatility for Delta

Volatility parameter for delta, also known as hedging volatility, affects the delta value and therefore the hedging result. Hedging error rises with the difference between the hedging volatility and the realized volatility of the underlying. If the difference between the hedge volatility rate and the realized volatility rate is zero, hedging error is also zero. (Jouini et al. 2001) Figure 6 illustrates how delta varies for call and put options with two different volatility inputs.

Figure 6. Option delta affected by implied underlying volatility

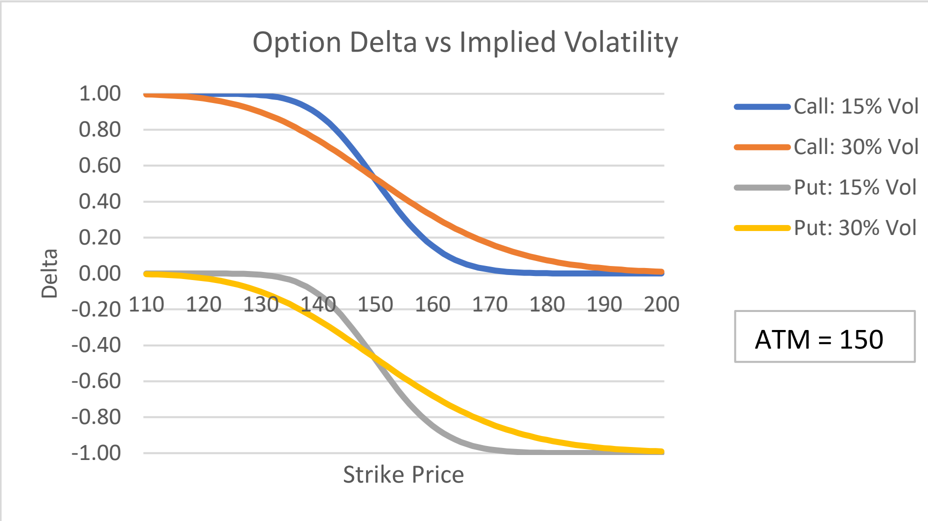


Figure 6 shows examples of call and put option deltas across strike prices with two different implied volatilities, 40 days to maturity, 1 % risk-free rate and with current stock price of 150. This is an illustration of a general process of the change in delta when stock or index price changes. Since the price change of an option is relatively highest for options close-to-money, delta of an option is also changing rapidly around the strike price. The slope of the change in delta diminishes for out-of-the-money and in-the-money options.

To make a profit regarding delta hedging knowing the actual volatility is not needed, instead for an option seller, knowing that the actual volatility will always be less than the implied volatility is enough to make a profit. From an option buyer's perspective, a profit could be generated if the actual volatility would always be higher than the implied volatility. Altogether, hedging with implied volatilities provides the smoothest mark-to-market profit behavior. (Ahmad & Wilmott, 2005)

The usual market practice for calculating delta is to use the options' implied volatility as the volatility parameter. This is called practitioner Black-Scholes delta. (Ahmad & Wilmott, 2005, Hull & White, 2017) Advantages of using implied volatility as a volatility parameter for delta are that the profit is deterministic and forecasting of volatility can be less accurate. Deterministic profit behaves better from a risk management perspective, although the present value of total profit is path dependent. (Ahmad & Wilmott, 2005)

Regardless of implied volatility delta hedging posing remarkable positives in hedging result, choosing the hedging volatility in complete generality is difficult. If the Black-Scholes assumptions excluding constant volatility are assumed to hold, profit can be made by choosing certain hedging volatility so that the true volatility stays on one side of the hedging volatility. However, uncertainty of the fulfilment of assumptions and the level of real volatility together with the changed variance of profits by hedge-volatility make choosing of volatility parameter for delta ambiguous. (Carr, 1999)

3 Literature Review

Reviewing the previous literature in this chapter is divided in two sections. Due to the limited amount of previous research literature on DAX options implied volatilities and trading profits, previous results from other index option markets are included as well. The starting section of this chapter is possessed for explaining the finding and selecting process of the related literature.

3.1 Search Process

Finna portal was used for the initial search of international scientific e-publications. Finna offers access to multiple databases with highly customizable search criteria, which made it a valid platform to find previous literature on this study. The following keywords were used separately and with multiple combinations to initialize the search: “implied volatility”, “volatility smile”, “term structure”, “put-call parity”, “option trading”, “trading profits”, “abnormal profits”, “index options” and “DAX index”.

Finna offers multiple options for limiting the search; for example, whether to find the keywords in titles, full texts, or abstracts. Finding studies with DAX option data was the most efficient by looking for the keywords in the full texts, since there is often no reference to the used data in the title of the study. Search results of Finna were complemented with search on Google Scholar with the above listed keywords. Research on different platform managed to provide some additional articles related to the subject.

The filtering of the search results led to excluding most of the articles. The most important determinant whether to include the article was the object of the research. In addition, due to the development in the option trading industry through the decades, articles using data sample including observations before year 1990 were excluded. After initial search and filtering of the articles backward reference searching was implemented. As a result, a total of 11 articles are reviewed in the upcoming sections.

3.2 Volatility Smile and Term Structure of DAX Options

Due to the limited amount of research on the term structure of stock index implied volatilities (Äijö, 2008) only two of the reviewed studies on DAX options include both dimensions of the volatility surface. Since different studies used option datasets with different filters and criteria, all the studies on DAX options market are discussed independently. Summary of reviewed studies in this section are listed in Table 3.

Table 3. Summary of reviewed literature on implied volatilities of DAX-index options

Author(s) and year	Object	Used data
Hafner & Wallmeier, 2000	Profile of DAX implied volatilities, explanators of volatility smile pattern	Intraday prices of DAX options from January 1995 to October 1999
Branger & Schlag, 2004	Pricing differences between index and stock options	Weekly prices of DAX options from 1998
Mixon, 2007	Testing of explanation hypothesis in context of term-structure	Weekly prices of DAX options from May 1994 to October 2001
Muzzioli, 2011	The skew pattern of implied volatility	Intraday prices of DAX options from July 1999 to December 2005
Da Fonseca & Grasselli, 2011	Calibration of volatility models	Daily prices of DAX options data from 28.8.2008

Hafner and Wallmeier (2000) used the Black-Scholes option pricing formula to extract implied volatilities for each option trade of DAX options recorded from January 1995 to October 1999. The purpose of the study was to model DAX IVs across moneyness categories and to find explanatory variables for volatility smiles. In order to compute implied volatilities, Hafner and Wallmeier measured time-to-maturities in calendar days, used DM-LIBOR and

EURIBOR rates as risk-free rate estimates and derived the underlying index level from the futures contracts on DAX.

Unlike other studies on DAX options (Branger & Shclag (2004), Mittnik (2000), Muzzioli (2011)), Hafner and Wallmeier (2000) report that dividends might still have an effect to DAX index value. They argued that the amount of cash dividends reinvested after deduction of the corporate income tax might differ due to possible differences in investors' tax rates. If the marginal investor's tax rate is lower than the corporate tax rate then extra dividend or difference dividend is received, which would cause declining effect on index on an ex-dividend day. Opposite effect would occur if the marginal investor's tax rate is higher than the corporate tax rate. Hafner and Wallmeier found this kind of causal relationship based on the options implied volatilities by moneyness of DAX 30 options. The implied volatilities on DAX-index options of puts were lower than the corresponding implied volatilities of calls and showed a downward concave shape, whereas call option implied volatilities showed a pattern of a smile. As a solution, Hafner and Wallmeier increased the DAX-index level by approximately 8 index points.

The main object in the study was DAX options that had a degree of moneyness between 0.95 and 1.05 and time-to-maturity of 45 days. The data was divided in two subperiods, from 1995 to June 1997, and from June 1997 to 1999, respectively. Also, options with above 150 % implied volatility were removed when the smile profile was estimated via regression analysis. Results in the context of volatility smile pattern showed that, on average, the smile pattern appeared as a skew. Implied volatilities decreased with increasing moneyness until the right border, where the function of implied volatilities rose slightly. On average, implied volatilities decreased by 4.2 percentage points when corresponding increase in moneyness was 0.1.

Other findings related to this study for options with TTM of 45 days were that the variation of moneyness explained about 95 % of the cross-sectional variation of DAX implied volatilities. Hypothesis of high demand for out-of-the-money put options held true, which pushes prices of those options. Referring to the existence of volatility smile and term structure, Hafner and Wallmeier (2000) also reported regressed implied volatility patterns for individual dates for time-to-maturities of 21, 28 and days. Implied volatility pattern across moneyness categories clearly developed from smile to skew as the TTM increased, which is widely recognized pattern for index options.

Branger and Schlag (2004) reported implied volatilities on pooled DAX options for both dimensions of volatility surface, using option data from year 1998. The objective of the study was to explain the downward sloping shape of the volatility smile and why the shape differs for individual and index options. Study shows the smile patterns for maturities of one, three and six months with moneyness ranges from 0.85 to 1.15. Implied volatilities of DAX options had a skew shape, which according to Branger and Schlag (2004) developed systematically less deep the longer maturity was considered. While options with one month to maturity had the highest IV for options with moneyness below approximately 0.96, options with the longest maturity of 6 months had the highest IVs in moneyness categories above approximately 0.96. Therefore, the term structure was dependent on the moneyness of options.

Branger and Schlag (2004) discussed the risk-neutral distribution as the source for volatility smiles and changing term structure. They concluded that the probability for market crash is a determinant for left-skewed risk-neutral distribution causing implied volatility skews. The changing shape of the smile or skew by maturity is explained by the underlying distribution approaching the lognormal case.

Muzzioli (2011) investigated the variation of option information content according to option moneyness and option type. Upon this, the study focused on comparing implied volatility-based forecasts with historical volatility of the DAX-index. The information content of call and put options was examined only for the most liquid at-the-money and out-of-the-money options. The study eliminated in-the-money put options, defined as having strike price to underlying price ratio above 1.03 and in-the-money call options, defined as having strike price to underlying price ratio below 0.97. OTM calls category consisted of calls with moneyness ratio above 1.03, whereas OTM puts had moneyness ratio below 0.97. Options included had time-to-maturity of approximately one month; maturities were between 17 to 22 days to the options expiry. The derived implied volatilities were compared to historical volatilities and forecasting performance of both option types and strike price categories were evaluated.

Data consisted of intraday data on DAX options during the period of July 1999 till December 2005. One-month Euribor rate was used as a proxy for the risk-free rate and intraday prices of the DAX-index served as underlying prices. Upon the filter for strike prices mentioned earlier, Muzzioli filtered options with price less than one euro, options with trading volumes of less than 100 options or one contract and options that violated arbitrage condition. Also, time-filter was incorporated, cleaned data set included only options recorded from 3.00 p.m. to 4.00 p.m.

Average results for ATM and OTM put and call implied volatilities across strike price categories formed a skew pattern, where implied volatility was a decreasing function of strike price. Based on the explanatory power measured by R squared, regression results suggested that ATM options had more information value as explanator for the volatility of the underlying.

In the context of term-structure, Mixon (2007) studied whether the expectations hypothesis holds true for the following indices: SPX, FTSE, DAX, CAC and NKY. Expectations hypothesis is based on the assumption that the slope of term structure behaves as a forecast for future short-term implied volatilities. The raw data were bid side implied volatilities for at-the-money DAX call options, collected at the end of each week over the period from May 1994 to October 2001. Data was given by a major investment bank, which eliminated the direct need for estimating risk-free rates and index values.

Resulting implied volatilities, computed with the Black-Scholes formula for option maturities of 1 month, 3 months, 6 months and 12 months showed upward sloping term-structure for at-the-money DAX options. The results also indicated that the slope of term-structure is a significant predictor of forthcoming short-dated implied volatility, although the predictions did not match with the expectations hypothesis, which was therefore rejected. Although implied volatilities have explanatory power on future volatilities, Mixon argued that the implied volatilities tend to be overpriced especially as the prediction horizon lengthens.

Upward sloping term structure caused the longer maturities to have higher differences to the actual volatilities of the underlying.

Da Fonseca and Grasselli (2011) investigated calibration properties of multi-factor stochastic volatility models by using vanilla options on DAX, FTSE and EuroStoxx50 indices. Data set of DAX call and put options was quoted on Auqust 28, 2008 and forward moneyness range from 0.8 to 1.2 was applied. Research concentrated on the most liquid options in similar fashion to Muzzioli (2011), and only OTM options were included. Options had maturities from three weeks to four years, which gives the reader an opportunity to also explore the term-structure part of the volatility surface.

As a result from interpreting the smile dimension of volatility surface, options with maturity of three weeks had a shape of smile rather than a smirk. The smile pattern faded away towards longer maturities and options with longer than four months to maturity did not show any kind of rise at the right end of moneyness range. For the term structure part of the study, Da Fonseca and Grasselli (2011) present similar results to Branger and Schlag (2004). Options with shorter maturities dominated the longer maturities in terms of IV below moneyness levels of approximately 0.92. Above the same level, longer maturities started to show higher implied volatilities. Exception to this were the 3-week options that had the highest IVs in both ends of the moneyness spectrum.

To conclude the literature review on DAX implied volatilities and the term structure, reviewed studies showed a similar pattern for both dimensions of the volatility surface. Studies reported that the implied volatilities are not constant across moneyness nor maturity categories. The shape of the volatility smile or preferably smirk was present in all studies. The term structure of the DAX options varied across moneyness categories, and on the other hand was dependent on market’s expectations on future volatilities.

3.3 Trading Profits of Index Options

Based on the amount of literature, volatility informed stock index option trading gained attention especially during the beginning of the 21st century. In 2004, The Journal of Finance published a paper of Bollen and Whaley who investigated reasons for volatility smile and trading profits of stock index options. This paper has been referred in many issues that have investigated the possibility for finding abnormal profits based on volatility smile or volatility surfaces, e.g see Chan et al., 2004. As research literature on trading profits of DAX options across moneyness and maturity categories is limited, findings from other index option markets will next be reviewed. Papers on other option markets are chosen based on their contribution in the research of option pricing and market location, so that the reviewed articles would cover multiple continents. Overview for discussed articles is presented in Table 4:

Table 4. Summary of reviewed literature on trading profits of stock index options

Author(s) and year	Object	Used data
Mittnik & Rieken, 2000	Validity of put-call parity for DAX-index options	Intraday prices of DAX options from February 1992 to September 1995
Blaskowitz, Härdle & Schmidt, 2004	Profitability of skewness and kurtosis trades	Daily closing prices of DAX options from June 1997 to March 2002
Bollen & Whaley, 2004	Net buying pressure, implied volatilities and option trading profits	Intraday prices of S&P 500 options from January 1995 to December 2000
Duan & Hung, 2010	Net buying pressure, implied volatilities and option trading profits	Intraday prices of TXO options from December 2001 to June 2005
Larkin, Brooksby & Zurbruegg, (2012)	Net buying pressure, implied volatilities and option trading profits	Intraday prices of S&P ASX 200 options from September 2006 to November 2008

Put-call parity violations and option-market efficiency can be evaluated by using tests that are solely based on the requirement of non-systematic arbitrage opportunities. The advantage of these non-model-based tests is that less assumptions are needed to be fulfilled. So called pure arbitrage tests do not assume particular pricing model, and therefore the validity of the pricing model and the correct specification of the model parameters are excluded, as well as the mentioned probability distribution for the underlying asset. (Mittnik & Rieken, 2000)

Mittnik and Rieken (2000) tested put-call parity and market efficiency with this kind of tests using regression analysis based on rearranged put-call parity relationship and nonparametric sign test. Sign tests were conducted by converting call prices into put prices using put-call parity and comparing the prices to market prices of put options. Economic significance of the results was judged by examining the possibility for systematic profit opportunities. Study included DAX option data from February 1992 to January 1995 with maturities up to six months. They used interbank bid and ask rates as risk-free rate estimates up to six months and screened the data for transactions under fair option value.

Based on regression results, Mittnik and Rieken reject put-call parity from statistical viewpoint. Put options on DAX were relatively overpriced, especially at the low strike price categories. To test whether specific types of options were responsible for the results, Mittnik and Rieken categorized the options for sign tests by moneyness ($m = \log{(I_t/Ke^{rT})}$) and maturity. Call options were divided in out-of-money ($0.9 \leq m < 0.98$), at-the-money ($0.98 \leq m \leq 1.02$) and in-the money ($1.02 < m \leq 1.1$) categories. Options farther from these were assigned in deep-in-the-money and deep-out-of-the-money categories. The moneyness classes for put options were analogous. Time-to-maturities of six maturity classes were the following: 0 to 10, 11 to 30, 31 to 60, 61 to 90 and 91 to 182 days to maturity.

The result was mostly repeated with sign tests; call options were relatively underpriced in all moneyness classes except the moneyness class consisting of out-of-the-money calls and in-the-money puts with maximum 10 days to maturity. In general, both type of tests suggested relative overpricing for put options from a statistical point of view. Ex-ante tests that reviewed the economic significance of the results were implemented by constructing long and short hedge portfolios with and without transaction costs and with different levels of execution lag. When ignoring the transaction costs, statistically significant profits could have been generated, especially with short hedge strategy. Including transaction costs decreased the mean portfolio profits for most part. Short strategy held on better on the profits when costs and execution lag was increased, but for the most recent sample year, both long and short strategies would have generated immensely significant losses.

Overall, put options maintained their higher profits in ex-ante tests, but the practical realization of these profits was questioned by Mittnik and Rieken (2000) due to transaction costs and restrictions for shorting at the time in Germany. They also remind that the ex-ante profits were not riskless, instead they are subject to immediacy risk. This could be seen as a risk premium for potential profits, Mittnik and Rieken (2000) concluded. Study also found that pricing of DAX options changed during the sample period. They conclude that the traders went through a learning process. This is something that is difficult to take into account, although trading nowadays is easier and on the other hand possibilities for abnormal profits may have vanished in 21st century.

Blaskowitz et al. (2004) conducted an option trading profit investigation on DAX options by generating portfolios that differed from the common practise of putting up and maintaining delta hedged portfolio. Instead of using the underlying as one instrument, Blaskowitz et al. set up skewness and kurtosis trades, skewness and kurtosis in this context referring to the skewness and kurtosis of the implied state price densities (implied SPD). They found violations of the Black-Scholes assumptions for the distribution of DAX returns and implemented risk-reversal and strangles in order to exploit possibilities for abnormal profits on DAX options. Data included call and put options with three months to maturity from June 1997 to March 2002. Data was considered as one overall period and separated into two subperiods, of which the first was from June 1997 to March 2000 and the second from June 2000 to March 2002.

Found evidence of left-skewed SPD resulted Blaskowitz et al. (2004) to sell all OTM puts ($K/S_t e^{rT}$ < 0.95) and buy all ITM calls ($K/S_t e^{rT}$ > 1.1). Kurtosis trade composed of shorting puts with moneyness below 0.90 and from 0.95 to 1.00. Puts were bought at moneyness levels between 0.90 to 0.95. Short position on calls was entered if moneyness level was between 1.05 and 1.10. Calls were bought at moneyness levels between 1.00 to 1.05 and above 1.10. Results from skewness and kurtosis trades signalled highly positive returns on the first subperiod, but on the second subperiod both strategies lost compared to risk-free investment. Similar to the paper of Mittnik and Rieken (2000), Blaskowitz et al. remark that the excess return could be a feature of the risk premium.

Bollen and Whaley (2004), Duan and Hung (2010) and Larkin et al. (2012) investigated implied volatilities and trading profits of index options with net buying hypothesis. Net buying hypothesis, introduced by Bollen and Whaley (2004) led to a better understanding of option pricing and has been the reason for the findings of abnormal option trading profits reported by Duan & Hung (2010) and Larkin et al. (2012). These studies investigated the role of supply and demand on option prices and found the net effect of supply and demand to have an influence on implied volatilities and trading profits across option types, moneyness levels and time-to-maturities.

The above-mentioned studies had a relatively uniform way of investigating implied volatilities and trading profits of index options. Studies separated options into moneyness and maturity categories, computed the implied volatilities and implemented option trading simulations by using naked trading and delta hedging strategies with and without transaction costs. Naked positions were set up by selling call and put options with different maturities and moneyness levels. Dynamically delta hedged portfolios consisted of either call or put options together with an appropriate position in futures of the underlying.

As an exception, Bollen and Whaley (2004) considered only moneyness dimension of volatility surface with total of six different trading simulation set-ups, including also delta-vega neutral portfolios. Since the results from Bollen and Whaley (2004) on S&P 500 options are separated for each trading simulation, in this study we concentrate on discussing only the delta-neutral strategies of the paper. The delta-neutral results are considered with and without trading costs for the subperiod from January 1995 till December 2000.

Although the data sets and parameter inputs such as risk-free rates, hedging volatilities and estimates for the trading costs differed between the studies, the results indicated the same kind of patterns for the implied volatility patterns, pricing of options and trading profits across moneyness and maturity categories. Bollen and Whaley (2004) found that the difference between implied and realized underlying volatilities was dependent on the moneyness of options, driven by the net buying pressure. Investigation whether the skew pattern of implied volatilities could generate abnormal trading profits was conducted for S&P 500 options with one month to maturity. Transaction costs were simulated from institutional investor's perspective.

Average profits for S&P 500 options (Bollen & Whaley, 2004) before trading costs were statistically significantly above zero for four out of five moneyness categories. Range for the positive returns ranged from 6.9 % for lowest strike price group (moneyness category 1) to 2.0 % for moneyness category 4. Moneyness category 5 had abnormal profits of 2.4 %. The pattern of abnormal returns corresponded to the difference between the implied and realized volatilities. Delta hedged strategy with trading costs generated the same pattern of profits with abnormal return range from 4.6 % to 1.4 %. These results are with hedging position in the underlying index but reproduced results with futures hedge remain mostly the same.

Duan and Hung (2010) reported that net buying hypothesis held true also on the Taiwan option market, which together with limits to arbitrage explain the behaviour of IVs and trading simulation results. Their results show that demand for especially OTM put options caused them to have higher implied volatilities over call options. Based on the put-call parity, call options were expected to mirror performance of puts, but both option types exhibited a negative skew according to moneyness of options.

Similar to Bollen and Whaley (2004), OTM put options produced the highest probabilities for profit, profits and profit ratios, including transaction costs. Generation of statistically significant abnormal profits were found mostly in out-of-the-money options for both option types. Trading simulation profits differed also across five different maturity groups from one week to two months. Longer maturities exhibited higher profits for delta-hedge strategy in index points, but profit ratios remained approximately same for all the maturity classes.

Larkin et al. (2012) investigated implied volatilities, option trading profits and net buying pressure around the financial crisis of 2008 on Australian S&P ASX 200 index options. The smile pattern of implied volatilities existed during the financial crisis for both call and put options, but in the bull market before the crisis, only put options exhibited an IV smile pattern. This proved that implied volatilities were time dependent. Call options generally had implied volatilities less than realized volatilities, whereas for puts IVs were higher than realized volatilities. The differences between implied and realized volatilities were used as indicators for mispricing in their regression for trading profits.

Upward biased OTM put implied volatilities yielded excess returns in trading simulations, which showed the connection between IVs and trading profits. Trading simulations were implemented by selling options with IVs above realized volatilities and by buying options with IVs less than realized volatilities. Trading simulations showed significant excess returns for multiple moneyness classes across option types and moneyness levels, although OTM put options performed noticeably better than others. Transaction costs were not included, which meant that the determined regression coefficient for the difference between implied and realized volatilities would need to exceed transaction costs so that the mispricing would generate economically significant results. (Larkin et al. 2012)

All the reviewed studies on index option markets found that finding abnormal profits by trading options is the most likely with out-of-the-money put options. Implied volatilities were often shown to be connected to the relative prices of options and this was often used as the base for trading strategies design. Excess returns were found on many option markets before transaction costs but incorporation of fees and other costs made realization of profits difficult. The main conclusions of this literature review are that the implied volatilities seem to be connected to the under- and overpricing of options and finding abnormal profits is the most likely among deep-out-of-the-money put options.

4 Data

In this chapter the used data sample is defined, the individual data observations are set in the categories according to the object of this study and the filtering of the raw data is explained. Last section focuses on the underlying index properties.

3.1 Sample Description

The original raw option data consists of options written on DAX 30 performance index from the beginning of 2018 to the end of 2019. DAX 30 performance index or DAX-index comprises 30 largest and the most actively traded German companies trading on the Frankfurt Stock Exchange. DAX-index used in this study is a capital-weighted performance index where dividends are assumed to be reinvested in the shares, meaning that dividends do not affect the index value and therefore dividend yield estimation is not needed (Muzzioli, 2010, Theissen, 2012). In addition, options for DAX-index can only be exercised at the expiration of an option, which together with missing dividend yield estimation makes the data less prone for measurement error (Muzzioli, 2010). As a clarification on the discussion in the literature review part of this paper, the assumption in this paper is that the dividends do not have effect on the DAX index level.

DAX options are European-style options expiring on the third Friday of the contract month. During the sample period DAX options were traded on the EUREX (German Options and Futures Exchange) between 8:50 and 17:30 CET, not including pre- or post-trading periods. Exercising of an option was possible on the final settlement day until 20:30 CET, settlement days being the two days after expiration of an option. (Eurex, 2020)

Prices of the options are in index points, contract value being EUR 5 per DAX-index point. Minimum tick size is 0.1 index points or EUR 0.50 and contract size is 100 shares. Options are offered with different time-to-maturities up to 60 months. Time-to-maturities up to 12 months are the three nearest successive calendar months and the three following quarterly months of the March, June, September and December cycle thereafter. From 12 months to 24 months maturities are the following semi-annual months of the June and December cycle thereafter. Longer than 24-month time-to-maturities are represented by annual December options. (Eurex, 2020)

Number of minimum strike prices and intervals between strike prices differ for different time-to-maturities. Number of minimum exercise prices for each maturity date with a term up to 24 months is 7, so that 3 of them are in-the-money, 1 is at-the-money and 3 are out-of-the-money. For maturities above 24 months minimum number of exercise prices is 5, divided similarly as options with shorter maturities. Option with 3 or less months to maturity have exercise price interval of 50 index points, maturities from 4 to 12 months have exercise price interval of 100 index points and options above 12 months to maturity have exercise price interval of 200, respectively. (Eurex, 2020)

Hedging instrument, Mini-DAX futures contracts on DAX-index are traded on the EUREX. Mini-DAX futures are used in this study instead of DAX futures because Mini-DAX futures can be traded in smaller slots. DAX futures are highly liquid cash-settled contracts that mature on the third Friday of March, June, September and December. Contract value for Mini-DAX futures is EUR 5 and the minimum tick size is 1 index point. (Eurex, 2020)

Data for put and call options is downloaded from Datastream database. The two time series datasets include option closing prices, bid and ask prices, strike prices, trading volumes and open interests. Data also includes daily implied volatilities and deltas for daily observations and the underlying closing prices for period 2.10.2017-31.12.2019. Observations from year 2017 are used for calculating realized volatilities to be used for delta hedging. Daily Euribor rates data offered by Thomson Reuters are used as risk-free rate estimates. The Euribor rates data include annual rates for one week, one month, three months and six months. The daily Euribor rates are converted into continuous rates and matched with the option observations. Options with time-to-maturity from six days to three months are matched with risk-free rates by using linear interpolation method. Options with time-to-maturity of one week or lower are set to use the Euribor rate of one week as risk-free estimate.

Criteria for choosing options is that they expire until the end of 2019 to avoid problems related to volatility changes during the option's life. Volatility risk is reduced by holding options until the maturity (Duan & Hung, 2010). Before filtering the daily data includes

176519 put option observations and 176519 call option observations. Strike prices in index points differ from 2000 to 20500 for both put and call options.

Data of the DAX-index closing prices are used for futures price estimation, following the work of Chan et al. (2004) and Mittnik and Rieken (2000). Reasoning for not using bid and ask prices of futures is complemented by findings from earlier studies: Mittnik and Rieken (2000) announce that the spread of futures bid and ask prices is difficult to estimate and trades also happen within the bid and ask prices, making the implementation of bid and ask prices less relevant. The main trading time for DAX 30 index was from 9:00 to 18:00 CET (EUREX, 2020).

Transaction costs and fees are dependent on the type of the market participant. Market makers are the least-cost traders, whereas this study tries to emulate option trading from private investor's viewpoint. Fee for private investor for trading DAX options at Eurex is EUR 0.54 per contract without assuming large contract volume (Eurex, 2020). Upon this, options are assumed to be sold at the bid price of each quote. Option positions are expected to be closed right before expiration and therefore, expiring of the options is not included in the transaction costs. In addition, this study aims to compare possible abnormal returns and following Duan and Huang (2010), an expectation is that the cost of capital on margin requirement does not affect the simulation results. The exchange fee for Mini-DAX futures for private investor is EUR 0.25 (Eurex, 2020). Transaction costs from bid-ask spreads for the index are assumed to be 0.011 %, according to study from Theissen (2012). Half of this spread is added in the transaction costs when each trade on the hedging instrument takes place.

4.2 Option Contract Categories

Basic factors affecting option prices and implied volatilities are those presented in section 2.1. In section 2.5.2.1 was discussed that option prices and implied volatilities can also be affected by transaction costs and demand-based matters. In the context of implied volatilities, Rouah and Vainberg (2007) among others perceive that the shape of the volatility smile is affected most by moneyness and the time-to-maturity of an option. In light of evidence of implied volatilities and option prices being affected by option's time-to-maturity and moneyness, the data will be categorized into time-to-maturity, and moneyness categories.

Option's maturity can be expressed either in terms of calendar days or trading days. The original dataset includes holidays but no weekends, and therefore the data is cleaned to include only the trading days. Time-to-maturities are calculated from the data as trading days to expiration. Eurex Exchange had 251 trading days both in 2018 and 2019 and therefore the whole period consists of 502 trading days (Eurex Exchange, 2020).

Most of previous literature investigating volatility surfaces combined with option trading profits consider either options with one specific time-to-maturity (for example Bollen & Whaley, 2004) or options with relatively short time-to-maturity. Chan, Cheng and Lung (2004) justifies the chosen maximum time-to-maturity of two months with low trading volume of longer options. Researches have been done also including longer time-to-maturities; for example, Larkin, Brooksby, Lin and Zurbruegg (2012) used six maturity categories up to six months. This paper uses options with up to 63 days to maturity, and by assuming 21 trading days per month this can be converted to 3 months. Options with above three months to maturity have relatively low trading volumes in the used dataset. Lower limit for time-to-maturity of an option is decided to be three days. Out-of-the-money options with less than three days to maturity are sometimes extremely cheap and have extremely noisy values of implied volatility. Therefore, options with less than three days to maturity are excluded to improve data quality.

Filters applied on the original data cut out most of the daily observations, which could make the investigation of narrow time-to-maturity categories unreliable. To ensure sufficient amount of observations in each maturity and moneyness group, options are divided into three maturity categories. Names and time-to-maturities of each maturity class are represented in Table 5.

Table 5. Maturity categories

Category no.	Category label	Maturity (DTM)
1	1-Month	3−21
2	2-Month	22−42
3	3-Month	43−63

After placing observations in three maturity categories next step is to categorise options by moneyness. Typically, moneyness is measured by the ratio of spot price to option's exercise price Duan & Hung (2010), or as the relative difference between the forward price of the underlying and the option's exercise price (Bollen & Whaley, 2004). Measure of spot price to strike price is often converted to a ratio of option's strike price to spot price.

However, moneyness defined as a probability for an option to end in the money at maturity is dependent also on the volatility of an underlying and option's remaining time-to-maturity (Bollen & Whaley, 2004), and therefore delta of an option is often used as a measure for moneyness. Although delta has its benefits, this paper takes a simpler approach and measure moneyness as a ratio of options strike price to the underlying price. By not using delta the problem of determining which volatility to use as an underlying volatility parameter for delta calculation is avoided. In addition, categorization of options by strike price to spot price ratio allows us to simply perform delta hedging strategies with different underlying volatilities with the same groups of options.

There is neither standard for the degree or the range of sub-categories regarding option's moneyness, but upper and lower limits are usually included since including extreme moneyness values can have multiple drawbacks. According to Ahn et al. (2008), extremely out-of-the-money options are cheap and therefore problems related to tick sizes can occur. The original data sample contains options with option prices that do not reach the minimum price change of 0.1 index points. In addition, Ahn et al. (2008) have found that extreme in-the-money options suffer from infrequent trading. Implementation of the Black-Scholes pricing formula in extreme OTM or ITM options are shown to produce unreliable estimations.

This study uses moneyness range following Blaskowitz et al. (2004) with upper limit of 1.10 and lower limit of 0.90 measured as option's strike price to spot price ratio (K/S_0). The moneyness range is divided in five moneyness categories following the work of Bollen and Whaley (2004) and Larkin et al. (2012). Moneyness categories for calls and puts are deep-out of-the-money (hereafter DOTM), out-of-the-money (hereafter OTM), at-the-money (hereafter ATM), in-the-money (hereafter ITM) and deep-in-the-money (hereafter DITM). Numerical values for categories are presented in Table 6.

Table 6. Moneyness categories for call and put options

	Put options		Call options	
Category label	Category no.	Moneyness range	Category no.	Moneyness range
DOTM	1	$0.90 \leq K/S_0 < 0.94$	5	$1.06 \leq K/S_0 \leq 1.10$
OTM	2	$0.94 \leq K/S_0 < 0.98$	4	$1.02 \leq K/S_0 < 1.06$
ATM	3	$0.98 \leq K/S_0 < 1.02$	3	$0.98 \leq K/S_0 < 1.02$
ITM	4	$1.02 \leq K/S_0 < 1.06$	2	$0.94 \leq K/S_0 < 0.98$
DITM	5	$1.06 \leq K/S_0 \leq 1.10$	1	$0.90 \leq K/S_0 < 0.94$

In the data used in this paper in-the-money put options with moneyness values above 1.10 and correspondingly in-the-money call options with moneyness values below 0.90 are suffering from low trading volumes. Limits for moneyness values are intended to ensure accurate price and IV estimates also at the boundaries.

4.3 Filtering the Option Data

Earlier presented tables 5 and 6 included limitations for options maturity and moneyness. Options only up to time-to-maturity of three months are included and options with moneyness values below 0.90 and above 1.10 are excluded. Effects of these limitations and effects of other applied filters applied in particular order are as follows:

1. Observations with time-to-maturity shorter than 3 days and longer than 63 trading days are excluded. After filter applied 74224 option price observations were excluded from call option data and correspondingly 74224 observations were excluded from put option data.

2. In order to use ask and bid offers for implied volatility calculations and trading simulations, only the observations with daily ask and bid offers can be included. This filter is also a liquidity filter, as without daily ask and bid offers the option liquidity could not be confirmed on each day. Consequently, 13150 observations from call option data and 7576 observations from put option data are removed.

3. Liquidity of options is crucial. Chou et al. (2011) found that there is a clear link between option's implied volatility and liquidity, implying that liquidity of an option is negatively correlated with option price. The original data also includes a large amount of deep-in-the-money and in-the-money observations that are not traded daily. Including these observations would not only include possible non-tradable pricing errors but also distort the relationship between the amounts of out-of-the-money, at-the-money and in-the-money options. Only daily observations with trading volume above zero are included. This filter removes 56186 options from call option data and 47723 options from put option data.

4. Observations with moneyness values below 0.90 and above 1.10 are removed because of possible distortions of price discreteness (Chan et al. 2004). Moneyness filter causes a removal of 4216 observations in call option data and removal of 16033 observations in put option data.

5. The original data includes a substantial amount of extremely cheap in-the-money observations that do not fulfil the non-arbitrage condition. This paper assumes that this is related to the quality of the option price data and to the assumption that each option could be traded at the midpoint of bid and ask prices. Lower boundary condition for both puts and calls are revised and after filtering observations that do not fulfil the non-arbitrage condition, 343 observations are removed from call option data and correspondingly, 1009 observations from put option data. Also the upper boundary condition is checked but none of the observations exceeded maximum price of an option.

4.3.1 Summary of the Filtered Option Data

After filtering the original data total of 28400 call option prices and total of 29954 put option prices with relevant trading information remain. Summary statistics for call- and put options together with continuously compounded risk-free interest rates are presented in Tables 7 and 8.

Table 7. Summary of filtered call option data and market variables

	Mean	Median	Min	Max
Option price [index points]	222.3	119.4	0.2	1385.5
Strike price [index value]	12396.2	12400.0	9500.0	14800.0
Time-to-maturity [days]	29.5	28.0	3.0	63.0
Risk-free interest rate, annual [%]	-0.37	-0.37	-0.50	-0.30

Table 7 shows that filtering of call option data removed the extreme minimum and maximum prices for out-of-the-money and in-the-money options. Also strike price range narrowed while mean time-to-maturity for call options is close to 30 trading days. Noteworthy is also that the risk-free interest rates are negative.

Table 8. Summary of filtered put option data and market variables

	Mean	Median	Min	Max
Option price [index points]	198.9	128.2	0.4	6352.9
Strike price [index value]	11802.3	11800.0	9350.0	14500.0
Time-to-maturity [days]	29.1	28.0	3.0	63.0
Risk-free interest rate, annual [%]	-0.37	-0.37	-0.50	-0.30

Filtered put option data (Table 8) has slightly lower mean option- and strike prices, but maximum option value is substantially higher compared to call option data in Table 7. Range for strike prices is approximately the same as for call options, as well as the mean time-to-maturity. Next, the option observations categorized in the three maturity- and moneyness categories are presented.

Table 9. Summary of call options in three maturity and five moneyness groups

	1-Month	2-Month	3-Month	Total
DOTM	1369	2181	1880	5430
OTM	3133	3399	2464	8996
ATM	3345	3151	2067	8563
ITM	1922	1379	567	3868
DITM	763	540	240	1543
Total	10532	10650	7218	28400

Table 9 shows that more than half of the options are out-of-the-money observations (50.80 %), whereas the portion of in-the-money observations is less than 20 percent (19.05 %). This is common for option markets, as out-of-the-money option contracts are traded more than in-the-money options contracts. Due to the filters employed, 1-Month maturity group including options with time-to-maturities from 3 days to 21 days has less observations than 2-Month maturity group that consists of observations with time-to-maturity between 22 and 42 days. The statistics for 3-Month maturity group verifies that option market offers more shorter-maturity contracts and that they are also more liquid.

Table 10. Summary of put options in three maturity and five moneyness groups

	1-Month	2-Month	3-Month	Total
DOTM	3091	3098	2036	8225
OTM	3428	3385	2213	9026
ATM	3264	3065	2082	8411
ITM	1206	1323	809	3338
DITM	339	354	261	954
Total	11328	11225	7401	29954

Table 10 for put option categories shows similar characteristics to call option categories. Most of the options are out-of-the-money (57.59 %) and the portion of in-the-money options is below 20 percent (14.33 %). For put options, the shortest 1-Month maturity group stays largest and again 3-Month maturity group has the fewest number of observations.

4.4 Descriptive Statistics of DAX 30 Returns and Realized Volatility

This section presents the descriptive statistics for the returns and the historical volatility of the underlying DAX 30 performance index. Logarithmic returns for the underlying are calculated as follows (Equation 27):

$$r_t = ln\left(\frac{p_t}{p_{t-1}}\right) \tag{27}$$

Where r_t is a continuously compounded return for trading day t and p_t and p_{t-1} are daily closing prices of the index for trading days t and $t-1$, respectively.

The sample for the underlying covers a time span 2.10.2017-30.12.2019, but descriptive statistics for daily returns are derived from the underlying price information from a time span 2.1.2018-30.12.2019. Daily closing prices and daily calculated returns of DAX 30 are plotted in Figure 7. The left plot on the figure shows that there was no clear overall trend for the index during the sample period. Overall, year 2018 was bearish on the German stock market, whereas year 2019 was bullish. Visual inspection of the logarithmic returns on the right of the Figure 7 indicates that the time series has a mean of approximately zero, but the volatility does not seem to be constant unlike expected by Black-Scholes model: the time series seems to have volatility clustering, which indicates that there are changes in the volatility. These kind of characteristics of the underlying returns have been recognized as one possible reason for the mispricing of the Black-Scholes option pricing model, discussed in section 2.5.2.

Figure 7. Daily closing prices and returns of DAX-index

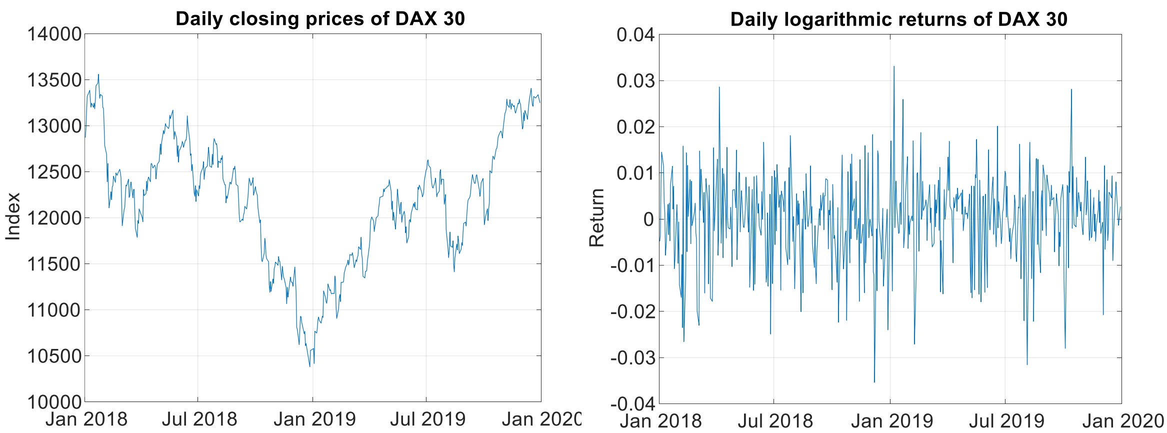
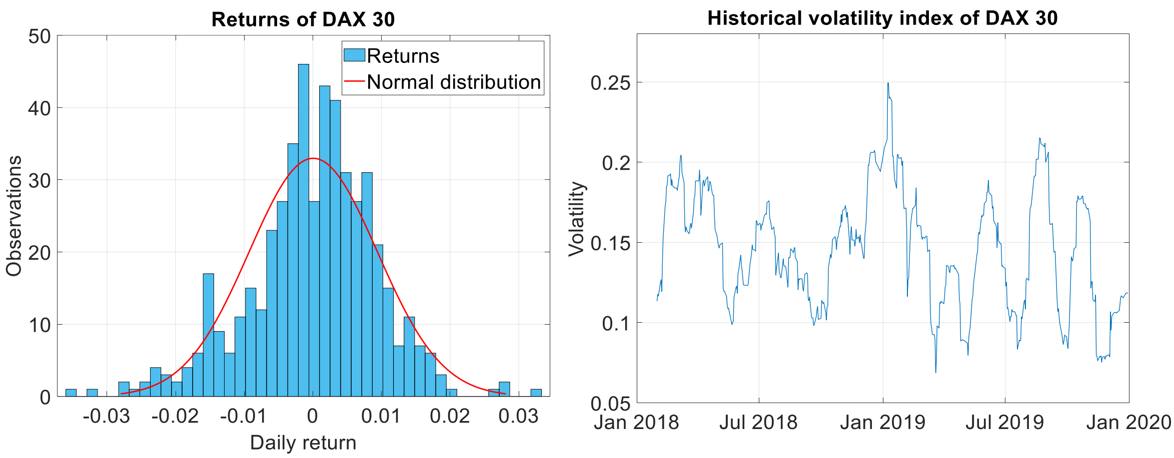


Figure 8 below presents the distribution of the returns and the historical, backwards looking time series volatilities of DAX 30 over the sample period. Graphical assessment of the returns plotted with normal distribution on the left of Figure 8 shows that the returns close to zero are more common than what normal distribution assumption expects. In addition, fat tail pattern especially on the negative side of the daily returns also indicates that the returns time series is not normally distributed. Backward looking daily rolling volatilities from the last 21 days of the historical returns plotted on the right of the Figure 8 strengthens the perception that the underlying volatility is not constant. In order to have a more accurate understanding of the characteristics of the DAX returns, numerical inspection for the returns and for the volatility of the returns is performed.

Figure 8. Histogram and historical volatility of daily logarithmic returns of DAX 30



Summary statistics of the DAX 30 returns are presented in Table 11. Kurtosis statistics confirms the leptokurtic distribution acquired from the visual evaluation of the Figure 8. Kurtosis above 3 is equal to the findings of Blaskowitz et al. (2011), who measured the kurtosis of DAX returns above 3 in every year for period 1997-2002. Negative skewness statistic affirms that the daily returns during the sample period are dominated by negative returns.

Table 11. Summary statistics of DAX 30 annualized logarithmic returns

Obs.	Mean	Median	Min	Max	Skewness	Kurtosis
502	1.03%	23.07%	-99.99%	358265.64%	-0.37635	3.96728

Table 12 presents the summary statistics for DAX 30 realized volatility from various holding periods. The realized volatility is proxied by the standard deviations of the DAX 30 returns from 2.1.2018-30.12.2019. The maturity in Table 12 describes the time span used for the realized volatility estimation, where 1 Month is 21 days, 2 Months is 42 days and 3 Months is 63 days.

Table 12. Summary statistics of DAX 30 annualized realized volatility

Maturity	Obs.	Mean	Median	Min	Max
1 Month	481	14.54%	14.58%	6.88%	24.95%
2 Months	460	14.83%	14.64%	9.36%	20.48%
3 Months	439	14.90%	14.51%	11.74%	19.10%

Table 12 confirms the variation of the underlying realized volatilities. Differences between the minimum and the maximum annual volatilities narrow, the longer the estimation window for volatility is.

5 Methodology

The methodology part is divided in three sub-sections, which explain the method used for implied volatilities extraction, chosen trading simulation designs and determinants for evaluating the trading profits. Also, implementation of statistical significance framework is presented.

5.1 Extraction of Implied Volatilities

This paper applies the standard Black-Scholes model (1973) to computing the implied volatilities. Inputs for implied volatility estimation are the underlying index price, strike price of an option, continuously compounded annual risk-free interest rate, time-to-maturity in years and the observed price of an option. All the inputs except risk-free rates and time-to-maturities are provided by the option dataset downloaded from Datastream. Derivation of the risk-free estimates were described in section 3.1 and time-to-maturities are calculated from the option dataset.

The Black-Scholes method for implied volatilities estimation finds the right underlying volatility parameter value so that the observed option price matches with the option price determined by Black-Scholes model (Hull, 318, 2012). Iterative search procedure is used to extract the implied volatility with termination tolerance of 0.000001. Implied volatility for each option price is estimated by solving the Equations presented in section 2.4. Following common practise option price is assumed to be the midpoint of an option’s ask and bid prices.

After determining implied volatilities, minimum, maximum and mean values are calculated for each time-to-maturity- and moneyness categories. Mean values of implied volatilities are used for evaluating possible differences in implied volatilities between the option categories. As a result, information of the possible existence of the volatility smile and the term structure on DAX 30 options is achieved.

5.2 Trading Simulation Design and Delta Determination

To assess the trading profits for different option categories, we conduct option trading simulations by selling all the option observations of the filtered option data. Selling options-strategy is chosen based on the earlier findings of trading profit differences between buying and selling options. The purpose of this study is to examine possible existence of abnormal profits on DAX 30 option market and therefore the focus is on choosing trading strategies that most likely can produce abnormal profits. Previous studies (Bollen & Whaley (2004), Chan et al. (2004), Green & Figlewski (1999) and Duan (2010)) have found that particularly writing of out-of-the-money put options has been abnormally profitable on numerous stocks and stock indices. The hypothesis known as net buying pressure is seen to produce margins on option prices especially out-of-the-money put options.

By trading all the option observations dilemma of subjective selection bias is avoided (Chan et al. 2004). Options are held to the expiration, which diminishes the volatility risk (Duan, 2010) and the vega hedging dilemma that rises if the hedged portfolios are updated with discrete time intervals (Green & Figlewski, 1999). In this study two different trading strategies with and without transaction costs are used to examine the existence of possible abnormal profits before and after transaction costs. Trading simulations are constructed as follows (Table 13):

Table 13. Trading simulation strategies

Strategy	Description
1	Delta-neutral hedge without transaction costs. Hedge-volatility equal to the implied volatility of an option. Options are sold at bid/ask midpoint and DAX-index is traded at the closing price of DAX 30.
2	Delta-neutral hedge without transaction costs. Hedge-volatility equal to the historical volatility of the underlying. Options are sold at bid/ask midpoint and DAX-index is traded at the closing price of DAX 30.
3	Delta-neutral hedge with transaction costs. Hedge-volatility equal to the implied volatility of an option. Options are sold at bid price and DAX-index is traded at the closing price of DAX 30 with trading cost equal to one-half of the DAX futures bid/ask spread. Fees for trading options and DAX futures are added.

4	Delta-neutral hedge with transaction costs. Hedge-volatility equal to the historical volatility of the underlying. Options are sold at bid price and DAX-index is traded at the closing price of DAX 30 with transaction cost equal to one-half of the DAX futures bid/ask spread. Fees for trading options and DAX futures are added.

Strategy number 1 incorporates delta-hedging with daily revision by using hedge-volatilities implied from option prices. Hedged call option position is opened by entering a short option position at a price equal to midpoint of bid and ask offers together with buying delta units of the underlying index. Put option strategy includes entering short position in put options and selling delta units of the underlying, delta being the absolute value of the option delta. Gains or losses from the opened positions are carried forward and realized at the expiration of the option position. By rebalancing the hedging position daily at the end of the trading day, gamma risk and the need for delta-gamma hedging is limited(Larkin et al. 2012).

The hedge-volatility parameter is kept constant during the option's life, reflecting the idea that the implied volatility of an option is the market's view of the underlying volatility over the remaining life of an option. Hull and White (2017) viewed this kind of delta with volatility parameter equal to the implied volatility of an option as "practitioner Black-Scholes delta". In addition, by keeping the volatility parameter constant instead of updating it based on the implied volatilities of the upcoming days, possible errors related to implied volatility estimations in the data are avoided. Since the original data may contain illiquid price observations, estimated implied volatilities and therefore volatility parameters for delta calculations might be biased.

Simulation Strategy 2 is delta hedging strategy constructed using historical volatility of the underlying as the volatility parameter for delta calculations. Most of the previous literature investigating option trading profits employ delta hedging strategies with deltas based on either implied volatility or volatility of historical returns. Bollen and Whaley (2004) used historical volatility from the last 60 days as a volatility parameter for delta calculations and kept it constant. Chan et al. (2004) employed implied volatilities, whereas Green and Figlewski (1999) used mean and weighted volatilities of past returns. Since volatility parameter affects the trading profits in a way described in section 2.6.1, Strategy 2 is performed by using historical volatility of returns as an input for delta calculation, and therefore, also as a forecast for the underlying future volatility.

Volatility estimates for delta hedging in Strategy 2 are drawn from past returns data, holding on to principle of using only information that would have been available at the time of initializing the hedge position. The chosen length of past returns data used for volatility estimations in Strategy 2 is derived from ideology presented by Green and Figlewski (1999). Green and Figlewski discuss that it is reasonable to deem that the most accurate estimates for future volatilities are achieved by using data sample with length as a function of the prediction horizon. In this study the maximum time-to-maturities of each option category are used as a measure for trading days incorporated in forecasting volatilities. Volatility estimates for 1-Month, 2-Month and 3-Month option categories are set to be the underlying volatilities from the last 21, 42 or 63 trading days. By not using the actual time-to-maturities of option observations some problems related to volatility estimation are avoided. If actual time-to-maturities were used for the realized volatility estimations, estimation errors could occur for 1-Month options due to short estimation period.

Simulation Strategies 3 and 4 are reproduced Strategies 1 and 2 with transaction costs. By incorporating transaction costs defined in section 3.1 economic significance of the results can be evaluated. Options are sold at bid price and following the work of Bollen and Whaley (2004) transaction cost of DAX-index is set equal to one-half of DAX futures bid/ask spread of 0.011 % (Theissen, 2012). Selling options at bid price differs from most of the previous literature. Most of the previous literature examined trading profits from viewpoint of an institutional or market maker perspective, whereas this paper focuses in the individual investor's perspective. Upon transaction costs incurred from spreads, fees for trading the options and Mini-DAX futures are added. Options are assumed to be sold once and the transaction is closed once by entering long position at the maturity of each option. Costs from daily delta-revision are dependent on the option's life.

Finally, after conducting simulation Strategy 4 robustness checks for the results are executed. Given that outliers for trading profit simulations might impact the empirical conclusion, following work from Chan et al. (2004) and Duan and Hung (2010), observations outlying more than two standard deviations from the mean of return rates are removed.

5.2.1 Profit Calculation and Statistical Significance of Results

Profit calculation for each portfolio includes the profit from short option position, the profits from opening the position in the underlying asset and the profits from the daily delta revision. First part of calculating trading profits, selling call and put options are presented in Equations 28 and 30 (Chan et al. 2004):

$$Call\_PPN = C_0 e^{rT} - C_T \tag{28}$$

where

$$C_T = max(S_t - X, 0) \tag{29}$$

$$Put\_PPN = P_0 e^{rT} - P_T \tag{30}$$

where

$$P_T = max(X - S_t, 0) \tag{31}$$

Where C_0 and P_0 are the initial option prices for call and put option, r is the continuously compounded risk-free rate at the time of opening the position, T is the time till expiration when options are sold, C_T and P_T are the option values for call and put options at the expiration calculated as the difference between spot rate S_t and the option's strike price X.

Calculation of profits in index points for delta-hedging strategies for call and put options are presented in Equations 32 and 33 (Chan et al. 2004):

$$Call_{PPD} = Call_{PPN} + \Delta_0(S_T - S_0 e^{rT}) + \sum_{t=0}^{T-1} \Delta_t(S_{t+1} - S_t)\, e^{r(T-t)} \tag{32}$$

$$Put_{PPD} = Put_{PPN} + \Delta_0(S_T - S_0 e^{rT}) + \sum_{t=0}^{T-1} \Delta_t(S_{t+1} - S_t)\, e^{r(T-t)} \tag{33}$$

Where $Call_{PPN}$ and Put_{PPN} are the profits for naked call and put positions, Δ_0 is the delta of an option when original positions are opened, S_T is the underlying value at the expiration of an option, S_0 is the underlying value when original positions are opened, r is the risk-free rate and T is the time to expiration when original positions are opened. Δ_t and S_t are the option delta value and the underlying value during the option's life between setting up the original positions and the expiration of the options.

The second term in the calculation of $Call_{PPD}$ and Put_{PPD} in Equations 31 and 31 represents the profit from the original position in DAX index that is opened to hedge the option position. The third term is the profit from the daily rebalancing of the position in DAX index.

Entering short positions are initially negative investments, which makes it arguable if a profit ratio calculation should be computed as a ratio of net cash inflow to initial investment (Chan et al. 2004). Following common practice, this paper computes the trading profits in percentages as a ratio of profits in index points to the absolute value of the initial investment. For the simulations 3 and 4 where transaction costs are included, costs of trading the options and the index are included in the selling price of an option. Profit in percentages for delta-hedged positions, calculated as a ratio of profits to the initial value of the portfolio is presented in Equations 34 and 35:

$$Call\_PERD = Call\_PPD \; / \; |\Delta_0 S_0 - C_0| \tag{34}$$

$$Put\_PERD = Put\_PPD \; / \; |\Delta_0 S_0 - P_0| \tag{35}$$

Where $Call\_PERD$ and Put_PERD are the profits from delta-hedged call and put option positions, Δ_0 is the delta of an option when original position is opened, S_0 is the underlying value when original positions are opened, C_0 is the value of initial investment for call options and P_0 is the value of initial investment for put options (Chan et al. 2004).

After calculating profits for each of traded options, mean values for the profits and for the profit ratios are calculated. Also, calculations for the probability that positive profit occurs are performed. Profit probability calculation is presented in Equation 36:

$$Profit\ propability = \frac{Number\ of\ positive\ trading\ profit\ simulation}{Number\ of\ all\ trading\ simulations} \tag{36}$$

Profit probability for each option group in Equation 36 is the ratio of number of positive trading profit simulations to number of all trading simulations.

Statistical significance for results of profit probability, profit in index points and profit ratio are tested. The significance test for profit probability show whether the probability of above zero profit is significantly greater than 50 % at the 10 %, 5 % or 1 % level. This paper performs upper-tailed binomial test following Chan et al. (2004), where the critical value for the test statistic is specified as (Equation 37):

$$T_U = N \times 0.5 + z_{1-\alpha} \times \sqrt{(N \times 0.5 \times (1 - 0.5))} \tag{37}$$

In Equation (36) N is the number of observations, z is the quartile observed of the normal distribution and α is the level of significance.

P-value calculation in this test is presented in Equations 38 and 39:

$$Probability(T_{run} \geq t_{observed}) \approx 1 - Probability(Z \leq z) \tag{38}$$

where

$$z = (t_{observerd} - N \times 0.5) / \sqrt{(N \times 0.5 \times (1 - 0.5))} \tag{39}$$

Where T_{run} = test statistic for the number of negative profits, $t_{observed}$ = observed value of positive profits.

P-value for hypothesis of 50 % probability for profit is observed from the standard distribution table. Significant test statistic for this upper-tailed binomial test at the 10 %, 5 % or 1 % level would mean that the profit is more likely to be positive.

Significance test for profit in index points and profit ratio indicates whether the profit or profit ratio is significantly greater than zero at the 10 %, 5 % and 1 % level. Since the distribution of profits from selling options is asymmetrical (Chan et al. 2004), the modified t-test introduced by Johnson in 1978 is used, following Bollen & Whaley (2004).

Johnson's modified t-test reduces the effect of skewness of the population so that tests considering the mean of the population can be more correctly calculated. In case skewness of the population is zero, Johnson's t-test statistic collapses to the basic t-test. (Johnson, 1978). Johnson's solution (1978) for the test statistic is presented in Equation 40:

$$t_{Johnson} = \left((\bar{x} - \mu_0) + \frac{\mu_3}{6\sigma^2 N} + \frac{\mu_3}{3\sigma^4} (\bar{x} - \mu_0)^2 \right) (s^2/N)^{-\frac{1}{2}} \tag{40}$$

Where $\bar{x}$ = sample mean, μ_0 = hypothesized sample mean, σ^2 and μ_3 are the second and third central moments of the population, N is the sample size and s^2 is the sample variance.

As Johnson (1978) suggest, σ^2 and μ_3 are estimated by the sample variance and skewness. The null hypothesis if rejected if the test statistic exceeds the critical value derived from the Student-t distribution. If the test statistic is higher at the 10, 5 or 1 percent significance level it would mean that the return or return rate is significantly above zero.

6 Results

In the first part of this chapter the empirical results for the volatility smiles and the volatility term structure of DAX options combined with the differences between the realized volatilities and IVs are presented. After this, the results of trading profits between option types, maturity- and moneyness categories are presented. Results for the implied volatilities are separated for call and put options. The trading profit results are divided in four chapters including two sections for the profits before transaction costs, one section for the profits after transaction costs and one section for the profits after outlier removal. Maturity- and moneyness classes in this section are as described in tables 5 and 6 in section 4.2. The focus of a study is to investigate mean values, but also minimum and maximum values for the implied volatilities and the trading profits are presented.

6.1 Volatility Smile and Term Structure of Call Options

First, results for the differences between the implied and the realized volatilities for call options are presented. Table 14 and Figure 9 show that the implied volatilities of individual call option classes are diversified around the realized volatility of the underlying. The realized volatilities are presented in Table 12 in section 4.4. The realized volatilities exceed deep-out-of-the-money and out-of-the-money implied volatilities in 4 out of 6 cases while the implied volatilities for deep-in-the-money and in-the-money options are higher than the realized volatilities in all moneyness and maturity categories. Overall, 9 out of 15 observations describing the difference between the implied and the realized volatilities for call options are positive.

Table 14. Implied minus realized volatilities for call options

	Maturity		
Moneyness	1-Month	2-Month	3-Month
DOTM	1.24%	-1.84%	-1.64%
OTM	-1.30%	2.04%	-1.14%
ATM	-0.42%	-0.35%	0.49%
ITM	3.70%	1.56%	2.45%
DITM	10.81%	3.81%	4.61%

Table 14 and Figure 9 shows that at-the-money IVs were closest to realized volatilities, and on average, 2-Month ATM implied volatility would have been the best estimate for the underlying volatility with difference of -0.35 %. Correspondingly, deep-in-the-money options have the highest differences to the realized volatilities, especially for 1-Month options (10.81 %). Based on the results, the null hypothesis for non-differentiating information content according to future volatilities must be relaxed for call options.

Figure 9. Implied minus realized volatilities for call options

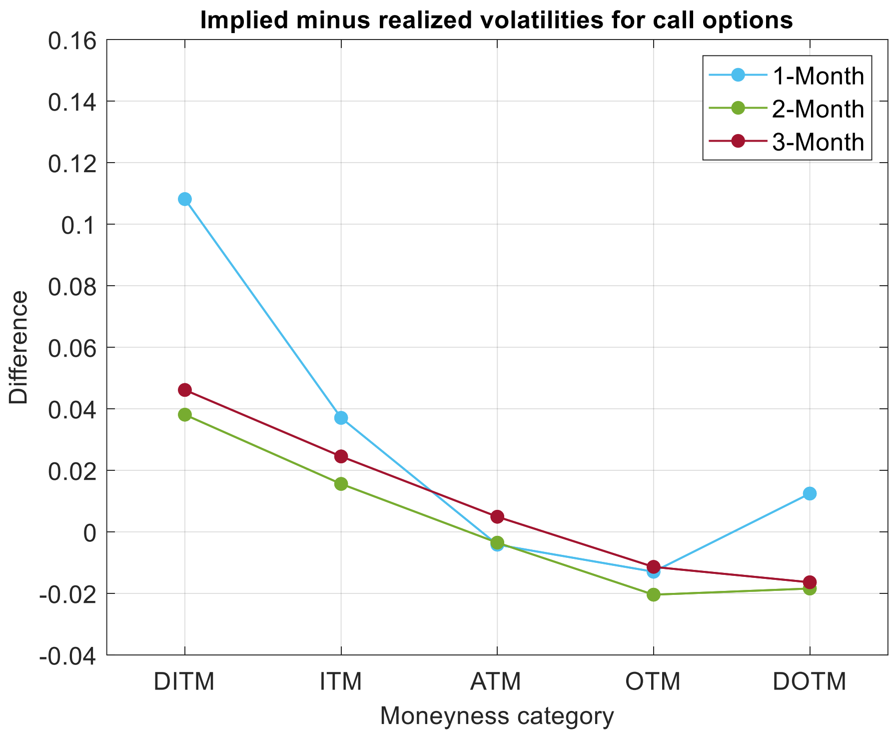


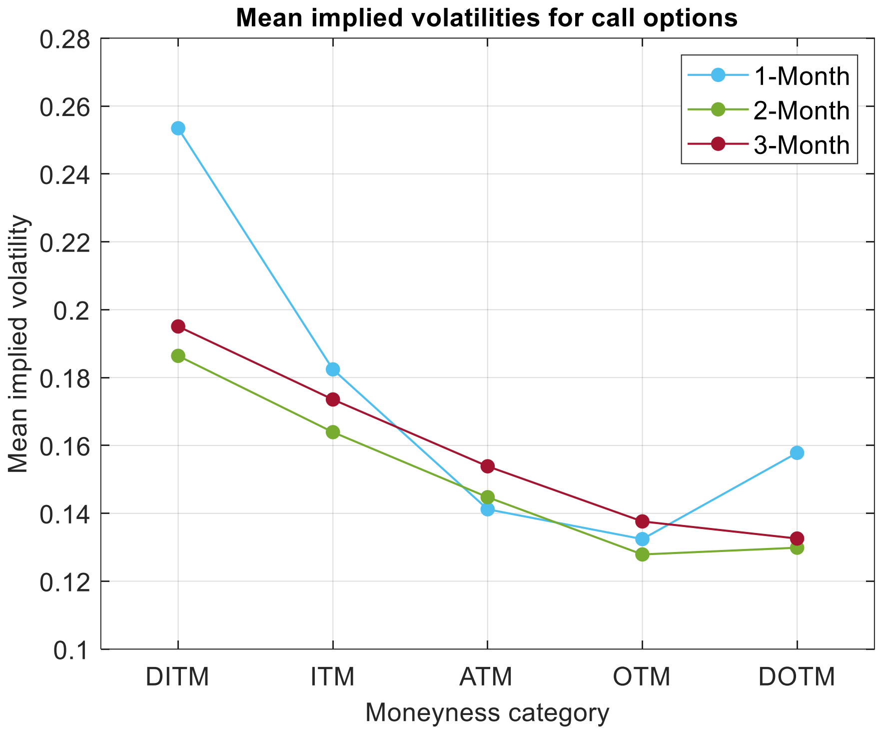
Table 15 below presents the descriptive statistics for the implied volatilities of call options through three maturity classes and five moneyness categories. Table 15 indicates that the DITM options have the highest mean implied volatilities in all three maturity classes. Mean implied volatility of DITM 1-Month maturity class options is 25.35 %, which is 6.71 % higher than the mean implied volatility of DITM 2-Month options and 5.84 % higher than the IV of DITM 3-Month options. However, the mean implied volatility gap between the maturity classes levels off when analyzing ATM options. 3-Month option class has the highest ATM mean implied volatility of 15.39 % while the maximum gap in mean IVs between ATM options is between 1-Month and 3-Month, 1.27 %.

Table 15. Descriptive statistics for implied volatilities of call options

Maturity	Moneyness	Obs.	IVmean	IVmin	IVmax
	DOTM	1369	15.78%	9.89%	76.08%
	OTM	3133	13.24%	7.98%	87.61%
1-Month	ATM	3345	14.12%	0.82%	73.06%
	ITM	1922	18.24%	0.06%	81.52%
	DITM	763	25.35%	1.45%	66.53%
	DOTM	2181	12.99%	9.46%	58.78%
	OTM	3399	12.79%	8.51%	54.22%
2-Month	ATM	3151	14.48%	4.51%	40.12%
	ITM	1379	16.39%	4.01%	28.13%
	DITM	540	18.64%	0.16%	33.55%
	DOTM	1880	13.26%	8.83%	36.26%
	OTM	2464	13.76%	8.02%	26.34%
3-Month	ATM	2067	15.39%	8.62%	28.86%
	ITM	567	17.35%	6.24%	28.75%
	DITM	240	19.51%	3.37%	26.88%

Mean implied volatilities indicate a smirk shape for 3-Month call options, indicating that the implied volatilities of call options are not constant across moneyness. Within 3-Month options, DITM options have the highest mean implied volatility of 19.51 %, whereas IVs keep degreasing reaching implied volatility of 13.26 % for DOTM options. Table 15 and Figure 10 show that the smirk pattern of 3-Month options starts to change towards some form of a smile the shorter time-to-maturities are considered. Implied volatilities for 2-Month and especially for 1-Month options start increasing at the right higher border of the moneyness spectrum.

Figure 10. Mean estimated implied volatilities for call options



Altogether, Figure 10 shows that the 2-Month and 3-Month mean IVs are relatively close to each other and represent a smirk pattern. 1-Month mean implied volatilities differ more across moneyness categories, representing more of a smile pattern than a smirk. The hypothesis for constant implied volatilities across moneyness or maturity is clearly not correct.

Minimum and maximum values for the implied volatilities (Table 15, Appendices 1, 2 and 3) illustrate that the estimated implied volatility observations are noisy for deep-out-of-the money and deep-in-the-money options. Especially, deep-in-the-money observations are highly scattered. Hentschel (2003) investigated errors in implied volatility estimations and found that options with short time-to-maturity produce extremely noisy and upward biased implied volatility estimates. Hentschel (2003) concludes that options close to maturity and deep-out-of-the-money or deep-in-the-money are particularly sensitive to measurement errors in option characteristics. Especially, measurement errors in option price and underlying price directly translate to errors in implied volatility measurements. The findings of Hentschel (2003) seem to apply at least on some degree on to the option data used in this study since extreme noisiness in implied volatilities (Table 15, Appendix 1) occur especially for 1-Month deep-in-the-money observations.

In addition, Hentschel (2003) found that as a consequence of measuring moneyness as a ratio of option strike price to underlying price, options close to maturity are always relatively far out-of-the-money or in-the-money-money if the strike price differs from the underlying price. These reasons are possible explanations for the dramatic increase of implied volatilities for options close to expiration. In this research, the used moneyness measure seems to be one factor making 1-Month call implied volatility observations higher and noisier than IVs for the longer maturities. Appendices 1, 2 and 3 show that the range and noisiness of the implied volatility observations rises, the further away from the money they are. As a conclusion, difference in noisiness between in-the-money options and out-of-the-money options is likely related to the liquidity of the options. Out-of-the-money options are traded more frequently, and price estimations are more likely to be reliable. All in all, the results for implied volatility estimations and trading profit simulations are more reliable for out-of-the-money options.

6.2 Volatility Smile and Term Structure of Put Options

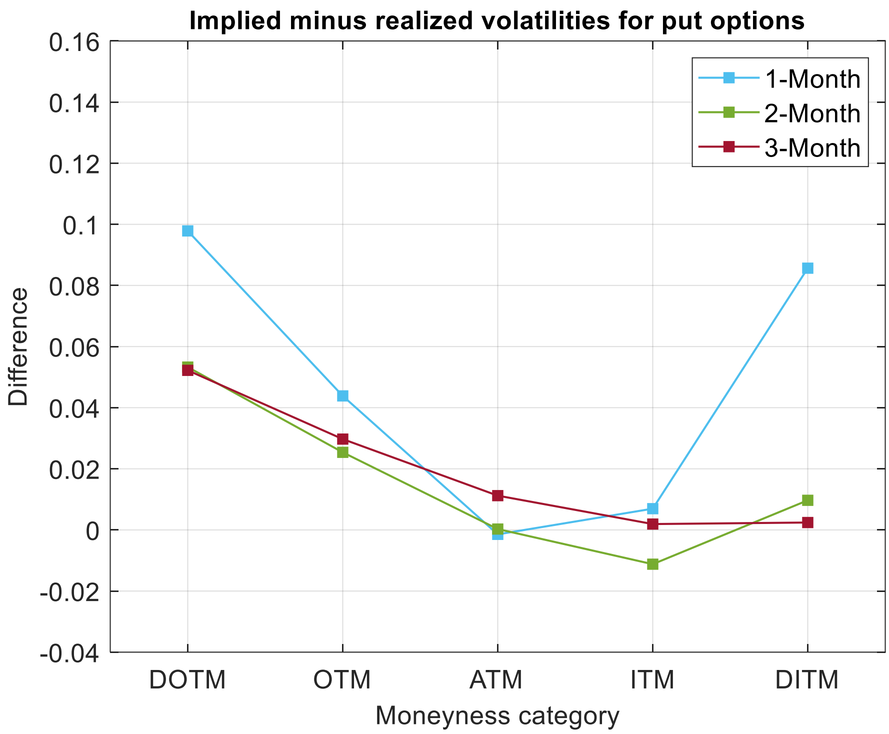
Results in Table 16 suggest that, on average, the implied volatilities for put options are higher than realized volatilities through the moneyness and maturity categories. 13 of the 15 individual differences are positive. The implied volatilities for DOTM, OTM and DITM 1-Month options and for OTM and DOTM options with longer maturities are substantially higher than the realized volatilities.

Table 16. Differences between implied and realized volatilities for put options

	Maturity		
Moneyness	1-Month	2-Month	3-Month
DOTM	9.78%	5.33%	5.22%
OTM	4.39%	2.54%	2.97%
ATM	-0.15%	0.03%	1.12%
ITM	0.69%	-1.12%	0.19%
DITM	8.57%	0.97%	0.24%

Numerical results from Table 16 together with visual interpretation from Figure 11 indicate that the mean ATM, ITM and DITM implied volatilities for 2-Month and 3-Month put options are relatively close to the realized volatilities. The best individual estimate for the underlying volatility on average would again have been the IVs of 2-Month ATM options. Overall, predictive power is higher for ATM options and hypothesis for constant predictive power is rejected. The results of this section combined with the results of call options suggest that the predictive power of DAX implied volatilities variate systematically across options moneyness.

Figure 11. Differences between implied and realized volatilities for put options



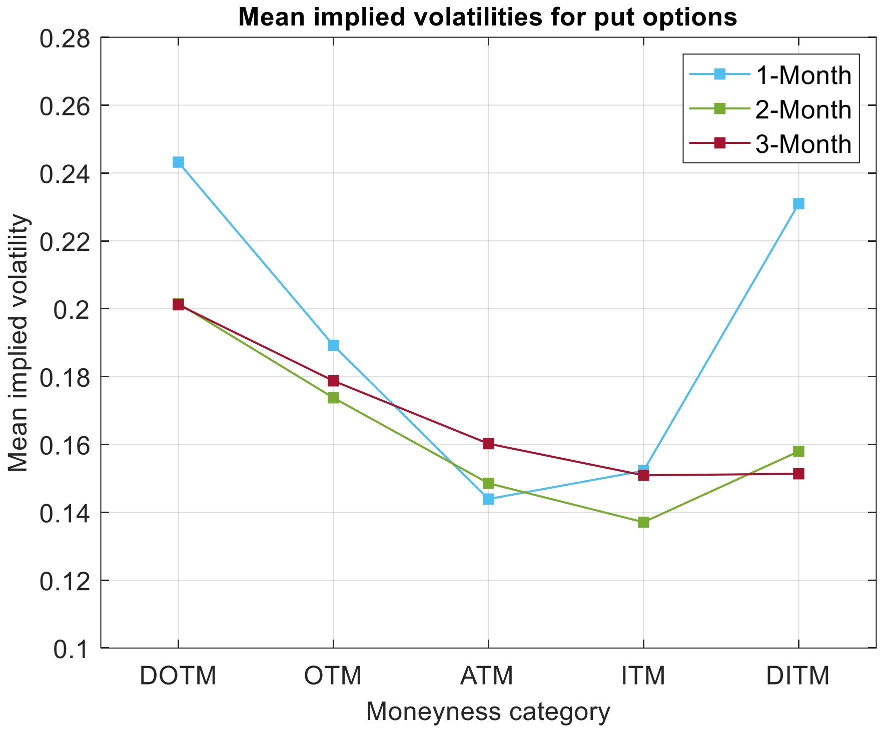
Descriptive statistics for the implied volatilities of put options (Table 17) show that the 1-Month DOTM and in DITM options have the highest mean implied volatilities of all the option classes while the lowest mean IVs were extracted from ATM and ITM moneyness categories. The Black-Scholes assumption for constant volatilities across moneyness and maturity categories is rejected.

Table 17. Descriptive statistics for implied volatilities of put options

Maturity	Moneyness	Obs.	IVmean	IVmin	IVmax
1-Month	DOTM	3091	24.32%	15.08%	196.75%
	OTM	3428	18.92%	11.74%	698.74%
	ATM	3264	14.39%	5.41%	110.98%
	ITM	1206	15.23%	6.08%	83.96%
	DITM	339	23.10%	11.94%	59.65%
2-Month	DOTM	3098	20.16%	14.01%	57.13%
	OTM	3385	17.37%	10.17%	88.81%
	ATM	3065	14.86%	5.26%	31.81%
	ITM	1323	13.71%	6.17%	27.43%
	DITM	354	15.80%	9.41%	29.80%
3-Month	DOTM	2036	20.12%	14.52%	30.66%
	OTM	2213	17.87%	11.89%	27.76%
	ATM	2082	16.02%	6.97%	32.29%
	ITM	809	15.09%	8.69%	31.39%
	DITM	261	15.14%	9.34%	21.83%

Both Table 17 and Figure 12 indicate a clear smile pattern for 1-Month put options, which again seems to fade away for the longer maturities. When the smile shape for call options completely vanishes for 3-Month category, for 3-Month DITM put options the mean IV is still slightly higher than for ITM options.

Figure 12. Mean estimated implied volatilities for put options



The noisiness pattern of the estimated implied volatilities follows the call options; Table 17 and Appendices 4, 5 and 6 show that noisiness decreases the longer maturities are considered. While call options included observations with small implied volatilities (Table 15), put options tend to have higher minimum IVs but higher maximum IVs. Implied volatilities up to 698.74 % are extremely high (Table 17), but since none of the price observations exceeded the upper boundary condition, this study follows the work from Chan et al. 2004 and do not exclude extraordinarily high implied volatility observations. In overall, the patterns of individual IVs for put options are similar to those of call options and discussed problems in section 6.1 probably hold true also for the put options.

6.3 Implied Volatilities of Calls vs Puts

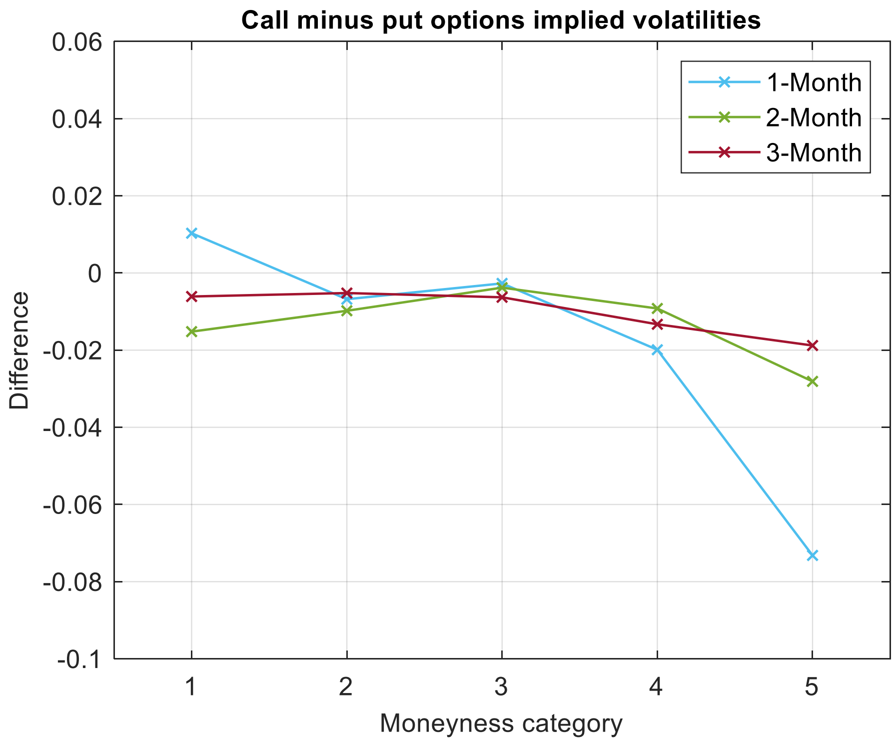
Next, more detailed results of the difference in implied volatilities between the call and the put options are presented. Comparison of IVs by option types is done for each moneyness and maturity group by subtracting the mean implied volatilities of puts from the mean implied volatilities of calls in the same strike price group (Table 6 in section 4.2). The differences of mean implied volatilities are presented in Table 18 and in Figure 13.

Table 18. Differences between call and put implied volatilities

	Maturity		
Moneyness	1-Month	2-Month	3-Month
1	1.03%	-1.52%	-0.61%
2	-0.68%	-0.98%	-0.52%
3	-0.27%	-0.38%	-0.63%
4	-1.99%	-0.92%	-1.33%
5	-7.32%	-2.81%	-1.88%

Table 18 and Figure 13 show that the implied volatilities of put options are higher in 14 out of 15 cases. Mean implied volatility of calls exceeded mean IV of puts only in moneyness category 1 of 1-Month options. Highest difference is in 1-Month category, DITM put option mean IV is 7,32 % higher than DOTM call option IV.

Figure 13. Differences between call and put option implied volatilities



The smallest difference between IVs of call and put options is for 1-Month ATM options (-0.27%). In overall differences grow the further options are from the money. Table 18 and Figure 13 confirm that the implied volatilities are not constant across option types.

6.4 Trading Profits with Implied Volatility Delta Hedging

The results of selling call and put options based on simulation Strategy 1 are presented in Table 19. Like discussed in section 5.2.1, sign test indicates the probability of positive profit, profits describe the mean profits in index points and the profits divided by the amount invested shows the return rate on initial absolute investment. *, ** and *** indicate that the result is statistically significant at the 10 %, 5 or 1 % level, respectively. Profits for both option types and for the different moneyness and maturity classes are expected to be zero and therefore also return rates should be zero. The hypothesis of zero profits would also imply that the implied volatilities could not be connected to the trading profits. In addition, the zero profit hypothesis means that the probability for profit is expected to be 50 %.

Table 19 shows that the probability for profit, returns in index points and return rates are not zero or constant through moneyness and maturity categories for either call or put options. Overall, 24 out of 30 sign test values across option types have statistically significant above 50 % probabilities for profit. 17 out of 30 profit in index values are significantly positive and 10 out of 30 return rates significantly exceed zero, respectively. The assumption for zero trading profits before transaction costs is relaxed for both option types.

A more detailed inspection of call options shows that DOTM and DITM call options have higher probabilities for profit, higher profits in index points and higher return rates throughout all maturity classes compared to other three moneyness categories. DOTM 1-Month call options have the highest probability for profit of 72.17 % with statistical significance at the 1 % level. Correspondingly, DITM 3-Month call options have the highest mean value (21.21) for the profit in index points at the 1 % significance level. 2-Month DOTM calls have the highest statistically significant mean profit ratio of 1.53 %.

Sign test- and profits in index points values from Table 19 indicate a smile pattern for profitability of call options across moneyness, suggesting systematic pattern in trading profits by moneyness. On the other hand, return rates are substantially higher for DOTM than DITM options and the smile shape for the return rates across moneyness attenuates more towards a smirk shape. However, DOTM profit ratio values are not statistically significant. ATM call options are the weakest performers in terms of trading profits based on all three profit indicators except for 3-Month return rate, which strengthens the perception of systematic fluctuation of trading profits.

Table 19. Results for trading simulation Strategy 1

Option	Moneyness	Obs.	Sign Test	Profits	Profits/|Initial Amount|
1-Month Call	DOTM	1369	72.17%***	6.42***	1.46%
	OTM	3133	65.21%***	0.84	0.25%
	ATM	3345	54.32%***	-10.77	-0.18%
	ITM	1922	59.37%***	-1.77	-0.01%
	DITM	763	69.46%***	7.00***	0.07%***
1-Month Put	DOTM	3091	59.43%***	8.01***	1.12%***
	OTM	3428	54.81%***	-1.12	-0.05%
	ATM	3264	52.67%***	-11.74	-0.21%
	ITM	1206	59.45%***	1.64	0.01%
	DITM	339	85.25%***	23.48***	0.21%***
2-Month Call	DOTM	2181	66.21%***	15.74***	1.53%
	OTM	3399	53.96%***	1.92	0.10%
	ATM	3151	48.46%	-13.70	-0.23%
	ITM	1379	55.47%***	-3.23	-0.03%
	DITM	540	67.78%***	15.63***	0.15%***
2-Month Put	DOTM	3098	57.97%***	18.19***	1.35%***
	OTM	3385	52.41%***	3.84**	0.15%***
	ATM	3065	47.83%	-11.02	-0.19%
	ITM	1323	56.24%***	8.02**	0.10%
	DITM	354	54.52%*	14.44***	0.14%***
3-Month Call	DOTM	1880	58.94%***	17.87***	1.26%
	OTM	2464	41.56%	-24.76	-0.83%
	ATM	2067	32.85%	-27.78	-0.50%
	ITM	567	33.33%	-4.25	-0.09%
	DITM	240	41.67%	21.21***	0.19%***
3-Month Put	DOTM	2036	92.39%***	37.62***	1.71%***
	OTM	2213	82.11%***	27.76***	0.75%
	ATM	2082	71.81%***	19.25***	0.33%
	ITM	809	62.55%***	26.14***	0.33%***
	DITM	261	73.95%***	59.98***	0.60%***

Trading profits in Table 19 for put options show a similar pattern to call option trading profits. However, positive profit from selling options together with delta-hedge is more likely for put option especially when longer maturities are considered. For 2-Month put options, profits in index points and profit ratios are above zero in 4 out of 5 cases, whereas corresponding indicators for call options are positive in 3 out of 5 cases. 3-Month put options are profitable in all moneyness classes, maintaining statistical significance at the 1 % level for all indicators except for OTM and ATM return rates. A decent performance of 3-Month put options can be confirmed also by viewing the individual best performers across moneyness and maturity classes. 3-Month DOTM puts have the highest probability for profit (92.39 %) and highest return rate (1.71 %), both being statistically significant at the 1 % level. DITM 3-Month put options have generated the highest profits in index points of 59.98 at the 1 % significance level.

Evidence from Table 19 suggests statistically significant above zero profits especially for DOTM and DITM options. Smile or smirk shape of returns according to moneyness classes is clear for both option types and maturities, and therefore, both hypotheses, that for zero trading profits as well as the one for constant profitability across moneyness categories are rejected.

6.5 Trading Profits with Historical Volatility Delta Hedging

Table 20 presents the trading simulation results for the simulation Strategy 2 where volatility input for delta-hedge is equal to the historical volatility of the underlying. Results differ from the results presented in section 6.4 due to the differences in hedging profits and initial amount invested in the opening of the hedging position. Also risks differentiate between Strategies 1 and 2 but the nature and amount of risks with different hedge-volatilities used are out of the scope of this paper.

Despite using different hedging tactics there are many similarities between the results presented in Table 20 and Table 19. Since the mean differences between the implied and historical volatilities are relatively small for at-the-money options, also the trading simulation results for ATM options are close to each other for the simulations 1 and 2. The largest difference compared to the results from trading simulation 1 is increased probabilities for profit: probability for profit for Strategy 2 increased in 28 out of 30 cases. Probabilities for profit increased relatively in the distance from at-the-money options. For both option types, the rise of the probabilities compared to Strategy 1 were highest for DOTM and DITM options. Increased probabilities of profit for other than ATM options makes the smile pattern of profits across moneyness categories clear, and therefore, the hypothesis of zero fluctuation in returns across moneyness categories is rejected.

Table 20. Results for trading simulation Strategy 2

Option	Moneyness	Obs.	Sign Test	Profits	Profits/|Initial Amount|
1-Month Call	DOTM	1369	80.28%***	8.76***	67.69%***
	OTM	3133	68.21%***	3.69***	2.63%***
	ATM	3345	54.38%***	-10.95	-0.17%
	ITM	1922	65.76%***	-1.15	-0.01%
	DITM	763	88.20%***	10.97***	0.10%***
1-Month Put	DOTM	3091	88.90%***	9.60***	30.65%***
	OTM	3428	68.20%***	-1.01	4.74%***
	ATM	3264	53.77%***	-11.82	-0.21%
	ITM	1206	63.76%***	3.06*	0.03%
	DITM	339	95.58%***	21.06***	0.18%***
2-Month Call	DOTM	2181	68.64%***	27.95***	2.26%***
	OTM	3399	58.61%***	9.79***	0.31%
	ATM	3151	48.46%	-12.54	-0.20%
	ITM	1379	54.82%***	-3.95	-0.05%
	DITM	540	75.56%***	15.34***	0.14%***
2-Month Put	DOTM	3098	71.98%***	17.99***	2.31%
	OTM	3385	52.70%***	1.54	0.17%
	ATM	3065	48.03%	-9.67	-0.17%
	ITM	1323	60.77%***	19.01***	0.22%
	DITM	354	66.95%***	20.41***	0.19%***
3-Month Call	DOTM	1880	63.30%	36.89***	1.99%
	OTM	2464	46.06%	-16.62	-0.36%
	ATM	2067	34.06%	-27.22	-0.47%
	ITM	567	39.15%	-5.42	-0.12%
	DITM	240	64.58%	29.01***	0.27%***
3-Month Put	DOTM	2036	82.91%***	29.98***	1.55%
	OTM	2213	71.58%***	21.10***	0.59%
	ATM	2082	69.98%***	18.22***	0.31%
	ITM	809	69.22%***	31.60***	0.39%***
	DITM	261	83.52%***	65.53***	0.65%***

Another clear difference compared to results from Strategy 1 is that Strategy 2 produced very high return rates for 1-Month DOTM call and put options. Return rates 67.69 % and 30.65 % are a result of the small initial amount required for the initial delta-hedge position. The sample of 1-Month options include observations with very small delta, caused by them being relatively far out-of-the-money or in-the-money combined with short time-to-maturity. This kind of options have very low probability to expire in the money. This causes extremely high individual return rates, which biases mean return rates for 1-Month DOTM options. The effect of very small deltas of DOTM options fades away towards at-the-money options.

Altogether, Table 20 confirms that the profits from selling call or put options are above zero for certain moneyness categories before incorporation of transaction costs. Trading profits exceeding zero are statistically significant for both option types for DOTM and DITM options. Systematic fluctuation for trading profits across moneyness categories is also confirmed.

6.5 Trading Profits including Transaction Costs

The results for Strategy 3 in Table 21 and for Strategy 4 in Appendix 7 show that abnormal trading profits disappear when transaction costs are included. Overall, including transaction costs makes delta-hedged shorting of options unprofitable. All the sign test values except for DITM 3-Month puts in Strategy 4 predict below 50 % probability for profit. Returns in index points and return rates are positive for 1-Month and 3-Month put options in Strategy 3 and for 3-Month put options in Strategy 4. However, as discussed before in sub-section 6.1, it is likely that the results for DITM options are biased. Before incorporating transaction costs, well-performed DOTM options have a high negative return rate due to the small initial investment required.

Table 21. Results for trading simulation Strategy 3

Option	Moneyness	Obs.	Sign Test	Profits	Profits/│Initial Amount│
1-Month Call	DOTM	1369	12.20%	-7.64	-7.67%
	OTM	3133	42.32%	-11.86	-1.39%
	ATM	3345	46.91%	-26.08	-0.46%
	ITM	1922	31.89%	-22.28	-0.23%
	DITM	763	18.61%	-14.93	-0.14%
1-Month Put	DOTM	3091	28.11%	-6.44	-1.19%
	OTM	3428	38.33%	-15.52	-0.80%
	ATM	3264	45.47%	-27.90	-0.49%
	ITM	1206	43.03%	-20.76	-0.20%
	DITM	339	46.02%	0.22	0%
2-Month Call	DOTM	2181	36.31%	-14.47	-4.42%
	OTM	3399	45.45%	-27.33	-1.34%
	ATM	3151	40.18%	-43.78	-0.76%
	ITM	1379	32.63%	-35.81	-0.41%
	DITM	540	20.56%	-18.93	-0.19%
2-Month Put	DOTM	3098	32.34%	-10.27	-0.72%
	OTM	3385	36.60%	-24.98	-0.82%
	ATM	3065	40.36%	-41.15	-0.70%
	ITM	1323	46.49%	-26.73	-0.27%
	DITM	354	22.88%	-21.87	-0.18%
3-Month Call	DOTM	1880	38.40%	-29.57	-3.07%
	OTM	2464	29.06%	-70.74	-2.38%
	ATM	2067	25.88%	-73.51	-1.32%
	ITM	567	26.81%	-53.20	-0.72%
	DITM	240	34.17%	-30.63	-0.38%
3-Month Put	DOTM	2036	25.10%	-7.36	-0.42%
	OTM	2213	31.45%	-17.27	-0.47%
	ATM	2082	33.33%	-26.95	-0.44%
	ITM	809	33.13%	-24.90	-0.26%
	DITM	261	50.57%	8.52	0.12%**

The earlier rejected hypothesis for non-positive profits and constant profits across moneyness categories before transaction costs seems to hold again when transaction costs are taken into account. Table 21 and Appendix 7 do not show constant patterns for profitability across moneyness categories. Profit probabilities variate inconsistently and return rates indicate that out-of-the-money options in general are the weakest performers. On the other hand, DOTM options are still the best performers in terms of returns in index points, followed by DITM and OTM options.

6.6 Results for Trading Simulations without Outliers

The results of all the trading simulations without outliers are presented in Appendices 8, 9, 10 and 11. Appendices 8 and 9 show the robustness checks for the simulation results before transaction costs (Strategies 1 and 2), whereas Appendices 10 and 11 present the robustness checks for simulation results after transaction costs (Strategies 3 and 4).

The robustness checks for the results of the Strategies 1 and 2 do not change the results obtained in sub-sections 6.4 and 6.5. Appendices 8 and 9 show that profits are significantly above zero especially for DOTM and DITM options in both option classes. Hypotheses for zero trading profits and non-fluctuating profitability across moneyness remain rejected. At-the-money options are closer to zero profits in index points and return rates for 1-Month and 2-Month options, but they still remain the worst performers.

Removing observations more than two standard deviations away from the mean profit rates for Strategies 3 and 4 results weak performance of ATM options being less severe for 1-Month and 2-Month options. The results for profitability of the different moneyness categories are still mixed. For robustness checked Strategy 3 (Appendix 10) DITM 1-Month and 3-Month put options show the only positive indicators for profits, whereas the robustness checked results for Strategy 4 (Appendix 11) show profitability also for 2-Month OTM calls and 2-Month ITM puts. All in all, robustness checks for Strategies 3 and 4 do not change the results obtained in sub-section 6.5.

7 Conclusion and Discussion

The purpose of this study was to extract price information from DAX options and evaluate possible inefficiencies in the DAX option market. The first research question along with two sub-questions aimed to investigate whether DAX implied volatilities follow the Black-Scholes model assumption for constant implied volatilities and constant information:

1. *Are extracted implied volatilities of DAX 30 call and put options constant through moneyness and maturity as the Black-Scholes option pricing model suggests?*
    a) *Are there differences in mean implied volatilities of DAX 30 options across call and put option types?*
    b) *Does information for future volatilities of the underlying vary systematically across option moneyness categories?*

The investigation of the information content of the prices of DAX options provided information against the hypothesis for constant implied volatilities. The results were consistent with the previous literature: implied volatilities were dependent on the moneyness and maturity of options. The volatility smile extracted from options with short time-to-maturities changed its shape towards a skew when longer maturities were considered.

Implied volatilities also differed between the option types. Put options exhibited higher implied volatilities for 14 out of 15 moneyness and maturity categories. The results also indicated strong evidence against constant information content of DAX implied volatilities. The information content for future volatilities differed systematically in different moneyness categories. At-the-money options provided the best estimates for the underlying volatilities across option types and maturity classes.

The second research question along with one subquestion were formed to investigate possibilities for gaining abnormal profits by selling DAX options. The strategy of systematic selling of options combined with delta-hedge was chosen based on the information content of implied volatilities and the results from the reviewed previous literature.

2. *Can systematic selling of DAX 30 options together with delta-hedge generate statistically significant profits before or after transaction costs?*
    a) *Do trading profits of DAX 30 options fluctuate systematically across option types or moneyness categories before or after transaction costs?*

The results from delta-hedged trading simulations before transaction costs showed statistically significant above zero profits for multiple moneyness categories and for both option types. Also, the hypothesis of constant trading profits across options' moneyness and option type was rejected. The profits before transaction costs for deep-out-of-the-money and deep-in-the-money options were substantially higher than the profits for the other moneyness categories. Overall, profits before transaction costs varied systematically by moneyness. At-the-money options were the worst performers while the trading profits of out-of-the-money and in-the-money options were in the middle of DOTM, DITM and ATM options. Excluding the outliers improved the mean performance of at-the-money options but the rejection of the two hypotheses before transaction costs remained valid.

Incorporation of the transaction costs changed the results essentially. On post-cost basis, the trading strategies did not generate positive profits and in addition, the earlier found relationships for the profitability across the moneyness categories disappeared. The removal of the outliers made the negative returns for at-the-money options less severe but all in all, the assumption of non-positive trading profits after transaction costs stayed valid.

The conclusions for the results after transaction costs make it clear that the costs related to trading and hedging of option positions play a large role on the final profits. From an individual investor's perspective, utilization of possible inefficiencies on DAX 30 option market is difficult due to transaction costs. Especially daily updating of delta-hedged portfolio makes profiting from small mispricing challenging.

The discussed results for implied volatilities and trading profits of DAX 30 options are based on data from 2018 to 2019. This relatively short time-span may affect the results and the conclusions might be different for a different dataset. Also, the type of the used data, daily closing bid and ask offers for the options and the daily closing prices of the underlying may weaken the reliability of the results. Ask and bid offers of the options should be matched with the time stamp of the underlying price, which might not be accurate in the used dataset. In order to fix this, intraday quotes should be used.

As discussed in section 6.1, the reliability of the results is highest for OTM and DOTM options. These options are substantially more liquid compared to ATM, ITM and DITM options categories. In addition, the original raw data included a remarkable amount of extremely cheap options in ITM and DITM categories. Hentschel (2003) discusses that bid-ask spreads along with precisions of the prices are potential factors causing estimation errors in implied volatilities. Naturally, wide spreads and imprecise price observations also affect the trading simulation results.

Another limitation is related to the possible trading risks from selling especially out-of-the-money options. Some studies discuss that the writing of options exposes the trader to vega risk, which is compensated with higher returns (Bollen & Whaley, 2004). On the other hand, Green and Figlewski (1999) mention that market makers compensate these kinds of risks by setting up an extra margin for sold options. However, in this study an extra margin set up by a market maker would show in the results as lower profits. This paper did not try to evaluate the risks of selling options and delta hedging, but the results were conducted with two different delta-hedge set-ups.

The contribution of this thesis is twofold; This thesis complemented the previous research on the results of DAX implied volatilities across moneyness and maturity categories. The results for the trading simulations provided information for trading of index options from an individual investor's perspective, which has not been a common subject in the previous research. With this different kind of research implementation, economically significant results for previously suggested overpricing of out-of-the-money put options were not found. The results of this paper showed that the Black-Scholes model assumption for constant implied volatilities on DAX 30 options is not valid, but the results after transaction costs also suggested that the DAX 30 option market is efficient with respect that an individual investor cannot generate abnormal profits by selling options combined with delta-hedge.

Future research on trading profits of options could incorporate different kind of trading strategies. The focus on the selling strategies in this paper was a rather big limitation and wider usage of different trading strategies might produce different results. For example, individual options could be sold when they exceed the mean historical implied volatility, and correspondingly, bought when the implied volatility is lower than the mean historical implied volatility. Implied volatility surfaces should be included in the strategy for calculating the differences for historical and current implied volatilities since implied volatilities tend to vary across moneyness and maturity of options.

References

Ahn, H. J., Kang, J., & Ryu, D. 2008. Informed trading in the index option market: The case of KOSPI 200 options. Journal of Futures Markets, vol. 28, no. 12, pp. 1118-1146.

Ahmad, R., & Wilmott, P. 2006. Which Free Lunch Would You Like Today, Sir?: Delta Hedging, Volatility Arbitrage and Optimal Portfolios. Wilmott Magazine, pp. 64-79.

Barry, M. J., & Taggart, R. A. 2007. Hedging Strategies for Exploiting Mispriced Options Using the Black-Scholes Model with Excel. Journal of Applied Finance, vol. 17, no. 2, pp. 5-12.

Bates, D. S. 2000. Post-'87 crash fears in the S&P 500 futures option market. Journal of Econometrics, vol. 94, no. 1-2, pp. 181-238.

Bentes, S. R. 2014. Measuring persistence in stock market volatility using the FIGARCH approach. Physica A, vol. 408, pp. 190-197.

Benzoni, L., Collin-Dufresne, P., & Goldstein, R. S. 2011. Explaining asset pricing puzzles associated with the 1987 market crash. Journal of Financial Economics, vol. 101, no. 3, pp. 552-573.

Black, F. & Scholes, M. 1973. The pricing of options and corporate liabilities. Journal of Political Economy, vol. 81 no. 3, pp. 637-654.

Blaskowitz, O. J., Härdle, W. K., & Schmidt, P. 2004. Skewness and kurtosis trades. Anastassiou, G. A. & Rachev, S., T. Handbook of Computational and Numerical Methods in Finance, pp. 1-14. Boston: Birkhäuser.

Bollen, N. P., & Whaley, R. E. 2004. Does net buying pressure affect the shape of implied volatility functions?. The Journal of Finance, vol. 59 no. 2, pp. 711-753.

Branger, N., & Schlag, C. 2004. Why is the index smile so steep? Review of Finance, vol. 8, no. 1, pp. 109-127.

Brooks, C. 2014. Introductory econometrics for finance. 3rd ed. Cambridge: Cambridge University Press.

Carr, P. 1999. FAQ’s in option pricing theory. Bank of America Securities, pp. 1-30.

Chan, K. C., Cheng, L. T., & Lung, P. P. 2004. Net buying pressure, volatility smile, and abnormal profit of Hang Seng Index options. Journal of Futures Markets, vol. 24, no. 12, pp. 1165-1194.

Chen, C. H., Chung, H., & Yuan, S. F. 2014. Deviations from put–call parity and volatility prediction: Evidence from the Taiwan index option market. Journal of Futures Markets, vol. 34, no. 12, pp. 1122-1145.

Christoffersen, P., Heston, S., & Jacobs, K. 2009. The shape and term structure of the index option smirk: Why multifactor stochastic volatility models work so well. Management Science, vol. 55 no. 12, pp. 1914-1932.

Chou, R. K., Chung, S. L., Hsiao, Y. J., & Wang, Y. H. 2011. The impact of liquidity on option prices. Journal of Futures Markets, vol. 31, no. 12, pp. 1116-1141.

Chung, S. L., & Shackleton, M. 2002. The binomial Black–Scholes model and the Greeks. Journal of Futures Markets, vol. 22, no. 2, pp. 143-153.

Corrado, C. J., & Su, T. 1997. Implied volatility skews and stock return skewness and kurtosis implied by stock option prices. The European Journal of Finance, vol. 3, no. 1, pp. 73-85.

Cox, J. C., Ross, S. A., & Rubinstein, M. 1979. Option pricing: A simplified approach. Journal of Financial Economics, vol. 7, no. 3, pp. 229-263.

Cremers, M., & Weinbaum, D. 2010. Deviations from put-call parity and stock return predictability. Journal of Financial and Quantitative Analysis, vol. 45, no. 2, pp. 335-367.

Cuthbertson, K., & Nitzche, D. 2001. Financial Engineering: Derivatives and Risk Management. Chichester: Wiley.

Da Fonseca, J. & Grasselli, M., 2011. Riding on the smiles. Quantitative Finance, vol. 11, no. 11, pp.1609-1632.

DAX Futures 2020. Eurex Exchange. Accessed 5.4.2020. Available: https://www.eurexchange.com/exchange-en/products/idx/dax/DAX-Futures-139902

DAX Options 2020. Eurex Exchange. Accessed 2.4.2020. Available: https://www.eurexchange.com/exchange-en/products/idx/dax/DAX-Options-139884

Mini-DAX Futures. 2020. Eurex Exchange. Accessed 5.4.2020. Available: https://www.eurexchange.com/exchange-en/markets/idx/dax/Mini-DAX-Futures-139894

Duan, C. W., & Hung, K. 2010. The Effect of Net Buying Pressure on Implied Volatility: Empirical Study on Taiwan's Options Market. International Review of Accounting, Banking and Finance, vol. 2, no. 2, pp. 51-84.

Dumas, B., Fleming, J., & Whaley, R. E. 1998. Implied volatility functions: Empirical tests. The Journal of Finance, vol. 53, no. 6, pp. 2059-2106.

Trading calendar. 2020. Eurex Exchange. Accessed 20.4.2020. Available: https://www.eurexchange.com/exchange-en/trade/trading-calendar/2800!tradingcalendar?state=H4sIAAAAAAAAAGWOzQoCMQyEX0Vy7kHFH-xNUE8Ki4rgsbRZLcQtJimyLPvu1oU9Oacw8w2ZDoJTPHB6gZ0tF6v5Zj2f_mSG4Jr-7Np5VAHb9eWOLHpEVWSwTSYy8IwqFXLlHliqQyM2nnLAS1SUEUsNtVWow-daOBA28M3ILFsAAo2TSW8TPCEtiLZnsyqKJE18g7-iU-QnkB9_32DP0Xb_G03cgAAAA&calMode=YEAR&calSheet=-1

Fouque, J. P., Papanicolaou, G., Sircar, R., & Solna, K. 2004. Maturity cycles in implied volatility. Finance and Stochastics, vol. 8 no. 4, pp. 451-477.

Ghufran, B., Awan, H. M., Khakwani, A. K., & Qureshi, M. A. 2016. What causes stock market volatility in Pakistan? Evidence from the field. Economics Research International, vol. 2016, pp. 1-9.

Green, T. C., & Figlewski, S. 1999. Market risk and model risk for a financial institution writing options. The Journal of Finance, vol. 54, no. 4, pp. 1465-1499.

Gutierrez, J. A., Martinez, V., & Tse, Y. 2009. Where does return and volatility come from? The case of Asian ETFs. International Review of Economics & Finance, vol. 18, no. 4, pp. 671-679.

Hamrita, M. E., & Trifi, A. 2011. The Relationship between Interest Rate, Exchange Rate and Stock Price: A Wavelet Analysis. International Journal of Economics and Financial Issues, vol. 1, no. 4, pp. 220.

Hentschel, L. 2003. Errors in implied volatility estimation. Journal of Financial and Quantitative Analysis, vol. 38, no. 4, pp. 779-810.

Hull, J. C. 2006. Options, Futures and Other Derivatives. 6th ed. Upper Saddle River: Prentice Hall.

Hull, J. C. 2012. Options, Futures and Other Derivatives. 8th ed. Upper Saddle River: Prentice Hall.

Hull, J. C. 2014. Options, Futures and Other Derivatives. 9th ed. Upper Saddle River: Prentice Hall.

Hull, J., & White, A. 2017. Optimal delta hedging for options. Journal of Banking & Finance, vol. 82, pp. 180-190.

Johnson, N. J. 1978. Modified t tests and confidence intervals for asymmetrical populations. Journal of the American Statistical Association, vol. 73, no. 363, pp. 536-544.

Jouini, E., Cvitanic, J., & Musiela, M. 2001. Option Pricing, Interest Rates and Risk Management. Cambridge: Cambridge University Press.

Kapner, K. 2006. Risk reversals. Global Financial Markets Institute, pp. 1-9.

Kim, B. & Kim, S. 2015. The Impact of Investor Sentiment on Risk Neutral Skewness: Around Financial Crisis. Journal of Derivatives and Quantitative Studies, vol. 23, no. 4, pp. 475-516.

Larkin, J., Brooksby, A., Lin, C. T., & Zurbruegg, R. 2012. Implied volatility smiles, option mispricing and net buying pressure: evidence around the global financial crisis. Accounting & Finance, vol. 52, no. 1, pp. 47-69.

MacKenzie, D. 2006 Is Economics Performative? Option Theory and the Construction of Derivatives Markets. Journal of the History of Economic Thought, vol. 28 no. 1, pp. 29-55.

Mittnik, S., & Rieken, S. 2000. Lower-boundary violations and market efficiency: Evidence from the German DAX-index options market. Journal of Futures Markets: Futures, Options, and Other Derivative Products, vol. 20, no. 5, pp. 405-424.

Mittnik, S., & Rieken, S. 2000. Put-call parity and the informational efficiency of the German DAX-index options market. International Review of Financial Analysis, vol. 9, no. 3, pp. 259-279.

Mixon, S. 2007. The Implied Volatility Term Structure of Stock Index Options. Journal of Empirical Finance, vol. 14, no. 3, pp 333-354.

Mo, D., Todorova, N., & Gupta, R. 2015. Implied volatility smirk and future stock returns: evidence from the German market. Managerial Finance, vol. 41, no. 12, pp. 1357-1379

Muzzioli, S. 2010. Option-based forecasts of volatility: an empirical study in the DAX-index options market. The European Journal of Finance, vol. 16, no. 6, pp. 561-586.

Muzzioli, S. 2011. The skew pattern of implied volatility in the DAX index options market. Frontiers in Finance and Economics, vol. 8, no, 1, pp. 43-68.

Mykland, P. A. 2000. Conservative delta hedging. Annals of Applied Probability, vol. 10, no. 2, pp. 664-683.

Lee, R. W. 2004. Implied volatility: Statics, dynamics, and probabilistic interpretation. Baeza-Yates, R., Glaz, J., Gzyl, H., Hüsler, J. & Palacios, J.L. Recent advances in applied probability, pp. 241-268. Boston: Springer Science+Business Media, Inc.

Pena, I., Rubio, G., & Serna, G. 1999. Why do we smile? On the determinants of the implied volatility function. Journal of Banking & Finance, vol. 23 no. 8, pp. 1151-1179.

Poon, S-H. 2005. A Practical Guide to Forecasting Financial Market Volatility. Chichester: John Wiley & Sons Ltd.

Poon, S-H., Granger, C.W.J 2003. Forecasting Volatility in Financial Markets: A Review. Journal of Economic Literature, vol. 41, no. 2, pp. 478-539.

Rouah, F. D. & Vainberg, G. 2007. Option Pricing Models and Volatility Using Excel-VBA. New Jersey: John Wiley & Sons.

Rubinstein, M. 1985. Nonparametric tests of alternative option pricing models using all diffusion approximations in financial markets. Review of Financial Studies, vol. 3, pp. 393-430.

Rubinstein, M. 1994. Implied binomial trees. Journal of Finance, vol. 49, no. 3, pp. 771-818.

Schoutens, W., Simons, E., & Tistaert, J. 2003. A perfect calibration! Now what? pp. 1-30. The best of Wilmott. John Wiley & Sons Ltd: Chichester.

Schwert, G. W. 1990. Stock market volatility. Financial Analysts Journal, vol. 46 no. 3, pp. 23-34.

Taleb, N. N. 1997. Dynamic hedging: managing vanilla and exotic options. 1st ed. New York: John Wiley & Sons.

Taylor, S. J. 2005. Asset Price Dynamics, Volatility and Prediction. Oxford: Princeton University Press.

Theissen, E. 2012. Price discovery in spot and futures markets: A reconsideration. European Journal of Finance, vol. 18, no. 10, pp. 969-987.

Äijö, J. 2008. Implied volatility term structure linkages between VDAX, VSMI and VSTOXX volatility indices. Global Finance Journal, vol. 18, no. 3, pp. 290-302.

Hafner, R., & Wallmeier, M. 2000. The dynamics of DAX implied volatilities, pp. 1-40.

Vähämaa, S. 2004. Delta hedging with the smile. Financial Markets and Portfolio Management, vol, 18 no. 3, pp. 241-255.

Wallmeier, M., & Hafner, R. 2000. The dynamics of DAX implied volatilities. Accessed 15.6.2020. Available: https://www.researchgate.net/publication/228241770_The_Dynamics_of_DAX_Implied_Volatilities

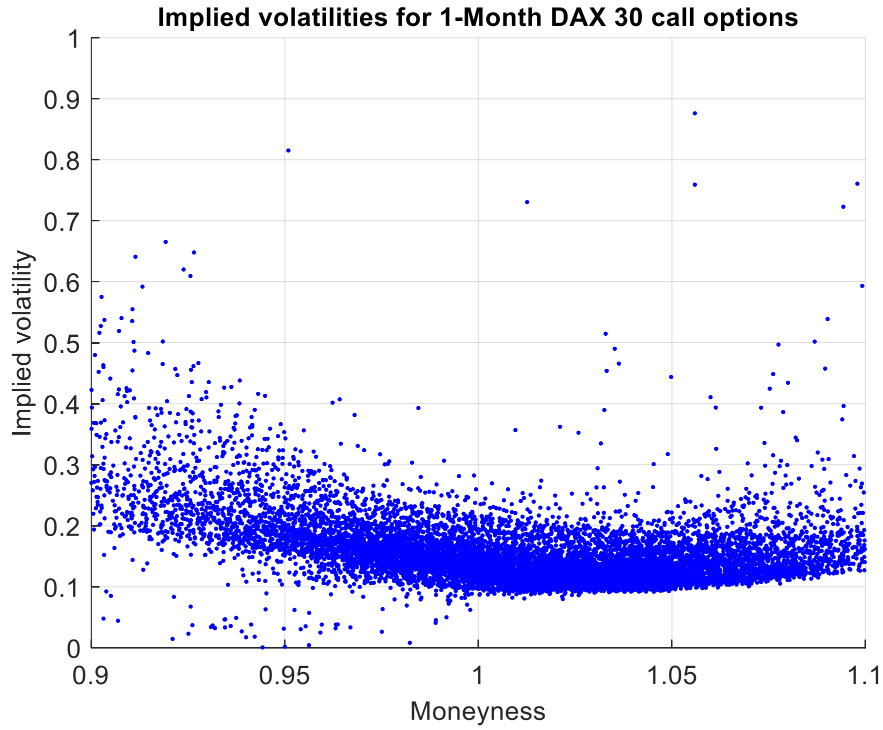
Weber, E. J. 2008. A Short History of Derivative Security Markets. Hafner, W. & Zimmermann, H. 2009. Vinzenz Bronzin's Option Pricing Models, pp. 431-466. New York: Springer.

Wilmott, P. 2007. Paul Wilmott introduces quantitative finance. Chichester: John Wiley & Sons.

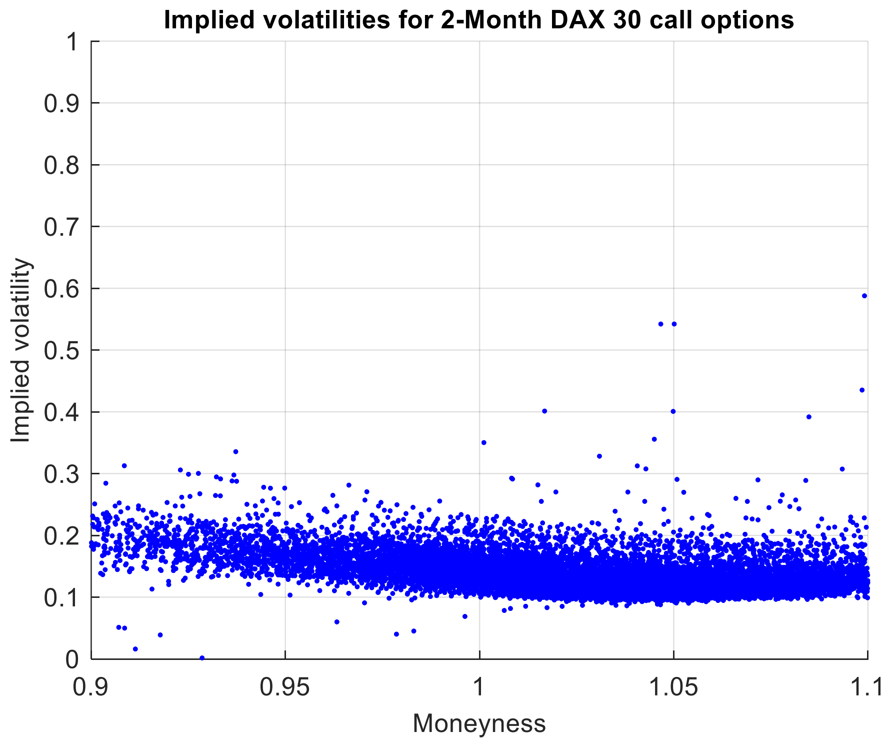
Xu, X., & Taylor, S. J. 1994. The term structure of volatility implied by foreign exchange options. Journal of Financial and Quantitative Analysis, vol. 29, no. 1, pp. 57-74.

Appendices

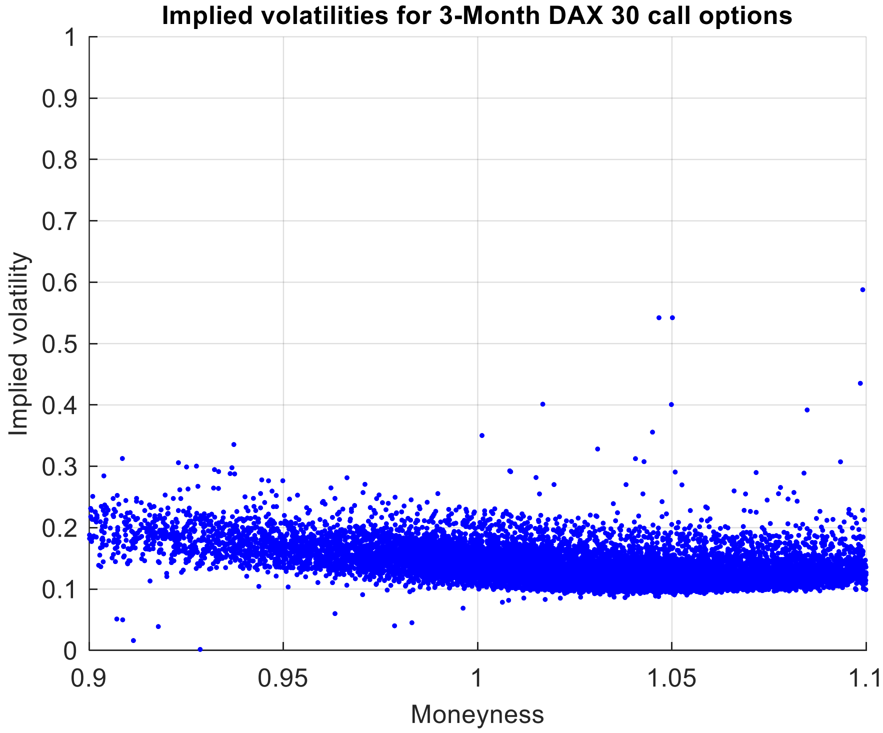
Appendix 1. Estimated implied volatilities for 1-Month DAX 30 call options



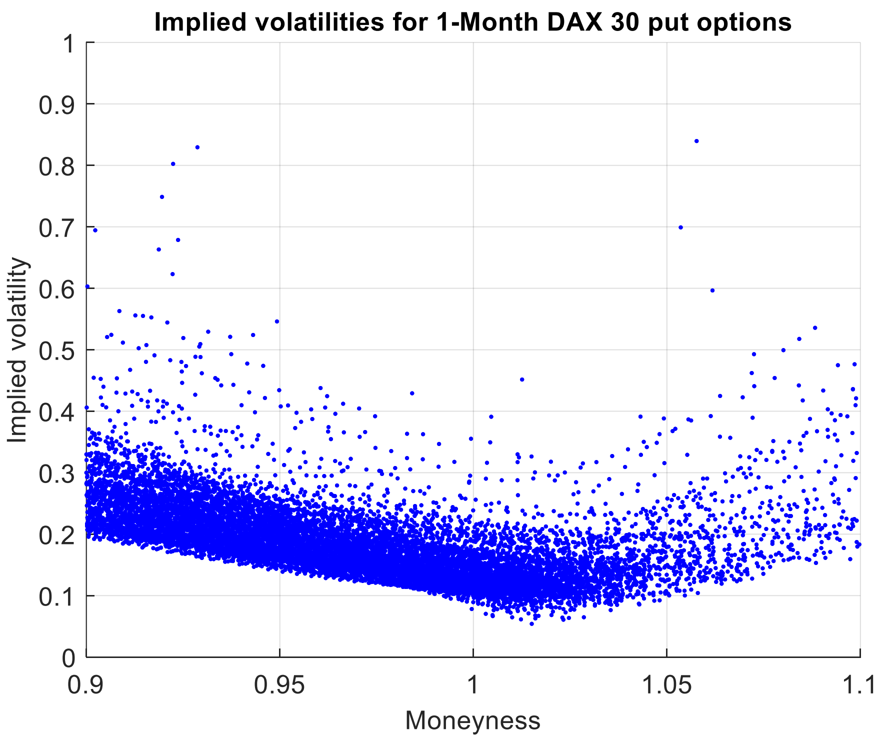
Appendix 2. Estimated implied volatilities for 2-Month DAX 30 call options



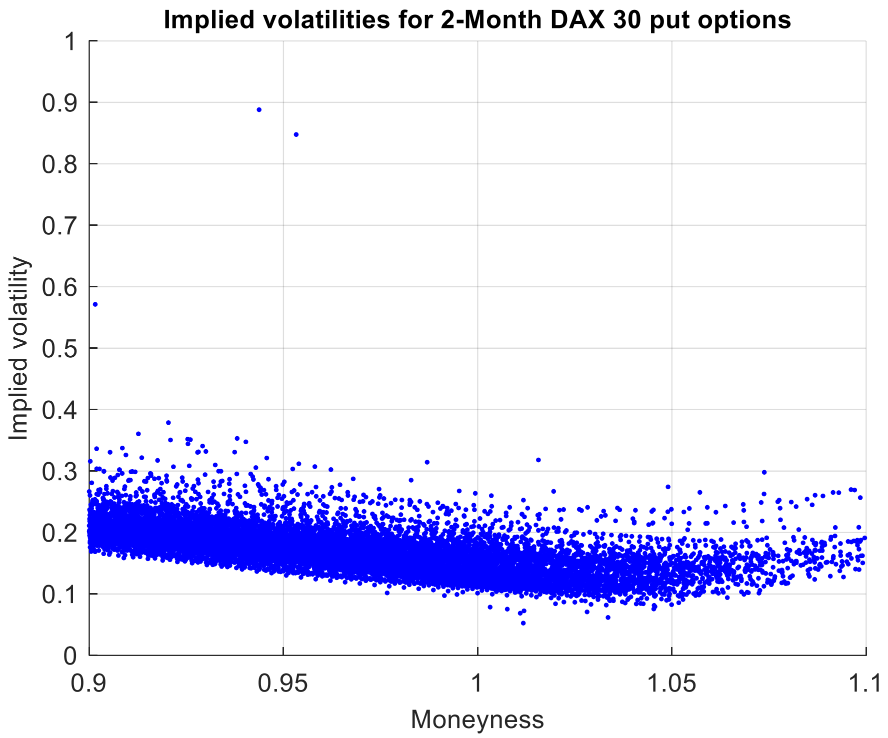
Appendix 3. Estimated implied volatilities for 3-Month DAX 30 call options



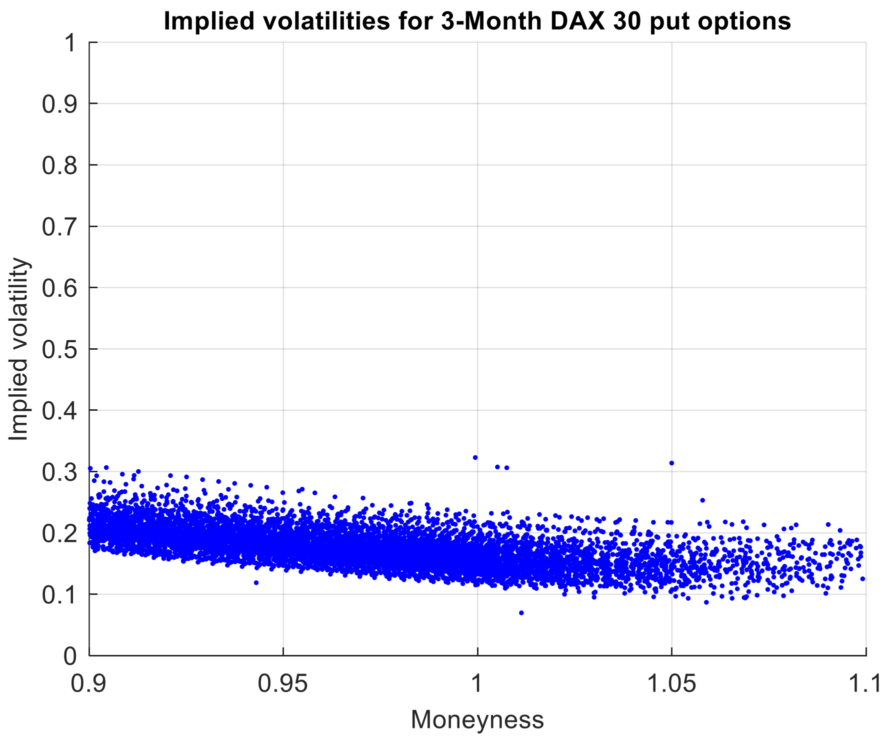
Appendix 4. Estimated implied volatilities of 1-Month DAX 30 put options (limited to 100 %).



Appendix 5. Estimated implied volatilities for 2-Month DAX 30 put options



Appendix 6. Estimated implied volatilities for 3-Month DAX 30 put options



Appendix 7. Results for trading simulation Strategy 4

Option	Moneyness	Obs.	Sign Test	Profits	Profits/|Initial Amount|
1-Month Call	DOTM	1369	21.40%	-4.35	-477.55%
	OTM	3133	45.13%	-8.79	-10.50%
	ATM	3345	48.10%	-26.17	-0.45%
	ITM	1922	40.01%	-21.47	-0.21%
	DITM	763	29.88%	-11.39	-0.10%
1-Month Put	DOTM	3091	35.00%	-4.85	-93.31%
	OTM	3428	48.86%	-15.41	-2.59%
	ATM	3264	47.58%	-27.98	-0.50%
	ITM	1206	41.13%	-19.34	-0.18%
	DITM	339	37.76%	-2.19	-0.01%
2-Month Call	DOTM	2181	50.48%	-0.65	-1.87%
	OTM	3399	50.72%	-19.09	-0.81%
	ATM	3151	40.72%	-42.47	-0.72%
	ITM	1379	35.24%	-36.62	-0.42%
	DITM	540	21.85%	-18.63	-0.18%
2-Month Put	DOTM	3098	34.47%	-10.47	-3.83%
	OTM	3385	37.61%	-27.28	-1.03%
	ATM	3065	40.72%	-39.81	-0.68%
	ITM	1323	50.49%	-15.74	-0.16%
	DITM	354	21.47%	-15.90	-0.13%
3-Month Call	DOTM	1880	49.68%	-8.84	-1.05%
	OTM	2464	34.42%	-61.82	-1.75%
	ATM	2067	26.08%	-72.85	-1.28%
	ITM	567	25.57%	-54.69	-0.73%
	DITM	240	30.42%	-23.05	-0.27%
3-Month Put	DOTM	2036	51.62%*	-15.01	-2.54%
	OTM	2213	40.89%	-23.93	-0.85%
	ATM	2082	33.14%	-27.98	-0.47%
	ITM	809	38.57%	-19.44	-0.20%
	DITM	261	61.30%***	14.07	0.17%***

Appendix 8. Results for trading simulation Strategy 1 without outliers

Option	Moneyness	Obs.	Sign Test	Profits	Profits/|Initial Amount|
	DOTM	1367	72.35%***	5.23***	3.32%***
	OTM	3037	66.18%***	4.37***	0.61%

1-Month Call	ATM	3128	56.91%***	1.23	0.06%
	ITM	1790	61.23%***	7.11***	0.07%
	DITM	719	75.38%***	7.96***	0.07%***
1-Month Put	DOTM	2883	58.90%***	4.12***	0.69%***
	OTM	3251	56.87%***	4.67***	0.30%
	ATM	3069	54.42%***	-3.76	-0.04%
	ITM	1126	61.90%***	8.63***	0.09%
	DITM	291	84.54%***	17.97***	0.15%
2-Month Call	DOTM	2116	67.53%**	18.93***	2.60%
	OTM	3286	54.99%***	9.67***	0.51%
	ATM	3014	49.24%	-7.71	-0.11%
	ITM	1268	58.23%***	2.94*	0.04%***
	DITM	493	72.21%***	10.44***	0.10%***
2-Month Put	DOTM	2849	54.48%***	7.83***	0.57%***
	OTM	3185	52.56%***	3.56**	0.10%***
	ATM	2884	49.24%	-2.78	-0.04%
	ITM	1258	58.35%***	21.45***	0.25%
	DITM	324	50.62%	6.68***	0.07%***
3-Month Call	DOTM	1774	61.78%***	24.78***	1.95%
	OTM	2343	43.06%	-13.38	-0.40%
	ATM	1972	32.20%	-27.47	-0.49%
	ITM	536	32.09%	-7.99	-0.13%
	DITM	226	42.04%	13.52***	0.11%***
3-Month Put	DOTM	1781	91.80%***	26.42***	1.19%
	OTM	1997	82.72%***	23.61***	0.63%
	ATM	1986	72.76%***	19.48***	0.34%
	ITM	746	59.79%***	16.96***	0.22%***
	DITM	214	68.22%***	38.05***	0.36%***

Appendix 9. Results for trading simulation Strategy 2 without outliers

Option	Moneyness	Obs.	Sign Test	Profits	Profits/|Initial Amount|
1-Month Call	DOTM	1358	80.12%***	8.81***	25.31%***
	OTM	3039	69.23%***	6.77***	1.08%***
	ATM	3141	57.40%***	2.00*	0.09%
	ITM	1789	69.14%***	9.80***	0.09%
	DITM	725	91.72%***	14.27***	0.13%
	DOTM	3027	90.62%***	11.85***	44.05%***
	OTM	3152	68.46%***	0.69	2.68%***

1-Month Put	ATM	3112	55.72%***	-2.50	0.02%
	ITM	1126	66.52%***	11.32***	0.11%
	DITM	295	95.93%***	16.20***	0.13%***
2-Month Call	DOTM	2135	69.88%***	32.05***	2.14%
	OTM	3315	59.55%***	15.62***	0.52%
	ATM	2997	49.42%	-6.48	-0.08%
	ITM	1277	56.77%***	3.82**	0.04%
	DITM	492	77.64%***	14.65***	0.13%
2-Month Put	DOTM	3002	73.52%***	20.00***	4.14%***
	OTM	3140	53.34%***	3.16**	0.27%***
	ATM	2882	49.27%	-2.25	-0.04%
	ITM	1264	62.97%***	31.93***	0.36%
	DITM	330	64.85%***	13.04***	0.11%***
3-Month Call	DOTM	1798	63.35%***	37.89***	1.93%
	OTM	2366	46.66%	-11.11	-0.21%
	ATM	1965	33.23%	-28.42	-0.50%
	ITM	539	38.03%	-9.41	-0.16%
	DITM	220	63.18%***	20.89***	0.19%***
3-Month Put	DOTM	1926	86.86%***	38.10***	2.90%
	OTM	2102	73.69%***	24.41***	0.73%
	ATM	1993	71.25%***	20.64***	0.35%
	ITM	738	67.07%***	24.17***	0.29%
	DITM	216	80.09%***	46.96***	0.45%

Appendix 10. Results for trading simulation Strategy 3 without outliers

Option	Moneyness	Obs.	Sign Test	Profits	Profits/│Initial Amount│
1-Month Call	DOTM	1247	13.39%	-7.12	-5.44%
	OTM	2951	44.87%	-6.43	-0.74%
	ATM	3103	50.11%	-10.97	-0.15%
	ITM	1776	33.61%	-9.99	-0.10%
	DITM	713	19.21%	-10.47	-9.62E-04
1-Month Put	DOTM	2849	27.06%	-5.94	-1.12%
	OTM	3247	39.85%	-6.51	-0.35%
	ATM	3061	47.73%	-15.58	-0.23%
	ITM	1120	45.71%	-9.29	-0.08%
	DITM	322	46.89%	2.66**	0%
	DOTM	2079	38.10%	-8.14	-2.94%
	OTM	3277	47.12%	-16.50	-0.83%

2-Month Call	ATM	2996	42.06%	-31.16	-0.50%
	ITM	1260	34.29%	-22.39	-0.25%
	DITM	483	19.67%	-16.88	-0.17%
2-Month Put	DOTM	2966	31.39%	-12.63	-0.83%
	OTM	3083	37.43%	-17.90	-0.62%
	ATM	2864	42.74%	-24.81	-0.41%
	ITM	1252	48.96%	-8.84	-0.07%
	DITM	333	21.62%	-22.98	-0.19%
3-Month Call	DOTM	1749	41.28%	-14.45	-1.82%
	OTM	2290	31.27%	-51.14	-1.69%
	ATM	1961	27.03%	-60.76	-1.06%
	ITM	532	27.63%	-44.42	-0.59%
	DITM	225	35.11%	-26.25	-0.32%
3-Month Put	DOTM	1817	22.89%	-8.99	-0.47%
	OTM	1987	33.37%	-10.43	-0.27%
	ATM	1929	35.93%	-18.61	-0.29%
	ITM	770	33.38%	-22.60	-0.23%
	DITM	247	47.77%	1.15	0.04%

Appendix 11. Results for trading simulation Strategy 4 without outliers

Option	Moneyness	Obs.	Sign Test	Profits	Profits/│Initial Amount│
1-Month Call	DOTM	1354	21.64%	-4.33	-206.51%
	OTM	3116	45.38%	-8.82	-3.38%
	ATM	3116	51.44%*	-10.27	-0.13%
	ITM	1777	42.32%	-7.64	-0.08%
	DITM	725	31.31%	-5.17	-0.05%
1-Month Put	DOTM	2980	36.31%	-3.72	-41.23%
	OTM	3357	49.57%	-14.34	-0.60%
	ATM	3105	49.95%	-15.38	-0.21%
	ITM	1118	43.65%	-6.66	-0.05%
	DITM	324	37.65%	-0.87	-2.19E-05
2-Month Call	DOTM	2069	53.21%***	5.12***	-0.84%
	OTM	3299	52.26%***	-9.92	-0.45%
	ATM	2987	42.55%	-29.76	-0.47%
	ITM	1265	37.47%	-21.80	-0.25%
	DITM	484	22.11%	-13.51	-0.13%
	DOTM	3022	35.34%	-8.07	-1.11%
	OTM	3148	38.44%	-21.64	-0.66%

2-Month Put	ATM	2855	43.01%	-24.07	-0.40%
	ITM	1255	53.15%**	1.80	0.04%
	DITM	328	19.21%	-18.39	-0.16%
3-Month Call	DOTM	1794	49.68%	-8.84	-1.05%
	OTM	2319	34.42%	-61.82	-1.75%
	ATM	1962	26.08%	-72.85	-1.28%
	ITM	538	25.57%	-54.69	-0.73%
	DITM	227	30.42%	-23.05	-0.27%
3-Month Put	DOTM	1922	54.68%***	-5.19	-0.66%
	OTM	2032	44.49%	-13.40	-0.39%
	ATM	1935	35.66%	-18.87	-0.30%
	ITM	777	39.38%	-16.12	-0.16%
	DITM	247	59.92%***	9.75***	0.12%***