Lappeenranta University of Technology
School of Engineering Science
Computational Engineering and Technical Physics
Technomathematics

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ENSEMBLE METHODS IN COMPUTATIONAL MARKET DYNAMICS

Master's Thesis

Examiners: Professor Tuomo Kauranne
D.Sc. Matylda Jablonska-Sabuka

Supervisor: Professor Tuomo Kauranne

ABSTRACT

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The dynamics of financial and commodity markets play a vital role in determining the prices of goods or services traded in these markets. Whereas numerous market models have been developed to study these dynamics, more findings find weaknesses in the existing models. For that cause, this study applies the Variational Ensemble Kalman Filter to simulate electricity spot prices for the Finnish and Swedish electricity markets. The model applies an approach to derive nonlinear system dynamics from Kalman filtering called Kalman Dynamics to simulate electricity spot prices by using traders as ensemble members. These traders make bids in perception of what they expect the prices to be in regard to other traders' bidding. We thus make them turn inward toward themselves and predict the behavior of future prices.We indicate that the dynamics in the electricity spot market influence the electricity spot prices making them to have an unstable movement.

PREFACE

With a grateful heart, I place on record my deepest gratitude to my supervisor Prof. Tuomo Kauranne, for his unending guidance, expertise and advice which have been of utmost importance during the course of doing this work. I also want to thank Dr. Matylda Jablonska for giving me an open access to the data that was used during the implementation of the methodology applied to this study and in the same sense, I reallly appreciate Martin Gunia for providing the *MATLAB* sample codes that guided me towards getting the results.

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Above all, I thank God, the Almighty, for the gift of life that He has given me for without Him, this work would not have been completed.

Lappeenranta, July 31, 2018

Kyabo Gloria Sharon

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ABBREVIATIONS AND SYMBOLS

KF	Kalman filter
EKF	Extended Kalman filter
EnKF	Ensemble Kalman filter
VEnKF	Variational Ensemble Kalman filter
LBFGS	Limited memory Broyden-Fletcher-Goldfarb-Shanno
KD	Kalman Dynamics
€/MWh	Euros per Megawatt hour
TWh	Terawatt hour
CHP	Combined Heat and Power

1 INTRODUCTION

Stochastic processes are widely used in financial and commodity markets. A specific example of a commodity market is the energy market where energy is traded and supplied. This energy can take the form of crude oil, electricity or gas. Among these, electricity prices are known to be one of the most unpredictable which makes the energy market a good target for financial and mathematical modeling. Due to their volatility, electricity prices are challenging to model in supplement to other features like seasonality and extremely large price movements which sometimes lead to negative prices. Thus for any researcher to work on such commodities, all these factors should be put into consideration. Since storage of electricity is challenging too, its price tends to behave in both a deterministic and a stochastic way. Demand and supply play a major role in determining market prices of a commodity in a given location and at a particular time.

Different models are utilized in the presentation of financial data. These include the autoregressive moving average models, stochastic time series models, geometric Brownian motion and autoregressive conditional heteroscedastic models among others. The proposed study will apply Kalman dynamics combined with an application of the Variational Ensemble Kalman Filter to simulate electricity spot prices for Finland and Sweden spot electricity markets.

1.1 Background

The Kalman Filter (KF) is an optimal estimator under which the parameters of interest are inferred from inaccurate, indirect and uncertain observations. It is recursive so that new measurements can be processed as they arrive [1]. In addition, to being easy to formulate and implement given a basic understanding, the Kalman Filter gives good results in practice. The term filter is used in this context in relation to the fact that it is a process of finding the best estimate from past to the current state of a dynamical system which amounts to filtering out the noise.

Kalman filters have been put into service in a number of studies ranging from time series at large to financial time series in particular. Grewal and Andrews (2010) studied the applications of Kalman filtering in Aerospace from 1960 to 2010. They concluded that the Kalman filter found early acceptance in the aerospace industry as the basis for modern estimation and control theory, not only for the theoretically optimal solution but as a practical and reliable solution [2]. As to applying Kalman filtering to financial markets, Bit-Kun et al. (2010) estimated the time-varying world integration of the Malaysian stock market and examined the paths of the time-varying integration in relation to the economic events of the country [3].

KF as an optimal method best suits methods for linear dynamic models. However, it can be extended to nonlinear models by linearization of both the non linear model and the non linear observation operator. The resulting technique is known as the Extended Kalman Filter (EKF). EKF extends the KF by linearizing a system dynamics' trajectory that is continually updated with state estimates resulting from measurements [4]. This technique has been known for a long time for its robust and accurate state estimates but it has been impossible for large scale problems. A full EKF has been always considered computationally unfeasible for numerical prediction purposes if there is a large number of model variables. The Ensemble Kalman Filter (EnKF) is another method primarily used to generate probabilistic state estimates.The EnKF has the advantage of not needing the tangent linear model and its computational requirements are affordable [5]. The method has been used successfully with a number of different dynamical nonlinear and chaotic models. More findings suggest that there is a modification required in the EnKF such that random noise is added to the observations at the analysis stage. This was realized in the study that emphasized the application of Ensemble Kalman filtering in the analysis scheme [6].

The KF, rooted in the state-space formulation of linear dynamical systems, provides a recursive solution to the linear optimal filtering problem. It applies to stationary as well as non-stationary environments. The solution is recursive in that each updated estimate of the state is computed from the previous estimate and the new input data, so only the previous estimate requires storage. In addition to eliminating the need for storing the entire past observed data, the KF is computationally more efficient than computing the estimate directly from the entire past observed data at each step of the filtering process [7].

In the study, we propose to apply Kalman dynamics to forecast electricity prices using the Variational ensemble Kalman filter (VEnKF). The method will be tested using the Lorenz95 model. Lorenz himself described the model as chaotic and periodic for some parameter values and therefore suitable to mimic the electricity market prices. For this purpose the Ensemble Kalman Filter must be made to turn inwards onto itself. That is to say it will use ensemble members as the observations. This property reflects the fact that spot market prices are mostly dependent on the traders' perception on how the other traders perceive price evolution and then bid accordingly. Ensemble members thereby represent virtual traders. The proposed study will employ and modify an existing VEnKF agorithm and use it to simulate electricity spot prices from the NordPool spot market.

1.2 Motivation

Financial time series draw the attention of financial analysts and researchers. The reports written in newspapers, watched on television and and other media give information about electricity prices, stock market index values and currency exchange rates. Investors and other people that are involved in international trade like the brokers and analysts who advise these people can all benefit from a deeper understanding of how prices change. This is done in order to reduce or avoid risks involved in the trading caused by fluctuating price changes. Forecasts of future price movements provide information about the risk involved, which information is then used to avoid unacceptable risks by hedging. Investigating financial time series gives an intuition of how prices behave in the market. A probability distribution is used to describe tomorrow's uncertain price. Therefore, statistical methods are a realistic way to investigate prices. Usually, one builds a model, which is a detailed description of how successive prices are determined. Secondly, knowing the behavior of prices reduces risk and leads to better decision making.

Kalman filtering as a mathematical tool is applied in many systems that deal with stochastic estimation and prediction. As a set of mathematical equations, it is used to execute a predictor- corrector type estimator. The tool reduces the estimated error covariance when the conditions presumed are met. The most important aspects of this filter include modelling the state vector of the process or system which is under study and processing of the noisy measurement data. The KF is one of the best and most effective estimators for a large class of problems, which makes it very easy to use alongside some conceptual tools. Since it was introduced, the KF has been the applied in many extensive areas of research and applications because of its relative simplicity and robust nature [4].

1.3 Hypothesis

Spot market prices are mostly dependent on the traders' perception about how the other traders perceive price evolution and then bid accordingly.

1.4 Objectives

The goal of the proposed research is to apply a modified version of an Ensemble Kalman Filter to simulate electricity spot market price evolution.

Specifically, the proposed research will determine how Kalman dynamics predicts electricity prices.

1.5 Structure of the thesis

The study covers electricity market dynamics using a Variational Ensemble Kalman Filter. The work is arranged in such a way that Section 2 covers detailed explanations of the electricity dynamics in the computational markets and NordPool electricity market. The section further gives a description of the data used in this study. In Section 3, the methodology of Kalman Dynamics is described. Price forecasting techniques are covered in Section 4 after which Section 5 indicates the results obtained and more discussions. The study ends with Section 6 which shows the conclusions and recommendations given.

2 PRELIMINARIES

In this section, we discuss an overview of commodity markets, with an emphasis on electricity markets and the NordPool electricity market.

2.1 Electricity Market

One of the most competitive markets around the globe is the electricity market. Unlike cases where consumers sometimes have a big say in the prices of goods, electricity markets are very dynamic. The consumers, apart from large scale customers, have little influence on the market. Major decisions are caused by the forces from those generating the electricity, the companies distributing it and regulatory bodies. These at times make decisions without the consent of consumers on the assumption that the consumers are to be served under all circumstances.

Categorized into two, wholesale electricity markets have shown a big progress over the last years in both Finland and Sweden unlike their counterparts, the retail electricity markets. Accordingly, fewer retail markets give opportunities to their customers to buy electrical energy at the spot market [10]. These electricity markets work under the presumption that electricity can also be treated as a commodity and it can be traded quantity wise. In their opinion they are right since electricity plays a major role in almost all sectors. Producers of consumable goods at times attach prices to them based on the production costs and in case these costs are high, the prices increase. For instance, assume the price of electricity increases, without prior knowledge of the producer of clothes. In case he realizes that the costs of production increased, he increases the prices of the clothes in order to maximize his profits. Similarly, if the producer knows that the prices are high before production, he can decide to reduce the quantity of clothes produced but leaves the cost of clothes unchanged. Whichever way, both to the consumer and to the producer, the market dynamics caused by the movement of electricity prices up and down affect them.

However, the relationship between demand and supply of electricity is influenced by many other factors. Additionally, the production cost for the case we have illustrated is not majorly governed by electricity prices. These prices contribute only a portion of the costs, likewise to the house holds, some consumers perceive electrical energy as a fraction of the costs of living that they incur. Nonetheless, electricity market, as competitive as it is, is also profitable since storage of electricity is hard.

In this study, we considered spot prices for Finland and Sweden electricity markets.

The spot prices of electricity are affected by a number of factors among which are the following [11].

- Characteristics of the market like generating capacity and imports or exports.
- Unforeseen circumstances or uncertainties like forecast load.
- Stochastic uncertainties like congestion index.
- Temperature and weather effects like seasonal changes for instance winter.
- Behavior characteristics like historical data.

Due to the unstorable nature of electricity [12], the dynamics of its price displays features that can not be completely captured by mathematical models which focus on the behavior of goods that can be stored. Unlike time series of financial derivatives like bond prices, electricity spot price time series display a formation that can not easily be applied for forecasting purposes. Forecasting is still based on available data or based on the information provided by the model.

2.2 Commodity markets and their modelling

The dynamics of the electricity market involve production of electricity based on existing demand and ability to produce it in a given quantity. It further involves distribution of electricity produced to the customers and planning of production for the new capacity. The increase in demand for electricity has turned the electricity industry to be more competitive in a way that electricity prices have to reflect the marginal costs of production [8]. Reshuffling the electricity market has mostly focused on the wholesale market performance and to a less extent the retail market. This is because in the wholesale market, electricity generating companies sell the produced electricity to other generators of electricity and to other service providers. The energy service providers then resell the electricity to other consumers in the retail market like households and other industries. The market requires both buyers and sellers to remain anonymous at times in order to allow free and fair participation. Furthermore, the market needs some constraints on trading that guide both sellers and buyers into maximizing total gains from the transactions.

A computational electricity market incorporates a trend where a given number of sellers and buyers submit their price offers for transactions to be carried out. The costs, payments and gains of both parties are kept away from other traders' view for confidentiality purposes. Moreover, price offers are frequently updated by both parties basing on their past experience. In other words, traders bid based on the profits, losses, or intuitive perception about what other traders will bid in a way that will make them earn profits.

These dynamics can be classified and studied under three models namely:

- Equilibrium models. These focus on the overall behavior of the market with an additional aspect of considering competition among all traders.
- Optimization models. These models aim at maximizing profits.
- Simulation models. These models are used under complex market dynamics which may not be addressed by equilibrium models.

Simulation models are a representation of each trader's decision dynamics based on sequential rules. Although the models are flexible in a way that any behavior of the market can be incorporated in them, care has to be taken to the assumptions made in each category of equations and these assumptions have to be justified theoretically by the model selected [9].

2.3 NordPool Dynamics

The Nordic electricity market traces its performance and establishment to way back in 1992 when the Norwegian energy act of 1991 allowed room for free trading between countries. Countries later on joined the market and this led to its expansion when Sweden joined in 1996, Finland in 1998 and Denmark in 2000 [13]. This made NordPool become the strongest power exchange market in the world. Later on in 2002, the *Elspot* which is the physical electricity market, was made independent of the derivatives market and it went by the name NordPool Spot and the market expanded further when the Baltic countries joined.

Currently, NordPool is Europe's leading power market which conveys intra-day known as Elbas and day-ahead also known as Elspot trading, clearing and settlement to its customers. However, the day-ahead market plays the biggest role for trading power among the shareholders and this is one of the reasons as to why it is the Elspot data that is used in this study.

The Elspot market works in such a way that the seller and the buyer make collections for the power that is to be delivered the following day. Planning made by the members drives the daily trading. For instance, the buyer has to determine how much energy in volume he will need to meet the electricity demand of the following day. He also estimates the amount he is willing to pay for the energy hour by hour. In a similar way, the seller of this energy has to establish how much energy he can convey and at how much he is to convey it hour by hour. All these obligations are mirrored through the orders that are formed by both sellers and buyers of electricity in the NordPool Elspot market. More so, 12:00 CET is the deadline for submitting bids for power which will be delivered the following day. The trading system feeds the information into a special computer system which calculates the price, based on an advanced algorithm.

Put simply, the price is set where the curves for sell price and buy price meet as shown in Figure 1. Thus the spot price is attained from a diverse number of bids that are presented to the administrator of the NordPool market until the time when there is no more auctioning. The offer given is done electronically through the internet with a price and volume of bids for each hour of the following day.

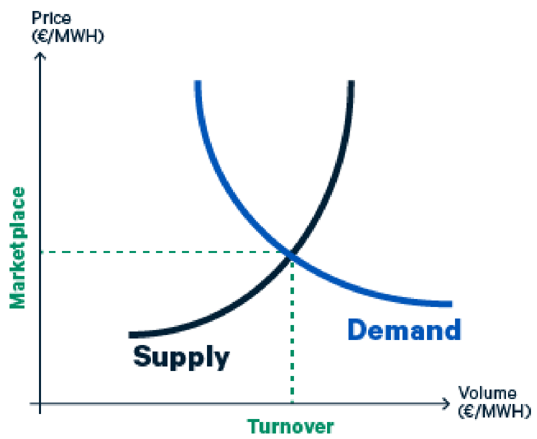

Figure 1. Demand and supply curve. *Source:* `https://www.nordpoolgroup.com/the-power-market/Day-ahead-market/`

Among other factors, unstable demand and supply play a major role in causing price fluctuations. The dynamics followed by the electricity prices are attached to how much electricity has been produced and how much can be produced at a given price at a given time. Figure 2 shows the structure of the supply of electricity in the NordPool market by various types of production in Terawatt hour (TWh).

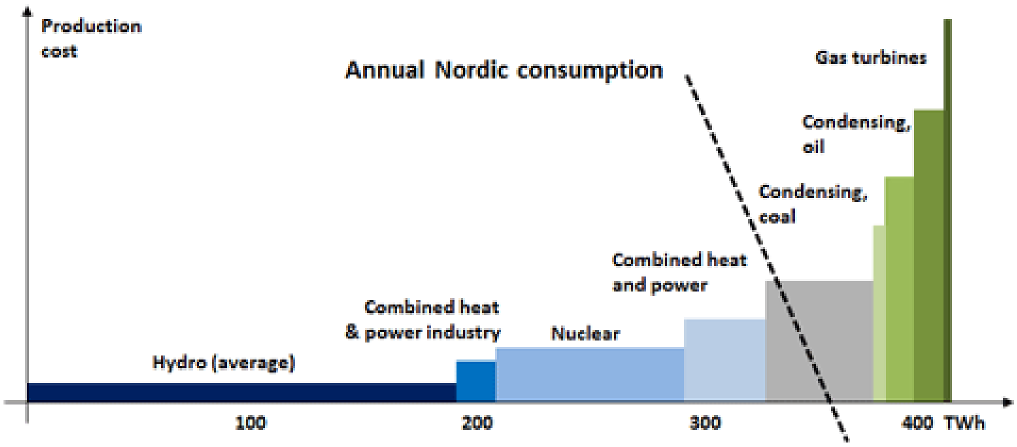

Figure 2. Supply of electricity in NordPool Market. *Source: https://www.nordpoolgroup.com/.*

However, some restrictions are advisable in such a way that the electricity market can be regulated. Consequently, one may be necessitated to pay a premium in form of a fine for the change or difference between the offered volume at the electricity spot market and the volume delivered. Regulating the market happens when the market is in excess of power or when it in deficit of power.

If the electricity market is directed into a deficit of power and the production of one producer is less than what is offered, other producers of electricity play a role by conforming to the situation. Thus they regulate (up) their production such that there is balance in the market. The one producer is subjected to a fine which can be in form of getting a price lower than the spot market price for the electricity he has produced.

Additionally, if production is more than the amount granted, the production units come in to reduce or avoid market deficit and thus they receive the market spot price for the excess production without being fined. Contrarily, under cases when the power produced in the market is in excess, other producers adjust their production by regulating (down) in order to keep the market in balance. The producer still gets a penalty as explained in the previous scenario. In case production is less than the bid in the market, the producer receives the market spot price with out getting a penalty [14]. The two cases are illustrated in Figure 3.

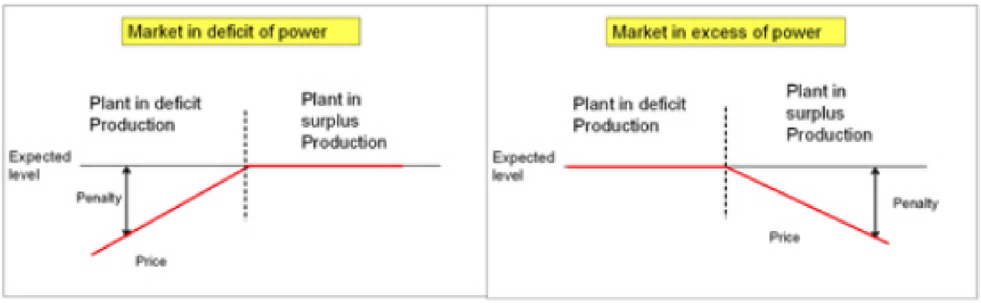

Figure 3. Regulating demand and supply. *Source*: `https://www.wind-energy-the-facts.org/power-markets-7.html`

Finland's electricity consumed by each sector from 1970 to 2015 is shown in Figure 4. Industry and construction used more electricity in TWh than the other sectors. It was followed by households and agriculture, services, public and other consumption and transmission and distribution losses which took the least TWh,

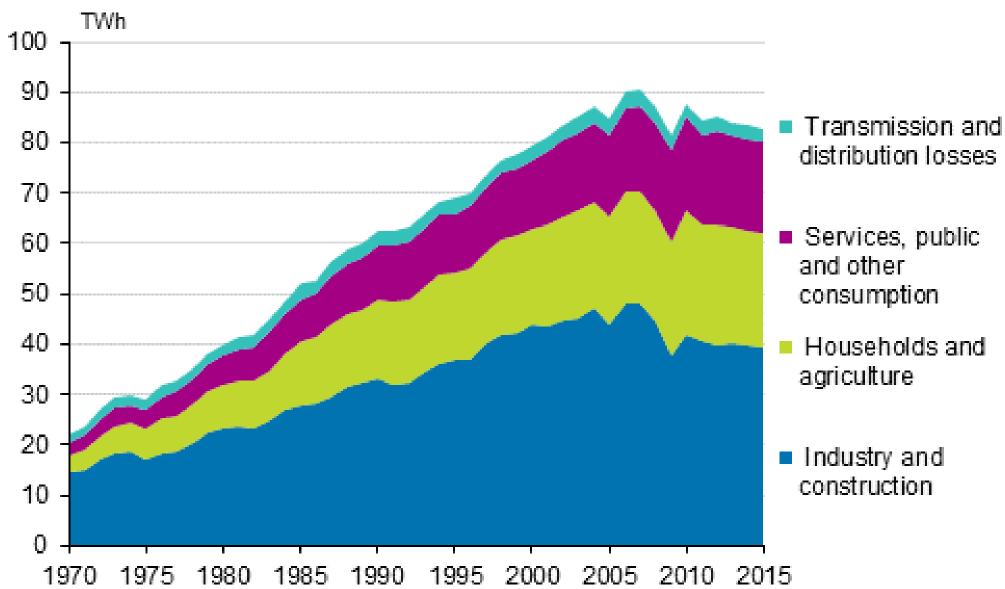

Figure 4. Finland's electricity consumption by each sector from 1970 to 2015. *Soure*: `http://www.stat.fi/til/ehk/2015/ehk_2015_2016-12-07_kuv_006_en.html`

Since electricity is supplied from numerous sources, Figure 5 indicates the amounts of electricity supplied by each sector in TWh for a period from 1970 to 2015. The digits show the amount of electricity obtained from hydro power, nuclear power, wind power, condensing power, Combined heat and power (CHP) industry, CHP district heat and net imports of electricity.

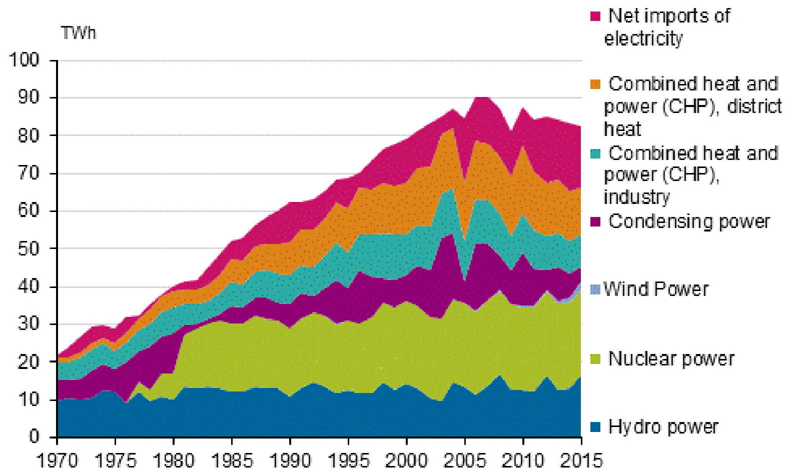

Figure 5. Finland's electricity supply from 1970 to 2015. *Source*: `http://www.stat.fi/til/ehk/2015/ehk_2015_2016-12-07_kuv_005_en.html`

Until the year 2004 when the the gas and electricity markets in the European union were liberated, Sweden had one of the lowest priced electricity markets in Europe [15]. The low prices of electricity in Sweden seem to have attracted domestic enterprises to use more electricity as compared to other forms of energy.

Sweden's electricity consumption by households and service sector from 1970 to 2006 is shown in Figure 6. The consumption is categorized into three, electricity for building service systems, domestic electricity and electric heating.

Electricity supplied in Sweden from 1970 to 2015 is shown in Figure 7. Hydro power is supplied most and round 1973, nuclear power started to be supplied in large quantities. Other forms of electricity supply in Sweden include the CHP industry, CHP domestic heating, wind power and other thermal power.

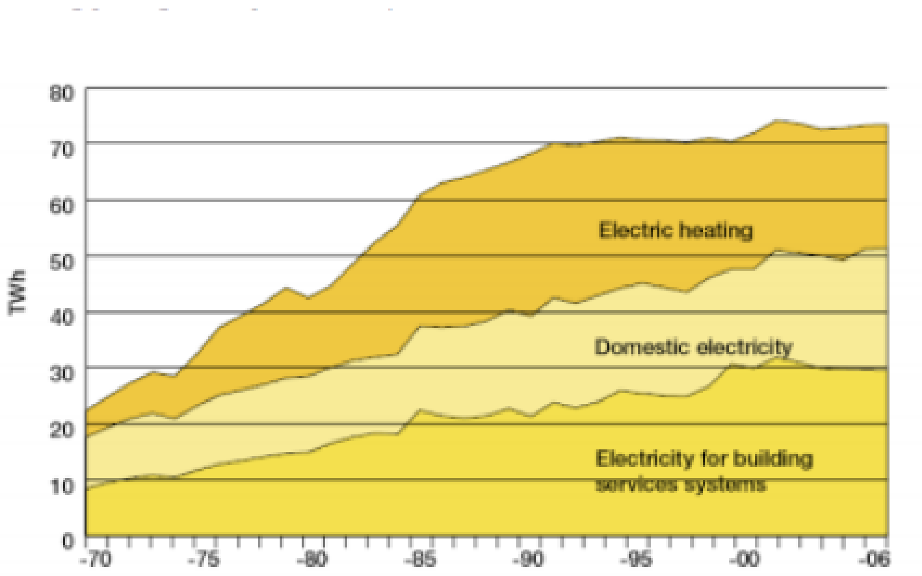

Figure 6. Sweden's electricity consumption by the household and service sector from 1970 to 2006. *Source*: `https://syracusecoe.org/conditions-of-behavioural-changes-towards-efficient-energy-use-article/`

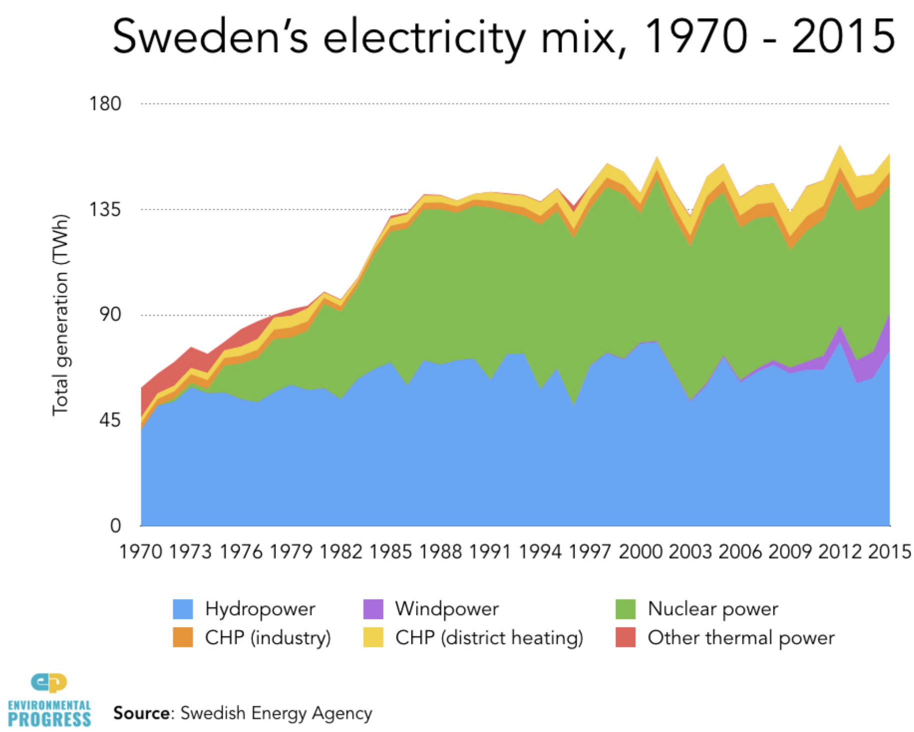

Figure 7. Sweden's electricity supply from 1970 to 2015. *Source*: `http://environmentalprogress.org/sweden/`

2.4 Description of Data

Data used in this study is hourly electricity spot price data in €/ MWh from Nord Pool electricity market with the case study of Finland and Sweden. The data consists of 13727 observations for each country starting from 28 November, 2013 to 03 June, 2015.

Table 1. Descriptive statistics of the data

	Finland	Sweden
Mean	34.4448	29.9865
Standard deviation	11.3609	6.5064
Skewness	2.8927	0.7534
Kurtosis	30.5609	9.2929
Minimum	1.95	0.59
Maximum	200.05	105.3900

Table 1 shows the summary of both sets of data. The average spot electricity prices for Finland and Sweden were 34.4484 $€/MWh$and 29.9865 $€/MWh$ respectively. Sweden had a smaller standard deviation of 6.5064 $€/MWh$ than Finland which had 11.3609 $€/MWh$. The skewness from both sets of data indicate that they are both moderately skewed right since the skewness is positive. In other words, the right tail is longer and most of the distribution is at the left tail of the Gaussian distribution curve. Higher values of the Kurtosis indicate that Finland had a higher sharper peak of the Gaussian distribution and Sweden exhibited a less distinct peak. Since the kurtosis of both Finland and Sweden are greater than 3 in comparison to a normal distribution, their tails are longer and fatter.

Figure 8, shows the trend followed by the electricity spot prices of Finland from 28/11/2013 to 03/06/2015 on an hourly basis. The price movement indicates that the electricity spot prices are not stable and they keep on moving up and down. A similar case is viewed in Figure 9 for Sweden. However, as noticed earlier, Sweden's prices are lower than Finland's prices during this time period.

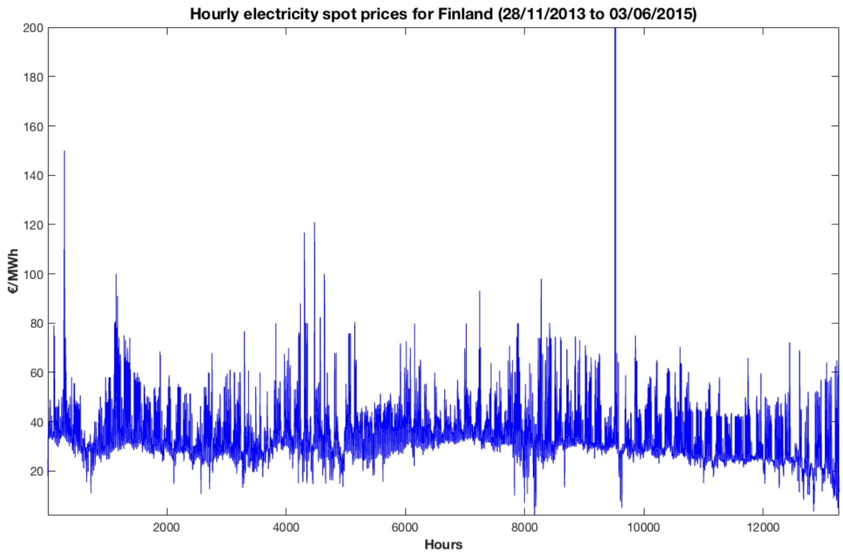

Figure 8. Finland's electricity spot prices from 28/11/2013 to 03/06/2015

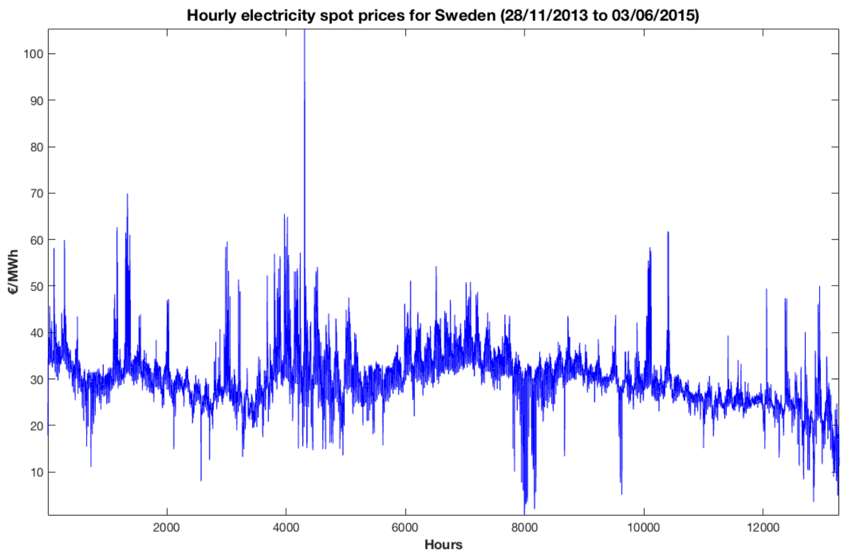

Figure 9. Sweden's electricity spot prices from 28/11/2013 to 03/06/2015

Figures 10 and Figure 11 indicate the histograms for the electricity spot prices for Finland and Sweden respectively. From the kurtosis results, Finland's histogram peak is evidenced to be higher than Sweden's peak.

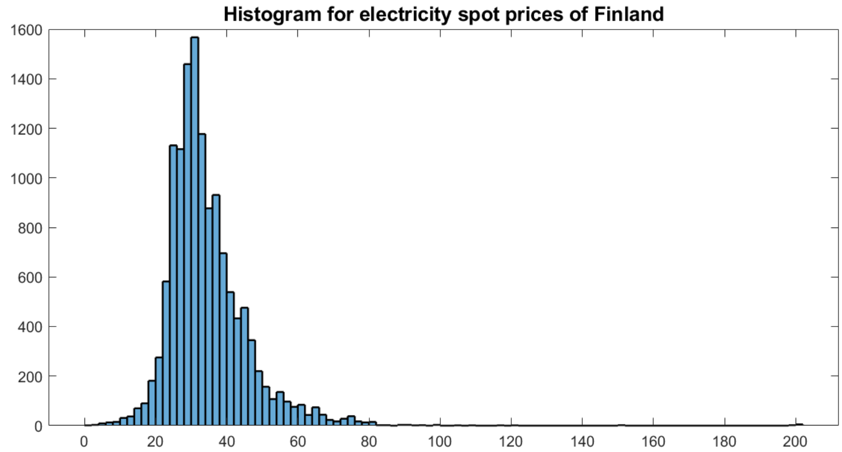

Figure 10. Histogram for Finland's electricity spot prices

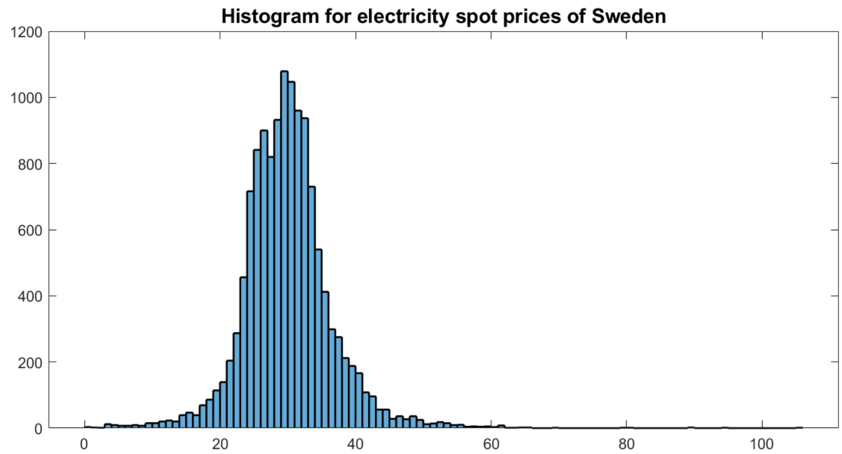

Figure 11. Histogram for Sweden's electricity spot prices

Since the data used is hourly data, we obtained the average prices for each hour from 28/11/2013 to 03/06/2015 for both Finland and Sweden.The outcome is evidenced in Figures 12 and 13. The average prices per hour take the same shape for both countries. The first hour represents the 0000 hours clock time. We observe from the figures that prices in the first three hours were very low and very high between the 6^{th} and 8^{th} hours. Around the 13^{th} hour, prices decrease sharply and then increase again up to the 17^{th} hour when they begin to drop again. These dynamics are brought about by the opening and ending hours for work places.

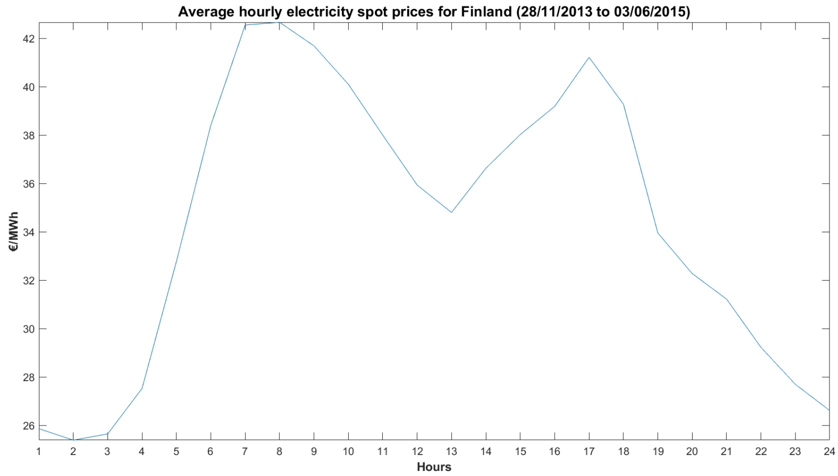

Figure 12. Finland’s hourly average electricity spot prices from 28/11/2013 to 03/06/2015

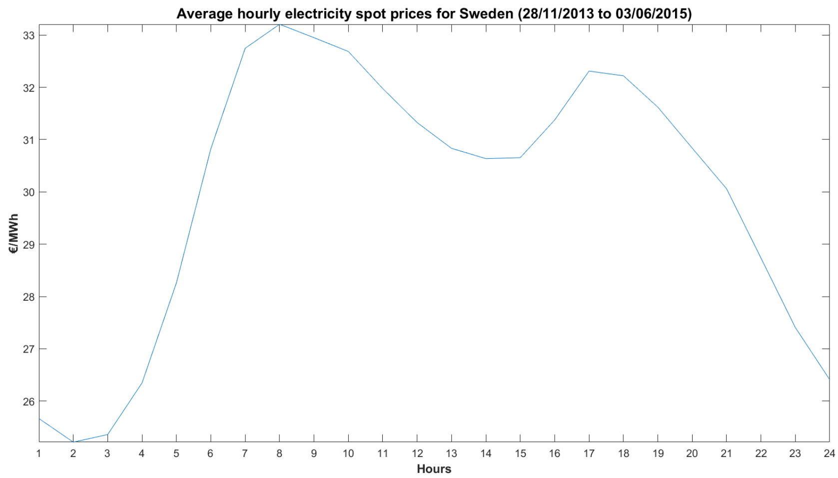

Figure 13. Sweden’s hourly average electricity spot prices from 28/11/2013 to 03/06/2015

3 METHODOLOGY

This section covers the description of data assimilation and its connection to the Kalman Dynamics. We further show the formulas for the Kalman filtering techniques.

3.1 Data assimilation

Data assimilation classifies methods that blend observed values and dynamic mathematical models for a better description of the system or to widen the knowledge known about the system. Data assimilation methods are motivated by the intrinsic fact that the mathematical model used does not give accurate depiction of the system under study. However, it tries to approximate the behavior of the system with some errors simply because some physical factors affecting the system are neglected by the model. Secondly, the measurements of the dynamical system are made with some errors hence can not alone give an accurate representation of the system [16]. Contrarily, data assimilation models are made to enhance the estimated variable by using in a combined way as much information as possible from the dynamic model and the measurements made. Thus data assimilation models provide data analysis and prediction or forecasting tools which do not deviate much form the observed variables.

Data assimilation is categorized into variational or adjoint assimilation and sequential assimilation. The former is described in such a way that for a given time interval, the initial conditions and the parameters that are unknown result in the trajectory of the model that best suits the measurements within that interval. Diversely, sequential assimilation involves commencing from a best guessed initial condition and updating the model results or solutions whenever measurements are available at a particular time step. Provided the conditions are favorable, the model solution approximately converges to the true observation. Knowledge about the system is increased since previous information is incorporated into present assimilation. Thus this kind of assimilation combines the solution of the model and the available measurements by applying an updating system to find a better estimation of the state variable.

Therefore the application of Kalman dynamics in this study falls into the later classification of data assimilation. The update of the state variables under Kalman dynamics is established on knowledge about the characteristics of the statistical errors in both the state of the model and the measurements. Hence, the covariance matrices for the errors of both the model state and measurements must be obtained.

When assimilating electricity prices, the model must capture the characteristics of the movement of the prices for instance their volatility and non linear nature. In this work, the VEnKF provides a realistic scheme to capture the behavior of the prices.

3.2 Description

To simulate electricity spot market price evolution, the EnKF must be made to turn inwards onto itself.

In order to describe simulation of electricity spot prices, we first analyze the Kalman dynamics with Lorenz 95 model. After this, we replace the Lorenz 95 model with a vector valued Brownian motion, that is to say, a multi-normally distributed random increment to each component, that in this new case is the collecting of ensemble members, with zero mean and a given covariance matrix. Initially we may use the identity matrix for this covariance matrix, but after testing that it works well, we take the posterior covariance matrix estimate from the previous time step as this new covariance matrix. This becomes the state model operator, usually denoted by M. VEnKF does this automatically in new ensemble generation, as described below. As the observations in this new case, we take a sample of other ensemble members from the previous time step, weighted by some normal distribution that has initially a standard deviation defined by the furthest member of the nearest members. Initially, we will start with some members and then increase the number later depending on the speed to each ensemble member. Then together we form a number of the observations in terms of weight, the other members are used to define the ensemble mode that becomes the next state of the system. This weighting is the "observation operator", denoted by K. VEnKF then generates a new ensemble with this mode as mean and the error covariance estimate matrix as its covariance matrix.

The ensemble in this case represents electricity traders distributed along a price bidding range. The initial range is got from electricity prices by looking at the distribution of this price history. We then study the impact of different forecast models and the standard deviation and weights of the "observation operator" to the ability of the Kalman Dynamics system to simulate the true price and possibly to forecasting it into the future.

3.3 Kalman Dynamics

In this section, we provide an overview of some of the existing Kalman dynamics that are of much relevance to our study. We begin by presenting how the basic KF and the EKF work and then proceed to ensemble filtering methods. For all the Kalman filters, the error in the analysis step is assumed to follow a Gaussian distribution.

The state of any system is bound to dynamically change. For example, in a chemical reaction the concentration for different compounds changes with time. Likewise, the price of electricity also changes due to many factors as discussed in Section 2. However, the state of the model of the system is not viewed wholly and thus requires new measurements as time proceeds. These measurements are then used to update the state estimates.

3.3.1 Filtering formulas

To estimate the system state x_i we use previous observations y_i at discrete times, $i = 1, 2, 3, \cdots$. Filtering under Kalman dynamics is generally stated in a pair of equations of state variables and observations as shown in Equation (1).

$$\begin{cases} x_i & = M_i(x_{i-1}) + \varepsilon_i, \\ y_i & = K_i(x_i) + \epsilon_i, \end{cases} \tag{1}$$

where $M : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the state or evolution model that evolves the state in time and $K : \mathbb{R}^n \rightarrow \mathbb{R}^p$ indicates the observation operator which maps from the state space to the observation space [17].

x is of dimension p,

y is of the observation vector of dimension $q, q \leq p$,

ε_i is the error of the model ,

ϵ_i is the error of the measurements.

ε_i and ϵ_i are normally distributed with zero mean and covariance matrices C_{ε_i} and C_{ϵ_i} respectively. If we note it in a probabilistic way, Equation 1 then becomes;

$$\begin{cases} x_i & \sim p(x_i|x_{i-1}), \\ y_i & \sim p(y_i|x_i). \end{cases} \tag{2}$$

In Kalman dynamics, we aim at finding a posterior distribution $p(x_i|y_{1:i})$ of the state variable given all the previous and current observations $y_{1:i}$.

Thus the main two stages which are prediction and update stages are dealt with. The state estimate and its uncertainty in the prediction stage are shifted to the time a head's prior by using a dynamic model. Then the prior of the current state can be considered as a predictive observation:

$$p(x_i|y_{1:i}) = \int p(x_i|x_{i-1})p(x_i|y_{1:i-1})\mathrm{d}x_{i-1} \tag{3}$$

The term $p(x_i|x_{k-1})$ includes the evolution model and describes the probability of having state x_i at time i, given that the state was x_{i-1} at the previous time step. Thus to get the predictive distributions of the observations y_i, the procedure is as shown in Equation (4).

$$p(y_i|y_{1:i}) = \int p(y_i|x_i)p(x_i|y_{1:i-1})\mathrm{d}x_i \tag{4}$$

To add to that, at the update stage, we use the Bayes' formula to get the posterior using the predictive distribution as the prior.

$$p(x_i|y_{1:i}) \propto p(y_{1:i}|x_i)p(x_i|y_{1:i-1}) \tag{5}$$

We then use the derived equations in implementing the Kalman dynamics results. Thus we seek to predict new values of x and correcting the state values by new measured observations y iteratively.

3.3.2 Kalman Filter

Consider a time-dependent process, the state vector x_i is observed at time points i which are defined as the minimal set of data that is sufficient to uniquely describe the unforced dynamical behavior of the system [7]. We begin by presenting how the basic KF and EKF work and proceed to ensemble filtering methods. Under Kalman filtering, the state estimate of the previous state estimate is $\hat{x}_{i-1}$ and the error covariance matrix $\hat{C}_{i-1}$ are moved forward to the next time step's prior x_i^p by using the linear model M_i. Hence the prior center point for the next time step i is given by the model prediction Equation (6).

$$x_i^p = M_i \hat{x}_{i-1} \tag{6}$$

The covariance, $cov(x_i^p) = C_i^p$ of the prior can be calculated as follows supposing $\hat{x}$ and ε are independent.

$$C_i^p = cov(M_i \hat{x}_{i-1} + \varepsilon_i) = M_i \hat{C}_{i-1} M_i^T + C_{\varepsilon_i}, \tag{7}$$

where C_{ε_i} is the model error covariance matrix. Using least squares, the formulas using the notation used in KF system Equations (6) and (7) can be collected as follows:

$$x_i^p = M_i \hat{x}_{i-1} \tag{8}$$

$$C_i^p = M_i \hat{C}_{i-1} M_i^T + C_{\varepsilon_i} \tag{9}$$

$$G_i = C_i^p K_i^T (K_i C_i^p K_i^T + C_\epsilon)^{-1} \tag{10}$$

$$\hat{x}_i = x_i^p + G_i(y_i - K_i x_i^p) \tag{11}$$

$$\hat{C}_i = S_i^p - G_i K_i C_i^p \tag{12}$$

Equations (8) to (12) form the Kalman Filter model where G is known as the Kalman Gain. We then implement this on the following algorithm shown in Table 2.

Table 2. Kalman Filter algorithm.

Select initial guess $\hat{x}_0$ and covariance $\hat{C}_0$
t ← 1;

repeat

Compute the evolution model estimate and covariance:

1: Compute $x_i^p = M_i \hat{x}_{i-1}$;

2: Compute $C_i^p = M_i \hat{C}_{i-1} M_i^T + C_{\varepsilon_i}$

Compute Kalman filter estimate and covariance:

3: Compute the Gain $G_i = C_i^p K_i^T (K_i C_i^p K_i^T + C_\epsilon)^{-1}$

4: Compute the filter estimate $\hat{x}_i = x_i^p + G_i(y_i - K_i x_i^p)$

5: Compute the estimate covariance $\hat{C}_i = S_i^p - G_i K_i C_i^p$

6: Update t: t ← t+1;

until the end of assimilation

When dealing with the KF, the model used for forecasting and the knowledge of its error characteristics are used to update the estimate of the error covariance that is forecasted. Earlier studies by Robert (1985) about the KF show that in the simulations about boundary value problems, combining the forecast model and the update made controlled the error to a certain extent in spite of the fact that the scheme used to statistically predict the boundary conditions was not accurate [18].

3.3.3 Extended Kalman Filter

The EKF is the extension of KF to nonlinear optimal filtering problems by forming a Gaussian approximation to distribution of states and measurements using a Taylor series expansion. Changes on the model evolution M and observation operator K are made. The change is the linearization. When linearizing a non-linear model M we replace the value of M by an approximation

$$M(x) \approx M(x_0) + \mathbf{M}(x - x_0) \tag{13}$$

where $\mathbf{M}$ is the Jacobian of M at state vector x_0.

$$\mathbf{M} = \frac{\partial M(x_0)}{\partial x}. \tag{14}$$

Thus both the model M and operator K have to be linearized at many different time instances I, we end up with the following definitions:

$$\mathbf{M}_i = \frac{\partial M(\hat{x}_{i-1})}{\partial x}, \tag{15}$$

$$\mathbf{K}_i = \frac{\partial K(x_{i-1})}{\partial x}. \tag{16}$$

To implement the EKF, we use non linear models in equations 8 to 12 and substitute their linearized version into the remaining equations. Hence we have that;

$$x_i^p = M_i \hat{x}_{i-1}, \tag{17}$$

$$C_i^p = \mathbf{M}_i \hat{C}_{i-1} \mathbf{M}_i^T + C_{\varepsilon_i} \tag{18}$$

$$G_i = C_i^p \mathbf{K}_i^T (\mathbf{K}_i C_i^p \mathbf{K}_i^T + C_\epsilon)^{-1} \tag{19}$$

$$\hat{x}_i = x_{ai} + G_i(y_i - K_i x_i^p) \tag{20}$$

$$\hat{C}_i = S_i^p - G_i \mathbf{K}_i C_i^p \tag{21}$$

The algorithm used in the simulation of this system is shown in Table 3. To provide an analysis the algorithm uses only past information up to the latest observations, which is necessary in a real time application such as predicting electricity prices. The EKF algorithm is optimal if the linearization hypotheses are verified exactly. If both the model and observation operator are not linear, the optimality is only approximately true to the extent that the tangent linear hypothesis is verified.

The nonlinearity affects error propagation in such a way that the dynamics of the error covariance propagation becomes more convoluted than when the model is linearized [16].

Table 3. Extended Kalman Filter algorithm.

Select initial guess $\hat{x}_0$ and covariance $\hat{C}_0$
t ← 1;

repeat

Compute the evolution model estimate and covariance:

1: Compute $x_i^p = M_i \hat{x}_{i-1}$;

2: Compute the linearized versions $\mathbf{M}$ and $\mathbf{K}$ using Equation 15 and Equation 16 respectively;

3: Compute $C_i^p = \mathbf{M}_i \hat{C}_{i-1} \mathbf{M}_i^T + C_{\varepsilon_i}$

Compute Kalman filter estimate and covariance:

4: Compute the Gain $G_i = C_i^p \mathbf{K}_i^T (\mathbf{K}_i C_i^p \mathbf{K}_i^T + C_\epsilon)^{-1}$

5: Compute the filter estimate $\hat{x}_i = x_i^p + G_i(y_i - K_i x_i^p)$

6: Compute the estimate covariance $\hat{C}_i = S_i^p - G_i \mathbf{K}_i C_i^p$

7: Update t: t ← t+1;

until the end of assimilation

3.3.4 Ensemble Kalman Filter

The (EnKF) presents a solution to the optimal state estimation problem which uses an ensemble of model states to approximate the mean state and covariance. Ensemble members can be generated through sampling from a probability distribution function.

In the standard EnKF [19], perturbations are added to the observations to generate a matrix consisting of an ensemble of observations. In this approach an ensemble of forecast, $\zeta_t^a, t = 1, 2, 3, \cdots, N$ is generated around the state value $x_{(i_0)}$, then the model is used to forecast the whole ensemble. Let $\varepsilon_t(i_k)$ denote the perturbation that corresponds to the t^{th} ensemble member obtained by either KF or EKF procedure. In the forecast step we move ensemble forward and perturb members with model error as shown in Equation 22.

We also calculate sample mean and covariance such that:

$$\zeta_t^f(i_k) = M\big(\zeta_t^a(i_{k-1})\big) + \varepsilon_t(i_k), \tag{22}$$

$$x^f(i_k) = \frac{1}{N}\sum_{t=1}^{N} \zeta_t^f(i_k), \tag{23}$$

$$\varepsilon^f = \big[\zeta_t^f(i_k) - x^f(i_k)\big], \tag{24}$$

$$P^f = \frac{1}{N-1}\varepsilon^f(i_k)^T, \tag{25}$$

when the measurements are ready, we proceed to obtain the Kalman gain such that;

$$G_i = P^f(i_k)K(i_k)\big[K(i_k)P(i_k)K(i_K)^T + R(i_k)\big]^{-1}, \tag{26}$$

$$\zeta_t^f(i_k) = \zeta_t^f(i_k) + G_i\big[y(i_k) - K(i_k)\zeta_t^f(i_k) + \epsilon(i_k)\big]. \tag{27}$$

where $\epsilon(i_k)$ is an optional additive measurement noise, used to make assimilation robust. Equations (26) and (27) compute the Kalman Gain and the ensemble members for the next round respectively. The EnKF can be a stochastic filter or a deterministic filter, depending on the added vectors. In the stochastic case, the EnKF uses Kalman Gain together with random perturbations while in the deterministic case the EnKF uses a non-random transformation on the forecast ensemble. Further more, a new ensemble member can be generated using individual previous ensemble members using the analysis equation of the model. The algorithm for this filtering is presented Table 4.

Table 4. Ensemble Kalman Filter Algorithm.

Select initial guess ensemble members $\zeta_t^a(i_0)$ for t = 1,2,3, $\cdots$, N
k← 1;

repeat

1: Push forward the particles and perturb $\zeta_t^f(i_k) = M(\zeta_t^a(i_{k-1}) + \varepsilon_t(i_k);$

2: Compute the forecast $x^f(i_k) = \frac{1}{N}\sum_{t=1}^{N}\zeta_t^f(i_k);$

3: Compute the difference $\varepsilon^f(i_k) = \left[\zeta_t^f(i_k) - x^f(i_k)\right];$

4: Compute the covariance matrix $P^f = \frac{1}{N-1}\varepsilon^f(i_k)^T;$

Compute Kalman filter estimate

5: Compute the Gain $g_i = P^f(i_k)K(i_k)\left[K)i_k)P(i_k)K(i_K)^T + R(i_k)\right]^{-1}$

6: Compute next ensemble members $\zeta_t^f(i_k) = \zeta_t^f(i_k) + G_i[y(i_k) - K(i_k)\zeta_t^f(i_k) + \epsilon(i_k)]$

7: Update k: k ← k+1;

until the end of assimilation.

3.4 Variational Ensemble Kalman Filter

The EnKF serves as a relief to the problems encountered when dealing with the EKF. Firstly, the error covariance can be computed easier than in the case of EKF since a few model states are viable for a statistically good convergence. Secondly when the ensemble sizes are sufficient, the noise in the statistical measurement dominates the errors. Thus the errors are not driven by the error variance that is unbounded. To acquire a Variational Ensemble Kalman Filter (VEnKF), a minimization part is added to the EnKF. Then the resulting model is used to check the robustness of the values. The VenKF works in such a way that we initialize the ensemble members and the background state after which we push forward the ensemble members. This is followed by perturbations of the forwarded ensemble members and then the forecast. We then compute the background ensembles for the next time.

We thus employ the ensemble mean as an estimate of the state that is used to minimise the root mean square forecast error. Further still, the choice of the ensemble size is vital. The size should be big enough such that information contained in the observed values is covered by the model variables. When the ensemble size is too small, poor approximations of the model parameters are observed which may result in inappropriate interpretation of the results and poor forecasting of the future behavior of the model [6].

Additionally, using the ensemble members as observations implies that the ensemble covariance and the error covariance of the ensemble mean are connected. Thus with an appropriate ensemble size, the the aforementioned covariances can be given by the VEnKF. The VenKF is a new approach of Kalman filtering which combines the variational Kalman filtering and ensemble filtering. The method uses the Limited memory Broyden-Fletcher-Goldfarb-Shanno (LBFGS) algorithm of optimization.

The prior ensemble covariance matrix of the VEnKF is approximated as follows:

$$C_i^p = Cov(M_i(\hat{x}_{i-1}) + \varepsilon_i), \tag{28}$$

$$= Cov(M_i(\hat{x}_{i-1})) + Cov(\varepsilon_i), \tag{29}$$

$$= X_i X_i^T + Q_i. \tag{30}$$

where

$$X_i = \frac{((x_{1,i} - x_i^p), \cdots (x_{N,i} - x_i^p))}{\sqrt{N}}. \tag{31}$$

X_i is a rectangular matrix.

We then assume that the state and the observations are not correlated. The cost function for obtaining the posterior distribution under the VEnKF is defined as in Equation 32.

$$J(x) = \frac{1}{2}(y - K_i(x))^T R^{-1}(y - K_i(x)) + \frac{1}{2}(x_i^p - x)^T (C_i^p)^{-1}(x_i^p - x). \tag{32}$$

In the update stage, Equation 32 is minimized using the LBFGS method to obtain the posterior estimate $\hat{x}_i$ and the covariance matrix $\hat{C}_i$. Thus the new ensemble is obtained by getting samples $\hat{x}_{i,k} \sim N(\hat{x}_i, \hat{C}_i)$.

We then obtain the values of $\hat{C}_i$ as follows.

$$\hat{C}_i = B_0 B_0^T + \sum_{t=1}^{d} b_t b_t^T, \tag{33}$$

where B_0 is and $n \times n$ matrix and b_t is an $n \times 1$ vector. We then sum over the number of vectors stored in the LBFGS algorithm. The VEnKF algorithm is shown in Table 5

Table 5. Variational Ensemble Kalman Filter Algorithm.

Select initial guess point $\hat{x}_i$ and initial ensemble members $\hat{x}_{i,0}, i = 1, \cdots, N$;
k← 1;

repeat

1: Push forward the point and ensemble using the forecast model M;

2: Minimize the cost function to obtain $\hat{x}_i$ and $\hat{C}_i$;

3: State new ensemble members $\hat{x}_{i,k} \sim N(\hat{x}_i, \hat{C}_i), i = 1, \cdots, N$;

4: Update k: k ← k+1;

until the end of assimilation

Hence we apply the Lorenz95 model to the chaotic and periodic for some parameter values and therefore enough to mimic the NordPool electricity market. Obtain the state estimate and its covariance by solving a minimization problem using the limited memory Broyden-Fletcher-Goldfarb-Shanno (LBFGS) method. The Ensemble of prices become the observations for the next step.

3.5 LBFGS method

Given a non linear function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ operating in a finite dimensional space, the large scale optimization seeks to minimize this function. In other words, for a large n, we seek to find $x \in \mathbb{R}^n$ such that:

$$\min_{x \in \mathbb{R}^n} f(x), \tag{34}$$

An assumption is made such that the gradient $g(x) = \nabla f(x)$ and the Hessian matrix $G(x) = \nabla^2 f(x)$ of f exist and they are continuous [20]. The LBFGS is mostly used on large scale unconstrained optimization. It is designed in such a way that it requires low storage in its computations. The method is very suitable for large scale problems since the user can control the amount of storage of the iterations or algorithms. Advantageously, any changes in the initially stated scaling matrix and the observation from the past iterations can be used in the new update at low operations costs. Studies carried out on a relatively large scale [21] indicate that the LBFGS is powerful and the inverse Hessian matrix is rescaled at every iteration. This works well when the scaling matrix used initially is a positive multiple of an identity matrix.

As one of the Quasi-Newton methods, the LBFGS is analogous to the Newton's method with a difference in the inverse of the Hessian matrix $(G(x_i))^{-1}$ that is replaced by symmetric matrix H_i which is an $n \times n$ dimensional matrix that satisfies the Quasi-Newton equation:

$$H_I y_{i-1} = s_{i-1}, \tag{35}$$

where

$$s_{i-1} = x_i - x_{i-1} = \lambda_{i-1} d_{i-1}, y_{i-1} = g_i - g_{i-1},$$

and $\lambda_{i-1} > 0$ is a step-length. If we assume H_i to be singular, then we can define $B_i = H_i^{-1}$. Therefore, the BFGS formula takes the form of Equation 36 such that;

$$B_{i+1} = .B_i - \frac{B_i s_i s_i^T B_i}{s_i^T B_i s_i} + t_I \frac{y_i y_i^T}{s_i^T y_i}, \tag{36}$$

where

$$t_i = \frac{2}{s_i^T y_i} \big[f(x_i) - f(x_{i+1}) + s_i^T g_i g_{i+1} \big]. \tag{37}$$

Hence the algorithm for the LBFGS is shown in Table 6

Table 6. LBFGS Algorithm.

1: Select $x_0, 0 < \beta\prime < \dfrac{1}{2}, \beta\prime < \beta < 1$ and $H_0 = I$;
Set $k = 0$;

2: Compute $d_i = -H_i g_i$ and $x_{i+1} = x_i + \lambda_i d_i$ where $\lambda = 1$ is tried first;

3: Let $\hat{m}$= min{i,m-1}. Update H_0 for $\hat{m} + 1$ times. In other words;

$$
\begin{aligned}
H_{i+1} = &(V_i^T \cdots V_{i-\hat{m}}^T) H_0 (V_{i-\hat{m}} \cdots V_i) \\
&+\rho_{k-\hat{m}} (V_i^T \cdots V_{i-\hat{m}+1}^T) s_{i-\hat{m}} s_{i-\hat{m}}^T (V_{i-\hat{m}+1} \cdots V_i) \\
&+\rho_{k-\hat{m}+1} (V_i^T \cdots V_{i-\hat{m}+2}^T) s_{i-\hat{m}+1} s_{i-\hat{m}+1}^T (V_{i-\hat{m}+2} \cdots V_i) \\
&\vdots \\
&+\rho_i s_i s_i^T;
\end{aligned}
$$

4: Update k: k $\leftarrow$ k+1;

repeat from step 2 until the end of assimilation

4 PRICE FORECASTING

Forecasting the prices of electricity is viewed as a complex task since the chances of it being accurate are minimal. However, recently, it has attracted the attention of both producers and consumers in the electricity market. This is due to the stochastic nature of the prices caused by the forces of demand and supply. Furthermore, when a consumer is willing to buy electricity in the spot market, he or she should have knowledge about the previous performance of the market so as to make a good decision. However, the factors that influence these prices are numerous and little information about these factors is incorporated in the forecasting scheme [10]. In order to maximize profits either in the spot market or otherwise, forecasting plays a big role in strategizing the bidding power for the market participants. Both producers and consumers rely on the price forecasted to make appropriate bidding decisions.

In order to forecast the day-ahead electricity prices, the following is followed [22]. For a given day d, its price forecast is given on the day before that is to say on day $d-1$ at an hour h_b. Additionally the prices for day $d-1$ are determined on the previous day which is day $d-2$ at hour for instance h_c. This implies that in order to forecast prices for day d, the actual forecasting occurs between hour h_c and h_b. Hence the prices of up to the 24^{th} hour of day $d-1$ are considered as illustrated in Figure 14.

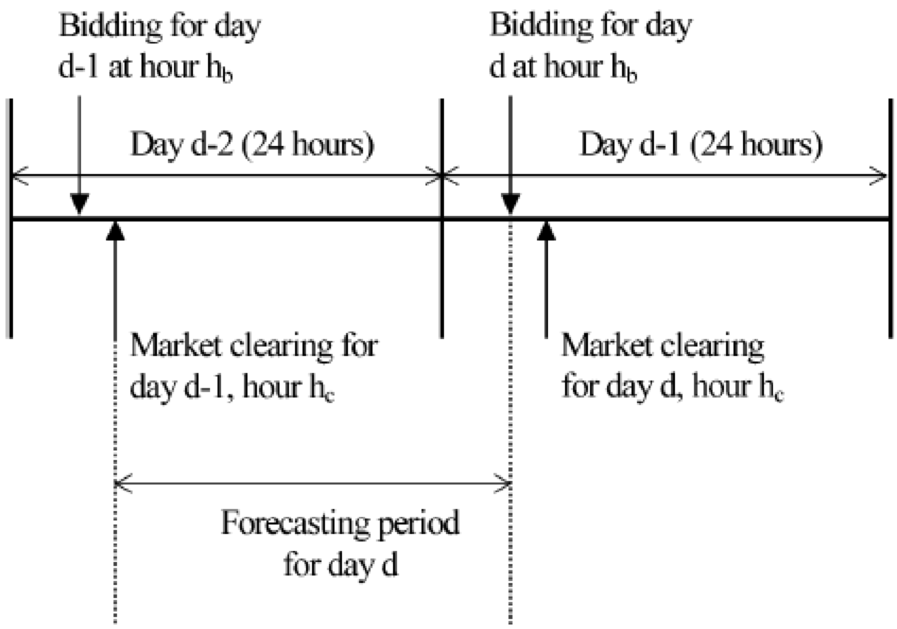

Figure 14. An illustration of day-ahead forecasting

In the electricity markets, a high level of competition can not be avoided, therefore price forecasts present relevant information for the sellers and buyers when strategizing their bidding in order to maximize their gains and utilities [23]. Thus the competition trend is based on the notion that a market with high competition provides services that are reliable and at low costs.

The root mean square error(RMSE) is which we use to asses the accuracy of the modal estimate $\hat{x}_i$ defined as;

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2}, \tag{38}$$

where $e_i, i = 1, 2, \cdots n$ are the model errors.

In order to get the impact of the forecast, we obtain the forecast skill of the model such that;

$$Forecast_{skill} = \frac{1}{\sigma}\sqrt{\frac{1}{n}\sum_{i=1}^{n} |M\hat{x}_i - x_{i+1}|^2}, \tag{39}$$

where σ is the standard deviation of the model state.

5 RESULTS AND DISCUSSION

In this section, we discuss the results obtained from this study. All simulations were done using *MATLAB* software.

5.1 Simulation results

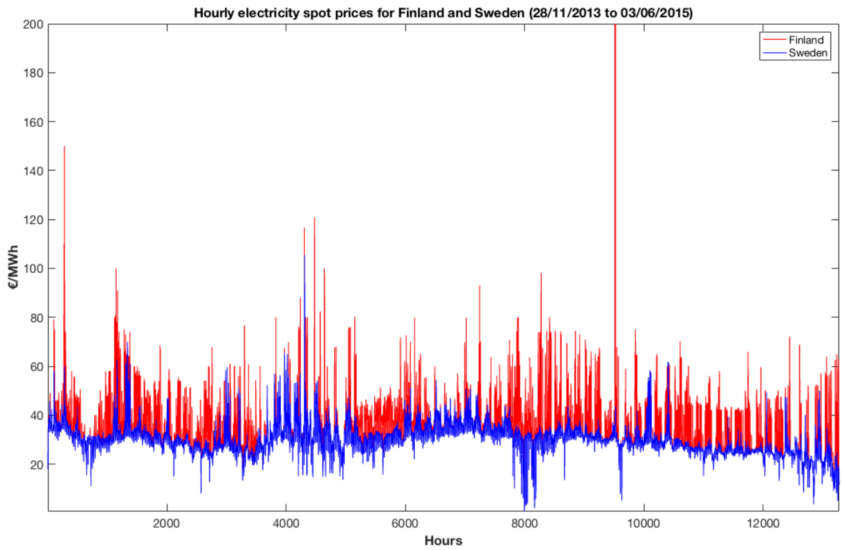

Figure 15. A comparison of Finland and Sweden electricity spot prices

Figure 15 shows electricity spot prices of Finland and Sweden combined. From the figure, we observe that Finland had relatively higher prices than Sweden for almost the entire period.

Using the VEnKF with the number of ensemble members equal to the length of the original data, the simulated prices for Finland and Sweden are shown in Figure 16 and Figure 18 respectively. In Figure 16, the ensemble almost simulates the trend of the original prices. However, the high spike shown by the simulated prices does not imitate any similar sequence from the real prices.

On the other hand, the simulated prices in comparison to Sweden's real prices in Figure 18 have a very sharp spike around 4500^{th} hour. In order to have a clear view of how the simulated prices behave in comparison to the real prices, we consider Figure 17 for

Finland and Figure 19 for Sweden respectively. It is observed that the simulated prices do not perfectly follow the trend of the real prices.

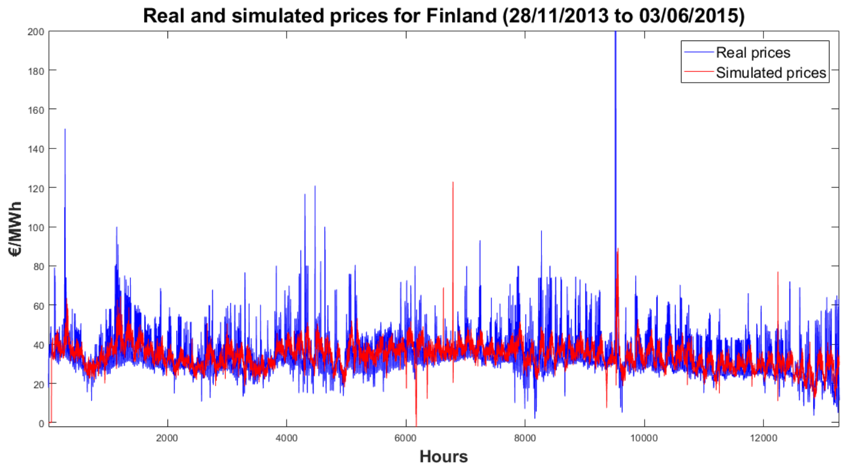

Figure 16. Finland's simulated and real electricity spot prices

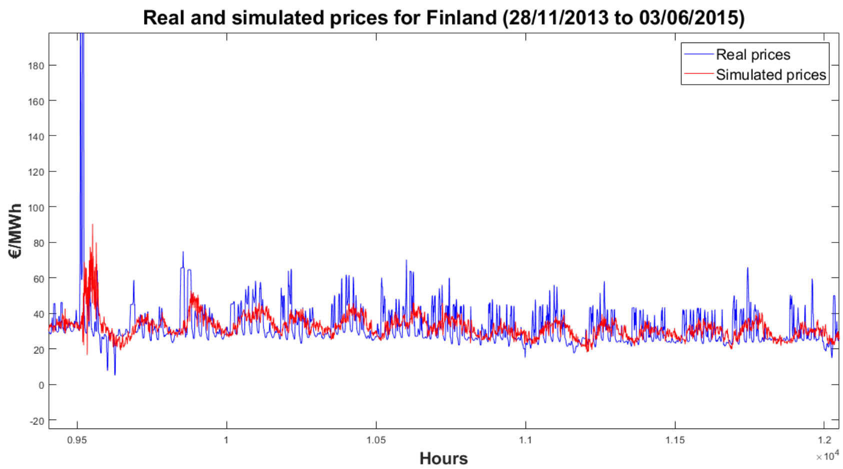

Figure 17. A cross section of Finland's simulated and real electricity spot prices

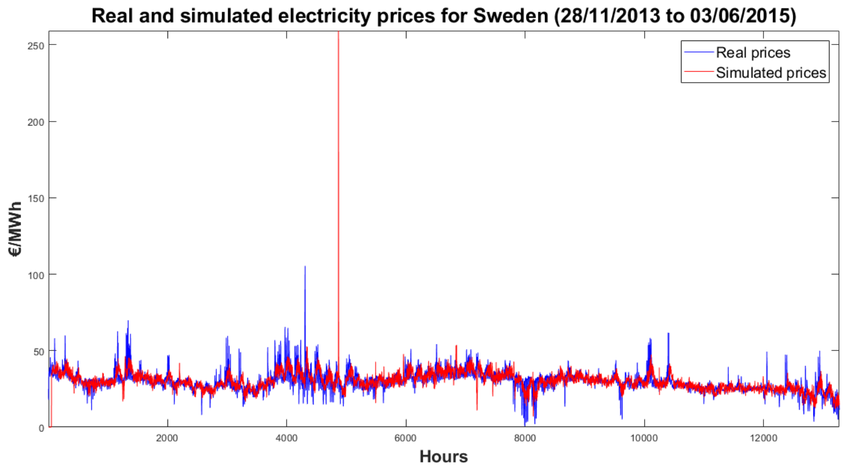

Figure 18. Sweden’s simulated and real electricity spot prices

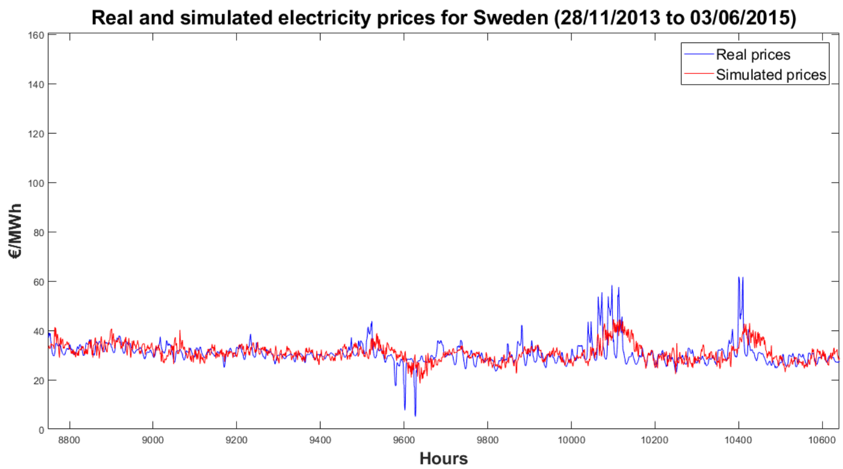

Figure 19. A cross section of Sweden’s simulated and real electricity spot prices

Figure 20 and Figure 21 are a representation of the histograms of the simulated prices obtained for both Finland and Sweden respectively.

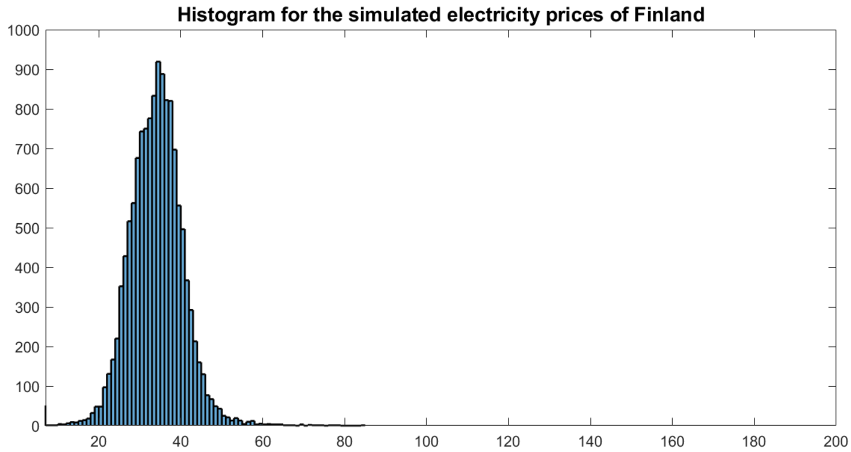

Figure 20. Histogram for Finland's simulated electricity prices

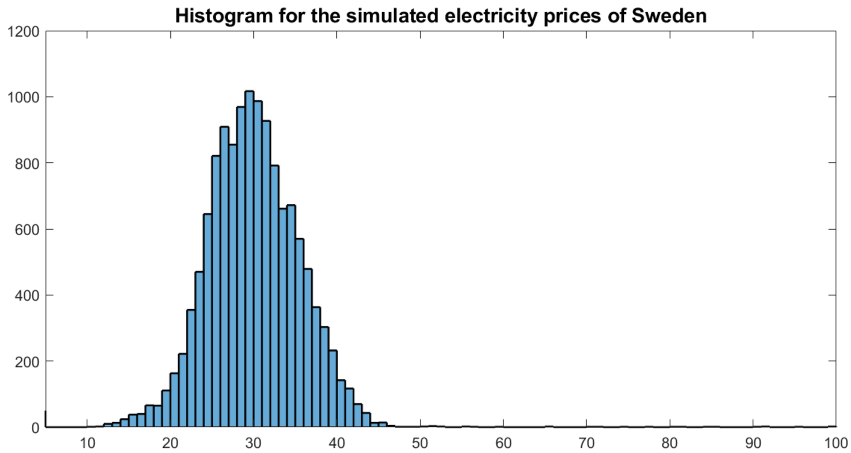

Figure 21. Histogram for Sweden's simulated electricity prices

By leaving the last 272 observations of the original prices out, we applied Kalman dynamics to forecast the prices. The results for Finland are shown in Figure 22. The forecast was done for a period of 272 hours. A comparison between the original prices and the predicted prices in this time period is shown in Figure 23. This figure is a plot of the original data that was not used used in the prediction and the predicted prices for the same period. Further more, although the predictions were not accurate, they showed a pattern almost similar to that of the original prices.

A similar case was done for Sweden and the results are shown in Figure 24 and the comparison of the forecasted prices to the real price is shown in Figure 25. The forecast is a strong tool that traders can base on to make bids for the following day.

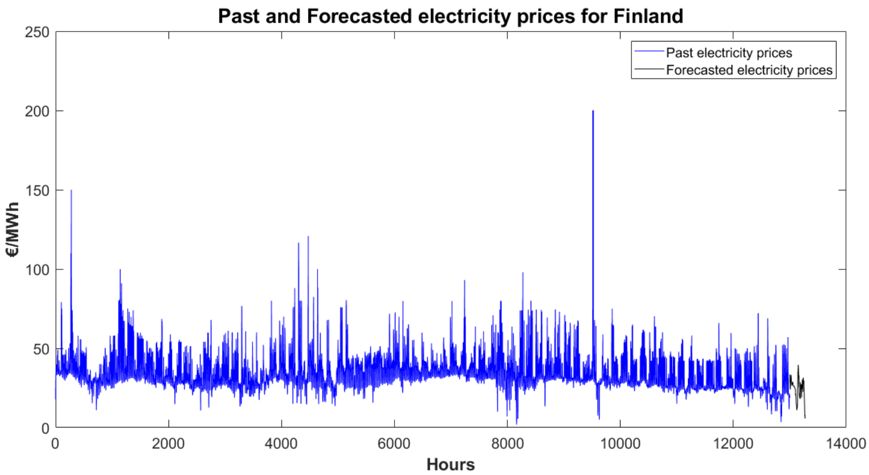

Figure 22. Past and forecasted prices for Finland

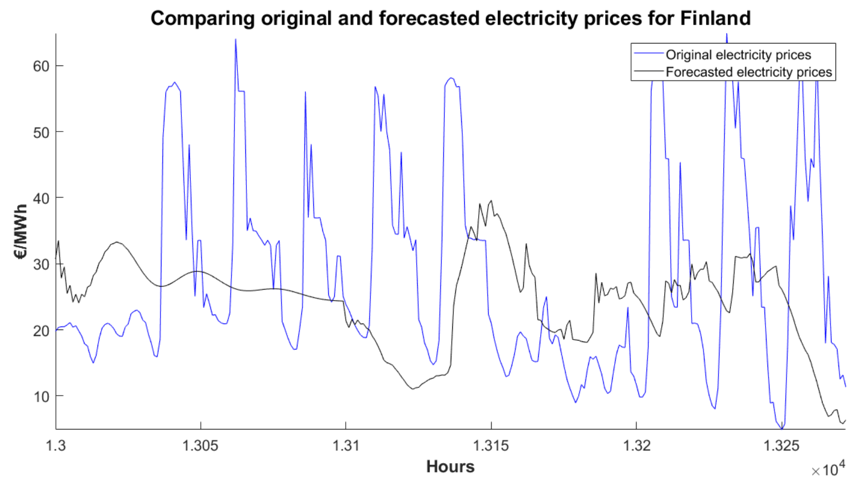

Figure 23. A comparison between original and forecasted prices for Finland

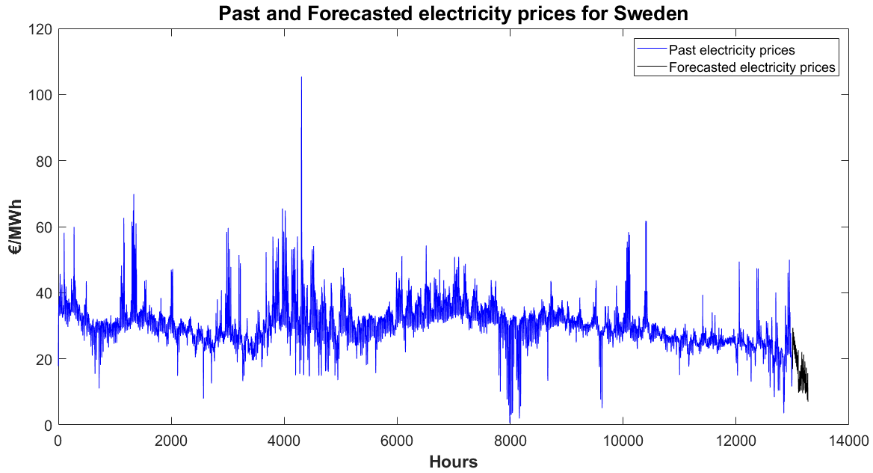

Figure 24. Past and forecasted prices for Sweden

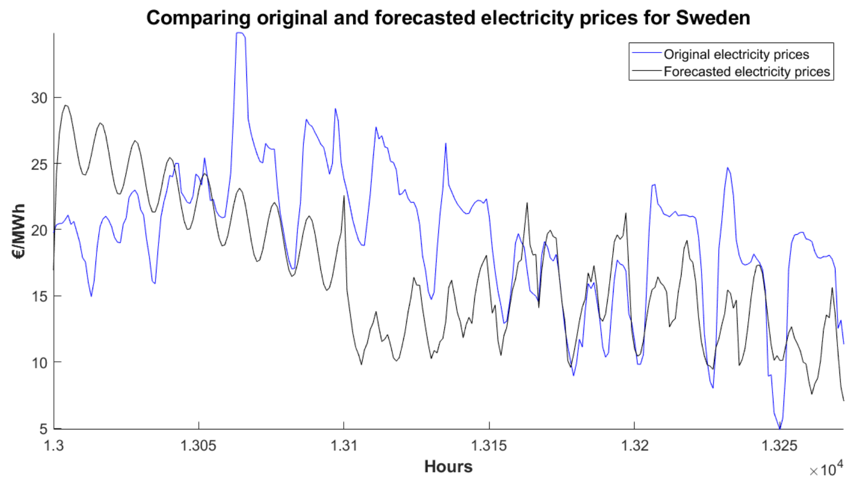

Figure 25. A comparison between original and forecasted prices for Sweden

5.2 RMSE and forecast skills for different ensemble sizes

Figures 26, 27, 28 and 29 show the RMSE and forecast skills obtained from the ensemble simulations. The major aim of these outputs is to compare the effect of ensemble sizes on the filtering skills of the VEnKF. The forecast skills too are almost the same with a small difference in the exact forecast values.

The ensembles are used to determine the RMSE therefore the RMSE indicates how the filter estimates the mean of the state variable. The square root of the difference between the filter estimate and the true solution give the model error which is used to determine the accuracy of the estimate. The forecast skill shows the error between the forecast state and true state without referring to the observations, we start at some estimate state and forecast it to different time points.

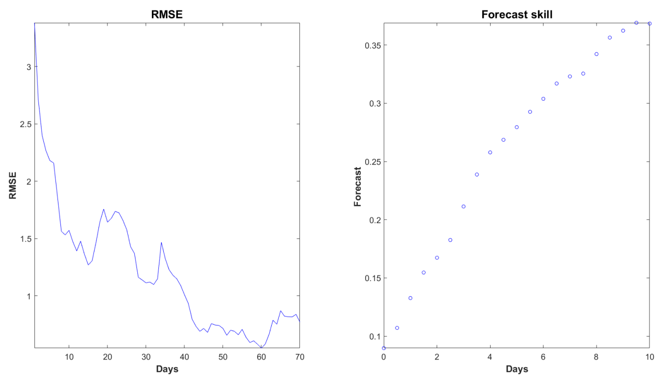

Figure 26. RMSE and forecast skill for ensemble size = 5

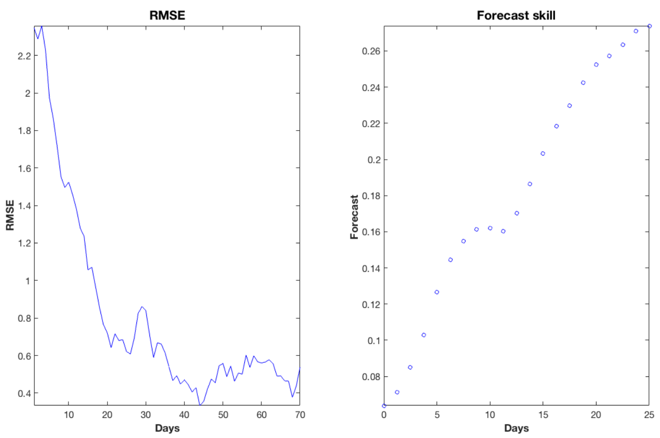

Figure 27. RMSE and forecast skill for ensemble size = 15

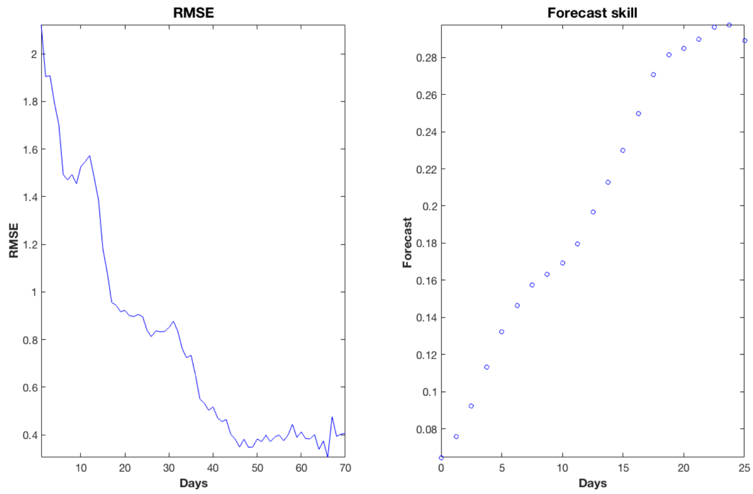

Figure 28. RMSE and forecast skill for ensemble size = 30

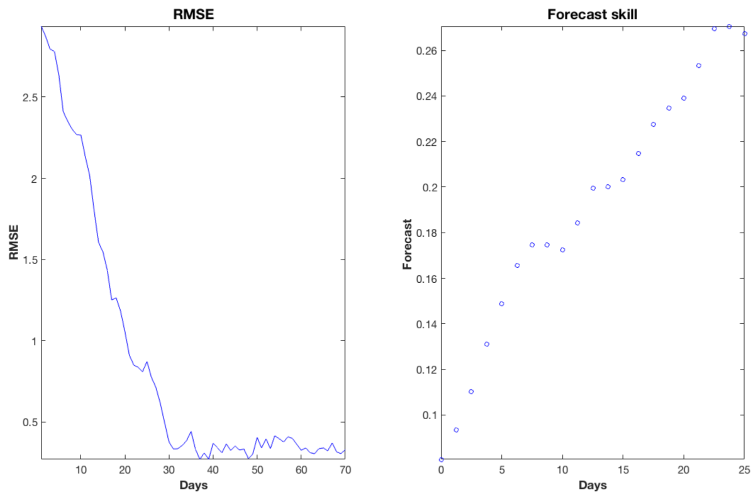

Figure 29. RMSE and forecast skill for ensemble size = 50

6 CONCLUSION

The study covered the application of Kalman dynamics in simulating electricity spot prices for Finland and Sweden for a period from $28/11/2013$ to $03/06/2015$. The simulated prices for both countries followed an almost similar trend like the real prices, however, a difference was observed in the exact values. In other words, although we were able to use the Kalman dynamics to obtain the simulations, these simulations only followed a particular trend which was not as exact as the trend followed by the original prices.

On the other hand, however, the predicted prices indicate that the algorithm can be used to independently forecast prices even though these prices are stochastic. The ensemble size determines the convergence of the standard deviation of the error in the state estimate. This implies that the higher the number of ensemble members the better the convergence. Since the model dynamics of the simulated electricity prices is expected to be closely related to the real prices, future prices were predicted and thus traders can bid with a knowledge of how the price evolution was in the past and use this to predict how it will be in the future.

We recommend that the VEnKF should be used in other areas of study with any other type of data. Secondly, we suggest that a researcher should test the model starting with a small number of ensemble members and increase them gradually because starting from a very large number seems to slow down the simulation speed.

REFERENCES

[1] Lindsay Kleeman. *Understanding and applying Kalman filtering*. 1996.

[2] Mohinder S Grewal and Angus P Andrews. Applications of kalman filtering in aerospace 1960 to the present [historical perspectives]. *IEEE Control systems*, 30(3):69–78, 2010.

[3] Bit-Kun Yeoh, Chee-Wooi Hooy, and Zainudin Arsad. Time-varying world integration of the malaysian stock market: A kalman filter approach. *Asian Academy of Management Journal of Accounting & Finance*, 6(2), 2010.

[4] Jinghe Zhang, Greg Welch, Gary Bishop, and Zhenyu Huang. A two-stage kalman filter approach for robust and real-time power system state estimation. *IEEE Transactions on Sustainable Energy*, 5(2):629–636, 2014.

[5] BR Hunt, E Kalnay, EJ Kostelich, E Ott, DJ Patil, T Sauer, I Szunyogh, JA Yorke, and AV Zimin. Four-dimensional ensemble kalman filtering. *Tellus A: Dynamic Meteorology and Oceanography*, 56(4):273–277, 2004.

[6] Gerrit Burgers, Peter Jan van Leeuwen, and Geir Evensen. Analysis scheme in the ensemble kalman filter. *Monthly weather review*, 126(6):1719–1724, 1998.

[7] Simon Haykin. *Kalman filtering and neural networks*, volume 47. John Wiley & Sons, 2004.

[8] James Nicolaisen, Valentin Petrov, and Leigh Tesfatsion. Market power and efficiency in a computational electricity market with discriminatory double-auction pricing. *IEEE transactions on Evolutionary Computation*, 5(5):504–523, 2001.

[9] Mariano Ventosa, Alvaro Baıllo, Andrés Ramos, and Michel Rivier. Electricity market modeling trends. *Energy policy*, 33(7):897–913, 2005.

[10] Daniel S Kirschen. Demand-side view of electricity markets. *IEEE Transactions on power systems*, 18(2):520–527, 2003.

[11] Sanjeev Kumar Aggarwal, Lalit Mohan Saini, and Ashwani Kumar. Electricity price forecasting in deregulated markets: A review and evaluation. *International Journal of Electrical Power & Energy Systems*, 31(1):13–22, 2009.

[12] Jesús Crespo Cuaresma, Jaroslava Hlouskova, Stephan Kossmeier, and Michael Obersteiner. Forecasting electricity spot-prices using linear univariate time-series models. *Applied Energy*, 77(1):87–106, 2004.

[13] Rafał Weron and Michał Zator. Revisiting the relationship between spot and futures prices in the nord pool electricity market. *Energy Economics*, 44:178–190, 2014.

[14] Klaus Skytte. The regulating power market on the nordic power exchange nord pool: an econometric analysis. *Energy Economics*, 21(4):295–308, 1999.

[15] Patrik Thollander, Magnus Karlsson, Mats Söderström, and Dan Creutz. Reducing industrial energy costs through energy-efficiency measures in a liberalized european electricity market: case study of a swedish iron foundry. *Applied Energy*, 81(2):115–126, 2005.

[16] Geir Evensen. Using the extended kalman filter with a multilayer quasi-geostrophic ocean model. *Journal of Geophysical Research: Oceans*, 97(C11):17905–17924, 1992.

[17] Janne Hakkarainen et al. On state and parameter estimation in chaotic systems. *Acta Universitatis Lappeenrantaensis*, 2013.

[18] Robert N Miller. Toward the application of the kalman filter to regional open ocean modeling. *Journal of Physical Oceanography*, 16(1):72–86, 1986.

[19] MRJ Turner, JP Walker, and PR Oke. Ensemble member generation for sequential data assimilation. *Remote Sensing of Environment*, 112(4):1421–1433, 2008.

[20] Leong Wah June and Malik Abu Hassan. Modifications of the limited memory bfgs algorithm for large-scale nonlinear optimization. *Mathematical Journal of Okayama University*, 47(1), 2005.

[21] Dong C Liu and Jorge Nocedal. On the limited memory bfgs method for large scale optimization. *Mathematical programming*, 45(1-3):503–528, 1989.

[22] Antonio J Conejo, Miguel A Plazas, Rosa Espinola, and Ana B Molina. Day-ahead electricity price forecasting using the wavelet transform and arima models. *IEEE transactions on power systems*, 20(2):1035–1042, 2005.

[23] Francisco Javier Nogales, Javier Contreras, Antonio J Conejo, and Rosario Espínola. Forecasting next-day electricity prices by time series models. *IEEE Transactions on power systems*, 17(2):342–348, 2002.

Appendix 1. Lorenz 95 Model

The Lorenz Equation

The Lorenz model describes a couple of differential equations published by E. Lorenz and thse are given as follows.

$$\frac{dx}{dt} = \sigma(y - x), \tag{40}$$
$$\frac{dy}{dt} = x(\rho - z) - y, \tag{41}$$
$$\frac{dz}{dt} = xy - \beta z. \tag{42}$$

where x, y, z are variables, non negative constants σ is called the Prandtl number, ρ is the Rayleigh number and β is a physical proportion. The ensemble size determines the convergence of the standard deviation of the error in the state estimate since the higher the number of ensemble members the better the convergence. The Lorenz equation consists of N differential equations with N variables; $X_1, X_2, X_3, \cdots, X_N$,

$$\frac{dx_k}{dt} = -X_k - 2X_{k-1} + X_{k-1}X_{k+1} - X_k + F. \tag{43}$$

where F is a constant which is independent of k. $X_{k-N} = X_k$ and $X_{k+N} = X_k$.

Appendix 2. Bayes' Estimation

When estimating parametes, parameters x are estimated based on certain measurements y. Thus for instance, using a least squares approach in Bayesian parameter estimation, x is interpreted as a random variable and the goal is to find the posterior distribution $\pi(x|y)$ of the parameters. This distribution gives the probability density for values of x, given measurements y. Using the Bayes' formula, the posterior density can be written as

$$\pi(x|y) = \frac{l(y|x)p(x)}{\int l(y|x)p(x)dx}.$$

The likelihood $l(y|x)$ contains the measurement error model and gives the probability density of observing measurements y given that the parameter value is x.

The prior distribution $p(x)$ contains all existing information about the parameters, such as simple bounds and other constraints. The integral $\int l(y|x)p(x)dx$ in the denominator is the normalization constant that makes sure that the posterior $\pi(y|x)$ integrates to one.