FACULTY OF THECHNOLOGY
DEPARTMENT OF MATHEMATICS AND PHYSICS

Regime Switching Models for Electricity Time Series that Capture Fat Tailed Distributions

The topic of this Thesis was approved by the Department of Technomathematics and Applied Physics on February 10, 2010.

March 16, 2010

Supervisors: Ph.D. Tuomo Kauranne.
Examiners: Prof. Ph.D. Heikki Haario and Ph.D. Tuomo Kauranne.

Lappeenranta, March 16, 2010

Jean-Paul MURARA
Rullatie 7 A
PARKKALILA
53200 Lappeenranta
FINLAND
Jean-Paul.Murara@lut.fi or mujeanpaul102@yahoo.fr

Abstract

Lappeenranta University of Technology
Department of Mathematics and Physics

Jean-Paul MURARA

Regime Switching Models for Electricity Time Series that Capture Fat Tailed Distributions

Master's thesis

March 16, 2010

pages 62, figures 61, tables 9, appendices 2

Key words: Regime-Switching, Electricity spot price, Stochastic Differential Equations.

In the power market, electricity prices play an important role at the economic level. The behavior of a price trend usually known as a structural break may change over time in terms of its mean value, its volatility, or it may change for a period of time before reverting back to its original behavior or switching to another style of behavior, and the latter is typically termed a ***regime shift*** or ***regime switch***. Our task in this thesis is to develop an electricity price time series model that captures fat tailed distributions which can explain this behavior and analyze it for better understanding. For NordPool data used, the obtained Markov Regime-Switching model operates on two regimes: regular and non-regular. Three criteria have been considered *price difference criterion*, *capacity/flow difference criterion* and *spikes in Finland criterion*. The suitability of GARCH modeling to simulate multi-regime modeling is also studied.

Acknowledgements

I would like to present many thanks to my supervisor, Dr. Tuomo Kauranne, for his guidance, encouragement and motivation, throughout the whole process of conducting this research. I am very thankful to have been able to work with him. I would never have believed this research was possible without his assistance.

Many thanks also go to Prof. Heikki Haario and Dr. Matti Heilio for their helpful financial support through Technomathematics Department. I would like to thank also Matylda Jabłońska for very helpful advices. She has been very helpful, cooperative, and prompt to listen and assist since the beginning of this thesis.

Many people have supported me during my study, especially; Kimmo Kerkkanen and Outi Kerkkanen, my beloved family in Rwanda, Tuija Maaret Pykäläinen, Pastor Sakari Kiiskinen, my classmates and friends here in Lappeenranta and also my beloved friend Marie-Claire Musabwamana ; I would not have been able to complete my master's study without her encouragement and inspiration. To all of you, I say: Thank you.

I dedicate this thesis to my late godmother Alice Mukamusoni who passed away this March 15, 2010. Dear mother, may your soul rest in peace.

Jean-Paul MURARA

UGUSHIMIRA

Mbikuye ku mutima, ndashimira mwebwe mwese mwagize uruhare kugira ngo mbashe gutunganya amasomo yanjye.

Mbere na mbere ndashimira, ishami ry'imibare muri kaminuza y'ikoranabuhanga ya Lappeenranta; cyane cyane mwarimu Tuomo Kauranne wanyoboye muri ubu bushakashatsi hamwe na marimu Heikki Haario wagize uruhare mu ikosorwa ry'iki gitabo. Ndashimira kandi abarimu bose banyigishije by'umwihariko mwarimu Matti Heilio wagize uruhare mu kuntera inkunga ngo mbashe gutangira amasomo.

Ndashimira kandi Matylda Jabłońska ku nama nyinshi z'ingirakamaro yampaye kugira ngo ubu bushakashatsi bubashe gutungana hamwe na bagenzi banjye twafatanyije amasomo ku nama nyinshi n'ubufatanye bangaragarije.

Ndashimira kandi incuti zose zambaye hafi ku bw'umubiri no ku mutima harimo; Kimmo na Outi Kerkkanen, Pasitoro Sakari Kiiskinen, Tuija Maaret Pykäläinen, umuryango wanjye hamwe n'incuti yanjye nkunda cyane Marie-Claire Musabwamana.

Iki gitabo ngituye umwe mu babyeyi banjye witabye Imana kuri uyu wa mbere tariki 15 Werurwe 2010 ariwe Alice Mukamusoni. Imana ikwiyereke iteka maze uruhukire mu mahoro.

Mwese ndabashimiye kandi Imana ibampere umugisha.

Jean-Paul MURARA

REMERCIEMENTS

Nous tenons à exprimer nos sentiments de profonde gratitude à toutes les personnes qui nous ont aidé à réaliser cette dissertation.

En premier lieu, nos sincères remerciements vont au Professeur Tuomo Kauranne, qui a proposé et accepté de diriger ce travail. Son encadrement et ses critiques judicieuses ont permis la réalisation de ce travail. Nous remercions aussi Professeur Heikki Haario qui a contribué dans la correction de cette thèse.

Nous remercions tous les professeurs de la Faculté de Technologie et spécialement ceux du Département des Mathématiques. En plus, un grand merci à Mlle Matylda Jabłońska pour ses conseils et aide.

Nous disons merci, aux collègues de promotion et tous les ami(e)s. Nous exprimons aussi nos sentiments de gratitude à Mr la famille Kimmo et Outi Kerkkanen, à Pasteur Sakari Kiiskinen , à Mlle M-Claire Musabwamana, à toute ma famille qui est au Rwanda et à toute personne qui de près où de loin nous a aidé dans la réalisation de ce travail.

Jean-Paul MURARA

Contents

List of Tables

List of Figures

1 Introduction

1.1 Motivation

Recently electric power industry has moved into a new stage of deregulation and market operation all over the world. In this industry, electricity is traded as a commodity, a process in which the prices have an important role. The price of electricity depends on the supply and demand on the market and the operating conditions of the transmission network, which are influenced by many factors, such as the climate, the economic situation, the planning for development and accidents and failure. The joint effect of these factors results in an extremely complicated random variation of the electricity price [6].

Many financial and economic time series seem to undergo episodes in which the behavior of the series changes quite dramatically compared to that exhibited previously. The behavior of a series could change over time in terms of its mean value, its volatility, or to what extent its current value is related to its previous value. The behavior may change once and for all, usually known as a structural break in a series. Or it may change for a period of time before reverting back to its original behavior or switching to yet another style of behavior, and the latter is typically termed a regime shift or regime switch [3].

A fat tail is a property of some probability distributions known also as heavy-tailed distributions which exhibits extremely large kurtosis particularly relative to the ubiquitous normal distribution which itself is an example of an exception being a very tailed distribution. Fat tail distributions have power law decay as a function of frequency. More precisely, the distribution of a random variable X is said to have a fat tail if:

$$P[X > x] \sim x^{-(1+\alpha)}$$

as

$$x \to \infty, \alpha > 0$$

Some reserve the term *fat tail* for distributions only where $0 < \alpha < 2$ this means only in cases with infinite variance.

If the distribution of the errors is *fat tailed*, in that it frequently produces relatively large errors, it turns out that linearity is unduly restrictive thus in the presence of fat tailed error distributions, although the ordinary least squares is BLUE, it is markedly inferior to some nonlinear unbiased estimators [15].

In the present thesis, the motivation behind is to develop an electricity prices time series model that captures fat tailed residuals and analyze it for better understandings.

1.2 Problem Statement

Generally, in the latest research, the electricity prices' behavior in different states appears to be rather different [19]. The task in this thesis is to try to find a model which can explain this behavior, analyze it and make simulations. In the first chapter, the introduction of our research is given and the second chapter is related to the theoretical background needed in this research. In chapter three, Nord Pool data used in this thesis are described and the fourth chapter is reserved to discuss a regime switching model for electricity prices with Nord Pool which is used in this research. Chapter five is the analysis of the obtained model and results. Finally, a conclusion is done in the sixth chapter and an appendix which contains some important mathematical derivations is given.

1.3 Some Definitions

1.3.1 Financial markets

Financial markets are markets where people easily buy and sell financial securities, commodities and other fungible items of value at low transaction costs and at prices that reflect the efficient-market hypothesis. The financial markets can be divided into different subtypes such as [26]:

- Capital markets (:Stock markets and Bond markets);
- Commodity markets facilitate the trading of commodities;
- Money markets provide short term financing and investment;
- Derivatives markets provide instruments for the management of financial risk;
- Insurance markets facilitate the redistribution of various risks and
- Foreign exchange markets facilitate the trading of foreign exchange.

In mathematical finance, the concept of a financial market is defined in terms of a continuous-time Brownian motion stochastic process and in this thesis we are interested in *Commodity markets*. Commodity markets are markets where raw or primary products are exchanged and they require the existence of agreed standards so that trades can be made without visual inspection. These raw commodities are traded on regulated commodities exchanges, in which they are bought and sold in standardized contracts. A forward contract is an agreement between two parties to exchange at some fixed future date a given quantity of a commodity for a price defined today and a fixed price today is known as a forward price while a futures contract has the same general features as a forward contract but is transacted through a futures exchange [26].

1.3.2 Markov Chain

A Markov process is a collection of random variables X_t $t=0,1,2,\ldots$ that have the property that, given the present state, the future is conditionally independent on the past states. In other words, a Markov process is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant [21]. Mathematically, we write this as the following:

$$P(X_t = j/X_0 = i_0, X_1 = i_1, \ldots, X_{t-1} = i_{t-1}) = P(X_t = j/X_{t-1} = i_{t-1}) \tag{1}$$

A simple random walk is a an example of a Markov chain and stock prices are usually assumed to follow a Markov process.

1.3.3 GARCH model

A linear combination of white noise processes, so that y_t depends on the current and previous values of a white noise disturbance term is a moving average model. We denote a moving average $MA(q)$ of order q as follows:

$$y_t = \mu + \sum_{i=1}^{q} \theta_i u_{t-1} + u_t.$$

When the current value of a variable, y , depends upon only the values that the variable took in previous periods plus an error term, we have in this case an autoregressive model. We denote an autoregressive model $AR(p)$ of order p as follows:

$$y_t = \mu + \sum_{i=1}^{p} \phi_i y_{t-i} + u_t.$$

A combination of these two models, $AR(p)$ and $MA(q)$ gives the $ARMA(p, q)$ model which can be written as follows:

$$y_t = \mu + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i u_{t-1} + u_t, \tag{2}$$

where $E(u_t) = 0$; $E(u_t^2) = \sigma^2$ and $E(u_t u_s) = 0$, $t \neq s$.

An (ARCH(q)) autoregressive conditional heteroscedasticity model is defined as one particular non-linear model often used in econometrics. This model considers the variance of the current error term to be a function of the variances of the previous time periods' error terms. Specifically, let ϵ_t denote the returns and assume that $\epsilon_t = \sigma_t z_t$ where $z_t \sim N(0, 1)$ and where the series σ_t^2 are modeled by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

and where $\alpha_0 > 0, \alpha_i \geq 0$.

An ARCH(q) model can be estimated using ordinary least squares. A methodology to test for the lag length of ARCH errors using the Lagrange multiplier test was proposed by Engle.

Often it requires many parameters to adequately describe the volatility and the ARCH(q) model simple to handle that; thus some alternative must be found. A useful extension known as the generalized ARCH (GARCH) model has been proposed [2] and GARCH(p,q) model is defined by the following equation:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{3}$$

This means that the current conditional variance is parameterized to depend upon q lags of the squared error and p lags of the conditional variance.

In general, a GARCH(1,1) model is sufficient to capture the volatility clustering in the data, and rarely is any higher order model estimated or even entertained in the academic finance literature [2].

It is preferred to use GARCH model instead of using ARCH model because ARCH is more parsimonious, and avoids overfitting. Consequently, the model is less likely to breach non-negativity constraints.

The mixture of ARMA-GARCH model is similar to the mixture of AR-GARCH model. Specifically, each component of the mixture model can be denoted as a normal ARMA series:

$$y_{t,j} = \sum_{i=1}^{r} b_{ij} y_{t-i,j} + \sum_{i=1}^{s} a_{ij} \epsilon_{t-i,j} + \epsilon_{t,j} \tag{4}$$

Furthermore, each residual term $\epsilon_{t,j}$ is assumed gaussian white noise with variance denoted by the GARCH model

$$\sigma_{t,j}^2 = \alpha_{0j} + \sum_{i=1}^{q} \alpha_{ij} \epsilon_{t-i,j}^2 + \sum_{i=1}^{p} \beta_{ij} \sigma_{t-i,j}^2 \tag{5}$$

where $\alpha_{ij} > 0$ for $i = 1, \cdots, q$ and $\beta_{ij} > 0$ for $i = 1, \cdots, p$.

Briefly, the ARMA-GARCH model can be defined by the following equations:

$$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{i=1}^{q} \psi_i \epsilon_{t-i} + \epsilon_t \tag{6}$$

where

$$\epsilon_t = \eta_t \sqrt{h_t}$$

and

$$h_t = \alpha_0 + \sum_{i=1}^{r} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{s} \beta_i h_{t-i}. \tag{7}$$

2 Theoretical background

2.1 Regime-Switching models

2.1.1 Introduction

In financial econometrics many time series data which can have many forms are used and represent different stochastic processes and from this we get different models caused by many characteristics. When we are modeling variations in the level of a process we can have one of the three broad classes of practical importance which are: *autoregressive (AR)* models, *integrated (I)* models and *moving average (MA)*models. Also when a sequence of random variables has constant variance it is called homoscedastic and it is called heteroscedastic when its random variables have different variables [3].

We have main types of models such as:

- Stationary time series models;
- Non-stationary time series models;
- Seasonal models;
- Time series regression models;
- Time series models of heteroscedasticity;
- Threshold models.

2.1.2 Threshold autoregressive models (TAR)

It is denoted AR(p) and defined an autoregressive model for *i=1,2,. . . ,p* a time series y_t given by the equation:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t \tag{8}$$

where ϕ_i *i=1,2,. . . ,p* are autoregressive coefficients, assumed to be constant over time and $\epsilon_t \sim N(0, \sigma^2)$ stands for white noise error term with constant variance σ^2.

The equation (8) can be written in a vector form.

Thus,

$$y_t = X_t \Phi + \sigma \epsilon_t$$

where

- $X_t = (1, y_{t-1}, y_{t-2}, \ldots, y_{t-p})$
- $\Phi = (\phi_0, \phi_1, \phi_2, \ldots, \phi_p)$

- $\epsilon_t \sim N(0,1)$
- σ is the standard deviation.

In order to allow higher degree of flexibility in model parameters through a regime switching behavior, threshold autoregressive (TAR) models are typically applied as an extension of autoregressive models. These models have been introduced by Howell Tong in 1977 and more fully developed in the seminal paper [26]. They can be thought of in terms of extension of autoregressive models, allowing for changes in the model parameters according to the values of weakly exogenous threshold variables z_t, assumed to be past values of y.

Threshold autoregressive model can be presented as follows:

$$y_t = X_t \phi^{(j)} + \sigma^{(j)} \epsilon_t$$

if $r_{j-1} < z_t < r_j$ where

$$X_t = (1, y_{t-1}, y_{t-2}, \ldots, y_{t-p}) \tag{9}$$

is a column vector of variables and

$$-\infty = r_0 < r_1 < \ldots < r_k = \infty$$

are $k-1$ non-trivial thresholds dividing the domain of z_t into k different regimes.

The self-exciting threshold autoregressive SETAR model is a special case of Tong's general threshold autoregressive models. The latter allows the threshold variable to be very flexible, such as an exogenous time series in the open-loop threshold autoregressive system ; a Markov chain driven threshold autoregressive model , which is now also known as the Markov switching model [26].

2.1.3 Markov regime switching models

Many economic time series occasionally exhibit dramatic breaks in their behavior, associated with events such as financial crises or abrupt changes in government policy [10].

Consider how we might describe the consequences of a dramatic change in the behavior of a single variable y_t. Suppose that the typical historical behavior could be described with a first-order auto regression,

$$y_t = c_1 + \phi y_{t-1} + \epsilon_t \tag{10}$$

with $\epsilon_t \sim N(0, \sigma^2)$, which seemed to adequately describe the observed data for $t=1,2,\ldots,t_0$.

Suppose that at time t_0 there was a significant change in the average level of the series, so that we would instead wish to describe the data according to:

$$y_t = c_2 + \phi y_{t-1} + \epsilon_t \tag{11}$$

for $t{=}t_0{+}1,\ t_0{+}2,\ \ldots$. This fix of changing the value of the intercept from c_1 to c_2 might help the model to get back on track with better forecasts, but it is rather unsatisfactory as a probability law that could have generated the data. We surely would not want to maintain that the change from c_1 to c_2 at date t_0 was a deterministic event that anyone would have been able to predict with certainty looking ahead from date $t = 1$.

$$y_t = c_{s_t} + \phi y_{t-1} + \epsilon_t \tag{12}$$

where s_t is a random variable that, as a result of institutional changes, happened in our sample to assume the value $s_t = 1$ for $t = 1, 2, \ldots, t_0$ and $s_t = 2$ for $t = t_0 + 1, t_0 + 2, \ldots$. A complete description of the probability law governing the observed data would then require a probabilistic model of what caused the change from $s_t = 1$ to $s_t = 2$. The simplest such specification is that s_t is the realization of a two-state Markov chain with:

$$P(s_t = j/s_{t-1} = i, s_{t-2} = k, \ldots, y_{t-1}, y_{t-2}, \ldots) = P(s_t = j/s_{t-1} = i) = p_{ij} \tag{13}$$

Assuming that we do not observe s_t directly, but only infer its operation through the observed behavior of y_t, the parameters necessary to fully describe the probability law governing y_t are then the variance of the Gaussian innovation σ^2, the autoregressive coefficient ϕ, the two intercept c_1 and c_2, and the two state transition probabilities, p_{11} and p_{22}.

The specification in (13) assumes that the probability of change in regime depends on the past only through the value of the most recent regime, though, as noted below, nothing in the approach described below precludes looking at more general probabilistic specifications. But the simple time-invariant Markov chain (13) seems the natural starting point and is clearly preferable to acting as if the shift from c_1 ti c_2 was a deterministic event. Permanence of the shift would be represented by $p_{22} = 1$, though the Markov formulation invites the more general possibility that $p_{22} < 1$. Certainly in the case of business cycles or financial crises, we know that the situation, though dramatic, is not permanent. Furthermore, if the regime change reflects a fundamental change in monetary or fiscal policy, the prudent assumption would seem to be to allow the possibility for it to change back again, suggesting that $p_{22} < 1$ is often a more natural formulation for thinking about changes in regime than $p_{22} = 1$ [10].

2.2 MCMC methods for continuous time series

2.2.1 Introduction

MCMC methods are the Markov chain Monte-Carlo methods. These methods are a set of algorithms for sampling from probability distributions based on constructing a Markov chain that has the desired distribution as its equilibrium distribution. The state of the chain after a large number of steps is then used as a sample from the desired distribution. The quality of the sample improves as a function of the number of steps [11].

2.2.2 Methods and theory of MCMC

Bayes' rule

In probability theory, Bayes'rule shows how one conditional probability depends on its inverse. This rule expresses the posterior probability of a hypothesis B in terms of the prior probabilities of B and A, and the probability of A given B. It implies that evidence has a stronger confirming effect if it was more unlikely before being observed. Bayes theorem is valid in all common interpretations of probability, and is applicable in science and engineering. However, there is disagreement between frequentest and Bayesian and subjective and objective statisticians in regards to the proper implementation and extent of Bayes´ rule.

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)} = \frac{P(B/A)P(A)}{\sum P(B/A_i)P(A_i)} \tag{14}$$

Bayesian methods for unknown parameters are often interested in forming the posterior distribution for the parameters. Since it is rarely possible to do it analytically, we are satisfied with a number of samples from the posterior distribution of the model parameters. To achieve this by applying the Bayes' rule (14). We have to integrate over the whole parameter space to calculate the normalizing constant for the posterior density.

Some MCMC algorithms to do this are [9] :

- Clifford-Hammersley Theorem
- Gibbs Sampling
- Metropolis-Hastings
- Convergence Theory

2.2.3 Financial application: Regime Switching models

Considering the Black-Scholes model, a model with a constant expected return and volatility [14]. In some situations, the models that relaxed the constant parameter specification, allowing the expected return and volatility to vary over time. In those models, expected returns or volatility are modeled as diffusions or jump-diffusions, where the jump component was independent and identically distributed.

In this thesis, an alternative is considered: the drift and diffusion are driven by a continuous-time, discrete state Markov Chain. The models are commonly called regime-switching models, Markov switching models or Markov modulated diffusions.

2.3 Stochastic differential equation

A differential equation in which one or more of its terms is a stochastic process is known as a Stochastic Differential Equation, this makes also its solution to be a stochastic process. Stochastic differential equations incorporate white noises which can be thought of as the derivative of the Wiener Process; however, it should be mentioned that other types of random fluctuations are possible, such as jump processes [16].

Consider the stochastic differential equation:

$$dX_t = \mu(X_t, t)dt + \sigma(X_t)dW_t \tag{15}$$

The process $X = (X_t)$ is called a strong solution of the stochastic differential equation above if for all $t > 0, X_t$ is a function $F(t, (W_s, s \leq t))$ of the given Wiener process W, integrals

$$\int_0^t \mu(X_s, s)ds$$

and

$$\int_0^t \sigma(X_s, s)dW_s$$

exist, and the integral equation

$$X_t = X_0 + \int_0^t \mu(X_s, s)ds + \int_0^t \sigma(X_s, s)dW_s \tag{16}$$

is satisfied.

An important example is the equation for geometric Brownian motion

$$dX_t = \mu X_t dt + \sigma X_t dB_t \tag{17}$$

or

$$dX(t) = f(t, X(t))dt + g(t, X(t))dW(t) \tag{18}$$

which is the equation for the dynamics of the price of a stock in the Black-Scholes options pricing model of financial mathematics. Mathematically and are the same.

The remind here is that the Gaussian stochastic process $W(t)$ is called Brownian motion or Wiener process if:

- $E(W(t_2) - W(t_1)) = 0;$
- $E((W(t_4) - W(t_3))(W(t_2) - W(t_1))) = 0, t_4 \geq t_3 \geq t_2 \geq t_1;$
- $E(W(t_2) - W(t_1))^2 = (t_2 - t_1), t_2 \geq t_1.$

We can replace $dW(t)$ by $\xi(t)dt$ where we assume that the stochastic process $\xi(t)$ is Gaussian.

Also the assumption is that $\xi(t)dt$ is a *white noise* and this means that it satisfies the following conditions:

- $E(\xi(t)) = 0;$

- $E(\xi(t_2)\xi(t_1)) = \delta(t_2 - t_1), t_2 \geq t_1$

where $\delta(t)$ is the Dirac function.

3 Nord Pool data

3.1 About Nord Pool

The Nordic Power Exchange (Nord Pool) is the single power market for Norway, Denmark, Sweden and Finland. It was the world's first multinational exchange for trading electric power. As of 2008, Nord Pool is the largest power derivatives exchange and the second largest exchange in European Union emission allowances (EUAs) and global certified emission reductions (CERs) trading.

In 1991 the Norwegian parliament decides to deregulate the market for power trading. In 1993 Statnett Marked AS (now Nord Pool ASA) was established as an independent company. Nord Pool was established in 1996, and was then owned by the two national grid companies, Statnett in Norway (50 per cent) and Svenska Kraftnät in Sweden (50 per cent).

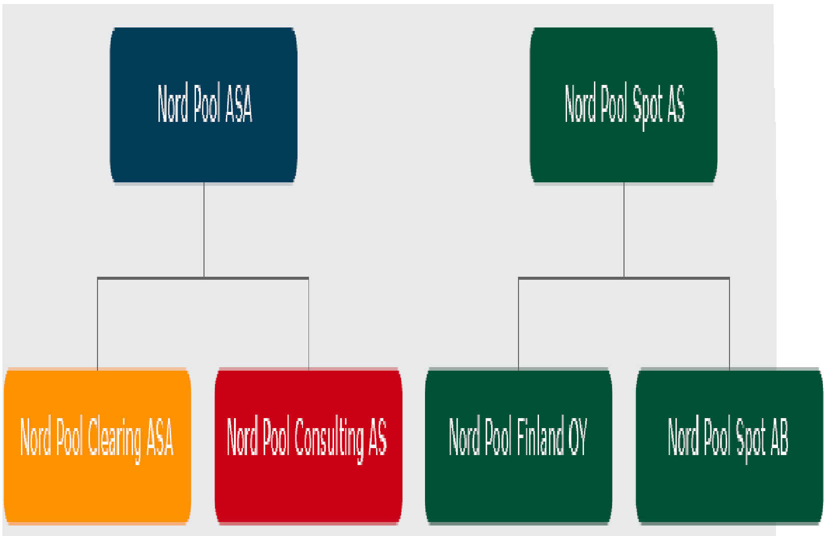

Figure 1: Part of the Nord Pool group.

On 21 December 2007, Nord Pool agreed to sell its two subsidiaries which are Nord Pool Clearing, which deals with clearing transactions, and Nord Pool Consulting to the global exchange operator NASDAQ OMX. The Nord Pool group comprises Nord Pool ASA and Nord Pool Spot AS. Nord Pool ASA comprises the wholly owned subsidiaries, Nord Pool Clearing ASA and Nord Pool Consulting AS. The national grid companies Svenska Kraftnät and Statnett holds 50 per cent each in Nord Pool ASA.

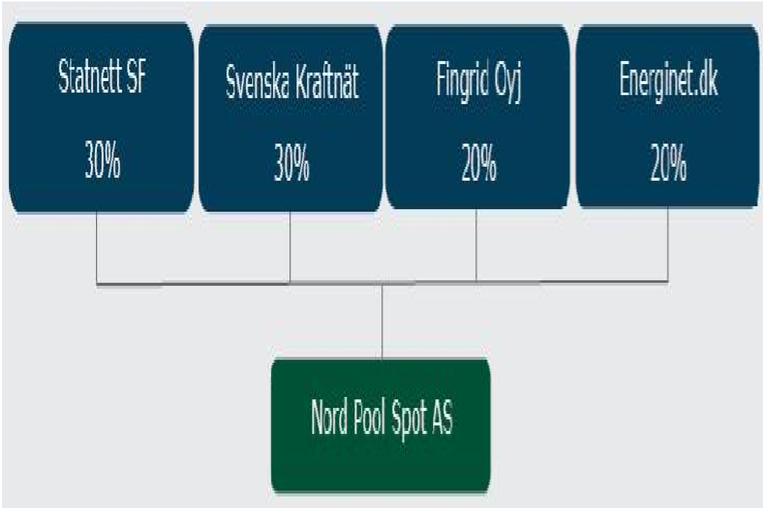

Figure 2: Ownership: Nordic Transmission System Operators.

Nord Pool Spot AS and its subsidiaries Nord Pool Finland Oy and Nord Pool Spot AB are owned by the national grid companies Fingrid, Energinet.dk, Statnett, Svenska Kraftnät and Nord Pool ASA. The Nord Pool group has offices in Lysaker (Oslo), Fredericia, Stockholm, Helsinki, Berlin and Amsterdam.

The Nord Pool group has more than 420 members in total, including exchange members, clearing clients, members and representatives in 20 countries. The customer base for Nord Pool ASA now includes 400 members. Nord Pool Spot AS has a customer base of 319 in Elspot and 55 in Elbas. The membership includes energy producers, energy intensive industries, large consumers, distributors, funds, investment companies, banks, brokers, utility companies and financial institutions [25].

3.1.1 Services in Nord Pool

The physical market is the basis for all electricity trading in the Nordic market. The spot price set here forms the basis for the financial market. Nord Pool Spot organizes the market place which comprises the Elspot and Elbas products. Elspot is the common Nordic market for trading physical electricity contracts with next-day supply. Elbas is a physical balance adjustment market for Sweden, Finland and Denmark. Both the Elspot and Elbas market also include the KONTEK area in Germany [25].

The physical market price on Nord Pool is set in the same way as on other commodity exchanges – through a market pricing. The big difference compared with other markets is that electricity is a commodity which must be generated and consumed simultaneously. This makes very special demands on output planning. The physical market's primary function is to establish a balance between supply of and demand for electricity on the following day. This job is extremely important because power shortages carry very substantial socio-economic costs. A well-functioning power market ensures that electricity gets generated wherever the cost of generation is lowest at any time of the day. Increases in demand will be balanced against more expensive modes of generation. That also gives the market an indication of what it would take to establish new generating capacity [25].

In socio-economic terms, this provides a clear indication of the cost society would have to bear to incorporate new output in the system. Where commercial considerations are concerned, generators will receive a good indication of where the break-even point lies for developing new generating capacity. Nord Pool's physical market ensures that power supply and demand are balanced right up to one hour before the time of consumption [25].

Nord Pool ASA provides a market place where the exchange members can trade derivative contracts in the financial market. Financial electricity contracts are used to guarantee prices and manage risk when trading power. Nord Pool offers contracts of up to six years' duration, with contracts for days, weeks, months, quarters and years. Nord Pool also trades European Union Allowances (EUAs).A European Union allowance (EUA) is the official name for European emission allowances, which is defined from 2008 as the official Kyoto allowance for countries within European Union. One EUA entitles the holder to emit one tonne of carbon dioxide or carbonequivalent greenhouse gas. By 28 February each year member states issue new EUAs to all companies covered by the EU ETS [25].

3.1.2 Transparency on Nord Pool

As long as Nord Pool performs the functions described above, generators, distributors and consumers in the Nordic region will derive great benefits from the market. But that depends on the trust held by players and society in general in price formation by the market – in other words, that this process is open and transparent. It is important to distinguish here between openness in trades and over the names of players [25].

All available information about offers/bids, volumes and prices is openly published from trading on Nord Pool. But the names of members making offers/bids remain confidential because, as in most other commodity markets, players operate under full anonymity in order to keep their positions confidential. However, Nord Pool's market surveillance department is always fully informed about which players are involved in any deal, so that investigations can be pursued and sanctions applied when necessary [25].

3.1.3 What influences Nordic power services

The Nordic system price is set in Nord Pool's physical market for the following day, hour by hour. This is based on registered offers/bids from generators and distributors respectively. The price is determined by Nord Pool's trading system, which matches offers/bids with Nordic transmission capacity. The electricity price is basically the same across the whole region, but with certain regional differences which reflect transmission bottlenecks. Prices paid by consumers can vary from country to country, however, primarily because of different tax levels and competition in the Nordic consumer market. Variations in the wholesale (spot price) for each country also play a minor role. In the financial market, the same buyers and sellers conclude forward contracts for power. This is because they want to hedge against price changes, which also provides a measure of predictability for consumers.

The main facts which influence Nordic power services are:

- Weather and temperature conditions;
- Electricity transmission;
- Economic factors;
- Prices for input factors in power generation;
- Nuclear energy and
- Power consumption over time.

.

3.1.4 Features of the financial market in NordPool

Risks associated with changes in physical market prices can be managed through the financial (forward) market for electricity. A buyer or seller of power can reduce the risk of future price changes by selling or buying the cost of future electricity to/from other players with the ability and willingness to accept this price risk. Players can thereby change their risk exposure as required, and are able to hedge the price of future output or consumption. Using the marketplace allows players to secure the greatest possible transparency around pricing, and reduce the risk of incorrect price formation in the future. The alternative is to do this bilaterally with limited information about the overall position [25].

3.2 NordPool data overview

3.2.1 Introduction

In this thesis, the data used are covering daily observations from 01 January 1999 to 11 March 2009 which give almost ten years data series; then they cover around 3700 observations. The concerned countries in our study are not the all NordPool counties because variables are; prices, flow and capacity for both Sweden and Finland only. Thus three variables are used for two countries: Finland and Sweden. The choice to lay our study on these two countries is that, because in the last researches their prices look almost similar and the the target is to know using our model why they are not similar totally.

3.2.2 Physical grid

NordPool Spot manages the capacity on the interconnections between the Nordic countries and the cables that connect the bidding areas in Norway. Nord Pool Spot uses the capacity for conducting power into high price areas and conducting power out of low price areas. Thereby, the price in high price areas is reduced whereas the price in low price areas is raised. This principle is right for society: the commodity ought to move towards the high price. This system also causes no market participant to be assigned privileges on any bottleneck, which is an important feature of a liberalized market. A privilege on a bottleneck could be abused by a commercial participant and it is therefore essential that the capacity is given to a neutral party.

Two commercial participants separated by a bottleneck in the grid cannot trade physical kWh with each other. This is impossible because the capacity of the bottleneck is given to the power exchange. Consequently, the two participants cannot exchange kWh across the bottleneck.

How do two participants like that then trade with each another? The answer is that they make a financial contract. The two participants trade the physical kWh with the exchange or a local market participant. Later, they settle with each other in accordance with the financial contract. The idea of this principle is as follows: kWh are always procurable. They can for example be bought via Elspot. What is interesting is, therefore, not to procure kWh, but only the price. However by means of a financial contract, the price is fixed. Participants who are not separated by any bottlenecks may also deal in physical kWh with each other. The exchange has no monopoly [25].

3.2.3 Capacity and Flow

In electricity market, the unit of measure for the capacity is MW. The maximum net transfer capacity available for market is 2050 MW from Sweden to Finland and it is 1650 MW from Finland to Sweden. The transmission capacity depends on power balance in the northern Finland.

Typically due to winter temperatures the deficit in northern Finland increases and this will be taken into account when defining the export capacity from Finland to Sweden. During night time the export capacity may be reduced according to the forecasted deficit volume.

In power market, the unit of measure for the flow is MWh/h. Now considering a border, where two power exchanges meet; we define market coupling as a process, where a cooperation between the two power exchanges ensures this: in case of congestion and during every hour of operation, all the available trading capacity is utilized with power flowing towards the high price area.

To illustrate how this is achieved, let us consider a simplified case with only two areas and one given hour of operation: Assume the cross-border trading capacity is 400 MW during the hour in question.

Then the cooperation between the power exchanges will proceed the following way to reach balance:

- There is a sales surplus in the low price-side of the border of 400 MW that the exchange can show as an own purchase surplus of 400 MW that will be transferred to the high-price side.
- There is an internal sales deficit on the high-price side of the border of 400 MW which is covered by the 400 MW exchange sales surplus that is caused by the transfer from the low price-side.

When the next day arrives at this hour, the exchange purchase surplus will, somewhat simplified, give rise to a production surplus of 400 MW at the low-price side of the border. Similarly, the exchange sale surplus will together with the internal sales deficit of 400 MW at the high-price side of the border.

Hence the laws of nature will ensure a power flow of 400 MW from the low-price area to the high-price area: The "cheap" power from the low-price area will flow into the high-price area.

3.2.4 Electricity Prices in Nord Pool

There many facts which influence electricity prices in Nord Pool as we mentioned such as:

- Weather and temperature conditions;
- Electricity transmission;
- Economic factors;
- Prices for input factors in power generation;
- Nuclear energy and
- Power consumption over time.

Also that there are three types of bids available in Elspot:

- Hourly Bid;
- Block Bid and
- Flexible Hourly Bid.

Participants submit their bids electronically to Nord Pool Spot on bidding forms. All bids are aggregated to form a curve for purchases and a curve for sales for each of the 24 hours of the day following the Elspot price determination. The point at which the two curves intersect within each hour determines the Elspot Price, which in turn establishes the trading result for each participant for that hour.

In Nord Pool prices are calculated based on an application of the social welfare criterium in combination with market rules mainly. SESAM is maximizing the value of the objective function subject to physical constraints; like volume constraints, area balances, transmission and ramping constraints [25].

In figures (3) and (4) where prices calculation is shown with and without bottlenecks respectively, PS represents System Price and as soon as the noon deadline for participants to submit bids has passed, all purchase and sell orders are aggregated into two curves for each delivery hour; an aggregate demand curve and an aggregate supply curve. The System Price for each hour is determined by the intersection of the aggregate supply and demand curves which are representing all bids and offers for the entire Nordic region. The System Price is also denoted the unconstrained market clearing price since the trading capacities between the bidding areas have not been taken into account in finding this price. The majority of the standard financial contracts traded in the Nordic region refers to the System Price. There are also standard financial contracts with reference to specific area prices [24].

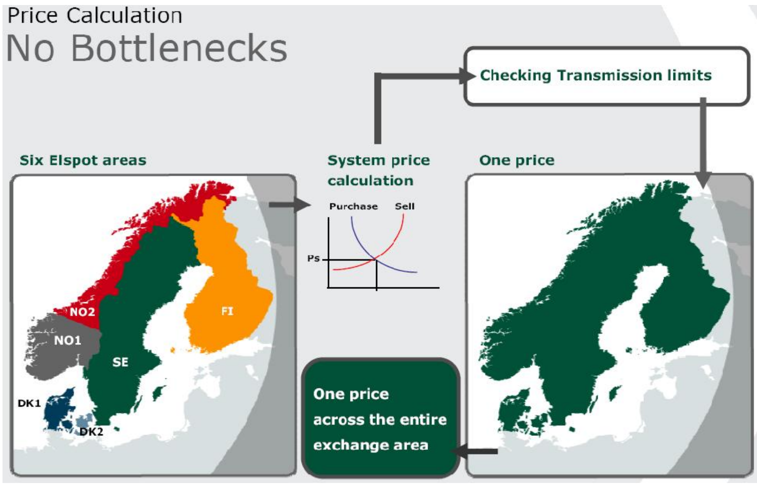

Figure 3: Price calculation without bottlenecks.

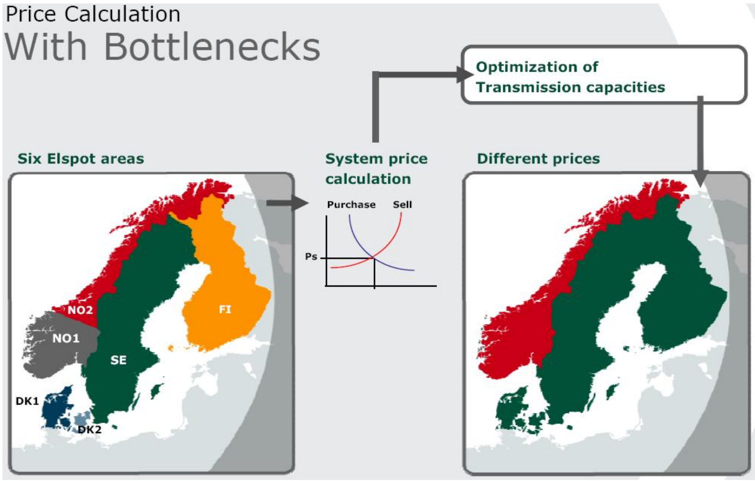

Figure 4: Price calculation with bottlenecks.

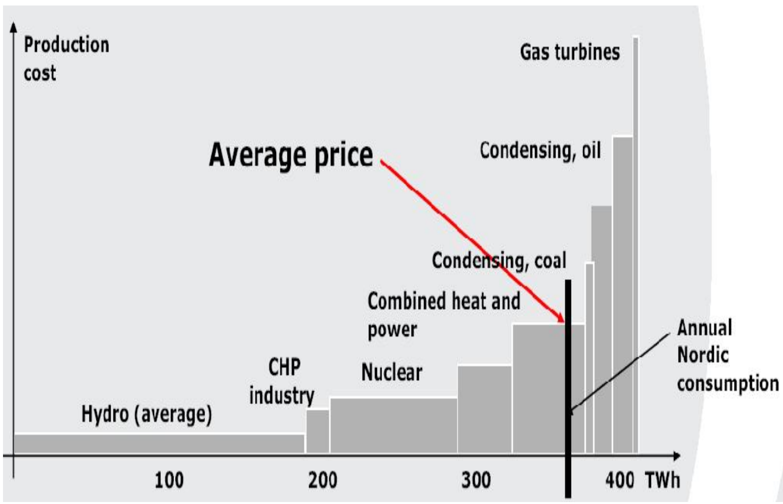

Figure 5: Price determination theoretically.

In figure (6) , it is to observe how were prices in Nord Pool from 1996 to the first term of 2008. The simulated prices for the other three months of 2008 are represented in blue and in red we have nothing for years 2009 to 2012 [24].

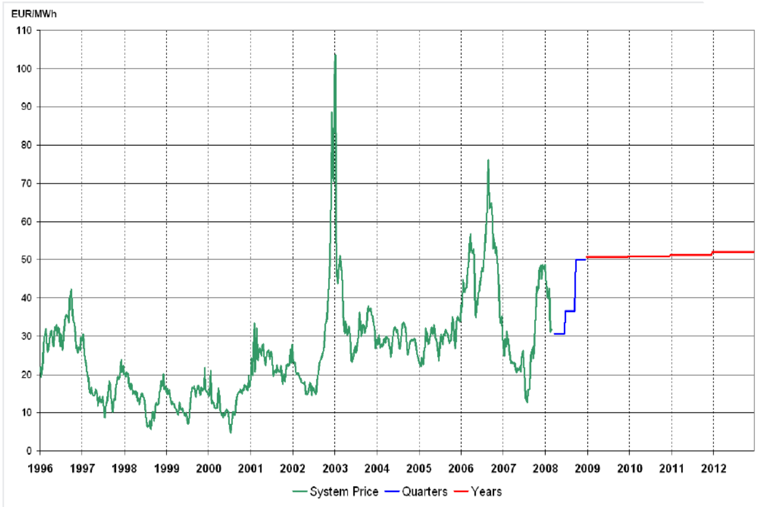

Figure 6: Nord Pool prices from 1996 to 2012.

4 A regime switching model for electricity prices with Nord Pool

4.1 Introduction

It has been argued in some studies, for instance, that under certain conditions time series variables can spuriously have long memory when measured in terms of their fractional order of integration , when in fact the series exhibit non-linear features, regime switching for instance [1]. Also, it has been long known that financial asset returns are not normally distributed. Rather, the empirical observations exhibit excessive kurtosis. This heavy-tailed character of the distribution of price changes has been repeatedly observed in various financial and commodity markets. There are also reports of heavy-tailed behavior of electricity prices [21].

In economics, one of the concepts is structural breaks and it appears when we see an unexpected shift in a time series. This can lead to huge forecasting errors and unreliability of the model in general [1]. In this chapter we develop a mathematical model which is used to describe the structural breaks occurring in electricity prices for Finland and Sweden. Depending on the main facts which influence Nordic power services especially in Sweden and Finland in which we are interested in this thesis; we know that the price behavior in different states appears to be rather different [12]. We prefer to use GARCH models because they are more persistent for structural breaks than ARCH models and also GARCH models are very flexible, thus they do better volatility forecasts [18]. The Markov regime-switching (MRS) models offer the best of the two worlds; they are a trade-off between model parsimony and adequacy to capture the unique characteristics of power prices [22]. The underlying idea behind the MRS scheme is to model the observed stochastic behavior of a specific time series by two (or more) separate phases or regimes with different underlying processes. In other words, the parameters of the underlying process may change for a certain period of time and then fall back to their original structure. Hence, regime-switching models divide the time series into different phases that are called regimes. For each regime one can define separate and independent underlying price processes. The switching mechanism between the states is assumed to be governed by an unobserved random variable [18]. So a regime switching GARCH model is developed and analyzed for Sweden and Finland.

4.2 Model development

4.2.1 Regime Switching GARCH model

Before developing our bivariate (price and volatility) Regime Switching GARCH which is relevant for our data let us first explain the Regime Switching GARCH model for the univariate (volatility) case. The standard GARCH(1,1) model of Bollerslev built in 1986 is defined as the process:

$$y_t = \mu_1 + \epsilon_t = \mu_1 + \sigma_t u_t \tag{19}$$

$$\sigma_t^2 = \omega_1 + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2, \tag{20}$$

where the error term u_t is independent identically distributed with zero mean and unit variance and the sum $\sigma_1 + \alpha_1$ measures the persistence of the volatility process. When this model, is estimated using daily or higher frequency data, the estimate of this sum tends to be close to one, indicating that the volatility is highly persistent and the process may not be covariance stationary. However, it was argued that the high persistence may artificially result from regime shifts in GARCH parameters over time.

This motivates specifying a model that allows for regime-switching in the parameters. We define for each t an unobserved state variable $s_t \in 1, 2, \ldots, n$, which selects the model parameters with probability $p_{jt} = Pr(s_t = j/T_{t-1})$ where T_t is an information set available at time t, which includes $(y_t, \sigma_t, \ldots, y_1, \sigma_1)$. This transition matrix $P = (p_{ij})$ contains the probabilities p_{ij} of switching from regime i at time t to regime j at time $t-1$, for i, $j = \{1, 2\}$. Here, as an example, when $s_t = 2$ it means that the process is in regime 2 at time t.

Thus we define the RS-GARCH model as

$$y_t = \mu_{s_t} + \epsilon_t = \mu_{s_t} + \sigma_t u_t \tag{21}$$

where $u_t \sim i.i.d.(0, 1)$

$$\sigma_t^2 = \omega_{s_t} + \beta_{s_t} \sigma_{t-1}^2 + \alpha_{s_t} \epsilon_{t-1}^2, \tag{22}$$

$$p_{jt} = P(s_t = j/T_{t-1}) = p_{jt}(y_{t-1}^2), j = 1, 2, \ldots, n, \tag{23}$$

where the function $p_{jt}(\cdot)$ can be a logistic or exponential link function and in this study j can be 1 or 2.

This function depends on the parameters not introduced explicitly at this stage and must be defined so that the probabilities are positive and sum to unity and here $\omega_{s_t}, \beta_{s_t}, \alpha_{s_t}$ are state dependent coefficients [5].

4.2.2 Regime Switching model for Nord Pool: Finland and Sweden

The main feature of regime-switching models is the possibility for some or all the parameters of the model to switch across different regimes or states according to a Markov process, which is governed by a state variable, denoted s_t. The logic behind this kind of modeling is having a mixture of distributions with different characteristics, from which the model draws the current value of the variable, according to the more likely state that could have determined such observation. The state variable is assumed to evolve according to a first-order Markov chain, with transition probability [5]:

$$P(s_t = j/s_{t-1} = i) = p_{ij} \tag{24}$$

that indicates the probability of switching from state i at time $t-1$ into state j at t.

Usually, these probabilities are grouped together into the transition matrix

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{25}$$

where for simplicity the existence of only two regimes has been considered. Because of the Markov property the current state S_t at time t of a Markov chain depends on the past only through the most recent value S_{t-1}.

Consequently, the probability of being in state j at time $t+m$ starting from state i at time t is given by

$$P(S_{t+m} = j | S_t = i) = (Q\prime)^m \cdot e_i,$$

where $Q\prime$ denotes the transpose of Q and e_i denotes the ith column of the 2×2 identity matrix [11].

The ergodic probability of being in state $s_t = 1$ is given by $\pi_1 = \frac{1-p}{(2-p-q)}$. We remind that the data set analyzed in this thesis is from Nord Pool and for our model we consider as variables: flow and capacity for only two countries, Sweden and Finland, and thus our model will be mainly based on two regimes described by respective two GARCH models.

In the following, we study three different ways to identify two regimes. One of the regimes is always the regular regime. The other one, generally called the non-regular regime, has three different names referring each to a different definition of non-regularity: the *spiky regime*, the *capacity-limited regime* and the *split regime*.

Now, an assumption is that there are two regimes. When the difference between Flow and Capacity is zero (*: Flow-Capacity = 0)*, we expect there congestions and thus possibly price change. According to the two regimes assumption, we have six parameters denoted by $P = (\mu_1, \mu_2, \beta_1, \beta_2, \alpha_1, \alpha_2)$.

In figures (7), (8) and (11), it is remarkable that the capacity for Finland and that for Sweden have trends which are almost constant for around the 3000 first observations. We do not know why here these trends are not constant for the remained observations but we expect to find the explanation by our Regime-Switching model.

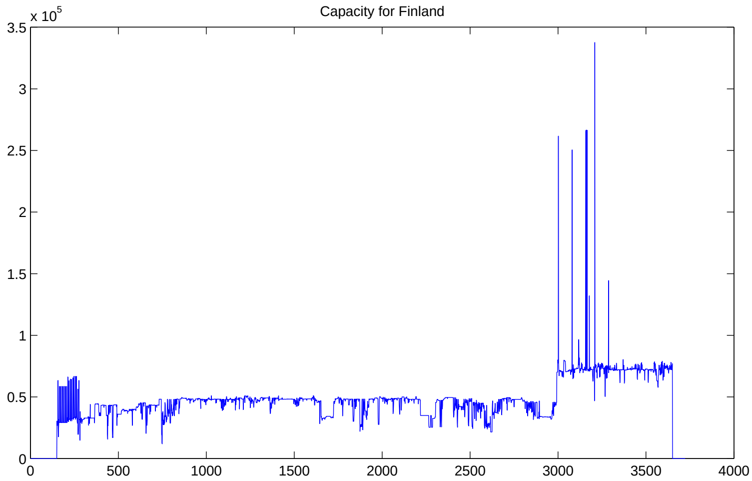

Figure 7: Capacity for Finland.

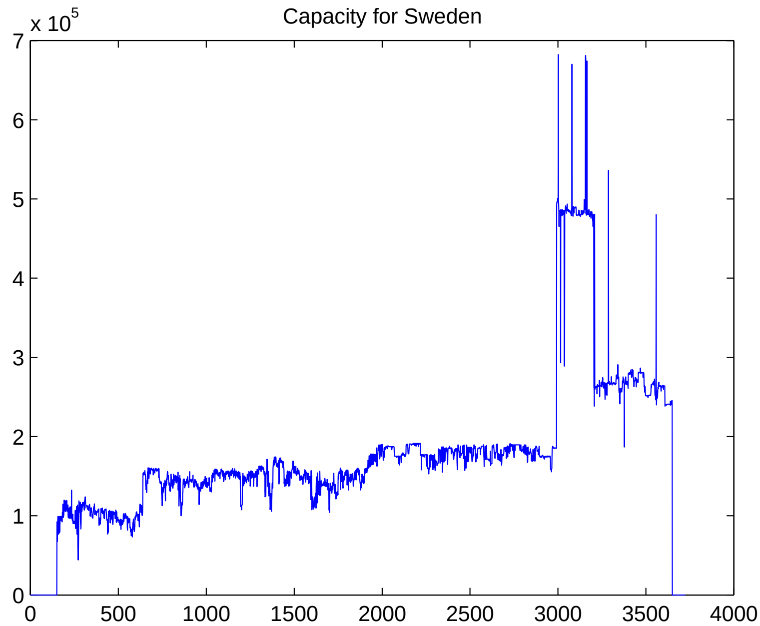

Figure 8: Capacity for Sweden.

In figures (9), (10) and (12), it is to remark that the flow for Finland and that for Sweden have trends which show that the flow is very volatile, this is because of the way electricity is generated.

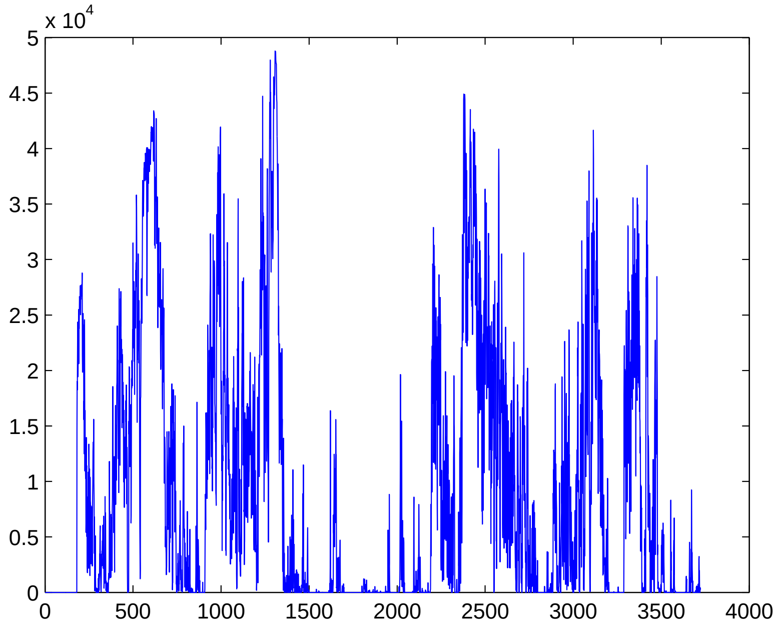

Figure 9: Flow for Finland.

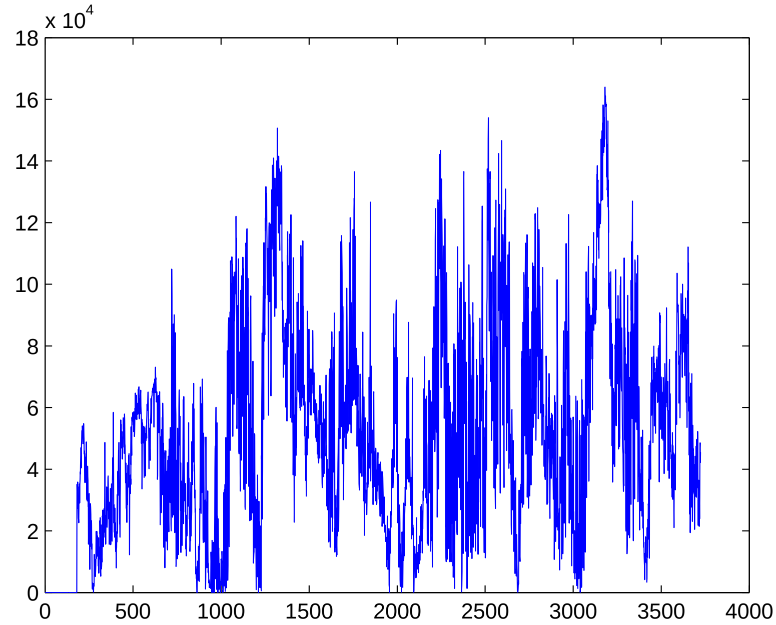

Figure 10: Flow for Sweden.

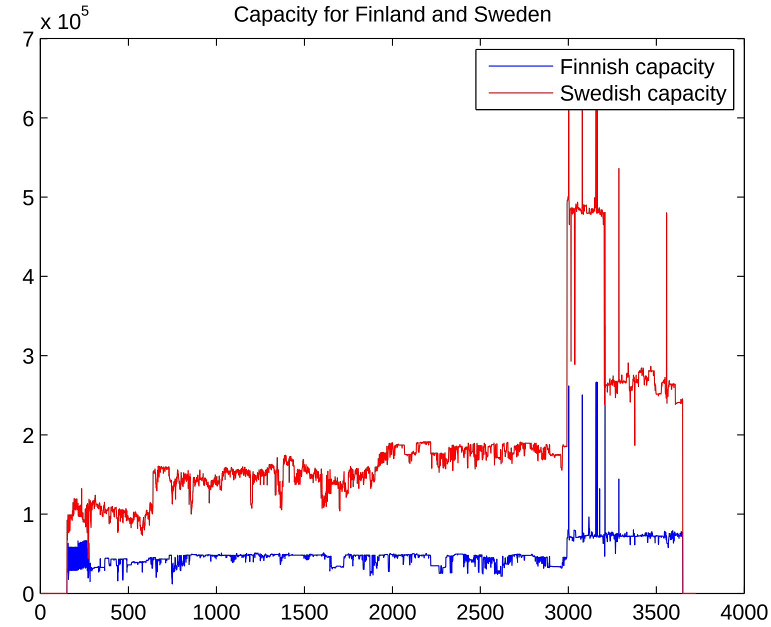

Figure 11: Capacity for Finland and Sweden.

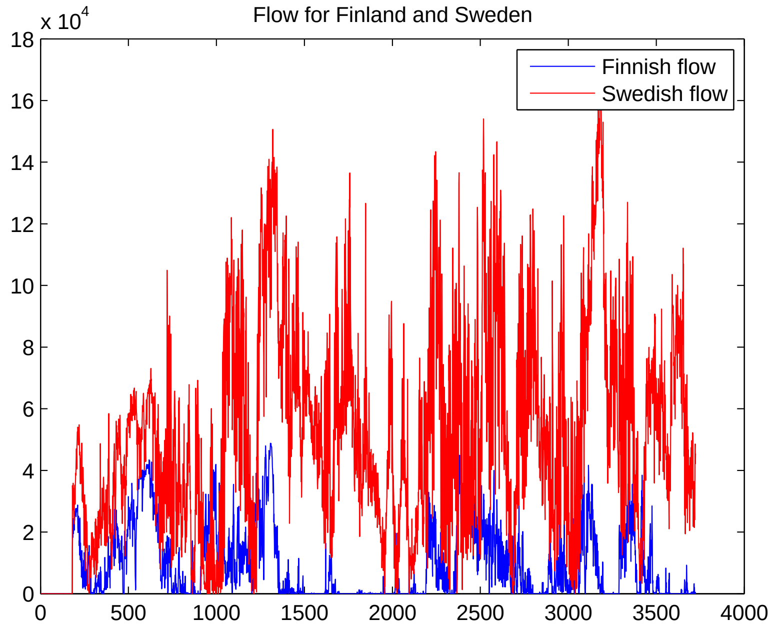

Figure 12: Flow for Finland and Sweden.

In figures (13), (14), (15), (16) and (17), the observations are how Finnish and Swedish flow or capacity vary but the most important thing which can be seen is how the differences between flow and capacity (*Flow-Capacity*) are varying for both countries. And from that difference there is expectation of price changes when this difference is equal to zero.

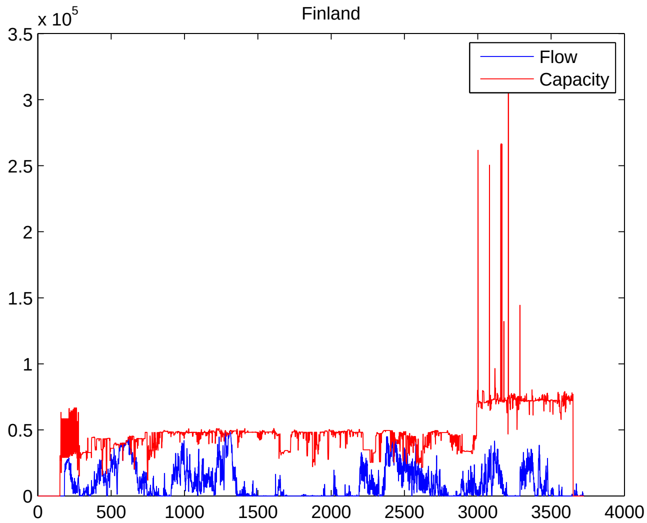

Figure 13: Flow and Capacity for Finland.

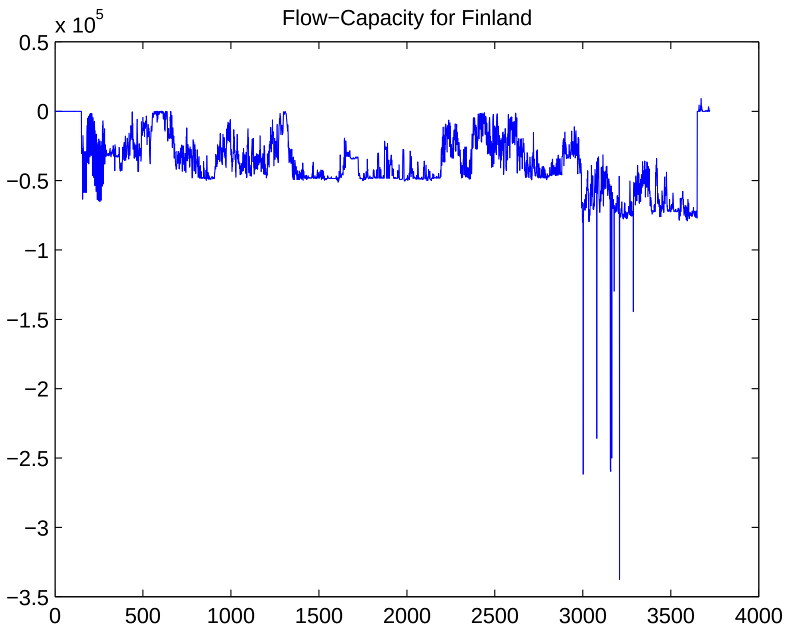

Figure 14: Flow-Capacity for Finland.

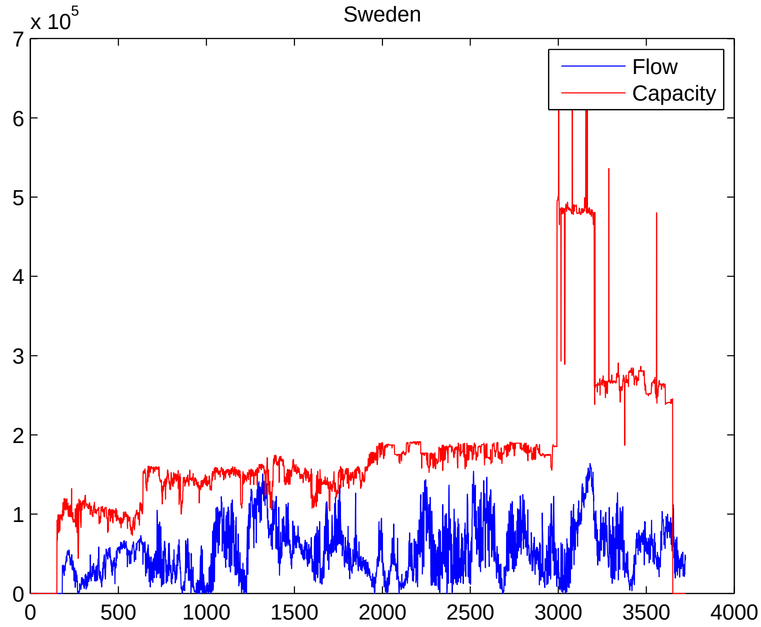

Figure 15: Flow and Capacity for Sweden.

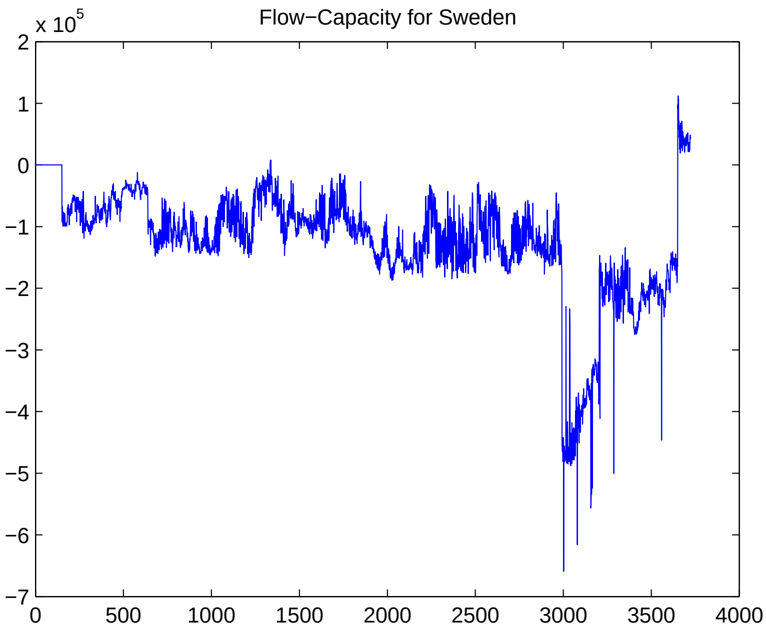

Figure 16: Flow-Capacity for Sweden.

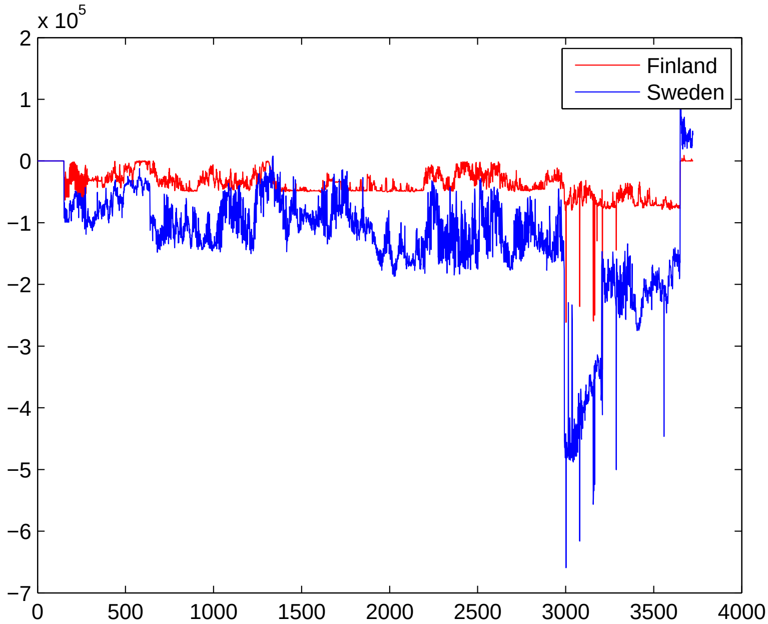

Figure 17: Flow-Capacity for Finland and Sweden.

In figures 18, 19 and 20, there are two important stylized facts of electricity prices for both Finland and Sweden: extreme volatility and price spikes, which lead to heavy-tailed distributions of price changes.

Finland and Sweden interest our study because a considerable amount of electricity which is used in these two countries comes from Norway. Then part is transmitted within Sweden and part is forwarded to Finland. Thus in case of any congestions in the power grid between Sweden and Finland we expect the Finnish area price to rise. In figures 18, 19 and 20, especially in figure 20 we remark that the prices for both Finland and Sweden look almost similar with few differences.

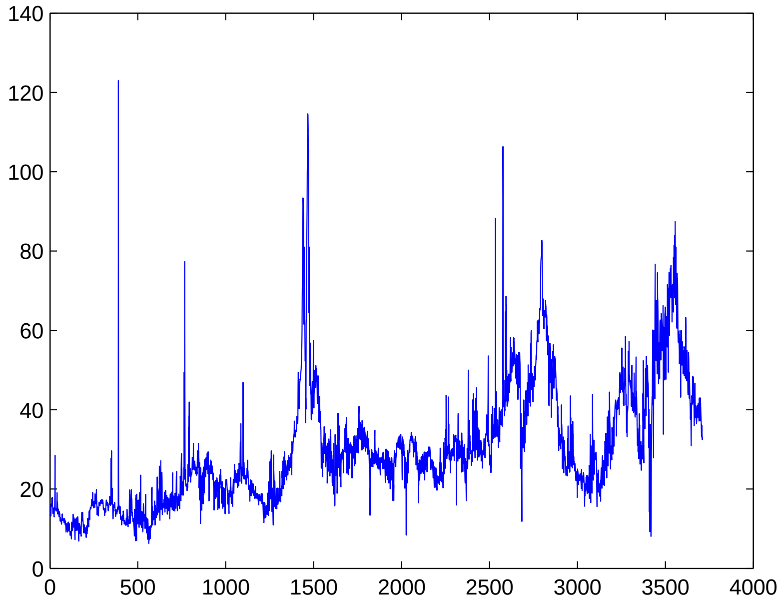

Figure 18: Finnish prices.

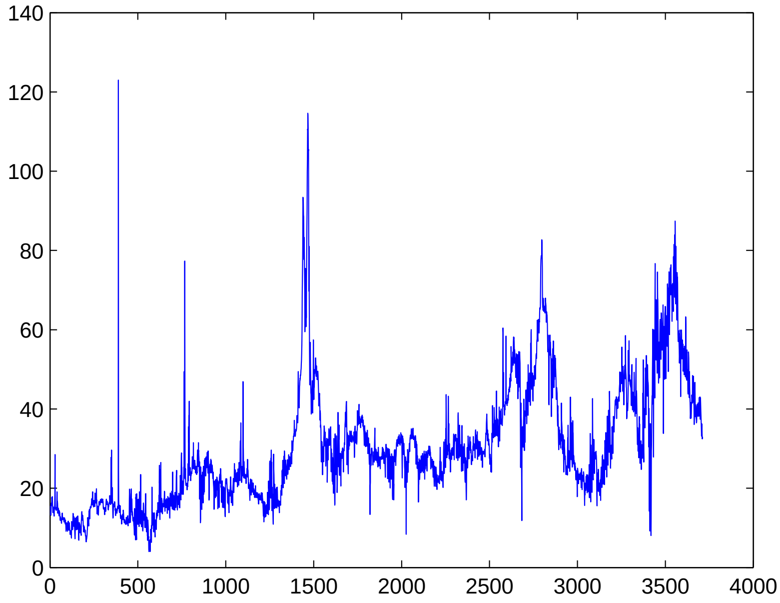

Figure 19: Swedish prices.

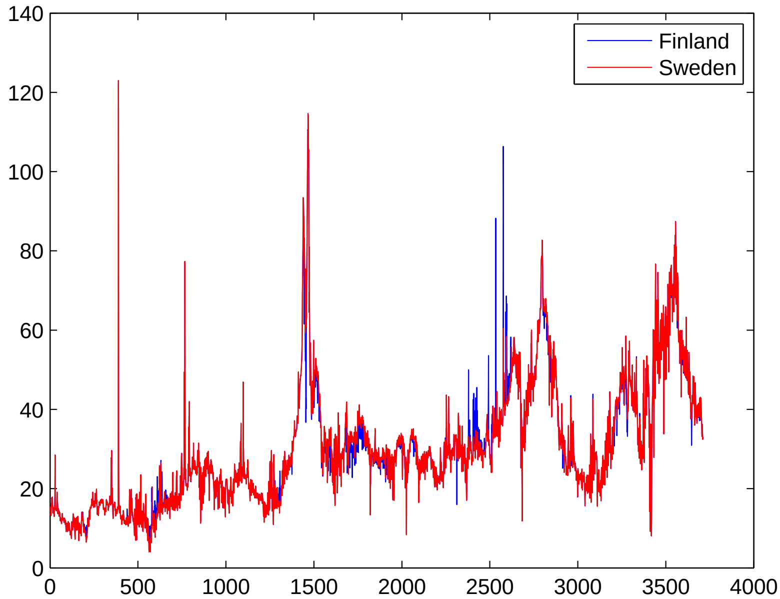

Figure 20: Finnish and Swedish prices combined.

In figures (21) and (22), we have the all 3712 observations while after separation of the equal and not equal prices in figures (23) and (24) we obtain, 2708 observations in figure (23) for equal prices and 1004 observations in figure (24) for non-equal prices.

This can be seen in figures (21), (22), (23) and (24) where red represents the 1004 not similar observations and blue the 2708 similar. Figure (21) represents the scatter plot of all the 3712 Finnish and Swedish prices.

From figures, we discover that for time instances when Flow is equal to capacity id est *Flow minus Capacity equals to zero*, the range is varying up to around 120 MW while for *Flow minus Capacity is different to zero*, the range is varying up to around 90 MW.

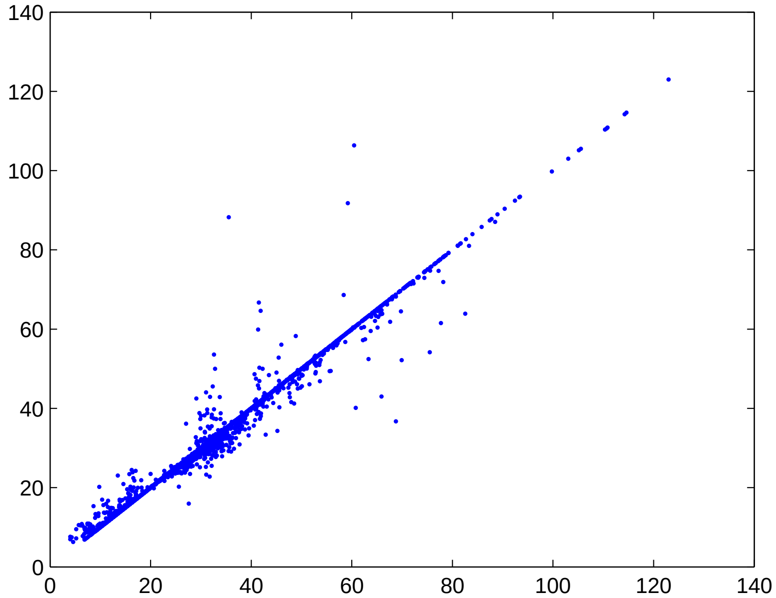

Figure 21: Scatter plot of Finnish and Swedish prices: x and y axes correspond to Sweden and Finland respectively.

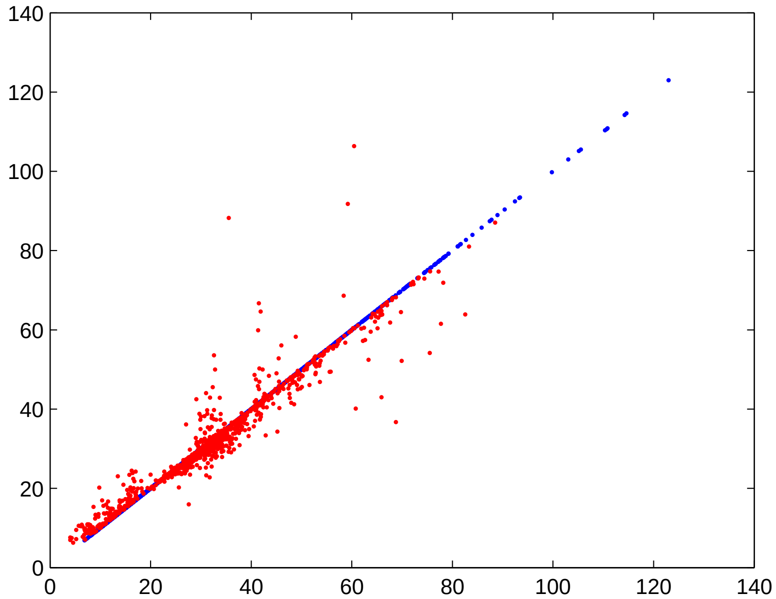

Figure 22: Scatter plot for regimes split by price difference: *Blue* represents regular regime and *Red* split regime.

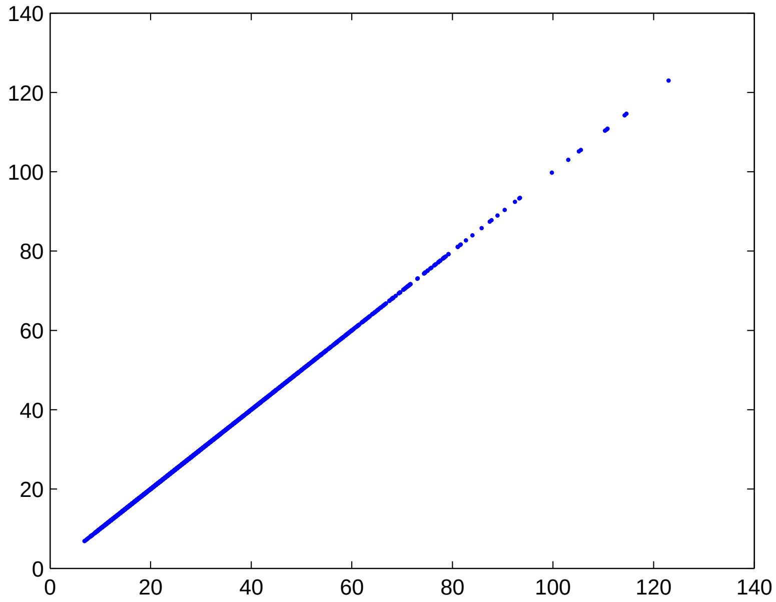

Figure 23: Scatter plot for regimes split by price difference: Regular regime.

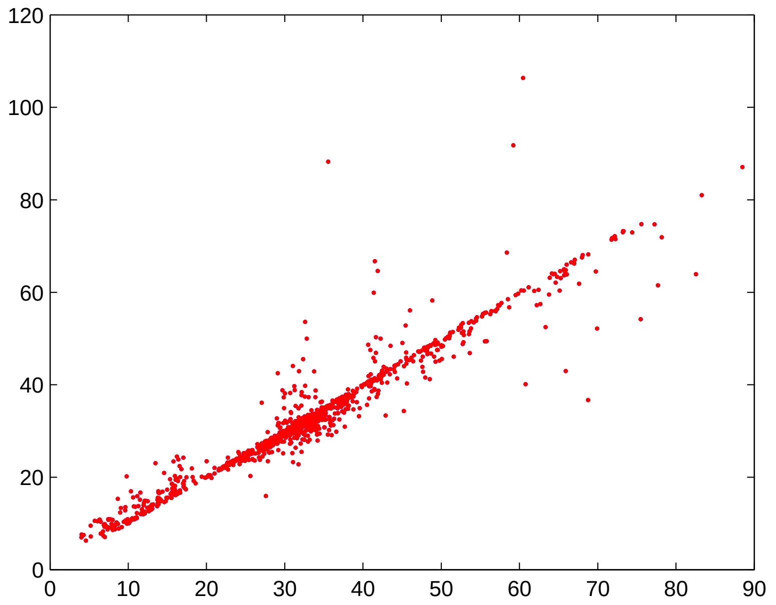

Figure 24: Scatter plot for regimes split by price difference: Split Regime.

From figure (25) to figure (35), we have *Scatter plots* for regimes split by differences between Swedish prices and Finnish prices where we use the absolute values of the differences to identify *REGULAR REGIMES* and *Non-Regular SPLIT REGIMES*.

Different values which are: 1 (figures (25) to (27)), 2 (figures (28) to (30)), 3 (figures (31) to (33)), and 4 (figures (34) and (35)) are checked and according to the figures the decision taken is to work with those corresponding to 4.

Figure 25: Scatter plot for regimes split by price difference: *Blue* represents regular regime $|Sp - Fp| \leq 1$ and *Red* split regime $|Sp - Fp| > 1$.

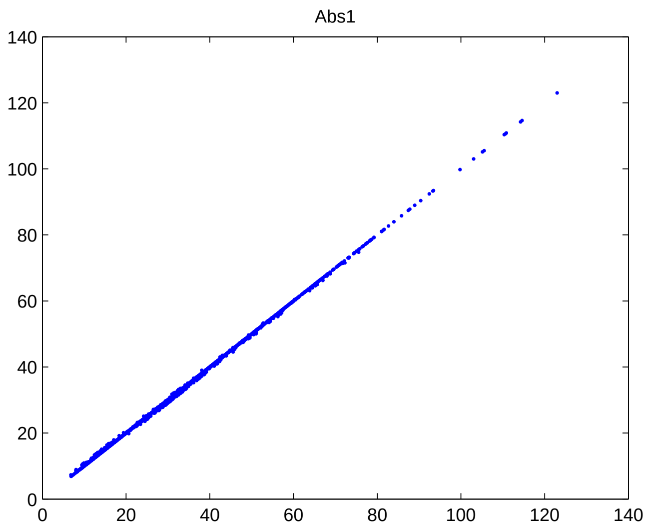

Figure 26: Scatter plot for regimes split by price difference: Regular Regime $|Sp - Fp| \leq 1$.

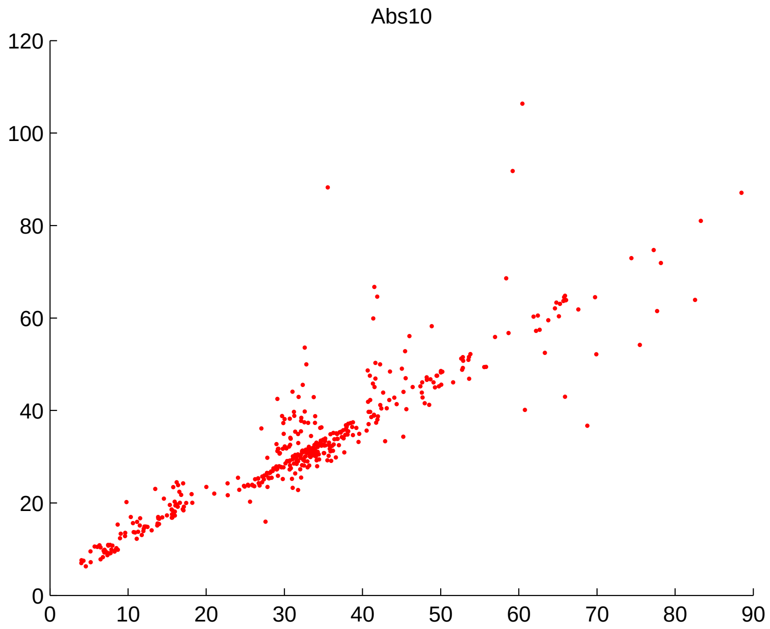

Figure 27: Scatter plot for regimes split by price difference: Split Regime $|Sp - Fp| > 1$.

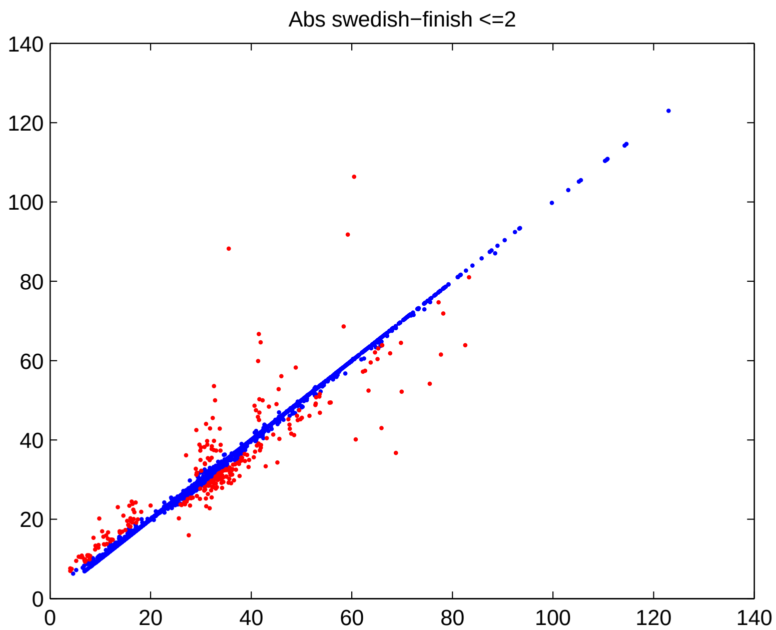

Figure 28: Scatter plot for regimes split by price difference: *Blue* represents regular regime $|Sp - Fp| \leq 2$ and *Red* split regime $|Sp - Fp| > 2$.

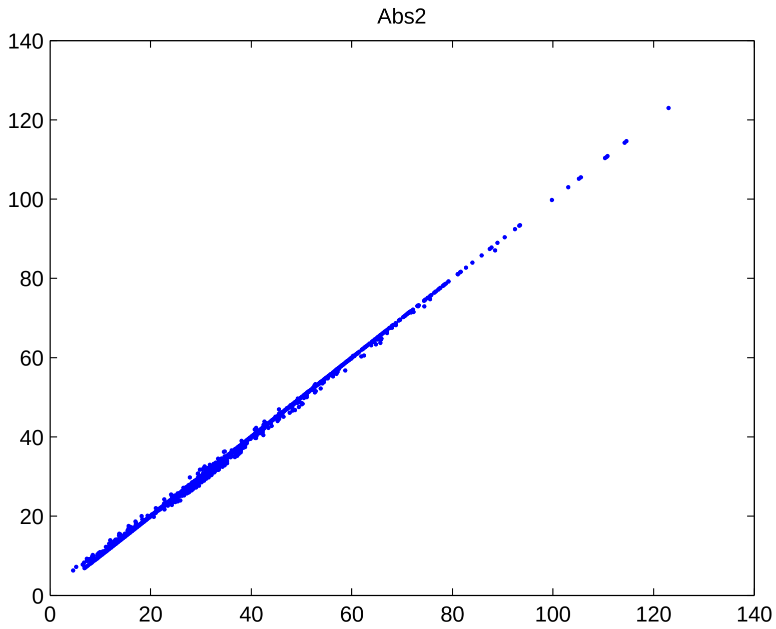

Figure 29: Scatter plot for regimes split by price difference:Regular Regime $|Sp - Fp| \leq 2$.

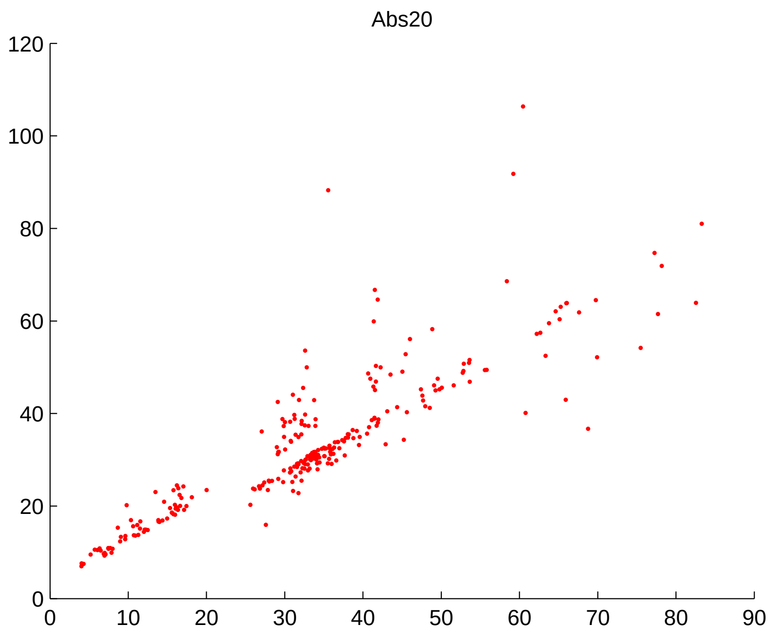

Figure 30: Scatter plot for regimes split by price difference: Split Regime $|Sp - Fp| > 2$.

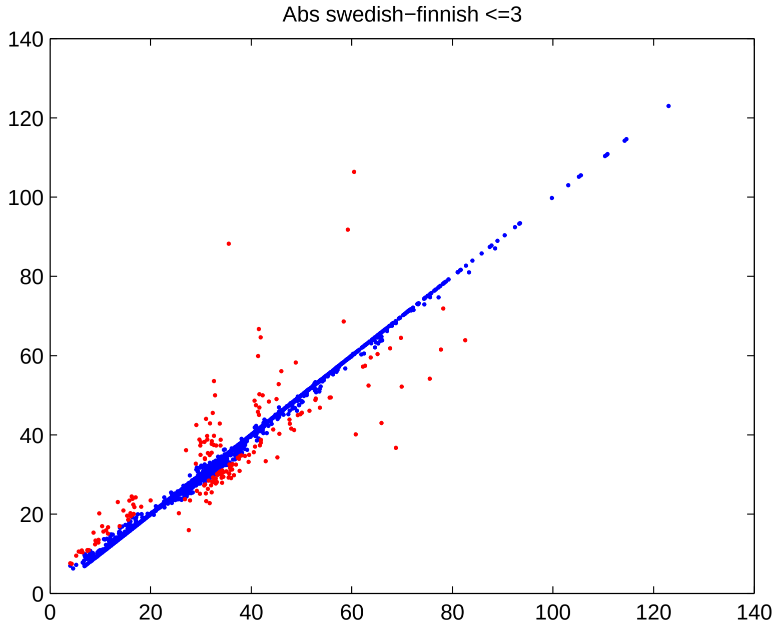

Figure 31: Scatter plot for regimes split by price difference: *Blue* represents regular regime $|Sp - Fp| \leq 3$ and *Red* split regime $|Sp - Fp| > 3$.

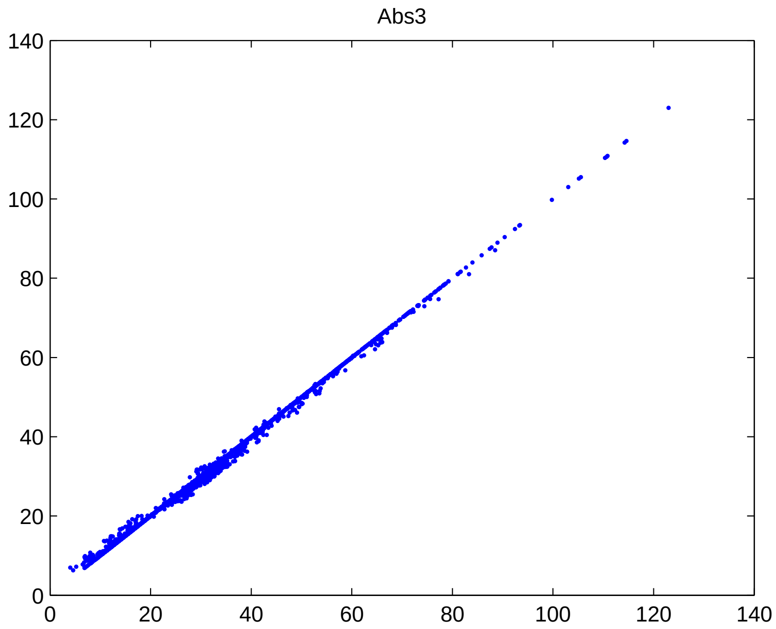

Figure 32: Scatter plot for regimes split by price difference:Regular Regime $|Sp - Fp| \leq 3$.

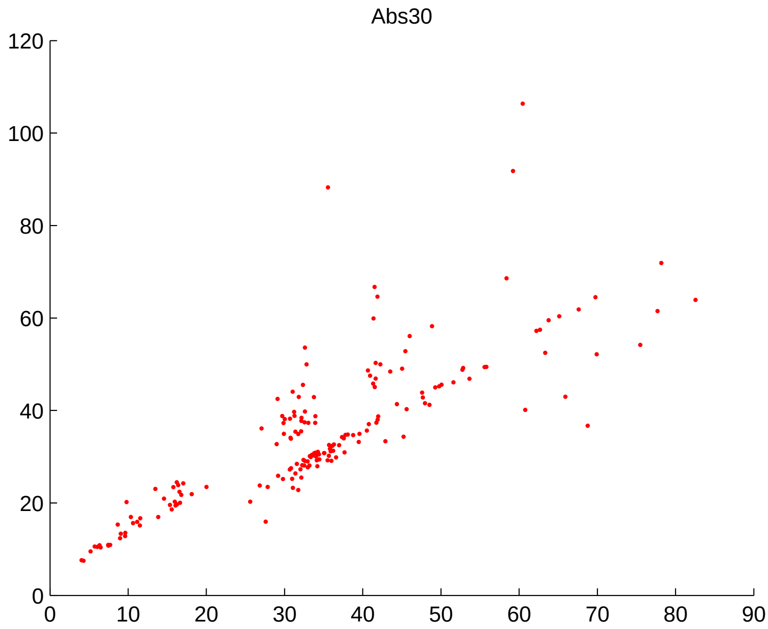

Figure 33: Scatter plot for regimes split by price difference: Split Regime $|Sp - Fp| > 3$.

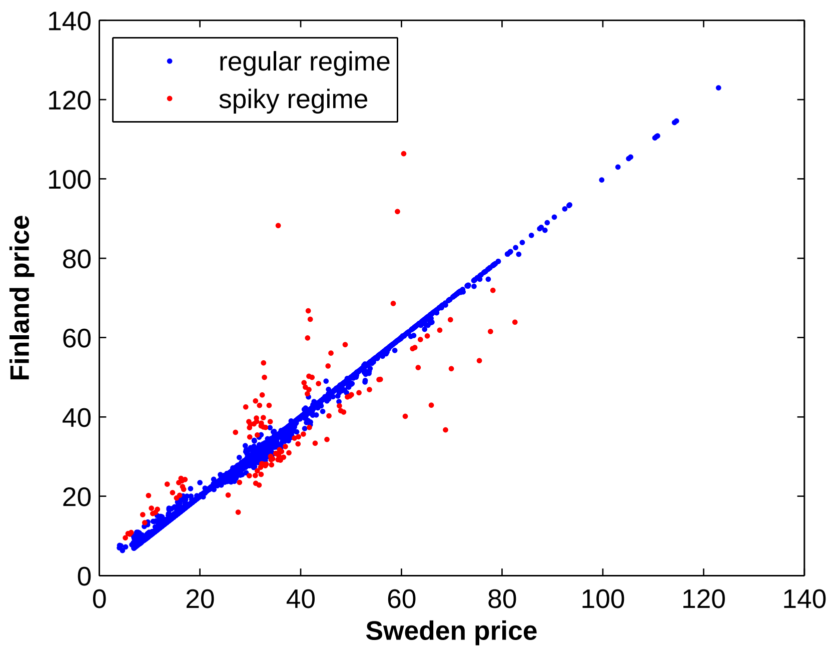

Figure 34: Prices scatter plot for regimes split by price difference.

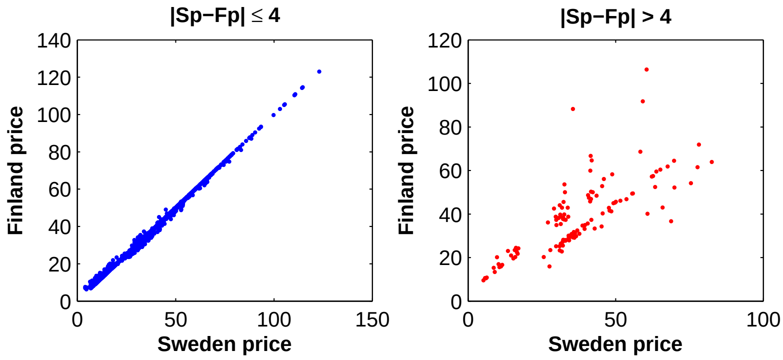

Figure 35: Price scatter plots for regimes split by price difference.

After choosing to work with *REGIMES* related to value 4 id est $|Swe_{pr} - Fin_{pr}| \leq 4$ for regular regime and $|Swe_{pr} - Fin_{pr}| > 4$ for split regime; we plot prices in regimes which are shown in figure (36).

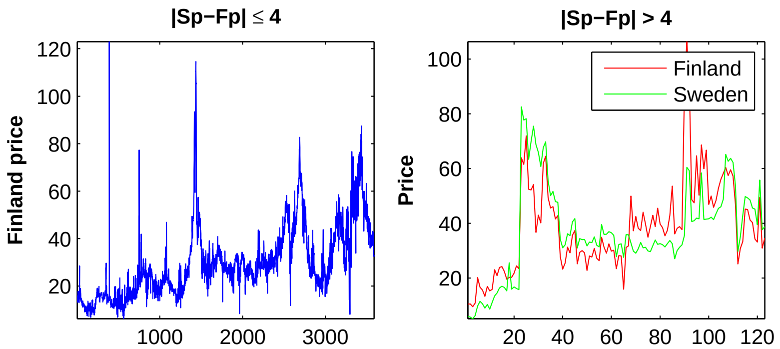

Figure 36: Prices in regimes split by price difference.

From figure (37) to figure (45), the plots are related to regime splitting by Finnish flow and Finnish capacity. We focused our study to Finland only, because its prices, flow and capacity are conditioned by the situation in Sweden.

These figures represent the two regimes depending on the difference between Finnish flow and Finnish capacity where $|Fin_{flow} - Fin_{capacity}| \leq 10 percent$ represents the *Capacity-limited Regime* and $|Fin_{flow} - Fin_{capacity}| > 10 percent$ represents *Regular Regime*; we have decided to work with $90 percent$ for determining regular or capacity-limited regimes because of the different figures we obtained when checked for 100 per cent, 99.5 per cent, 99 per cent, 90 per cent, 80 per cent and 50 per cent where we observed that 90 per cent, in figure (46), is the most realistic to the others.

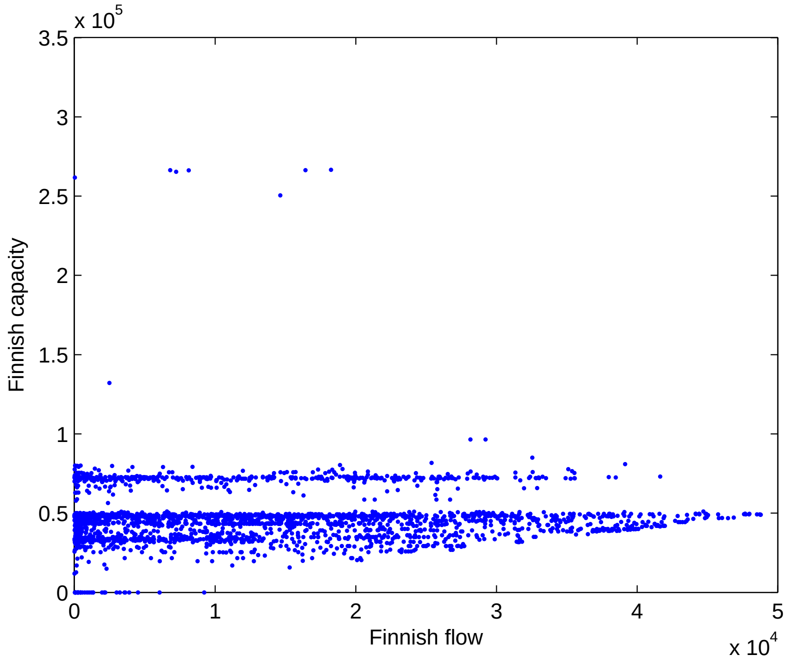

Figure 37: Scatter plot of Finnish flow and capacity.

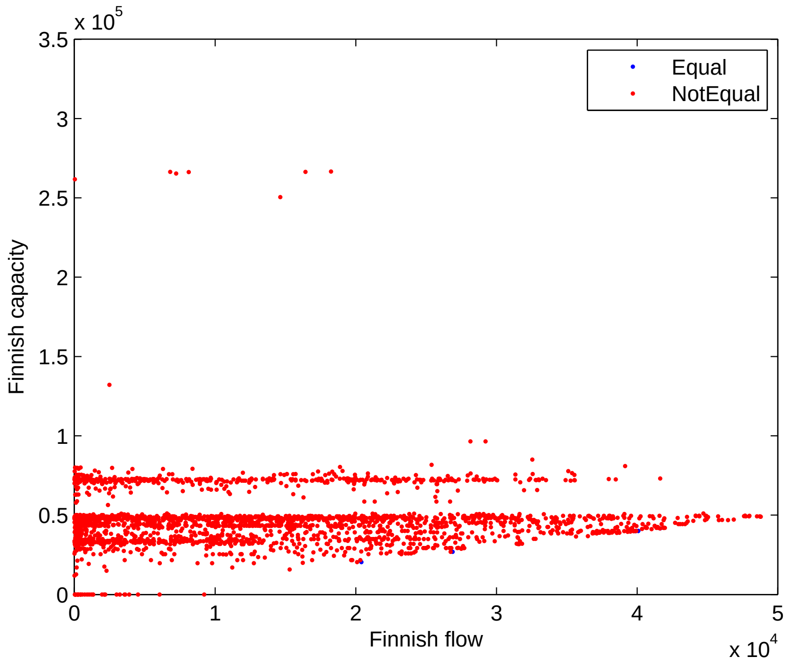

Figure 38: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 100 per cent.

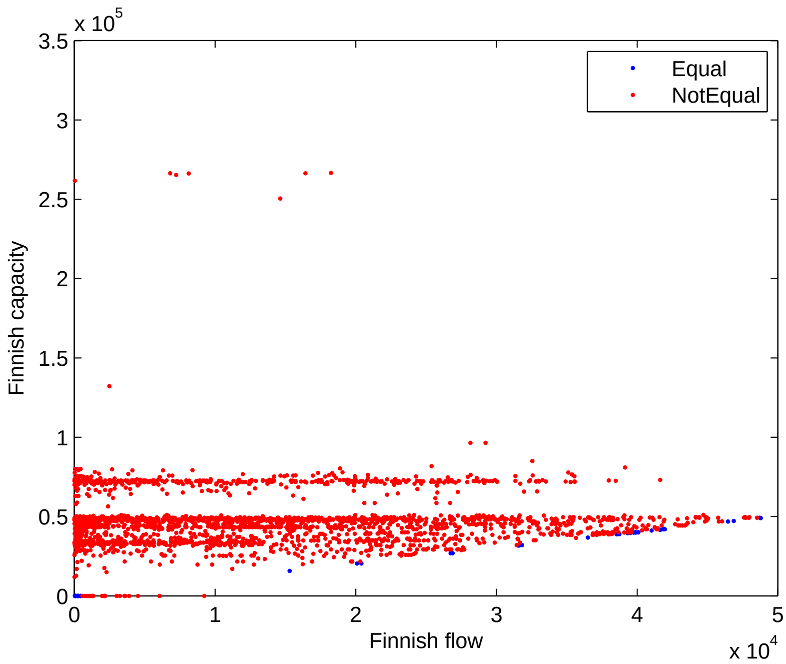

Figure 39: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 99.5 per cent.

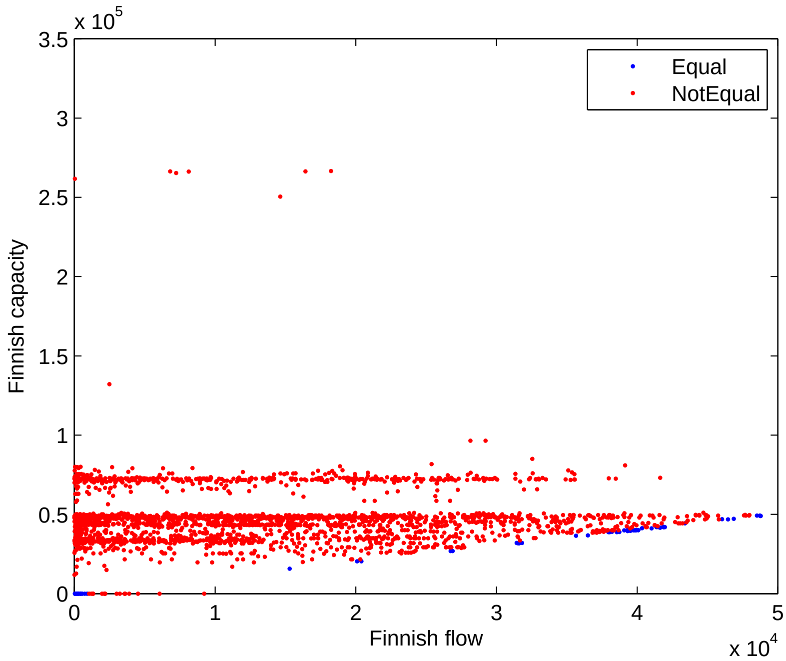

Figure 40: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 99 per cent.

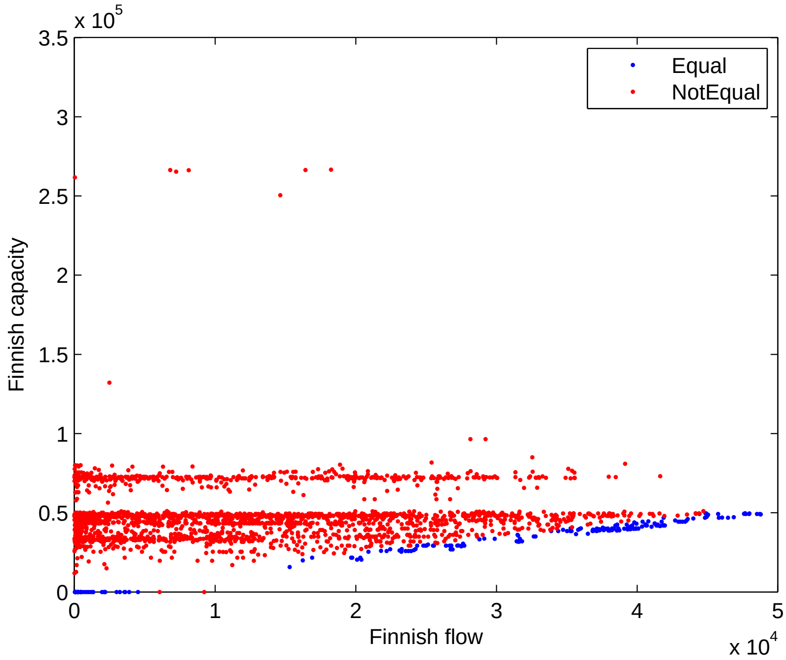

Figure 41: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 95 per cent.

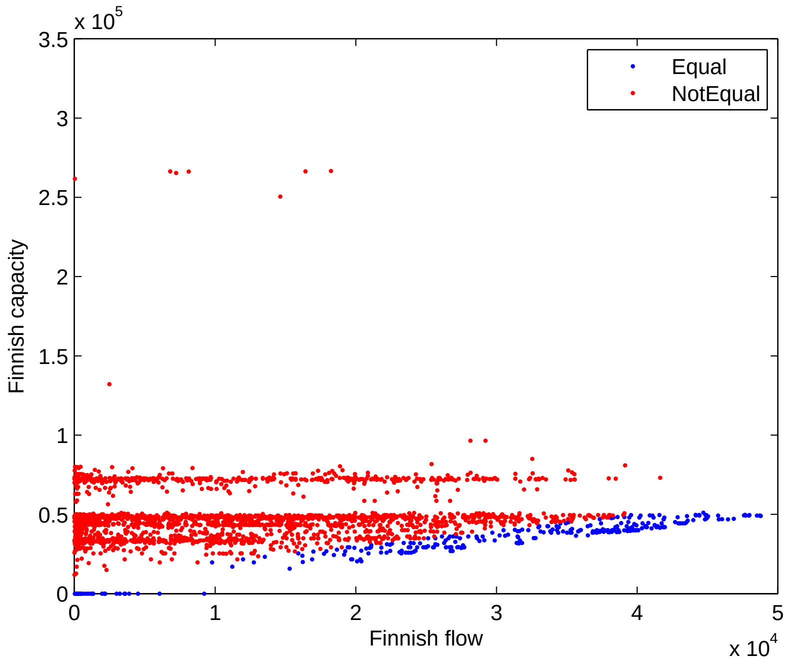

Figure 42: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 90 per cent.

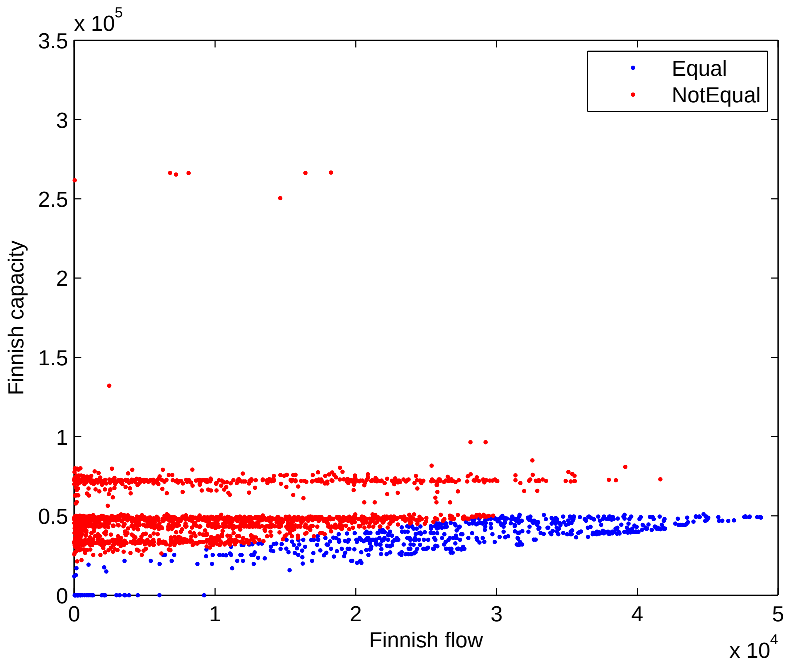

Figure 43: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 80 per cent.

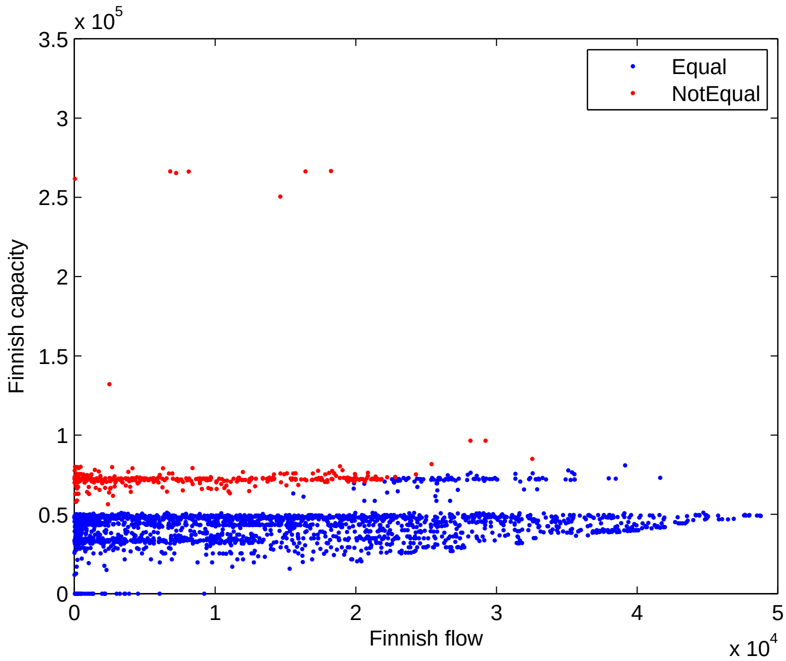

Figure 44: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 50 per cent.

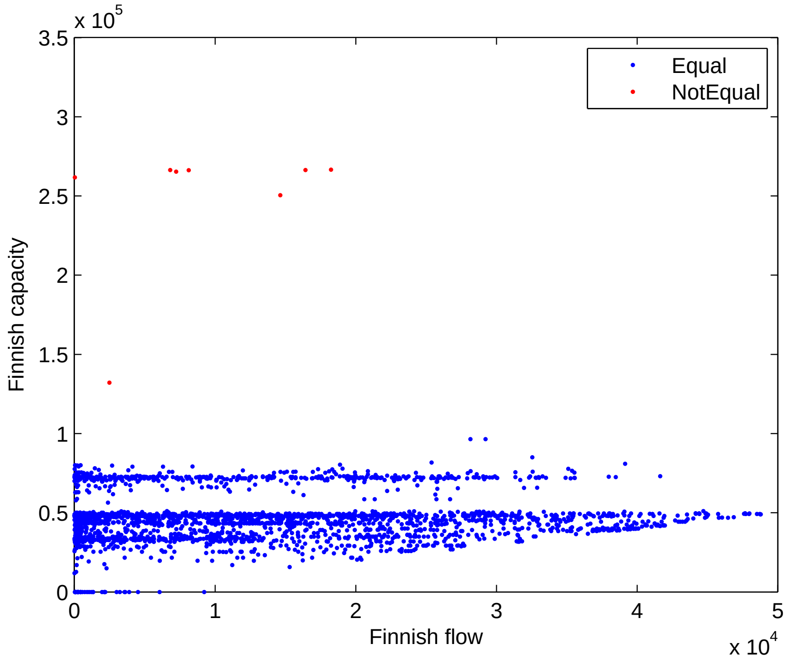

Figure 45: Scatter plot of Finnish flow and capacity: *Red* Regular regime and *Blue* Capacity-limited regime 0 per cent.

The analysis of prices correspond to the difference between Finnish flow and capacity at 90 per cent. In table (1), we have the amount of observations in different regimes for the case of 90 per cent and 80 per cent. We remind that in our study, we decided to work with 90 per cent and in this table *Less/equal to* corresponds to *Capacity-limited Regime* while *Greater than* corresponds to *Regular regime.*

Table 1: Sizes in prices.

	count
Less/equal at 90 pct	3238
Greater than 90 pct	474
Less/equal at 80 pct	2919
Greater than 80 pct	793

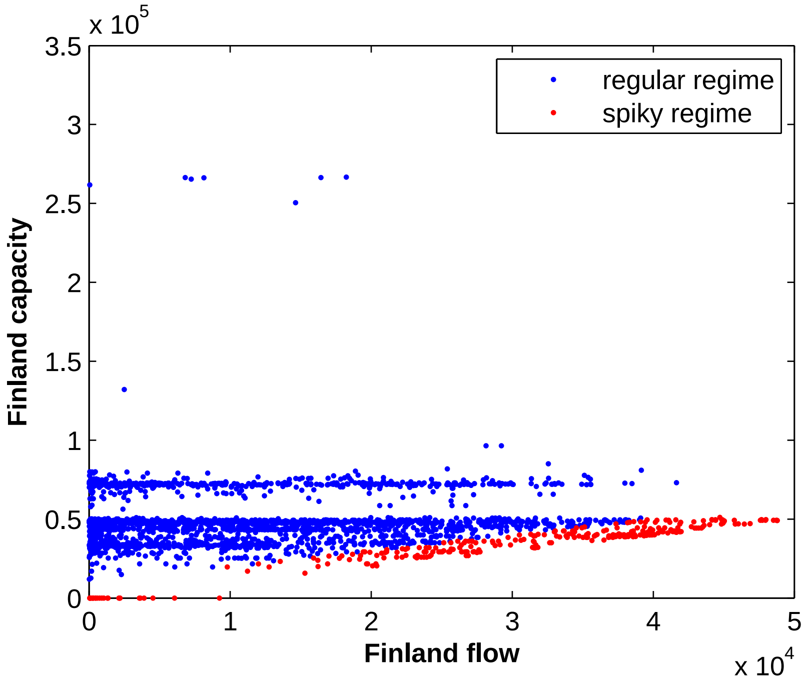

Figure 46: Flow and capacity in Finland scatter plot for regimes split by capacity-flow difference at 90 per cent.

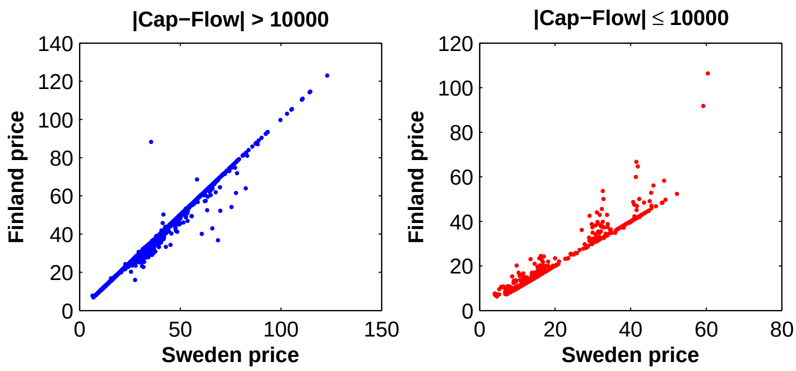

Figure 47: Price scatter plots for regimes split by capacity-flow difference.

In figure (48), a portion of Finnish prices is more spiky and look higher than Swedish, which confirms that closeness of electricity flow limit in Finland correlates with price separation from Swedish price level. It has been seen that this is 474 observations for spiky regime, 3238 observations for regular regime and in total we have 3712 at a level of 90 percent.

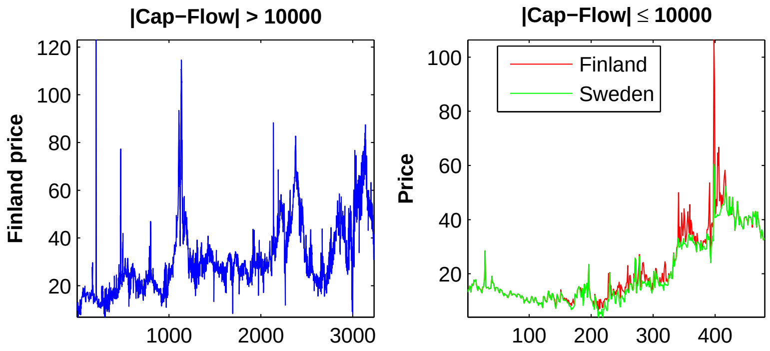

Figure 48: Prices in regimes split by capacity-flow difference.

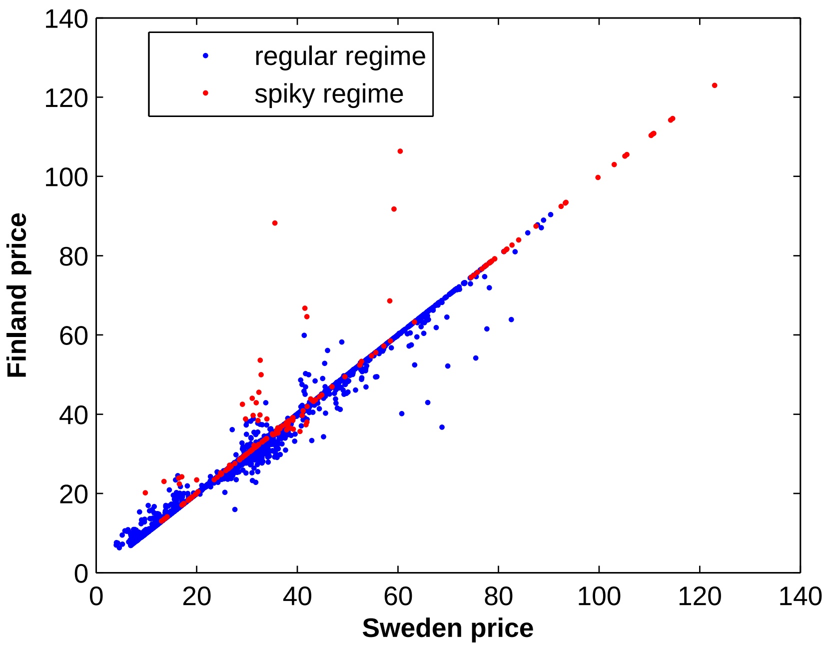

Figure 49: Prices scatter plot for regimes split by spike occurrence in Finland.

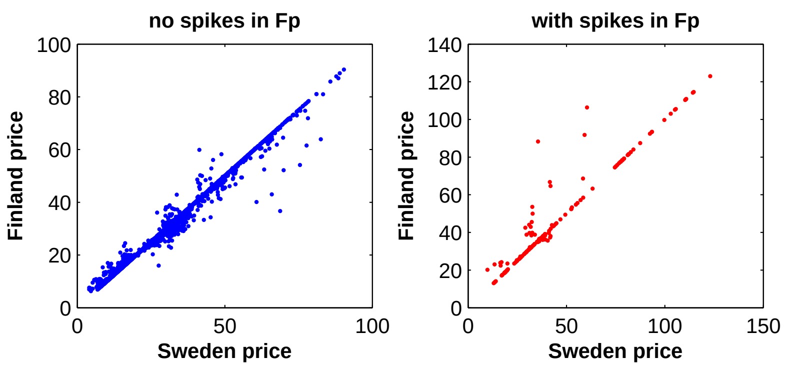

Figure 50: Price scatter plots for regimes split by spike occurrence in Finland.

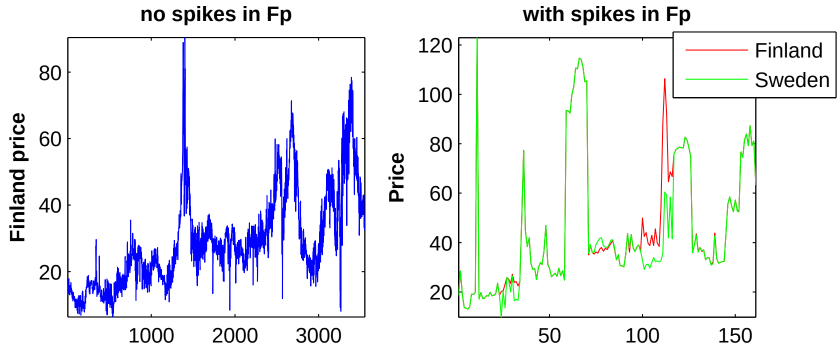

Figure 51: Prices in regimes split by spike occurrence in Finland.

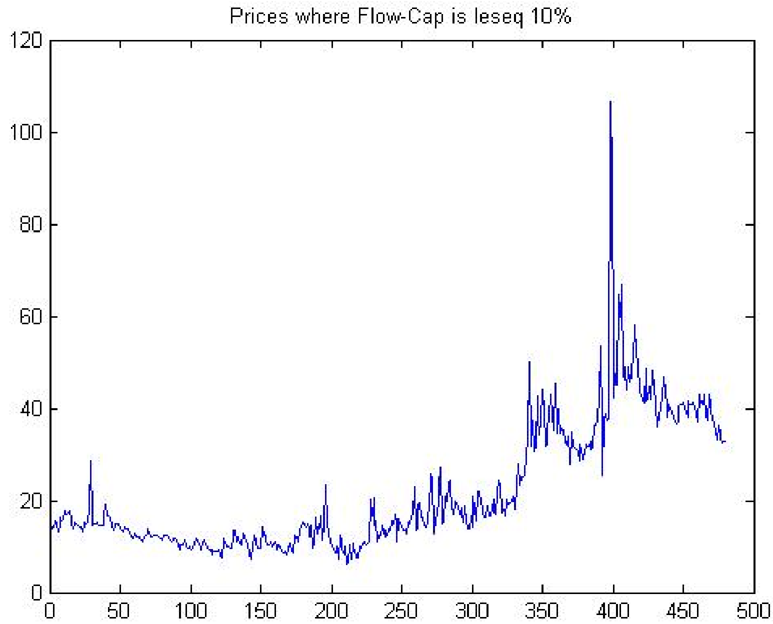

Figure 52: Finnish prices which correspond to capacity-limited regime at 90 per cent.

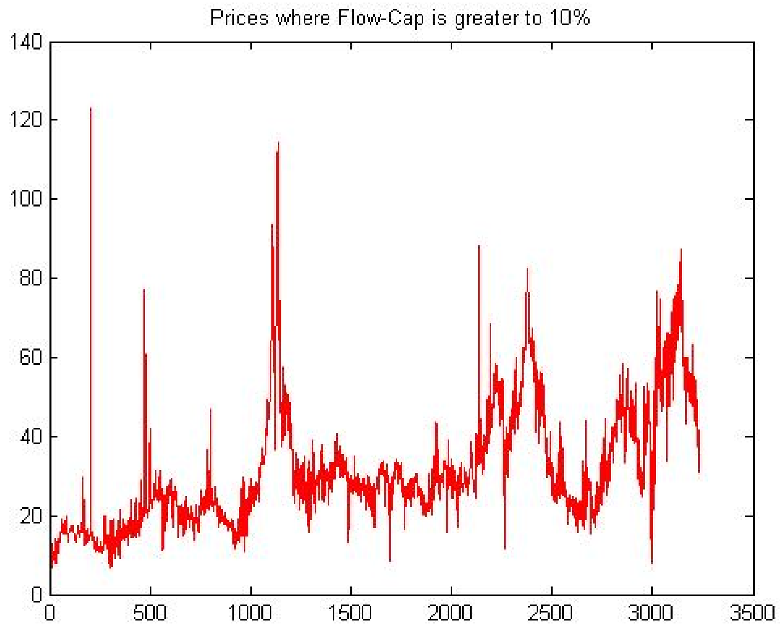

Figure 53: Finnish prices which correspond to regular regime at 90 per cent.

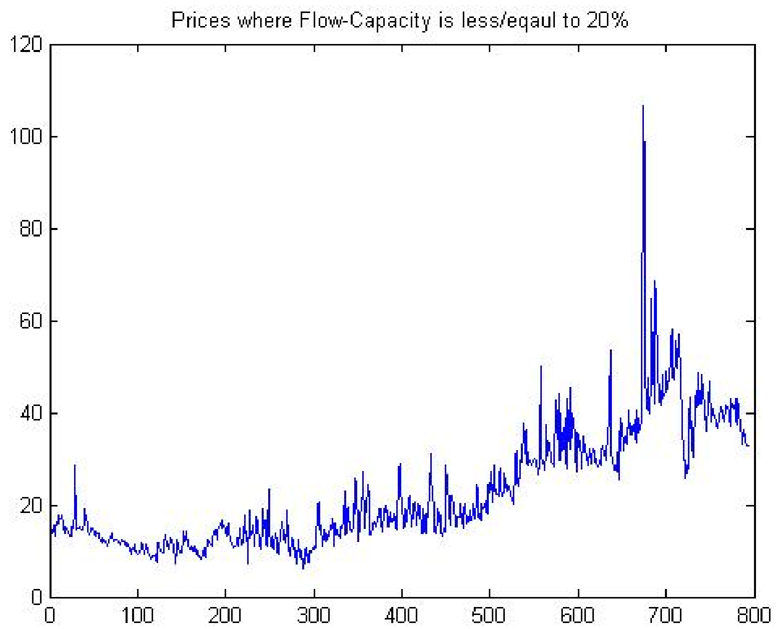

Figure 54: Finnish prices which correspond to capacity-limited regime at 80 per cent.

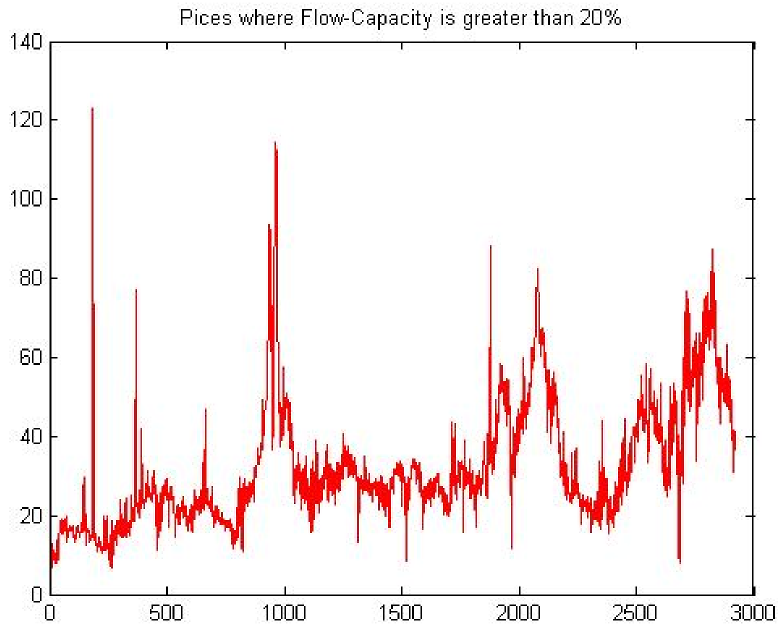

Figure 55: Finnish prices which correspond to regular regime at 80 per cent.

After realizing that ARMA(0,0)-GARCH(1,1) does not give good results ([20]), we converted series related to prices into returns series because these last are mean-stationary while prices are not.

Prices can be transformed into logarithmic returns using the following formula ([12]):

$$r_t = \ln \frac{P_t}{P_{t-1}}. \tag{26}$$

After getting the univariate returns series, the parameters of a conditional mean specification of ARMAX form are estimated ([18]), and conditional variance specification of GARCH form. The estimation process infers the residuals from the return series, and fits the model specification to the return series by maximum likelihood.

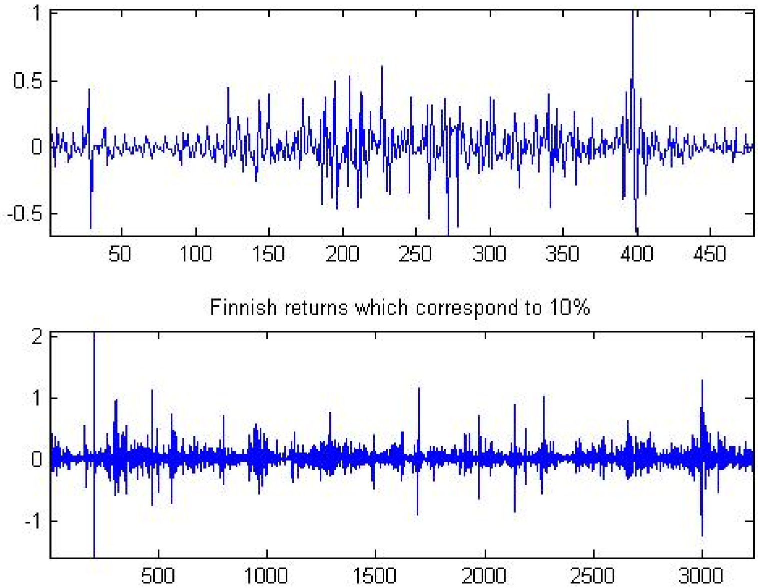

Figure 56: Finnish returns which correspond to capacity-limited and regular regimes at 90 per cent.

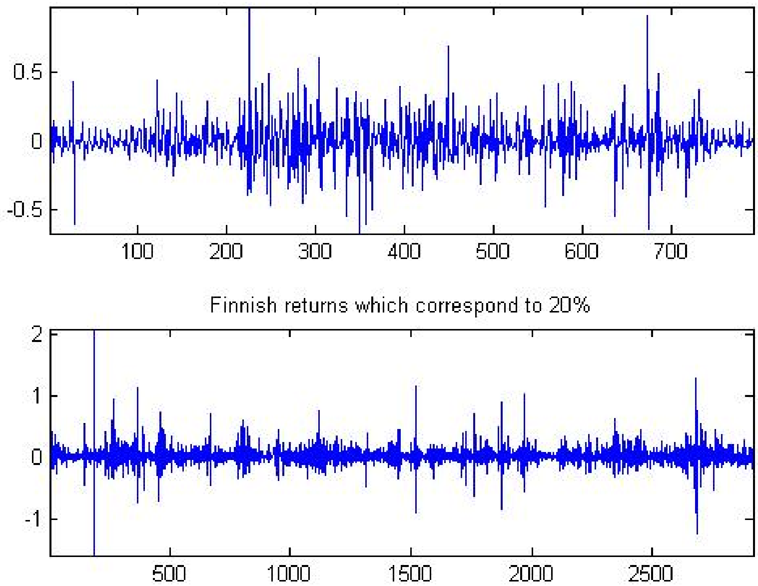

Figure 57: Finnish returns which correspond to capacity-limited and regular regimes at 80 per cent.

In figures (58) and (59) we observe how are the results after fitting GARCH(1,1) model to returns for both regimes and for the two cases: 90 per cent and 80 per cent.

We do not plot the results related to the simulated returns for the fitted model because they change every time we run our codes and every time they look totally different to the previous ones. This happens also when we change the simulated returns into prices and the reason lies in randomness of the simulation.

Our model is theoretically correct but practically there is something wrong to change, the remark is that GARCH model does not seem realistic for electricity price returns, which can be handled maybe in other further researches.

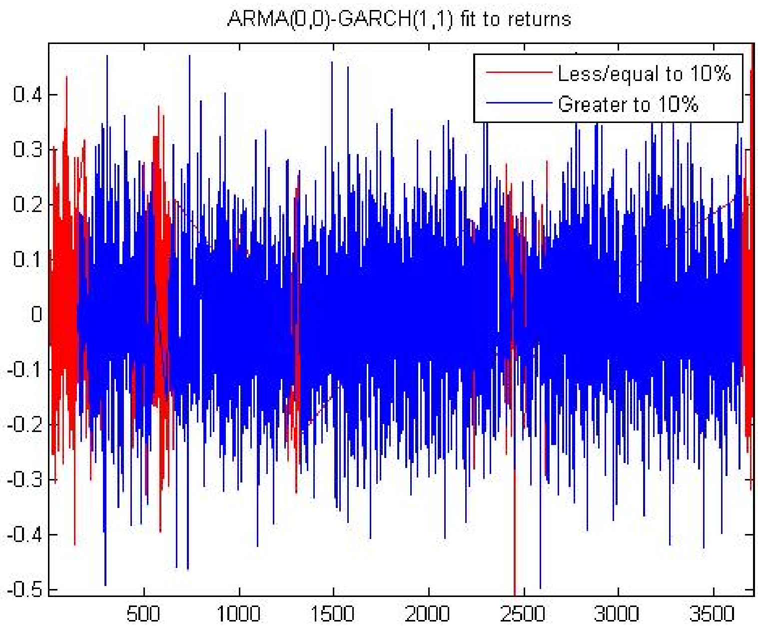

Figure 58: Finnish returns fitted to ARMA(0,0)-GARCH(1,1) at 90 per cent.

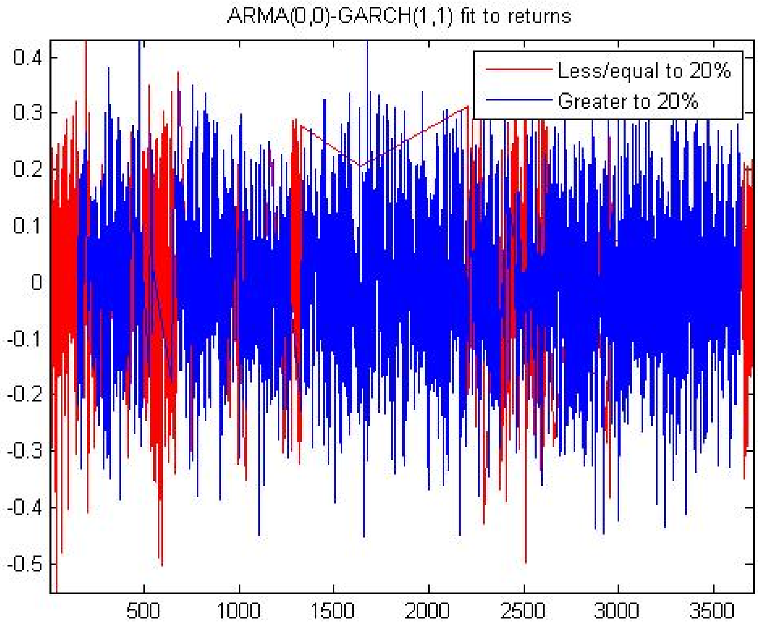

Figure 59: Finnish returns fitted to ARMA(0,0)-GARCH(1,1) at 80 per cent.

4.3 Empirical results

The following tables show some basic statistics and calculations done in our study.

Table 2: Basic statistics for Finland and Sweden electricity Capacity and Flow.

	Finnish Capacity	Swedish Capacity	Finnish Flow	Swedish Flow
count	3723	3723	3723	3723
mean	$4.5922 \cdot 10^4$	$1.7451 \cdot 10^5$	$9.2334 \cdot 10^3$	$5.2298 \cdot 10^4$
std	$2.0349 \cdot 10^4$	$49.8305 \cdot 10^4$	$1.1684 \cdot 10^4$	$3.4587 \cdot 10^4$
skewness	2.7701	1.6594	1.2056	0.5589
kurtosis	38.3507	7.4624	3.4684	2.8270
max	337630	6825484	$44.8791 \cdot 10^4$	$1.6395 \cdot 10^4$
min	0	0	0	0

Table 3: Basic statistics for Finnish and Swedish Flow-Capacity.

	Finnish F-C	Swedish F-C
count	3723	3723
mean	$-3.6689 \cdot 10^4$	$-1.2221 \cdot 10^5$
std	$2.2684 \cdot 10^4$	$9.2210 \cdot 10^4$
skewness	-2.2024	-1.7133
kurtosis	25.7467	7.4062
max	$9.2323 \cdot 10^3$	$1.1210 \cdot 10^5$
min	-337630	$-6.5922 \cdot 10^5$

Table 4: Basic statistics for Finland and Sweden electricity prices.

	Finnish Prices	Swedish Prices
count	3712	3712
mean	30.3270	30.3642
std	15.3192	15.4189
skewness	1.3409	1.2774
kurtosis	5.7001	5.4572
max	122.9892	122.9892
min	6.3017	4.0200

Depending on the results we have in table , we assume that there are two regimes $s_t = 1, s_t = 2$ where $s_t = 1$ characterizes when the process is in the first regime at time t and $s_t = 2$ characterizes when the process is in the second regime at time t. These regimes will be split from our data by taking the data from the computations of difference between Flow and capacity ($Flow - Capacity$) and for the difference close to 0 we expect congestions and possibly price change.

Table 5: Gaussian Number of Model Parameters Estimated: ARMA-GARCH.

	90 per cent		80 per cent	
	-	-	-	-
C	0.0012	0.0070	0.0013	0.0071
P	1	1	1	1
Q	1	1	1	1
K	$4.8725 \cdot 10^{-4}$	0.0040	$8.6516 \cdot 10^{-4}$	0.0041
GARCH	0.7941	0.5449	0.8266	0.5568
ARCH	0.2059	0.2805	0.1668	0.2586

4.4 Is the model statistically adequate ?

To our best knowledge, Ethier and Mount were the first to apply MRS models to electricity prices. They proposed a two state specification in which both regimes were governed by AR(1) price processes and concluded that there was strong empirical support for the existence of different means and variances in the two regimes [21].

Huisman and Mahieu proposed a regime-switching model with three possible regimes in which the initial jump regime was immediately followed by the reversing regime and then moved back to the base regime. Consequently, their model did not allow for consecutive high prices and hence did not offer any obvious advantage over jump-diffusion models [13].

In the model we have built we possess two regimes: *Regular regime* and *Spiky regime*. The probabilities to stay in one regime or to move from one regime to another can be seen in tables (6) and (7). Regarding to what is said in theory we remark that our Markov Regime Switching GARCH model satisfied what is required but surprisingly, the results obtained in the above figures, especially in figures (58) and (59) shown that there is something to change because when we converted the fitted returns to prices we obtained something totally different to the real prices. The reason lies in the fact that in real returns always after a prominent positive value there will come a negative one, similar in the absolute magnitude, to bring back the price jump (spike) to the previous level, whereas in the GARCH simulation, a number of consecutive upward movements can occur causing the prices to explode. Then they will not come back to a regular level as long as there will be no sequence of negative returns, sufficiently big enough.

5 Analysis of the model and results

5.1 Model

From the previous chapter the obtained model can be summarized and written like this:

$$y_t = \mu_1 + \epsilon_t = \mu_1 + \sigma_t u_t \tag{27}$$

$$\sigma_t^2 = \omega_1 + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2, \tag{28}$$

for the first regime and

$$y_t = \mu_2 + \epsilon_t = \mu_2 + \sigma_t u_t \tag{29}$$

$$\sigma_t^2 = \omega_2 + \beta_2 \sigma_{t-1}^2 + \alpha_2 \epsilon_{t-1}^2, \tag{30}$$

for the second regime; where $u_t \sim i.i.d.(0,1)$ and

$$p_{jt} = P(s_t = j/T_{t-1}) = p_{jt}(y_{t-1}^2), j = 1, 2. \tag{31}$$

This makes the probability function a 2 x 2 matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{32}$$

where

- p_{11} is the probability to stay in regime 1,
- p_{12} is the probability to move from regime 1 to regime 2,
- p_{21} is the probability to move from regime 2 to regime 1 and
- p_{22} is the probability to stay in regime 2.

From the two tables (6) and (7), we remark that p_{22} is the highest value for both cases which means that the probability to stay in regime two (: regular regime) is the highest and thus, by our model we can affirm that the prices do not change a lot to equality. This is because regime two represents the case where the prices are not changing for the when the difference between Finnish flow and capacity is almost zero.

Table 6: Probabilities for 10 per cent.

	p_{11}	p_{12}	p_{21}	p_{22}
Value	0.1121	0.0170	0.0167	0.8542

Table 7: Probabilities for 20 per cent.

	p_{11}	p_{12}	p_{21}	p_{22}
Value	0.1846	0.0288	0.0286	0.7580

5.2 Co-occurences

When talking about correlation of binary variables it is not as informative to use the classical Pearson correlation, as in case of continuous variables. The reason is that the classical correlation tells about linear dependance between two variables, whereas the scatter plot of two binary variables would cover only the four corners of a unit square, simply with different numbers of hits in each of the vertices. However, we do compute those numbers to get a sufficient view on the regime dependencies. The results are presented in table (8). The three groups of numbers (in different rows) come from the following observation space criteria: The first coefficient (**coef1**) is considered in space of observations when for a given pair of regimes at least one of those two is active. Results of **coef2** are found from the whole data set and, finally, **coef3** is analogical to the first one, but the space of considered time instances is now always containing days when either of the three regimes is ON.

Table 8: Classical Pearson correlation coefficients for occurrences of ON states of regimes with respect to the three criteria.

	r1-r2	r1-r3	r2-r3
coef1	-0.6937	-0.8276	-0.8691
coef2	0.0832	0.0357	0.0461
coef3	0.1798	0.1379	0.0401

We can see that for all three pairs the correlation seems to be strongly negative, rather than expectedly positive. The correlations on the whole data set are slightly positive mostly due to the many simultaneous OFF states in all regime criteria. Finally, when for each pair we consider the space indicated by all three regimes, the values though higher for cases r1-r2 and r1-r3, still remain statistically insignificant.

In figure (60), we can see how regime time instances are occurring when considering the three criteria which are:

- ***price difference criterion***,
- ***capacity/flow difference criterion*** and
- ***spikes in Finland criterion***

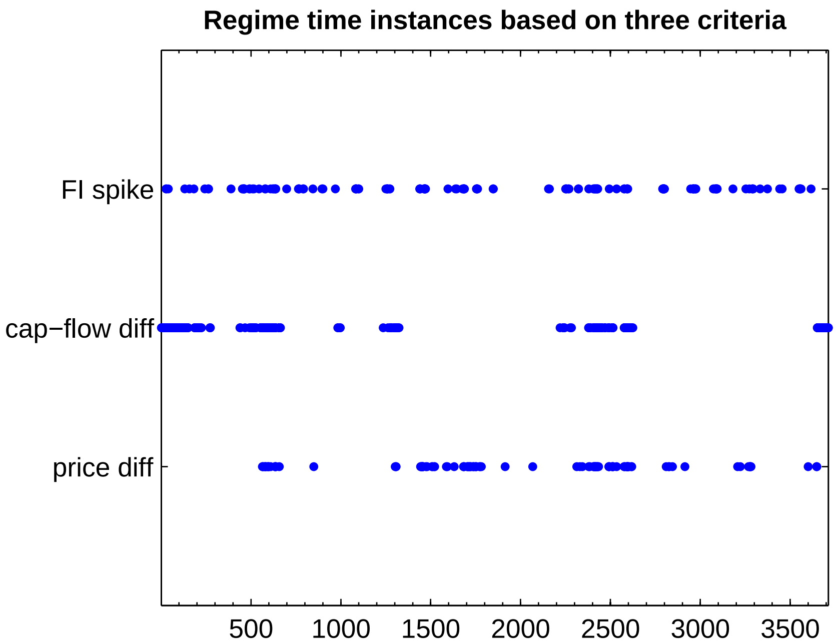

Figure 60: Time instances of regime 2 ON with respect to different criteria.

5.2.1 Co-occurence results and the meaning

As the considered regime variables are binary, we decide to measure their co-occurrence in a form of respective ratios presented below.

Notations used:

r1 – regime with price difference criterion,

r2 – regime with cap-flow difference criterion,

r3 – regime with spike-in-Finland criterion,

1 – regime takes value 1 when it's ON,

0 – regime takes value 0 when it's OFF,

N(...) – number of observations satisfying condition (...) and

N – total number of observations = 3712.

- Correlation corr1 is computed as a ratio between cases when both/all regimes are ON with respect to number of time instances when either of them is ON. So we neglect here the majority of cases when both/all regimes are OFF, but for each coefficient only those respective to regimes being analyzes at the moment
  $corr1_{r1-r2} = \frac{N(r1=1 \wedge r2=1)}{N(r1=1 \vee r2=1)}$,

$corr1_{r1-r3} = \frac{N(r1=1 \wedge r3=1)}{N(r1=1 \vee r3=1)}$,
$corr1_{r2-r3} = \frac{N(r2=1 \wedge r3=1)}{N(r2=1 \vee r3=1)}$,
$corr1_{r1-r2-r3} = \frac{N(r1=1 \wedge r2=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$

- Correlation corr2 is computed as a ratio between cases when both/all regimes are concordant with respect to the total number of observations
  $corr2_{r1-r2} = \frac{N(r1=r2)}{N}$,
  $corr2_{r1-r3} = \frac{N(r1=r3)}{N}$,
  $corr2_{r2-r3} = \frac{N(r2=r3)}{N}$,
  $corr2_{r1-r2-r3} = \frac{N(r1=r2=r3)}{N}$

- Correlation corr3 analogous to corr1, but now number of all cases
  $corr3_{r1-r2} = \frac{N(r1=1 \wedge r2=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$,
  $corr3_{r1-r3} = \frac{N(r1=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$,
  $corr3_{r2-r3} = \frac{N(r2=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$,
  $corr3_{r1-r2-r3} = \frac{N(r1=1 \wedge r2=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$

It is seen that comparison of time points when any of the 'spiky' regimes is active gives small correlations. For corr2 the results are high because of many OFF simultaneous cases for all regimes (regime 2 with price criterion had 123 ON instances, with cap-flow criterion it had 480 ON cases and with spikes in Finland there were 161 occurrences, all are out of 3712 observations, so with many simultaneous zeroes we get high corr2 values).

In the table (9), it is remarkable as suspected that, it turns out that visually the two conditions for two regimes seem correlated (id est price separation versus capacity-flow) when we compare the corresponding time series for Finland and Sweden prices. But their correlation coefficient, even when correctly normalized, remains at 0.10.

Table 9: Co-occurrences which correspond to the three criteria.

	$r_1 - r_2$	$r_1 - r_3$	$r_2 - r_3$	$r_1 - r_2 - r_3$
corr1	0.1024	0.0923	0.0508	0.0297
corr2	0.8677	0.9364	0.8440	0.8241
corr3	0.0832	0.0357	0.0461	0.0297

Co-occurrences between either of these and spikes are mildly negative visually, but it is true that price separation occurs at relatively high average price level. To see the correct co-occurrence, vision alone cannot ne trusted, because data sets are of quite different sizes. And co-occurrence coefficients between either criterion and spikes remains below 0.1.

5.3 Lorenz' butterfly

The concept of the butterfly effect is frequently referred to in popular culture in terms of the novelty of a minor change in circumstances causing a large change in outcome [26].

The butterfly emerges from the Lorenz' equations [23], a system of three non-linear ordinary differential equations. The phase space of this system possesses two wings as it is shown in figure (61) that represent two different regimes of behavior. Between the wings, there is a transaction zone in which regime switch may take place. The topology at this transition zone is fractal and thus it is very different to predict when a regime may occur. However, regime-switches never occur outside the transition zone. It seems prices in electricity market display analogous behavior defining regular regime and non-regular regimes.

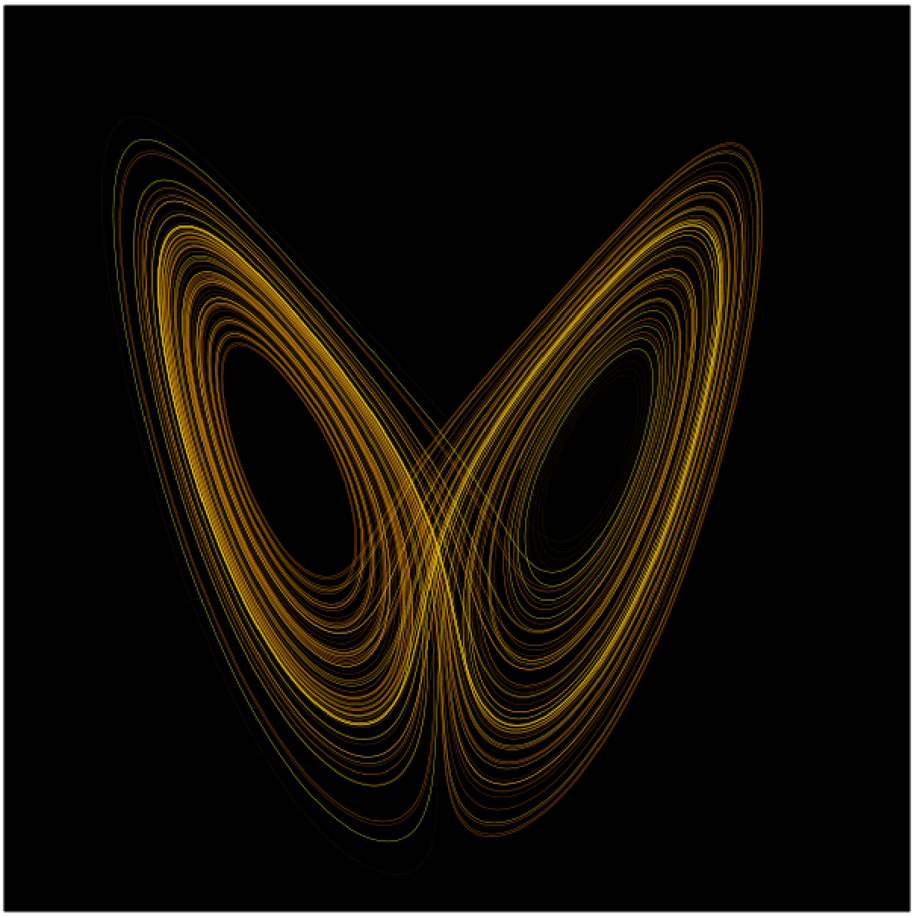

Figure 61: Plot of the trajectory Lorenz system for values $\rho = 28$, $\sigma = 10$, $\beta = \frac{8}{3}$

6 Conclusion

In this thesis, the main problem was to try to find a model which can explain the structural break behavior of electricity prices. The used data are Nord Pool daily observations from January 1, 1999 to March 11, 2009 and the variables are prices, flow and transmission capacity for only Finland and Sweden.

Two regimes *Regular regime* and *Non-regular regime* have been considered through three criteria: *price separation* , *spikes in Finland separation* and *capacity-flow*. Based on these three conditions for the two regimes, two conditions: *price separation* and *capacity-flow* seem correlated when we compare the corresponding time series for Finland and Sweden prices; but their correlation coefficient, even when correctly normalized, remains at 0.10.

Prices have been converted into returns because prices are not stationary and the results related on time instances based on the three conditions used show that the difference capacity/flow presents more clearly the different two regimes than the other two criteria.

The results related to the simulated returns for the fitted model are not plotted because they change every time we run our codes and every time we run they look totally different to the previous ones. This happens also when a change is done for the simulated returns into prices. Theoretically, the built Markov Regime Switching GARCH model is verified but the remark is that GARCH does not seem the best to use for these kind of data and the recommendation is to use any other for further researches. The probabilities to stay in one regime or to move from one regime to another show that p_{22} is the highest. The conclusion to do here is that it is more probable to stay in regular regime when being there than in non-regular regime or moving from one regime to the other.

7 APPENDICES

7.1 Probability distribution

A probability distribution is a table or an equation that links each outcome of a statistical experiment with its probability of occurrence. In other words, a probability distribution identifies either the probability of each value of an unidentified random variable and this is when the variable is discrete, or the probability of the value falling within a particular interval for the case where the variable is continuous. The probability distribution describes the range of possible values that a random variable can attain and the probability that the value of the random variable is within any *measurable* subset of that range [25].

7.2 Black-Scholes model

A mathematical model of the market for an equity, in which the equity's price is a stochastic process.

The Black-Scholes model consists of 2 assets with dynamics:

$$dB(t) = rB(t) \cdot dt$$

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW_t$$

where r, α and σ are deterministic constants.

Assumptions:

- Risk-free asset (bond) B_t with constant continuously compound interest rate r.
- Risky asset, the stock S_t governed by GBM.
- Trading takes place continuously.
- Short-selling of the assets allowed.
- Positivity, Divisibility, Liquidity.
- No transaction costs (No friction).
- No arbitrage opportunity.
- The stock does not pay a dividend.
- Returns from the security follow a Log-normal distribution.
- Options use the European exercise terms.

References

[1] Ardia D. , (July 2007) ; *Bayesian Estimation of the Markov-Switching GARCH(1,1) Model with Student-t Innovations*, Department of Quantitative Economics University of Fribourg, Switzerland.

[2] Bauwens L. et al. ; (July 17, 2006)*Regime Switching GARCH Models*; JEL Classification:C11, C22,C52.

[3] Brooks, C. (2002) ; *Introductory econometrics for finance*; Cambridge University Press, United Kingdom.

[4] Cochrane John H. , (January 2005) ; *Time Series for Macroeconomics and Finance*;Graduate School of Business; University of Chicago.

[5] Enders, W. (2004) ; *Applied Econometric Time Series*, second edition. Wiley,United States

[6] Evarest E., (May 2008) ; *Princing of energy by mean of stochastic model: Dissertiation for Master of Science*;University od Dar es Salam.

[7] Granger, C, W, J. (2001) ; *Forecasting Economic Time Series*, 2nd edition. Academic Press, INC, United States.

[8] Greene W. H. , (2003) ; *Econometric Analysis; 5th Edition*; New York University; Prentice Hall.

[9] Haario H. ; *Statistical Analysis in Modelling: MCMC Methods; Lecture material*; Lappeenranta University of Technology; Lappenranta 2008.

[10] Hamilton James D. , (May 18 2005) ; *Regime-Switching Models*; Department of Economics, University of California.

[11] Him T. et al. (January 2005) ; *Finite Mixture of ARMA-GARCH Model for Stock Price Prediction*; The Chinese University of Hong Kong.

[12] Jabłońska, M (2008) ; *Analysis of outliers in electricity spot prices with example of New England and New Zealand markets* . Masters Thesis , Lappeenranta University of Technology.

[13] Janczura J. and Weron R. (April 2009) ; *Regime-switching models for electricity spot prices: Introducing heteroskedastic base regime dynamics and shifted spike distributions*; Hugo Steinhaus Center, Wroclaw University of Technology.

[14] Johannes M. and Polson N. , (December 22, 2003) ; *MCMC Methods for Continuous-Time Financial Econometrics.*

[15] Kennedy P. , (2003) ; *A Guide to Econometrics*; 5th Edition.

[16] Mahera W. C. ; *Stochastic Differential Equations: Lecture Material*; Lappeenranta University of technology; Lappeenranta 2009.

[17] Marcucci J. , (March 2005)*Forecasting Stock market Volatility with R-S GARCH models*; Department of Economics, University of California.

[18] Marcucci J. , (December 5, 2001) ; *Regime-Switching GARCH in the analysis and forecasting of stock-market volatility and the effects on option evaluation*; Department of Economics, University of California; JEL Classification: C22, C52, C53.

[19] Niels H. and Morten N. (2004) ; *A regime switching long memory model for electricity prices*; University of Aarhus-Danmark.

[20] Tsay, Ruey S. (2002) ; *Analysis of Financial Time Series,*; Wiley-Interscience, John Wiley and Sons, Inc.

[21] Weron R. , (May 09, 2008) ; *Heavy-tails and regime-switching in electricity prices*; Hugo Steinhaus Center, Wroclaw University of Technology.

[22] Yang Zijian (June 28, 2004);*Estimation of Markov Regime-Switching Model- CCFEA PROJECT*; Supervisor: Dr. Sheri Markose.

[23] http://bcev.nfrance.com/Lorenz/equations.htm

[24] http://www.lpc.lt/repository/skelbimai/Seminaras/ (*NPS company presentation* 2009 *January* 2*.pdf*)

[25] http://www.nordpool.com/en/

[26] http://www.wikipedia.org