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Financial Management

An Empirical Comparison of Option Pricing Model Performance in the DAX Index Option Market

Bachelor's Thesis
Jaakko Salminen
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Instructor: Sheraz Ahmed

Contents

1. Introduction

The fast growth of derivatives markets has made them a major part of today's finance. This makes them an interesting and important subject for research as well. Financial innovation has brought a myriad of new products to the market over the past decades. In the pace these products have spread to the market there has not been a lot of time to gain experience in what kind of effects their wide adoption will have. (Sveiby 2012) This is of course a problem for regulators who have a lot of work to do to keep up with the innovation. In the past there has been failures in the financial industry that demonstrate the need for effective regulation. Extensive research is also called for before the wide adaption of new products to prevent the mishaps like the Long Term Capital Management case and the subprime crisis. A lot of research is already done in this space and the work that the researchers do today is as important as ever.

This study is done to showcase the development of option pricing models and to test their performance compared to the original Black-Scholes Model (1973). The Black-Scholes model was revolutionary at its time of introduction and was a big step forward but has later been proven to have notable flaws that make its use in its original form questionable. Advanced models have been introduced since, that in the light of previous studies perform better. The complexity of the models has also grown which makes them more unpractical for professionals to use leaving practitioners to balance between usability and accuracy.

Previous studies have shown the traditional Black-Scholes model to have problems estimating prices of certain options because of the constant volatility assumption (Hull and White 1987; Derman and Kani, 1994; Dupire, 1994; Rubinstein, 1994). Option pricing models have later been developed to take different approaches to volatility estimation, which should give better results in estimating option prices.

The Black-Scholes model has been later modified to deal with volatility with more flexibly and these new versions are being used in practice today. At the same time focus in option pricing studies has drifted from the original Black-Scholes model, to the more recently introduced models. In addition to the original Black-Scholes model this study focuses on an ad hoc procedure of the Black-Scholes model introduced by Dumas, Fleming & Whaley (1998), that smooths the Black-Scholes implied volatilities across exercise prices and times to maturity, and a model created by Heston and Nandi (2000) that uses the GARCH process to estimate volatility. These models are used to estimate future option prices using DAX index options.

The major difference between the models is the way they estimate volatility. Volatility is a key variable in all the option pricing models and has a significant effect on the estimated theoretical option prices. The original Black-Scholes model assumes volatility to be constant which has later been proven to be a major problem in the model. Realized volatility is often used in the model but according to studies it might not give a good estimate about future volatility. The ad hoc Black-Scholes model uses implied volatility to estimate a specific volatility for each option depending on its exercise price and time to maturity. The model simply smooths the implied volatilities to gain a more accurate estimate. (Dumas et al. 1998) This approach is commonly used among practitioners since it is fairly simple but gives more precision to option pricing (Berkowitz 2009). The GARCH model used in the study assumes that volatility is a stochastic process that has the tendency to return to its long run average. Because the model assumes that volatility is a random process and not constant it should be able to price options with more precision than the Black-Scholes model. However GARCH based models are not that commonly used outside academic studies since they are quite demanding.

It will be interesting to see how the three fundamentally different models perform in comparison. The Black-Scholes model is well-known and has had a great impact to option pricing, but the fact is that its questionable assumptions make it vulnerable for mispricing of options. The ad hoc Black-Scholes model is more commonly used in practice and is expected to perform better. It is still merely simple and easy estimation process compared to the GARCH model. The GARCH model however has been to produce accurate and steady estimates but it is interesting to find out wheatear its more demanding estimation process makes it notably more precise.

The main research question of the study is:

- Which option pricing model predicts option prices most accurately in the DAX option market?

To help answer the main research question a supporting questions is formed, which is:

- How much does time to maturity and moneyness affect the accuracy of the models?

The study is constructed in the following way. First, in the theoretical section, the most important aspects of options and option pricing are presented and then the latest relevant option pricing studies are reviewed. After the theory section the empirical study is presented. The empirical section starts with an introduction of methodology and data in addition to a closer look to how the option pricing models were used in practice and after that the results of the study are presented and the paper ends to a conclusions section.

2. Theoretical framework

In the theory section of this paper we review the most important theoretical knowledge related to option pricing. Firstly, since options are derivatives, there is a short overview of different derivatives and derivatives markets. It is important to review the whole asset class to see what are the parts of this growing and dynamic field. After that we get to the center of the subject and take a closer look at options and how they are valuated and used in practice. In the end of the theoretical section we look at one integral part in option pricing, volatility. As the last part of the theoretical section we take a closer look at the three option pricing models used in this study.

2.1 Derivatives

Derivatives are an important part of modern finance. They are often used in risk management and can be used e.g. as an insurance against price movements. Derivatives are also used to get access to assets or markets when trading would be difficult otherwise. There are a myriad of different derivatives but they all have one thing in common, their value is derived from the underlying asset. The underlying asset is often a simple security like a stock but it can also be something a lot more complicated like weather conditions. The most common derivatives are swaps, futures, options and forwards. (Hull 2005)

2.1.1 Derivatives markets

First derivatives were developed as early as the 17^{th} century but the derivatives market didn't see much growth until the 1970's due to lack of a suitable pricing model, low volatility and high regulation. In the 70's the growth in derivatives markets was pushed forward by a number of reasons. Volatility in interest rates and exchange rates increased rapidly which created more need for hedging against the changes. Deregulation and the growth in international trade and finance also contributed to the need of new products for risk management. One of biggest developments in the use of derivatives was the introduction of the Black-Scholes formula for pricing options in 1973. Black-Scholes formula immediately became widely used in option pricing but it was also utilized to create and price new types derivatives. All of this resulted in fast growth of the derivative markets. (Stulz 2004)

The trade of derivatives is divided in to two different markets, the exchange markets and over-the-counter (OTC) markets. The exchange market is where individuals trade standardized products listed for public trading. Exchange traded derivatives consist mostly of options and futures (BIS 2014). The first exchange was the Chicago Board of Trade, established in 1848 (CBOE 2014a). Trading in the over-the-counter market is done between financial institutions or the financial institutions and their clients. The trading is done privately between the participants without an actual exchange. Derivatives traded in OTC markets are forwards, options and swaps (BIS 2014). The benefit of the OTC trading is that the traded contracts do not have to be standardized and the participants are free to agree on whatever deal they see fitting.

Despite the slow economic growth since the latest financial crisis the derivatives markets have still grown at a fast pace. According to Bank for International Settlements (2014) size of the OTC market measured in notional amounts of outstanding contracts is 691 trillion US dollars, which means a growth of 214% in the past 10 years. The exchange traded derivatives market is considerably smaller at notional amount of outstanding contracts of 73 billion dollars. The options that are used in this study are exchange traded index options and their amount in the market is 6,5 billion dollars. (BIS 2007;2014)

2.2 Options

Option is a contract that gives the buyer the right to buy or sell an asset at a specific price (exercise price) on a specific time (exercise time). An option that gives the buyer the right to buy or in other words take a long position on an asset is called a call option. An option that gives the right to sell or take a short position on an asset is called a put option. Unlike in futures contracts the buyer of an option is not obligated to exercise the option but has a choice. If the buyer of an option chooses to exercise the option the seller is obligated to take the other side of the transaction. There are two types of options, American options and European options. An American option can be exercised at any point of maturity and a European option is exercised only at the time of expiration. (Ross, Westerfield, Jaffe 2002 p. 614)

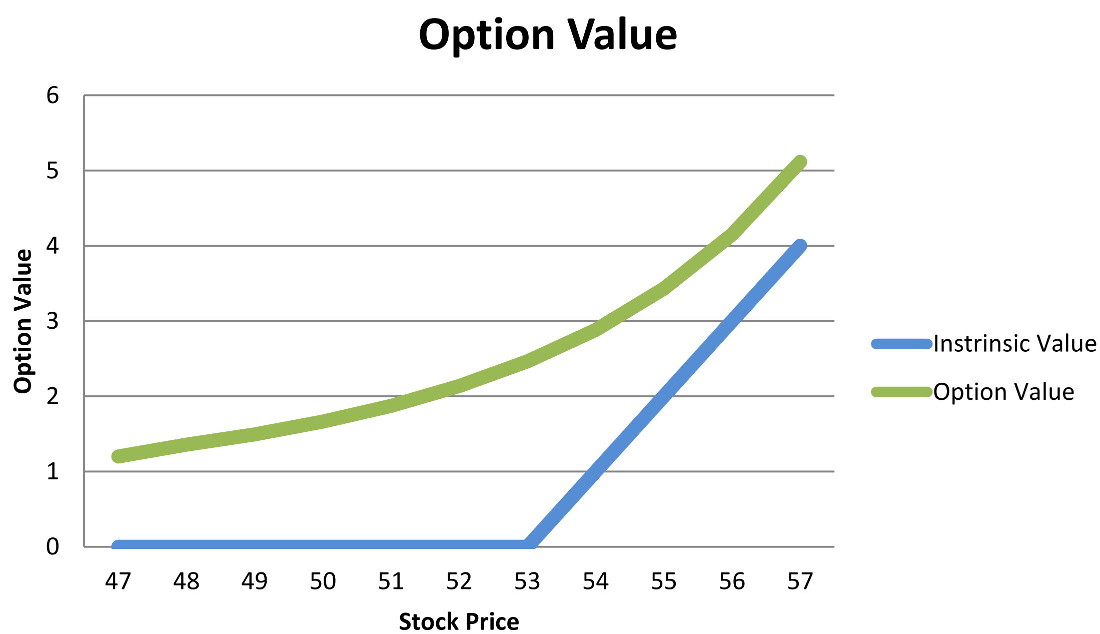
The buyer of the option pays a premium (price of the option) for the seller, this is guaranteed income for the seller, but the profits and losses depend on the relationship between the exercise price and the asset price. If the price of the underlying asset is higher than the exercise price at the time of expiration, the buyer of a call option makes a profit. On the contrary, a buyer of a put option makes a profit if the price of the asset is lower than the exercise price at expiration. The trade of options is a zero-sum-game, the buyers profit is the sellers loss, and vice versa. (Ross et al. 2002 p. 616) When the exercise price of a call option is lower than the current price of the underlying asset the option is in-the-money and when the exercise price is higher, the option is out-the-money. Put options on the other hand are in-the-money when the exercise price is above the current market price of the underlying asset and out-the-money when the exercise price is below the current price of the underlying asset. If the exercise price and the asset price are at the same level the option is at-the-money. (Ross et al. 2002 p. 614) In research papers these three stages are commonly referred to as the moneyness of the option.

Exchange traded equity options are physically settled which means if the option is exercised there is an actual equity transaction between the buyer and the seller (CBOE 2014b). But with some options this is not possible due to high transaction costs or the type of underlying asset. Then the option will be cash settled. This means that if the option is exercised the buyer simply receives the difference between current underlying price and the exercise price from the seller in cash. One common example of a cash settled option is index options, like the DAX index options used in this study.

2.3 Option value

The value of a sock option is a sum of two parts, its intrinsic value and extrinsic value. The intrinsic value is the fundamental value of a financial asset, in this case the payoff of the option. The payoff is the difference between the exercise price and the current stock price. (Hull 2005 p. 186) Generally the value of a call option increases as the price of the underlying asset increases and the value of a put option will increase when the value of the underlying asset decreases. Figure 1 represents the value of a call option which exercise price is 53 €. As usual the payoff is positive if the stock price is below the exercise price. At the stock price of 53 € and under the payoff is zero, so the intrinsic value is zero. However the option has still value because the option has extrinsic value left. Extrinsic value is commonly known as time value. It represents the potential that the payoff will be positive before time of maturity. Time value can be measured by subtracting intrinsic value from the price of the option. In the diagram time value is the space between the two lines. Time value is the premium that the investor pays for the current exercise value. Time value will decrease over time as maturity nears. When there is less time to maturity the probability of a better exercise value decreases so the time value decreases. The decrease does not happen steadily during the maturity but at an accelerating rate so the decrease in time value gets faster closer to maturity.

Figure 1. Option intrinsic and time value



To understand the theoretical option valuation models let's build a simple example for a theoretical option pricing model based on expected returns. A stock is currently trading at 100 € and at a point in time in the future there are five possibilities for the price: 80 €, 90 €, 100 €, 110 €, or 120 €. Each possibly has 20 % probability of happening. If an investor opens a long position in the stock at a price of 100 € what will be the excepted return?

-20 € x 20 % -10 € x 20 % + 0 € x 20 % + 10 € x 20 % +20 € x 20 % = 0 €

Since the possible profits and losses offset each other the expected return for a stock investment is zero. Now if the investor had opened a long position in a call option with an exercise price of 100 € in the same stock. What would be the expected return? If the premium paid for the option is not accounted for and the possible stock prices and the probabilities are still the same the expected return for a call option is:

0 € x 20 % +0 € x 20 % + 0 € x 20 % + 10 € x 20 % +20 € x 20 % = 6 €

If the stock price is under the exercise price the payoff for the buyer of the option is zero because the option will not be exercised and it will expire worthless. This means that the expected return for an option can never be under zero but the at same time the whole investment can be lost when the options become worthless where as in the stock investment the investor would still have possession of the stocks. Taking this risk gives the investor an expected return of 6 € which is higher than the stock investments expected return.

In this simplified approach a theoretical price for the option can be estimated by using expected returns. However this model simplifies things a lot and does not account for many factors that affect option prices. For one the probabilities for all the possible outcomes of the stock price would not be evenly distributed but would be something closer to a normal distribution. Also to develop a realistic pricing model more variables should be added to the formula.

How different variables affect the price of an option are listed in Table 1. Naturally the price of the underlying asset affects the options intrinsic value and the therefore the price of the option, and as demonstrated above the effect is opposite for calls and puts. Time to maturity obviously affects the time value of an option. As the time to maturity decreases so does the value of the option. Like in any financial valuation the interest rate have to be accounted for because of the opportunity costs that have a small but still measurable effect. Since the estimated payoff will be in the future the owner of the underlying asset will have interest costs or a loss of interest income for owning the underlying asset, which will reflect to the option price. The seller of a call option will get compensation for the costs of carry in a higher option premium as the buyer of a put option will get compensation in a lower premium. This is why the effect of changes in interest rate is reverse for call and put options. Possible dividends would have to be taken in to consideration because the price of a stock usually drops the amount of the dividend at the ex-dividend date and a change in the price of the underlying asset naturally affects the option price. All of the first four variables in the table are easy to obtain for all most any investor but the last variable, volatility, is more difficult to estimate. This is point is also a major difference between the pricing models. Volatility is very important in option pricing because after all the changes in price are the reason why options exist. The more volatile the price of the underlying asset, the better the odds are, that the underlying price will end up further away from the current price by the time of expiration. Therefore rise in volatility of the underlying asset's movements drives up both call and put option prices. (Ross et al. 2002 p. 622-628; Natenberg 1994 p. 44-49)

Table 1. Variables affecting option prices

Variable	Change in Variable	Call Option Value	Put Option Value
Price of the underlying asset	↑	↑	↓
Time to maturity	↓	↓	↓
Interest rates	↑	↑	↓
Future dividends	↑	↑	↓
Volatility of the underlying asset	↑	↑	↑

2.4 How options are used?

Options are versatile derivatives that can be used to both speculation and risk management. Options may be used in hedging against changes in asset prices. For example if an investor has a substantial position in a stock he might want insurance against unexpected changes in the price. Options offer a way of managing risk of sudden fluctuations in stock prices. An investor can insure himself against losses by buying put options of a stock that the investor has a long position in. For this insurance the investor has to pay a fee the amount of the option premium. Many companies use options for risk management and hedging is very common in the corporate world and by using options for hedging companies can also enjoy the possible upside in the risk they are hedging against. With futures or forwards this would not be possible but because with options the company has the right but not the obligations to exercise the options they can choose not to exercise. Options also offer possibilities to return structures that could not be achieved by a direct investment in the underlying asset. With a simple stock investment the only possibility to gain is if the stock goes up but with a put option investment an investor can gain if the stock goes down. There are multiple strategies available for an option trader to gain from stock price movements. By combinations of buying call and put options investors can even gain weather the stock goes up or down. (Ross et al. 2002 p.618)

2.5 Volatility

Volatility expresses the uncertainty of the returns. It can be defined as the standard deviation of the returns over a certain time. If volatility is high the probability that the stock will perform exceptionally well or exceptionally poor is high. Therefore the probability that an option is in-the-money at the time of expiration is higher when the volatility is high. As it can be seen there is a distinct connection between volatility of the underlying asset and the returns of the option. This also leads to the fact that the volatility of the underlying asset has a major impact on the option price.

Historical or realized volatility can be calculated from historical data but future volatility is more difficult to estimate. Calculation using the realized volatility is the simplest way but the problem is how long the sample should be. Longer sample might make the estimate more accurate but since volatility is not constant over time the past returns might not be a good estimate of the future volatility. There are numerous ways to estimate volatility statistically in addition to realized volatility. Moving average models smooth the random price fluctuation and form a lagged trend for volatility. Two well-known moving averages are SMA (Simple moving average) and EWMA (exponentially weighted moving average) which gives more weight to the latest observations. Autoregressive models have become popular in academic research due to their good future volatility forecasting ability. Some common autoregressive models are ARMA (autoregressive moving average), ARCH (autoregressevie conditional heteroscedasticity) and GARCH (generalized autoregressive conditional heteroscedasticity). The motivation behind ARCH and GARCH models is that, there is a relationship between todays squared returns and past squared returns. ARCH and GARCH models utilize this relationship to estimate volatility. (Abdella & Winker 2012) This way they can forecast future volatility very accurately by using the past returns.

It is difficult to estimate future volatility because it is impossible to know exactly what will happen in the future. Statistical methods only work with historical data and don’t have knowledge of up coming events that might effect volatility. Investor expectations of future volatility often differ from statistical estimations. What the investors in the market expect for future volatility is known as implied volatility. This markets estimate of the future volatility can be estimated using the Black-Scholes formula. Implied volatility can be obtained by setting the Black-Scholes price estimate to match the current market price of the option, and keeping all the other variables constant and then solving volatility from the formula. Option traders can use this information to evaluate if an option is under or overvalued. (Natenberg 1994 p.73-74)

2.6 Option pricing models

During the history of option pricing there have been numerous pricing models introduced. Large number of these models was motivated by the original Black-Scholes model and its strict assumptions of constant volatility, and normal distribution of returns. To relax these assumptions multiple new models have been created that also perform better than the Black-Scholes model. It is difficult to divide the pricing models in to groups since a lot of the models have similarities but also differences at the same time. In this section we will go through the most important option pricing models and after that take a deeper look in to the three models used in this study.

2.6.1 Stochastic models and Binomial Trees

Since the introduction of the Black-Scholes model, a large part of the studies has focused on stochastic models. Constant elasticity of variance (CEV) model was one of the first models that assumed volatility to be a stochastic process (Cox 1975) and it also allowed correlation of the underlying asset and volatility. The model was not very successful but better stochastic models have been introduced since. The results of stochastic volatility models in option pricing have been studied by Hull and White (1987), Amin and Ng (1993) and Heston (1993) and many others who all formed their own stochastic volatility models. These stochastic models differ from each other in their volatility estimation process. Hull and White used geometric Brownian motion, Scott (1987) used a mean reverting process and Wiggins (1987) used a Wiener process. Melino and Turnbull (1990, 1991) have also reported that stochastic models are successful in currency options. A major problem with many stochastic models is that they are not in a closed-form which means that there is no analytical solution available and their use requires Monte-Carlo simulation or another form of numerical solution to estimate option prices. This means they are very demanding to use but it is difficult to say which type of solution is more accurate in option pricing (Chistoffersen, Jacobs & Mimouni 2006). In addition some of the models very questionably assume there is no correlation between returns of the underlying asset and volatility (Nandi 1996). However Heston (1993) have created a closed-form stochastic model which allows correlation between underlying asset and volatility. Heston's model was originally applied to bond and currency options but Nandi (1996) showed that it can also be used to price index options successfully. Hestons model later had a significant influence in the development of GARCH based option pricing models.

Most of option pricing models have been created for European options but one type of pricing models is known for its easy application to American options, the binomial three approach. First binomial three models where motivated by the original Black-Scholes model. Cox, Ross and Rubinstein (1979) created a simplified alternative for the Black-Scholes model using binomial threes. Their model was also easier to adjust to different situations and for this reasons binomial threes are also used in practice. However one drawback of binomial threes is that in some cases their numerical solutions can be very demanding. (Korn & Müller 2010)

2.6.2 ARCH and GARCH models

Most recent studies have focused on models that use more advanced methods to estimate volatility and GARCH models have been a common subjects in research papers. ARCH model suggested by Engel (1982) is the base of the GARCH model. ARCH models assume non-constant volatility and allow conditional variance to change over time as the function of past errors while the unconditional variance stays constant. This meaning that the model identifies that the uncertainty of variation often changes over time because volatility often appears in clusters where large changes in returns tend to be followed by large changes, and small changes are often followed by small changes. Due to the non-negativity constrains of the conditional variance in the ARCH model Bollerslev (1986) extended the model to have a more flexible lag structure so the non-negativity constrains would not be met so easily and created the first GARCH volatility estimation model. Option values are affected greatly by volatility of the underlying asset and GARCH process seems to be a very good fit to estimate volatility in time series so naturally the GARCH process was applied to option pricing as well. Heston and Nandi's

(2000) closed-form GARCH model is one of the well-known option pricing models that are based on the GARCH process. Another GARCH model that has been often seen in empirical studies is Duan's (1995) NGARCH model that is based on the work of Engle and Ng (1993) on asymmetric volatility. However Duan's NGARCH model is more difficult to implement and requires numerical solution to estimate option prices, whereas the closed-form Heston and Nandi's GARCH model can be solved analytically.

2.6.3 Black-Scholes model

In 1973 Fischer Black and Myron Scholes introduced the option pricing model known as the Black-Scholes model. In their study they successfully used the formula for dynamic hedging and showed how risk could be eliminated from a portfolio of stocks and options. This formula was a major step forward in stock option pricing and lead to the beginning of fast expansion in derivatives markets. (Ray 2012)

The Black-Scholes formula:

$$Call\ option\ price = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$Put\ option\ price = Ke^{-rT} N(d_1) - S_0 N(d_2)$$

where,

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

S is price of the underlying asset, K is exercise price, r is risk-free interest rate, σ is volatility, T is time to maturity and function $N(x)$ is the cumulative probability distribution function for a standardized normal distribution.

The Black-Scholes model is very straightforward and easy to implement. It only requires five input variables that, with the exception of volatility, are all available for investors and are objective figures that do not require human judgment. Under the assumptions made in the model, the value of an option depends only on the price of the underlying asset and time to maturity and on variables that are taken to be constants. It also provides a closed-form solution. The model was originally created for European call and put options for non-dividend-paying stocks but he formula has been later adjusted to dividend paying stocks as well (Hull 2005 p.281; Natenberg 1994 p.44).

The Black-Scholes model is the cornerstone of derivatives pricing theory but there is a common understanding that it exhibits two pricing errors systematically. One of the models assumptions is that the underlying asset price returns follow a geometric Brownian motion with constant volatility. (Black & Scholes 1973) It has been later noticed that this is not an accurate description of reality. According to the assumptions of the Black-Scholes model implied volatilities inferred from the market price using the Black-Scholes formula for options that have the same underlying asset and expire on the same date but have different exercise should have same implied volatility. But it has been empirically proven that in-the-money options and out-of-money options have higher implied volatilities than at-the-money options (Rubinstein 1985, 1994; Derman & Kani, 1994; Ederington & Guan 2002). This phenomenon is known as the volatility smile/skew and has been recognized in many option markets but the reason for the smile is still under debate. A common opinion is that the flawed assumptions of the Black-Scholes model create the smile but other reasons have also been proposed like inefficiency of the option markets. Another documented phenomenon is that implied volatilities for options on the same underlying asset with different maturities are different (Black 1975). This second known pricing error is known as volatility term structure of implied volatilities. (Duan & Zhang 2001)

2.6.4 Ad hoc Black-Scholes model

Dumas et al. (1998) presented an ad hoc Black-Scholes model in which volatility is estimated using a deterministic volatility function (DVF). The ad hoc approach is one of the most common methods among practitioners and traders and therefore it is also known by the name Practitioners Black-Scholes model. The model smooths implied volatilities across exercise prices and times and uses those volatilities in the Black-Scholes model. This way the model adapts to the implied volatilities and takes in to consideration that volatilities are not constant. In the model volatility is seen as a function of exercise price and time to maturity. Because of its impressive performance in empirical tests it is often used as a benchmark in studies for forecast accuracy of other models. (Berkowitz 2009)

Dumas et al. (1998) presented multiple different possibilities for a deterministic volatility function. One of their functions was also chosen for this study. In this function volatility is estimated as function of both exercise price and time to maturity in the following matter:

$$\sigma = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 T + \beta_4 XT$$

Where σ is the implied volatility of an option with exercise price X and time to maturity of T and β_0, β_1, β_2, β_3, β_4 are model parameters.

2.6.5 Heston and Nandi's GARCH model

In the year 2000 Heston and Nandi introduced their version of an option pricing model that uses the GARCH process. The model defines option prices as functions of asset price and the historical asset prices. This is one of the differences to the Black-Scholes model in which option prices are functions of current asset prices. It also recognizes the stochastic nature of volatility and correlation between volatility and asset returns. The model is similar to Heston's (1993) stochastic volatility model but easier to apply. Easier application for a GARCH model was part of the motivation behind the model (Heston & Nandi, 2000). Which can also bee seen in that the model is closed-form and does not require a numerical solution like many other GARCH models.

Heston and Nandi's GARCH process presented in risk-neutral form is the following:

$$\log\left(R_i(t)\right) = \log(R_i(t_{-1})) + i + \lambda h_t + \sqrt{h_t z_t}$$

where,

$$h_t = \omega + \beta_1 h_{t-1} + \alpha_1 (z_{t-1}) - \gamma_1 \sqrt{h_{t-1}})^2$$

R_i is the underlying assets return at time t, i is continuously compunded interest rate, z_t is a standard normal variable and h_t is the conditional variance of log returns. $\alpha, \beta, \omega, \gamma$ are model parameters determined by maximum likelihood estimation. α determines the kurtosis of the distribution, γ controls the skewness or the asymmetry of the distribution of the log returns and λ is a risk premium parameter.

The value of a European call option is solved from the formula:

$$C = e^{-i(T-t)} E_t[max(S_t - K, 0)]$$

The value of the call option C, with expiration price of K, that expires at time T is the expected payoff calculated using risk neutral probabilities and discounted by the risk free interest rate.

3. Previous Studies

The ad hoc procedure of Black-Scholes has been compared multiple times with GARCH models. The general result of these studies has been that the ad hoc Black-Scholes model fits the data well in-sample but typically underperforms in out-of-sample comparison to GARCH type approaches. Here we will take a closer look at some of the more relevant studies.

In Heston and Nandi's (2000) empirical study with S&P 500 index options showed that their GARCH model's out-of-sample estimations were much more accurate than an ad hoc Black-Scholes model's. Their GARCH model beat the ad hoc model in accuracy even when the ad hoc models parameters where updated every period and the GARCH model parameters were hold constant. When the GARCH model was also updated every period, the out-of-sample accuracy got even better. Hsieh and Ritchken (2000) have also compared the models and came to a similar result that Heston and Nandi's GARCH model is more accurate in out-of-sample comparison. In the study they compared the ad hoc Black-Scholes, Heston and Nandi's GARCH and Duans (1995) NGARCH models in their out-of-sample pricing performance. According to them GARCH models are able to explain the maturity and exercise price biases (volatility smile and volatility term structure) very well which makes them more precise in out-of-money and in-the-money options than the ad hoc Black-Scholes model. Like Heston and Nandi, Hsieh and Ritchken state that GARCH models are fairly accurate even when parameters haven't been re-estimated in a long time, especially in relation to the ad hoc Black-Scholes model.

Majority of the studies are done in the bigger markets like the S&P 500 index option market and therefore it is difficult to find relevant studies done in the DAX index option markets. One of the studies done with DAX index options is Lehnerts study from 2003. Like many others Lehnert used the ad hoc Black-Scholes as a benchmark as he studied the performance of GARCH models, including Heston and Nandi's GARCH model. The result was that Heston and Nandi's GARCH model's pricing errors where smaller than the ad hoc Black-Scholes model's in the DAX index option market. In the out-of-sample valuations for different moneyness and maturity categories Heston and Nandi's GARCH model was also superior thus confirming the findings of Heston and Nandi (2000) in the DAX market. Lehnert also did a comparison of future option price forecasting ability for different forecasting time periods. Heston and Nandi's model's pricing errors stayed quite steady as the period got longer, which gives and indication of good forecasting ability. The ad hoc Black-Scholes models pricing errors on the other hand varied between the different forecasting lengths, which makes its forecasts more unstable. Another important observation that can be made from Lehnerts study is that his result strongly indicate a volatility smile in the DAX index options.

The ad hoc Black-Scholes and GARCH models have also been tested under economical crisis, first by Duan and Zhang (2001) and then by Moyaert and Petitjean (2011). These studies are relevant because just like during economic crisis also in the sample used in this study the volatility changes a lot during the test period. Duan and Zhang studied the models under the Asian financial crisis using Hang Seng index options. The main conclusion they made was that their GARCH model outperformed the ad hoc Black-Scholes model both before and during the financial crisis when there were drastic changes in market volatility. GARCH models were also more accurate than the ad hoc Black-Scholes model both in-sample and out-of-sample. The Moyaert and Petitjean's study was done in a similar setting to Duan and Zhangs study, during the subrime crisis, but the data consisted of Eurostoxx 50 index options and the results reported were somewhat different. According to Moyaert and Petitjean's results the ad hoc Black-Scholes model outperformed the Heston and Nandi's GARCH model. The ad hoc Black-Scholes was better in the in-sample period when volatility was still lower, but also out-of-sample, when volatility got higher due to the crisis. The results are not completely comparable since they studies used different GARCH models and where done in different markets but they still show that the difference in pricing accuracy between the models is very small and is depended on the models and the data used.

Moyaert and Petitjeans study is also relevant because it compared all of the models used in this study. The Black-Scholes model is rarely used these days in its original form but Moyaert and Petitjeans study can give us a preview of how it did in comparison to the ad hoc Black-Scholes and Heston and Nandi's model. The results of the study show that the original Black-Scholes model was clearly outperformed by the other models. Both of the known pricing biases of the model are visible in the results and the model has problems especially with short maturity and out-of-money options. The other two models don't seem to have so much of a problem pricing options with different maturity and moneyness'.

Another recent study that used the Black-Scholes model in its original form was done by Berkowitz in 2009. In his study he compared the original model to several ad hoc models with different deterministic volatility functions. Again the original model was outperformed by the ad hoc approach. These result indicate that the original Black-Scholes model really is outdated.

Also the pricing biases of the original model seem to be present but with a simple adjustment in the ad hoc model they can be corrected quite well and with not much effort.

4. Empirical study

4.1 Methodology and data

The empirical tests are done by first estimating the optimal parameter values for each model using the in-sample data, which is the first six months of the year. Then using those values forecasts of future option prices in the out-of-sample (last six months of the year) data are estimated. Option pricing models are evaluated by using a loss function (Christoffersen, Jacobs 2004). There is no consensus on what loss function should be used. Heston and Nandi (2000) used RMSEs calculated from the dollar errors ($RMSE) and Lehnert (2003) used %RMSEs. The loss function chosen for this study is root mean square percentage error (%RMSE). %RMSEs are calculated from the difference between the theoretical prices and market prices. Root mean percentage square error is the average of the squared percentage errors between the option price estimates and the market prices. By squaring the errors there is no need to deal with negative values and it also gives more weight to larger errors. The %RMSE’s are calculated for different maturities (short, medium, long) and moneyness (in-the-money, at-the-money, out-of-money) categories. The results are presented for both in-sample and out-of-sample periods and also for each month of the out-of-sample period separately. In the end the pricing errors for the whole period are assessed.

Empirical research was done by using DAX index call options traded in the EUREX derivatives exchange in the time 3.1.2011-30.12.2011. Market prices of the options were compared to the theoretical price estimates every week. The weekly observations are primarily Wednesday closing prices for each week but if Wednesday’s price were not available Thursday’s price was used. If Thursday’s price was not available the week’s price is from Tuesday. This is a common practice in studies of this field. Daily DAX index prices were obtained for volatility estimation.

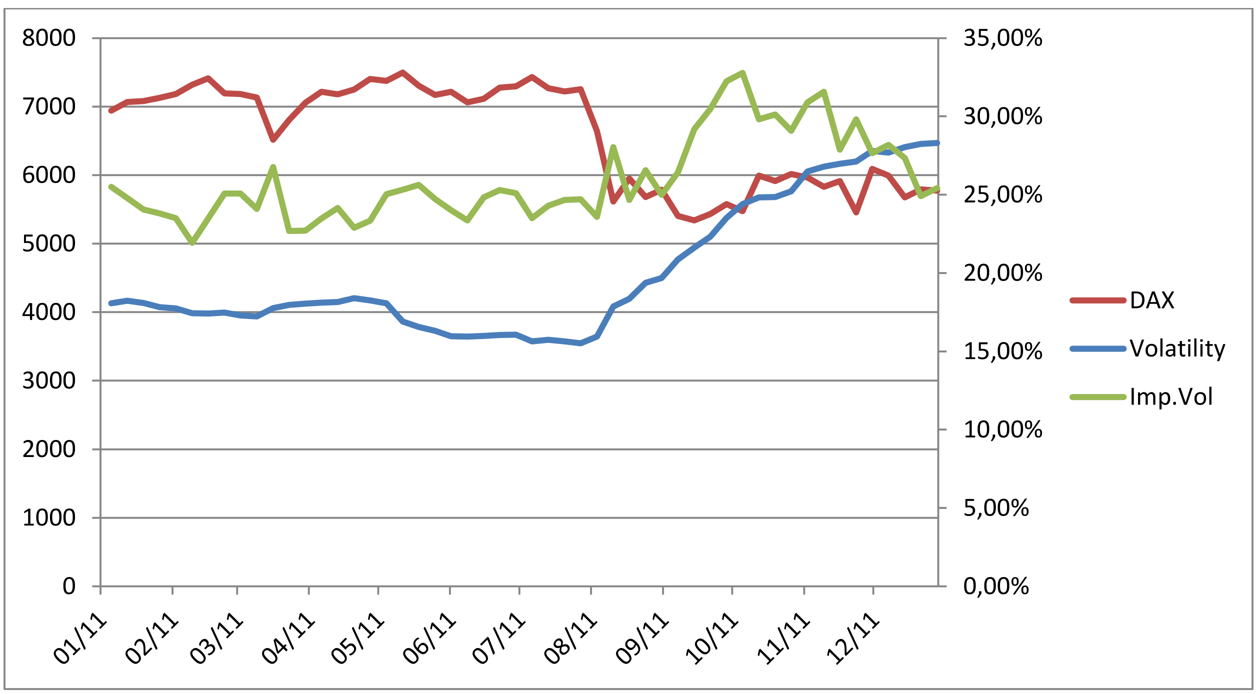
The index returns and option market prices were gathered from Datastream. The data consists of 544 weekly option market price observations. Amount of observations divided in to maturities and moneyness categories are in table 2. All options that had maturity of under 30 days were removed from the data because in some cases the percent errors were unusually high due to very low prices, especially in deep-out-of-money options. Euribor was used as the risk free interest rate for the Black-Scholes and ad hoc models. The correct interest rates were chosen based on the maturity of the option. As it has been done in some studies the risk free interest rate in the GARCH model was set to zero in order to simplify the estimation process since its effect is very minimal in the model (Duan 1995, Schmitt 1996).

Table 2. Amount of observation in the maturity and moneyness categories

	Maturity			
Moneyness	*Short*	*Medium*	*Long*	*Total*
Out-of-money	*62*	*78*	*12*	*152*
At-the-money	*31*	*41*	*48*	*120*
In-the-money	*17*	*110*	*145*	*272*
Total	*110*	*229*	*205*	*544*

To gain perspective of the data the DAX index returns, volatility and implied volatility during the test period were presented in a chart that can be seen in figure 2. As mentioned the economic conditions changed during the sample period quite drastically. In the first half of the year the index level stays around the 7000 point mark but in the beginning of the second half drops quit rapidly to about 6000 index points. At the same time volatility was also stable around 15 % during the first half but rose to 20 % when the index fluctuated and volatility had nearly doubled by the end of the year. This type of change in the volatility between in-sample and out-of-sample will probably affect the pricing errors and but the volatility estimation processes of the models to a test. The empirical results will give a good indication which of the models are able to handle changes in volatility after the model parameters have been optimized. In the chart implied volatility was higher than realized volatility during almost the whole period and it also rose after the drop in the index level. Comparing realized volatility to implied volatility it can be inferred that the market participants expect the volatility to be higher in the future than the realized volatility is.

Figure 2. DAX index returns, volatility and implied volatility during the test period



4.2 Model optimization

Realized volatility, which was calculated from the past returns of the DAX index during the in-sample period, was used as a volatility estimate in the Black-Scholes model for the out-of-sample forecasts. Annualized realized volatility for the in-sample data was 16,42 % but as it could be seen from figure 2 above, implied volatilities were a lot higher during the test period so the volatility estimation method for Black-Scholes model might develop in to a problem during the option price estimations. As mentioned before realized volatility can also differ a lot from implied volatility. Implied volatilities obtained from the Black-Scholes formula for each test category are presented in table 3. There is obviously a big difference in the realized volatility and what the volatility is assumed to be in the future by the market. The table also shows, that just like researchers Ederington and Guan (2002) have noticed, when they studied the volatility smile, in-the-money call options have higher implied volatilities. This is also visible in the table 3 and indicates that in the DAX index market the volatility smile is in more of a skewed form.

Table 3. Implied volatilities in maturity and moneyness categories

		Maturity	
Moneyness	*Short*	*Medium*	*Long*
Out-of-money	*27,73 %*	*25,57 %*	*23,79 %*
At-the-money	*31,84 %*	*26,34 %*	*21,23 %*
In-the-money	*40,28 %*	*27,72 %*	*25,26 %*

Ad hoc Black-Scholes option prices estimates were done using a four-step process that started with the estimation of volatility. First the implied volatilities for each option were obtained from the Black-Scholes formula by setting the market price as the price estimate and then solving implied volatility from the function. After that the regression model presented below is created where the implied volatility was set as dependent variable σ and independent variables were exercise price, exercise price squared, time to maturity and exercise price times time to maturity. Parameters β_0, β_1, β_2, β_3, and β_4 were estimated using the ordinary least squares method and used in the deterministic volatility function to estimate volatility for each of the options. The last step was to estimate the theoretical option prices using the Black-Scholes pricing formula.

The regression model:

$$\sigma = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 T + \beta_4 XT + \epsilon$$

The regression model was done using the in-sample data and the model fit the data well. The parameter estimates of the model are presented in table 4 and the whole results can be found in appendix 1.

Table 4. Ad hoc Black-Scholes regression parameter estimates.

X	**X^2**	**T**	**XT**
-0,000092704900	*0,000000001704*	*-0,295372881000*	*0,000039474500*

According to the results independent variables explain 89 % of variability in implied volatility. How the exercise prices and time to maturity affect implied volatility can be seen in the parameters. Both exercise price and time to maturity have a negative effect on implied volatility. This seems logic since as mentioned in-the-money options have higher volatilities as do short maturity options. Meaning that when the exercise prices are higher, options are further away from being in-the-money and have lower volatilities, thus the parameter is negative. And when, when time to maturity is shorter, implied volatility is higher and therefore the parameter value for time to maturity in the model is also negative.

For the GARCH model parameter estimation a sample size of more than 500 observations is recommended (Hwang, Pedro 2006). The in-sample period does not have enough observations for a reliable GARCH model so daily closing prices for DAX index from 2.1.2009-30.6.2011 were used for the parameter estimation of the Heston and Nandi's GARCH. Number of observations in this data is 640 which is enough to have good GARCH estimates. Maximum likelihood estimation was used to estimate the model parameters. The MLE estimates for parameters λ, α_1, ω, β_1, annualized stationary volatility Θ and log-likelihood value can be found in table 5.

Table 5. Maximum likelihood estimation of GARCH model parameters

λ	α_1	ω	β_1	γ_1	Θ	Log-Likelihood
6,916	*1.316e-22*	*1.074e-05*	*0,9423*	*0*	*13,64 %*	*3526,67*

5. Empirical results

The results are presented in three parts. First the errors from the in-sample period are presented for each of the categories of time to maturity and moneyness. This will give an understanding of how the maturity and moneyness of the options affect the pricing accuracy of the models when the models parameters are optimal. After that the forecasting ability of the models in the out-of-sample period is assessed. The out-of-sample errors are also divided in the maturity and moneyness categories to see how well the models performance without optimization of the parameters. Future option price forecasting ability of the models is also compared by dividing the out-of-sample period in to months, which will give a view of how consistent the accuracy of the forecasts is when the forecasting period gets longer. In the end the whole combined sample period is evaluated.

5.1 In-sample results

As the results from the in-sample period presented in table 6 indicate the ad hoc Black-Scholes model proved to be the most accurate during the in-sample period, just like in the previous studies. It had the smallest errors in all of the categories, which means the used volatility function fit the data well. This could also be predicted from the regression results where the deterministic volatility function was able to explain 89 % of the changes in the implied volatilities during the in-sample period. Time to maturity seemed to have only some effect on the accuracy of the model and it priced both medium and long maturity options very well. Conclusions about the effect is however difficult to make from the in-sample period because there was no options with short time to maturity in the period. Moneyness' effect can be assest better. The ad hoc model was very good at pricing in-the-money options but had larger errors for at-the-money and out-of-money options.

The performance of Heston-Nandi's GARCH model was surprisingly poor compared to the ad hoc model. It's errors in the in-sample period were over four times larger than the ad hoc model's errors. It had major difficulties in pricing out-of-money options and its estimates for at-the-money options were also quite inaccurate. Time to maturity had less effect on the accuracy than with the ad hoc model but pricing errors in the moneyness categories varied a lot more than the ad hoc models.

The original Black-Scholes model was the least accurate of the models. It had the largest pricing errors for all the categories except out-of-money options but its overall accuracy did not fall far from the GARCH model. The Black-Scholes models relatively small pricing errors may be explained by the rather small variation in volatility during the in-sample period.

The effect of the changes in the maturity and moneyness categories that are due to different implied volatilites in the categories is also visible in the results. The ad hoc model that takes the different exercise prices and maturities in to consideration is able to handle these changes a lot better than the other two models. This is possibly the reason for its better performance in the in-sample period. The original Black-Sholes model was, as expected, affected by the implied volatilites but the GARCH models performance was unexpectedly poor in the light of the previous studies that had stated otherwise.

Table 6. In-sample RMSEs

Maturity	**Black Scholes**	**Ad Hoc-Black Scholes**	**Heston-Nandi GARCH**	**Observations**
Short				*0*
Medium	*8,23*	*1,34*	*5,90*	*66*
Long	*11,39*	*2,28*	*9,32*	*163*
	10,58	*2,05*	*8,47*	*229*

Moneyness	**Black Scholes**	**Ad Hoc-Black Scholes**	**Heston-Nandi GARCH**	**Observations**
Out-of-the-money	*27,72*	*6,03*	*29,94*	*2*
At-the-money	*15,91*	*3,42*	*15,64*	*50*
In-the-money	*8,04*	*1,32*	*3,70*	*177*
	10,58	*2,05*	*8,47*	*229*

Percentage RMSE's for the in-sample period are calculated from the weekly pricing errors between DAX option market prices and model estimates. Maturity is time to maturity in days (Short is 30-120 days, Medium is 120-240 days, Long is 240-360 days) and Moneyness is spot prices of the option divided by exercise price (Out-of-money is <0,95, At-the-money is 0,95-1,05, In-the-money is >1,05). Observations column shows the amount of weekly price observation in the data.

5.2 Out-of-sample results

Out-of-sample comparison gives an understanding of the forecasting ability of the models. This measures the models' future volatility estimation ability. By processing this information conclusions of how well the models could actually work in practice can be made. Therefore the information of out-of-sample performance is more significant than the previous in-sample results. The out-of-sample RMSEs are presented in table 7.

During the forecasting period errors for all of the models expectedly got larger since the model parameter values were not optimal for this period. As it can be seen from the figure 2 the index level stayed fairly still during the in-sample period and volatility did not change much during the period either. Coming to the out-of-sample period DAX index level dropped and volatility got higher. This change in conditions gives us information about how well the models adjust to these changes after they have been optimized using only the in-sample data.

The ad hoc model and the GARCH model seem to be quite evenly accurate in their forecasting ability and the original Black-Scholes model has now fallen to the predicted levels of inaccuracy compared to the newer models in the way the previous studies indicated. This probably is because of the change in the volatility compared to the in-sample period that the models volatility estimation process was not able to adapt to.

The GARCH model is the most accurate in predicting future option prices but only by a thin margin. Its pricing errors are smallest when maturity is long and its pricing errors are larger when maturity gets shorter. This is in contrast with the in-sample results were the pricing errors for options with medium time to maturity were smaller than options that had long time to maturity. However this is more in line with the other models. Moneyness of the options appears to have similar effect to its price forecasting accuracy than it did in the in-sample price estimates. The GARCH model estimates in-the-money options most accurately and accuracy drops when the options get further out-of-money, just like in the in-sample period.

The ad hoc Black-Scholes performs well in out-of-sample estimation. Its overall errors are very close to the GARCH model in most of the maturity categories and it performs far better in pricing of at-the-money and in-the-money options were its errors are about half of the GARCH errors. Ad hoc models accuracy also depends in the maturity of the options similarly than the GARCH models. It also has more trouble pricing options with short time to maturity.

The Black-Scholes model predicts future option prices very poorly. Its overall errors during the out-of-sample period are double the errors of either of the other models. This can be explained by the changes in volatility over the out-of-sample period that the model fails to handle. Just like the ad hoc model and the GARCH model, the Black Scholes model prices short time to maturity options and out-of-money options the most inaccurately and long time to maturity options with the most precision.

It is clear in the results that all the errors change as the function of exercise price and time. This indicates that the changes in the implied volatilities are affecting the models. The effect is especially large with the Black-Scholes model but the other models seem to be affected as well.

Table 7. Out-of-sample RMSEs

Maturity	**Black-Scholes**	**Ad hoc Black-Scholes**	**Heston-Nandi GARCH**	**Observations**
Short	*68,86*	*32,92*	*31,58*	*110*
Medium	*54,53*	*23,83*	*22,02*	*163*
Long	*37,35*	*15,30*	*15,80*	*42*
	58,13	*26,52*	*25,13*	*315*

Moneyness	**Black-Scholes**	**Ad hoc Black-Scholes**	**Heston-Nandi GARCH**	**Observations**
Out-of-money	*77,35*	*37,46*	*32,42*	*150*
At-the-money	*44,53*	*11,36*	*20,86*	*70*
In-the-money	*16,69*	*4,53*	*10,55*	*95*
	58,13	*26,52*	*25,13*	*315*

Percentage RMSE's for the out-of-sample period are calculated from the weekly pricing errors between DAX option market prices and model estimates. Maturity is time to maturity in days (Short is 30-120 days, Medium is 120-240 days, Long is 240-360 days) and Moneyness is spot prices of the option divided by exercise price (Out-of-money is <0,95, At-the-money is 0,95-1,05, In-the-money is >1,05). Observations column shows the amount of weekly price observation in the data.

5.2.1 Monthly out-of-sample %RMSEs

This part of the empirical study was done to see how the length of the forecasting time period affects the forecasting accuracy of the models. For this the out-of-sample period was divided in to months so the forecasting accuracy could be compared between different forecasting times. The results are presented in the table 8 where *n* means how many months ahead the forecast are done from the point when the model parameters where optimized. As it can be expected the original Black-Scholes model is again the worst performer and all ready two months in to the forecasting period its accuracy is very poor. This inaccuracy continues all through the out-of-sample period and the model is the least accurate in each month. The errors also vary a lot between different period lengths which makes the models predictions unstable.

The ad hoc models' forecast one month ahead were extremely accurate and it was the most accurate in its projections two months ahead as well. It seems that during the first three months of the out-of-sample estimates the model gets more inaccurate as the estimates are done further in to the future.

As it has already been seen form the results the GARCH model out performs the ad hoc model in its overall accuracy during the out-of-sample period. This is due to its more consistent performance between different estimation period lengths. All though when the estimation periods length grows the accuracy of the GARCH model gets worse a lot faster than the ad hoc model's. The difference between accuracy of the forecasts done one month and two months ahead is already significantly larger than the ad hoc models. But after that the GARCH models predictions are steadier.

Table 8. Monthly out-of-sample RMSEs

n	Black-Scholes	Ad hoc Black-Scholes	Heston-Nandi GARCH	Observations
1	*9,43*	*1,15*	*6,09*	*56*
2	*48,52*	*13,98*	*27,04*	*66*
3	*75,56*	*36,71*	*31,90*	*55*
4	*64,46*	*32,70*	*26,64*	*56*
5	*39,09*	*19,93*	*12,80*	*52*
6	*65,37*	*22,35*	*30,71*	*30*
	58,13	*26,52*	*25,13*	*315*

Percentage RMSE's are calculated from the weekly pricing errors between DAX option market prices and model estimates. Column n shows the length of the forecast period in months. Observations column shows the amount of weekly price observations in the data.

5.3 Results for the whole time period

When all observations during the time period were considered the GARCH model was still the most accurate but just like in the out-of-sample the margin to the ad hoc model was very small.

The original Black-Scholes model was expectedly the most inaccurate. Looking at the results for the whole period there is not much data that can be gathered from the table for comparing the model performance that couldn't be seen in the in-sample and out-of-sample results but since there are now a lot more observations in each of the maturity and moneyness categories conclusions about how time to maturity and moneyness affect the models can be made with more reliability.

As it is clearly visible in the results, time to maturity has an effect on the accuracy of the estimations and the errors seem to change as the function of the time to maturity and moneyness, just like the implied volatilites. All of the models seem to be the less accurate when time to maturity is short. This is possibly because when time to maturity is shorter implied volatilities tend to be higher which is also visible in table 3. This insufficiency to price short maturity functions due to their different implied volatility is in large part the fault of each models volatility estimation processes. The ad hoc Black-Scholes and the GARCH models that have more sophisticated volatility estimation processes have a difference of roughly about ten %RMSE's between the medium and short maturity options where the Black-Scholes model posts double that. This means that the ad hoc model and the GARCH models explain large part of the pricing biases associated with the Black-Scholes model just the results in previous studies suggested.

As expected pricing errors also change due to moneyness. All of the models post very large errors for out-of-money options and the errors get smaller when the options are in-the-money. A reason for these changes could also be found in implied volatility, as it is known implied volatilities change due to moneyness. Again the original Black-Scholes model was the worst with the changes in moneyness and had the biggest differences between the categories. The two other models had also high errors but had relatively small errors for at-the-money and in-the-money options.

The errors for the Black-Scholes model seem to change between all of the moneyness categories just like the implied volatilities. But for the ad hoc and the GARCH models the errors are relatively small except for the out-of-money options. Besides the implied volatilities there might be another factor that affects the errors. That is the loss function. The errors between the theoretical prices and the market prices are considerably higher on short maturity and out-of-money options than the other categories. The reason for this could be in the way the errors are estimated. The %RMSEs calculated from the percentage errors between the theoretical prices and the market prices might distort the results. This could indicate that the loss function favors certain options. (Moyaert and Petitjean 2011) One possible reason is that the option values for short maturity options tend to be lower because of there is not much time value left. Out-of-money options on the other hand have low values because there is not much intrinsic value. The loss function possibly favors high value options, which puts more weight to the errors on low value options and making their %RMSEs larger. This could explain the relatively high out-of-money option values of the GARCH and the ad hoc Black-Scholes models.

Table 9. RMSEs for the whole year

Maturity	**Black-Scholes**	**Ad hoc Black-Scholes**	**Heston-Nandi GARCH**	**Observations**
Short	*68,86*	*32,92*	*31,58*	*110*
Medium	*46,22*	*20,12*	*18,85*	*229*
Long	*48,61*	*21,03*	*19,86*	*229*
	43,11	*19,74*	*18,76*	*544*

Moneyness	**Black-Scholes**	**Ad hoc Black-Scholes**	**Heston-Nandi GARCH**	**Observations**
Out-of-money	*76,90*	*37,22*	*32,39*	*152*
At-the-money	*35,73*	*8,98*	*18,91*	*120*
In-the-money	*11,81*	*2,88*	*6,91*	*272*
	43,11	*19,74*	*18,76*	*544*

Percentage RMSE's for the out-of-sample period are calculated from the weekly pricing errors between DAX option market prices and model estimates. Maturity is time to maturity in days (Short is 30-120 days, Medium is 120-240 days, Long is 240-360 days) and Moneyness is spot prices of the option divided by exercise price (Out-of-money is <0,95, At-the-money is 0,95-1,05, In-the-money is >1,05). Observations column shows the amount of weekly price observation in the data.

7. Conclusions

The purpose of this study was to see how option pricing models have advanced since a notable brake in option pricing, the Black-Scholes formula. In this purpose the study was very successful and it was clear from the empirical results that the option pricing models have come a long way since introduction of the Black-Scholes model. In the empirical study, three different models where put to a test in their accuracy and the original Black-Scholes model was far from the newer models which indicates that there has been some progress made in option pricing. In the in-sample period the Black-Scholes models performance was quite close the other models but because of its miserable out-of-sample results it came clear that the two other models were better. This result however was not a surprise since previous studies have come to the same results. The more difficult question was that how would the other two models compare with each other. After all Heston and Nandi's GARCH model came on top in overall pricing accuracy but there was very little difference between the models. In the in-sample part the ad hoc model was very accurate and outperformed Heston and Nandi's GARCH model but in the more important out-of-sample period and overall the GARCH model was more accurate. Even though Heston and Nandi's GARCH model was more accurate in its future estimation accuracy in the out-of-sample period the ad hoc model was more accurate in making short term option price predictions. So it can be argued that it is best of the models in practice when only short term predictions are needed. The two models where in fact so close to each other in many ways that it is very difficult to make any definitive conclusions about which one of the models was actually the best one.

All of the models were very similarly affected by time to maturity and had trouble with short maturity options. The two newer were better in pricing at-the-money and in-the-money options but all of the models seemed to have problems pricing out-of-money options. Again it is hard to distinguish the difference between the GARCH model and the Black-Scholes model but what is clear is that they both were better than the Black-Scholes model in dealing with the changes in implied volatilities. It is difficult to say how much the choice of the loss function affected the results but because the loss function was same for all the models they can still be compared (Christoffersen, Jacobs 2004). There is still a lot of improvements to be made so the models can effectively deal with volatility term structure and volatility smile. The ad hoc approach does already take these phenomena in to account but if the implied volatilities can ever be completely dealt with remains to be seen as the reasons for these phenomena are still under debate. Future volatility is also impossible to predict with complete accuracy but the GARCH model that utilizes a more advanced volatility processing method did a good job in making steady future price predictions, just by using historical index prices.

Based on the empirical results it can be said that the original Black-Scholes model was definitely the worst performer in every way. The ad hoc approach and the GARCH model both had their advantages and disadvantages. The results for the ad Hoc Black-Scholes model and Heston and Nandi's GARCH model were promising. Both of the models use a different approach to volatility estimation. The ad hoc Black-Scholes model factors implied volatility to its estimation process, which makes its in-sample estimates and short term future option price forecasts more accurate. Heston and Nandi's model uses the GARCH process, which gives steadier and more accurate estimates overall. In the future it would be interesting to see if implied volatilities could somehow be combined with the GARCH process in option pricing. Another major concern in the development of new option pricing models is practicality. New models should be usable among professionals and outside of academics. This does not mean only creating models that are practical but also educating practitioners in the use of the new models.

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Appendices

Appendix 1. Ad hoc Black-Scholes regression results.

Regression Statistics	
Multiple R	0,944600329
R Square	0,892269781
Adjusted R Square	0,890363051
Standard Error	0,011280747
Observations	231

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	0,238200554	0,059550139	467,9582292	4,592E-108
Residual	226	0,028759685	0,000127255		
Total	230	0,266960239			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0,788045538	0,039078241	20,1658396	2,00096E-52
X	-9,27049E-05	1,09543E-05	-8,462885233	3,29811E-15
X^2	1,70433E-09	9,93005E-10	1,716333869	0,087471187
T	-0,295372881	0,04682597	-6,307885946	1,47253E-09
XT	3,94745E-05	7,5991E-06	5,194620422	4,5752E-07