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Modelling the stochastic dynamics of transitions between states in social systems incorporating self-organization and memory

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ABSTRACT

This conceptual research presents a new stochastic model of the dynamics of state-to-state transitions in social systems, the Zhukov–Khvatova model. Employing a mathematical approach based on percolation theory the model caters for random changes, system memory and self-organisation. Curves representing the approach of the system to the percolation threshold differ significantly from the smooth S-shaped curves predicted by existing models, showing oscillations, steps and abrupt steep gradients.The modelling approach is new, working with system level parameters, avoiding reference to node-level changes and modelling a non-Markov process by including self-organisation and the effects (memory) of previous system states over a configurable number of time intervals. Computational modelling is used to demonstrate how the percolation threshold (i.e. the share of nodes which allows information to spread freely within the network) is reached.Possible applications of the model discussed include modelling the dynamics of viewpoints in society during social unrest and elections, changing attitudes in social networks and forecasting the outcome of promotions or uptake of campaigns. The easy availability of system level data (network connectivity, evolving system penetration) makes the model a particularly valuable addition to the toolkit for social sciences, politics, and potentially marketing.

1. Introduction

Conducting theoretical and practical research into processes of information transmission, state changes and clustering in social systems is essential for developing new approaches to forecasting people's behavior in society, their demands and needs, as well as for forecasting and trying to prevent the spread of undesired viewpoints.

This becomes especially important when new information technologies are developed and introduced, for example, social networks. When measures to prevent the transmission of undesirable information/views are absent, using technologies based on an understanding of the transfer process becomes the only option to counter undesired effects.

Such an understanding is similarly of interest in relation to the spread of public moods and emotional states in people networks. Public moods and emotional states can be positive or negative, and can be connected with certain people, products, politics and/or social issues (Rahn et al, 1996; Kotler, Kartajaya, and Setiawan, 2010; Peter and Olson, 2010; Bell, 2011). This research explores information adoption and consequent dissemination, wherein each participant of a social system (or in a particular case, a user of a social network) is ready to transmit onward a mood or viewpoint they have adopted. However, we are not looking at the viewpoints and states of particular participants of a social system. For us, only the change in the whole system which is influenced by individuals' interactions matters.

As an example of the model's applicability, social networks will be used for modelling the spread of viewpoints. The spread of moods in social media networks has been proposed as a valuable heuristic in predicting a wide variety of phenomena ranging from stock market movements (Bollen et al. 2011) to social values, acculturation and group identity (Broniatowski, 2012; Choudhary et al., 2019) and its importance is well reported by Kramer et al. (2014).

To gain further relevant understanding, it is important to investigate the essence of the processes operating in social systems and to develop adequate models for monitoring and managing their states.

The objective of the current research is to develop a new stochastic model of the dynamics of how moods and viewpoints change within social systems. With this new model, it will become possible to define the

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probable timing of a system reaching a critical threshold. To reach the overall objective, the research analyses the application of the stochastic model to social networks using specially developed computer software (Zhukov et al., 2018). To demonstrate the practical applicability of the model, the authors then propose an algorithm for monitoring the state of a social network structure. Social networks are chosen as the example for demonstrating the applicability of the model because we are able to calculate the percolation threshold (i.e. the share of nodes which allows information to spread freely within the network) by using the dependencies between the percolation threshold and the network density we obtained in previous works (Block et al., 2015; Zhukov and Lesko, 2015a,b).

Percolation refers to regular movement of media (whether information, liquid, epidemics, etc.) within a random environment (Tarasevich, 2002; Zhukov and Aleshkin, 2008). Percolation occurs if the neighboring active nodes of a network, with a certain probability, connect and group together to form a cluster which gradually becomes large enough to connect two “sides” of the net or two chosen nodes. This is called the “percolation threshold” (PT): the proportion of transmitting (non-blocked) nodes when conductivity between two randomly chosen nodes in a network takes place (Zhukov et al., 2018). For the purpose of this research, when observing the transmission of moods and viewpoints, the PT is a defined proportion of people having certain moods (emotions) and viewpoints. If this proportion is reached in a network, the information (moods, views, emotions) can be transmitted across the whole social system without any barriers.

In previous papers, the present authors developed new approaches for describing the state of viewpoints and moods in social networks based on percolation theory (Zhukov et al., 2018; Block et al., 2015). Computer modelling was used to demonstrate how the density of a random network - defined as the average number of connections per node - influences its ability to transfer information from which percolation thresholds were estimated and a dependency equation was obtained. This equation enables us to model questions such as: how does the density of a popular social network (for example, supporters of a football club) affect the optimal level of influence for changing people's opinions about a product or service? In the current paper we will use the equations obtained earlier to define the PT in social networks while demonstrating how the newly developed model can be applied.

The model presented in this research (the Zhukov-Khvatova model) is novel due to several features: *first*, the model takes account of memory about previous states of the system, i.e. not only about the closest previous state (as in the Markov processes), but also those states which occurred before the immediately previous state. *Secondly*, the uniqueness of this model lies in the fact that, while deriving the differential equation, self-organization processes are considered. This is described in the current paper where the logical meaning of each element of the new equation is discussed in Section 3.1. *Thirdly*, formulating and solving the boundary value problem enables us to obtain the probability density function for social network transitions from one state to another without making a priori assumptions about the process. Therefore, this paper presents not only a new model, but also a new approach to modelling incorporating stochasticity, self-organisation and memory.

A fundamental characteristic and benefit of our approach to modelling is that it is a macro statistical approach to describing processes in complex social systems. For our model, it is important that each network has a macro characteristic, which describes the system as a whole – this is chosen to be the percolation threshold (PT), and this depends on the network density. Our model can describe not only the processes of reaching the PT, but also how the given critical value of the share of people with a certain state of mind (viewpoint) can be reached.

The paper is structured as follows: Section 2 presents a literature review of existing models that describe information distribution in society along with their strengths and weaknesses; and a gap in the research is identified. Section 3 presents our newly developed model of the stochastic dynamics of state-to-state transitions in social systems and social networks (as the most obvious example of the model applicability) combined with results previously obtained by the authors. Section 4 presents simulation results wherein the new stochastic model is compared to the Blackman model which has been one of the key models describing the kinetics of state changes within a social system, and with other models. Finally, in Section 5, an algorithm for monitoring the states of a social network structure is proposed and further examples of practical applications of the proposed stochastic model in analysing and managing processes in complex social and economic systems are discussed. The Conclusions section summarises the results obtained, discusses limitations and suggests avenues for further research.

2. Literature review

The first models to describe social dynamics and information distribution processes in society, both negative and positive, were mainly models borrowed from communication theories in the fields of Biology and Economics (Minaev et al., 2013). These include, for example, the models of Griliches, Kalman, Blackman, Bass, Gompertz, Bertalanffy, etc. In this research, they present an S-shaped logistic curve of how the number of followers of a specific idea will change over time.

Blackman (1905) was the first to create the equation for modelling biotechnological processes. It has become an important theoretical tool which helps in the understanding of many natural biological systems, and industrial and social processes. The Blackman model has become one of the central models describing the dynamics of state change in a social system and will be described in more detail in Section 5.3 when the model developed here is compared with it and other models.

The Gompertz model is another traditional model which presents a function describing growth as being slowest at the start and end of a given time period: a special case of a generalized logistic function. The function was originally designed to describe human mortality, but now it is also used to model market impacts in finance, new product uptake such as mobile telephone diffusion (Sultanov et al., 2016), etc.

In certain very simple cases, Blackman and Gompertz models can produce good results. However, they will generally not correctly describe the dynamics of such processes, which manifest themselves in an avalanche growth of the number of followers of a new idea, gradually reaching the threshold value before declining when interest in the new idea decreases, or when the idea is replaced by another one. It is also important to note that neither the Blackman nor Gompertz models consider the opportunity of change from one state to another, and then back, for example, the transfer of a supporter to a loyal state then back to being disloyal.

Griliches proposed a logistic growth function (an S-shaped curve) for the Rogers diffusion-of-innovation framework (Griliches 1957). This model has been very influential in marketing and management sciences (Pannhorst and Dost, 2019, Fernández-Durán, 2014, etc).

As is evident above, models of processes in complex social systems have evolved considerably over time - from early models described by smooth S-shaped curves in which the inverse dependencies were not taken into account, to more recently developed sophisticated curves which account for positive and negative inverse dependencies among variables (Van Oorschot, 2018).

The distinctive feature of the model developed in this paper (making it different from all existing models) is that, while describing complex social processes, it not only takes account of inverse dependencies, but also of the potential in the system for self-organization and the presence of memory about previous states. In order to emphasize the particular features of the suggested model and point out its place alongside the variety of existing models, a brief overview of some modern dynamic models which describe processes in social networks is included later in this paper.

It is important to note that there are certain analogies between the dynamics of processes taking place in social networks, the spread of epidemics and some physical diffusion (Barrat et al., 2008; Easley and Kleinberg, 2010; Jackson, 2008; Lesko et al., 2019), and chain processes appearing in paired interactions (Barrat et al., 2008). Therefore, theoretical approaches to describe such social systems have much in common with describing physical systems, which to a considerable extent helps in understanding their behavior (Watts, 2002; Gleeson and Cahalane, 2007, Liu, He, and Liu, 2018;).

Social processes are usually characterized by sophisticated mechanisms of chain reactions and stochasticity, wherein various multiple conditions of the nodes depend on the influence of their neighbors, and their conditions can also vary (Easley and Kleinberg, 2010; Karsay et al., 2014; Centola and Macey, 2007). For example, in a youth environment it is essential to consider how peer pressure will influence the dynamics and final outcome of the process (Centola and Macey, 2007).

Dynamic models, which use threshold mechanisms, mainly focus on cascade events, where the share of network nodes in a certain condition develops quickly from a certain microscopic state, which seizes more and more new nodes. These approaches were created in earlier social theories (Granovetter and Soong, 1983) and described in detail by Watts in his model of cascade events (Watts, 2002). This model is certainly current (Watts, 2002, Watts, D. J. & Dodds, 2007; Gleeson, 2008, 2011, 2013, Nematzadeh et al., 2014; Singh et al., 2013; Sreenivasan et al., 2013; Piedrah´ıta et al., 2013). However, observing the real processes in social networks reveals its limitations. In particular, Watts' model focuses on the appearance of momentary global cascades initiated by single localized disturbances, whereas examples can be seen wherein it is the threshold mechanisms of social network process development which play an important part. Furthermore, the stochastic component of the processes is in question.

In many cases, dependency on time is applied to describe the processes in social networks using a stochastic approach. For example, in Airoldi et al. (2008) a model of mixed membership in stochastically formed groups is discussed. This model aims to explain pairwise changes, such as the presence or absence of links among pairs of objects. Analysis of probabilistic changes among the pairs requires specific assumptions, for example, about independence or instability of the link (of mixed membership in stochastically formed groups). Such a model enables us to model the changes in the number of members in groups and their clustering.

Du, Wang, and Faloutsos (2010) analysed large multimodal social networks. The essence of their approach was that interactive social networks often simultaneously include multiple relations, people can build a social network by adding each other as friends and they can also form several implicit social networks (multimodality). The method used by the authors was structure analysis of a multimodal social network graph developing over time. Applying this approach to real structures reveals temporary online regularity in people's social interactions. The phenomenological (or descriptive) nature of this model can be seen as a drawback. Like the previously discussed models, it does not consider self-organization of processes and the presence of memory of previous states affecting the system participants.

Some progress in describing social processes was also achieved by cellular automaton models, which consider the dynamics of change in states over time. However, they present a number of significant disadvantages (for example, they consider only general scenarios). Models based on graph theories and matrices of Markov chain transitions are also not entirely suitable for researching the dynamics of social processes due to their static nature.

In summary, all the models described have strengths and weaknesses, and each has a specific field of application, so none are universal. Arguably, the authors of the aforementioned models do not consider a number of important circumstances: *first* that the human factor leads to stochasticity in processes occurring in complex systems; and *secondly* that account should be taken of self-organization present in the system as well as of node-level memory about previous system states. No evidence of such research has been found among existing works, and this specifically motivated our research.

3. Developing a new model of stochastic dynamics of state-to-state transitions in social systems and social networks (as an example of the model's applicability)

3.1. Constructing probabilistic differential schemes of state-to-state transitions

In the approach developed, we can consider society as a system whose states at any moment in time can be described by certain parameters, for example, the share of people inclined against (or in favor of) the ruling party, or people holding extremist views; such parameters can possess continuous random values and follow a non-deterministic distribution law. It is important to emphasize that our model is not about the individual interactions among people or states (conditions) of certain people. We describe macroscopic changes in the state of the whole social system using the language of the theory of stochastic, but non-Markov processes.

Quite often, when applying probabilistic models, researchers make assumptions in advance about the characteristics of events and processes they are researching. For example, assumptions can be made about the value of an observed process (or a parameter describing this process) according to a certain distribution law (for example, Poisson's law). However, the result of this is that the result obtained depends on the preliminary assumptions. Non-determinacy (of our model), on the contrary, assumes that a priori nothing is known about the characteristics of an observed random value and its distribution law; this removes the dependency of the final result on preliminary assumptions inherent in the model.

We can call the whole set of a social system's states X. The state observed at a certain moment of time t can be denoted as x_i $(x_i \in X)$.

The observed states may be related to, for example, economic and political processes taking place in society. We can also introduce the time interval τ, in which the state x_i can change. In this case, any value τ of current time $t = h\tau$, where h is the number of transitions, or stage of transition, between states (the process of transition between steps becomes quasi-continuous, with an infinitely small time interval τ), $h = 0,1,$ 2, 3, …, N. The current state x_i at stage h, after transition to stage $h+1$ can either increase by a given value ε, or decrease by a given value ξ under the influence of randomly appearing factors, and become equal to $x_i+\varepsilon$, *or* x_i-ξ. The values of ε and ξ are the parameters of the processes being modelled. Beyond this, the following limitations should be placed on $x_i+\varepsilon$, and x_i-ξ: $x_i+\varepsilon \le L1$ (L1 is the upper limit of the set X) and $x_i-\xi \ge L2$ (L2 is the lower limit of the set X). In the most straightforward scenario ε and ξ are certain constant values for every stage h and could depend on the activities of mass media.

We can furthermore introduce the notion of the probability of the system being in this or that state, with the assumptions that, for successive stages h, the described system can be characterized by:

$P(x\text{-}\varepsilon,h)$ – the probability that the system is in state $(x\text{-}\varepsilon)$;
$P(x, h)$ – the probability that the system is in state x;
$P(x+\xi,h)$ – the probability that the system is in state $(x+\xi)$.

After each stage, the state x_i (hereafter the index i is omitted to save space), can change by ε or by ξ.

The probability P(x, h + 1) that in the next stage (h + 1) the system (or process) will shift to state x is

$$P(x, h+1) = P(x-\varepsilon, h) + P(x+\xi, h) - P(x, h) \tag{1}$$

The expression (1) and schema in Fig. 1 can be explained as follows: the probability *P*(x, *h* + 1) of transition to state *x* in stage h is defined by the sum of the probability of transition into this stage from state (*x-ε*) and the probability *P(x + ξ, h)* of transition from state (*x* + *ξ*). In these two states (*x-ε*) and (*x* + *ξ*), the system was in stage h. Then, the probability *P(x,h)* of system transition from state *x* (which it was in in stage h) into any other state in the stage is *h* + 1. In this case, it is assumed that transitions occur with a probability which is equal to 1.

Let us note that, in this case, h increases by m = 1.

If *ε(x,h)* and *ξ(x,h)* are 0, then: *P(x,h + 1) = P(x,h)* and the state does not change, which is logical. In this case we consider a continuous Markov process in which the system does not have memory of former states. However, in reality, in such a system as human society, there is always memory of the former composition or state. The model should therefore take this into account. To account for memory let us define the probabilities *P(x-ε,h), P(x + ξ,h)* and *P(x,h)* accounting for the states of the system at previous steps. The schemes of transitions can be depicted in a similar way as in Fig.1. Considering that ε and ξ are certain permanent values, and *t* = *h*•*τ*, where *t* is the time of the process, *h* is the number of a step, *τ* is the length of one step, let us change over from h to t, and after some transformations, obtain the differential equation of our model which describes the dynamics of changes in the states in a social system:

$$\begin{aligned} &m\tau \frac{dP(x,t)}{dt} + \frac{(m\tau)^2}{2!}\frac{d^2P(x,t)}{dt^2} \\ &= \frac{1}{2}\left\{m^2\varepsilon^2 - 2m(m-1)\varepsilon\xi + m^2\xi^2\right\}\frac{d^2P(x,t)}{dx^2} \\ &\quad - m[\varepsilon - \xi]\frac{dP(x,t)}{dx} \end{aligned} \tag{2}$$

The detailed derivation of this equation is presented in the Appendix.

After deriving Eq. (2) with x, let us advance to the dependency of the probability of the system being in state *x* time *t* and the depth of memory *m* (*m* ranges from 1 to ∞):

$$\begin{aligned} \frac{d\rho(x,t)}{dt} &= \frac{m\varepsilon^2 - 2(m-1)\varepsilon\xi + m\xi^2}{2\tau}\cdot\frac{d^2\rho(x,t)}{dx^2} \\ &\quad - \frac{\varepsilon-\xi}{\tau}\cdot\frac{d\rho(x,t)}{dx} - \frac{m\tau}{2}\cdot\frac{d^2\rho(x,t)}{dt^2} \end{aligned} \tag{3}$$

The member of equation $\frac{d\rho(x,t)}{dx}$ describes the ordered transition either into a state where the described value is increasing (ε > *ξ*), or when it is decreasing (ε < *ξ*); the member of equation $\frac{d^2\rho(x,t)}{dx^2}$ describes the random change in state (*uncertainty of change*). The member of equation $\frac{d\rho(x,t)}{dt}$– can be identified as the speed of general change of state of the system over the course of time; the member of equation $\frac{d^2\rho(x,t)}{dt^2}$ describes the process wherein the states become sources of other states emerging (self-organization and acceleration of the ordered ($\frac{d\rho(x,t)}{dx}$) and random ($\frac{d^2\rho(x,t)}{dx^2}$) transitions).

From the point at which the model becomes applicable in equations (3), it is important to consider the limitation imposed on the coefficient $a = (m\varepsilon^2 - 2(m-1)\varepsilon\xi + m\xi^2)/2\tau$ before the second derivative of x, which considers the possibility of a random change of state. The following condition should be fulfilled: $(m\varepsilon^2 - 2(m-1)\varepsilon\xi + m\xi^2) \geq (l - x_0)^2$, and this means that the transition from the starting state x_0 across the threshold of the event occurring cannot happen faster than the time taken to complete one stage τ. If $(m\varepsilon^2 - 2(m-1)\varepsilon\xi + m\xi^2) < (l - x_0)^2$, then the system crosses the threshold of event occurrence in just one stage.

3.2. Formulating and solving the boundary-value problem: an example of social networks

As was noted earlier, research into the processes of state changes in social systems is important for developing new approaches to forecasting people's behavior in society, as well as for forecasting and trying to prevent the spread of undesired viewpoints. This means that there should be a criterion, which tells the system (the society) that the situation is getting dangerous and some preventive steps should be taken. Such a criterion for decision-making can be the share of people with undesired viewpoints in a social system when the administrative bodies must start acting to change the situation. In the context of our research, this criterion can be naturally represented by the percolation threshold (PT).

We can demonstrate that the results of our previous research (Khvatova et al., 2017; Zhukov et al., 2018), on modelling percolation thresholds depending on network density, can be explored in combination with the new model developed in the current research, in order to create efficient algorithms for monitoring and managing the states of social networks.

The previously obtained results are important for the current research in which we define the probable time of reaching the percolation threshold in the newly developed model of stochastic dynamics of state-to-state transitions in social systems in our example of social network structures. It is important to note that the dependence on time of the probability of reaching the PT has not been researched before.

The proportion of people who hold certain views, beyond which proportion these views can unrestrictedly spread among all susceptible people in society, can be called the percolation threshold (PT) of the social network. People are nodes within this network and edges of this network are represented by communication connections; the number of connections is random. In such a network, information transmission can happen simultaneously via many channels and through various nodes.

By illustration we might assume that it is important to monitor the share of negatively-oriented citizens or consumers in society and aim to make sure that this share does not rise above a certain value, i.e. make sure that this share does not exceed the PT. This would mean that the share of people with an undesired viewpoint must remain between 0 and the PT for the given social network. Let PT = *l*. If, for example, the economic situation in the country (society) is having a negative impact,

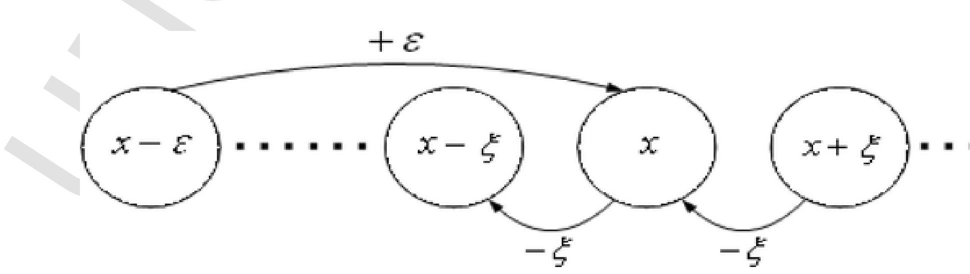

Fig. 1. The schema of potential transitions between states of the system (process) in stage h + 1 (Zhukov, Khvatova and Zaltcman, 2017).

then as it continues the share of negatively-inclined citizens will increase because the value of ε is bigger than the value of ξ at each stage of the process. Hence the state z will come closer to the percolation threshold PT. According to the present model applied in this way, stochastic dynamics are described by the change in the state of society as the parameters ε and ξ change, and their values are defined by the general economic situation or the other external pressure acting on the network.

First boundary condition. We can choose the first boundary condition on the basis of the following assumptions: that condition $x = 0$ defines the complete absence of any processes with particular and measurable parameters in the IS (for example, there are no negative views in society and the share of negative viewpoint-transmitters is zero). The probability of identifying such a state can be different from 0 (although it should be close to 0). However, the probability density which defines the flow in the condition when $x = 0$, should be set to *0* (the conditions of the system cannot enter an area of negative values (the reflecting boundary condition is realised), i.e.:

$$\rho(x,t)_{x=0} = 0 \quad (\boldsymbol{a})$$

Second boundary condition. We can consider such condition of the IS when the value of the vector is close to the boundary limit of its possible values (we can describe this limiting value as *L*). The probability of discovering such a state will not be zero. However, it is important that the probability density defining the probability flow in state $x = L$ be set at zero (the states of the system cannot enter the range of values beyond the maximum possible value), i.e.:

$$\boldsymbol{\rho(x,t)_{x=L}} = 0 \quad \text{(b)}$$

Given that at the moment of time $t = 0$, the system conditions can already be equal to a certain value x_0, the initial condition should be set as follows:

$$\rho\left(x, t=0\right) = \delta\left(x - x_0\right) = \begin{cases} \int \delta\left(x - 0\right) dx = 1, x = x_0 \\ 0, \quad x \neq x_0 \end{cases}$$

As the initial condition contains the delta function, the solution for $\rho(x,t)$ is split across two areas - when $x > x_0$ and when $x \leq x_0$. The presence of the δ–function leads to the fact that the solution, remaining continuous at point $x = x_0$, has derivative discontinuity at this point. For solving Eq. (3), in which the second derivative of t is present, we need to set the second initial condition: $\left.\frac{d\rho(x,t)}{dt}\right|_{t=0}$, i.e. the condition which sets the speeds of change of probability density for any amplitude. The second initial condition is not as obvious as the first, but in this case, we can use the function discontinuity for any moment of time. Let us choose the second initial condition as: $\left.\frac{d\rho(x,t)}{dt}\right|_{t=0} = 0$. This is the condition of having zero speed for the change of the probability density for the time $t = 0$.

Using the methods of operator calculus to solve Eq. (3), for the probability density $\rho_1(x,t)$ and $\rho_2(x,t)$ of discovering the state of the system in one of the values on the interval from 0 to *L*, it is possible to derive the following system of equations:

When $x \geq x_0$:

$$\rho_1\left(x, t\right) = -\frac{2}{L}\alpha\left(x\right)\cdot e^{-\frac{t}{m\tau}}\sum_{n=1}^{\infty}\frac{\sin\left(\pi n\frac{x_0}{L}\right)\sin\left(\pi n\frac{L-x}{L}\right)}{\cos\left(\pi n\right)}\cdot\beta\left(n,t\right)$$

When $x < x_0$

$$\rho_2\left(x, t\right) = -\frac{2}{L}\alpha\left(x\right)\cdot e^{-\frac{t}{m\tau}}\sum_{n=1}^{\infty}\frac{\sin\left(\pi n\frac{L-x_0}{L}\right)\sin\left(\pi n\frac{x}{L}\right)}{\cos\left(\pi n\right)}\cdot\beta\left(n,t\right)$$

where

$$\alpha\left(x\right) = e^{\frac{(x-x_0)(\varepsilon-\xi)}{m\varepsilon^2-2(m-1)\varepsilon\xi+m\xi^2}}$$

$$\beta\left(n,t\right) = ch\left(\frac{t}{\tau}\sqrt{\frac{2\varepsilon\xi}{m^2\left(m\varepsilon^2 - 2\left(m-1\right)\varepsilon\xi + m\xi^2\right)} - \frac{\pi^2 n^2}{L^2}\cdot\frac{m\varepsilon^2 - 2\left(m-1\right)\varepsilon\xi}{m}}\right.$$

The integral *P(l,t)* can be calculated:

$$P\left(l, t\right) = \int_0^{x_0}\rho_2\left(x,t\right)dx + \int_{x_0}^{l}\rho_1\left(x,t\right)dx \qquad (4)$$

Then, the *P(l,t)* function will set the probability that the system state by moment *t* will be along the interval from *0* to *l*, i.e. the *threshold l* will not be reached.

Respectively, the probability $Q_i(t)$ that the *threshold l* will have been reached or exceeded by moment *t*, can be defined as follows:

$$Q\left(l, t\right) = 1 - P\left(l, t\right). \qquad (5)$$

4. Solving the boundary-value problem through dynamics and self-organization of social states in society, and discussion

Analysis of social processes shows that if society is in a stable stationary economic and political state, then the share of people with strictly negative views (viewpoints) is comparatively small, ranging between 0.05 and 0.1, and this provides stability (Sharan 1995). All other individuals (these can be called neutral or pendulous) compose a share of approximately 0.9–0.8. N.B. here we refer to people's true internal convictions.

4.1. Computation experiment and discussion of the results

In order to analyse the proposed model, it is important to set the appropriate values of percolation thresholds of random networks, which were defined in previous works (Khvatova et al., 2017, Zhukov and Lesko, 2015a, b). The values of percolation thresholds depend on the network density (average number of connections per node). The density of a social network can be determined experimentally and then, by using the dependencies of percolation threshold on the average number of connections per node, possible percolation thresholds can be computed (Khvatova et al., 2017, Zhukov and Lesko 2015a, b).

The most common modelling proposed in percolation theory is that of nodes or of bonds (Grimmet, 1989). In the field of bond modelling, researchers attempt to discover the proportion of bonds which must be removed in such a way that the net would fall into two parts in order to prevent percolation. For node modelling, researchers investigate how many nodes must be blocked in order to break the network

For node modelling, the equation $y = 4.39z - 2.41$ is used (Zhukov et al., 2018), wherein $z = 1/\ \omega$, where ω is the network density; y is the natural logarithm of percolation threshold PT (PT = the share of nodes at which transmissivity occurs). When $z = 1/5 = 0.2$ we obtain y = *ln*P = –1.532 and the PT $P = e^{-1.532}$ equals 0.22 (in this case, this is a ratio of the unblocked or 'open' nodes required for information transfer). For bond modelling, the equation $y = -6.581z - 0.203$ is applied (Zhukov et al., 2018). When $z = 0.2$ we obtain a ratio of broken bonds which cause the percolation within the whole network to disappear as being equal to 0.22. Therefore, percolation will occur if the ratio of transmissive connections is equal to 0.78.

To model the process, let us suppose that the share of alternatively inclined citizens x_0 in stable stationary economic and social conditions does not exceed 5% ($x_0 = 0.05$, see Fig. 2) or 10% (see Fig. 3); the value τ is equal to one conditional unit of time ($\tau = 1$), $\varepsilon = 0.02$ (2%)

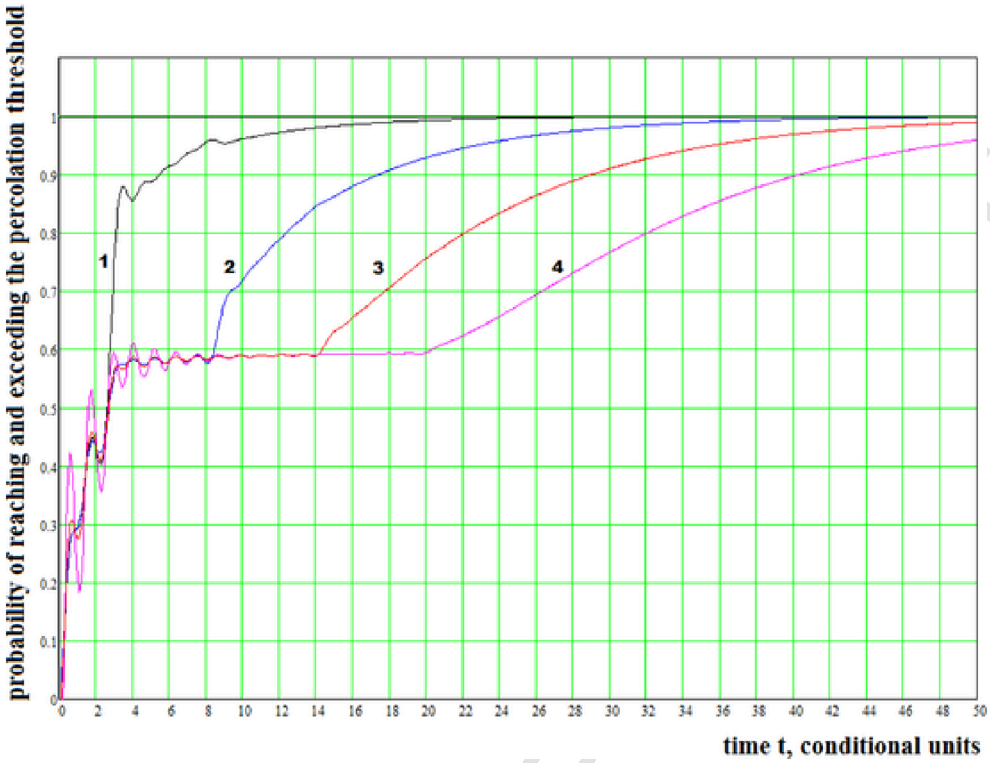

Fig. 2. Graphic representation of modelling results for exceeding the percolation thresholds of negative viewpoints in society (the initial share of negatively-inclined citizens is given as 0.05; this share can decrease by 0.01 and increase by 0.02 with every unit of time).

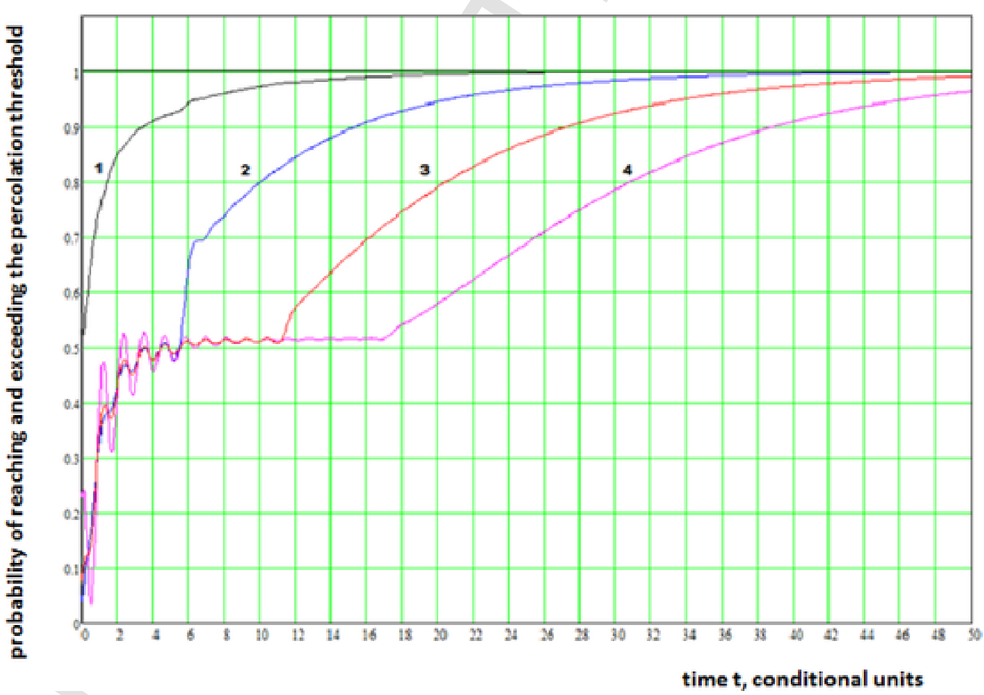

Fig. 3. Graphic representation of modelling results for exceeding the percolation thresholds of negative viewpoints in society (the initial share of negatively-inclined citizens is given as 0.10; this share can decrease by 0.01 and increase by 0.02 with every unit of time).

and $\xi = 0.01$ (1%). Remember that if the situation in the country (in society) is not favourable, the share of alternatively inclined individuals will increase because the value of ε is greater than the value of ξ at every stage of the process. Moreover, the values of ε and ξ may depend on the actions undertaken by mass media.

Let us consider a case when the depth of memory is not too large, for example, when $m = 2$, i.e. information is preserved not only about the previous state, but also about the state before that.

The results of modelling the time of reaching the percolation threshold (Eq. 5) using Eq. (4) and the above example set of parameters, are represented graphically in Fig. 2 and Fig.3.

Curve 1 in Fig.2 and Fig. 3 is constructed for the PT equal to 0.1; curve 2 is made for the PT equal to 0.2; curve 3 – for the PT equal to 0.3; and curve 4 – for the PT equal to 0.4.

Results of the model analysis represented in Figs. 2 and 3 demonstrate the capacity of the system's self-organization which resides in the following: the values of change in state x in one stage ε (increase of x) and ξ (decrease of x) are, in themselves, random. A simple arithmetic approach shows that the number of stages (we can call this q) at which the PT = l might be reached cannot be smaller than $q = (l - x_0)/(\varepsilon - \xi)$. For percolation thresholds $l = 0.1$; 0.2; 0.3; 0.4; the initial state $x_0 = 0.05$, at $\varepsilon = 0.02$ and $\xi = 0.01$ for q will obtain respectively 5; 15; 25 and 35. However as the modelling results show (see Fig. 2) the probability of exceeding the PT is already not equal to zero after the first stage, and this grows significantly faster over time, as the simple arithmetic calculations show. This can be explained by the system's self-organization (not only do ε and ξ define the change in the state of x, but also the states of x themselves are sources of change) and by including memory in the developed model.

Furthermore, Figs. 2 and 3 demonstrate the following:

1. The closer the value of the initial state of the system to the value of the percolation threshold, the faster the probability of it being reached approaches 1.
2. The increase of the probability of PT occurrence has a jump-like character, the length of a step depends on how close the initial share of negatively-inclined individuals x_0 is to the percolation threshold.
3. The probability of reaching the PT has an oscillating character. The farther the initial value of the share of negatively-inclined individuals x_0 is from the PT, the stronger the oscillations will be.

This can be illustrated in Fig. 3: for Curve 1 – x_0 is closest to the percolation threshold and there are almost no oscillations observed. The farther x_0 is from the percolation threshold, the stronger the oscillations. This is especially obvious for Curves 3 and 4 (from the beginning until the first horizontal area. However, this is only true for the given values of ξ and ε. For other ξ and ε the curves will look different, but the general conclusion will be the same. It is important to consider that the model has several parameters, so the dependence of the probability of reaching the percolation threshold on these can be graphically represented as a hypersurface within the space of these parameter values. The number of dimensions of this space is equal to the number of parameters.

This hypersurface will contain particular areas of interest, but this is beyond the scope of this paper and requires further research. The notable characteristic of the process of reaching the PT in stochastic dynamics is the presence of a plateau, which is lengthy in time. Its value depends on the initial share of negatively-inclined citizens x_0.

The special feature of the process of reaching the PT in stochastic dynamics is that if the initial value of negatively-inclined individuals x_0 is equal to the PT (see curve 1 in Fig. 3), the probability of reaching the PT is not equal to 1, but it quickly converges to this value, and this does not contradict our theory. This can arguably be explained as follows: during any sufficiently small time period τ_0 the condition of the system can decrease by values ε = 0.02 and ξ = 0.01 and move away from the percolation threshold 0.10.

When the values of ε and ξ are equal (for example, ε = ξ = 0.02) the character of curves describing the probability of reaching the PT changes slightly (see Fig. 4), in particular, the lengthy plateau with smoothly growing probability of exceeding the PT until it reaches 1, is not observed anymore. The increase of the probability of transition has a sharply pronounced jump-like character. This is connected with the fact that the coefficient $b = \frac{\varepsilon - \xi}{\tau}$ in Eq. (3) $\frac{d\rho(x,t)}{dt} = a\frac{d^2\rho(x,t)}{dx^2} - b\frac{d\rho(x,t)}{dx} - c\frac{d^2\rho(x,t)}{dt^2}$ is equal to 0, and the equation itself looks as follows (when m = 2):

$$\frac{d\rho(x,t)}{dt} = \frac{\varepsilon^2}{\tau}\frac{d^2\rho(x,t)}{dx^2} - \tau\frac{d^2\rho(x,t)}{dt^2}. \tag{6}$$

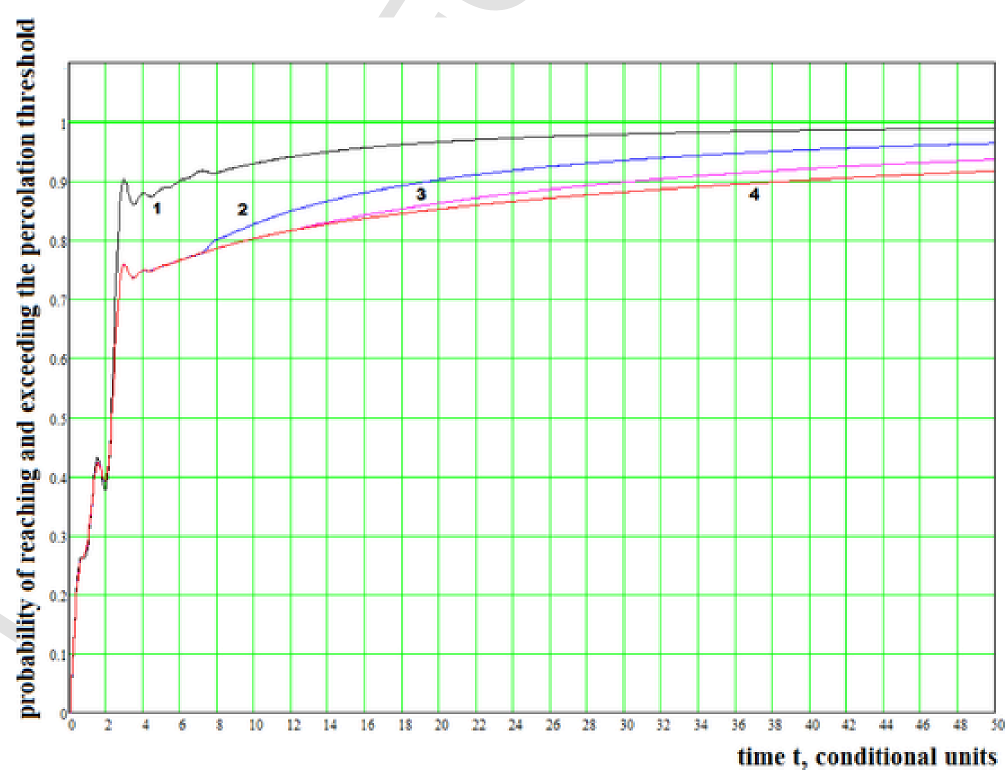

Fig. 4. Graphic representation of modelling results for exceeding the percolation thresholds of negative viewpoints in society (the initial share of negatively-inclined citizens is given as 0.05; this share can decrease by 0.02 with every unit of time; and it can increase by 0.02).

Regular transitions are not possible because the term of Eq. (6) $\frac{d\rho(x,t)}{dx}$ disappears. The term of the equation $\frac{d^2\rho(x,t)}{dx^2}$ remains and defines the random change (only random transitions). In this case, the term of Eq. (6) $\frac{d^2\rho(x,t)}{dt^2}$ defines the acceleration only of random transitions, while the regular transitions are not accelerated (the term of Eq. (6) $\frac{d\rho(x,t)}{dx}$ is missing).

Increases in the values of ε and ξ (when ε > ξ) change the size of the plateau (the horizontal part of the correlation of probability of exceeding the percolation threshold to the second part of sharp increase) in Figs. 2 and 3, however the general correlation between the probability of transition and time does not change in nature.

Using this stochastic dynamic model of changes in the states of a social network which accounts for memory of previous states and processes self-organization, it is possible to investigate any number of steps (m) of the process wherein memory about previous states is preserved, and analyse the model's behaviour. However, this is beyond the scope of the present research and provides a promising avenue for future research.

4.2. Comparing the new stochastic model with the Blackman model

Our next step is to compare the results obtained with the Blackman model, which provides a logistic curve of change over time of the number of supporters of a given idea. The Blackman model was, for a long time, one of the central models describing the dynamics of state change in a social system and is itself developed from many existing models. This model can be obtained from the following statements:

- Any randomly chosen node of the social network is negatively inclined with the probability $\frac{R_h}{Q}$;
- Any randomly chosen node of the social network is positively or neutrally inclined with probability $\left\{1-\frac{N_k}{Q}\right\}$ (Q – the number of people in the described system).

The process of transmission of the negative influence can be viewed as the chain of steps, each step having a length τ. The number of nodes in the social network which happen to be in negative state in step (h + 1) are denoted as R_{h+1}, the number of nodes in negative state in step h - as R_h.

The increase in the number of negatively-inclined citizens (contamination) caused owing to negatively-inclined ones on the (h + 1) will be equal to: $n_0\Omega R_h\left\{1-\frac{R_h}{Q}\right\}$, (Rh of negatively-inclined individuals with n0 connections, with the coefficient Ω influence on neutrally or positively-inclined people, at the probability $\left\{1-\frac{R_h}{Q}\right\}$ of the event that one connection leads to such a node of a social network). Therefore $R_{h+1}-R_h=n_0\Omega R_h\left\{1-\frac{R_h}{Q}\right\}$.

The length of every step τ0, when all the lengths t of the process and the number of steps h are connected to each other is as follows: t = h τ. Substituting the number of steps h and k by the time of the process t, we obtain: $R(t+\tau)-R(t)=n_0\Omega R_h\left\{1-\frac{R_h}{Q}\right\}$.

Decomposing the obtained equation into the Taylor row and retaining only the first derivative, we get:

$$\tau\frac{dR(t)}{dt}=n_0\Omega R(t)\left\{1-\frac{R(t)}{Q}\right\} \quad (7)$$

where τ is the length of one step in conditional units of time, n_0 is the average number of connections of every node of a social network, Ω is the coefficient of proportionality (of influence) which is an empirical value, R(t) is the number of negatively-inclined citizens, L is the total number of citizens.

Let us divide the right and the left parts of Eq. (7) by the total number of people in the given society, and we get:

$$\frac{dr(t)}{dt}=\alpha\, r(t)\{1-r(t)\} \quad (8)$$

where $\alpha=\frac{n_0\Omega}{\tau}$, and r(t) is the share of negatively-inclined citizens in the moment of time t. After integrating the equation (8), considering that at the moment of time t = 0, the share of negatively-inclined people is r_0, we derive the key equation of the Blackman logistic model:

$$r(t)=\frac{r_0\, e^{\alpha t}}{1+r_0\left(e^{\alpha t}-1\right)} \quad (9)$$

As the results in Figs. 2 and 3 show, the model developed in the current research is considerably different from Blackman's model (see Fig. 5) and provides significantly new results. It is important to note that the Blackman logistic model in Fig. 5 is created not for the probability of reaching the percolation threshold for negative influence depending on time, but for the relation of the share of negatively-inclined individuals with time (when r_0 = 1% and k = 0.25 – curve 1, k = 0.30 – curve 2, k = 0.35 – curve 3, k = 0.40 – curve 4, k = 0.45 – curve 5 in Eq. (6)).

In a logistic model, the growth in the number of negatively-inclined individuals starts from the proportion of such individuals in the initial moment of time and then reaches 1 demonstrating an S-shaped character of the process (Fig.5).

The Blackman logistic model has a fundamental drawback: it does not consider the possibility of individuals' transition from a negative to a non-negative state.

The Gompertz model can also help us to better understand the results of this research. If the speed of growth is described by the following differential equation: $\frac{dr(t)}{dt}=k\, r(t)\ln\frac{\alpha}{r(t)}$, wnere k and ∝ are numeric parameters of the model, then after this equation is integrated on the time parameter t, the Gompertz equation is obtained: $r(t)=\alpha\cdot exp\left\{-\beta\cdot e^{-kt}\right\}$, where β is a numeric parameter defined by the integrating conditions. The graph of the r(t) function depending on t has the shape of an S-curve (as does the Blackman model), however, it is not symmetrical in relation to the inflection point.

Among other S-shaped models, it is important to note the Bertalanffy model which is rooted in biology (Bertalanffy, 1941). $r(t)=\left\{\alpha^{1-m}-\theta\cdot e^{-kt}\right\}^{\frac{1}{1-m}}$, where α, θ, k and m are the numeric parameters of the model which should be estimated. If m = 2 and θ = β/α, the Bertalanffy equation turns into a logistic function; if m→1, the curve tends to look like the Gompertz model.

Richardson's model is based on the following system of differential equations:

$$\begin{cases}\frac{dx}{dt}=\alpha y-\beta x+\theta\\ \frac{dy}{dt}=\gamma x-\epsilon y+\varphi\end{cases}$$

where $\frac{dx}{dt}$ is the speed of change of military expenditures of a state, and $\frac{dy}{dt}$ is the speed of military expenditures of another state, α, β, γ, ϵ, θ and φ are numeric parameters of the model. In the first equation, the parameter ∝ defines the increase in the rate of military expenditure of the first state ($\frac{dx}{dt}$) when the expenditure of the second state equals y; the parameter β accounts for the decrease in the rate of military expenditure considering the fact that the higher the current level of expenditure is, the lower the rate of growth will be (inverse negative dependency); the parameter θ defines the level of constant expenditure, whether there is a military threat or not.

When t →∞, Richardson's model may behave in three ways:

1. Endless military race: x →∞ and y →∞.
2. Mutual disarming: x → 0, y → 0.

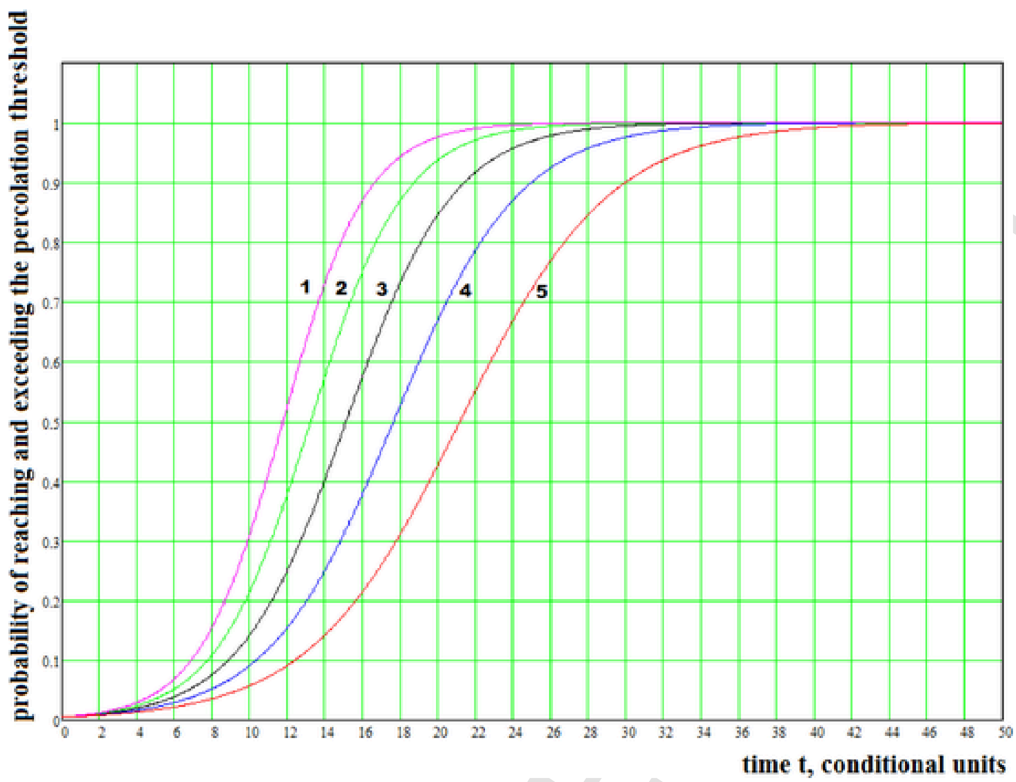

Fig. 5. Graphic representation of the results of the modelling the share of negatively-inclined citizens in the Blackman model

3. Balance of armament: x → x*, y→ y*, where y*, x* > 0.

In Table 1 we summarize the characteristics of some existing models and compare them to the newly developed model of stochastic dynamics of transitions between states in social systems which considers self-organisation and the presence of memory (the Zhukov-Khvatova model).

As is evident above, the traditional models proposed by Blackman, Gompertz and Bertalanfi result in smooth S-shaped curves, while the presently developed model results in more sophisticated non-smooth step-like S-shaped curves. These traditional models do not account for inverse dependencies among the variables, while the present model does, and this is important for social processes. Being based on the system of differential equations, Richardson's model considers inverse positive and negative dependencies, but does not consider the system's self-organization processes and the potential for memory. These features are important when a human factor is present in the process. And the Zhukov-Khvatova model developed here incorporates all features: inverse dependencies, self-organization and the presence of memory about previous states.

4.3. Summary of the features of the stochastic model

In the presently developed model, the probability of reaching and exceeding the PT of information distribution in a social network with a given number of connections per node (the network density) is simulated. The PT depends greatly on this parameter. The shape of curves in Figs. 2 and 3 show the capacity for an increase of the probability of exceeding the PT as from the start of the process. This model takes into account the existence of the system's memory of former states and enables us to describe the system's self-organization as it considers the second order derivative $\frac{d^2\rho(x,t)}{dt^2}$ in the differential equation. It is important to note that the model presented can be applied to any social systems where a human factor is present.

The second feature of this model is the opportunity to monitor (and anticipate) several jump-like changes of the probability of exceeding the PT; this is in accordance with the observed processes in revolutions, where, as revolutionary viewpoints increase, over time, a plateau may be observed (for example, curves 3 and 4 in Figs. 2 and 3), before the rapid approach to exceeding the PT occurs.

Table 1
Comparing selected existing models and the newly developed Zhukov-Khvatova model.

Name of the model	Number of parameters used in the model	The graphic representation of the modelled dependency of time	Presence of inverse dependencies	Accounting for memory about previous states	Considering self-organization
Blackman	1	S-shaped curves	No	No	No
Gompertz	3	S-shaped curves	No	No	No
Bertalanfi	4	S-shaped curves	No	No	No
Richardson	6	S-shaped curves	Yes	No	No
Zhukov - Khvatova	3	Non smooth S–shaped curves, having an oscillating character and several steps	Yes	Yes	Yes

It is worth mentioning here that the jump-like transition occurs within a very short period of time without any external influence and is defined by the system's self-organization. Such behaviour of a social system is hard to deal with in sociological research. To give an example, such a jump-like transition could account for what happened in the United Kingdom during the events surrounding the referendum on leaving the European Union ("Brexit"), wherein, on the eve of the referendum vote, all opinion surveys predicted that the British would vote to remain in Europe.

The third feature of the proposed model is the presence of oscillations in the probability of reaching the PT; this is also very coherent with how viewpoints really do change during revolutions, preparations for referendums and presidential elections. The approach developed provides a tool which is more nuanced and may therefore be better able to describe social dynamic processes. Going farther would require much more empirical evidence. In particular, if sociological data on the average number of connections per person in a given society exists, then it is possible to find the percolation threshold of that society's transition into a negative state (Khvatova et al, 2017, Zhukov and Lesko, 2015, Zhukov et al., 2018). Moreover, using the proposed stochastic dynamic model, it is possible to forecast the time at which a negative situation or any suggested scenario will probably occur. However, this is true only if a stable trend in ε and ξ is assumed, as described below. This assumes that efforts to affect opinion will not succeed and the trend will simply continue irrespective of these.

5. Applying the stochastic model in social systems

5.1. An algorithm for monitoring the states of a social network structure

The model currently developed enables us to create an algorithm for monitoring social states which can be easily put into practice. The essence of this algorithm is as follows:

1. Using sociological monitoring, the average number of connections per person in a given society is defined; then, the share of alternatively inclined individuals is defined for a given moment of time x_0 ($t=0$).
2. Based on the data on average number of connections, the PT of the researched social network is calculated (Zhukov et al., 2018).
3. After one chosen conditional unit of time ($\tau=1$, for example, one week or one month) the share of alternatively inclined individuals is re-calculated at another given moment of time ($t=0+\tau$). The change of the share enables us to define the value of an upward or downward trend. Furthermore, the value ε = - is calculated, and the value of ξ is set at 0. If ε < 0, then the value ξ = x1- x0, and ε = 0.
4. By using Eq. (5) according to the values of x_0, ε, ξ defined in steps 1-3 of this algorithm, and according to the values of parameters and PT l, the behaviour of the probability of exceeding the PT is modelled; then, the acceptable limit of time for changing the situation is defined.
5. Furthermore, the model makes it possible to mirror what might be described as 'manipulation', in cases where efforts are made to delay the PT being reached, for example by isolating a set of the network nodes (people with undesired views), by decreasing their number of connections, or by decreasing the values defining the trend by using external influence (for example, agitation, campaigning, etc).

The percolation threshold and how it is reached is an illustration of one of the possible applications of our new model. As a 'toy' example, we can look at the following example. Let us imagine a wish to control the dynamics of changes in people's opinions about the leader of a country. The perception is that the share of negative views in society should not be higher than 0.15 (15 %). Just to simplify, let us call this share a critical threshold, i.e. the level when certain action should be taken to improve the public opinion; it may in fact be the percolation threshold for uncontrollable diffusion of the negative views, but in policy terms it is seen as that "tipping point". Polling data is gathered on a regular basis and the implications for decisions are considered. Let us imagine that one week ago from 1000 randomly chosen people 30 people had a negative attitude (3%), but today there are 50 people like that (5%). Further analysis indicates that of the original 30 people, 10 people converted their opinion from negative to positive. Correspondingly the number of people who have become negative is 30 (taking into account the 10 converts who are no longer negative). In applying the model the unit of time of the process is τ = 1 week, the parameters are: $\varepsilon=50\text{-}(30\text{-}10)=30$ ($\varepsilon=0{,}03$), and $\xi=10$ (0,01). If starting the modelling from today $x_0=50/1000=0{,}05$ (this is the starting point for modelling).

A simple extrapolation process will produce an assessment of the time after which the critical threshold of 0.15 will be reached and appropriate action can be decided on. However, a simple extrapolation ignores the network effects. In real life the existence of a given level of dissatisfaction (say the 5% in this case) is itself a factor affecting progression the next week - over and above any trend as a result of other factors. The prevailing level of dissatisfaction may make some negatively-minded people nervous and cause them to switch to positive and may make others, previously positive, more willing to "join the bandwagon". Such complex influences are represented only at first-order level, if at all, in prevailing models; in reality the effects extend to second, third and higher orders and create complex interactions – furthermore stochastic characteristics at each level make detailed micro-modelling impractical.

For forecasting, working with a conventional modelling approach the observations will be plotted on either a straight line for simple extrapolation or an S curve according to the model concerned. When the results for the following week(s) are available and the results do not match what was forecast, there are two options: the model may be abandoned or belief in the model may lead to a search for features that explain the deviation. Where there is strong belief in a particular model, particularly an S-curve model, a normal "explanation" is achieved by finding a section of the curve where the data points fit well enough. The consequence of that is forecasts based on the remaining shape of the curve arguing, for instance, that the current observations of rapid change correspond to "take off" or conversely that reduced change reflects to "flattening out" or near saturation in the case of an S-curve. These interpretations have direct consequences for the prediction of what will happen over future time periods and there are potentially serious consequences if the curve is not correct.

The model described here shows that the developing levels of the monitored variable can display oscillations and plateaux instead of a smooth straight line or a smooth S-curve. It none the less does have an "end point" in that it addresses the probability of having reached the critical level and for many practical purposes it is the percolation threshold that is of most interest. Observations which appear to be "flattening out" or "take off" therefore need to be interpreted in terms of the modelled curve for the ε and ξ values and network density in the particular case. If a close fit can be found and the ε and ξ values remain constant, then the mathematics assures us that a more accurate prediction of the future development can be made.

As a further toy example, we can caricature an application in relation to entertainment. Consider an audience witnessing an event which has a series of stages at which the audience satisfaction level can be considered/'measured' (e.g. after performances of successive musical pieces or comedy acts). It is a normal expectation that the audience response (clapping or booing) will be a group reaction (individuals being influenced by how others respond) and in this sense there is self-organization. We may also postulate that there is system memory, for example that the effect on audience satisfaction level is affected by the progress of the event – the audience satisfaction level at earlier stages and critics' reviews in the newspaper. The model addresses the issue of predicting when the audience satisfaction level will reach a threshold level (perhaps the trigger for critics to decide to report that the audience was satisfied).

In such a case, and as a toy example, the impresario who finances such events can estimate ε, ξ (from long experience). They are increments and decrements in the *level of satisfaction of the audience as a whole*. A figure for the number of links is also required, but since this is a toy example, we can assume that this is also known (perhaps the audience is a group for which it is known). We then have to follow the logic as set out in the paper:

1. *The average number of connections is R (given and we want to apply the model starting from an x_0 ($t=0$) of 0.25 = 25% satisfied audience)*
2. *The PT is calculated (say it comes out as 45%)* [1]
3. *After 1 act/performance/unit of time the percentage satisfaction is, let us say, 28%. $\varepsilon = x_1 - x_0$ is calculated – in this case 3 percentage points. ξ is set at 0.*
4. *"By using equation (5) according to the values of x_0, ε, ξ defined in steps 1-3 of this algorithm, and according to the values of parameters and PT l, the behaviour of the probability of exceeding the PT is modelled; then, the acceptable limit of time for changing the situation is defined."*

"Respectively, the probability Qi(t) that the threshold l will have been reached or exceeded by moment t, can be defined as follows:

$$Q(l,t) = 1 - P(l,t)\,.\,(5)$$

and

The integral P(l,t) can be calculated:

$$P(l,t) = \int_0^{x_0} \rho_2(x,t)\,dx + \int_{x_0}^{l} \rho_1(x,t)\,dx$$

Inspecting the resulting graph, we can see that if the measured satisfaction level is say 44% then it is not certain that the PT will be reached one act later. Linear extrapolation and smooth S curve models will indicate that the next act will increase audience satisfaction to over 45%, whereas our model will allow for the possibility of a plateau in the increase in satisfaction rate. In real life this might arise from a "memory" effect such as a realisation by the audience that they had been laughing more than was justified by the average of the earlier acts. Similarly when the measured level is 48% the other models would indicate that there is still a need for several more acts/units of time to push the percentage to well over 50% in order for the critics' positive reports to be assured, but the present model, because it indicates that the PT has probably been reached, would give more assurance that the 50% plus will arise naturally and the critics would report satisfaction without addition input.

5.2. Discussing potential practical implications

The stochastic dynamic model of influence expansion and transitions between states in social networks developed here has many potential practical applications. In the current research it is applied to analysis of processes in complex social and economic systems. It is too early to fully draw conclusions regarding its strengths and weaknesses; however, several important practical implications can be discussed.

[1] although we may imagine that the critics will only give the positive report we want if the satisfaction level is 65%, the percolation theory result means that once 45% is reached there is nothing else we can do – or need to.

First, there are processes in the business world, for example, managing an advertising campaign in social networks. Given stiff competition, each marketing company wants their product/brand to be easily recognizable, and information about it to be freely transmissible among existing and potential customers. It is now a commonplace that social networks can play a significant role in this. At a given moment in time, the nodes of such networks (users, customers) can have various attitudes towards a given brand (product or service), ranging from "know nothing about the product/service" to "highly recommend this brand". The goal of every advertising campaign in a social network is to (with minimal investment of time and funds) achieve the state wherein favourable information about the product starts to spread freely throughout the network without any additional external influence. This is the state where the PT has been achieved. In other words, the share of users holding a good opinion about the brand and ready to share this opinion with other users will be equal to or higher than the PT of the given social network. After having analysed the social network structure using SNA, it is possible to define its density (average number of connections per node). Then, using the dependencies of PT in random networks on the density obtained in this research, it is possible to calculate the PT of the given social network in which the advertising campaign is planned. Furthermore, by analysing the share of some existing nodes within the social network (let us call them the initial share of influential individuals selected to promote the product) over a certain period of time (for example, one week or one day) positive information about the acceptance of the promoted brand is obtained and the resulting transition in the state of the network is tracked. In our model of stochastic dynamics, the network state corresponding to the initial share of influential individuals is x_0, and the time interval of influence upon the network is the parameter τ.

Then, after the period τ, using for instance automated text message analysis, the change in the social network nodes (i.e. changes in people's opinions about the brand) is measured and linked to the altered state of the network. Modern approaches using computer linguistics enable us to define not only the change in frequency of a brand being mentioned, but also the sentiment of users' messages. Based on the impact of the pool of positive and negative responses about the brand collected during the interval τ, it is possible to define the parameters ξ and ε (upward and downward trends correspondingly) of the stochastic dynamic model.

Furthermore, this new model enables us to assess the time needed to reach the PT after which a majority of consumers will be ready to adopt the brand. If the market has to be prepared faster, then decisions will have to be taken to increase the group of influential individuals. However, this will inevitably lead to increased expenditures in advertising/promotion and control of such additional expenditure requires insight into the trajectory of progress toward reaching the PT. Applying this model can provide such an insight and thereby help to optimize the necessary expenditures of time and money according to the priorities of the marketing company. It is important to note that the example described above is only one example of potential applications of this model in business systems.

Secondly, this model can be used to analyse processes in social systems. Electoral campaigns of various levels and referendums on significant societal issues (such as Brexit) can serve as good examples. In this case the 'brand' is represented by a candidate or an idea (for example, to remain in or leave the EU). The goal of the 'advertising campaign' is to increase the level of support for a candidate, an idea or political party. Otherwise, the approach is identical to that described above.

An example of a negative application of the proposed model could be adverse publicity generated with the aim of decreasing the support for a candidate or an idea. In this case, influential people will create and spread negative opinions in a social network, and in so doing, decrease the share of individuals positively-inclined towards the candidate/idea until this share is lower than the PT. After this occurs, the political influence is reduced, which can potentially lead to the victory of another candidate, idea or party.

6. Conclusions

In this research a new stochastic model of the dynamics of state-to-state transitions in social systems was developed. The processes taking place within social structures were described using a mathematical approach based on percolation theory. The features of the suggested new model were illustrated using the example of social networks which can be easily modelled. These networks have been considered because while describing network characteristics as a whole, we are able to discuss parameters such as the percolation threshold that are important from the point of information transfer. This allowed us to connect the suggested model of processes happening in networks over time, with the opportunity to estimate the time by which the percolation threshold would have been reached. In previous research, we modelled the dependency of the percolation threshold of a viewpoint, allegiance or mood in the social network on the network density, i.e. the average number of connections per node. In this paper, the relationship between the percolation threshold and time was modelled. It is important to note that such a relationship has not been researched before. A new second order differential equation based on differential schemes for probabilities of transitions was obtained. Based on this, a boundary problem was formulated and resolved. This enabled us to describe the dynamics of state-to-state transitions within a system with memory and self-organization. Using the model developed in the current research, it is possible to define the probable time of reaching the percolation threshold in a social network with a given density in the context of measured impacts of factors determining trends. The implications of the new model were tested using specially developed computer software (Zhukov et al., 2018). Furthermore, the new stochastic model was compared to the Blackman model which is currently one of the key models describing the kinetics of state change in social systems. The Blackman logistic model was found to have fundamental drawbacks: it does not consider either the strength of the factors affecting individuals' transition from a negative to a non-negative state or the system's capacity for self-organization and the presence of memory. The model we developed in the current research overcomes these drawbacks. Finally, an algorithm for assessing the state of a social network structure was developed which proves the practical importance of the stochastic model.

The peculiarity of our approach to modelling is that we are taking a macroscopic approach to describing processes in complex systems. Also, for us it is not important how the system is built, or whether it has a regular structure or not. It is not important whether each node has a random number of connections with a random number of other nodes (as for example in a random network) or whether the system is symmetrical (i.e. square, triangular, etc.). Similarly, our model does not require the number of connections of a node to follow the power law. For our model, however, it is important that each network (regular or random) has a macro characteristic whose behaviour describes the state of the system as a whole – our reference to percolation theory is because that macro characteristic emerges in the form of the system's evolution toward the percolation threshold, which, following earlier work, we note can be determined by reference to the network density.

Our model of stochastic dynamics with self-organisation and memory describes how this percolation threshold can be reached over time, after which the system transfers into either a) a transmissive state or b) a quiet state. Our new Zhukov-Khvatova model can be interpreted not only as describing the processes of reaching the percolation threshold, but also as describing how the given critical value of the share of people with the characteristic relevant to the system state (e.g. the percentage supporting a given position, the percentage using a particular facility) can be reached. For our macro dynamic approach, it is not important exactly which node is in which condition at any given time. Nor is it important how the nodes communicate with one another. Only the total share of nodes which are in this or that condition matters. The advantage of the macro model is that it considers only macro parameters, for example, the percolation threshold.

Our model *is new and different*, and we feel is more advanced and superior, because it allows us to deal with more sophisticated process dynamics. It is more sophisticated compared to what can be described by S-shaped smooth curves; and poor experience in forecasting the dynamics of social systems using such curves indicates the need for a new approach in some cases. This model is more advanced because it addresses features believed to be significant in social systems (self-organisation and system memory) and approaches these through a macro model that uses probability to deal with stochastic aspects

- The advantage of the stochastic model is that it considers macro parameters. It does not require assumptions about the micro level processes involved in state change. The model incorporates stochastic changes, memory and self-organisation.
- Accepted probability theory is used by the representation of stochastic changes in the initial differential equation by the factor $\frac{d^2P(x,t)}{dx^2}$. This factor is responsible for self-organisation.
- The model allows us to estimate the period until given states such as the percolation threshold will be reached.

Our model also demonstrates *a new approach* to modelling, in which the following key features emerge from our work:

1. The model demonstrates the potential increase in the probability of reaching the percolation threshold event in the social system as from the start of the process, taking into account both memory of previous states of the system and its capacity for self-organization. The proposed model shows the possibility of jump-like changes in the probability of exceeding the percolation threshold and models the presence of oscillations in how this probability behaves.
2. The closer the value of the initial state of the system to the value of the percolation threshold, the faster the probability of it being reached approaches.
3. The increase in probability of exceeding the threshold has a step-like aspect, in which the length of a stage in time depends on how close the initial value of the system state was to the threshold. The post-stage transition takes place over a very short period of time without any external influence and is defined by the system's self-organization. It is worth mentioning that such behaviour of a social system is quite difficult to identify through sociological research alone.
4. The notable characteristic of the process of reaching the PT in stochastic dynamics is the presence of a plateau, which is lengthy in time. Its value (in probability terms) depends on the initial network state (e.g. corresponding to the share of nodes (citizens) in the relevant state).

Our research has the following *limitations*. The model developed uses a macro-kinetic approach, i.e. the social system/network can be described as a whole, as one object considering collective characteristics – the network density and the percolation threshold. However, the model cannot consider at the same time the micro-level of process description inside social networks, i.e. the description at the level of certain nodes and clusters. Parameters used in this model, such as the share of nodes being in a given condition and their changes within each stage of interaction, require further research involving social science approaches possibly including techniques such as e.g. psycholinguistic analysis of users' messages in social networks.

A limitation on practical applicability, as opposed to a limitation on the model structure, is that while using the new stochastic dynamic model it is possible to forecast the time at which a negative situation or any suggested scenario will occur, this is true only if a stable trend, reflected in ε and ξ is assumed, as described earlier. This assumes that additional efforts to affect opinion will not be effective and the trend will simply continue irrespective of these. A similar limitation, however, applies to most other available modelling techniques.

A possible avenue for *future research* is to develop applications of our newly presented stochastic model of state-to-state node transitions. One idea is to apply the present approach to describe the dependence of amplitudes of voter preference oscillations in election campaigns on the time intervals for which they are calculated. Preliminary research demonstrates that the polling data observed, for example, during the 2008 US Presidential election campaign, reflects fluctuations that are very much in keeping with our new model. However, this requires further investigation.

Author statement

Dmitry Zhukov: Conceptualization, Methodology, Software **Tatiana Khvatova**: Conceptualization, Methodology, Writing-Original draft preparation, Reviewing and Editing. Carla Millar: Conceptualization, Supervision, Reviewing and Editing. **Anastasia Zaltcman**: Software, Validation.

Uncited References:

, , , , , Bertalanffy, 1941,

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.techfore.2020.120134.

Appendix

Deriving the differential equation for the model

We can define probabilities P(x–ε,h), P(x+ξ,h) and P(x,h) via states in stage h–1.

Considering that ε and ξ are certain constants, we can write for every stage h as follows:

$$P(x-\varepsilon,h) = P(x-2\varepsilon,h-1) + P(x-\varepsilon+\xi,h-1) - P(x-\varepsilon,h-1) \tag{1}$$

$$P(x+\xi,h) = P(x+\xi-\varepsilon,h-1) + P(x+2\xi,h-1) - P(x+\xi,h-1) \tag{2}$$

$$P(x,h) = P(x-\varepsilon,h-1) + P(x+\xi,h-1) - P(x,h-1) \tag{3}$$

If (1), (2) and (3) are inserted into Eq. (1), we obtain the following equation:

$$P(x,h+1) = \{P(x-2\varepsilon,h-1) + P(x-\varepsilon+\xi,h-1) - P(x\cdot$$
$$+\{P(x+\xi-\varepsilon,h-1) + P(x+2\xi,h-1) - P(x+\xi,h-1)\} - P(x-\varepsilon,h-1)$$

Note that in the left part of Eq. (4) the number of stages is (h+1), while in the right part it is (h–1). In order to avoid Taylor decomposition of the right part of Eq. (4) in the vicinity of the number of stages h (or in time) and decompose only in the vicinity of point x, we can transform (4) as follows:

$$P(x,h+2) = \{P(x-2\varepsilon,h) + P(x-\varepsilon,h) + \xi,h) - P(x-\varepsilon,h)\} + \{P(x-\varepsilon+\xi,$$
$$-P(x-\varepsilon,h) - P(x+\xi,h) + P(x,h)$$
$$P(x,h+2) = P(x-2\varepsilon,h) + P(x+2\xi,h) + P(x,h) + 2\{P(x-[\varepsilon-\xi],h$$

Note that, in this case, h increases by m = 2. Furthermore, we can express the members of the right side of Eq. (2) in terms of probabilities of the corresponding transitions on the previous step; and after certain transformations when m = 3, we will obtain:

$$P(x,h+3) = P(x-3\varepsilon,h) + P(x+3\xi,h) - P(x,h) + 3\{P(x-[2\varepsilon-\xi],h) + P$$
$$-2P(x-[\varepsilon-\xi],h) - P(x-2\varepsilon,h) - P(x+2\xi,h) + P(x-\varepsilon,h) + P($$

For m = 4, the following will be obtained:

$$P(x,h+4) = P(x-4\varepsilon,h) + P(x+4\xi,h) + P(x,h) + 4\{P(x-[3\varepsilon-\xi],h) + P$$
$$-P(x+3\xi,h) - P(x-\varepsilon,h) - P(x+\xi,h) + 3\{P(x-[\varepsilon-\xi],h) - P(x-[2\varepsilon$$
$$+3\cdot 2\{P(x-[2\varepsilon-2\xi],h) + P(x-2\varepsilon,h) + P(x+$$

After analyzing Eqs. (4) – (7), for any m, it is possible to derive the following recurrent equation for the probability where P(x,h + m), that for a certain time (h + m), the state will be x:

$$P(x,h+m) = \begin{cases} \sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!\cdot P(x-[k\cdot\varepsilon-l\cdot\xi],h)}{k!\cdot l!(m-k-l)!}, & \text{when } k+l-m \le 0, \\ 0, & when\ k+l-m>0 \end{cases}$$

Where m is the number of steps considered while studying the change in the system's state (m can be called 'depth of memory'). Furthermore, considering that $t = h\bullet\tau$, where t is the time of the process, h is the number of the step, τ is the length of one step, let us advance from h to t.

$$P(x,t+m\tau) = \begin{cases} \sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!\cdot P(x-[k\cdot\varepsilon-l\cdot\xi],h)}{k!\cdot l!(m-k-l)!}, & \text{when } k+l-m \le 0, \\ 0, & when\ k+l-m>0 \end{cases} \tag{8}$$

Let us do Taylor transformations of the members of Eq. (8), and then, considering derivatives no higher than of second order, we can obtain the following differential equation for the probability that the system will be in a certain state x depending on time t and depth of memory m:

$$P(x,t) + m\tau\frac{dP(x,t)}{dt} + \frac{(m\tau)^2}{2!}\frac{d^2P(x,t)}{dt^2} =$$
$$= \sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!}{k!\cdot l!(m-k-l)!}\left\{P(x,t) - [k\cdot\varepsilon - l\cdot\xi]\frac{dP(x,t)}{dx} + + \frac{[k\cdot\varepsilon-l\cdot\xi]^2}{2!}\frac{d^2P(x,\right.$$

Furthermore, considering that:

$$\sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!}{k!\cdot l!(k+l-m)!} = 1$$

$$\sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!\cdot k}{k!\cdot l!(k+l-m)!} = \sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!\cdot l}{k!\cdot l!(k+l-m)!} = m$$

$$\sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!\cdot k^2}{k!\cdot l!(k+l-m)!} = \sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!\cdot l^2}{k!\cdot l!(k+l-m)!} = m^2$$

$$\sum_{k,l=0}^{m} (-1)^{(m-k-l)} \frac{m!\cdot k\cdot l}{k!\cdot l!(k+l-m)!} = m(m-1),$$

We obtain:

$$m\tau \frac{dP(x,t)}{dt} + \frac{(m\tau)^2}{2!}\frac{d^2P(x,t)}{dt^2} = \frac{1}{2}\left\{m^2\varepsilon^2 - 2m(m-1)\varepsilon\xi + m^2\xi^2\right\}\frac{d^2P(x,t)}{dx^2} - m[\varepsilon - \xi]\frac{dP(x,t)}{dx}. \quad (9)$$

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