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**ANALYZING HYDRAULIC HEAD OF FLUID FLOW
IN UNDERGROUND POROUS MEDIUM**

Examiners: Professor Jari Hämäläinen
Professor Matti Heiliö

ABSTRACT

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Keywords: Hydraulic head, Fluid flow, Porous medium, Underground, Richard's equation.

Hydraulic head is distributed through a medium with porous aspect. The analysis of hydraulic head from one point to another is used by the Richard's equation. This equation is equivalent to the groundwater flow equation that predicts the volumetric water contents.

COMSOL 3.5 is used for computation applying Richard's equation. A rectangle of 100 meters of length and 10 meters of large (depth) with 0,1 m/s flux of inlet as source of our fluid is simulated. The domain have Richards' equation model in two dimension (2D). Hydraulic head increases proportional with moisture content.

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May my wife Valentine Mukundwa, my little angel Kundwa Ishimwe Ella Divine, my father Valens Fayida, Sister Aline Uwababyeyi, relatives and friends receive here the expression of my gratitude for what they did for me during my training. To these and to many others whose kindness has touched the depth of my soul, I warmly express my gratitude.

Lappeenranta, December, 2013.

Germain Ishimwe

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1 INTRODUCTION

The passage of fluid from one distance to another seems interesting specially through a medium with porous aspect. In this paper, we discuss the fluid flow underground porous analyzing its hydraulic head distribution using Richard's equation according to a specific time. Natural ground fluid like ground water contains some impurities, even if it is unaffected by human activities. The types and concentrations of natural impurities depend on the nature of the geological material through which the fluid (groundwater) moves and the quality of the recharge water. Fluid (Groundwater) moving through sedimentary rocks and soils may pick up a wide range of compounds such as magnesium, calcium, and chlorides. Some aquifers have high natural concentration of dissolved constituents such as arsenic, boron, and selenium. The effect of these natural sources of contamination on groundwater quality depends on the type of contaminant and its concentrations. Apart for those natural impurities, there are others artificial ones caused by the human activities. At or near the land surface: municipal waste landspreading, salt for de-icing streets, streets and parking lots chemicals: storage and spills, fuels: storage and spills, mine tailing piles, chemical spills, fertilizers, livestock waste storage facilities and landspreading, pesticides, fertilizers, homes, cleaners, detergents, motor oil, paints, pesticides or below the land surface (landfills, leaky sewer lines, underground storage, tanks, wells: poorly constructed or abandoned, septic systems, wells: poorly, constructed or abandoned). For this study we analyze the fluids coming from the waste disposal or industrial discharge.

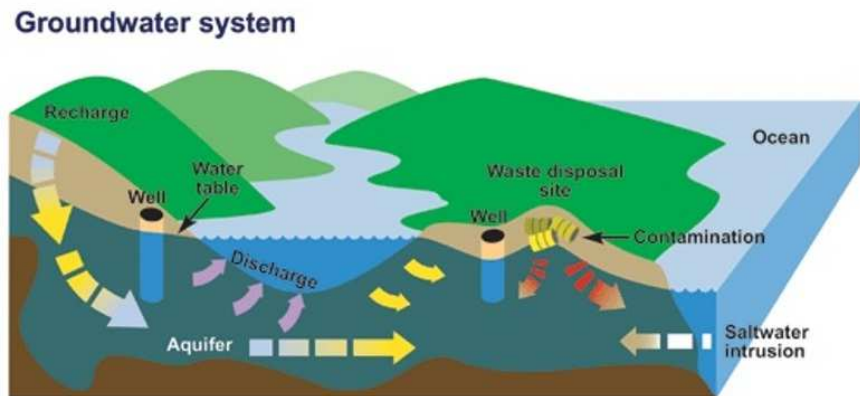


Figure 1: Groundwater system

The theory of ground water was previously discussed by Fowler (1997) where several mathematical equations were applied to describe the seepage, consolidation, solute dispersivity and heterogeneous porous media. However, the hydraulic head distribution underground was not clarified and it is interesting especially in the field where there are different layouts with different soil properties. We will analyze the fluid transport theories and fluid flow underground water. The mathematical model equations implementation with Comsol software will allow us to analyze hydraulic head underground from a point A on the ground and a point B underground after T time and L distance. The software COMSOL 3.5a will be used in our computation.

The Richards' equation is the most often used model. This equation is equivalent to the groundwater flow equation, which is in terms of hydraulic head (h), by substituting $h = \psi + z$; where h is the hydraulic head, ψ is the pressure head and z is the elevation, and changing the storage mechanism to dewatering. The reason for writing it in the form above is for convenience with boundary conditions (often expressed in terms of pressure head, for example atmospheric conditions are $\psi = 0$). It has been introduced by Richards (1931) who has suggested that the Darcy's law originally devised for saturated flow in porous media is also applicable to unsaturated flow in porous media. For experiments on water transport in soil horizontal columns, Richards' equation predicts that volumetric water contents should depend solely on the ratio (distance)/(time)^q where $q = 0.5$. Substantial experimental evidence shows that value of q is significantly less than 0.5 in some cases. Donald Nielsen and colleagues in 1962 related values of $q < 0.5$ to 'jerky movements' of the wetting front, i.e. occurrences of rare large movements. The physical model of such transport is the transport of particles being randomly trapped and having a power law distribution of waiting periods. The corresponding mathematical model is a generalized Richards' equation in which the derivative of water content on time is a fractional one with the order equal or less than one.

The structure of this study is organized as follows: In Chapter 1 we give general introduction to the underground water, its pollution and the mathematical equations used by previous researcher to describe fluids flow in porous media. Mathematical background on fluids flow and porous medium is presented in Chapter 2, even the software description COMSOL 3.5a will be given in this chapter. In Chapter 3, we have methodology, discuss and formulate our particular problem specifying the limits, boundary conditions and properties. Computation, simulations and different scenarios will be presented in the 4th chapter. We will conclude and propose some recommendation in the last chapter, Chapter 5.

1.1 OBJECTIVES

The objectives of this study are to analyze, evaluate by Richard's equation the hydraulic head of the underground and its transmission from a point A of

porous ground to a point B according the time.

For this work, we create the rectangle of 100 meters of length and 10 meters of large (depth) with 0,1 m of inlet as source of our fluid. The domain will have Richards' equation model in two dimension (2D)with properties of sand. We compute and analyze the model with three non homogeneous levels. We consider the fluid as homogeneous element with exact fixed properties.The density of used fluid is $1000kg/m^3$ (water).

2 LITERATURE REVIEWS

The fluid is flowing underground what we qualified as the porous media and that is why in the following chapter we will discuss the properties of the medium and the mathematical equations describing the area and the fluids in general in the prescribed ground in general. These mathematical descriptions will allow us to determine the aspect of that fluid being infiltrated and flowing underground.

2.1 The description of fluid flow through a porous medium

The fluid in the porous medium obey the Darcy's law. Darcy's law is a simple proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given distance.

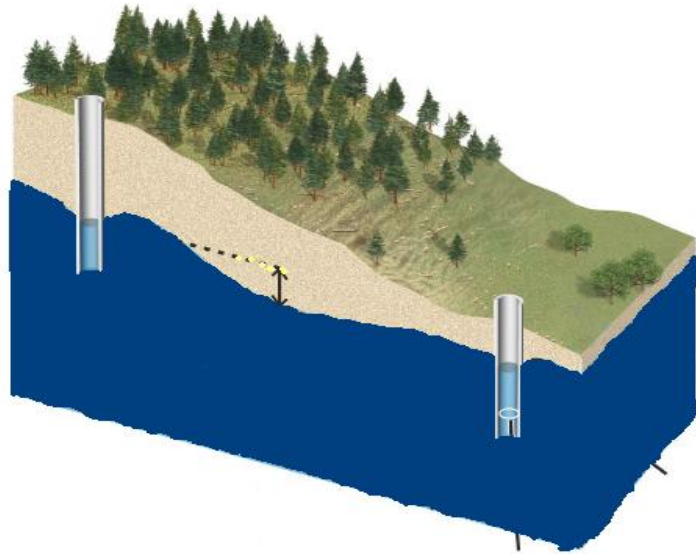


Figure 2: Ground and underground aspect

The rate of flow (= volume of water passing per unit time), Q

$$Q = KA \frac{h_1 - h_2}{L} \quad (1)$$

is proportional to the cross-sectional area, A , of the column, proportional to the difference in water level elevations, h_1 and h_2 , in the inflow and outflow

reservoirs of the column, respectively, and inversely proportional to the column length, L , where K is a coefficient of proportionality called hydraulic conductivity.

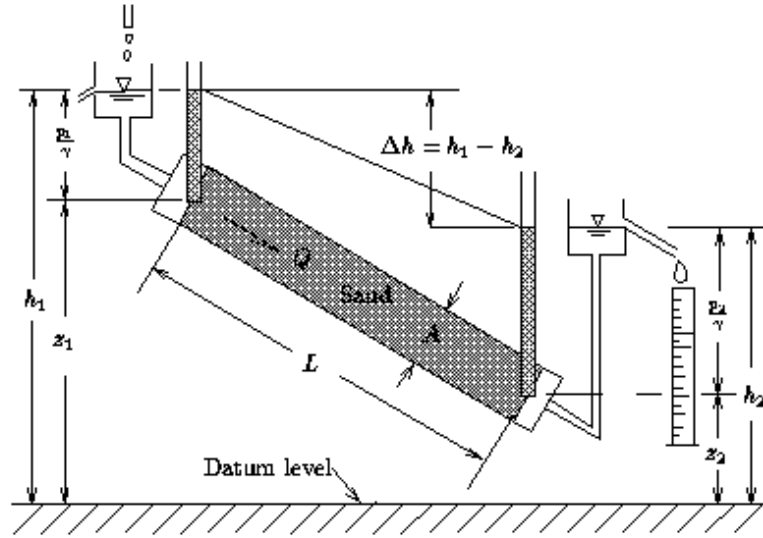


Figure 3: Darcy's experiment

In this experiment, h (hydraulic head or head)

$$h = Z + \frac{P}{\rho g} \quad (2)$$

where z is the elevation, p and ρ are the fluid's pressure and mass density, respectively, and g is the gravity acceleration.

2.1.1 Hydraulic head

Hydraulic head is a measurement of the amount of energy available in ground-water due to pressures in a water table or the height of the water level in the ground. Ground water flow occurs because of the difference in energy of the water from one point to another. Ground water flows from a point of higher energy to a point of lower energy. The energy of water at a particular point in the ground water system consists of potential energy, elastic energy and kinetic energy. The kinetic energy can be ignored in most cases, however, because the ground water flow velocity is typically very low; kinetic energy is usually considered negligible compared to the potential and elastic energy. The permeability of the unsaturated zone varies with moisture content, hydraulic head increases proportionately with moisture content.

2.2 A real porosity

Soils are made of particles of different types and sizes. The space between particles is called pore space that determines the amount of water that a given volume of soil can hold. Porosity refers to how many pores, or holes, a soil has. Porosity is the open space in a rock divided by the total rock volume (solid + space or holes). Mathematically, Porosity is normally expressed as a percentage of the total rock which is taken up by pore space.

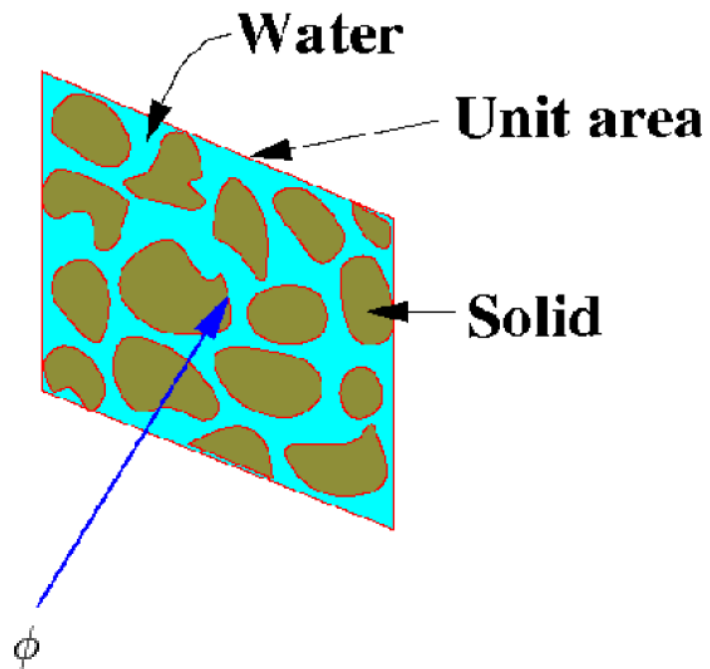


Figure 4: Area porosity

$$\phi = \frac{V_v}{V} \quad (3)$$

where

ϕ =porosity (percent)

V_v volume of voids

V the total volume

Soil porosity values range from 0 to 1. Soils with a high bulk density have low total porosity because empty pores do not have any mass

2.2.1 Unsaturated Flow through Pavements

The unsaturated zone is located above the water table. In this zone, the pore spaces are usually only partially filled with water, the reminder of the voids are taken up by air. Therefore, the volumetric water content is lower than the soil porosity. Due to the fact that water in this zone is held in the soil pores under surface-tension forces, negative pressures or suction pressures are developed. Both the volumetric water content and the hydraulic conductivity are, in soil porosity, also function of this suction pressure. The soil volumetric water content is held between the soil grains under surface-tension forces that are reflected in the radius of curvature of each meniscus. The higher the volumetric water content, the larger the radii of curvature and the lower the tension heads. The hydraulic conductivity is not constant because of the change in volumetric water content. The hydraulic conductivity content increases with increasing the volumetric water content. (Freeze and Cherry, 1979).

2.3 Mathematical equations for the fluids flow underground water

2.3.1 Derivation of Richards' equation

Barari et al.(2009) describe the derivation of Richards' equation in the journal "*Hydrology and Earth System Sciences Discussions*". Darcy's law and the continuity equation are given by

$$q = -K \frac{\partial H}{\partial z} = -K \frac{\partial (h + z)}{\partial z} = -K \left(\frac{\partial h}{\partial z} + 1 \right) \quad (4)$$

and

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial z} \quad (5)$$

Where K is hydraulic conductivity, H is head equivalent of hydraulic potential, q is flux density and t is time. The mixed form of Richards' equation is obtained by substituting equation 4 in equation 5

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[K \left(\frac{\partial h}{\partial z} + 1 \right) \right] \quad (6)$$

Equation 6 has two independent variables: the soil water content(θ) and pore water pressure head (h). Obtaining solutions to this equation therefore requires constitutive relations to describe the interdependence among pressure, saturation and hydraulic conductivity. However, it is possible to eliminate either θ or h by adopting the concept of differential water capacity, defined as

the derivative of the soil water retention curve:

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left(K \frac{\partial h}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (7)$$

This equation is used for modeling flow of water through unsaturated soils. Introducing "pore water diffusivity" D defined as the ration of the hydraulic conductivity and the differential water capacity, we obtain the θ - based form of Richards' equation. D can be written as:

$$D = \frac{K}{C} = \frac{K}{\frac{\partial \theta}{\partial h}} = K \frac{\partial h}{\partial \theta} \quad (8)$$

For this Equation 8, D and K are highly dependent on water content. Combining equation 8 with equation.6 . We get Richards' equation as:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D \frac{\partial \theta}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (9)$$

In order to solve equation.9 , we must first properly address the task of estimating D and K , both of which are dependent on water content. Several models have been suggested for determining these parameters. The Van Genuchten model (Van Genuchten, 1980) and Brooks and Corey's model (Brooks and Corey, 1964, Corey, 1994) are the more commonly used models. The Van Genuchten model uses mathematical relations to relate soil water pressure head with water content and unsaturated hydraulic conductivity, through a concept called "relative saturation rate". This model matches experimental data but its functional form is rather complicated and it is therefore difficult to implement it in most solution schemes. Brooks and Corey's model on the other hand has a more precise definition and is therefore adopted in the present research. This model uses the following relations to define hydraulic conductivity and water diffusivity:

$$D(\theta) = \frac{K_s}{\alpha \lambda (\theta_s - \theta_r)} \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{2 + \frac{1}{\lambda}} \quad (10)$$

$$K(\theta) = K_s \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{3 + \frac{2}{\lambda}} \quad (11)$$

where K_s is saturated conductivity, θ_r is residual water content, θ_s is saturated water content and α and λ are experimentally determined parameters. Brooks and Corey determined λ as pore-size distribution index (Brooks and Corey, 1964). A soil with uniform pore-size possesses a large λ while a soil with varying pore-size has small λ value. Theoretically, the former can reach infinity and the latter can tend towards zero. Further manipulation of Brooks and Corey's model yields the following equations (Witelski, 1997; Corey, 1986; Witelski, 2005):

$$D(\theta) = D_0 (n + 1) \theta^m \quad m \geq 0 \quad (12)$$

$$K(\theta) = K_0 \theta^k \quad k \geq 1 \quad (13)$$

where K_0 , D_0 and k are constants representing soil properties such as pore-size distribution, particle size, etc. In this representation of D and K , θ is scaled between 0 and 1 and diffusivity is normalized so that for all values of m , $\int D(\theta)d\theta = 1$ (after Nasser et al., 2008). Several analytical and numerical solutions to Richards' equation exist based on Brooks and Corey's representation of D and K , replacing $n=0$ and $k=2$ in equation 12 and 13 and yields the classic Burgers' equation extensively studied by many researchers (Basha, 2002; Broadbridge and Rogers, 1990; Whitman, 1974). The generalized Burgers' equation is also obtained for general values of k and m (Grundy, 1983). As seen previously, the two independent variables in equation (7) are time and depth. By applying the traveling wave technique (Wazwaz, 2005; Abdoul et al., 2008; Elwakil et al., 2004), instead of time and depth, a new variable which is a linear combination of them is found. Tangent-hyperbolic function is commonly applied to solve these transform equations (Soliman, 2006; Abdou, and Soliman, 2006). Therefore the general form of Burgers' equation in order of $(n, 1)$ is obtained as (Wazwaz, 2005):

$$\theta_t + \alpha \theta^n \theta_z - \theta_{zz} = 0 \quad (14)$$

It's exact solution is

$$\theta(z, t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2} \tanh((A_1(z - A_2 t))) \right)^{\frac{1}{n}} \quad (15)$$

$$A_1 = \frac{-\alpha n + n|\alpha|}{4(1+n)} \gamma \quad (n \neq 0) \quad (16)$$

$$A_2 = \frac{\gamma \alpha}{1+n} \quad (17)$$

2.3.2 Richards' equation with Green's function

Richard's equation was studied by D. Crevoisier (2006) applying the Green's function in his paper "*Analytical approach predicting water bidirectional transfers: application to micro and furrow irrigation*". Water transfers are submitted to Richards' equation considering the following domain:

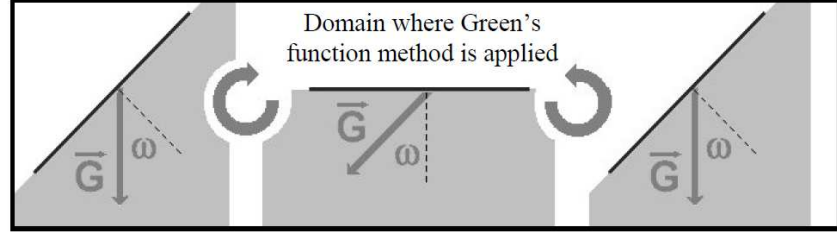


Figure 5: Domain where Green's function method is applied

with ω the angle between gravity force and vertical axis. The Richards's equation is written

$$\nabla (k \nabla (h - \sin(\omega) x - \cos(\omega) z)) = \partial_t \theta + S \quad (18)$$

where k is the hydraulic conductivity ($cm.h^{-1}$), h the pressure head (cm), θ the water content ($cm^3 cm^{-3}$), z the vertical coordinate taken positive downward (cm) and S a sink or source term, usually the plant uptake (h^{-1}). This equation is highly non-linear and its writing has to be simplified to allow its resolution using Green's function. The following three equations allow the linearization of Richards' equation by applying the Kirchhoff transformation defined in equation 19 and by choosing θ and K relationships suited to the problem, respectively linear soil model defined in equation 20 and used by Warrick and Gardner model defined in equation 21.

$$\phi(h) = \int_{-\infty}^h k(h) dh \quad (19)$$

$$\theta(h) = \theta_r + \frac{k(h)}{K} \quad \text{with} \quad K = \frac{k_s}{(\theta_s - \theta_r)} \quad (20)$$

$$k(h) = k_s e^{\alpha h} \quad (21)$$

where k_s is the saturated hydraulic conductivity ($cm.h^{-1}$), α the inverse of the capillary length (cm^{-1}), θ_r and θ_s , the retention and saturated water content ($cm^3 cm^{-3}$). ϕ is the flux potential ($cm^2.h^{-1}$). The resulted linear PDE

is then submitted to two transformations. First, dimensionless variables are introduced and a function change is used. The Richards' equation become:

$$\partial_T \Psi = \Delta \Psi \quad (22)$$

with the following dimensionless variables

$$\frac{X}{x} = \frac{Z}{z} = \frac{\alpha}{2}; \quad \frac{T}{t} = \frac{\alpha k}{4}; \quad \frac{\Phi}{\phi} = \frac{\alpha}{K_s} \quad (23)$$

and the following function change

$$\Psi = e^{-X \sin \omega - Z \cos \omega + T} \Phi \quad (24)$$

The initial condition is

$$[\Psi]_{T=0} = e^{-X \sin \omega - Z \cos \omega} \Phi_i \quad (25)$$

Green's function method gives analytical solutions to PDE with complex boundary conditions. It involves multiplying the initial PDE by the Green's function G and integrating the result. The use of Green's function is fully developed by Greenberg [5]. This function $G(X_s, Z_s, T_s)$ is the solution to the initial PDE submitted to an infinite pulse at the point (X_s, Z_s) and time T_s as the initial condition. Green's function depends on the type of boundary conditions considered in the PDE but is, in both cases, the linear combination of functions $G_{1D}(X, X_s, T, T_s)G_{1D}(Z, Z_s, T, T_s)$ defined in equation 26

$$G_{1D}(U, U_s, T, T_s) = \frac{1}{\sqrt{4\pi(T - T_s)}} e^{\frac{(U - U_s)^2}{4(T - T_s)}} \quad (26)$$

Thanks to the Green's function, the solution of the PDE considered in the eqn(22) can be analytically written

$$\Psi = \int_0^\infty \int_{-\infty}^\infty [G\Psi]_{T_s=0} dX_s dZ_s + \int_0^T \int_{-\infty}^\infty [\Psi \partial_{z_s} G - G \partial_{z_s} \Psi]_{z_s=0} dX_s dT_s \quad (27)$$

where the first integral accounts for the initial condition and the second for the boundary condition at the soil surface.

2.4 SOFTWARE DESCRIPTION: COMSOL Multiphysics

3.5a

COMSOL Multiphysics is a powerful interactive environment for modeling and solving all kinds of scientific and engineering problems based on partial differential equations (PDEs). You can easily extend conventional models for

one type of physics into multiphysics models that solve coupled physics phenomena and do so simultaneously. Using the application modes in COMSOL Multiphysics, you can perform various types of analysis (stationary and time-dependent analysis, linear and nonlinear analysis, eigenfrequency and modal analysis). The software runs the finite element analysis together with adaptive meshing and error control using a variety of numerical solvers. COMSOL Multiphysics is used in many application areas: acoustics, bioscience, chemical reactions, diffusion, electromagnetics, fluid dynamics, fuel cells and electrochemistry, geophysics, heat transfer, microelectromechanical systems (MEMS), microwave engineering, Optics, photonics, porous media flow, quantum mechanics, radio-frequency components, semiconductor devices, structural mechanics, transport phenomena, wave propagation, etc. The COMSOL 3.5a product family includes the following modules:

- AC/DC Module
- fluids:Acoustics Module
- Chemical Engineering Module
- Earth Science Module
- Heat Transfer Module
- MEMS Module
- RF Module
- Structural Mechanics Module

With this work, we apply "Earth Science Module".

2.4.1 Earth Science Module

The earth and planets are giant laboratories that involve all manner of physics. The Earth Science Module combines application modes for fundamental processes and links to COMSOL Multiphysics and the other modules for structural mechanics and electromagnetics analyses. New physics represented include heating from radiogenic decay that produces the geotherm, which is the increase in background temperature with depth. The variably saturated flow

application modes analyze unsaturated zone processes (important to environmentalists) and two-phase flow (of particular interest in the petroleum industry as well as steam-liquid systems). Important in earth sciences, the heat transfer and chemical transport application modes explicitly account for physics in the liquid, solid, and gas phases. Available application modes are:

- Darcy's law for hydraulic head, pressure head, and pressure. Also part of a predefined interface for poroelasticity (requires the Structural Mechanics Module or the MEMS Module).
- Solute transport in saturated and variably saturated porous media
- Richards' equation including nonlinear material properties using van Genuchten, Brooks and Corey, or user-defined parameters.
- Heat transfer by conduction and convection in porous media with one mobile fluid, one immobile fluid, and up to five solids
- livestock waste storage facilities and landspreading
- Brinkman equations
- Incompressible Navier-Stokes equations

3 METHODOLOGY AND COMPUTATION

3.1 GEOMETRY

We create our geometry using COMSOL software in space dimension of 2D with 100 meters of length and 10 meters of depth, after we make three layout of different depth R1, R2, R3. We position 2 point (P1: 30;0 and P2: 30,1;0). This figure is considered as a portion of soil on a hill where the fluid will be flowing.

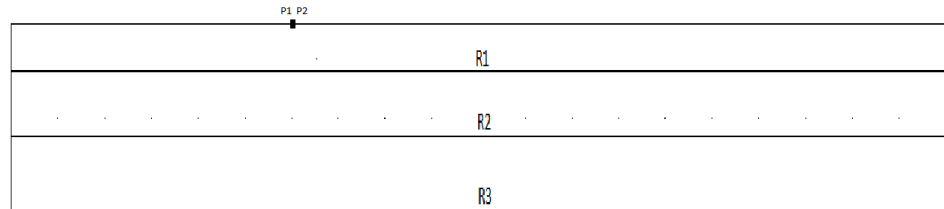


Figure 6: Geometry

Let us fix our boundary properties;

Our fluid is coming continuously at the position between our fixed 2 point (P1: 30;0 and P2: 30,1;0) what is our inlet (boundary 8). The fluid is free to move out (outlet) from left and right side of our figure (boundaries 1,2,3,5,10,11 and 12) as it is shown on Figure 7. The bottom of our Figure (boundary 2) is considered as the impermeable rock, that means that the porous properties do not allow the passage of our fluid. At the top (boundaries 7 and 9), there is no other entrance except our fixed Inlet. Next paragraph we fix physics subdomain and boundary settings related with our Richard's equation.

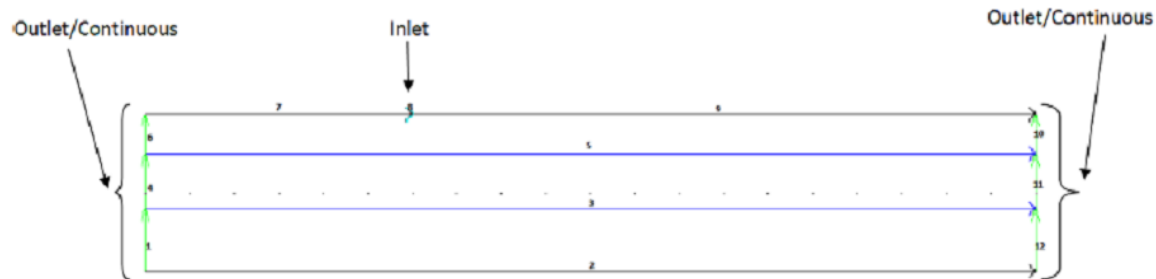


Figure 7: Geometry with boundary

3.2 Application Mode: Richards' Equation

3.2.1 Domain equation

The Richard's equation is applied to our model for each level as following:

$$\frac{\partial}{\partial t}(\rho\varepsilon_p) + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (28)$$

$$\varepsilon_p = p\left(\frac{C_m}{\rho g} + S_e S\right) \quad (29)$$

$$\frac{\partial}{\partial t}(\rho\varepsilon_p) = \rho\left(\frac{C_m}{\rho g} + S_e S\right)\frac{\partial p}{\partial t} \quad (30)$$

and

$$\mathbf{u} = -\frac{k}{\mu}(\nabla p + \rho g \nabla D), \text{ with } k = k_s k_r(\varepsilon_p) \quad (31)$$

Where

t is time (s)

ρ is the density (kg/m^3)

Q_m is Liquid source (1/s)

C_m is Specific moisture capacity (1/Pa)

g is Gravity (m/s^2)

S_e is Effective saturation(1)

S is Storage (1/m)

p is pressure (Pa)

μ is Dynamic viscosity (Pa.s)

D is Elevation (m)

k_s is Saturated hydraulic conductivity (m/s)

k_r is Relative permeability (1)

We resume the settings and variables for each layout in the following tables

Richards' Equation Model 1

Figure 8: first level: domain 1

Description	Value
Density	User defined
Density	1
Permeability model	Hydraulic conductivity
Saturated hydraulic conductivity	{{0.294, 0, 0}, {0, 0.294, 0}, {0, 0, 0.294}}
Retention model	Brooks and Corey
Saturated liquid volume fraction	0.46
Residual liquid volume fraction	0.01
Constitutive relation constant	2
Constitutive relation constant	1
Storage	$21/(esvr.g*esvr.rho)$
Storage	User defined

Table 1: equation settings domain 1

Richards’ Equation Model 2

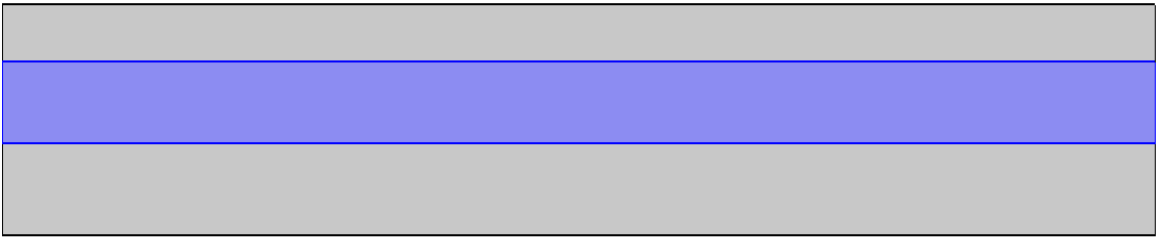


Figure 9: second level: domain 2

Description	Value
Density	User defined
Density	1
Permeability model	Hydraulic conductivity
Saturated hydraulic conductivity	{{0.0294, 0, 0}, {0, 0.0294, 0}, {0, 0, 0.0294}}
Retention model	Brooks and Corey
Saturated liquid volume fraction	0.46
Residual liquid volume fraction	0.05
Constitutive relation constant	2
Constitutive relation constant	1
Storage	$24/(esvr.g*esvr.rho)$
Storage	User defined

Table 2: equation settings domain 2

Richards’ Equation Model 3

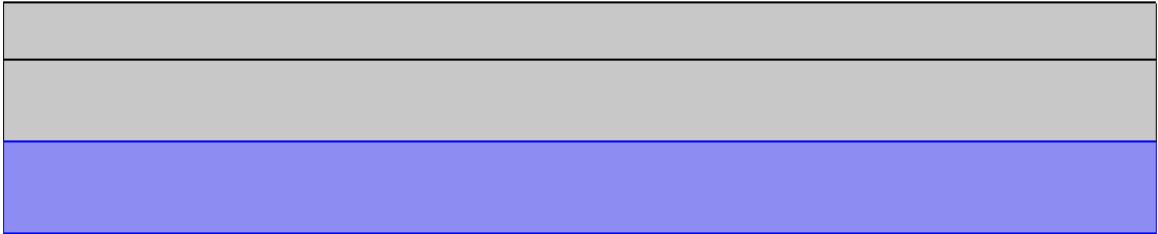


Figure 10: third level: domain 3

Description	Value
Density	User defined
Density	1
Permeability model	Hydraulic conductivity
Saturated hydraulic conductivity	{{0.00294, 0, 0}, {0, 0.00294, 0}, {0, 0, 0.00294}}
Retention model	Brooks and Corey
Saturated liquid volume fraction	0.46
Residual liquid volume fraction	0.1
Constitutive relation constant	2
Constitutive relation constant	1
Storage	28/(esvr.g*esvr.rho)
Storage	User defined

Table 3: equation settings domain 3

3.2.2 Boundary equation

We have three boundaries aspect:

No flow This is corresponding to the boundaries 2,7 and 9, they are called *Zero flux/Symmetry*. For those boundaries hydraulic head (H_0) = 0, pressure head (H_{p_0}) = 0, pressure (P_0) = 0, inward flux (N_0) = 0, external head (H_b) = 0, external pressure (P_b) = 0, external conductance (R_b) = 0 and elevation (D_b) = 0.

$$-\mathbf{n} \cdot \rho \mathbf{u} = 0 \quad (32)$$

Hydraulic Head

$$p = \rho g(H_0 - D) \quad (33)$$

The boundaries 1,3,5,10,11 and 12 correspond to our outlet, this means that our fluid is free to move out, at initial stage hydraulic head (H_0) = 0, pressure head (H_{p_0}) = 0, pressure (P_0) = 0, inward flux (N_0) = 0, external head

$(H_b) = 0$, external pressure $(P_b) = 0$, external conductance $(R_b) = 0$ and elevation $(D_b) = 0$.

Mass Flux

$$-\mathbf{n} \cdot \rho \mathbf{u} = N_0 \quad (34)$$

On our Inlet at the boundary 8 there is *Flux discontinuity*. Hydraulic head $(H_0) = 0$, pressure head $(Hp_0) = 0$, pressure $(P_0) = 0$, inward flux $(N_0) = 0.1$, external head $(H_b) = 0$, external pressure $(P_b) = 0$, external conductance $(R_b) = 0$ and elevation $(D_b) = 0$.

Boundary		2, 7, 9	1, 3, 5, 10,11,12	8
Type		Zero flux/Symmetry	Hydraulic head	Flux discontinuity
name				
Hydraulic head (H_0)	m	0	0	0
Pressure head (Hp_0)	m	0	0	0
Pressure (p_0)	Pa	0	0	0
Inward flux (N_0)	m/s	0	0	0.1
External head (H_b)	m	0	0	0
External head (H_{pb})	m	0	0	0
External pressure (p_b)	Pa	0	0	0
External conductance (R_b)	1	0	0	0
External elevation (D_b)	m	0	0	0

Table 4: Boundary settings

Number of degrees of freedom	12329
Number of mesh points	3157
Number of elements	6016
Triangular	6016
Quadrilateral	0
Number of boundary elements	532
Number of vertex elements	10
Minimum element quality	0.594
Element area ratio	0

Table 5: Mesh statistics

Next step we generate mesh for our graph

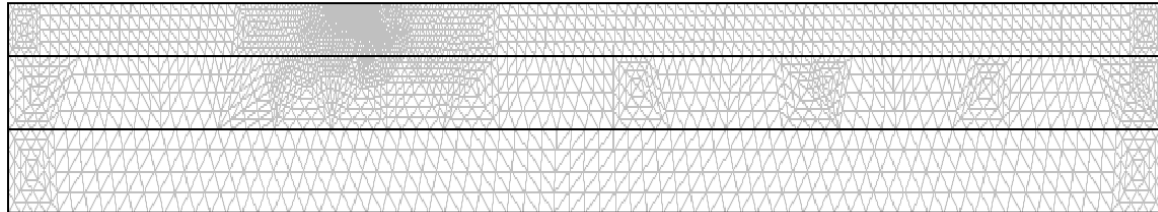


Figure 11: Mesh

We use time dependent for our model in order to analyze hydraulic head according to time t . Our time range are $0; 86400; 86400000$, where $86400s$ correspond to **one day**. Let us look how the results come out in the following chapter.

4 RESULT AND DISCUSSION

The hydraulic head coming from the inlet with $0.1m$ is distributed its self in our simulated medium trying to reach output infiltrating the porous soil. The figures below show us by iso-contours the value of hydraulic head at any place of the medium and at specific time. We realize that the hydraulic head is increasing with time because there is a constant continuity from inlet. There is a small deviation from one level to another because of differences of their properties.

The hydraulic head is high at a the point close to the inlet and it decreases as long as receding from the inlet.

Ten hours

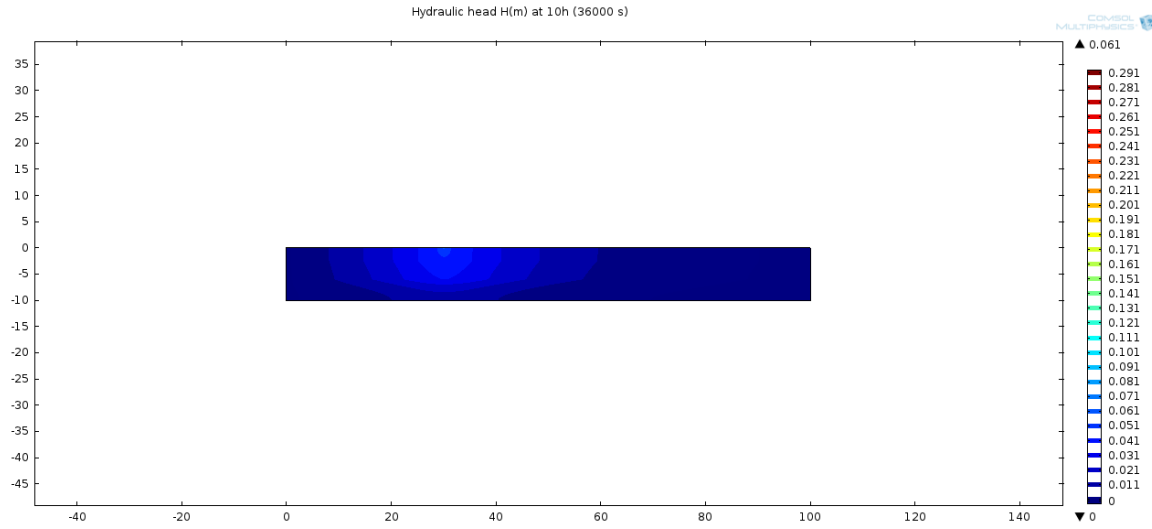


Figure 12: After ten hours

The figure above shows us the aspect after ten hours the fluid from inlet started constantly to flow in.

One day

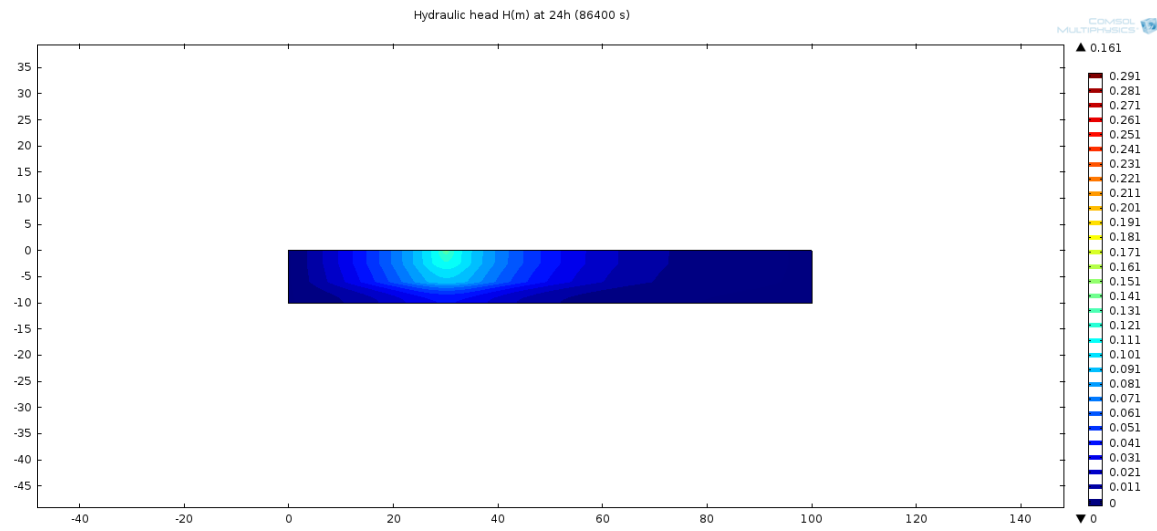


Figure 13: After one day

One week

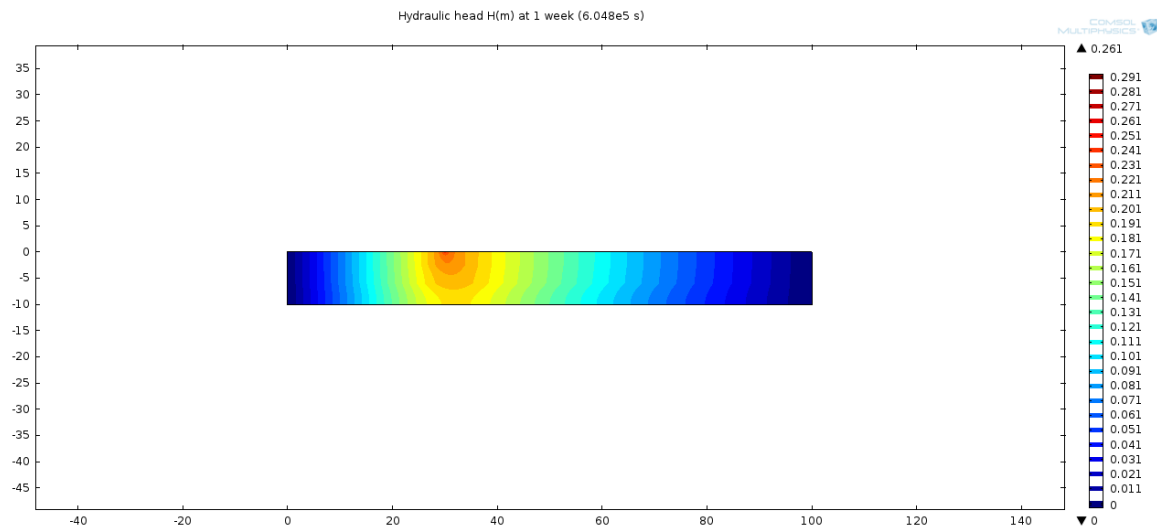


Figure 14: After one week

The shape of our contours is getting modified, as long as the time increase, trying to become the simple lines even if these three media are different physical.

One month

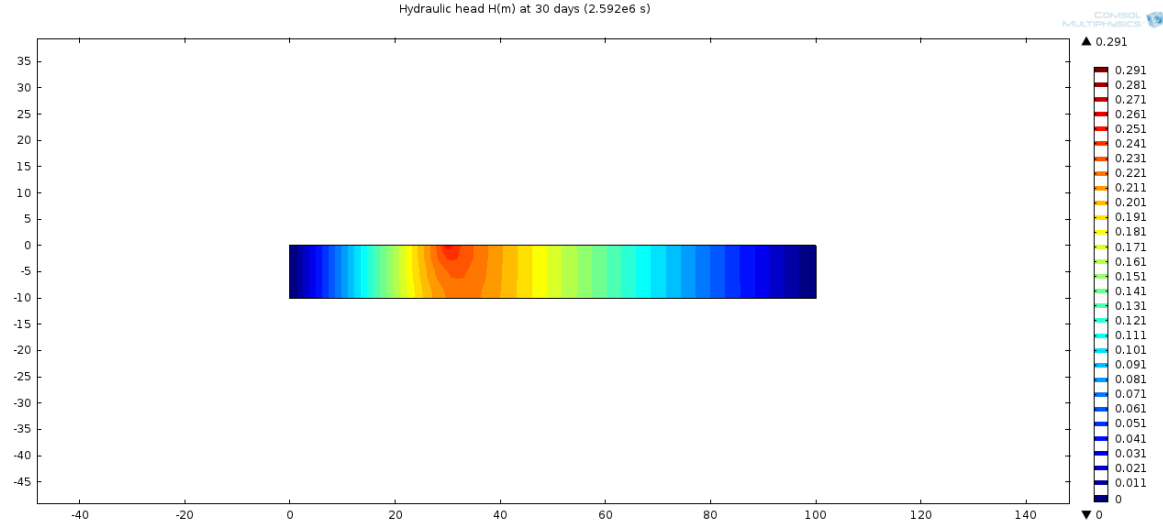


Figure 15: After one month

Let analyze the whole section plotting on the same figure the hydraulic head for $t_1 = 86400s = \text{one day}$, $t_2 = 6.048e5s = \text{seven days} = \text{one week}$ and $t_3 = 2.592e6s = \text{thirty days} = \text{one month}$.

Cross-section A

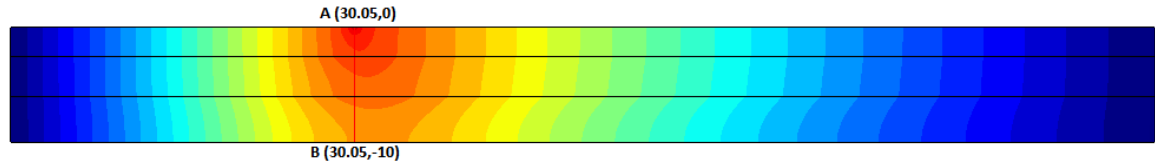


Figure 16: Cross section at A section (30.05,0;30.05,-10)

This section is situated at the perpendicular of our inlet section (30.03)

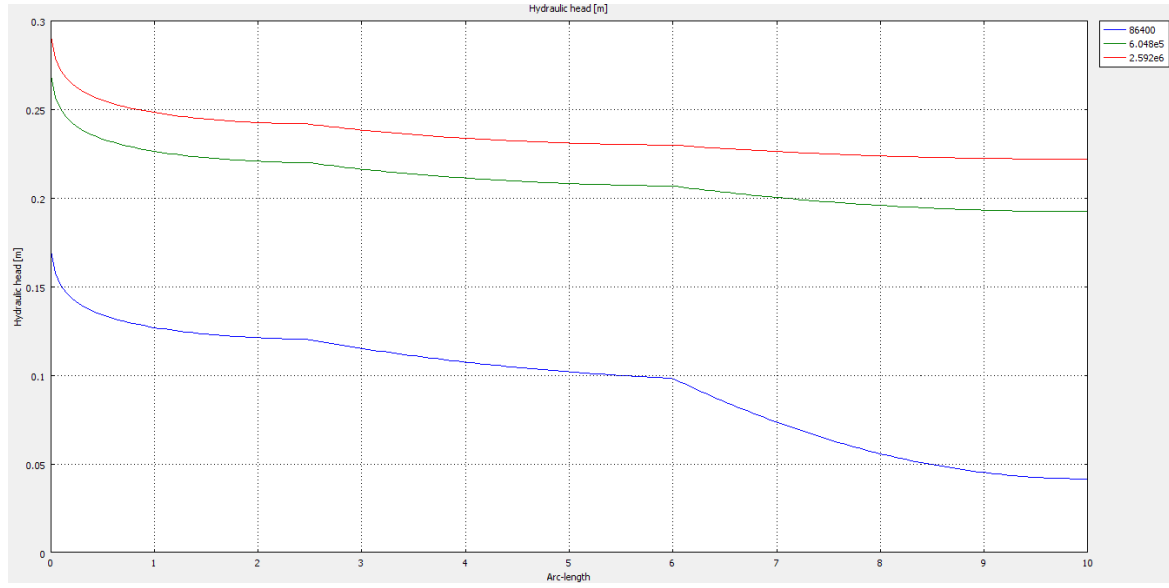


Figure 17: Hydraulic Head at point A (30.05;0;30.05,-10)

It is not easy to realise the passage from one level to another at t_3 while at t_1 there is change in inclination of our curve. This is due to that according to our inlet, after one month of the process the hydraulic head is trying to become identic in all levels.

Let us look how the process appear at another section situated at (80,0;80,-10).

Cross-section B

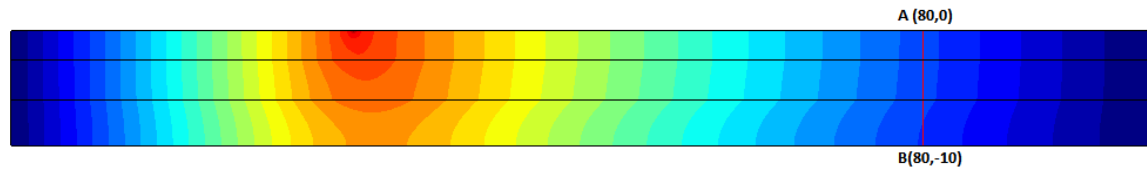


Figure 18: Cross section at B section (80,0;80,-10)

At the section above, after one month, the hydraulic head is identic in all three levels even if its value is less than the one at the inlet section ($0.07 < 0.28$). The big difference of hydraulic head between those two sections depends on the difference of distance.

We see that, on the figure below, after one day and one week, there is a small change in inclination of our curve.

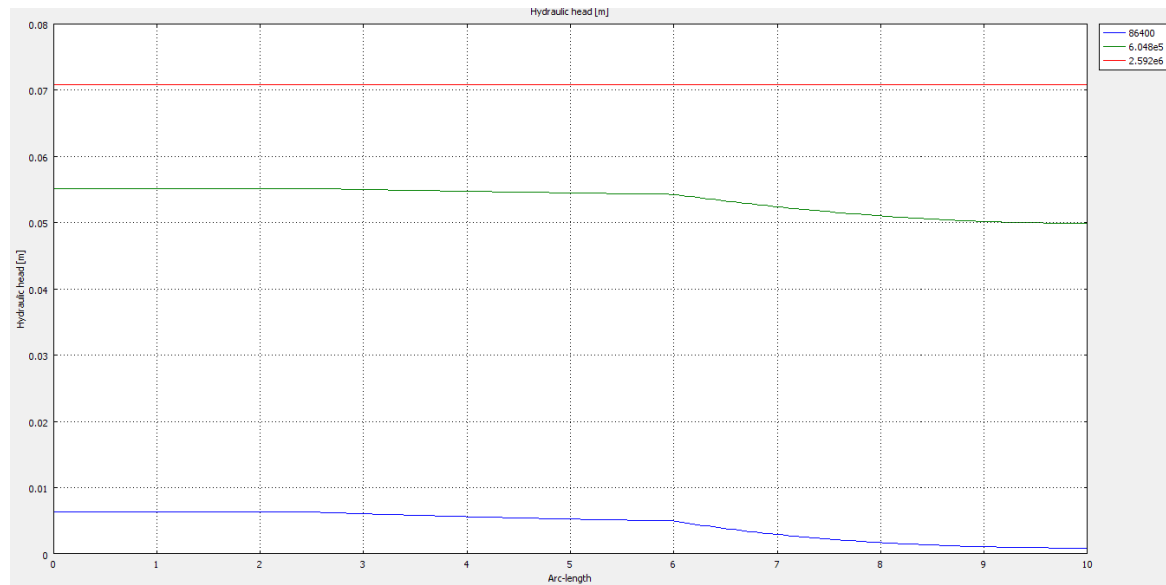


Figure 19: Hydraulic Head at point B (80,0;80,-10)

The curiosity allows us to look how hydraulic head is being distributed from a point A (30,05) at the top of inlet and a point B (80,-10) at the bottom of section B

Cross-section A-B

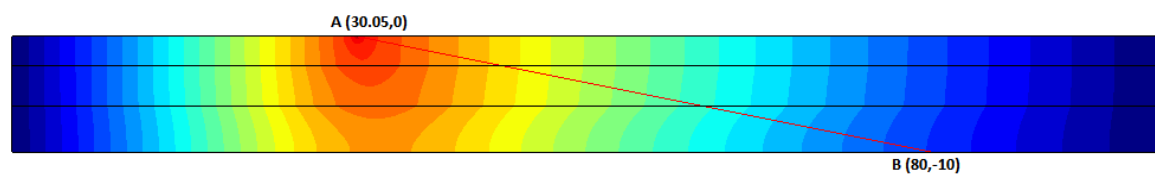


Figure 20: Cross section between the points A and B (30,05;80,-10)

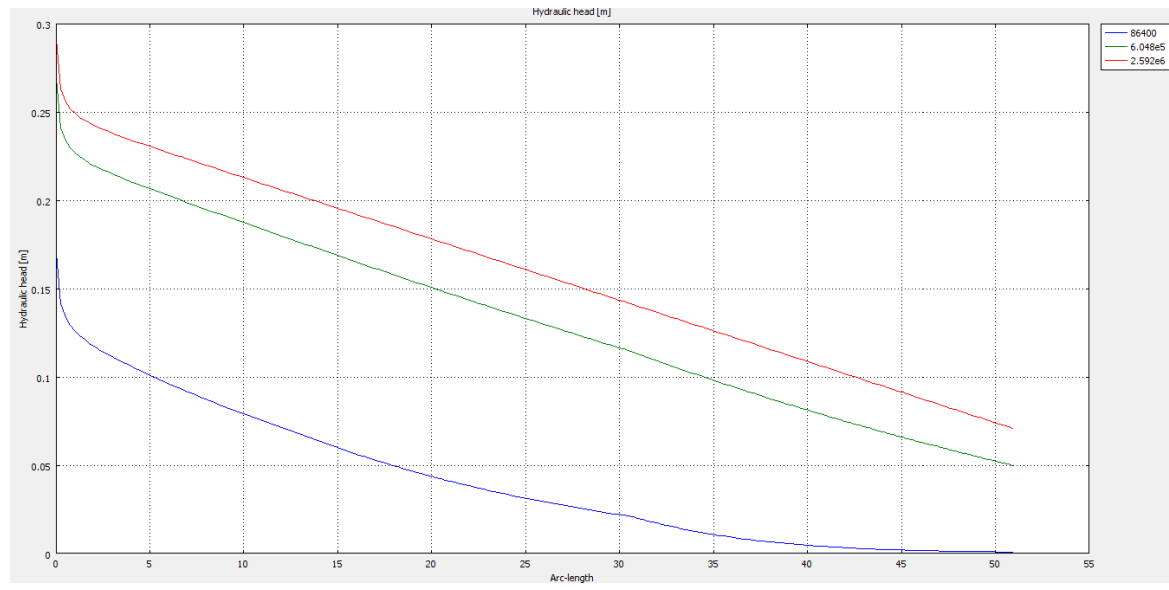


Figure 21: Hydraulic Head between the points A and B (30,05;80,-10)

5 CONCLUSION AND RECOMMENDATION

The results from COMSOL software applying Richard' equation allowed us to analyze the hydraulic head for underground porous medium, the medium created consisted of three levels with different physical properties. Our liquid was flowing from inlet passing through porous property from one level to another for reaching the outlet. Hydraulic head was increasing proportional with moisture content.

The hydraulic head become identic with small value of hydraulic head for all three level at B cross section after t_3 , while at A cross section perpendicular to our inlet the hydraulic head is high but not identic.

I recommend several studies and computations in such domain using three dimension (3D) and considering fluid as composed mixed material. The values of soil property should be tested and specific to the area of study. The study of randomly inlet is interesting in normal life case.

I recommend that The idea of fluid heterogeneity and theory of transported harmful material through non regular shape might be discussed in the futures topics and researches.

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