THE IMPACT OF CHANGING UNCERTAINTY PARAMETERS IN PROJECTS

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ABSTRACT

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The impact of changing uncertainty parameters in projects

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Uncertainty can come in different forms and for different reasons and can have an impact on projects. This requires project managers to know the different methods for planning projects under uncertain conditions, and at best different mathematical models, in order to make predictions and, if necessary, further measures about the course of a project. This study investigates the effect of changing uncertainty parameters in projects through the method of Monte Carlo simulation.

The study was conducted using quantitative methods by experimentally simulating input uncertainty, combining the results with literature reviews. The results show that partially clear stochastic effects can be observed when comparing different parameter changes induced by uncertainty, with other stochastic variables showing no correlation. Furthermore, clear correlations can be shown by which parameter settings the simulation provides more accurate and reliable results. It was also found that it is possible to simulate uncertainty in a meaningful way and that minor changes to the parameters before the simulation, which can arise due to uncertainty, still lead to usable results.

Abbreviations

CAAN	Controlled Alternative Activity Networks
CBS	Cost Breakdown Structure
CCPM	Critical Chain Project Management
CPM	Critical Path Method
GERT	Graphical Evaluation and Review Technique
MCS	Monte Carlo Simulation/Method
NPD	New Product Development
NPV	Net Present Value
OBS	Organizational Breakdown Structure
PERT	Programme Evaluation and Review Technique
PMBOK	Project Management Body of Knowledge
PMI	Project Management Institute
PRAM	Project Risk Analysis and Management
RCPSP	Resource-Constrained Project Scheduling
RQ	Research Question
SRCPSP	Stochastic Resource-Constrained Project Scheduling
TCT	Time-Cost Trade-off
WBS	Work Breakdown Structure

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1 Introduction

Recently, global crises such as Covid-19 and problems in interconnected supply chains, exacerbated by the Ukraine crisis, have made every economist aware once again that uncertainty in planning is and will always be present. For this recurring reason, attempts have been made since the 1960s to model uncertainty - in whatever form - in order to draw conclusions and make predictions.

In project management in particular, one deadline follows the next, which leads to very tight and less flexible planning in resource and time management. Especially in the area of the critical main path, deviations from the optimum can quickly jeopardize the entire project. Therefore, it is a proven method to use mathematical models to find the best possible solution for such industrial applications that optimally takes into account an uncertainty factor and is equally effective.

Scheduling is a process of assigning specific activities to specific resources over time. Similarly, in reality, these can be machines and jobs that take different forms, such as machines in a production workshop and operations in a manufacturing process. Tasks can be distinguished from each other by priorities, release dates or processing times, among other things (Mundi *et al.*, 2019).

Mathematical scheduling problems are analyzed in scheduling theory, consisting of basically two parts, which are either based on stochastic or deterministic models. On a more general level, scheduling settings have been constructed using a stochastic model, whereby the processing time of a task is considered to be a random variable with known probability spread. In this way, difficulties can emerge in reality. On the one hand, one may not have sufficient prior information to properly describe the probability spread of a coincidental processing time. Conversely, while the probability distributions of all random processing times may be known in advance, these distributions are only useful for a significant number of implementations of similar planning environments, while in practice they may be less useful with respect to a single or a limited number of such implementations. Deterministic models got established for scheduling environments wherein the processing time of each (machine) order is given before the utilization of a scheduling process and is considered a constant during the implementation of a schedule. However, in reality, accurate numerical data is not known beforehand, and challenges occur when certain job processing times, previously assumed to be known, fluctuate as a result of modifications in a dynamic environment. And even if all machining times are known before planning, the degree of accuracy of the equipment used to determine machining times, possible errors in the implementation of a plan, machine failures, rounding errors in the computerized calculation of a plan or the receipt of additional orders must be considered with high priority (Sotskov *et al.*, 2010).

1.1 Background

The focus of this master's thesis is on the Monte Carlo Simulation (MCS), investigating the effect of changing various input parameters that may arise due to uncertainties on a project deadline.

Researchers can use the results of the parameter influence experiments in this work and apply them to further experimental research on uncertainty. In addition, the theoretical knowledge explored can be directly used for industrial applications to be informed about the stochastic effects of uncertainty in projects. In this way, project managers can keep in mind the possible uncertainty impact on their project from the beginning and use it for project duration estimations and assessments.

Over time, a variety of mathematical models have evolved that can be used to make uncertainty tangible, depending on the situation and the need for analysis. This thesis summarizes the most important mathematical models in this respect, as well as typical management scheduling methods that can be used when uncertainty occurs. In addition, to make the origin and classification of uncertainty understandable, an overview of the types of uncertainty in projects is provided.

1.2 Research gap

Since the 1960s, mathematical models for project and operations management have been created, studied, and developed, leading to new forms and areas of research. Research also addresses the origin of uncertainty in projects, with current literature focusing on the intricacies and actual origin of cause relationships. Various approaches to managing uncertainty in projects are also presented in the literature.

However, what is not really presented in the literature is concrete research on the stochastic effects of parameter selection and input conditions, which can be considered as uncertainty simulation already at the input of a project, in a MCS of planning problems. Nor are the concrete stochastic effects of uncertainty on critical path changes discussed.

1.3 Objectives, research questions and scope of the research

The main objective of this master thesis is to provide a clear picture of how uncertainty factors and their modeling stochastically affect the outcome of mathematical analysis methods, specifically MCS. The results and conclusions obtained through this thesis using quantitative experimental (empirical) studies have a clear benefit for researchers as they provide a researched insight into the types of uncertainty, mathematical models, and the influence of parameters in the simulation of uncertainty. With this knowledge, further research can be conducted with other parameters and simulation methods, and the results can be used to evaluate the impact of input parameters due to uncertainty. Also, the work provides project managers with a good overview of the facets of uncertainty in projects, as well as the stochastic effects of uncertainty can be better understood prior to a project, which can help, for example, in estimating the expected project completion range.

With the help of the research questions (RQ), an overview of the impact of uncertainty factors on management applications can be provided. In this context, the following table 1 shows the RQs and their specific objectives.

Table 1. Overview of the RQs with their objectives

Research question	Objective
1. How does uncertainty affect input parameters in project management?	To assess the impact of input uncertainty factors on an operation and project management tool.

2. Do some parameters in mathematical simulation modeling (e.g., MCS) provide more accurate and reliable results than others?	To give an evaluation of which parameters are better or worse suited to obtain more accurate results for uncertainty prediction.
3. Is it possible to simulate uncertainty in a useful way or does every parameter change lead to completely new results?	Evaluation of the usefulness of uncertainty simulations based on the distribution of results.

The results of the literature review are generally applicable to project-related topics, with classifications and overviews based on different types of literature and sources. The results of the experimental evaluation are clearly only safe to use for MCSs and similar problems. Problem modifications require re-examination and may be the subject of further research.

1.4 Research realization and structure of the thesis

This paper is mainly divided into a literature review and the empirical part. The literature search provides a scientific background that allows the reader to obtain coherent information on the topic. In addition, the theoretical part is important in order to be able to correctly classify and partly answer the RQs. The research was conducted in the search databases of Google Scholar, Semantic Scholar, ResearchGate and the university's own LUT Primo in order to be able to find and process as much relevant literature as possible. Literature on mathematical models in operation and project management since their origins in the 1960s was included, whereby in the area of uncertainty research, the search was mainly for more recent scientific findings from the last few years. However, in all cases, the literature was reviewed and used on the basis of its relevance, regardless of age. Where possible, the focus was on peer-reviewed literature to ensure reliability and to identify established models. As (combined) search keywords, mainly uncertainty, mathematical modelling, MSC and scheduling were investigated.

The empirical part deals with the investigation of the influence of input parameter changes and fluctuations in various project schedules, which are statistically examined with the help of a MCS, and conspicuous features or correlations are recorded.

After the introduction, the second chapter gives an overview of internal and external sources of uncertainty in management. The third chapter then deals with how these uncertainties are dealt with and which scheduling methods resulting from this problem exist specifically in project management.

The fourth chapter gives an introduction to the common mathematical models in project management that are also used to model uncertainty problems. Among these models is the MCS, which is examined again in the fifth chapter with regard to its advantages and limitations, since it was also used as an analysis method in this thesis.

The methodology of the master's thesis is then explained, whereby the research context, the choice of methods, the data collection and explanations of the data quality are examined. Subsequently, the results of the experimental part are given in relation to project scheduling and critical path analysis, which are analyzed and discussed in the following chapter. The conclusion is a summary of the research results and an answer to the RQs.

2 Uncertainty in project management

According to Hazır and Ulusoy (2020) , uncertainty in projects is divided into external and internal, which are further classified into subcategories (see Fig. 1).

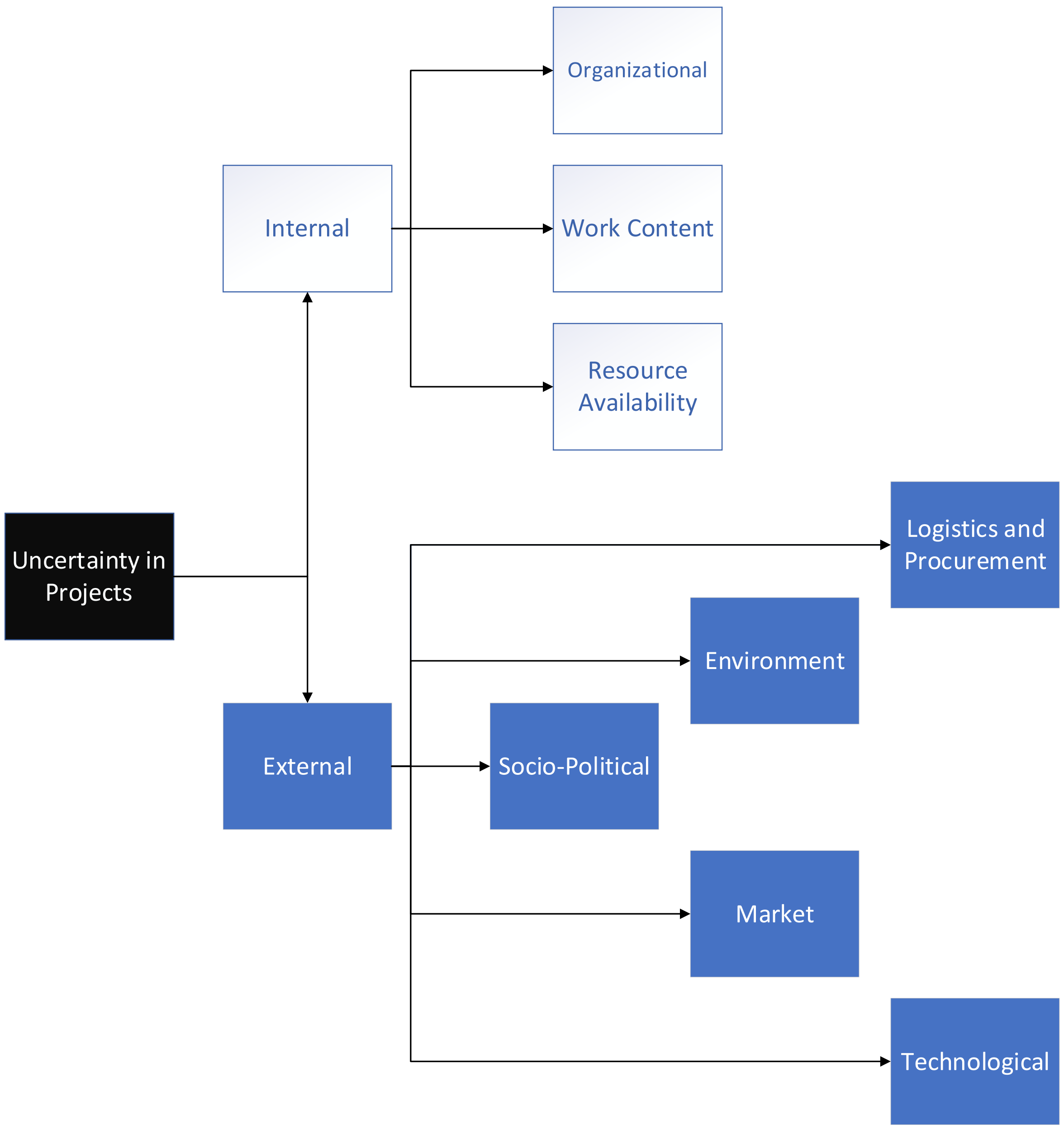


Figure 1. Types and classification of uncertainty in projects, adapted overview (Hazır and Ulusoy, 2020).

Here, the factors of internal can be controlled and resources as well as systems have a direct relation to the project. If the control factor is not given, the external factors are referred to, whereby a clear distinction and classification is not always unambiguous, since the uncertainty factors interact or cause others.

An example of such an unclear classification is that the development of the market situation or economic parameters is externally imposed and thus external. As the project manager has no direct control over them, this could lead to a change of priorities in the project, which in turn is an internal source of uncertainty (organisational uncertainty). Furthermore, economic uncertainty can affect the availability of resources and influence the budget. The subcategories are presented in more detail below.

2.1 Internal

Internal factors refer to the respective project and can be controlled and influenced by the organisation. Subcategories are the nature of the organization, the work content, and the availability of resources.

2.1.1 Organizational

Factors that decisively shape the course of the project are organisational goals and priorities, the structure, management concepts, responsibilities and administrative competences, information channels and cooperation with stakeholders and other companies.

Structural factors may shift in the course of project implementation. As a result of such deviations, the client may re-evaluate its objective and priorities with regard to the project in question. A project's priorities could be reprioritised, more resources made available or the project manager asked to speed up project delivery. An important task of the manager is to communicate this time target and review the overall approach (Petit and Hobbs, 2010).

The parallel handling of different processes can lead to complexity in companies. Excessive commitment of resources and momentous missteps in design can lead to an unjustified shortfall of funds in other projects and, in general, to uncertainty in resource management.

In project planning, the prerequisites and scope of tasks are first defined and then split into smaller sub-projects (Engwall and Jerbrant, 2003).

2.1.2 Work Content

From an operational management standpoint, the focus of project management is on setting the objectives, usually in the form of time or/and cost objectives, the process of developing the Work Breakdown Structure (WBS) and the strategy for reaching these objectives. Most research on scheduling either assumes deterministic process parameters or considers only the activity duration as random. Effectively, the work content is beside the point, and may be further characterised as the overall effort needed to accomplish the totality of the activities in the project (Tereso, Araújo and Elmaghraby, 2004).

Aggregate input necessary to perform an activity can be estimated in terms of hours worked or machinery hours operated by multiplying the input of resources with the length of the activity. There is a correlation between these two variables, and the activity duration depends on the resource input. Consequently, the activities can be carried out in several ways, resulting in processing with alternating technologies, resource inputs and changing times and costs.

The duration of the activities does not match the forecasts for many projects. Usually, the activities require more time or additional resources because these parameters were not set correctly, or unexpected circumstances occurred. Sometimes rework may also be necessary, but this still does not reduce workload in cases. However, for other tasks, the processing sequence may need to be adjusted, especially if technical requirements are modified, which requires replanning both the project network and time schedule.

The main variables contributing to work content uncertainties can be grouped into four subgroups: Task duration, resource utilization, requirement changes, and quality issues (Hazır and Ulusoy, 2020).

- Activity duration: The main reason for changes in activity duration is inaccurate forecasting.

- Resource requirements: the number of resources required to perform an activity is not always accurately estimated.
- Changes in requirements: Changing organizational, customer, or technical process needs may entail adjusting the content of the work. This may require a correction of the project network. In addition, new activities or hierarchical relationships can either be deleted or added to the network. Deadlines and dates may shift accordingly.
- Quality issues: In the case of quality issues, rework is possible, which in turn can lead to a postponement.

The main causes of inaccuracy in estimating both activity duration and resource use are the absence of detailed analysis, explicit specifications or expert knowledge, and the difficulty of the accurate estimation procedure (Ward and Chapman, 2003).

Project managers are not only concerned with estimation problems, but also with quality issues. Changes in customer needs, on the other hand, usually entail adjustments in project scope and require restructuring. Project parameters must therefore be reformulated: the quality requirements, relationships of priorities, activity durations, resource requirements, and the project network. Subsequently, attention must be paid to the uncertainty of these parameters.

Project managers usually allocate additional resources to the activities, for example in the form of additional machines or additional labour, when the appropriately time-managed execution is considered risky or when experiencing variances to the schedule. This leads to fluctuations in the quantity assigned to each operation and in the aggregate utilisation of them. Resorting to additional resources to reduce the duration of the activity, for example, by terminating the project, similarly introduces cost uncertainty (Nozick, Turnquist and Xu, 2004).

During the project duration, the funding situation is not influenced through the selection of the cost function or the assignment of the activity duration, but through the schedule of payments (Zhang and Elmaghraby, 2014).

Difficulties in implementing projects on behalf of engineers due to planning uncertainties, which include changes in specifications, are frequently perceived. Engineers' information uncertainties are transmitted onto the project network, although activities are created to deal with them. These postpone the making of design related decisions and are common in alternative designs (Vaagen, Kaut and Wallace, 2017).

Not to be neglected are contract-related uncertainties that usually occur due to changes in requirements. Schedule fluctuations can occur due to requirements from internal departments or stakeholders. Missed activities can lead to inevitable delays, and schedule slippage can lead to penalties, as well as bonuses, should the project be completed ahead of schedule (Bordley, Keisler and Logan, 2019).

2.1.3 Resource Availability

Failures are among primary sources of uncertainty in the availability of resources. In the context of resource availability, liquidity planning is of key importance (Aytug *et al.*, 2005).

It is of great importance for the project manager to ensure the availability of cash during the project period in order to avoid cash flow irregularities. In this context, one objective would be to minimize the maximum cumulative difference between payments received and payments made by the contractor. The lack of capital can be compensated by the appropriate use of time buffer systems and the use of lower-cost service types (Ning *et al.*, 2017).

From a risk perspective, budgeting is critical. During execution, project support resources may vary from initial estimates. When managing many projects, organizations must weigh various funding options, so budget uncertainties should be known and accounted for. The greater the uncertainty, the tighter the funding constraints and the more time must be allowed for contingencies (Yang, 2005).

2.2 External

In particular, externally caused uncertainties involve logistical and procurement-related, technological, socio-political, environmental and market factors (see figure 1).

2.2.1 Procurement and logistics factors

Supply disruptions caused by vendor difficulties and variations in procurement are not uncommon in projects. The availability of materials, equipment and personnel and their timely delivery during the project cycle can significantly impact execution. Many companies prefer to use subcontractors, and the procurement process is increasingly confusing and fraught with uncertainty. Coordination problems between different subcontractors can lead to project delays (Hazır and Ulusoy, 2020).

2.2.2 Environmental Factors

The effects of weather and environmental problems are crucial to a number of industries. Thus, in the building industry, changes in plans induced by weather conditions are among the main causes of delays.

Cancellation of projects can be an obvious consequence. In some large construction projects, environmental damage was not adequately assessed during the design phase or social and economic risks were underestimated. In the field of project evaluation, the real options theory has gained acceptance, which allows the possibility of cancellation due to a catastrophic event to be included (Schwartz and Zozaya-Gorostiza, 2003).

2.2.3 Socio-Political Factors

Among the components defining the project environment within which the project life cycle operates, including those likely to vary through time, are legislation, political and social influences. Modifications to state environmental or fiscal laws, or health and labour regulations, may impact achieving the project's objectives. The planned progress of projects can be influenced and disrupted by a variety of events. Among these are politically or socially related events which include governmental requirements and labour strikes. Thus, it can be expected a disturbing event of this nature will interrupt all immediate labour related activities by a certain duration, but the indirect and overall works will proceed and the corresponding expenses will be incurred (Klastorin and Mitchell, 2012).

2.2.4 Market Factors

During the implementation of the project, the market situation, producer prices and market demand may change. Changes in exchange rates and factor prices can lead to uncertainties in the costs incurred. Fluctuations in exchange rates or material costs, for example, can cause a significant increase in total costs.

Considering the high level of uncertainty in product development's early phases, companies spend a great deal of effort to determine which features will be successful in the market. Equally high is the uncertainty in the "make-or-buy" decision that is made in the start-up phase of a new product development (NPD). These kinds of uncertainties add to uncertainties in supplier partnerships and may affect the market (Petit and Hobbs, 2010).

2.2.5 Technology factors

As a project is implemented, new product and process variations can evolve. When the usage of new processes and materials gets implemented while the project is being carried out, for example, the plan can be decisively changed or modified. According to Shenhar and Dvir (1996), there are two dimensions of projects, which are complexity and uncertainty. Regarding uncertainty, the authors looked at technological uncertainty and distinguished between low-tech and super-high-tech projects. Low-tech projects, such as basic road construction, apply incumbent technologies and best practices, while super-high-tech projects exclusively use new technologies. In terms of complexity, a distinction is made between the levels array, system, and assembly. Arrays are the extension, construction or design of a widely distributed and large set of systems that cooperate to accomplish a shared objective. Systems comprise a set of interaction elements and sub-systems (developed or built) that fulfil a broad range of tasks or functions. An assembly aggregates a set of components to form a single common unit.

Shenhar (2001) concludes that project management techniques differ according to the level of uncertainty and complexity, where uncertainty refers to the solution path for technical issues, while complexity refers to aspects of project management that are administrative in nature.

3 Managing Project Uncertainty

The following planning approaches are being discussed within the literature to mitigate the consequences of unforeseen changes on project performance (see figure 2): Reactive/ Robust/ Stochastic/ Fuzzy Scheduling and Sensitivity Analysis (Herroelen and Leus, 2005).

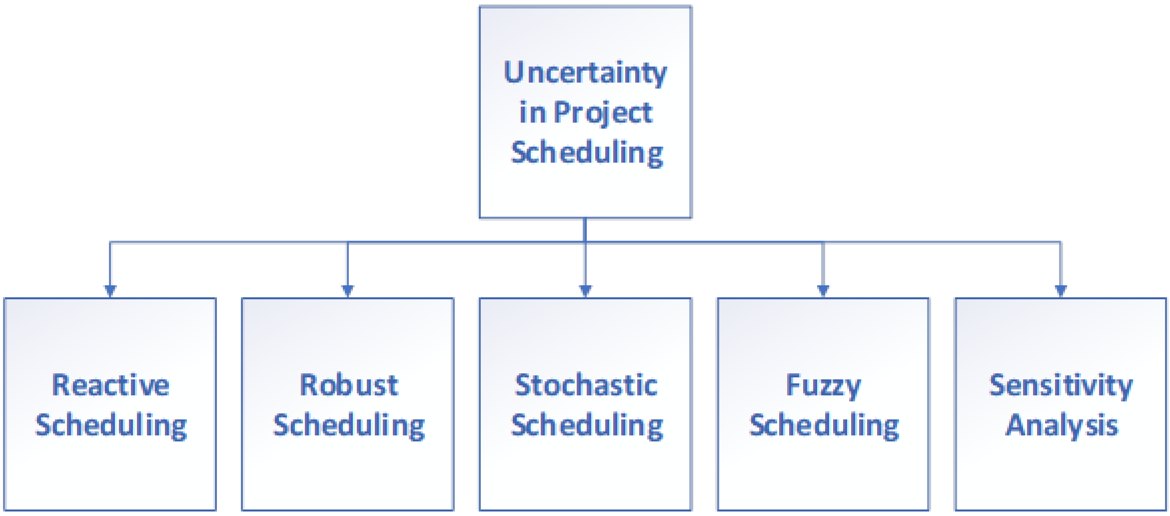


Figure 2. Project scheduling under uncertainty taxonomy, adapted overview (Herroelen and Leus, 2005).

In this section, the individual sub-areas of uncertainty in project scheduling are explained in more detail and characteristics are described.

3.1 Reactive Project Scheduling

Reactive planning is when a schedule has to be modified or changed in response to problems. The creation of a baseline plan prior to actual execution is referred to as forward-looking, reactive planning. However, the plan can also be created in a dynamic manner (dynamic planning) (Aytug *et al.*, 2005).

Reactive planning is primarily concerned with making timely decisions about how and when to reschedule. There are two ways of planning. With periodic planning, the planning is newly planned at the start of each cycle, whereas with event-driven planning, the planning is renewed if an unforeseen incident happens. When disruptions occur, correction measures in the form of either partial or complete rescheduling can take place. With partial rescheduling, there is only a partial actualisation of the ongoing schedule, while with full rescheduling, there is a rescheduling of every potential task.

Reactive project scheduling often uses the method of simulation. Furthermore, it has been experienced that a heuristic of simulated annealing was more efficient than a simple planning rule in developing schedules. In addition, it was found that the impact of rescheduling is related to the intensity of the constraints in terms of urgency and resources, and that the frequency of rescheduling has a significant impact on completion time. In case these restrictions are severely loose or tight, the effects of rescheduling are vanishingly small (Yang, 1996).

3.2 Robust planning of projects

In robust or proactive scheduling, the aim is to have a schedule that is less subject to disruption, and modelling incorporates variation. The robustness of the schedule can be divided into two categories: quality robustness and solution robustness. Robustness of quality is understood to be the insensitivity of schedule output, such as project length, to disruptive influences. Whereas solution robustness is conceived as the insensitivity of task launch dates against changes in their input information (Herroelen and Leus, 2005).

The purpose of quality robust scheduling is to design the schedule in such a way that the performance value is influenced by disruptions to the minimum possible extent. Critical Chain Project Management (CCPM, see also chapter 4.4 for explanations of Critical Chain Scheduling), originating from Goldratt (1997), remains the most famous approach in project management based on the robustness of quality.

CCPM focuses on managing and identifying system constraints, thereby improving overall performance. Safety buffers are used to monitor project performance and protect the project from uncertainty. Factors of safety are deducted from individual activities and summarised together towards the end in the form of a project buffer. This allows shifts within an activity to be mitigated and risks to be combined by deducting safety factors from another activity. Thus, project buffers can be adapted through analysing the extent of resource interconnection among activities and resource constraints (Zhang, Song and Díaz, 2016).

Mathematical programming models were formulated to create solution robust project schedules or linear programming models. These in turn allowed for a single activity interruption, which made it possible to increase the length of an activity while the schedule was running. In addition, the stability framework was adapted to resource-constrained networks by implementing resource flow networks, which expressed the quantity of resource units transferred between tasks in terms of the flow of resources from one activity to another (Herroelen and Leus, 2004).

3.3 Stochastic Project Scheduling

Stochastic scheduling covers planning issues with random properties such as production times, deadlines or stochastic machine failures. The objective of stochastic scheduling problems is either a conventional task, such as reducing the total lead time, time span, or total cost of delay for missed due dates, or it is an irregular objective, like the minimisation of follow-up costs of premature or delayed completion of tasks in operational management (Cai, Wu and Zhou, 2014).

Since the duration of the activities is considered random, the result of these strategies is not a deterministic schedule but a variable schedule (Möhring, Radermacher and Weiss, 1984).

It was further shown that deterministic proximity methods such as the Programme Evaluation and Review Technique (PERT) produce biased results (see also chapter 4) (Mitchell and Klastorin, 2007).

3.4 Fuzzy Project scheduling

Bellman and Zadeh (1970) are considered pioneers of fuzzy programming, which serves as an alternative paradigm designed to solve uncertainty driven planning problems. Limitations become definable with the help of membership functions and fuzzy sets, and rather than random variables, parameters of uncertainty become modulated as fuzzy numbers. Membership functions provide a measure of the extent to which the constraints are satisfied and might permit certain constraint violations. According to the fuzzy approach, the probability distributions for the length of activity are usually not known because, for example, accurate historical data are not available. In addition, it is believed that activity durations approximated by human experts tend to be inaccurate (Zimmermann, 2001).

In this context, corporate resources are used differently depending on the position in the company. According to Gang, Xu and Xu (2013), decision-makers at the upper level, e.g., company managers, try to distribute company resources across several projects to achieve minimum total costs. In contrast, on the lower level, the project manager allocates resources in order to minimise the length of the project he or she is in charge of.

3.5 Sensitivity Analysis

Data analyses can be used to formulate the development and probability of success of a project, which is also referred to as project sensitivity. In addition, risks are listed, classified according to their probability of occurrence and a quantified assessment of their impact is given. It is presented through mathematical modelling and a written analysis. The model is based on existing data, whereby the project duration and the average duration of the individual sections are estimated using hypothesis models (Razavi *et al.*, 2021).

While consideration can be given to the entire project, it is also possible to assess only some (critical) sections and/or building blocks, such as project implementation, by means of sensitivity analysis. The general objective is to find a suitable approach to the problem at hand. By looking at the big picture, components that hinder the project goal can be identified, sorted by threat level and addressed (Hall and Posner, 2004).

4 Standard mathematical models in project management

Mathematical models can accurately replicate problem situations and help decision-makers make more accurate and faster decisions. As a rule, they offer convenience and cost advantages over other ways of obtaining the necessary information about reality. When used successfully, large and complex problems can be easily represented and solved, conveying information and implications to others under changing conditions (*Advantages of mathematical modelling - quantitative techniques for management*, 2020).

Since the full introduction of the Critical Path Method (CPM) and PERT in the early 1960s (first mentions in 1959), the scientific field of project planning has evolved in two directions, as shown in Figure 3. Exploring more general and complex network structures and contingencies (Graphical Evaluation and Review Technique (GERT)), and explicitly modelling resource constraints like Resource-Constrained Project Scheduling (RCPSP), have become widely known. In other words, uncertainty and resource constraints have been researched on separately in the context of PERT/GERT and RCPSP for nearly 50 years (Moore and Clayton, 1976; Elmaghraby, 1977; Demeulemeester and Herroelen, 2002).

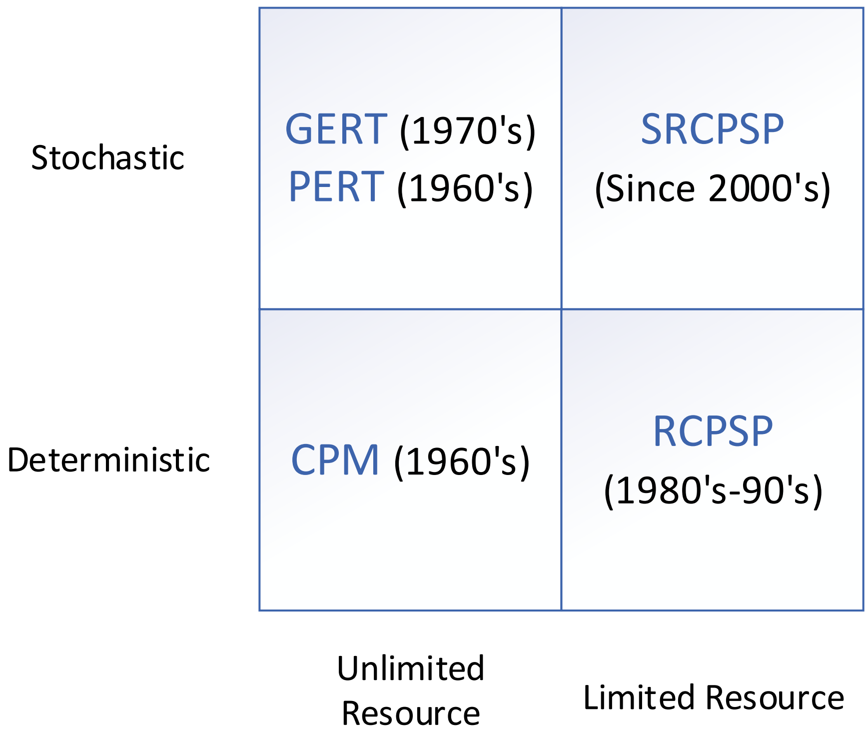


Figure 3. Evolution of the project scheduling methods, adapted overview by (Li and Worner, 2011).

In the following, the standard models for describing single projects, especially the standard project management model, deterministic network models, stochastic cost models, stochastic time-network models and stochastic resource-constrained project planning are further elaborated through the following subchapters.

4.1 The standard project management model

The Project Management Body of Knowledge (PMBOK), first published in 1996 and now in its seventh edition in 2021, provides world-wide accepted and standardised guidelines and terminology regarding project management. Many widely used and well-known methods, such as the mathematical model of the CPM, have been able to significantly increase their level of awareness through inclusion in the guide, which is released by the Project Management Institute (PMI) (Project Management Institute, 2021).

Operational scientists create models to evaluate what will happen, has already happened or is happening in a project, meaning that the analysis needs to consider different circumstances and their effects on project output. Indeed, controlling the project towards meeting these objectives and defining the goals of a project is a fundamental aspect of project management. A project can be described as an organisation formed by people committed to a certain objective or purpose. Further, projects are usually unique, large, risky, or expensive ventures that have to be accomplished within a specific timeframe, with a specific anticipated performance level, and for a specific monetary investment (Steiner, 1969).

The trinity of meeting performance, schedule and cost targets (see figure 4) emerged to be the standard criterion for success being generally considered the significant task of project managers (Barnes, 1988).

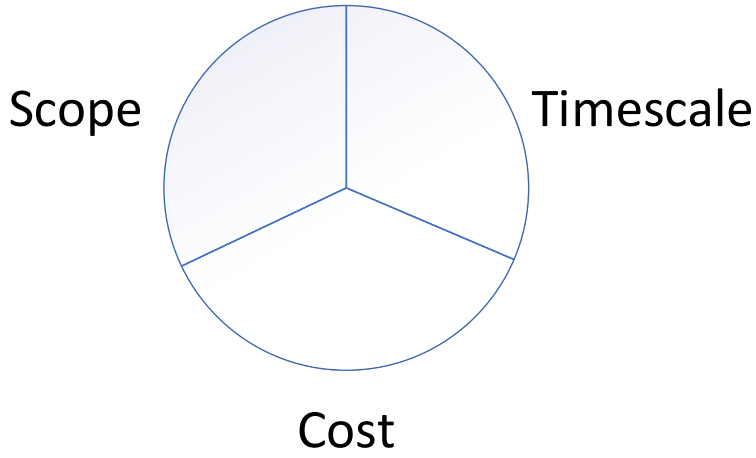


Figure 4. Threefold principle of a project; provide the basis for the typical trade-off relationships.

The threefold principle includes the key trade-offs which the project manager has to address, and which models have to adequately reflect. Meeting either of the objectives tends to take place at the expense of both others. Since measurements can include more than one dimension, the criteria are not adherence to budget, schedule, and specifications, rather adherence to performance, cost and schedule objectives. This means not only understanding whether the total cost of the project is on budget, but also knowing the piece cost of the finished product, the cash flow and oftentimes the lifetime cost of the end product. (Turner, 1995).

However, this fails to do justice to the complexity of identifying success criteria, so that in some cases more sophisticated objective functions are required in the models.

Project management methods are premised on decomposing the project into its constituent parts and monitoring each of them individually. Therefore, various methods are built on the control of the individual parts according to the triplicate criterion, which will be described in more detail below:

The first area of the threefold principle is cost. When preparing cost estimates, a breakdown structure like the WBS is frequently used, namely the Cost Breakdown Structure (CBS). Breaking down the organisational structure of the project through a related framework, the Organizational Breakdown Structure (OBS), will result in a three-dimensional matrix, commonly referred to as a cost control cube (Farid and Karshenas, 1986).

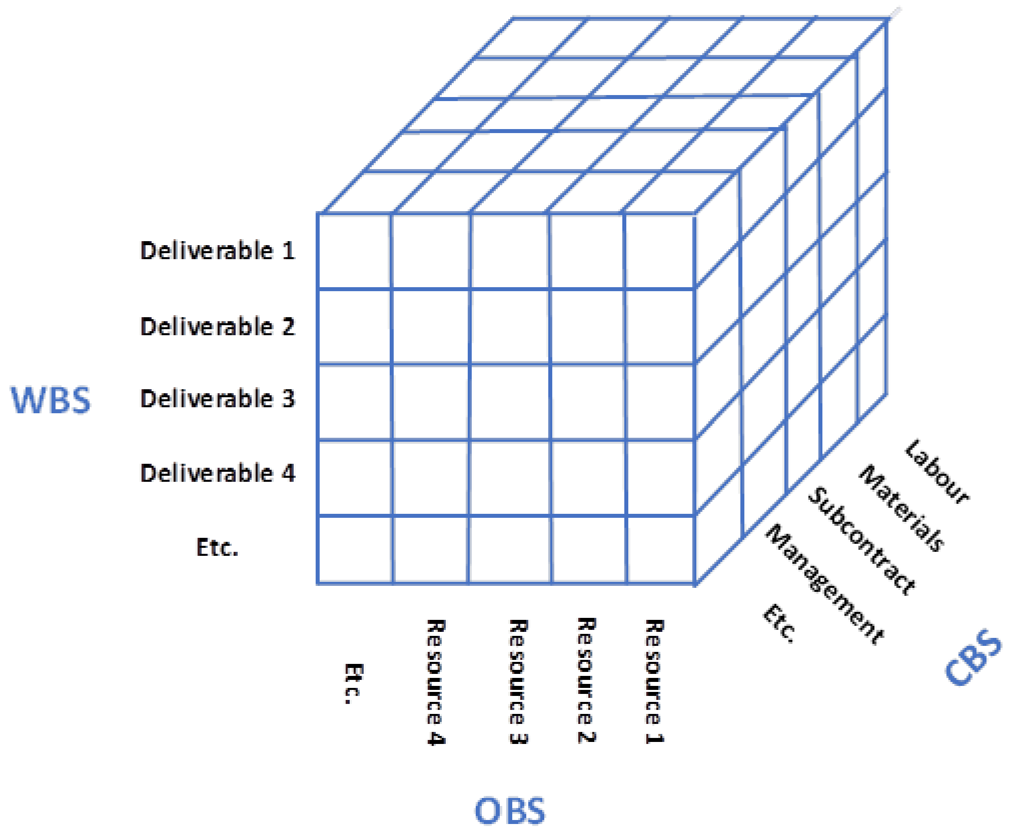


Figure 5. Cost control cube, based on Farid and Karshenas (1986).

As shown in the figure 5, a cube of WBS (ordinate, element to be delivered), OBS (abscissa, resource allocation) and CBS (applicate, e.g., labour costs, material costs, etc.) is formed. This forms many smaller cubes that have clearly definite structures properties and affiliations.

The CBS not only gives an early indication of project expenditure but can also be used to monitor the development of expenditure over time and the progress of the project. The prerequisite is that the activities have been planned in a baseline schedule, applying the principles of acquired value analysis by considering not only the total budgeted cost of the actual completed activities, but also the budgeted work, in order to determine deviations in both the schedule and the costs (Fleming and Koppelman, 2000).

The second section of the threefold principle is the timescale. Project networks are the main basis of the timescale analysis. They are visualised in the form of an activity on arrow (see figure 6), with the activities represented as arrows and the nodes as events. Here, the networks begin with a start event and ends with a finish event. Further, before an activity can be completed, it is necessary for the event preceding that activity to be completed. Finally, to complete an event, all activities leading to that event (node) are required to be finished. (Kent, 2016; Lester, 2021).

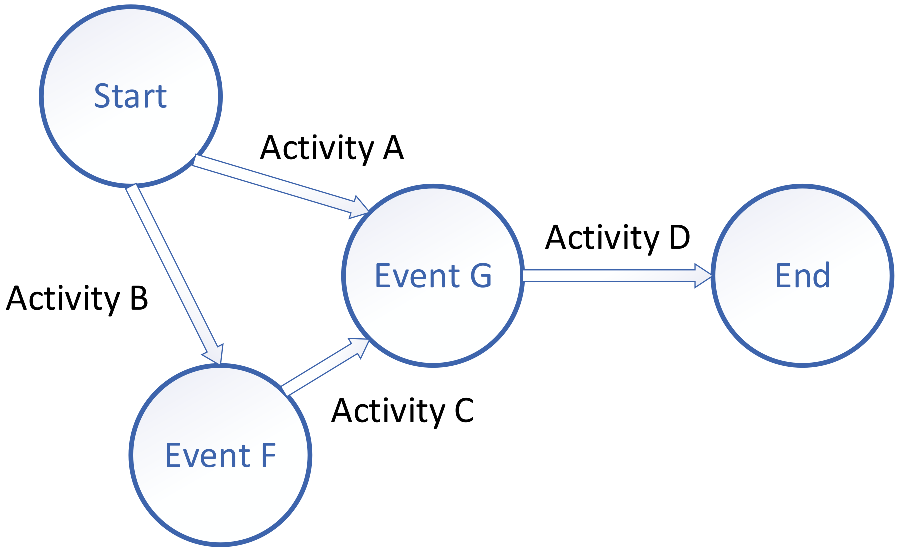


Figure 6. Exemplary activity on arrow project network (Rajapakse Ruwan, 2017).

Alternatively, the project network can be mapped in the form of an activity on a node (see figure 7), where the links from one activity to another are represented by using the node as the activity field and their linkage by lines, with the duration written to the activity field or node, which has the advantage of not requiring separate dummy activities. Both examples of the formats have random concrete content purely for visualisation purposes. These were the basis for the development of the PERT in 1958 (Malcolm *et al.*, 1959).

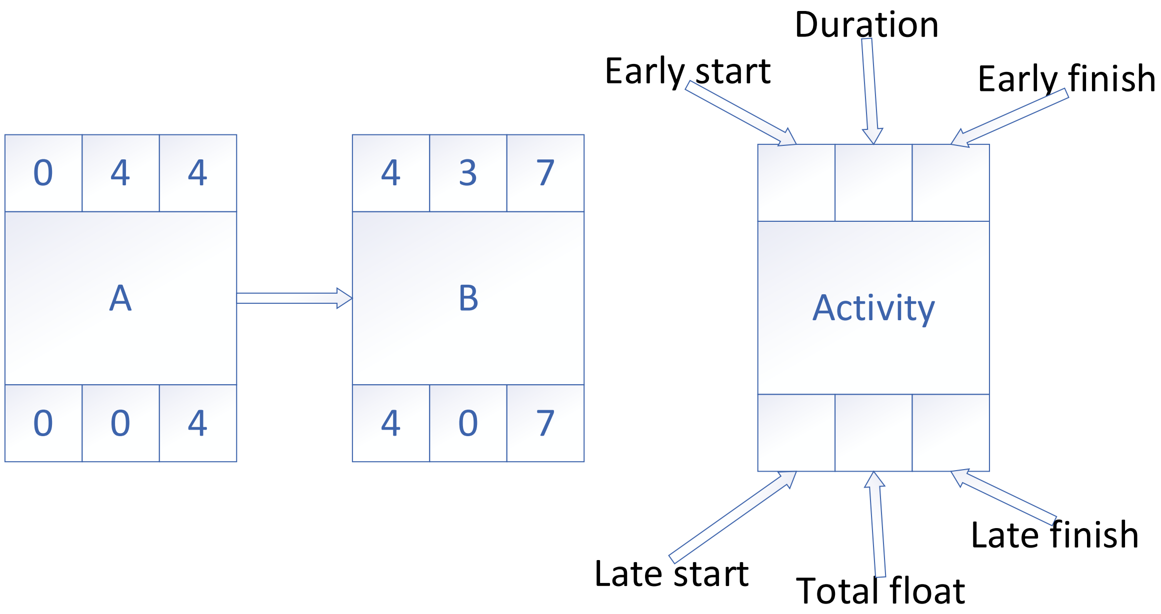


Figure 7. Exemplary activity on node format with explanation of the columns.

The third and final section of the threefold principle is the scope. Mathematical modelling's contribution during the beginnings of project management concentrated around project timescales built around the idea of a network. Summing up the costs in an outline structure is generally trivial. Although assessing the overall length of a basic network needs some elementary sums, when resource constraints and uncertainty are included and the key/critical activities are determined, there is plenty of scope within which modelling can be applied for a better insight (Williams, 2003).

Here, the main procedure is the application of the WBS along with visualisation forms similar to the milestone plan, which presents the milestones of one or more projects in chronological order, improving visualisation, facilitating understanding and highlighting particularly important events (Turner, 1995).

4.2 Deterministic network models

The first appearance of papers on PERT as well as the CPM was in 1959. Nevertheless, solving the basic CPM model, which is not resource-constrained but deterministic, is rather trivial, while including constraints or specifying broader definitions offers the possibility of generating mathematically rich and better-defined problems.

Three components are found in the basic versions of the network models (Kolisch and Padman, 2001). Whereas a project is made up of a sequence of activities, where inputs can be processed in one of various modes, meaning different ways of carrying out the activity, the time required for each activity is determined by its mode. Thus, the project generally takes the form of a directed graph, where the priority correlation that exists within two activities is expressed by a directed arc, and an activity is depicted through a node. Accordingly, the activities require resources that are either renewable, i.e., the number of resources available is fixed for every period, or non-renewable, implying a constraint on the overall demand throughout the project.

There are a number of common problems associated with these definitions, most of which involve minimizing either the time span, or more precisely the total duration of the project, or the cost represented by money or resource input. Deterministic network models exist in many different forms and accordingly cover a wide range of mathematical modelling possibilities and problem-solving options.

One of these models is the Net Present Value (NPV) maximisation without resource constraints. In case resource constraints do not exist, at that point the fundamental problem of minimizing the period of time is well known and rather straightforward. With large projects, though, there is the additional issue of seeking to maximize net present value with the goal of postponing activities with negative cash flow and giving preference to those with positive cash flow (Elmaghraby and Herroelen, 1990).

However, if resources are subject to constraints when planning a project, the problem can be described as RCPSP, which is the fundamental problem type in project planning and targets to keep the total project time to a minimum. Indeed, it is among the most researched project scheduling problems within the literature and has led to a large number of papers providing solution methods for the problem (Carlier, Moukrim and Xu, 2009).

Moreover, the traditional Time-Cost Trade-off (TCT) is a deterministic network model. In TCT analysis, the activity length is shortened by assigning a greater number of resources, which leads to reduced indirect costs incurred at the expense of higher direct costs and a reduced project length. As a result, project planners can conduct a TCT analysis to determine the least-cost project length. Still, the outcome of the TCT analysis is not a feasible selection without also addressing financing costs and constraints on the availability of cash. During construction, for example, assuming in the TCT analysis that there is unlimited access to cash throughout the project duration, will not be viable if intermediate payments are subject to delay and if the contractor incurs a retainage at the time of intermediate payments. Consequently, contractors frequently require supplemental funds to prevent shortfalls (Alavipour and Arditi, 2019).

The figure 8 shows such a TCT. The abscissa reflects the project duration and the ordinate the costs in US dollars. The course of the "direct costs" decreases exponentially with the project duration, the "indirect costs" increase exponentially with the project duration and the sum of these two lines is the "total cost" line, which has a global minimum (here optimum cost-time point). When the indirect and direct cost lines cross, the optimal project duration is given based on the minimum cost. The optimum can also be slightly before or after the intersection of the two lines. According to this model, project durations that are either shorter or longer than the optimum inevitably lead to an increase in project costs.

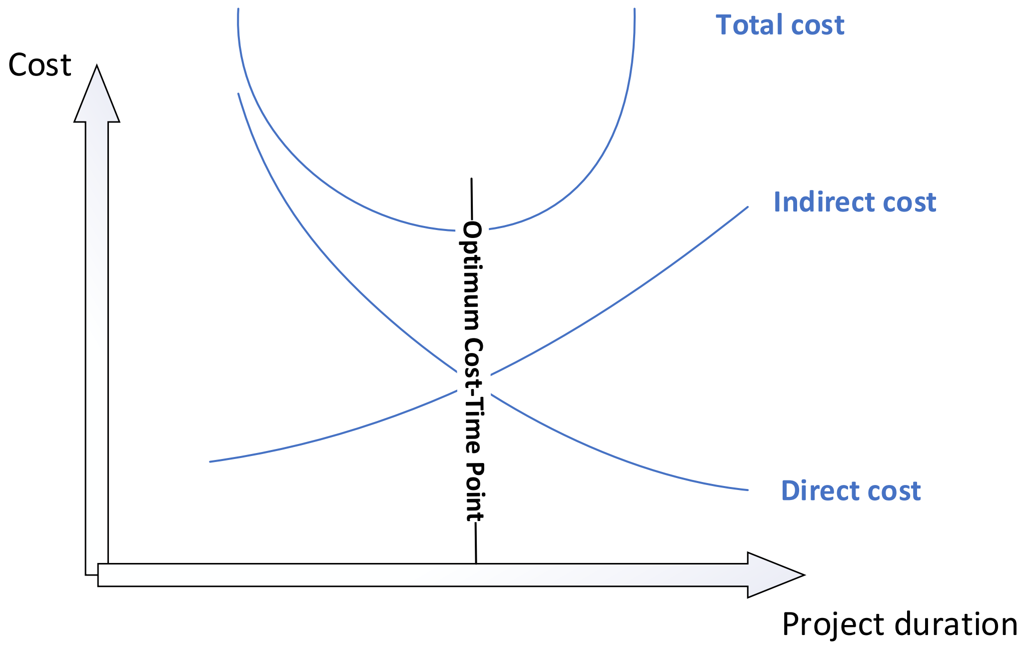


Figure 8. Graphical visualization of the TCT, based on Alavipour and Arditi (2019).

Often, the duration of the work can be shortened at the cost of consuming more resources, be it a renewable resource like machines or people or a non-renewable one like money.

This poses two possible problems for modelling, either finding the realisation that minimises the duration in a way where the aggregate cost is neither higher than a fixed budget limit, or finding the realisation that minimises the aggregate cost (Brucker *et al.*, 1999).

4.3 Stochastic cost models

The summation of the stochastic costs for the CBS is obviously a fairly uncomplicated matter of summing up expense distributions, with more complicated modelling required only when these costs are associated with a time profile. In this context, straightforward approaches such as those developed by Ho and Pike (1992) can be used to develop a cost allocation plan for the project as a whole.

An example of the application of stochastic cost models is the study of the impact of high wind penetration on the operation of the electricity system, which is curcial given the intermittent nature of wind energy and the ever-increasing number of wind turbines.

Scheduling solutions in the power grid are commonly decided one day ahead in order to cover the peak demand and meet any net load. Nevertheless, the vast variability of wind during the forecasting period makes the planning of the power generators a difficult undertaking. For best utilisation of generators and optimal planning, it is essential to estimate wind penetration levels with a sufficient degree of reliability. Furthermore, its very volatile nature requires additional reserves to operate the wind-integrated electricity network at the required span of stability. The electricity grid operators require a robust load commitment system that can accommodate the irregular intermittent character of wind generation. Swaroop *et al.* (2009) used a stochastic cost model which considers the effects of uncertainties regarding demand and generation of wind to run the power system in the best possible way. Based on particle swarm optimisation, the suggested uncertainty modelling is capable of handling a large number of possible settings. The choice of ideal settings is built on the technique of swarm intelligence. As the mitigation process is not subject to comparison on the basis of 1:1, the method is calculational efficient. With the ability to manage a high level of branching, this scenario mitigation technique is capable of modelling the full stochastic nature of the uncertainties.

However, MCSs (see chapter 5) are often used to identify reasonably good distributional approximations instead of using more analytical methods.

4.4 Stochastic time-network models

As described in chapter 4.2, solutions to the basic CPM model, which is without resource constraints and deterministic, tend to be rather trivial. The assumption of infinite resources is an unrealistic aspect of the base model, although the addition of resource constraints turned out to add mathematical interest to the problem. A further unrealistic aspect of the base model involves the assumption of deterministic activity periods, whereby loosening this assumption equally yields a mathematically interesting problem. Including uncertainty in activity durations was sought in the original PERT, but this method did not catch on (MacCrimmon and Ryavec, 1964).

As with deterministic network models, stochastic time-network models use resource-unconstrained and constrained models.

In the case of non-resource-constrained, the main problem with the network technique is, as always, the estimation of the project duration. In this case, however, the distribution of project span has to be assessed on the basis of the hypothesis that the length of activities is both stochastic and that these individual activities are initially assumed to have independent distributions (Ritchie, 1985).

By contrast, in resource-constrained time-network problems, allowing for activity durations is problematic because they complicate the problem (Brucker *et al.*, 1999).

Special consideration should be given to the concept of criticality. Management wants to know what the most important or "critical" operations are. While management wants this question resolved, no single standard definition of these terms is in place. The widespread and well-established definition of criticality, which has been in use for several decades, states that the criticality level of an activity is the likelihood that the activity is on the critical path (Williams, 1995).

Yet it has become apparent over time that this is actually an unhelpful definition, primarily due to the fact that it does not convey the type of information a manager is intuitively likely to expect. Moreover, the concept of critical path for a limited resource network remains to be defined. It has therefore been shown that the correlation between the length of an activity and the overall length of the project yields more meaningful information, and this parameter has been referred to as the criticality of an activity (Elmaghraby, Fathi and Taner, 1999).

However, this is no substitute for the prior criticality definition, but the process of decision-making demands taking both metrics into account: criticality to assign relevance to monitoring the uncertainty of the activity's length, coupled with criticality to assign relevance to monitoring the activity's length (Elmaghraby, 2000).

In the context of stochastic time network models and criticality, simulations play a crucial role. Determining the time span for stochastic networks even under constrained conditions is challenging and, certainly, determining criticalities (of any definition) involves such calculation beforehand. That said, the main issues with analytical frameworks are the compelling assumptions required by all of them, hampering their applicability in almost all real-world situations. As such, there are limitations in commissioning, as the majority presume a certain, sometimes unrealistic, length distribution and are consequently valid only if that distribution is met by all activities (Williams, 1999).

Of greater significance are the omission constraints, which are of great importance. Aside from the resource constraints that naturally make analytical approaches inconvenient, many complex elements need to be integrated into the analysis of a project model creator to ensure that the results are both valid and relevant. These are impacts that span multiple activities and resources, including third party or common cause impacts, where the assumption that the length of activities are autonomous, identically dispersed random variables is generally not plausible in practical terms. In this context, the unusual distributions of activity lengths are also related. Here, examples include the effects of goal setting as in Management by Objectives, a strategic model designed to enhance an organization's output by establishing explicit goals that are agreed to by executive management as well as employees (Williams, 1995).

Similarly, Parkinson's Law, a model that outlines the trend for the effort required to perform a given activity to rise and take up more and more of the time available to do it (see Figure 9). The abscissa represents the allocated time, and the ordinate indicates the effort. The optimal time to complete a task is marked with a dashed line and is at the level of the maximum effort value. More time than the optimum corresponds to wasted time and less time than the optimum corresponds to an increase in productivity. The generalized concept refers to the trend of using all the capacities available in a given system (Parkinson and Osborn, 1957).

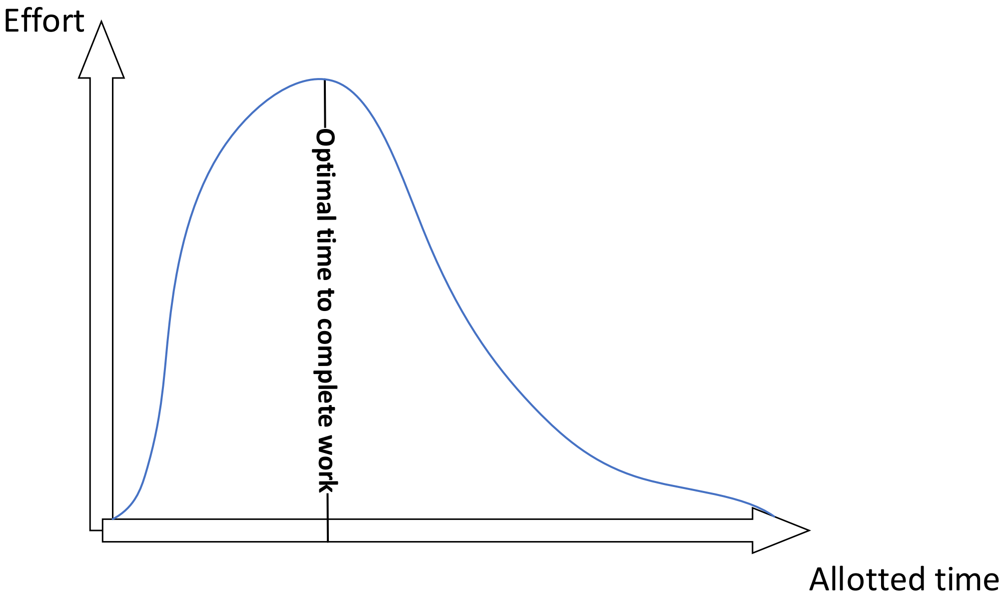


Figure 9. Visualization of Parkinson’s law, based on Parkinson and Osborn (1957).

In addition, there are uncertainties in the structure of the project network, because although classical network planning techniques all consider the network itself to be fixed, there may be branching conditionally or probabilistically in practice.

Another thing to consider is that several analysis methods cover only certain points in time during the project. In practice, though, a major use of this analysis is to offer assistance in solving problems involving the project length at a chosen confidence interval or the likelihood of the project deadline being met. Such distributions cannot be adopted within standard models, and problems involve the need for estimation of the overall duration distribution. Even for a basic stochastic network, the total period of time is the result of the assumption of maxima and a mixture of convolutions. Consequently, the duration can take complicated forms in the presence of more complex uncertainties (Mehrotra, Chai and Pillutla, 1996).

Therefore, in practice, MCSs are almost exclusively used for real applications (see chapter 5). Network simulation will be considered the standard method and has replaced the PERT method. According to PMI, schedule simulation ought to be utilised for any major or complicated project, as conventional mathematical analysis tools such as CPM and PERT do not take path constraints into account and therefore tend to underestimate the project length (Project Management Institute, 2021).

This view was confirmed in the 1997 Project Risk Analysis and Management (PRAM) Guide, which noted that the PERT technique had been substituted by the better-performing MCS (see in more detail chapter 5) and that PERT was no longer regarded as an appropriate risk analysis technique (Peter, Hillson and Newland, 1997).

An issue with many real-world network simulation outcomes is that the scatter of results is frequently very large since the simulations themselves run unwisely past every iteration in the absence of management intervention. Obviously, in a real system, management will take steps to keep a delayed project under control, yet many simulations do not consider this.

One approach to incorporating management measures into the models comes from Golenko-Ginzburg and Gonik (1998), which outlines a range of work on GERT that results in Controlled Alternative Activity Networks (CAAN) and attempts to make decisions to upgrade the networks, albeit that the decisions do not depend on the advancement of the project at that point in time. Such methods do not appear to be broadly applied in the real world, as they are very complex and not general and transparent enough to be widely recognised by practitioners. However, only straightforward control measures are analysed, and straightforward results are modelled, while the effects of these kinds of measures are frequently not evident in practice. Therefore, multiple impacts interact to produce the oftentimes counter-intuitive effects.

Finally, in connection with stochastic time-network models, the critical chain method should not be forgotten. Buffer management or critical chain scheduling is the application of Goldratt's Theory of Constraints principles to the management of projects, referring to his book Critical Chain mentioned in chapter 3.2 on the robustness of project scheduling (Goldratt, 1997). Essentially, the thinking is to pinpoint the critical chain, adopt the 50% likely time estimates for the tasks to minimise the cumulative length of the project, and utilise resource buffers to keep the critical chain tasks from starting late due to the unavailability of resources. Indeed, it offers a simplified and uncomplicated planning tool compared to many other techniques, which are quite challenging to execute and complex, although the creation of a proper, realizable plan, with the goal of finding the critical chain, might not be easy. However, it is important when the method is applied in an oversimplified way, resulting in plans that are far below the optimum (Herroelen and Leus, 2001).

4.5 Stochastic Resource-Constrained Project Scheduling (SRCPSP)

In the early 2000s, the SRCPSP was developed to consider resource constraints and uncertainty simultaneously (Herroelen and Leus, 2005).

An SRCPSP can include either non-structural contingencies, such as stochastic operation duration and resource capacity, or GERT-type structural contingencies, such as uncertain operation outcomes, which can potentially change the underlying network structure.

An example of the application of SRCPSP can be found in military applications. For example, a military mission can be modelled as a project composed of a queue of tasks as encountered in the WBS of project management. Several resources such as equipment, material and manpower might be critical to the accomplishment of a mission. When considering a mission planning problem in the naval ship context, for instance, in emergency scenarios survival often requires that the tasks are properly planned in the correct order, and according to the mission, the task list to be done on board differs depending on the period. Additionally, the tasks have to be completed with limited resources, namely a multi-skilled and permanent crew. Therefore, the deterministic version of the challenge might be modelled as a project planning challenge involving multi-purpose resources, being a variation of the RCPSP. Within this environment, non-structural as well as structural contingencies can appear, leading to an SRCPSP. Thus, a task may succeed or fail and the task length can be stochastic (Li and Worner, 2011).

5 Monte Carlo Simulation for Project Scheduling

Simulations are applied to gain an understanding of a deterministic problem and statistical sampling is applied to assess the uncertainties within the simulations. MCSs invert this type of approach, solving deterministic problems by probabilistic analogies. Back in 1930, Enrico Fermi was the first to experiment with the MCS while investigating neutron diffusion, although none of his papers were published. At a later stage, Stanislaw Ulam, a mathematician, used it in the 1940s when developing the Manhattan Project for nuclear arms during the Second World War (Bagal and Kulkarni, 2019).

In fact, it was given its name based on the famous casino city of Monaco, as the likelihood element is key to the modelling process, much like the game of roulette. Ever since their implementation, MCSs have evaluated the impact of uncertainty in a wide range of realistic scenarios, such as revenue forecasting, stock prices, artificial intelligence, pricing, and project management. In addition, they deliver a number of advantages over predictive models based on fixed inputs, which include the ability to calculate the correlation of inputs or to perform sensitivity analyses. The correlation enables an understanding of the relationships among any input variables, while the sensitivity analysis allows decision-makers to determine the impact of certain inputs on certain results (IBM Cloud Education, 2020).

Through the application of MCSs to scheduling, the traditional approach of a critical path and an end date in a schedule (deterministic approach) is instead substituted with a set of possible critical paths and end dates with corresponding probabilities (probabilistic approach). Although the project schedule is on the whole determined by a single critical path, being the main continuous path in the network, the MCS identifies and evaluates several critical paths. Figure 10 illustrates the approach differences of the two techniques.

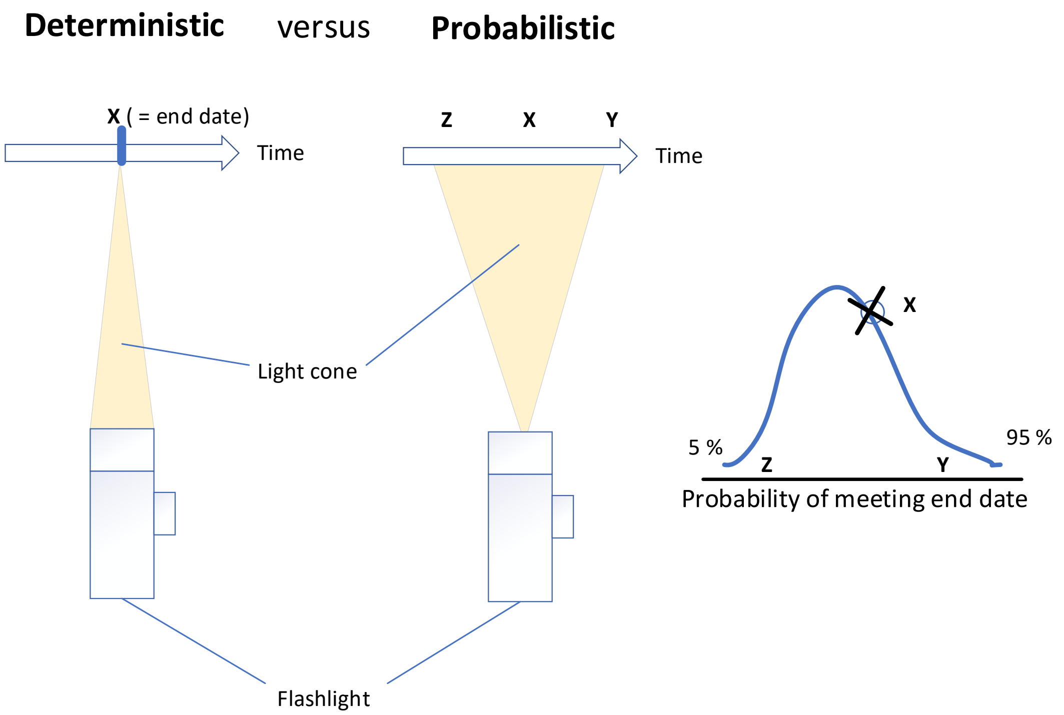


Figure 10. Visualization of a deterministic and probabilistic situation; based on (Verschoor and Eng, 2005).

A MCS allows modelling of the potentially changing duration of tasks, dynamic date ranges can be set, and multiple schedule variants can be simulated using a probabilistic approach, as opposed to a deterministic schedule consisting of a single period. As a result, many different schedules and variants of the critical path can be simulated. Using the outcomes of each of these simulations, data may be interpreted that provides an understanding of which activities are most likely to affect the final deadline, relative to the activities along the actual critical path. Furthermore, this may be used to determine a reasonable end date that can be mutually agreed upon by the owner and contractor (Verschoor and Eng, 2005).

5.1 Benefits

The benefit of applying a MCS to projects is that it is a very powerful simulation technique: Indeed, without considering uncertainty in both schedules and project costs, there is a risk that the project will overrun the project targets. The MCS assists in providing justification and quantifying reasonable project contingency to manage the risk activities that arise during the project cycle. An added feature of a MCS compared to competing methods of project evaluation lies in the fact that it attempts to incorporate uncertainty. Many analytical methods are available for project planning, although the issue with these analytical approaches is the constraining set of assumptions they each demand, effectively disabling them in all real-world situations. The analytical methods frequently yielded particular moments of project duration only rather than distributions of project duration, which actually would be far more beneficial in providing answers to questions regarding the degree of confidence in project completion dates. Yet while the programme evaluation and verification technique has been the past method for evaluating project schedule networks, this methodology statistically fails to account for path convergence and therefore normally leans towards underestimating project lifespans. Fortunately, the MCS, which repeats project cycles n times depending on the default settings, takes these path convergence situations into account (Bagal and Kulkarni, 2019).

5.2 Limitation

The disadvantages of MCSs in the past included the heavy computational power consumption and the high time and resources needed to carry out the simulation activity. Shortage of user-friendly software-based tools to conduct more complex simulations based on project plans has also been a constraint. These concerns are almost outdated by the drastic enhancements in processing capabilities and the addition of MCSs as an additional functional extension of the software to common project management tools. Similarly, another disadvantage is that a MCS reveals a very wide spread of project length. Indeed, this is attributable to the fact that the simulations simply proceed unintelligently through every iteration with no management intervention. However, in reality, it is highly probable that management takes steps to rescue projects that are seriously lagging behind schedule and taking some of these steps might contribute to returning the project back to an agreeable time frame. While models have emerged that include management actions in the simulation, these models have proven to be highly complex and nonetheless do not deliver adequate levels of generality with adequate transparency for acceptability in real-world applications. While the MCS is an exceptionally strong performing tool, a simulation is considered to be only as useful as the model it is fed with to represent the information it is simulating. If the project model or network does not keep up, the simulations will not closely represent even the real activities. Likewise, unless the task duration distributions for simulating the project duration are inadequate, the simulation will be flawed. Forecasting project activity length typically involves expert insight, and feedback even provided when a three-point estimation (as in the PERT estimate) is undertaken to include uncertainty in a model still leaves partial uncertainty latent in the three-point estimation. Experience and advance knowledge from previous projects of a comparable nature are valuable in reducing this estimation uncertainty, even though this data is often lacking (Bagal and Kulkarni, 2019).

6 Methodology

Throughout this section, the methodology used in the study will be described and the factors that influenced the choice of these research methods will be highlighted. Equally, the chapter gives an overview of how the analysis was conducted and the information obtained from it was evaluated.

6.1 Research context

The purpose of the analysis conducted in this study is to find answers to the RQs posed in the introductory chapter:

- How does uncertainty affect input parameters in project management?
- Do some parameters in mathematical simulation modelling (e.g., MCS) provide more accurate and reliable results than others?
- Is it possible to simulate uncertainty in a useful way or does every parameter change lead to completely new results?

The thesis uses quantitative research methods by experimentally simulating uncertainty to achieve the objectives of the thesis and provide answers to the RQs. The end result of the thesis should provide a clear understanding of how uncertainty stochastically affects project planning and what conclusions can be drawn.

6.2 Methodological choices

The aim is to investigate how uncertainty stochastically affects the project length. Furthermore, it is to be investigated how uncertainty as an influencing variable affects the result of these applications and thus to be able to give an assessment of whether a simulation of uncertainty makes sense.

To investigate this, a quantitative analysis method was chosen in which the data was examined and generated experimentally. In this way, possible correlations and influences of uncertainty on the range of project end dates can be identified. Scheduling problems and uncertainty-induced changes to the critical path are investigated, as this does happen in the real project world.

MCS was chosen as the mathematical analysis method, which provides a good overview of stochastics, is easy to perform and comprehensible.

6.3 Data collection

A project plan has been created in MS Project, which ideally represents a typical project with all its imponderables. The basic model of the study on scheduling consists of several task sections that differ in duration. In order to include the factor of uncertainty, a possible variable temporal variance was also inserted for each individual section. In addition, the number of task sections was varied, and the location of the temporal variances was examined.

The basic Gantt chart from MS Project was transferred to MS Excel, with the addition that the individual sections contain a random factor. Thus, after each update of the spreadsheet, the sections output a new random value in the previously entered time span. However, the relevant value for further simulations is only the project end date.

This end date, which changes after each update, is the starting point and input value for the MCS, which was also carried out in MS Excel.

The simulation is started by a VBA macro and the spreadsheets are updated preset desired n times, each time recording the end date. A histogram was now formed from this data list, which was used for analysis purposes enabling statements to be made about the uncertainty influence. In addition, stochastic parameters such as central tendency, dispersion, shape, and quartiles were plotted. All these data were then compared with each other depending on the desired analysis in order to be able to make statements and possible correlations about the influence of uncertainty on the project planning tool.

Furthermore, in a second area of investigation, the stochastic effects of uncertainty on the change of the critical path in parallel processes were investigated. Here, simple critical paths without branching were compared with double and triple parallel paths.

6.4 Data quality of the research

Since uncertainty can occur in many different forms, the focus was placed specifically on the uncertainty of deadlines in the planning of projects. This is to ensure the validity of the results, as variations in the input parameters and thus the realization of the uncertainty factor also provide comprehensible and measurable results. However, the origin of the uncertainty does not play a role in the investigations, as only the actual resulting time delay is relevant and underestimated.

The reliability of the results depends on the parameters used for the experiment and in this case for the simulation. In fact, the problem of uncertainty research is exactly this: that reproducibility is difficult and problem concretization makes uncertainty disappear. Since uncertainty does not occur consistently, this point is challenging, and in the following section some parameters turned out to be more useful than others. This allows other researchers to use the more reliable parameters from the evaluation of this work and thus reproduce the results if needed. Nevertheless, the complete generalizability of the results is not available and need to be re-examined for each new problem and change in the project data, although the results of this work can give a good preliminary estimate.

7 Results

The results presented are divided into project planning and critical path change analysis. The network plans of the project under investigation and the experimental parameters in general are described first, followed by the experimental results of the MCS. The results are rounded to a maximum of one decimal place and correspond to the average values of ten runs per parameter setting.

The results should provide the basis for answering the RQs such as the influence of uncertainty on input parameters in project management, the usefulness of simulation under uncertainty conditions, and reliability. The actual discussion and interpretation of the results takes place in the following chapter 8.

The results of the mean, median, standard deviation, range, kurtosis and skewness are based on the usual mathematical definitions (for example in Serfozo (2009)). The standard error is the uncertainty associated with the estimated mean and is an estimate of the standard deviation of the sample mean for repeated MCSs. The IQ range is the difference between the upper and lower quartiles (Q (0.75) - Q (0.25)).

7.1 Project scheduling

The basic model consists of 15 task sections, each lasting from 2 to a maximum of 63 days, resulting in a total duration of 413 days. In order to include the factor of uncertainty, the individual sections have a possible temporal variance of ± one to three days. In addition, the number of task sections was varied, and the location of the temporal variations was examined.

Appendix 1 illustrates a Gantt diagram of the basic scheduling parameters without uncertainty.

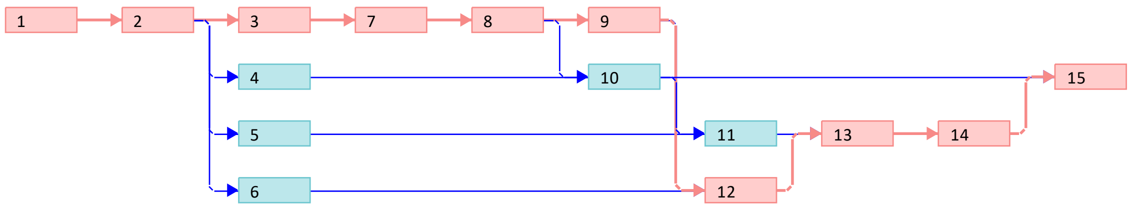


Figure 11. Basic network diagram for the series of project scheduling experiments.

The network diagram (see figure 11) has thus depicted typical project scheduling management issues. For example, the 11th section can only take place after the 5th, 9th and 10th sections have been completed. The 10th, 12th and last sections each have two predecessors. Since four process steps can start at the same time after the completion of the second section, this creates a parallelism of tasks. The critical path consists of sections 1, 2, 3, 7, 8, 9, 12, 13, 14 and 15, which are marked in red in the network diagram.

When simulating uncertainty, four specifications in particular were compared with each other (unless otherwise stated, the simulation runs were set to 1000):

- First, the uncertainty level was simulated by having each section represent a random variation of ± one, two or three days.
- The second variation concerns the project scope. To keep the results comparable, the project model was doubled to 30 sections by running the basic model of 15 sections twice, connected by the original last 15th section with the theoretically originally first and now 16th section.
- The third comparison examines how a variation of the uncertainty location, or more precisely whether the location of the temporal uncertainties plays a role. For this purpose, the base was doubled as in the previous experiment, with temporal variances first applied only to the first half (i.e., the original base) and then only to the second half. Thus, it was investigated whether an uncertainty rather at the beginning of the project or at the end of the project has an impact on the project end date.
- The last investigation directs the consideration to the influence of the simulation runs. Here, a temporal uncertainty of ± 3 days was specified for each section, and then the

statistical behavior of the final time distribution was examined for 100, 1000, and 10000 runs.

The analysis was always carried out **10 times** per new parameter setting and minimum, maximum and average values were noted, which were then compared with other parameter settings.

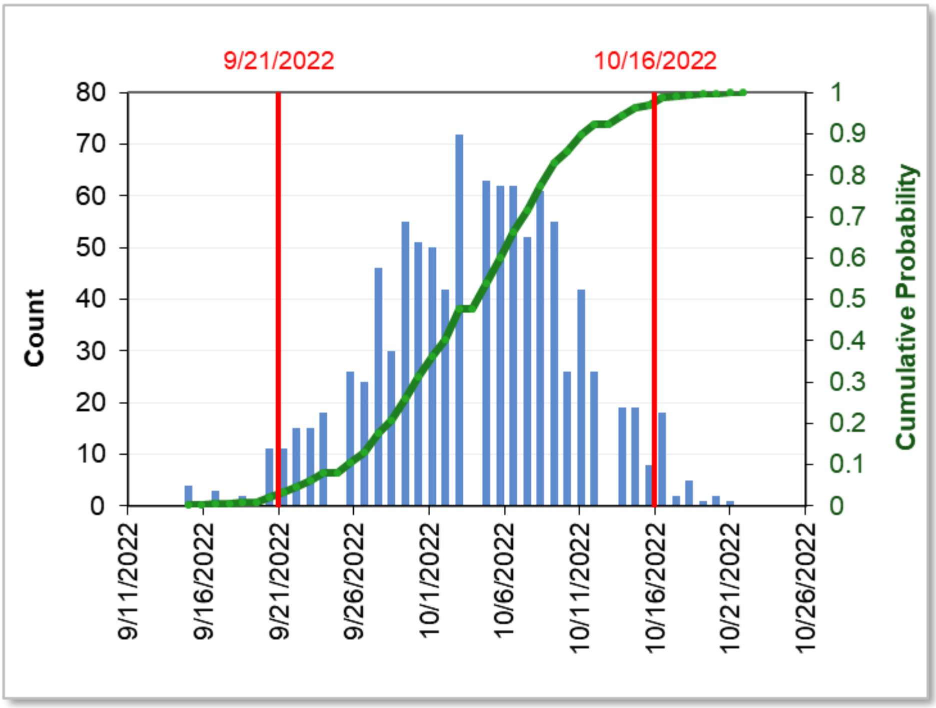


Figure 12. Exemplary histogram from a test series (iterations: 1000, ± 3 days of uncertainty on all tasks, basis model).

The histograms generated after each iteration give an overview of the results (see figure 12). These depend on the preset parameters and change with each iteration. The abscissa values are simulated project end data, whereas those of the ordinate reflect the absolute frequency of these. It follows that the longer a blue bar is, the more often this date was simulated as the end of the project. This results in a probability distribution of the end dates, whereby it flattens out towards the edge and less frequent end dates are found there. The solid red lines represent the 95% interval (0.025 and 0.975 quantiles). This means that 2.5% of the results are to the left of the left line and 2.5% to the right of the right line and could be hidden depending on the analysis desired in order to minimize outliers. In this case, the lower quantile is 9/21/22 and the upper quantile is 10/16/22. The right ordinate also shows the cumulative probability. The green line thus shows how the final data are distributed. The average of the ten simulation runs (of which the histogram shows one as an example due to similarity) has a rounded value of 0 for skewness and -0.2 for kurtosis, which makes the simulated distribution slightly flatter than the normal distribution.

7.1.1 Variation of the uncertainty level

The following Table 2 (data overview in Appendix 2) summarizes the results of varying the uncertainty level.

Table 2. Results of variation of the uncertainty level

Category	Subcategory	Result Description
Spread	Range	Highest average values for three-day uncertainty level (38.4), followed by two-day (30.7, -20%) and one-day (24.3, -21 % to the previous level).
	Standard Deviation	The standard deviation increases from 96 h at one-day level, to 121 h (+26%) at two-day level, up to 152 h (+26 % to the previous level) at three-day level.
Central Tendency (Location)	Mean	The mean is highest for one-day fluctuations (10/4/22 9:27) and about the same for the remaining two (± 2 days: 10/3/22 18:53 and ± 3 days: 10/3/22 19:55).
	Standard Error	The standard error increases from 3h 2min for one-day fluctuations, to 3h 50min (+26%) for two-day fluctuations, to 4h 49min (+26% to previous level) for three-day fluctuations.
	Median	The average median falls from the one-day uncertainty level (10/4/22 14:24) to a similar level at

		the other two levels (± 2 days: 10/3/22 21:36 and ± 3 days: 10/4/22 0:00).
Quartiles	IQ Range	The average IQ range increases from 6.5 at one-day to 7.3 (+12%) at two-day to 8.9 (+22% to previous level) at three-day uncertainty level.
Shape	Skewness	On average, this is in the range of -0.1 to 0.1 for all around 0.
	Kurtosis	Here, the average values of -0.1 for one day, -0.1 for two days and -0.2 for three days are obtained in relation to the uncertainty level.

When comparing the histograms from the test series, it is noticeable that the scaling of the ordinate decreases from a maximum of 140 at the one-day uncertainty level, to 120 at the two-day uncertainty level, to 80 at the three-day uncertainty level. In addition, the largest deviations from the respective mean of the abscissa are found at the uncertainty level of ± 3 days and the least at the uncertainty level of one day. At the three-day uncertainty level, the histogram is visually more detailed than compared to the spottier one-day level.

The cumulative probability curve is similar for all three uncertainty levels, although the one-day curve appears with more plateaus than the three-day curve, which is more "smoothed".

7.1.2 Variation of the project size

A summary of the results of the variation in project size is given below (see table 3, data overview in Appendix 3).

Table 3. Results of variation of the project size

Category	Subcategory	Result Description
Spread	Range	The average value is about 50% higher when the doubled is compared to the base model (base: 38.4, doubled: 57.1)

	Standard Deviation	Standard deviation average is about 42% higher when the doubled is compared to the base model (base: 152 h, doubled: 216 h)
Central Tendency (Location)	Mean	Here, a minimally fluctuating value is observed in each case (base: 10/3/22 19:55, doubled: 7/7/23 22:43). The dates are 277 days apart.
	Standard Error	The standard error grows by about 42% when the base is doubled (base: 4h 49min, doubled: 6h 51min).
	Median	The values are quite constant on average (base: 10/4/22, doubled: 7/8/22) and, as with the mean, 277 days apart.
Quartiles	IQ Range	The IQ range increased by 42% when the base was doubled (base: 8.9, doubled: 12.5).
Shape	Skewness	The values are around 0.
	Kurtosis	This has a base value of -0.2 to -0.1 when doubling.

The course and visual appearance of the two histograms show similarities. The ordinate of the histogram with twice as many task sections is slightly larger with a maximum of 100 counters than that of the basic histogram with a maximum of 80. The abscissa of the basic histogram is somewhat spottier because fewer different end date values were assumed.

The cumulative probability therefore has fewer plateaus in the histogram with the doubled base and thus appears somewhat smoother.

7.1.3 Variation of uncertainty localization

The results of the variation of the uncertainty localization are shown below (Table 4, data overview in Appendix 4).

Table 4. Results of variation of uncertainty localization

Category	Subcategory	Result Description

Spread	Range	The average range values do not differ greatly (40.4 to 38.6), although with temporal uncertainty in the second half the values fluctuate more (difference between maximum and minimum 7 to 2).
	Standard Deviation	The average standard deviation is comparable in both analyses (156 h to 155 h).
Central Tendency (Location)	Mean	The mean of the two analyses is slightly two days apart (7/7/23 8:17 to 7/9/23 19:56).
	Standard Error	This value is almost the same for both evaluations (4h 57 min to 4h 55min).
	Median	The median is about three days apart (7/6/23 to 7/9/23), with temporal uncertainty in the second half causing the values to fluctuate by about one day.
Quartiles	IQ Range	The IQ range dropped by an average of about 11 % when the temporal uncertainty was localized in the second half, from 10 to 8.9.
Shape	Skewness	The values range around 0 for both versions from -0.1 to 0.1.
	Kurtosis	The values range around -0.1, tending to be slightly lower when uncertainty is present in the second half.

A comparison of the two exemplary histograms reveals clear differences. The ordinate scaling is many times higher if temporal uncertainties were only set in the first half (scaling comparison 300 to 80 with uncertainties in the second half). The abscissa when the first half is occupied by uncertainty factors consists of only a fraction of possible end data values compared to the other histogram.

The cumulative probability curve appears in the histogram with the variable time data in the first half with clear long plateaus, whereas the histogram of the variable second half is without visible plateaus.

7.1.4 Variation of simulation accuracy/runs

Table 5 (data overview in Appendix 5) shows the results of the variation of the simulation runs.

Table 5. Results of variation of simulation accuracy/runs

Category	Subcategory	Result Description
Spread	Range	The average range increased from 32 at 100 runs, to 38 at 1000, to 44 at 10000, an increase of +19% at 1000 and +38% at 10000 runs.
	Standard Deviation	The values of the standard deviation first decreased slightly and then remained almost unchanged (158 h at 100, to 152 h at 1000, to 153 h at 10000).
Central Tendency (Location)	Mean	The average mean value of the three test series differs only in the exact time of day. (10/3/22 14:57 at 100, 19:55 at 1000, 18:22 at 10000).
	Standard Error	The standard error drops sharply from 100 to 1000 runs by 328% from 15h 50min to 4h 49min. When comparing 1000 and 10000 runs, it drops further by 315% from 4h 49min to 1h 32min.
	Median	The median is 10/3/22 for 100 runs and 10/4/22 for the other two.
Quartiles	IQ Range	The average IQ range of the three runs was almost unchanged. (9:8.9:9)
Shape	Skewness	With 100 runs this varies from -0.6 to 0.2. With 1000 from -0.1 to 0.1 and with 10000 from -0.1 to 0.
	Kurtosis	This is between -0.8 and 1.1 for 100 runs, between -0.4 and -0.1 for 1000 runs, and between -0.2 and 0 for 10000 runs.

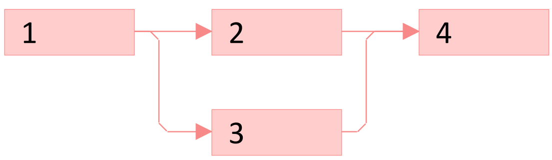
The three exemplary histograms of the test series show clear differences. The ordinate scaling grows from 12 at 100 runs, to 80 at 1000, to 700 at 10000. This corresponds to a factor of 1:7:59. The number of different values is greatest at 10000 runs and least at 100. The interruptions or jumps between the different end dates are greatest at 100 runs and least at 10000.

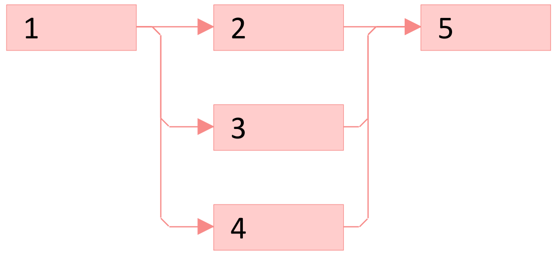
The cumulative probability curve has clear and longer plateaus at 100 runs, whereby these have already decreased at 1000 and are no longer visually recognizable at 10000.

7.2 Critical path analysis

Three different project sequences were simulated, which are shown in the form of the network diagram in Table 6. The first and last task of each of the three test series are constant in time (5 days) and are not subject to fluctuations. On the other hand, the tasks in the middle have a random duration in the range of 3 to 9 days, i.e., the values fluctuate with equal probability by ± 3 days for a duration of 6 days. Again, the analysis was carried out 10 times per new parameter setting and minimum, maximum and average values were noted (see graphical summary with example histograms Appendix 6).

Table 6. Visualization of the critical path analysis as a network diagram, created in MS Project

Number of paths	Visualization in the network diagram
1	
2	

3	

When comparing the three example histograms, smaller, visually not too significant differences (under default graph settings) are noticeable. The ordinate scaling increases from 300 for one path to 350 and 450 for three parallel paths. The end dates for one path seem about equally probable, whereas for three parallel paths a clearer maximum value is reached, with values occurring more rarely the further away they are from the maximum value. The cumulative probability curve begins with only one variable section with a higher initial value (here 30%). The first cumulative value with three parallel variable values, on the other hand, is about 5%. Table 7 shows the results of the analysis of the influence on critical paths changing due to uncertainty.

Table 7. Results of variation of number of parallel paths

Category	Subcategory	Result Description
Spread	Range	The value for all three test series is 7 with no fluctuations.
	Standard Deviation	The approximate average standard deviation is almost identical for the first two series of experiments (one and two paths: 55h) and decreases slightly to 51h for three parallel paths.
Central Tendency (Location)	Mean	The mean increases from 1/16/22 via 1/17/22 to 1/18/22 by one day each time an additional variable path was added.
	Standard Error	As with the standard deviation, the average standard error values are almost constant for the first two test

		series (one path: 1 h 45min, two paths: 1 h 44 min) and drop slightly to 1 h 38 min for three paths.
	Median	The median increases by one day for each additional path from 1/16/22 via 1/17/22 to 1/18/22.
Quartiles	IQ Range	The average IQ range has no fluctuations and first increases from 4 to 5 and then decreases again to 4 with three parallel paths.
Shape	Skewness	The values are on average 0.8 for one path, 0.2 for two paths and 0 for three paths.
	Kurtosis	The average values are -0.4 for one path, -1.1 for two pages and -1.2 for three paths.

It is important to mention that these are the results of a very specific problem and cannot be generalized.

8 Discussion

This section, like the previous one, is divided into two subsections (project scheduling and critical path analysis). For each statistical category of analysis, salient correlations are presented and for each series of experiments, the major anomalies are generally noted. In addition, further potential research ideas are noted.

8.1 Project scheduling

The first sub-category of the discussion section deals with the four experiments on project scheduling.

8.1.1 Variation of the uncertainty level

It can be seen from the histograms that an increase in the uncertainty level leads to a broader distribution of potential final data. This is expressed by the fact that the absolute frequency of individual end dates increases with each uncertainty level and more potential end dates emerge. Furthermore, the cumulative probability curve becomes smoother as the uncertainty level increases, as more possible end dates are theoretically reached. Table 8 summarizes further discussions on this subtopic.

Table 8. Discussion of the results of the variation of the uncertainty level in project scheduling

Category	Subcategory	Result Description
Spread	Range	The average range decreases by about 20% each time the uncertainty level is increased by one day.
	Standard Deviation	The average standard deviation increases by 26% when the uncertainty level increases by one day.

Central Tendency (Location)	Mean	The mean value for all three levels lies approximately in the night from 10/3/22 to 11/3/22. No decrease or increase can be detected, also due to partially overlapping error bars.
	Standard Error	This value increases by 26% with each additional day of uncertainty.
	Median	Like the mean, the median fluctuates around the transition from 10/3/22 to 10/4/22 regardless of the uncertainty level. However, the fluctuations around the mean seem to decrease with a higher uncertainty level.
Quartiles	IQ Range	The IQ range increases as the uncertainty level increases, and this rises even more for larger uncertainty levels than for the transition from smaller ones.
Shape	Skewness	The uncertainty level does not seem to have any influence on the skewness, as it differs only minimally for all of them and is around 0.
	Kurtosis	The kurtosis seems to decrease slightly with increasing uncertainty level.

The values of the range, standard deviation, standard error and IQ range increase proportionally with the increase of the uncertainty value of one day and thus the variation of the uncertainty range here has a clear influence on the stochastic quantities. This influence is due to the fact that the range of possible end dates increases with the increase of the possible uncertainty level.

In contrast, the mean, median and both shape categories are less influenced by the variation of the uncertainty level. With no change in the probability of each possible end date, the mean and median remain fairly constant. The stochastic shape (skewness, kurtosis) of the histogram also shows no changes due to the remaining probabilities.

8.1.2 Variation of the project size

From a purely visual point of view, the change in project size has no great influence on the histogram. Logically, the abscissa values vary because the basic project was doubled in size and therefore the original values cannot be reached. Also, when the project volume is doubled, the distribution appears somewhat denser with fewer interruptions. A detailed examination of the statistical parameters nevertheless reveals some more differences (see table 9).

Table 9. Discussion of the results of the variation of the project size in project scheduling

Category	Subcategory	Result Description
Spread	Range	The doubling of the project scope has the effect of increasing the range by 50%.
	Standard Deviation	A similar effect is observed for the standard deviation, where a doubling of the project size increases it by 42%.
Central Tendency (Location)	Mean	The mean values are not really comparable, since linking two "basic projects" also inevitably shifts the position of the mean value. What is comparable, however, are the fluctuations around the mean value, which take up about half a day in both test series and are thus independent of the project size.
	Standard Error	As with the standard deviation, doubling the project size has an effect of increasing the standard error by 42%.
	Median	As with the mean, the location of the median is not really comparable. However, the doubling has led to slight fluctuations of ± 1 day around the average median value.
Quartiles	IQ Range	The IQ range increases by 42% with the doubling of the project scope.

Shape	Skewness	The values differ only minimally, from which it can be concluded that the project size has no influence on skewness.
	Kurtosis	The kurtosis increases minimally as the project size doubles but is still slightly in the negative range.

For the subcategories, range, standard deviation, standard error and IQ range, an increase of 42-50% was observed when doubling the project size. This is due to the fact that more possible end dates are possible and therefore the standard deviation error also increases.

In this constellation, the project size has no significant influence on the shape categories, as the probability distribution has not changed.

8.1.3 Variation of uncertainty localization

Visually, the location of the uncertainty factors has a very large influence on the histogram (see for example appendix 4). For instance, the number of possible end dates changes greatly, which has an influence on the ordinate scaling and the cumulative probability curve. Since this result was rather unexpected, the exact cause of this development should be investigated in further test series. Thus, the basic project could be tripled and then only simulate the uncertainty factors in one third each. Other parameters such as the range of uncertainty and their influence on the statistical evaluation could also be investigated.

For the present series of experiments, however, it can be stated that the location of the uncertainty factors in the second half produces significantly more different project end data. Further discussion regarding the variation of uncertainty localization is presented in Table 10.

Table 10. Discussion of the results of the variation of uncertainty localization in project scheduling

Category	Subcategory	Result Description

Spread	Range	The uncertainty towards the end of the project leads to more fluctuations in the range and in comparison, with the range at the beginning of the project it decreases slightly.
	Standard Deviation	The location of the uncertainty did not affect the standard deviation, as the values for both test series were statistically unobtrusive.
Central Tendency (Location)	Mean	The mean value changes at a later point in time (in this case approx. +2.5 days) when the uncertainties tend to take place towards the end of the project.
	Standard Error	The standard error is, like the standard deviation, not statistically distinguishable for both test series.
	Median	As with the mean value, the uncertainty towards the end of the project leads to a shift of the median backwards by two to three days.
Quartiles	IQ Range	The IQ range decreases slightly when the uncertainties are localized towards the end of the project, as well as the fluctuations of the IQ range during the simulations, decreases in the second half.
Shape	Skewness	The position of the uncertainty does not seem to have any influence on the skewness.
	Kurtosis	Due to the minimal differences, no influence of the uncertainty localization on the kurtosis can be stated.

When looking at the influence where the uncertainty impact on the project in terms of temporal variation in either the first half or second half (both halves have the same basic parameters) is shown by a shift in the mean and median when the uncertainty is localized in the second half by two to three days. Whereby this is quite small when viewed over the entire project duration (approx. 400 days excluding uncertainty fluctuations) and thus no clear general influence can be determined.

In the subcategories such as standard deviation, standard error, skewness and kurtosis, no differences could be found in the comparisons within this test series, as these cannot depend on the location of the uncertainty under these conditions.

8.1.4 Variation of simulation accuracy/runs

By comparing the histograms, it can be concluded that increasing the number of simulations runs leads to a visually more spread-out normal distribution. This is also expressed by a cumulative probability curve with less recognizable plateaus for higher simulation runs. The scaling of the ordinate also increases with higher simulation runs, but not linearly, as the values are statistically more evenly distributed. This means that a tenfold increase in simulation runs does not cause a tenfold increase in the ordinate scaling, and this scaling will increase less and less rapidly with further tenfold increases. Table 11 provides a summary of the interpretation of the results for the variation of the simulation runs.

Table 11. Discussion of the results of the variation of simulation accuracy/runs in project scheduling

Category	Subcategory	Result Description
Spread	Range	The range increases by 19% for each tenfold increase in the number of simulations runs.
	Standard Deviation	The standard deviation remains almost unchanged regardless of the number of runs, with the fluctuation around the average standard deviation becoming smaller with a larger number of simulations runs. Namely, it first decreases by 56% and with a further tenfold by 81%. This means that the results are more stable and reliable (same result with a new run) with a higher number of simulations runs.
	Mean	The average mean value is approximately the same and thus independent for all simulation runs.

Central Tendency (Location)		However, the fluctuation of the individual test series around the mean value decreases significantly the higher the preset number of simulations runs.
	Standard Error	A tenfold increase in the number of simulations runs leads to a decrease of just over 300% of the standard error, as well as much smaller fluctuations in the results from the different test series. With 10000 runs, this is even almost constant (± 1 min; to be compared with over 3 h with 100 runs).
	Median	The median stabilizes at higher runs, whereby this effect is already visible at 1000 runs.
Quartiles	IQ Range	The number of simulation runs does not seem to have any influence on the IQ range, as the average values are almost the same. However, the fluctuation decreases noticeably and is even constant or non-existent at 10000 runs.
Shape	Skewness	It becomes clear that the fluctuations of the maximum and minimum values decrease the more runs the simulation has. The more runs, the closer to the normal distribution (i.e., value 0).
	Kurtosis	Similar to skewness, the fluctuations decrease significantly with increasing simulation runs and approach the normal distribution value (= 0).

The observation of this section that the dispersion of the results decreases as the number of simulation runs increases is thus consistent with the result of a study by Farrance and Frenkel (2014). One of the results of the study is shown in Figure 13, which shows the spread of the standard deviation in relation to the number of MCSs runs. More specifically, the scatter of the standard deviation calculated by 100, 1000, 10.000, 100.000 and 1.000.000 Monte Carlo trials is shown, where a small number of trials per simulation leads to a larger scatter of results. Thus, the values for the standard deviation fluctuate approximately between 2.1 and

2.5 for 100 simulation runs and, in contrast, are almost point-like at approx. 2.27 for 100000 runs.

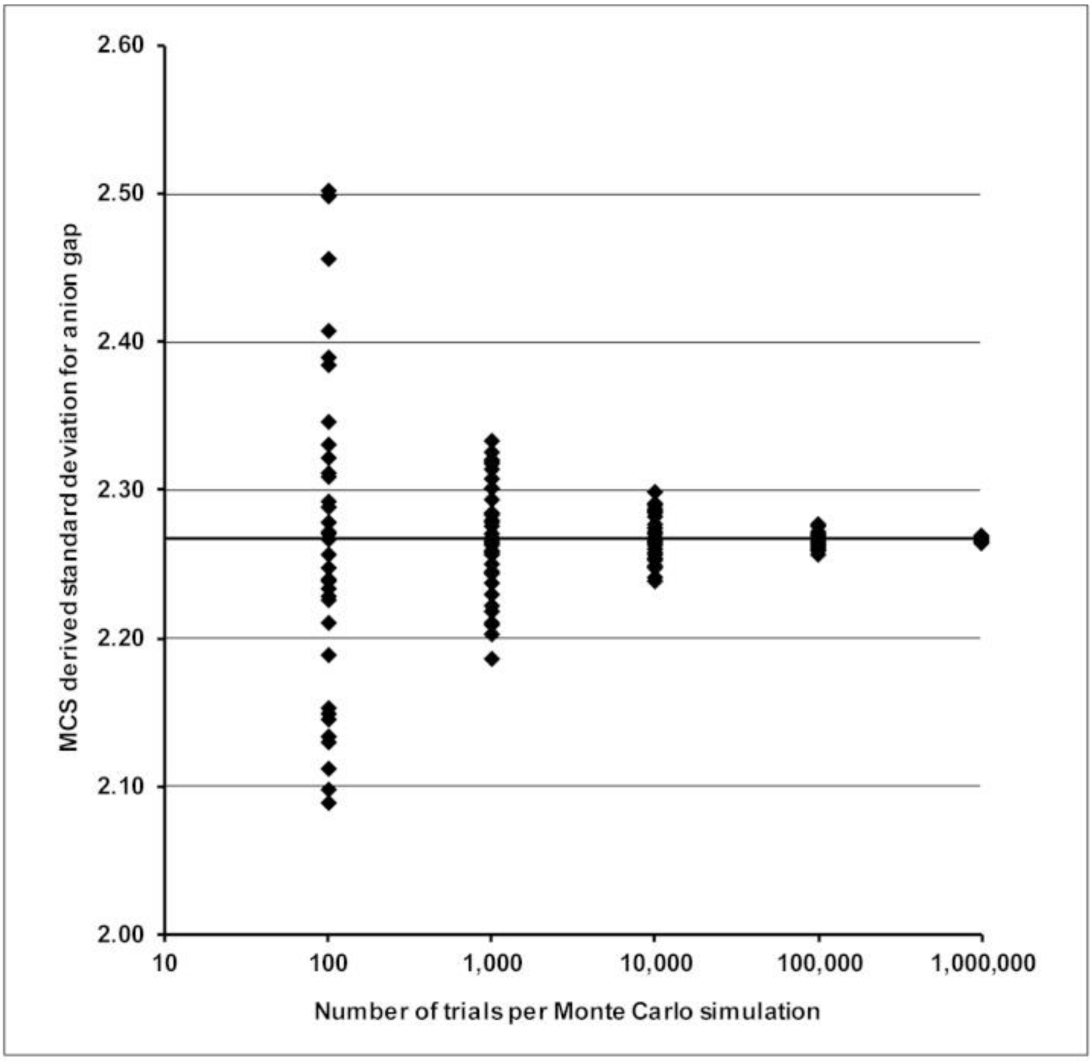


Figure 13. Results of a study of the dispersion of the standard deviation in relation to the number of MCS runs (Farrance and Frenkel, 2014).

It should be noted that the required computing power and computing time for higher simulation runs increases significantly, which is why it should be determined beforehand which scattering interval is acceptable. Ultimately, the project manager must decide, depending on the situation, whether the simulated results are sufficiently accurate or whether variations are acceptable.

8.2 Critical path analysis

The more parallel paths there are, the more a most probable end value emerges, which is the last possible end date. This finding results from the comparison of the three example histograms of the respective test series. Thus, the scaling of the ordinate also increases with more paths. The cumulative probability curve confirms this observation, as the jumps of the plateaus between the individual end dates become larger and larger (from the first date to the end date) the more parallel paths there are. Table 12 combines the interpretation of the results for the variation in the number of possible critical paths.

Table 12. Discussion of the results of the critical path analysis

Category	Subcategory	Result Description
Spread	Range	The number of paths appeared to have no influence on the range, as it was consistently constant across all three test series.
	Standard Deviation	The standard deviation remains constant at one and two paths and decreases slightly at three parallel paths.
Central Tendency (Location)	Mean	The mean value increases with more paths.
	Standard Error	The standard error behaves like the standard deviation, i.e., it remains constant at first and then decreases slightly with three parallel paths.
	Median	The median, like the mean, increases with more paths.
Quartiles	IQ Range	The IQ range first increases slightly and then drops back to the initial value with three parallel threads.
Shape	Skewness	With more paths, the skewness seems to converge to the normal distribution.
	Kurtosis	The kurtosis seems to decrease slightly with more threads.

The subcategories of the shape category show correlations in the increase of paths, as the probability distribution of the possible end dates is changed with each new parallel path. Specifically for this constellation, the categories such as range, standard deviation, standard error and IQ range remain constant, or have only small fluctuations without patterns.

However, as further potential experiments, investigations of more complex project sequences and even more branches would be interesting.

9 Conclusion

In order to draw a connection to the RQs formulated at the beginning, the RQs are answered below, and their objectives are also taken into account. In addition, the limitations of the thesis results are discussed, as well as possible follow-up research opportunities.

The first RQ deals with the question of how uncertainty affects the input parameters in project management? Stochastic correlations are recognisable, but these depend on the comparative variables chosen. Concrete correlations such as the reduction of the scatter with an increase of simulation runs are clearly recognisable, whereas the variation of the localisation of the uncertainty at different task sections has shown fewer clear deviations. In general, each statistical variable must be examined individually for abnormalities, whereby e.g., the standard deviation and the standard error, or also the median and mean value produced similar and coherent results within the test series.

The second RQ, on the other hand, examines the issue of whether some parameters in mathematical simulation modelling (e.g., MCS) provide more accurate and reliable results than others? When varying the uncertainty level, it has been shown that logically a larger temporal uncertainty results in larger end date deviations (e.g., +20% increase in range with ± one additional day of uncertainty in all task sections). Also, increasing the scope of the project has a direct impact on the stochastic results of the uncertainty analysis, e.g., an increase of 42% of the standard deviation when doubling the scope of the project. The localisation of uncertainty within the project, on the other hand, did not show any significant differences in the evaluation. However, the most significant differences were found when the number of simulation runs was increased, with fluctuations in the simulation results decreasing significantly with an increase in the number of runs.

When analysing the change in the critical path due to temporal variation of task sections caused by uncertainty, it was shown that with more parallel paths, a most likely end date emerged. In contrast, with only one possible critical path, the distribution of theoretical end dates produced several equally likely project end dates.

Finally, the third RQ addresses the question of whether it is possible to simulate uncertainty in a meaningful way or does every parameter change lead to completely new results? The

MCS provides fast and usable results for various parameter settings, which remain comparable even with the proportional change of input parameters caused by uncertainty. From this, e.g., a project manager can gain important information about his project flow and take precautions. However, it should be noted that even comparatively small changes in the input parameters (increasing the uncertainty of 15 tasks by ± one day for a project over a total of 400 days, can lead to a 20% larger range of possible project end dates) can produce different simulation results, which is why several parameter settings should always be simulated in order to avoid inferring incorrect estimates.

Probably the clearest and most generalizable results of the study are those of the influence of the simulation runs on the stochastic values of the MCS, with results in line with a comparable study. Here, the recommendation can be made to set a high number of simulations runs, as these make the results of the simulation more accurate. However, the noticeable increase in computing power and simulation duration should be kept in mind, as well as what the desired error interval should be, since normally quite useful results can be delivered in real project situations even with fast simulation times.

On the other hand, the results of the other test series refer to very specific problems and may give different results for other parameter settings of the project plan. Furthermore, the results of the study all refer to the MCS, although consideration of other mathematical models, especially for more complex problems, may lead to generalizable results. Moreover, in the real world, different sources of uncertainty may act at once, resulting in different probabilities of project delays, and thus may be subject for further investigation, as the present study only covers equally probable delays.

In general, though, it is important to know the source types of the forms of uncertainty discussed in chapter 2 in projects so that precautions can possibly be taken to minimize uncertainty.

However, since these will always be part of the planning in real projects, project managers should be aware of the different types of scheduling discussed in chapter 3. This will allow you to use the method that best suits your needs as a solution approach. Finally, the mathematical models presented in chapter 4 can be used to solve the respective problem.

Optimally, future research would focus on the impact and reduction possibilities of individual sources of uncertainty on projects and develop methodologies for this. This could specifically increase planning certainty and, desirably, make studies on the impact of input uncertainty on project processes obsolete. In addition, for complex problems, it is advisable to use one of the mathematical scheduling solution algorithms, which are constantly being further developed, since they can take into account more input parameters.

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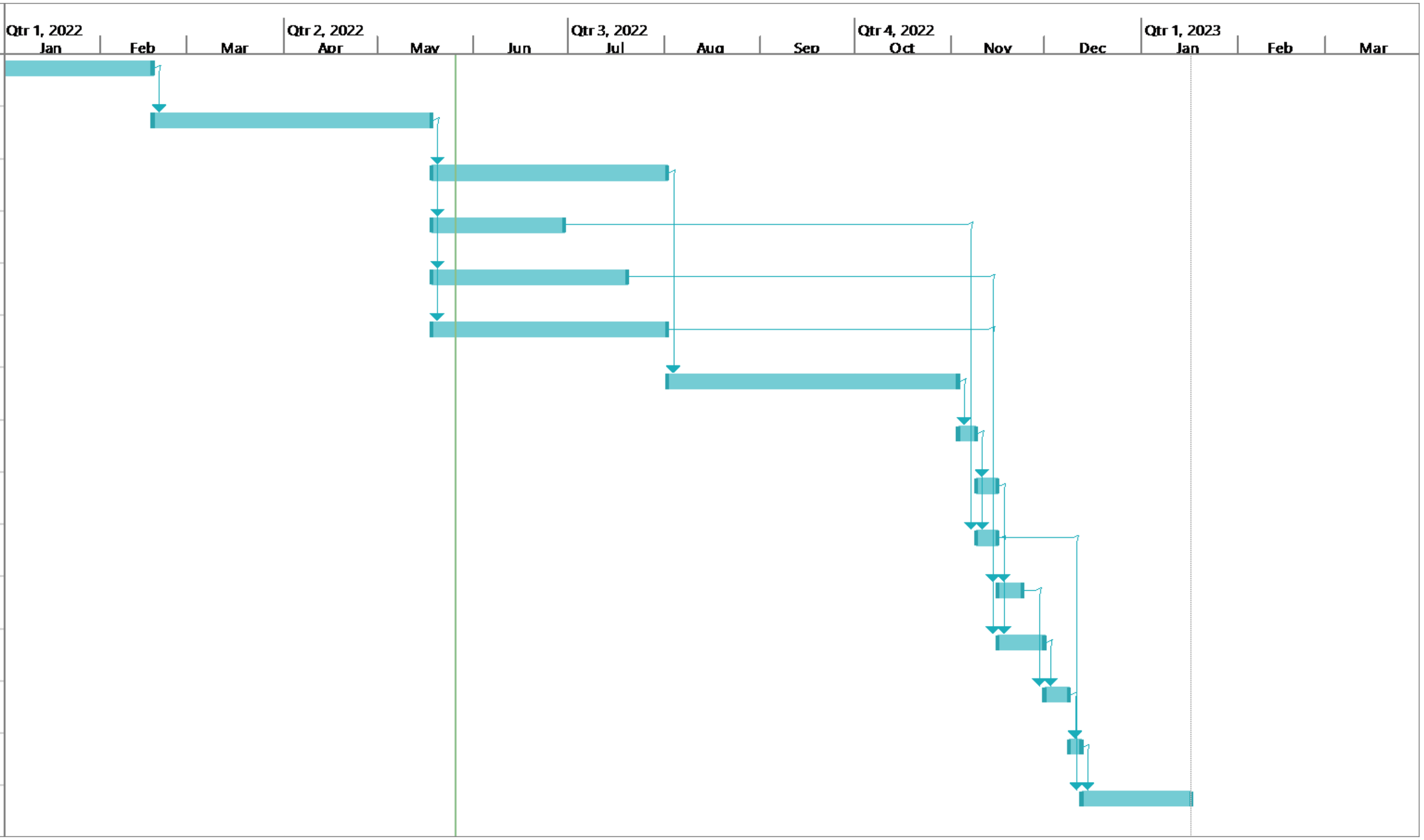
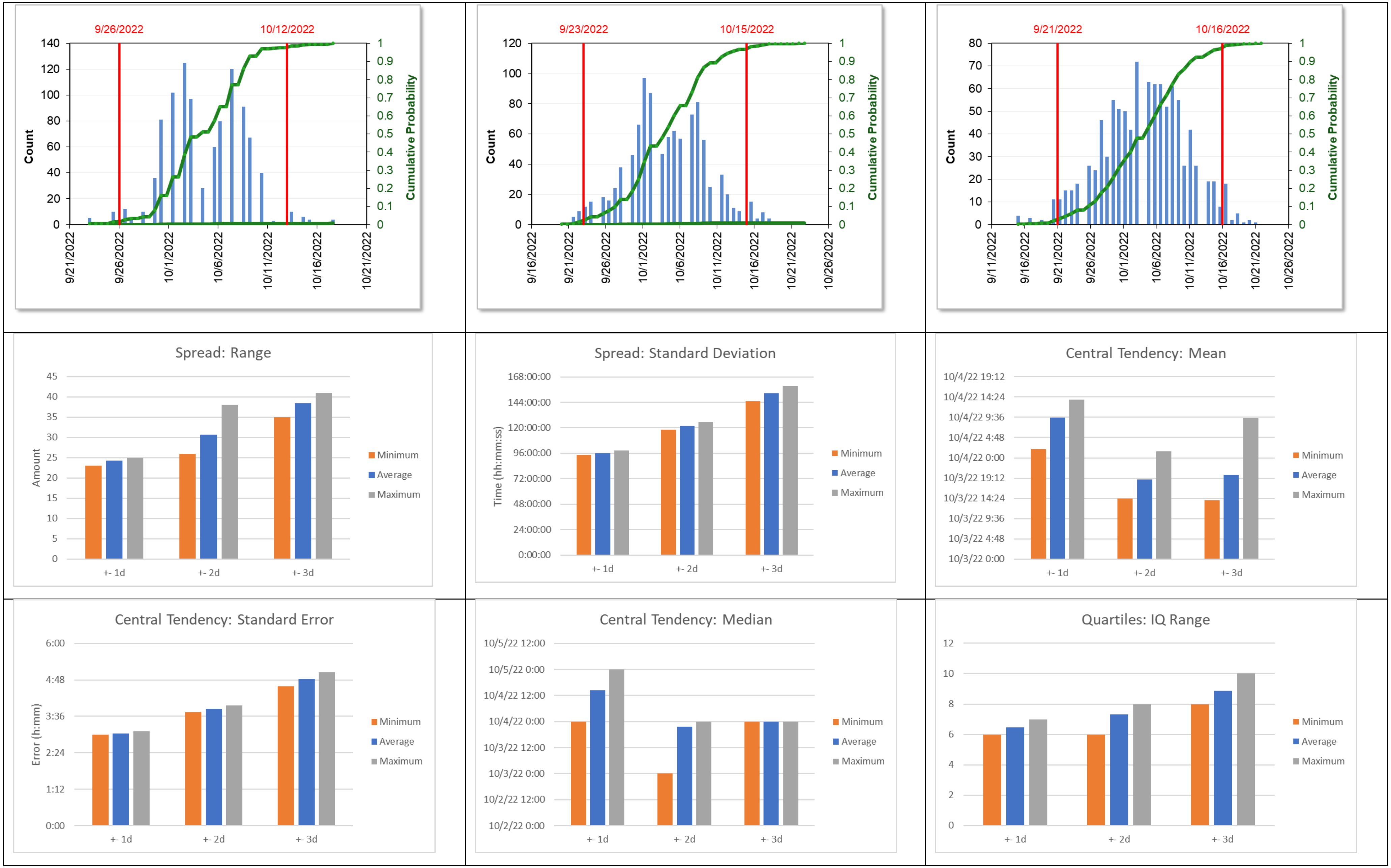
Appendix 1. Gantt chart of the basic scheduling parameters without uncertainty parameters

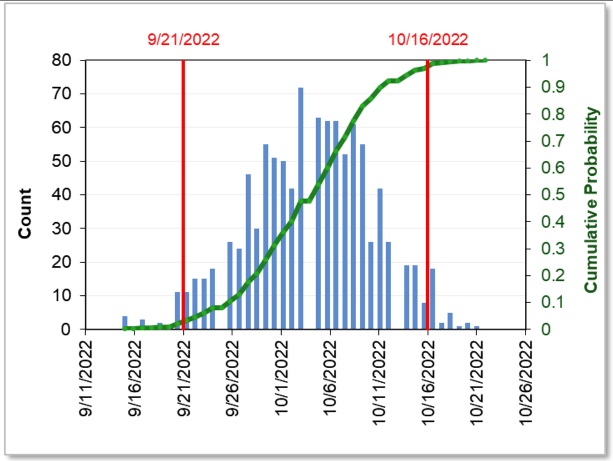
ID	Task Name	Duration	Start	Finish	Predecessors
1	A	35 days	Sat 1/1/22	Thu 2/17/22	
2	B	63 days	Fri 2/18/22	Tue 5/17/22	1
3	C	54 days	Wed 5/18/22	Mon 8/1/22	2
4	D	31 days	Wed 5/18/22	Wed 6/29/22	2
5	E	45 days	Wed 5/18/22	Tue 7/19/22	2
6	F	54 days	Wed 5/18/22	Mon 8/1/22	2
7	G	67 days	Tue 8/2/22	Wed 11/2/22	3
8	H	4 days	Thu 11/3/22	Tue 11/8/22	7
9	I	5 days	Wed 11/9/22	Tue 11/15/22	8
10	J	5 days	Wed 11/9/22	Tue 11/15/22	4,8
11	K	6 days	Wed 11/16/22	Wed 11/23/22	5,9,10
12	L	11 days	Wed 11/16/22	Wed 11/30/22	6,9
13	M	6 days	Thu 12/1/22	Thu 12/8/22	11,12
14	N	2 days	Fri 12/9/22	Mon 12/12/22	13
15	O	25 days	Tue 12/13/22	Mon 1/16/23	10,14

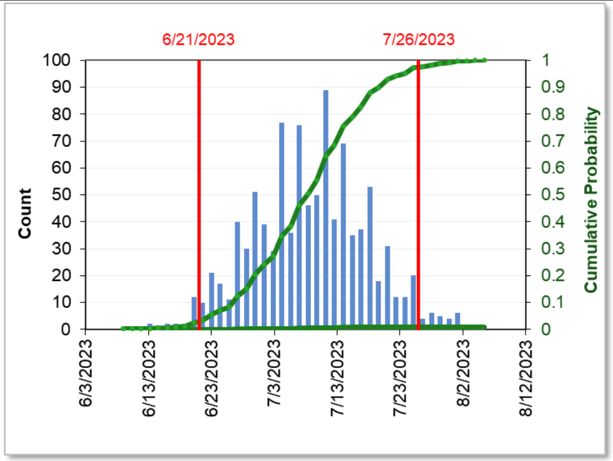


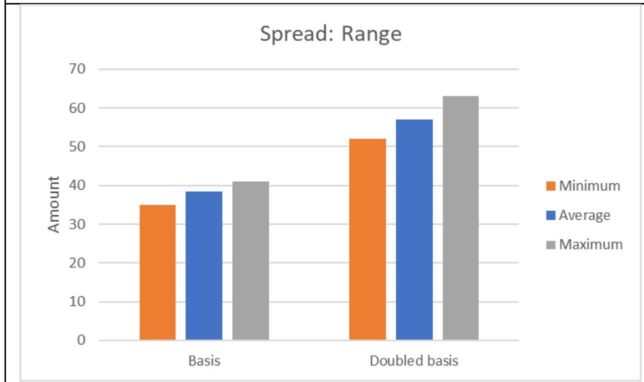
Appendix 2. Variation of the uncertainty level (Upper left: ± 1d, upper middle: ± 2d, Upper right: ± 3d)

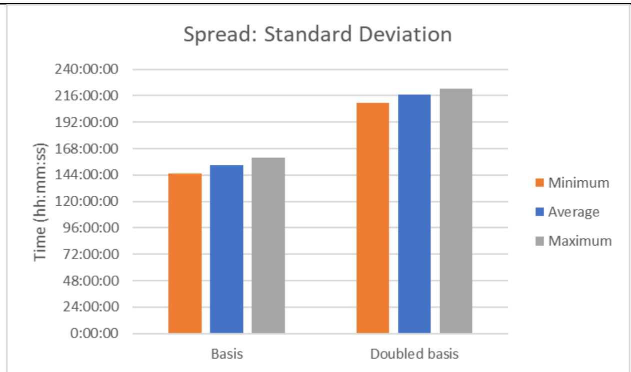


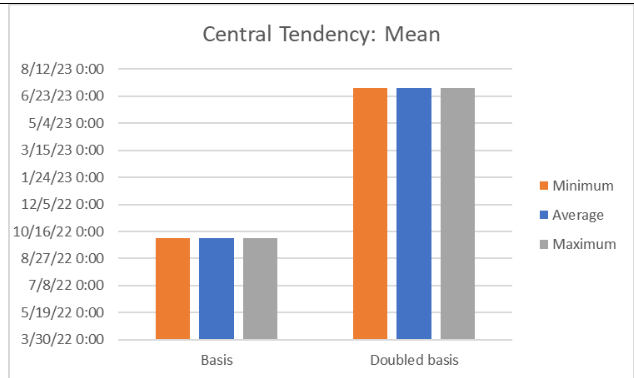
Appendix 3. Variation of project size (Upper left: basis size, upper middle: doubled basis size)

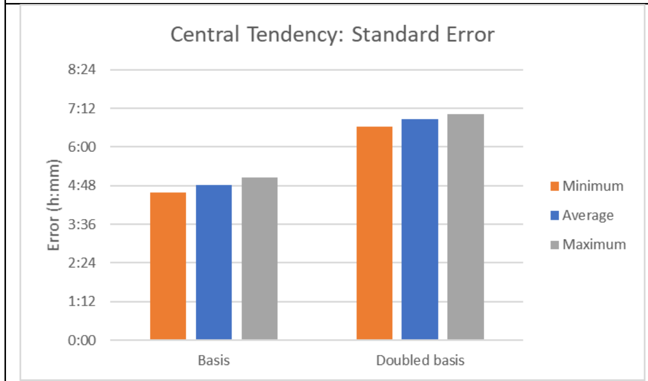


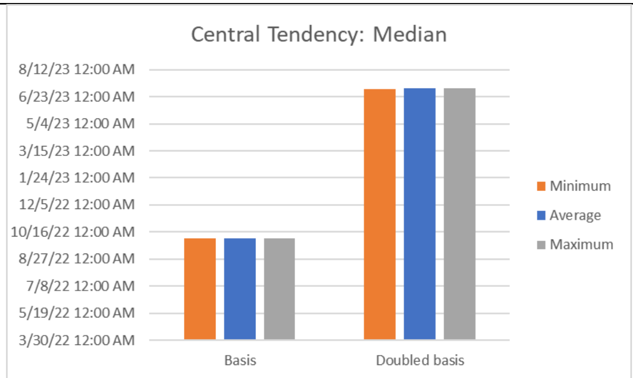


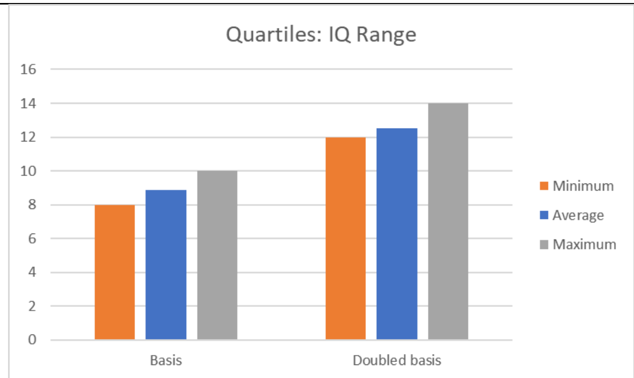




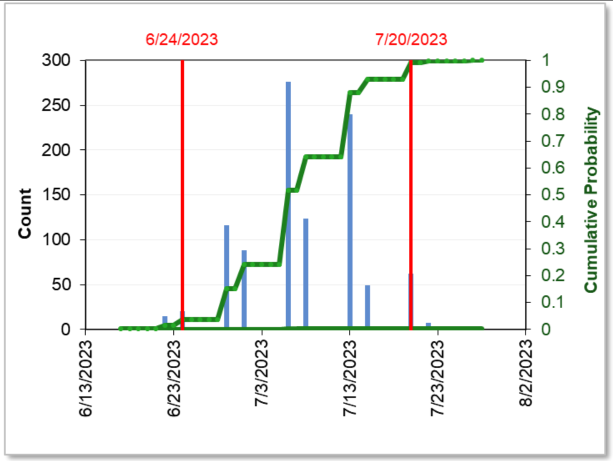


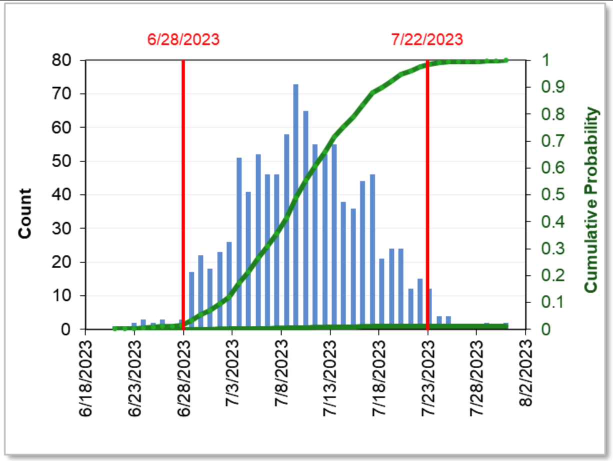


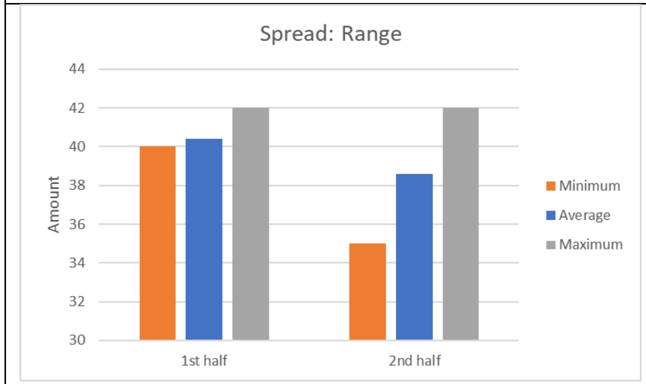


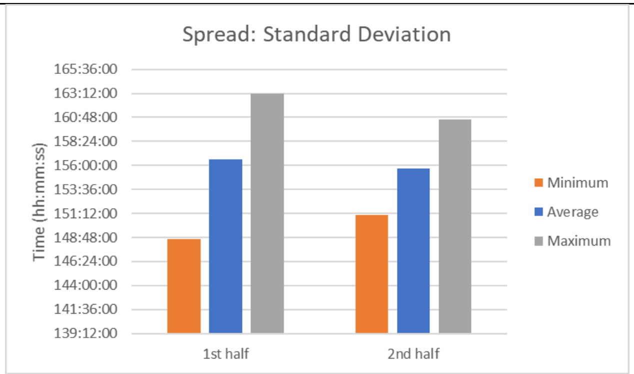


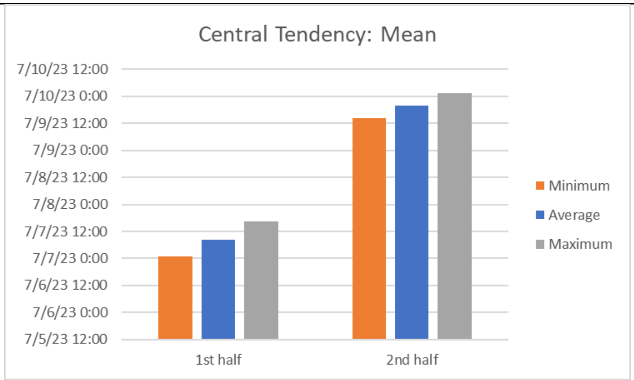
Appendix 4. Variation of uncertainty localization (Upper left: Uncertainty in first half; upper middle: Uncertainty located in second half)

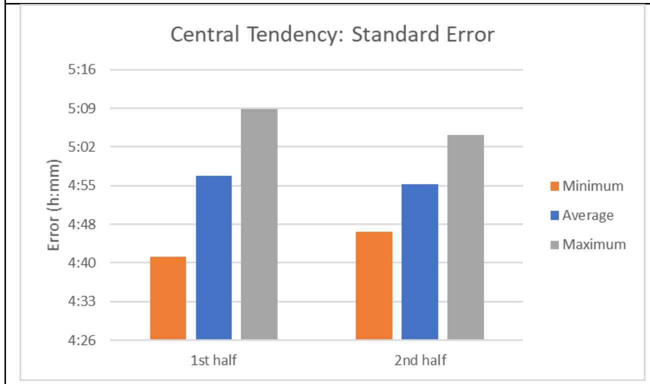


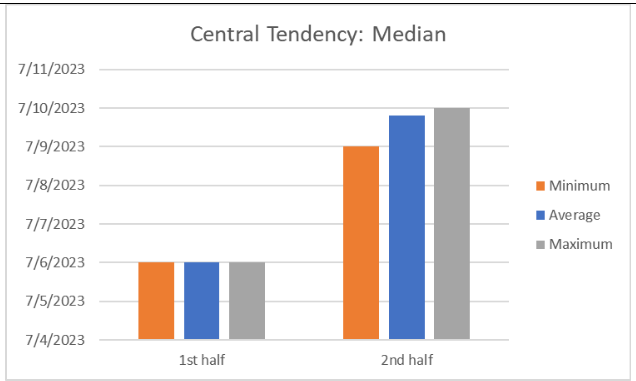


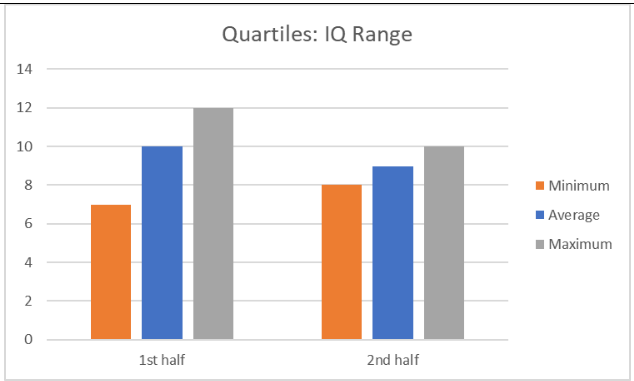




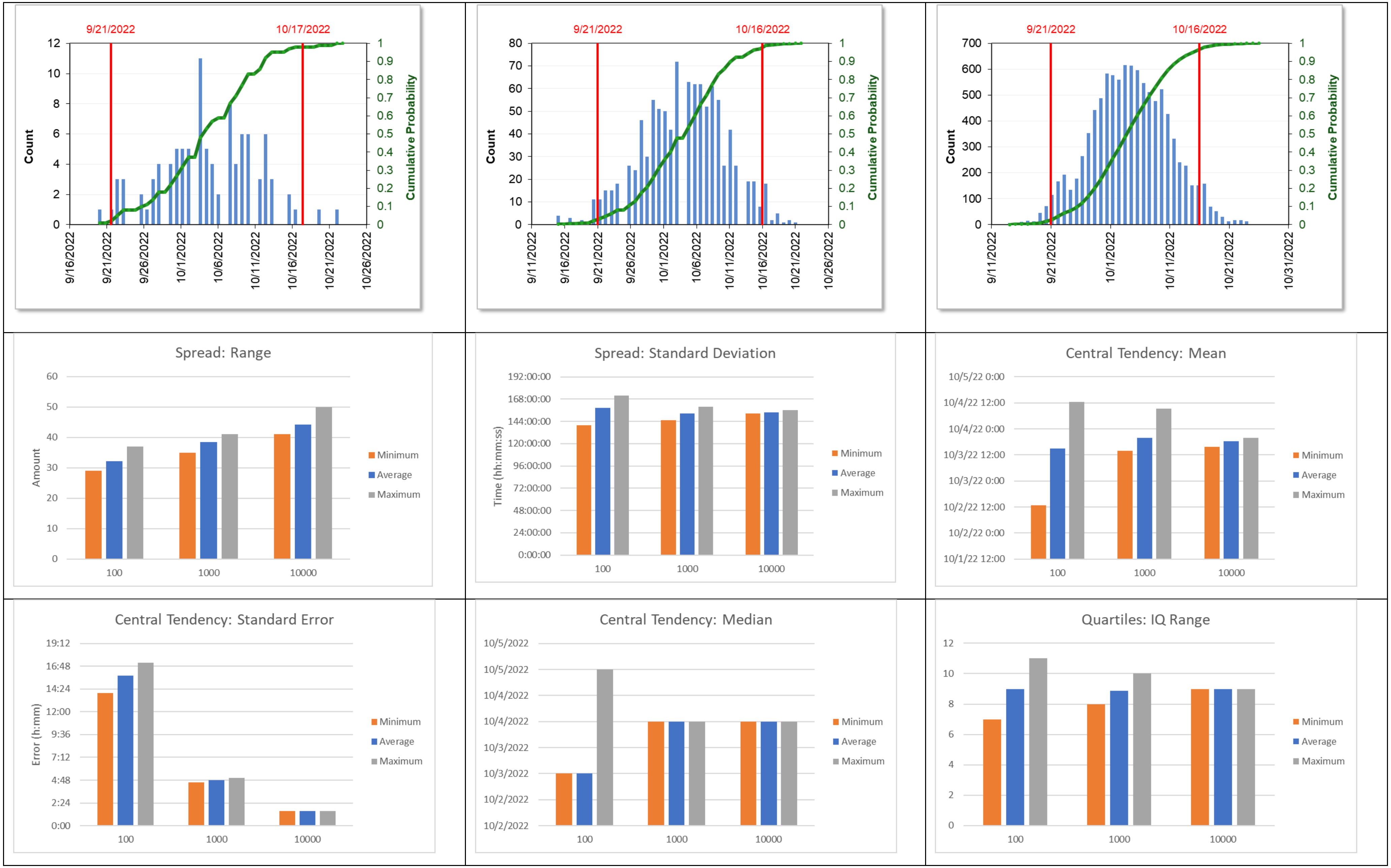








Appendix 5. Variation of simulation accuracy/runs (Upper left: 100, upper middle: 1000, Upper right: 10000)



Appendix 6. Variation of parallel paths (Upper left: 1 path, upper middle: 2 paths, Upper right: 3 paths)