THE LINKAGE BETWEEN MACROECONOMIC VARIABLES AND EXCHANGE RATE FLUCTUATIONS FROM THE EUROZONE PERSPECTIVE: A COMPARISON BETWEEN THE LINEAR REGRESSION AND THE ARMA MODEL

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Examiners: University Lecturer, Ph.D., Roman Stepanov
Associate Professor, Ph.D., Sheraz Ahmed

ABSTRACT

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Keywords: Forecasting exchange rates, purchasing power parity, interest rate parity, forecast accuracy

Exchange rates affect several market participants in the global economy, although predicting them has proven difficult. Regardless of the model, currency pair, or period, no consensus has been reached on the possibility of predicting them.

This thesis aims to investigate the predictability of exchange rates using several machine learning models. The paper examines whether the prevailing differences in interest rates, inflation levels, and income levels between two different currency areas statistically affect the exchange rate fluctuations using time series data consisting of monthly observations from 2009 to 2020. Secondly, the work investigates whether the forecast's accuracy can be improved with a standardized regression model, which includes lambda in the equation, shrinking the coefficients of explanatory variables. Thirdly, the work challenges the traditional regression model with an autoregressive moving average model based on monthly observations from 1975 to 2023.

Based on the results, macroeconomic variables (interest rate differences and differences in inflation and income levels) have no statistically significant impact on exchange rate fluctuations, excluding one minor case. Of the three variables for two different currency pairs, only the interest rate difference between the Eurozone and Japan statistically impacted the exchange rate fluctuation between the euro and the yen. In addition, the prediction accuracy remained high, and the standardized regression improved the accuracy by very little. The autoregressive moving average considerably improved forecast accuracy, although the results were inconsistent and dependent on the currency pair. Forecast accuracy was improved when predicting the euro and the yen exchange rate. Still, the predictability between the euro and the dollar with an autoregressive moving average was not possible.

TIIVISTELMÄ

Lappeenrannan–Lahden teknillinen yliopisto LUT
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Kauppatieteet

Joonas Knuutinen

Makrotalouden indikaattoreiden ja valuuttakurssien kehityksen välinen yhteys euroalueen näkökulmasta: Vertailu lineaarisen regressioanalyysin ja ARMA-mallin välillä

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Valuuttakurssit vaikuttavat globalisoituneessa maailmassa moneen markkinaosapuoleen, joskin niiden ennustaminen on osoittautunut hankalaksi tehtäväksi mallista, valuuttaparista tai ajanjaksosta riippumatta, eikä niiden ennustettavuudesta ylipäätänsä ole päästy konsensukseen.

Tämän työn tarkoituksena on tutkia valuuttakurssien ennustettavuutta usealla koneoppisella mallilla. Työssä tarkastellaan vaikuttavatko kahden eri valuutta-alueen välillä vallitsevat erot korkotasoissa, inflaatiotasoissa tai tulotasoissa tilastollisesti merkitsevästi valuuttakurssien kehitykseen käyttäen aikasarja-aineistoa, joka koostuu kuukausittaisista havainnoista vuodesta 2009 vuoteen 2020. Toiseksi työssä selvitetään voiko ennustetarkkuutta parantaa vakinaistetulla regressiomallilla, joka tuo yhtälöön mukaan lambdan pienentäen selitettävien muuttujien kertoimia. Kolmanneksi työssä haastetaan perinteinen regressiomalli autoregressiivisellä liukuvalla keskiarvolla, joka perustuu kuukausittaisiin havaintoihin vuodesta 1975 vuoteen 2023.

Tulosten perusteella makrotaloudellisilla muuttujilla (korkoeroilla ja eroilla inflaatio- sekä tulotasoissa) ei lähtökohtaisesti ole tilastollisesti merkitsevää vaikutusta valuuttakurssien vaihteluun yhtä poikkeusta lukuun ottamatta. Kolmesta muuttujasta kahdelle eri valuuttaparille ainoastaan euroalueen ja Japanin välinen korkoero vaikutti tilastollisesti merkitsevästi euron ja jenin välisen valuuttakurssin kehitykseen. Lisäksi ennustetarkkuus pysyi suurena eikä vakinaistettu regressio parantanut ennustetarkkuutta kuin nimellisesti. Autoregressiivinen liukuva keskiarvo tarjosi huomattavan parannuksen ennustetarkkuuteen, joskin myös sen kohdalla tulokset olivat ristiriidassa keskenään ja valuuttaparista riippuvaisia. Ennustetarkkuutta saatiin parannettua euron ja jenin valuuttakurssia ennustaessa, mutta euron ja dollarin välinen ennustettavuus autoregressiivisellä liukuvalla keskiarvolla sen sijaan ei ollut mahdollista.

Abbreviations

ARMA	Auto Regressive Moving Average
ECB	European Central Bank
EFFR	Effective Federal Funds Rate
EONIA	Euro Overnight Index Average
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
IFE	International Fisher Effect
IRP	Interest Rate Parity
OLS	Ordinary Least Squares
PPP	Purchasing Power Parity
RMSE	Root Mean Squared Error
TONAR	Tokyo Overnight Average Rate

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1. Introduction

As of 2023, the world economy consists of 180 currencies circulated over 190 countries (Eurochange 2023). Currencies are being priced continuously towards each other, meaning that the price of each currency simultaneously appreciates or depreciates against other currencies as time passes. This causes significant issues and opportunities for all market participants, such as governments, companies, banks, and individual investors.

Currency exchange rate fluctuations are a topic of significant interest in financial research, and many previous studies have tried to explain how the price of currencies fluctuates over time, such as Engel & West (2005), Meese & Rogoff (1983), Rogoff (1996), and Engel et al. (2007). Currencies are always being valued in pairs, and therefore, the macroeconomic environment, and more precisely, the difference between macroeconomic environments in two different currency areas, should determine how the exchange rates fluctuate. For example, Madura (2016), Sarno & Taylor (2002), and James, March & Sarno (2012) cover broadly in their textbooks that the difference in interest rates, inflation levels, and income levels between two currency areas have an impact on exchange rates.

Based solely on textbooks that cover exchange rate determination – interest rates should affect the exchange rates positively while inflation levels and income levels negatively (Madura 2016). The issue with the theory is that the empirical research from the field does not always confirm the relationship. One of the most frequently cited papers on the weak linkage between exchange rates and macroeconomic fundamentals is from 1983 when Meese and Rogoff wrote their paper about *The Exchange Rate Disconnect Puzzle*. The paper has been challenged by several researchers, such as Engel & West (2005), Horioka & Ford (2017), and Engel, Mark & West (2007), who have tried to prove the linkage between exchange rates and macroeconomic fundamentals. For example, Engel, Nelson & Kenneth (2007, 381) mention that researchers have a consensus that monetary variables such as prices and interest rates are not connected. For that reason, many models have been created that aim to explain the connection between macroeconomic indicators and exchange rates.

The research has evolved towards more complex machine learning models such as neural networks, GARCH models, and fuzzy expert systems to resolve the issues surrounding exchange rate movements. Leskovec, Rajaraman & Ullman (2014, 463) explain that machine learning aims to exact information from data and use the information to predict the future. To mention a few papers that have used complex machine learning modeling for exchange rate forecasting, for example, Das, Bisoi & Dash (2018) evaluated whether currency exchange rates can be forecasted with a hybrid forecasting model. On the other hand, Lin et al. (2008) evaluated whether early warnings for currency crises could be predicted with fuzzy expert systems. Biswas et al. (2023) used deep learning models to examine macroeconomic factors influencing the exchange rates.

Various research suggests that currency exchange rate fluctuations can be predicted using several techniques. Forecasting techniques can be divided into four different categories: technical forecasting, fundamental forecasting, market-based forecasting, and mixed forecasting (Madura 2018, 299). Because exchange rates are still important, and consequently, there is still no consensus among experts on whether it is even possible to forecast them, this paper aims to re-evaluate the fundamentals of exchange rate forecasting and challenge the results with a more complex machine learning model. Therefore, this study covers three machine-learning techniques to forecast future currency exchange rate fluctuations, with the ultimate goal of finding a method that minimizes the absolute forecast error. The techniques used in this study range from a simple linear regression analysis and regularized regression to an autoregressive moving average model.

This thesis covers two different currency pairs. The first currency pair is the euro and United States dollar, hereafter EURUSD, and the second is the euro and Japanese yen, hereafter EURJPY. Therefore, this thesis prices euros in two foreign currencies, USD and JPY. Figures 1 and 2 show how the prices of selected currency pairs have fluctuated over time. Figure 1 shows the time series from January 1975 to July 2022 of the spot rate of EURUSD; as seen from the graph, it could be more stable. The euro was at its weakest against the dollar in January 1985, being worth approximately 0,64 dollars, and at its strongest right before the 2008 financial crisis, being worth approximately 1,60 dollars.

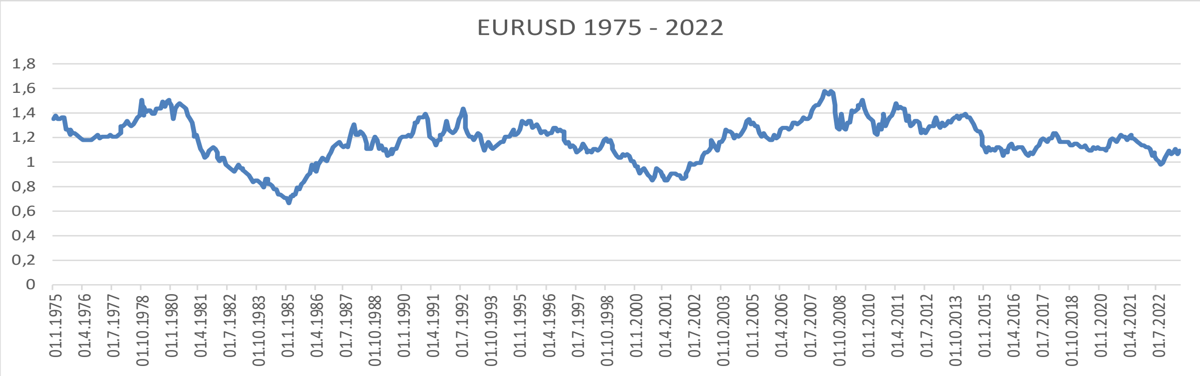

Figure 1. The FX spot rate for EURUSD from January 1975 to July 2022 (Refinitiv Eikon 2023)

Figure 2 shows a similar time series for EURJPY from January 1975 to September 2022. In 1975, the euro was at its strongest, worth approximately 400 yen, and at its weakest right before the 21st century, worth approximately 100 yen.

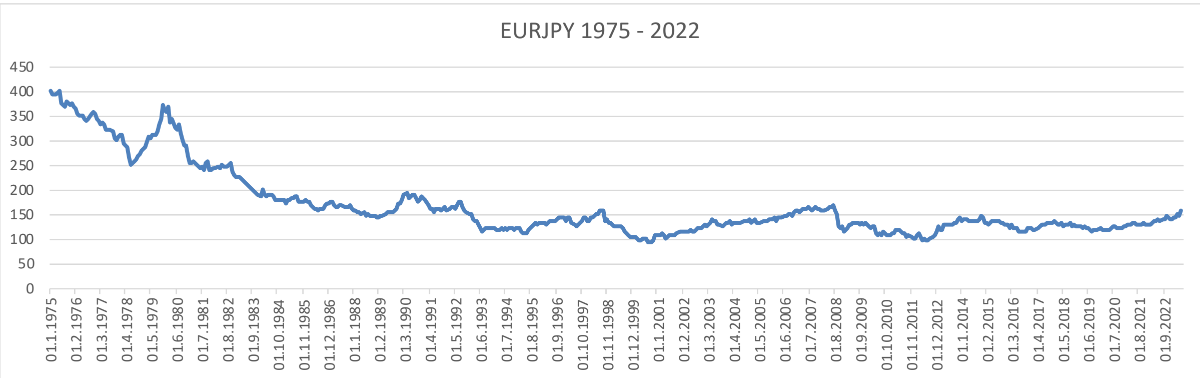

Figure 2. The FX spot rate for EURJPY from January 1975 to September 2022 (Refinitiv Eikon 2023)

1.1. Previous Research

The issue of forecasting exchange rates dates back to 1983 when Meese and Rogoff wrote their paper on the exchange rate disconnect puzzle (Sarno & Schmeling 2014). The exchange rate disconnect puzzle explains the weak linkage between nominal exchange rates and monetary fundamentals (Chou 2018). Bacchetta & van Wincoop (2006) mention in their paper that Rogoff and Meese, in 1983, and the subsequent literature, found the random walk to predict exchange rates more accurately than macroeconomic models in the short run. Sarno & Taylor

(2002, 136) conclude the issue by mentioning that even though the theory of exchange rate determination has produced plenty of reasonable models, it still needs to create a statistically satisfactory model to be reliable and robust both in-sample and out-of-sample.

However, not all research agrees with Meese and Rogoff's paper and the subsequent analysis that denies the connection between macroeconomic indicators and exchange rates. For example, Horioka & Ford (2017) argue that the exchange rate disconnect puzzle arose because the correct fundamentals were not used. Indeed, some other research also supports the possible linkage between macroeconomic fundamentals and exchange rates. Starting from 2005, for example, Engel & West found evidence for the link between fundamentals and the exchange rates in the opposite direction – exchange rates can help forecast the fundamentals. Thus, the discounted sum of expected fundamentals could be incorporated with exchange rates and be used for forecasting. However, they also mention that they do not find much evidence that the observable fundamentals purely explain the exchange rates.

Later, in 2007, Engel, Mark & West found that the PPP model significantly forecasts exchange rates more accurately than the random walk model, thus implying evidence that the connection might exist after all. In 2015, Bahmani-Oskooee, Hosny & Krishor used the same data as Engel & West in 2005. With a different approach, they found the long-run relationship between exchange rates and fundamentals. Similar results were achieved by Tawadros (2017) when revisiting the issue of the disconnect puzzle. He also states that there is a relationship between exchange rates and fundamentals in the long run. Successful studies that have proved the long-run relationship have used mainly the approach presented, for example, by Johansen in 1991 of cointegration vectors in Gaussian vector autoregressive models.

Additionally, several papers examine exchange rate fluctuations with complex machine-learning models. For example, Abdulcileem, Abdulqudus, and Abdulqadir (2021) discussed in their article the forecasting of the US dollar and the British pound and tried to find the best ARMA model for creating predictions of future spot rates. They found that the average forecast error in the ARMA model was for any dataset below 0,2%. They also found that small ARMA models, such as the ARMA (2,3) model, were generally the most accurate (depending on the case). In other words, the value from two periods ago and the moving average of order of 3 achieve reliable results.

Similar results were achieved by Bis et al. (2023), who studied how the exchange rate of the US dollar and Bangladeshi taka could be forecasted with a deep-learning model. They achieved the RMSEs of 0,1984 and 0,2059, depending on the approach. However, not all research has been as promising. For example, Laurent, Lecourt, and Palm (2016) studied whether Gaussian ARMA-GARCH models could predict the jumps in exchange rates. They could detect fewer than 1% of the jumps for the three exchange rates, although they clarified that they included short-memory models. Mammadova (2010) concludes that, excluding some minor cases, ARMA models do not outperform the random walk.

1.2. Research Questions and Restrictions

The topic of this thesis is divided into three different questions. The first research question tries to answer the fundamental question of whether the differences in interest rates, inflation levels, and income levels between two currency areas have a statistically significant impact on exchange rates:

Question 1. Does the linkage between exchange rates and differentials in interest, inflation, and income levels exist?

Research is then extended to evaluate whether a regularized regression model could be more accurate compared to the classical linear regression model and if technical analysis could compete with fundamental forecasting:

Question 2. Can quantitative forecasting reduce the forecast error compared to fundamental forecasting?

Lastly, as this paper covers two different currency pairs, all the different methods are applied to both datasets and, thus, the third research question focuses on whether the results are consistent for different currency pairs:

Question 3. Are the results consistent for different currency pairs?

To explore the three research questions of this study, some restrictions need to be presented. This thesis aims to find a method to forecast currency exchange rates that float, in other words, exchange rates that are determined freely in the market. Therefore, other exchange rate systems, such as dirty float or fixed exchange rate regimes, are outside the scope of this thesis. The countries and the Eurozone as a currency area selected for this thesis are developed countries. Therefore, the results from the research in this study might apply only to developed currencies. Currencies from developing countries are, therefore, outside this thesis's scope.

The reason for selecting major currencies (EURUSD and EURJPY) is that they are the most traded currencies in the world. For example, the Bank for International Settlements (2022) shows in their statistics that euros, dollars, and yen are the three most traded currencies on the FX market. The high volume in the FX market reduces the risk of price manipulation. On the other hand, the US and Japan will offer two perspectives regarding the macroeconomic environment, as figures 4-8 will show. Japan has much lower interest rates and inflation than the Eurozone or the US. Also, the availability of macroeconomic data is better in these developed countries than in developing countries, which makes fundamental and quantitative forecasting easier.

As stated in this introduction section, this thesis evaluates the differentials in interest rates, inflation, and income levels. However, as government controls and expectations are two critical components of exchange rates, it is reasonable to explain why they are not included in this thesis. Governments can, in general, influence the equilibrium exchange rate by, for example, imposing trade barriers, intervening in the foreign exchange markets, and affecting macro variables like inflation, interest rates, and income levels. The speculating trading activity causes the effect on exchange rates from future expectations. (Madura 2016, 112) As these factors are not easy to empirically interpret due to the availability of measurable data, they are outside the scope of this research. However, it is notable that they might affect the variables examined in this paper and, thus, the achieved results.

1.3. Paper Structure

This study is divided into six different sections. The introduction section has now explained the reasons behind the entire research and introduced the most relevant previous studies from the field. After introducing prior research, the research questions that will be evaluated later in this paper were presented and explained, yet briefly introducing some restrictions that should be kept in mind. Next, following the introduction section, a complete theoretical framework will be introduced, focusing on the different forecasting techniques, fundamental factors of exchange rates, and the three theories associated with exchange rates. The theoretical framework is then extended to cover ordinary least squares (OLS) and autoregressive moving averages (ARMA). After the theoretical framework, section four will focus on the data and how this paper's actual research is conducted, briefly introducing the results from different models. Finally, a deep dive into the results will occur in chapter five, comparing the findings with previous research and evaluating the possible differences. Section six will conclude the findings from this thesis.

2. Literature Review and Theories of Exchange Rates

This section covers the theories of exchange rates and different forecasting techniques. The theoretical background is introduced by following the textbooks of Madura (2016), Sarno & Taylor (2002), and James, Marsh & Sarno (2012). In addition to books, this section also shows the literature review and compares the findings to those mentioned in common textbooks. The section is divided into three parts, and the first covers the market equilibrium in general terms and how it can be modeled.

The second part focuses on the different theories that affect the demand and supply of currencies and change the market equilibrium. Interest rate parity (IRP) aims to explain arbitrage opportunities related to interest rate differentials and exchange rates. It links the two variables, whereas purchasing power parity (PPP) introduces the relationship between inflation rate differentials and exchange rates. Lastly, the international fisher effect (IFE) connects inflation and interest rates more deeply.

Finally, in the third part of this theory section, the theoretical background of FX forecasting is introduced, starting from fundamental forecasting and continuing to quantitative forecasting and briefly explaining market-based and mixed forecasting. However, they are outside the scope of this thesis.

2.1. Market Equilibrium

To be able to explain forecasting techniques and underlying theories, it is reasonable first to explain the market equilibrium that should be the prevalent price of each currency. Several models have developed surrounding exchange rates, and it is impossible to cover them all in this paper. However, to mention a few essential and cited models, James, Marsh & Sarno (2012, 46) mention the Mundell-Fleming model, which focuses on capital account and current account flows. Moreover, they also mention a more recent model, the monetary model, which has been the most common approach in the exchange rate literature. James, Marsh & Sarno (2012, 46) explain that the monetary model views exchange rates as the relative price of currencies, where the relative price depends on the relative demand and supply of money. Rapach & Wohar

(2002) mention in their paper that the monetary model of exchange rate determination draws a strong link between the nominal exchange rate and certain monetary fundamentals. Therefore, it is also the primary approach in this thesis.

Market equilibrium is visually modeled through supply and demand curves. The equilibrium exchange rate is the spot price where the supply and demand are equal for a particular currency (Madura 2016, 106-107). Figure 3 shows that the demand curve (D) is downward-sloping, meaning that the demand (x-axis) for currency is higher when the price (y-axis) is lower. Consequently, the supply curve (S) is upward-sloping, meaning that the supply is higher when the currency's price is higher. Market equilibrium is at the intersection of these two slopes and represents the price of currency where the supply and demand are equal.

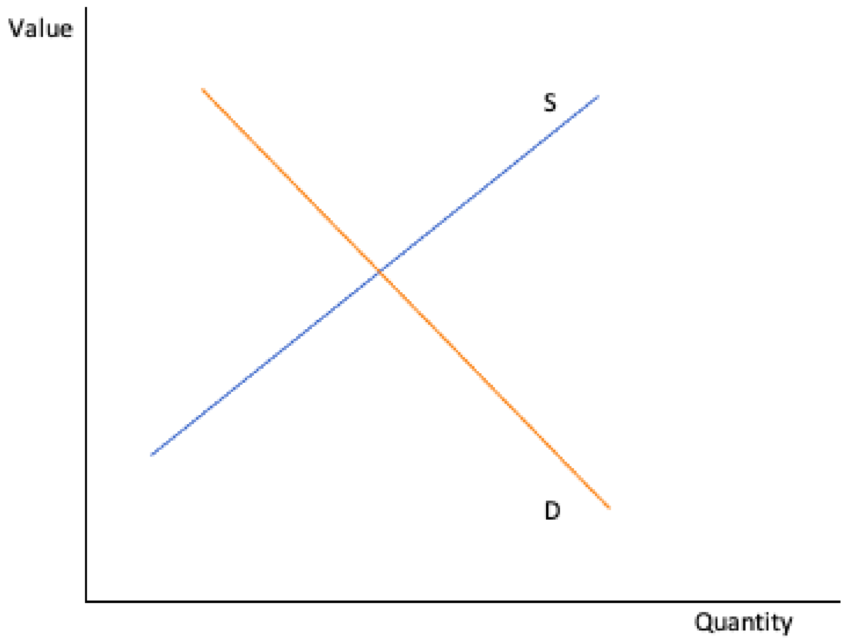

Figure 3. Market equilibrium, following Madura (2016, 107)

Currencies can be bought through financial intermediaries that have reserves of different currencies. The market equilibrium fluctuates as financial intermediaries might have sudden shortages or excesses of currencies, and they, therefore, need to adjust the price to balance their reserves (Madura 2016, 107). According to Madura (2016, 107-108), it can happen in four ways. As this thesis focuses on two different currency pairs, EURJPY is an example in this section.

The first scenario where market equilibrium might change is when the demand for foreign currency increases. If the demand for the Japanese yen increases in the Eurozone and the supply of the Japanese yen remains the same, it leads to a shortage of yen at the prevailing price. Therefore, as intermediaries do not have enough yen to sell, they increase the price, limiting the demand and increasing the supply. Intermediaries will increase the exchange rate to the level where the number of yen demanded equals the number of yen supplied.

The second scenario when market equilibrium might change is when the demand for a foreign currency decreases. If, for example, the Japanese yen's supply is constant even though it is demanded less in the Eurozone, it leads to a situation where intermediaries have excess yen. Therefore, intermediaries will decrease the price of a yen so that the demand increases and supply decreases and will keep doing so until the supply and demand are equal.

The first two scenarios covered the change in the demand for a foreign currency, but the changes might also occur towards the supply of a foreign currency. Suppose that the demand for euros increases in Japan and people start to sell more yen for euros. This leads to a situation where there is an oversupply of yen at the prevailing price, and intermediaries have surpluses of yen. Therefore, intermediaries start to decrease the price of yen, increasing the demand in the foreign exchange market. Intermediaries will again keep doing so until the demand and supply are equal.

Similarly, the supply of yen can also decrease. Suppose that the demand for euros decreases in Japan and, therefore, the supply of yen decreases, leading to a situation where intermediaries have shortages of yen. Intermediaries need to adjust the price higher so that the supply increases in the foreign exchange market and the price reaches market equilibrium.

2.2. Theories Explaining the Changes in Market Equilibrium

Madura (2016, 302) suggests that currency exchange rate fluctuations should be mainly caused by five leading factors that affect the supply and demand curves, as explained in the previous subsection. Percentage change in the currency's spot price is impacted by the change in the differential between the home country's inflation and the foreign country's inflation (ΔINF), by the change in the differential between the home country's interest rate and the foreign country's interest rate (ΔINT), by the change in the differential between the home country's income levels and the foreign country's income levels (ΔINC), by the change in government controls (ΔGC) and by the shift in expectations of future exchange rates (ΔEXP).

The relationship between currency exchange rate fluctuations and economic indicators can, therefore, be shown as a function of:

$$e = f\,(\Delta INF, \Delta INT, \Delta INC, \Delta GC, \Delta EXP). \quad (1)$$

As the previous sub-section explained how the market equilibrium changes through the increased or decreased demand and supply, the following three subsections further cover the reasons behind the changes in demand and supply and connect the existing theories to the variables of interest rates, inflation levels, and income levels. To clarify the discussion behind supply and demand, it is reasonable to introduce the theories of IRP, PPP, and IFE.

2.2.1. Interest Rate Parity (IRP)

Interest rates affect the allocation of foreign money as it flows towards higher interest rates– aiming to earn the maximum profit. For example, Madura (2016, 110) explains that if the interest rates are higher, for instance, in the USA than in the UK, the demand for pounds will decrease in the USA. On the other hand, the supply of pounds will increase as British investors change currencies and allocate money towards higher interest rates, thus causing the dollar to appreciate against the pound.

Therefore, a positive relationship exists between interest and exchange rates, ceteris paribus. However, for example, James, March & Sarno (2012, 48) mention in their text that the relationship between interest rates and exchange rates is sometimes more complex, and research shows varying results.

The theory of interest rates is closely related to interest rate parity (IRP). IRP is discussed in two different forms, in uncovered and covered versions. To further discuss the covered interest rate parity (the version of the theory used in this study), the intuition behind uncovered interest rate parity (UIP) must be discussed briefly. Sarno & Taylor (2002, 11) note that when the riskneutral efficient market hypothesis holds, the gain from the expected exchange rate must be enough to offset the interest rate differential between the same currencies.

Therefore, mathematically, the condition can be expressed as (Sarno & Taylor 2002, 5):

$$\Delta kS^{e}{}_{t+k} = i_t + i*_t \qquad (2)$$

where s_t = logarithm of the spot exchange rate (domestic price of foreign currency)
i_t = the nominal interest rates in domestic currency
$i*_t$ = the nominal interest rates in foreign currency

However, usually, the discussion appears to be around the covered interest rate parity, which is related to forward agreements. Madura (2016, 236) explains that covered interest rate parity (CIP) is the equilibrium state where the covered interest rate arbitrage is no longer feasible. Covered interest rate arbitrage refers to a trading strategy where one capitalizes the difference in interest rates between two countries while covering the exchange rate risk with a forward contract (Madura 2016, 233). At the equilibrium (where the arbitrage is no longer feasible), the forward rate differs from the spot rate to offset the interest rate differential between the two currencies (Madura 2008, 237).

Therefore, the equation to illustrate CIP algebraically by following Sarno & Taylor (2002) is:

$$\frac{F^{(k)}{}_t}{S_t} = \frac{1+i_t}{1+i_t} \qquad (3)$$

where s_t = the spot exchange rate of domestic currency
$F^{(k)}{}_t$ = the k-period forward rate
i_t = the nominal interest rates in domestic currency
$i*_t$ = the nominal interest rates in foreign currency

Therefore, in the absence of risk, currencies should appreciate in case they have lower interest rates. For example, Bonga-Bonga (2009) mentions in her paper how forward rate should be an unbiased estimator of future spot rate but clarifies that several studies have doubted its validity. Also, Božović (2021) discusses similar findings of how low interest rates should lead to currency appreciation. However, he also mentions that empirical study has usually rejected this hypothesis.

In their text, James, Marsh & Sarno (2012, 283) discuss that high-interest rate currencies are more likely to appreciate than depreciate against low-interest rate currencies. The research around IRP concludes that disconnection is usually the “forward rate puzzle” or the “Fama puzzle”. Therefore, for further study, this paper follows the null hypothesis that interest and exchange rates have a positive correlation.

2.2.2. Purchasing Power Parity (PPP)

Similarly, like interest rates, inflation rates also affect international trade activity. Madura (2016, 109) explains that if the inflation rate in the USA increases while staying constant in the UK, consumers in the USA should start to buy more British goods, causing the demand for pounds to increase. Additionally, Madura (2016, 109) explains that at the same time, the price hike in the US should decrease their demand for dollars, causing the supply of pounds to decrease. Therefore, the equilibrium market price needs to shift downwards, and the dollar depreciates against the pound, which means a negative relationship exists between inflation rates and exchange rates, ceteris paribus.

Income levels have a similar effect on exchange rates as inflation rates, which come through the demand for imports. Madura (2016, 111) explains that if income levels increase in the US, for example, and remain unchanged in the UK, the demand for the pounds is expected to increase due to increased demand for imports in the US. At the same time, the supply of the pounds should remain the same, and therefore, the dollar depreciates against the pound. Therefore, a negative relationship exists between income levels and exchange rates, ceteris paribus.

One of the most influential theories related to exchange rates is purchasing power parity (PPP). Technically, PPP exists in two forms–absolute and relative. The idea behind PPP is quantifying the relationship between inflation and the exchange rates. Absolute form PPP assumes the absence of international barriers, and consumers will channel their demand towards the currency area where prices are the lowest. As the assumption of no international barriers is unrealistic, relative form PPP also considers market imperfections such as transportation costs, tariffs, and quotas. (Madura 2016, 257-258)

For example, Mark, Taylor & Peel (2001) explain the rationale behind PPP to be that a consumer should be indifferent on whether to buy products locally or from another country as purchasing power should be the same. Madura (2016, 257) further explains that if products are cheaper in one country, consumers will shift their demand towards that country, increasing the demand for the foreign currency and supply of the home currency, causing the currencies to appreciate and depreciate.

James, March & Sarno (2012, 190) introduce the most straightforward statement of PPP that can be expressed as:

$$s_t \, {p^*}_t = p_t \tag{4}$$

where s_t = exchange rate
p^*_t = foreign price
p_t = domestic price

However, James, March & Sarno (2012, 55) clarify that researchers have no consensus on whether PPP holds. In general, the disconnection is usually referred to as the PPP puzzle. For example, Mark, Taylor & Peel (2001) discuss in their paper that several studies have found deviations from PPP, and reasons seem to thrive from nonstationary and, more particularly, from unit root behavior for exchange rates, implying no clear long-run connection to PPP. For example, Rogoff (1996) studied the same issue broadly in his paper and concluded that there is a large buffer in which nominal exchange rates may move without an immediate effect in response to relative domestic prices. He mentions that this is most likely due to the segmented international goods market where tariffs, transportation costs, and lack of labor mobility pose barriers.

PPP is closely related to the monetary model introduced at the beginning of subsection 2.1. The monetary model assumes PPP to hold either continuously (flexible prices version) or in the long-run (sticky prices version), and when imposing PPP to the monetary model, James, Marsh & Sarno (2012, 47) model the relationship as:

$$s_t = (m_t - m*_t) - \varphi(y_t - y*_t) + \lambda(i_t - i*_t) \qquad (5)$$

where m = log nominal money stock
y = log income
i = short-term interest rate

The monetary model yields two outcomes – relatively higher income induces stronger currency, and relatively higher interest rates induce weaker currency. The issue of interest rates was already evaluated in the last subsection. Still, suggesting a positive relationship between income levels and exchange rates contradicts the previous conclusion. The previous assumption was that income levels and exchange rates are negatively correlated. Thus, relatively higher income levels lead to the depreciation of the home currency. James, Marsh & Sarno (2012, 47) mention in their book that the suggestion posed by the monetary model is the opposite of the Mundell-Fleming model, where higher income leads to higher imports. In the monetary model, higher income leads to higher money demand relative to the supply.

Therefore, despite all the skepticism around the subject, this paper takes the null hypothesis that income and inflation levels are negatively correlated with exchange rates.

2.2.3. International Fisher Effect (IFE)

The third and final theory of this paper combines IRP and PPP. As it is commonly known, interest rates and inflation levels are correlated. Usually, high inflation leads to high interest rates as central banks try to adapt their monetary policy. The International Fisher Effect (IFE) aims to take a more comprehensive approach and include the correlation between interest rates and inflation levels when interpreting exchange rates.

Madura (2016, 266) explains that IFE suggests that the nominal interest rate in one country can be used to derive its expected inflation level and how the differential in inflation levels can predict the exchange rate movements. The core idea of Irving Fisher's theory is that the minimum return saver demand from local savings institutions is the rate that is higher than the expected inflation rate. Therefore, assuming the real interest rate (interest rate deducted by inflation rate) to be constant over time, any change in expected inflation should lead to a change in the nominal interest rates as saving institutions need to offer higher yields to attract savers.

Madura (2016, 267) further explains that modifying the equation of

$$\textit{real interest rate} = \textit{nominal interest rate} - \textit{expected inflation rate} \qquad (6)$$

will turn it into

$$\textit{expected inflation rate} = \textit{nominal interest rate} - \textit{real interest rate} \qquad (7)$$

and further taking into consideration the differential in expected inflation between countries A and B, the final form is introduced as

$$\textit{expected inflation differential} = (i_A - Real_A) - (i_B - Real_B). \qquad (8)$$

Assuming the real interest rate required by the savers to be the same for all countries, the expected inflation differential is reduced to

$$\textit{expected inflation differential} = i_A - i_B. \qquad (9)$$

Madura (2016, 267) says the final formula is extremely powerful as the differential between expected inflation rates can now be calculated from the difference between their nominal interest rates. Once the expected inflation rates are derived from the nominal interest rates, PPP can be applied to illustrate the future movements in exchange rates between these two countries (Madura 2016, 267).

Like IRP and PPP, IFE is a common subject in economic research, and results vary. For example, Koráb & Svatopluk (201) mention in their paper that the research around IFE is debatable, and no clear results can be discerned. They wrote their paper on countries that decided to shift from fixed arrangements to floating exchange rate regimes and achieved differing results for different countries. They found that IFE was mainly for a subgroup of countries with a rigid exchange rate policy. Anokye & Ofori (2021) further studied the IFE in the West African Monetary Zone, and they found a long-run relationship between the relative changes in exchange rate and nominal interest rate differentials for fifteen out of twenty countries. However, they clarify that the assumptions for validation were met only for two countries at a 5% significance level. Joksimović & Joksimović (2020) studied the IFE in the European Union with linear regression. Their paper concludes that even though they found a connection, the R2 of coefficients was only at 3,3%.

2.3. Forecasting Techniques

So far, the theory section has covered different theories related to exchange rates and introduced the previous research on the subject. Therefore, the third and last section of the theory chapter briefly introduces the different forecasting techniques. According to Madura (2016, 299 – 307), exchange rate movements can be predicted in several ways. The four methods are technical forecasting, fundamental forecasting, market-based forecasting, and mixed forecasting.

Technical forecasting uses historical data to predict future exchange rate fluctuations. Madura (2016, 299) explains it relies on the assumption that there may be a trend in the exchange rate adjustment that could lead to a continuation of the specific trend. The methodology of technical forecasting conducted in this thesis will be presented in sub-section 4.3 when the ARMA model is introduced. Madura (2016, 299) further mentions that technical analysis is usually done only for a very short period and is unreliable in the long run.

For example, Panopoulou & Souropanis (2019) mention in their paper that even though many technical indicators are older than the most prominent macroeconomic models, research has paid less attention to them. Hsu, Taylor & Wang (2016) studied technical trading rules in developed countries and emerging markets. They found out that technical analysis has predictive power in both concepts, and quite interestingly, the predicting power was more accurate in emerging market currencies. Zarrabi, Snaith & Coakley (2017) applied the universe of 7650 trading rules to six different currencies, and they found for their long-term in-sample the predictive ability of up to 75 %.

The second forecasting technique of this thesis is fundamental forecasting. Madura (2016, 300) explains it based on the fundamental relationships between economic variables (such as interest, inflation, and income) and exchange rates. Madura (2016, 301) further explains that fundamental forecasting may account for the lagged impact, meaning that the changes in variables in the previous period affect the exchange rate movement in the current period. However, Madura (2016, 302) mentions that the regression model's flexibility is limited as explanatory variables' lagged impact may not be limited to a -1 lag, and the regression model needs to be adjusted accordingly. The methodology of fundamental forecasting will be presented in subsections 3.1 and 4.2 when introducing the linear regression.

Gramacy, Malone & Horst (2014) mention several papers that have tried to prove the predicting power of linear models, non-linear models, non-parametric kernel regressions, and Markov switching models to challenge Meese & Rogoff (1983). The most promising models from past research are the generalized autoregressive conditional heteroskedasticity (GARCH) models (Gramacy, Malone & Horst, 2014). The issue of small predicting power in fundamental forecasting is not likely to arise from the lack of information in the fundamentals but rather from the poor model selection (Sarno & Valente, 2009).

The last two forecasting techniques are market-based forecasting and mixed forecasting. The second one (mixed forecasting) combines several forecasting techniques, and it is usually the most frequently used as no single forecasting method outperforms the others (Madura 2016, 306). Madura (2016, 303) mentions the first method to generate forecasts from market indicators which is usually based either on the spot or the forward rate. Market-based forecasting does not have much earlier research; for example, Moosa (2004) has studied the matter in her paper. She mentions that market-based forecasting is based on two hypotheses that are the simple random walk hypothesis (changes in the currency spot rate are random and unpredictable) and the unbiased efficiency hypothesis (the forward rate is an unbiased estimator of the currency spot rate at the time of the maturity of the forward contract). She also mentions that neither of these hypotheses is supported by empirical evidence and concludes in the findings that the forward rate is a faulty forecaster.

3. Theoretical Framework for OLS and ARMA

This section briefly describes the theoretical background for ordinary least squares and autoregressive moving averages. The results for both analyses are presented in chapter 4.

3.1. Theory for Ordinary Least Squares (OLS)

The regression analysis of this research is conducted as ordinary least squares (OLS). Hill, Griffiths, and Lim (2011, 51) explain that OLS aims to minimize the sum of squared errors of estimated and actualized y-values. Brooks (2008, 32) denotes the estimated value as $\hat{y}_t$ for the given value of x of observation t, whereas the actualized value is y_t. Therefore, he explains $\hat{u}$ to describe the error term (residual), which can be denoted as ($\hat{y}_t$ - y_t). Following the further approach of Brooks (2008, 33), $\hat{y}_t$ - y_t is squared to avoid the positive and negative values canceling each other out. Brooks (2008, 33) explains that minimizing the sum of squared distances (as explained above) is the same as minimizing the residual sum of squares (RSS). OLS will then compute the best-fitted line where the RSS is at its smallest and determine each variable's coefficient (Brooks 2008, 33).

Coefficients can then be used to predict the response in the dependent variable when observations for independent variables are known. However, the estimated y-value is usually not the same as the actualized y-value, thus constituting the residual as explained above. After taking the mean of the residual sum of squares (RSS) and squaring it, the result describes the root mean squared error (Christie & Neill 2022). Christie and Neill (2022) further explain RMSE as a good measure of accuracy in comparing forecasting errors of different models. Also, Brooks (2008, 257) mentions RMSE as a useful method to forecast errors. Therefore, as this thesis aims to find the most exact model to predict exchange rates, it is reasonable to use RMSE as an accuracy indicator for each model.

In this research, regression is done twice for two dependent variables (y): the change in the FX spot rate for EURUSD and EURJPY. Independent variables for both dependent variables are the difference in the Eurozone’s interest rates, inflation levels, and income levels compared to foreign countries’ (the US and Japan) interest rates, inflation levels, and income levels.

Regression for the currency pair EURUSD is modeled as follows:

$$e_{\text{EURUSD},t} = b_0 + b_1\,\Delta INF_{t-1} + b_2\,\Delta \text{IR}_{t-1} + b_3\,\Delta INC_{t-1} + \varepsilon_t, \qquad (10)$$

where
$\Delta INF_{t-1} = INF_{\text{EUR},t-1} - INF_{\text{USD},t-1}$
$\Delta I\text{R}_{t-1} = \text{IR}_{\text{EUR},t-1} - \text{IR}_{\text{USD},t-1}$
$\Delta INF_{t-1} = I\text{NC}_{\text{EUR},t-1} - IN\text{C}_{\text{USD},t-1}$
ε_t = residual

Regression for the currency pair EURJPY is modeled as follows:

$$e_{\text{EURJPY},t} = b_0 + b_1\,\Delta INF_{t-1} + b_2\,\Delta \text{IR}_{t-1} + b_3\,\Delta INC_{t-1} + \varepsilon_t, \qquad (11)$$

where
$\Delta INF_{t-1} = INF_{\text{EUR},t-1} - INF_{\text{JPY},t-1}$
$\Delta I\text{R}_{t-1} = \text{IR}_{\text{EUR},t-1} - \text{IR}_{\text{JPY},t-1}$
$\Delta INF_{t-1} = I\text{NC}_{\text{EUR},t-1} - IN\text{C}_{\text{JPY},t-1}$
ε_t = residual

Therefore, for every three independent variables in two different regressions, the statistical software calculates the squared distance for each of approximately 140 observations and finds the best-fitted line where RSS is at its smallest. Following this procedure, section 4.2. introduces further the results from EURJPY and EURUSD regressions.

3.2. Theory for Autoregressive Moving Average (ARMA)

In univariate time series models, the future values try to be predicted with information contained only in their past values and possibly with current and past values of the error term (Brooks 2008, 206). ARMA(p, q) is a combination of AR(p) and MA(q) processes. Therefore, the y-value in the ARMA model is dependent on its previous values in addition to a white noise error term.

AR(p) process refers to the autoregressive process of order p where the current value of y depends on the previous values of y plus an error term. AR process constitutes the stationarity requirement in the ARMA model as non-stationarity will cause the previous values of the error terms to have a non-declining effect on the current value of y as time progresses. (Brooks 2008, 215-216 & 223-224) In general terms, stationarity is a stochastic process that is not dependable on time (Hill, Griffiths & Lim 2011, 475-482).

The second part of the ARMA model is from the MA(q) process, which is the moving average of order q. MA process is a linear combination of white noise processes (sequence of random, unpredictable numbers), meaning that y is dependent on the current and previous values of white noise disturbance. Therefore, the MA process requires the white noise process to have constant mean and variance and autocovariance of zero except at lag zero to be used reliably in the ARMA model. (Brooks 2008, 207-215) Notably, the MA process discussed here differs from the commonly known moving average for measuring technical analysis, such as stock movements.

ARMA model can be investigated graphically from correlograms and white noise. Correlograms can be used to roughly overview the order of the ARMA model and white noise to evaluate if white noise has a constant mean and variance. Brooks (2008, 225) explains that an AR process should have a geometrically decaying autocorrelation function (ACF), and the number of non-zero points is the order of partial autocorrelation function (PACF) and thus, the order of the AR process. PACF measures the correlation between a current observation and an observation in the past (Brooks 2008, 222). The MA process should have a geometrically decaying PACF, and the number of spikes represents the order of the ACF and, thus, the order of the MA process. Furthermore, the ARMA model combination should have a geometrically decaying PACF and ACF.

Although correlograms offer a rough estimate of the order in the ARMA model, there are more accurate ways to describe them. Brooks (2008) explains in his book that the ARMA model is usually associated with the Box-Jenkins methodology, which Box and Jenkins pioneered in 1976 and were the first to estimate an ARMA model systematically. The approach involves three steps: identification, estimation, and diagnostics checking. The steps in detail are determining the model's order, estimating the model's parameters, and checking whether the model is adequately specified and assessed.

Selecting the order of the model is done via information criteria. Brooks (2008, 232-233) explains that the information criteria consist of two factors: the residual sum of squares and the penalty term for losing degrees of freedom from adding extra parameters. The goal is to minimize the information criteria, meaning that the residual sum of squares is minimized to the point where the fall in the residual sum of squares is no longer more significant than the increased value of the penalty term (Brooks 2008, 233).

The estimation of parameters can be done, for example, by using least squares or maximum likelihood (Brooks 2008, 231). Model checking, on the other hand, can be done with, for example, the Ljung-Box test (Brooks 2008, 231). In the contest of Box-Jenkins, it measures autocorrelation, and any P-value above the chosen threshold (here 0,05) results in not rejecting the null hypothesis of no autocorrelation (Brooks 2008, 231 & 235). Following the graphical identification and Box-Jenkins methodology, sub-chapter 4.3 later shows results from the ARMA model used in this paper.

4. Data and Methodology

This section covers the research of this thesis. Section 4 can be divided into several subsections where the first describes the dataset comprehensively and evaluates possible issues that might affect the results. The second subsection introduces the regression results, and the third introduces results from the ARMA model. The research of this study has been done via two different statistical software. Classical linear regression among its assumption testing was done in StataSE 16, and Ridge regression and the ARMA model were done in MATLAB R2021a.

4.1. Dataset

The dataset is divided into two different data tables. The first data table has one dependent variable, the change in the FX spot price for EURUSD, and three independent variables: the difference between the Eurozone's and the US interest rate, inflation, and income levels. The second data table includes the same information. Still, it has EURJPY as a currency pair in the dependent variable, and the independent variables are the difference between the Eurozone's and Japan's interest rate levels, inflation levels, and income levels. Both datasets include approximately 140 monthly observations for each variable, and the time interval is from January 2009 to December 2020.

All the independent variables have a t-1 lag compared to the dependent variables as, according to the theory, their changes in the previous period should affect the dependent variable in the next period.

Both exchange rates are exported from the Refinitiv Eikon database and represent the average ask price of EURUSD and EURJPY on the last date of the month. Monthly changes in the currency pairs are then calculated as $EURUSD_t$ / $EURUSD_{t-1}$ – 1 and $EURJPY_t$ / $EURJPY_{t-1}$ – 1.

Interest Rates

To further evaluate interest rates, it is essential first to find appropriate benchmark interest rates that would describe the general interest rate level in the specific area as accurately as possible. This paper's benchmark interest rate for the Eurozone is the Euro Overnight Index Average (EONIA), the predecessor of the Euro Short Term Rate (ESTER). For the US, the benchmark interest rate is the Effective Federal Funds Rate (EFFR), and for Japan, the benchmark interest rate is the Tokyo Overnight Average Rate (TONA). Interest rates are also imported from the Refinitiv Eikon database and converted into monthly changes in the same way as currency pairs.

European Central Bank (ECB) has three key interest rates: main refinancing, marginal lending, and deposit rates. The main refinancing operations (MRO) provide liquidity to the banking system. At the same time, the deposit facility shows at which rate banks may place overnight deposits, and the marginal lending facility shows at which rate banks can apply for overnight credit. (ECB 2023) EONIA, on the other hand, is the rate at which banks provide loans to each other with a one-day duration (Euriborrates 2023). Paavola from the Bank of Finland (2020) explains that EONIA settles between the marginal lending facility and the deposit facility, thus trailing the main refinancing rate (MRO) in the Eurozone.

Therefore, intuitively, EONIA is quite a useful benchmark interest for the Eurozone. For example, if one bank has excess cash and the central bank has increased its deposit facility to offer 1% interest, the same bank should be willing to lend money to other banks at a higher rate. For a bank that needs more money, the marginal lending facility from the central bank might be, for example, 3%. The same bank should be willing to borrow money for a lower rate from other banks. Therefore, EONIA should settle between the marginal lending and deposit facilities. Figure 4 shows how EONIA has changed from 2001 to 2021.

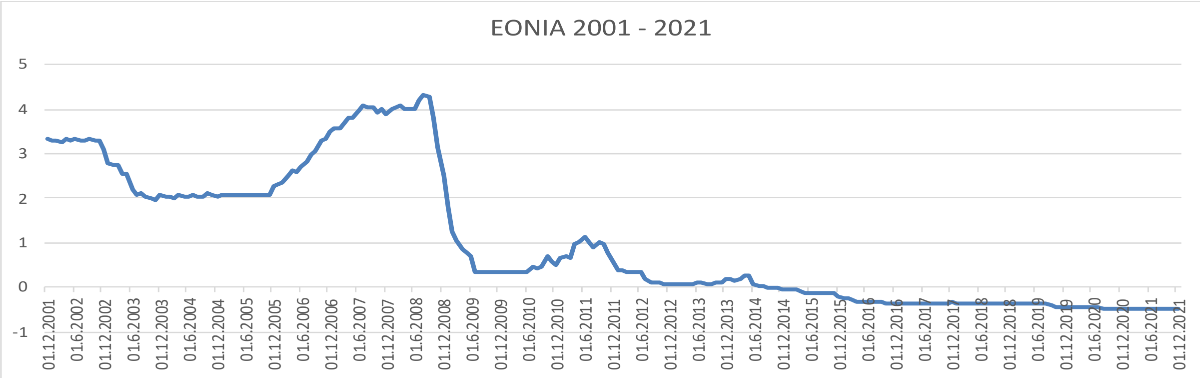

Figure 4. EONIA from December 2001 to December 2021 (Refinitiv Eikon 2023)

The effective federal funds rate is a volume-weighted median of overnight federal funds transactions (Federal Reserve Bank of New York 2023). It is the rate at which deposit institutions trade balances held at the Federal Reserve Bank with each other overnight. In other words, banks with surplus cash balances in their balance sheets lend money to the banks that need short-term liquidity. Thus, the federal funds rate is determined in the market, but in the end, it is influenced by open market operations to reach the federal funds target rate. The Federal Open Market Committee (FOMC) affects the rate by buying and selling government bonds, thus increasing and decreasing the market's liquidity. With less liquidity, the interest rates will rise and vice versa. (St. Louis Fed 2023)

Additionally, according to Baughman and Carapella (2019), the Federal Reserve has affected the rates through the floor system as the Fed pays interest on excess reserves at the IOER rate (interest on excess reserves). Baughman and Carapella (2019) mention that this pushes EFFR towards the IOER rate. Figure 5 shows how EFFR has changed over time.

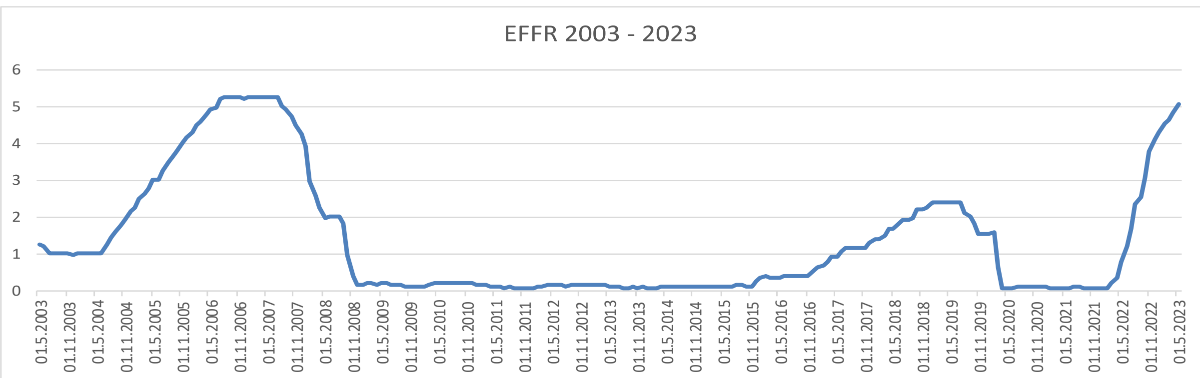

Figure 5. EFFR from January 2003 to May 2023 (Refinitiv Eikon)

Thus, it can be easily understood that, like EONIA, neither EFFR is formed solely at the market. However, the main purpose of this comparison is to find as comparable interest rates as possible for all three currencies and with enough data to build a time series model. Therefore, new interest rates such as ESTER (Euro Short-Term Rate) and SOFR (Secured Overnight Financing Rate) are not possible to examine yet due to a lack of data and, therefore, ineligibility to form a proper regression model. LIBORs (London Interbank Offered Rate), on the other hand, have been burdened by scandals and crises. For example, Lanchester (2013) reminds us that in 2012, major London-based banks responsible for producing estimates for borrowing costs that LIBOR uses as a basis were caught manipulating the index to their advantage.

Tokyo Overnight Average Rate (TONA or TONAR) measures the cost of borrowing in JPY, thus being a benchmark interest rate for Japan. TONA is the volume-weighted average of the rates of all transactions settled on the same day as the trade date and maturing the following business day. (Bank of Scotland 2021) Figure 6 shows how TONAR has developed from 2003 to 2023.

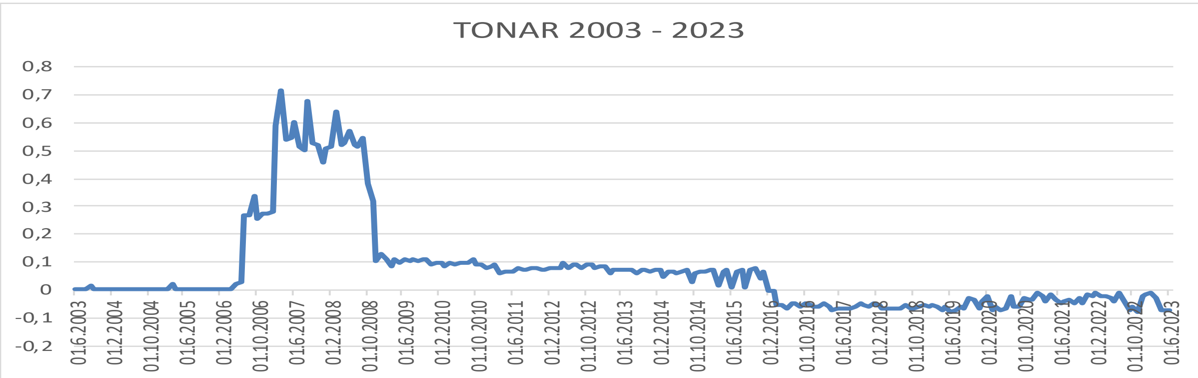

Figure 6. TONAR from June 2003 to June 2023 (Refinitiv Eikon)

<u>**Inflation Rates**</u>

Inflation levels for all three currency areas are non-seasonally adjusted as it should be the primary interest of consumers about the prices they pay. Inflation indexes include all items, so no specific industries are excluded. Inflation levels are also imported from the Refinitiv Eikon database, and monthly inflation level changes are calculated similarly to the two previous variables. Figure 7 describes how inflation levels have developed in the Eurozone, the US, and Japan.

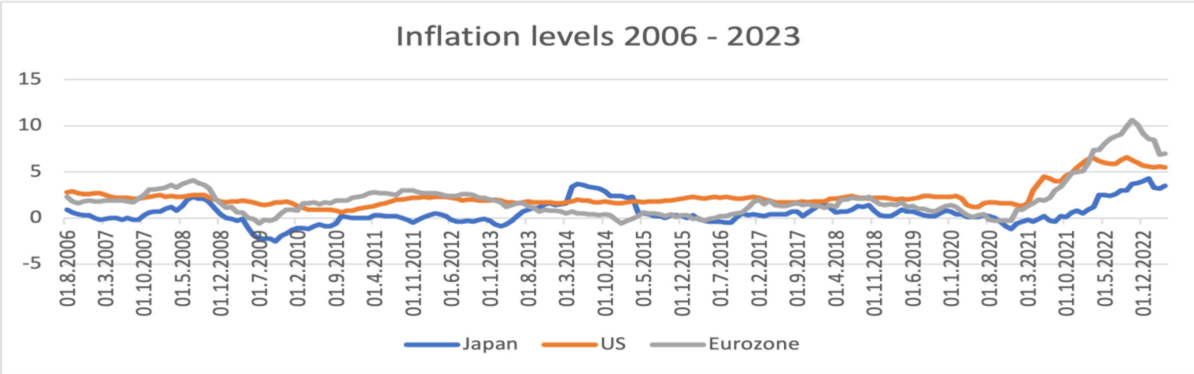

Figure 7. Inflation levels from August 2006 to April 2024 (Refinitiv Eikon 2023)

Income Levels

Lastly, income level data was collected from CEIC DATA, which focuses on producing data from economic industries. It is notable to mention in this context that the availability of income level data for monthly frequency is very limited, and therefore, it was not available from common databases. In the CEIC dataset, income level data for the US and Japan was initially ready for regression analysis, but the dataset for the Eurozone was modified. The Eurozone's income levels for this research are calculated by taking the average income of countries in the dataset that use the euro as a currency. Therefore, the income level for the Eurozone is the average monthly income levels from Germany, Austria, Belgium, Cyprus, Estonia, Finland, Greece, Ireland, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, and Spain.

Therefore, for example, the population of these countries and the contribution to consuming foreign goods are not equal, so the dataset is not perfect. Nevertheless, the observations in the dataset are converted into dollars, which can also give a wrong indication of the actual income. However, as mentioned earlier, the availability of income level data for this frequency is an issue. Thus, the regression later is conducted with this imperfection regarding the income level data.

Figure 8 shows how income levels have developed in the Eurozone, the US, and Japan.

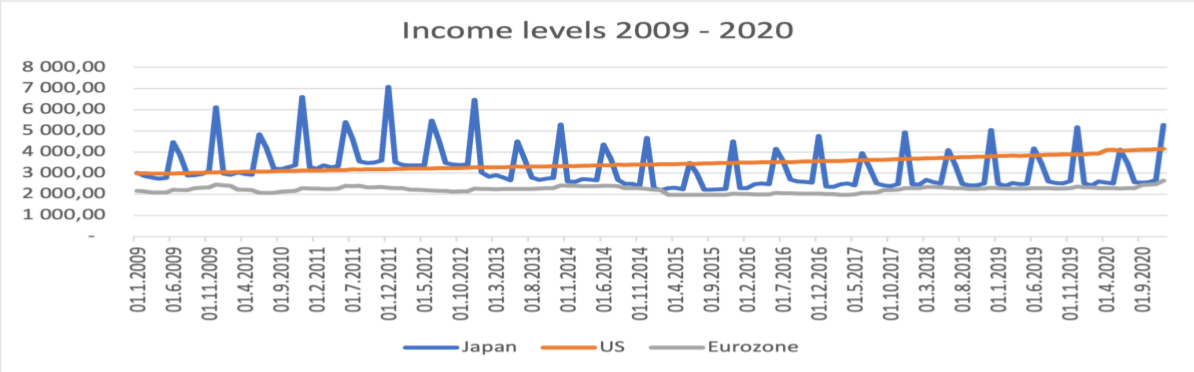

Figure 8. Income levels from January 2009 to September 2020 (CEIC DATA)

4.2. Results from Ordinary Least Squares (OLS)

Table 1. Dataset statistics

Stats	eEURUSD	EURUSDIR	EURUSDINF	EURUSDINC
N	143	142	141	142
mean	0,00002430	0,03151610	0,03049380	0,00625380
sd	0,02619640	0,43301440	0,73050950	0,05309030

Stats	eEURJPY	EURJPYIR	EURJPYINF	EURJPYINC
N	143	142	140	142
mean	0,00118690	- 0,05417420	- 0,00897810	0,07811770
sd	0,03296570	1,12847200	1,06772000	9,25395700

Table 1 introduces the basic statistics of EURJPY and EURUSD datasets. Each variable contains approximately 140 observations, as some variables have null cells. It is now notable that the table does not describe absolute values of variables but monthly changes.

<u>EURJPY Regression</u>

Figures 9, 10, and 11 illustrate the relationship between the dependent variable and each independent variable. Figure 9 shows that a positive change in the difference in interest rates has a declining effect on the EURJPY exchange rate. If the Eurozone experiences higher interest rate levels than Japan, the EUR will depreciate against the JPY. Figure 10 further visualizes the results on the relationship between the EURJPY exchange rate and inflation levels. As the fitted line shows, a positive change in the difference between inflation levels has a negative impact on the EURJPY exchange rate. Figure 11 shows the relationship between the EURJPY exchange rate and income levels. As the fitted line suggests, like with inflation levels, there is a negative relationship between income levels and the EURJPY exchange rate.

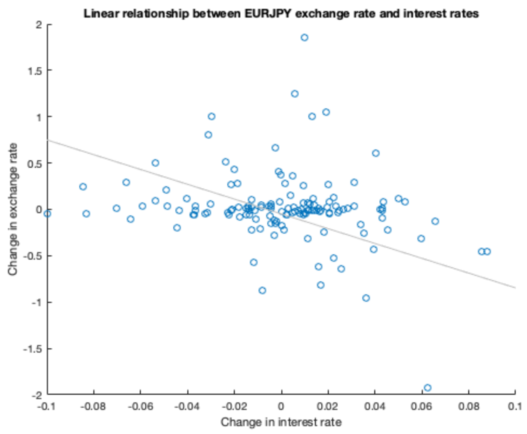

Figure 9. Linear relationship between EURJPY exchange rate and interest rates

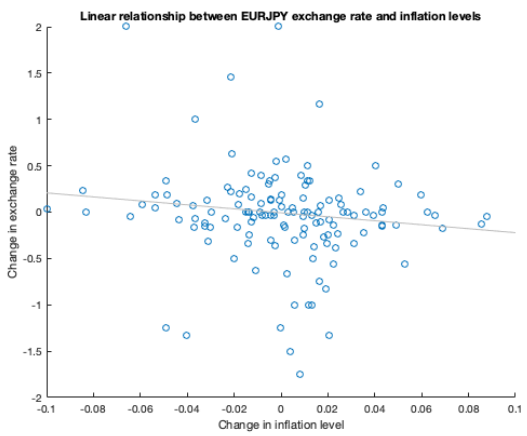

Figure 10. Linear relationship between EURJPY exchange rate and inflation levels

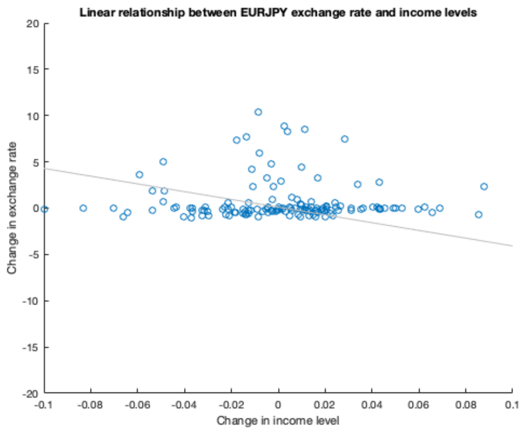

Figure 11. Linear relationship between EURJPY exchange rate and income levels

Table 2 further introduces the results from EURJPY regression. For EURJPY, the number of observations in the dataset is 140. The f-value of the regression model is at 0,01090, which means that the model is statistically significant at a 95% confidence level, below the threshold of 0,05. Root mean square error (RMSE) is at 0,03104, which is quite high, and R-squared, which measures how well the independent variables explain the dependent variable, is only at 7,86%.

Results show that from three independent variables and constant, only the interest rate has a p-value below 0,05, being statistically significant at a 95% confidence level. This means the null hypothesis that the coefficient is zero can be rejected. In other words, this regression's results show that neither inflation nor income levels statistically affect the EURJPY exchange rate. This means the null hypothesis that the coefficients are zero cannot be rejected.

Table 2. EURJPY regression results for ordinary least squares (OLS)

Number of observations	140		
F-value	3,86		
Prob > F	0,01090		
R2	0,07860		
RMSE	0,03104		
eEURJPY	**Coeff.**	**Std. Err.**	**P-value (95 %)**
EURJPYIR	- 0,0064294	0,0023190	0,006
EURJPYINF	- 0,0017546	0,0024668	0,478
EURJPYINC	- 0,0004803	0,0002828	0,092
Constant	0,0005528	0,0026270	0,834

EURUSD Regression

Figures 12, 13, and 14 illustrate the relationship between the dependent variable and each independent variable. Figure 12 shows that a positive change in the difference in interest rates has a declining effect on the EURUSD exchange rate, which means that if the Eurozone experiences higher interest rate levels than the US, the EUR will depreciate against the USD. This finding aligns with the previous finding regarding the EURJPY exchange rate. Figure 13 further visualizes the results on the relationship between the EURUSD exchange rate and inflation levels. As the fitted line shows, a positive change in the difference between inflation levels positively impacts the EURJPY exchange rate, which contradicts the previous finding regarding the EURJPY exchange rate. Figure 14 shows the relationship between the EURUSD exchange rate and income levels. As the fitted line suggests, like with inflation levels, there is a positive relationship between income levels and EURUSD exchange rates, which also contradicts the previous finding in the EURJPY regression.

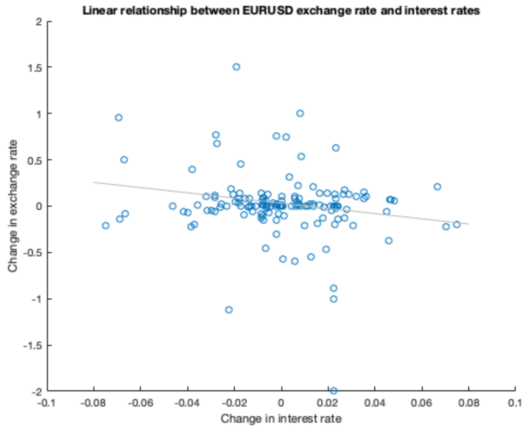

Figure 12. Linear relationship between EURUSD exchange rate and interest rates

Figure 13. Linear relationship between EURUSD exchange rate and inflation levels

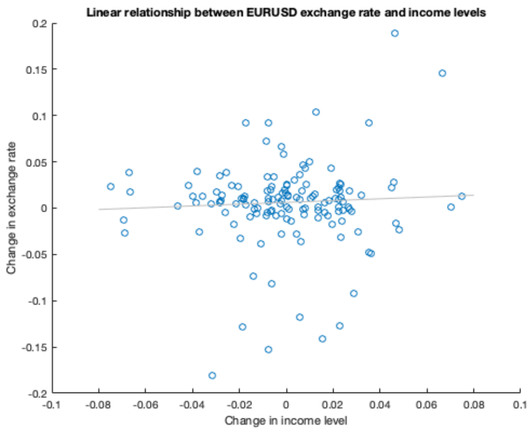

Figure 14. Linear relationship between EURUSD exchange rate and income levels

Table 3 further illustrates the results from the EURUSD regression. For EURUSD, the number of observations in the dataset is 141. The f-value of the regression model is at 0,11230, which means that the model is not statistically significant at a 95% confidence level as it is above the threshold of 0,05. The root mean square error is at 0,02605, which is quite high, and R-squared is only at 4,26%.

Results show that none of the variables has a p-value below 0,05 from three independent variables and constant. Thus, none are statistically significant at a 95% confidence level. In other words, results from this regression show that interest rates, inflation levels, and income levels have no statistically significant effect on EURUSD exchange rates. Thus, it means again that the null hypothesis that the coefficients are zero cannot be rejected.

Table 3. EURUSD regression results for ordinary least squares (OLS)

Number of observations	141		
F-value	2,03		
Prob > F	0,11230		
R2	0,04260		
RMSE	0,02605		
eEURUSD	**Coeff.**	**Std. Err.**	**P-value (95 %)**
EURUSDIR	- 0,0098131	0,0051104	0,057
EURUSDINF	0,0041043	0,0030305	0,178
EURUSDINC	0,0195207	0,0418584	0,642
Constant	0,0000351	0,0022214	0,987

As the results vary and are unexpected, further evaluation is needed to understand why the results contradict each other and with theories. OLS has been explained roughly above, but it comes with several assumptions that cannot be violated. Assumptions for both currency pairs are presented in tables 4 and 5 with null hypothesis and p-values.

Assumptions

According to Brooks (2008, 129-178) and Hill, Griffiths & Lim (2011, 172-173), the assumptions for OLS to be the Best Linear Unbiased Estimators (BLUE) are listed from 1 to 5. Additionally, for the OLS to be as accurate as possible, assumptions 6 and 7 should be fulfilled. “Best” in BLUE means that it has minimum variance among the class of linear unbiased estimators (Brooks 2008, 45), and it thrives from the Gauss-Markov theorem after its pioneers, Gauss and Markov. In The Concise Encyclopedia of Statistics (2008), it is explained that Gauss provided proof in his work "*Theoria combinationis observationum erroribus minimis obnoxiae*" in 1821 that the method of least squares is to minimize the variance, and each error is assumed to be zero with same unknown variance. Based again on The Concise Encyclopedia of Statistics (2008), Markov rediscovered the theorem in 1900, and Graybill wrote it to its modern notation in 1976. According to Hill, Griffiths & Lim (2011, 75), the assumptions (1-4 from the list below) must be fulfilled for the Gauss-Markov theorem to hold.

1. $E(u_t) = 0$	The mean value of errors is zero
2. $var(u_t) = σ2 < \infty$	Homoscedasticity, errors have constant variance
3. $cov(u_i, u_j) = 0$	No autocorrelation, errors are linearly independent of one another
4. $cov(u_t, x_t) = 0$	No correlation between errors and independent variables
5. $u_t \sim N(0, σ2)$	Errors are normally distributed

Additionally,

6. No multicollinearity
7. Functional form is correct

The first assumption is that the mean value of errors is zero. Brooks (2008, 131) mentions that this assumption is never violated when a constant is included in the regression equation. As tables 2 and 3 showed, both regressions include a constant term. The second assumption for homoscedasticity was tested with White's and Breuch-Pagan's tests, while the third assumption for autocorrelation was tested with Breusch-Godfrey's test.

The fourth assumption states that x_t is non-stochastic and is not an exact linear function of the other explanatory variables (Hill, Griffiths & Lim 2011, 174), which means that the observations are not random. However, the variables used in this research are random, being revealed simultaneously with the dependent variables. As this fact fails to correspond to the OLS assumptions, it raises an issue regarding endogeneity, which is covered more in-depth later in this chapter.

The fifth assumption that residuals are normally distributed was tested with Shapiro-Wilk's test, and the sixth assumption of collinearity was tested with VIF-test. Finally, the last assumption of correct functional form was tested with Ramsey's RESET test. The normal distribution of residuals can be divided into skewness and kurtosis, where the first one measures the unsymmetric of the distribution's mean value, and the second one measures how fat the tails are (Brooks 2008, 161). A normal distribution is not skewed and has a kurtosis of 3. The Shapiro-Wilk's test takes a null hypothesis of normal distribution, and the null hypothesis was confirmed as the p-value was above 0,05.

OLS assumes that the independent variables are not correlated, meaning there is no relationship between them, and thus they are orthogonal. In case the explanatory variables are correlated with each other, they are said to be multicollinear. (Brooks 2008, 170-174) To test multicollinearity, the VIF-test (variance inflation factor) was run on Stata, and VIF-values were below the threshold of 10. However, intuitively, it is hard to disregard the connection between interest rates and inflation levels. Usually, the interest rates are adjusted retrospectively.

The last assumption of OLS assumes that the model's functional form is linear. In other words, the model should be linear in parameters, and the visual representation of y and x variables is a straight line (Brooks 2008, 177). Ramsey's RESET test showed p-values above the threshold of 0,05, indicating that the parameters' linear relationship is fulfilled. Tables 4 and 5 show the results from EURJPY and EURUSD regressions.

Table 4. The results for OLS assumptions from EURJPY regression

	Assumption	Null hypothesis H_0	P-value	H_0 confirmed
2	$var(u_t) = σ2 < ∞$	Constant variance	0,9026	Yes
3	$cov(u_i, u_j) = 0$	No serial autocorrelation	0,1071	Yes
5	$u_t \sim N(0, σ2)$	Normally distributed	0,1952	Yes
6	Collinearity	No multicollinearity	VIF<10	Yes
7	Functional form	Model has no omitted variables	0,5094	Yes

Table 5. The results for OLS assumptions from EURUSD regression

	Assumption	Null hypothesis H_0	P-value	H_0 confirmed
2	$var(u_t) = \sigma 2 < \infty$	Constant variance	0,8346	Yes
3	$cov(u_i, u_j) = 0$	No serial autocorrelation	0,4652	Yes
5	$u_t \sim N(0, \sigma 2)$	Normally distributed	0,0720	Yes
6	Collinearity	No multicollinearity	VIF<10	Yes
7	Functional form	Model has no omitted variables	0,8906	Yes

As the results suggest, for both regressions, all the null hypotheses for assumptions 2,3,5, and 7 can be confirmed as p-values are above 0,05, and for the sixth assumption, VIF is below 10. Therefore, the unexpected regression results seem not to thrive from any violations regarding at least these OLS assumptions. However, one issue still arises – x variables are stochastic and, thus, cause a problem with endogeneity.

Limitations

Endogeneity refers to a situation where an independent variable and the error term are correlated, and it can be tested via the Hausman test (Hill, Griffiths & Lim 2011, 402 & 420). However, testing for endogeneity is subject to instrumental variables that are not easy to interpret, and therefore, endogeneity is out of the scope of this thesis.

The process is said to be stationary if it is non-dependable on time and non-stationary if its features depend on time. Non-stationarity can lead to spurious regressions meaning that two unrelated items seem to have a connection. (Hill, Griffiths & Lim 2011, 475-482) Stationarity is not evaluated further in the scope of OSL, but it will be revisited when evaluating the ARMA models.

4.2.1. Regularization

As previous sections imply that linear regression does not show perfect results for either of the currency pairs and in addition, the results contradict each other, it evokes concern about whether OLS is the best model to predict exchange rates. Regression analysis is not limited to linear regression; it can be improved with other ML models.

In their articles, Dhrubjun (2021) and Negi (2020) mention that the linear regression model can either be overfitted or underfitted to the data. Gutta (2020) explains that underfitting refers to the situation when a linear model does not have the flexibility to accurately describe the true relationship of variables, thus leading to high bias and low variance. On the other hand, he explains that overfitting refers to the situation where the model is too complex and leads to low bias and high variance. Gutta (2020) explains that linear regressions generally have high bias and low variance and, thus, are prone to underfitting. Therefore, the optimal trade-off between overfitting and underfitting is mandatory to find the best model for a given dataset.

In exchange rate analysis for currency pairs EURUSD and EURJPY, the RMSEs are 2,6% and 3,1%, respectively. The results can be seen as having low accuracy. Suppose one says the euro will appreciate against the United States dollar by 5% during the next month but clarifies that the exact appreciation may fluctuate by 2,6 points. In that case, it makes the prediction much less useful for anyone. Therefore, evaluating further alternatives for OLS to lower the RMSEs might be practical. One possibility is to use Ridge regression. Lasso regression is not considered at this point as it might shrink coefficients to zero, and as the regression already includes only three independent variables, feature selection is not a desired property.

Ridge regression is a widely used regularization technique in regression analysis. Ridge regression aims to find a line that does not fit as perfectly with the training data, adding a small amount of bias to the model (Ray 2015). This is done by adding a penalty to the regular regression formula, the dependent variable is therefore increased or decreased by multiplying the squared coefficient by lambda. Therefore, in Ridge regression, the aim is to minimize the RMSE plus the Ridge regression penalty term.

To further evaluate Ridge regression and its performance on exchange rate forecasting, the dataset used in the previous section was divided into several subsets of data. At first, 70% of the original dataset was divided into training dataset 1 and 30% into the test dataset. Then for a further split, 80% of the training dataset 1 was divided into training dataset 2, and 20% was divided into the validation dataset. Afterward, using the training dataset 2, five different models were trained with different values of lambda, and the performance of each model was tested over the validation dataset. Based on the lowest RMSE, the most accurate model was selected for training dataset 1. After training the training dataset 1, the performance was evaluated over the test dataset to achieve the final RMSE for the model.

The left curves of figures 15 and 16 first illustrate how the RMSEs of the Ridge models react to different lambda values. As the results suggest, for both currency pairs, a higher value of lambda yields lower RMSE. Thus, a more regularized model can predict exchange rates more accurately. The second curves of figures 14 and 15 compare the results from the best Ridge model (Ridge model with the lambda yielding to the lowest RMSE) and the results from normal OLS regression. For that comparison, it seems that the results contradict each other. For EURJPY, the Ridge regression with a lambda of 5 can produce lower RMSE than classical OLS regression. In contrast, for EURUSD, the OLS regression survives better, resulting in lower RMSE than the Ridge regression.

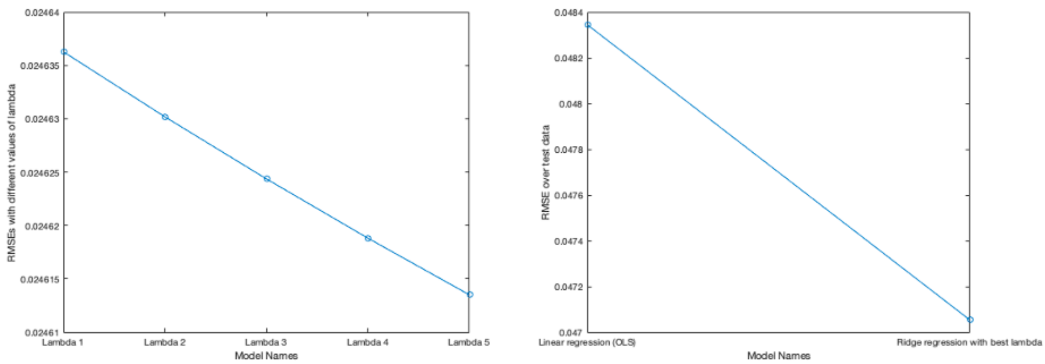

Figure 15. RMSEs for different values of lambda for EURJPY

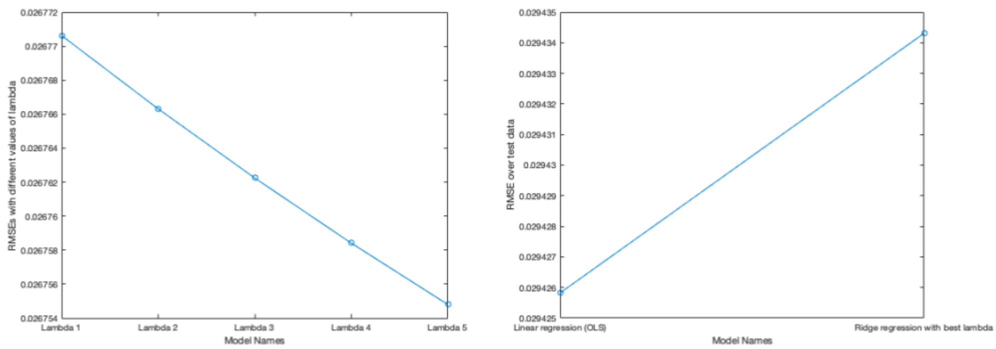

Figure 16. RMSEs for different values of lambda for EURUSD

4.3. Results from Autoregressive Moving Average (ARMA)

The theoretical background of the ARMA model suggested that only the values of the predictable variable are needed to generate an ARMA model. This section, therefore, disregards interest rates, inflation levels, and income levels, focusing on only the information of previous EURJPY and EURUSD exchange rates. The dataset is formed slightly differently, and it now includes monthly spot prices as observations for each currency pair from January 1975 to June 2023. Therefore, each dataset includes approximately 582 observations calculated to percentual changes to see how the spot price has changed between the previous month and the following month.

Figures 17 and 18 illustrate both components – the white noise for both currency pairs and the autocorrelation (ACF) and partial autocorrelation (PACF) plots to evaluate the connection of variables in terms of how far past the correlation is found.

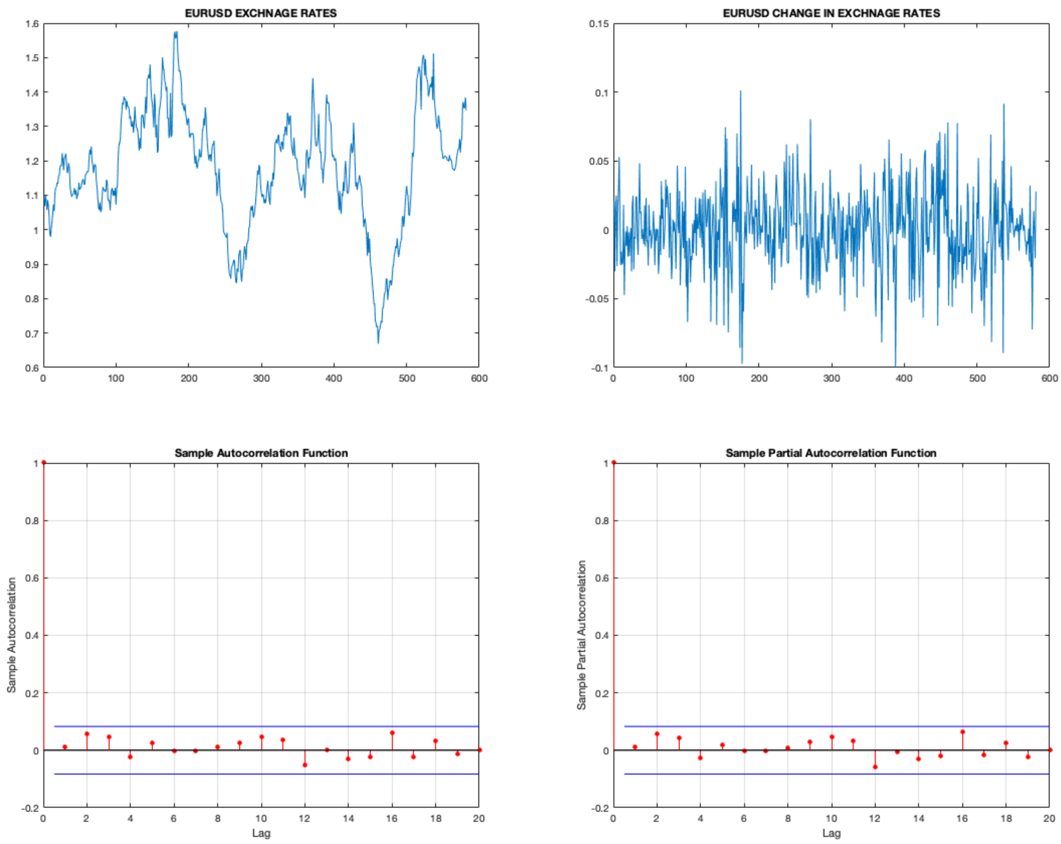

Figure 17. White noise, ACF, and PACF for EURUSD

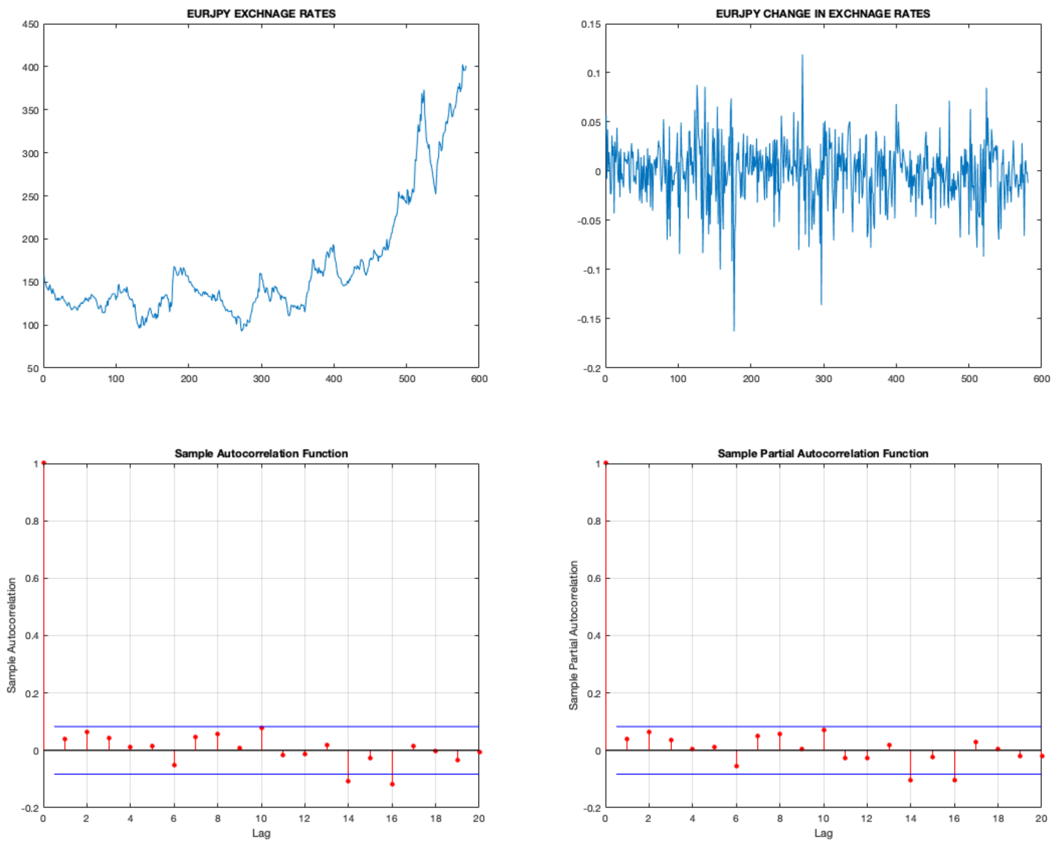

Figure 18. White noise, ACF, and PACF for EURJPY

The AR process is subject to stationarity, as mentioned earlier, and stationarity as a restriction was mentioned already among OLS assumptions. Considering figures 22 and 23, it can be noted that neither of the currency pairs seems to be fully non-time dependent. However, as spot prices are converted into monthly changes, figures 17 and 18 also show the white noise for percentual changes for both currency pairs, and from those figures, no time-dependent trend can be confirmed. Therefore, the percentual monthly changes in exchange rates seem to be stationary. The MA process is subject to a constant mean and variance and autocovariance of zero except at lag zero in the white noise process, which can be confirmed based on figures 17 and 18.

The plots from autocorrelation and partial autocorrelation are not encouraging for either currency pair. Plots from autocorrelation and partial autocorrelation look almost identical, which might indicate that the AR process has no pattern. Secondly, at a 95 % confidence interval, there are no clear spikes to statistical significance except barely at a lag of 16 for EURUSD and at lags of 10, 14, and 16 for EURJPY.

However, for the ARMA model, graphical identification is not the typical way to identify the order, and therefore it is reasonable to use the Box-Jenkins methodology. Following the example of Brooks (2008, 230), the Box-Jenkins approach was tested over the data. The first part of the Box Jenkins methodology is to identify the order of the model, which is done via information criteria.

Several different information criteria are available, and here Akaike's information criteria (AIC) is used. Complete tables for the information criteria for both currency pairs can be viewed in appendices 17 and 18, and based on the tables, MATLAB suggests ARMA (1,2) and ARMA (0,0) models for EURJPY and EURUSD, respectively. ARMA (0,0) means the model has no autoregressive or moving average terms.

The second part of the Box Jenkins methodology involves estimating the parameters. Applying the least squares to the parameters from the first step, the comprehensive results are in appendices 19 and 20. Comparing the predicted values against the actualized values, the model yields RMSEs of 0,000091588 and 0,000252 for EURUSD and EURJPY, respectively. However, the third and final step of Box Jenkins methodology is model checking which determines whether the model specification and estimation are adequate. Appendices 19 and 20 show the results for the Ljung-Box test, which can be used to evaluate if the model can capture features of the data. For EURUSD and EURJPY, the p-values for the Ljung-Box test are 0,9090 and 0,1848, respectively. P-values above 0,05 mean that the null hypothesis of no autocorrelation is not rejected, meaning that no autocorrelation is left in the residuals. Therefore, it looks like the model has captured the autocorrelation, and based on the Ljung-Box test, both models are adequate.

5. Results and Discussion

The previous section introduced results from the different models trying to predict and explain exchange rate movements. In contrast, the introduction and theory sections introduced prior research on exchange rate predictions. Therefore, this section combines previous research and the results achieved in the fourth section of this thesis. As section four showed, exchange rates for both currency pairs, EURUSD and EURJPY, were first studied with linear regression and, more precisely, with ordinary least squares. After the results from OLS, the RMSEs were tried to be reduced with Ridge regression and the ARMA model. Results were varied as some contradicted previous research and theories, and on the other hand, results differed depending on the currency pair.

Predicting the exchange rates has been challenging for financial research, and plenty of research has been done to achieve reliable results. In 1983 Meese and Rogoff named the issue *The Exchange Rate Puzzle,* and it explained the weak linkage between nominal exchange rates and monetary fundamentals. Based on the textbook written by Madura (2016), there was explained to be a positive connection between the difference in interest rates between two countries with different currencies and exchange rate movements and a negative correlation between the difference in inflation and income levels between two countries with different currencies and exchange rate movements. However, as *The Exchange Rate Puzzle* may imply, the connection has proven not as straightforward. In addition to Meese and Rogoff (1983), several other studies have disclaimed the connection, and some studies, on the other hand, have been more optimistic about the linkage between monetary variables and exchange rate movements (papers covered later in this section). Therefore, this section consists of two objectives – it presents the results from section 4 more in-depth, and on the other hand, it compares the findings with prior results from other studies and common textbooks.

The core of this thesis is based on a classical linear regression model that uses interest rate differentials, inflation level differentials, and income level differentials as explanatory variables for currency exchange rate fluctuations. Regression was done for two different currency pairs, EURUSD and EURJPY. As shown in table 2, the results for EURJPY regression were not straightforward. The p-value for interest rate differential was 0,006, meaning it is statistically significant at a 95% confidence level. The coefficient was -0,0064, meaning that an increase in the interest rate differentials leads to the depreciation of EURJPY, so if the Eurozone experiences higher interest rates than Japan, the euro will depreciate against the yen. Notably, such a finding contradicts the theory introduced in the textbook, where interest rate differentials were explained to impact currency exchange rate fluctuations positively. Still, on the other hand, it is in line with the IRP.

The same regression was run on EURUSD, and the results were similar. The coefficient was -0,0098, and the p-value was 0,057. However, the p-value higher than 0,05 means it is not significant at a 95% confidence level, so it is not a statistically significant finding. If neglecting the high p-value, the finding is the same as with EURJPY, so if the Eurozone experiences higher relative interest rates than the US, the euro will depreciate against the dollar.

For example, Madura (2016, 110) explained that higher relative interest rates should attract foreign investors as money should flow towards higher profits. Intuitively, it sounds reasonable, and other textbooks such as James, Marsh & Sarno (2012, 283) agree that high-interest rate currencies are more likely to appreciate than depreciate against low-interest rate currencies. The core theory of this thesis for interest rate differentials constituted of two forms of IRP - UIP and CIP, and like Sarno & Taylor (2002, 11) explained, the expected gain from holding one currency over another should offset the interest rate differential of the same currencies. Therefore, IRP found a negative correlation between exchange rate fluctuations and interest rate differentials, which is exactly what this thesis's results suggested.

Charles et al. (2019) mention in their paper that studied the interest rate parity puzzle that when including an inflation variable in the regression of UIP, there was a statistically strong impact on the exchange rates for inflation but not interest rates. Also, Božović (2021) mentions that the interest rate differential is not statistically significant in his results. Therefore, the results from this study, in terms of interest rates, align with at least part of the previous research. In addition, it finds that, at least for EURJPY, there is a statistically significant relationship between the interest rate differential of EONIA and TONA and the exchange rate fluctuations, and more precisely, the connection is negative.

It is notable that there are several interest rates available. This thesis decided to use EONIA, EFFR, and TONA as benchmark interest rates, and as reasoned in sub-chapter 4.1, they are not perfect. Therefore, using different benchmark rates, such as government bonds, could offer alternative results and statistically significant p-values. Another possible reason might be a lagged impact not limited to t-1 lag. Interest rate differentials can take more than one month to affect the exchange rates.

The second independent variable of the regression analysis focused on the inflation level differentials. The regression results for EURJPY show that the inflation level differential yields a p-value of 0,478 above the threshold of 0,05, meaning that it does not significantly impact exchange rates. The regression results for EURUSD show a similar p-value of 0,178 which is also above the threshold of 0,05. Moreover, the inflation differential coefficient for EURJPY is -0,0017, and for EURUSD, it is 0,0041. Even though the high p-values fail statistical significance, it is notable that the coefficients contradict each other. For EURJPY, the coefficient suggests a negative relationship between the inflation level differentials and exchange rates. If the Eurozone experiences higher inflation than Japan, the euro will depreciate against the yen. For EURUSD, the coefficient, on the other hand, suggests that if the Eurozone has higher inflation than the US, the euro is to appreciate against the dollar.

As the theory section showed, inflation levels are closely associated with PPP. Additionally, the effect of inflation levels on exchange rates was covered broadly, for example, in the textbook by Madura (2016, 109) where he explains that inflation level differentials should affect the exchange rates negatively as consumers should prefer cheaper goods, and thus consume foreign goods in case domestic goods become more expensive. PPP explained that consumers should be indifferent about buying foreign or domestic goods. Thus it is intuitively easy to understand that a hike in price levels needs to be adjusted by the currency's depreciation (Mark, Taylor & Peel 2001). Therefore, this research's findings are not in line with PPP. For EURJPY, the coefficient conforms to the PPP (again neglecting the high p-value). For EURUSD, the coefficient contradicts PPP (again, not statistically significantly due to the high p-value).

However, the incoherent results are not surprising. As mentioned earlier, for example, James, March & Sarno (2012, 55) discuss that there is no consensus on whether the PPP holds. The issue was usually referred to as the PPP puzzle among researchers. Taylor (2003) mentions that the issue with research that fails to meet PPP is usually due to low-frequency data and linear model specifications, which were both used in this thesis. He further explains that if the adjustment horizon is of the order of days, then monthly (or even less frequent data) is not expected to reveal it. Rogoff (1996) mentions that nominal exchange rates adjusted for differences in national price levels tend towards the PPP in the very long run, at least based on several recent studies. Rogoff (1996) also concludes that there must be a large buffer wherein nominal exchange rates can move without producing an immediate response in relative domestic prices. Thus, to conclude the result for both regressions in terms of inflation level differentials, this study finds no statistical significance for inflation level differentials affecting the exchange rates, which conforms to the PPP puzzle and the findings by Rogoff (1996). A third possible reason for such a finding is the possible multicollinearity between interest rates and inflation, although assumptions testing showed collinearity. The presence of multicollinearity could affect the p-values.

The third and last regressor for exchange rate fluctuations was income levels. The regression results for EURJPY show a p-value of 0,092, indicating that the regressor is not statistically significant at a 95% confidence level. The coefficient is -0,00048, which shows a negative correlation between exchange rates and income levels. If the income levels in the Eurozone are higher than in Japan, the euro should depreciate against the yen. For the EURUSD regression, the p-value is 0,642, indicating that income levels have no statistically significant impact on exchange rates. The coefficient is 0,0195, which indicates a positive relationship between exchange rates and income levels. Therefore, if the Eurozone experiences higher income levels than the US, the euro should appreciate against the dollar.

As explained in the theory section, for example, Madura (2016, 111) mentions that income level differentials should have a similar effect on exchange rates than inflation differentials as it should affect the demand for imports (yet increase the demand for foreign currency and supply of home currency). Marsh & Sarno (2012, 47) showed that the monetary model, conversely, yielded that relatively higher income induces stronger currency. They also noted that the idea posed by the monetary model is the opposite of the Mundell-Fleming model, where higher income leads to higher imports. Therefore, the results from this research correspond with both frameworks depending on the currency pair (again neglecting the high p-values). For EURUSD, the results align with the monetary model, and for EURJPY, the results align with the suggestion by Madura (2016). Possible reasons might be, for example, a lagged impact that is not limited to t-1 lag. It might be that the income level differential takes more time than one month to affect the exchange rates. As figure 8 shows, the income level dataset also has plenty of noise.

Finally, before moving on to the results from the Ridge regression and ARMA model, it is reasonable to point out a few facts regarding the EURJPY and EURUSD regression models. Even though each regressor has now been explained separately, the complete models need more evaluation. The EURJPY regression model shows an R2 of 7,86%, which detects less than ten percent of EURJPY exchange rate movements. The RMSE is 3,1%, representing the forecasting accuracy, and as the high percentage indicated, the model is not too accurate. The f-value is 0,0109, which indicates that the entire model is statistically significant. The EURUSD regression is much less promising. The f-value is 0,1123, meaning the entire model does not show statistical significance. The R2 is even lower at 4,26%, and the RMSE as an accuracy indicator is at 2,605%. Therefore, to conclude the results from EURJPY and EURUSD regressions, the results evoke concern about whether the exchange rate movements can even be modeled linearly with OLS.

Ridge regression was done to reduce the RMSEs and improve the prediction accuracy of the models. Different lambda values were applied to the models to shrink coefficients for both currency pairs. Based on figures 15 and 16, a higher lambda value for both currency pairs yielded lower RMSEs, indicating that a more regularized model can predict exchange rates more accurately. However, the results varied when applying the regularized models with lambda of 5 to the data. For EURJPY, the Ridge regression yielded lower RMSEs but only for less than a tenth of a decimal. For EURUSD, the OLS regression was still able to compete better. It is also notable that the RMSEs from comparing Ridge and OLS regressions are slightly different than in the linear regression model in sub-section 4.2 due to the data set split described in sub-section 4.2.1. Overall, the results from Ridge regression still show no great improvement in the prediction accuracy, and the RMSEs remain quite high.

The last sub-section of the data and methodology chapter introduced the ARMA model, which was also the approach for technical forecasting in this thesis. The results from the ARMA model were dependent on the currency pair. They showed that for EURJPY, the Box-Jenkins methodology was able to yield an ARMA model of orders AR (1) and MA (2), which means that the value from the previous period influences the value in the current period (AR process) in addition to a moving average of order 2. For EURUSD, the Box-Jenkins methodology suggested the ARMA (0,0), which was a surprising finding. The ARMA model of order zero for both the MA, and AR parts, means that the model has no correlation among the errors over time. In other words, based on the monthly observations dataset, the rate changes of EURUSD are not predictable with an ARMA model. Possible reasons might stem from the frequency of the data as there might not be structure in monthly observations.

The Ljung-Box test showed adequate models; no autocorrelation was left in the residuals. The RMSE for the EURUSD ARMA model is not important anymore as the model did not show any autoregressive structure. Still, for the EURJPY, the ARMA (1,2) model yielded an RMSE of 0,000252, much lower than the RMSEs from OLS or Ridge regression.

As the introduction section showed, the ARMA model is a researched subject in the field of finance. To compare the results from this paper, the ARMA model was used, for example, in papers by Abdulcileem, Abdulqudus, Abdulqadir (2021), and Mammadova (2010). In the paper by Abdulcileem, Abdulqudus, and Abdulqadir (2021), they evaluated the exchange rate between the US dollar and the British pound, and they found that relatively small ARMA models were selected by the AIC such as ARMA (2,3), ARMA (4,5) and ARMA (4,1). They also found that the ARMA model yielded a forecast error (again case dependent) of about 0,2%. Notably, they measured the exchange rate fluctuation by taking the first difference of its logarithmic transformation.

Mammadova (2010) used several ARMA models for the exchange rate of the Brazilian real, and neither could she find an improving performance by the ARMA models. Moreover, she found that neural networks, on the other hand, outperform the ARMA model. As this research showed varying and exchange rate dependable results, it might be that the ARMA model is indeed not the best tool for technical analysis of exchange rates. However, EURJPY outperforms the fundamental forecasting technique conducted by OLS and Ridge regressions.

6. Conclusions

The introduction section showed that the global economy has 180 currencies circulating through approximately 190 countries, and the former research has still not reached a consensus on how to predict them. The literature review showed weak links between monetary fundamentals and exchange rates. Different models can beat the random walk only in the long run, and theories are subject to different puzzles, such as the exchange rate or Fama. Therefore, financial research around exchange rates has shifted toward more complex models that use purely the information of FX spot prices.

This thesis revisited the issue surrounding the linkage of monetary fundamentals and exchange rates and compared the results from classical fundamental forecasting with the results from quantitative forecasting. To conclude the results from fundamental forecasting conducted with OLS, table 6 shows the results for both regressions in terms of statistical significance and the direction of coefficients (whether the impact of the regressor is positive or negative for exchange rates).

Table 6. Combined results for fundamental forecasting

	= Statistically significant
	= Not statistically significant

	Interest rates	Inflation levels	Income levels
EURJPY	-	-	-
EURUSD	-	+	+

Therefore, given the results in table 6, the first research question can finally be answered.

Question 1. Does the linkage between exchange rates and differentials in interest, inflation, and income levels exist?

Given the results from two separate regressions that included information from 11 years, the statistical significance was found only for the interest rate differential between the Eurozone and Japan. Therefore, it can be stated that the linkage between exchange rates and differentials in interest rates, inflation levels, and income levels does not exist, excluding the one minor case. Therefore, based on this thesis, the exchange rate puzzle still holds.

Question 2. Can quantitative forecasting reduce the forecast error compared to fundamental forecasting?

The ARMA model showed that the results were dependable on the currency pair, and for EURUSD, no results were achieved. However, with EURJPY, the results yielded a very low RMSE (0,000252) which means that, on average, the ARMA (1,2) model for EURJPY can forecast exchange rates with an absolute forecast error of 0,0252 %. Therefore, the answer to research question two is dependable on the currency pair. For EURUSD, quantitative forecasting cannot reduce the forecast error compared to fundamental forecasting, but for EURJPY, it can yield much more accurate results.

Question 3. Are the results consistent for different currency pairs?

Finally, as this thesis has two different currency pairs, evaluating whether the results are case-dependent or consistent for both currency pairs is reasonable. Results from the fundamental forecasting show that the direction of coefficients (whether the impact of the regressor is positive or negative for exchange rates) is the same in interest rates for both currency pairs but different for inflation and income level differentials. The ARMA model was also successful only on EURJPY but not on EURUSD. Therefore, for research question three, whether the results are consistent for different currency pairs, the answer is no.

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Appendices

Appendix 1. Correlation table for EURJPY

	eEURJPY	EURJPYIR	EURJPY~F	EURJPY~C
eEURJPY	1.0000			
EURJPYIR	-0.2351	1.0000		
EURJPYINF	-0.0642	0.0120	1.0000	
EURJPYINC	-0.1508	0.0423	0.0209	1.0000

Appendix 2. Regression results for EURJPY

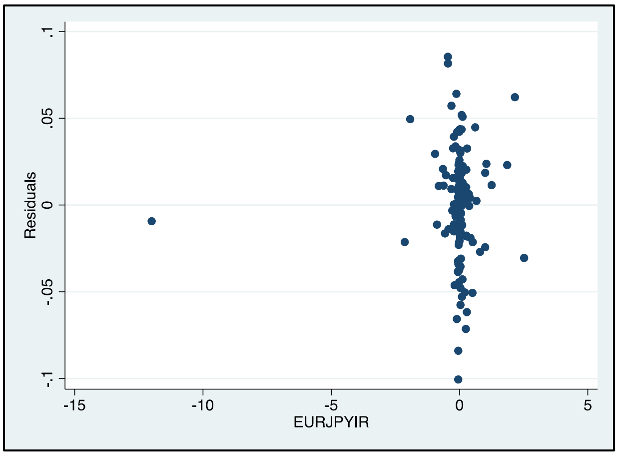
Source	SS	df	MS			
				Number of obs	=	140
				F(3, 136)	=	3.86
Model	.011174398	3	.003724799	Prob > F	=	0.0109
Residual	.131069206	136	.000963744	R-squared	=	0.0786
				Adj R-squared	=	0.0582
Total	.142243604	139	.001023335	Root MSE	=	.03104

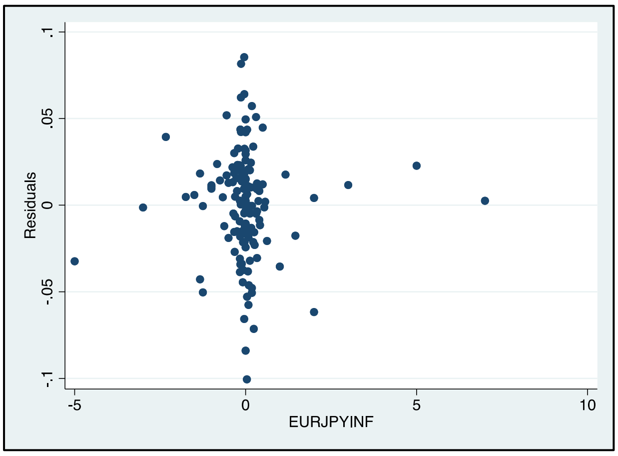
eEURJPY	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
EURJPYIR	-.0064294	.002319	-2.77	0.006	-.0110154	-.0018434
EURJPYINF	-.0017546	.0024668	-0.71	0.478	-.0066329	.0031237
EURJPYINC	-.0004803	.0002828	-1.70	0.092	-.0010396	.000079
_cons	.0005528	.002627	0.21	0.834	-.0046423	.0057479

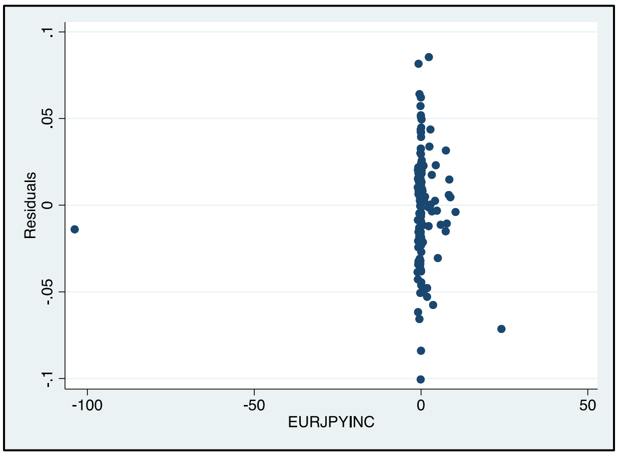
Appendix 3. Ramsey RESET test (EURJPY)

```
Ramsey RESET test using powers of the fitted values of eEURJPY
       Ho:  model has no omitted variables
                 F(3, 133) =      0.78
                  Prob > F =      0.5094
```

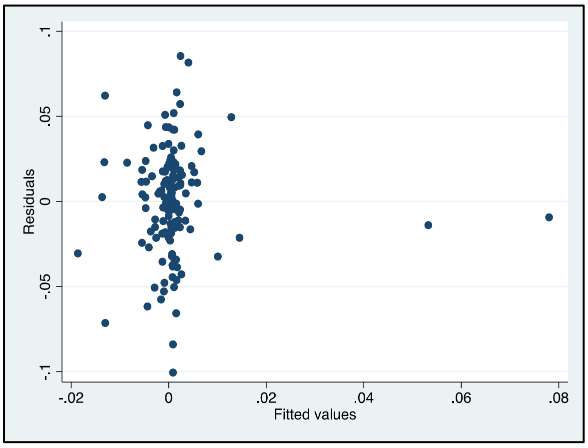
Appendix 4. Residual scatterplots for EURJPY regression







Appendix 5. Testing for heteroscedasticity: White's test, Breuch-Pagan-test, and scatterplot (EURJPY)




```
White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(9)      =      4.13
         Prob > chi2  =    0.9026

Cameron & Trivedi's decomposition of IM-test
```

Source	chi2	df	p
Heteroskedasticity	4.13	9	0.9026
Skewness	1.64	3	0.6510
Kurtosis	2.70	1	0.1002
Total	8.47	13	0.8117

```
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of eEURJPY

         chi2(1)      =     0.78
         Prob > chi2  =   0.3779
```

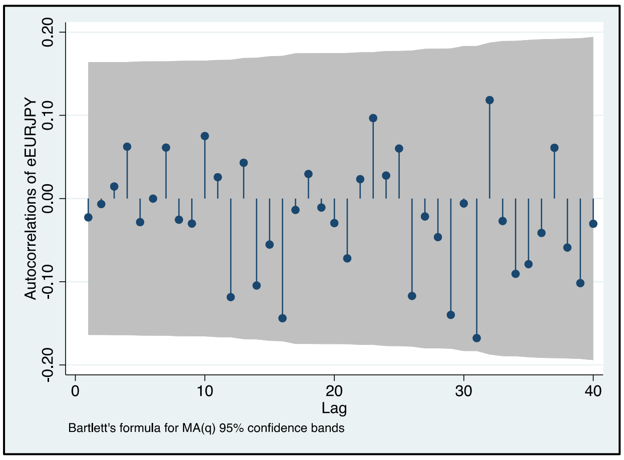
Appendix 6. Testing for autocorrelation: Breusch-Godfrey-test and correlograms (EURJPY)

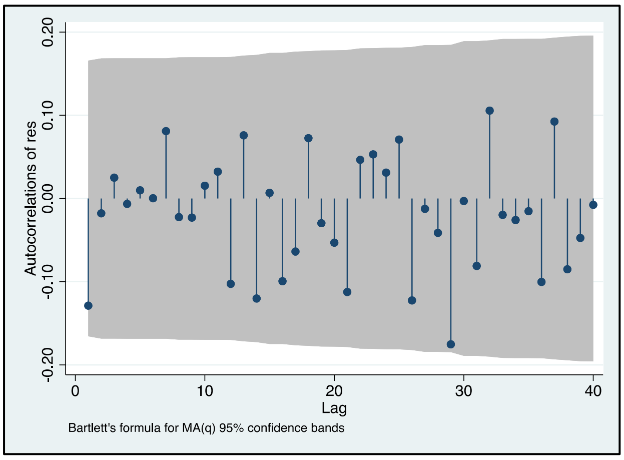
Number of gaps in sample: 2

Breusch-Godfrey LM test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	2.597	1	0.1071

H0: no serial correlation

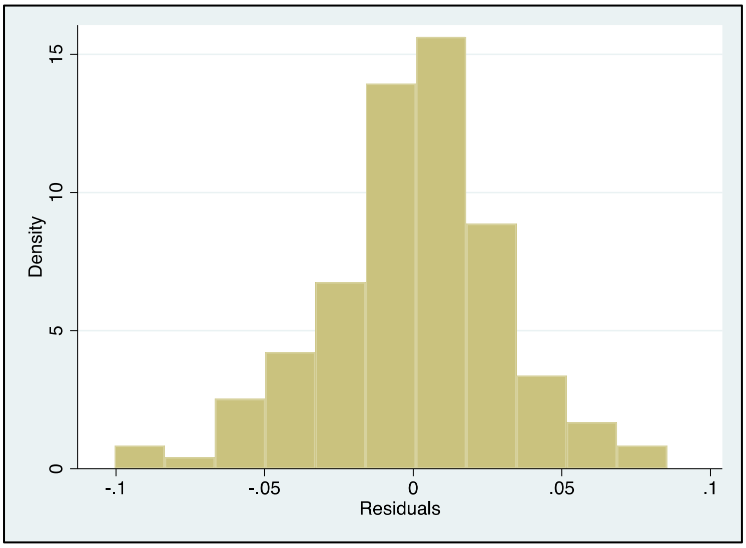




Appendix 7. Testing for multicollinearity: VIF-test (EURJPY)

Variable	VIF	1/VIF
EURJPYINC	1.00	0.997795
EURJPYIR	1.00	0.998085
EURJPYINF	1.00	0.999441
Mean VIF	1.00	

Appendix 8. Testing for the normal distribution of residuals: Shapiro-Wilk-test and histogram (EURJPY)



Shapiro-Wilk W test for normal data

Variable	Obs	W	V	z	Prob>z
res	140	0.98668	1.461	0.857	0.19582

Appendix 9. Correlation table for EURUSD

	eEURUSD	EURUSDIR	EURUSD~F	EURUSD~C
eEURUSD	1.0000			
EURUSDIR	-0.1703	1.0000		
EURUSDINF	0.1151	-0.0311	1.0000	
EURUSDINC	0.0487	-0.1238	-0.0955	1.0000

Appendix 10. Regression results for EURUSD

Source	SS	df	MS
Model	.00413717	3	.001379057
Residual	.092964847	137	.000678576
Total	.097102017	140	.000693586

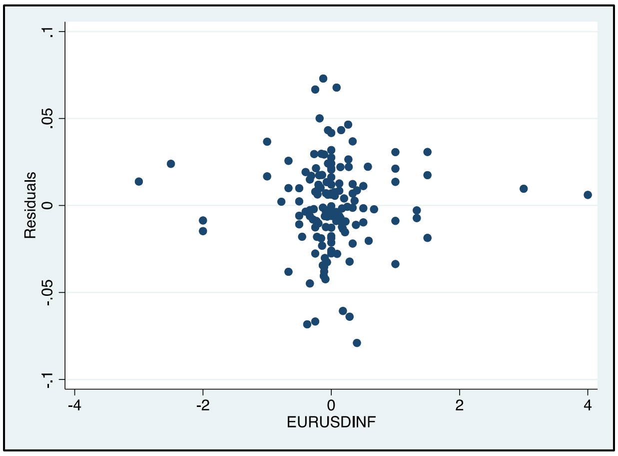
Number of obs	=	141
F(3, 137)	=	2.03
Prob > F	=	0.1123
R-squared	=	0.0426
Adj R-squared	=	0.0216
Root MSE	=	.02605

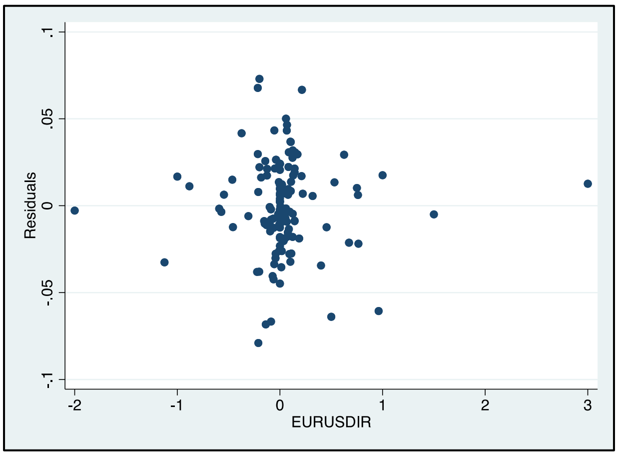
eEURUSD	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
EURUSDIR	-.0098131	.0051104	-1.92	0.057	-.0199185	.0002923
EURUSDINF	.0041043	.0030305	1.35	0.178	-.0018883	.0100968
EURUSDINC	.0195207	.0418584	0.47	0.642	-.0632514	.1022929
_cons	.0000351	.0022214	0.02	0.987	-.0043576	.0044279

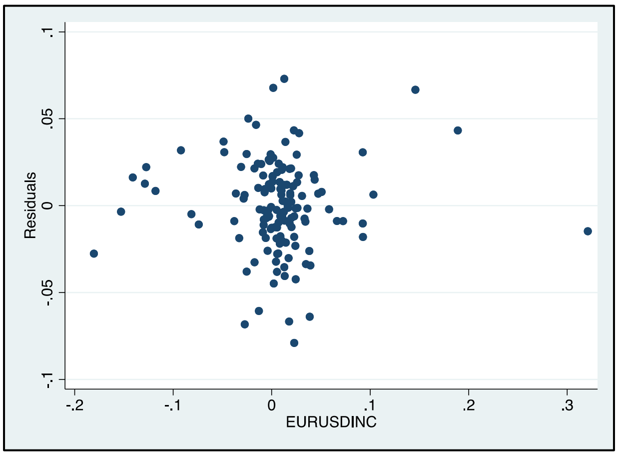
Appendix 11. Ramsey RESET test (EURUSD)

```
Ramsey RESET test using powers of the fitted values of eEURUSD
       Ho:  model has no omitted variables
                 F(3, 134) =      0.21
                  Prob > F =      0.8906
```

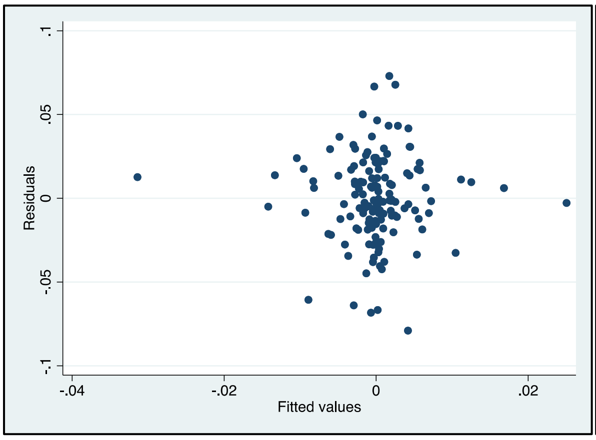
Appendix 12. Residual scatterplots for EURUSD regression







Appendix 13. Testing for heteroscedasticity: White's test, Breuch-Pagan-test, and scatterplot (EURUSD)




```
White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(9)      =     5.00
         Prob > chi2  =   0.8346

Cameron & Trivedi's decomposition of IM-test
```

Source	chi2	df	p
Heteroskedasticity	5.00	9	0.8346
Skewness	2.23	3	0.5270
Kurtosis	5.54	1	0.0186
Total	12.76	13	0.4662

```
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of eEURUSD

         chi2(1)      =     0.00
         Prob > chi2  =   0.9964
```

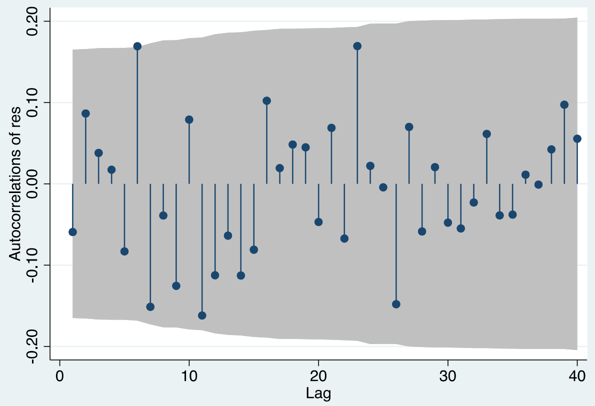
Appendix 14. Testing for autocorrelation: Breusch-Godfrey-test and correlograms (EURUSD)

Number of gaps in sample: 1

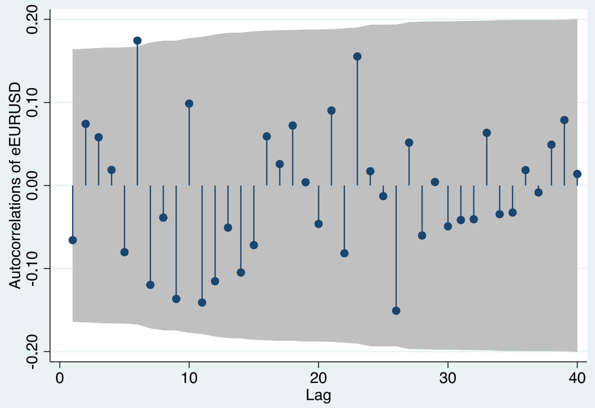
Breusch-Godfrey LM test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	0.533	1	0.4652

H0: no serial correlation



Bartlett's formula for MA(q) 95% confidence bands

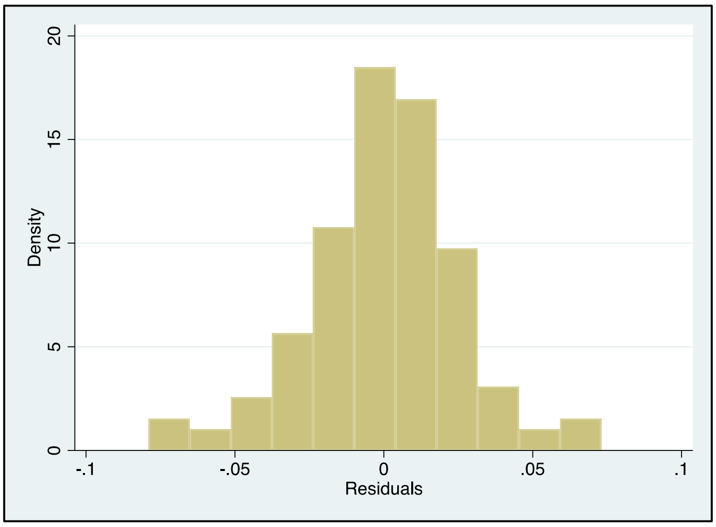


Bartlett's formula for MA(q) 95% confidence bands

Appendix 15. Testing for multicollinearity: VIF-test (EURUSD)

Variable	VIF	1/VIF
EURUSDINC	1.03	0.974806
EURUSDIR	1.02	0.982828
EURUSDINF	1.01	0.989016
Mean VIF	1.02	

Appendix 16. Testing for the normal distribution of residuals: Shapiro-Wilk-test and histogram (EURUSD)



Shapiro-Wilk W test for normal data

Variable	Obs	W	V	z	Prob>z
res	141	0.98270	1.909	1.461	0.07204

Appendix 17. AIC for EURUSD

```
AIC = 6x6
10^3 ×
   -2.4565   -2.4546   -2.4546   -2.4537   -2.4520   -2.4504
   -2.4546   -2.4526   -2.4533   -2.4528   -2.4500   -2.4488
   -2.4545   -2.4532   -2.4515   -2.4524   -2.4480   -2.4509
   -2.4537   -2.4530   -2.4510   -2.4522   -2.4510   -2.4492
   -2.4522   -2.4510   -2.4527   -2.4509   -2.4440   -2.4438
   -2.4504   -2.4490   -2.4508   -2.4491   -2.4420   -2.4477
```

Appendix 18. AIC for EURJPY

```
AIC = 6x6
10^3 ×
   -2.3765   -2.3754   -2.3756   -2.3746   -2.3728   -2.3708
   -2.3755   -2.3734   -2.3773   -2.3755   -2.3708   -2.3688
   -2.3759   -2.3773   -2.3755   -2.3706   -2.3688   -2.3668
   -2.3747   -2.3755   -2.3773   -2.3686   -2.3668   -2.3648
   -2.3727   -2.3674   -2.3717   -2.3666   -2.3648   -2.3628
   -2.3708   -2.3654   -2.3723   -2.3646   -2.3628   -2.3608
```

Appendix 19. ARMA model for EURUSD

```
    ARIMA(0,0,0) Model (Gaussian Distribution):

                  Value        StandardError    TStatistic      PValue
                 __________    _____________    __________    _________

    Constant     6.1956e-05      0.0012092       0.051239       0.95914
    Variance     0.00084783     4.1563e-05         20.399     1.721e-92
h = logical
   0
P_val = 0.9090
```

Appendix 20. ARMA model for EURJPY

```
    ARIMA(1,0,2) Model (Gaussian Distribution):

                    Value        StandardError    TStatistic      PValue
                 ___________    _____________    __________    ___________

    Constant     -0.00034922     0.00044916       -0.77749        0.43687
    AR{1}            0.74101         0.1547         4.7901     1.6674e-06
    MA{1}           -0.70691        0.15856        -4.4583      8.262e-06
    MA{2}           0.022512       0.039255        0.57347        0.56633
    Variance      0.00096161     4.0742e-05         23.602    3.6509e-123
h = logical
   0
P_val = 0.1848
```