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# **Balancing of a Rotor With Active Magnetic Bearing System: Comparison of One- and Two-Plane Balancing Procedures**

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## **Keywords**

«Diagnostics», «Magnetic Bearings», «Simulations»

## **Abstract**

Rotor balancing is a procedure used to reduce the unbalance vibration of rotating machinery. Unbalance vibration creates forces to the bearings and causes harmful effects such as reducing their lifespan. Unbalance is caused simply because of the difference of the rotors axis of inertia and the rotating axis. In this paper, two balancing techniques are presented to reduce the unbalance vibration: one-plane balancing procedure used for two-plane balancing and a two-plane balancing procedure. Both balancing procedures take into account the cross effect between the planes. Rotor levitated with active magnetic bearings is balanced. Magnetic levitation is unstable so the xy-position of the rotor is measured on both rotor ends for the control purposes. The unbalance vibrations are analyzed from these position measurements so no additional hardware is required for the rotor balancing. Results are presented from a simulation model of an active magnetic bearing system. Comparison of the one-plane and two-plane balancing is provided.

## **Introduction**

Unbalance is an unwanted property of any rotating machinery. To some degree it exists on all rotating machines [1]. The unbalance causes vibration of the machines [1],[2]. These vibrations create forces

to the bearings and increase their chance of fatigue failure and reduce their lifespan [2]. Vibrations also cause intensification of stresses at other receivers [3]. The force that an unbalance mass causes can be calculated with [3]

$$F = m\omega^2 r \cdot \exp(i\omega t) \quad (1)$$

where  $m$  is the unbalance mass,  $\omega$  is the angular frequency,  $r$  is the radius from the rotating center and  $t$  is time.

In general unbalance is caused by the difference between the rotor's rotating axis and the axis of inertia. This is due to the uneven mass distribution along the rotor's rotating axis. [3] There are many causes for this uneven mass distribution. During manufacturing process the material problems occur such as the density variation, porosity, voids and blowholes [1],[4]. Also human error during assembly such as misaligned assembly and misshapen castings [4]. The keys and keyways added during component balancing, bolts, nuts, rivets, welds, cranks and other necessary features increase the likelihood of unbalance [3],[4]. The operation conditions such as humidity, temperature, corrosive environment, wear and deposit build up also play an important role in the unbalance [1],[3],[4]. Thus some rotors should be rebalanced as they age.

There are four types of unbalance: static, coupled, quasi-static and dynamic unbalance. Static unbalance is caused by a point mass at certain distance from the center of the gravity of the rotor on one plane. In coupled unbalance two equal point masses are positioned symmetrically around the center of gravity of the rotor but positioned  $180^\circ$  from each other. [1],[2] Quasi-static is the type of unbalance that is a combination of static and coupled unbalances where one of the couple moments is in line with the static unbalance [2]. Dynamic unbalance is the most common type of unbalance. This is similar to quasi-static unbalance but now the static unbalance is not in line with either of the couple moments [1],[2]. Static, coupled and quasi-static unbalances can be considered as a special case of dynamic unbalance.

The process of minimizing the unbalance thus to improve the mass distribution of a rotor is called balancing. The balancing is done by adding correction weights at prescribed locations or removing fixed quantities of material from rotor with for example drilling [2]. Balancing techniques can be divided into three groups: one-plane, two-plane and multi-plane. For rigid rotors one-plane and two-plane balancing are typically used. The choice between one- and two-plane balancing can be made with for example the rotor's length to diameter (L/D) ratio and the rotational speed and is not based on the unbalance type [1]. However this should only be treated as a guideline and two-plane balancing is always the preferred choice. In [1] the one-plane balancing is suitable if L/D ratio is less than 0.5 and rotational speed is below 1000 rpm and if the L/D ratio is higher than 0.5 and rotational speed is below 150 rpm. In other cases the two-plane balancing should be used. Multi-plane balancing is mostly used on very complex rigid or flexible rotors. Example of a flexible rotor balancing is shown in [5]. Flexible rotor in this case means a rotor which has a service speed of higher than 70% of the first critical speed. Some flexible rotors however can be balanced with one- and two-plane balancing methods [1]. In this paper a rigid rotor and one- and two-plane balancing with vector methods are discussed. Balancing is done with adding correction weights.

## Balancing of Rotors

For the balancing procedure information about the vibration amplitudes and the corresponding angles are needed. Vibration amplitude refers to the amplitude at the rotation frequency only. The locations of the rotor where this data is collected are called measurement planes. For each measurement plane there exist a balancing plane where the correction and trial weights are attached.

In this case an active magnetic bearing (AMB) is utilized to get the data from the measurement planes. Both ends of the rotor are supported with AMBs and the xy-positions of both ends are needed for the control because magnetic levitation is unstable without the control. This allows using either x- or y-position of each end as a measurement plane. The measurements planes are called Drive (D) plane and non drive (N) plane. Fig. 1. shows an example of a rotor supported with AMBs showing the locations

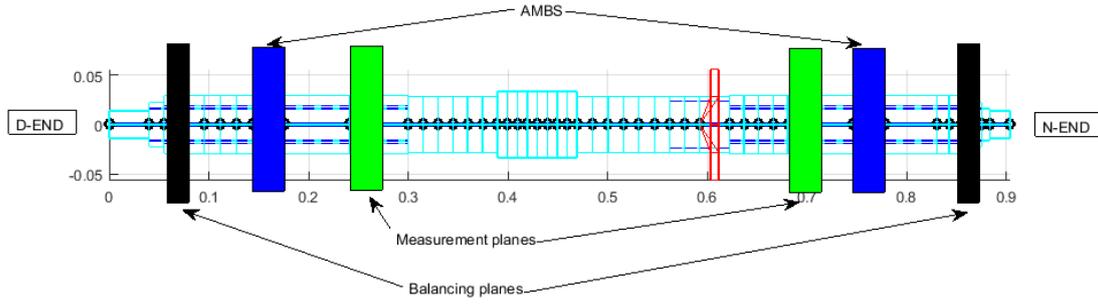


Fig. 1: Example of a rotor supported with AMBs showing the locations of the measurement- and balancing planes and AMBs.

of the AMBs, measurement- and balancing planes. The angle is measured as a lag between the rotor  $0^\circ$  angle and the position (vibration) signal peak. No additional hardware is thus needed in the balancing and balancing can be done in the field without a balancing machine. An example of a AMB rotor balancing procedure is presented in [6]. In [7],[8] online methods for the unbalance compensation of AMB rotor systems is presented.

### 1-Plane balancing

The well-known one-plane balancing procedure has been presented by numerous researchers [1],[3],[4]. Here the balancing procedure is based on the two-plane balancing procedure presented in [3] and is adapted for one-plane balancing.

Original measured unbalance vibration of the rotor is denoted with  $\mathbf{V}_D$ . After attaching a trial weight  $\mathbf{W}_D$  to the rotor unbalance vibration becomes  $\mathbf{V}_{D1}$ . Now the idea is to find the original unbalance mass  $\mathbf{U}_D$  and then the balancing mass  $\mathbf{B}_D$ . The original unbalance vibration can be written as

$$\mathbf{V}_D = \mathbf{A}_{DD}\mathbf{U}_D \quad (2)$$

where  $\mathbf{A}_{DD}$  is the effect of the mass to the vibration on the bearing. Respectively for the unbalance vibration  $\mathbf{V}_{D1}$

$$\mathbf{V}_{D1} = \mathbf{A}_{DD}(\mathbf{U}_D + \mathbf{W}_D) = \mathbf{A}_{DD}\mathbf{U}_D + \mathbf{A}_{DD}\mathbf{W}_D \quad (3)$$

Substituting (2) to (3) yields

$$\mathbf{V}_{D1} = \mathbf{V}_D + \mathbf{A}_{DD}\mathbf{W}_D \quad (4)$$

Solving the (4) with respect to  $\mathbf{A}_{DD}$

$$\mathbf{A}_{DD} = (\mathbf{V}_{D1} - \mathbf{V}_D)/\mathbf{W}_D \quad (5)$$

Substituting (5) to (2) yields

$$\mathbf{V}_D = ((\mathbf{V}_{D1} - \mathbf{V}_D)/\mathbf{W}_D) \cdot \mathbf{U}_D \quad (6)$$

Now the unbalance mass  $\mathbf{U}_D$  is solved from (6)

$$\mathbf{U}_D = (\mathbf{W}_{D1}\mathbf{V}_D)/(\mathbf{V}_{D1} - \mathbf{V}_D) \quad (7)$$

The required balancing mass  $\mathbf{B}_D$  is simply

$$\mathbf{B}_D = -\mathbf{U}_D \quad (8)$$

Equation (8) shows that the balancing mass  $\mathbf{B}_D$  has the same magnitude as the unbalance mass  $\mathbf{U}_D$  but is

180° out of phase. The one-plane balancing procedure requires three start/stop cycles of the rotor. First the original vibration is measured, then the trial weight is attached and the vibration is measured again. After that the calculated balancing mass is added and the final unbalance is measured.

## 2-Plane balancing with 1-plane balancing method

When balancing two or more planes the cross effect between these planes has to be taken into account. The simplest way would be to balance each plane individually and only one at time. [1] Here the one-plane balancing procedure presented previously is used for the two-plane balancing. Steps of the procedure are presented in [1].

First the amplitude and phase of unbalance vibration is measured from both planes and the plane with the higher unbalance vibration is balanced first. Then the one-plane balancing procedure is used for this plane. Next new original unbalance vibration is measured for the second plane because of the cross effect. Then the second plane is balanced and the balancing procedure is completed. The process needs five start/stop cycles of the rotor.

However, it is likely that the unbalance in the first plane has changed because of the cross effect. If the unbalance vibration in the first plane has increased to unsatisfactory level the balancing procedure must be repeated. Depending on the rotor the procedure may have to be repeated several times. [1]

## 2-Plane balancing

The cross effect mentioned previously has to be taken into account when balancing two- or more planes. Here a two-plane balancing procedure presented in [3] is used. It follows similar structure than the 1-plane balancing procedure in previous section.

Original measured unbalance vibration of the rotor in the first plane is denoted with  $\mathbf{V}_D$  and on the second plane with  $\mathbf{V}_N$ . After attaching a trial weight  $\mathbf{W}_D$  to first plane on the rotor unbalance vibration on the first plane becomes  $\mathbf{V}_{D1}$  and on the second  $\mathbf{V}_{N1}$ . Attaching a trial weight  $\mathbf{W}_N$  to second plane on the rotor unbalance vibration on the first plane becomes  $\mathbf{V}_{D2}$  and on the second  $\mathbf{V}_{N2}$ . The idea here is to find both original unbalance masses  $\mathbf{U}_D$  related to the first plane and the unbalance mass  $\mathbf{U}_N$  related to the second plane and the corresponding balancing masses  $\mathbf{D}_D$  and  $\mathbf{B}_N$ . The original unbalance vibrations can be written as

$$\mathbf{V}_D = \mathbf{A}_{DD}\mathbf{U}_D + \mathbf{A}_{DN}\mathbf{U}_N \quad (9)$$

$$\mathbf{V}_N = \mathbf{A}_{ND}\mathbf{U}_D + \mathbf{A}_{NN}\mathbf{U}_N \quad (10)$$

Where  $\mathbf{A}_{DD}$ ,  $\mathbf{A}_{DN}$ ,  $\mathbf{A}_{ND}$  and  $\mathbf{A}_{NN}$  are the effects of the mass to the vibration on the bearing. Equation (9) and (10) in the matrix form is

$$\begin{bmatrix} \mathbf{V}_D \\ \mathbf{V}_N \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{DD} & \mathbf{A}_{DN} \\ \mathbf{A}_{ND} & \mathbf{A}_{NN} \end{bmatrix} \begin{bmatrix} \mathbf{U}_D \\ \mathbf{U}_N \end{bmatrix} \quad (11)$$

Similarly for the unbalance vibrations  $\mathbf{V}_{D1}$ ,  $\mathbf{V}_{N1}$ ,  $\mathbf{V}_{D2}$  and  $\mathbf{V}_{N2}$

$$\mathbf{V}_{D1} = \mathbf{A}_{DD}(\mathbf{U}_D + \mathbf{W}_D) + \mathbf{A}_{DN}\mathbf{U}_N \quad (12)$$

$$\mathbf{V}_{N1} = \mathbf{A}_{ND}(\mathbf{U}_D + \mathbf{W}_D) + \mathbf{A}_{NN}\mathbf{U}_N \quad (13)$$

$$\mathbf{V}_{D2} = \mathbf{A}_{DD}\mathbf{U}_D + \mathbf{A}_{DN}(\mathbf{U}_N + \mathbf{W}_N) \quad (14)$$

$$\mathbf{V}_{N2} = \mathbf{A}_{ND}\mathbf{U}_D + \mathbf{A}_{NN}(\mathbf{U}_N + \mathbf{W}_N) \quad (15)$$

and in the matrix forms

$$\begin{bmatrix} \mathbf{V}_{D1} \\ \mathbf{V}_{N1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{DD} & \mathbf{A}_{DN} \\ \mathbf{A}_{ND} & \mathbf{A}_{NN} \end{bmatrix} \begin{bmatrix} \mathbf{U}_D \\ \mathbf{U}_N \end{bmatrix} + \begin{bmatrix} \mathbf{A}_{DD} \\ \mathbf{A}_{ND} \end{bmatrix} \mathbf{W}_D \quad (16)$$

$$\begin{bmatrix} \mathbf{V}_{D2} \\ \mathbf{V}_{N2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{DD} & \mathbf{A}_{DN} \\ \mathbf{A}_{ND} & \mathbf{A}_{NN} \end{bmatrix} \begin{bmatrix} \mathbf{U}_D \\ \mathbf{U}_N \end{bmatrix} + \begin{bmatrix} \mathbf{A}_{DN} \\ \mathbf{A}_{NN} \end{bmatrix} \mathbf{W}_N \quad (17)$$

Substituting (11) to (16) and (17)

$$\begin{bmatrix} \mathbf{V}_{D1} \\ \mathbf{V}_{N1} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_D \\ \mathbf{V}_N \end{bmatrix} + \begin{bmatrix} \mathbf{A}_{DD} \\ \mathbf{A}_{ND} \end{bmatrix} \mathbf{W}_D \quad (18)$$

$$\begin{bmatrix} \mathbf{V}_{D2} \\ \mathbf{V}_{N2} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_D \\ \mathbf{V}_N \end{bmatrix} + \begin{bmatrix} \mathbf{A}_{DN} \\ \mathbf{A}_{NN} \end{bmatrix} \mathbf{W}_N \quad (19)$$

Solving (18) for  $\mathbf{A}_{DD}$  and  $\mathbf{A}_{ND}$  and (19) for  $\mathbf{A}_{DN}$  and  $\mathbf{A}_{NN}$

$$\begin{bmatrix} \mathbf{A}_{DD} \\ \mathbf{A}_{ND} \end{bmatrix} = \begin{bmatrix} (\mathbf{V}_{D1} - \mathbf{V}_D)/\mathbf{W}_D \\ (\mathbf{V}_{N1} - \mathbf{V}_N)/\mathbf{W}_D \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} \mathbf{A}_{DN} \\ \mathbf{A}_{NN} \end{bmatrix} = \begin{bmatrix} (\mathbf{V}_{D2} - \mathbf{V}_D)/\mathbf{W}_N \\ (\mathbf{V}_{N2} - \mathbf{V}_N)/\mathbf{W}_N \end{bmatrix} \quad (21)$$

Substituting (20) and (21) to (11)

$$\begin{bmatrix} \mathbf{V}_D \\ \mathbf{V}_N \end{bmatrix} = \begin{bmatrix} (\mathbf{V}_{D1} - \mathbf{V}_D)/\mathbf{W}_D & (\mathbf{V}_{D2} - \mathbf{V}_D)/\mathbf{W}_N \\ (\mathbf{V}_{N1} - \mathbf{V}_N)/\mathbf{W}_D & (\mathbf{V}_{N2} - \mathbf{V}_N)/\mathbf{W}_N \end{bmatrix} \begin{bmatrix} \mathbf{U}_D \\ \mathbf{U}_N \end{bmatrix} \quad (22)$$

Now solving (22) for  $\mathbf{U}_D$  and  $\mathbf{U}_N$

$$\begin{bmatrix} \mathbf{U}_D \\ \mathbf{U}_N \end{bmatrix} = \begin{bmatrix} (\mathbf{V}_{D1} - \mathbf{V}_D)/\mathbf{W}_D & (\mathbf{V}_{D2} - \mathbf{V}_D)/\mathbf{W}_N \\ (\mathbf{V}_{N1} - \mathbf{V}_N)/\mathbf{W}_D & (\mathbf{V}_{N2} - \mathbf{V}_N)/\mathbf{W}_N \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{V}_D \\ \mathbf{V}_N \end{bmatrix} \quad (23)$$

Finally the required balancing masses  $\mathbf{B}_D$  and  $\mathbf{B}_N$  are simply

$$\mathbf{B}_D = -\mathbf{U}_D \quad (24)$$

$$\mathbf{B}_N = -\mathbf{U}_N \quad (25)$$

The two-plane balancing procedure requires four start/stop cycles of the rotor. First the original vibrations are measured, then the trial weight is attached to the first plane and the vibrations are measured again. Then the trial weight is attached to the second plane and the vibrations are measured again. After that the calculated balancing masses are added and the final unbalances are measured.

## Simulations and results

A simulation model of an AMB system was used to simulate and test the introduced one and two-plane balancing approaches. Drive-end x-axis (DX) position and non drive-end x-axis (NX) position are used as the measurement planes. Balancing planes and initial unbalances are simulated with added disturbance forces to the rotor. The rotor was balanced in 10 000 rpm speed. First critical speed of the rotor is 16 200 rpm, around 62% of the rotating speed, so the rotor is considered as rigid. Initial unbalance mass for the DX was 10 g with radius of 0.01 m and 46° angle and force (1) of 110 N. For the NX, respectively, mass of 8 g, radius of 0.01 m, 327° angle and force of 88 N. For the position, the initial unbalance is on drive-end x-axis 11.82 μm on 175° angle and on the non drive-end x-axis 10.18 μm on 20.60° angle respectively. In the balancing planes the radius for the trial and correction weights is constant 0.01 m.

### 2-plane balancing with 1-plane balancing method

When balancing 2-planes with 1-plane balancing method, first one plane is balanced and the correction weights are left in this plane when balancing the other plane. Based on the initial unbalance measurements, first the D-plane is balanced and then the N-Plane.

The D-plane balancing is done with adding a trial weight of 10 g in 100° angle on the D-plane. This

results in  $22.46 \mu\text{m}$  unbalance on  $183^\circ$  angle. Based on this a correction weight is calculated of  $10.9 \text{ g}$  and angle of  $263^\circ$  degrees. After adding this correction weight an unbalance of  $0.284 \mu\text{m}$  on  $52.2^\circ$  angle is observed on the D-plane. On the N-plane this results an unbalance of  $4.685 \mu\text{m}$  on  $60.3^\circ$  angle.

The initial unbalance on the N-plane is the previously mentioned  $4.685 \mu\text{m}$  on  $60.3^\circ$  angle. Balancing the N-plane is done similarly than the D-plane. A trial weight of  $10 \text{ g}$  is added on  $120^\circ$  angle on the N-plane. This results an unbalance of  $7.54 \mu\text{m}$  on  $197^\circ$  angle. A correction weight of  $4.11 \text{ g}$  on  $147^\circ$  angle is added on the N-plane. This results a final unbalance of  $0.0079 \mu\text{m}$  on the N-plane and on the D-plane  $3.906 \mu\text{m}$  respectively. The cross effect is clearly noticed as the unbalance on D-plane has increased from  $0.284 \mu\text{m}$  to  $3.906 \mu\text{m}$ . The initial unbalances, correction weight locations and final unbalances are shown in Table I.

Table I: Balancing results with 2-plane balancing using 1-plane balancing method

Condition	D-plane		N-plane	
	Value	Angle [ $^\circ$ ]	Value	Angle [ $^\circ$ ]
Initial ub [ $\mu\text{m}$ ]	11.82	175	10.18	20.6
Final ub [ $\mu\text{m}$ ]	3.906	N/A	0.0079	N/A
Balancing weight [g]	10.9	263	4.11	147

## 2-plane balancing

Using the 2-plane balancing method both planes are balanced simultaneously and the effect of the trial weight added on one plane is measured and taken into account on both planes. Adding a trial weight of  $10 \text{ g}$  in  $100^\circ$  angle on the D-plane results in the D-plane an unbalance of  $22.46 \mu\text{m}$  on  $183^\circ$  angle and on the N-plane an unbalance of  $16.76 \mu\text{m}$  on  $17.9^\circ$  angle respectively. Adding a trial weight of  $10 \text{ g}$  in  $120^\circ$  angle on the N-plane results in the D-plane an unbalance of  $7.359 \mu\text{m}$  on  $127^\circ$  angle and on the N-plane an unbalance of  $2.686 \mu\text{m}$  on  $271^\circ$  angle respectively. Based on this information correction weight of  $10.1 \text{ g}$  on  $229^\circ$  angle is added on the D-plane and a correction weight of  $7.64 \text{ g}$  on  $147^\circ$  angle is added on the N-plane. This results a final unbalance of  $0.391 \mu\text{m}$  on the D-plane and  $0.146 \mu\text{m}$  on the N-plane. The initial unbalances, correction weight locations and final unbalances are shown in Table II.

Table II: Balancing results with 2-plane balancing method

Condition	D-plane		N-plane	
	Value	Angle [ $^\circ$ ]	Value	Angle [ $^\circ$ ]
Initial ub [ $\mu\text{m}$ ]	11.82	175	10.18	20.6
Final ub [ $\mu\text{m}$ ]	0.391	N/A	0.146	N/A
Balancing weight [g]	10.1	229	7.64	147

## Comparison of the 2-plane and 1-plane balancing methods on 2-plane balancing

From the results presented in Table I and Table II it can be noted that overall the 2-plane balancing methods performs better than the 1-plane balancing method. But on the N-plane the 1-plane balancing method performs slightly better. Both methods are able to reduce the initial unbalance on both the D- and N-plane. This can also be seen from recorded rotor xy-positions when rotor is rotating at the balancing speed of  $10\,000 \text{ rpm}$ . The positions of the D- and N-end are shown on Fig. 2.

## Conclusion

To conclude both the 1-plane balancing method used to balance 2-planes and the 2-plane balancing were found to be suitable for the rotor balancing in AMB systems. However, 2-plane balancing performed better overall. Also the 2-plane balancing has the advantage that it requires one start/stop cycle of the rotor less than the 1-plane balancing. Authors would propose of using 2-plane balancing whenever possible. Because of the unstable nature of the magnetic levitation, the active magnetic bearings require

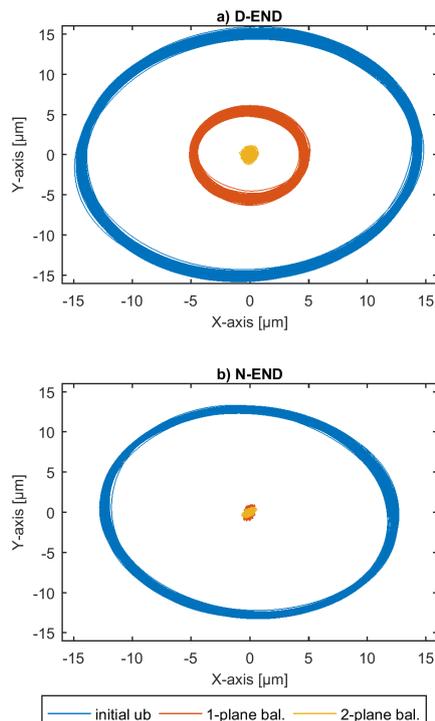


Fig. 2: Rotor D(a)- and N(b)-end xy-positions before balancing and after 1- and 2-plane balancing.

continuous control and data from the rotor position. Based on this the balancing procedure can be carried out without additional hardware or without the need of a balancing machine. For the field balancing of existing systems this is a clear advantage.

## References

- [1] R. L. Fox, "Dynamic balancing," Proc. of the 9th Turbomach. Symp., Texas, USA, pp. 151-183, 1980.
- [2] M. MacCamhaoil, "Static and dynamic balancing of rigid rotors," Bruel and Kjaer application notes, BO 0276-12.
- [3] C. C. Ozoegwu, C. C. Nwangwu, C. F. Uzoh, and A. V. Ogunoh, "zPure analytical approach to rotational balancing," J. of Saf. Eng., Vol. 1, Issue 4, pp.50-56, 2012.
- [4] J. Lyons, Primer on dynamic balancing, "Causes, corrections and consequences," presented at MainTech South 1998, pp.1-11, Dec. 1998.
- [5] Y. A. Khulief, M. A. Mohiuddin, and N. El-Gebeily, "A new method for field-balancing of high-speed flexible rotors without trial weights," J. of Rot. Mach., Vol. 2014, pp.1-11, 2014.
- [6] C. Liu, and G. Liu, "Field dynamic balancing for rigid rotor-amb system in a magnetically suspended fly-wheel," IEEE/ASME Trans. Of Mech., Vol. 21, No. 2, pp.1140-1150, Apr. 2016.
- [7] J. Kejian, Z. Changsheng, and C. Liangliang, "Unbalance compensation by recursive seeking unbalance mass position in active magnetic bearing-rotor system," IEE Trans. Of Ind. Elec., Vol. 62, No. 9, pp. 5655-5664, Sep. 2015.
- [8] K.-Y. Lum, V. T. Coppola, and D. S. Bernstein, "Adaptive virtual autobalancing & adaptive autocentering for on-line balancing of magnetic rotors," In Proc. of 14th Int. Modal Ana. Conf., pp. 1396-1402.