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LOW VOLATILITY ANOMALY IN THE JAPANESE STOCK MARKET

Lappeenranta–Lahti University of Technology LUT

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Examiner: University lecturer, Henri Huovinen

ABSTRACT

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Low volatility anomaly in the Japanese stock market

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This bachelor's thesis examined the existence of the low volatility anomaly in the Japanese stock market during the period 1.1.2003-30.12.2022. To examine the anomaly, a low volatility portfolio and a high volatility portfolio were constructed, and their performance followed. A market index was additionally formulated consisting of all the stocks in the Japanese stock market. This thesis also examined the differences in returns between small- and large market capitalization companies. For this study, portfolios were formed based on their market capitalizations and volatilities. This research was conducted as a quantitative study, where portfolio performance was compared using volatility, annual returns, beta, Jensen's alpha, and Sharpe ratio. Companies monthly market capitalizations were used in calculating the returns and volatilities of stocks, and the data was collected from Refinitiv Workspace database.

Study results showed that there are some signs of the low volatility anomaly in the Japanese stock market, as risk-adjusted metrics demonstrated better outcomes for low volatility portfolios. However, portfolios with high volatility outperformed in both overall and annual returns when compared to low volatility portfolios and the market index. As for company market capitalization differences, small market capitalization portfolios excelled both in annual returns and in risk-adjusted methods. The results of this study achieved similar results compared to previous research to some degree, as previous research asserted the low volatility anomaly to be much stronger in the Japanese market.

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Tämä kandidaatintutkielma tutki alhaisen volatiliteetin anomalian esiintymistä Japanin osakemarkkinoilla ajalla 1.1.2003-30.12.2022. Anomalian tarkastelua varten muodostettiin matalan volatiliteetin ja korkean volatiliteetin portfoliot, joiden suoriutumista seurattiin. Lisäksi luotiin markkinaindeksi, joka koostui kaikista Japanin osakemarkkinoiden osakkeista. Tässä tutkielmassa tutkittiin myös pienten ja suurten markkina-arvojen omaavien yritysten eroja suoriutumisessa. Tätä tutkimusta varten muodostettiin portfolioita yritysten markkina-arvojen ja volatiliteetin perusteella. Tämä tutkimus toteutettiin kvantitatiivisena tutkimuksena, jossa portfolioiden suoriutumista vertailtiin volatiliteetin, vuosituottojen, beetan, Jensenin alfan ja Sharpen luvun suhteen. Yritysten kuukausittaisia markkina-arvoja käytettiin osakkeiden tuottojen ja volatiliteettien laskemiseen, ja data kerättiin Refinitiv Workspace -tietokannasta.

Tutkimustulokset osoittivat, että Japanin osakemarkkinoilla on joitain merkkejä alhaisen volatiliteetin anomaliasta, sillä riskikorjatut mittarit osoittivat parempia tuloksia matalan volatiliteetin portfolioille. Kuitenkin korkean volatiliteetin portfoliot ylittivät sekä kokonaistuotoissa että vuosituotoissa matalan volatiliteetin portfoliot ja markkinaindeksin. Yritysten markkina-arvojen pohjalta muodostettujen portfolioiden osalta pienten markkina-arvojen portfoliot menestyivät parhaiten sekä vuosituotoissa, että riskikorjatuin menetelmin. Tämän tutkimuksen tulokset vastasivat likimäärin aiempien tutkimusten tuloksia. Aiemmissa tutkimuksissa alhaisen volatiliteetin anomalian on kuitenkin havaittu olevan Japanin markkinoilla vahvempi verrattuna tämän tutkimuksen havaintoihin.

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1 Introduction

Whether we like it or not, none of us can escape risks. We take risks willingly or unwillingly every single day. From these risks we expect rewards that are at least equal to the risks that we take, as there would be no rationale for undertaking these risks otherwise. This is the concept of risk and return and in the world of finance it's known as the risk-return tradeoff. When an ordinary person thinks of this relationship it seems quite simple. The less risk you take the smaller the return and vice versa, but this isn't always the case.

For decades, people in the market have found the relationship between risk and return fascinating, and multiple studies have been conducted on this subject. One key study related to the risk-return tradeoff is the modern portfolio theory. Harry Markowitz (1952) the founder of modern portfolio theory wrote in his book about an idea of an efficient frontier. The idea of the theory was for the investor to take into account how a security co-moved with other securities. When this is considered, a portfolio can be built that has less risk but the same expected return as a portfolio that disregards the interactions between securities. (Elton & Gruber 1997)

Another key study related to the risk-return tradeoff is the capital asset pricing model (CAPM). The capital asset pricing model provided framework for answering the question of how the expected return of an investment should be influenced by its level of risk. This model was developed in the early 1960s and gave valuable information on what type of risk is related to return (Perold 2004).

The early studies related to risk-return tradeoff, such as the capital asset pricing model supported the argument that with greater risk comes greater reward. This was the consensus until the year 1972 when Black, Jensen & Scholes (1972) released a paper that found inadequacies in the capital asset pricing model. In the study they showed proof that an investments expected excess return is not strictly comparable to its Beta (β) in the New York Stock Exchange during the study period (1931-1965). Thus, it is possible for a low-risk (low volatility) investment to outperform a riskier one.

The possibility of low-risk securities being able to outperform risker ones gave a foundation to what is today called as the low volatility anomaly. This anomaly was given more empirical evidence by Robert Haugen & James Heins (1972), as they wrote a paper *On the Evidence Supporting the Existence of Risk Premiums in the Capital Market.* In this study the researchers arrived at the conclusion that stock portfolios with lower variance exhibited greater average monthly returns than their more volatile counterparts in the New York Stock Exchange in 1926-1971. This paper was one of the first to document the absence of a positive correlation between risk and return in empirical analysis of stock market returns across multiple sectors. In more recent studies it has become very clear that there is an anomaly between low-risk and high-risk assets in various markets.

Volatility is used to measure the risk of an investment. It is the standard deviation of returns for a security in a certain period. Higher volatility (standard deviation) means that the investment's value changes in a wider range of values. This usually means that if the volatility of a security is higher, it is considered riskier.

There are many definitions for the word "anomaly", but in financial economics anomalies are defined as irregularities. They are something that differs from what is thought to be the standard or norm. Empirical findings show a deviation from what is expected based on theoretical predictions. (Frankfurter & McGoun 2001) The low volatility anomaly is based on research showing that stocks with lower volatility have outperformed stocks with higher volatility. This kind of anomaly offers opportunities for investors to gain excess profits. Anomalies appear in many ways in the finance industry and have appeared for a long time. For example, the anomaly of stock returns depending on the day of the week. On Fridays stock prices tend to rise more often compared to other days of the week and Mondays are when they have risen the least often (Cross 1973).

In the stock market anomalies often disappear quickly once they are noticed. Eugene Fama presented the Efficient Market Hypothesis (EMH) in 1965 that suggests that in efficient markets, all relevant information is already priced in. According to him when new information is available prices adjust rapidly, which makes it hard for anomalies to persist. This is not the case when it comes to the low volatility anomaly, as this anomaly has been around for decades and is still something that investors can benefit from.

1.1 Research questions and the objective of the thesis

The main objective of this thesis is to examine if there is a low volatility anomaly in the Japanese stock market. Most of the studies regarding the low volatility anomaly have focused on the United States markets or the European stock market. It is interesting to examine this phenomenon in the Asian market, more specifically Japan. There are also other reasons to examine this anomaly in Japan, such as the relevance of its market to investors. These reason will be more thoroughly discussed later in the thesis. This thesis aims to discover if the performance of low volatility stocks has been better than high volatility stocks in the Japanese market during the study period. This research problem will be addressed through the following research questions.

The main research question of the thesis:

- *Does the low volatility anomaly appear in the Japanese stock market?*

This thesis will also answer the following sub question:

- *What is the difference in return between the small and large size company portfolios?*

1.2 The delimitations of the thesis and research material

The theme of this study is quite large and research about the low volatility anomaly can be constructed on every single index in the world. To keep this thesis compact the delimitation of this study is that it only focuses on one specific market. It examines if the low volatility anomaly appears in Japanese stock market and from it gives valuable information on the opportunities that may arise from this anomaly.

This thesis is conducted with quantitative methods and data used for this research is company market capitalization data from Japanese stocks. The data for these stocks is collected from the Refinitiv Workspace (Eikon) database. The time period from which the stock data is gathered is 20 years, from the start of 2003 to the end of 2022. This long timeframe is chosen to get a more accurate knowledge on the existence of the low volatility anomaly.

1.3 Structure of the thesis

The structure of the thesis will consist of six chapters, which each will have an important role in this paper. The first chapter is the introduction of the thesis consisting of a short background, research questions and objectives, delimitations and research materials and the structure of the thesis. The second chapter is the theoretical framework concentrating on the previous theories and literature related to the subject. The second chapter also discusses the reasons for Japan being an interesting research market. The third chapter is also a part of the theoretical framework, but this chapter focuses entirely on the low volatility anomaly, its history, and the reasons for its existence.

The fourth chapter will consist of describing the data and elucidating the research method employed in this thesis. This chapter also goes through the methods and mathematical expressions needed to conduct the study. These include volatility and the risk-adjusted methods of Sharpe ratio and Jensen's alpha. The fifth chapter goes through the empirical results of the study and discussion related to those results. Chapter six is the summary chapter where research questions are answered and final thoughts on the thesis are provided.

2 Financial theory and concepts

This chapter is the first part of theoretical framework in this thesis. The chapter itself is a summary of essential financial theories and concepts, and how they are associated with the low volatility anomaly. The key theories discussed in this chapter are essential to know before diving deeper into the anomaly itself.

2.1 Modern portfolio theory

In 1952 an article was published in *The Journal of Finance* titled *Portfolio Selection*. The author of this article was Harry Markowitz and the paper itself was not very known during the time of its publication. Despite that, throughout the years the ideas presented in the study have laid the groundwork for what is now referred to as Modern Portfolio Theory (MPT). (Fabozzi, Gupta & Markowitz 2002) Today modern portfolio theory is one of the key theories in the financial industry.

Markowitz presented the idea that investors should diversify their investments into various asset classes to achieve a better risk-return tradeoff. Combining securities (such as stocks) into a portfolio reduces risk. Risk in his paper was measured by variance and investors were encouraged to minimize variance while seeking maximum returns. (Markowitz 1952) This idea of diversifying a portfolio into multiple assets can be expressed mathematically, since the expected return of a portfolio can be presented as a simple formula:

$$E\left(r_p\right) = \sum_{i=1}^{n} w_i \; E\left(r_i\right) \tag{1}$$

In this formula $E(r_p)$ is the expected return of the portfolio, n is the number of assets in the portfolio, w_i is the weight of an asset in the portfolio and $E(r_i)$ is the expected return of an asset. The expected return of a portfolio can be changed by adjusting the weight of assets in the portfolio or by changing the quantity of the assets. The most interesting question from the investor's viewpoint is how many assets the portfolio should contain for it to be sufficiently diversified. A lot of research has been done on this question and it has led to many different conclusions. Evans & Archer (1968) used a rule of

8-10 stocks but other researchers such as Chong & Phillips (2013) suggested a diversification into 30 or more stocks. Some researchers even investigated that 100 stocks or more are required to attain optimal diversification (Diyarbakirlioglu & Satman 2013). It is hard to determine what the appropriate number of assets should be because of the idiosyncratic risk of a portfolio. This topic will be more thoroughly discussed in the theory section related to the capital asset pricing model.

Combining assets into portfolios has the potential to lower the standard deviation (risk) to a level lower than what can be obtained through a straightforward weighted average calculation. This is made possible by correlation coefficients. To minimize risks, an investor must consider how a security correlates with the portfolio. It is in the investors' best interest to find assets with low correlation to the rest of the portfolio (Markowitz 1952). In terms of reducing a portfolio risk, it is also important to incorporate assets with a negative covariance between them. Covariance is a statistical metric that quantifies the directional relationship between the prices of two assets (Investopedia 2023). Modern portfolio theory employs this statistical measure to mitigate the overall risk within a portfolio. The formula for calculating the covariance between assets is as follows (Banton 2022):

$$cov\,(x,y) = \frac{\sum_{t=1}^{n}\left(R_{xt} - \overline{R}_x\right)\cdot\left(R_{yt} - \overline{R}_y\right)}{n-1} \tag{2}$$

Where R_{xt} is the return of the asset x at time t, $\bar{R}_{\mathrm{x}}$ is the average return of the asset x during the period, R_{yt} is the return of asset y at time t, $\bar{R}_{\mathrm{y}}$ is the average return of asset y during the period, and n is the sample size. When the covariance between assets is negative, it indicates that when one asset's return is positive, the other tends to move in the opposite direction, resulting in a negative return (Sands 1977). The covariance is an important metric, but it is much more useful when combined with correlation. There are various ways of calculating correlation. One method is that once the covariance is known, the correlation (ρ) between two assets can be calculated as follows (Banton 2022):

$$\rho = \frac{cov\,(x,y)}{\sigma_x \sigma_y} \tag{3}$$

Where the *cov(x,y)* is the covariance of x and y assets, σ_x is the standard deviation of asset x and σ_y is the standard deviation of asset y. Covariance and correlation are most useful when used together since covariance can indicate how assets move in relation to each other, while correlation evaluates the strength of this relationship (Banton 2022).

The idea of trying to find the optimal point of a portfolio risk and return led to what is called the efficient frontier. The efficient frontier is an investment tool that provides a representation of the optimal level of diversification that aligns with an investor's specified risk tolerance, aiming to maximize their potential returns. When a portfolio lands on the frontier it is considered that the portfolio is efficient, at least in theory. (Behan 2022) Portfolios that are outside of the frontier have excessive risk relative to their return.

A frontier can be formed from a portfolio with two or more securities. As mentioned before the efficient frontier is formed by the investors' individual risk tolerance. This means that there is no "one and only" efficient frontier. The shape of the frontier is also dependent on how the securities inside the portfolio correlate.

Markowitz and his modern portfolio theory have had a profound impact on the financial industry, despite that the theory has received some criticism. One area of criticism is the behavior of MPT during financial crises. The diversification of portfolio risk was ineffective in 2008 when every asset class was in a declining "bear" market (Warner 2010). The assumption of investors willing to accept elevated levels of risk only if compensated with greater returns has also been criticized. Often, investment strategies necessitate institutional investors to engage in what might be seen as a risky investment, with the aim of lowering overall risk without necessarily resulting in an increase in expected returns (Mangram 2013).

Jon Lukomnik and James Hawley (2021) criticized MPT in their book *Moving beyond Modern Portfolio Theory: Investing that Matters* for it only mitigating idiosyncratic risk. MPT does not take market risk into consideration, nor new systemic risks, such as climate change. In addition, they claim that in the real-world systematic risk has a much more significant impact on returns compared to risks associated with any specific security or firm. They specifically highlight the MPT lacking the necessary tools to address the modern real-world systemic risks effectively. This being an urgent issue for investors today. (Eccles 2021)

In spite of criticism, Harry Markowitz is considered to be one of the most important pioneers of financial economics. In his work he made concepts such as portfolio risk and return and diversification more popular. By doing this, he moved attention away from just the performance of individual assets.

2.2 Efficient Market Hypothesis

The capital market's main role is to distribute capital between suppliers and investors to find the most effective use for that capital. In 1970 Eugene Fama introduced the theory of "efficient markets" to the world. The article released in the *Journal of Finance* discussed mainly about the efficiency of capital markets and gave more insight to his earlier paper released in 1965. According to Fama when markets work efficiently, prices will completely reflect all available information. In his paper he concluded that the empirical investigation of the efficient market hypothesis (EMH) can be categorized into three groups based on the specific subset of information under consideration: *weak form*, *semi-strong form,* and *strong form*. (Fama 1970) These forms will be more thoroughly discussed in the following paragraphs.

In the *weak form* of market efficiency, the information set is restricted by historical prices and the current price incorporates all available information from previous ones. To verify this kind of efficiency, demonstrating statistical independence of prices or returns is adequate. Statistical independence means that prices are random variables that are independent. In conjunction with the identical distribution of prices this constitutes a random walk model. The weak form of the efficient market hypothesis is often described with this random walk model. To put it differently the market possesses a random walk characteristic. (Khrapko & Vladimir 2013)

Random walk theory suggests that asset prices move completely at random, and future prices cannot be predicted based on historical prices. This theory challenges the idea of timing the market or using trends to profit from future stock prices, since tomorrow's prices are independent from today's prices. It also implies that outperforming the market is unattainable without accepting extra risk. (Smith 2023) Random walk theory is interesting from the viewpoint of the low volatility anomaly. The theory of random walk is in contradiction to the anomaly since low volatility assets have been consistently outperforming assets with higher volatility.

The *semi-strong form* presumes that current asset prices quickly adjust to the release of all new public information. In semi-strong form all public information has been priced in and moves that are occurring in the market are reflections of that information. Of all the EMH hypothesis, this form is appraised to be the most practical. (Chen 2022) This form also concludes that technical analysis cannot be used to predict future prices. In efficient markets, the reaction to new public news occurs immediately and accurately making the price rise (or fall) instantly and reaching precisely the level required by the news. In the case of markets reacting slowly, investors may not immediately understand the value of the new information and prices will eventually settle to the accurate price. Figure 1, where the horizontal axis presents time, and the vertical axis presents return, provides an example of a stock price's efficient and slow reaction to market-positive information.

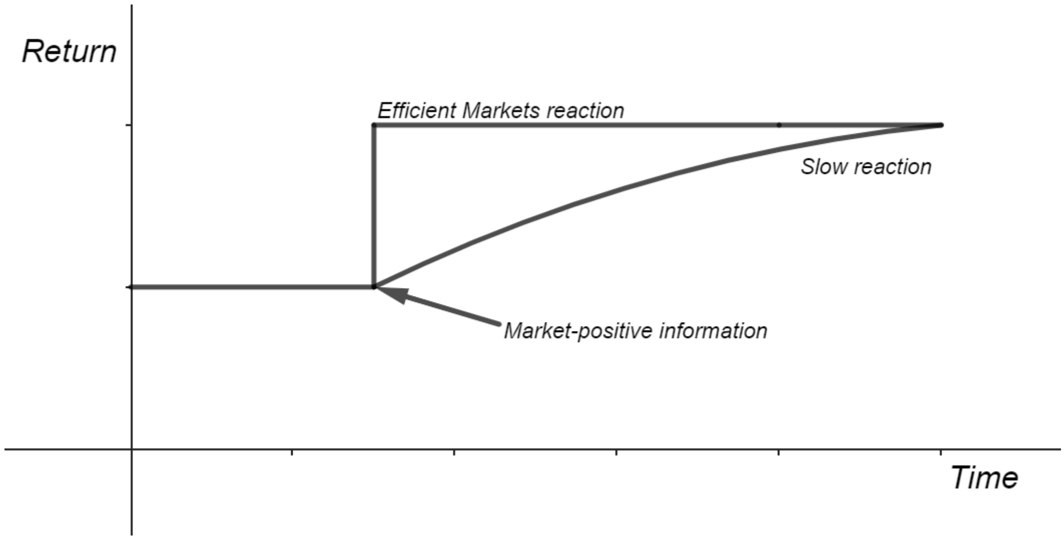

Figure 1. Different reactions to market-positive information.

The *semi-strong forms* weakness is that it cannot explain material nonpublic information (MNPI) that is deemed advantageous for trading. (Chen 2022) For a better understanding, a simple example is provided to understand how MNPI is useful to investors. Company X stock is trading at 20€ a day before earnings. On that day negative (public) news about the company comes out, lowering the stock price to 18€. Despite this, tomorrow after the earnings release company X stock is trading at 21€ because they outperformed the predicted earnings by making an effective strategy to increase revenue. In this case if material non-public information of the revenue strategy had been public for investors, they would have made a good surplus.

The *strong form* of the EMH assumes that all publicly available information, historical information and non-public information is entirely reflected in the security's price. (Finnerty 1976) This means that no individual has an advantage in the market, as all information is available to everyone at the same time. Even insider information cannot be used to achieve profits as all information is already priced in the market.

Like all theoretical constructs, the Efficient Market Hypothesis also rests upon a set of underlying assumptions. One assumption being that investors are assumed to be rational and make choices based on maximum expected utility. If this is not the case and investors behave irrationally it should be presumed that their market behavior is random, balancing any impact on prices. (Naseer & Bin Tariq 2015) EMH also assumes that information in the market is perfect and that there are no transaction costs (Gilson & Kraakman 2003). These three key assumptions of the EMH are based on the markets being perfect when in fact they aren't. That is why EMH has also received its fair share of criticism.

One argument against the Efficient Market Hypothesis is that some investors, for example Warren Buffet and George Soros have outperformed the market consistently by investing in undervalued assets (Becker 2021). This is something that is not supposed to be possible. Another area of criticism is the fact that investors don't always behave rationally in the markets, as behavioral science has a large impact on investing (Schleifer 2000). The availability of information has also been criticized since information can be costly. Prices cannot flawlessly reflect the accessible information, as doing so would leave investors who invested resources in obtaining that information without any recompense (Grossman & Stiglitz 1980).

Another criticism of EMH is the assumption of the market not having transaction costs, since in the real market transaction costs cannot be avoided. Total transaction costs including execution costs and commissions have a noticeable effect on the market (Berkowitz, Logue & Noser, Jr. 1988). The most important criticism from the point of this thesis is the appearance of anomalies. According to EMH, anomalies shouldn't appear in the markets, at least not for long time periods. Yet, various empirical studies have proved the existence of multiple anomalies such as the low volatility anomaly or the January effect that have been in the market for decades.

Despite criticism, the efficient market hypothesis provided substantial framework and insight for understanding asset prices and led to a significant amount of further research into their behavior. Additionally, it had a major impact on the field of behavioral finance, as the underlying assumptions of EMH are in contradiction to behavioral psychology. The EMH also has various empirical studies supporting it. Even Michael Jensen (1978) claimed to believe that there is no other proposition in economics that has more reliable empirical evidence in support of it than the Efficient Market Hypothesis.

2.3 Capital Asset Pricing Model

In his work on the modern portfolio theory, Markowitz (1952) introduced the concept of investors diversifying their holdings across multiple assets to enhance their risk-return tradeoff. It was also acknowledged that combining securities into a portfolio is an efficient way to decrease risk. Historical financial literature has mainly segmented total risk into two distinct categories: idiosyncratic risk and market risk (systematic risk) (Shahzad, Fareed & Wang 2020).

Idiosyncratic risk is something that investors can dispose of by diversifying their portfolio efficiently, leaving the portfolio risk to consist purely of market risk. This diversification of idiosyncratic risk is presented in Figure 2, where the variance of the portfolio decreases as the assets of the portfolio increase. Covariances and correlations between assets are assumed to be optimized.

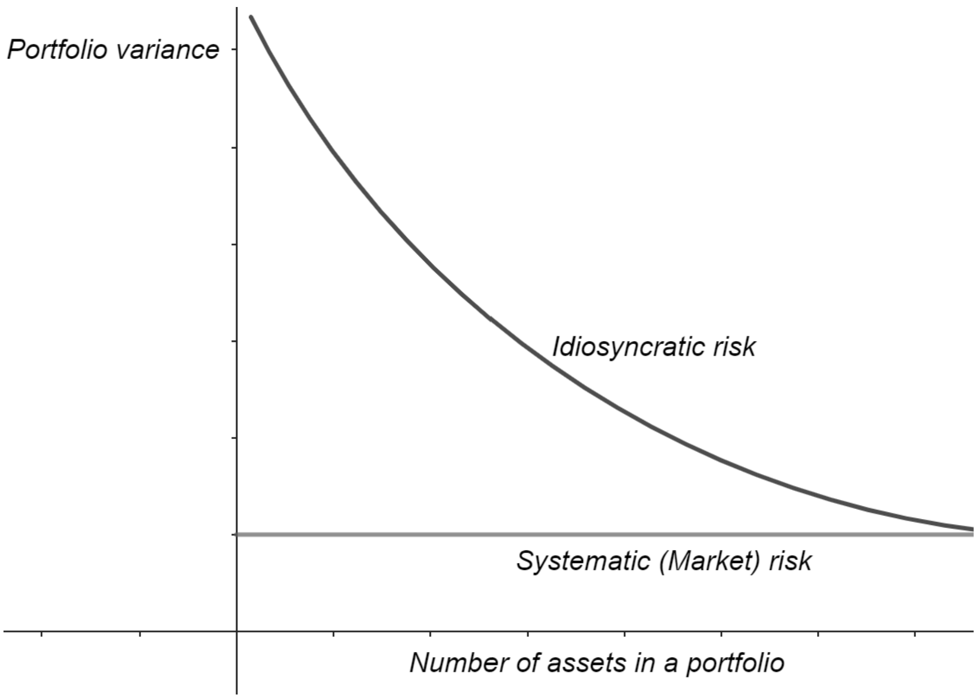

Figure 2. Behavior of portfolio-risk, as number of assets increase.

As investors were able to eliminate the idiosyncratic risk, interest arose in how to measure the systematic risk. There was a demand to develop a risk metric that measures the portion of risk that remains with the asset after diversification. This risk metric was developed and is known as beta (β). Beta is used to measure systematic risk of a portfolio or a security in relation to the broader market (Kenton 2022). The beta of a stock can be calculated with the following formula:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \tag{4}$$

In this formula σ_{im} is the covariance of the return of the stock *i* and the return of the market, and σ^2_m is the volatility of the market portfolio. If a stock is a part of the S&P 500, then S&P 500 is used as a benchmark, meaning that its beta is 1 (Kenton 2022). When a stocks beta is 1,1 this means that the stock price changes 1,1 times that of the S&P 500 (DeNicola 2022). A beta greater (less) than 1 means that the investment is more (less) volatile than the market (Insana 2022).

Investors demand returns based on systematic risk, as it is the only source of risk after diversification. This is the idea on which the capital asset pricing model is based on. The capital asset pricing model (CAPM) was a groundbreaking theory that had a major effect on the financial world. This theory was introduced in the early 1960s by the works of John Lintner, Jack Treynor, William Sharpe, and Jan Mossin. (Perold 2004) The primary outcome of the model is an assertion regarding the connection between the anticipated risk premiums associated with individual assets and their "systematic risk" (Black, Jensen & Scholes 1972). According to the model, the investment's required return is determined as follows:

$$E\left(r_i\right) = r_f + \beta\left[E\left(r_m\right) - r_f\right] \tag{5}$$

Where $E(r_i)$ is the expected return of investment *i*, r_f is the risk-free rate of return, β is the market beta of asset *i* and $E(r_m)$ is the return of the market (Elbannan 2015). The risk-free rate used by academics and professionals is usually the rate of long-term Treasury bonds or short-term Treasury bills (Mukherji 2011). It is clear to see from the formula that when beta increases, so does the expected return of an asset. Therefore, the capital asset pricing model establishes that risk (beta) and return have a positive correlation.

The capital asset pricing model was a breakthrough in finance, and decades later it is still widely used. In MBA level investing courses, CAPM is frequently the sole asset pricing model taught. Today the model is applied in various contexts, such as appraising the effectiveness of managed portfolios and approximating the cost of equity capital for companies. (Fama & French 2004)

Despite CAPM being widely used and respected, it has received plenty of criticism in numerous studies. An assumption of the model is that investors can borrow or lend money at a risk-free rate, which is uniform for all investors and unaffected by the lent amount (Elbannan 2015). Black (1972) criticized the possibility of borrowing unlimited money at a risk-free rate, calling it unrealistic. Market betas have also faced criticism for being imperfect and inadequate in accounting for fluctuations in expected returns (Ball 1978). Fama & French (2004) criticize the assumption of investors focusing purely on risk and return of one-period portfolio returns rather than considering how their portfolio's return aligns with their income and future investment prospects.

Studies have revealed that relying solely on the S&P 500 or other simplified proxies for the true market portfolio has resulted in persistent pricing inaccuracies within the CAPM framework. Consequently, certain stocks and stock portfolios consistently yield returns that deviate from what is predicted by CAPM (Berk, DeMarzo & Harford 2019). The low volatility anomaly is an example of one of these deviations that challenges the CAPM's predictions.

According to CAPM, expected returns on assets are directly related to their beta (systematic risk), implying that riskier assets should offer higher returns. However, the low volatility anomaly contradicts this prediction since lower risk stocks have sometimes provided higher risk-adjusted returns than expected by the CAPM. This observation has raised concerns about the model's accuracy in describing real-world stock market behavior, as it fails to account for some empirical phenomena, one of them being the low volatility anomaly.

In spite of its criticism CAPM is a commonly employed return model that can be readily computed and subjected to stress testing. Additionally, CAPM takes systematic risk into account, an aspect omitted by other return modes, such as the dividend discount model. (Zucchi 2021)

2.4 Japan as a research market

When discovered, the low volatility anomaly was mainly studied in the United States markets. After studies in the U.S., researchers were interested in finding out if the anomaly existed in other markets as well. In 2007 empirical evidence for the anomaly in different markets was presented, as David Blitz and Pim van Vliet (2007) observed the low volatility effect in European, U.S. and Japanese markets.

Japan is an interesting market to study this anomaly for multiple reasons. One reason is its relevance today as a well performing market. The history of returns in the Japanese market has been devastating since the 1990s, as Japan experienced its "lost decades" in returns. The Nikkei 225 index that is a stock index consisting of 225 publicly traded Japanese companies reached its peak on the last trading day of 1989. Since that day, it took the index 32 years to reach its all-time high again, after the asset price bubble in Japan collapsed in 1990. (Zhang 2022) This means that investors that invested in the Japanese market in the end of 1989 were on negative returns for over 30 years.

Despite its history, Japan has been performing very well compared to the U.S. and European markets in 2023. The Nikkei 225 has risen almost 30% from January to August providing a substantial return, as the U.S stock market has an average annualized nominal return of around 9 – 10%. (Murai & Schnabl 2023; Zhang 2022) This performance can be partially explained by Prime Minister Fumio Kishida's "new capitalism" initiative in 2022. The endeavour, titled *Grand Design and Action Plan for a Novel Capitalistic Approach,* set forth ambitious objectives, advocating for heightened investments in start-ups, people, science, digital and green transformations, technology, and innovation. (Okina 2023)

One reason can be the U.S. and European interest rate rises compared to Japan, as the Japan's average credit interest rate in June 2023 was 0.7% and 10-year government yield was under 0.5% (Murai & Schnabl 2023). Warren Buffet's visit to Japan and Berkshire Hathaway's purchase of more shares in major Japanese trading houses in 2023 could be seen as an endorsement for other investors to look at the investment opportunities in Japan. As a whole, the awaking of the Japanese market is a promising indicator for investors, suggesting that there might be better times ahead for a market that has endured decades of challenges.

Japan is one of the biggest markets in the world, being the second biggest country behind United States in total world equity market value (7.4%). Japan ranks even ahead of China (5.1%) and the United Kingdom (4.1%). (Dimson, Marsh & Staunton 2021) Despite this, the primary focus of this anomaly has been on the U.S. and European markets. The overall interest towards Japan's market has risen substantially among institutional investors and for that reason it is also important to study the existence of the low volatility anomaly. U.S. institutional investors have started to investigate Japan's market more closely and wanting more exposure to Japan, as it might offer better expected returns than the U.S. (McElhaney 2023).

Japan is an interesting market to conduct the research also for its high average volatility. United States being one of the most studied markets is also one of the least volatile, having a standard deviation of 19.9% in 1900-2022, according to Credit Suisse. During that period Japan ranked being one of the highest volatile countries in the world with a standard deviation of 28.9%. (Dimson, Marsh & Staunton 2023) All of these reasons provide a strong foundation for investigating the existence of the low volatility anomaly in the Japanese market.

3 The low volatility anomaly

This chapter is the second part of the framework for this thesis. It investigates the existence of the anomaly and reasons behind it, such as the behavioral aspect of investors. Investment strategies related to the anomaly will also be discussed in this chapter.

The phenomenon known as the low volatility (or low beta) anomaly was first observed in the early 1970s. Robert Haugen & James Heins (1972) produced a paper in which they concluded that stock portfolios with lower variance in monthly returns demonstrated higher average returns compared to their "riskier" counterparts. After this, interest in the subject arose and multiple empirical studies on the anomaly have proved its existence across diverse stock markets globally, including both developed and emerging markets.

The low volatility anomaly refers to lower risk (volatility) assets outperforming higher risk assets over an extended duration (Dunn 2019). It is an anomaly, as it defies traditional finance theories, such as the capital asset pricing model. Black, Jensen & Scholes (1972) also discovered the anomaly as they found average returns of low beta portfolios to be greater than predicted by the capital asset pricing model. This means that the security market line of the CAPM exhibited a flatter slope in empirical studies than was anticipated.

In 1981 Rolf Banz discovered that small companies' stocks demonstrated greater returns compared to large companies' stocks, even when accounting for the fact that small stocks on average carry higher risk than large stocks. This marked the first significant challenge to the CAPM. (Banz 1981) The problems with CAPM started to become clearer in the 1990s, when Fama & French (1992) discovered that when market beta and size are adequately separated, the market beta has no pricing influence in the cross-section of stock returns. Anomalies like value or size show that the CAPM needs to be increased and slightly modified with other factors, whereas the low volatility anomaly challenges the whole concept of CAPM.

A year after the findings of Fama & French, Fischer Black (1993) discovered that the relationship of return and beta had become flat in the years after Black, Jensen & Scholes initial study in 1972. The flatness meaning that greater beta did not result in more return. Falkenstein (1994) went as far as reporting this relationship as being negative, particularly when implementing the necessary control for the effect of size.

One of the most influential studies in the history of the low volatility anomaly is the paper *The Volatility Effect* by Blitz & van Vliet (2007). They noticed that the connection between return and risk is not just flat but, in fact, inverted. This study had a timeframe of 20-years, and it discovered the anomaly to also be present in other markets than just the United States. The importance of this paper was the evidence that this anomaly exists across countries and industries. This study, being conducted during the period from 1986 to 2006, affirmed that the anomaly has become stronger than it was in previous studies. (Blitz & van Vliet 2007)

Baker, Bradley & Wurgler (2011) later concluded that there has been a substantial underperformance in high volatility stocks compared to lower volatility stocks. Their study consisted of 1000 U.S stocks and their performance over the 1968-2008 period. The researchers also concluded the same as Blitz & van Vliet (2007), that during recent years the anomaly has become stronger. Multiple other academics, such as Baker & Haugen (2012) came to the same conclusion, even when using different interpretations for risk.

In 2016 the anomaly was tested and found in the stock market of India. Joshipura & Joshipura (2016) proved the anomaly existing while using a timeframe from 2001 to 2015. They defined risk with standard deviation and beta, and took into account factors of momentum, size, and value.

Two years later the anomaly was also discovered in the Chinese market by Han, Li & Li (2018). The researchers concluded that the security market line of CAPM is descending in China. This result was fascinating, as it was different from the flat line that was observed in the empirical tests conducted in the United States market. They also claimed that because of this finding, investing in low beta assets would have been more beneficial in China comparatively to United States in the timeframe from 1996 to 2016.

The low volatility anomaly has also been studied using data from the early 1900s. A study done in the Brussels stock exchange by Annaert & Mensah (2014) used stock data starting from 1873 and ending in 1914. The study unveiled the anomaly clearly existing in the market even before the First World War. During that time Brussels exchange was one of the largest stock markets in the world.

The phenomenon of low volatility and low beta is evident even among highly liquid and large stocks in the United States. This discovery was made by Auer & Schuhmacher (2015) as they studied the existence of the low beta anomaly in the most liquid stocks of the Dow Jones Industrial Average spanning from 1926 to 2013. In terms of these assets' performance, they uncovered compelling evidence pointing to an inverse relationship between risk and return. This result stayed the same even when considering transaction costs. The evidence showing the anomaly existing in large cap stocks makes it easier for investors to benefit from it. Numerous anomalies are associated with small cap stocks, posing challenges for investors seeking to capitalize on them.

Research on the low volatility anomaly has been ongoing since the 1970s, as it remains a persistent phenomenon in financial markets. That is why multiple studies regarding this subject have also been conducted in recent years. There is a scarcity of studies that specifically use the Japanese market as a research market, despite its immense impact on the global economy. This provides a compelling reason to study the anomaly in the Japanese stock market.

3.1 Explanations for the anomaly

There are multiple explanations for the existence of the low volatility anomaly. One reason being the borrowing constraint. Black (1972) interpretated that leverage restrictions as margin requirements can result in low beta assets outperforming. Investors may not have access to leverage, as it is costly and unachievable for some. This directs investors to invest into higher volatility assets in the hopes of higher returns. As a result of this, lower volatility assets become underpriced. Blitz & van Vliet (2007) also claimed leverage constraints to be a plausible explanation for the anomaly.

The investors' desire to beat the index is a substantial reason for the existence of the anomaly. Alongside individual investors are institutional investors (professional investors) that have a significant effect on the markets. Institutional investor's typically have a mandate to outperform a fixed benchmark, discouraging investments in low beta stocks (Baker, Bradley & Wurgler 2011). As a result, high beta and high volatility stocks are preferred, making the lower volatile assets underpriced. Karceski (2002) observed that mutual fund investors often pursue returns over time, prioritizing their desire to outperform during bull markets over avoiding underperformance in bear markets, resulting in an increased demand for high beta stocks.

Baker & Haugen (2012) believe the low volatility anomaly to be mainly caused by "agency issues" and "manager compensations", which on average compel institutional investors to maintain a greater proportion of volatile stocks. They presented manager compensation and agency issues being (1) between clients and professional investors and (2) among investment managers within an organization. Managers get compensated with a bonus when the performance of their portfolio is acceptably high. This drives managers to recommend a more volatile portfolio to a client. Baker & Haugen (2012) represented the option-like manager compensation as the following figure:

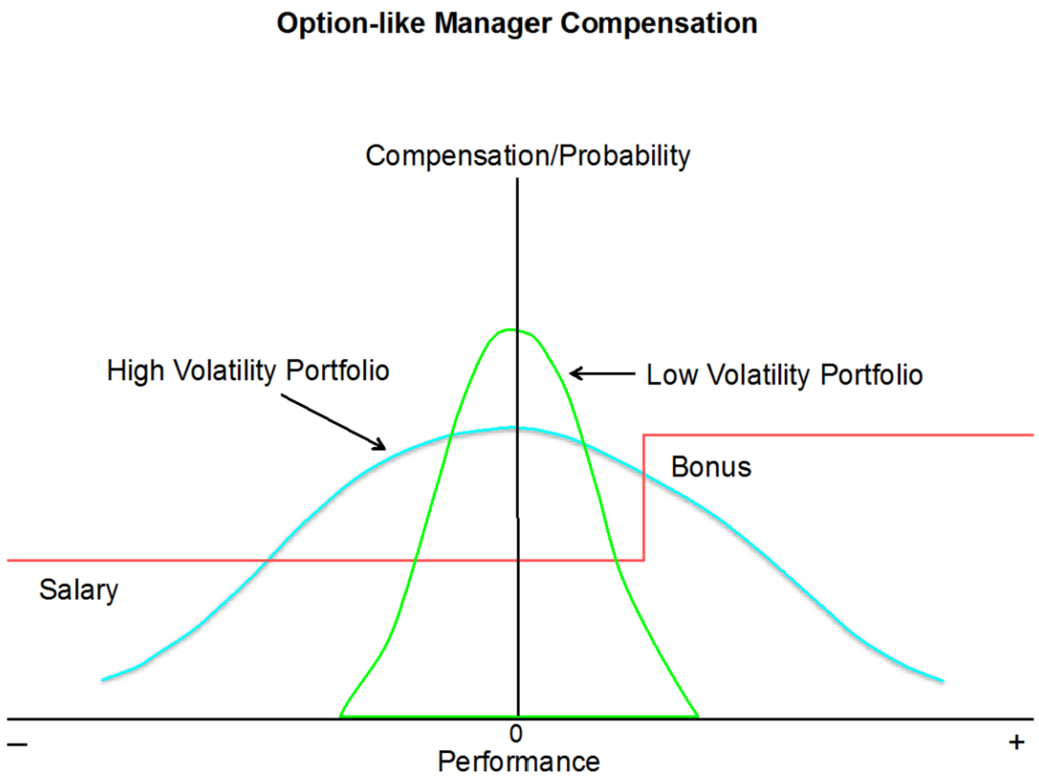

Figure 3. Option-like Manager Compensation model. (Baker & Haugen 2012)

Where probability distributions for low volatility and high volatility portfolios are shown. This figure shows the compensations expected value increasing when the manager addresses the high volatility portfolio to the client.

The central issue of compensation among investment managers lies in the dynamics within an organization. During periodic investment committee meetings, analysts present their stock recommendations to the Chief Investment Officer, and from these a model portfolio is designed as a guide for client portfolios. To make a strong impression and secure their position, analysts tend to favor high-profile stocks with substantial media coverage. Due to the influx of new information regarding these stocks, they tend to display above-average levels of volatility. (Baker & Haugen 2012) These stocks are later on recommended to the clients. Consequently, lower volatility stocks are once again being undervalued.

3.2 The perspective of behavioural science

Behavioral science has an important role in finance, and it has been listed as a reason for the existence of the low volatility anomaly by nearly all researchers that have studied the anomaly. Behavioral science studies the psychological factors that influence individuals' financial decisions. Most financial theories, such as the CAPM and EMH, expect investors to behave rationally, when in fact they do not. The irrational behavior of investors is one of the main reasons for the existence of the low volatility anomaly. Baker, Wurgler & Bradley (2011) presented that there are three investor biases leading to an irrational preferencing towards volatile stocks: overconfidence, a preference for lotteries, and representativeness.

Overconfidence is when investors believe they have exceptional abilities in selecting individual securities. Assets that overconfident investors choose tend to be highly volatile, as the top winners and biggest returns are among those assets. This behavior results in highly volatile securities being overpriced. (Cornell 2009) According to research, overconfident investors also tend to underestimate risks associated with some investments, leading them to invest into high volatile assets without understanding the consequences (Hayes 2023).

A preference for lotteries is when investors exhibit a desire for assets that resemble lottery-like payoffs (highly skewed returns). By virtue of this eagerness for fast and greater profits, investors tend to overweight small probabilities and underweight large probabilities. (Barberis & Huang 2008) This contributes to the outperformance of low volatility assets, as they are generally more stable.

Purchasing a low-priced, volatile stock resembles buying a lottery ticket. There is a chance of it tripling, quadrupling, or much more within weeks, but a considerably higher likelihood that it will decrease in worth. (Baker et al. 2011) Baker et al. (2012) also concluded that a significant segment of investors still tends to favor risky stocks despite probabilities. They believe that this lottery-like investing leads to an overvaluation of high volatility stocks, ultimately resulting in lower future returns.

In behavioral science and finance "representativeness" refers to the tendency of individuals to make decisions based on how closely an event resembles a stereotype (Tversky & Kahneman 1974). Representativeness can lead to stock selection bias, where investors have a mental idea of what a "profitable" and "good" stock looks like. These stocks are often high growth and high volatility, as they are the ones that get the most attention. For instance, investors may be willing to invest in new highly volatile Electric Vehicle (EV) stocks, such as NIO, as they categorize the stock with the success of Tesla. In this example NIO is the event and Tesla is the stereotype.

One behavioral explanation for the anomaly is the fact that investors today are in an even bigger hurry to be wealthy. This is fueled by the effect of media and social media, as stocks and other securities that have made people millionaires in a matter of days are discussed or "hyped" to retail (non-professional) investors. This is made worse by the possibility of anyone without qualifications recommending stocks to other investors through different social media platforms. As a result, multiple investors are investing in extremely volatile assets without knowing what they are doing.

Over the past seven years, a massive shift in trading volume has occurred from the cash market to the options market. In addition, the data shows that in 2022, the count of distinct individual trades in the derivatives market rose by more than 500% in comparison to 2019. (Modi 2023) This indicates that highly leveraged and volatile financial instruments have increased in attraction. Consequently, investors that do not have access to derivatives tend to seek greater returns by investing in higher volatility assets.

3.3 Investment strategies related to the anomaly

As anomalies in the market exist, there are always investors trying to benefit from them once noticed. This is also the case with the low volatility anomaly, as investors have made strategies around this phenomenon. The most known investment strategies regarding the anomaly are betting against beta (BAB) and minimum volatility investing.

Betting against beta is a financial strategy, which involves investing in assets with low beta. This strategy derives from the fundamental concept of Blacks' (1972) beta-arbitrage, and it was first studied by Frazzini & Pedersen (2014). They presented a betting against beta factor that consists of two parts: (1) long leveraging low beta assets and (2) shorting high beta assets. This strategy is based on the empirical findings that low beta assets have outperformed high beta assets in the market.

One explanation for the outperformance of low beta assets is leverage-constrained investors overweighting assets with higher betas to obtain a higher expected return than the market. This action increases the low beta assets future returns and decreases the future return of high beta assets. (Frazzini & Pedersen 2014) Betting against beta is related to the low volatility anomaly, as it also claims that low-risk assets outperform high-risk assets. The difference in this strategy is that it uses beta as the measure of risk.

Robert Novy-Marx & Mihail Velikov (2022) found fault in the BAB factor for it using a “rank-weighting” strategy instead of weighting based on market capitalization, as is the standard. In addition, they criticized the hedging by leveraging method that Frazzini and Pedersen (2014) utilized in their study. The strategy has also been criticised for not considering transaction costs. When taking transaction costs into account, the profitability of BAB experiences a substantial reduction (Swedroe 2022). Despite criticism, the BAB investment strategy has offered more insight into the relationship between risk and return. It is also a strategy being used by a large hedge fund “AQR” that has over a 140 billion dollars in assets under management (Swedroe 2022).

Another strategy related to the anomaly is minimum volatility investing. This strategy seeks assets to invest in that have volatility lower than the market (Smart 2023). Investors can have their own criteria on how they define minimum volatility and how a minimum volatility portfolio is constructed. For example, BlackRock emphasizes the examination of individual stock volatility and correlations when evaluating a minimum volatility portfolio (Smart 2023). As a whole, there are various securities that invest in low volatility assets that are available to investors, such as low volatility ETFs for multiple sectors and markets in the world.

4. Data and research method

This chapter will consist of discussing the market data that is used in conducting the research. The construction of the portfolios is also explained in this chapter. This chapter will also elucidate the research method applied, and the different metrics used to conduct the research. These methods include volatility and the risk-adjusted methods of Sharpe ratio and Jensen's alpha.

4.1 Market data

This thesis uses the monthly market capitalizations (market caps) of companies that are listed in the Japanese exchange during the timeframe of 1.1.2002- 31.12.2022. As this study is for a 20-year period, the data from the year 2002 volatilities is needed to construct the portfolios for the year 2003. This means that the portfolios will be constructed from the year 2003 onward, and this study will follow the performance of the constructed portfolios in the period of 1.1.2003- 31.12.2022.

This research contains in total 1974 stocks market caps from the 20- year period. The market cap data for these stocks is gathered from the Refinitiv Workspace database. The market cap data gathered consists of each companies every month's last days closing market cap. Portfolios are reconciled annually after the close on the last trading day of the year. The settlement is done by using the volatility of last year's stocks. This methodology makes it possible to examine the performance of the low volatility investment strategy. Monthly data is used instead of daily data due to the extensive number of stocks under consideration and the prolonged duration of the study. Using daily market cap data would result in an unwieldy volume of data points (approximately 11 million). The market capitalization data obtained from Refinitiv Workspace exhibits various missing datapoints, yet these have been organized and removed.

Company market caps are utilized in this research instead of stock prices. The reason for this is that the stock price data may not take stock splits into consideration. This problem does not occur when using market capitalization for calculating returns and volatilities. Following a stock split, the stock price undergoes a reduction due to the increased number of outstanding shares. In a 10-for-1 split, for instance, the share price is one-tenth of its price after the split. Despite the alterations in the share price and the number of shares outstanding, the company's market cap remains constant. This is because market capitalization is calculated by multiplying the stocks share price by the total number of shares outstanding (Fernando 2023). Using market capitalization also considers stock dividends.

Cash dividends are not taken into consideration in this study, as market capitalization does not account for cash dividends shared by companies. This is not a significant concern, given the substantial number of stocks included in this study. Consequently, the individual companies that share cash dividends should not have an immense impact on the overall results of the data. This problem would also occur if using stock price data, as it similarly fails to incorporate cash dividends.

A risk-free rate is needed to calculate the Sharpe ratios in this thesis. There are different options for selecting a risk-free rate in research and Sharpe ratio calculations. This study employs the U.S. 10-year treasury rate, as it is the one of the most widely traded bonds in the world. The development of the 10-year treasury rate during the study period is displayed in Appendix 1.

4.2 Research method

The empirical part of this study is conducted by using quantitative methods. The employed research methodology will examine and compare the distinctions among the constructed portfolios. Evaluation criteria will include the portfolio returns and risk-adjusted returns. Additionally, a market index will also be constructed from all the available stocks in the Japanese market, and comparative analyses will be conducted between this index and the portfolios.

The following discussion will focus on the construction of the portfolios. The data collected from the Refinitiv Workspace database displayed the Japanese universe consisting of approximately 4000 publicly traded companies (stocks). Starting from the year 2002, most of these stocks did not show any datapoints, meaning that they were N.A values. After filtering and removing the N.A values from the data, the number of stocks with available market capitalizations decreased to 2072.

In this research the smallest market cap stocks (micro caps) are removed due to their liquidity issues and the lack of accuracy of the data. This was done by firstly ranking the 2072 stocks by their market capitalization in a descending order. After this, 5% of the smallest market capitalizations were removed. After removal, the number of stocks decreased to 1974, with the smallest market cap of these stocks being around 1.1 billion yen (between 7 and 10 million dollars depending on the JPY to USD rate).

Consequently, the number of stocks from which the portfolios are constructed is 1974 stocks. This amount is fixed throughout the research period. Therefore, even if there would be more stock data available, for example for the year 2020, the number of stocks that are a part of this study in 2020 is still 1974 stocks. Thus, the stocks can change every year, but the number of stocks stays the same. This allows the microcaps to be discarded from the research for all the study years. The market capitalization ranking is done every single year at the beginning of the year. This also makes sure that the stocks used for this research are relatively liquid.

As the number of stocks stays the same throughout the period, but the stocks are ranked by market capitalization, the smallest market cap stock changes (grows) every year. In 2002 the smallest market cap considered is slightly under 10 million dollars, and in 2022 it is approximately 100 million dollars. This indicates that the research also includes all small market capitalization companies. For instance, Nasdaq Composite categorizes "small market caps" to be within the range of 300 million to 2 billion dollars (Nasdaq 2023).

From the volatilities of the 1974 stocks, two portfolios are constructed: a low volatility portfolio and a high volatility portfolio. The volatilities of all stocks are calculated once a year at the end of the year, and from these volatilities the two portfolios are constructed for the next year. The portfolios are formed by using the 30th and 70th percentiles as breakpoints, as this method has been used in several prior studies. As a result, 30% of the stocks with the lowest volatility make the low volatility portfolio and 30% of stocks with the highest volatility make the high volatility portfolio.

The sub question of this thesis is to examine if there are differences in the performance of portfolios constructed from small- and large market capitalization companies. To study the difference between market capitalization and volatilities, four portfolios were formed. These portfolios are also constructed with nearly the same premises that were used in the making of the previous portfolios. The difference being that the stocks were first market cap ordered and divided using the 30th and 70th percentiles into a small cap portfolio and a large cap portfolio. Inside these portfolios a split was made based on the volatilities of the companies. This meant that the large cap portfolio was further divided using the 30th and 70th percentile method into large cap high volatility and large cap low volatility. The same was done with the small cap portfolio. A visual representation of this division is presented in Figure 4.

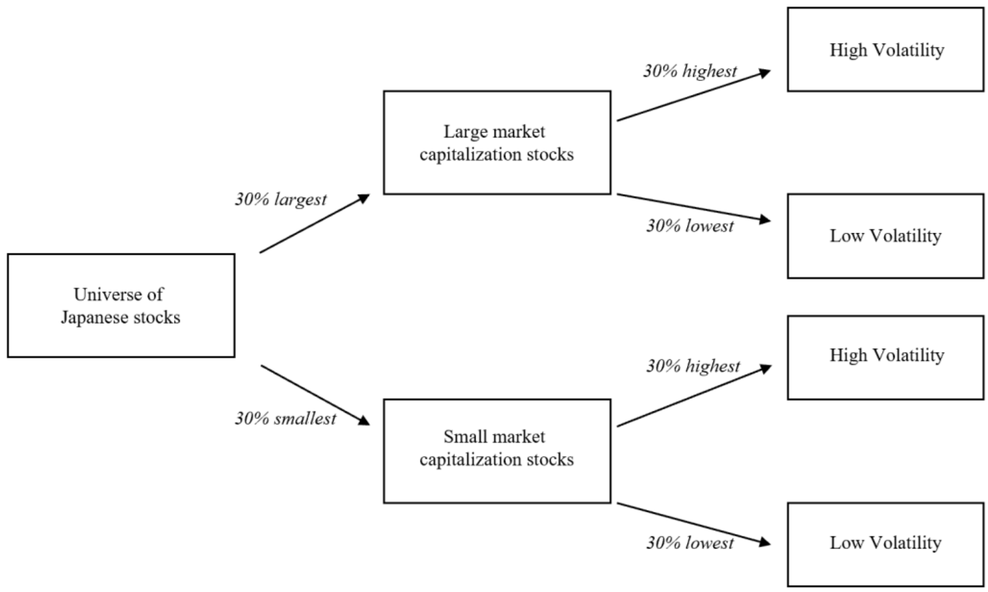

Figure 4. Portfolios constructed based on market capitalization

The majority of indices are market capitalization weighed, meaning that the companies with a higher market cap have more weight in the index. To not give some stocks too much dominance, an equally weighted portfolio must be constructed. As a result, the index and the portfolios formed in this thesis are equally weighted. The performance of these portfolios will be measured by comparing their returns, Sharpe ratios, volatilities, alphas, and betas. These metrics will be elucidated in the following subchapters.

4.2.1 Volatility

Volatility is a statistical measure of risk that describes how a securities price moves around its mean price. It is often presented as standard deviance, but it can also be presented with other means. For that reason, it is always important to mention what instrument is used for "volatility" in a study. In this thesis, standard deviation is used to describe volatility. If an assets volatility is higher, it is considered riskier.

Volatility can be divided into implied (future) volatility and realized (historical) volatility. The difference between these types of volatility is that realized volatility measures historical price changes, while implied volatility measures how volatile the future market will be. Implied volatility is important for some investment strategies, such as options trading, but it cannot be used to measure investment strategies based on historical volatility. This is why historical volatility is utilized in this thesis to be able to study how the low volatility investment strategy performs. The formula for standard deviation is the following (Hargrave 2023):

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \tag{6}$$

Where σ is the standard deviation, x_i is the value of point i in the set of data, $\bar{x}$ is the data sets mean value, and n is the number of data points in the entire data set. This thesis uses monthly datapoints, meaning that the volatility calculated using the market capitalizations is monthly volatility. To convert the monthly volatility into annual volatility, the monthly volatility is multiplied by the square root of twelve, as there are twelve months in a year. This method of transforming monthly volatility to annual volatility is also utilized by Morningstar.

4.2.2 Sharpe ratio

Sharpe ratio is a performance measure used to evaluate risk compared to investments return. This ratio is used on portfolios to measure the ratio of a portfolio's return to its risk. It was introduced to the world by William Sharpe in 1966 and was first known as the *reward-to-variability* measure. (Sharpe 1994) The mathematical expression for calculating the Sharpe ratio (S_i) is the following:

$$S_i = \frac{r_p - r_f}{\sigma_p} \tag{7}$$

Where r_p is the return of the portfolio, r_f is the risk-free rate and σ_p is the portfolio's standard deviation. The Sharpe ratio is a good performance measure as it takes both systematic and idiosyncratic risk into consideration. When comparing portfolios, a higher Sharpe ratio means that the portfolio has been performing better.

4.2.3 Jensen's alpha

Jensen's alpha is a risk-adjusted metric that measures the performance of a portfolio. This measurement was first introduced by Jensen (1967), and it is based on the capital asset pricing model. Jensen's alpha measures how portfolios return deviates from the prediction of the capital asset pricing model. (Jensen 1967) Jensen's alpha is mathematically presented as the following formula:

$$\alpha = R(i) - R(f) + \beta (R(m) - R(f)) \quad (8)$$

Where *R(i)* is the return of a portfolio, *R(m)* is the return of the market, *R(f)* is the risk-free rate, and *β* is the beta of the portfolio. If a portfolio's alpha is positive, then the portfolio is outperforming the capital asset pricing model, and it has a return even higher than the risk adjusted return. The higher the alpha, the better the outperformance.

5. Empirical results

This chapter provides a summary of the performance of the constructed portfolios and discusses the results derived from empirical research. The performance of the low volatility and high volatility portfolios are displayed first, followed by an examination of the performance of portfolios based on different market capitalizations. In this chapter all the constructed portfolios are analyzed based on their overall returns, annual returns, volatility, Sharpe ratios, betas, and alphas.

Table 1 illustrates the results of the performance of low volatility and high volatility portfolios alongside the market index in the 2003-2022 period. These portfolios concentrate on how the low volatility investment strategy performed during the study years. Of these two portfolios, the high volatility portfolio performed better than the low volatility portfolio in overall returns, having a return of 753,07% from the 20-year period. During this period the low volatility portfolio achieved an overall return of 300,14%, underperforming the market index that achieved an overall return of 451,52%.

When comparing the annual returns (compound annual growth rate) between the portfolios and the market index, the high volatility portfolio had the highest annual return of 11,31%. The market index had the second highest return of 8,91%, and the low volatility portfolio had the lowest annualized return of 7,18%.

The outperformance of the high volatility portfolio can be partially explained by the "bullish" performance of the overall market during the study period. This means that during this period, the market was mainly experiencing a strong upward trend. The years 2000-2002 the Japanese market was in a downtrend, alongside various other markets due to the burst of the "dot-com bubble" in the year 2000. In addition, the Japanese market was in a recession starting from march of 2000 (Magnier 2000). The market experienced a substantial decline over these years but stabilized at the start of 2003.

Starting from the second quarter of 2003 the market gained large returns that it had lost in the previous years. This is also visible from the results of this study, as the market index rose 48,45% in 2003, 32,38% in 2004, and 63,85% in 2005. To gain a better understanding of how the market performed, the performance of Nikkei 225 is displayed in Appendix 2. The Nikkei 225 consists of 225 high trading volume stocks listed on the Tokyo Stock Exchange. While it is not the representative of the entire market, it serves as a good example, as it is one of the most monitored stock indices in Asia.

This made high volatility portfolios perform exceptionally well, benefiting from the strong upward trend in market performance, coupled with the aggressive beta associated with the high volatility portfolio. The compounding effect of years with very high returns resulted in an immense overall return for the high volatility portfolio. On the other hand, the low volatility portfolio with a defensive beta had much lower returns than the market during this time. After the year 2006, the market stabilized and exhibited a sustained upward trend for the duration of the remaining study period, notwithstanding occasional years of negative returns.

Table 1. Performance of low volatility and high volatility portfolios alongside the market index 2003-2022.

	Low volatility	**High volatility**	**Market Index**
Overall return (2003-2023)	300,14 %	753,07 %	451,52 %
Annual return	7,18 %	11,31 %	8,91 %
Volatility	7,56 %	25,65 %	15,34 %
Sharpe ratio	0,56	0,33	0,39
Beta	0,64	1,30	1,00
Alpha	0,99 %	0,93 %	0,00

As for the volatilities of the portfolios, the low volatility portfolio exhibited a volatility of 7,56%, the high volatility portfolio registering 25,65%, and the market index displaying a volatility of 15,34%. The volatility for the low volatility portfolio is quite low when compared to the empirical evidence of real-world stock indices. There are a few explanations for the observed volatility being this low.

The market capitalization data used in conducting this study showed that some companies' market caps did not change for some months. This means that their returns were zero percent, also making their volatility zero percent. Some of these observations were excluded, but not all could be taken into account. Most of the companies which data showed a zero percent change were small capitalization stocks. These stocks are in reality more likely to be considered higher volatility stocks, as small cap stocks are often more volatile than large cap stocks due to the capital required to make substantial movements in these stocks' prices.

Another reason for the volatility being low is that the volatility was calculated from monthly data instead of daily data. Volatility computed from monthly data tends to be lower than when derived from daily data. The liquidity of stocks is also an important factor when considering volatility. The companies used in this study have no liquidity requirements (beyond stock market criteria for liquidity and float), meaning that companies with low liquidity are also included in the study results. This contributes to a reduction in the overall volatility, particularly for the low volatility portfolio. The volatility of the portfolio is also considerably lower than the volatility of individual stocks, due to reasons discussed in the theory section of Modern Portfolio Theory.

As mentioned in the theory part of this thesis, it is crucial for investors to factor in the risk associated with investments when seeking potential opportunities. This is why risk-adjusted metrics are important. The results from the metric of Sharpe ratio were interesting, as the low volatility portfolio exhibited a much higher Sharpe ratio than the market index and the high volatility portfolio. This means that the low volatility portfolio has performed better when taking risk (volatility) into consideration.

When comparing the portfolios Jensen's alphas, the low volatility portfolio has a slightly higher alpha than the high volatility portfolio. Both the low volatility portfolio and the high volatility portfolio have positive alphas. This implies that the portfolios surpassed the performance predicted by the capital asset pricing model, resulting in excess returns. The beta of high volatility was quite aggressive, being 1,3. The elevated beta led to the portfolio's good performance, as it benefited from the prevailing uptrend in the market. Conversely, the low beta being defensive at 0,64 slowed down the positive performance of the low volatility portfolio.

The sub question for this thesis is to find out how large market capitalization companies differ from small market capitalization companies in their performance. Table 2 displays the performance of different portfolios constructed based on companies' market capitalizations. In overall returns the small cap high volatility portfolio had the best performance, resulting in a gain of 870,57%. The second-highest return was attained by the large cap high volatility portfolio, amounting to 385,73%. Despite variations in company size, both low volatility portfolios exhibited lower overall returns when contrasted with their high volatility counterparts.

When considering companies' market capitalizations, small cap portfolios had higher returns than large cap portfolios. This phenomenon has also been noticed to exist in the markets. The small minus big (SMB) factor constructed by Fama & French (1993) was also derived from the observations that smaller companies tend to outperform larger companies over the long-term.

Table 2. Performance of portfolios formed based on market capitalization 2003-2022.

	Large Cap (Low Volatility)	Large Cap (High Volatility)	Small Cap (Low Volatility)	Small Cap (High Volatility)
Overall return (2003-2023)	236,30 %	385,73 %	323,78 %	870,57 %
Annual return	6,25 %	6,98 %	7,49 %	12,03 %
Volatility	8,42 %	24,48 %	7,46 %	27,99 %
Sharpe ratio	0,40	0,17	0,61	0,33
Beta	0,57	1,07	0,69	1,40
Alpha	0,74 %	-0,65 %	0,95 %	1,10 %

The small cap high volatility had the highest volatility of these portfolios at 27,99% and the smallest volatility was attained from the small cap low volatility portfolio 7,46%. The factors mentioned in the analysis of Table 1 contributing to the overall volatility being low similarly apply to Table 2. Consequently, the volatility of both low volatility portfolios remains lower than normal. When comparing the volatility of the small cap high volatility portfolio and the large cap high volatility portfolio, small cap stocks are more volatile than large cap stocks. This is logical, as empirical observations indicate that small cap stocks tend to exhibit higher price volatility than large cap stocks in markets worldwide.

When comparing the risk-adjusted method of Sharpe ratio between the portfolios, the low volatility portfolios have performed the best. This implies that low volatility has exhibited superior performance when considering risk, consistent with the performance of the low volatility portfolio in Table 1. As for portfolios betas, both small cap portfolios exhibit higher betas compared to their large cap counterparts. This result is in line with the small cap stocks carrying higher risk than large cap stocks, as beta and volatility are both measures of risk.

Regarding alpha, when comparing the small cap portfolios, high volatility had a higher alpha, while both portfolios' alphas were positive. This was not the case with the large cap portfolios, as the high volatility had a negative alpha, while the low volatility had a positive alpha. When comparing the different market capitalization portfolios to the market index, the small cap high volatility portfolio was the only portfolio to outperform the market index. When comparing alphas, it is important to address their statistical significance. In these study results, none of the portfolios alphas are statistically significant at a five percent risk level. The alphas and betas were computed using linear regression, and all the p-values for these alphas and betas are presented in Appendix 3.

The performance of the portfolios is visually shown in Figure 5 that demonstrates how the portfolios perform in regard to the capital asset pricing model. A Security Market Line (SML) is formed from the market index annualized return and the risk-free rate of the 10-year treasury rate. The portfolios constructed in this thesis are placed in the graph according to their returns and betas.

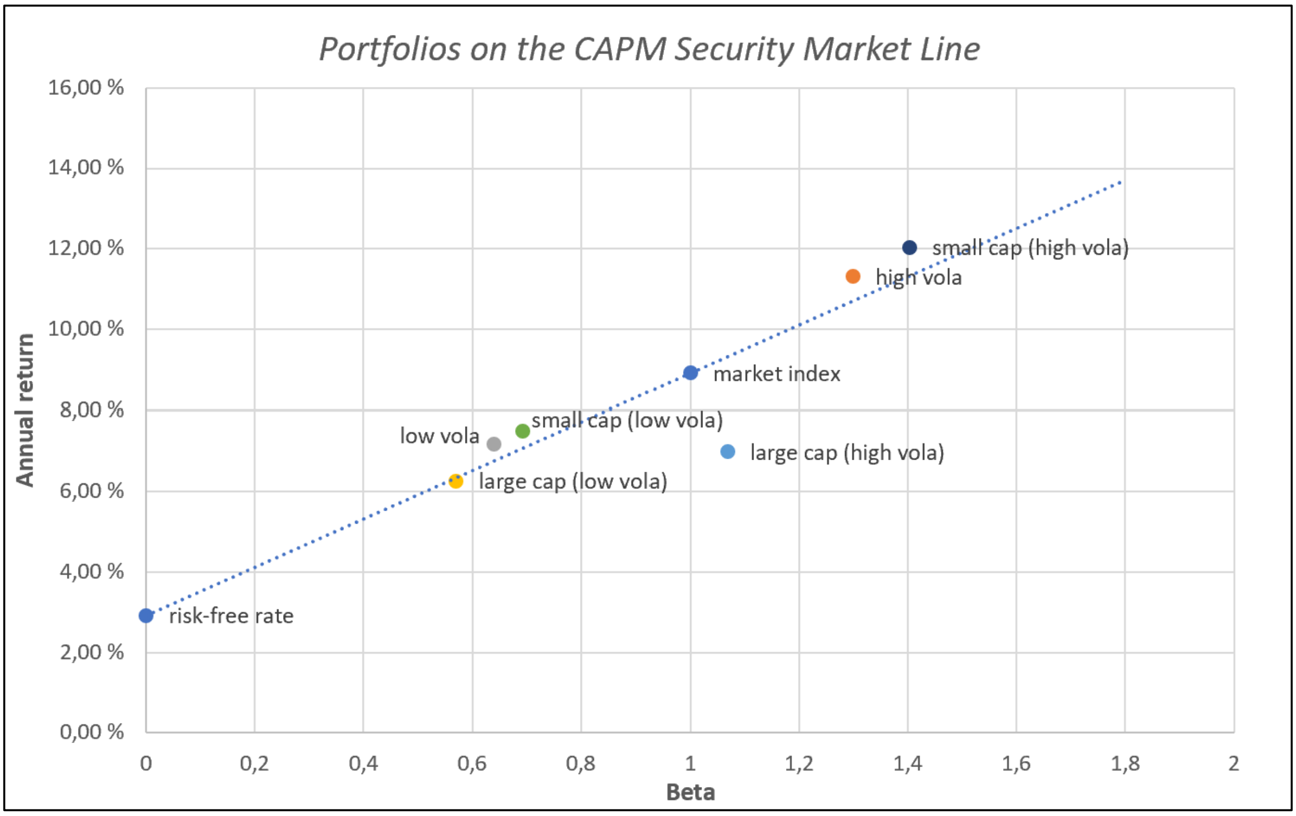

Figure 5. Portfolios constructed from the Japanese market on the Security Market Line

This illustration indicates that the portfolios have nearly behaved in the way outlined by the theory of the capital asset pricing model. Figure 5 also aids in determining whether the portfolio offers a favorable return compared to its level of risk. The figure displays that the low volatility, high volatility, small cap low volatility and small cap high volatility portfolios are above the SML. This means that these portfolios are undervalued based on their risk.

The large cap low volatility portfolio is closely aligned with the SML, whereas the large cap high volatility portfolio falls below it. Thus, in large cap companies, as the risk increases, the returns do not rise at a proportionally similar rate. From these portfolios the large cap high volatility portfolio is the only one considered to be overvalued. When comparing different market capitalization portfolios from the graph, both of the small cap portfolios outperformed the large cap portfolios.

6. Conclusions

The objective of this thesis was to examine the existence of the low volatility anomaly in the Japanese stock market. Japan was chosen for this research due to its significance in the global economy and the risen interest towards its stock market. The scarcity of studies on the anomaly in the Japanese market also provided a strong reason for its investigation. The existence of the low volatility anomaly was studied in this thesis during a 20-year period, spanning from the start of 2003 to the end of 2022.

The data used in this thesis was monthly company market capitalization data from Japanese companies in 1.1.2002-31.12.2022. This data was gathered from the Refinitiv Workspace database. The empirical research was conducted by forming two portfolios from Japanese stocks. One of these portfolios comprised low volatility stocks, while the other consisted of high volatility stocks. These portfolios were reconciled once a year after the close on the last trading day of the year. An overall market index was also formed from all the stocks in the data. The risk-free rate utilized in this study was the U.S. 10-year treasury rate.

In addition, this thesis examined the differences in performance between large market capitalization and small capitalization stocks. This study was done by dividing companies based on their market capitalization into a large cap portfolio and a small cap portfolio. These portfolios were further divided into low- and high volatility portfolios. The performance of the portfolios was measured in overall returns as well as employing risk-adjusted measures of Sharpe ratio and Jensen's alpha. The portfolios alphas and betas were also derived by performing linear regressions. This thesis aimed to answer two research questions, the main research question (1) and the sub question (2) being the following:

- *(1) Does the low volatility anomaly appear in the Japanese stock market?*
- *(2) What is the difference in return between the small and large size company portfolios?*

The empirical results show that high volatility stocks outperformed low volatility stocks in the Japanese market during the research period. The low volatility portfolio demonstrated the poorest performance in terms of overall returns, even underperforming the market index. The portfolios constructed in the empirical research behaved in a way that the traditional financial theory of capital asset pricing model would predict. This behavior is visible from Figure 5.

Despite the outperformance in overall returns, the high volatility portfolio resulted in an inferior Sharpe ratio than the low volatility portfolio. The low volatility portfolio even exhibited a superior Sharpe ratio compared to the market index. This indicates that the risk-adjusted performance of the low volatility portfolio was higher. When comparing different market capitalization portfolios, the same occurs. Sharpe ratios are better for low volatility portfolios. If Sharpe ratios of low volatility stocks are consistently higher than high volatility stocks, investors achieve a greater risk-adjusted return by favoring lower risk assets. Based on these research findings there are signs of the low volatility anomaly appearing in the Japanese stock market.

Similar results were achieved by David Blitz & Pim van Vliet (2007) when they examined the low volatility anomaly in the Japanese market during 1986-2006. They also found that Sharpe ratios of low volatility portfolios were higher than high volatility portfolios. The researchers also concluded that monthly excess returns (beyond the local risk-free return) were higher with low volatility portfolios than high volatility portfolios. This is in contradiction with the results of this thesis, as in overall returns the high volatility portfolio performed better. The differences in these results can be partially explained by the market conditions, as the Japanese market was in a steep uptrend during the research period of this thesis. This made high volatility stocks with aggressive betas perform exceptionally well.

The difference in returns between the small and large company portfolios observed in the empirical research was quite substantial. Portfolios constructed from small capitalization stocks outperformed the portfolios with large cap stocks. The risk-adjusted metrics of Sharpe and Jensen's alpha were also better for small cap stocks compared to their large cap alternatives. The small cap high volatility portfolio had the highest return, and the highest alpha. Yet this alpha was not statistically significant.

When comparing these portfolios with the market index, the index outperformed all the portfolios except for the small cap high volatility in overall returns. When taking risk into account, the small cap low volatility and large cap low volatility performed better than the index. The outperformance of the small cap portfolios aligns with previous studies claiming that small cap stocks have higher returns than large cap stocks on a prolonged basis. This "small firm effect" has been noticed by various researchers, such as Fama & French (1993).

It is important to note that the results from the study conducted in this thesis are not flawless. In further studies, it would be interesting to conduct this research by using split adjusted price data instead of market capitalization data to calculate returns and volatilities. The use of daily data instead of monthly data could also result in a slightly different outcome. It would also be intriguing to use liquidity requirements when gathering data and testing the anomaly using stocks with higher liquidity. This could result in higher volatilities and in slightly different results.

As mentioned before, the empirical research of this thesis did not include cash dividends shared by companies. If dividends were included, it could have a great impact on the performance of large market capitalization companies. Large cap companies are the ones to often share cash dividends with investors, as they are in a better position compared to small cap companies to share excess profits.

According to the theory of the Efficient Market Hypothesis it would be optimal to try to avoid the selling and buying of stocks due to transaction costs. In this thesis, this was not avoided. The portfolios constructed in this research were reconciled once a year, meaning that the stocks in this study changed during the study period. This would mean selling the stocks, which would result in transaction costs, and if the stocks were profitable, tax payments. This was not taken into consideration in this research and is a factor that should be considered in further studies.

It would be interesting to examine the low volatility anomaly by categorizing stocks into deciles based on their volatility, similar to the methodology employed by Blitz & van Vliet (2007) in their research. Employing this method could provide more insight into the existence of the anomaly. This approach would also make it easier to identify if there is a trendline in Sharpe ratios decreasing by deciles when volatility rises.

This research could also be further developed by having a longer study period, as it is important for the investor to consider how their investment performs over the long term. As an illustration, Haugen & Heins (1972) utilized a study period of over 40 years. The extended study period would likely include more years characterized by increased market stability and downward trends in the market. A prolonged timeframe would offer a more comprehensive understanding of the low volatility anomaly in the Japanese stock market.

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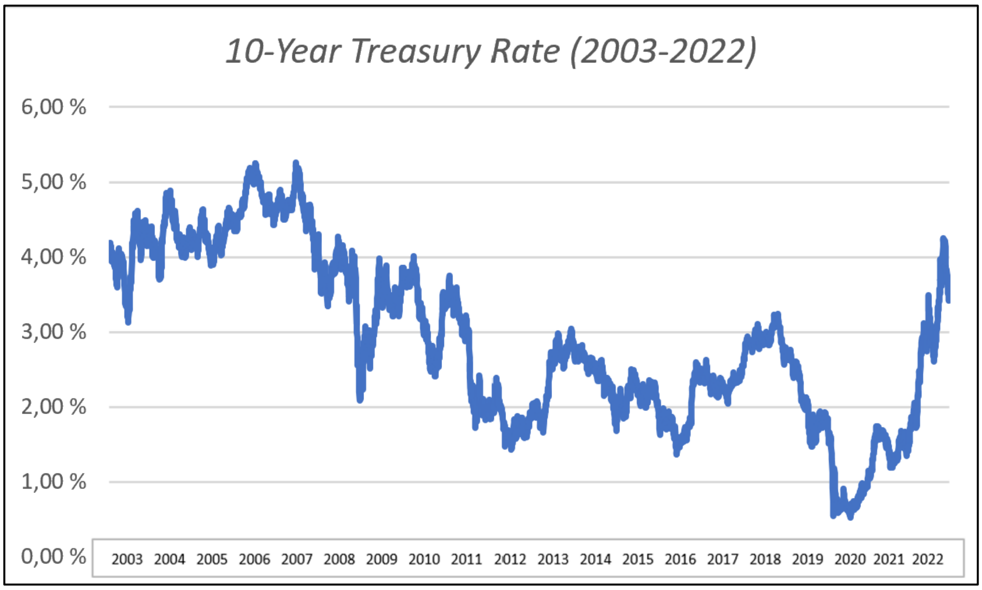
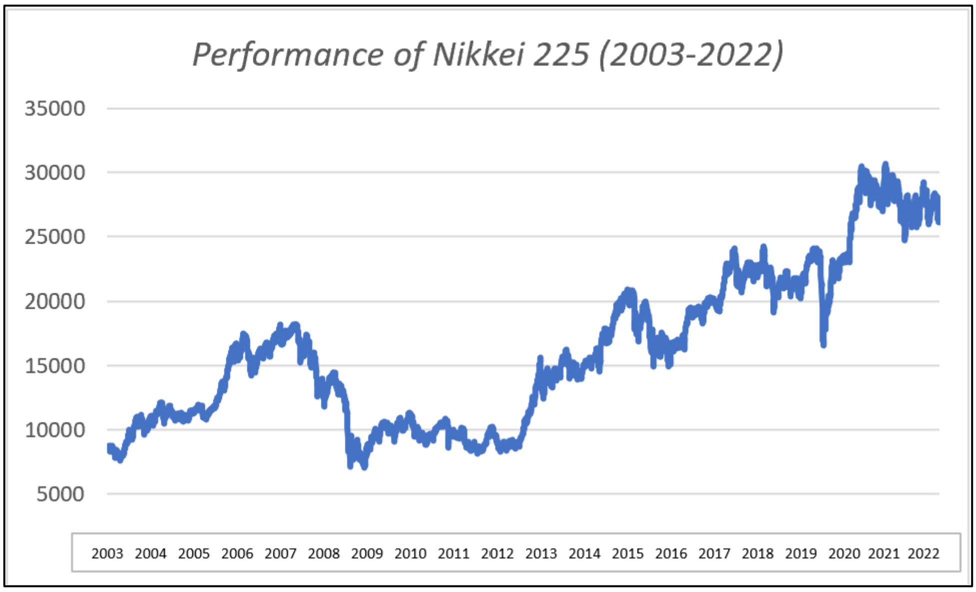
Appendices

Appendix 1. The development of the U.S. 10-year Treasury Rate during the study period.



Appendix 2. Performance of Nikkei 225 during the study period.



Appendix 3. Portfolios alpha and beta p-values.

Portfolio	alpha (α)	p-value	Beta (β)	p-value
Low Volatility	0,009893	0,439563	0,639895	4,01E-11
High Volatility	0,009347	0,607242	1,298513	1,03E-13
Large Cap Low Volatility	0,007437	0,665005	0,570427	2,91E-08
Large Cap High Volatility	-0,00652	0,753824	1,067941	2,91E-11
Small Cap Low Volatility	0,009486	0,556771	0,692427	5,19E-10
Small Cap High Volatility	0,01097	0,597668	1,40319	2,71E-13