Agatha Augustine Msola.

MATHEMATICAL MODELLING OF MALARIA BASED ON UGANDA DATA.

Master’s Thesis

Examiner: N.N.
Supervisor: Professor Heikki Haario.
ABSTRACT

Lappeenranta University of Technology
School of Engineering Science
Computational Engineering and Technical Physics
Technomathematics

Agatha Augustine Msola.

Mathematical modelling of malaria based on Uganda data.

Master’s Thesis

2018

40 pages.

Examiner: N.N.

Keywords: malaria, malaria cases, mosquito, population dynamics, climate, life-cycle.

Malaria infection remains a major health burden in many parts of the world, especially sub-Saharan Africa. Long lasting insecticide nets (LLIN) and indoor residual sprays (IRS) are the most recommended malaria control tools, and they have dramatically reduced indoor malaria transmission. Anopheles gambiae is one of the most important species in malaria transmission in Africa, Uganda in particular. Climatic and weather factors become important determinants of vector-borne diseases transmission like malaria. This study assess the likely impact of LLINs on the feeding cycle dynamics of Anopheles gambiae, modelling the seasonal human-mosquito population level as a variable of rainfall and the impact of rainfall on the malaria cases pattern in Uganda.
PREFACE

Firstly, I would like to express my sincere gratitude to Almighty God for his guidance throughout my studies at Lappeenranta university of technology. Also my sincere gratitude to my supervisor, Professor Heikki Haario for the continuous support of my master's thesis, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my thesis.

My sincere thanks also goes to Lappeenranta university of technology committee for providing me this wonderful opportunity to join this university, and for the provision of the full scholarship. Without their precious support it would not be easy financially to stay in Finland.

Sincere gratitudes to Betty Kivumbi Nannyonga and Mr John Kissa from Ministry of health in Uganda for the provision of the malaria data.

I thank my fellow double degree students (special thanks to Bob Kyeyune) for the good relationship we had, for the studying discussions, for the sleepless nights we were working together before deadlines, and for all the fun we have had during our stay at LUT. In particular, I am grateful to Associate Professor, Tuomo Kauranne for his supervision during the presentation periods.

Last but not the least, I would like to thank my family: my mother (Elenor Msola), my brother (Valerian Msola) and my sister (Elizabeth Msola) for supporting me spiritually throughout writing this thesis and my life in general.

Agatha Augustine Msola.
CONTENTS

1 General introduction. 6
  1.1 Background of study. ........................................... 6
      1.1.1 Malaria. ........................................... 6
      1.1.2 Malaria transmission. ................................ 6
      1.1.3 Effective malaria control strategies. ............... 8
  1.2 Malaria in Uganda. .......................................... 8
  1.3 Statement of the problem. ................................. 9
  1.4 Objectives of the study. .................................. 9
  1.5 Scope of the study. ....................................... 9
  1.6 Structure of the report. .................................. 10

2 Literature review. 11

3 Study site and methods. 13
  3.1 Study site. .................................................. 13
      3.1.1 Nakasongola district. ................................ 14
      3.1.2 Kalangala district. .................................. 15
      3.1.3 Wakiso district. .................................... 16
  3.2 Methods. ................................................... 16
      3.2.1 Seasonal trend decomposition by LOESS (LOcally Estimated Scatterplot Smoothing). .. 16
      3.2.2 Mathematical model to describe life cycle of mosquito. ................ 17
      3.2.3 Population prediction with seasonality. ............. 18
      3.2.4 Assessing the relationship between rainfall and malaria cases. ... 19
      3.2.5 Modelling the impact of LLIN use on adult mosquito population. 19

4 Study design and data analysis. 23

5 Results. 24
  5.1 Time series plot and malaria cases patterns. .......... 24
  5.2 Mosquito system behaviour with seasonality. .......... 26
  5.3 Rainfall related to malaria transmission. .............. 28
  5.4 Relationship between rainfall and malaria cases. .... 28
  5.5 Effect of LLIN on some model parameters. ............ 35

6 Conclusion and future work. 36
  6.1 Conclusion. ................................................ 36
  6.2 Future work. .............................................. 36
REFERENCES
1 General introduction.

In this chapter, we discuss malaria as the global threat, the feeding cycle of mosquitoes and their role in malaria transmission, malaria incidence in Uganda and the topic of this study. We further go on to statement of the problem that need to be solved, and what we expect to achieve in this study. Finally, we give a roadmap to the rest of the project in the structure of the report section.

1.1 Background of study.

1.1.1 Malaria.

Malaria is one of the most common and serious disease in Sub Saharan Africa and the cause of most malaria cases occurring in Sub Saharan Africa, approximately 3000 lives are lost each day. Specific population risk groups are young children who have not developed protective immunity, especially below 5 years old and pregnant women (both non-immune, semi-immune and semi-immune HIV-infected pregnant women). The most common symptom of malaria is fever, accompanied by weakness, shakiness, loss of energy, excessive sleeping and difficulties in waking up in the mornings[1]. Malaria if not treated, can affect the brain and cause cerebral malaria which affects approximately 575000 children a year in Africa, and kills 10-40% of patients[3],[4],[5]. 5-20% of the survivals experience neurological problems which includes learning disabilities and inability to carry out some tasks such as initiating, planning and executing tasks[6],[7]. Malaria research is of a great contribution to the socio-economic improvement of Africa since the groups most vulnerable to this disease are of great economic importance to the development of Africa.

1.1.2 Malaria transmission.

Malaria is caused by protozoan parasite belonging to genus Plasmodium and transmitted by female Anopheles mosquito [4]. There are more than hundred species but just four of them are responsible for almost all human infections: P falciparum, P vivax, P malariae and P ovale. In Africa, P falciparum is the one responsible for infections as well as most severe disease and mortality [19]. The parasites in human body replicate in the liver and
then cause infections to red blood cells [20]. Figure 1 illustrates the life cycle of malaria parasite in human body.

![Malaria parasite life cycle](image)

**Figure 1.** Malaria parasite life cycle adopted from The open university, 2003, Infectious Disease Book 5: Evolving Infections.

Natural ecology of malaria involves malaria parasites infecting successively two types of hosts, human and female anopheles mosquitoes. In humans, parasites grow inside and multiply in the liver cells and then in red blood cells. In blood, parasites grow inside the red cells and destroy them releasing daughter parasites known as merozoites which continue the cycle by invading other red cells. The parasites in blood stage are known as gametocytes and they cause symptoms of malaria. During a blood meal, female Anopheles mosquito pick up the gametocytes where they start another different cycle of growth and multiplication in the mosquito. After 10-18 days, the parasites are found as sporozoites in mosquito’s salivary glands. The parasites are injected into human body through the mosquito saliva and cause human infection. The mosquito is acting as a host vector by carrying a disease from one human to another.

Malaria is very sensitive to climatic conditions which is why it is most prevalent in tropical climates, where the breeding sites are enough and favourable temperature for mosquito. Mosquitoes are cold blooded, the protozoan itself survives in certain favourable temperature. Hence, a slight change in temperature can drastically affect the lifespan and population of mosquitoes. Water is another factor which contributes to the spread of malaria due to the fact that mosquitoes breed in pools of water. More rainfall leads to the increase of possible breeding sites for mosquito, which results in increase of more vectors to spread malaria. Little rainfall leads to few breeding sites for mosquitoes.
1.1.3 Effective malaria control strategies.

Many malaria vector control interventions have been proposed and adopted in order to control the spread of malaria. Majority of the interventions focus on the adult mosquito vectors. They are designed so as to kill mosquito, targeting all life stages, discourage adult mosquitoes from a region of space and irritate mosquito that come into contact with the interventions. World Health Organisation (WHO) have promoted the use of Long Lasting Insecticided treated Nets(LLINs) and Indoor Residual Spraying(IRS) as among the control interventions in endemic areas so as to control and possibly eliminate malaria [21]. It is also important to control the mosquito population by examining the weather parameters such as temperature and rainfall which are imperative in determining the disease epidemics. Accurate seasonal climate forecasts of weather parameters make it possible to utilize malaria models that account for early warning systems in endemic regions [24].

1.2 Malaria in Uganda.

The republic of Uganda is situated in East Africa. It is bordered by Kenya in the east, Tanzania in south, Sudan in the north, Democratic Republic of Congo in the west and Rwanda in the southwest. Uganda occupies a total area of 241038 sq km, out of which 82% is land. It is a tropical country receiving rain throughout the year with two dry seasons which are December-February and June-August.

Malaria is the major cause of morbidity and mortality in Uganda especially in children below five years with a high frequency(25%). The most common malaria parasite in Uganda is Plasmodium faciparum and the malaria vectors are Anopheles gambiae and Anopheles funestus which are both indoor feeders. Statistics from the Ministry of Health in Uganda show that malaria is still the leading cause of death in Uganda, accounting for over 27% of deaths. The statistics also show that Uganda has the world’s highest malaria incidence, with a rate of 478 cases per 1,000 population per year. Uganda ranks as 6th among African countries with high malaria-related mortality rates. According to the 2013/2014 report by the parliamentary committee of health, it showed that 50% of deaths caused by malaria in Uganda are among children below five years.
1.3 Statement of the problem.

Despite malaria being preventable and treatable, it remains one of the health problems for developing world and Uganda bears particularly large burden of disease. Over the years, Anopheles gambiae has played an increased role in malaria transmission in Uganda. The rapid adaptability of the species to changing environmental conditions makes it resistant to many forms of interventions developed to combat mosquito populations. As a result, it continues to play a major role in residual malaria transmission. Insecticide treated nets (ITNs, LLINs) are among the control interventions which have been promoted for use in malaria endemic regions, Uganda in particular. What are the effects of LLIN coverage on Anopheles gambiae populations? What is the impact of rainfall on malaria transmission mediated by Anopheles gambiae?

1.4 Objectives of the study.

We focus on the following questions:
• Identifying the patterns of malaria cases in Uganda (Wakiso, Nakasongola and Kalangala).
• Modeling malaria cases based on the level of rainfall.
• Testing the impact of control intervention (LLINs) on the population of adult female Anopheles gambiae for some model parameters.

1.5 Scope of the study.

In this study, we model the population dynamics of infected adult mosquito using classical Ross malaria mathematical model. The effects of mosquito populations on malaria transmission are studied through the entomological inoculation rate (EIR) which is calculated using the formula:

\[ EIR = maS. \]  (1)

Where m is the ratio of mosquitoes to humans, a is the man-biting rate of mosquitoes and S is the proportion of adult mosquitoes that are infected by malaria. Species considered in this study are Anopheles gambiae since they are responsible for most malaria
cases in Uganda. Moreover, we identify the patterns of malaria cases for the study areas (Kalangala, Wakiso and Nakasongola) and model the cases based on rainfall.

1.6 Structure of the report.

This project report consists of six chapters. Chapter 1 is a general introduction to the report, and has six sections. Section 1.1 gives the general background of the study. Section 1.2 gives information about malaria in Uganda. The specific questions we wish to answer and what we hope to achieve at the end of the study are given in section 1.3 and section 1.4. Section 1.5 states the scope of this work. Finally, this section (section 1.6) gives a roadmap to the arrangement of the entire report. Chapter 2 provides a literature review acknowledging related works done previously. It also identifies existing gaps in knowledge. Chapter 3 provides information about the study sites and methods used in this study. Section 3.1 provides information about the three districts. Section 3.2 has five subsections and it gives information about the methods. Subsection 3.2.1 talks about seasonal decomposition of timeseries data by LOESS method, subsection 3.2.2 gives a description of mosquito life cycle model and the basic outputs of the model, subsection 3.2.3 explains about mosquito population prediction with seasonality using the basic Ronald Ross model, subsection 3.2.4 is for assessing the relationship between rainfall and malaria cases and finally subsection 3.2.5 gives general information about LLINs and proceeds to describe how to model LLIN impact on adult mosquitoes. Chapter 4 presents the study design and data analysis. Chapter 5 discusses the results obtained from the models and the graphs. It discusses the model projections from chapter 4 and how they relate to real life. Then, chapter 6 it gives a general conclusion to the study and future work.
2 Literature review.

Mathematical models have been useful in malaria control efforts and they become even more important when malaria elimination is in the foreseeable future [11]. Sir Ronald Ross was the first person to prepare a scientific mathematical models which aimed at studying malaria transmission and control [9]. He developed the model in 1916 which had a basic deterministic formula [8]. The models have been employed in malaria control studies and they have been very helpful in controlling malaria [10]. Malaria transmission models play a significant role in understanding the dynamics of the disease [25]. They have long been applied to assess the possible means of intervention [26, 27]. In many studies, the dynamics have investigated through deterministic models [28, 29] and some through stochastic models [30, 31]. Some of the studies either abandon the impact of climate or incorporate it through the force of infection. In 1957, MacDonald advanced on Ronald Ross’ work. He introduced the latency period in human and the mosquito populations respectively [14], which is known as the Ross-MacDonald model.

An ecological model of Anopheles gambiae sensu lato population was developed by White [21]. He used the model to compare the impact of vector control interventions (LLINs, IRS, Larvicides and Pupacides) directed against adult mosquito stages and also aquatic mosquito stages. The model incorporated a rainfall-dependent carrying capacity and density-dependent regulation of mosquito larvae in breeding sites.

Kileen and Chitnis have done a work on modelling the impact of LLINs on Anopheles arabiensis species. They used mathematical models to evaluate the consequence of behavioural adaptation to the success of controlling Anopheles arabiensis with LLINs [12]. Another study was done by Eckhoff in 2013 using a mathematical model to study Anopheles population dynamics and malaria transmission [13]. Another study was done by Nyachae, he used a deterministic mathematical model to analyse the spread of malaria in Nyamira in Kenya. The model consists of seven compartments with seven non-linear differential equations that describe the spread of malaria with three state variables for mosquitoes (\(S_m, E_m, I_m\)) and four state variables for humans (\(S_h, E_h, I_h, R_h\)) [15].

Mathematical, compartmental model to forecast the population dynamics of mosquito (Aedes aegypti) and its life cycle in relation to seasonal variations of rainfall and temperature was introduced by Aditya Vaidya. The populations within the compartments were expressed in the form of a system of coupled differential equation (DEs) which describes changes in the mosquito population through process of maturation and mortality. Variations in the mosquito population due to seasonal temperature variations were predicted for Buenos
Aires, Rio de Janeiro and Dallas and they matched actual mosquito trap data. Mosquitoes need stagnant water to lay eggs and reproduce [32]. In the model, integration of rainfall as a determining factor of growth was a natural extension. Variation of the carrying capacity of the model with rainfall was included. The model is sufficiently flexible to be used with other mosquito species such as Culex and Anopheles that spread nile virus and malaria. In this study, we are going to adopt this model since it can be used to predict effects of newer weather patterns on the spread of mosquitoes by modeling the vectors and the disease they spread. Furthermore, we adopt the methods used by Tsegahum Worku Brhanie in Modelling rainfall and malaria cases in North West Ethiopia so as to assess the relationship between rainfall and malaria cases.
3 Study site and methods.

3.1 Study site.

Uganda is one of the countries in East Africa bordering Lake Victoria in South East. It is bordered by South Sudan in North, Democratic Republic of the Congo in West, Kenya in East and by Rwanda and Tanzania in South. Uganda occupies an area of 241,551km square. The country has a population of 34.8 million people which makes it the world’s second most populous landlocked country after Ethiopia. Uganda is divided into 80 districts across 4 administrative towns. In this study, we consider only 3 districts which are Kalangala, Wakiso and Nakasongola since they are the only ones with full annual malaria cases data. The study sites are presented by red color on the maps given below; The first map shows Kalangala district, the second map shows Nakasongola district and the last map shows Wakiso district.
Figure 5. Study sites presented by red color; Kalangala district, Nakasongola district and Wakiso district respectively.

3.1.1 Nakasongola district.

Nakasongola is a town in Nakasongola District in the Central Region of Uganda. It is approximately 125 km north of Kampala. The coordinates of the town are $1^\circ 18'54.0''$N, $32^\circ 27'54.0'\text{E}$ (Latitude: 1.3150; Longitude: 32.4650). The average annual temperature is 22.5 degree centigrade and the rainfall peak falls on October with the average of 1229 mm in a year.

Figure 6. Average monthly rainfall at Nakasongola district.
3.1.2 Kalangala district.

Kalangala is situated in Lake Victoria. It covers an area of 9066.8 km square of which only 432.1km square is land and the rest is water. Kalangala district is entirely made up of 84 islands widely scattered in Lake Victoria. Kalangala experience a typical humid climate. The annual rainfall ranges from 1125 to 2250mm and the rainfall peaks are in March-May and October-November.

Figure 7. Average monthly rainfall at Kalangala district.
3.1.3 Wakiso district.

Wakiso District lies in the Central Region of Uganda. The coordinates of the district are:00 24N, 32 29E. It covers a total area of 2704km square. The rainfall in Wakiso is bi-modal. There are two wet seasons from April-May and October-November. The dry seasons are from January-February and July-August.

![Monthly average rainfall at Wakiso district](image.png)

Figure 8. Average monthly rainfall at Wakiso district.

3.2 Methods.

3.2.1 Seasonal trend decomposition by LOESS (LOcally Estimated Scatterplot Smoothing).

Trend decomposition is implemented in the Seasonal Trend decomposition by LOESS method (STL) [18]. This method is based on a locally weighted regression smoother (LOESS), which generates an output consisting of a non-linear trend line, a seasonal component, and a remainder (residual). This is a method of extracting components which was used to assess the seasonal pattern of the malaria cases data. Decomposition methods generally separate a time series into a seasonality- patterns repeating in fixed time, trends-Underlying trends, increasing or decreasing and random fluctuation-noise[17]. LOESS is
applied iteratively to the observations in moving time windows to filter the time series in a way that results in estimates of trend and seasonality that are robust to aberrant behavior in the time series [18]. Basically STL method helps us to get the clear picture of how the series vary.

3.2.2 Mathematical model to describe life cycle of mosquito.

In this study, we adopt a continuous, compartmental model by Aditya Vaidya which was used to describe the life cycle of the mosquito. The compartments correspond to one stage of the life cycle of a mosquito. The stages are: egg, larva, pupa, and adult mosquito. Male mosquitoes were not modeled in the population balance since the only responsible vectors for malaria transmission are female mosquitoes. This model paradigm reduces computational complexity if more environmental factors are to be taken into account. It has been assumed that the populations within each compartment are homogeneous [32]. The model is written as follows:

\[
\begin{align*}
\frac{dE}{dt} &= b\rho A - (\mu_p + \rho_E)E, \\
\frac{dL}{dt} &= \rho_E E - \alpha L^2 - (\mu_L + \rho_L)L, \\
\frac{dP}{dt} &= \rho_L L - (\mu_p + \rho_p)P, \\
\frac{dA}{dt} &= \rho_p \frac{P}{2} - (\mu_A)A.
\end{align*}
\] (2)

The model is expressed in the system of differential equations to represent the change of the population of each of the four compartments. The definition of the variables in the system of differential equations are described below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>time(days)</td>
</tr>
<tr>
<td>\rho</td>
<td>maturation rate</td>
</tr>
<tr>
<td>\mu</td>
<td>mortality rate</td>
</tr>
<tr>
<td>\alpha</td>
<td>carrying capacity</td>
</tr>
<tr>
<td>b</td>
<td>number of eggs per oviposition</td>
</tr>
<tr>
<td>E</td>
<td>number of eggs</td>
</tr>
<tr>
<td>P</td>
<td>number of pupae</td>
</tr>
<tr>
<td>L</td>
<td>number of larvae</td>
</tr>
<tr>
<td>A</td>
<td>number of adults</td>
</tr>
</tbody>
</table>
Rainfall has been shown to be a key factor in mosquito reproduction since it determines the availability of breeding sites [32]. The availability of breeding sites determines the carrying capacity of vector population. There is high competition between larvae for food if the feeding sites are less.

### 3.2.3 Population prediction with seasonality.

The method in which variable rainfall is to be used involves a sinusoidal regression of rainfall during a time period, making the rainfall a smooth function of time. This then causes seasonal variations in mosquito densities in a manner similar to what you would expect in a setting with rainy seasons per year. Ross model is the earliest attempt to quantitatively describe the dynamics of malaria transmission at a population level. Hence we adopt this model in modelling the artificial seasonal variations of mosquito population depending on rainy seasons for each district, it is given by the following system of ordinary differential equations;

\[
\frac{dI_h}{dt} = abmI_m(1 - I_h) - rI_h, \tag{3}
\]

\[
\frac{dI_m}{dt} = acI_h(1 - I_m) - \mu I_m.
\]

Where \( I_h \) and \( I_m \) denote the fractions of infected humans and mosquitoes respectively, \( a \) is the mosquito-human biting rate, \( m \) stands for ratio of number of female mosquito to that of human, \( b \) is the proportion of bites that produce infection in human, \( c \) is the proportion of bites by which one susceptible mosquito becomes infected, \( \mu \) denotes the mosquito mortality rate and \( r \) stands for the human recovery rate. The model considered the infection at the low, medium and high level using high parameter values. A summary of the model parameters alongside their sources is given in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>mosquito-human biting rate</td>
<td>0.5</td>
<td>[33]</td>
</tr>
<tr>
<td>b</td>
<td>proportion of bites causing infection in human</td>
<td>0.4</td>
<td>[33]</td>
</tr>
<tr>
<td>c</td>
<td>proportion of bites which susceptible mosquito becomes infected</td>
<td>0.4</td>
<td>[33]</td>
</tr>
<tr>
<td>m</td>
<td>mosquito human ratio</td>
<td>3.9</td>
<td>[33]</td>
</tr>
<tr>
<td>r</td>
<td>average human recovery rate</td>
<td>0.05</td>
<td>[33]</td>
</tr>
<tr>
<td>( \mu )</td>
<td>mosquito mortality rate</td>
<td>0.05</td>
<td>[33]</td>
</tr>
</tbody>
</table>
3.2.4 Assessing the relationship between rainfall and malaria cases.

We use matlab to find the matches that will originally bring meaningful statement in real life, between the malaria cases and the average monthly rainfall levels of the three districts. The malaria cases are plotted on the left y axis while the rainfall on right y axis and the months on x axis from 2006 to 2011.

3.2.5 Modelling the impact of LLIN use on adult mosquito population.

This section has two parts. The first part gives information about long lasting insecticide-treated nets in general and how they protect humans from mosquitoes. The second part presents the mathematical modelling of the impact of LLINs on adult mosquitoes.

- **Background of LLINs.**
  The intervention method whose effect we are interested on is long lasting insecticide nets (LLINs), a form of insecticide-treated mosquito nets which are efficacious over a longer period of time; about three years, and maintain their insecticidal efficiency up to twenty washes. They are available on the market. Insecticide nets and indoor residual sprays (ITNs) are the intervention methods that have been promoted for protection against malaria in malaria-endemic areas [16]. Mosquito nets are relatively affordable and can be used by all groups of people. Research has shown that most malaria transmitting mosquito bites occur indoors during the night time. Therefore, the use of nets provide an effective way of protecting humans from such bites when they sleep. Nets can either be treated with insecticides or used untreated. Untreated nets only provide a physical roadblock against mosquitoes which attempt to bite humans. However, insecticide nets provide protection through chemical action on mosquitoes as well as presenting a material barrier against the malaria vectors. Thus, the use of insecticide-treated nets is more advantageous than the use of untreated nets. Nets are assumed to have four effects: direct killing of a mosquito landing on them, repellency which results in a longer gonotrophic cycle and possible diversion to a non-human blood host, a direct protective effect for the individual sleeping beneath the net, and a reduction in transmission from infected individuals sleeping under the net to susceptible mosquitoes.

- **Modelling the impact of LLIN use on adult mosquito population.**
  The model used in evaluating the impact of LLIN use is adopted from Arnaud Le Menach [22]. Here we consider the effect on the feeding cycle, the mosquito feed-
The feeding cycle is described by 2 stages which are host-seeking time to successful feeding, and resting through to oviposition. We take into consideration the possibilities that arise when a mosquito comes across a LLIN in the process of feeding. Finally, we combine all this information into the adult mosquitoes mortality rate and study the outcome on some model parameters.

During the host seeking process, proportion $\kappa O(0)$ of mosquito finds human and $1 - \kappa O(0)$ finds other vertebrate host. Fraction using LLIN are protected. If a mosquito finds a protected human, these things can happen: It can successfully feed regardless of the treated net with probability $s$(if the net has holes or not properly deployed), it dies from contact with insecticide with probability $d$ or it leaves to search for another host with probability $r=1-s-d$.

Without the use of LLIN, the host seeking process takes $\tau_1(0)$ days and mosquito survives with probability $\rho_1(0)$. After successful meal, mosquito rest, finds larval habitat and oviposit. The process lasts $\tau_2(0)$ days with probability $\rho_2(0)$. The feeding cycle is described in the figure below; We assume that a surviving repelled mosquito can successfully feed after several attempts. The mosquito can repeat the attempts as many times as necessary to complete the feeding cycle. The parameter values used for projecting the model are presented in the table below.

![Mosquito feeding cycle flow chart](image)

**Figure 9. Mosquito feeding cycle flow chart.**
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>frequency of feeding</td>
<td>0.33</td>
<td>[23]</td>
</tr>
<tr>
<td>$\tau_1(0)$</td>
<td>host seeking time</td>
<td>0.69</td>
<td>[23]</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>resting time</td>
<td>0.31</td>
<td>[23]</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>probability of surviving at zero net coverage</td>
<td>0.91</td>
<td>[23]</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>probability of resting at zero net coverage</td>
<td>0.82</td>
<td>[23]</td>
</tr>
<tr>
<td>$\mu_M$</td>
<td>mosquito mortality at zero net coverage</td>
<td>0.096</td>
<td>[23]</td>
</tr>
<tr>
<td>$\kappa o(0)$</td>
<td>preference to human blood</td>
<td>0.95</td>
<td>[22]</td>
</tr>
<tr>
<td>$\phi_T$</td>
<td>proportion human protected by LLIN</td>
<td>0-1</td>
<td>This paper</td>
</tr>
<tr>
<td>$d_T$</td>
<td>probability a mosquito is killed by LLIN</td>
<td>0.41</td>
<td>[23]</td>
</tr>
<tr>
<td>$S_T$</td>
<td>probability a mosquito feeds successfully with LLIN</td>
<td>0.03</td>
<td>[23]</td>
</tr>
<tr>
<td>$r_T$</td>
<td>probability a mosquito is repelled by LLIN</td>
<td>0.56</td>
<td>[23]</td>
</tr>
<tr>
<td>$\gamma T$</td>
<td>proportion of human covered by LLIN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_T$</td>
<td>probability a mosquito successfully survive feeding attempt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z_T$</td>
<td>probability a mosquito repeats the feeding cycle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_T$</td>
<td>daily mosquito survival probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_T$</td>
<td>daily mosquito mortality probability</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the flow chart in Figure 2, a mosquito feeds successfully by either feeding on animals, feeding on a human not covered by LLINs, feeding on a human who is covered by LLIN. The probability that a mosquito succeeds the feeding attempt is presented as.

$$\omega_T = 1 - \kappa o(0) \phi_T (1 - S_T).$$  \hspace{1cm} (4)

A mosquito which survives the feeding attempt despite the LLIN but fails to obtain a blood meal repeats the search all over again with the probability given below.

$$Z_T = \kappa o(0) \phi r_T.$$  \hspace{1cm} (5)

The host seeking time at LLIN coverage is given as

$$\tau_1 = \frac{\tau_1(0)}{1 - Z_T}.$$  \hspace{1cm} (6)

The probability of a mosquito surviving a day at LLIN coverage is calculated in equation given below;

$$\rho_T = \left(\frac{\rho_1(0) \rho_2 w_T}{1 - Z_T \rho_1(0)}\right)^{f(\phi)}.$$  \hspace{1cm} (7)
Finally, adult mosquito mortality at LLIN net coverage is calculated as:

\[ \mu_M = -\log \rho_T. \]  

(8)
4 Study design and data analysis.

The data available and used in this study are monthly malaria cases for the three districts in Uganda which are Kalangala, Wakiso and Nakasongola and their respective average monthly rainfall in a year. The source of malaria cases data is ministry of health in Uganda. The data includes age (under 5 years and 5 years plus), gender, malaria cases and place of residence (districts). The monthly rainfall data for the three districts were obtained from meteoblue website. This paper will examine the seasonal pattern of malaria cases using time series method and explore the relationship between malaria cases and one potential driver which is rainfall. Malaria case data was compiled in a monthly format in Excel and analysed using time series methods. It is often difficult to draw conclusions on seasonal patterns as case data usually exhibits noise based on the case data itself. Hence there is a need to extract the seasonal pattern from the data. In particular, Seasonal decomposition of Time series by LOESS (LOcally Estimated Scatterplot Smoothing) method of extracting components was used to assess the seasonal pattern of the data.
5 Results.

5.1 Time series plot and malaria cases patterns.

Figure 13. Time series plots of Kalangala, Nakasongola and Wakiso districts respectively.

The above figures present the seasonal patterns of malaria cases for the three districts from 2006 to 2011. It is difficult to draw conclusions from the graphs hence trend decomposition by LOESS (Locally Estimated scatterplot smoothing) method was adopted to decompose the time series plot for each district for all children below five years old and adults five years old and above, for both male and female so as to get the clear picture of the series variations. The decomposed time series plots are given below:

Figure 18. Decomposed time series plots for Kalangala district.
The malaria cases in all districts are shown to have seasonality with different levels of variation given on the sides of the windows, that is the cases change every year. The trends also are varying from time to time, Sharp increases of malaria cases and no/low malaria cases are observed. The level of residuals is as well shown on the plots. It wouldn’t be easy/possible to observe all this on the time series plots without decomposing.
5.2 Mosquito system behaviour with seasonality.

Population level of infected mosquitoes varies over time. Environmental factors such as temperature, pollution and rainfall can affect the population. In this section, we begin by modelling the population levels as a function of rainfall depending on rainy seasons in a year for the three districts in Uganda.

![Graph showing seasonal mosquito population variation in Wakiso district.](image1)

**Figure 29.** Seasonal mosquito population variation in Wakiso district.

The rainfall peaks in Wakiso district fall on April-May and October-November. Hence the graph above shows the mosquito population of mosquitoes have peaks twice in a year similar to the rainy seasons.

![Graph showing seasonal mosquito population variation in Kalangala district.](image2)

**Figure 30.** Seasonal mosquito population variation in Kalangala district.

Kalangala district experience two rainy seasons in a year as well. The peaks fall on March-May and October-November. Therefore, the population mosquito peaks follows the rainy seasons.
Figure 31. Seasonal mosquito population variation in Nakasongola district.

The rainfall peak in Nakasongola district falls on October-November. Hence the peak of mosquito population is observed once a year as shown on the graph.
5.3 Rainfall related to malaria transmission.

Climate is claimed to be a major driving force behind malaria transmission. Climate data are often used to account for the spatial, seasonal and interannual variation in malaria transmission [34]. Climatic factors modify the abundance of mosquito populations, the length of parasite cycle in the mosquito and the malarial dynamics [35]. Rainfall affects malaria transmission because Anopheles mosquitoes’ breeding habitat depends on standing water. Standing water is however influenced within-season temporal rainfall pattern and hence the mosquito population can be affected [36]. During the wet seasons, mosquito population increases due to increase of the breeding sites and hence malaria transmission rate is also high during this season.

5.4 Relationship between rainfall and malaria cases.

We compared malaria cases patterns for the three districts from 2006 to 2011 with the average monthly rainfall in millimeters. The year 2006 is at the top left followed by the other years till 2011 at the bottom right. Below are the results we obtained;

![Graphs showing relationship between rainfall and malaria cases](image)

**Figure 32.** Kalangala district below 5 years cases.

The above graphs are for the malaria cases for children below five years old in Kalangala
district and rainfall levels for six years starting from 2006 at the top left to 2011 at the bottom right. The malaria cases data on 2007 matches with rainfall due to the fact that on March the cases increase and that is exactly the time when the rainy season starts, hence the rain creates breeding sites for mosquitoes. On October 2007, the cases also increases because it is the beginning of the second rainy season. The cases lowers when the rain increases on November. Furthermore the malaria cases data in 2010 matches well with the level of rainfall as the cases starts to increase on March when the rainy season starts. The cases decreases just when the rain increases on May because when it rains too much the breeding sites can be flooded away. The rainy season ends on May, and therefore there is an increase of the cases because that is when the breeding sites are plenty and they can not be flooded.
The graphs above represent the malaria cases for adults who are five years and above in Kalangala district and rainfall levels for six years starting from 2006 at the top left to 2011 at the bottom right. The rainy seasons starts on March, then the level increases on April and lowers on May. Again the rainy season starts on October and level increases on November then lowers on December. The malaria cases data in 2006 matches well with the rainy seasons as the cases start to increase on March just as the rain starts. When the level of rainfall increases on April, the cases lowers due to the fact that too much rain can flood away the breeding sites of mosquitoes. The cases pattern increases again on May when the rainy season is ending. Another rainy season starts on October and increases on November, therefore the cases are observed to increase on November. During the start of rain on October, the cases seem to lower down and this could be may be people took note of the incoming rains and decided to protect themselves and clear the breeding sites. Another years which makes the similarity between the cases and rainfall are April-2008, April-2009 and April-2010 because during this time the rain increases and the cases decrease and the reason could be floods which clear the breeding sites.
Figure 34. Wakiso district below 5 years cases.

The above graphs are for malaria cases for children below five years old and the average monthly rainfall for Wakiso district. The data of 2009 matches well especially the male cases as the rainfall level in April is a bit much, so the cases decreases. The rainfall level reaches maximum level on october, the cases decreases as well since too much rainfall wash away the breeding sites. April 2010 and April 2011 cases also matches because the rain is much and the cases reduces.
Figure 35. Wakiso district 5 years and above cases.

The above graphs are for Wakiso district for adults who are five years old and above compared with the monthly average rainfall. The rainfall season starts in April, but the level of rainfall is a bit much compared to other months except for the October when the level is at maximum. The male cases on April 2009 reduces. Both male and female cases on April 2010 and 2011 decreases due to much rainfall. Hence this is a good match. The female cases reduces on October 2009, this is as well a good match simply because the rainfall level is at maximum level hence flooding the breeding sites.
Figure 36. Nakasongola district below 5 years cases.

The above graphs are for Nakasongola district for children below five years. In Nakasongola the maximum rainfall level is observed on October then ends on November. This means that Nakasongola faces a quite long dry seasons. Most high malaria cases are observed on June when the rain is not even much especially in the years 2006, 2007, 2010 and 2011. The cases then reduces until October when they start to slightly increase. This could be because of the long dry season so when the rain starts and when it is at maximum level, it creates a lot of breeding sites for mosquitoes.
Figure 37. Nakasongola district 5 years and above cases.

The above graphs are for the adults who are five years and above for Nakasongola district. The results are quite similar to the ones for children below five years old. The cases start to increase on October when the rainfall level is at maximum. This could be due to long dry period. Some peaks of the cases are observed at low rainfall level. There could be another reason for that case.
5.5 Effect of LLIN on some model parameters.

**Figure 38.** Effects of LLIN coverage on host seeking time, probability of feeding successfully and probabilities of both daily survival and mortality of adult *Anopheles gambiae* mosquitoes. Results generated using values from and equations provided above.

LLIN use increases both the time required to search for a host and the daily mortality of adult mosquitoes. In contrast, the probabilities of both feeding successfully and surviving a day are decreased.
6 Conclusion and future work.

6.1 Conclusion.

We observed patterns of malaria cases from the malaria data of three districts. Both peaks, low/no malaria cases have been observed as well. On matching the average monthly rainfall for every district with the recorded malaria cases, we observed that some of the data in certain years match well and can be related to real life. For instance, when the rain period just begins, it creates an environment for mosquitoes to breed as they depend on stagnant water. On the other side, if the rain level is at maximum level the mosquitoes’ breeding sites can be flooded hence the mosquito population reduces so do the malaria transmission. Moreover, too much rain can also lead to increase in mosquito population if and only if the area faces a long dry period, so this means there won’t be any flooding of the breeding sites. Age and sex of infected human don’t seem to be a factor for malaria cases patterns as majority of the cases follows the same patterns. There could be other factors that contribute such as temperature because there are limits of temperature for the growth of mosquitoes. Migration could be one of the factors also, people with malaria can migrate from one area to another and spread the disease (imported cases).

Long lasting treated bed nets is one of the methods which is highly recommended so as to control malaria transmission. On modeling the impact of LLIN we found that LLIN use increases both the time required to search for a host and the daily mortality of adult mosquitoes. In contrast, the probabilities of both feeding successfully and surviving a day are decreased.

6.2 Future work.

The future work will be on working with real rainfall data as for this study we just used average monthly rainfall data for the three districts in a year. Additionally, checking on the impact of the migration as in imported cases depending on the availability of data will also be the wise and most important thing to do in identifying the cases pattern.
REFERENCES


