

# Loss Development Analysis of a Micro-Scale Centrifugal Compressor

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## Abstract

The ever-increasing demand for more efficient energy conversion has placed designers under increasing pressure to develop processing equipment that can meet contemporary needs. It has long been known that a decreasing Reynolds number has a negative effect on centrifugal compressor efficiency. The drop in efficiency can be accounted for relatively easily in the design process using various empirical correlations. However, the correlations only account for a reduction in performance; they do not offer any consideration of the extent to how the drop in efficiency can be countered in the design process. To identify potential methods by which it is possible to improve the performance of centrifugal compressors operating at low Reynolds numbers, the loss development in centrifugal compressors with a reducing Reynolds number must be studied. Recent works on loss development, in general, have focused on the overall performance deterioration, and the differentia-

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tion of the losses originating from different causes with the reducing Reynolds number has been studied only in an axial compressor. The present paper examines loss development in a centrifugal compressor with a vaneless diffuser with respect to the Reynolds number and differentiates between the losses that originate from different causes. A new hybrid method is used to calculate the boundary layer thickness inside a complex flow field. The results show that the diffuser plays a significant role in the performance deterioration of centrifugal compressors with a low Reynolds number and should be included in the loss development analysis. A study of the boundary layers, flow fields and loss development indicates that growth in the impeller hub and diffuser boundary layers should be reduced to improve the performance of the compressor.

*Keywords:* boundary layer thickness, CFD, correction equation, low Reynolds number, tip clearance, transition

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## 1 Nomenclature

### 2 Latin alphabet

3	$A$	area	[m <sup>2</sup> ]
4	$a$	fraction of Reynolds-number-independent losses in Eqn. (2)	[-]
5	$a$	speed of sound	[m/s]
6	$b$	blade height	[m]
7	$b$	fraction of Reynolds-number-dependent losses in Eqn. (4)	[-]
8	$B_{\text{ref}}$	coefficient in Eqns. (5) and (6)	[-]
9	$c$	absolute velocity	[m/s]
10	$c$	chord length	[m]

11	$c$	coefficient in Eqn. (3)	[-]
12	$c_f$	friction coefficient	[-]
13	$C_{pr}$	pressure recovery coefficient	[-]
14	$c_p$	specific heat capacity at constant pressure	[J/kgK]
15	$D$	diameter	[m]
16	$f$	friction factor	[-]
17	$h$	specific enthalpy	[J/kg]
18	$K_p$	total pressure loss coefficient	[-]
19	$Ma_U$	tip speed Mach number	[-]
20	$n$	Reynolds-number-ratio exponent in Eqns. (2) and (4)	[-]
21	$n$	rotational speed	[rpm]
22	$N_s$	specific speed	[-]
23	$p$	pressure	[Pa]
24	$q_m$	mass flow rate	[kg/s]
25	$q_v$	volume flow rate	[m <sup>3</sup> /s]
26	$R$	specific gas constant	[J/kgK]
27	$r$	radius	[m]
28	$Re_c$	chord Reynolds number	[-]
29	$T$	temperature	[K]
30	$t$	tip clearance	[m]
31	$U$	tip speed	[m/s]
32	$U_\delta$	velocity at the boundary layer edge	[m/s]
33	$U_\infty$	free-stream velocity	[m/s]
34	$w$	relative velocity	[m/s]
35	<b>Greek alphabet</b>		

36	$\alpha$	flow angle	[°]
37	$\delta$	boundary layer thickness	[m]
38	$\eta$	efficiency	[-]
39	$\mu_0$	work input coefficient	[-]
40	$\nu$	kinematic viscosity	[m <sup>2</sup> /s]
41	$\omega$	angular velocity	[rad/s]
42	$\phi$	flow coefficient	[-]
43	$\pi$	pressure ratio	[-]
44	$\psi$	pressure coefficient	[-]
45	$\rho$	density	[kg/m <sup>3</sup> ]

#### 46 **Abbreviations**

47	DES	design point
48	FB	full blade
49	LE	leading edge
50	NC	near choke
51	NS	near stall
52	PE	peak efficiency point
53	PS	pressure side
54	SB	splitter blade
55	SF	scaling factor
56	SS	suction side
57	TE	trailing edge

#### 58 **Subscripts**

59	1	impeller inlet
60	2	impeller outlet

61 3 diffuser outlet  
62 ave average  
63 crit critical  
64 r radial  
65 ref baseline case  
66 s isentropic, static  
67 t total

## 68 1. Introduction

69 The sustainable development goals of the United Nations aims at reduc-  
70 ing greenhouse gas emissions, improving energy efficiency and increasing the  
71 share of renewable energy sources [1]. Additionally, the European Union has  
72 similar goals [2]. Finland has committed to the EU targets and aims at in-  
73 creasing self-sufficiency in energy [3]. The industrial sector accounts for, on  
74 average, 50% of the overall electricity consumption [4]. A cost-effective way  
75 to achieve the international and national targets involves improving energy  
76 efficiency [5]. The improvement of compressor performance, in particular,  
77 plays an important role in improving energy efficiency and reducing the end-  
78 use electricity demand, as compressors alone account for 15% of the overall  
79 electricity consumption within industry [4].

80 Micro-scale centrifugal compressors (impeller outlet diameter less than 30  
81 mm [6]) have great potential for efficiency improvement due to their clearly  
82 low performance. The performance of micro-scale centrifugal compressors  
83 is worse than that of the larger compressors due to the losses caused by  
84 low Reynolds numbers, the larger relative blade thickness, surface roughness

85 and tip clearance [7]. The effect of Reynolds number on the compressor  
86 performance was discovered e.g. by Yang et al. [8].

87 The improvement in the efficiency of the micro-scale centrifugal compres-  
88 sors could result in e.g. the increased technological feasibility of micro-scale  
89 gas turbines [9]. Micro-scale gas turbines (less than 100-1,000 kW [10]) could  
90 represent a potential solution for combined heat and power applications to  
91 cut greenhouse gas emissions [11]. These machines are both flexible and scal-  
92 able [12]. Therefore, they could also increase the share of renewable energy  
93 sources and self-sufficiency in energy [9]. In addition to distributed energy  
94 generation, micro-scale gas turbines also hold potential in applications that  
95 require a compact, portable power source due to high power density; e.g., un-  
96 manned aerial vehicles [13]. A micro-scale centrifugal compressor could also  
97 replace a displacement compressor in small refrigeration systems to achieve  
98 lower power consumption and weight [14].

99 The effect of the Reynolds number on the compressor efficiency can be  
100 accounted for relatively easily in the design process with empirical correction  
101 equations; however, these equations do not consider whether the efficiency  
102 drop can be countered somehow. Thus, in order to find potential ways to  
103 improve the performance of low-Reynolds-number compressors, loss develop-  
104 ment in centrifugal compressors with reducing Reynolds number is studied  
105 in this paper.

106 Recent works on loss development in low-Reynolds-number compressors  
107 have, in general, focused on the overall performance deterioration in the com-  
108 pressor stage. In a centrifugal compressor, the results of Schleer and Abhari  
109 [15] showed a 0.5% decrease in the total-to-static pressure ratio. In addition,

110 the results of Zheng et al. [16] showed a 6.9% decrease in the total-to-total  
111 isentropic efficiency of a centrifugal compressor. In an axial compressor, the  
112 study of Choi et al. [17] indicated approximately a 69% increase in the total  
113 pressure loss coefficient. In addition to the total pressure loss, Choi et al. [17]  
114 investigated the differentiation of losses originating from different causes with  
115 the reducing Reynolds number in the axial compressor. To the author's best  
116 knowledge, the differentiation of losses with the reducing Reynolds number  
117 has not previously been investigated in centrifugal compressors apart from  
118 the previous work by the authors, where the loss development was studied  
119 in the downscaled centrifugal compressors [18]. And later in the centrifugal  
120 compressors with varying inlet conditions [19].

121 The above-mentioned recent works on the differentiation of losses in low-  
122 Reynolds-number centrifugal compressors have focused on the impeller, while  
123 considerably less attention has been placed on the diffuser. This is because,  
124 according to Dietmann and Casey [20], more losses occur in the impeller  
125 than in the diffuser due to higher velocities. The hypothesis of this work  
126 is that the diffuser plays a marked role in the performance deterioration  
127 of the compressor. Thus, the first novel aspect of this study is that the  
128 role of a vaneless diffuser in the loss development is analysed. The second  
129 novel aspect of the study is that it demonstrates how the hybrid method [21]  
130 for calculating the boundary layer thickness inside the complex flow field of  
131 a centrifugal compressor enables a more sophisticated analysis of the losses  
132 associated with the blade and endwall boundary layers from the impeller inlet  
133 to the diffuser outlet than in previous works by the authors. Additionally,  
134 the question of whether the transition model should be used when modelling

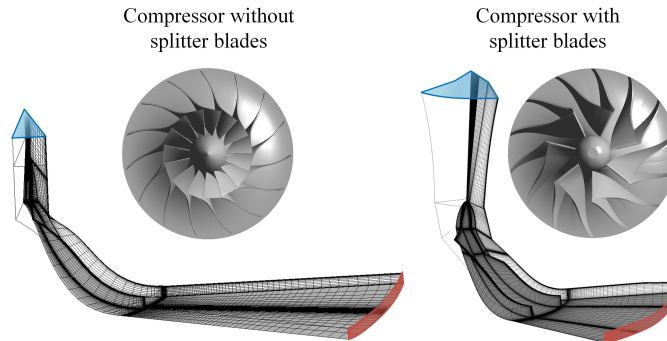


Figure 1: Compressor geometries and computational domains

135 the low-Reynolds-number centrifugal compressors is addressed in this paper.

## 136 2. Methods

137 The effect of the Reynolds number on centrifugal compressor performance  
 138 and losses were assessed in two centrifugal compressors: one with splitter  
 139 blades and the other without. The compressor geometries and computa-  
 140 tional domains are shown in Fig. 1. Both compressors included a vaneless  
 141 diffuser. The compressor with splitter blades was studied experimentally and  
 142 numerically at Lappeenranta University of Technology, Finland [22]. The  
 143 compressor without splitter blades is the test case Radiver, for which the  
 144 measurements were carried out at the Institute of Jet Propulsion and Tur-  
 145 bomachinery at RWTH Aachen, Germany. Part of the research was funded  
 146 by the Deutsche Forschungsgemeinschaft (DFG) [23]. The compressor with  
 147 splitter blades was studied at the design point and the compressor without  
 148 splitter blades at the peak efficiency point at a reduced speed,  $n/n_{DES} = 0.8$ .  
 149 Details of the compressor geometries and the significant dimensionless per-



Table 1: Technical data of the compressors

	With splitter blades	Without splitter blades
Number of blades	7 + 7	15
Relative blade height ( $b_2/D_2$ )	0.058	0.041
Relative tip clearance ( $t/b_2$ )	0.052	0.045
Chord Reynolds number ( $Re_c = \frac{w_{1c}}{\nu_1}$ )	$17 \cdot 10^5$	$16 \cdot 10^5$
Flow coefficient ( $\phi = \frac{q_v}{U_2 D_2^2}$ )	0.065	0.051
Pressure coefficient ( $\psi = \frac{\Delta h_s}{U_2^2}$ )	0.520	0.450
Specific speed ( $N_s = \frac{\omega \sqrt{q_v}}{\Delta h_s^{0.75}}$ )	0.830	0.830
Tip speed Mach number ( $Ma_U = \frac{U_2}{a_1}$ )	0.920	1.170

150 formance parameters at the design/peak efficiency point are shown in Table  
 151 1.

152 Both compressors were modelled at three different operating points: the  
 153 one with splitter blades at the design operating point ( $q_m/q_{m,DES} = 1.0$ ,  
 154  $n/n_{DES} = 1.0$ ), near choke ( $q_m/q_{m,DES} = 1.3$ ,  $n/n_{DES} = 1.0$ ) and near stall  
 155 ( $q_m/q_{m,DES} = 0.6$ ,  $n/n_{DES} = 1.0$ ); and the one without splitter blades at the  
 156 peak efficiency point ( $q_m/q_{m,PE} = 1.0$ ,  $n/n_{DES} = 0.8$ ), near choke ( $q_m/q_{m,PE} =$   
 157  $1.2$ ,  $n/n_{DES} = 0.8$ ) and near stall ( $q_m/q_{m,PE} = 0.8$ ,  $n/n_{DES} = 0.8$ ). The op-  
 158 erating points near stall and choke were chosen by comparing the measured  
 159 operating maps and typical values used in the literature. The minimum  
 160 normalised near stall mass flow rate found in the literature was 0.70 [24].  
 161 The maximum normalised near stall mass flow rate was 0.91 [25]. The min-  
 162 imum normalised near choke mass flow rate was 1.05 [26]. The maximum  
 163 normalised near choke mass flow was 1.30 [27]. The near stall point does

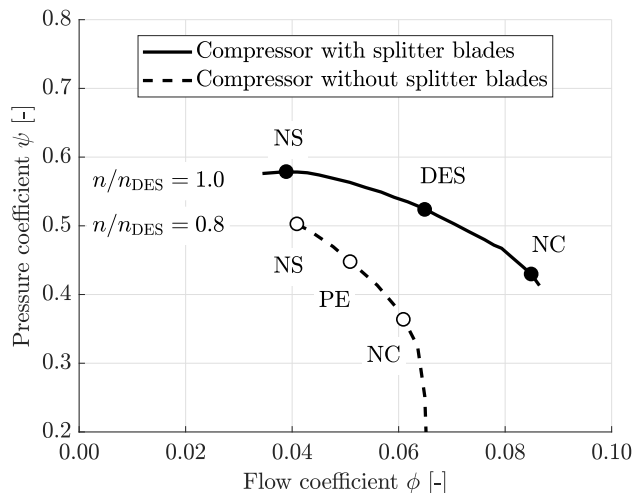


Figure 2: Dimensionless compressor map

164 not represent the real stall point, but is the point at a low flow rate that  
 165 converges stably when modelled.

166 The modelled operating points are shown in Fig. 2. All the compressor  
 167 performance curves were provided by Jaatinen-Värri et al. [28] for the com-  
 168 pressor with splitter blades. For the compressor without splitter blades, the  
 169 compressor performance curves were provided by Ziegler et al. [23].

170 In addition to three operating conditions at the baseline Reynolds num-  
 171 ber,  $Re_{\text{ref}}$ , a low Reynolds number case was also studied. Three operating  
 172 conditions at the baseline Reynolds number were used to validate the numer-  
 173 ical results against experimental data.

174 The Reynolds number can be varied by changing either the compressor  
 175 size or the compressor inlet conditions. As demonstrated in a previous pa-  
 176 per by the authors [19], the Reynolds number variation method does not  
 177 affect the loss generation. In the present study, low Reynolds numbers were

178 achieved by downscaling all geometric dimensions of the compressors with  
179 the same scaling factor as the impeller outlet diameter

$$SF = \frac{D_{2,\text{scaled}}}{D_{2,\text{baseline}}}. \quad (1)$$

180 Also, the same ideal gas properties of air were used for the downscaled com-  
181 pressors as those employed for the baseline compressor. All of the dimen-  
182 sionless numbers (flow coefficient  $\phi$ , pressure coefficient  $\psi$ , and impeller tip  
183 speed Mach number  $Ma_U$ ) were kept constant, except for the Reynolds num-  
184 ber, which decreased as the compressor was downscaled. The studied chord  
185 Reynolds number ( $Re_c = \frac{w_{1c}}{\nu_1}$ ) varied from 1,700,000 to 80,000, with the scal-  
186 ing factor varying from 1 to 0.05. The downscaled compressors were modelled  
187 at the design/peak efficiency points.

### 188 3. Numerical Model

189 The commercial software ANSYS CFX 17.0 was employed for the numer-  
190 ical calculations. The total pressure and total temperature were specified  
191 at the inlet boundary, and the mass flow rate at the outlet boundary. The  
192 computational domains are shown in Fig. 1, on which the inlet is marked  
193 with blue and the outlet with red. Turbulence was modelled using the two-  
194 equation  $k - \omega$  shear stress transport (SST) model developed by Menter [29].  
195 This model is widely used and has been validated for turbomachinery ap-  
196 plications [30]. The values of the non-dimensional wall distance were below  
197 unity on most of the surfaces, with the most challenging region for meshing  
198 being the stagnation point at the blade leading edge.

199 In Fig. 3, the non-dimensional wall distance is shown in both compres-  
200 sors and the values above unity are clipped. The regions with the values

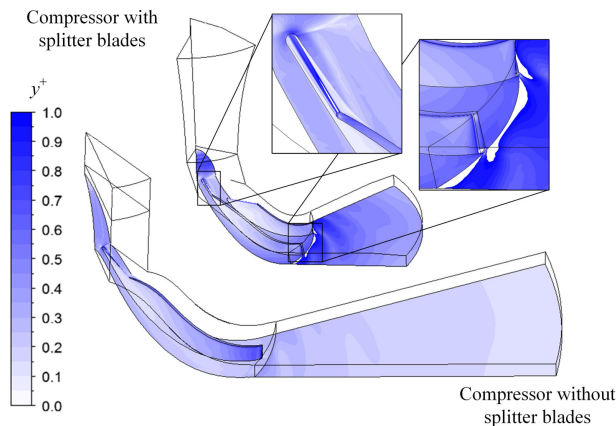


Figure 3: Values of non-dimensional wall distance ( $y^+$ ) on the compressor surfaces. Values above unity are clipped and highlighted in the compressor with splitter blades.

201 above unity are highlighted in the compressor with splitter blades, the max-  
 202 imum value being 35 on the blade surface and two in the diffuser. Overall,  
 203 more than ten mesh cells were located inside the boundary layer. The turbu-  
 204 lence model was used because it switches automatically from a low-Reynolds-  
 205 number treatment to wall functions if the mesh is not dense enough locally  
 206 for a low-Reynolds number treatment [31], and it combines the advantages  
 207 of  $k - \epsilon$  and  $k - \omega$  models being robust and reasonably accurate in complex  
 208 flow fields as inside centrifugal compressors.

209 The frozen rotor approach was used to model the transition between the  
 210 rotating and stationary domains. The target values for numerical conver-  
 211 gence were the efficiency and mass imbalance between the inlet and outlet.  
 212 Convergence was achieved when the change in the target values was below  
 213 0.1%, and the change in the normalised residuals of energy, mass, momentum,  
 214 and turbulence parameters was stabilised.

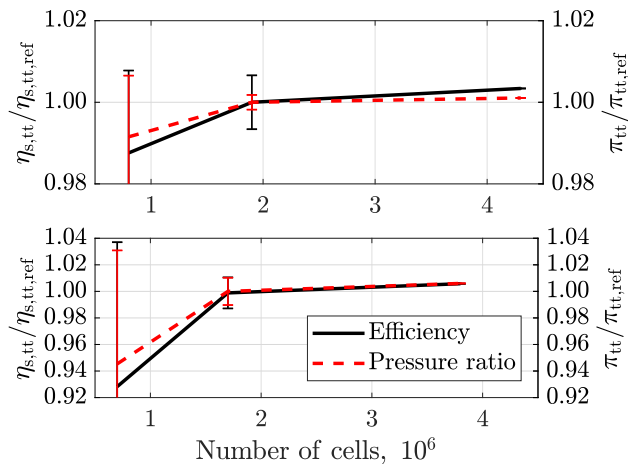


Figure 4: Mesh independence of the compressors with splitter blades (top) and without splitter blades (bottom). The ordinate is heavily scaled to show variation.

### 215 3.1. Mesh Independence Study

216 For the mesh independence study, three structured meshes with 0.8, 1.9,  
 217 and 4.3 million computational cells were used for the compressor with splitter  
 218 blades, and three meshes with 0.7, 1.7, and 3.8 million cells for the compres-  
 219 sor without splitter blades. As a result of the mesh independence study,  
 220 the meshes with 1.9 and 1.7 million cells were chosen for the compressors  
 221 with and without splitter blades respectively. The target values regarding  
 222 mesh independence were the total-to-total efficiency and total-to-total pres-  
 223 sure ratio between the computational domain inlet and diffuser outlet. The  
 224 discretisation error was estimated using the procedure presented by Celik et  
 225 al. [32]. The estimated discretisation error is shown in Fig. 4, which presents  
 226 the results of the mesh independence study for the compressors with splitter  
 227 blades (top) and without splitter blades (bottom). The meshes of the base-  
 228 line compressors were scaled for the downscaled compressors such that they

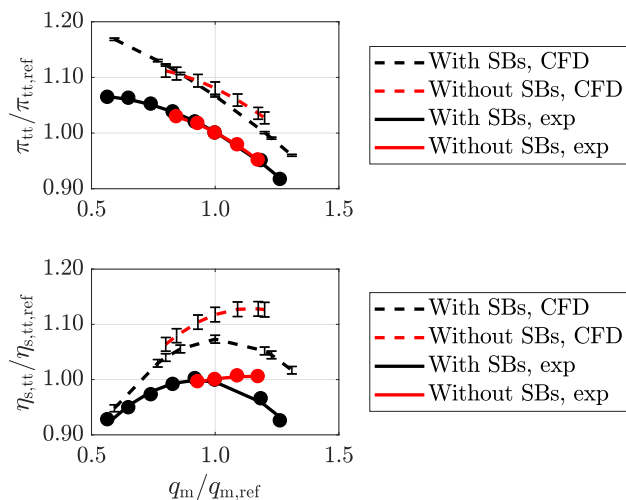


Figure 5: Validation of computational results for the pressure ratio (top) and efficiency (bottom) against the experimental data

229 had the same number of cells in both the baseline and the downscaled cases.

### 230 3.2. Validation Against Experimental Data

231 The numerical results for the baseline, non-scaled compressors were compared to the experimental results. The computational and measured total-to-total pressure ratios and efficiencies with discretisation errors are shown 232 as functions of the normalised mass flow rate in Fig. 5. The efficiency and 233 pressure ratio were normalised by the measured value at the design/peak 234 efficiency point, and the mass flow rate was normalised by the design/peak 235 efficiency mass flow rate. 236 237

238 The validation of the numerical model shows an over-prediction of the 239 efficiency and pressure ratio in both cases, but still the trend is captured. It 240 must be noted that the computational efficiency and pressure ratio were calculated between the computational domain inlet and diffuser outlet, whereas 241

242 the measurements were conducted between the compressor inlet and outlet  
243 for both compressors. Therefore, the computational results do not account  
244 for the pressure loss in the volute or in the exit cone, which can be seen  
245 as part of the difference between the computational and measured values  
246 (approximately 1.5 – 6% in the investigated compressors). The estimation is  
247 based on the total pressure loss coefficient of 0.4 – 0.85 for the volute and exit  
248 cone measured by Hagelstein et al. [33], and in the compressor with splitter  
249 blades, the experimental results indicated that the volute and the exit cone  
250 were responsible for approximately 4% of the additional losses at the design  
251 point. The losses due to disk friction, leakage flow through the backside  
252 cavity, or surface roughness were also neglected in the computational model.  
253 According to Sun et al. [34], leakage through the backside cavity can be re-  
254 sponsible for approximately 1% of additional losses in the pressure ratio and  
255 efficiency. Part of the difference between the computational and measured  
256 results was also due to the inability of the two equation models to predict all  
257 the losses.

258 Despite the over-prediction of the efficiency and pressure ratio, the com-  
259 putational model predicted the flow field fairly accurately; e.g., the relative  
260 differences between the area-averaged measured and modelled values of the  
261 absolute velocity, relative velocity and absolute flow angle (from the radial  
262 direction) in the compressor without splitter blades at  $r/r_2 = 0.99$  were -  
263 2.5%, -1.7% and +1.0%, respectively (Fig. 6). A similar numerical approach  
264 to that used in this study was employed by Bareiß et al. [35], and the com-  
265 parison of their numerical results against the experimental ones showed that  
266 the model overpredicted the total-to-total pressure ratio by 7.4% and the

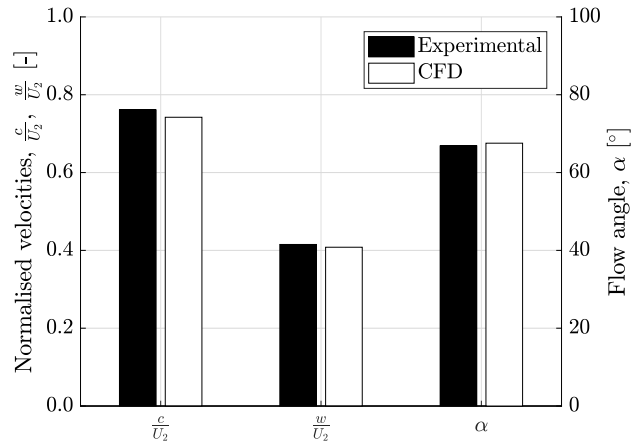


Figure 6: Validation of computational results of normalised absolute velocity, normalised relative velocity, and flow angle (from the radial direction) in the compressor without splitter blades at  $r/r_2 = 0.99$  against experimental data

267 total-to-total isentropic efficiency by 8.9% at the design point, the values  
 268 being similar to those employed in this study.

#### 269 4. Correction Equations

270 The numerical results for the downscaled, low-Reynolds-number compres-  
 271 sors were compared to the empirical correction equations, which are presented  
 272 in Table 2. The empirical correction equations cannot replace measurements;  
 273 however, because they are based on experimental data, they represent an ac-  
 274 ceptable alternative to experiments and can be used to validate the trends  
 275 of the numerical results. To validate the numerical results in full detail, ex-  
 276 perimental data of the flow fields inside a low-Reynolds-number compressor  
 277 should be available for comparison to the flow fields inside a high-Reynolds-  
 278 number compressor.



Table 2: Summary of the efficiency correction equations published in the literature

Reference	Equation
Old empirical formula [36]	$\frac{1-\eta}{1-\eta_{\text{ref}}} = a + (1-a) \left[ \frac{Re_{\text{ref}}}{Re} \right]^n$ (2)
Casey (1985) [37]	$\Delta\eta = -\frac{c}{\mu_0} \Delta f$ (3)
Heß & Pelz (2010) [38]	$\frac{1-\eta}{1-\eta_{\text{ref}}} = (1-b) + b \left( \frac{Re_{\text{ref}}}{Re} \right)^n$ (4)
Casey & Robinson (2011) [7]	$\Delta\eta = -\frac{B_{\text{ref}}}{f_{\text{ref}}} \Delta f$ (5)
Dietmann & Casey (2013) [20]	$\Delta\eta = -\frac{B_{\text{ref}}}{f_{\text{ref}}} \Delta f$ (6)
Pelz & Stonjek (2013) [39]	$\Delta\eta = -\frac{1-\eta_{\text{ref}}}{c_{f,\text{ref}}} \Delta c_f$ (7)

279 The results in Fig. 7 indicate that the compressor’s total-to-total isen-  
 280 tropic efficiency decreased as the Reynolds number decreased, as predicted  
 281 by the correction equation published by Dietmann and Casey [20]. Accord-  
 282 ing to Eqn. (6), which was provided by Dietmann and Casey [20], a change  
 283 in the Reynolds number causes a change in the friction factor, which results  
 284 in a change in the efficiency. The term  $B_{\text{ref}}$  refers to inefficiency due to  
 285 friction losses, while  $f_{\text{ref}}$  refers to the friction factor at the reference condi-  
 286 tions. The term  $B_{\text{ref}}$  is based on experimental data from over 30 compressors  
 287 ( $Re_c = 50,000 \dots 100,000,000$ ), and depends on the flow coefficient as fol-  
 288 lows:

$$B_{\text{ref}} = 0.05 + \frac{0.002}{\phi + 0.0025}. \quad (8)$$

## 289 5. Results

290 The results are shown for the baseline compressor at the design/peak effi-  
 291 ciency point (DES/PE), near stall (NS) and near choke (NC) conditions and

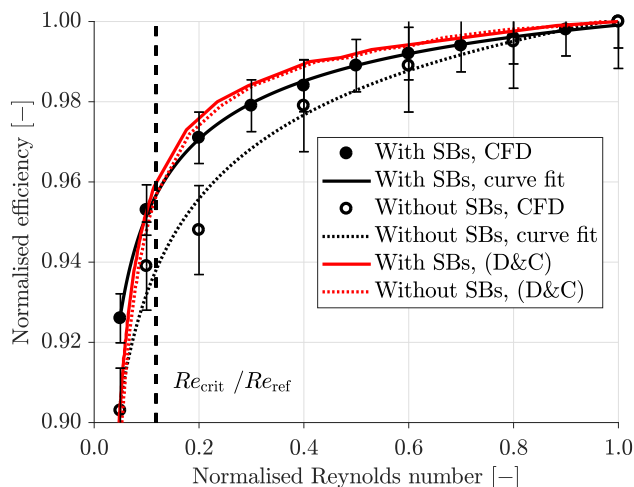


Figure 7: Change in total-to-total isentropic efficiency with a varying Reynolds number

292 for the smallest downscaled compressor (SF=0.05) at the design/peak effi-  
 293 ciency point. The impeller outlet diameter of the smallest downscaled com-  
 294 pressor (16.6 mm with splitter blades and 13.5 mm without splitter blades) is  
 295 of the same order of magnitude as that of the centrifugal compressor manufac-  
 296 tured by Isomura et al. [40] (10 mm). Additionally, a small-scale compressor  
 297 (12 mm) was manufactured by Kang et al. [41]. In this study, the manufac-  
 298 turing tolerances of the blade thickness or tip clearance were not accounted  
 299 for in order to purely investigate the Reynolds number losses.

300 The preliminary results presented previously by the authors [18] showed  
 301 that the largest fraction of the losses was generated in the tip clearance  
 302 and blade boundary layers of the compressor with splitter blades. However,  
 303 the analysis suffered due to the difficulty calculating the boundary layer  
 304 thickness inside the blade passage of a centrifugal compressor. The previous  
 305 results by the authors [21] showed that the constantly increasing boundary

306 layer thickness of a flat plate has its weaknesses in the case of a centrifugal  
 307 compressor due to the jet-wake flow structure and locally increasing relative  
 308 velocity. Thus, the authors proposed a hybrid method for calculating the  
 309 boundary layer thickness inside a complex centrifugal compressor flow field  
 310 [21]. This approach was employed in this study because it made it possible  
 311 to analyse loss generation with a more sophisticated approach.

312 When the hybrid method is employed, the velocity profile on the line  
 313 perpendicular to the investigated surface is exported for the post-processing  
 314 purposes. The number of lines in the meridional direction is specified by the  
 315 user. Firstly, the flow is assumed attached, and the free-stream velocity and  
 316 boundary layer thickness are calculated as follows:

$$\frac{dU}{dn} = 0.005 \Rightarrow U_{n-1} = 0.995U_n. \quad (9)$$

317 Hence, the boundary layer thickness is the distance between the surface and  
 318 the location where the velocity is 99.5% of the velocity of the adjacent data  
 319 point. Secondly, the average value of for the free-stream velocities in the  
 320 meridional direction is calculated as follows:

$$U_{\infty,ave} = \frac{1}{N} \sum_1^N U_n. \quad (10)$$

321 Thirdly, the velocity profile is plotted and flow separation on the blade suc-  
 322 tion side near the leading edge is qualitatively analysed from the plot by  
 323 the user. Fourthly, the locations of flow separation are specified by the user.  
 324 Finally, the free-stream velocity and boundary layer thickness are calculated  
 325 for attached flow by using Eqn. (9) and for separated flow, as follows:

$$U_{\delta} = 0.995U_{\infty,ave}. \quad (11)$$

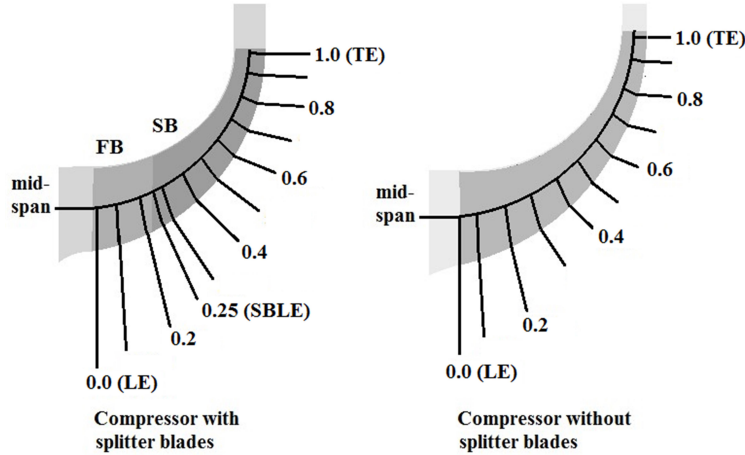


Figure 8: Observation planes along the meridional direction from the full blade (FB) leading edge (LE) to the trailing edge (TE)

326 *5.1. Boundary Layer Losses at the Blade Surfaces*

327 The hybrid method described above was used to calculate the blade  
 328 boundary layer thickness. The observation planes in the meridional direction  
 329 are shown in Fig. 8. Figures 9 and 10 provide a sum of the boundary layer  
 330 thickness near the blade surfaces ( $\delta_{\text{FBPS}} + \delta_{\text{FBSS}} + \delta_{\text{SBPS}} + \delta_{\text{SBSS}}$ ) from the  
 331 blade leading edge (0.0) to the trailing edge (1.0) in the compressors without  
 332 and with splitter blades respectively. The boundary layer thickness was nor-  
 333 malised by the pitchwise length of the modelled compressor blade passage at  
 334 the impeller outlet. Based on the sensitivity analysis [21], the relative ve-  
 335 locity values of 10,000 data points in the pitchwise direction were analysed.  
 336 The data points were located at the mid-span in order to exclude the effect  
 337 of the endwall boundary layers on the blade boundary layers.

338 The results in Figs. 9 and 10 indicate no change in the boundary layer

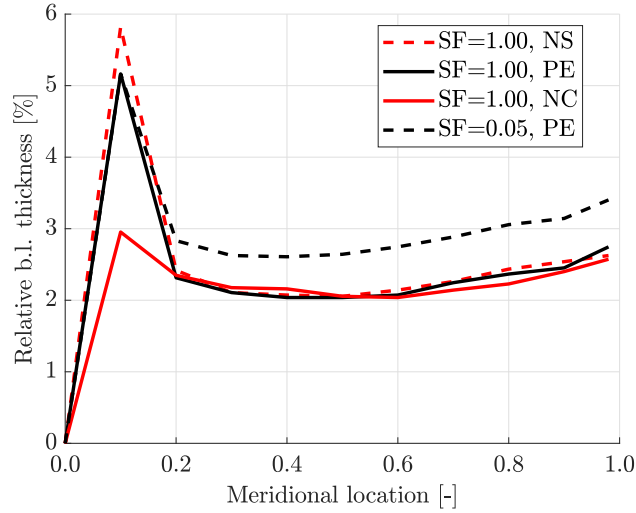


Figure 9: Sum of the relative boundary layer thickness near the full blade pressure and suction surfaces ( $\delta_{\text{FBPS}} + \delta_{\text{FBSS}}$ ) in the compressor without splitter blades

339 thickness under different operating conditions at a high Reynolds number  
 340 (SF=1.00), except for a separation point at 10% chord length from the leading  
 341 edge. However, the boundary layer thickness increased, on average, by 30  
 342 and 36% at a low Reynolds number (SF=0.05) in the compressors without  
 343 and with splitter blades, respectively. In the compressor with splitter blades  
 344 (Fig. 9), the summarised boundary layer thickness increased downstream to  
 345 the separation point due to the increased boundary layer thickness around  
 346 the splitter blade (the splitter blade leading edge at a meridional location  
 347 of 0.25); this stands in contrast to the other compressor. Table 3 shows  
 348 how the boundary layer thickness, averaged over the blade length, changed  
 349 in the near-stall, near-choke and low-Reynolds-number cases compared to  
 350 the baseline case (SF=1.00, DES); the change was insignificant under the  
 351 off-design conditions, but remarkable in the low-Reynolds-number case.

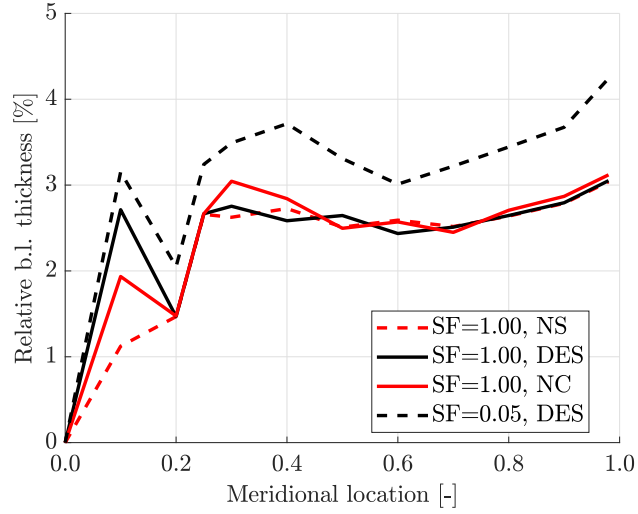


Figure 10: Sum of the relative boundary layer thickness near the full and splitter blade pressure and suction surfaces ( $\delta_{FBPS} + \delta_{FBSS} + \delta_{SBPS} + \delta_{SBSS}$ ) in the compressor with splitter blades

Table 3: Relative increase in the average blade boundary layer thickness in the blade passage compared to the baseline case at the design/peak efficiency point (SF=1.00, DES/PE)

	SF=1.00, NS	SF=1.00, NC	SF=0.05, DES/PE
Without splitter blades	+3%	-5%	+30%
With splitter blades	-5%	$\pm 0\%$	+36%

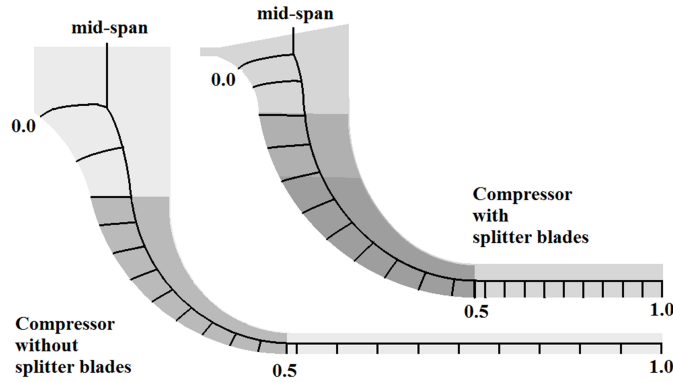


Figure 11: Observation locations in the meridional direction from the impeller inlet (0.0) to the diffuser outlet (1.0)

352 *5.2. Boundary Layer Losses at the Endwalls*

353 When calculating the boundary layer thickness at the impeller and dif-  
 354 fuser endwalls, the relative and absolute velocities, respectively were anal-  
 355 ysed. In total, 22 locations in the meridional direction from the impeller  
 356 inlet (0.0) to the diffuser outlet (1.0) were investigated. The observation lo-  
 357 cations in the meridional direction are shown in Fig. 11. The velocity was  
 358 averaged in the pitchwise direction based on ten investigated locations, and  
 359 the boundary layer thickness from the spanwise distribution was calculated.  
 360 The method was less sensitive to the number of data points in the spanwise  
 361 direction than in the pitchwise direction. For this reason, based on the sen-  
 362 sitivity analysis, in this study, 160 points were analysed in the impeller and  
 363 100 points in the diffuser.

364 Increased boundary layer thickness was clearly visible near the diffuser  
 365 hub and diffuser shroud of the investigated compressors (Figs. 12 and 13). In  
 366 the compressor without splitter blades (Fig. 12), the maximum thickness of

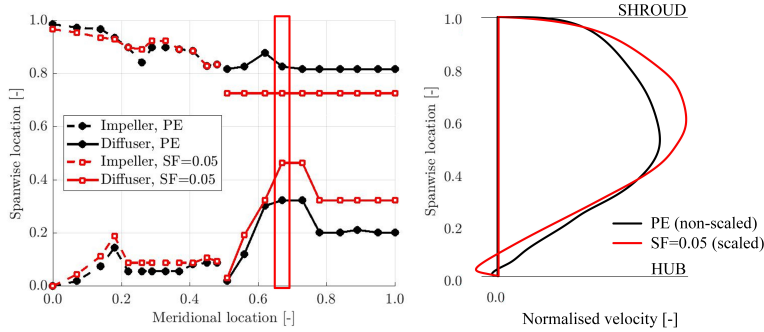


Figure 12: Left: Boundary layer thickness normalised by the passage height near the hub ( $b/b_{\text{shroud}} = 0.0$ ) and shroud ( $b/b_{\text{shroud}} = 1.0$ ) in the compressor without splitter blades. Right: Velocity profile projected in the radial direction at the meridional location, marked with a red rectangle.

367 the boundary layer near the diffuser hub was located at a meridional location  
 368 of 0.67 ( $r/r_2 = 1.52$ , marked with a red rectangle) as a result of reverse flow.  
 369 The velocity profile on the right-hand side in Fig. 12 illustrates the reverse  
 370 flow near the hub.

371 In the compressor with splitter blades (Fig. 13), the boundary layer thick-  
 372 ness increased near the hub and shroud from the impeller inlet to the diffuser  
 373 outlet. In the low-Reynolds-number compressor (SF=0.05), the reversed flow  
 374 near the hub at the diffuser outlet led to an increase of 200% in the boundary  
 375 layer thickness compared to the baseline compressor (DES). The reverse flow  
 376 is illustrated by the velocity profile on the right-hand side in Fig. 13.

377 Table 4 presents the relative increase in the average endwall boundary  
 378 layer thicknesses in the smallest downscaled compressor compared to the  
 379 baseline case at the design/peak efficiency point. The comparison of the re-  
 380 sults in Tables 3 and 4 indicates greater thickening of the boundary layer, on



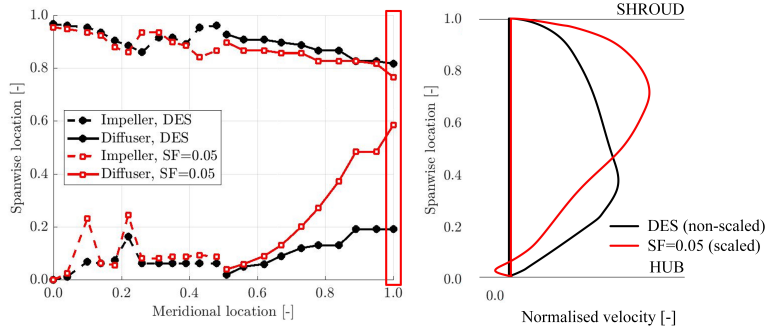


Figure 13: Left: Boundary layer thickness normalised by the passage height near the hub ( $b/b_{\text{shroud}} = 0.0$ ) and shroud ( $b/b_{\text{shroud}} = 1.0$ ) in the compressor with splitter blades. Right: Velocity profile projected in the radial direction at the meridional location, marked with a red rectangle.

Table 4: Relative increase on the average endwall boundary layer thicknesses in the smallest downscaled compressor (SF=0.05, DES/PE) compared to the baseline case at the design/peak efficiency point (SF=1.00, DES/PE)

	With splitter blades	Without splitter blades
Endwall average	+62%	+45%
Impeller average	+54%	+35%
Impeller shroud	+54%	+21%
Impeller hub	+54%	+48%
Diffuser average	+69%	+55%
Diffuser shroud	+29%	+59%
Diffuser hub	+108%	+50%

381 average, near the endwalls than near the blade surfaces. In both compressors,  
382 the boundary layer thickness increased more, on average, in the diffuser than  
383 in the impeller. The greater thickening of the boundary layer near the im-  
384 peller hub than that near the blade surfaces might result from the secondary  
385 flow, which shifts the low-momentum fluid towards the impeller hub and fur-  
386 ther along the blade surfaces to the wake located in the shroud suction side  
387 corner- of the blade passage [42]. In the diffuser, the low-momentum fluid  
388 from the boundary layers is not shifted to the wake as it is in the impeller,  
389 resulting in greater thickening of the boundary layers.

390 More detailed investigation of the relative increase in the boundary layer  
391 thickness indicated that the endwall boundary layer thickness increased more  
392 at the impeller hub than at the impeller shroud in the compressor without  
393 splitter blades, whereas in the compressor with splitter blades, the increase  
394 was equal at the impeller hub and shroud. The greater increase of the im-  
395 peller shroud boundary layer thickness in the compressor with splitter blades  
396 might be due to the larger relative tip clearance than in the compressor with-  
397 out splitter blades.

398 The relatively thicker boundary layers result in increased blockage, which  
399 is observed as increased radial velocity. The velocity profiles on the right-  
400 hand side of Figs. 12 and 13 indicate increased velocity due to the increased  
401 blockage and Table 5 shows the average increase in the radial velocity at  
402 the diffuser inlet and outlet in the low-Reynolds-number case compared to  
403 the baseline case. The radial velocity is calculated from the mass flow rate  
404 through the computational domain, pitchwise-averaged density distribution  
405 from the numerical simulation, and cross-sectional area of the computational

Table 5: Relative change in the radial velocity component at the diffuser inlet ( $r/r_2 = 1.04$ ) and outlet ( $r_3$ ) compared to the baseline case at the design/peak efficiency point (SF=1.00, DES/PE)

		SF=0.05, DES/PE
Without splitter blades	Diffuser inlet, $r/r_2 = 1.04$	+7.1%
Without splitter blades	Diffuser outlet, $r_3/r_2 = 2.48$	+8.6%
With splitter blades	Diffuser inlet, $r/r_2 = 1.04$	+5.1%
With splitter blades	Diffuser outlet, $r_3/r_2 = 1.68$	+5.2%

406 domain as follows:

$$c_r = \frac{q_{m,\text{domain}}}{\rho A_{\text{domain}}}. \quad (12)$$

407 The normalised radial velocity is averaged in the spanwise direction in  
408 order to calculate the relative increase. The increased radial velocity increases  
409 the wall shear stress and decreases the static pressure, resulting in greater  
410 friction losses and weaker compressor performance. The results presented  
411 in this subsection indicate that the method that exhibits the most potential  
412 to decrease the losses due to low Reynolds numbers involves controlling the  
413 boundary layers near the impeller hub and diffuser surfaces. The result of  
414 the diffuser's significant role in the performance deterioration is in contrast  
415 to previous knowledge; i.e., most of the losses occur in the impeller due to  
416 the high flow velocities [20].

### 417 5.3. Losses Associated with Tip Clearance

418 The losses associated with the tip clearance are difficult to distinguish  
419 from the boundary layer losses near the impeller shroud due to the tip leakage  
420 flow. However, in many compressors, the blade boundary layer, endwall

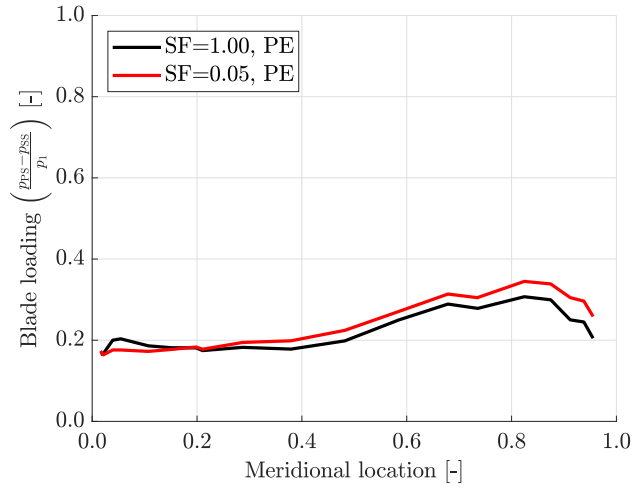


Figure 14: Blade loading defined as a normalised pressure difference across the blade at the 95% span of the compressor without splitter blades

421 boundary layer and tip leakage losses are of the same order of magnitude [43].  
 422 To analyse the effect of the decreased Reynolds number on the tip leakage  
 423 losses, the blade loading is investigated. In Fig. 14, the blade loading at  
 424 the 95% span is defined as a pressure difference across the blade normalised  
 425 by the pressure at the compressor inlet. The blade loading increased by 7%  
 426 on average in the smallest downscaled compressor compared to the baseline  
 427 case. As the pressure difference across the blade drives the tip leakage flow  
 428 from the pressure side to the suction side and the increased blade loading  
 429 corresponds to the strengthened tip leakage flow [44], the results presented  
 430 in Fig. 14 indicate that the tip leakage strengthened with the decreased  
 431 Reynolds number when the relative tip clearance remained constant.

432 However, in micro-scale centrifugal compressors, the relatively larger tip  
 433 clearances due to the manufacturing and controlling reasons would result

434 in further increased tip leakage losses. The numerical results of this work  
 435 indicated that a 100% larger relative tip clearance (from 25  $\mu\text{m}$  to 50  $\mu\text{m}$ )  
 436 in the smallest downscaled compressor without splitter blades resulted in a  
 437 less than 1% additional decrease in the efficiency. This result agrees with the  
 438 results presented elsewhere [45].

#### 439 5.4. Overall Losses

440 The increased friction losses of the low-Reynolds-number compressor re-  
 441 sult in an increased total pressure loss coefficient:

$$K_p = \frac{p_{t,2} - p_{t,3}}{p_{t,2} - p_{s,2}}, \quad (13)$$

442 which is presented in Fig. 15 as a function of the Reynolds number for both  
 443 compressors. At the critical chord Reynolds number (200,000), the increase  
 444 in the total pressure loss was approximately 40% in the compressor without  
 445 splitter blades and 30% in the compressor with splitter blades.

446 Additionally, Figure 16 presents the pressure recovery coefficient

$$C_{pr} = \frac{p_{s,3} - p_{s,2}}{p_{t,2} - p_{s,2}} \quad (14)$$

447 as a function of the Reynolds number. At the critical chord Reynolds number,  
 448 the decrease in the pressure recovery coefficient was approximately 10% for  
 449 both compressors. Figure 17 shows both the impeller and compressor stage  
 450 efficiencies, which were calculated using Eqns. (15) and (16), respectively,  
 451 and based on the adiabatic assumption,  $T_{t2} = T_{t3}$ .

$$\eta_{s,t1-t2} = \frac{\left(\frac{p_{t2}}{p_{t1}}\right)^{\frac{R}{c_p}} - 1}{\frac{T_{t2}}{T_{t1}} - 1} \quad (15)$$

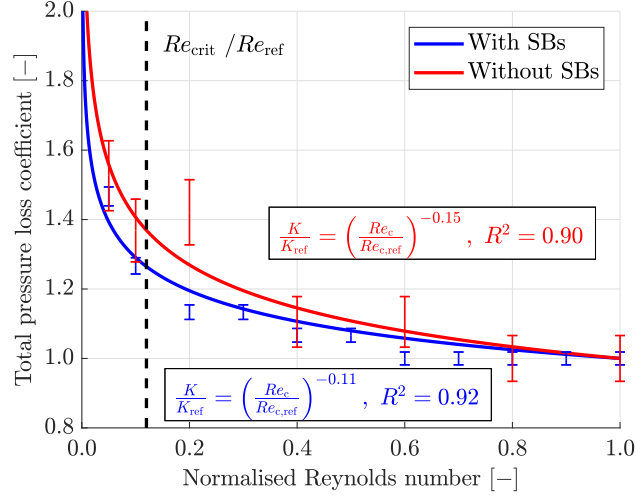


Figure 15: Change in a normalised total pressure loss coefficient with a varying Reynolds number.  $R^2 = 0.92$  with splitter blades (SBs) and  $R^2 = 0.90$  without splitter blades.

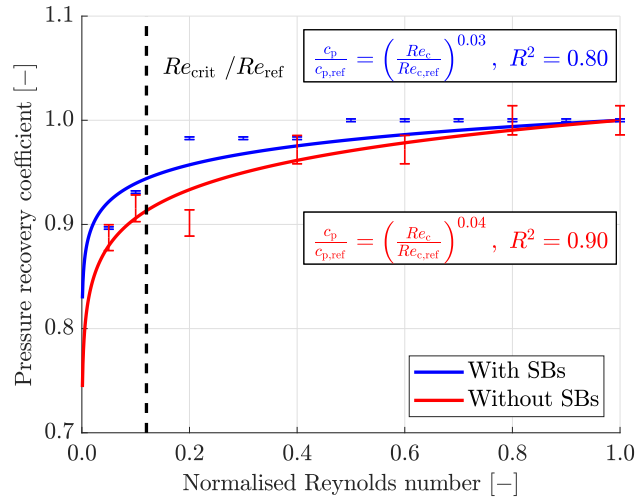


Figure 16: Change in a normalised pressure recovery coefficient with a varying Reynolds number.  $R^2 = 0.80$  with splitter blades (SBs) and  $R^2 = 0.90$  without splitter blades.

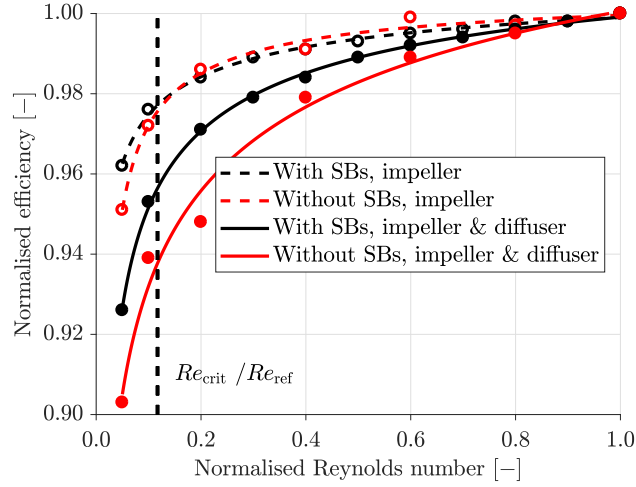


Figure 17: Change in impeller efficiency (dash line) and compressor stage efficiency (solid line) with a varying Reynolds number

452

$$\eta_{s,t1-t3} = \frac{\left(\frac{p_{t3}}{p_{t1}}\right)^{\frac{R}{c_p}} - 1}{\frac{T_{t3}}{T_{t1}} - 1} \quad (16)$$

453 The results shown in Figs. 15 – 17 indicate that the diffuser had a signifi-  
 454 cant influence on the performance deterioration of the low-Reynolds-number  
 455 compressors. When the Reynolds number decreased by 90% (from the base-  
 456 line to the critical Reynolds number), the impeller caused 54% and the  
 457 diffuser 46% of the efficiency deterioration in the compressor with splitter  
 458 blades. In the compressor without splitter blades, the respective values were  
 459 50% (impeller) and 50% (diffuser). And, as shown in Table 4, the boundary  
 460 layer growth was greater near the diffuser endwalls than near the impeller  
 461 endwalls. The calculation of the changes in the mass-flow-averaged specific  
 462 entropy of the impeller and diffuser strengthens this argument; e.g., in the

463 compressor with splitter blades at the baseline Reynolds number, 58% of  
464 the total specific entropy increase occurred in the impeller and 42% in the  
465 diffuser, whereas in the smallest downscaled compressor, the fraction of the  
466 diffuser increased to 47%. In the compressor without splitter blades, 36%  
467 of the total specific entropy increase occurred in the diffuser at the baseline  
468 Reynolds number, and 46% at the lowest investigated Reynolds number (in  
469 the smallest downscaled compressor). These results indicate that the role of  
470 the diffuser on the performance deterioration strengthens with the decreasing  
471 Reynolds number. Therefore, the diffuser should be included in the loss  
472 development analysis even though recent investigations have mainly focused  
473 on the impeller.

474 The deterioration in performance observed as the decreased efficiency in  
475 the present study exhibited a similar trend (Fig. 7) as predicted by the  
476 correction equation of Dietmann and Casey [20]. In the present study, the  
477 fully turbulent flow field was assumed, and the transition from laminar to  
478 turbulent flow was not accounted for. However, it is not obvious whether the  
479 transition model should be used or not when modelling the low-Reynolds-  
480 number flows in centrifugal compressors. Previously, different approaches  
481 have been employed to model small-scale centrifugal compressors. For exam-  
482 ple, the standard  $k - \epsilon$  model [46], Chien's low-Reynolds-number  $k - \epsilon$  model  
483 [41], a model proposed by Spalart and Allmaras [47] and even a laminar ap-  
484 proach [48]. In the present study, the influence of transition on turbulence  
485 modelling was investigated using the SST  $k - \omega$  model in combination with  
486 the  $\gamma - Re_\theta$  transition model proposed by Langtry and Menter [49]. Accord-  
487 ing to the results of this study, accounting for the transition did not change



488 the results from the fully turbulent solution. The transition model could  
489 capture the laminar separation; however, because the flow separation near  
490 the blade leading edge resulted from the centrifugal force and not from the  
491 laminar separation, the modelling of transition in the centrifugal compressor  
492 did not add value.

493 Since the efficiency correction equation proposed by Dietmann and Casey  
494 [20] is based on the experimental data of over 30 compressors, the numerical  
495 results in this study were compared to their experimental data. Simply  
496 measuring a micro-scale compressor would not have yielded original results  
497 because there is already data available on low-Reynolds-number compressors  
498 [20]. In addition, experimental data of micro-scale compressors exists [41].  
499 While information about the flow field inside the micro-scale compressor  
500 would introduce a certain degree of novelty, as long as the lack of micro-scale  
501 measurement instruments continue to limit the experiments, the flow fields  
502 still needs to be studied with the help of computational fluid dynamics.

503 The results presented above provide an overview of the loss generation  
504 due to low Reynolds numbers.

## 505 **6. Discussion**

506 Is there a way to counter the reduction in efficiency of a compressor that  
507 is caused by the low Reynolds number? Based on the numerical results, the  
508 increased boundary layer thickness most strongly affects the diminished ef-  
509 ficiency in the low-Reynolds-number compressors. Therefore, the efficiency  
510 could be increased by controlling the boundary layer. Controlling the im-  
511 peller hub and diffuser boundary layers would result in the most significant

512 reduction in losses.

513 To decrease the boundary layer thickness, the low-momentum fluid near  
514 the surface should be either accelerated or removed. The difficulties in using  
515 the boundary layer acceleration or suction inside a centrifugal compressor  
516 stem from the requirement for pressurised air and a control device. The flow  
517 from the blade pressure side to the suction side through small holes could  
518 accelerate the boundary layer flow on the blade suction side, but the holes  
519 would decrease the mechanical strength and loading. The boundary layer  
520 control would be easier on the stationary diffuser endwalls, but implementing  
521 the control device would be challenging, especially in the case of micro-scale,  
522 low-Reynolds-number compressors; their advantages include added savings  
523 in size and weight.

524 In a vaned diffuser, researchers have proved that a sufficiently simple flow  
525 control method called the porous throat diffuser, which links the throats  
526 of the diffuser passages via a side cavity, broadens the operating range due  
527 to a more uniform pressure distribution [50]. The applicability of this kind  
528 of simple configuration for a vaneless diffuser should be investigated in the  
529 future. To conclude, the improvement in the low-Reynolds-number com-  
530 pressor's performance would require an innovative means for boundary layer  
531 control without the need for external control devices.

## 532 **7. Conclusions**

533 At a low Reynolds number:

- 534 • Blade boundary layer thickness increases, on average, from approxi-  
535 mately 30% to 36%;

- 536 • Endwall boundary layer thickness increases, on average, from approxi-  
537 mately 45% to 62%;
- 538 • Tip leakage strengthens;
- 539 • Blockage due to thicker boundary layers increases the radial velocity  
540 component at the diffuser outlet from 5% to 9%. The increased radial  
541 velocity increases the wall shear stress and decreases the static pressure;
- 542 • The 90% decrease in the Reynolds number results in a 30–40% increase  
543 in the total pressure loss coefficient and in a 10% reduction in the  
544 pressure recovery coefficient;
- 545 • The impeller accounts for 50 – 54% and the diffuser 46 – 50% of the  
546 efficiency deterioration, and the role of the diffuser in the performance  
547 deterioration strengthens with the decreasing Reynolds number;
- 548 • The 100% increase in the relative tip clearance (from 0.045 to 0.091)  
549 results in an additional 1% reduction in efficiency;
- 550 • Modelling transition in the centrifugal compressor does not add value  
551 compared to the computational cost of the process, as the flow separates  
552 near the leading edge due to centrifugal force, regardless of whether or  
553 not the laminar separation occurs.

554 To improve the performance of the centrifugal compressors that operate  
555 at low Reynolds numbers, the boundary layers near the impeller hub, and  
556 especially in the diffuser, should be suppressed. However, the use of control  
557 devices for boundary layer acceleration or suction is problematic because their

558 small size and low weight make micro-scale gas turbines the more attractive  
559 alternative to batteries and piston engine-based power modules; however,  
560 the control devices required for these would increase the overall weight of the  
561 compressors.

562 Future work should focus on the development of an innovative means of  
563 boundary layer control without the need for external control devices in order  
564 to improve the low-Reynolds-number compressor's performance.

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