LAPPEENRANTA UNIVERSITY OF TECHNOLOGY
School of Engineering Science
Computational Engineering

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Modeling a Network of Heat Exchangers

Examiners: Associate Prof. Matti Heiliö
Prof. Heikki Haario

ABSTRACT

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ABSTRACT

This research work addresses the problem of building a mathematical model for the given system of heat exchangers and to determine the temperatures, pressures and velocities at the intermediate positions. Such model could be used in finding an optimal design for such a superstructure. To limit the size and computing time a reduced network model was used. The method can be generalized to larger network structures. A mathematical model which includes a system of non-linear equations has been built and solved according to the Newton-Raphson algorithm. The results obtained by the proposed mathematical model were compared with the results obtained by the Paterson,s approximation and Chen's Approximation. Results of this research work in collaboration with a current ongoing research at the department will optimize the valve positions and hence, minimize the pumping cost and maximize the heat transfer of the system of heat exchangers.

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Lappeenranta, May 14, 2015.

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Alphebatical Conventions

A	Heat transfer area	$[m^2]$
a	Cross sectional area of the pipe	$[m^2]$
$\dot{C}_c$	Rate of heat capacity of cold fluid	$[W/K]$
$\dot{C}_h$	Rate of heat capacity of hot fluid	$[W/K]$
c_{pc}	Specific heat capacity of the cold fluid	$[J/(kgK)$]
c_{ph}	Specific heat capacity of the hot fluid	$[J/(kgK)$]
C_v	Valve Sizing constant	$[m^3/h]$
C^*	Heat capacity rate ratio	$[Dimensionless]$
D	Diameter of the pipe	$[m]$
D_1	Diameter of the pipe used at the terminals	$[m]$
F	Correction factor	$[Dimensionless]$
g	Acceleration due to gravity	$[m/s^2]$
h_f	Head loss of the pipe	$[m]$
$K_{i,j}$	Loss-coefficient for flow cming from branch i to j	$[Dimensionless]$
L	Length of the pipe	$[m]$
L_1	Length of the pipe used at the terminals	$[m]$
$\dot{m}$	Mass flow rate	$[kg/s]$
NTU	Number of trancfer units	$[Dimensionless]$
P	Pressure	$[Pa]$
Q	Heat transfer rate	$[W]$
Re	Reynold's number	$[Dimensionless]$
T	Temperature	$[^\circ C, K]$
ΔT_{cm}	Chen approximation to LMTD	$[^\circ C, K]$
ΔT_{lm}	Logarithmic mean temperature difference	$[^\circ C, K]$
ΔT_{pm}	Patterson approximation to LMTD	$[^\circ C, K]$
U	Overall heat transfer coefficient	$[W/m^2K]$
v	Velocity	$[m/s]$
Z	Elevation above the reference level	$[m]$

Greek Conventions

ϵ	Effectiveness	$[Dimensionless]$
ρ	Density	$[kg/m^{-3}]$
τ	Shear stress	$[kg/ms^2]$
μ	Dynamic viscosity	$[kg/mK]$

List of Abbreviations

AMTD	Arithmatic mean temperature difference
GMTD	Geometric mean temperature difference
CS	Total number of cold process streams
HS	Total number of hot process streams
LMTD	Logarithmic mean temperature difference
NTU	Number of transfer units
PHE	Plate heat exchanger
ST	Stages of the network

1 Introduction

1.1 Background

Saving energy is a major problem in many chemical process industries. A large amount of heat energy is normally wasted through cooling utilities. It is possible to reuse the heat energy of hot process streams to warm up the cold process streams by using additional heat exchangers and building a network consisting of heat exchangers, dividers, mixers or other equipment for heat transfer between process streams which is known as synthesis of heat exchanger networks (HEN). This network can be used for the cold process stream to be heated by the hot streams and vice-verse. In this way the heating and cooling loads from external sources and in result the utility costs can be reduced.

Mathematical modeling of a chemical process symbolizes an important issue for the process design and for the operations of the chemical processes. Recently, Patrascioiu and Marinoiu [2010] have proposed a mathematical model to solve for a single shell and tube heat exchanger for the temperature outlets. In this effort, the proposed superstructure is based on a stage wise demonstration analogous to the network proposed by Yee and Grossmann [1990] for the synthesis of HEN and to reduce the complexity occurring with the number of equations and computation time a simplified version of the superstructure has been proposed in this work. This work is a part of a joint work in which this work will complete the requirements of building a system of non-linear equations to solve for three variables: temperature, pressure and velocity at the intermediate positions of the proposed network. Furthermore, the other part of the work will be to optimize the divider valve positions in order to minimize the pressure drop hence, to minimize the pumping cost and to maximize the thermal performance by minimizing the entropy generation.

The architecture of the superstructure unless it is a trivial series system, means that the system model cannot in theoretical sense solved by serial manner solving the input output relationship of one unit at the time. In practice some engineering approaches have been developed for such approach and in actual complex real scale systems such approximate approach may be the only realistic way.

In this work the attempt was to build a mathematical model that will deal with the system as a coupled system where all the active variables are solved simultaneously. To carry out this task in a reasonable time and avoiding the growth of the system too big and computationally heavy several simplifying assumptions has been done.

The determination of intermediate temperatures and pressures of a network of heat exchangers is rather a challenging task due to the complex architecture of systems of heat exchangers in real world applications. In the computations of engineering aspects it is therefore, desirable to use an alternative method named as effectiveness-NTU method (NTU-method) which will solve the complex problems approximately closer to the real outputs.

The objective of this study was to model a network of heat exchangers by modeling the mathematical formulas for the temperatures, pressures and velocities at the intermediate junctions of the system. As a simplification to the complex system the problem is simplified to a simple network architecture of two heat exchangers with one hot fluid flow and one cold fluid flow. Moreover, it will include two divider elements and two mixer elements where in the division of fluid flows they are assumed to be having the behavior similar to a control valve and the mixers to be having the behavior of a T-junction.

Non-linear system of equations is modeled to compute the outlet temperatures and pressures of a counter-flow heat exchanger and the results obtained by using the system of non-linear equation formed with the LMTD term [Hewitt et al., 1994] were compared with the solutions obtained with the Paterson's method [Edvardsen, 2011] and Chen's approximation method [Chen, 1987].

The model equations of the form non-linear are formed according to the Logarithmic mean temperature difference term [Hewitt et al., 1994], mass balance of the junctions [Ati, 2009], energy balance of the junctions and in mixers and dividers [Ati, 2009], pressure drops according to Darcy-Weisbach equation [Crowe et al., 2010] and Bernoulli equation [Sleigh and Noakes, 2009]. System of non-linear equations are solved and hence the intermediate temperatures and pressures are determined by the solutions of the system of non-linear equations. To solve the system of non-linear equations Newton-Raphson algorithm has been used [Patrascioiu and Marinoiu, 2010].

1.2 Organization of the study

In order to give a clear idea to the reader about the research that has been carried out, this text has been divided into several sections and content of each section can be explained as follows.

Section 2, provides an overview of the fundamentals of heat exchangers, including the

principles of heat transfer, classification of heat exchangers and the use and purpose of heat exchangers in real world applications. Section 3, explains the derivation of the heat exchanger model equation, derivation of its components, two methods of heat exchanger analysis (LMTD and $\epsilon - \text{NTU}$ method) and two approximations to the LMTD term to avoid complexity of using LMTD has been explained. Moreover, Section 3, includes an introduction to the complex network of the heat exchangers and the problem related to the simplified system of heat exchangers on which the current study has been carried out.

Section 4, is based on the pressure drop analysis of heat exchanger elements, mixers and the divider elements. Section 5, explains the formation of system of non-linear equations to compute the outlet temperatures and pressures for a unit heat exchanger and the network of heat exchangers of the interest. Further, it explains how the system of non-linear equations are solved using the Newton-Raphson algorithm and the numerical evaluation of Jacobean matrix.

Section 6, represents the physical properties of the devices used in computations and computations related to a single heat exchanger to compute the outlet temperatures and pressures by using the method of non-linear equation solving method so called Newton-Raphson method. Moreover, the results obtained by the system of non-linear equations formed by using the LMTD term has been compared with some results obtained by few approximations to the LMTD term and in addition results of the network of heat exchangers are presented. Section 7, will present a summary of the work and the discussion about the remaining future work of this joint research work.

2 Basics of Heat Exchangers

2.1 Principles of Heat Transfer

Heat transfer is a study of thermal energy transportation inside a medium or among adjacent media by the interaction of the molecules (conduction), motion of the fluid particles (convection) and electromagnetic waves (radiation) causing from a spatial variation in temperature [Holman, 2001]. Variation in temperature is governed by the principle of energy conservation, which when applied to a control system, states that the summation of the flow energy and heat across the system, the work performed on the system and the amount of energy stored and transformed inside the system is zero [Holman, 2001].

Conduction

The conduction is defined as the transfer of energy from one point of a medium to another under the influence of temperature gradients [Incropera et al., 2007]. Transfer of the kinetic energy (exchange of kinetic energy between the particles in higher temperature and lower temperature regions) from one molecule to an adjacent molecule is involved in conduction. There are solids such as glass and quartz where the thermal energy is transmitted both by conduction and radiation. Inside a flow of a fluid in streamline (laminar motion) heat transferred by conduction occurs at right angles to the direction of fluid flow, e.g. metal shell of a heat exchanger or a boiler.

Convection

Convection involves the transfer of heat from one place to another by the motion of fluids [Incropera et al., 2007]. Dominant method of heat transfer by the circular motion of currents from one region to another in liquids and gases is usually the convection. In practical sense, flow of the heat by conduction through fluids is limited to certain issues of laminar flow or heat flow through thin films of metals. Only the convection of the heat flow unaccompanied by conduction is under no circumstances encounter because of laminar layer (i.e, tube wall) through which the heat must be transferred by both conduction and convection. Both conduction and convection occurs simultaneously and it is relatively difficult to distinct them nor is it advantageous. The motion of the fluid may be exclusively the result of the differences in densities occurred from the differences in temperature as in natural convection or the motion induced by the means of mechanics such as stirrer, agitator as in forced convection.

Radiation

Radiation is a process of transfer of energy through space by electromagnetic waves such as; light, infrared radiation, ultra-violet radiation or microwaves. For the propagation of the electromagnetic radiation existence of a medium is not required [Incropera et al., 2007].

2.2 Classification of Heat Exchangers

In industries and their products a variety number of heat exchangers are used. In this section, a brief mention is made to illustrate the classification of heat exchangers. Heat exchangers are classified according to the constructional features, transfer processes, number of fluids, degree of surface compactness, flow arrangement and heat transfer mechanism [Shah and Sekulic, 2003].

2.2.1 Classification According to Construction Features

i **Plate Heat Exchanger**

Plate-type heat exchangers (PHE) are manufactured of thin plates which transfer heat between two fluids. Major advantage of PHEs is the contact of the fluid with much larger surface area than the conventional heat exchangers. The plates might be smooth or corrugated. Further, PHEs are sub categorized into gasketed, welded or brazed heat exchangers [Shah and Sekulic, 2003].

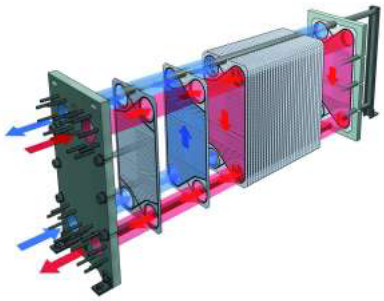

Figure 2.1: Plate heat exchanger

ii **Tubular Heat Exchanger**

Tubular heat exchangers are generally constructed of circular tubes. For some particular applications elliptical, rectangular or flat tubes are also been used.

Tubular heat exchanger is designed and specifically manufactured for the flow of high viscosity fluids with particles, pulp and fiber. Also, they are designed to tolerate high pressure differences between fluids. Further, tubular heat exchangers are classified as double-pipe, shell-and-tube and spiral tube exchangers [Shah and Sekulic, 2003].

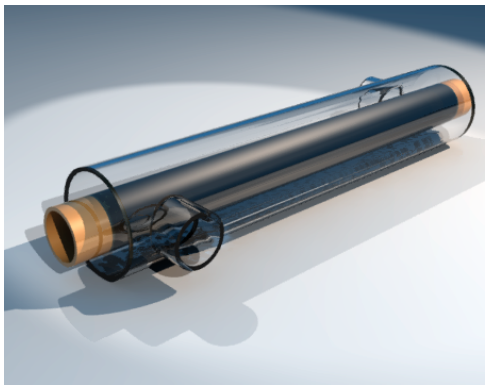

Figure 2.2: Tubular heat exchanger

iii **Extended Surface Heat Exchanger**

Previously described plate heat exchangers and tubular heat exchangers are all primary heat exchangers in which the heat exchanger effectiveness is usually 60 percent or below [Incropera et al., 2007]. For advanced applications, much higher exchanger effectiveness is required. By adding extended surface fins the surface area and exchanger compactness can be increased. The exchanger is therefore, known as extended surface exchanger. The most common type of extended heat exchangers are the tube-fin and the plate-fin exchangers [Shah and Sekulic, 2003].

Figure 2.3: Extended surface heat exchanger

iv **Regenerators**

Re-generator intermittently stores heat from the hot fluid in a thermal storage medium before it transfers to the cold fluid. The advances of this type of heat exchanger is that it has a much higher surface area for a given volume. It provides reduction in exchanger volume for a given energy density, pressure drop and effectiveness. Common types of re-generators are rotary regenerators, fixed matrix regenerators and micro scale regenerative heat exchangers [Shah and Sekulic, 2003].

2.2.2 Classification According to Flow Arrangement

According to the flow arrangement, heat exchangers are classified into parallel flow, counter-flow and cross flow heat exchangers [Shah and Sekulic, 2003].

Parallel flow heat exchanger: Both hot and cold fluids enter the heat exchanger at the same end, flow through in the same direction and leaves at the same end. This is also called as concurrent heat exchanger [Incropera et al., 2007].

Figure 2.4: Parallel flow heat exchanger

Counter-flow heat exchanger: Hot and cold fluids enter at the opposite ends of the heat exchanger, flows through in opposite directions and leaves the heat exchanger at the opposite ends [Incropera et al., 2007].

Figure 2.5: Counter-flow heat exchanger

Cross flow heat exchanger: Hot and cold fluids flow at a right angle to each other. Further, cross flow can be divided into unmixed flow and mixed flow in unfinned tubular heat exchangers and finned tubular heat exchangers respectively [Incropera et al., 2007].

Figure 2.6: Cross flow heat exchanger

2.2.3 Classification According to Heat Transfer Process

Heat exchangers are classified as indirect and direct contact type according to the heat transfer process [Shah and Sekulic, 2003].

Indirect contact type heat exchangers: In indirect contact type heat exchangers the fluid flows are separated by a wall and heat transfers continuously through the dividing wall. There is no direct contact between the fluids. This type of heat exchangers are also known as surface heat exchangers [Shah and Sekulic, 2003].

Direct contact type heat exchangers: The hot and cold streams of fluid in two phases which are not separated by a wall, contact directly, exchange heat and then separates. Regularly in direct contact heat exchangers, the process of heat transfer is also associated with mass transfer [Shah and Sekulic, 2003].

2.2.4 Classification According to Heat Transfer Mechanism

Basic heat transfer mechanisms that takes place within the heat transfer between two fluids are: single phase convection, two phase convection by forced and radiation or combined convection. Depending on the phase change mechanisms heat exchangers are classified as condensers and evaporators [Shah and Sekulic, 2003].

Condensers: Heat generated from condensing liquids or gases may be used for heating fluid. Usually, the fluid which is condensing is circulated outside the tubes with a water-cooled stream condenser or inside the tubes with gas cooling such as in refrigerators

and air-conditioners [Shah and Sekulic, 2003].

Evaporators: Evaporators are primarily used in air-conditioning systems together with the condensers to evaporate from liquid state to gaseous status while absorbing heat in procedure. Moreover, it can also be used to remove water or other liquids from mixtures. Apart from air conditioners, evaporators are used in food and beverage industries, desalination of sea water, pharmaceutical industry and etc [Shah and Sekulic, 2003].

2.2.5 Classification According to Number of Fluids

The most common type of heat exchangers which is classified according to the number of fluids is the two-fluid heat exchanger. Three-fluid heat exchangers are used in cryogenics, and in chemical process such as: air separation systems, ammonia gas synthesis and in purification and liquefaction of hydrogen. In advanced chemical process applications heat exchangers with as many as twelve fluid steams are used [Shah and Sekulic, 2003].

2.3 Counter-flow Heat Exchanger

Heat exchangers are mechanical devices designed for efficient heat transfer from one fluid matter to another via a solid surface. The solid surface separate the fluids from each other without mixing. There are many applications of heat exchangers which will be further discussed under Section 2.4. For the modeling and optimization of the network of heat exchangers in this research a simple generic form of counter-flow heat exchanger has been used in order to avoid dealing with the technical and geometric complexities of more realistic specific devices. In a counter-flow heat exchanger as shown in Figure 2.7, the two fluids flow parallel to each other but in opposite directions [Kakaç et al., 2012]. In actual engineering purposes the model which was used in this work should be replaced by more accurate device specific models taking into account the type, geometric design, dimensions, and exact flow regimes, possible phase changes etc. Moreover, for the simplicity some of the essential features such as: losses due to height variation, exact description of surface roughness and etc., has been neglected since the concentration was on the challenge of modeling a network superstructure.

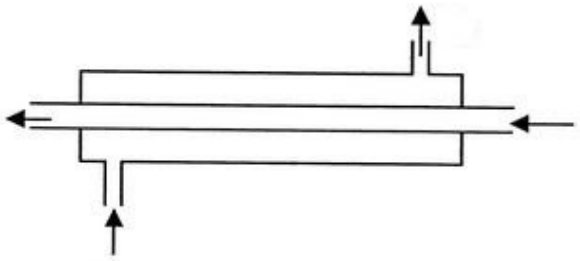

Figure 2.7: Direction of fluid flow in a counter-flow heat exchanger

The variation of the temperature of the two fluids can be represented as shown in Figure 2.8. The temperature along the wall fluctuates periodically as shown between the wall limits. Here $\dot{C}_h = (\dot{m}c_p)_h$ is the rate of heat capacity of the hot fluid, $\dot{C}_c$ is the rate of heat capacity of the cold fluid, and c_p are the specific heats which are considered to be constants. The countercurrent flow preparation is thermodynamically more advanced than any other flow arrangements. The counter-flow heat exchanger is the most efficient model of heat exchangers, generating the highest temperature difference in each fluid compared to any other type of fluid flow arrangements for a given overall thermal conductance (UA), heat capacity rates, and fluid inlet temperatures [Incropera et al., 2007]. Furthermore, the maximum difference of the temperature across the exchanger between the surfaces of the wall exposed to the hot and cold sides at the terminals is the lowest. Moreover, it generates the minimum thermal stresses in the wall compared to a similar performance to any other flow arrangements.

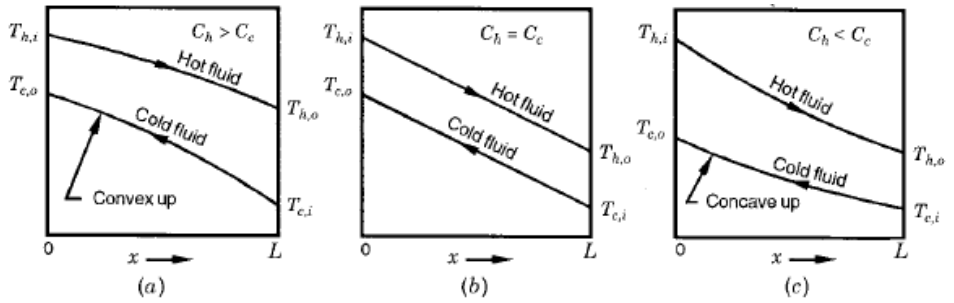

Figure 2.8: Temperature distributions in a counter-flow heat exchanger

2.4 Use and Purpose of Heat Exchangers

Heat exchangers control a system's temperature by adding or removing thermal energy. Although, there are different types of heat exchangers in different sizes and different level of sophistication, they all include a thermal conducting element to separate two fluids, such that the thermal energy can be transferred from one fluid to the other. To meet a variety of highly demanding requirements modern heat exchangers are manufactured to ensure maximum heat transfer while keeping the size to a minimum.

Most common use of heat exchanger is the home heating system. The system use a heat exchanger to transfer combustion gas heat to water or air, which is circulated through the house. Moreover, heat exchanger is considered to be a necessary piece of device in the modern technological advanced chemical plants. Heat exchanger is given a major role and importance in the reaction system. In the purification systems, the distillation column in reality is a direct contact heat exchanger [Kakaç et al., 2012].

Moreover, heat exchangers are used in a wide variety of applications such as in power production, process, food industries, cryogenics, waste heat recovery, electronics, environmental engineering, manufacturing industry and in space applications. Process industry use two-phase flow heat exchangers for the purpose of vaporizing, condensing, and freezing in crystallization [Shah and Sekulic, 2003]. Various types of nuclear steam generators, fossil boilers, condensers, cooling towers and regenerators are used in the power industry. In refrigerators and in air conditioners heat exchangers are used in an opposite way from general heating systems.

In generally heat exchangers should satisfy some requirements to be efficient such as higher thermal effectiveness, low pressure drop, high life expectancy and reliability, compatibility of the materials with the fluids used, easy to install and appropriate size, high quality invention with high safety, convenient maintenance, low initial cost, low weight but strong enough to withstand the high pressures and vibrations and simple to manufacture [Kakaç et al., 2012].

3 Heat Exchanger & Heat Exchanger Network Model

3.1 Derivation of Heat Exchanger Equation Using LMTD

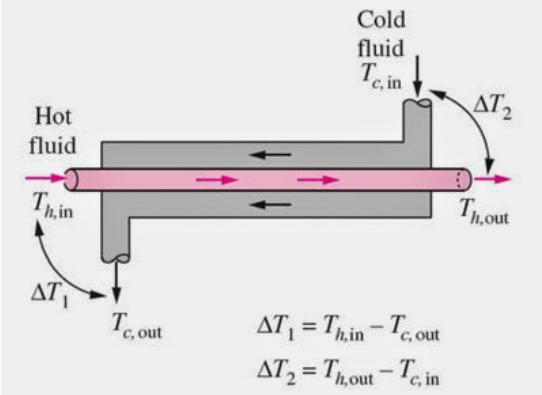

Figure 3.1: Temperature difference at terminals of a counter-flow heat exchanger

Let the total rate of heat transfer between the hot and cold fluids be $\dot{Q}$ and assume that there is negligible heat transfer between the exchanger and its surroundings. Moreover, the potential energy and the kinetic energy changes are assumed to be negligible. The energy balance for the both hot and cold streams are given by [Ati, 2009]:

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) \tag{3.1}$$

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) \tag{3.2}$$

where the temperatures appearing in the expressions refer to the inlet and outlet temperatures of hot and cold streams.

Furthermore, the relationship between the total heat transfer rate $\dot{Q}$ and the temperature difference ΔT_{lm}, is:

$$\dot{Q} = UAF\Delta T_{lm} \tag{3.3}$$

where ΔT_{lm} is the logarithmic mean temperature difference given by the Equation 3.4 and F is a correction factor. For the counter-flow configuration F is assumed to be equal to 1 [Incropera et al., 2007]. The correction factor, F, is a function of the ratio of mass flow rates, heat exchanger configuration and the temperature differences. The logarithmic mean temperature difference is given by [Hewitt et al., 1994]:

$$\Delta T_{lm} = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln\left[\dfrac{(T_{h,in} - T_{c,out})}{(T_{h,out} - T_{c,in})}\right]} \tag{3.4}$$

The energy balance equation with the LMTD is given by:

$$\dot{Q} = UA\frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln\left[\frac{(T_{h,in} - T_{c,out})}{(T_{h,out} - T_{c,in})}\right]} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) \tag{3.5}$$

3.2 Logarithmic Mean Temperature Difference and Approximations

The logarithmic mean temperature difference (LMTD) is a basic concept used when designing and analyzing heat exchangers [Habimana et al., 2009]. The term LMTD (logarithmic mean temperature difference) is an expression that results when solving a pair of partial differential equations describing the transfer (by Fourier's law) of heat from hot to cold stream in a generic counter flow device through a wall with given heat conductivity. Although the temperature difference between the two fluids is changing in each point of the surface of heat exchanger, it can be shown that the mean temperature difference of the two fluids throughout the entire counter-flow heat exchanger can be solved by the equation 3.6.

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \tag{3.6}$$

where; $\Delta T_1 = T_{h,in} - T_{c,out}$ and $\Delta T_2 = T_{h,out} - T_{c,in}$ are the temperature difference between the inlets temperature differences of the heat exchangers and the outlet temperature differences of the heat exchangers and are shown in the Figure 3.2.

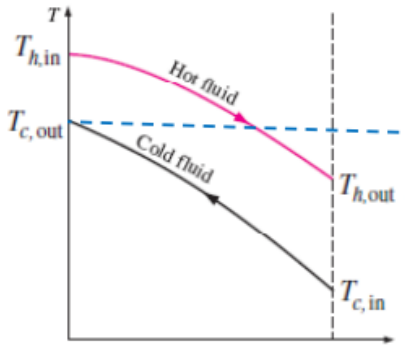

Figure 3.2: Temperature distribution of counter-flow heat exchanger

The main issue in solving heat exchanger equation is that it sometimes approaches to a division by zero. Therefore, optimization problems might sometimes lead to in determined solutions. To overcome this issue few approximation methods are proposed to eliminate the LMTD term such as Paterson approximation [Edvardsen, 2011] and Chen approximation [Chen, 1987].

Paterson Approximation (ΔT_{PM})

Paterson approximation [Edvardsen, 2011] is constructed on the circumstance that LMTD is bounded by arithmetic mean temperature difference (AMTD) and geometric mean temperature difference (GMTD) and is expressed as below.

$$AMTD = \frac{1}{2}(\Delta T_1 + \Delta T_2) \tag{3.7}$$

$$GMTD = (\Delta T_1 \Delta T_2)^{1/2} \tag{3.8}$$

$$\Delta T_{PM} = \frac{2}{3} GMTD + \frac{1}{3} AMTD \tag{3.9}$$

The energy balance equation with the Paterson's approximation is given by:

$$\dot{Q} = UA\Delta T_{PM} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) \tag{3.10}$$

Where;

$\Delta T_1 = T_{h,in} - T_{c,out}$ and $\Delta T_2 = T_{h,out} - T_{c,in}$

Chen Approximation (ΔT_{CM})

The Chen approximation [Chen, 1987] provides a slight overestimate for the area while the Paterson approximation underestimates. If both temperature differences are zero both approximations give zero unlike LMTD. The numerical difficulties which occur when solving equations with LMTD will overcome by the both approximations.

$$\Delta T_{CM} = (\frac{1}{2} \cdot \Delta T_1^{0.3275} + \frac{1}{3} \cdot \Delta T_2^{0.3275})^{\frac{1}{0.3275}} \tag{3.11}$$

The energy balance equation with the Chen's approximation is given by:

$$\dot{Q} = UA\Delta T_{CM} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) \tag{3.12}$$

Where;

$\Delta T_1 = T_{h,in} - T_{c,out}$ and $\Delta T_2 = T_{h,out} - T_{c,in}$

3.3 The Effectiveness-NTU Method

When the inlet fluid temperatures are known and the outlet fluid temperatures are indicated by using the energy balance equations it is easier to use the logarithmic mean temperature method (LMTD) and hence the value of ΔT_{lm} can be determined [Saari, 2010]. However, if only the fluid inlet temperatures are the only known factors it will need iterative process to perform computations. Hence it is desirable to use an alternative method known as effectiveness-NTU method.

The NTU-method is constructed on three dimensionless parameters: the effectiveness of the heat exchanger ϵ, heat capacity ratio C^*, and the number of heat transfer units NTU.

3.3.1 Effectiveness of the Heat Exchanger ϵ

Effectiveness ϵ measures the thermal performance of a heat exchanger and is expressed as the ratio of actual heat transfer rate of the heat exchanger $\dot{Q}$ to the maximum heat transfer rate possible through the heat exchanger $\dot{Q}_{max}$ according to the second law of thermodynamics [Saari, 2010].

$$\epsilon = \frac{\dot{Q}}{\dot{Q}_{max}} \tag{3.13}$$

The actual amount of heat transferred in the heat exchanger if there is no phase change takes place, can be determined by the product of change in temperature ΔT and the rate of heat capacity $\dot{C}$ of both fluids as follows:

$$\dot{Q} = (\dot{C}\Delta T)_h = (\dot{C}\Delta T)_c \tag{3.14}$$

The maximum amount of heat transferred is determined by the product of the difference of the inlet hot and cold temperatures $T_{h,in} - T_{c,in}$ and the minimum of the heat capacity of both the fluids $\dot{C}_{min} = min(\dot{C}_h, \dot{C}_c)$.

$$\epsilon = \frac{\dot{C}_c(T_{c,out} - T_{c,in})}{\dot{C}_{min}(T_{h,in} - T_{c,in})} = \frac{\dot{C}_h(T_{h,in} - T_{h,out})}{\dot{C}_{min}(T_{h,in} - T_{c,in})} = \frac{\mid T_{in} - T_{out} \mid_{\dot{C}_{min}}}{(T_{h,in} - T_{c,in})} \tag{3.15}$$

Hence, the total amount of heat transferred can be represented as:

$$\dot{Q} = \epsilon \dot{C}_{min}(T_{h,in} - T_{c,in}) \tag{3.16}$$

3.3.2 Heat Capacity Rate Ratio C^*

Heat capacity rate ratio expresses the ratio of the smaller to larger heat capacity rate for the both fluids. Value of C^* will necessarily be less than 1 [Thulukkanam, 2013].

$$C^* = \frac{\dot{C}_{min}}{\dot{C}_{max}} = \frac{(mc_p)_{min}}{(mc_p)_{max}} \tag{3.17}$$

Where;

$\dot{C}$: heat capacity rate of the fluid (product of mass and specific heat of the fluid)

3.3.3 Number of Transfer Units (NTU)

NTU describes a dimensionless parameter which comprises of design variables in engineering such as overall heat transfer rate and heat transfer area. NTU is defined as the ratio of the product of overall heat transfer coefficient (U) and heat transfer area (A) (also product UA is known as heat conductance) to the minimum of the heat capacity rate [Thulukkanam, 2013].

$$NTU = \frac{U \cdot A}{\dot{C}_{min}} \tag{3.18}$$

3.3.4 Effectiveness-NTU relationships

The effectiveness-NTU relation for a counter-flow heat exchanger is defined by:

$$\epsilon = \frac{1 - \exp(-NTU(1 - C^*))}{1 - C^* \exp(-NTU(1 - C^*))} \tag{3.19}$$

The energy balance equations enrolled with ϵ-NTU method can be expressed as follows:

$$Q = \dot{m}_h c_{p,h}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{p,c}(T_{c,out} - T_{c,in}) = \epsilon \dot{C}_{min}(T_{h,in} - T_{c,in}) \tag{3.20}$$

3.4 Overall Heat Transfer Coefficient

In a direct contact type of heat exchanger, the heat is transferred by convection from hot fluid to the tube wall, by conduction through the wall, and again by convection from the tube wall to the cold fluid [Liptak, 2005]. Overall heat transfer coefficient is used to determine the total heat transfer that takes place through a pipe wall from

hot fluid to the cold fluid. This coefficient depends on the type of the heat exchanger, thickness and thermal conductivity of the mediums through which heat is transferred. The relationship between the overall heat transfer coefficient (U) and the heat transfer rate ($\dot{Q}$) can be established as follows:

$$\dot{Q} = U \cdot A \cdot \Delta T_{lm} \tag{3.21}$$

According to the equation (3.21) it can be seen that the overall heat transfer coefficient is directly proportional to the heat transfer rate

$$\frac{1}{U} = \frac{1}{h_1} + \frac{L}{\lambda} + \frac{1}{h_2} \tag{3.22}$$

Where;

h = convective heat transfer coefficient, $W/(m^2\,°\mathrm{C})$

L = thickness of the wall, m

λ = thermal conductivity, $W/(m\,°\mathrm{C})$

3.5 Heat Exchanger Network

The structure of the network of heat exchangers as shown in the Figure 3.3 is constructed based on the stage wise structure of the heat exchanger network (HEN) as introduced by Yee and Grossmann [1990] for the synthesis of HEN. Either fresh or cold flow is used as the cold utility in order to obtain a network with minimum cost. The superstructure as shown in Figure 3.3, contains of three hot process streams and three cold process streams. The number of stages of the system is equal to the number of hot streams and connected nodes represents a unit heat exchanger arrangement [Ponce-Ortega et al., 2007].

The term superstructure refers to the architecture of the network, which is consisant of several lines, coupled in series or parallel or both, hence generating a complex coupled system. Designing an ideal superstructure architecture may be a challenging discrete optimization problem in itself. Once the superstructure architecture is given, the switching of the coolant flow in the network by divider valves should be optimized. Modelling the flow and heat transfer in the network was the challenge of this thesis work. The system model is needed in order to derive the optimal control of the coolant flow.

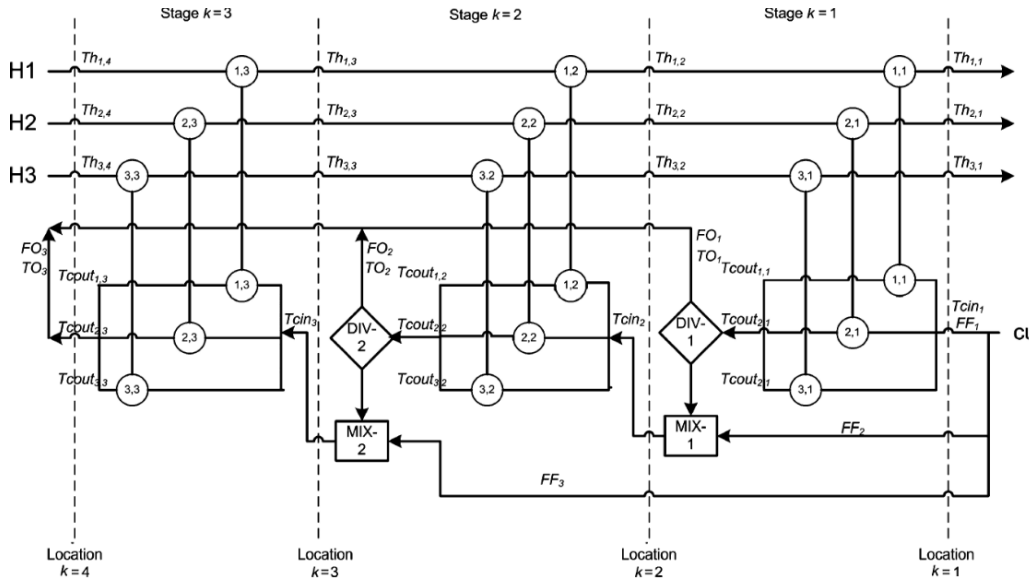

Figure 3.3: Superstructure of the network of heat exchangers

The outlet temperatures and pressures of each heat exchanger is considered as the inlet temperature and pressure to the consecutive heat exchanger. $DIV1$ and $DIV2$ represents the stream splits and in a divider one major pipe is divided into several pipes with specific flow rates and also the temperatures and enthalpies are conserved within the network. In this research for the stream division a control valve has been used. $MIX1$ and $MIX2$ represents the fluid flow mixers and mixer is constructed in a way pipe branches combines to form a single pipe. The mass flow of the main pipe is equal to addition of the branched pipe mass flows and the balance of enthalpy can be used to calculate the enthalpy and the temperatures.

For the simplification of the network as shown in Figure 3.3, a simpler structure of a HEN has been proposed by the Figure 3.4, and the objective of the research work is to model a system of non-linear equations comprising of equations to determine the intermediate temperatures, velocities and pressures of the network when the inlet temperatures and pressures of both the hot utility and cold utility are given. The objective of simplifying the network is to avoid the complexity occurs with many equations when solving the equations as a system of non-linear equations. Moreover, the true scale complex network would have been much too heavy to compute and even write the model equations.

However, it should be noted that the same approach can be easily generalized to more complex and more accurate system by replacing component models with more accurate

versions, by including minor effects etc. This will add more of the number of variables and equations and increase computing time but essentially the same method would work.

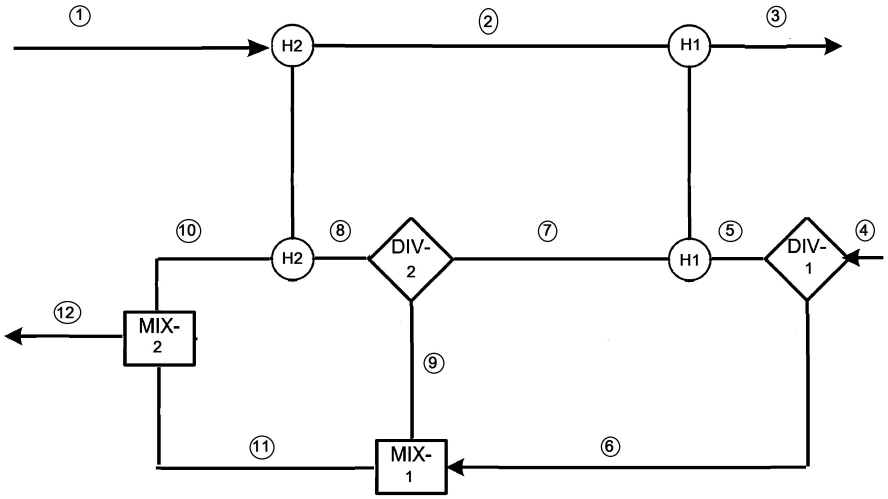

Figure 3.4: Proposed simplified structure for the network of heat exchangers

The simplified model has one hot process stream and one cold process stream with two heat exchangers, two mixers and two dividers. The input temperature and the pressure of hot inlet and cold inlet will be known (position 1 and position 4). The outlet hot tube and outlet cold tube will be connected to a long pipe with rough interior surface in order to make the pressure at the end to be zero.

3.6 Mass Balance for the Divider and Mixer

The physics of the mixer and divider will be discussed in the sections below [Ati, 2009]. A mixer with two fluid flows and one outlet flow has been used in this research work. Pipe setting of a T-junction has been used in the mixing device.

Mass Balance Equations for a Mixer

The mass balance equation describes that the total mass inlet is equal to the total mass outlet. If there are two inlet flow branches $(in, 1)$, $(in, 2)$ and one outlet branch $(out, 1)$ then the mass balance can be expressed as follows [Ati, 2009]:

$$m_{(in,1)} + m_{(in,2)} - m_{(out,1)} = 0 \tag{3.23}$$

Where,
$m_{(in,1)}, m_{(in,2)}$ Mass flow-rate of inlet streams
$m_{(out,1)}$ Mass flow-rate of outlet streams

Mass Balance Equations for a Divider

As mentioned under topic 3.5, divider is a device with one inlet flow and two outlet flows where the outlet split fractions are controlled by a control valve. The mass balance equations for the divider model are introduced below [Ati, 2009].

$$m_{(in,1)} - m_{(out,1)} - m_{(out,2)} = 0 \tag{3.24}$$

$$x_1 \cdot m_{(in,1)} - m_{(out,1)} = 0 \tag{3.25}$$

$$x_2 \cdot m_{(in,1)} - m_{(out,2)} = 0 \tag{3.26}$$

Where,
$m_{(in,1)}$ Mass flow-rate of inlet stream
$m_{(out,1)}, m_{(out,2)}$ Mass flow-rate of outlet streams
x_1, x_2 Split fractions of outlet branches

4 Pressure Drop Analysis of the System

Head loss also known as pressure loss is used as a measure to compute total energy per unit weight above a particular point of reference. Usually, pressure loss is the summation of three constituent; elevation head (relative potential energy in terms of an altitude), velocity head (kinetic energy generated by the motion of fluid) and, pressure head (corresponding gauge pressure of a column of water at the base of the piezometer where piezometer is a well with a small diameter used to measure the hydraulic pressure of underground water). The pressure loss occurs in a pipe is the loss of flow energy due to the friction or turbulence.

Moreover, the pressure loss is divided into two components namely major loss and minor loss [Crowe et al., 2010]. Both major and minor head losses depend on the properties of the fluid and the material of the pipe. The total head loss is the combination of pipe head loss and component head loss. The viscous or the shear stress that act on the fully developed flowing fluid is known as the pipe head loss. Component head loss is occurred when the fluid flows through pipe such as valves, bends, and tees. Sometimes the major head loss is known as the pipe head loss and minor head loss is known as component head loss. Major losses are easily calculated by using Darcy-Weisbach equation as expressed in the Section 4.1 [Crowe et al., 2010].

For example when the pipes are long such as in water distribution pipes in a city the major pressure loss might be large and moreover, the minor pressure loss will also be dominant because of the large number of bends and valves in the network. Therefore, in reality the pressure drop is inevitable as the pipes are not perfectly smooth with no friction and no fluid exists without turbulence.

Pressure loss of a fluid flow is directly proportional to the length of the pipe, the square of the velocity of the fluid flow and a proportionality constant known as a friction factor while diameter is inversely proportional.

4.1 Pressure Drop Accross the Heat Exchanger

Fluids are required to be pumped through the heat exchangers in most of the industrial applications. It is significant to determine the fluid pumping power required as a part of designing the system and in cost operation analysis. Pumping power of the fluid in a device is directly proportional to the fluid pressure drop, which is associated

with friction of the fluid and other pressure drop influences along the path of fluid flow. Pressure drop of the fluid has a direct association with heat exchanger heat transfer, process, dimension, mechanical features and other relevant aspects including economic considerations. Objective is to outline the methods for pressure drop analysis in heat exchangers and related flow devices. Determination of pressure drop across a heat exchanger is essential for many applications for at least two reasons: The heat transfer rate can be influenced significantly by the saturation temperature change for a condensing/evaporating fluid if there is a large pressure drop associated with the flow. This is because saturation temperature changes with changes in saturation pressure and in turn affects the temperature potential for heat transfer [Saari, 2010].

In this section Darcy-Weisbach equation [Crowe et al., 2010], which is one of the most useful equations in fluid mechanics is presented. It is used to calculate the head loss which occurs in a fluid in a straight flow pipe. Darcy-Weisbach equation can be used if the flow is completely developed and steady. It is used for either laminar flow or turbulent flow and for either circular pipes or non circular pipes such as a rectangular pipe. Purpose of using the Darcy-Weisbach equation is to calculate a value for the friction factor. As stated in the equation the friction factor, the pipe-length-to-diameter ratio, and the mean velocity squared influence the head loss.

$$h_f = f.\frac{L}{D}.\frac{V^2}{2g} \tag{4.1}$$

h_f is the head loss due to friction which will be in meters in SI units. Therefore, it is converted into a form with the term ΔP which will give the pressure drop in Pascals.

$$\Delta P = \rho \cdot g \cdot h_f \tag{4.2}$$

By combinning Equation (4.1) and (4.2) an equation to compute the pressure drop across the heat exchanger can be dervied as follows.

$$\Delta P = f \cdot \frac{L}{D} \cdot \frac{\rho \cdot V^2}{2} \tag{4.3}$$

Where;
ΔP : Pressure loss due to friction across the heat exchanger
f : Darcy friction factor
ρ : Density of the fluid
V : Velocity
D : Diameter of the pipe
L : Length of the pipe

The simplification to the pressure drop across the heat exchanger can be derived by the combination of Equation 4.3 and Equation 4.7 as follows:

$$\Delta P = 0.1582 \cdot \rho^{3/4} \cdot L \cdot D^{-5/4} \cdot \mu^{1/4} \cdot V^{7/4} \tag{4.4}$$

4.1.1 Darcy Friction Factor

Friction factor is the ratio between the shear stress acting at the wall and the kinetic pressure as shown in the Equation (7.11).

$$f \equiv \frac{4 \cdot \tau_0}{\rho \cdot V^2/2} \tag{4.5}$$

Friction factor is also known as Darcy friction factor, Darcy-Weisbach friction factor and the resistance coefficient is a dimensionless quantity. Moreover, friction factor for laminar flow depends only on the Reynolds number as shown in below [Crowe et al., 2010].

$$f = \frac{64}{Re} \tag{4.6}$$

There are many correlations such as: Colebrook-White equation, Haaland equation, Swameed'-Jain equation, Serghides's solution which were determined by experimental data to obtain formulas for friction factor and one such important correlation which is called Blasius correlation is of interest as follows.

$$f = \frac{0.3164}{Re^{1/4}} \tag{4.7}$$

Where;

$$Re = \frac{\rho V D}{\mu} \tag{4.8}$$

The Blasius correlation [Nunn, 1989] is valid for turbulent flows with up to Reynolds number 10^5.

4.2 Pressure Drop at Dividers (Valves)

Figure 4.1: Cross-section of a control valve

The dividers of the system is considered to have a control valve for the division of the fluid flow into given proportions. Control valve is a component that controls or regulates a flow of fluid by opening, closing or partially obstructing. There are many types of valves such as hydraulic valve, pneumatic valve, manual valve, solenoid valve and motor valve. Control valves are important in applications such as in controlling water for irrigation, industrial processes, taps, oil refineries, gas, petroleum processes and etc [Liptak, 2005].

Daniel Bernoulli, introduced a relationship to express the relation between the pressure drop occurs at a valve and the velocity by using the principle of conservation of energy. The pressure drop across the valve is directly proportional to the square of the velocity while the specific gravity of the fluid is inversely proportional. If the velocity of the flowing fluid across the valve is higher the pressure drop is higher and greater the density the lower the velocity and hence pressure drop. The valve sizing coefficient (C_v) computed by experiments is dependent on the size and the type of the valve. The pressure drop across the valve is as follows [McAllister, 2013] [Liptak, 2005]:

$$Q = C_v \sqrt{\frac{\Delta P}{G}} \tag{4.9}$$

By rearranging the terms,

$$\Delta P = G \cdot \left(\frac{Q}{C_v}\right)^2 \tag{4.10}$$

Where;

G :Specific gravity of fluids (Water is normally 1.0000)
Q : Capacity in Gallons per minute
C_v: Valve sizing constant determined experimentally
ΔP: Pressure difference in psi $1psi = 6894.75729Pa$

4.3 Pressure Drop at Mixers (T-junction)

The mixers of the heat exchanger network are considered to have T-junctions and to compute the pressure drop at the mixers formulas modeled for the T-junctions are used. When computing pressure loss at a T-junction head loss coefficient for T-junction plays an important role.

The pressure loss which occurs at T-junction depends on few factors, such as velocity of incoming and outgoing fluid at the junction, pipe diameters and the angle of the

branches at the junction. Some classical formulas for the pressure drop at a T-junction have been derived by Benedict [1980].

There are few assumptions made at the T-junction to compute pressure loss. A T-junction can be considered as a combination of two pipe components such as two elbows or an elbow and a sudden contraction [Vasava, 2007]. This section will state the classical formulas and the relevant formulas that were derived based on the above assumptions [Vasava, 2007]. Possible pipe arrangement for a mixer is illustrated in Figure 4.2.

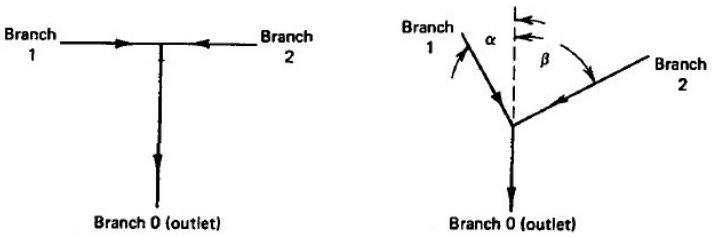

Figure 4.2: Pipe arrangement of a mixer

Loss coefficient for combining fluid flows

The formula in this section was derived by considering the combination of the fluid flows. Fluid flow may combine, combining from two or more pipes at the inlet to one pipe at outlet. The formula for the loss coefficient of the fluid within the branch 1 and 0 is given by,

$$K_{1,0} = \lambda_3 \left(\frac{V_1}{V_0}\right)^2 + 1 - 2\left[\left(\frac{V_1}{V_0}\right)\left(\frac{a \cdot V_1}{a \cdot V_0}\right)\cos\alpha' + \left(\frac{a \cdot V_2}{a \cdot V_0}\right)\left(\frac{a \cdot V_2}{a \cdot V_0}\right)\cos\beta'\right], \quad (4.11)$$

where K depends on the kinetic energy of the combined flow in branch 0, V defines the velocities of the respective branches, λ_3 is defined by Figure 4.3, to be equal to 0.61 at a T-junction, and α' and β' are defined by the same graph as described by equation (4.12) [Benedict, 1980],

where;

$$\begin{aligned} \alpha' &= 141\alpha - 0.00594\alpha^2 \\ \beta' &= 141\alpha - 0.00594\beta^2 \end{aligned} \quad (4.12)$$

Data from T-junction states that there is no variation of the loss coefficient for the fluids with Reynolds number $R_D > 1000$.

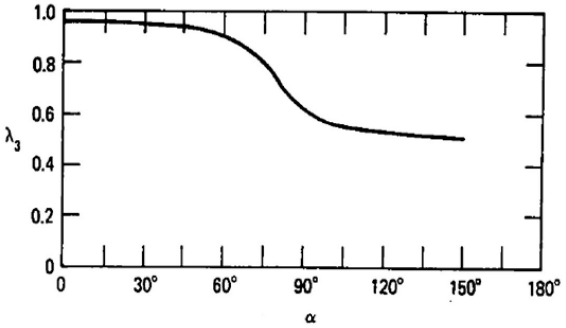

Figure 4.3: Plot of value of λ_3 across the pipe angle

Pressure drop for combining fluid flows

As shown in the Figure 4.2, branch 1 and branch 2 will combine together to form branch 0. Let the velocities and volumetric flow rates be V_1, V_2, V_0 and $a \cdot V_1, a \cdot V_2, a \cdot V_0$ respectively. The loss coefficient along the branch 1 to 0 is given by:

$$K_{1,0} = \lambda_3 \left(\frac{V_1}{V_0}\right)^2 + 1 - 2\left[\left(\frac{V_1}{V_0}\right)^2 \cos\alpha' + \left(\frac{V_2}{V_0}\right)^2 \cos\beta'\right], \tag{4.13}$$

Where;

$\lambda_3 = 0.61$, and $\alpha = \beta = 90°$, Hence; $\alpha' = \beta' = 78.7860°$. Therefore, the pressure drop along branch 1 to 0 is given by:

$$P_1 - P_0 = \frac{2 \cdot g \cdot K_{1,0}}{(V_1^2)} \tag{4.14}$$

Similarly, pressure drop along branch 2 to 0 is given by:

$$P_2 - P_0 = \frac{2 \cdot g \cdot K_{2,0}}{(V_2^2)} \tag{4.15}$$

4.4 Pressure Drop according to Bernoulli Equation

Bernoulli equation is an important principle in fluid dynamics and is derived from the principle of conservation law of energy. Further, it states that, in a steady state fluid flow, the combination of energy appearing in all forms remains constant along the streamline. Bernoulli equation includes the energy in forms of potential energy, kinetic energy and internal energy. In addition, it includes some limitations of the fluid such

as: steady state, incompressible fluid with negligible friction losses [Sleigh and Noakes, 2009].

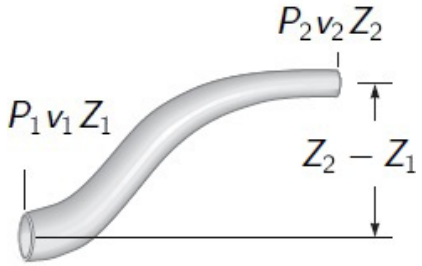

Figure 4.4: Bernoulli Effect

$$Z_1 + \frac{P_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} = Z_2 + \frac{P_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} \tag{4.16}$$

Where;

$Z_{1,2}$: Elevation above the reference level and in this work all the pipes are considered to be at the same level for the simplification and simplified form of the Bernoulli equation to compute the pressure drop can be stated as follows.

$$P_1 - P_2 = \frac{\rho}{2} \cdot (V_2^2 - V_1^2) \tag{4.17}$$

5 System of Non-linear Equations

This section will express how the mathematical model equations (to express temperature, pressure and velocity) for the three main devices heat exchanger, divide and mixer have been formed. Furthermore, the system of non-linear equations formed for a single heat exchanger and for the simplified network will be presented.

5.1 Mathematical Model Equations for the Devices

Model of temperature, pressure and velocity for a heat exchanger

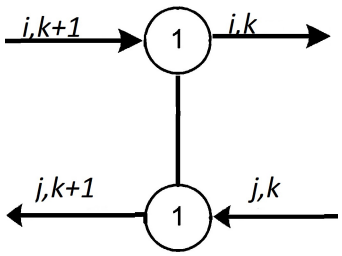

Figure 5.1: Interpretation of unit heat exchanger

The set HS is used to indicate the total number of hot process streams, CS to indicate total number of cold process streams and ST to indicate the total number of stages in the super structure [Ponce-Ortega et al., 2007]. The number of stages is denoted by k . The energy balance model equations for the hot stream i, and cold stream j of a heat exchanger unit as written according to the Equation 3.5 is as follows [Patrascioiu and Marinoiu, 2010]:

$$V_{i,k+1} \cdot a_{i,k+1} \cdot \rho \cdot c_{ph}(T_{i,k+1} - T_{i,k}) - V_{j,k} \cdot a_{j,k} \cdot \rho \cdot c_{ph} \cdot (T_{j,k+1} - T_{j,k}) = 0$$
$$V_{i,k+1} \cdot a_{i,k+1} \cdot \rho \cdot c_{ph}(T_{i,k+1} - T_{i,k}) - U \cdot A \cdot \frac{(T_{i,k+1} - T_{j,k+1}) - (T_{i,k} - T_{j,k})}{\ln\left[\frac{(T_{i,k+1} - T_{j,k+1})}{(T_{i,k} - T_{j,k})}\right]} = 0 \quad (5.1)$$

Where, $i \in HS \quad k \in ST$ and $a_{i,k+1} = a_{j,k}$ (Assuming that throughout the system pipe diameter remains constant). Temperature feasibility to confirm a monotonous decrease of temperature along the stages of the network of heat exchangers, the constraints as

follows are used:

$$\begin{aligned} T_{i,k} \leq T_{i,k+1} \quad i \in HS \quad k \in ST \\ T_{j,k} \leq T_{j,k+1} \quad j \in CS \quad k \in ST \end{aligned} \tag{5.2}$$

The pressure drop model equations across the heat recovery unit i for the hot flow and j for the cold hot flow is expressed accordingly by the Equation 4.4 and by Equation 4.17 are as follows:

$$\begin{aligned} P_{i,k+1} - P_{i,k} - 0.1582 \cdot \rho^{3/4} \cdot L \cdot D^{-5/4} \cdot \mu^{1/4} \cdot V_{i,k+1}^{7/4} = 0 \\ P_{j,k} - P_{j,k+1} - 0.1582 \cdot \rho^{3/4} \cdot L \cdot D^{-5/4} \cdot \mu^{1/4} \cdot V_{j,k}^{7/4} = 0 \end{aligned} \tag{5.3}$$

$$\begin{aligned} P_{i,k+1} - P_{i,k} - \frac{\rho}{2} \cdot (V_{i,k}^2 - V_{i,k+1}^2) = 0 \\ P_{j,k+1} - P_{j,k} - \frac{\rho}{2} \cdot (V_{j,k}^2 - V_{j,k+1}^2) = 0 \end{aligned} \tag{5.4}$$

Model of temperature, pressure and velocity for a divider

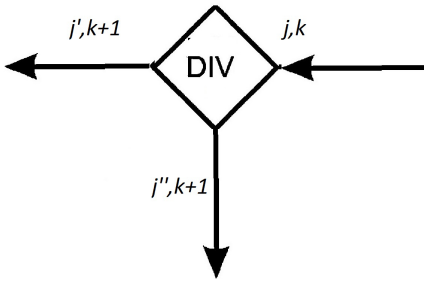

Figure 5.2: Interpretation of a divider

Temperature after the stream splitting will be the same to the incoming flow and it can be represented mathematically as follows:

$$\begin{aligned} T_{j,k} - T_{j',k+1} = 0 \\ T_{j,k} - T_{j'',k+1} = 0 \end{aligned} \tag{5.5}$$

The pressure drop relationship across the divider in the cold sream j can be expressed accordingly to the Equation 4.10 is as follows:

$$\begin{aligned} P_{j,k} - P_{j',k+1} - k_1 \cdot (y_p \cdot V_{j,k}^2) = 0 \\ P_{j,k} - P_{j'',k+1} - k_1 \cdot ((1 - y_p) \cdot V_{j,k}^2) = 0 \end{aligned} \tag{5.6}$$

Where;

$k_1 = \left(\frac{0.0104 \cdot a_{j,k}^2 \cdot \rho}{C_v^2}\right)$, $p = 1, 2$ and assuming that y_p propotion of the incoming flow will pass across the branch $j', k+1$ and the remaining amount $1 - y_p$ will pass through the branch $j'', k+1$.

Model of temperature, pressure and velocity for a mixer

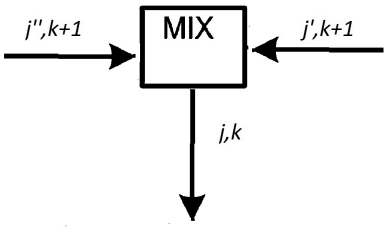

Figure 5.3: Interpretation of a mixer

Temperature balance in a divider can be expressed as follows:

$$T_{j,k} \cdot V_{j,k} - T_{j',k+1} \cdot V_{j',k+1} - T_{j'',k+1} \cdot V_{j'',k+1} = 0 \tag{5.7}$$

The pressure drop relationship across the divider in the cold sream j can be expressed accordingly as represented in the Section 4.3 is as follows:

$$\begin{aligned} P_{j'',k+1} - P_{j,k} - \frac{2 \cdot g \cdot K_{(j'',k+1),(j,k)}}{V_{j'',k+1}^2} \\ P_{j',k+1} - P_{j,k} - \frac{2 \cdot g \cdot K_{(j',k+1),(j,k)}}{V_{j',k+1}^2} \end{aligned} \tag{5.8}$$

Where;

$K_{(j'',k+1),(j,k)}$ and $K_{(j',k+1),(j,k)}$ are the loss coefficents along the branches $j'', k+1$ to j, k and $j', k+1$ to j, k respectively.

5.2 Mathematical Model for a Counter-flow Heat Exchanger

The single unit of heat exchanger is characterised by hot and cold inlet temperatures and pressures $T_{h,i}$, $T_{c,i}$, $P_{h,i}$, and $P_{c,i}$. A system of four non-linear equations are formed in order to compute the four outputs: hot stream and cold stream temperatures and

pressures. The mathematical model for the temperatures are formed with the Equation 6.1 while the non-linear pressure equations are formed with model Equation 5.3.

$$\begin{aligned}
V_{h,i} \cdot a \cdot \rho \cdot c_{ph} \cdot (T_{h,i} - T_{h,o}) - V_{c,i} \cdot a \cdot \rho \cdot c_{pc} \cdot (T_{c,o} - T_{c,i}) &= 0 \\
V_{h,i} \cdot a \cdot \rho \cdot c_{ph} \cdot (T_{h,i} - T_{h,o}) - U \cdot A \cdot \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\ln\left[\frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right]} &= 0 \\
P_{h,i} - P_{h,o} - 0.1582 \cdot \rho^{3/4} \cdot L \cdot D^{-5/4} \cdot \mu^{1/4} \cdot V_{h,i}^{7/4} &= 0 \\
P_{c,i} - P_{c,o} - 0.1582 \cdot \rho^{3/4} \cdot L \cdot D^{-5/4} \cdot \mu^{1/4} \cdot V_{c,i}^{7/4} &= 0 \qquad (5.9)
\end{aligned}$$

By solving the system of equations 5.9 with the Newton-Raphson method the desired solutions for the outlet temperatures and pressures obtained are discussed in Table 6.3.

5.3 Mathematical Model for the Network of Heat Exchangers

As shown in the Figure 3.4, the proposed simplified network of heat exchangers consists of 12 positions with 36 variables (12 each for pressure, temperature and velocity). The known variables will be the inlet temperature and pressure at the hot and cold process inlets (position 1 and 4). All the other 32 intermediate temperature, pressures, and velocities are to be determined by the analysis of heat exchanger network. Therefore, a system of non-linear equations with 32 equations were modeled to reach the goals. Heat exchanger equations for the two heat exchangers modeled using the temperature analysis term LMTD together with the equations formed by the energy balance equations are presented in Equation 5.10.

$$\begin{aligned}
V_1 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_1 - T_2) - V_8 \cdot a \cdot \rho \cdot c_{pc} \cdot (T_{10} - T_8) &= 0 \\
V_1 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_1 - T_2) - U \cdot A \cdot \frac{(T_1 - T_{10}) - (T_2 - T_8)}{\ln\left[\frac{T_1 - T_{10}}{T_2 - T_8}\right]} &= 0 \\
V_2 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_2 - T_3) - V_5 \cdot a \cdot \rho \cdot c_{pc} \cdot (T_7 - T_5) &= 0 \\
V_2 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_2 - T_3) - U \cdot A \cdot \frac{(T_2 - T_7) - (T_3 - T_5)}{\ln\left[\frac{T_2 - T_7}{T_3 - T_5}\right]} &= 0 \\
V_{11} \cdot a \cdot \rho \cdot T_{11} - V_9 \cdot a \cdot \rho \cdot T_9 - V_6 \cdot a \cdot \rho \cdot T_6 &= 0 \\
V_{12} \cdot a \cdot \rho \cdot T_{12} - V_{10} \cdot a \cdot \rho \cdot T_{10} - V_{11} \cdot a \cdot \rho \cdot T_{11} &= 0 \qquad (5.10)
\end{aligned}$$

Mathematical model equations to calculate the pressure drops at heat exchanger 1 (H1)

and heat exchanger 2 (H2) at both hot and cold streams can be represented as folows:

$$\begin{aligned} P_1 - P_2 - f_1 \cdot v_1^{7/4} &= 0 \\ P_2 - P_3 - f_1 \cdot v_2^{7/4} &= 0 \\ P_5 - P_7 - f_2 \cdot v_5^{7/4} &= 0 \\ P_8 - P_{10} - f_2 \cdot v_8^{7/4} &= 0 \end{aligned} \tag{5.11}$$

Where;

$f_1 = 0.1582 \cdot \rho^{3/4} \cdot L \cdot D^{-5/4} \cdot \mu_h^{1/4}$
$f_2 = 0.1582 \cdot \rho^{3/4} \cdot L \cdot D^{-5/4} \cdot \mu_c^{1/4}$

Model equations to calculate the pressure drops at the dividers which are assumed to be control valves are given as follows:

$$\begin{aligned} P_4 - P_5 - k \cdot (y_1 \cdot v_4 \cdot a \cdot \rho)^2 &= 0 \\ P_4 - P_6 - k \cdot ((1 - y_1) \cdot v_4 \cdot a \cdot \rho)^2 &= 0 \\ P_7 - P_8 - k \cdot (y_2 \cdot v_7 \cdot a \cdot \rho)^2 &= 0 \\ P_7 - P_9 - k \cdot ((1 - y_2) \cdot v_7 \cdot a \cdot \rho)^2 &= 0 \end{aligned} \tag{5.12}$$

Where;

$k = \dfrac{0.0104}{C_v^2}$, C_v: Control valve coeffficient

y_1: Propotion of fluid towards H1

y_2: Propotion of fluid towards H2

Model equations to calculate the pressure drops at the mixers which are assumed to be T-junctions are given as follows:

$$\begin{aligned} P_6 - P_{11} - \frac{2gh_{6,11}}{v_6^2} &= 0 \\ P_{10} - P_{12} - \frac{2gh_{10,12}}{v_{10}^2} &= 0 \end{aligned} \tag{5.13}$$

Where;

$h_{6,11}$, $h_{10,12}$ are the head loss coefficients as defined in the Equation 4.13.

To compute the intermediate velocities at the internal pipes both mass balance equations and Bernoulli's equations are used.

$$\begin{aligned}
v_2 - \sqrt{\frac{2}{\rho} \cdot (P_1 - P_2) + v_1^2} &= 0 \\
v_3 - \sqrt{\frac{2}{\rho} \cdot (P_2 - P_3) + v_2^2} &= 0 \\
v_5 - y_1 \cdot v_4 &= 0 \\
v_6 - (1 - y_1) \cdot v_4 &= 0 \\
v_7 - \sqrt{\frac{2}{\rho} \cdot (P_5 - P_7) + v_5^2} &= 0 \\
v_8 - y_2 \cdot v_7 &= 0 \\
v_9 - (1 - y_2) \cdot v_7 &= 0 \\
v_{10} - \sqrt{\frac{2}{\rho} \cdot (P_8 - P_{10}) + v_8^2} &= 0 \\
v_{11} - v_9 - v_6 &= 0 \\
v_{12} - v_{11} - v_{10} &= 0
\end{aligned} \tag{5.14}$$

Finally, as the end conditions at the end of position 3 and position 12 special pipe with a rough internal surface has been used in order to make the pressure drops at the two terminals to be equal to P_3 and P_{12}. Furthermore, by using the pressure drop relation of a pipe and with different physical properties the following equations were modeled to satisfy end conditions.

$$\begin{aligned}
P_3 - f_3 \cdot v_3^{7/4} &= 0 \\
P_{12} - f_4 \cdot v_{12}^{7/4} &= 0
\end{aligned} \tag{5.15}$$

Where;

$f_3 = 0.1582 \cdot \rho^{3/4} \cdot L_1 \cdot D_1^{-5/4} \cdot \mu_h^{1/4}$
$f_4 = 0.1582 \cdot \rho^{3/4} \cdot L_1 \cdot D_1^{-5/4} \cdot \mu_c^{1/4}$

By the combination of the Equations, 5.10, 5.11, 5.12, 5.13, 5.14 and 5.15 will form the system of non-linear equations for the simplified network in the form represented by equation 5.16: as follows with n to be the total number of unknown variables. Furthermore, the total system of equations together will be represented in the Appendix.

$$\begin{cases} f_1(x_1, x_2, \cdots, x_n) = 0 \\ f_2(x_1, x_2, \cdots, x_n) = 0 \\ \vdots \\ f_n(x_1, x_2, \cdots, x_n) = 0 \end{cases} \tag{5.16}$$

5.4 Algorithm of Solving Non-linear Equation Systems

If the system of non-linear equations is expressed as given by the system of Equation 5.16, it can be further written in the form below respectively.

$$F(X) = 0 \tag{5.17}$$

Where the X vector denotes the non-linear system of equations.

$$X^T = [x_1, x_2, \cdots, x_n] \tag{5.18}$$

The vector F consists of n functions interpreted into the system of non-linear equations given by equation 5.16.

$$F^T = [f_1, f_2, \cdots, f_n] \tag{5.19}$$

The solution of the system of equations is usually approximated using the successive calculations starting with some initial conditions.

$$X^{(0)T} = [x_1^{(0)}, x_2^{(0)}, \cdots, x_n^{(0)}] \tag{5.20}$$

By using the Newton Method the solution of the non-linear system given by equation **??** can be determined. Newton method is constructed on the Taylor linear approximation formula.

$$f(X) \approx f(X^{(0)}) + \sum_{j=1}^{n} \left[\left. \frac{\partial f}{\partial x_j} \right|_{X=X^{(0)}} \cdot \Delta x_j \right] \tag{5.21}$$

The generalized form of the Taylor linear approximation on entire n functions of the system of equation5.16 will be formed as follows:

$$f_i(X) \approx f(X^{(0)}) + \sum_{j=1}^{n} \left[\left. \frac{\partial f_i}{\partial x_j} \right|_{X=X^{(0)}} \cdot \Delta x_j \right] \quad i = 1, 2, \cdots, n \tag{5.22}$$

A new form of equation is formed by using the equation (5.18), equation(5.19) and equation(5.22).

$$F(X) = F(X^{(0)}) + J(X^{(0)})\Delta X \tag{5.23}$$

$J(X)$ represents the Jacobean matrix coupled to the system of non-linear equations(5.16).

$$J(X) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{5.24}$$

Newton method is further used in Newton-Raphson method and Broyden method.

5.5 The Newton-Raphson Algorithm

The Newton-Raphson method contains into novel aproximation of the solution of the system of non-linear equations and the aproximation is then clarified by the equation(5.25).

$$X^{(k+1)} = X^{(k)} + \Delta X^{(k)} \tag{5.25}$$

The vector $\Delta X^{(k)}$ can be obtained using the equation (5.23) and $F(X) = 0$ respectively.

$$J(X^k)\Delta X^{(k)} = -F(X^{(k)}) \tag{5.26}$$

If $J^{(k)}$ is a non-singular matrix, the solution of equation (5.26) will be of form:

$$\Delta X^{(k)} = -J(X^{(k)})^{-1}F(X^{(k)}) \tag{5.27}$$

Using the Gauss algorithm numerical solutions of equation(5.27) is obtained. The stopping criteria of the Newton-Raphson method is as folows:

$$\mid f_i(X^{(k)}) \mid \leq \epsilon_i \quad i = 1, 2, \cdots, n \tag{5.28}$$

The entries of the Jacobian matix corresponding to the system of equations (**??**) are as follows:

$$\frac{\partial f_1}{\partial T_{h,o}} = -\dot{m}_{h,i} \cdot c_{ph} \tag{5.29}$$

$$\frac{\partial f_1}{\partial T_{c,o}} = -\dot{m}_{c,i} \cdot c_{pc} \tag{5.30}$$

$$\frac{\partial f_2}{\partial T_{h,o}} = -\dot{m}_{h,i} \cdot c_{ph} - U \cdot A \cdot \frac{-\left(\ln \frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right) - \frac{T_{h,i} - T_{h,o} + T_{c,i} - T_{c,o}}{T_{h,o} - T_{c,i}}}{\left(\ln \frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right)} \tag{5.31}$$

$$\frac{\partial f_2}{\partial T_{c,o}} = -U \cdot A \cdot \frac{-\left(\ln \frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right) - \frac{T_{h,i} - T_{h,o} + T_{c,i} - T_{c,o}}{T_{h,i} - T_{c,o}}}{\left(\ln \frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right)} \tag{5.32}$$

Newton-Raphson method has a disadvantage of evaluating n^2 entries of the Jacobean matrix $J(X^k)$ and the n functions of $F(X^{(k)})$.

6 Numerical Solutions

6.1 Physical Properties and Stream Data of the Used Heat Exchangers

Table 6.1 and Table 6.2 will represent the physical properties of the heat exchangers used along with the inlet temperature and pressure of the hot and the cold stream respectively. The divider positions of the network of heat exchangers will be fixed in this research work to be 0.8 and 0.9 for heat exchanger H1 and heat exchanger H2 respectively. That means towards H1 0.8 of the incoming fluid flow will pass while 0.9 of the incoming flow will pass through H2.

Property	Symbol	Value
Specific heat capacity of the hot and cold process	c_{ph}, c_{pc}	4200 (J/K)
Overall heat transfer coefficient	U	1400(W/m^2)
Length of the pipe	L	10 (m)
Flow coefficient	C_v	5
Gravitational acceleration	g	9.81 (m/s^2)
Diameter of the pipe	D	0.025 (m)
Density of water	ρ	999.97 (Kg/m^3)
Dynamic viscosity of hot water (Average value)	μ_h	0.315×10^{-3} (Kg/ms)
Dynamic viscosity of cold water (Average value)	μ_c	0.798×10^{-3} (Kg/ms)

Table 6.1: Physical Properties of the counter-current heat exchanger used in the thesis

Property	Hot Stream	Cold Stream
Temperature (C°)	95	5
Pressure (Bar)	1	1

Table 6.2: Stream data for the hot and cold inlet

6.2 Numerical Results for a Single Heat Exchanger

6.2.1 Comparison of LMTD Method with Approximations

The outlet temperatures and pressures obtained by the solution of non-linear equations formed with LMTD method and pressure equations were compared with, Paterson's approximation method and Chen's approximation method for a unit heat exchanger and the outlet temperatures and pressures are represented in Table 6.3.

Method	$T_{hout}(C^\circ)$	$T_{cout}(C^\circ)$	P_{hout} (Bar)	P_{cout} (Bar)
LMTD	55.60	83.80	0.9967	0.9987
Paterson's Approximation	58.32	78.36	0.9967	0.9987
Chen's Approximation	63.09	68.83	0.9967	0.9987

Table 6.3: Comparison of the LMTD Method with Approximations

Table 6.3 gives the results which are given for a single heat exchanger and the resuls include the outlet temperature and the pressure of the respective hot and cold streams. As shown by the results the results given by the Paterson's approximation are closely related while Chen's approximation is a bit deviated from the results given with the solutions with the non-linear equations formed with the LMTD term. Here, the pressure outlets remain the saim despite of the method use as the pressure equations are independent from the temperature terms.

6.2.2 Comparison of the Newton-Raphson Method with NTU-method

The outlet temperatures obtained by solving the system of non-linear eqauations by using the Newton-Raphson Algorithm were compared with the results obtained by the $\epsilon - \text{NTU}$ Method and compared results are represented in Table 6.4. As shown the results are similar to each other despite the method used.

Outlet Temperature	Newton-Raphson Method	$\epsilon - \text{NTU}$ Method
Hot Outlet (C°)	55.60063785257	55.600637852571
Cold Outlet (C°)	83.79872429486	83.7987242948581

Table 6.4: Comparison of outlet temperatures of LMTD method with $\epsilon - \text{NTU}$ Method

6.3 Numerical Results for the Network of Heat Exchangers

6.3.1 Solution for Intermediate Temperature, Pressure and Velocity

Position	Pressure (Bar)	Temperature (C°)	Velocity (m/s)
1	1.000000000	95.00	3.1
2	0.970810778	71.65	3.9
3	0.926656834	54.29	4.9
4	1.000000000	05.00	2.6
5	0.999992523	05.00	2.3
6	0.999999907	05.00	0.3
7	0.976567984	33.48	3.2
8	0.976559304	33.48	2.5
9	0.976567441	33.48	0.6
10	0.94986725	55.53	3.4
11	0.99749999	25.16	0.9
12	0.94986049	49.20	4.3

Table 6.5: Solution for Intermediate Temperature, Pressure and Velocity

The results obtained by solving the system of non-linear equations modeled for the simplified network as shown in Figure 3.4, is represented in the Table 6.5. The unknown intermediate temperature, pressure and velocities at the 12 positions were given as the outputs. In this table the first three positions from position 1 to position 3 shows the hot stream while from position 4 to 12 shows he colder streams. It can be seen that the temperature gets cooled down from position 1 to position 3 along the hot process stream as exxpected. Moreover, it can be seen that the temperature of the cold stream gets warmed when moving from position 4 to 12.

6.3.2 Comparison of the Temperature obtained by the Model with the Approximations

Position	LMTD	Paterson's approximation	Chen's approximation
1	90.00	90.00	90.00
2	66.24	62.57	59.18
3	54.69	52.59	51.00
4	05.00	05.00	05.00
5	05.00	05.00	05.00
6	05.00	05.00	05.00
7	61.10	53.49	44.71
8	61.10	53.49	44.71
9	61.10	53.49	44.71
10	70.18	63.98	56.49
11	54.65	47.92	40.14
12	69.08	62.84	55.33

Table 6.6: Comparison of the Temperature obtained by the Model with the Approximations

The model equations built using the Log mean temperature difference term was replaced by the Paterson's approximation to the LMTD and Underwood-Chen's approximation to the LMTD and results obtained are represented in the Table 6.6. Basically the aproximations were used to determine an accurate set of initial conditions to the system of equations.

Viscosity measures the resistance of a fluid and it varies with the temperature. Therefore, the results are compared with the variation of the viscosity with temperature and are represented in the Figure 6.1.

Position	LMTD Method			Varying Viscosity with Temperature		
	Temperature	Pressure	Velocity	Temperature	Pressure	Velocity
1	90.00	1.0000000000	0.11	90.00	1.00000000000	0.10
2	66.24	0.9990804993	0.44	45.17	0.99929926133	0.38
3	54.69	0.9899305512	1.42	35.20	0.99137455077	1.31
4	05.00	1.0000000000	0.10	05.00	1.00000000000	0.12
5	05.00	0.9999999999	0.09	05.00	0.99999999998	0.11
6	05.00	0.9999999999	0.01	05.00	0.9999999999	0.01
7	61.10	0.99927447423	0.39	39.40	0.9989630276	0.47
8	61.10	0.9992744741	0.31	39.40	0.9989630274	0.37
9	61.10	0.9992744742	0.07	39.40	0.9989630276	0.09
10	70.18	0.9930295329	1.16	51.44	0.9916100883	1.27
11	54.65	0.9183635744	0.08	35.36	0.9451947014	0.10
12	69.08	0.9929491866	1.24	50.20	0.9915437526	1.37

Figure 6.1: Comparison with visocity variation

7 Summary & Discussion

One of the most common problems in industry is the excessive consumption of energy. Studies in heat recovery systems has been increased after the first world energetic crisis during seventies. Therefore, minimization in consumption of energy has increased recently. A large amount of heat energy is used in cooling systems. It is often possible to collect and reuse part of the heat energy by Heat Exchangers and this produces savings. Instead of using a single HE-unit one may couple them together in a network. Such a system is built by combining hot and cold process streams, several heat exchangers and adding mixers, dividers or other equipment for heat transfer between the streams. The utility costs can be minimized by optimizing the performance of such a system of heat exchangers by controlling how the cold flow is running through the system. This control is done by optimal configuration of the divider valves. In general such a network may contain several HE-devices and several divider valves. In this work a simple reduced model was studied having only two divider components. The same approach can be easily generalized in to a bigger system. This research work will be a part of a current ongoing research at the department and the results of this work has been used for further research under optimization of the heat exchanger analysis. The scope of this research work was to build a system of non-linear equations to the given network of heat exchangers and hence, determine the temperatures, pressures and velocities at the intermediate positions. Moreover, the work will be extended to optimize the best suitable position of the existing divider valves in order to minimize the pressure drop hence to minimize the pumping cost and to maximize the thermal performance by maximizing the heat transfer. The objective function (utility function) will be formed by a combination of the respective two objectives.

To find the optimal position of the valve, there are several approaches that can be used. The first methodology is to follow classical approaches, multivariate optimization methods, for example gradient based algorithms such as the method of Steepest Descent. Also gradient free-optimization approach (fminsearch) will be a possible approach.

The second possible methodology is the evolutionary based algorithms such as Genetic Algorithm and Differential Evolution.

Finally the third approach that one could try is Metropolis-type methods of stochastic search. At the optimization stage the feasibility of different approaches should be tried. Especially in the case where instead of the reduced model one would be searching the optimal control of a large scale problem with higher number of variables, equations and a complex design of the network, the choice of optimization method may turn out

to be crucial.

Finally in the optimization phase an important question will be the robustness of the method and sensitivity of the solution. If there are errors in the input data or model parameters, how much variation will appear in the solution of the state equations would be computed. Similarly when the optimal valve positions are computed, how sensitive is the optimal solution with regard to the initial values or the model parameters are computed.

These are the natural questions in this extended research and they will require approaches of uncertainty quantification. Monte Carlo Markov Chain (MCMC) is used to obtain the distribution of optimal values and measure the sensitivity of the solutions. This approach of MCMC has been studied at LUT deeply and will be the main approach studied in the uncertainty analysis part in the continuation of this thesis.

The optimization task can be described as follows: The input vector of the system is given by,

$$(X, \theta) = (P_1, T_1, P_4, T_4, y_1, y_2) \tag{7.1}$$

Where; the valve settings are denoted by $\theta = (y_1, y_2)$. The response vector (output vector) $Y = (V_1, \cdots, V_{12}, P_2, P_3, P_5, \cdots, P_{12}, T_2, T_3, T_5, \cdots, T_{12})$ consists of the values of state variables computed at the intermediate positions of the network of heat exchangers.

The system of non-linear algebraic equations can be written in vector notation as follows:

$$F(Y, X, \theta) = 0 \tag{7.2}$$

By using Matlab the solution of Y for given (X, θ) can be obtained. The matlab function is denoted by $Y = F^*(X, \theta)$.

The objective of the optimization will be to obtain the best performance for the heat exchanger network by maximizing heat transfer and minimizing pressure loss. Choosing suitable weights/prices the objective function to be maximized can be expressed as follows

$$u(Y) = u(Y, \theta) = c_T(T_1 - T_3) - c_P(P_{12} - P_4) \tag{7.3}$$

The objective is to compute the valve setting $(\theta = y_1, y_2)$ for a given input X such that the state Y of the system $F(Y, X, \theta) = 0$ produces maximal value for the utility u(Y).

Referring to the Matlab function F^* the valve setting $\theta = (y_1, y_2)$ which maximizes the

value of $u(\theta) = U[F^*(X, \theta)]$ is computed.

On the other hand, the optimization task can also be written in the form:

Minimize with respect to (Y, θ) the function

$$W||F(Y, X, \theta)||_2 - u(Y)$$

Where; W is the weight factor.

There are few assumptions that have been used in building the system of non-linear equations to model the network such as: that there will be no phase change involved in the system of heat exchangers and the pipe flows will only contain liquids (water), all the connecting pipes of the system will have a constant diameter, the heat exchangers used here will only be of counter-flow heat exchangers with pipes inside. Moreover, initially it was considered that the viscosity of the two fluids are constant(average value depending on the temperature variance) for both hot and cold processes but finally, with the variance of viscosity the results were obtained and represented in Table **??**. Furthermore, in building equations to reduce the complexity it was considered that the pressure loss in straight pipes between the devices are negligible. If the pipe lengths of the connecting pipes were taken into account the pressure drop equations should be changed (the amount of drop in pressure to a given length of the pipe should be added) according to the Equation 4.4. Since, the pressure drop along the pipes is rather a small number (0.091 %) compared to the inlet temperature it is logical to consider them to be negligible. Moreover, it is assumed that there is no variation in height hence, the potential energy is omitted from the loss formulas.

References

Uma Maheshwar Kiran Ati. Heat exchanger network design, monitoring and optimization. Master's thesis, McMaster University Hamilton, Ontario, Canada., 2009.

Robert P. Benedict. *Fundamentals of Pipe Flow*. John Wiley and Sons, 1980.

J. J. Chen. Comments on improvements on a replacement for the logarithmic mean. *Chemical Engineering Science*, 42(10):2488–2489, 1987.

Clayton T. Crowe, Donald F. Elger, Barbara C. Williams, and John A. Roberson. *Engineering Fluid Mechanics*. John Wiley and Sons, 2010.

Daniel Greiner Edvardsen. Optimal operation of heat exchanger networks. Master's thesis, Norwegian University of Science and Technology, 2011.

Christodoulos A. Floudas. *Nonlinear and Mixed-Integer Optimization*. Oxford University Press, Princeton, New Jersey, 1995.

Dominique Habimana et al. Statistical optimum design of heat exchangers. 2009.

Geoffrey F. Hewitt, G. L. Shires, and Theodore R. Bott. *Process heat transfer*. CRC Press, 1994.

J.P. Holman. *Heat Transfer*. McGraw-Hill, 2001.

Frank P. Incropera, David P. Dewitt, Theodore L. Bergman, and Adrienne S. Lavine. *Fundamentals of Heat and Mass Transfer*. John Wiley and Sons, Notre Dame, Indiana, 2007.

S. Kakaç, H. Liu, and A. Pramuanjaroenkij. *Heat Exchangers: Selection, Rating, and Thermal Design, Third Edition*. Taylor & Francis, 2012.

B.G. Liptak. *Instrument Engineers' Handbook, Fourth Edition, Volume Two: Process Control and Optimization*. CRC Press, 2005.

John H Mathews and Kurtis D Fink. *Numerical methods using MATLAB*, volume 31. Prentice hall Upper Saddle River, NJ, 1999.

E.W. McAllister. *Pipeline Rules of Thumb Handbook: A Manual of Quick, Accurate Solutions to Everyday Pipeline Engineering Problems*. Elsevier Science, 2013.

Robert H. Nunn. *Intermediate fluid mechanics*. Taylor & Francis, 1989.

Cristian Patrascioiu and Cristian Marinoiu. The applications of the non-linear equations systems algorithms for the heat transfer processes. In *Proceedings of the 12th WSEAS international conference on mathematical methods, computational techniques and intelligent systems Kantaoui-Sousse, Tunisia*, pages 30–35, 2010.

José M Ponce-Ortega, Medardo Serna-González, and Arturo Jiménez-Gutiérrez. Minlp synthesis of optimal cooling networks. *Chemical Engineering Science*, 62(21):5728–5735, 2007.

Jussi Saari. Heat exchanger thermal design guide, 2010. Lappeenranta University of Technology, Finland.

Ramesh K. Shah and Dusan P. Sekulic. *Fundamentals of Heat Exchanger Design*. John Wiley and Sons, Hoboken, New Jersey, 2003.

Andrew Sleigh and Cath Noakes. An introduction to fluid mechanics @ONLINE, 2009. URL `http://www.efm.leeds.ac.uk/CIVE/FluidsLevel1/Unit00/T1.html`.

K. Thulukkanam. *Heat Exchanger Design Handbook, Second Edition*. CRC Press, 2013.

VDI-Gesellschaft Verfahrenstechnik und Chemieingenieurwesen and VDI Gesellschaft. *VDI Heat Atlas*. Springer Science & Business Media, 2010.

Paritosh R. Vasava. Fluid flow in t-junction of pipes. Master's thesis, Lappeenranta University of Technology, Finland, 2007.

Ari Vepsalainen, Janne Pitkanen, and Timo Hyppanen. Fundamentals of heat transfer, 2012. Lappeenranta University of Technology, Finland.

T.F. Yee and Grossmann. Simultaneous optimization model for heat integration ii heat exchanger network synthesis. *Computers and Chemical Engineering.*, 14:1165–1184, 1990.

APPENDIX

7.1 Darcy-Weisbach Equation

In this section Darcy-Weisbach equation, which is one of the most useful equations in fluid mechanics is presented. It is used to calculate the head loss which occurs in a fluid in a straight flow pipe.

Derivation of the Darcy-Weisbach Equation

Assume that the fluid is fully developed and steady in a circular tube with a constant diameter D as shown in the Figure 7.1. A cylindrical shaped control volume of diameter D and length ΔL is considered. A cylindrical coordinate system is defined with an axial coordinate in the streamwise direction s and the radial coordinate in the direction r. The momentum equation is applied to the control volume which is shown in Figure7.1.

$$\sum F = \frac{d}{dt}\int v\rho dV + \int v\rho V \cdot dA \tag{7.4}$$

The equation (7.4) expresses that the net forces are equal to the summation of momentum accumulation rate and the net efflux of momentum respectively. The three terms of equation (7.4) is analyzed along the stream wise direction and the net flux of momentum is zero since velocity at position 2 is similar to the velocity at position 1. Since the flow is steady the momentum accumulation is also zero and the equation (7.4) reduces to $\sum F = 0$. The net forces in the streamwise direction is given by,

$$F_{pressure} + F_{shear} + F_{weight} = 0 \tag{7.5}$$

$$(p_1 - p_2)\frac{\pi D^2}{4} - \tau_0 \pi D \Delta L - \gamma \frac{\pi D^2}{4}\Delta L \sin\alpha = 0 \tag{7.6}$$

Figure 7.1 shows that $\sin\alpha = \dfrac{\Delta z}{\Delta L}$ and then,

$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \frac{4\Delta L \tau_0}{D} \tag{7.7}$$

Next, the energy equation(7.8) reduces to equation after applying the energy equation to the control volume with $h_p = h_t = 0$, $V_1 = V_2$, and $\alpha_1 = \alpha_2$.

$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \gamma h_L \tag{7.8}$$

Combining Equation (7.7) and Equation (7.8) and in addition replacing ΔL by L and by introducing a new symbol h_f for the head loss in pipe,

$$h_f = \text{head loss in a pipe} = \frac{4L\tau_0}{D\gamma} \tag{7.9}$$

Rearranging the Equation(7.9),

$$h_f = \left(\frac{L}{D}\right)\left\{\frac{4\tau_0}{\rho V^2/2}\right\}\left\{\frac{\rho V^2/2}{\gamma}\right\} = \left\{\frac{4\tau_0}{\rho V^2/2}\right\}\left(\frac{L}{D}\right)\left\{\frac{V^2}{2g}\right\} \tag{7.10}$$

A dimensionless new friction factor f that gives the ratio of wall shear stress τ_0 to kinetic pressure$\rho V^2/2$ is defined.

$$f \equiv \frac{4 \cdot \tau_0}{\rho V^2/2} \approx \frac{\text{shear stress acting at the wall}}{\text{kinetic pressure}} \tag{7.11}$$

Friction factor is generally known as: friction factor, Darcy friction factor, Darcy-Weisbach friction factor, and the resistance coefficient. Darcy-Weisbach Equation is derived by the combination of Equation(7.10) and Equation(7.11) as follows.

$$h_f = f.\frac{L}{D}.\frac{V^2}{2g} \tag{7.12}$$

Darcy-Weisbach equation can be used if the flow is completely developed and steady. It is used for either laminar flow or turbulent flow and for either circular pipes or non circular pipes such as a rectangular pipe. Purpose of using the Dacy-Weisbach equation is to calculate a value for the friction factor. As stated in the equation the friction factor, the pipe-length-to-diameter ratio, and the mean velocity squared influence the head loss.

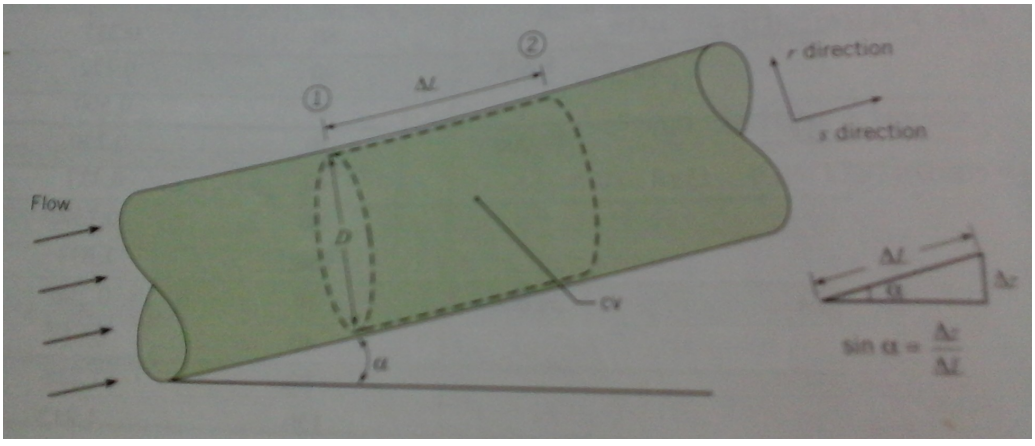

Figure 7.1: Control unit for the derivation of Darcy-Weisbach equation

System of Non-linear Equations for the Simplified Network

$$
\begin{aligned}
v_1 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_1 - T_2) - v_8 \cdot a \cdot \rho \cdot c_{pc} \cdot (T_{10} - T_8) &= 0 \\
v_1 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_1 - T_2) - U \cdot A \cdot \frac{(T_1 - T_{10}) - (T_2 - T_8)}{\ln\left[\frac{T_1 - T_{10}}{T_2 - T_8}\right]} &= 0 \\
v_2 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_2 - T_3) - v_5 \cdot a \cdot \rho \cdot c_{pc} \cdot (T_7 - T_5) &= 0 \\
v_2 \cdot a \cdot \rho \cdot c_{ph} \cdot (T_2 - T_3) - U \cdot A \cdot \frac{(T_2 - T_7) - (T_3 - T_5)}{\ln\left[\frac{T_2 - T_7}{T_3 - T_5}\right]} &= 0 \\
V_{11} \cdot a \cdot \rho \cdot T_{11} - V_9 \cdot a \cdot \rho \cdot T_9 - V_6 \cdot a \cdot \rho \cdot T_6 &= 0 \\
V_{12} \cdot a \cdot \rho \cdot T_{12} - V_{10} \cdot a \cdot \rho \cdot T_{10} - V_{11} \cdot a \cdot \rho \cdot T_{11} &= 0 \\
P_1 - P_2 - f_1 \cdot v_1^{7/4} &= 0 \\
P_2 - P_3 - f_1 \cdot v_2^{7/4} &= 0 \\
P_5 - P_7 - f_2 \cdot v_5^{7/4} &= 0 \\
P_8 - P_{10} - f_2 \cdot v_8^{7/4} &= 0 \\
P_4 - P_5 - k \cdot (y_1 \cdot v_4 \cdot a \cdot \rho)^2 &= 0 \\
P_4 - P_6 - k \cdot ((1 - y_1) \cdot v_4 \cdot a \cdot \rho)^2 &= 0 \\
P_7 - P_8 - k \cdot (y_2 \cdot v_7 \cdot a \cdot \rho)^2 &= 0 \\
P_7 - P_9 - k \cdot ((1 - y_2) \cdot v_7 \cdot a \cdot \rho)^2 &= 0 \\
P_6 - P_{11} - \frac{2gh_{6,11}}{v_6^2} &= 0 \\
P_{10} - P_{12} - \frac{2gh_{10,12}}{v_{10}^2} &= 0 \\
v_2 - \sqrt{\frac{2}{\rho} \cdot (P_1 - P_2) + v_1^2} &= 0 \\
v_3 - \sqrt{\frac{2}{\rho} \cdot (P_2 - P_3) + v_2^2} &= 0 \\
v_5 - y_1 \cdot v_4 &= 0 \\
v_6 - (1 - y_1) \cdot v_4 &= 0 \\
v_7 - \sqrt{\frac{2}{\rho} \cdot (P_5 - P_7) + v_5^2} &= 0 \\
v_8 - y_2 \cdot v_7 &= 0 \\
v_9 - (1 - y_2) \cdot v_7 &= 0 \\
v_{10} - \sqrt{\frac{2}{\rho} \cdot (P_8 - P_{10}) + v_8^2} &= 0 \\
v_{11} - v_9 - v_6 &= 0 \\
v_{12} - v_{11} - v_{10} &= 0 \\
P_3 - f_3 \cdot v_3^{7/4} &= 0 \\
P_{12} - f_4 \cdot v_{12}^{7/4} &= 0
\end{aligned}
$$

7.2 Derivation of log mean temperature difference

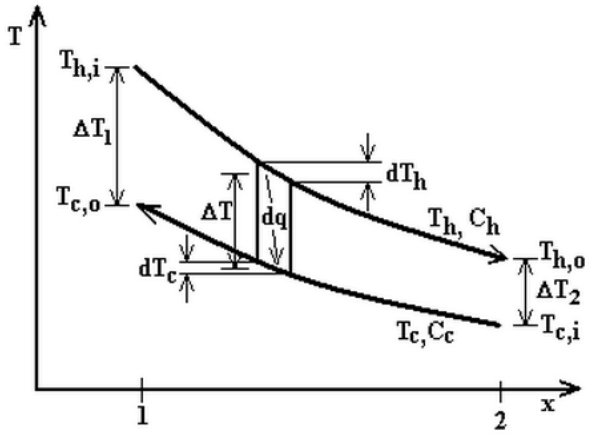

Figure 7.2: Temperature distribution of a counter-flow Heat Exchanger

By applying the energy balance to each of the differential elements of Figure XX,

$$dq = -\dot{m}_h c_{p,h} dT_h = -C_{p,h} dT_h = -\dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) \tag{7.13}$$

and

$$dq = -\dot{m}_c c_{p,c} dT_c == -C_{p,c} dT_c = \dot{m}_h c_{p,h}(T_{c,o} - T_{c,i}) \tag{7.14}$$

The amount of heat transferred across the surface area dA can be represented as:

$$dq = U \Delta T dA \tag{7.15}$$

$\Delta T = T_h - T_c$ is the temperature difference between both hot and cold fluids.

$$d(\Delta T) = dT_h - dT_c \tag{7.16}$$

Where; $C_h = \dot{m}_h c_{p,h}$ and $C_c = \dot{m}_c c_{p,c}$.

By substituting Equation 7.13 and 7.14 in 7.16:

$$d(\Delta T) = -dq\left(\frac{1}{C_{ph}} - \frac{1}{C_{pc}}\right) \tag{7.17}$$

By integrating Equation 7.17 across the heat exchanger from 1 to 2:

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \int_1^2 dA \tag{7.18}$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \tag{7.19}$$

$$\begin{aligned}\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) &= -UA\left(\frac{T_{h,i} - T_{h,o}}{q} + \frac{T_{c,o} - T_{c,i}}{q}\right) \\ &= -\frac{UA}{q}[(T_{h,i} - T_{c,i}) - (T_{h,o} - T_{c,o})]\end{aligned} \tag{7.20}$$

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \tag{7.21}$$

$$q = UA\Delta T_{TM} \tag{7.22}$$

Where;

$$\Delta T_{LM} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \tag{7.23}$$