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Development of Data Sheets for Statistical Evaluation of Fatigue Data

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ABSTRACT

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To enable a mathematically and physically sound execution of the fatigue test and a correct interpretation of its results, statistical evaluation methods are used to assist in the analysis of fatigue testing data. The main objective of this work is to develop step-by-step instructions for statistical analysis of the laboratory fatigue data. The scope of this project is to provide practical cases about answering the several questions raised in the treatment of test data with application of the methods and formulae in the document IIW-XIII-2138-06 (Best Practice Guide on the Statistical Analysis of Fatigue Data). Generally, the questions in the data sheets involve some aspects: estimation of necessary sample size, verification of the statistical equivalence of the collated sets of data, and determination of characteristic curves in different cases. The series of comprehensive examples which are given in this thesis serve as a demonstration of the various statistical methods to develop a sound procedure to create reliable calculation rules for the fatigue analysis.

Keywords: fatigue, verification, statistical equivalence, data sheets

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Lappeenranta, March, 2007.
Jia Yu

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NOMENCLATURE

α	Significance level
A	Intercept of the regression line
β	Shape parameter of Weibull distribution
η	Scale parameter of Weibull distribution
γ	Location parameter of Weibull distribution, significance level of confidence limits
C	Fatigue capacity
C_μ	Confidence level applied on the population mean
C_σ	Confidence level applied on the population variance
Cr	Criteria of hypothesis test
E	Parameter of Multiple non-linear regression,
F	Percentage point of the F distribution, Parameter of Multiple non-linear regression
h, χ^2	Independent variable in the chi-square distribution
k	One sided tolerance limit factor
k_c	Confidence limit factor
m	Slope exponent of the S-N curve
λ	Independent variable in the standardized normal distribution
μ	Population mean of a continuous random variable
n	Number of specimens
N	Life in number of cycles from the S-N curve
N_p	Life related to failure probability P
P	Probability of failure

R	Reliability, stress ratio, variance ratio
s	Sample standard deviation
S	Stress range
S_a	Stress amplitude
S_m	Mean stress
$S_{\max}$	Maximum stress equal to the sum of the mean and the amplitude of an alternating stress
$^{s}\mathrm{S}$	Standard deviation
ΔS	Stress range
σ	Stress, population standard deviation
$\hat{\sigma}^2$	The best estimate of the variance of the data about regression line
σ_a	Stress amplitude
σ_e	Estimate of the common variance of two samples
t	Independent variable in the Student's t distribution
T	Difference between two estimated slops
x	Continuous random variable
$\overline{x}$	Sample mean
x_k	Characteristic value
X	Difference between two estimated intercept
$\Phi(x)$	Distribution function. Probability density integral
δ	Specified precision of the estimate
z_α	Ordinate on the normal curve corresponding to significant level

1 INTRODUCTION

Components of machines are frequently subjected to cyclic loads, and the resulting cyclic stresses can lead to microscopic physical damage. The process of damage and failure due to cyclic loading is called fatigue. In practice, Mechanical structures often have defects or stress concentrations which cause cracks to nucleate and propagate. Even very small stress concentration areas can lead to fatigue failure of dynamically loaded structures. Therefore, the fatigue failures have been highly concerned in engineering design.

Fatigue tests are made with the objective of determining the relationship between the stress range and the number of stress cycles applied before causing failure. It is implemented to determine basic material or material joining characteristics such as: fatigue limit, S-N fatigue strength curves and crack propagation curves. It based on the relationship between the fatigue resistance of a given material, component or structural detail and cyclic load. The testing samples are usually subjected to cyclically varying stresses which may be tension, compression, torsion and bending or a combination of these stresses.

In fatigue tests, different test specimens and testing conditions make the observed results distinct and invariably scattered. Consequently, statistical methods are used to establish the required relationship between applied load and the number of cycles to failure. Statistical methods are available to assist in the analysis of fatigue testing data. For the non-expert, these are often difficult to apply.

With the increasing use of fatigue testing to supplement design rules, the IIW commission of fatigue of welded components and structures has developed a Best Practice Guide on Statistical Analysis of Fatigue Data (Schneider and Maddox, 2006). It focuses on fatigue endurance test results obtained under constant amplitude load. The analyses are concerned purely with the experimental data. They are independent of the tested material.

This current work is based on this Best Practice Guide. The goal has been to develop data sheets to solve a series of practical questions in the analysis of fatigue. The objective of this work is to provide step-by-step recommendations on the use for analyzing fatigue data. The scope of this guidance is to provide practical cases to answer several important questions raised regarding the treatment of test result. Generally, the questions involved the estimation of necessary sample size, verification of the statistical equivalence of the collated sets of data, and determination of characteristic curves in different cases.

Working group 1 of Commission XIII in the International Institute of Welding is currently developing a series of working sheets in order to help non-experts apply the statistical principals outlined in the best practice guidance document [IIW.XIII-WG1-121r3-06] (International institute of welding, 2006). The goal is that works sheets on the following questions should be developed:

1. Can two data sets be merged?
2. Are the variances of 2 data sets statistically equivalent?
3. Are the means of 2 data sets statistically equivalent?
4. Are the data normal distributed using a normal probability graph (Henry graph)
5. Are the data normal distributed using a likelihood test?
6. Are the data Weibull distributed using a Weibull probability graph?
7. Does a standard S-N curve fit with a data set?
8. How many results are necessary to validate a selected S-N curve?
9. What S-N curve can be selected from a set of non cracked samples results?
10. How to determine a design S-N curve, slope fixed (prediction limits)?
11. How to determine a design S-N curve, slope estimated (prediction limits)?
12. How to determine a design S-N curve, slope fixed (tolerance limits)?
13. How to determine a design S-N curve, slope estimated (tolerance limits)?
14. How to determine a mean S-N curve, Bastenaire equation?
15. Are 2 experimental design S-N curves statistically equivalent?

Additionally, Lappeenranta University of Technology is interested in post weld improvement technologies; therefore, one additional question has been added.

16. How to determine the degree of improvement produced by a post-weld treatment process?

Draft versions of worksheets for questions 1 through 5 and question 7 have been developed previously. The specific goal of this thesis is to develop draft worksheets for questions 6, 8 through 13, 15 and 16.

2 FATIGUE TESTING

2.1 The relationship between stress range and cycles

The results of fatigue tests from a number of different stress levels can be plotted to obtain a stress-life curve, also called an S-N curve. The amplitude of local stress or nominal stress, σ_a or S_a is commonly plotted versus the number of cycles to failure N_f.

In most cases, test results are obtain at constant stress ratios, R. The S-N data is presented in a graph showing logarithm of endurance in cycles as the abscissa and logarithm of stress range as the ordinate. There is an underlying linear relationship between $\log S$ and $\log N$ of the form:

$$\log N = \log A - m \log S \tag{2.1}$$

where: m is the slope and $\log A$ is the intercept

Another rewritten form is commonly used to describe S-N curves in design rules:

$$S^m N = A \tag{2.2}$$

The lower limit on S is determined by the fatigue endurance limit. Commonly, it is chosen on the basis of endurance that can be achieved without fatigue cracking, typically between $N = 2 \times 10^6$ and 10^7 cycles. While, the upper limit on S is taken to be the maximum allowable static design stress.

2.2 The log-normal distribution

In statistics, a probability distribution describes how probabilities are distributed on events. The probability distribution is used to help us estimate the population by using limited samples. According to different types of random variables, the probability distribution is categorized to discrete distributions, continuous distributions, and joint distributions (Marray, 1990).

For statistical evaluation of fatigue testing data, it is often assumed that fatigue life in the finite life regime is log-normally distributed. The log-normal distribution which is one of basic continuous distributions provides a lot of benefits to researchers due to its wide applicability to reliability problems, especially in the area of maintainability and to certain fracture problem.

Mathematically, if Y is log-normally distributed, then $\log Y$ is normally distributed. If a variable can be thought as the multiplicative product of many small independent factors, it is able to be regarded as log-normally distributed.

A probability density function (pdf) serves to represent a probability distribution in terms of integrals. For continuous random variables, the expression for the log-normal probability density function (pdf) and cumulative distribution function (CDF) are given as follow (Mann, Schafer, and Singpurwalla, 1974).

pdf $$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\left(-\left[\frac{\ln(x)-\mu}{\sigma}\right]^2/2\right)} \qquad 0 \le x \le +\infty \qquad (2.3)$$

CDF $$F(x) = \frac{1}{2} + \frac{1}{2} erf\left[\frac{\ln(x)-\mu}{\sigma\sqrt{2}}\right] \qquad 0 \le x \le +\infty \qquad (2.4)$$

where: x is the continuous random variable

μ is the population mean

σ is the population standard deviation

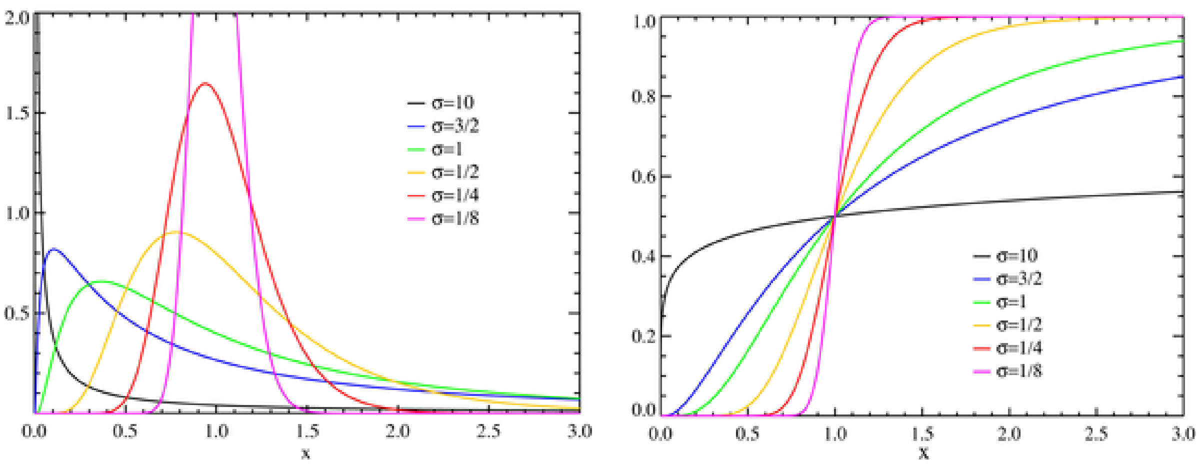

Figure 2.1 Log-normal pdf and CDF (Stahel, Limpert, and Abbt, 2001)

2.3 Statistical independent results

In probability theory, statistical independence means that the occurrence of events makes it neither more nor less probable that the others occur. In fatigue testing, individual specimen is usually tested for the basic research of fatigue strength. The test specimen is usually simple welded and is known the weakest part in terms of the fatigue resistance. For example, a longitudinal fin in a base plate or a base plate welded with longitudinal fins on both sides. Various strain gauges are often implemented in tests and the loading can be multi-axial. Final failure appears in the form of crack growth which is caused by numerous stress ranges (Niemi, Marquis and Poutiainen, 2005). The number of endurance cycles at different stress range level is recorded. Each test is carried out separately and does not affect the outcome each other. In other words, the result from each test is statistically independent of the others.

The above mentioned characteristics are foremost assumptions in the statistical evaluation of fatigue data. The testing data must be match for them in order to truly represent the population. Commonly, the above assumptions are available in practice. There are relative statistical tests which are able to justify their validity.

3 FITTING A S-N CURVE

Simply, the S-N curve is composed of n data points according to stress range and the endurance in cycles. The standard approach in curve fitting is to assume that the parameter on the x-axis is the independent variable and the one on the y-axis is dependent variable.

In the S-N curve, the constant amplitude fatigue limit (CAFL) is defined as the stress range below which failure will not occur. For design purposes, it is assumed that the S-N curve extends down to the CAFL and then turns sharply to become a horizontal line. Typically, the endurance of smooth specimens is around 2×10^6 cycles. For notched specimens, 10^7 cycles is commonly used. Moreover, it is important to note that the test results which lie in region approaching the CAFL should not be used to estimate the best-fit linear S-N curve. These suggestions are from the Best Practice Guide (Schneider and Maddox, 2006).

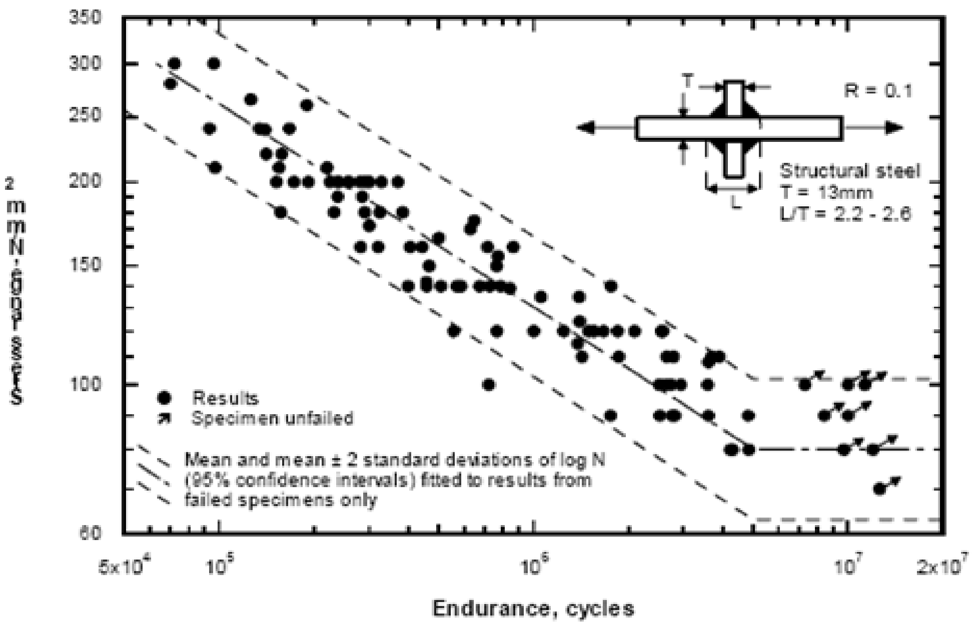

Figure 3.1 Example of S-N curve with data (Maddox, 1993)

3.1 Fatigue data

Statistical evaluation relies extensively on the testing data to make estimations. Hence, the supplied data affects the accuracy of the prediction greatly. The fatigue life data can be separated into two categories: complete data or censored data.

Complete data means that the value of each sample unit is observed or known. In the case of life data analysis, the data set would be composed of the times-to-failure of all units in the sample. For instance, if five samples are tested and all failed, we would have complete information as the time of each failure in the sample. (Reliasoft, 1992)

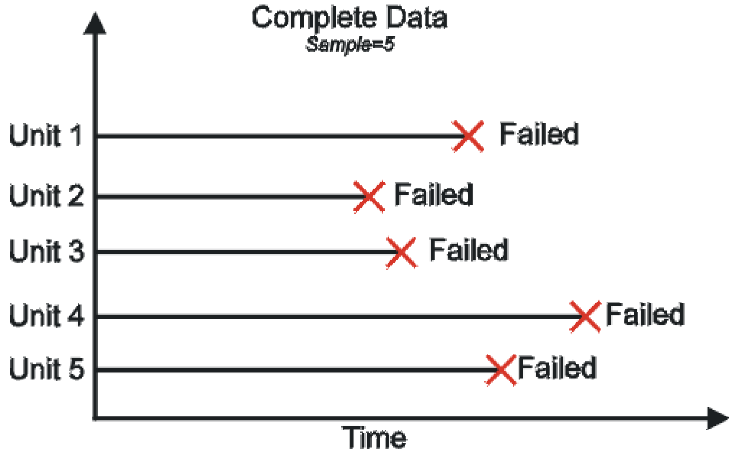

Figure 3.2 Complete data (Reliasoft, 1992)

In fatigue testing, all of the samples may not have failed when the testing finishes. We do not make sure each specimen under fatigue test yielded an exact failure endurance. Therefore, it is possible to obtain results from the specimens or parts of specimens which are unfailed. These unfailed specimen are often termed 'run-outs'. This type of data is commonly called censored data. There are three types of possible censoring schemes, right censored (suspended data), interval censored and left censored.

Right censored is common in the fatigue testing. These data sets are composed of units that did not fail. The failure would yields at some time on the right on the time scale.

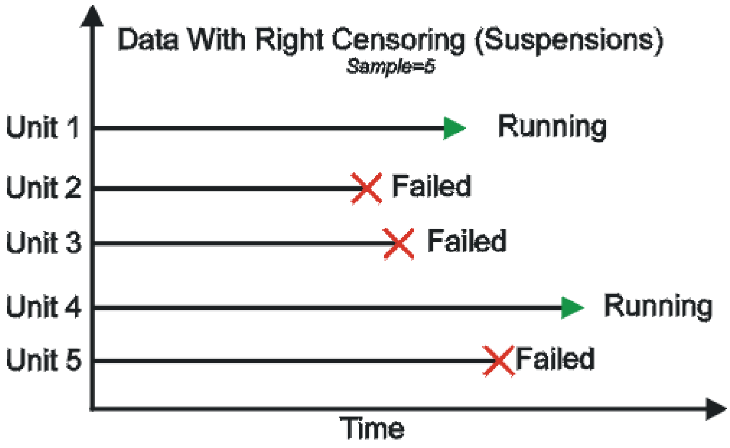

Figure 3.3 Right censoring data (Reliasoft, 1992)

Interval censored data reflects uncertainty as to the exact times the units failed within an interval. We concern about whether the samples are failed in a certain interval of time.

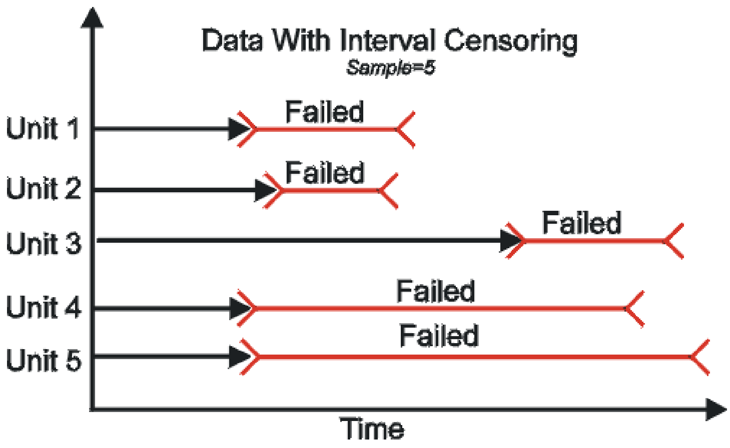

Figure 3.4 Interval censoring data (Reliasoft, 1992)

In left censored data, a failure time is only known before a certain time. For example, we know that a certain unit failed sometime before an interval time.

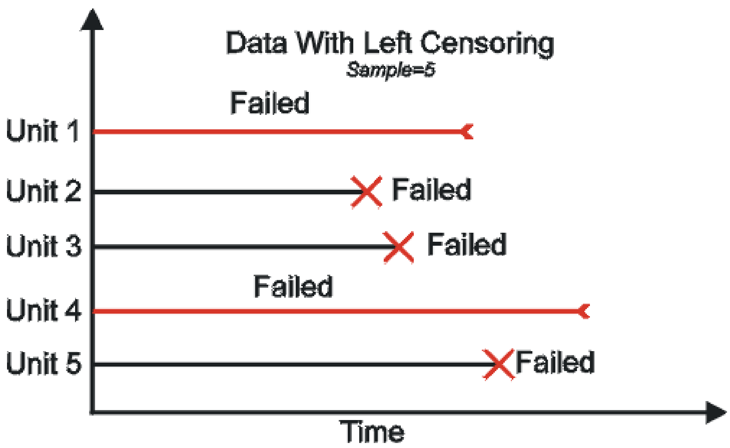

Figure 3.5 Left censoring data (Reliasoft, 1992)

3.2 Methods of estimating the S-N curve parameters

The general problem of finding equations of approximating curves which fit given sets of data is called curve fitting. One of the main purposes of curve fitting is to estimate dependent variable from the other independent variable. The process of estimation is termed as regression. (Spiegel, 1990)

Many methods of statistical evaluation are available to determine best-fitting line and the statistical parameters. Generally, those methods are used to model the univariate data with a specific probability distribution.

Table 3.1 Method of estimating the S-N curve parameters (International Organization for Standardization, 2000)

Method of Analysis	**S-N curve equation**	**Method and Assumptions**	Parameters
(A)	$\log N = \log C - m \log S$	Mean line bisecting the two regression lines	$C, m,$ ^{s}logS
(B)	$\log N = \log C - m \log S$	Linear regression of log N on log S ignoring run-out's	$C, m,$ ^{s}logS
(C)	$\log N = \log C - m \log S$	Maximum likelihood (includes run-out's)	$C, m,$ ^{s}logS
(D)	$\log N = \log C - m \log S$	Linear regression of log N on log S run-out's being included	$C, m,$ ^{s}logS
(E)	$N = \frac{A}{S-E}\left[-\frac{(S-E)C}{B}\right]$	Multiple non-linear regression including censored data (run-out's) Stress response curves: sigmoid normal	A, B, E ^{s}S, F

It is noted that methods A and B do not take unbroken specimens into account. While other three methods consider the run-outs into account in the analysis and which have a bearing on the final results.

Whether the results from unfailed specimen are used in the statistical analysis of the data depends on the circumstances. But when the fatigue data include either run-outs or suspended data, the appropriate statistical analyses become more complicated. Fortunately, many statistical software packages can perform the required calculations according to (International Organization for Standardization, 2000).

3.21 Least squares regression method

Considering only the results from truly failed specimens, the ordinary linear regression, called ‘least square estimation’, is used to estimate the intercept $\log A$ and slope m of the best-fit line through the data. This method attempts to minimize the sum of the squares of the dependent variables between points generated by the function and corresponding points in the data. Note that the implicit requirement of the least squares method to work is the errors in each measurement should be randomly distributed.

Calculate the intercept $\log A$ and slope m for best-fit S-N curve by using least square regression method (Spiegel, 1990):

$$Y = A + mX \tag{3.1}$$

$$Y = \log N \qquad X = \log S \tag{3.2}$$

$$\hat{m} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \tag{3.3}$$

$$\hat{A} = \frac{\sum y_i}{n} - \hat{m}\frac{\sum x_i}{n} \tag{3.4}$$

Linear least square regression is widely used in process modeling because of its effectiveness and completeness. Moreover, linear least squares regression makes very efficient use of the data. Good results can be obtained within relatively small data sets. However, when the data sets include the unusual data points, the estimates will be affected greatly.

Least squares estimation for linear models is notoriously weak to outliers. If the distribution of outliers is skewed, the estimates maybe biased. This problem affects the efficiency extremely. When outliers occur in the data, another method such as robust regression is more appropriated to be used.

3.22 Maximum likelihood regression method

We suggested a method of curve fitting the probability density function to the test data in a maximum likelihood sense. It has the advantage of considering every valid observation of the sample. This method expressed the probability of the combined events failures and runouts to find the optimum estimates of the sample mean and variance.

Simply, when the fatigue life data satisfied with the assumptions mentioned in section 2 and are neither run-outs nor suspended data, the following maximum likelihood estimators of intercept $\log A$ and slope m are recommended to use (ASTM international, 2004):

The S-N relationship is described by the linear model:

$$Y = A + mX$$

$$Y = \log N \qquad\qquad X = \log S$$

$$\hat{A} = \overline{Y} - \hat{m}\overline{X} \tag{3.5}$$

$$\hat{m} = \frac{\sum_{i=1}^{k}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{k}(X_i - \overline{X})^2} \tag{3.6}$$

Maximum likelihood Method can be developed for a large variety of estimation situations. As the sample size increases, it offers minimum variance unbiased estimators. The approximate normal distributions and sample variances can be used to generate confidence bounds and hypothesis tests for the parameters. However, the estimates might be heavily biased for small samples. Additionally, the calculation is laborious without computer assistant.

3.3 Sample size and population

In practice we are interested in drawing valid conclusions about a large group of individuals or objects. Instead of examining the entire group, called the population, which may be difficult to do, one may arrive at the idea of examining only a small part of this population, which is called a sample (Spiegel, 1990).

In fatigue testing, we face a finite population which is sampled with replacement. "How many measurements should be included in the sample?" It is one of the most frequent questions which need to be considered for statistical evaluation. Sample size determines the precision of that estimate. Larger sample size gives smaller error bounds of estimation. In order to determine the sample size we need to consider a series of factors such as cost of sampling, variability of the population, the precision of the final estimation. Fortunately, the prior information derived form previous study or research can be used to reduce the sample size.

Commonly, there is no correct answer without additional information or specific assumptions. But the following steps can help us basic idea how to solve the problem. First, determine what we are trying to estimate and how precise we want. Second, find some equations that connect the desired precision of the estimate with the sample size. Third, get unknown properties in the equations. Final, review the final sample size to make sure it is acceptable.

Actually, the researcher should make sampling decisions based on the data. The basic sample size determinations can be categorized according to the type of continuous data and categorical data. Continuous data is measured on a scale and can be meaningfully subdivided into finer and finer increments. Categorical variables represent types of data which may be divided into groups. Before proceeding with sample size calculations, assuming continuous data, it is important to determine if a categorical variable will play a primary role in data analysis. For fatigue analysis, the fatigue data are continuous data because they are continuous in stress range.

In this work, I study this question: How many results are necessary to validate a selected S-N curve? The assumption is that the slope and standard deviation of the mean S-N curve of the new test results are the same as those parameters of the selected S-N curve. Then the null hypothesis that the new test results belong to the same population as the main data base is made (Schneider and Maddox, 2006)

$$\overline{\log A_{test}} \geq \overline{\log A} + \frac{Z_{p\%}\sigma}{\sqrt{n}} \tag{3.7}$$

where: $\overline{\log A_{test}}$ is the mean logarithm of interception from new tests

$\overline{\log A}$ is the logarithm of the interception from the mean for the selected S-N curve

$Z_{p\%}$ is standard normal probability for a probability

The above equation can be transformed into following form to determine the sample size:

$$n \geq z_{\alpha}^{2}\left(\frac{\sigma^{2}}{\delta^{2}}\right) \tag{3.8}$$

where: σ is the population standard deviation

z_{α} is the ordinate on the normal curve corresponding to α

δ is the specified precision of the estimate. For instance, $\delta = \overline{\log A_{test}} - \overline{\log A}$.

It is indicates that the essential thing of determination sample size for fatigue testing is to determine a sample size that is large enough to guarantee the risk. In other hand, sample along with high quality data collection efforts will result in more reliable, valid, and generalizable results. It is able to do resource saving greatly.

4 ESTABLISHING A DESIGN CURVE

4.1 Mean, characteristic and design values

Generally, a structure can be damaged in various ways. Calculation of a single value such as strength value for a structure does not ensure that the structure itself will not fail. Numbers of values are needed to be obtained from the alternated failure modes. Then the mean value is often to be calculated to report central tendency. The mean value is not appropriate for describing skewed distributions. Fortunately, samples from fatigue testing are normal distributed.

In practice, there are some load events which are not known in advance. Therefore, designer often uses limit state design to avoid failures due to unforeseen events. A limit state is a condition beyond which a structure is less than suitable to perform the required function. Fundamental ideas of limit state design include statistically based definitions of characteristic strength, characteristic loads and characteristic stress. By using those characteristic values associated with partial safety factor, the design values are able to be calculated. (Niemi, Marquis and Poutiainen, 2005).

Commonly, on the load side, the characteristic value is determined that it is larger than the mean value by two times the standard deviations. The design value can be calculated by using the characteristic value multiply the partial safety factor for fatigue actions. On the resistance side, the characteristic value is defined that the mean value subtracted twice standard deviation. The design value can be obtained by using the characteristic value divided by the partial safety factor for fatigue resistance. Moreover, using the twice standard deviations is based on the 95% probability of survival to reach a reliability of 75%.

The figure 4.1 indicates that, on resistance side, the mean value and characteristic value according to 95% probability of survival.

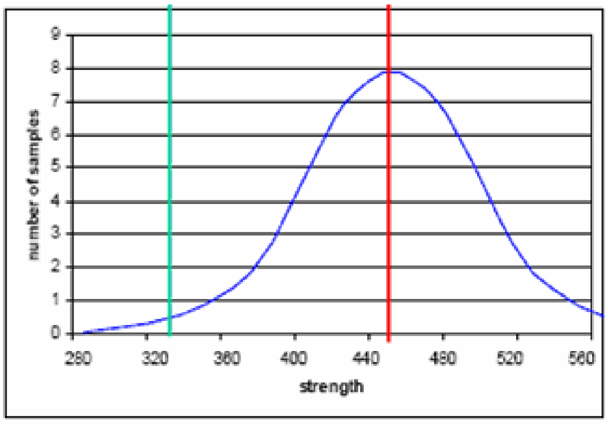

Figure 4.1 Mean and characteristic strength

In most cases for normal fabrication quality and regular inspections, the partial safety factor is selected to be 1. Therefore, a characteristic or design curve is established by adopting characteristic values that lay a certain number of standard deviations below the mean S-N curve as the figure 3.1. Fatigue life verification of fail safe structures depends largely on the design parameters of a structure. Three different methods have been developed to determine the design curve.

4.2 Confidence limits

When we take a random sample from a population to approximate the mean of the population, we concern about how well the sample statistically estimates the population value. The confidence limits, also called as confidence interval, which provides a range of values which is likely to contain the population parameter of interest. Confidence intervals with a confidence level mean that if the population is sampled on numerous occasions and interval estimates are made on each occasion, the resulting intervals would bracket the true population parameter in approximately the certain percentage of the confidence level (NIST/SEMATECH, 2003). For example, a 95% confidence interval means that if many samples are collected and the confidence interval computed, in the long run about 95% of these intervals would contain the true mean.

A fatigue design or characteristic curve is established by adopting characteristic values that lay a certain number of standard deviations below the mean S-N curve. It has to be accepted that only a small part of the samples can be tested in practice. It is normally accepted that the sample mean of normal distribution is characterized by the Student's t distribution and the sample variance is characterized by the chi-square distribution.

When the confidence is applied to the sample mean and variance, the sample mean and variance will be limited without the influence from number of tests. These values with confidence are used as the mean value respective the variance for the whole population. In principle, the characteristic values are values at a $\alpha = 95\%$ survival probability associated to confidence interval of the mean and the standard deviation:

$$x_k = \overline{x} - k_c \cdot s \tag{4.1}$$

The factor k_c considers the effects of variance of data and deviation evaluated. It corresponds to the minimum value of the mean confidence interval and maximum value of the variance confidence interval. Taking into account the probability distribution of the mean corresponds to student t-distribution and the variance corresponds to Chi-square distribution, it can be calculated by using following equation (Hobbacher, 2005):

$$k_c = \frac{t_{(p,n-1)}}{\sqrt{n}} + \Phi^{-1}_{(alpa)} \sqrt{\frac{n-1}{\chi^2_{(\frac{1+\beta}{2},n-1)}}} \tag{4.2}$$

where: t value of the two sided t-distribution

Φ distribution function of the Gaussian normal distribution probability

χ^2 Chi-square for a probability at $n-1$ degrees of freedom

Normally we use 95% confidence level to analysis the test results. The following figure shows the typical confidence limits.

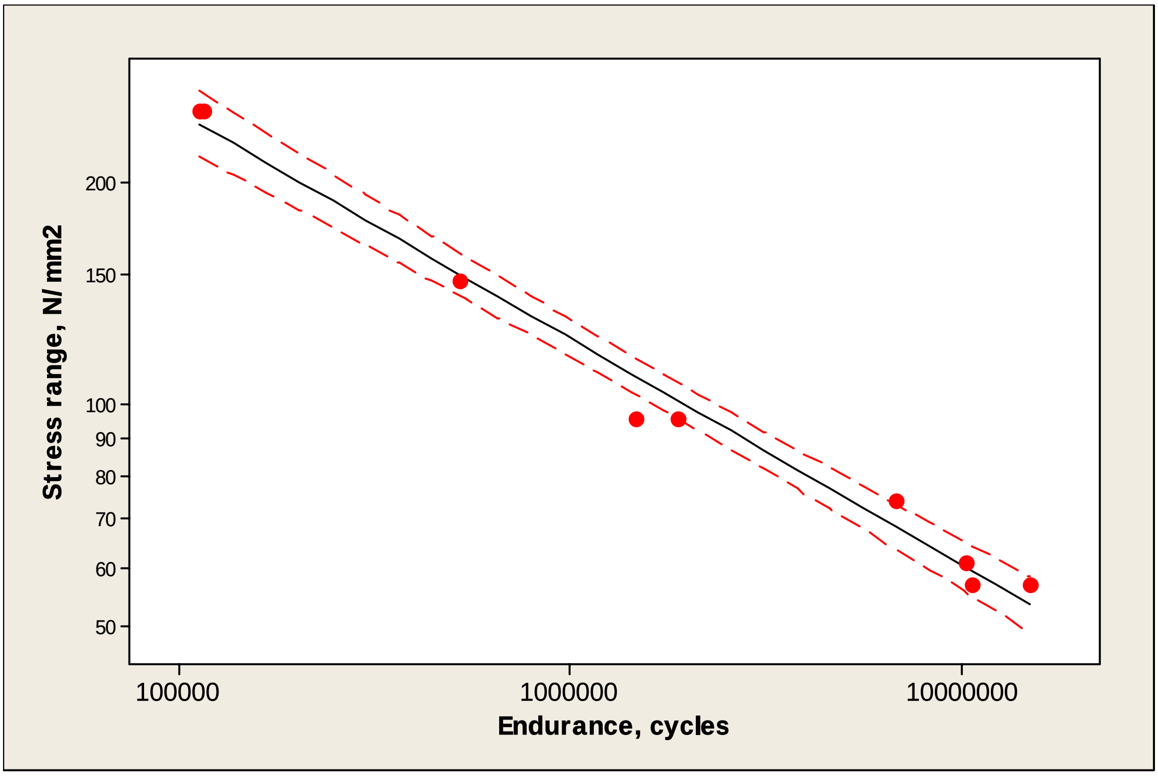

Figure 4.2 Regression line and design curve based on 95% confidence limits

4.3 Prediction limits

The prediction limit describes the probability that an individual point will be above or below a certain value. A two-sided prediction limit sets an interval for the data while a one-sided prediction limit describes the probability that a point will exceed a certain value.

For the normally distributed fatigue data sets, the prediction limits at stress range S can be expressed explicitly in the form (Schneider and Maddox, 2006):

$$\log N^{\pm}_{P\%} = (\log A + m \log S) \pm t\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(\log S - \overline{\log S})^2}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2}} \tag{4.3}$$

where: log A and m are the coefficients of the regression line through the n data points

$\overline{\log S}$ is the mean of the n values of $\log S_i$

t is the appropriate percentage point of Students t distribution

$\hat{\sigma}$ is the best estimate of the variance of the data about the regression line,

f is degrees of freedom, $f = n - 2$ in case where the two coefficients of the regression line have been estimated from the data

When the slope is chosen to take a fixed value, the expression under the square roots is exactly equal to one. It is recommended to use when the sample size is less than ten. But when the sample sizes are smaller than 4, the following method of tolerance limits is regard as a more conservative design curve compared with prediction limits.

4.4 Tolerance limits

According to the "Best Practice Guide on Statistical Analysis of Fatigue Data" (Schneider and Maddox, 2006), the tolerance limits yield a more conservative design curve. The disadvantages of tolerance limits also could be ignored. They are inherently more complicated and harder to implement. They are more sensitive to deviations from the assumed normality.

Generally, the tolerance limits are defined by lower and upper tolerance limits which are calculated from a series of results. For fatigue testing, the lower one-sided $P\%$ tolerance limit takes general form:

$$\log N_{P\%}^{-} = \hat{\mu} - ks \tag{4.4}$$

where: $\hat{\mu}$ is an estimate of the mean log of the endurance at stress S, based on n observations

s is an estimate of the standard deviation of the log of the endurance at stress S, based on degree of freedom

k is a one-sided tolerance limit factor

Estimating lower confidence limits of the form $\hat{\mu} - ks$ on the prediction limit is available to avoid the sampling uncertainties. This statement means that at least a proportion

$P\%$ of the population is greater than $\hat{\mu} - ks$ with confidence $\gamma\%$.

For fatigue analysis, if the slope of the regression line is estimated from the data, tolerance limit factors for the normal distribution are not easy to obtain. It needs the evaluation of both $P\%$ percentage points of the normal distribution and the $\gamma\%$ percentage points of the noncentral t distribution. Based on the handbook of statistical tables (Owen, 1962), k is determined by:

$$\Pr\left\{(noncenral\ t\ with\ \delta = K_P\sqrt{n}) \le k\sqrt{n}\ \right\} = \gamma \tag{4.5}$$

where: the noncentral t has f degrees of freedom

K_P is defined by $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{K_p} \exp(-x^2/2)dx = P$

The main trouble of calculation is to obtain the critical values t_0 of the noncentral t -distribution, because its calculation involved several parameters such as η, δ, λ. Tables of factors for computing critical values of the noncentral t-distribution in (Owen, 1962) are used to calculate value of λ according to η, δ. After obtaining corresponding value λ, critical value t_0 is obtained by the formula:

$$t_0 = \frac{\delta + \lambda\left(1 + \frac{\delta^2}{2f} - \frac{\lambda^2}{2f}\right)}{1 - (\lambda^2/2f)} \tag{4.6}$$

Then k value can be obtained by equation:

$$k = \frac{t_0}{\sqrt{n}} \tag{4.7}$$

For the case of degree of freedom $f = n - 2$, the relative laborious calculation has been done in this thesis. The common used k values are able to be find from the table of one-sided tolerance limit factor in appendixⅪ

While, if the slope is fixed, the tolerance factors for several cases which are interested in fatigue analysis are offered in appendix X from (Schneider and Maddox, 2006).

4.5 Comparison

Typically, confidence limits are based on sample mean and sample standard deviations. Therefore, it is usually recommended to use when we have larger sample sizes. Prediction intervals are especially powerful because they can predict a future compliance point. Its requirement of sample size is low. The tolerance limits is a way to determine a range that will contain a certain percentage of the population. It is recommended to be used as a means of justifying design curves that are based on small samples, especially for critical applications.

Figure 4.3 is the design curve based on 95% confidence limits and prediction limits for the same regression line. It indicates prediction limits are more conservative than confidence limits.

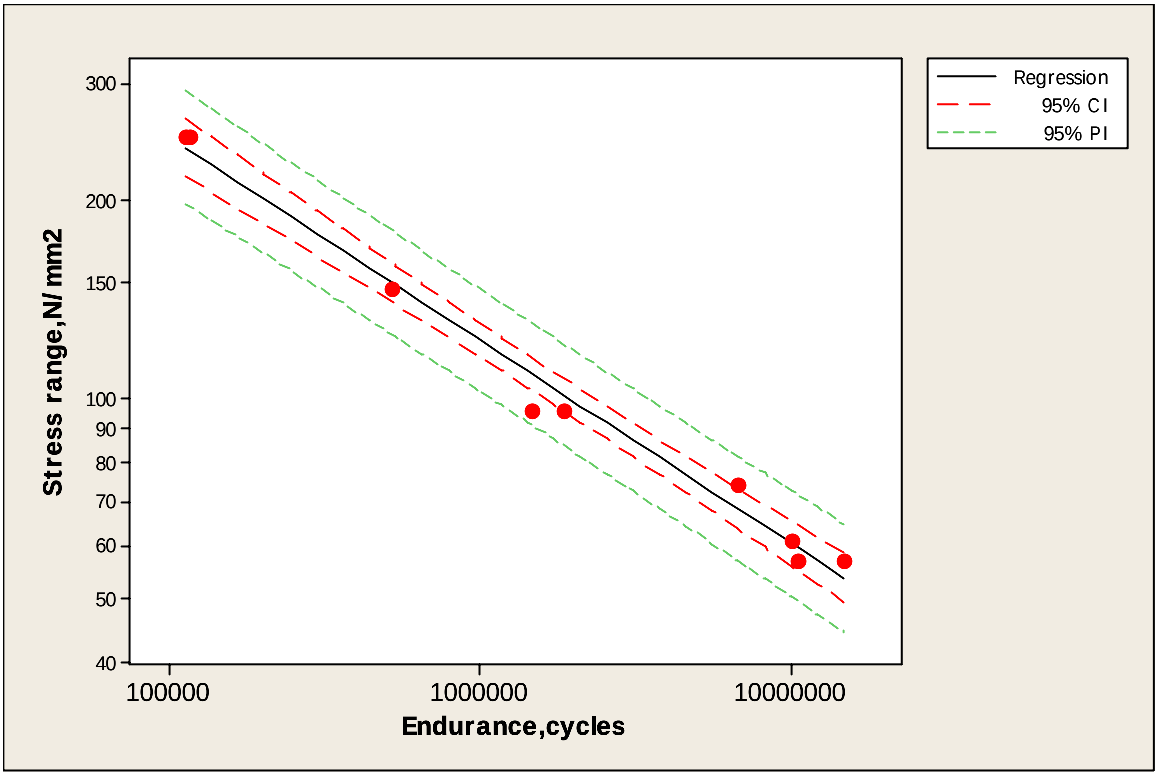

Figure 4.3 Design S-N curve based on confidence and prediction limits

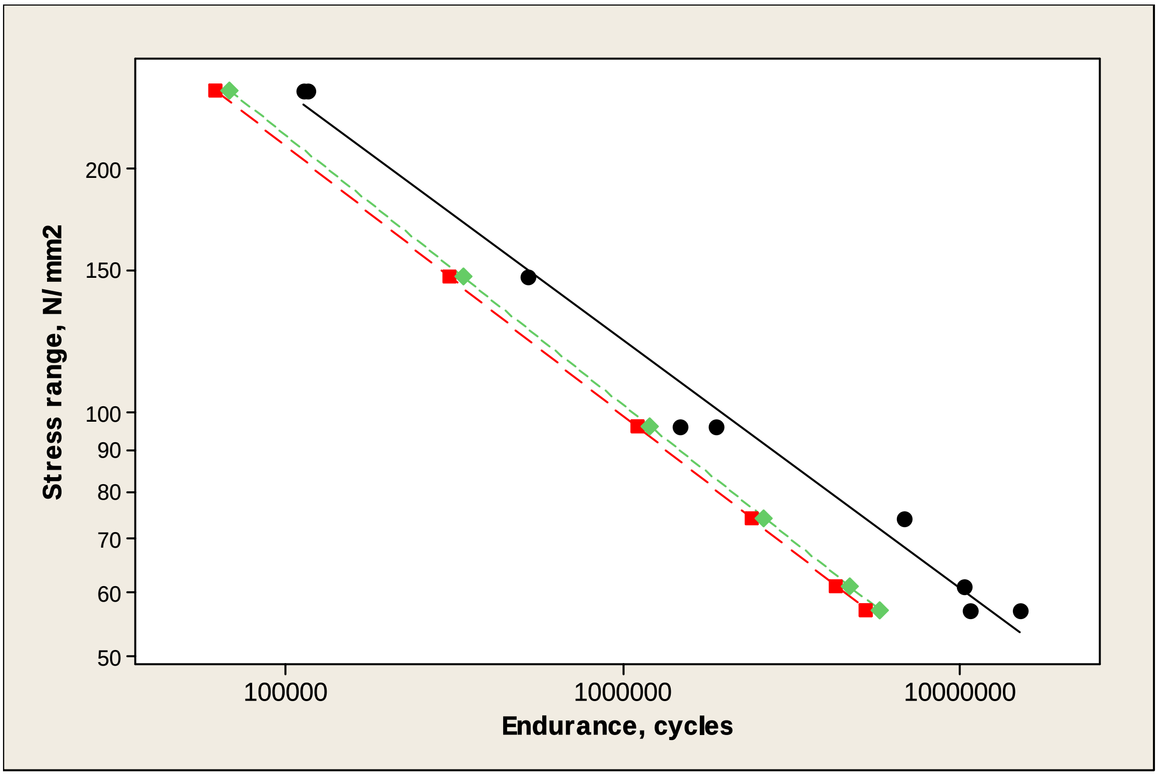

Figure 4.4 Design S-N curve based on prediction and tolerance limits

The figure 4.4 shows the comparison of the results from data sheets 10 and 12. The green and red line respectively expresses the design curves based on prediction and tolerance limits. It is obvious that the tolerance limits offers a lower design curve. It proves that tolerance limits are more conservative than prediction limits.

5 VERIFICATION OF THE STATISTICAL EQUIVALENCE FOR TWO S-N CURVES

In manufacturing field, when the manufacturing process has been changed or new processing technology is used, we want to determine whether a significant change has been produced. In statistical evaluation of fatigue data, this problem is transformed to justify the significant difference between two sets of S-N data or whether two sets of S-N data from the same population.

A null hypothesis is often used to check this problem. In statistics, a null hypothesis is a hypothesis set up to be nullified or refuted in order to support an alternative hypothesis. This is the basis for regarding the null hypotheses as plausible, and for rejecting the alternative hypothesis that two data sets of test results belong to different populations. In this thesis, it is assumed that both sets of S-N data are exact data even though this approach can be extended to the case of censored data in principle. (Spiegel, 1990)

The limitation of this a null hypothesis is that it is only useful if it is possible to calculate the probability of observing a data set with particular parameters form it. It is much harder to be precise about how probable the data would be if the alternative hypothesis is true. The theory underlying the idea of a null hypothesis is closely associated with the frequency theory of probability. A failure to reject the null hypothesis is meaningful only in relation to an arbitrarily large population from which the observed sample is supposed to be drawn.

5.1 Tests performed to an S-N curve

When we consider the statistical equivalence of the two S-N curves, the residual standard deviation, the intercepts and slopes of the two S-N curves are need to be taken into account. These formulas have been given by (Schneider and Maddox, 2006):

Test the residual standard deviation

Null hypothesis: two sets of results belong to populations having the same standard deviation.

$$\frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} \le F_{f_1}^{f_2} \tag{5.1}$$

where: $F_{f_1}^{f_2}$ is the $P\%$ percentage point of the F distribution

$\hat{\sigma}^2$ is the best estimated variances. $\hat{\sigma}_1^2 > \hat{\sigma}_2^2$

Test the intercepts of the two S-N curves

Null hypothesis: two sets of results belong to populations having the same intercept.

$$\left|\overline{\log A_1} - \overline{\log A_2}\right| \le t\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{(\overline{\log S_1})^2}{\sum_{i=1}^{n_1}(\log S_{1,i} - \overline{\log S_1})^2} + \frac{(\overline{\log S_2})^2}{\sum_{j=1}^{n_2}(\log S_{2,j} - \overline{\log S_2})^2}\right)\sigma_e^2} \tag{5.2}$$

where: $\overline{\log A_1}$, $\overline{\log A_2}$ are the estimated intercepts of the regression lines of two data sets

t is the appropriate two-sided percentage point of Student's t distribution, with degrees of freedom $n_1 + n_2 - 4$

σ_e is an estimate of the common variance of the two samples, $\sigma_e^2 = \dfrac{f_1\hat{\sigma}_1^2 + f_2\hat{\sigma}_2^2}{f_1 + f_2}$

Test the slopes of the two S-N curves

Null hypothesis: two sets of results belong to populations having the same slope.

$$\left|m_1 - m_2\right| \le t\sqrt{\left(\frac{1}{\sum_{i=1}^{n_1}(\log S_{1,i} - \overline{\log S_1})^2} + \frac{1}{\sum_{j=1}^{n_2}(\log S_{2,j} - \overline{\log S_2})^2}\right)\sigma_e^2} \tag{5.3}$$

where: m_1, m_2 are the estimated slopes of the regression lines of the two data sets

In this case, the numbers of degree of freedom should be equal to $n-2$, because two coefficients of intercept and slope have been estimated to obtain the S-N curves. At the same time, the significance level of the null hypothesis should be noted. For instance, each of the three null hypotheses is tested at 5% significance level; the combination of the probability is 15%. Therefore the guild (Schneider and Maddox, 2006) is recommended choosing 1.7% significance level for each individual test in order to get a 5% significance level of the composite hypothesis.

Here, I need to emphasize the definition of the significance level. In this hypothesis testing, the significance level of a test is the maximum probability, assuming the null hypothesis, that the statistic would be observed. Therefore, the significance level is the probability that the null hypothesis will be rejected in error when it is true. That means when we use significance level 1.7% instead of 5%, we have more opportunities to accept the null hypothesis.

For example, test whether two sets of results belong to populations having the same slope. The null hypothesis at a significance level $\alpha\%$ is accepted if the equation (5.3) is satisfied. When we reduce the significance level, the critical region is increased due to growing value of t. However, a smaller significance level is possible to give greater risks of failing to reject a false null significance.

6 VERIFICATION OF WEIBULL DISTRIBUTED DATA

6.1 Weibull probability graph

Weibull probability plot is a graphical technique for assessing whether or not a data set follows the Weibull distribution. The data are plotted against a theoretical distribution in such a way that the points should form approximately a straight line. In this plot, the departures from this straight line indicate departures from the Weibull distribution.

The Weibull distribution has a relatively simple distribution form, while, the shape parameter allows it to assume a wide variety of shapes. This combination of simplicity and flexibility in the shape of the Weibull distribution has made it to be an effective distributional model in reliability applications. Depending on the parameters' values, the Weibull distribution can approximate an exponential, a normal or a skewed distribution. The standard form of any distribution is the form that has location parameter zero and scale parameter one. Therefore, the Weibull plot is a graphical technique for determining if data sets from a population that would logically be fitted by a 2-parameter Weibull distribution. The following graphs show the Weibull distribution function with shape parameters: 0.5, 1.0, 2.0 and 5.0. (Mann, Schafer and Singpurwalla, 1974)

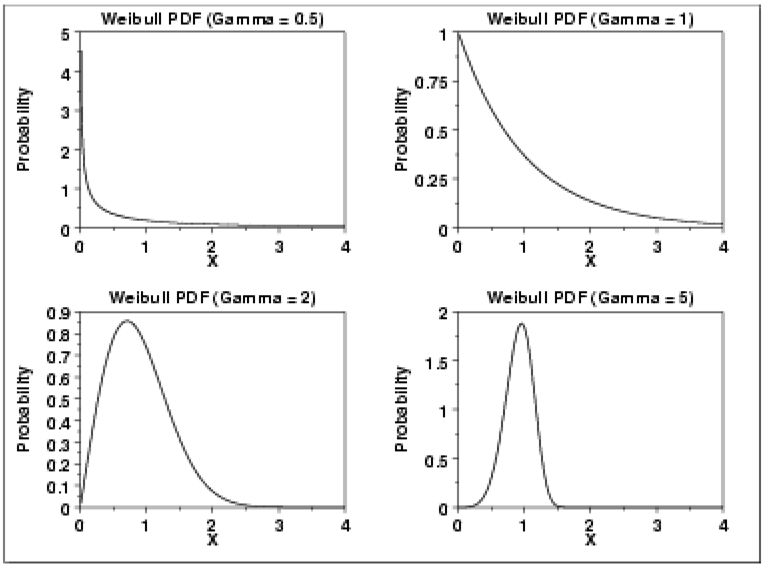

Figure 6.1 Weibull distribution function with different shape parameters (NIST/SEMATECH, 2003)

Generally, the Weibull shape parameter indicates whether the failure rate is increasing, constant or decreasing. The value of shape parameter $\beta < 1.0$ indicates that a decreasing failure rate. The $\beta = 1.0$ indicates a constant failure rate and $\beta > 1.0$ indicates an increasing failure rate.

The weibull distribution density function is given by (Mann, Schafer and Singpurwalla, 1974):

$$f(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}, \beta > 0, \eta > 0, x > \gamma \geq 0 \tag{6.1}$$

The cumulative Weibull distribution function is given by (Mann, Schafer and Singpurwalla, 1974):

$$F(x) = 1 - e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}} \tag{6.2}$$

where: β is the shape parameter, η is the scale parameter, and γ is the location parameter.

Letting γ =0, we change the form of the cumulative Weibull distribution function equation:

$$\ln\left(\ln\left[\frac{1}{1-F(x)}\right]\right) = \beta \ln x - \beta \ln \eta \tag{6.3}$$

Formula for reliability assuming a Weibull distribution is given in (Dorner, 1999):

$$R(t) = e^{-\left(\frac{\chi}{\eta}\right)^{\beta}} \tag{6.4}$$

Then the Weibull probably graph can be plotted corresponding to above equations. The probability plot is formed by: Vertical axis: Ordered reliability of responsible values and Horizontal axis: Order endurance cycles. It is possible to visualize the reliability of each design for multiple cycle values.

6.2 Determine the shape parameter and scale parameter

For distribution with shape parameters, the shape parameters are necessary to generate the Weibull probability graph. For Weibull distribution which has only single shape parameter, the shape parameter can be estimated by probability plot correlation correlation coefficient plot (PPCC). PPCC is a graphical technique for identifying the shape parameter for a distribution family. It is suitable for Weibull distribution perfectly.

The PPCC plot is used first to find a good choice for estimating the shape parameter of Weibull distribution. Then find estimates of the location and scale parameter. First of all, estimate the $F(x_i)$ from following methods. Commonly, we choose the median rank.

Table 6.1 Methods of estimating $F(x_i)$ (Al-Fawzan, 2000)

Method	$F(x_i)$
Mean Rank	$\frac{i}{n+1}$
Median Rank	$\frac{i-0.3}{n+0.4}$
Symmetrical CDF	$\frac{i-0.5}{n}$

Then use the least squares method to estimate the shape parameter β in equation 6.3. When we perform the linear regression, we will find the estimate for Weibull β parameter comes directly form the slope of the line. The estimate for the scale parameter η must be calculated as follows:

$$\eta = e^{-\left(\frac{A}{\beta}\right)} \tag{6.5}$$

where: A is estimated for the linear regression $Y = A + \beta X$

6.3 A goodness of fit test: Anderson-Darling

When we assume the fatigue data follow a specific distribution, e.g. Weibull distribution, it takes a serious risk. If our assumption is wrong, the results obtained from research could be invalid and useless.

According to document: Anderson-Darling: A Goodness of Fit Test for Small Samples Assumptions from DoD Reliability Analysis Center (Volume 10, Number 5), there are two main approaches to checking distribution assumptions. One involves empirical procedures and is based on the intuitive and graphical properties of the distribution. Another one is more formal. It is the goodness of fit test. The results form latter approach is more reliable than those form the empirical procedure. At the same time, Goodness of fit test is essentially based on either of two distribution elements: the cumulative distribution function (CDF) or the probability density function (pdf). The Anderson-Darling (AD) test which use CDF is suitable for small samples. Moreover, this AD test is also used in the Minitab software.

In AD test, we assume a pre-specified distribution to estimate the distribution parameters. This process generates a distribution hypothesis. When the assumed distribution is correct, the theoretical CDF closely follows the empirical step function CDF, as conceptually illustrated in Fatigue 6.2. The data are given as an ordered sample and the assumed distribution has a CDF. We will compare the theoretical and empirical results. If they agree, the data supports the assumed distribution.

The following equation is used to calculate the AD value for fitting Weibull distribution:

$$AD = \sum_{i=1}^{n} \frac{1-2i}{n} \left\{ \ln(1-\exp(-Z_{(i)})) - Z_{(n+1-i)} \right\} - n \tag{6.6}$$

where: $Z_{(i)} = [x_{(i)} / \eta]^{\beta}$

$$AD^{*} = (1 + 0.2/\sqrt{n})AD \qquad \text{(Corresponding to the estimation)} \tag{6.7}$$

The OSL is given:

$$OSL = 1/\left\{1 + \exp[-0.1 + 1.24\ln(AD^*) + 4.48(AD^*)]\right\} \tag{6.8}$$

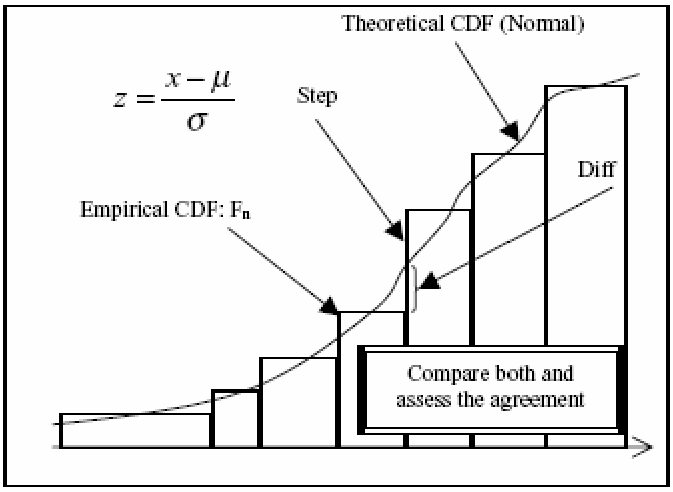

Figure 6.2 Distance goodness of fit test conceptual approach

Then the observed significance level (OSL) probability is used for testing the Weibull assumption. If this p-value is less then 0.05, Weibull assumption is rejected and the error committed is less than 5%. In other hand, the null hypothesis is accepted.

7 DETERMINE THE IMPROVEMENT FROM POST-WELD TREATMET

7.1 post-weld treatment

The dominant intention in fatigue design of load-carrying structures in mechanical engineering is to prevent fatigue failure. During cyclic load, it has been turned out that the weakest point in fabricated structures is normally the welded joints. In the weld area, it presents high stress concentration and high tensile residual stress.

The post-weld treatment is a good method to improve fatigue resistance of welded joints. The principles of this method are: (a) to modify the stress distribution near the weld to produce beneficial compressive residual stress. (b) to modify the local geometry of the weld toe to eliminate the initial defects and decrease the local stress concentration.

For the most improvement techniques such as needle peening, hammer peening and ultrasonic impact treatment, the magnitude of the improvement depends on the joint severity and base material strength. The benefit for steel can only be achieved in design class FAT 90 or lower in the IIW notation for S-N curves according to (Lihavainen Marquis and Statnikov, 1990).

7.2 Statistical analysis of the test data

The null hypothesis also can be used to analyze this problem to justify the significant difference between the data sets from as-weld and the data sets form post-weld. But it is much harder to be precise about how probable the data would be if the alternative hypothesis were true as we mentioned in section 5. Therefore, the fatigue class (FAT) is calculated to compare the difference of them in order to derive the degree of improvement.

The FAT indicates the characteristic stress range, which gives a fatigue life of two million cycles at 95% survival probability. The method of statistical analysis is developed within the International Institute of Welding (Niemi, Marquis and Poutiainen, 2005):

$$\triangle\sigma_i^m \cdot N_i = C_i = FAT^m \cdot 2000000 \quad (7.1)$$

Mean fatigue capacity $$\log C_{50\%} = \frac{\sum \log C_i}{n} \quad (7.2)$$

Standard deviation $$s = \sqrt{\frac{\sum (\log C_i - \log C_{50\%})^2}{n-1}} \quad (7.3)$$

$$\log C_{95\%} = \log C_{50\%} - s(1.64 + \frac{1.15}{\sqrt{n}}) \quad (7.4)$$

Characteristic Fatigue capacity $$FAT_{95\%} = \sqrt[m]{\frac{C_{95\%}}{2000000}} \quad (7.5)$$

Note that: due to the relatively small sample size and the convenience of evaluation, the slope is choose to be 3 for welded steel joint which the fatigue life is dominated by crack growth. Using m =3 to do the calculation is a conservative estimate.

When we estimate the slope form the obtained fatigue data to do this statistical analysis, we will find the value of degree of improvement is higher than that value obtained by using fixed slope.

8 DATA SHEETS: APPLICATION OF STATISTICAL ANALYSIS OF FATIGUE

In this data sheets, the familiar questions of statistical analysis in fatigue design are studied. In addition, the instructions of solution are shown with examples. The relative fatigue testing data is from (Marquis, 1995) and Laboratory of Fatigue and Strength in Lappeenranta University of Technology. The data form laboratory is shown in appendixⅫ. The data sheets are in the appendixes.

In the above chapters, the background and basic knowledge for each questions in the data sheets has been presented. The following words state the application of them:

Data sheets 6: Weibull distribution is often used in place of the normal distribution due to it is able to be generated easily without typically variates as normal distribution. That is the reason to determine whether the data set is Weibull distributed.

Data sheets 8: The validation of a selected S-N curve is based on a limited number of new fatigue tests. The tests of hypotheses and significance are applied to determine the sample size.

Data sheets 9: In the estimation of parameters of S-N curve, the suitable method needs to be selected according to the type of fatigue data. If the data sets include censored data, the appropriate statistical analysis will be more complicated.

Data sheets 10-13: Characteristic curve are developed to ensure the safety in design. Two methods are introduced to establish the design S-N curve with different numbers of estimated coefficients from the data.

Data sheets 15,16: When different fabrication techniques or procedures are implemented, it is necessary to ensure the improvement is obtained not only due to the sampling. Usually, we verify the means of data sets. For S-N curve, additional parameters are needed to be taken into account.

9 CONCLUSION AND RECOMMENDATIONS

In this thesis project, it illustrates the basic frame of how to use statistical methods to evaluate fatigue testing data and the solution of problem in practice. I think the well planed statistical procedures in the first several sections from section 2 to section 7 and the specific instructions for special practical cases in appendixes are helpful in fatigue design.

In this work, the precondition which assumes that the fatigue life N for a given stress range S is log-normally distributed has been emphasized. Both of least squares method and maximum likelihood method are widely used to obtain the linear regression line of S-N curve. The maximum likelihood estimation of the sample density function has many advantages and is recommended for analyzing censored data.

The sample mean and standard deviation for the normal distribution are random variables. The sample mean is related to the student's t distribution and the sample standard deviation is characterized by the chi-square distribution. There are more or less differences between the evaluation of the sample mean and the sample variance from the test data with a curve fitting method and that from theoretical distributions. It is recommended to apply a confidence level to the sample mean and deviation to get the population mean and population standard deviation. The confidence limits, prediction limits and tolerance limits are also generated to obtain characteristic values and design values of the results. They are able to be chosen according to the sample size. When we separately use the prediction limits and tolerance limits to determine the design curve base on the testing data, the comparison indicates that the second methods are more conservative.

Weibull probability graph is one of useful approaches to checking the Weibull distributional assumption. The advantage of this method is the simplicity and speed. The parameters of the distribution can be obtained easily, because the intercept and slope estimate of the fitted line are in fact estimates for the location and scale parameters of the distribution. At the same time, the Anderson-Darling test is illustrated in this work. This distance test is available to check the distribution assumption and is suitable for Weibull distribution.

The importance of determination of the significance of two fatigue data sets has been mentioned in this work. We need to pay more attention to the significance level of the composite hypothesis. However, we ignore that null hypothesis has always been controversial about the likelihood of rejecting. The theoretical calculation of fatigue class is proved to be an available method to approximate the degree of improvement.

As development of data sheets, there are still some questions need to be done. For example, how to determine a mean S-N curve by using Bastenaire equation? However, I wish this work can give people some conveniences to do statistical evaluation of fatigue data.

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APPENDIXES

Appendix Ⅰ

IIW/IIS Com XIII-WG1	Are the data Weibull distributed using a Weibull probability graph?	Sheet 6

General

1. Context

The Weibull distribution is an important distribution especially for reliability and maintainability analysis. The graphical methods are used because of their simplicity and speed. However, it involves a great probability of error.

2. Principle

This test is based on the verification of the linearity of the cumulative frequency distribution using a graph with adapted scales.

The cumulative Weibull distribution function is given by: $F(x) = 1 - e^{-(\frac{x-\gamma}{\eta})^{\beta}}$

where: β is the shape parameter, η is the scale parameter, and γ is the location parameter.

Letting γ=0, we change the form of the cumulative Weibull distribution function equation: $\ln\left(\ln[\frac{1}{1-F(x)}]\right) = \beta \ln x - \beta \ln \eta$

Anderson-Darling statistic is used to measure the nonparametric step function (based on the plot points). The smaller Anderson-Darling values indicate that the distribution fits the data better. Then observed significance level (OSL) probability (p-value) is obtained to check the assumption. If OSL<0.05, the hypothesis of Weibull distribution is rejected.

3. Condition of application

The necessary data to determine the Weibull probability graph are the following:

The data set, n values x_i (cycles)

$F(x_i)$, Commonly, we choose the median rank $\frac{i-0.3}{n+0.4}$ to estimate the $F(x_i)$

Shape parameter β, Scale parameter η

Determine the survival probability and reliability of i^{th} failure

IIW/IIS Com XIII-WG1	Are the data Weibull distributed using a Weibull probability graph?	Sheet 6

Procedure

1. Criteria

The selected level of risk is set to α , 0.05% probability to reject a correct hypothesis

2. Calculation

Calculate the shape parameter β by using least square method:

$$y = A + Bx$$

$$Y = \ln\left(\ln[\frac{1}{1-F(x)}]\right) \qquad X = \ln(x_i)$$

Estimate $F(x_i)$ of i failure- $F(x_i) = \dfrac{i-0.3}{n+0.4}$

$$B = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \qquad A = \frac{\sum y_i}{n} - B\frac{\sum x_i}{n}$$

$B =$ ………… $\qquad A =$ …………

Shape parameter- $\beta = m$ …………

Scale parameter- $\eta = e^{-\left(\frac{A}{\beta}\right)}$ …………

Reliability estimate- $R(t) = e^{-(\frac{x}{\eta})^{\beta}}$

Survival probability- $1 - R_{(t)}$

A goodness of fit test: Anderson-Darling

$$AD = \sum_{i=1}^{n} \frac{1-2i}{n}\left\{\ln(1-\exp(-Z_{(i)})) - Z_{(n+1-i)}\right\} - n$$

$$Z_{(i)} = [x_{(i)} / \eta]^{\beta}$$

$$AD^* = (1 + 0.2/\sqrt{n})AD$$

$$OSL = 1/\left\{1 + \exp[-0.1 + 1.24\ln(AD^*) + 4.48(AD^*)]\right\}$$

3. Conclusion

If OSL<0.05, the correct hypothesis of Weibull distribution is rejected.

Example

1. Data and formula

Samples data

n=29

cycles to failure			
521,382	112,910	1,133,000	689,973
1,879,752	6,766,000	3,675,000	958,318
115,816	6,195,271	22,082,998	13,475,000
10,204,041	4,310,000	7,154,785	20,511,538
14,910,395	1,015,824	664,000	95,982
10,646,018	1,580,669	3,816,199	19,048,838
1,475,769	11,541,520	787,894	605,721
			9,232,000

2. Calculation

Calculate the shape parameter β by using least square method:

$$Y = A + BX$$

$$Y = \ln\left(\ln[\frac{1}{1-F(x)}]\right) \qquad X = \ln(x_i)$$

Rank	$F(x_i)$	$\ln(\ln(\frac{1}{1-F(x_i)}))$	x_i	$\ln(x_i)$
1	0.023809524	-3.725645038	95,982	11.47191595
2	0.057823129	-2.820733108	112,910	11.63434632
3	0.091836735	-2.33996397	115,816	11.659758
4	0.12585034	-2.006163702	521,382	13.16423826
5	0.159863946	-1.747600408	605,721	13.31417476
6	0.193877551	-1.534703301	664,000	13.40603743
7	0.227891156	-1.352357777	689,973	13.44440775
8	0.261904762	-1.191772815	787,894	13.57711884
9	0.295918367	-1.047365219	958,318	13.77293494
10	0.329931973	-0.915351077	1,015,824	13.83121066

11	0.363946	-0.79303	1,133,000	13.94038
12	0.397959	-0.6784	1,475,769	14.20469
13	0.431973	-0.56989	1,580,669	14.27336
14	0.465986	-0.46628	1,879,752	14.44665
15	0.5	-0.36651	3,675,000	15.11706
16	0.534014	-0.26971	3,816,199	15.15477
17	0.568027	-0.17508	4,310,000	15.27645
18	0.602041	-0.08185	6,195,271	15.6393
19	0.636054	0.010694	6,766,000	15.72742
20	0.670068	0.10334	7,154,785	15.78329
21	0.704082	0.196941	9,232,000	16.03819
22	0.738095	0.292501	10,204,041	16.13829
23	0.772109	0.39129	10,646,018	16.1807
24	0.806122	0.495018	11,541,520	16.26146
25	0.840136	0.60619	13,475,000	16.41635
26	0.87415	0.728834	14,910,395	16.51757
27	0.908163	0.870349	19,048,838	16.76252
28	0.942177	1.047448	20,511,538	16.8365
29	0.97619	1.318462	22,082,998	16.91032

$$B = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \qquad A = \frac{\sum y_i}{n} - B\frac{\sum x_i}{n}$$

$B = 0.7244$ $\qquad$ $A = -11.22$

Shape parameter- $\beta = m$ 0.7244

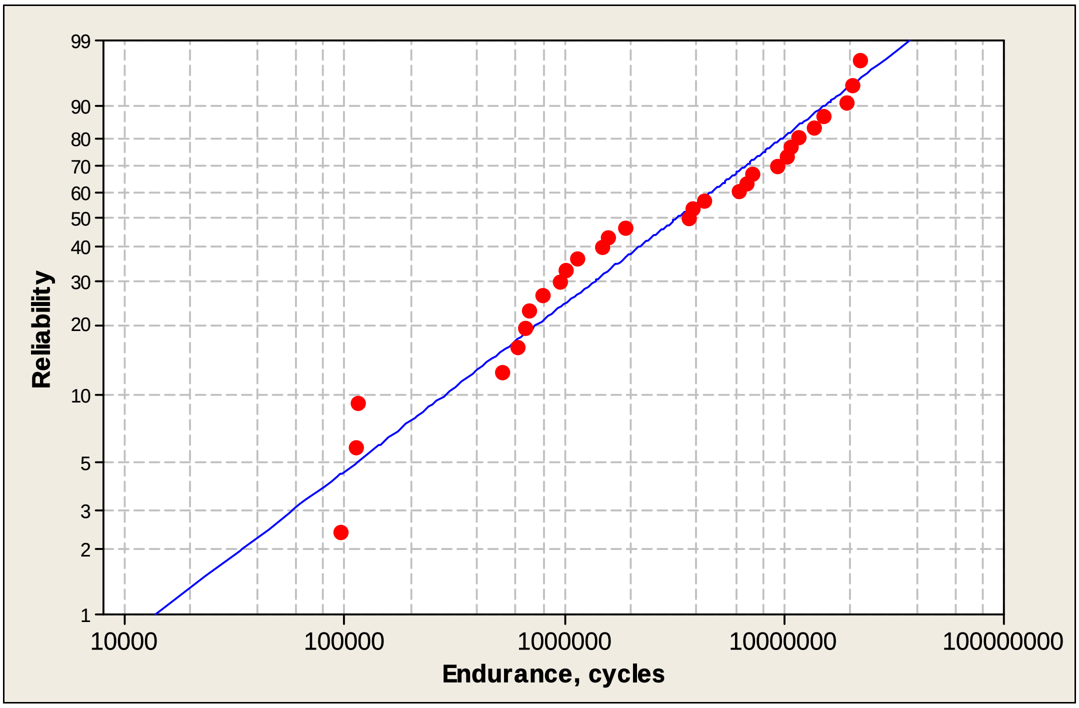
Scale parameter- $\eta = e^{-\left(\frac{A}{\beta}\right)}$ 5329032

Reliability estimate- $R(t) = e^{-(\frac{x}{\eta})^{\beta}}$

Survival probability-$1 - R_{(t)}$

IIW/IIS Com XIII-WG1	Are the data Weibull distributed using a Weibull probability graph?	Sheet 6

Cycles	Reliability	Survival probability	Cycles	Reliability	Survival probability
521,382	0.830554006	0.169445994	3,675,000	0.465804	0.534196
1,879,752	0.624950498	0.375049502	22,082,998	0.060773	0.939227
115,816	0.939476587	0.060523413	7,154,785	0.289992	0.710008
10,204,041	0.20171019	0.79828981	664,000	0.801553	0.198447
14,910,395	0.121586325	0.878413675	3,816,199	0.456054	0.543946
10,646,018	0.191884736	0.808115264	787,894	0.778497	0.221503
1,475,769	0.674013375	0.325986625	689,973	0.796571	0.203429
112,910	0.940547031	0.059452969	958,318	0.749349	0.250651
6,766,000	0.30458684	0.69541316	13,475,000	0.141119	0.858881
6,195,271	0.327823034	0.672176966	20,511,538	0.070315	0.929685
4,310,000	0.424223692	0.575776308	95,982	0.946968	0.053032
1,015,824	0.740083541	0.259916459	19,048,838	0.080762	0.919238
1,580,669	0.660587425	0.339412575	605,721	0.813049	0.186951
11,541,520	0.173717071	0.826282929	9,232,000	0.225612	0.774388
1,133,000	0.721977173	0.278022827			



Weibull probability graph

A goodness of fit test: Anderson-Darling

$$AD = \sum_{i=1}^{n} \frac{1-2i}{n} \left\{ \ln(1-\exp(-Z_{(i)})) - Z_{(n+1-i)} \right\} - n$$

$$Z_{(i)} = [x_{(i)} / \eta]^{\beta}$$

AD =29.4244-29=0.4244

$AD^* = (1+0.2/\sqrt{n})AD$ =0.4402

$$OSL = 1/\left\{1+\exp[-0.1+1.24\ln(AD^*)+4.48(AD^*)]\right\}$$

$$OSL = 0.425 > \alpha = 0.05$$

3. <u>Conclusion</u>

As OSL>0.05 then the data is Weibull distributed.

Appendix Ⅱ

IIW/IIS Com XIII-WG1	How many results are necessary to validate a selected S-N curve?	Sheet 8

General

1. Context

In fatigue testing, we would like to take enough observations to obtain reasonably precise estimates of the interested parameters, but we also want to do it within a practical resource budget.

2. Principle

We estimate the mean of population to determine the minimum sample size. When sample data is collected and the sample mean is calculated. The margin of error δ is the maximum difference between the observed sample mean and the population mean.

$$n = (z_\alpha + z_\beta)^2 \left[\frac{\sigma}{\delta}\right]^2 \qquad \text{One-sided test [1]}$$

where: the positive z value is at the vertical boundary in the right tail of the standard normal distribution. To control the risk of accepting a false hypothesis, β is the probability of accepting the null hypothesis. σ is the population standard deviation; n is the sample size.

Validating tests performed to S-N curve:

$$\overline{\log A_{test}} \geq \overline{\log A} + \frac{1.645\sigma}{\sqrt{n}} \qquad [2]$$

where: $\overline{\log A_{test}}$ is the mean logarithm of interception from the tests. $\overline{\log A}$ is the logarithm of the interception from the mean for the selected S-N curve and the value 1.645 is obtained from standard normal probability tables for a probability of 0.95.

3. Condition of application

- The slope of the mean S-N curve for the new test results is the same as m of the selected curve.
- The standard deviation σ_{test} of log N about the mean S-N curve for the new tests is the same as that for the main database.

Procedure

1. Criteria

For other significance levels, different values are obtained from the table:

10%	Level of significance: 1.285
5%	Level of significance: 1.645
2.5%	level of significance: 1.96
1%	Level of significance: 2.33

Note that the corresponding level of significance of 5% is commonly considered to give a sufficiently low probability of concluding that the populations are different in the case where they are actually that same.

2. Data and formula

Validation the use of Class D at the 5% level of significance:

Mean S-N curve for Class D- $S^3 N = 3.99 \times 10^{12}$

Standard deviation of $\log N$ - $\sigma = 0.2097$

Slop of design curve- m ………...

Sum of values- $\sum \log N$ ………...

Mean value- $\overline{\log N} = \dfrac{\sum \log N}{n}$ ………...

Sum of values- $\sum \log S$ ………...

Mean value- $\overline{\log S} = \dfrac{\sum \log S}{n}$ ………...

According to $\overline{\log A} = \overline{\log N} + m\overline{\log S}$, mean value- $\overline{\log A}$ …………

Transform the Eq.[2] into the form as Eq.[1]:

$$n \geq (z_{\alpha/2} + z_{\beta})^2 \left(\frac{\sigma}{\overline{\log A_{test}} - \overline{\log A}} \right)^2$$

Example

1. Data and formula

Samples data

n =29

Stress range	cycles to failure	Stress range	cycles to failure
147	521,382	74	3,675,000
96	1,879,752	53	22,082,998
250	115,816	53	7,154,785
61	10,204,041	136	664,000
57	14,910,395	75	3,816,199
57	10,646,018	136	787,894
96	1,475,769	136	689,973
250	112,910	147	958,318
74	6,766,000	53	13,475,000
74	6,195,271	53	20,511,538
74	4,310,000	265	95,982
136	1,015,824	54	19,048,838
136	1,580,669	176	605,721
53	11,541,520	74	9,232,000
136	1,133,000		

2. Calculation

Validation the use of Class D S-N curve at the 5% level of significance:

Mean S-N curve for Class D- $S^3 N = 3.99 \times 10^{12}$

Standard deviation of $\log N$ - $\sigma = 0.2097$

Slop of design curve- m 3

Mean value- $\overline{\log A}$ 12.601

Sum of values- $\sum \log N_{test}$ 185.401

Mean value- $\overline{\log N_{test}} = \dfrac{\sum \log N_{test}}{n}$ 6.393

Sum of values- $\sqrt{\log S_{test}}$ 57.437

Mean value- $\overline{\log S_{test}} = \dfrac{\sum \log S_{test}}{n}$ 1.981

According to $\overline{\log A} = \overline{\log N} + m\overline{\log S}$, mean value- $\overline{\log A_{test}}$ 12.335

Transform the Eq.[2] into the form as Eq.[1]:

$$n \geq (z_\alpha + z_\beta)^2 \left(\frac{\sigma}{\overline{\log A_{test}} - \overline{\log A}} \right)^2 \qquad \alpha = 5\% \qquad \beta = 10\%$$

$$n \geq (1.645 + 1.285)^2 \left(\frac{0.2097}{12.335 - 12.601} \right)^2$$

$$n \geq 5.335$$

3. <u>Conclusion</u>

According to above calculations, the minimum number of sample size to validate the selected class D S-N curve should not be less than 6.

AppendixⅢ

IIW/IIS Com XIII-WG1	What S-N cure can be selected from a set of non cracked results?	Sheet 9

General

1. <u>Contest</u>

Commonly, we assumed that each specimen in the test yielded an exact failure. Actually, the results or parts of specimens which are obtained from specimens have not failed. Those unfailed or non cracked specimen are often termed ‘run-outs’.

It is known that under constant amplitude loading there is a fatigue endurance defined as the stress range below which failure will not occur. For design purpose, we make the S-N curve extend down to the constant amplitude fatigue limit and turn to a horizontal line. However, in practice fatigue test results usually follow an S-N curve that gradually changes slope in the region of the constant amplitude fatigue limit.

2. <u>Principle</u>

Various statistical methods can be used to analyze the endurance test results, determined statistical parameters. Based on the working group 1 of Commission XII of the International Institute of Welding, those statistical methods can be sorted as the following table according to the each main characteristic.

Method of Analysis	S-N curve equation	Method and Assumptions	Parameters
(A)	$\log N = \log C - m \log S$	Mean line bisecting the two regression lines	$C, m,$ ^{s}logS
(B)	$\log N = \log C - m \log S$	Linear regression of log N on log S ignoring run-out's	$C, m,$ ^{s}logS
(C)	$\log N = \log C - m \log S$	Maximum likelihood (includes run-out's)	$C, m,$ ^{s}logS
(D)	$\log N = \log C - m \log S$	Linear regression of log N on log S run-out's being included	$C, m,$ ^{s}logS
(E)	$N = \frac{A}{S-E}\left[-\frac{(S-E)C}{B}\right]$	Multiple non-linear regression including censored data (run-out's) Stress response curves: sigmoid normal	A, B, E ^{s}S, F

Method A

Definition of the S-N curve by drawing bisecting line of the straight lines plotted from the regression of $\log N$ against $\log S$ and of $\log S$ against $\log N$ respectively.

Method B

Conventional plotting of $\log N$ - $\log S$

Method C

By using the maximum likelihood method, straight lines can be plotted in a $\log S - \log N$ coordinate system. It is an appropriate tool for solving the general problem of estimating the 'best fit' line through censored test data. In general case, numerical iteration is required to derive maximum likelihood estimates. In special case of exact data, the maximum likelihood method leads to the least squares function on which linear regression is generally based.

This method amounts to curve fitting in compliance with the formula:

$$N \times S^n = K$$

Method D and E

BASTENAIRE's method provides for fitting particularly any model of S-N curve by supplementing a basal computer program with sub-programs. Two models may be considered:

$$N \times S^m = C \qquad \text{Method D}$$

$$N(S-E) = A\exp[-(S-E)/B]^c \qquad \text{Method E}$$

where: S is the nominal stress and N the corresponding number of cycles to failure, A, B, C and E are statistically assessed parameters.

3. Condition of application

- In statistical analysis, we are able to decide whether the results from unfailed specimens can be used according to the circumstance.
- Test results from either failed or unfailed specimens that lie in this transition region approaching the constant amplitude fatigue limit should not be combined with those obtained at higher stresses in the estimation of the best-fit linear S-N curve.
- Depending on this guide, notched or welded specimens that give $N < 2 \times 10^6$ cycles could be fall into this category, for smooth specimen $N < 10^6$.
- In addition, fatigue test results from unfailed specimens can be used to estimation of the best-fit S-N curve in two other situations:
    a) The test is stopped deliberately.
    b) The test specimen contains more than one site for potential fatigue failure and fails from just one of them.

4. Conclusion

The methods A and B do not take unbroken specimens into account. While other three methods consider the run-outs in the analysis and which have a bearing on the final results. But when the fatigue data include either run-outs or suspended test, the appropriate statistical analyses become more complicated. Fortunately, many statistical software packages can perform the required calculations.

At the same time, methods based on the formula $N \times S^m = C$ yield strongly variable results if parameters C and m are taken into consideration; while the deviation between the S values is smaller for a given fatigue life.

When we test a number of specimens which contain the same number of nominally identical welds and any of which might fail first. If each specimen is tested until it fails at exactly one of the potential locations, then the S-N curve for a single weld can be established by using least squares estimation, together with the tabulated extreme value statistics for normal distribution.

Appendix Ⅳ

IIW/IIS Com XIII-WG1	How to determine a design S-N curve, slope fixed (prediction limits)?	Sheet 10

General

1. Contest

For design purpose, it is necessary to establish limits between which a given proportion (typically 95%) of the data lies. Prediction limits are used to avoid confusion with the confidence limits on the coefficients of the regression line. The interval between the upper and lower prediction limits is called a prediction interval.

2. Principle

Prediction limits at stress range S can be expressed explicitly, in the form:

$$\log N_{p\%}^{\pm} = (\log A - m\log S) \pm t\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(\log S - \overline{\log S})^2}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2}}$$

where: $\log A$ and m are the coefficients of the regression line through the n data points

$\overline{\log S}$ is the mean of the n values of $\log S_i$

t is the appropriate percentage point of Student's t distribution

$\hat{\sigma}$ is the best estimate of the variance of the data about the regression line,

f is degrees of freedom

3. Condition of application

- It is often assumed that design curves will only be applied to values of $\log S$ that are not far from the mean value. In this case the third term under the square root in the equation can be ignored.
- When slope m is chosen to take a fixed value. The number of degrees of freedom should be $n-1$. Whenever the sample sizes is less than ten it is recommended. The expression under the square root is exactly equal to one.
- When the sample size is larger than 20, the second term under the square root is able to be ignored.

Procedure

1. Criteria

The design curves will only be applied to values of $\log S$ that are not far removed from the mean value $\overline{\log S}$.

In general welding case, we use a gradient of $m=3$.

2. Data and formula

Using the number of stress range, load cycles, and $\log A = \overline{\log N} + m\overline{\log S}$ to obtain:

Mean of Intercept of regression line- $\log A$

Slop of design curve- m ....

Sample size- n ....

Sum of values- $\sum \log S_i$ ....

Mean values- $\overline{\log S} = \dfrac{\sum \log S_i}{n}$ ....

Sum of values- $\sum \log N_i$ ....

Mean values- $\overline{\log N} = \dfrac{\sum N_i}{n}$ ....

Estimated mean of Intercept of regression line- $\log \hat{A}$ ....

Estimated value- $\log \hat{N}_i = \log \hat{A} - 3 \times \log S_i$

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n} (\log N_i - \log \hat{N}_i)^2}{f}$ ....

Degree of freedom- $f = n-1$ ....

$$\hat{\sigma} = \sqrt{\frac{\sum(\log N_i - \log \hat{N}_i)^2}{f}}$$ ....

Appropriate percentage point of two sided Student's t distribution- t ....

Design S-N curve: $\log N^{\pm}_{p\%} = (\log A - m \log S) \pm t\hat{\sigma}$

Example

1. Data and formula

Samples data

$n = 9$

Stress range	cycles to failure
147	521,382
96	1,879,752
250	115,816
61	10,204,041
57	14,910,395
57	10,646,018
96	1,475,769
250	112,910
74	6,766,000

2. Calculation

Proportion 95%

Using the number of stress range, load cycles, and $\log A = \overline{\log N} + m\overline{\log S}$ to obtain

Mean of Intercept of regression line- $\log A$

Slop of design curve- m 3

Sample size- n 9

Sum of values- $\sum \log S_i$ 18.094

Mean values- $\overline{\log S} = \frac{\sum \log S_i}{n}$ 2.010

Sum of values- $\sum \log N_i$ 56.317

Mean values- $\overline{\log N} = \frac{\sum N_i}{n}$ 6.257

Estimated mean of Intercept of regression line- $\log \hat{A}$ 12.2889

Estimated value- $\log \hat{N}_i = \log \hat{A} - 3 \times \log S_i$

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n} (\log N_i - \log \hat{N}_i)^2}{f}$ 0.0117

Degree of freedom- $f = n - 1$ 8

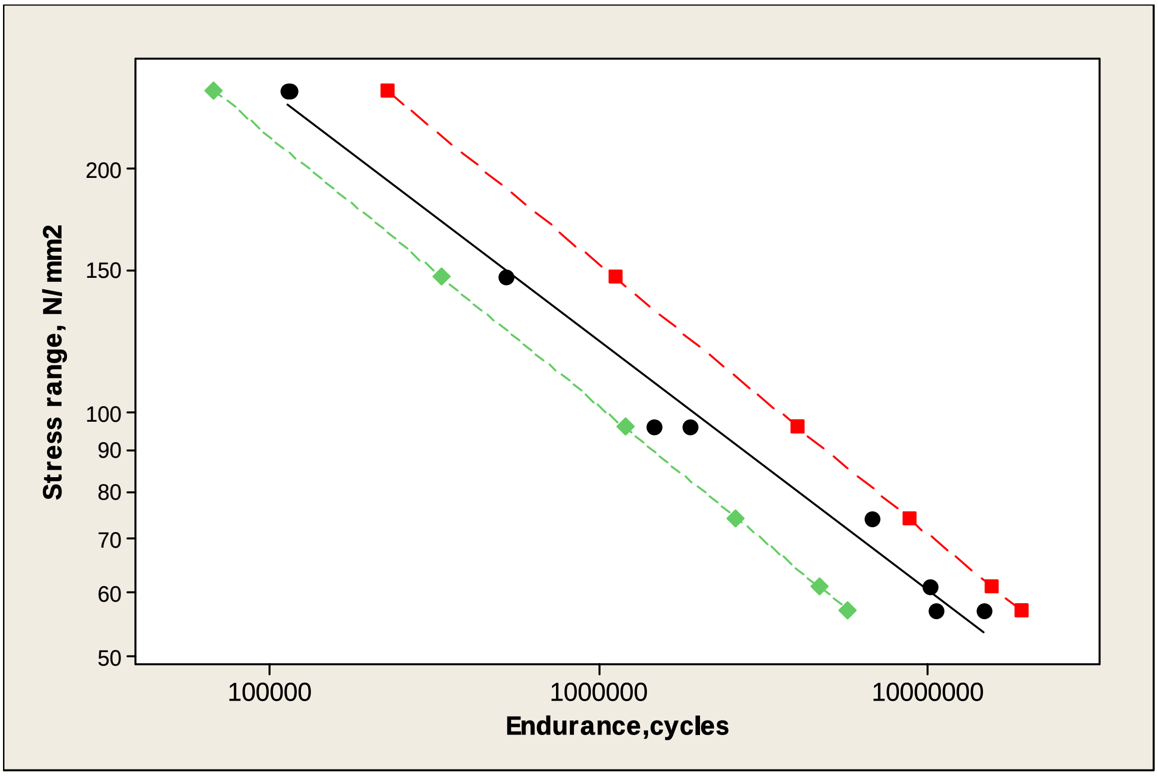
$\hat{\sigma} = \sqrt{\dfrac{\sum (\log N_i - \log \hat{N}_i)^2}{f}}$ 0.108

Appropriate percentage point of two sided Student's t distribution- t 2.306

Design S-N curve: $\log N^{\pm}_{p\%} = (\log A - m \log S) \pm t\hat{\sigma}\sqrt{1 + \dfrac{1}{n}}$

$$\log N^{\pm}_{95\%} = (12.2889 - 3 \log S) \pm 2.306 \times 0.108 \times \sqrt{1 + \frac{1}{9}}$$

$$\log N^{\pm}_{95\%} = (12.2889 - 3 \log S) \pm 0.2625$$



Design curve based on prediction limits (slope fixed)

Appendix V

IIW/IIS Com XIII-WG1	How to determine a design S-N curve, slop estimated (prediction limits)?	Sheet 11

General

1. Contest

For design purpose, it is necessary to establish limits between which a given proportion (typically 95%) of the data lies. The instruction for determination of a design S-N curve based on prediction limits has been introduced in Data Sheet10. But it focuses on the slop fixed case. When both of the slop and intercept are needed to be estimated, some changes are needed to be emphasized.

2. Principle

Prediction limits at stress range S can be expressed explicitly, in the form:

$$\log N_{p\%}^{\pm} = (\log A - m \log S) \pm t\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(\log S - \overline{\log S})^2}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2}}$$

where: $\log A$ and m are the coefficients of the regression line through the n data points

$\overline{\log S}$ is the mean of the n values of $\log S_i$

t is the appropriate percentage point of Student's t distribution

$\hat{\sigma}$ is the best estimate of the variance of the data about the regression line,

f is degree of freedom, $f = n - 2$ in case where the two coefficients of the regression line have been estimated from the data

Additionally, the Maximum likelihood estimation is used to estimate the slope and intercept coefficients.

3. Condition of application

- In general, it is often assumed that design curves will only be applied to values of log S that are not far from the mean value $\overline{\log S}$.In this case the third term under the square root of equation can be ignored
- When the sample size larger than 20. Then the second term under the square root is able to be ignored.

Procedure

1. Criteria

The design curves will only be applied to values of $\log S$ that are not far removed from the mean value $\overline{\log S}$

2. Data and formula

The correct values for the constants A and m are obtained from the following two equations by using the maximum likelihood method:

$$\log A = \overline{\log N} + m\overline{\log S}$$

$$m = \frac{\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2}$$

Sample size- n …………

Degree of freedom- $f = n-2$ …………

Sum of values- $\sum \log S_i$ …………

Mean values- $\overline{\log S} = \frac{\sum \log S_i}{n}$ …………

Sum of values- $\sum \log N_i$ …………

Mean values- $\overline{\log N} = \frac{\sum N_i}{n}$ …………

$\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})$ …………

$\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2$ …………

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Estimated slope- $\hat{m}$ ....

Estimated mean of Intercept of regression line- $\log\hat{A}$ ....

Estimated value- $\log\hat{N}_i = \log\hat{A} - \hat{m}\times\log S_i$

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n}(\log N_i - \log\hat{N}_i)^2}{f}$ ....

$$\hat{\sigma} = \sqrt{\frac{\sum(\log N_i - \log\hat{N}_i)^2}{f}}$$ ....

Appropriate percentage point of two sided Student's t distribution- t ....

Design S-N curve: $\log N^{\pm}_{p\%} = (\log A - m\log S) \pm t\hat{\sigma}\sqrt{1+\frac{1}{n}}$

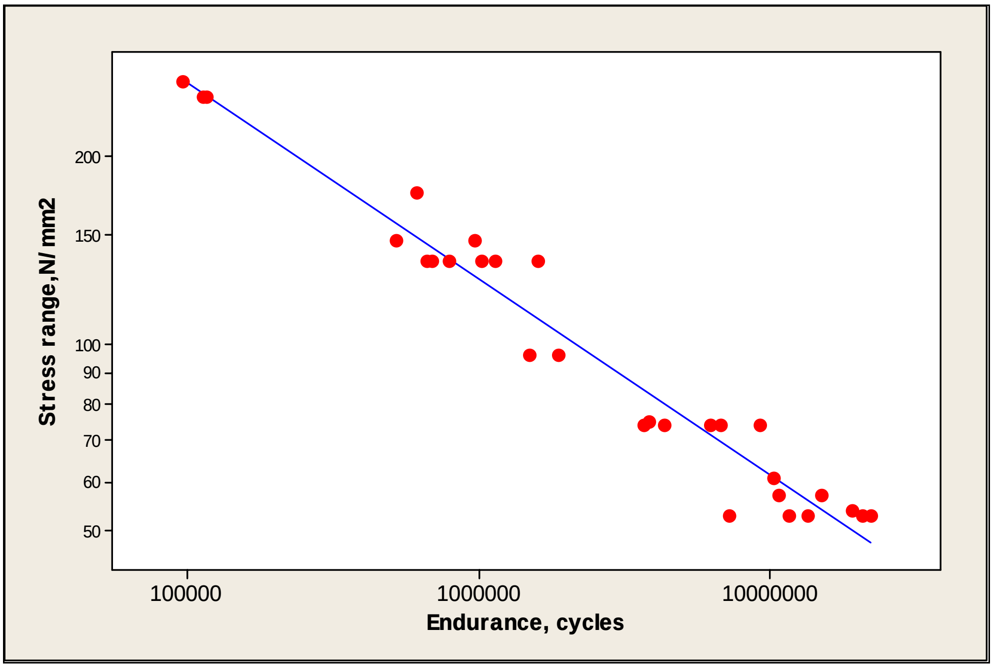
Example

1. Data and formula

Samples data

$n = 29$

Stress range	cycles to failure	Stress range	cycles to failure
147	521,382	74	3,675,000
96	1,879,752	53	22,082,998
250	115,816	53	7,154,785
61	10,204,041	136	664,000
57	14,910,395	75	3,816,199
57	10,646,018	136	787,894
96	1,475,769	136	689,973
250	112,910	147	958,318
74	6,766,000	53	13,475,000
74	6,195,271	53	20,511,538
74	4,310,000	265	95,982
136	1,015,824	54	19,048,838
136	1,580,669	176	605,721
53	11,541,520	74	9,232,000
136	1,133,000		



Linear regression S-N curve

2. Calculation

Proportion 95%

The correct values for the constants A and k are obtained from the following two equations by using the maximum likelihood method:

$$\log \hat{A} = \overline{\log N} + m\overline{\log S}$$

$$\hat{m} = -\frac{\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2}$$

Sample size- n	29
Degree of freedom- $f = n-2$	27
Sum of values- $\sum \log S_i$	57.437
Mean values- $\overline{\log S} = \frac{\sum \log S_i}{n}$	1.9806
Sum of values- $\sum \log N_i$	185.401
Mean values- $\overline{\log N} = \frac{\sum N_i}{n}$	6.3931
$\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})$	-4.332
$\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2$	1.427
Estimated slope- $\hat{m}$	3.036
Estimated mean of Intercept of regression line- $\log \hat{A}$	12.4062

Estimated value- $\log \hat{N}_i = \log \hat{A} - \hat{m} \times \log S_i$

IIW/IIS Com XIII-WG1	How to determine a design S-N curve, slop estimated (prediction limits)?	Sheet 11

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n} (\log N_i - \log \hat{N}_i)^2}{f}$ 0.0215

$\hat{\sigma} = \sqrt{\dfrac{\sum (\log N_i - \log \hat{N}_i)^2}{f}}$ 0.1465

Appropriate percentage point of two sided Student's t distribution- t 2.052

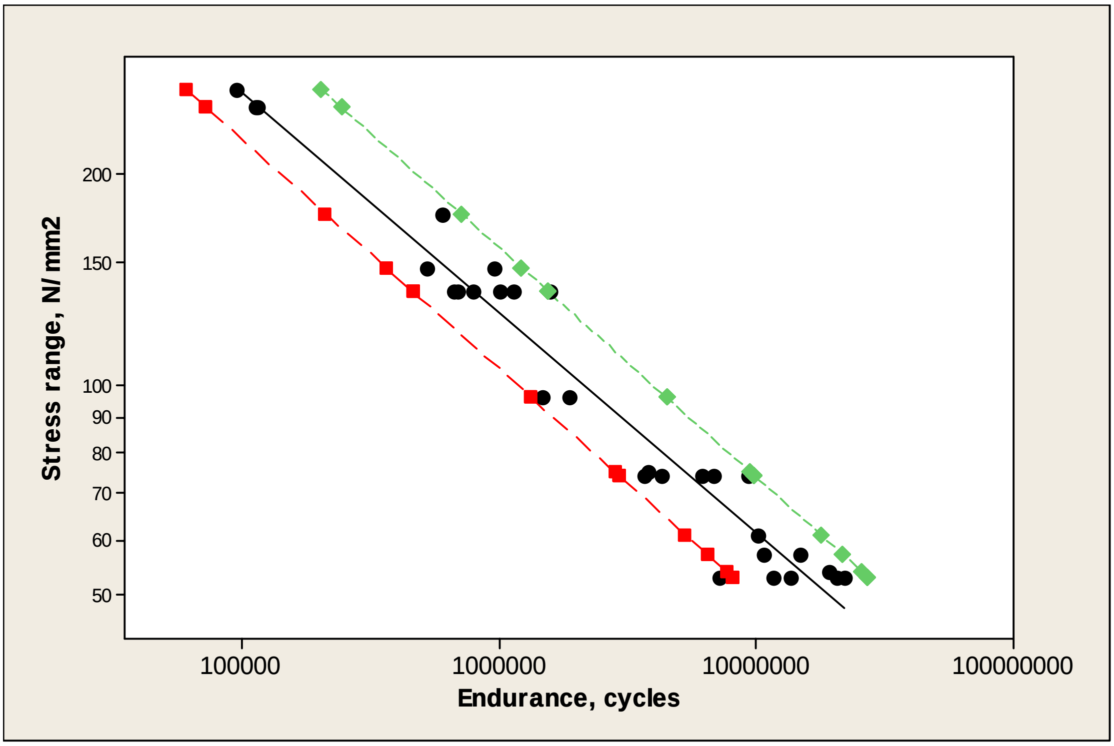
Design S-N curve: $\log N^{\pm}_{p\%} = (\log A - m \log S) \pm t\hat{\sigma}\sqrt{1+\dfrac{1}{n}}$

$$n = 29 > 20$$

$$\log N^{\pm}_{p\%} = (\log A - m \log S) \pm t\hat{\sigma}$$

$$\log N^{\pm}_{95\%} = (12.4062 - 3.036 \log S) \pm 2.052 \times 0.1465$$

$$\log N^{\pm}_{95\%} = (12.4062 - 3.036 \log S) \pm 0.301$$



Design curve based on prediction limits (slope estimated)

Appendix Ⅵ

IIW/IIS Com XIII-WG1	How to determine a design S-N curve, slope fixed (tolerance limits)?	Sheet 12

General

1. Contest

The use of tolerance limits rather than prediction limits would yield a more conservative design curve and they have the advantage that they explicitly allow for uncertainty in estimates of population statistics from a small sample.

2. Principle

This statement is made on the basis of a sample of n independent observations. A tolerance limit can be regarded as a confidence limit on a prediction limit.

$$\log N_{p\%}^{-} = \hat{\mu} - ks$$

where: $\hat{\mu}$ is an estimate of the mean log of the endurance at stress S

s is an estimate of the standard deviation of the log of the endurance at stress S, based on f degree of freedom

k is a one-sided tolerance limit factor

Tolerance limits stated that at least a proportion of normal population is greater than $\hat{\mu} - ks$ with confidence γ.

In this slope fixed case, the one sided tolerance limit factor k has been tabulated in the appendix X table 1.

3. Condition of application

- It is assumed that design curves will only be applied to values of $\log S$ that are not far removed from the mean value $\overline{\log S}$.
- When slope m is chosen to take a fixed value. The number of degrees of freedom should be increased by one. Whenever the sample sizes is less than ten it is recommended.

Procedure

1. <u>Criteria</u>

A normal distribution for $\gamma\%$ =90%, Proportion 95%

The design curves will only be applied to values of $\log S$ that are not far removed from the mean value $\overline{\log S}$

2. <u>Data and formula</u>

Using the number of stress range, load cycles, and $\log A = \overline{\log N} + m\overline{\log S}$ to obtain mean of Intercept of regression line- $\log A$

Slop of design curve- m ....

Sample size- n ....

Sum of values- $\sum \log S_i$ ....

Mean values- $\overline{\log S} = \dfrac{\sum \log S_i}{n}$ ....

Sum of values- $\sum \log N_i$ ....

Mean values- $\overline{\log N} = \dfrac{\sum N_i}{n}$ ....

Estimated mean of Intercept of regression line- $\log \hat{A}$ ....

Estimated value- $\log \hat{N}_i = \log \hat{A} - 3 \times \log S_i$

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n} (\log N_i - \log \hat{N}_i)^2}{f}$ ....

Degree of freedom- $f = n - 1$ ....

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(\log N_i - \log \hat{N}_i)^2}{f}} \qquad \ldots\ldots\ldots\ldots$$

One-sided tolerance limit factor- k ....

Design S-N curve: $\log N^{-}_{p\%} = \hat{\mu} - ks$

Example

1. **Data and formula**

Samples data

n =9

Stress range	cycles to failure
147	521,382
96	1,879,752
250	115,816
61	10,204,041
57	14,910,395
57	10,646,018
96	1,475,769
250	112,910
74	6,766,000

2. **Calculation**

Proportion 95%

Using the number of stress range, load cycles, and $\log A = \overline{\log N} + m\overline{\log S}$ to obtain

Mean of Intercept of regression line- $\log A$

Slop of design curve- m 3

Sample size- n 9

Sum of values- $\sum \log S_i$ 18.094

Mean values- $\overline{\log S} = \dfrac{\sum \log S_i}{n}$ 2.010

Sum of values- $\sum \log N_i$ 56.317

Mean values- $\overline{\log N} = \dfrac{\sum N_i}{n}$ 6.257

Estimated mean of Intercept of regression line- $\log \hat{A}$ 12.2889

Estimated value- $\log \hat{N}_i = \log \hat{A} - 3 \times \log S_i$

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n} (\log N_i - \log \hat{N}_i)^2}{f}$ 0.0117

Degree of freedom- $f = n - 1$ 8

$\hat{\sigma} = \sqrt{\dfrac{\sum (\log N_i - \log \hat{N}_i)^2}{f}}$ 0.108

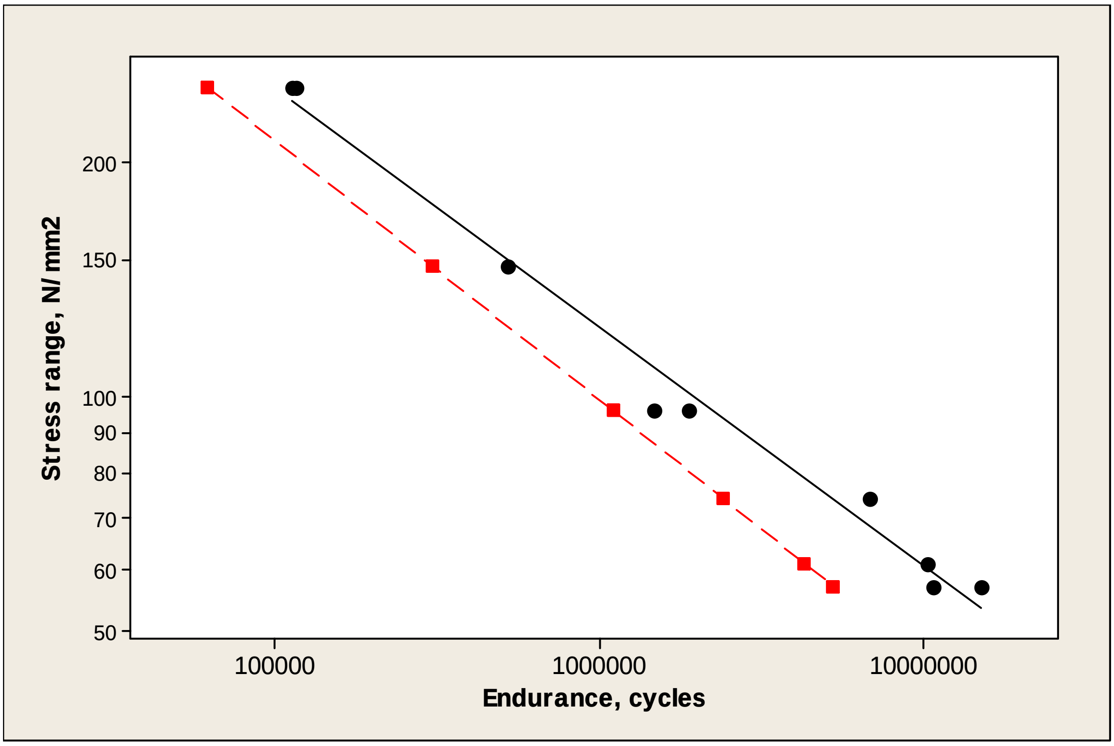
One-sided tolerance limit factor- k 2.650

Design S-N curve: $\log N^{-}_{p\%} = \hat{\mu} - ks$

$$\log N^{-}_{p\%} = (\log A - m \log S) - k\hat{\sigma}\sqrt{1 + \frac{1}{n}}$$

$$\log N^{-}_{95\%} = (12.2889\text{-}3\log S)\text{-}2.65 \times 0.108 \times \sqrt{1 + \frac{1}{9}}$$

$$\log N^{-}_{95\%} = 11.9869 - 3 \log S$$



Design curve based on tolerance limits (slope fixed)

Appendix Ⅶ

IIW/IIS Com XIII-WG1	How to determine a design S-N curve, slop estimated (tolerance limits)?	Sheet 13

General

1. Contest

In Data Sheet12, the instructions for determining a design S-N curve with fixed slope value based on tolerance limits have been illustrated. In the general case where the slope of the regression line is estimated from the data, the tolerance limit factor is different from the previous example.

2. Principle

This statement is made on the basis of a sample of n independent observations. A tolerance limit can be regarded as a confidence limit on a prediction limit.

$$\log N^{-}_{p\%} = \hat{\mu} - ks$$

where: $\hat{\mu}$ is an estimate of the mean log of the endurance at stress S

s is an estimate of the standard deviation of the log of the endurance at stress S, based on f degree of freedom

k is a one-sided tolerance limit factor

The involved calculation to determine the one-side tolerance limit factor k is laborious. Mathematically, k is determined by the following equation:

$$\Pr\{(noncenral\ t\ with\ \delta = K_p\sqrt{n}) \le k\sqrt{n}\ \} = \gamma$$

where: K_p is defined by $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{K_p}\exp(-x^2/2)dx = P$

If δ and f are given, the critical value of noncentral t-distribution t_0 can be calculated, then k value can be obtained by equation: $k = \frac{t_0}{\sqrt{n}}$.

3. Condition of application

- It is assumed that design curves will only be applied to values of $\log S$ that are not far removed from the mean value $\overline{\log S}$.

Procedure

1. Criteria

A normal distribution for $\gamma\%$ =90%, Proportion 95%

The design curves will only be applied to values of $\log S$ that are not far removed from the mean value $\overline{\log S}$

2. Data and formula

The correct values for the constants A and m are obtained from the following two equations by using the maximum likelihood method:

$$\log A = \overline{\log N} + m\overline{\log S}$$

$$m = \frac{\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2}$$

Sample size- n ....

Degree of freedom- $f = n-2$ ....

Sum of values- $\sum \log S_i$ ....

Mean values- $\overline{\log S} = \frac{\sum \log S_i}{n}$ ....

Sum of values- $\sum \log N_i$ ....

Mean values- $\overline{\log N} = \frac{\sum N_i}{n}$ ....

$\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})$ ....

$\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2$ ....

Estimated slope- $\hat{m}$ …………

Estimated mean of Intercept of regression line- $\log \hat{A}$ …………

Estimated value- $\log \hat{N}_i = \log \hat{A} - \hat{m} \times \log S_i$ …………

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n} (\log N_i - \log \hat{N}_i)^2}{f}$ …………

$\hat{\sigma} = \sqrt{\dfrac{\sum (\log N_i - \log \hat{N}_i)^2}{f}}$ …………

Upper critical value of standard normal distribution- $K_{95\%}$ …………

Value of $\delta = K_p \sqrt{n}$ …………

Value of $\eta = \dfrac{\delta}{\sqrt{2f}} \left(1 + \dfrac{\delta^2}{2f}\right)^{-1/2}$ …………

Value of λ based on η, f …………

Critical values of the Noncentral t -distribution- $t_0 = \dfrac{\delta + \lambda \left(1 + \dfrac{\delta^2}{2f} - \dfrac{\lambda^2}{2f}\right)}{1 - (\lambda^2 / 2f)}$ …………

According to $\Pr\left(noncenral\ \ t \le k\sqrt{n}\right) = \gamma\%$

One-sided tolerance limit factor- $k = t_0 / \sqrt{n}$ …………

Design S-N curve: $\log N_{p\%}^{\pm} = (\log A - m \log S) \pm k\hat{\sigma}\sqrt{1 + \dfrac{1}{n}}$

IIW/IIS Com XIII-WG1	How to determine a design S-N curve, slop estimated (tolerance limits)?	Sheet 13

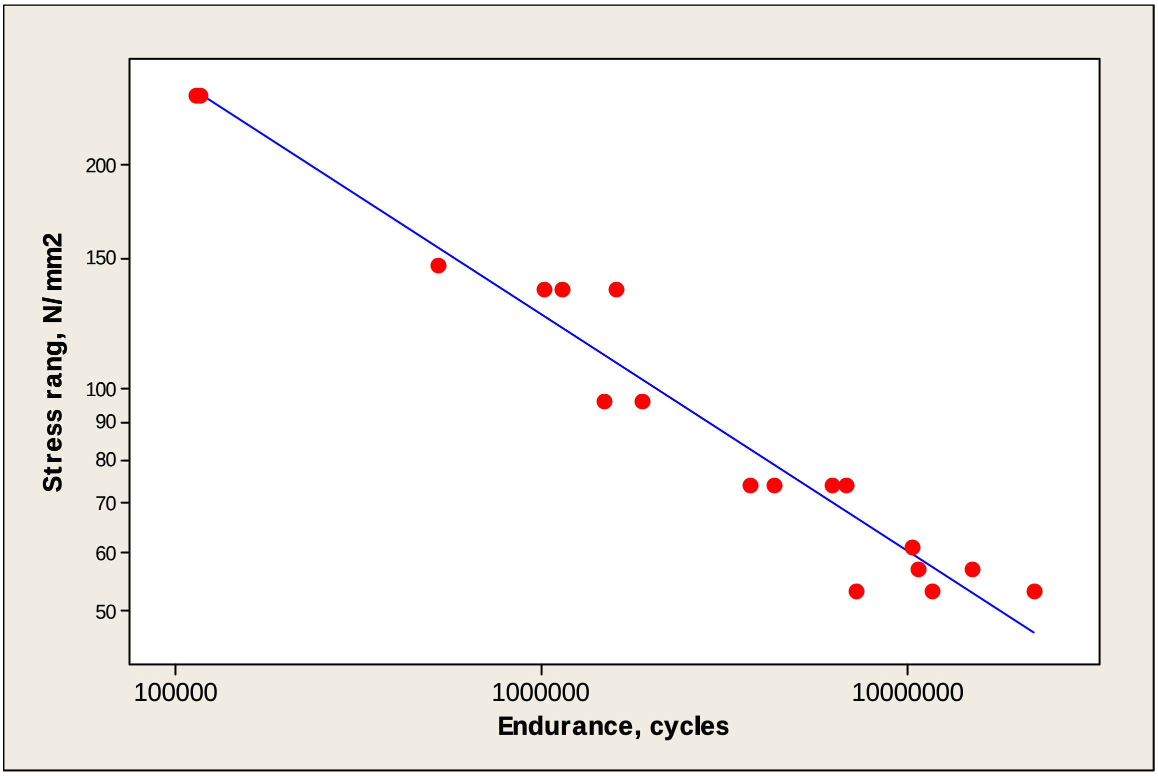
Example

1. Data and formula

Samples data

$n = 18$

Stress range	cycles to failure	Stress range	cycles to failure
147	521,382	74	6,195,271
96	1,879,752	74	4,310,000
250	115,816	136	1,015,824
61	10,204,041	136	1,580,669
57	14,910,395	53	11,541,520
57	10,646,018	136	1,133,000
96	1,475,769	74	3,675,000
250	112,910	53	22,082,998
74	6,766,000	53	7,154,785



Linear regression S-N curve

2. Calculation

Proportion 95%

The correct values for the constants A and k are obtained from the following two equations by using the maximum likelihood method:

$$\log \hat{A} = \overline{\log N} + m\overline{\log S}$$

$$\hat{m} = -\frac{\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2}$$

Sample size- n	18
Degree of freedom- $f = n - 2$	16
Sum of values- $\sum \log S_i$	35.275
Mean values- $\overline{\log S} = \frac{\sum \log S_i}{n}$	1.9597
Sum of values- $\sum \log N_i$	115.829
Mean values- $\overline{\log N} = \frac{\sum N_i}{n}$	6.4350
$\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})$	-2.477
$\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2$	0.8314
Estimated slope- $\hat{m}$	2.979
Estimated mean of Intercept of regression line- $\log \hat{A}$	12.2729

Estimated value- $\log \hat{N}_i = \log \hat{A} - \hat{m} \times \log S_i$

Estimated the variance- $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n}(\log N_i - \log \hat{N}_i)^2}{f}$ 0.0232

$\hat{\sigma} = \sqrt{\dfrac{\sum(\log N_i - \log \hat{N}_i)^2}{f}}$ 0.1522

Upper critical value of standard normal distribution- $K_{95\%}$ 1.65

Value of $\delta = K_{95\%}\sqrt{n}$ 7.0

Value of $\eta = \dfrac{\delta}{\sqrt{2f}}\left(1+\dfrac{\delta^2}{2f}\right)^{-1/2}$ 0.778

Value of λ based on η, f 1.338

Critical values of the Noncentral t-distribution- $t_0 = \dfrac{\delta + \lambda\left(1+\dfrac{\delta^2}{2f}-\dfrac{\lambda^2}{2f}\right)}{1-(\lambda^2/2f)}$ 10.923

According to $\Pr\left(noncenral\ \ t \le k\sqrt{n}\right) = 90\%$

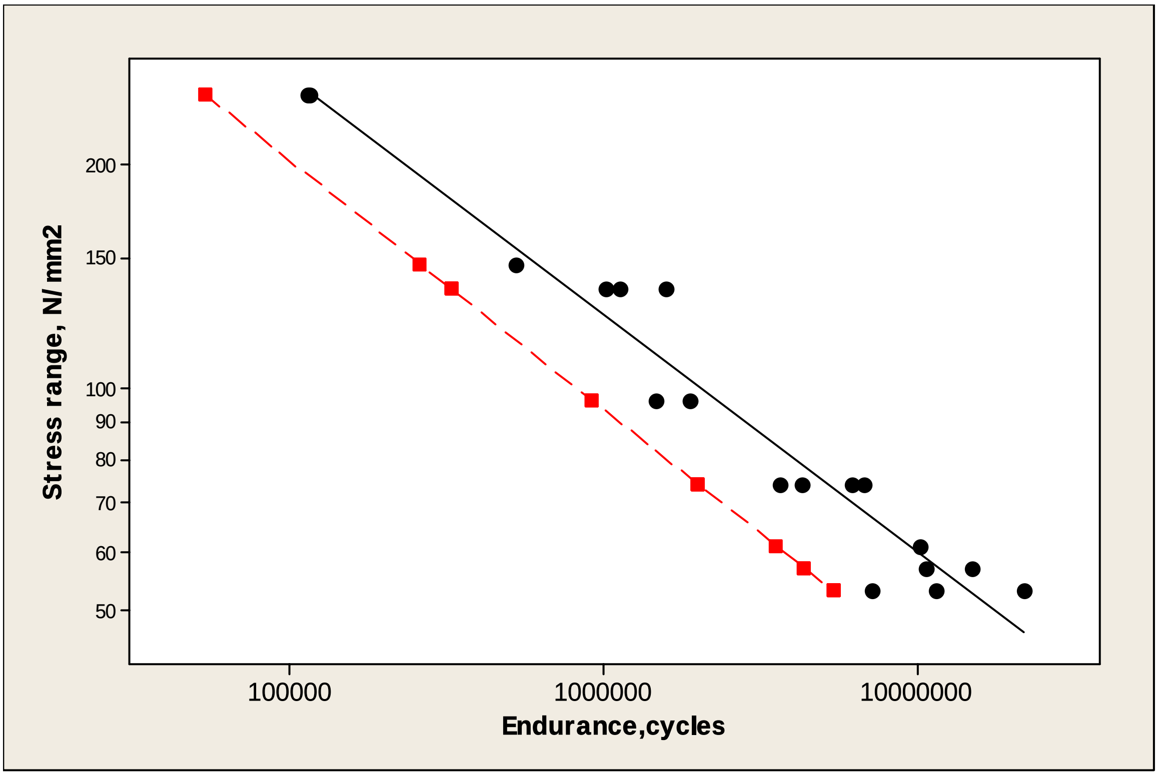
One-sided tolerance limit factor- $k = t_0/\sqrt{n}$ 2.575

Design S-N curve: $\log N_{p\%}^{\pm} = (\log A - m\log S) - k\hat{\sigma}\sqrt{1+\dfrac{1}{n}}$

$$\log N_{95\%}^{\pm} = (12.2729 - 2.979\log S) - 2.575\times 0.1522\sqrt{1+\frac{1}{18}}$$

$$\log N_{95\%}^{\pm} = 11.8699 - 2.979\log S$$

IIW/IIS Com XIII-WG1	How to determine a design S-N curve, slop estimated (tolerance limits)?	Sheet 13



Design curve based on tolerance limits (slope estimated)

Appendix Ⅷ

IIW/IIS Com XIII-WG1	Are 2 experimental design S-N curves statistically equivalent?	Sheet 15

General

1. Contest

The problem is likely to be of interest where the two data sets have been collected under different conditions, but are expected to give comparable fatigue performance. The following methods are used to justify the conclusion that there is a significant difference between them.

2. Principle

Whether two experimental design S-N curves are statistically equivalent can be determined by using a hypothesis test. Because the design curve is established by adopting characteristic values lay a certain number of standard deviations below the mean S-N curve, the question can be solved to determine the statistical equivalence of the mean S-N curve.

A lower significance level is often used for each individual hypothesis test. For instance, a significance level of 1.7% for each individual test would roughly correspond to a 5% significance level for the composite hypothesis.

3. Condition of application

Design curves will only be applied to values of $\log S$ that are not far removed for the mean value. The necessary data to perform the equivalence verification are the following:

- The two data sets, $\sigma 1_i$, $n1$ values and $\sigma 2_i$, $n2$ values

  It is assumed here that both sets of S-N data are exact data

  If the data sets do not follow a normal distribution, the number of $n1$ and $n2$ shall be greater or equal to 30.

- The estimated means of the 2 data sets
- The estimated variances of the 2 data sets

Procedure

1. **Criteria**

The one of applied tests is the two-sided Snedecor F-test with unpaired samples.

The selected level of risk is set to α , $\alpha\%$ probability to reject a correct hypothesis.

2. **Data and formula**

Test that residual standard deviation are consistent

The correct values for the constants A and m are obtained from the following two equations by using the maximum likelihood method:

$$\log A = \overline{\log N} + m\overline{\log S} \tag{1}$$

$$m = \frac{\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2} \tag{2}$$

	Sample No.1	Sample No.2
Sample size- n_i	…………	…………
Sum of values- $\sum \log N_i$	…………	…………
Mean value- $\overline{\log N_i} = \frac{\sum \log N_i}{n_i}$	…………	…………
Sum of values- $\sum \log S_i$	…………	…………
Mean value- $\overline{\log S_i} = \frac{\sum \log S_i}{n_i}$	…………	…………
$\sum_{i=1}^{n}(\log S_{i,i} - \overline{\log S_i})^2$	…………	…………
Estimated value of the slop- $\hat{m}$	…………	…………
Estimated value of the intercept- $\log \hat{A}$	…………	…………
Degree of freedom- $f_i = n_i - 2$	…………	…………

Estimated value- $\log \hat{N}_i = \log \hat{A} - \hat{m} \log S_i$

Estimated variances- $\hat{\sigma}_i^2 = \dfrac{\sum(\log N_i - \log \hat{N}_i)^2}{f}$ ………… …………

Variance ratio- $R = \dfrac{\hat{\sigma}_i^2}{\hat{\sigma}_j^2}$ …………

Critical value for one-sided Snedecor F-distribution- $F(1-\alpha, f_i, f_j)$ …………

Criteria- $Cr = R - F(1-\alpha, f_i, f_j)$ …………

Conclusion:

If Cr >0 the hypothesis of equivalence has to be rejected

Test that the intercepts of the two S-N Consistent

Two-sided percentage point of Student's t distribution- $t(1-\alpha, n_1 + n_2 - 4)$ …………

An estimate of the common variance of the two samples- σ_e

$\hat{\sigma}_1^2 =$ ………… $\hat{\sigma}_2^2 =$…………

$\sigma_e^2 = \dfrac{f_1\hat{\sigma}_1^2 + f_2\hat{\sigma}_2^2}{f_1 + f_2}$ …………

Estimated intercepts of the regression lines- $\overline{\log A_i}$ ………… …………

Mean difference of estimated intercepts- $X = \left|\overline{\log A_1} - \overline{\log A_2}\right|$ …………

Criteria= $X - t\sqrt{\left(\dfrac{1}{n_1} + \dfrac{1}{n_2} + \dfrac{(\overline{\log S_1})^2}{\sum_{i=1}^{n_1}(\log S_{1,i} - \overline{\log S_1})^2} + \dfrac{(\overline{\log S_2})^2}{\sum_{j=1}^{n_2}(\log S_{2,j} - \overline{\log S_2})^2}\right)\sigma_e^2}$

Conclusion:

If Cr >0 the hypothesis of equivalence has to be rejected

Testing that the slopes of the two S-N curves are consistent

Estimated slopes of the regression lines- m_i ………… …………

Difference of estimated slopes- $T = |m_1 - m_2|$ …………

$$\text{Criteria} = T - t\sqrt{\left(\frac{1}{\sum_{i=1}^{n_1}(\log S_{1,i} - \overline{\log S_1})^2} + \frac{1}{\sum_{j=1}^{n_2}(\log S_{2,j} - \overline{\log S_2})^2}\right)\sigma_e^2}$$

Conclusion:

If $Cr > 0$ the hypothesis of equivalence has to be rejected

3. Conclusion

If one of above hypothesis is rejected, the composite hypothesis can not be accepted.

Example

1. Data and formula

Samples data

No.1 n_1=14

Stress range	Cycles to failure
147	521382
96	1879752
250	115816
61	10204041
57	14910395
57	10646018
96	1475769
250	112910
74	6766000
74	6195271
74	4310000
136	1015824
136	1580669
53	11541520

No.2 n_2=15

Stress range	Cycles to failure
136	1133000
74	3675000
53	22082998
53	7154785
136	664000
75	3816199
136	787894
136	689973
147	958318
53	13475000
53	20511538
265	95982
54	19048838
176	605721
74	9232000

2. Calculation

The risk level is fixed to α =1.7%

Test that residual standard deviation are consistent

The correct values for the constants A and m are obtained from the following two equations by using the maximum likelihood method:

$$\log A = \overline{\log N} + m\overline{\log S} \tag{1}$$

$$m = \frac{\sum_{i=1}^{n}(\log S_i - \overline{\log S})(\log N_i - \overline{\log N})}{\sum_{i=1}^{n}(\log S_i - \overline{\log S})^2} \tag{2}$$

	Sample No.1	Sample No.2
Sample size- n_i	14	15
Sum of values- $\sum \log N_i$	89.011	96.390
Mean value- $\overline{\log N_i} = \dfrac{\sum \log N_i}{n_i}$	6.3579	6.4260
Sum of values- $\sum \log S_i$	27.824	29.613
Mean value- $\overline{\log S_i} = \dfrac{\sum \log S_i}{n_i}$	1.9874	1.9742
$\sum_{i=1}^{n} (\log S_{i,i} - \overline{\log S_i})^2$	0.6714	0.7543
Estimated value of the slop- $\hat{m}$	3.062	3.008
Estimated value of the intercept- $\log \hat{A}$	12.444	12.365
Degree of freedom- $f_i = n_i - 2$	12	13
Estimated value- $\log \hat{N}_i = \log \hat{A} - \hat{m} \log S_i$		
Estimated variances- $\hat{\sigma}_i^2 = \dfrac{\sum (\log N_i - \log \hat{N}_i)^2}{f}$	0.0171	0.0283
Variance ratio- $R = \dfrac{\hat{\sigma}_i^2}{\hat{\sigma}_j^2}$		1.6550
Critical value for one-sided Snedecor F-distribution- $F(1-\alpha, f_i, f_j)$		3.4808
Criteria- $Cr = R - F(1-\alpha, f_i, f_j)$		-1.8258

Conclusion:

As Cr <0 the hypothesis of equivalence has no reason to be rejected and therefore the residual standard deviations are statistically equivalent.

Test that the intercepts of the two S-N Consistent

Two-sided percentage point of Student's distribution- $t(1-\alpha, n_1+n_2-4)$ 2.5572

An estimate of the common variance of the two samples- σ_e

$\hat{\sigma}_1^2 = 0.0171$ $\hat{\sigma}_2^2 = 0.0283$

$$\sigma_e^2 = \frac{f_1\hat{\sigma}_1^2 + f_2\hat{\sigma}_2^2}{f_1 + f_2} \qquad 0.0382$$

Estimated intercepts of the regression lines- $\overline{\log A_i}$ 12.444 12.365

Mean difference of estimated intercepts- $X = \left|\overline{\log A_1} - \overline{\log A_2}\right|$ 0.079

$$\text{Criteria} = X - t\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}+\frac{(\overline{\log S_1})^2}{\sum_{i=1}^{n_1}(\log S_{1,i}-\overline{\log S_1})^2}+\frac{(\overline{\log S_2})^2}{\sum_{j=1}^{n_2}(\log S_{2,j}-\overline{\log S_2})^2}\right)\sigma_e^2}$$

$$=0.079-2.5572\times\sqrt{\left(\frac{1}{14}+\frac{1}{15}+\frac{3.95}{0.6714}+\frac{3.90}{0.7543}\right)0.0382}$$

$= 0.079 - 1.672 =$-1.593

Conclusion:

As Cr <0 the hypothesis of equivalence has no reason to be rejected

Testing that the slopes of the two S-N curves are consistent

Estimated slopes of the regression lines- m_i 3.062 3.008

Difference of estimated slopes- $T = |m_1 - m_2|$ 0.054

$$\text{Criteria}=T-t\sqrt{\left(\frac{1}{\sum_{i=1}^{n_1}(\log S_{1,i}-\overline{\log S_1})^2}+\frac{1}{\sum_{j=1}^{n_2}(\log S_{2,j}-\overline{\log S_2})^2}\right)\sigma_e^2}$$

$$\text{Criteria}=0.054-2.5572\sqrt{\left(\frac{1}{0.6714}+\frac{1}{0.7543}\right)0.0382}$$

$$\text{Criteria}=0.054-0.839=\text{-}0.785$$

Conclusion:

As Cr <0 the hypothesis of equivalence has no reason to be rejected

3. <u>Conclusion</u>

The above three individual null hypothesizes with a significance level 1.7% are accepted. Now, we can draw the conclusion this composite hypothesis with a significance level 5% is accepted.

Appendix Ⅸ

IIW/IIS Com XIII-WG1	How to determine the degree of improvement produced by a post-weld treatment process?	Sheet 16

General

1. Context

A post-weld treatment is implemented to remove welding residual stresses in order to pursue improvement in fatigue behavior. Commonly used residual stress methods are hammer peening, wire bundle, shot peening and ultrasonic peening.

The magnitude of improvement depends primarily on the joint severity and base material. The pose-weld treatment can be regarded as a means of improving the fatigue strength of welded joints.

2. Principle

The term fatigue class (FAT) indicates the characteristic stress range, which gives a fatigue life of two million cycles at 95% survival probability. The data is able to be evaluated according to the statistical methods outlined by the Huther.

In order to prove that a process has been improved, we must measure the process capability before and after improvements are implemented. The difference of FAT between the as-weld and post-weld indicates the degree of improvement produced by the post-weld treatment.

3. Condition of application

- It is noticed that the benefit of peening of steel components can only be claimed for details in design Class FAT 90 or lower in the IIW notation for S-N curves.
- The effect of specimen thickness on fatigue strength was only slight.

4. Conclusion

The conclusion is able to be drawn after the comparison based on the statistical analysis. Generally, there is a definite increase in the fatigue life of the as-weld versus post-weld in these tests.

Procedure

1. Criteria

A failure probability of 97.5 percent can be used in the standard since the results are based on a large number of test pieces.

If the sample size is small, characteristic fatigue strength is $FAT_{95\%}$ and not $FAT_{97.5\%}$

2. Data and formula

	No.1 AW	No.2 UIT
Sample size- n	………..	………..
Slop of curve- m	………..	………..

Calculate the $FAT_{95\%}$ by following equations:

$$\triangle\sigma_i^m \cdot N_i = C_i = FAT^m \cdot 2000000$$

Sum of values- $\sum \log C_i$	……….	……….
Mean fatigue capacity- $\log C_{50\%} = \dfrac{\sum \log C_i}{n}$	………..	………..
Standard deviation- $s = \sqrt{\dfrac{\sum (\log C_i - \log C_{50\%})^2}{n-1}}$	……….	……….
$\log C_{95\%} = \log C_{50\%} - s(1.64 + \dfrac{1.15}{\sqrt{n}})$	………..	………..
Characteristic Fatigue capacity- $FAT_{95\%} = \sqrt[m]{\dfrac{C_{95\%}}{2000000}}$	………..	………..

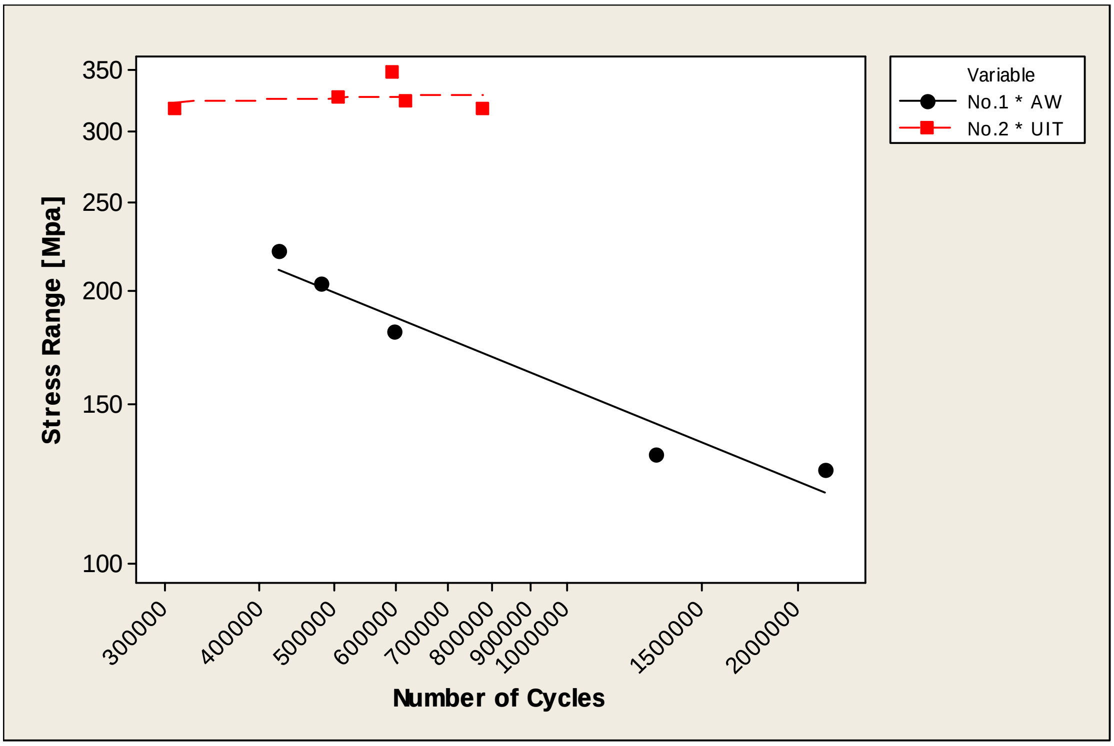
Example 1

Note: The slope of the S-N curve m is fixed.

1. Data and formula

Samples data No.1

	t	R	$\Delta\sigma$	N	FAT(m=3)	Observation
AW-11	5	0.1	180	599377	120	WT
AW-12	5	0.1	221	422755	131	WT
AW-13	5	0.1	127	2173795	131	WT
AW-14	5	0.1	132	1313035	115	WT
AW-15	5	0.1	204	480284	127	WT
UIT-1	5	0.1	349	596082	233	UITG
UIT-2	5	0.1	318	310170	171	UITG
UIT-3	5	0.1	324	620074	219	UITG
UIT-4	5	0.1	327	505913	207	UITG
UIT-5	5	0.1	318	781200	232	UITG



The comparison between as-welded and UI-treated test series, t =5mm, R =0.1

2. Calculation

	No.1 AW	No.1 UIT
Sample size- n	5	5
Slop of curve- m	3	3

Calculate the $FAT_{95\%}$ by following equations: $\Delta\sigma_i^m \cdot N_i = C_i = FAT^m \cdot 2000000$

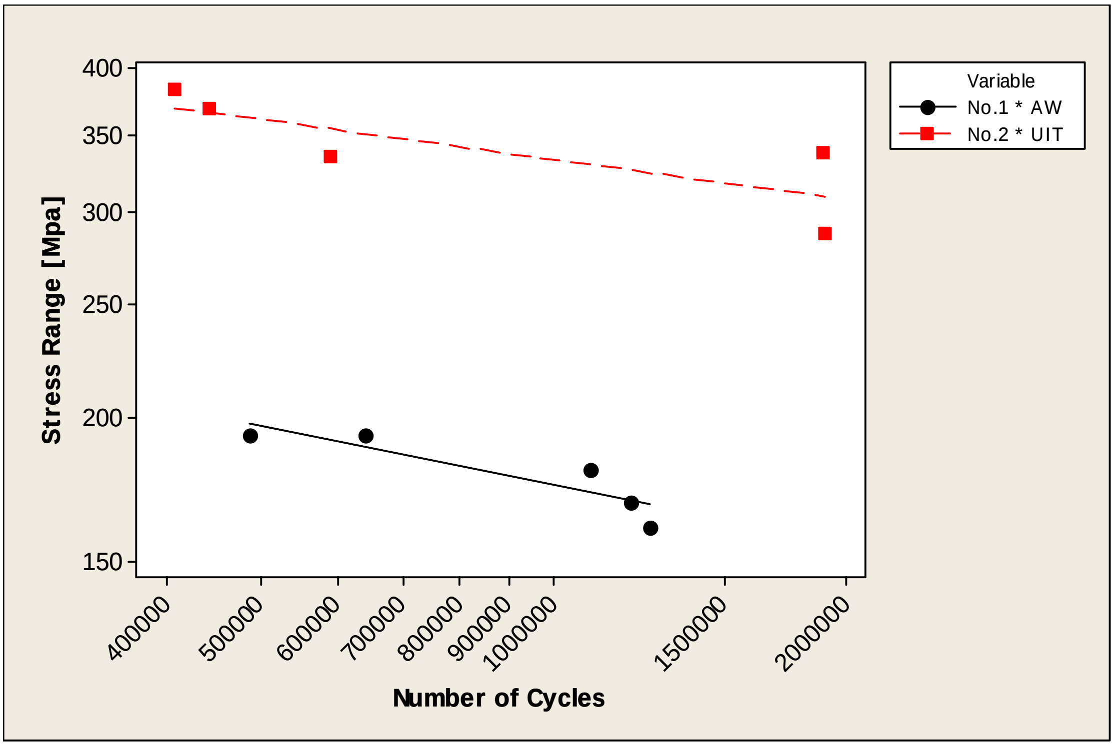
	No.1 AW	No.1 UIT
Sum of values- $\sum \log C_i$	62.9418	66.3745
Mean fatigue capacity- $\log C_{50\%} = \dfrac{\sum \log C_i}{n}$	12.5884	13.2749
Standard deviation- $s = \sqrt{\dfrac{\sum (\log C_i - \log C_{50\%})^2}{n-1}}$	0.07566	0.16704
$\log C_{95\%} = \log C_{50\%} - s(1.64 + \dfrac{1.15}{\sqrt{n}})$	12.4254	12.915
Characteristic Fatigue capacity- $FAT_{95\%} = \sqrt[m]{\dfrac{C_{95\%}}{2000000}}$	110.0162	160.1978

Example 2

1. Data and formula

Samples data No.2

	t	R	$\Delta\sigma$	N	FAT(m =3)	Observation
AW-28	8	0.1	193	485897	120	WT
AW-29	8	0.1	193	640024	132	WT
AW-30	8	0.1	161	1257193	138	WT
AW-31	8	0.1	180	1091393	147	WT
AW-32	8	0.1	169	1199013	143	WT
UIT-18	8	0.1	288	1902884	283	UITG
UIT-19	8	0.1	369	441958	223	UITG
UIT-20	8	0.1	383	407610	225	UITG
UIT-21	8	0.1	335	588203	223	UITG
UIT-22	8	0.1	338	1892369	332	UITG



The comparison between as-welded and UI-treated test series, t =8mm, R =0.1

2. Calculation

	No.2 AW	No.2UIT
Sample size- n	5	5
Slop of curve- m	3	3

Calculate the $FAT_{95\%}$ by following equations: $\triangle\sigma_i^m \cdot N_i = C_i = FAT^m \cdot 2000000$

	No.2 AW	No.2UIT
Sum of values- $\sum \log C_i$	63.4923	67.5723
Mean fatigue capacity- $\log C_{50\%} = \frac{\sum \log C_i}{n}$	12.6984	13.5145
Standard deviation- $s = \sqrt{\frac{\sum(\log C_i - \log C_{50\%})^2}{n-1}}$	0.1013	0.2364
$\log C_{95\%} = \log C_{50\%} - s(1.64 + \frac{1.15}{\sqrt{n}})$	12.4802	13.0052
Characteristic Fatigue capacity- $FAT_{95\%} = \sqrt[m]{\frac{C_{95\%}}{2000000}}$	114.7422	171.6814

3. Calculation

Table: Comparison fatigue classes $FAT_{95\%}$ between welded and treated series, m is fixed

	post-weld treatment	as-weld treatment	Post-weld/as-weld
FAT95%(t=5mm)	160	110	1.45
FAT95%(t=8mm)	172	115	1.50
FAT95%(8mm)/FAT95%(5mm)	1.075	1.045	

The fatigue strength of post-weld welds is 45%-50% higher than the fatigue class for as-welded specimens. The fatigue strength of 8mm thick specimen is higher than 5mm thick specimen. It also turns out that the thickness of specimen does not affect the degree of improvement in fatigue strength greatly.

Example 3

Note: The slope of the S-N curve m is estimated.

1. Data and formula

Samples data No.2

	t	R	$\Delta\sigma$	N	FAT(m =3)	Observation
AW-28	8	0.1	193	485897	120	WT
AW-29	8	0.1	193	640024	132	WT
AW-30	8	0.1	161	1257193	138	WT
AW-31	8	0.1	180	1091393	147	WT
AW-32	8	0.1	169	1199013	143	WT
UIT-18	8	0.1	288	1902884	283	UITG
UIT-19	8	0.1	369	441958	223	UITG
UIT-20	8	0.1	383	407610	225	UITG
UIT-21	8	0.1	335	588203	223	UITG
UIT-22	8	0.1	338	1892369	332	UITG

2. Calculation

The correct values for the constants A and m are obtained by using least squares regression method:

$$Y = A - mX$$

$Y = \log N$ $\qquad\qquad$ $X = \log S$

$$A = \frac{\sum y_i}{n} + m\frac{\sum x_i}{n} \tag{1}$$

$$m = -\frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \tag{2}$$

$$m_{AW} = \frac{5\times 66.888 - 334.555}{5\times 25.367 - 126.812} \qquad \text{AW}$$

$$m_{AW} = 4.754$$

$$m_{UIT} = \frac{5\times 74.8701 - 374.609}{5\times 32.082 - 160.366} \qquad \text{UIT}$$

$$m_{UIT} = 5.63$$

	No.2 AW	No.2 UIT
Sample size- n	5	5
Slop of curve- m	4.754	5.630

Calculate the $FAT_{95\%}$ by following equations: $\triangle\sigma_i^m \cdot N_i = C_i = FAT^m \cdot 2000000$

	No.2 AW	No.2 UIT
Sum of values- $\sum \log C_i$	83.244	100.878
Mean fatigue capacity- $\log C_{50\%} = \frac{\sum \log C_i}{n}$	16.6488	20.1755
Standard deviation- $s = \sqrt{\frac{\sum(\log C_i - \log C_{50\%})^2}{n-1}}$	0.0806	0.2001
$\log C_{95\%} = \log C_{50\%} - s(1.64 + \frac{1.15}{\sqrt{n}})$	16.475	19.744
Characteristic Fatigue capacity- $FAT_{95\%} = \sqrt[m]{\frac{C_{95\%}}{2000000}}$	138.0658	244.196

Table Comparison fatigue classes $FAT_{95\%}$ between welded and treated series, m **is estimated**

	post-weld treatment	as-weld treatment	Post-weld/as-weld
$FAT_{95\%}$(t=8mm)	244	138	1.77

3. Calculation

When the fatigue class based on free slope, there is 77% improvement can be obtained by using post-treatment. The improvement of fatigue strength based on free slope is larger than the one based on a fixed slope.

Appendix X

Table 1 One-sided tolerance limit factors k for a normal distribution for $\gamma\%$ =90%

Degree of freedom $f = n-1$ (Schneider and Maddox, 2006)

Sample size n	Value of k for p %=95%	Value of k for p %=97.5%
2	13.090	15.586
3	5.311	6.244
4	3.975	4.637
5	3.401	3.983
6	3.093	3.621
7	2.893	3.389
8	2.754	3.227
9	2.650	3.106
10	2.568	3.011
11	2.503	2.936
12	2.448	2.872
13	2.403	2.820
14	2.363	2.774
15	2.329	2.735
16	2.299	2.700
17	2.272	2.670
18	2.249	2.643
19	2.228	2.618
20	2.208	2.597
21	2.190	2.575
22	2.174	2.557
23	2.159	2.540
24	2.145	2.525
25	2.132	2.510
30	2.080	2.450
35	2.041	2.406
40	2.010	2.371
45	1.986	2.344
50	1.965	2.320
60	1.933	2.284
70	1.909	2.257
80	1.890	2.235

90	1.874	2.217
100	1.861	2.203
120	1.841	2.179
145	1.821	2.158
300	1.765	2.094
500	1.736	2.062
∞	1.645	1.960

Appendix Ⅺ

Table 2 One-sided tolerance limit factors k for a normal distribution for $\gamma\%$ =90%

Degree of freedom $f = n - 2$

Sample size n	Value of k for p %=95%	Value of k for p %=97.5%
3	13.080	15.587
4	5.251	4.193
5	—	—
6	4.204	5.226
7	3.754	4.636
8	3.467	4.259
9	3.267	4.000
10	3.117	3.807
11	3.000	3.655
18	2.575	3.113
38	2.221	2.668
146	1.917	2.290
∞	1.650	1.960

AppendixⅫ

Table3 fatigue testing data from Laboratory of Fatigue and Strength in LUT

	t	R	$\Delta\sigma_{SR}$	N	FAT（$m=3$）	Observation
UIT-1	5	0.1	349	596082	233	UITG
UIT-2	5	0.1	318	310170	171	UITG
UIT-3	5	0.1	324	620074	219	UITG
UIT-4	5	0.1	327	505913	207	UITG
UIT-5	5	0.1	318	781200	232	UITG
UIT-6	5	Ohta	333	298108	177	UITG
UIT-7	5	Ohta	338	473704	209	UITG
UIT-8	5	Ohta	327	980692	258	UITG
UIT-9	5	Ohta	297	333199	163	UITG
UIT-10	5	Ohta	295	1163070	246	UITG
AW-11	5	0.1	180	599377	120	WT
AW-12	5	0.1	221	422755	131	WT
AW-13	5	0.1	127	2173795	131	WT
AW-14	5	0.1	132	1313035	115	WT
AW-15	5	0.1	204	480284	127	WT
AW-16	5	0.1	173	6814655	-	RO
AW-17	5	0.1	134	3086407	-	RO
UIT-18	8	0.1	288	1902884	283	UITG
UIT-19	8	0.1	369	441958	223	UITG
UIT-20	8	0.1	383	407610	225	UITG
UIT-21	8	0.1	335	588203	223	UITG
UIT-22	8	0.1	338	1892369	332	UITG
UIT-23	8	Ohta	360	256226	181	UITG
UIT-24	8	Ohta	392	393186	230	UITG
UIT-25	8	Ohta	367	247240	183	UITG
UIT-26	8	Ohta	379	205424	177	UITG
UIT-27	8	Ohta	336	254817	169	UITG
AW-28	8	0.1	193	485897	120	WT
AW-29	8	0.1	193	640024	132	WT
AW-30	8	0.1	161	1257193	138	WT
AW-31	8	0.1	180	1091393	147	WT
AW-32	8	0.1	169	1199013	143	WT
AW-33	8	0.1	153	3453562	-	RO