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Master's Degree in Strategic Finance and Business Analytics

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**Artificial Intelligence in investing: Stock clustering with  
Self-organizing map and return prediction with model  
comparison**

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## **ABSTRACT**

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This study presents an analysis of artificial intelligence (AI) methods in investment and further comparing them to classical methods. Bearing in mind the limited coverage by academic literature using these methods in one study to form an investment strategy and especially in the Finnish market, this study aims to analyse the process of using these methods to form an investment strategy for an individual investor.

The methodology available in research representing artificial intelligence is comprehensive. For the purpose of this study two artificial intelligence methods and two classical methods were utilized by using Matlab® and Microsoft Excel®. To begin with a Self-organizing map, representing AI, was utilized to form portfolios. The Self-organizing map showed that portfolios can be clustered based on their financial characteristics to answer investors' different needs. The second step was further optimizing the portfolio weights with a minimum variance portfolio. Furthermore, this step proved to be valuable, as it provided higher returns than an equally weighted portfolio. The third step in the study was utilizing ARMA models to forecast the returns of the portfolios and index. The results for all four portfolios and index showed to be white noise time-series, which cannot be predicted. For the purpose of this study and to show how analysis would be if the time series data was not white noise, the study was continued with the models. The fourth step was conducting a similar forecast for NAR models, representing AI. The results proved to be more accurate than for the white noise time-series based models. However, neither NAR nor ARMA models proved to be that accurate compared to the real returns, but in whole the NAR models were more accurate. This result was not surprising as the comparison models were random. As this study is quite specific, so is the contributions it provides. The study contributes to the available academic literature by providing insight to investment options, confirming that white noise cannot be forecasted and highlighting that AI methods provide better forecast results than random time-series models.

## TIIVISTELMÄ

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Tämä tutkimus esittää analyysin tekoälyn menetelmistä sijoittamisessa ja niiden vertaamisesta klassisiin menetelmiin. Ottaen huomioon akateemisen kirjallisuuden rajallisen kattavuuden näiden menetelmien käytöstä yhdessä tutkimuksessa sijoitusstrategian muodostamiseksi ja erityisesti Suomen markkinoilla, tutkimus pyrkii analysoimaan prosessia, jolla näitä menetelmiä käytetään sijoitusstrategian muodostamisessa yksittäiselle sijoittajalle.

Käytettävissä olevat tekoälyä vastaavat menetelmät ovat kattavat. Tämän tutkimuksen tarkoitukseen käytettiin kahta tekoälyn menetelmää ja kahta klassista menetelmää käyttämällä Matlab® ja Microsoft Excel®. Aluksi itseorganisoituvaa karttaa, joka edustaa tekoälyä, käytettiin osakesalkkujen muodostamiseen. Itseorganisoituva kartta osoitti, että salkut voidaan ryhmitellä niiden taloudellisten ominaisuuksien perusteella vastatakseen sijoittajien erilaisiin tarpeisiin. Toinen vaihe oli salkun painojen optimointi vähimmäisvarianssisalkun avulla. Tämä vaihe osoittautui hyödylliseksi, koska se tuotti korkeamman tuoton kuin tasan painotettu salkku. Tutkimuksen kolmas vaihe oli ARMA-mallien hyödyntäminen salkkujen ja indeksin tuottojen ennustamiseksi. Kaikkien neljän salkun ja indeksin tulokset osoittivat olevan valkoisen kohinan aikasarjoja, joita ei voida ennustaa. Tätä tutkimusta ja analyysia siitä, jos aikasarjojen data ei olisi valkoista kohinaa varten, tutkimusta jatkettiin malleilla. Neljäs vaihe oli samanlaisen ennusteen suorittaminen tekoälyä edustaville NAR-malleille. Tulokset osoittautuivat tarkemmiksi kuin satunnaiset, valkoisen kohinan aikasarjan mallit. Kumpikaan NAR- tai ARMA-malleista ei kuitenkaan osoittautunut olevan niin tarkka todellisiin tuottoihin verrattuna, mutta kaiken kaikkiaan NAR mallit olivat tarkempia. Tämä tulos ei ollut yllättävä, koska vertailumallit olivat satunnaisia. Koska tämä tutkimus on melko tarkka, samoin ovat sen antamat kontribuutiot. Tutkimus myötävaikuttaa saatavissa olevaan akateemiseen kirjallisuuteen tarjoamalla oivalluksen sijoitusvaihtoehdoista, vahvistamalla, että valkoista kohinaa ei voida ennustaa, ja korostamalla, että tekoälymenetelmät tarjoavat parempia ennustetuloksia kuin satunnaiset aikasarjamallit.

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*“Surround yourself with the dreamers and the doers, the believers and thinkers, but most of all with those who see greatness within you, even when you don’t see it yourself.” -Edmund Lee*

With the above quote I would foremost like to thank LUT for giving me the opportunity and belief to study new and exciting subjects. Several professors went above and beyond to help me reach my full potential and for that I will always be grateful. The knowledge gained during my studies does not culminate in this thesis but will be highly utilized further in my career.

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In Vantaa, 28.7.2019

Ayan Mohamed

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## **ABBREVIATIONS**

AI	Artificial Intelligence
EMH	Efficient Market Hypothesis
SOM	Self-Organizing Map
AR	Auto-Regressive
MA	Moving Average
ARMA	Auto-regressive Moving Average
ANN	Artificial Neural Network
NAR	Nonlinear Autoregressive Network
NARX	Nonlinear Autoregressive Networks with Exogenous Input
MSE	Mean Squared Error
MPE	Mean Percentage Error
MVP	Minimum Variance Portfolio



# 1 INTRODUCTION

“Those who do not remember the past are condemned to repeat it” (Graham, 2006). This statement has been repeated by many in different fields, but it especially carries weight in investment. It is especially applicable when the goal is to forecast market movement, stocks and their returns, index movements, bonds values and when conducting portfolio optimisation. It has been a continuous goal to try and understand what’s to come by developing different methods. The goal is not to definitively state the future, but to understand to which direction the future is going. This is quite useful when trying to, for instance, decide on investment.

Some of the most popularly used classical forecasting methods include ARMA, AR and MA models. These models being based on mathematics and statistics have for decades provided useful applications to time series forecasting and are continuously used still. Nevertheless, they have been proved to be not as accurate as other types of models. In addition to accuracy, development of Big Data and development in computing have opened the need and opportunity for more advanced methods. This has led to Artificial intelligence (AI) rising as an alternative method used in investment. Research conducted by Deloitte (2018) highlighted that these AI tools used in analyst forecasting and decision-making provide workers swiftness, large-scale data processing and time management in their operations. J.P.Morgan (2017) agrees on this research and also emphasizes the potential to use AI in investment decisions and strategies. The potential received from using AI is not only limited to forecasting as the need in investment decisions is multifaceted. Classifying or clustering data has been a useful method on comprehending possibly complex data and even simpler ones. Moreover, it provides a way to understand customers, market movements and differences in investment targets.

Whichever form the use of AI in investment may take, it is apparent that there is a need, tools and methods to perform it with practical results. As AI is enhancing the field of investment with continuously improving methods, this study is dedicated to analyzing some of these methods and comparing them to classical methods when trying to form a solid investment plan.

## 1.1 Purpose of study

The use of Artificial intelligence in investment has garnered a lot of attention and as it is a wide field with vast possibilities, this study will only capture a fraction of it. It is a hot topic not only for researchers but for the everyday person also. The goal to optimize processes and simplify practises is in everyone's mind but what applications and room for progress there are available is not that widely known. Hence exploring that in an understandable way is important to this study and the author. The field of investment is growing, and many applications are available, but mostly for professionals or for individuals through payment. Accordingly, the main aim would be to bring practical solutions in forming an investment strategy to everyone.

Furthermore, the goal of this study is to provide means and purpose in using artificial intelligence-based tools when making investment decisions and conducting an investment plan. As the need for knowledge and easier access grows, so should the available methodologies. Hence this study aims to showcase that AI does provide an alternative method when trying to classify, cluster or predict returns. In addition to this the goal of the study is to research the differences of forecasting models when applied to stock returns. Moreover, researching the possibility of artificial neural network models providing a more accurate model than mathematical, statistical forecasting models.

## 1.2 Research focus and questions

The research focus for this study is constructed by defining four steps. These steps are presented in Figure 1. These steps helped this study to find a research gap that can be utilized in future research. The first step is to look at the research area. The research area consists of four areas; Stock Market, Finland, Forecasting returns and Clustering of stocks. These four areas constitute the base for this study. Especially the Finnish market is important, as a similar study has not been conducted with the Finnish stocks. The second step was finding the research objective based on the chosen areas. The objective of this study is to forecast and optimize portfolios with

AI and classical methods. Several methods were explored, and the most suitable for this study were chosen to continue the research with. The third step was to find the perspective this study will take. As the main interest is forecasting stock returns, whom other than investors would this information mostly benefit. Corporations could for example use the forecasting models for other data, but this study will focus on the investors and the possibilities these methods provide them. In conclusion, with these steps, the research focus is determined. The focus is on comparing AI and classical forecasting methods for Finnish stock returns.

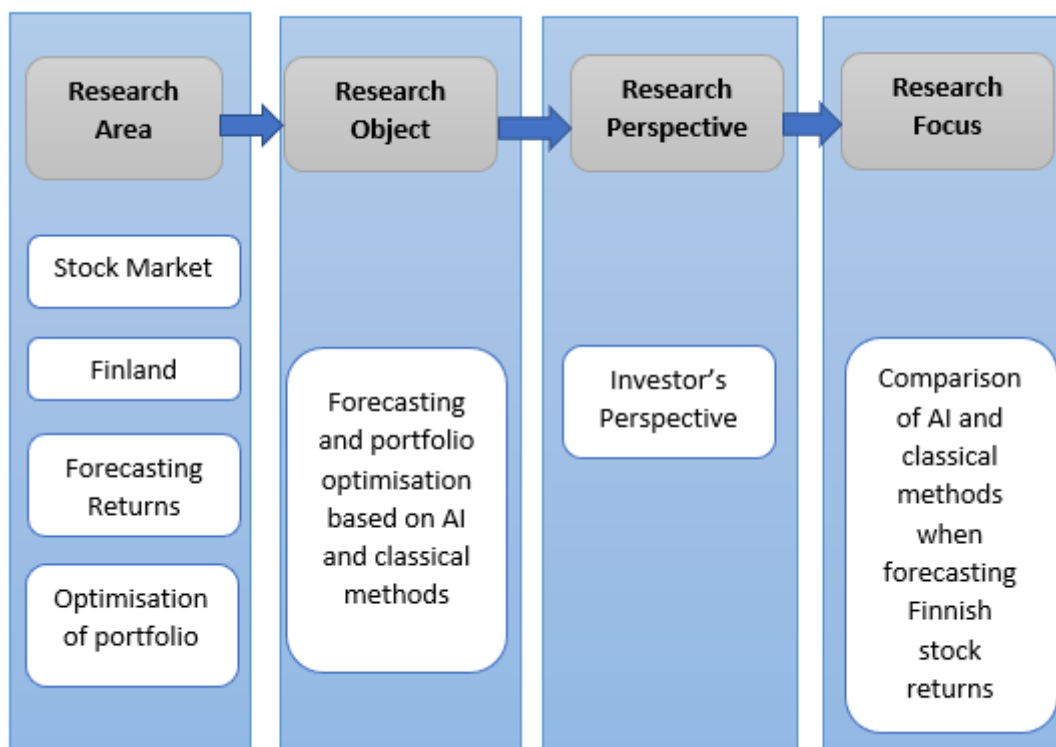


Figure 1. Research Focus

As this study has various consecutive steps, the main research question has been divided into seven sub questions. These sub questions provide a cohesive and gradual path to gaining a solution for the main question. Figure 2 has visualized the structure of the study questions and their relationships. The main question is divided into four main parts that form 7 sub questions. The first part includes the clustering and optimisation of stocks, the second part the classical forecast and comparison,

the third part the ANN forecast and comparison and finally the fourth part of comparing the classical and ANN forecast models.

Firstly, the main research question is ***Can artificial neural networks be used to form an investment strategy in the Finnish stock market and would the ANN stock return forecast results be more accurate than mathematical statistical models ARMA/MA/AR.*** This epitomises the heart of this study and aims to find out through empirical research if forming an investment strategy is a possibility by using several models acting as proxies to artificial intelligence in this study.

Then more specifically the sub-questions. The initial step in answering the main research question and forming an investment plan is to first form portfolios.

1. *What type of portfolios can be formed with SOM clustering technique by using 9 financial characteristics of target stocks?*

After initially choosing the most suitable method for this study to cluster the stocks, Self-organizing map will be used to cluster 25 stocks included in the OMXH25 index. The clustering is initialized with chosen financial characteristics.

2. *How can the optimization tool be used to minimize the risk in each portfolio?*

When the Self-organizing map has finalized the clustering, the study will continue by optimizing the formed portfolios by a built optimisation tool in Microsoft excel ®. This optimisation will be based on obtaining the lowest possible risk for the expected return.

3. *Compared to the real returns, which ARMA forecast for portfolio/index gives the closest forecast value? Which ARMA model has the smallest MSE?*

After the optimisation of portfolios is finalized, the next step is to forecast the returns of formed portfolios and index with classical methods. The results will be compared based on return prediction.

4. *What differences can be detected between the portfolios ARMA forecasted returns?*

Following the return comparison, the portfolio differences will be also compared based on their characteristics.

5. *Compared to the real returns, which ANN forecast for portfolio/index gives the closest forecast value? Which ANN model has the smallest MSE?*

A neural network prediction will also be performed for the formed portfolios and index. The forecasted returns are to be compared also.

6. *What differences can be detected between the portfolios ANN forecasted returns?*

In addition to the return prediction, the differences of each predicted portfolio return will be compared.

7. *Which model provides the most accurate forecast for each portfolio/index? What common factors/financial characteristics do they have?*

The last step of the study will compare the ANN models and the classical models with each other and determine the value of the performed forecasts.

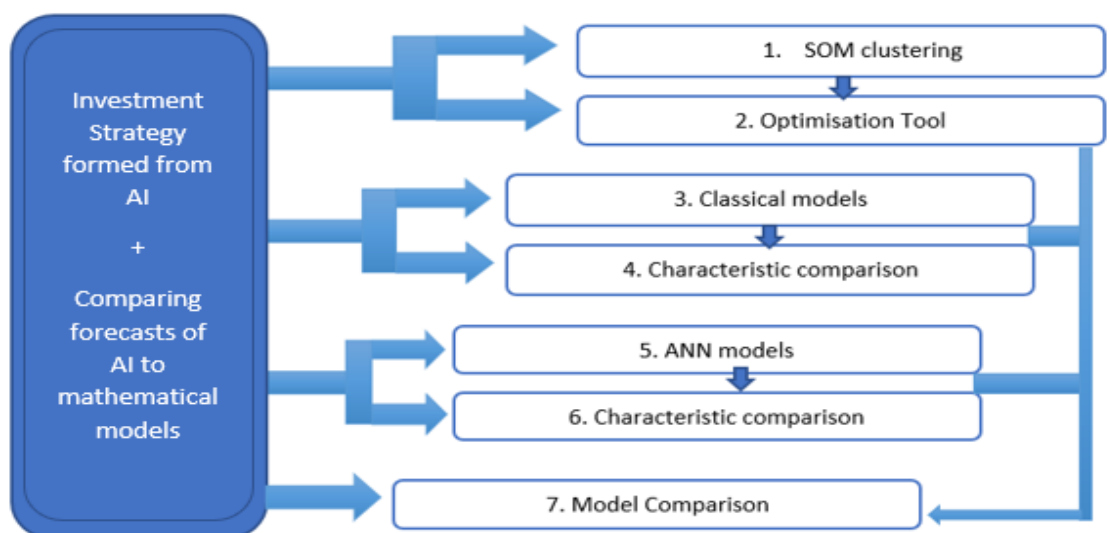


Figure 2. Connections between sub-questions and main question.

### 1.3 Methodology structure

As the research questions have been defined, in order to find a solution for them different tools and methods are used. Table 1 lists each method and tool used for each sub question. The main tools implemented in this study are Matlab ® and Microsoft excel ®.

Table 1. Methodology tools

<b>Sub question:</b>	<b>Method:</b>	<b>Tool:</b>
<b>1</b>	Self-organizing map	Matlab ®
<b>2</b>	Optimisation Tool	Microsoft Excel ®
<b>3</b>	ARMA/MA/AR Forecasting	Econometric Toolbox. Matlab ®
<b>4</b>	MSE error term	Matlab ®
<b>5</b>	Neural network (NAR) Forecasting	Neural Network Toolbox. Matlab ®
<b>6</b>	MSE error term	Matlab ®
<b>7</b>	MPE error term and forecast comparison	Microsoft Excel ®

### 1.4 Contribution of study

Various research has been conducted regarding statistical forecasting, neural network forecasting and Self organizing map clustering. They have been used to forecast different types of time series i.e. returns and GDP. There are several papers on comparing the statistical forecasting methods and neural network models, which have provided a great base for this study. Further self-organizing maps also interest a lot of researchers and it has been implemented before as a portfolio forming method. However, none of these studies have utilized them by using these methodologies together as an investment strategy in one study. Especially in the Finnish market, which is a unique place for a study to be conducted as completely similar study has not been explored. Therefore, this study fills a research gap and does provide an interesting point of view.

## 1.5 Study Structure

The structure of this study is divided into three main sections. The first section includes the introduction and the background research the study is based on. The second part begins the empirical research with conducting a two-part analysis to form optimal portfolios. The third part of the study consists of forecasting and analysis of the formed models. Finally, this part concludes with conclusion and discussion. The three main sections have been divided into seven chapters, that are introduced in Figure 3 and explored next.

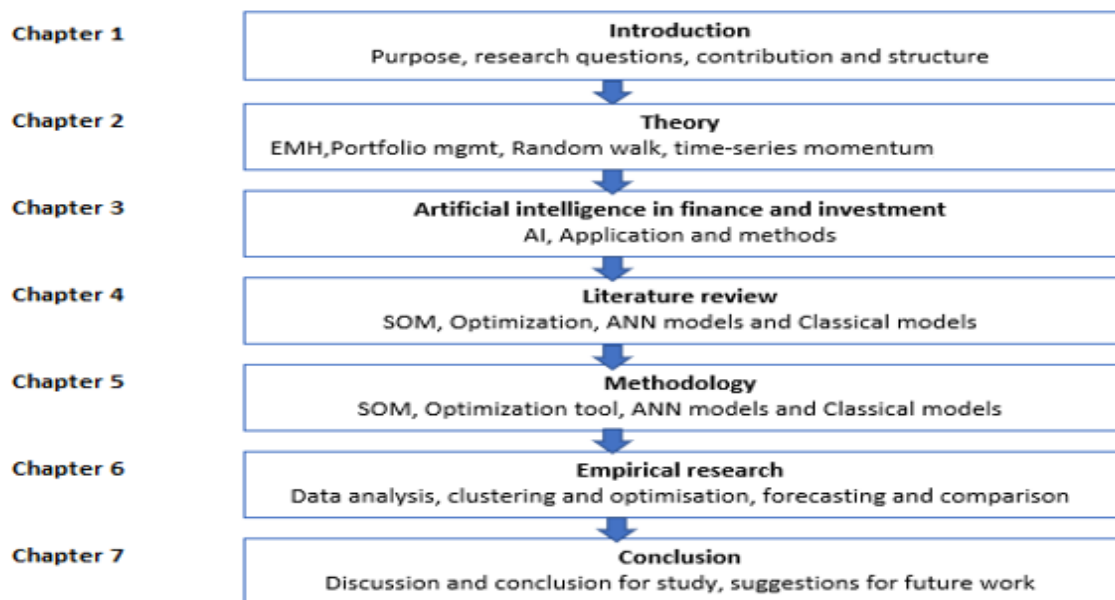


Figure 3. Chapter structure of study

The first chapter is the introduction of this study. It covers the aim and motivation for conducting this study. Then it moves into introducing the main research question and the corresponding sub questions. This chapter concludes with the contribution it gives in this field and the structure of the whole study.

The second chapter presents the theory base for this study. It includes finance theories that provide the base for this study and comprehension for the analysis processed.

The third chapter will also comprise of theory, but it will focus on Artificial intelligence. Due to AI being a broad concept, the focus will be on the meaning it

has in this study. It will also explore some of the applications AI has in investment and their applicable methods.

The fourth chapter moves on from the theory and outlines all the relevant research done regarding using AI methods in clustering and using them and classical models for forecasting.

The fifth chapter continues from the theory and introduces the methodology used. As this study includes several main methodologies, with this chapter the comprehension for them is fulfilled.

The sixth chapter is the main part of the study as it is the Empirical research part. This chapter introduces the analysis performed for the chosen data set. The study will start with forming stock portfolios. Instead of hand picking the stocks, self-organizing map is used to classify the stocks and then form portfolios based on the similarities of the stocks. In addition to this, a portfolio optimisation tool is used to optimize the portfolios based on the risk level.

The next step of the study is to predict returns based on historic returns of year 2018. In addition to the neural network-based portfolios, the Helsinki stock market index OMXH25 is used for the prediction. The first move in this is to predict the returns with the mathematical prediction model ARMA for 7 days and 30 days. This part will form a suitable model for each portfolio and index. After that the same portfolios and index will have their returns predicted with an artificial neural network for the same time periods. Finally, the formed prediction models will be compared based on accuracy and returns.

The seventh chapter is the conclusion of this study. This chapter summarizes all that have been analysed, the results and the different models. It will also discuss possible implications for such study, possible future venues and changes that would be done in this study if circumstances were different.



## 2 FINANCIAL THEORIES

This chapter will be introducing theories and models that this study is based on, and that are used as a benchmark to explain the results. As there are many financial concepts and models close to this study, it was important to narrow them down to those most closely connected to the area of this study. As visualized in Figure 4. The theory review consists of 4 financial theories. The chapter will start with defining the Efficient Market Hypothesis, as it is one of the essential theories concerning stocks. The next part will explain the Modern Portfolio theory, as the EMH, integral for stocks and more specifically forming portfolios. The third part entails the Random Walk hypothesis, a theory close to the Efficient Market Hypothesis. The fourth part introduces the Time-series momentum Theory that gives a contradictory perspective to the Random Walk hypothesis. It is important to highlight both theories, as forecasting is a core part of this thesis.

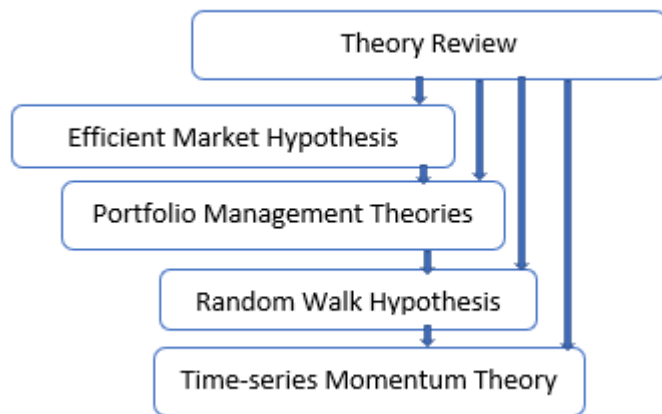


Figure 4. Structure of chapter 1.

### 2.1 Efficient Market Hypothesis

The Efficient market hypothesis was initiated in the 1960s from the work of Eugene Fama, an economist. The main hypothesis is that the market cannot be beat since prices in the market have considered all information that may have an impact on any stock. In practise this would mean that buying or selling a security would not

need skill but would rather be based on chance. According to this hypothesis, for the market to be efficient it will always reflect the most precise price for every security. This would enable anyone to buy securities at a reduced price. (Corporate finance institute - Understanding and Testing EMH, 2019)

Figure 5. has demonstrated all the variations of Efficient market hypothesis. There are altogether three variations: Weak form, Semi-strong form and Strong form. **The weak form**, presented in the middle of the figure, is limited compared to the other forms. It only includes information regarding historical prices. According to Fama's (1970, p.388) research wide tests were performed and most of them supported the hypothesis. However, it is important to note that this level only takes historical prices into consideration. **The Semi-strong form**, presented as the middle ring, takes into consideration in addition to the historical prices also all public information. As testing continued to this level of available information, the highest concern to rise was the swiftness of price change. Meaning that how fast would the stock price react to for example an announcement of a stock split. The last and final variation is **The Strong form**. This variation contains information mentioned in the two previous forms and including all private information. The concern for this level of a fully reflective market was if any individual or a group would have access before anyone else to certain information. These days this type of monopolistic information and profiting from that is highly regulated. (Fama, 1970.)

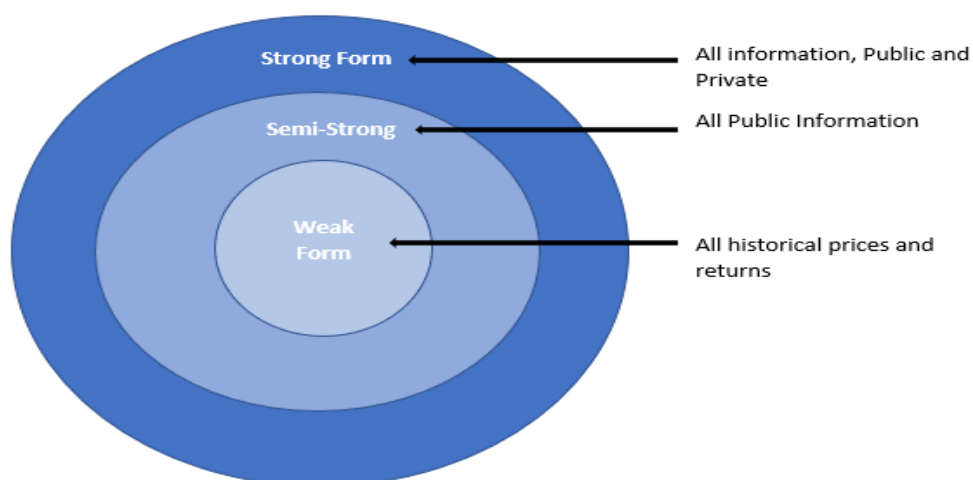


Figure 5. Efficient market hypothesis variations (Fama, 1970)

Since Fama gave the initial hypothesis of an efficient market, Fama (1991) has updated it such that it will take into consideration transaction costs and the incentive following their absence. Also, Grossmann and Stiglitz (1980) have stated that for sophisticated investors, information is reflected by the prices only partially. Henceforth paying for information gains compensation. They also continue to state that if prices would fully reflect all information, no one would be financially interested on gaining information. In 1998, Fama further added to the theory that “taking chance” is the reasoning for overreaction and underreactions in different conditions. (Fama, 1998)

Lekovic have researched in 2018 all available research and tried to summarize information of five decades. Lekovic also concluded that even after this period of time, there is no consensus on the validity of this hypothesis. There has been a lot of financial research regarding the efficient market hypothesis, but it is quite clear that there isn't one clear consensus for or against it in the literature. However, it is a highly important financial theory that should be considered in any financial paper, such as this. (Lekovic, 2018)

## 2.2 Portfolio Management Theories

As this study explores forming a few portfolios, different portfolio theories will be presented next. In this study the theory that will be looked at further and implemented in the research is the Modern Portfolio theory. However, it is important to note that the other theories are accessible but will not be used for the purpose of this thesis.

A main modern approach in the portfolio theories is called the Markowitz **Modern Portfolio Theory**. This theory was introduced by Harry Markowitz in 1952 in his article about Portfolio selection. Markowitz introduced the basics of the diversification of portfolios in conjunction with how an investor may reduce standard deviation of the returns of the portfolio by picking stocks that move differently. According to Markowitz (1952), there are two stages in selecting a portfolio. The first stage consists of observing and experiencing followed by having beliefs about future

performance of securities available. The second stage starts where the first one ended, with beliefs of future performances and the finishes in choosing the portfolio. The modern portfolio theory focuses on the second stage, where the portfolio and weights of the securities in the portfolio are chosen.

This theory in summation is a way for risk-averse investors to compile a portfolio to maximize or optimize expected return of the portfolio based on the level of given risk. This draws to the attention that in order to achieve higher reward, the risk level is indeed significant. Furthermore, the portfolio desired by the risk-averse investor can be constructed by either choosing the desired risk level and maximizing the return for that or choosing the desired return and minimizing the risk. This type of portfolio is also called a **Mean variance portfolio**. (Markowitz, H. 1952)

The modern portfolio theory reasons that instead of looking at an individual investment and its risk and return, what matters is its effect on the risk and return of the portfolio. In addition, as this theory assumes a risk averse investor, it is implied that an investor will only assume a higher risk level, if the return expected is also higher. So, the level of risk and return would be explored for the portfolio. (Markowitz, H. 1952)

In addition to the mean-variance portfolio described above, the modern portfolio theory also enables to form a **minimum variance portfolio (MVP)**. A minimum variance optimisation portfolio works by assigning weights independent from expected returns. Henceforth, the portfolios are formed by using the estimated stock covariance matrix by excluding forecasted returns. (Clarke et al, 2006) As only the measures of risk is used for the construction of minimum variance portfolios, this is an optimal optimisation method when forming portfolios as future returns in the stock market are always hard to estimate.

Several researchers have concluded that when comparing the market portfolio and mean variance portfolio to the MVP, that the MVP performs the best. Bednarek & Patel (2018) conjectured that on a risk-adjusted basis the MVP appeared to perform better than a mean variance optimized portfolio. Haugen & Baker (1991) on the other hand compared the performance of the MVP to the market portfolio and concluded in the same result for the benefit of the MVP. The reason for the

outperformance can be explained by the fact that the MVP tends to detect risk-based anomalies. Furthermore, “the MVP overweighs low beta assets and under weighs assets with high idiosyncratic risk”. (Scherer, 2011)

### 2.3 Random Walk Hypothesis Vs. Time-series Momentum Theory

Time series Momentum was published by Moskowitz et al (2012) as an asset pricing anomaly. According to their research they found strong evidence that securities past returns can be used to form predictions. This anomaly particularly was strongest in a short-term prediction, more specifically for predictions under one year ahead. After the first year the accuracy of the predictions went down and ultimately the momentum effects reversed. However, sound this theory is, there is a contradicting theory called the Random Walk Hypothesis. This theory states that past movement of a security does not indicate future movement. For example, if a price of a security went down in the past or went up, this information cannot be used to inform if it will rise or fall again in the future. Both theories are crucial in time-series issues and depending on the data and market behaviour, can both appear in practice. (Moskowitz, 2012)

### 3 ARTIFICIAL INTELLIGENCE IN FINANCE AND INVESTMENT

As the main finance theories related to the research area have been explained, the next chapter will be exploring the other theory base related to the study. This would be Artificial intelligence. As artificial intelligence, henceforth mentioned as AI, is a concept that is used everywhere from movies to company boards, it is important to understand the concept of it that will be explored in this study. As this study is interested on how to implement AI in the field of finance and more specifically investing, we will research the possible models that could represent AI in this context.

As presented in Figure 6, this chapter is structured by first explaining AI. After the meaning is clear and specifically the meaning in this study, this chapter will look at how AI is used in investment. Then it will move to defining the application of AI in this thesis. Furthermore, why some AI models were used instead of others in clustering and in forecasting.

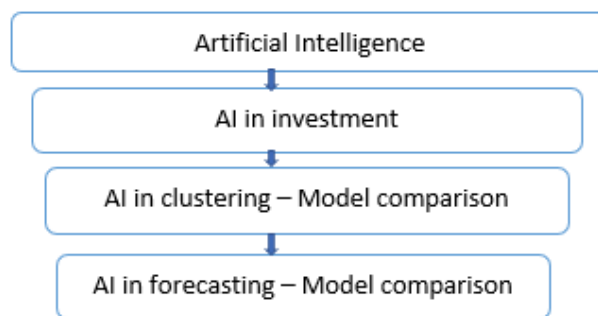


Figure 6. Structure of chapter 3

#### 3.1 Artificial intelligence

As stated previously AI is a concept that is widely interpreted and presented in vast amount of different ways. For this reason, it is important that the different definitions are shortly presented and for the reader of this study to have the same concept in

mind as the author of this study. Hence the study will be more comprehensible to whomever the audience may be.

Russell and Norvig (2009) have organized definitions of AI into four categories: Thinking Humanly, Thinking Rationally, Acting Humanly and Acting Rationally. This has been presented in Table 2.

Table 2. Definitions of AI organized into four categories (Russell and Norvig, 2009)

<p><b>1.Thinking Humanly</b></p> <ul style="list-style-type: none"> <li>• Haugeland, 1985</li> <li>• Bellman, 1978</li> </ul>	<p><b>2.Thinking Rationally</b></p> <ul style="list-style-type: none"> <li>• Charniak and Mcdermott, 1985</li> <li>• Winston, 1992</li> </ul>
<p><b>3. Acting Humanly</b></p> <ul style="list-style-type: none"> <li>• Kurzweil, 1990</li> <li>• Rich and Knight, 1991</li> </ul>	<p><b>4. Acting Rationally</b></p> <ul style="list-style-type: none"> <li>• Poole et al., 1998</li> <li>• Nilsson, 1998</li> </ul>

The first category: Thinking Humanly, represents the thought of humans and how that process works and develops. This category is also known as the cognitive modelling approach. After researchers had this process fully observed, it was possible to form a theory base that could be expressed as a program run by a computer. (Russell et al, 2009) One of the earliest definitions for AI in this category was presented by Bellman (1978). According to this, AI is thinking of humans that have been automated. For example, how humans think when making decisions or solving different problems. Another definition in the Thinking Humanly category is one presented by Haugeland (1985). The idea and concept of AI was presented as “Machines with minds”, which corresponds with the idea Bellman had of automating the thinking process.

The second category: Thinking Rationally, adds to the Thinking humanly category by presenting logic. This signifies that problems can be solved by computers like humans would if the correct premise is available. Meaning that guidance for the necessary steps to take first to solve and rationalize the problem are present. (Russell et al 2009) Accordingly, Charniak and McDermott (1985) presented AI as “the study of mental faculties through the use of computational models”. This also

regards logic and how the full thought process would be implemented in computers. Close to this is the definition by Winston (1992) that explains AI as a way of computers to take in information, understand it and act accordingly. This also consists of the whole thought process a human would have if faced with a similar problem.

Moving to the third category that moves from the thought process to the actions taken: Acting Humanly. Also presented as the Turing Test approach, which states that a computer to be AI it must have the following capabilities: Natural language processing, knowledge representation, automated reasoning and machine learning. (Russell et al 2009) Machine learning is important in this thesis and will be presented in the following sub-chapter. When Kurzweil (1990) presented AI as the capability of machines performing with the capabilities of humans, it summarized the concept of them acting humanly. In addition to this a year later, Rich and Night (1991) said that AI aims to do things much better than humans. As this is a growing field with continuous improvement, this may become reality often.

The fourth category: Acting rationally, also presented as the rational agent approach, sets forth the notion of a computer program being a rational agent. This indicates that a computer must aim to achieve the best outcome or if the inputs had uncertainty then the best outcome expected. (Russell et al 2009) This is also a term used by Poole et al. (1998) to explain AI. In other words, "intelligent agents" is the basis of studying AI. Nilsson (1998) on the other hand did not use agents in the definition but summarized AI with being "concerned with intelligent behaviour in artefacts". Furthermore, all these definitions take into account rationality with different approaches but with the notion of it dictating the path to the required result.

All the definitions set forth above present a valid and thorough explanation of what AI means to that specific author and time. They have a lot of similarities but have taken different approaches to the term AI, which have presented a more thorough understanding of it.

In conclusion, out of these four categories, the Turing Test approach (Acting Humanly) would be the one closest to the AI definition used in this study. More specifically machine learning is implemented in different forms. In addition to the



Turing test approach, the author defines AI to be an operation similar to the human brain but processed by a machine. In other words, machines mimicking the human brain as best as possible. This includes recognizing patterns and making conclusions based on them. This culminates in this study by using artificial neural networks. However, it is important to take into consideration that as time progresses and processes develop, AI will change and most likely will have more definitions set forth.

### 3.2 Application in investment

In the past decade artificial intelligence in Finance, and more specifically in investing, has seen major advances and the research is ongoing. In the next several years AI applications in this field will most likely be a main component in the development of investing and act as a disruptive force in it. Research done by Deloitte (2019) determined that AI in investment management enables among other things automated insights, powering risk performance, growth opportunities, operations intelligence and relationship mapping. These are huge aspects for any investment firm but necessary in order to transform with the market. PwC (2018) also researched that AI is the next step in the field of investment. AI is used in executing trades, managing portfolios and in client service. In executing trades, machine learning is used in high frequency trading. Decision are made this way in split seconds, which would not be possible if a Human was making similar decisions. In portfolio management, the role of AI is to analyse markets systematically based on the information available. The AI based signals work as the foundation of the investment process, and they try to find above average returns. In Client service, one firm in PwC's study used AI to free employees to focus on client service. So, AI was not used to service clients, but to minimize the need of employees in more routine workflows.

### 3.3 Application in thesis

AI in investing is rapidly growing and will provide investors with a lot of options and freedom to focus on other things. However, in this thesis, the focus on AI will be

narrower. The first AI method will be used in clustering of the stocks. The second part in which AI is used is forecasting the returns with artificial neural networks. Furthermore, the Methodology chapter will explain the chosen models more thoroughly and the process of application.

### 3.3.1 Clustering methods and Forecasting Methods

The first part of the empirical research is clustering stocks. In artificial intelligence, and more specifically machine learning as a branch of AI, there is presented several ways to cluster data. When researching clustering methods two methods have proven to be implemented and researched a lot: K- means and Self-organizing Maps.

K-means clustering method, published 1955, is one of the oldest and simplest methods, which may explain its popularity. This method works as a partitional clustering algorithm that finds clusters from the data concurrently. (Jain, 2010) It is quite similar to Self-organizing maps as they also seek clusters without putting them in any order. They define the uniqueness of every cluster.

Mingoti et al (2006) did a comparison of clustering methods: SOM neural network, Fuzzy c-means, K-means and traditional hierarchical clustering algorithms. They concluded that Fuzzy K-means showed, in its simplicity, good performance. SOM also performed well depending on the data but needs more attention than the K-means method. On the other hand, Self-organizing maps have also been proven to be a good substitute to K-means method as they possess the same final stages in the training procedure. (Bacao et al, 2005)

In addition to these two methods, it is important to also note that Brentan et al, (2018) have further developed these models and formed a hybrid model using both methods to cluster data. They determined this model to be effective, however, hybrid models won't be implemented in this study.

A Self-organizing map was used instead of the K-means clustering method in this study because it is very effective, visually clear and as it is a direct substitute to K-means. It also provides clearer visual results that help the reader to comprehend

the performed analysis. Self-organizing map clustering method will be explored further in Methodology chapter.

When dealing with forecasting methods, this study solely focuses on forecasting financial data and more over time series data. A lot of models are available for this purpose and all provide a solid forecast. When researching time series forecasting in artificial neural networks, there were two neural networks used often for forecasting. The first one is the recurrent neural network (RNN), which works by using feedback connections. Henceforth allowing information to move laterally or backwards. The second one is called nonlinear autoregressive networks with exogenous input (NARX) and its other form, nonlinear autoregressive network (NAR). They also are a type of RNNs. As the RNNs more conventional model, the NARX and NAR models usually provides better results. (Wunsch et al, 2018) Due to this and the extensive amount of research on the NARX and NAR models, they are used in this study to act as a proxy for artificial intelligence for prediction. Especially the NAR model as it suits the data in hand. If for example the goal would be to forecast with more than one input, the NARX model would be the most suitable one. Hybrid models are also possible and used more often, but this study focuses only on non-hybrid models. The forecasting methods will be further explored in the methodology chapter.

## 4 LITERATURE REVIEW

This chapter consist of the literature review for this study. It is related to the questions and methodologies implemented and explores state-of-the-art academic literature. Figure 7 visualizes the content and structure of this section of the study. The literature review is divided into three sections to provide the most comprehensive background. The first section explores literature on clustering methods used for stocks. The second section includes literature of forecasting methods, the focus being on classical methods and available artificial neural network models. The third part focuses on research conducted on comparing the classical forecasting methods and artificial neural network models.

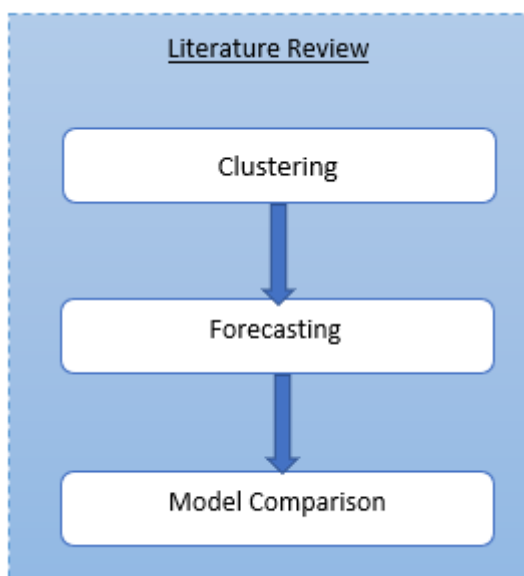


Figure 7. Structure of literature review

### 4.1 Clustering

Artificial intelligence and especially neural networks have been implemented in several ways in investing. One of the main aspects when investing, is knowing and identifying the possible subjects to invest in, and this is where clustering has been utilized. A good clustering tool is Kohonen Self-Organizing map. SOM has been proven to be a good and visual clustering tool by many researchers. Research conducted in the Asian stock market by Khan et al. (2010), Nanda et al. (2010) and

Widiputra et al. (2012), in India and Indonesia respectively, concluded that SOM can be used successfully on classifying stocks, whether it be forming a portfolio, checking for liquidity or even picking stocks for high returns. This is where this thesis will come in and further study forming a portfolio with SOM and more specifically the possibility of it in the Finnish stock market.

However, some limitations to SOM being used in classification, clustering and variable selection have been detected. Lasri (2016) distinguished that “in the normalization of the inputs space, the classifications lose their precision and the neurons cannot differentiate between the original inputs.” In his research this has been overpassed by preparing the inputs with a principal component analysis. So even though there are limitations, fortunately they can be overcome.

## 4.2 Forecasting

When SOM has been utilized to classify the stocks, the investment strategy is verified by forecasting the returns. Forecasting returns has been done for a long time and through time the technique available has evolved. One of the classical models is the ARMA (autoregressive moving average) model. This model has been used to forecast a vast number of variables in different fields, which indicate the usability and popularity of the model. Some examples of the versatility of SOM is presented by Datta (2011) that forecasted inflation, Siregar et al. (2017) in forecasting plastic factory production and also by Al-Shiab (2016) in forecasting the movements of the Amman stock exchange. In this study the model will be used to forecast stock returns.

In addition to the ARMA model, forecasting techniques have evolved and one of the newest ways is by using an artificial neural network model. Artificial neural networks have recently been implemented a great deal in forecasting. Selmi et al. (2015) used it to forecast stock market returns such as in this thesis. Furthermore, ANN's have also gained popularity in timeseries prediction. One ANN model that has been used, called NARX (Nonlinear autoregressive exogenous) model, is more in depth and will provide the data in this study a valuable model as the study by Hang et al.

(2009) shows. Some research has been done with this model, and especially in forecasting time series. For example, Wunsch et al, (2018) used the model to forecast groundwater levels half a year ahead and Ozoegwu (2019) used it to forecast daily solar radiation. As this study focuses on financial time series, it would be a great addition to the already available research base.

### 4.3 Forecasting Comparison

As this study focuses on comparing the ARMA and ANN models, a good research base is available. However, they have mostly focused on comparing the model performance based on error terms and not checking the accuracy from the real returns. For instance, Safi (2016) and Ayodele et al. (2014) have focused on timeseries prediction instead of using the forecasting methods in an investment strategy, as is the aim of this study.

Several researchers have conducted studies using hybrid models formed from ARIMA and ANN to forecast returns. One good example is the research conducted by Manish et al. (2012). In their research they compared a hybrid ARIMA-ANN model with the performance of ARIMA and ANN models in forecasting stock market index returns. The advantage of the hybrid model being the ability to combine the benefits of a linear and non-linear model. Thus, being able to also perform better than the models separately. This is an interesting, but not a surprising find and has potential for further research but in this thesis the focus will be the difference of the models. There were a lot of research that included classification models and return forecasting, but interestingly not both implemented in the same investment strategy research. This study will try to fill that gap.

## 5 METHODOLOGY

In order to answer the research questions for this study, methodology is needed. The construct of the Methodology part of the study is presented in Figure 8. The methodology will start with explaining Self-organizing maps and their applications. Their flaws and possible improvements in SOM are also discussed. The next part will undertake the optimization tool used to assign weights for each stock in the portfolio. This will be followed by demonstration of the forecasting models. First the neural network model and then the econometric forecasting models. This chapter concludes by presenting the forecasting accuracy methods used for the comparison.

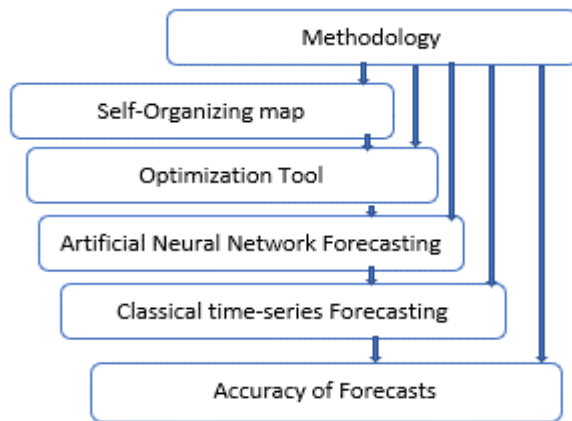


Figure 8. Construct of chapter 5

### 5.1 Self-Organizing Map

Self-organizing map (SOM) was introduced in the 1980s by Teuvo Kohonen. It is sometimes referred to as also the Kohonen Map. It is a method to analyse data automatically and presenting them visually to ease the comprehension of the data in hand. Furthermore, it offers insight by showing topographic associations of data. Since its fruition it has been widely used to cluster and understand data in different fields such as finance, linguistics, industry and natural sciences. (Kohonen, 2013) For instance, finding a relationship between credit rationing and leasing (Severin, 2010), clustering time-series (Cherif et al, 2011) and financial forecasting (Huan et al, 2010) (Nair et al, 2017).

A SOM is an artificial neural network with a single layer based on unsupervised learning. The neurons in the network have been set in a n-dimensional grid which usually is a 2-dimension rectangular grid. It has also been implemented as hexagonal or toroidal grid, but these are not relevant for this study. (Resta, 2012) The neighbourhood relations between the neurons are the base of the structure for the grid and the neurons link to each other from the input layer to the output layers. (Nanda et al, 2010)

According to Kohonen (2013) the SOM model can be implemented by two different algorithms. The first type is called “a recursive, stepwise approximation process”. In this type of algorithm, the SOM works by inserting the input data to the algorithm separately by random or periodic sequence. These steps are repeated until the algorithm reaches a stable state. The second type is called “the batch-type process”. Contrary to the recursive, stepwise approximation process this function works by inserting the input data at once to the algorithm, leading the models being updated at the same time. This type of algorithm usually needs to be reiterated until it stabilizes. When running this algorithm, it is usual to get different cluster amounts, so it needs to be run several times until the cluster amount stops at one number. The batch-type process is the most commonly used and is the one implemented in this study to cluster the stocks. The algorithm for the SOM in this study has been executed by using MATLAB ®.

The process of training a SOM algorithm can be categorized into three steps. The first step is to “evaluate the distance between  $X$  and each neuron of the SOM”. ( $X$ = input data). The second step is “to select the neuron (node) with the smallest distance from  $X$ . This is also referred to as the Best matching unit. The third and final step is “to correct the position of each node according to the results of Step 2., in order to preserve the network topology”. (Resta, 2012)

As visualized in Figure 9., the SOM grid consists of the input data ( $X$ ) and how it is broadcasted into a set of models ( $M_i$ ). All the smaller circles represent models.  $M_c$  represent the model best matching the input data  $X$  and all the models in the larger circle in the grid match better with the best model  $M_c$  than with the other models.



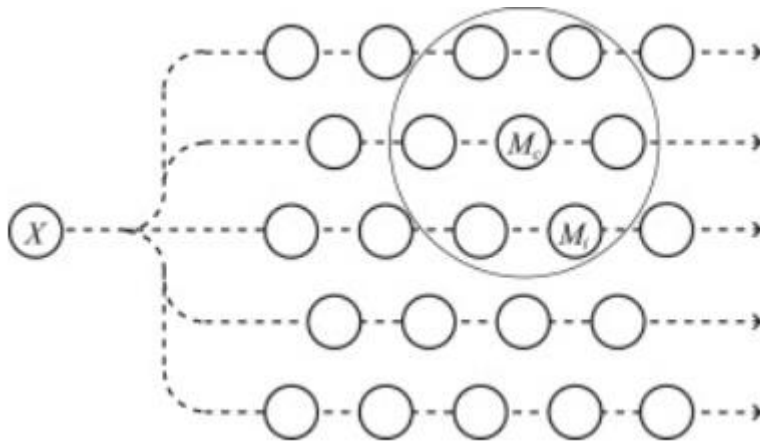


Figure 9. Illustration of a SOM. (Kohonen, 2013)

Results of performing a SOM are usually visualized in a U-matrix, labels and cluster amount. These are presented in Figure 10. The U-matrix (unified distance matrix) visualizes the distance between the nodes. The darker the blue, the closer they are and yellow indicates that they are further apart. The Labels present how many of the input data are in each neuron in the grid. Meaning that similar types of data are grouped together. This helps to analyse the divide and differences between the data. The last figure shows what the optimal number of clusters would be for that specific input data with the lowest data point. In the example it would be 3 clusters. As stated previously, the amount may change by every run, but it stabilizes by time.

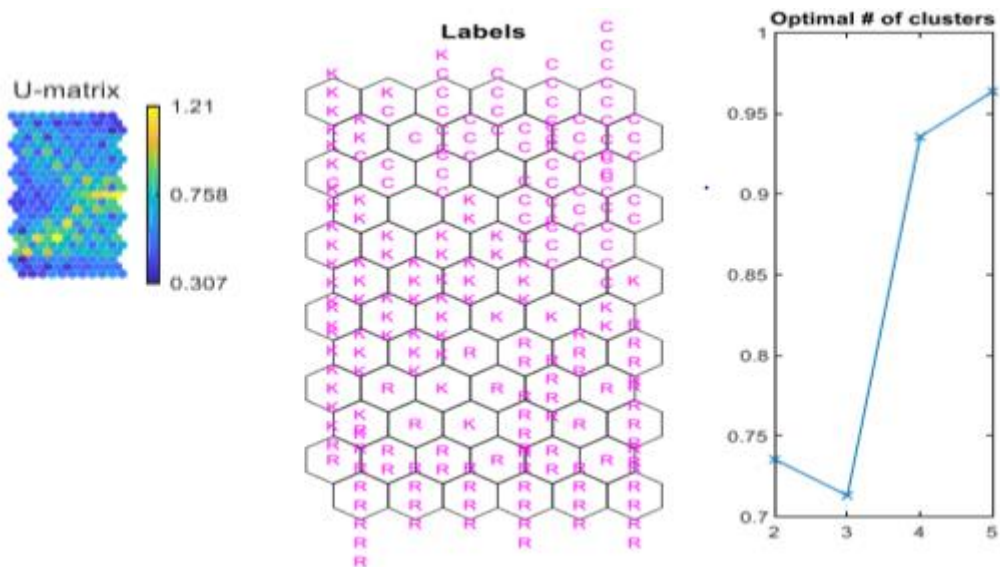


Figure 10. Visual results of SOM.

### 5.1.1 Benefits and Drawbacks

One of the largest benefits of using SOM as a clustering method is that very large data sets can be clustered in good time. It will save the users data management time and produce analysis that can be implemented to production or other use. (Kohonen, 2013) SOM is also simple, understandable and visual. It enables visualization of complicated multidimensional data which is its main application area. (Vesanto, 1999)

However, some drawbacks have been noted. In Pampalks (2001) study the limitations experienced included that SOM cannot be used if information on existing clusters is present. This means if existing clusters are present, they cannot be used in defining the new data. Also, the absence of an automatic function to calculate the quality of clustering is seen as a drawback.

With the benefits weighing more, the SOM is used in this study to cluster stocks and form portfolios based on the cluster results.

## 5.2 Optimization Tool

An optimization tool was constructed using Microsoft Excel ®. This tool helped to assign weights for the stocks in the portfolios formed by the SOM analysis. The tool was constructed based on a minimum variance portfolio because, as an optimisation analysis, it performed better than a mean variance portfolio.

The steps in using the portfolio optimisation tool are:

- Calculating daily returns for each stock from stock prices
- Calculating Standard deviation, Variance, Mean, Expected daily return, Expected yearly return and Beta for the portfolio
- Using Microsoft Excel Built-in tool called Solver:
  - Using the portfolio variance and covariance matrix to minimize the risk.
  - With the above in mind and the maximum weight being 1 or 100%, Solver assigns the optimal weights for each stock.

- Equation (1) of the minimum variance portfolio weights is presented below. The  $q = (q_1, q_2, \dots, q_N)^T$  in the formula states that the portfolio is a vector, where  $q_1$  represents the weight invested in asset 1 and  $T$  represents the transpose operation. The  $V$  in the formula represents the covariance matrix of the returns. (Jian et al 2019)

$$q_{MVP} = \frac{V^{-1}\mathbf{1}}{\mathbf{1}^T V^{-1} \mathbf{1}} \quad (1)$$

### 5.3 Artificial Neural Network models

In essence, artificial neural networks work by trying to emulate the human brain activity. They are a group of nonlinear and flexible models that work by finding patterns adaptively within the given data deprived of the underlying connections in a problem. (Zheng et al, 2011)

Neural network (NN) models, pictured in Figure 11, are typically formed by three layers. The input layer, hidden layer and the output layer. The input is the data in hand, and it is processed in the hidden layer which then provides the output for the problem, or in this case forecast. (Mane et al, 2018)

The input layer functions as the condition the neural network is trained for. This layer presents a pattern to the neural network based on the external environment. The next layer is called the hidden layer and it is between the input and output layers. This layer is where the training takes place before proposing a solution to the output layer. Furthermore, this layer is where the formed pattern is presented to the external environment. (Karsoliya, 2012)

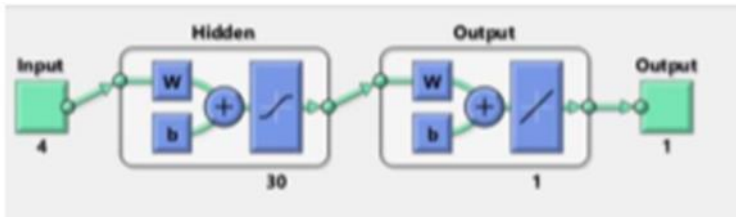


Figure 11. Neural network model (Matlab, 2019)

Figure 12 presents the NAR (Nonlinear autoregressive) model. This model works as a closed loop network. It works by providing the output layer as a response to the input layer. Forecasting is done solely on the time series by implementing the past values. The figure 1:2 in the hidden layer represent the delay, usually 1:2 being the default in Matlab. And the 30 under the Hidden layer represent the hidden neuron amount. (Mane et al, 2018)



Figure 12. NAR model (Matlab, 2019)

Figure 13 showcases the structure of the NARX (Nonlinear autoregressive with external inputs) model. This model works also as a closed loop network. The feedback connection within the model is “from the output layer to the input layer”. Meaning that information is moved between the input and output layers in order to provide a forecast. The mathematical equation for this model is expressed in Equation 2. This equation basically expresses that response variable  $y$  is gained from two values: previous values of response variable and previous values of predictor variable. (Mane et al, 2018) As this model is usually used with several input values, it would not be the most suited for this kind of study.

$$y(t) = f[x(t-1), x(t-2), \dots, x(t-d), y(t-1), y(t-2), \dots, y(t-d)] \quad (2)$$

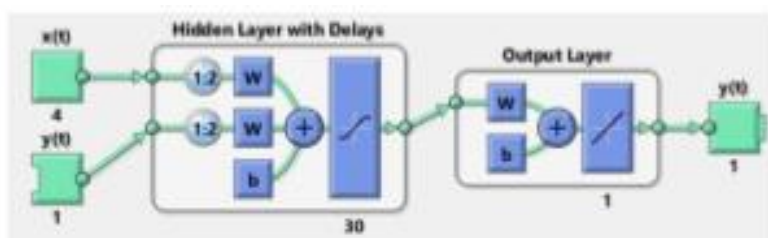


Figure 13. NARX model (Matlab, 2019)

There are three steps in processing an artificial neural network model: Training, Validation and Testing. Training was conducted with 70% of the data, Validation with 15% of data and Testing with 15% of data. For this study the chosen training algorithm for all the NAR models was the Levenberg-Marquart algorithm. (Matlab, 2019) This is a hybrid algorithm that combines the Gauss and Newton algorithms and it is one of the most efficient training algorithms for neural networks. It functions by deciding on the step size by taking large values first and then the small values. (Puig-Arnavat et al, 2015)

As the data chosen for this study consist of past returns, the NAR model was chosen to act as a proxy for artificial intelligence.

## 5.4 Econometric Forecasting

This study focuses on using univariate time series forecasting models to represent the classical econometric models. These models can be used to predict, among other things, financial variables based on their own past values. They are often described being a-theoretical, which implies that there is no underlying theoretical model for the behaviour of any variable used. Henceforth they operate by capturing “empirically relevant features of the observed data that may have arisen from a variety of different (but unspecified) structural models”. (Brooks, 2008)

Before defining the types of forecasting models, it is important to understand stationarity. Stationarity in time series or the lack thereof has great influence on its behaviour and properties. “A strictly stationary process is one where, for any

$t_1, t_2, \dots, t_T \in Z$ , any  $k \in Z$  and  $T = 1, 2, \dots$

$$F_{y_{t_1}, y_{t_2}, \dots, y_{t_T}}(y_1, \dots, y_T) = F_{y_{t_1+k}, y_{t_2+k}, \dots, y_{t_T+k}}(y_1, \dots, y_T) \quad (3)$$

Where  $F$  denotes the joint distribution function of the set of random variables.”

It shows values remaining the same with progression of time, henceforth implying that the value interval for  $y$  (for example stock returns) is most likely the same now than in the past or future.

A weakly stationary process is present, if a series will satisfy three below equations (4) – (6) for  $t = 1, 2, \dots, \infty$ ,

$$(1) E(y_t) = \mu \quad (4)$$

$$(2) E(y_t - \mu)(y_t - \mu) = \sigma^2 < \infty \quad (5)$$

$$(3) E(y_{t_1} - \mu)(y_{t_2} - \mu) = \gamma_{t_2 - t_1} \quad \forall t_1, t_2 \quad (6)$$

The first equation states a stationary process should have a constant mean, the second that a constant variance must be present and the third that a constant autocovariance structure is present. The autocovariances determine for the time-series how  $y$  is related to previous values. This relationship is visualised in an autocorrelation function graph more clearly explained in Table 3. (Brooks, 2008)

The four most common models in this category are Auto-regressive Models (AR), Moving average models (MA), Autoregressive Moving Average models (ARMA) and Autoregressive integrated Moving Average Models (ARIMA). (Brooks, 2008)

One of the basic models for time series is the Moving average (MA) process. This model constitutes of white noise processes, in a way that the value of  $y_t$  (chosen variable i.e. stock return) is determined by the white noise disturbance terms present and past values. The equation (7) for this process is expressed below:

$$y_t = \mu + \sum_{i=1}^q \theta_i u_{t-i} + u_t \quad (7)$$

In an Autoregressive model (AR) the current value of  $y$  is based on its previous values and an error term. This error term is represented by the white noise disturbance term  $u_t$ . Equation (8) expresses the autoregressive process. (Brooks, 2008)

$$y_t = \mu + \sum_{i=1}^p \phi_i y_{t-i} + u_t \quad (8)$$

The Autoregressive Moving Average model (ARMA) is basically the combination of AR(p) and MA(q) models. This model states that not only it is dependent on its previous values but also the current and previous values of a white noise error term. Equation (9) expresses the ARMA process. (Brooks, 2008)

$$\begin{aligned} \phi(L)y_t &= \mu + \theta(L)u_t \\ \text{where} \\ \phi(L) &= 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p \quad \text{and} \\ \theta(L) &= 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q \end{aligned} \quad (9)$$

The Autoregressive integrated Moving Average Model (ARIMA) has a one letter difference to the ARMA model. The letter I represents integrated, as in its “characteristic equation has a root on the unit circle”. Brooks (2008) explains that in ARMA and ARIMA models the Box-Jenkins approach is often used. This approach includes three steps to estimate and ARMA model:

- (1) Identification: Determining the order of the model by using ACF +PACF and information criteria
- (2) Estimation: estimation of parameters of the model by using least squares or maximum likelihood
- (3) Diagnostic checking: determining of the model specified and estimated is satisfactory by using methods called overfitting and residual diagnostics.

In Table 3 the behaviour of these four models have been logged. After running the models, the Autocorrelation function (ACF) and the Partial Autocorrelation function (PACF) with information criteria is used to determine the model order.

From the visualization of the PACF and ACF, the appropriate model for the data can be determined according to the behaviour. For example, if the PACF spikes then decays to zero and the ACF spikes till  $p^{\text{th}}$  lag then cuts off to zero, the model in question would be a Moving average model.

Table 3. Behaviour of time series models (Momin et al 2017) (Brooks, 2008)

<b>Model</b>	<b>Characteristic</b>	<b>PACF correlogram</b>	<b>ACF correlogram</b>	<b>Data Characteristic</b>
<b>White noise</b>	Cannot be used for Time series modelling	No spikes	No spikes	Random in nature
<b>AR (p)</b>	1. $y_t$ depends on its own past values 2. p is computed using PACF function	a number of non-zero points of pacf = AR order.	a geometrically decaying acf	Data should be stationary in nature
<b>MA (q)</b>	1. $y_t$ depends on error term which follows a white noise process 2. q is computed using ACF function	a geometrically decaying pacf	number of non-zero points of acf = MA order	Data should be stationary in nature
<b>ARMA (p,q)</b>	1. ARMA = AR+MA 2. Value of p and q are determined using AIC and BIC criteria	a geometrically decaying pacf	a geometrically decaying acf	Data should be stationary in nature
<b>ARIMA (p,d,q)</b>	1. Data is made stationary by differencing it 2. Box-Jenkins approach is used to determine model	a geometrically decaying pacf	a geometrically decaying acf	Data is made stationary by differencing it

#### 5.4.1 Benefits and drawbacks

According to Chu (1978), ARIMA models have been proven to be quite useful for short term forecasting, but that they lose accuracy in long term predictions.

Subsequently, if the goal would be to forecast for example 5 years ahead, these



models would not be suitable for that. In addition to time restrictions, these models are quite limited in forecasting uncommon movements in prices. Zhao et al (2017) also researched short term forecasting with ARIMA models and concluded on the same point that for short term forecast, this model was the most suitable.

## 5.5 Accuracy of Forecasting

Accuracy of these time series models can be estimated with Root mean square error (RMSE) or Mean Absolute error (MAE) or Mean Squared error (MSE). These error measures received from the model results can be compared to those of other models and receive the most accurate model by finding the lowest value of the error measure. (Brooks, 2008) For this study the MSE is used to measure accuracy of ARMA and NAR forecasting models. However, as the results of the forecasts are also compared to the real returns achieved in the time period, another error term is needed. For this comparison the Mean Percentage Error (MPE) is utilized. Equation 10 shows how the MPE is calculated for the returns. Actual represents the actual return and predict the forecasted return. (Salim et al, 2009)

$$MPE = \frac{(\sum \frac{ACTUAL - PREDICT}{ACTUAL})}{N} \times 100\% \quad (10)$$

## 6 EMPIRICAL RESEARCH

This chapter focuses on the research conducted based on the theory and methodology introduced previously. The construct of this chapter is visualized in Figure 14. It will begin by introducing the data used for this study and how it has performed in the past. The next step is using SOM to cluster the stocks into different portfolios and optimizing the weights of the stocks in the portfolios. After the portfolios are ready, forecasting is implemented based on their past returns. Initially with neural network models and finally with the econometric forecasting models. Once forecasting is done and appropriate models are found, the differences between the forecasting models is explored and the financial characteristics of the portfolios.

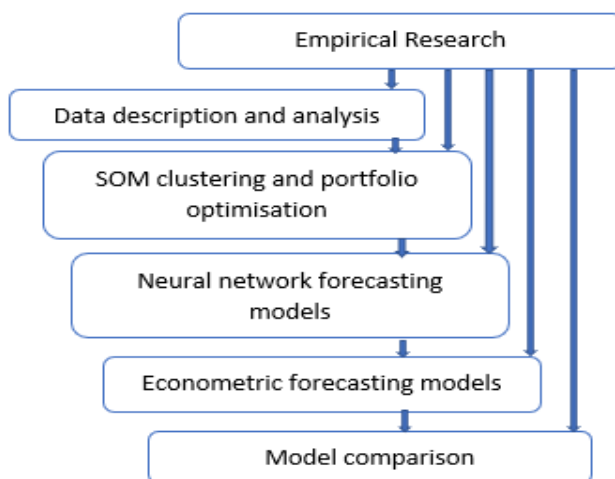


Figure 14. Construct of chapter 6.

### 6.1 Data description

The first part of the empirical research consists of introducing the data and presenting it in a comprehensible manner. Data used for this study has been gathered from Nasdaq OMX Nordic and Thomson Reuters database.

OMXH25 index was chosen for this research because it consists of 25 of the most active trading stocks in the Helsinki Stock Exchange. In other words, the most

desirable stocks in the eyes of the investors. This index is also very often used as a benchmark index for management of diversified Finnish stock portfolios, and for this reason will be the most suitable comparison index for the portfolios formed in the SOM analysis. The daily stock prices for these stocks are from January 2018 to December 2018 and represent a full year of development. As this study focuses on short term movements, a year of prices to cluster and forecast is adequate. Furthermore, as the OMXH25 is used as the main data and the 25 stocks it includes are clustered to form several portfolios. Table 4 includes the 25 stocks included in the benchmark index.

Table 4. OMXH25 Stocks (Nasdaq OMX Nordic, 2019)

Amer Sports	Cargotec	DNA	Elisa	Fortum
Huhtamäki	Konecranes	Kesko B	Kone	Metsä Board B
Metso	Nordea Bank	Neste	Nokia	Nokian Renkaat
Orion B	Outotec	Outokumpu	Sampo A	Stora Enso R
Telia	UPM-Kymmene	Valmet	Wärtsilä B	YIT

As the stock prices and returns are used in this study, the total data amount used is  $26 \text{ (index + stocks)} \times 249 \text{ (daily values)} = 6474$ .

In order to perform the clustering with SOM, 9 financial characteristics for 2018 were chosen to determine the differences of these stocks and their attractiveness to any investor. These chosen variables, explored in Table 5, were Beta, Volatility, Price/Earnings, Dividend Yield, ROE %, Earnings per share, Current Ratio, Quick Ratio and Operating Profit margin. These variables were mainly chosen because each one of them expresses an important aspect to any investor interested in these stocks. They were obtained from Reuters ® database.

Table 5. Financial values for SOM analysis (Wei, 2014)

<b>Term</b>	<b>Measure</b>
<b>Beta</b>	“Systematic risk of security. The sensitivity of a security’s return to the overall market’s movements”
<b>Volatility</b>	“Riskiness of a stock. High volatility means great price fluctuations”

<b>Price / Earnings</b>	“Ratio of stock price over the recent annual earnings per share”
<b>Dividend Yield</b>	“The total amount of per-share dividends received during the year divided by the share price”
<b>Return on Equity %</b>	“Net income divided by the book value of equity. It reflects a firm’s overall financial performance”
<b>Earnings per Share</b>	“Total earnings divided by the total number of shares outstanding”
<b>Current Ratio</b>	“Ratio of current assets over current liabilities. measures a firm’s ability to satisfy the claims of short-term creditors using exclusively current assets such as cash and marketable securities.”
<b>Quick Ratio</b>	“The ability of a company to pay its current liabilities when they come due with only quick assets”
<b>Operating profit Margin</b>	“the percentage of profit a company produces from its operations”

As this study is, among other things, interested in the return development for these stocks and index, it is important to look at how they have performed in the past year. Figure 15 visualizes the daily returns for the index and stocks. There has been up and down movement the whole year, which is expected. But it seems that at the end of the year overall downward returns are to be seen.



Figure 15. Returns for the OMXH25 index and the 25 stocks.

## 6.2 SOM clustering and portfolio optimisation

In this chapter of the study, the second part of the empirical research will be introduced. The goal is to cluster stocks that are included in the OMXH25 index and to find the optimal weights for each stock in the formed portfolios.

### 6.2.1 SOM

The data included the 9 variables for each stock for the year 2018, concluding the data as  $9 \times 25$  (=225) items. The goal is to form clusters based on the financial characteristics of the stocks. The analysis starts by initialisation that creates the data structure for the model and normalisation with unit variance. Once the initialisation is done, the SOM is constructed by training the map. This step was repeated until the model stabilized and found the most suitable number of clusters. Fortunately for this data set it did not take many retries to settle on an optimal cluster number.

Figure 16 represents the clusters. The first figure on the right side visualizes the optimal number of clusters, which is for this data four. The distance matrix also denotes the sizes of the nodes and how different they are from the surrounding points.

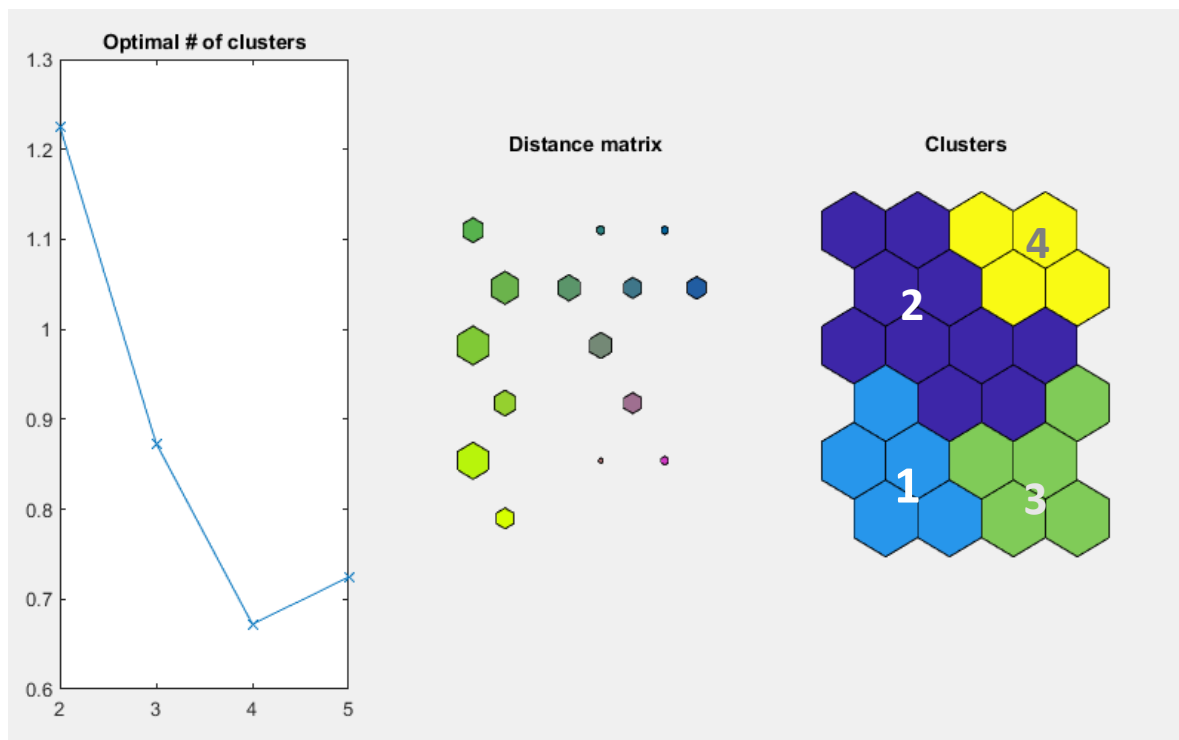


Figure 16. Cluster amount for SOM model

To continue with the SOM results Figure 17 represents each stock in each node. It is clear and as intended that each stock has found a node representing its financial characteristics. For example, Amer Sports and YIT reside in the same node, which expresses that these stocks are similar based on the 9 variables. Neste is close to their node, but it is on its own due to differences.

Based on the colour coded cluster nodes in Figure 16 and labels in Figure 17, the stocks can be assigned to their own clusters. As the SOM analysis provided ready clusters with similar stocks based on the financial characteristics given, instead of picking stocks from different clusters to form portfolios, one cluster represents one portfolio. Diversification in forming these portfolios is not based on the financial differences of the companies but moreover their different fields and business. The formed portfolios represented next provide investors with different needs options. It was important to use the SOM clustering to form different portfolios that also have enough diversification. For example, a risk-averse investor can choose the one suited for them best.



Figure 17. Labels in SOM grid

Therefore, presenting groups of stocks with similar financial characteristics that can be used to form 4 different portfolios. The portfolios formed from this SOM model have been categorized in Table 6.

Table 6. Cluster portfolios from SOM

Portfolio 1 /Cluster 1	Portfolio 2 /Cluster 2	Portfolio 3 /Cluster 3	Portfolio 4 /Cluster 4
DNA	Amer Sports	Telia	Outokumpu
Wärtsilä B	YIT	Fortum	Outotec
Nordea Bank	Neste	Nokian Renkaat	Metsä Board B
Elisa	Huhtamäki	Orion B	Nokia
Kone	Cargotec		
Sampo A	Stora Enso R		
	Metso		
	UPM-Kymmene		
	Konecranes		
	Kesko B		
	Valmet		

The U-matrix and component planes for the SOM model are presented in Figure 18.

This matrix shows what financial characteristic each formed portfolio contains.

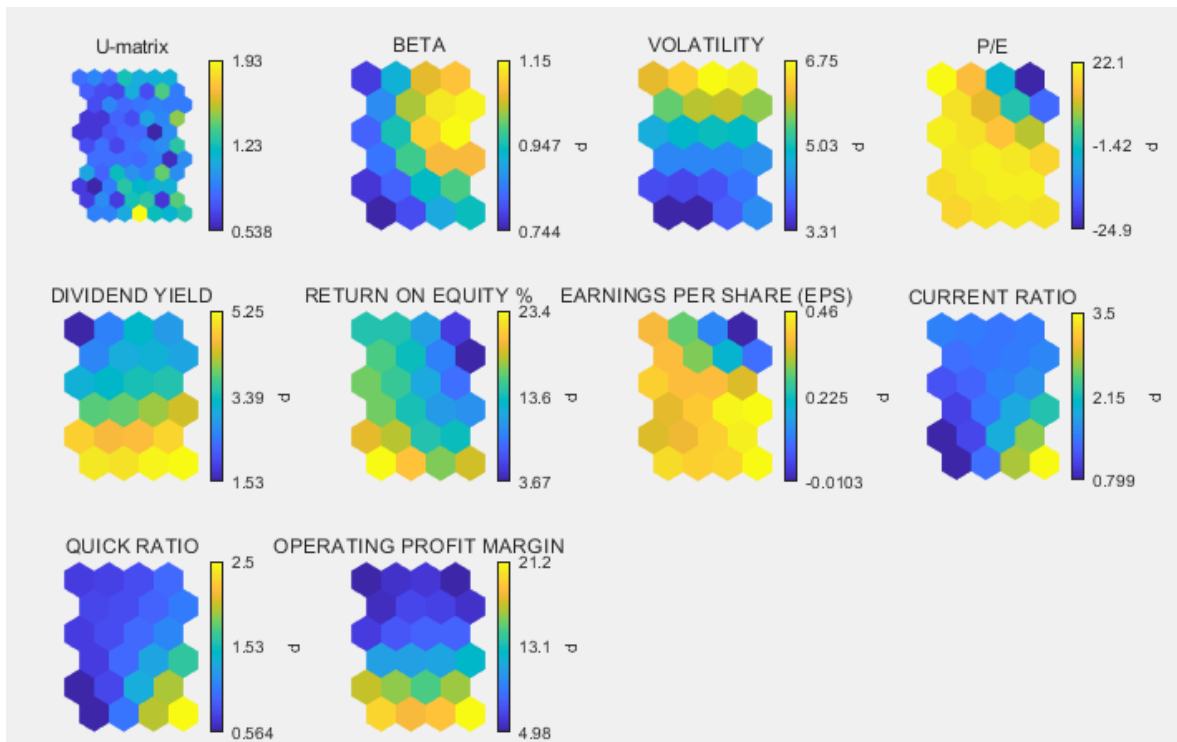


Figure 18. U-matrix

### Financial Characteristics of Portfolio 1:

Starting with Beta for the portfolio, it has the lowest Beta of the four portfolios amounting 0.6. Meaning that the systematic risk for this portfolio compared to the market is almost half. Also, the Volatility is the lowest of the group. Compared to the benchmark index, this portfolio movement has been at half pace. This would indicate that the price movements would not be so vast. The P/E shows that the relative value of these shares would be high. Furthermore, averaging at 20.4 for this portfolio, investors are willing to pay 20.4 EUR for 1 EUR of current earnings. This could indicate that some of these stocks are overvalued, but that higher growth is expected by the investors. The average Dividend yield for the portfolio received in 2018 was 3.78 EUR. So, all the stocks would provide dividend. The ROE% and EPS are both positive which shows the investor how profitably the equity used by the investor to buy the shares have been used. The Quick ratio and the current ratio for these companies is less than 1 meaning, that at the moment of the study data, they would not have been able to pay all of their liabilities in the short term. The stocks in this portfolio have the lowest quick and current ratios. The operating profit margin is also negative, but only approximately 4%.



### **Financial characteristics of Portfolio 2:**

The second portfolio, consisting of the highest number of stocks of 11, has the second lowest beta and volatility of the portfolios. The average Beta for the is 0.7, which gives it a higher systematic risk than portfolio 1, but lower than the market. This also indicates that the price fluctuations are steadier than the other stocks. The volatility for this portfolio also supports this point. Moving to the P/E ratio for this portfolio, it also shows that the relative value of these shares would be high. The averaging P/E is 15.6 for this portfolio, which means that investors are willing to pay 15.6 EUR for 1 EUR of current earnings. This is lower than in portfolio one, but not lowest among all the portfolios. The dividend yield is averaging at 4.6 EUR, which would guarantee investors some yearly returns per share. ROE% and EPS show also high results, which indicates profitability from the companies. The current and quick ratio for the portfolio are averaging at 1. This means that companies in portfolio 2 have been able to pay all short-term liabilities. Furthermore, the average operating profit margin is the highest for all the portfolios, however as all these stocks represent different industries, this value cannot be used to compare the companies.

### **Financial characteristics of Portfolio 3:**

The third portfolio, consisting of 4 stocks, has a Beta of 0.97. This indicates that the systematic risk is very close to the market and that the price movements also mimic what happens in the market almost fully. So, the volatility matches the market, which would bring similar returns than the market in whole. The P/E for this portfolio is 19.6, which would be higher than the 15% benchmark used by investors. According to this benchmark, it would not be suitable to invest in this, but it is important to note that this is not the only indication on the attractiveness of a stock or portfolio. As the other portfolios, this also carries a dividend yield amounting to 3.7 EUR. The EPS and ROE% give high profitability as the other portfolios. The current ratio and quick ratio are both a bit above 1, which gives the companies more than enough to pay for any short-term liabilities. Finally, the operating profit margin indicates that a 12% margin.

### **Financial characteristics of Portfolio 4:**

The fourth portfolio, consisting of 4 stocks also, is almost identical to market movement with a beta of 1. Therefore, there is no further risk or return compared to the market to be expected for this portfolio. This is the Only portfolio with negative P/E ratio. It shows that in 2018, these companies were losing money or had negative earnings. This does not however indicate impending bankruptcy but could be a result of company changes or changes in the market trend. EPS also negative, this can stem from having experienced net loss instead of net profit. This portfolio provides dividend yield, but lower than the other portfolios amounting approximately 1.8 EUR.

All these four portfolios provide individual characteristics on which an investor can decide the most suitable for their needs. However, for any investment strategy diversification is important. If the goal is to find for example the riskiest or the least risky stocks for the portfolio, SOM analysis provides an easy way to cluster and visualize the characteristics of these stocks. Based on the portfolio characteristics portfolio 1 can be indicated as the *low risk, high return* portfolio, portfolio 2 as *low risk, high return*, portfolio 3 as *high risk, high return* and portfolio 4 as *high risk, low return*.

### 6.2.2 Optimisation

After conducting the SOM analysis, this study aimed to further optimize the weights of each stock in the portfolios. This was conducted by forming an Optimisation tool in Microsoft Excel®.

As explained in the theory chapter, the minimum variance portfolio proved to have the best results, so it was chosen as the optimal analysis method for this next step. However, an equally weighted portfolio is also examined to see which weighted portfolio would bring higher returns for the portfolios formed. Keeping in line with the minimum variance portfolio, the optimisation tool aimed to provide the lowest risk level. This would provide the investor based on the historic returns, the lowest risk without compromising the return the stock has had in the past. The tool assigned

different weights based on this criterion. In some portfolios, not all the stocks made the optimal portfolio and in most portfolios the stocks received unique weights.

Table 7 represents the optimization tool result received for all portfolios. In **portfolio 1** optimization started with 6 stocks and all them made the optimal portfolio. Sampo stock took the biggest share in the portfolio with 52% and the smallest went to Wärtsilä amounting only 1%. **Portfolio 2** had before optimisation the largest amount of stocks, amounting 11, out of the portfolios. After the optimisation, 4 of these stocks did received 0% weights, so they are removed from the portfolio. Cargotec, Stora Enso R, Metso and Konecranes have been removed from the portfolio, leaving only 7 stocks. Out of these stocks the highest weight received Kesko B with 28% and the lowest to UPM-Kymmene with 2%. The **portfolio 3** started the optimisation process with 4 stocks in the portfolio and all of them remained after. The highest weights were assigned to Telia 32% and Fortum 34%. None of the weights were under 10%, with the lowest assigned weight to Orion B being 11%. Henceforth, the stocks in this portfolio are quite balanced in weights. The **portfolio 4** also started with 4 stocks, and after running the optimisation tool, all of them remained. This portfolio is quite unbalanced in weights represents Nokia has 65% of the total portfolio. The lowest weight is assigned to Outotec with 6%.

Portfolio 1	Weights	Portfolio 2	Weights
DNA	0,04 → 4%	Amer Sports	0,16 → 16%
Wärtsilä B	0,01 → 1%	YIT	0,12 → 12%
Nordea Bank	0,08 → 8%	Neste	0,09 → 9%
Elisa	0,17 → 17%	Huhtamäki	0,15 → 15%
Kone	0,18 → 18%	Cargotec	0,00
Sampo A	0,52 → 52%	Stora Enso R	0,00
<b>Total:</b>	<b>1 → 100%</b>	Metso	0,00
<b>Portfolio 3</b>	<b>Weights</b>	UPM-Kymmene	0,02 → 2%
Telia	0,32 → 32%	Konecranes	0,00
Fortum	0,34 → 34%	Kesko B	0,28 → 28%
Nokian Renkaat	0,23 → 23%	Valmet	0,18 → 18%
Orion B	0,11 → 11%	<b>Total:</b>	<b>1 → 100%</b>
<b>Total:</b>	<b>1 → 100%</b>	<b>Portfolio 4</b>	<b>Weights</b>
		Outokumpu	0,12 → 12%
		Outotec	0,06 → 6%
		Metsä Board B	0,17 → 17%
		Nokia	0,65 → 65%
		<b>Total:</b>	<b>1 → 100%</b>

Table 7. Portfolio Weights

After optimizing the portfolio based on the minimum variance portfolio, that was deemed to perform better than otherwise weighted portfolios by Bednarek & Patel (2018) and Haugen & Baker (1991), the minimum variance portfolio is compared to an equally weighted portfolio. The comparison of the weighted portfolios is conducted by comparing the real returns for each portfolio for 1-month based on the assigned weights. The timeline for the comparison is the same as the longer period for the forecast. For the equally weighted portfolio, the optimization tool is not needed as the weights are equal for all stocks in the portfolio. For portfolio 1, the MVP gave lower loss with -0,24%, the equally weighted having -0,49%. For portfolio 2 the trend was similar with the MVP having 0,14% return and the equally weighted having 0,12% return. For portfolio 3 both returns were negative, but the MVP had a smaller loss with -1,52% compared to -2,81%. Portfolio 4 did not also divert from the trend that the MVP gave lower loss with -1,41% compared to the equally weighted -2,84% return. From the results for all four portfolios, the minimum variance portfolio does indeed present the holder of the portfolios with higher returns.

All in all, the optimisation tool was able to further develop the portfolios formed by the SOM analysis. Instead of all stocks carrying equal weights (i.e. 25%, 25%, 25% and 25%) the weights are based on the goal wanted to be attained from the portfolio. In this case that would be to get the highest possible return, with the lowest possible risk level.

### 6.3 Forecasting

Now that the portfolios have been optimized forecasting is generated for all four portfolios and market index, so that 1 year of daily returns are used to predict returns 1 week (7 days) and 1 month (30 days) ahead. Returns for each portfolio have been calculated based on the optimized weight amounts. The weighted return for each portfolio and Index for the year 2018 is used for both forecasting methods, classical and neural network. This amounts to 249 daily returns to be used in each analysis.

Figure 19 presents the structure of this Forecasting chapter. The forecasting analysis starts with classical model forecasting. The first step in this is forming the models for each portfolio and index and then performing the forecasts. The results will then be compared with each other based on return and MSE. The final part for the classical forecasting would be to compare the forecasts to the real returns to give a more realistic approach to the study. The next part of this chapter is processing the same steps with the artificial neural network models. Finally, the third part includes comparing the results received from classical forecasting and neural network forecasting.

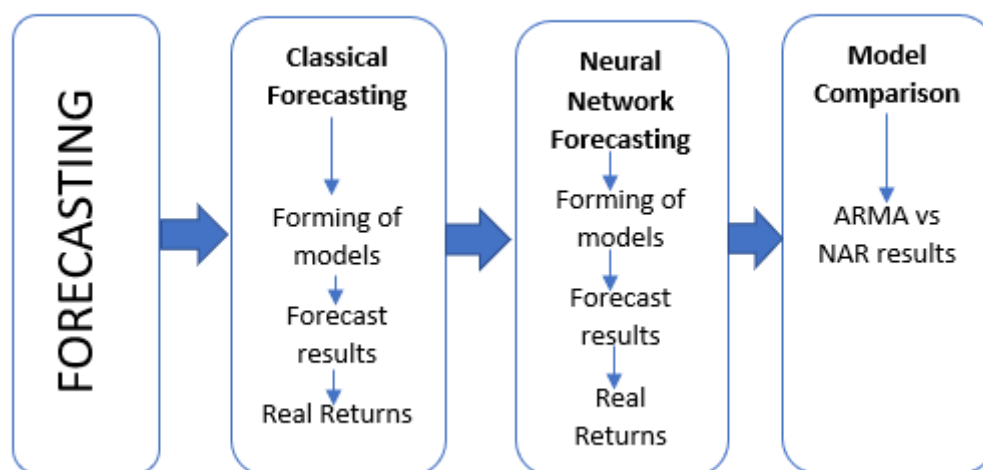


Figure 19. Structure of chapter 6.3

### 6.3.1 Classical forecasting – model and forecast

For the classical forecasting the Box-Jenkins framework is used to build the ARMA models. For the build both graphical methods and information criteria is used. Starting with **Portfolio 1** the model estimation for classical forecasting starts by defining the best suited model. The first step in this is checking the Sample Autocorrelation Function (ACF) and Sample Partial Autocorrelation Function (PACF). These figures indicate if the best suited time series model is AR, MA or

ARMA. As presented in Figure 20, there are no visible lags and both ACF and PACF seem to trail off gradually. The figures do not indicate to be AR ( $p$ ) or MA ( $q$ ) models. The figures would indicate a white noise, but testing is continued by calculating information criteria. The graphical model if looking at the ACF and PACF for this data seem support the Random Walk hypothesis, that future value of an asset cannot be forecasted by past data. However, information criteria is further used to determine the validity of this.

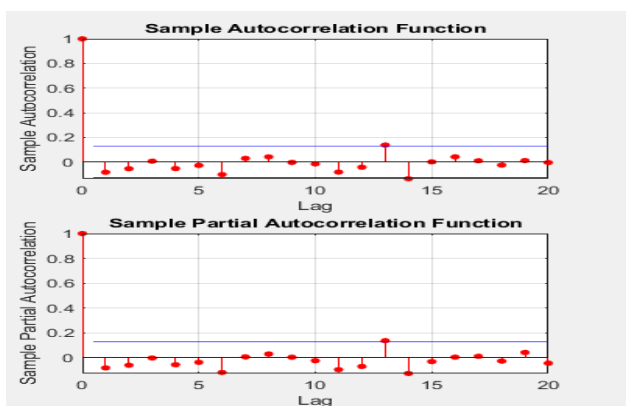


Figure 20. ACF and PACF of portfolio 1

As the order of an ARMA model is quite difficult to estimate straight from the ACF and PACF graphs, the estimation of the order is processed further in Matlab. The estimation returned values AIC: ARMA (1,2) and BIC: ARMA (0,0). Looking at the ACF and PACF and also the BIC (0,0) value, the model would indicate towards the data being white noise. However, to further confirm this a Ljung-box test is conducted. The Ljung-box test resulted with  $h = 0$ , meaning that data in hand is indeed white noise.

For portfolio 1 the model results showed that the data is white noise. This means that the data is random and cannot be forecasted. However, as one aim of this study is to compare ARMA and NAR models the study continues to use the results of the white noise time-series model in order to showcase how the analysis would have been if the results were not white noise. It is interesting to see if forecast of a white noise done by ARMA model will be any closer than the NAR model to the real returns. Nevertheless, it is important to keep in mind as the model is indeed based on white noise time-series, to be careful in trusting the results gained from it.

As the AIC model tends to overestimate the order of the model and BIC underestimate, AIC would be chosen out of these two as an ARMA model is more complex than and AR or MA model. The next step was fitting model ARMA (1,2) to the dataset by evaluating the residuals and their Autocorrelation further with the Ljung-Box test. The results indicated that  $h = 0$ , meaning that we should not reject the null hypothesis that the residuals are not autocorrelated, which as a white noise data was not correlated to begin with. As we are dealing with white noise, the fit of the model cannot be said to be good, but it is used for the purpose of this study.

As the order and fitness have been assessed the next step was to forecast with the chosen model and dataset. The forecast is for 7 days and 30 days ahead to see the situation of portfolio 1 after 1 week and 1 month of not selling or buying any other assets. The forecasted return for 1 week was -0,15%. This indicates that investing for 1 week in this portfolio would not be worth it for any investor. The forecast after 30 days showed 0 % return. This indicates that in that period any gain and loss in returns for this portfolio would be balanced. This is an interesting situation as it is ideal for a risk averse investor that does not think the returns in short term are that important. The possible returns would not possibly hold that much value if the losses are minimal or non-existent in the short term. As the difference in one week for portfolio 1 is positive, it could indicate that on a longer time period positive returns could be obtained.

The next model estimation is for **Portfolio 2**. Estimation of the suitable model for this dataset also begins by presenting the PACF and ACF in Figure 21.

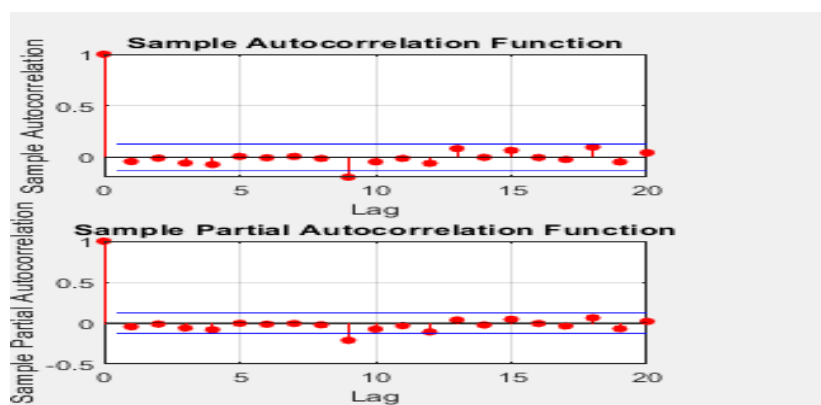


Figure 21. ACF and PACF of portfolio 2

Comparable to Portfolio 1, these figures do not have visible lags and seem to trail off gradually with couple of visible spikes. Again, the figure does not show to be an AR (p) or MA (q) model. However, it could indicate to be white noise or an ARMA (p,q) model. This also further seems to support the random walk hypothesis. For this reason, estimation needs to be further investigated by calculating information criteria. This is conducted with the AIC and BIC. The AIC proposed an ARMA (2,2) and the BIC and ARMA (0,0) model. In this portfolio also, checking the ACF and PACF and also the BIC (0,0) value, point towards white noise. To confirm further if it really is white noise a Ljung-box test is conducted. The Ljung-box test resulted with  $h = 0$ , meaning that data in hand is indeed white noise.

The model for portfolio 2 has similar conditions as portfolio 1 and conducting the forecast with a white noise timeseries is also implemented. Again, considering the randomness of the model, care is implemented in trusting the results.

As the ARMA model is complex, it is safer to continue with the higher value ARMA (2,2) model. The next step evaluated the residuals and their Autocorrelation with the Ljung-Box test. The results indicated that  $h = 0$ , meaning that the acceptance of the null hypothesis that the residuals are not autocorrelated.

As the model is now chosen and fitted to the data, the forecast is performed for 7 days and 30 days ahead. The interest is mainly in short term forecast and especially 30 days because professionals in the finance field in Finland are usually prohibited on trading for 30 days from buying a stock. The initial 7-day forecast showed positive return of 0,21%. Even though the return % is quite small, for an impatient investor it could be ideal. The 30 day return for portfolio 2 was 0,05%. This is a positive return. The investor with this portfolio would gain some return, if they would sell the assets and would not experience loss in this time period. However, there is slight indication of the return going down in the longer time period, so this portfolio may not be the most suitable for longer time periods.

The next estimation is performed for **Portfolio 3**. Figure 22 shows the ACF and PACF for the returns for this portfolio. Neither graphs have visible lags and seem to trail off gradually. These graphs are quite similar of those in portfolio 1 and 2 which indicates to an ARMA (p,q) model or white noise and supports random walk



hypothesis. These models are known to be more complex than AR (p) and MA (q), so the order of the model cannot be estimated straight from these graphs. The estimation of the model order is checked with AIC and BIC values of the information criteria. AIC presented an ARMA (3,2) model and BIC an ARMA (0,0) model. In this portfolio also, checking the ACF and PACF and also the BIC (0,0) value, point towards white noise. To confirm further if it really is white noise a Ljung-box test is conducted. The Ljung-box test resulted with  $h = 0$ , meaning that data in hand is indeed white noise.

With this portfolio also forecast is continued for the purpose of the study, but efficiency and performance will be discussed further in the results. Again, it is important to keep in mind the randomness of the white noise time series data when analysing the results.

Again, the higher one is chosen for the next step. The Ljung Box test is performed yet again for residual and autocorrelation estimation. The results indicated that  $h = 0$ , meaning that the acceptance of the null hypothesis that the residuals are not autocorrelated, not surprising as they were not autocorrelated to begin with.

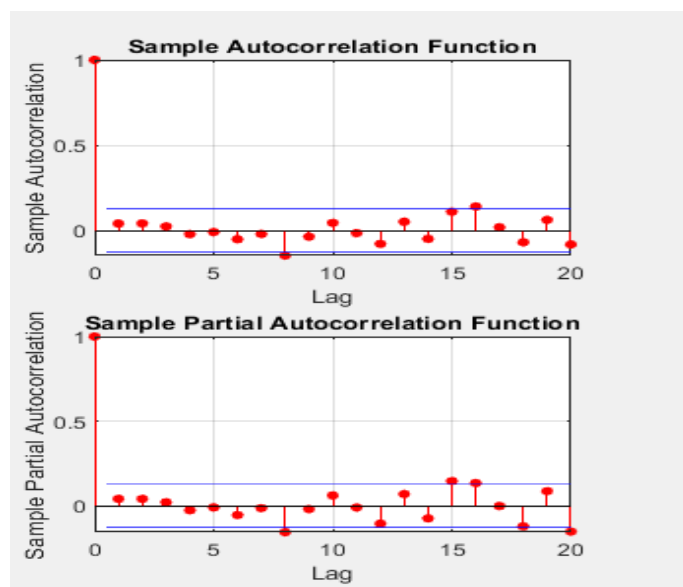


Figure 22. ACF and PACF of portfolio 3.

As the model is now settled on, the forecast is performed 7 day and 30 day ahead. After 1 week of obtaining this portfolio, the forecasted return is  $-0,11\%$ .

Furthermore, the forecast indicates that after a month the portfolio would present a 0,1% return. Once more, a positive return is predicted, but not a high one.

The last of the portfolios, **Portfolio 4** is also first presented with the ACF and PACF graphs to estimate the order of the model. This is presented in Figure 23. Once more, there are no visible lags and seem to trail off gradually. This indicates at an ARMA (p,q) model or white noise and also seem to support the random walk hypothesis. The order has to be estimated with the information criteria as it cannot be visually determined from the ACF and PACF graphs. The AIC results propose an ARMA (3,3) model and the BIC results and ARMA (0,0) model. Again, looking at the ACF and PACF and also the BIC (0,0) value, the model would indicate towards the data being white noise. As the data sets are similar, with same length of time it is not that surprising to receive another indication of with noise. Furthermore, this is confirmed with Ljung-box test. The Ljung-box test resulted with  $h = 0$ , meaning that data in hand is indeed white noise. Portfolio 4 has similar conditions as portfolios 1-3 with a white noise time-series ARMA model. Nevertheless, the model is used for analysis for the purpose of this study.

Once more, the higher value is chosen and estimated. The Ljung Box test is performed, and the results indicated that  $h = 0$ , meaning that the acceptance of the null hypothesis that the residuals are not autocorrelated.

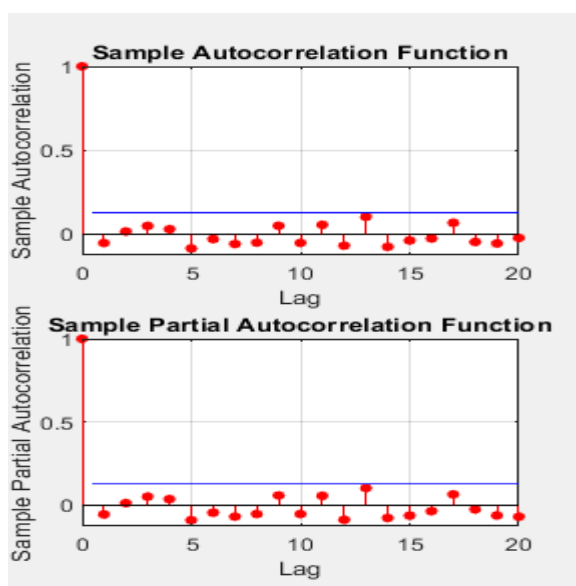


Figure 23. ACF and PACF of portfolio 4

Moreover, as the model is chosen, the forecast is performed 7 days and 30 days ahead. After 1 week, the forecasted return is -0,14% and after owning the portfolio for 30 days 0,09% return. Again, a very small return, but clearly it is better to have this portfolio for the whole 30 days than sell it after 7 days.

After the portfolios are forecasted, the **OMXH25 index** is treated the same way. Figure 24 presents the ACF and PACF graphs. Yet again, there are no visible lags and they trail off gradually. This indicates an ARMA (p,q) or white noise and further seems to support the random walk hypothesis. Detection of the correct model or lack of is continued. However, it is no surprise that all of the portfolios and Index would have a similar model as the time series are quite similar. Furthermore, the AIC and BIC proposed and ARMA (3,3) and (0,0) respectively. Similar to the 4 portfolios the index data also points to white noise and again was tested with the Ljung-Box test with positive results showing white noise. The forecasting is continued and the aspect of all of the models for the classical forecasting indicating to be based on white noise time-series will be further discussed in the results.

The higher (3,3) model is implemented to get more accurate results. The Ljung box test indicates that  $h = 0$ , meaning that the acceptance of the null hypothesis that the residuals are not autocorrelated.

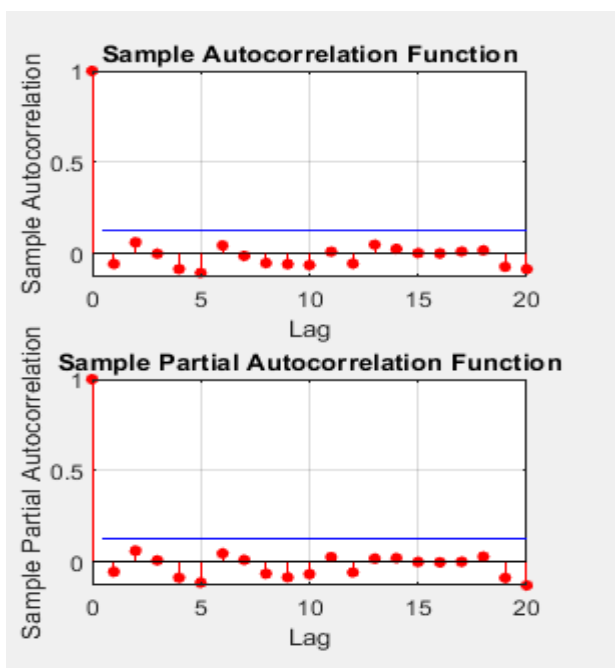


Figure 24. ACF and PACF of OMXH25

The next step for the index, is forecasting 7 days and 30 days ahead returns. After 1 week of trading the forecasted return for the index would be -0,5%, so there would be a slight decrease in the short run. Furthermore, after 1 month has passed the return forecasted for the index would be 0,12%. The return was able to turn to positive and rise from the 1-week investment period. This is a good amount for this short time period and again any positive return is good for the risk averse investor. All of the other visual results gained from running these models can be found in Appendix 1.

As all of the portfolios and index have found a suitable model and the returns forecasted for 7 and 30 days, the differences are inspected. Table 8 compiles the return % and MSE for the portfolios and index. Out of these five the OMXH25 index provides the highest return for a 30 day period and for a 7 day period the highest return is predicted for Portfolio 2. This is not that surprising the Index has highest in 30 days as it has all 25 stocks in it and in such provides the most diversification. It entails risky and less risky stocks that balance each other out. However, in a shorter investment period it has the lowest return %. This could be that the diversification of the index only takes effect in the long term.

Table 8. ARMA model results

<b>ARMA MODEL RESULTS</b>	<b>1 month Return %</b>	<b>1 month MSE</b>	<b>1 week Return%</b>	<b>1 week MSE</b>
<b>OMXH25</b>	0,0012 → 0,12%	0,00041	-0,0050 → -0,5%	0,0003947
<b>Portfolio 1</b>	0,0000 → 0%	0,00008088	-0,0015 → -0,15%	0,00007943
<b>Portfolio 2</b>	0,0005 → 0,05%	0,00008247	0,0021 → 0,21%	0,00008133
<b>Portfolio 3</b>	0,0010 → 0,1%	0,00009280	-0,0011 → -0,11%	0,00009074
<b>Portfolio 4</b>	0,0009 → 0,09%	0,0002049	-0,0014 → -0,14%	0,0002028

Portfolio 1 is presented with 0% return in the first month, which is a rise from the 1 week forecast. This portfolio has the lowest beta and lowest volatility of the portfolios. The slow movement and less risk can explain the non-existent return achieved from this portfolio in the 30 days. However, the same performance cannot be seen in the 7-day returns. The second lowest forecasted return is with portfolio 2 for 30 days. Secondary to portfolio 1, this portfolio has a low beta and volatility. Interestingly the portfolios with lowest volatility and beta are forecasted to have the

lowest returns after a 1-month period. Therefore, there is a connection on the returns and the financial characteristic stocks of portfolios have. However, it does not seem to work for the 1-week time period. The returns for the 1-week period indicate an opposite reaction based on financial characteristics the shorter the time period seems to be.

Portfolio 3 and Portfolio 4 have forecasted 1 month 0,1% and 0,09% returns respectively. These returns are almost double than in portfolio 2. Furthermore, both portfolios have beta and volatility almost identical to the market. This is visible in the return % as they are very close to the index return%. If the goal would be to imitate the index by investing in separate stocks instead, portfolio 3 and 4 would be the most suited proxies. Or on the other hand, only invest in the index. For the shorter 1-week period both of these portfolios have negative returns, but much lower than the index has. Therefore, the imitation does not act the same for the shorter time period.

In addition to the return %, the MSE of the ARMA models can be compared. This measures how well MSE can explain the data set, so the closer to 0 the better. Out of these five models all of the MSE values are relatively small and close to 0. However, Portfolio 1 has the smallest value and would be based on that have the best fit out of the models in both time periods. The index has the largest of the MSE, but then again with a very small difference.

From the investors point of view portfolio 3 would be the most desirable based on the return % prediction and MSE of the portfolios for the 1-month investment period and for the 1-week period, Portfolio 2 would be the most desirable. As these all forecasts, the accuracy or fit of the model is important in determining how believable the formed forecast is.

### 6.3.2 Classical forecasting – forecast and real returns

After comparing model results and returns, it is important to further test the accuracy of the forecasted returns. This is conducted by comparing them to the real returns comparable to the time period forecasted and measuring the differences with Mean

percentage error. This influences the desirability of each portfolio afterwards, as the real returns expose which of the forecasted results can be trusted the most. Once more, the data was collected from Nasdaq OMX Helsinki and the real returns represent the real trading days.

Figure 25 has visualized the differences between the 1-week ARMA forecasts and real returns. Starting with Portfolio 1, the forecasted return is -0,15% and real return 0,32% with the calculated MPE is standing at 147%. The error margin between the forecast and real return is high but the real return is higher than forecasted with the ARMA model. Thus, having portfolio for 1 week and anticipating a slight slump due to the forecast would in reality bring the investor positive returns. Portfolio 2 on the other hand has an opposite relationship between the real and forecasted return, -0,19% and 0,21% respectively. The MPE is 209%, so almost the polar opposite return. If the investor would have acted based on the forecast for portfolio 2, they would have encountered loss, instead of the expected gain. For portfolio 3 the real return proved to be higher than the forecasted return, with 0,20% and -0,11% respectively. The MPE is again high with 154%, but just looking at the return %, the difference is not that high. Portfolio 4 had negative forecasted returns and the real returns turned to be negative too with 80% MPE. The index had also a positive outcome, when looking at the return %. With the forecast being -0,5%, the real return turned out to be 0,06%. A small return, but a positive one.

The lowest forecast error of the ARMA models was for Portfolio 4. This means that portfolio 4 has the best forecast accuracy. However, the difference is still enough to rethink the accuracy of forecast with ARMA models for the 1-week period.

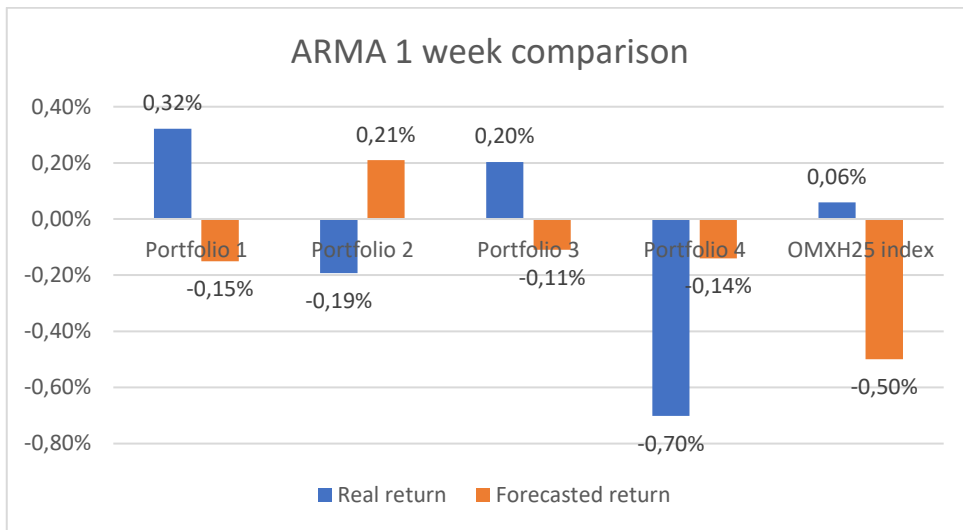


Figure 25. ARMA 1-week comparison

Figure 26 showcases the 1-month comparison for ARMA models and real returns. The corresponding MPE's are for portfolio 1 = 100%, portfolio 2 = 65%, portfolio 3 = 107%, portfolio 4 = 106% and OMX25 index = 140%. Only portfolio 2 has higher real return than forecasted return and the rest have all negative real returns. The lowest accuracy is for the index and the most accurate forecast is for portfolio 2.

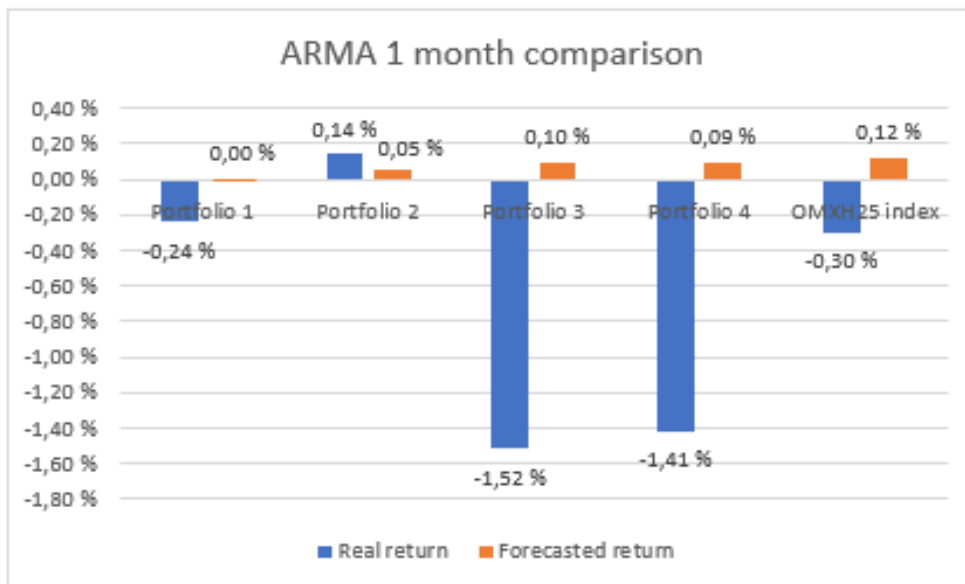


Figure 26. ARMA 1-month comparison

Comparing the 1-week MPE's and the 1-month MPE's, it is clear that more accuracy is gained with the longer time period. This result is not that much of a surprise

because if investors could accurately predict returns for 1-week period or shorter, the trading market would be immensely more different.

All in all, as the ARMA models were used to forecast a white-noise process (or a process very close to a white noise one), the predictions obtained might as well be considered random.. And as visible in Figure 25 and 26 almost all the predictions missed the sign of the real returns. When the real returns are negative, the ARMA forecast is positive and vice versa.

### 6.3.3 Neural network forecasting – model and forecast

NAR forecasting was conducted in Matlab®. For all the models performed the same conditions were used, so that they could be comparable. Also, as the data is similar, implementing the same conditions was a necessity. The first step was choosing the ANN model. As only one input (return time series) is used, NAR model is chosen. The next step was choosing the training, validation and testing amounts. For all of the 5 data sets, for Training 70% of the data was used, which was 175 data points. For validation and testing the amount was for both 15%, representing 37 data points each. Once these were logged in, different number of neurons and delays were tested to get the most accurate model. For all the portfolios and index, the best accuracy was with 10 neurons and the number of delays for the model being 2. Furthermore, the model is trained by performing the Levenberg-Marquardt training algorithm. After the training is finished the model forecasts returns 7 days and 30 days ahead. Figure 27 represents the NAR model implemented for forecasting for the portfolios and index. Appendix 2 has also all the visual results for each model.

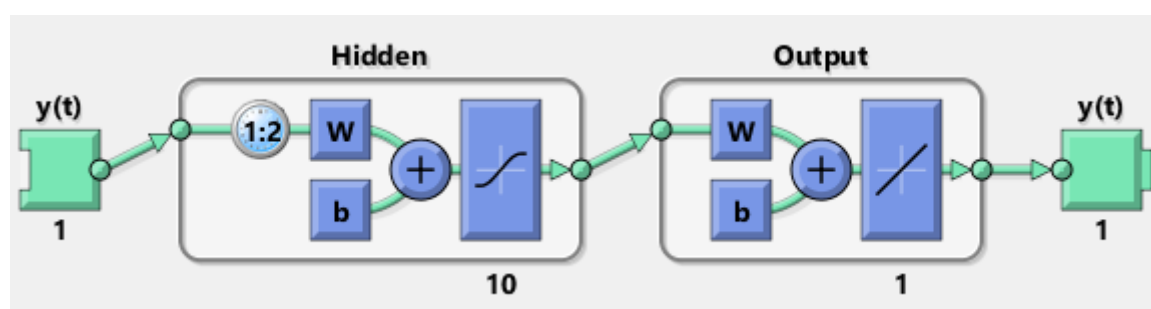


Figure 27. NAR Neural network



As the above steps are performed and validated the results for the 7 day and 30 day ahead forecasts are available. After conducting the return prediction for the next week and month, portfolio 1 was predicted to have 0,02% and 0,018876% return, respectively. This states that the return after one week and one month for this portfolio would go slightly up. A positive return is forecasted but quite minimal. As the time period for the forecast is short, massive returns are not to be expected. Interestingly, Portfolio 2 has over four times higher forecasted return % than portfolio one with 0,08585% for both time periods. Interestingly the returns are quite dissimilar regarding the similarities between the financial characteristics of these portfolios. Both have similar volatility and beta and current and quick ratio. So, there aren't a similar clear connection than with the ARMA models. Portfolio 3 and 4 have returns of 0,17% and 0,16% for 1 week and 0,18% and 0,14% return predictions for 1 month respectively. As these portfolios both have similar financial characteristics, it is not a surprise that the returns are close to each other. It is however apparent that portfolios 1 and 2 that have the lowest beta and volatility also have lowest returns, excluding the index. So, regarding the all of the portfolios and index there are some apparent indication to the similarities of the financial characteristics, but not as clear than in the ARMA models. The OMXH25 index is the only one with a negative return% prediction of - 0,18% for both time periods.

Table 9 presents all of the NAR model results for each portfolio and index. Starting with the return %, the highest return was predicted for Portfolio 3 followed by portfolio 4. Again, both portfolios have quite similar financial characteristics. They both have betas and volatility close to market performance, so the systematic risk taken with these portfolios would be same than the market situation. However, it is interesting that the OMXH25 index forecast with the NAR model presented a negative return. As portfolio 3 and 4 should mimic the performance of the index based on the beta and volatility, it seems that with the NAR model this is not realized. In addition to that the most accurate fit of NAR model is for portfolio 3, with portfolio 1 coming as a close second. The fit of the model does not seem to have any connection to the financial characteristics of each portfolio and index, as the MSE is close to similar in very opposite portfolios 1 and 3.

From the investors point of view portfolio 3 would be the most desirable based on the return % prediction and MSE of the portfolios.

Table 9. NAR model results

<b>NAR MODEL RESULTS</b>	<b>1-month Return %</b>	<b>1-week Return %</b>	<b>Model MSE</b>
<b>OMXH25</b>	-0,0018 → -0,18%	-0,0018 → -0,18%	0,0002142
<b>Portfolio 1</b>	0,00018876 → 0,018876%	0,0002 → 0,02%	0,000090796
<b>Portfolio 2</b>	0,00085852 → 0,085852%	0,00085855 → 0,085%	0,00011161
<b>Portfolio 3</b>	0,0018 → 0,18%	0,0017 → 0,17%	0,000082094
<b>Portfolio 4</b>	0,0014 → 0,14%	0,0016 → 0,16%	0,00021359

#### 6.3.4 Neural network forecasting – forecast and real returns

Furthermore, to further investigate the processed forecasts, it is important to look at the real returns gained for these portfolios and index during this time period. This is done by comparing the MPE results for each return. Figure 28 visualizes 1-week comparison of NAR real and forecasted returns. The MPE's of the returns are portfolio 1 = 94%, portfolio 2 = 145%, portfolio 3 = 16%, portfolio 4 = 123% and OMXH25 index 403%.

For portfolio 1, portfolio 3 and OMXH25 index the real return % turned out to be higher than the forecasted return, which is a positive surprise for any investor. However, the differences are notable, even though they are positive. The most accurate forecast based on the MPE is portfolio 3. The forecast and the real return are so close to each other that any investor looking for a short-term investment would have been satisfied. Furthermore, it is important to keep in mind that short term forecasting cannot be done accurately, so to receive a forecast this close is quite good.

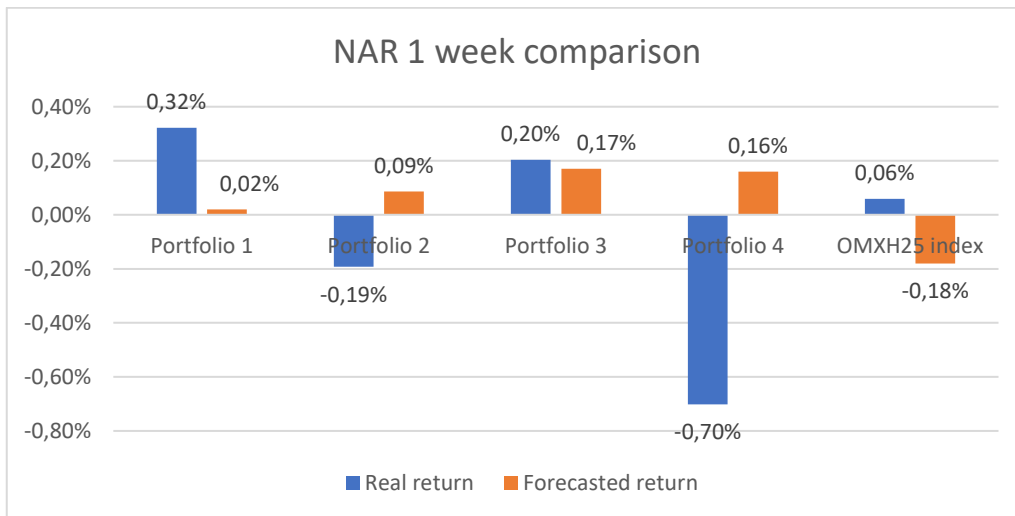


Figure 28. NAR 1-week comparison

Figure 29 represents the 1 month real and forecasted return for the NAR models. The MPE's are from portfolio 1 to index, 108%, 40%, 112%, 110% and 40% respectively. These are much more accurate than the 1-week errors in total. This is not surprising, as again, short-term forecast of 1-week is quite unpredictable. The most accurate forecast for 1-month was for portfolio 2 and the index. Looking at the financial characteristics of the portfolios, there isn't a clear connection if there would be a difference between 1-week and 1-month prediction accuracy based on them.

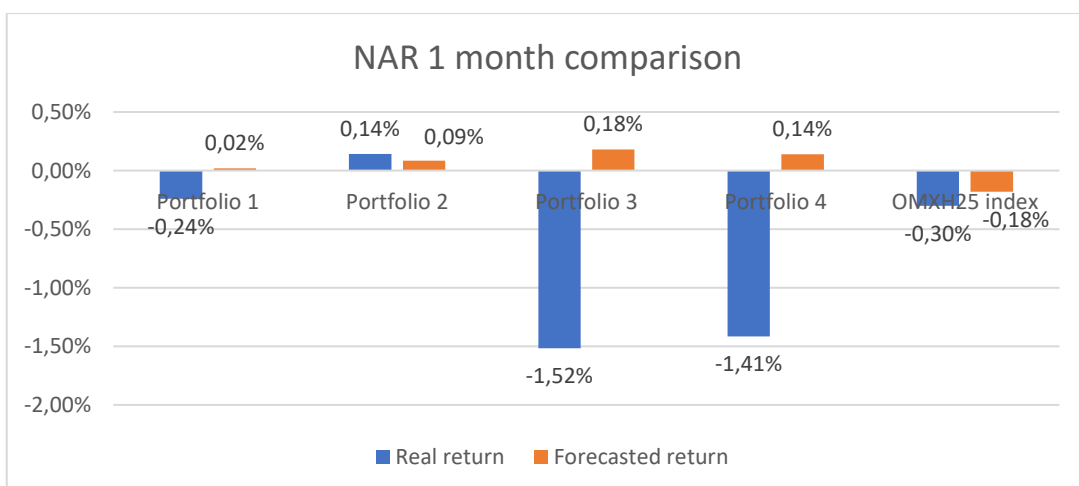


Figure 29. NAR 1-month comparison

### 6.3.5 Model comparison

As both classical and ANN forecasting is performed, they can be both compared to each other. The comparison is based on the forecasted return%, the real return %

and the error terms for each model. Figure 30 represents the forecasted return % and real return % for portfolio 1 for each model.

Firstly, looking at the 1 week returns for portfolio 1, the NAR model is much closer to the real return and it also has a better MPE. However, neither forecast is quite accurate compared to the real return. But if an investor were to choose the more accurate forecast for 1 week, it would be the NAR model. Furthermore, the return % for 1 month is also a bit different. As the ARMA model presented a 0% return, the NAR model gave a 0,018876% return with the real return % being negative. As an investor the interest would be to trust the one with higher return, but the MSE of the NAR model is a bit higher. This would indicate that for portfolio1 returns the ARMA model is more fitted and thus would provide more accurate forecasted returns.

The results showed that for portfolio 1, NAR model was more accurate for the shorter time period of 1 week and ARMA was more accurate for the longer time period of 1 month. Further investigation to the other portfolios and index will show if this is the same for all of forecasts.

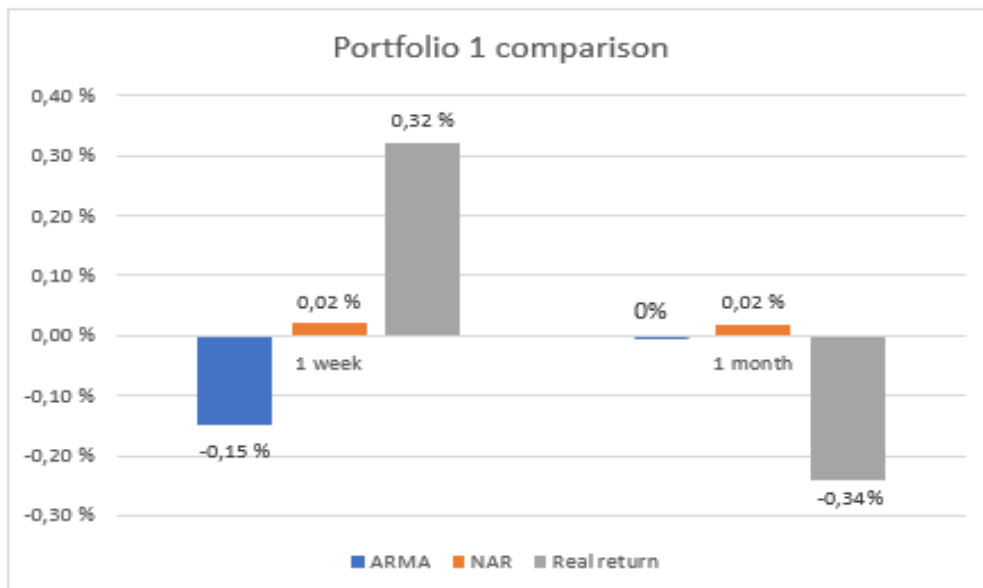


Figure 30. 1 -week and 1-month comparison for portfolio 1.

Figure 31 has visualized both model results and real return for portfolio 2. For the 1-week time period, both forecasts are positive when the real return % is negative with visible difference. The MPE of NAR is a bit smaller with 145% to 209%, so it would make the NAR model more accurate for portfolio 2.

The higher return is also predicted with the NAR model for the 1-month period. The difference is slight, but enough to garner the interest of a potential investor. When looking at the fit of the model, the ARMA model has lower MSE. This would indicate that the ARMA is better fitted to this data set. However, looking at the accuracy of the forecast from MPE, the NAR model presents the lower value. Thus, for portfolio 2, the NAR models gave more accurate forecasts for both time periods.

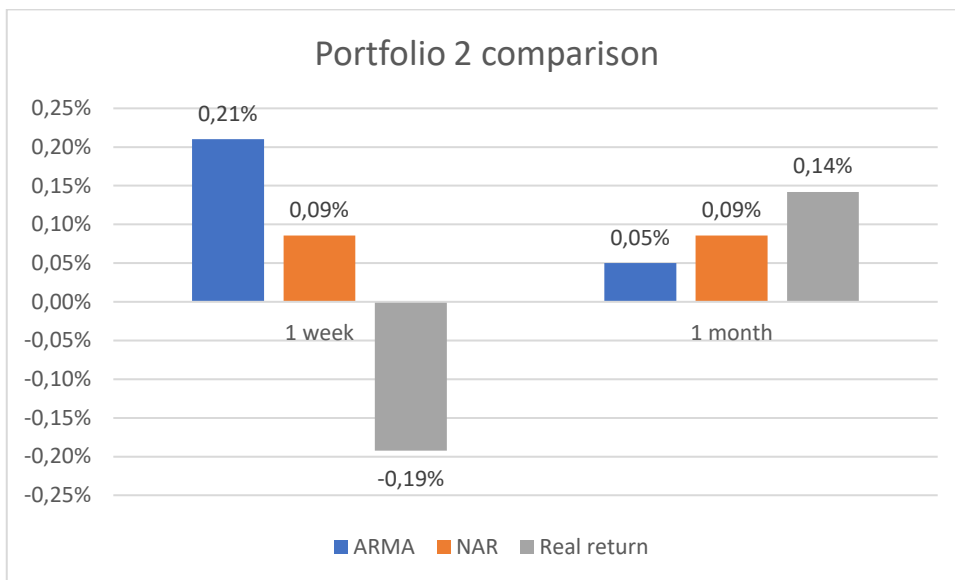


Figure 31. 1-week and 1-month comparison for portfolio 2.

The results for each model performed for portfolio 3 returns are presented in Figure 32 with the real return%. For the 1-week forecasts, the NAR model is clear winner in accuracy as it represents a 16% MPE. The lowest of all the forecasts. For the 1-month figures, the higher return is yet again predicted with the NAR model with 0,08% difference. However contrary to the previous portfolios, the MSE is lower with the NAR model. This indicates that the NAR model is a better fit for the portfolio 3 returns. In addition to this, the MPE shows that the ARMA model is slightly more accurate than the NAR model. As accuracy is important in forecasting, the ARMA model would be more suited. All in all, for portfolio 3, the NAR model gives more accurate forecasts for the shorter time period and ARMA for the longer time period.

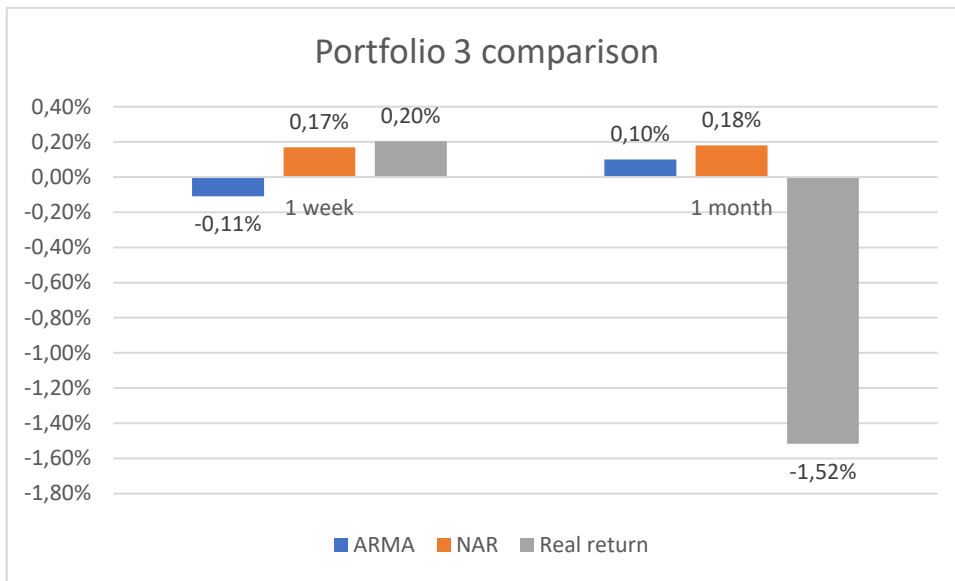


Figure 32. 1-week and 1-month comparison for portfolio 3

Figure 33 presents the NAR and ARMA model results for portfolio 4 with the real return %. For the 1-week time period, the higher return is predicted with the NAR model. However, the MPE shows that the more accurate model is the ARMA model with 80%.

With the 1-month time period, the higher return is also forecasted with the NAR model. This continues the similar pattern to the other portfolios. The return % difference between the two models is 0,05%. From the investor's perspective the NAR model result would be the most ideal, as the forecasted return is higher. However, the ARMA model has a lower MSE, making it a better fit. This is also supported by the accuracy measure MPE, that is lower with the ARMA model. Nevertheless, the difference in the MSE and MPE between these models is quite small and could be debatable if that small of a difference really gives a better fit or not. For the sake of comparison, even the smallest difference counts. In conclusion when looking at the values for portfolio 4, it is clear that the ARMA model prediction is the most accurate.

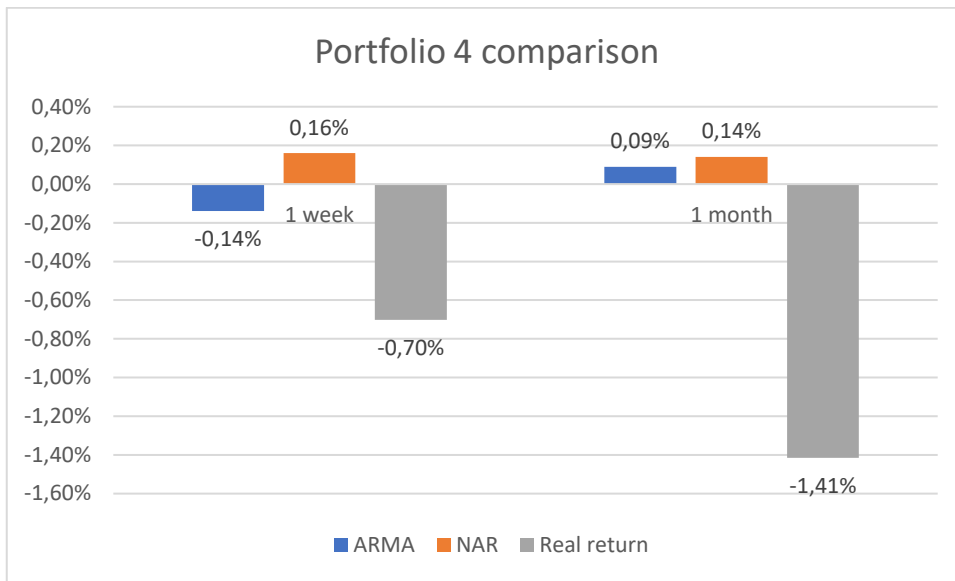


Figure 33. 1-week and 1-month comparison for portfolio 4

The OMXH25 model results are presented in Figure 34. Starting with the 1-week figures, it is clear that the ARMA model forecast is quite off base. The MPE is really high 940% and the return is negative compared to the real return. The NAR model is closer to the real return and thus gives more accurate forecast.

For the 1-month period the difference between the ARMA forecast and real return is quite big. The MPE again indicates that the ANN would be the more accurate model. Out of all the portfolios these is the highest return% difference in the models. Furthermore, for the 1-week period, the real return is a positive surprise as the forecast are both negative. For the 1-month period it is quite contrary because the real return turned out to be much lower than forecasted. The MSE is smaller with the NAR model, which indicates that the more accurate forecast for this data is performed with the NAR model. In conclusion both time periods for the index are more accurately forecasted with the NAR model.

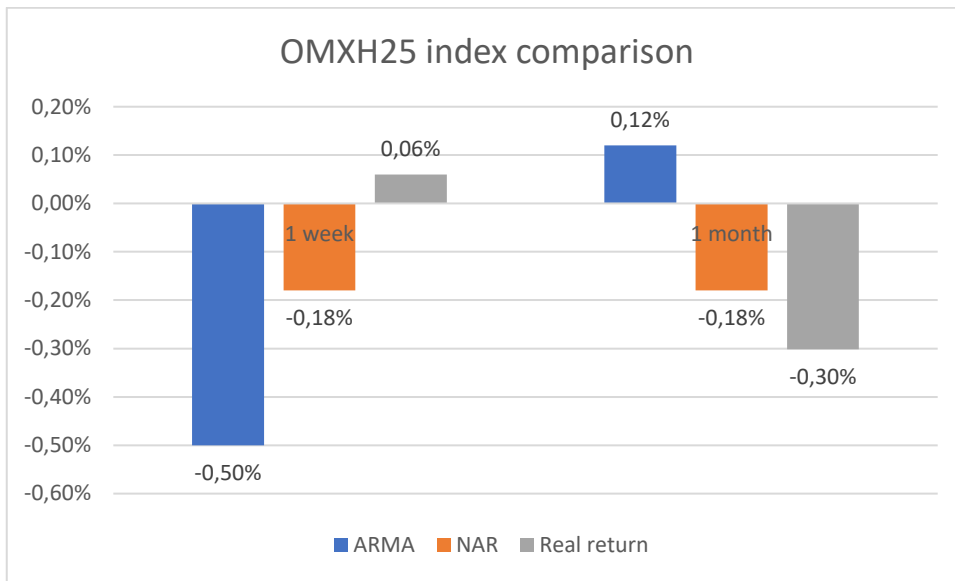


Figure 34. 1-week and 1-month comparison for OMXH25

In conclusion, all the ARMA models and NAR models were not that accurate compared to the real returns. When looking at the financial characteristic of each portfolio and index and the best suited model, it seems that there are inconclusive results. Firstly, looking at the 1-week results. Portfolio 1 and 2 have similar financial characteristics and for both the most accurate forecast model is the NAR model. Then again for portfolio 3 and 4 that are also similar have respectively NAR and ARMA as the most accurate. For the 1-month period the results were the opposites. For portfolio 1 and 2 the most accurate model was respectively the ARMA and NAR models. Thus, for portfolio 3 and 4 the most accurate model being the ARMA model.

Additionally, when comparing the return % of portfolios and index in ARMA models, it seems that there is a connection between the similarities if financial characteristics and return %. In NAR models similar connection was not visible or clear but could not be definitively ruled out.

Difference in MSE is very small between all of the models. But as the goal was to find the most accurate, every difference count. Based on only the MSE, it is clear that for these portfolio and index returns the ARMA model for portfolio 1 would be the best. Furthermore, based on only the return %, NAR model for portfolio 3 would be the best. In both NAR and ARMA models, based on the return % and MSE, the best portfolio for an investor was deemed to be portfolio 3.



However, the accuracy is as important than the fit and forecasted return amount. Based on the MPE of the forecasts presented in Table 10, when comparing the two models, ARMA and NAR, the most accurate models for almost all portfolios and index is with NAR models in the shorter time period. The only standout is portfolio 4, that was presented with more accurate results with ARMA models, in both time periods. However, as the ARMA models were all based on a white noise time series, this result seems to be just coincidence. For the 1-month time period the accuracy is quite even, with the ARMA model being the most accurate for 3 portfolios and the NAR model being the most accurate for portfolio 2 and index. Looking at both time periods, the NAR model turned out to be the most accurate, with it representing most of the accurate results. As expected, the NAR model gave more accurate results than the forecasts from the ARMA models.

From an investors point of view, based on only the accuracy compared to the real returns, the best option would be portfolio 3 for 1 week. For a 1-month investment period, the best options are portfolio 2 or the index.

Table 10. Model MPE's

	<b>1 WEEK ARMA MPE</b>	<b>1 WEEK NAR MPE</b>	<b>1 MONTH ARMA MPE</b>	<b>1 MONTH NAR MPE</b>
<b>Portfolio 1</b>	147 %	<b>94 %</b>	<b>100 %</b>	108 %
<b>Portfolio 2</b>	209 %	<b>145 %</b>	65 %	<b>40 %</b>
<b>Portfolio 3</b>	154 %	<b>16 %</b>	<b>107 %</b>	112 %
<b>Portfolio 4</b>	<b>80 %</b>	123 %	<b>106 %</b>	110 %
<b>OMXH25 INDEX</b>	940 %	<b>403 %</b>	140 %	<b>40 %</b>

## 7 CONCLUSION AND DISCUSSION

This final chapter of the thesis is divided into three parts: The first part is focused on summarizing the study results by answering the sub questions, the second part summarises the main question and resulting deductions, and finally the third part will conclude by discussing the limitations and possible future research on this topic.

### 7.1 Study results for sub questions

As this study is finalized, it has comprehensive research and detail implemented in conducting a viable investment plan using AI method. There were a lot of steps and methods used to achieve the goal, but none were unnecessary in answering the sub questions and finally the main question of this study. This sub chapter aims to answer the 7 sub questions by summarizing the study results obtained in the Empirical research chapter.

#### ***What type of portfolios can be formed with SOM clustering technique by using 9 financial characteristics of target stocks?***

Chapter 6.2.1 of this study explored the forming of portfolios based on chosen financial characteristics. As it is an explored research field, it was interesting to see how the chosen 25 stocks would divide into groups/portfolios. Based on the chosen financial characteristics of the stocks/companies, running the SOM settled on four separate clusters. Based on their similarity or dissimilarity running the SOM could have led to unclear clusters, which would have made the premise for this study unattainable. Luckily a difference between the stocks was noticed and SOM as an AI proxy was able to form 4 separate portfolios. This AI clustering method is seen, based on this study and several before that, as a viable method in forming portfolios, without having to go through every stock individually through. Especially when diversification is based on having stocks from different fields and having financially similar stocks in one portfolio.

#### ***How can the optimization tool be used to minimize the risk in each portfolio?***

Chapter 6.2.2. introduced an enhancement tool for the SOM formed portfolios. Even though the formed portfolios have similar type of stocks in them, it is not practical to

have them carry equal weights. The comparison between the equally weighted and minimum variance portfolio proved that. The return and risk would be evenly distributed between the stocks and this would not be an ideal investment goal. The optimisation tool formed in Excel Solver tool was a fascinating addition to the study. It shows the investor a way to choose the amount of stocks to add to the portfolio in order to have minimum risk without compromising the return level.

Performing the optimisation tool for the portfolios based on their historic returns fortunately brought forth vast results. For portfolio 1, that had 6 stocks to begin with was assigned by the optimisation tool 52% for Sampo and the rest evenly for the other three stocks. In portfolio 3 with 4 stocks, the optimal situation was achieved by assigning all the portfolios almost equal weights and for portfolio 4 with 4 stocks majority of 65% was assigned to Nokia. All these three portfolios achieved the optimal situation by keeping all the stocks with different weights. Interestingly in portfolio 2 the optimisation led to 4 stocks being removed from the portfolio. This portfolio was to begin with the largest of the four with 11 stocks, so optimizing also levelled out the stock amount to be closer to the other portfolios. Furthermore, even though SOM had clustered these stocks in one portfolio, the optimisation tool showed that in order to achieve minimal risk not all stocks should be in the portfolio. This is an interesting addition to the SOM, as it showed not only the different weights but that the optimal situation can change from the starting point.

So, in finality the optimisation tool was able to further optimize the portfolio and the results showed that it was vital. As the investment field has always used optimisation tools, it was interesting to implement one that was manually formed and operated making it a possibility and free to every interested investor.

***Compared to the real returns, which ARMA forecast for portfolio/index gives the closest forecast value? Which ARMA model has the smallest MSE?***

Chapter 6.3.1 introduced the part of the study focused on the forecasting with ARMA models. As the forecasting was conducted on the optimized portfolios returns, the models were constructed with a Matlab script. The script was able to find the most suited models for each portfolio/index considering all the models proved to be based on white noise time-series. Furthermore, performing a forecast based on that for 7

and 30 days ahead. The fit of the models based on the MSE was also deemed good. The MPE acted as measure for the accuracy of the models compared the real returns. The most accurate 1-week return % was predicted for portfolio 4 with 80% MPE and for 1-month for portfolio 2 with 65% MPE. None of the ARMA forecasts gave very accurate forecasts, however the % difference is quite small. This is as expected as the study implemented white noise time series for the ARMA model forecasts.

***What differences can be detected between the ARMA forecasted returns?***

In addition to the forecast accuracy and model fit, the financial characteristics and forecasted returns were compared. The highest return for the ARMA models was predicted for the index for both time periods, which was not a surprise as it has the most diversification with high and low risk stocks and acts as a benchmark for the market. Interestingly when comparing all the forecasted returns, it was apparent that for portfolios that had similar financial characteristics, the return % was also similar. It seemed that for low risk portfolios 1 and 2 the return % was similar and for high risk portfolios 3 and 4 they were similar. It was a clear difference between them and amazing to see that the ARMA models reflect the underlying stocks and their returns so clearly in the predictions. However, as white noise time series were used, this could have been a coincidence.

***Compared to the real returns, which ANN forecast for portfolio/index gives the closest forecast value? Which ANN model has the smallest MSE?***

Chapter 6.3.2 introduced the part of the study which showcased on forming the ANN models and forecasting the returns 7 and 30 days ahead. This was also performed in Matlab and with the Neural Network toolbox, a suitable model was found for each portfolio/index return. For the 1-week period portfolio 3 was the most accurate with 16% MPE. This prediction was also the most accurate of all the forecasts. For the 1-month prediction the most accurate was a tie between the index and portfolio 2. None of the forecasts are however accurate enough to be deemed a great forecast.

***What differences can be detected between the ANN forecasted returns?***

The highest NAR return for the portfolios/index was forecasted for portfolio 3. However, when looking at the differences between the results for these models, not a clear similarity was detected as in the ARMA models. To begin with, the low risk vs high risk portfolios did not have clear similarities in the return % forecast. For example, portfolio 2 had over four times higher predicted returns as the similar low risk portfolio 1. In addition to this the OMXH25 index had negative forecasted return %, when portfolio 3 and 4 that had almost the same beta on volatility had a positive forecasted return%. According to these results, the differences between the portfolios/index are quite random and do not clearly indicate a connection. This is a fascinating result, as the models gave good fit results, but would not base the forecasted returns clearly on the stocks financial characteristics as the ARMA models. The reason for such vast difference between the models would be an interesting study and could be done as a continuation for this study.

***Which model provides the most accurate forecast for each portfolio/index?  
What common factors/financial characteristics do they have?***

Chapter 6.3.3 compared the accuracy of the models and their return prediction. The main assumption before starting the study was that the AI models would present a more accurate forecast. Both ARMA models and NAR models provided accurate forecasts. For the shorter time period of 1-week, the NAR model was the most accurate based on the MPE for 3 portfolios and the index. The only differencing portfolio was portfolio 4, that had ARMA as the more accurate model. For the longer time period the difference between the accuracies of portfolios and index was not that clear. The NAR model was the more accurate model for portfolio 2 and index and the ARMA model more accurate for portfolio1, portfolio 3 and portfolio 4. Thus, it would seem that the NAR model worked better for the shorter time period and for the longer both performed well. It is important to note that forecasting the ARMA models indicated to be based on white noise time-series, so in that essence the NAR model can be deemed to be the more accurate model. It was surprising to see that there was not much difference in the accuracies, but that there were enough to provide a comparison base for the purpose of this study. The NAR models predicted the highest returns and coincidentally the ARMA model provided the best accuracy.

As the return % and MSE were overall compared, Portfolio 3 received the highest predicted return % and lowest MSE. When comparing the accuracy with MPE, portfolio 3 with ANN forecast was also the most accurate in the 1-week period. For the 1-month period portfolio 2 or index with ANN forecast were the most accurate. When looking at the financial characteristics of each portfolio, in ARMA models the predicted % were similar with those that had similar financial characteristics and in NAR models the connection was not clear. Consequently, based on that there isn't a clear indication if the financial characteristics would definitively have same predicted results, as the models gave contradicting results.

## 7.2 Study results for main question

The main question and purpose of the study was to find a solution for forming an investment strategy for Finnish stocks with AI models and would it form a better forecast as the mathematical classical models. Through several steps and different methods, the question was clearly answered. An investment strategy can be formed with AI methods and those can be compared to classical forecasting methods. SOM especially turned out to be quite useful when forming portfolios. The investor can choose based on the results what type of portfolios correspond to their needs. However, when looking at the differences between AI models and classical models, there wasn't a clear "winner". Especially as the classical models were all modelling a white noise time-series. As both models did not give that accurate forecasts, they cannot be considered as the best methods for forecasting in the short term. However, the NAR models gave the investors a forecast closer to the right direction than the ARMA models in most cases. It also turned out to be the more accurate model in the shorter time period. In some portfolios/index the differences between forecasting model was visible. It was interesting to see that based on the model type, the forecasted returns were opposites, especially in the OMXH25 index prediction. In the other portfolios, such difference was not noted. Also, as some differences in the financial characteristics was noted, but not enough to specially specify a clear and sound connection. As this study is quite specific and Finnish market orientated the results are not directly proportional to other markets, but it can be easily used as a benchmark or comparison study in other stock markets.

### 7.3 Limitations and suggestions for future research

As this study was a master's thesis study, some limitations were to be expected when restricting the study area. Implementing the AI models and classical models in the Finnish stock market, is a very specific area of research and due to the small size of the market could perform quite differently than other markets. However, it was important to implement it in the Finnish market, as not many researches have been done in this field. Another limitation to this study was the time period chosen. The goal was to find returns for a short time period and in real time check the real returns. Only a year of daily historic returns was used, and forecast was done 7 days and 30 days ahead. To further develop this study, it would be interesting to see if the performance of the models would change with larger data sets, for example doing the forecasts based on 3 years data and then doing a three-month forecast. A lot of aspects can be changed regarding the time period, but due to time restrictions and study limit it was important to choose the most interesting in the eyes of the investor.

This study has a lot of potential for the research to be continued. Firstly, as stocks and returns are a high interest points in the field of investment, any method to ease investment decision is a welcome one. This study can be implemented in a larger market i.e. the United States and compare to find differences based on market size and volatility. Another aspect to further develop this study would be to use Hybrid forecasting methods to possibly receive more accurate results. Also forming an automated tool installed with all of the method used to ease the usage and understanding for the user would be an interesting task to take.

All in all, this study filled a needed research gap and provides any interested reader an alternative way to explore stocks. Even though the study has negative results, they provide a good base for further research and a way for another study to avoid gaining the negative results. The study also provides the investor with opportunities and a different point of view in investing. The methods presented may not be used as is, but if an inspiration to implement any part of this study is risen, the ultimate goal is achieved.

## REFERENCES

Al-Shiab, M. 2016. The Predictability of the Amman Stock Exchange using the Univariate Autoregressive Integrated Moving Average (ARIMA) Model. *Journal of Economic and Administrative Sciences*. Volume 22, Issue 2. pp.17-35.

Ayodele, A., Adewumi, A. & Ayo, C. 2014. Comparison of ARIMA and Artificial Neural Networks Models for Stock Price Prediction. *Journal of Applied Mathematics*.

Bacao, F., Lobo, V., Painho, M. 2005. Self-organizing Maps as substitutes for K-means clustering. *Computational Science – ICCS 2005*. *ICCS 2005. Lecture Notes in Computer Science*, vol 3516. pp 476-483

Bellman, R.E. 1978. *An introduction to Artificial intelligence: Can computers think?* Boyd & Fraser publishing company.

Bednarek, Z., Patel, P. 2018. Understanding the outperformance of the minimum variance portfolio. *Finance Research Letters*. Volume 24, pp 175-178.

Brentan, B. Meirelles, G. Luvizotto, E. Izquierdo, J. 2018. Hybrid SOM+k-means clustering to improve planning, operation and management in water distribution systems. *Environmental modelling & software*. Volume 106. pp. 77-88

Brooks, C. 2008. *Introductory Econometrics for Finance*. Second Edition. Cambridge. Cambridge University press.

Charniak, E. and McDermott, D. (1985). *Introduction to Artificial Intelligence*. Addison-Wesley.

Cherif, A. Cardot, H. Bone, R. 2011. SOM time series clustering and prediction with recurrent neural networks. *Neurocomputing*. Volume 74, issue 11. pp 1936-1944.



Chu, K.Y. 1978. Short-Run Forecasting of Commodity Prices: An Application of Autoregressive Moving Average Models. IMF Staff Papers 25. pp 90-111.

Clarke, R., De Silva, H., Thorley, S. 2006. Minimum-Variance Portfolios in the U.S. Equity market. The Journal of portfolio management. Volume 33. pp. 10.24.

Datta,K. 2011. ARIMA Forecasting of Inflation in the Bangladesh Economy. IUP Journal of Bank Management. Volume X, issue 4. pp. 7-15.

Deloitte. Artificial Intelligence. The next frontier for investment management firms. 2019. [Online] Available at:

<https://www2.deloitte.com/content/dam/Deloitte/global/Documents/Financial-Services/fsi-artificial-intelligence-investment-mgmt.pdf> [Accessed 20.3.2019]

Efficient Markets Hypothesis - Understanding and Testing EMH. [Online] Available: at:

<https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/efficient-markets-hypothesis/> [Accessed 18.2. 2019].

Fama, E., 1970. Efficient Capital Markets: A Review of Theory and Empirical Work. The Journal of Finance, 2(25), pp. 383-417.

Fama, E., 1991. Efficient Capital Markets: II. The Journal of Finance, 46(5), pp. 1575-1617.

Fama, E., 1998. Market Efficiency, Long-Term Returns and Behavioral Finance. Journal of Financial Economics, 49, pp. 283-306.

Graham, B. 2006. The intelligent investor. Fourth Revised Edition. Harper Collins. pp 1.

Grossman, S. J. & Stiglitz, J. E., 1980. On the Impossibility of Informationally Efficient Markets. *American Economic Review*, 70(3), pp. 393-408.

Hang, X., Tang, H & Liao, Y. 2009. Time series prediction based on NARX neural networks: An advanced approach. 2009 International Conference on Machine Learning and Cybernetics. 12-15 July 2009. Hebei, China

Haugen, R.A., Baker N.L. 1991. The efficient market inefficiency of capitalization-weighted stock portfolios. *J. Portfolio Manage.*, 17 (3), pp. 35-40.

Haugeland, J. 1985. *Artificial Intelligence: The very idea*. MIT press.

Huan, S. Wu, T. 2010. Integrating recurrent SOM with wavelet-based kernel partial least square regressions for financial forecasting. *Expert Systems with Applications*. Volume 37, issue 8. pp 5698-5705.

J.P.Morgan. 2017. Informing Investment Decisions Using Machine Learning and Artificial intelligence. [Online] Available at: <https://www.jpmorgan.com/global/cib/research/investment-decisions-using-machine-learning-ai> [Accessed 1.5.2019]

Jain, A. Data clustering: 50 years beyond K-means. 2010. *Pattern Recognition Letters*. Volume 31, Issue 8. pp 651-666.

Jian, C., Du, J., An, Y. 2019. Combining the minimum-variance and equally weighted portfolios: Can portfolio performance be improved. *Economic modelling*. Volume 80. pp 260-274

Karsoliya, S. 2012. Approximating number of hidden layer neurons in Multiple Hidden layer BPNN architecture. *International journal of engineering trends and technology*. Volume 3, Issue 6. pp 714-715

Kataria, R. Dannemiller, D. 2018. Will artificial intelligence transform investment research. Deloitte. [Online]Available at:  
<https://www2.deloitte.com/us/en/pages/financial-services/articles/will-artificial-intelligence-transform-investment-research.html>[Accessed 1.5.2019]

Khan, A., Bandopadhyaya, T.K. & Sharma, S. 2010. SOM and Technical Indicators Based Hybrid Model Gives Better Returns on Investments as Compared to BSE-30 Index. Third International Conference on Knowledge Discovery and Data Mining. pp. 544-547.

Kohonen, T. 2013. Essentials of the self-organizing map. Neural networks. Volume 37. Pp 52-65.

Kurzweil, R. (1990). The Age of Intelligent Machines. MIT Press.

Lasri, R. 2016. Clustering and Classification Using a Self-Organizing MAP. SAI Computing Conference 2016. 13-15 July 2016. London, UK.

Leković, M., 2018. Evidence for and Against the Validity of Efficient Market Hypothesis. Economic Themes, 56(3), pp. 369-387

Mane, A. Pulugurtha, S. 2018. Link level Travel time prediction using artificial neural network models. 21st International Conference on Intelligent Transportation Systems (ITSC). pp 1487 – 1492.

Manish, K. & Thenmozhi, M. 2012. Stock Index Return Forecasting and Trading Strategy Using Hybrid ARIMA-Neural Network Model. International Journal of Financial Management. Volume 123, Issue 1. pp. 1-14

Markowitz, H. Portfolio Selection. The journal of finance, Vol. 7, No. 1. (Mar.,1952) pp.77-91.

Matlab. 2019.

Mingoti,S.A, Lima, J.O. 2006. Comparing SOM neural network with Fuzzy c-means, K-means and traditional hierarchical clustering algorithms. European Journal of Operational Research. Volume 174, Issue 3. pp 1742-1759

Momin, B. Chavan, G. 2017. Univariate Time series Models for forecasting stationary and Non-stationary Data: A Brief Review. Information and Communication Technology for Intelligent Systems. Volume 2. pp 219-226.

Moskowitz, T. Ooi, Y. Pedersen, L. Time series momentum. 2012. Journal of financial economics. Volume 104. Issue 2. p. 228-250.

Nair, B.B. Kumar, P.K. Sakthivel. N.R. Vipin, U. 2017. Clustering stock price time series data to generate stock trading recommendations: an empirical study. Expert systems with applications. Volume 70. pp 20-36.

Nanda, S.R. Mahanty, B. Tiwari, M.K. 2010. Clustering Indian stock market data for portfolio management. Expert systems with Applications. Volume 37, issue 12. pp 8793-8798.

Nanda,S.R., Mahanty, B. Tiwari,M.K. 2010. Clustering Indian stock market data for portfolio management. Expert Systems with Applications. Volume 37, Issue 12. pp.8793-8798.

NasdaqOMXNordic. 2018. [Online] Available at :[www.nasdaqomxnordic.com](http://www.nasdaqomxnordic.com) [Accessed 31.12.2018]

Nilsson, N. J. (1998). Artificial Intelligence: A New Synthesis. Morgan Kaufmann

Ozoegwu, C.G. 2019. Artificial neural network forecast of monthly mean daily global solar radiation of selected locations based on time series and month number. Journal of cleaner production. Volume 216. pp 1-13.

Pampalk, E. 2001. Limitations of the SOM and the GTM. University of Vienna. Department of medical cybernetics and Artificial Intelligence. pp 5-9.

Poole, D., Mackworth, A. K., and Goebel, R. (1998). Computational intelligence: A logical approach. Oxford University Press.

Puig-Arnavat, M. Bruno, J. 2015. Artificial neural networks for thermochemical conversion of biomass. Recent advances in thermo-chemical conversion of biomass. pp 133-156.

PwC. Smart Money: AI transitions from fad to future of institutional investing. 2018. [Online] Available at:

<https://www.pwc.com/us/en/industries/financial-services/library/pdf/pwc-fsi-whitepaper-artificial-intelligence-investing.pdf> [Accessed 30.3.2019]

Resta, M. 2012. Graph Mining Based SOM: A tool to Analyze Economic Stability. Applications of Self-organizing maps. Intech. pp 3-8.

Reuters. 2018.

Rich, E. and Knight, K. (1991). Artificial Intelligence (second edition). McGraw-Hill.

Russell, S. Norvig, P. 2009. Artificial Intelligence: A modern Approach. Third Edition. pp. 1-5

Safi, S. 2016. A Comparison of Artificial Neural Network and Time Series Models for Forecasting GDP in Palestine. American Journal of Theoretical and Applied Statistics Volume 5, Issue 2. pp. 58-63.

Salim, N.A., Rahman, T.K., Jamaludin, M.F., Musa, M.F. 2009. Case study of short term load forecasting for weekends. IEEE Student conference on Research and Development. Malaysia.

Scherer, B. 2011. A note on the returns from minimum variance investing. *J. Empir. Finance*, 18 (4), pp. 652-660

Selmi, N., Chaabene, S. & Hachicha, N. 2015. Forecasting returns on a stock market using Artificial Neural Networks and GARCH family models: Evidence of stock market S & P 500. *Decision Science letters*. Volume 4, Issue 2. pp. 203-210.

Severin, E. 2010. Self organizing maps in corporate finance: Quantitative and qualitative analysis of debt and leasing. *Neurocomputing*. Volume 73, issues 10-12. pp 2061-2067.

Siregar, B., Nababan, E., Yap, A., Andayani, U. & Fahmi. 2017. Forecasting of raw material needed for plastic products based in income data using ARIMA method. 2017 5th International Conference on Electrical, Electronics and Information Engineering (ICEEIE). 6-8 Oct. 2017. Malang, Indonesia.

Vesanto, J. 1999. SOM-based data visualization methods. *Laboratory of computer and information science*. Helsinki University of Technology. pp 2. Volume 2014.

Wei, J. 2014. *A Layman's Guide to Financial terms*. University of Toronto Scarborough. pp 1-112.

Widiputra, H. & Christianto, L. 2012. Indonesia stock exchange liquid stocks identification using self-organizing map. 2nd International Conference on Uncertainty Reasoning and Knowledge Engineering. 14-15 Aug. 2012. Jalarta, Indonesia.

Winston, P. H. (1992). *Artificial Intelligence (Third edition)*. Addison-Wesley.

Wunsch, A., Liesch, T., Broda, S. 2018. Forecasting ground water levels using nonlinear autoregressive networks with exogenous input (NARX). *Journal of Hydrology*. Volume 567. pp 743-758.

Zhao, Z. Wang, C. Nokleby, M. Miller, C. 2017. Improving short-term electricity price forecasting using day-ahead LMP with ARIMA models. *IEEE Power & Energy Society General Meeting*. Chicago. pp 1-5.

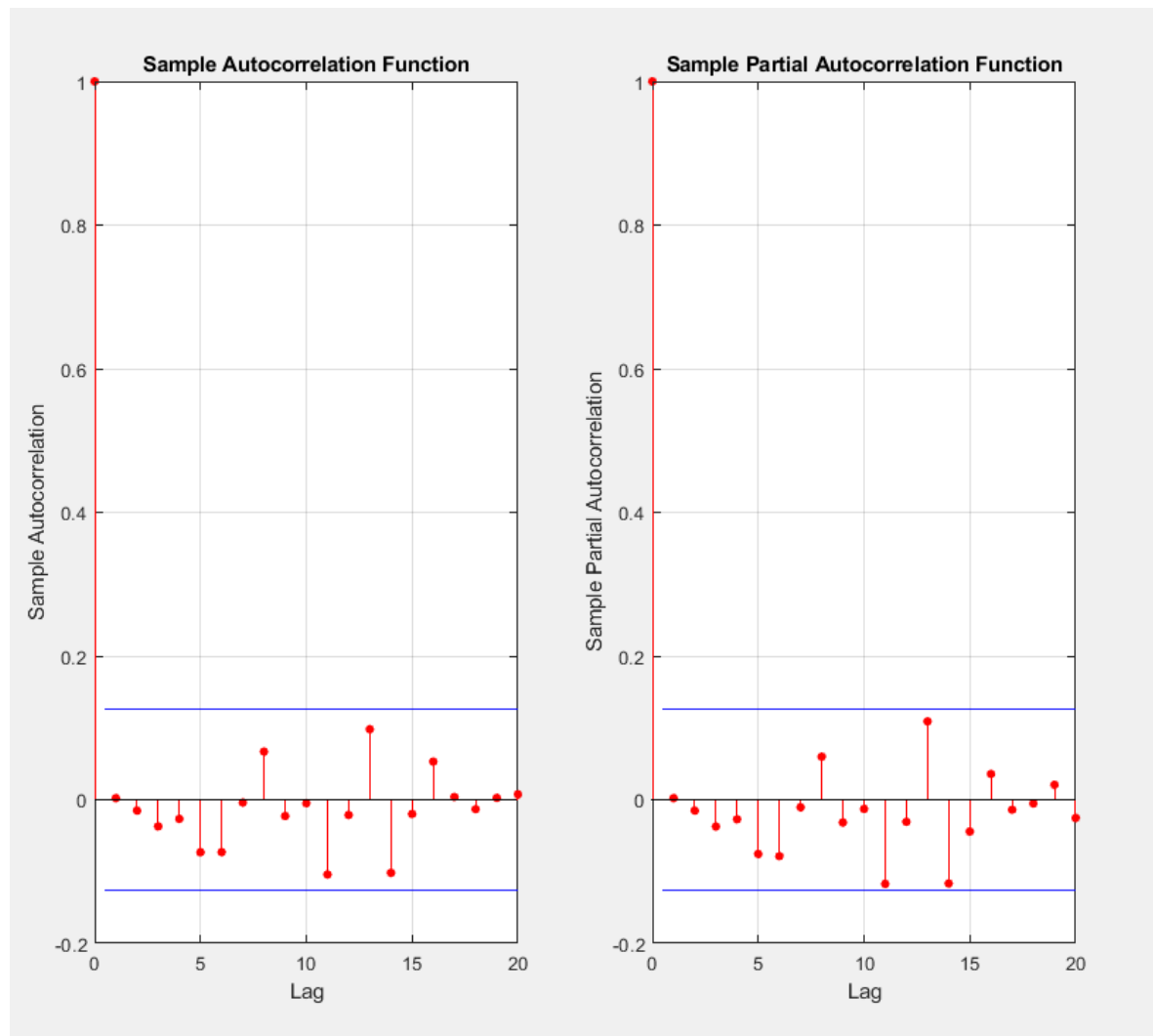
Zheng, F. Zhong, S. 2011. Time series forecasting using an ensemble model incorporating ARIMA and ANN based on combined objectives. *2<sup>nd</sup> International Conference on Artificial Intelligence, Management Science and Electronic Commerce*. pp 2671-2674.

# APPENDIX 1- ARMA model results

Portfolio 1 ARMA model results:

ARIMA(1,0,2) Model (Gaussian Distribution):

	Value	StandardError	TStatistic	PValue
Constant	-0.00045681	0.00090043	-0.50732	0.61193
AR{1}	-0.93333	0.025434	-36.697	8.2432e-295
MA{1}	0.8718	0.064373	13.543	8.7144e-42
MA{2}	-0.1282	0.06048	-2.1197	0.034035
Variance	7.7634e-05	5.6051e-06	13.85	1.2642e-43

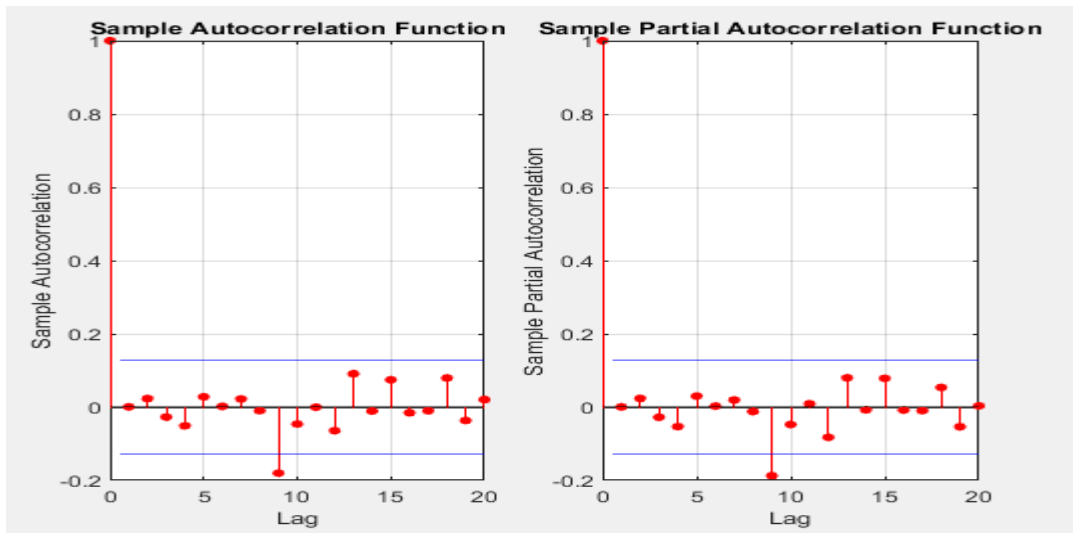




**Portfolio 2 ARMA model results:**

ARIMA(2,0,2) Model (Gaussian Distribution):

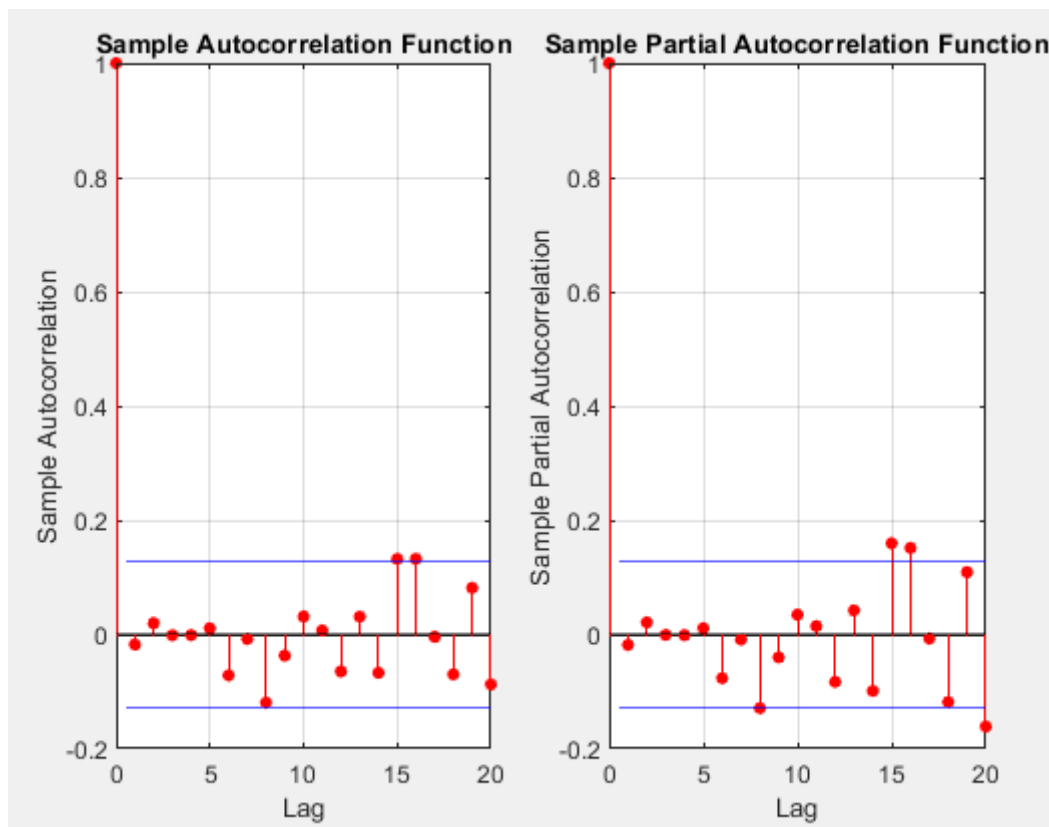
	Value	StandardError	TStatistic	PValue
Constant	8.8015e-05	2.8883e-05	3.0473	0.0023089
AR{1}	-0.055835	0.045756	-1.2203	0.22236
AR{2}	0.85254	0.0489	17.434	4.52e-68
MA{1}	-0.041897	0.045107	-0.92885	0.35297
MA{2}	-0.9581	0.045455	-21.078	1.261e-98
Variance	7.8176e-05	6.8605e-06	11.395	4.4253e-30



**Portfolio 3 ARMA model results:**

ARIMA(3,0,2) Model (Gaussian Distribution):

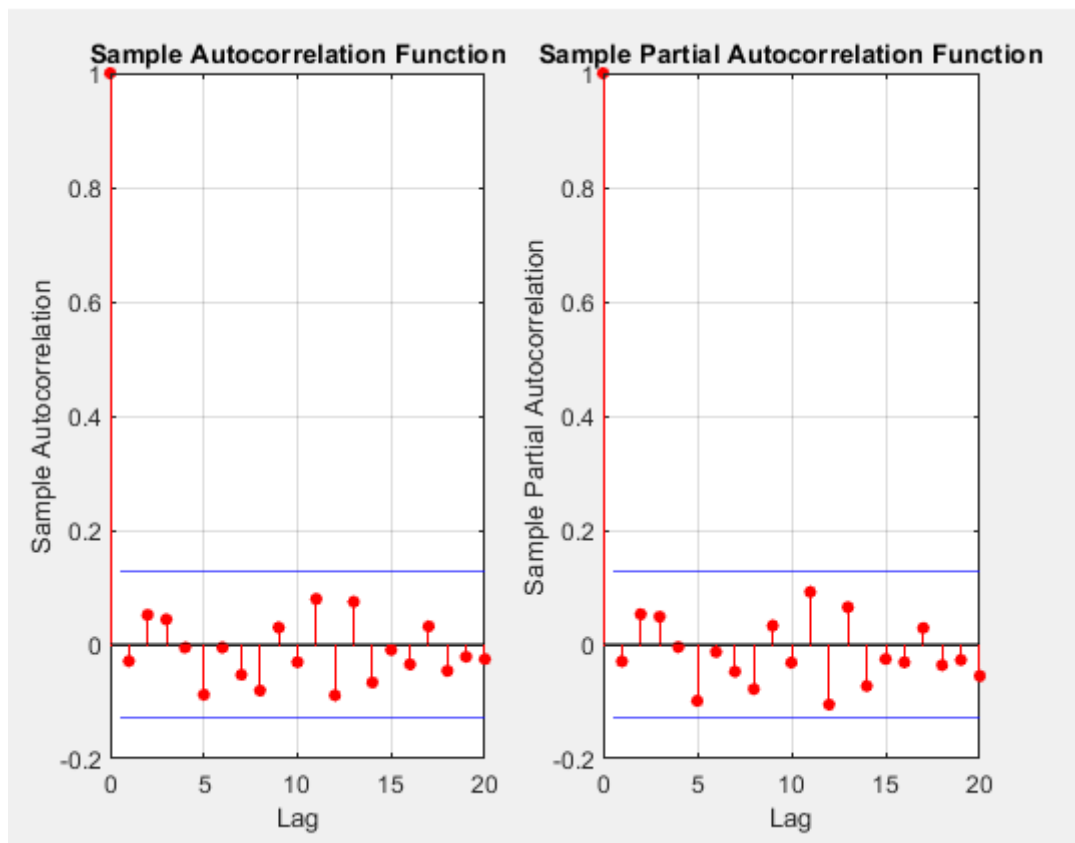
	Value	StandardError	TStatistic	PValue
Constant	0.00017159	0.0014279	0.12016	0.90435
AR{1}	-0.38	0.065042	-5.8424	5.1467e-09
AR{2}	-0.89061	0.024159	-36.864	1.7279e-297
AR{3}	0.07932	0.05756	1.378	0.16819
MA{1}	0.45052	0.025455	17.699	4.2962e-70
MA{2}	1	0.024628	40.605	0
Variance	8.9033e-05	7.8061e-06	11.406	3.9243e-30



**Portfolio 4 ARMA model results:**

ARIMA(3,0,3) Model (Gaussian Distribution):

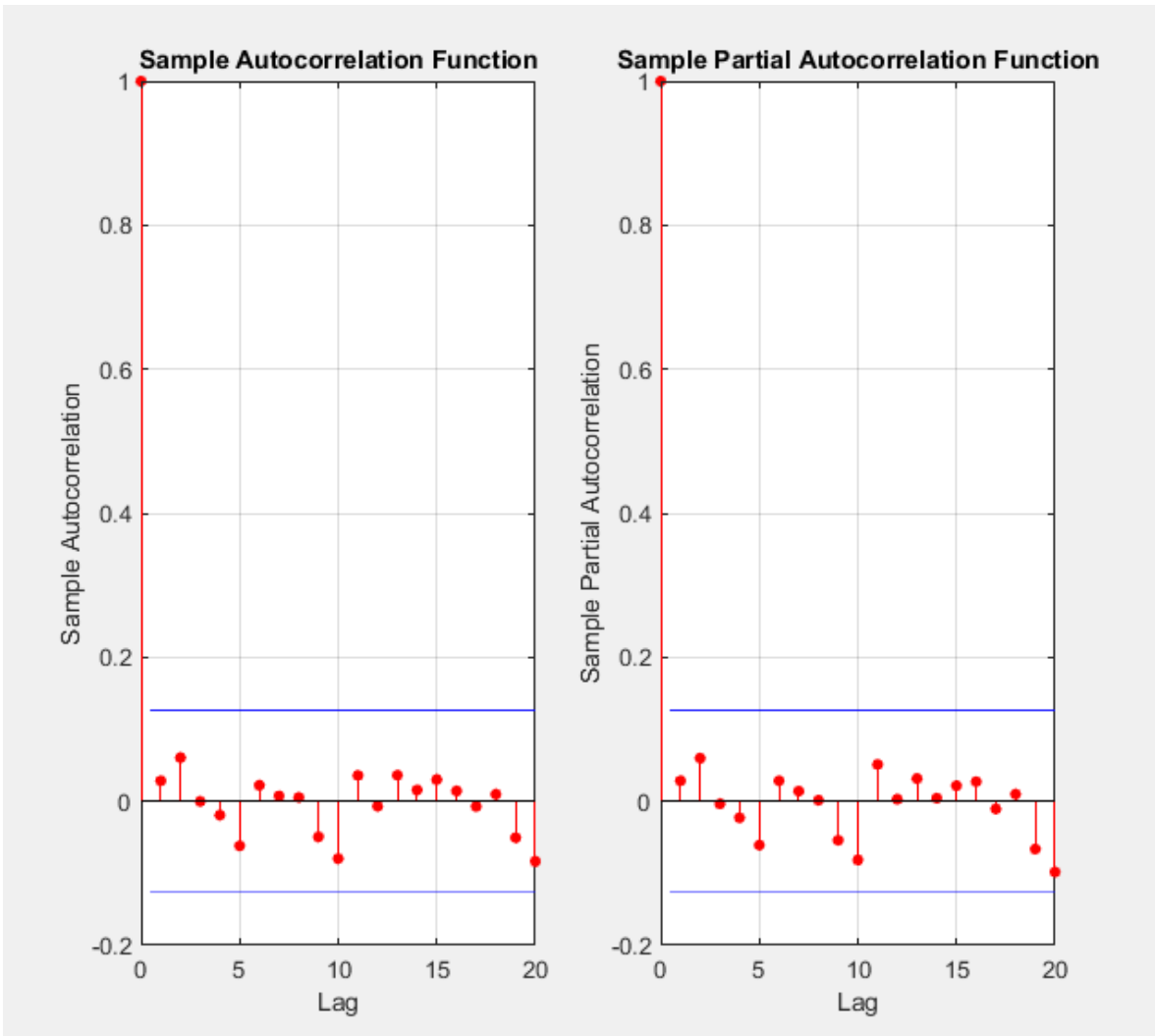
	Value	StandardError	TStatistic	PValue
Constant	-4.9892e-05	0.00085348	-0.058457	0.95338
AR{1}	0.69282	0.21879	3.1666	0.0015423
AR{2}	-1.0672	0.049436	-21.587	2.3697e-103
AR{3}	0.47052	0.19787	2.3779	0.01741
MA{1}	-0.72543	0.22471	-3.2283	0.0012454
MA{2}	1.1136	0.060578	18.383	1.8122e-75
MA{3}	-0.49679	0.21031	-2.3622	0.018167
Variance	0.00020202	1.5057e-05	13.417	4.8234e-41



**OMXH25 ARMA model results:**

ARIMA(3,0,3) Model (Gaussian Distribution):

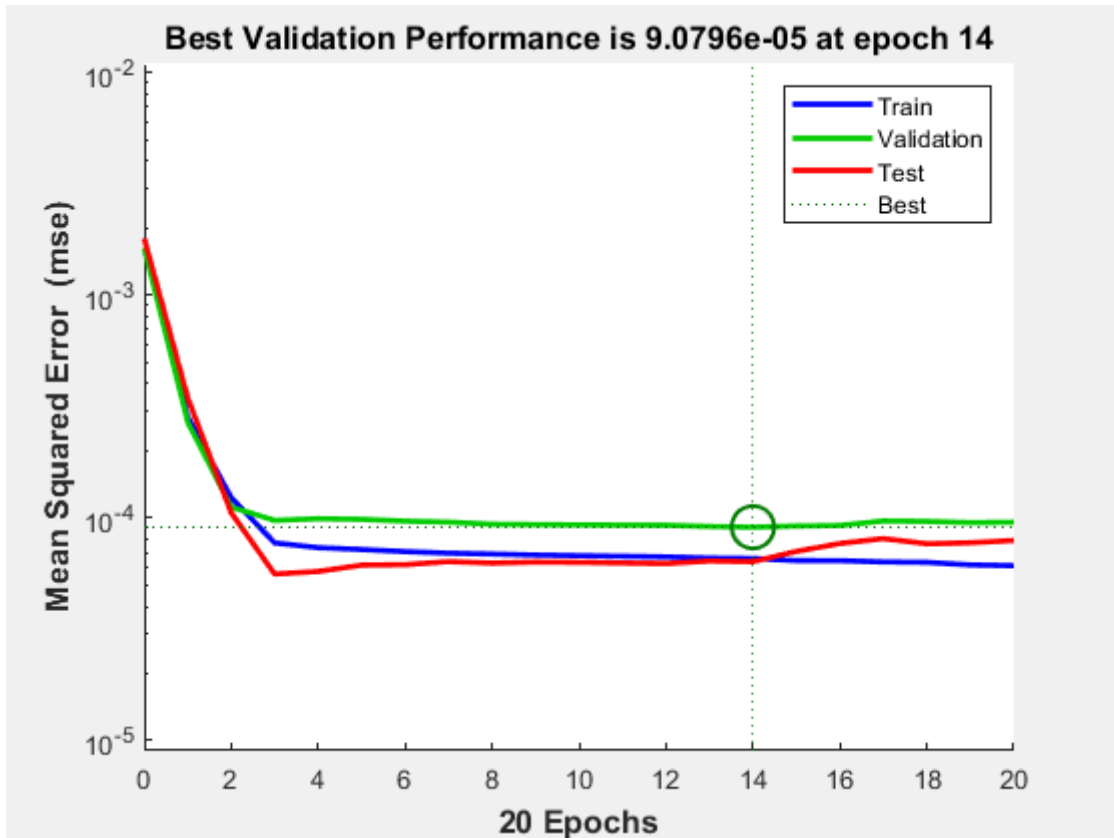
	Value	StandardError	TStatistic	PValue
Constant	0.0003822	0.00014506	2.6347	0.0084211
AR{1}	0.65322	0.057569	11.347	7.704e-30
AR{2}	-0.66951	0.052803	-12.679	7.6851e-37
AR{3}	0.84784	0.036879	22.989	5.9442e-117
MA{1}	-0.77364	0.028666	-26.988	2.0342e-160
MA{2}	0.75145	0.034742	21.629	9.4911e-104
MA{3}	-0.97781	0.029431	-33.224	4.8587e-242
Variance	0.00038017	2.4419e-05	15.568	1.1979e-54



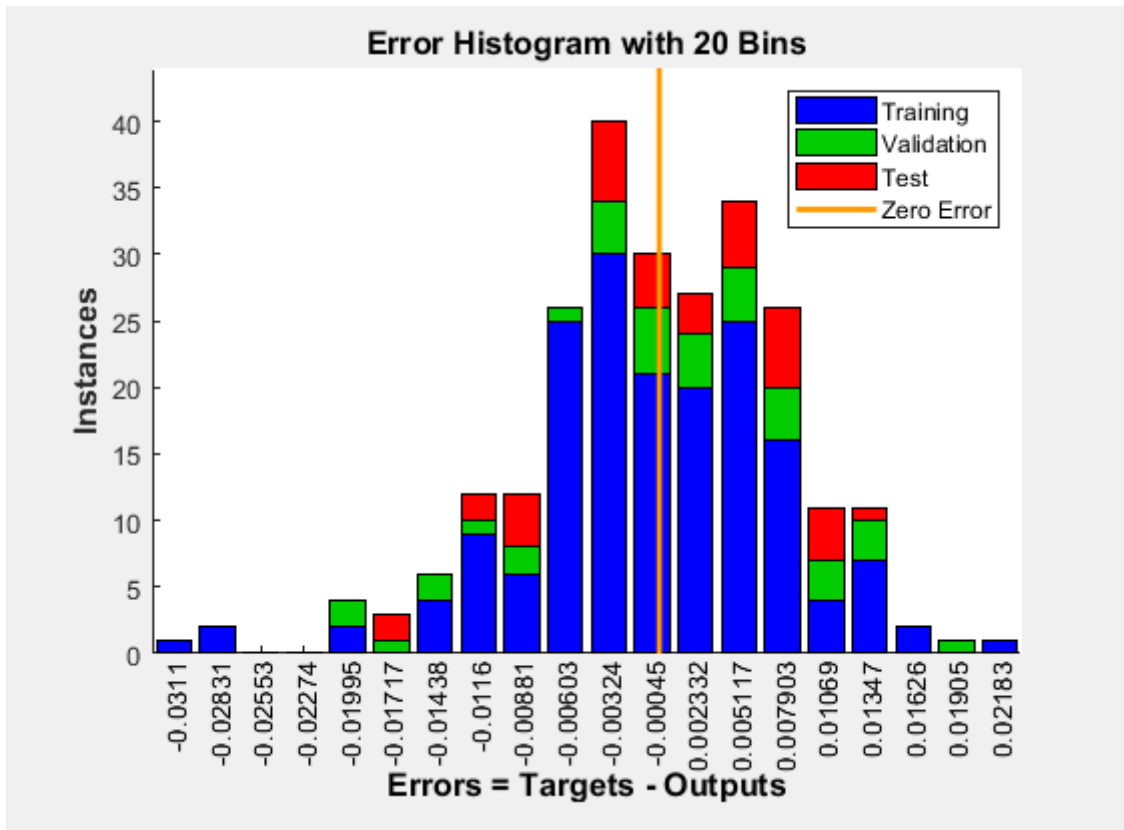
## APPENDIX 2- NAR model results

Portfolio 1 NAR results:

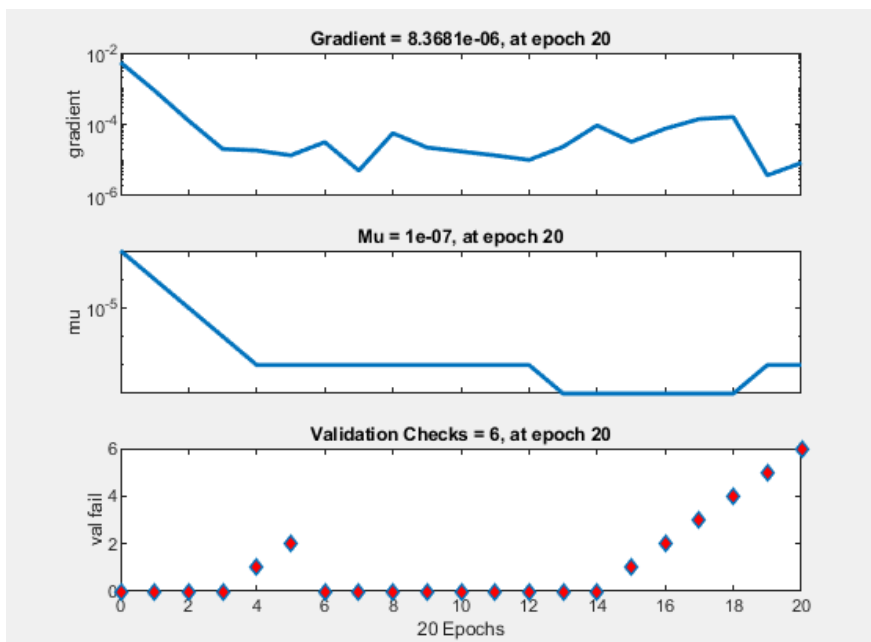
Performance plot:



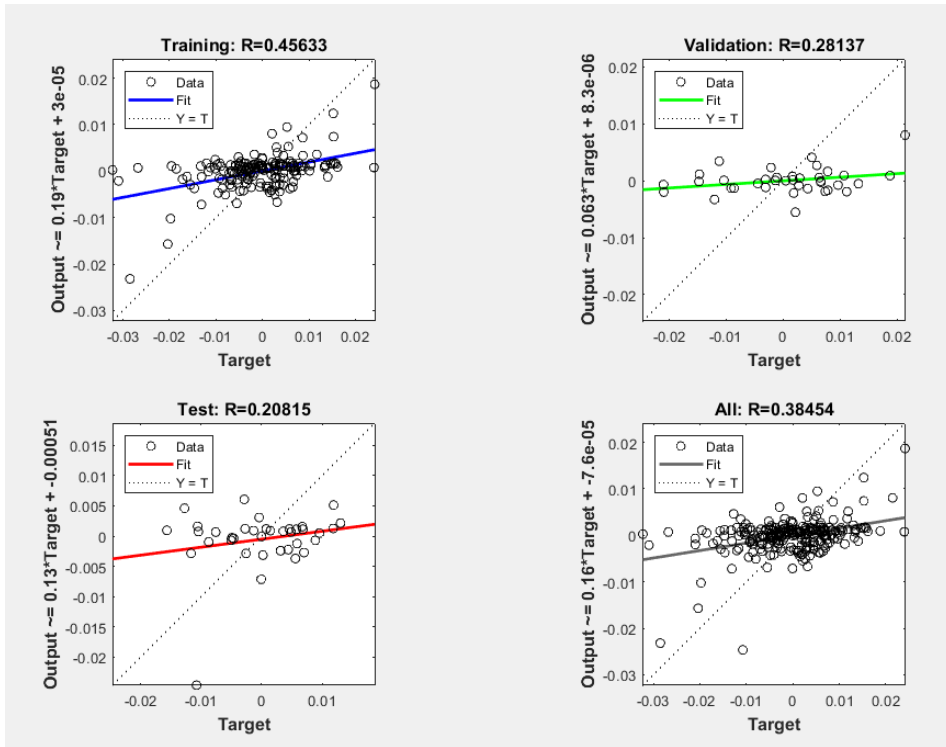
Error histogram:



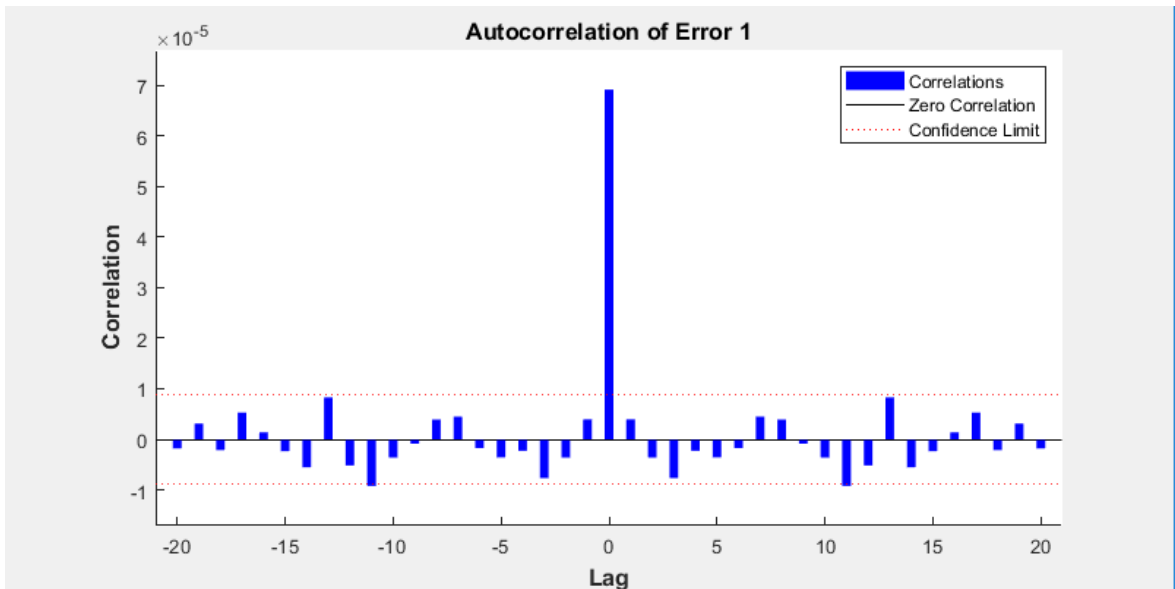
Training state:



Regression:

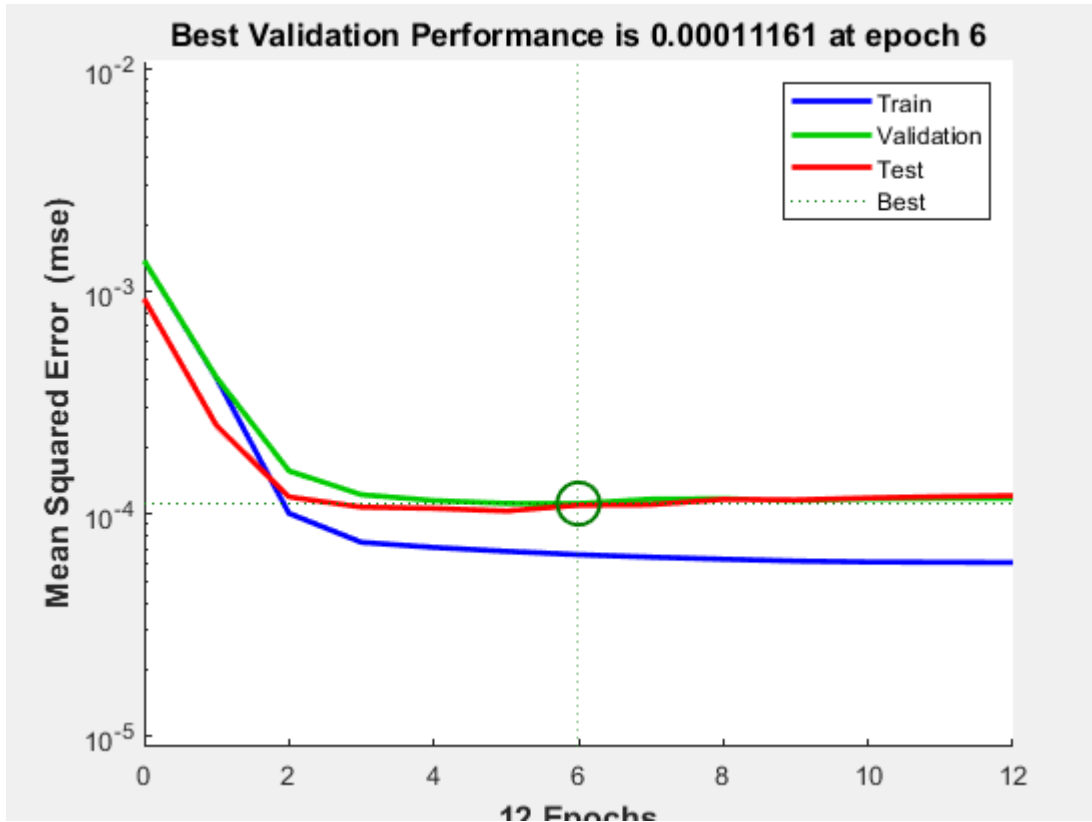


Error autocorrelation:

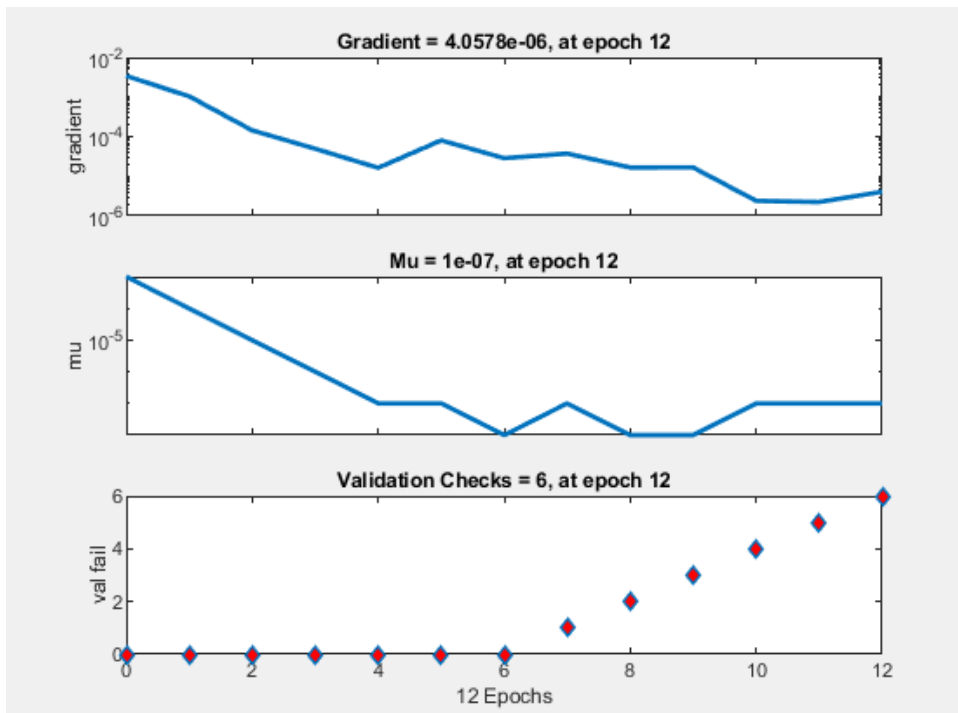


Portfolio 2 NAR results:

Performance:

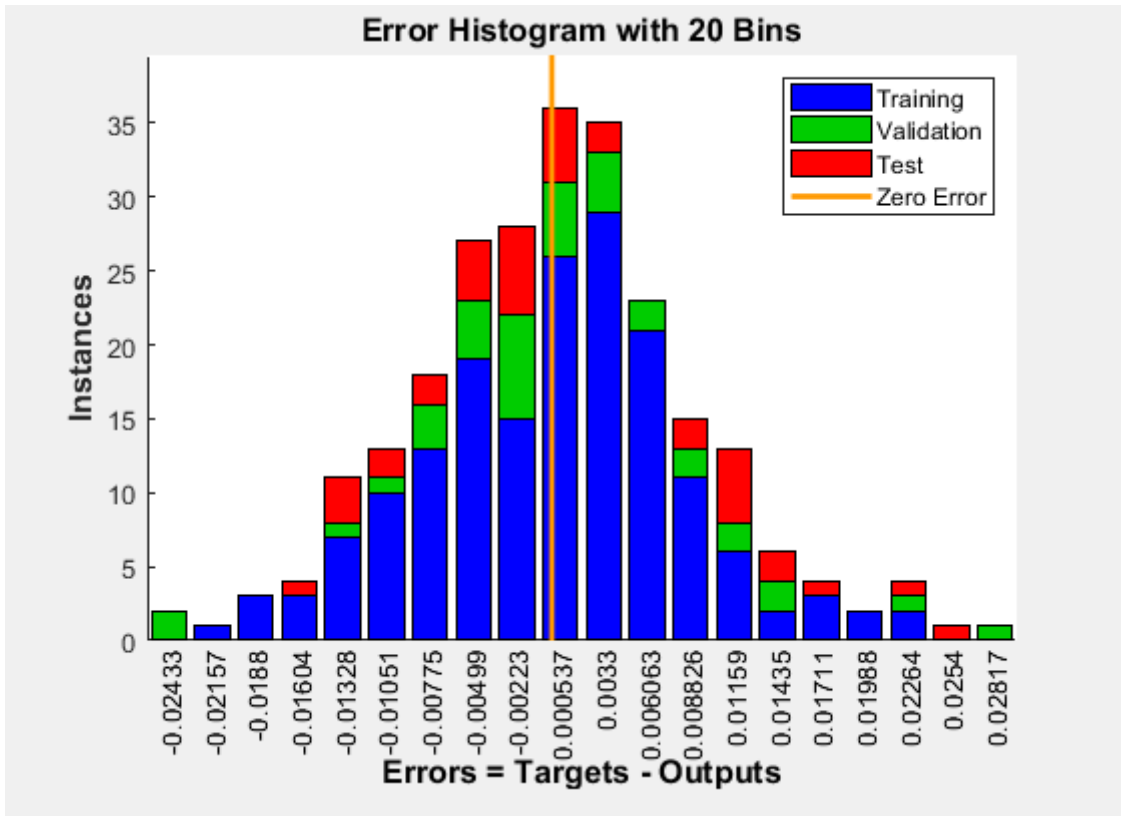


Training state;

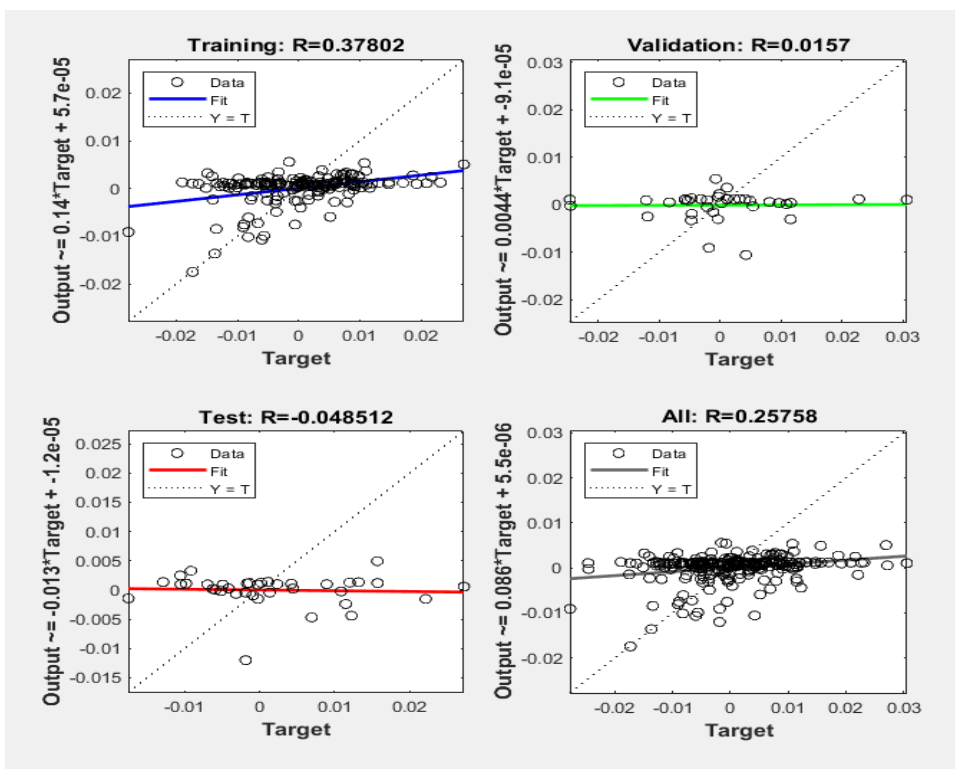




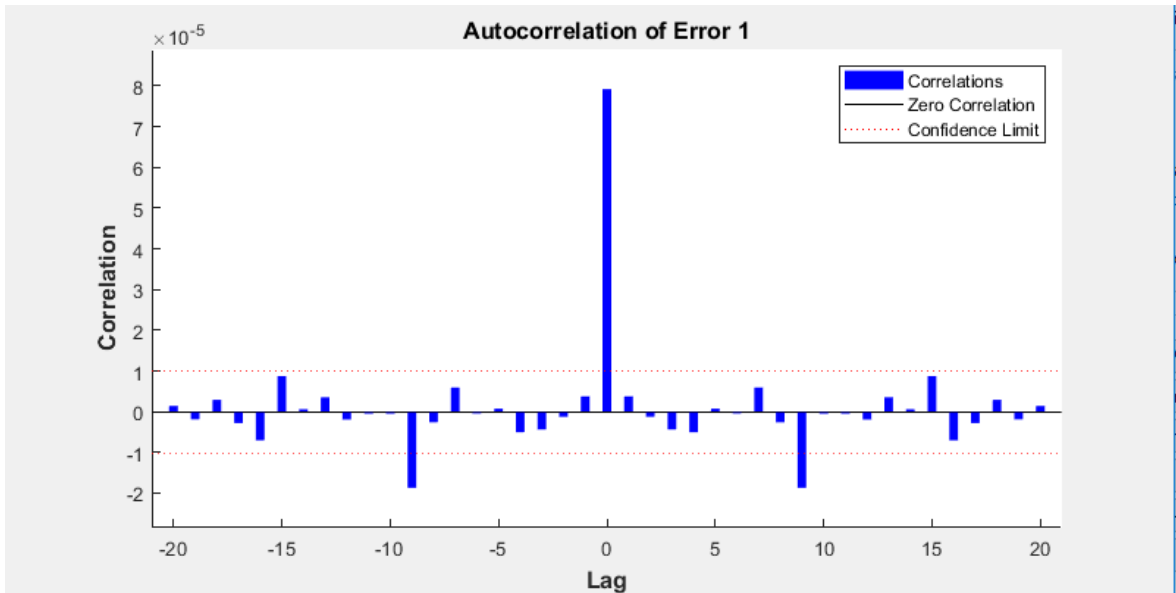
Error histogram:



Regression:

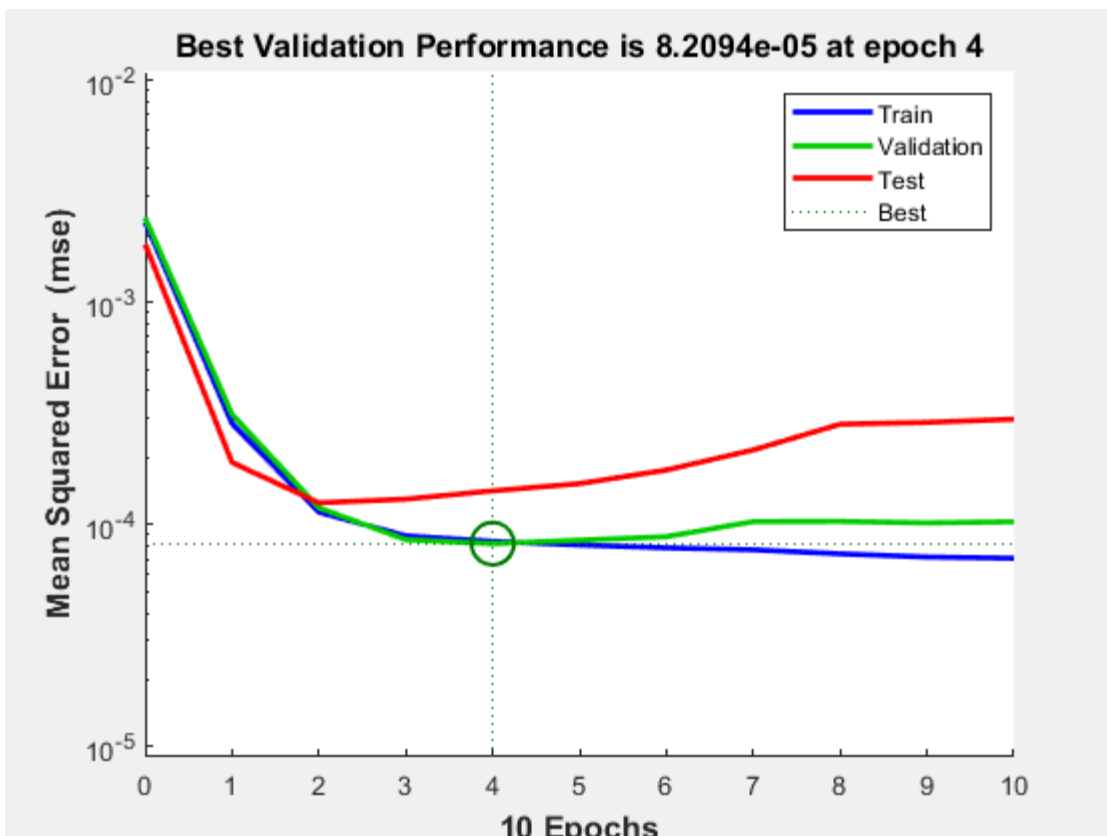


Error autocorrelation:

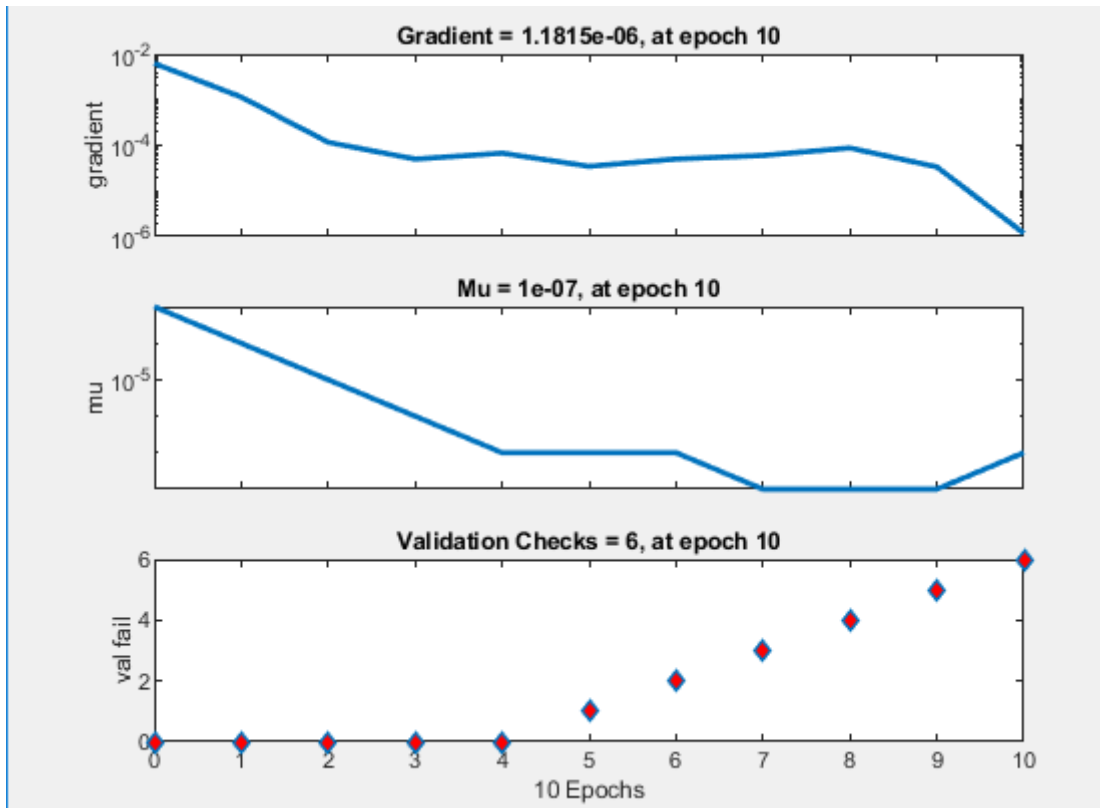


Portfolio 3 NAR results:

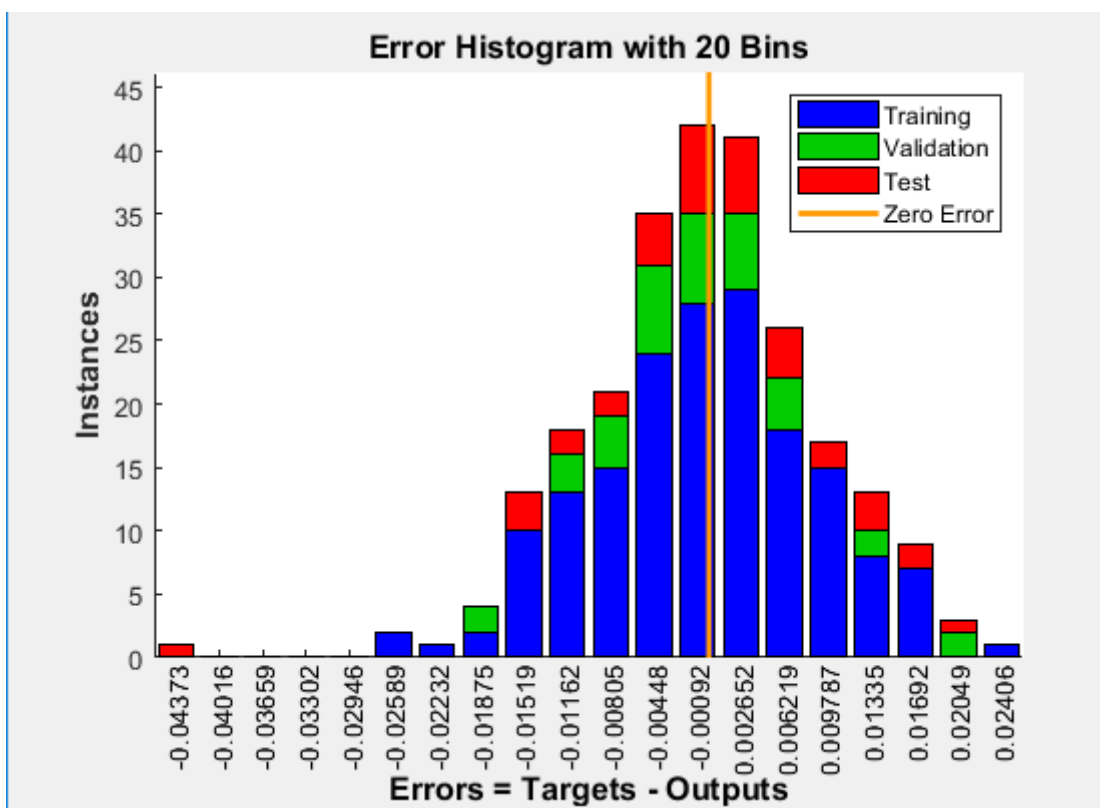
Performance:



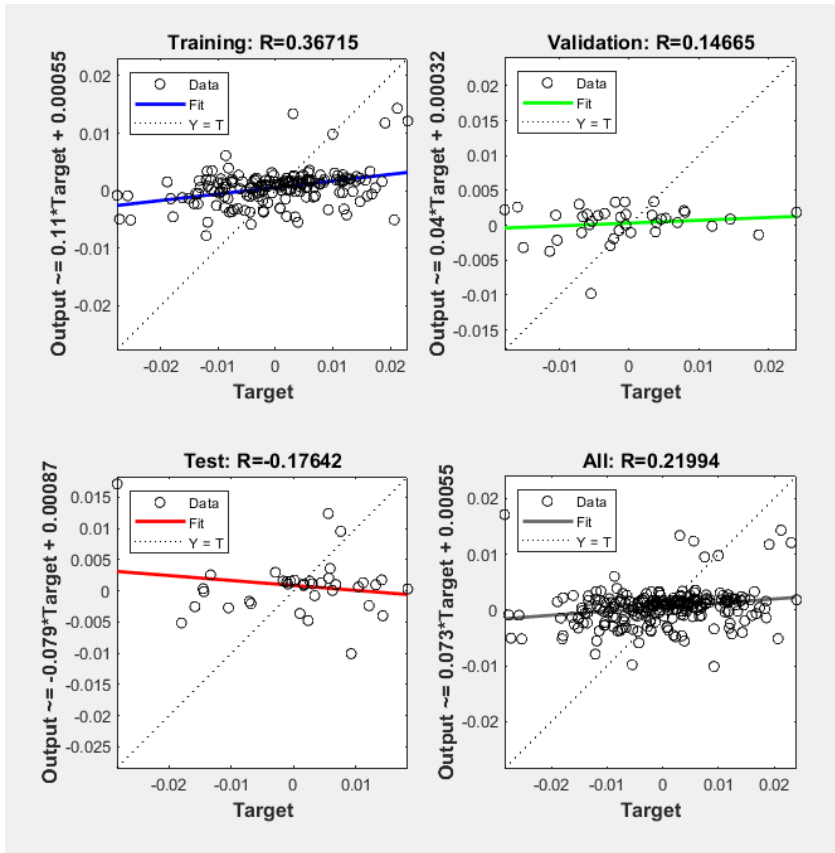
Training state:



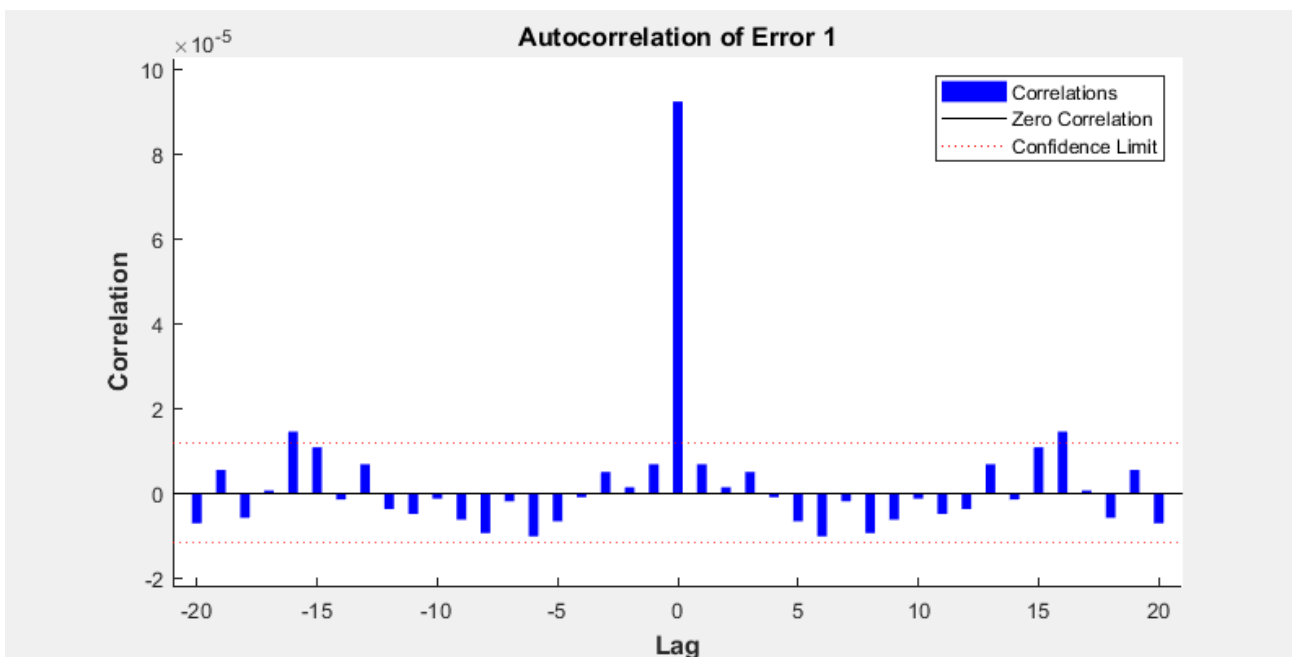
Error histogram:



Regression:

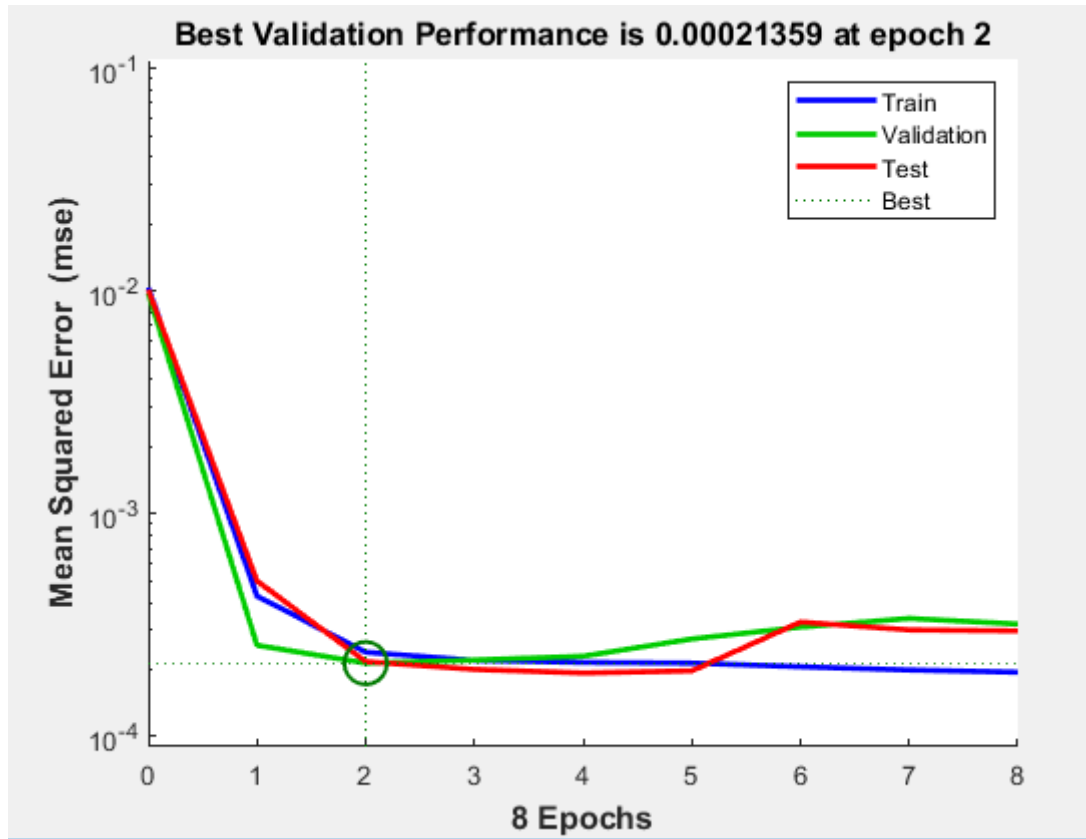


Error autocorrelation:

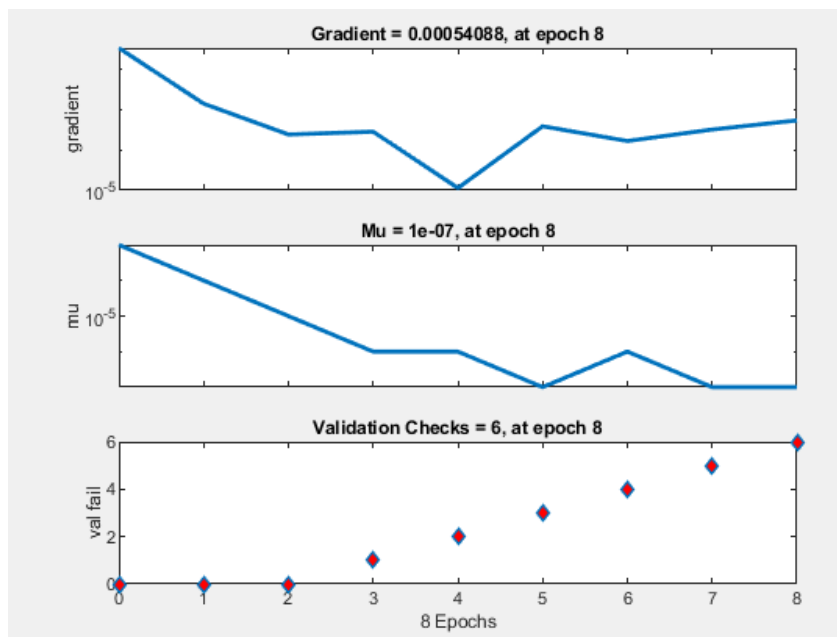


**Portfolio 4 NAR results:**

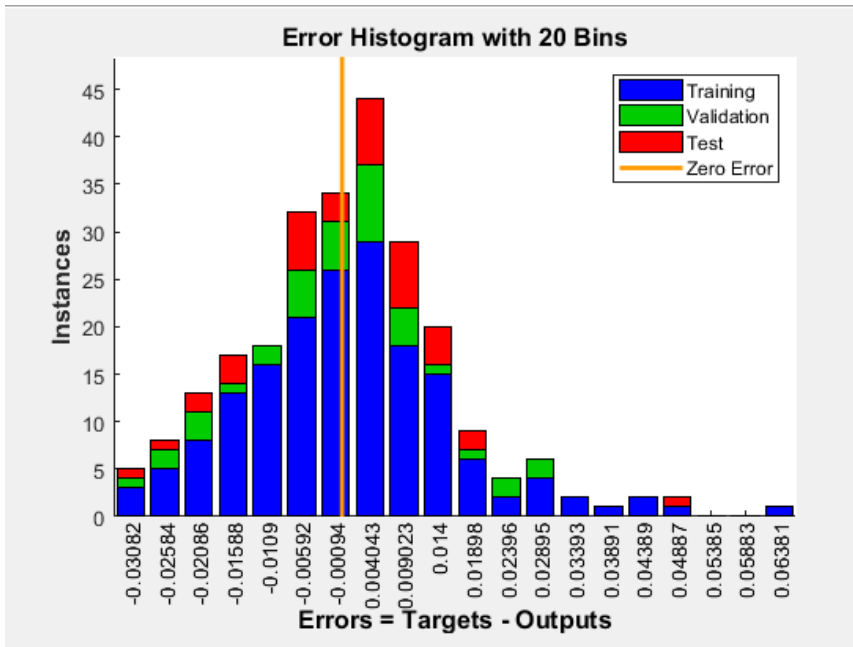
PERFORMANCE:



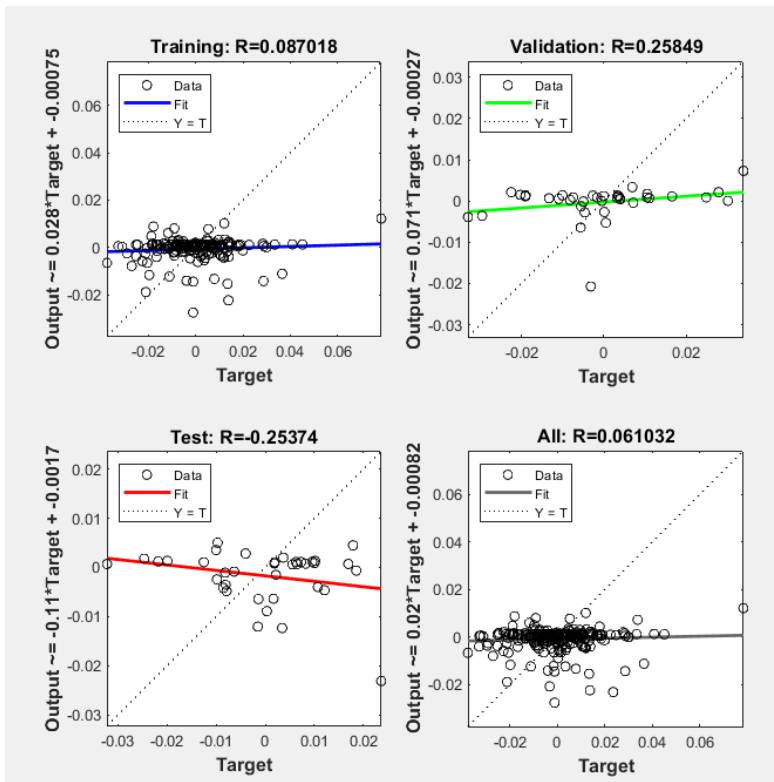
TRAINING STATE:



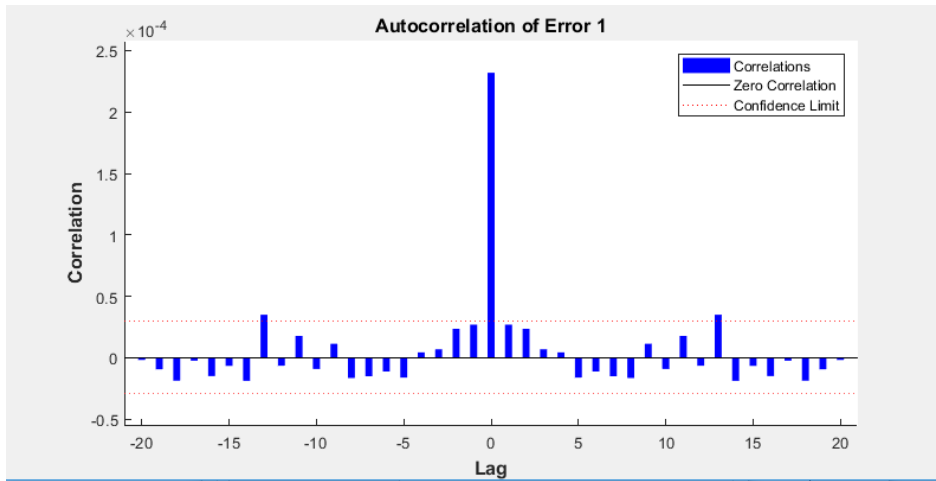
ERROR HISTOGRAM:



REGRESSION:

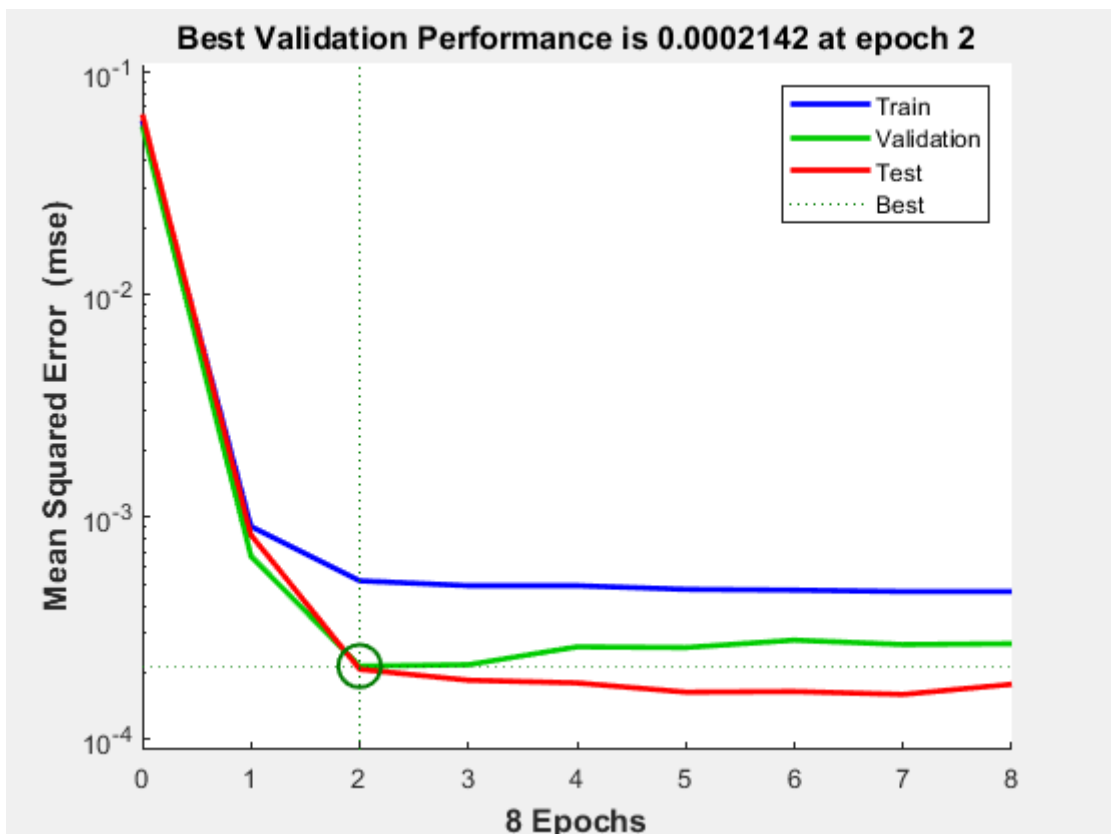


AUTOCORRELATION:

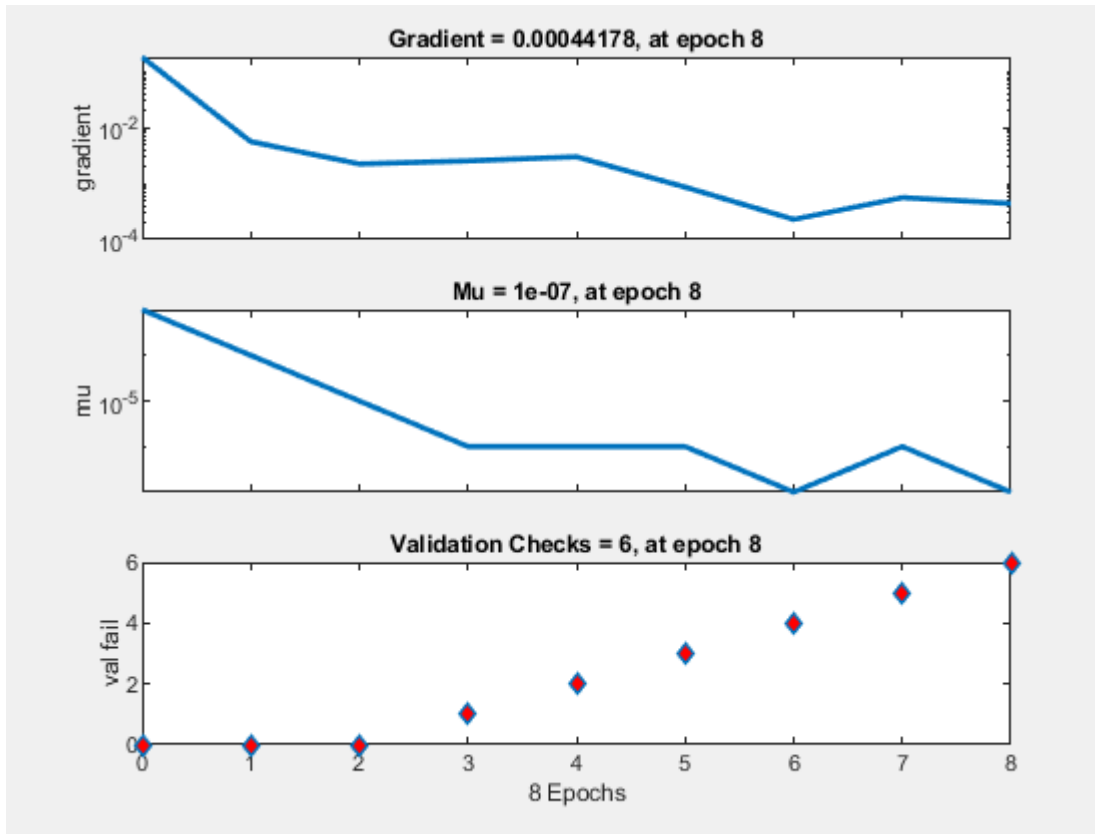


OMXH25 NAR results:

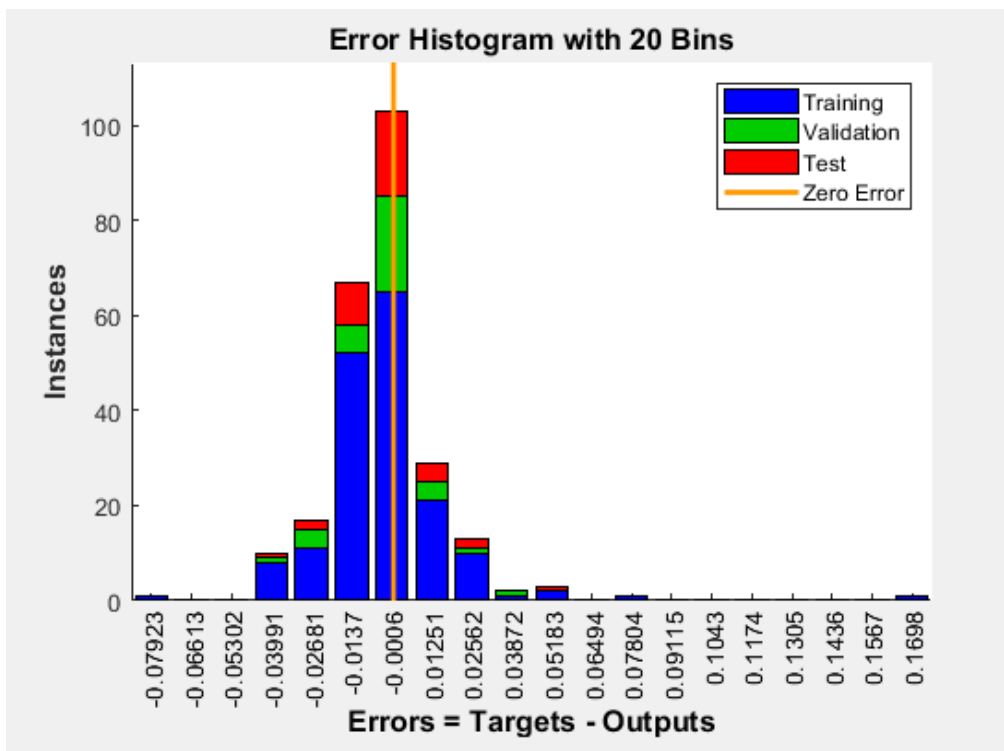
Performance:



Training state:

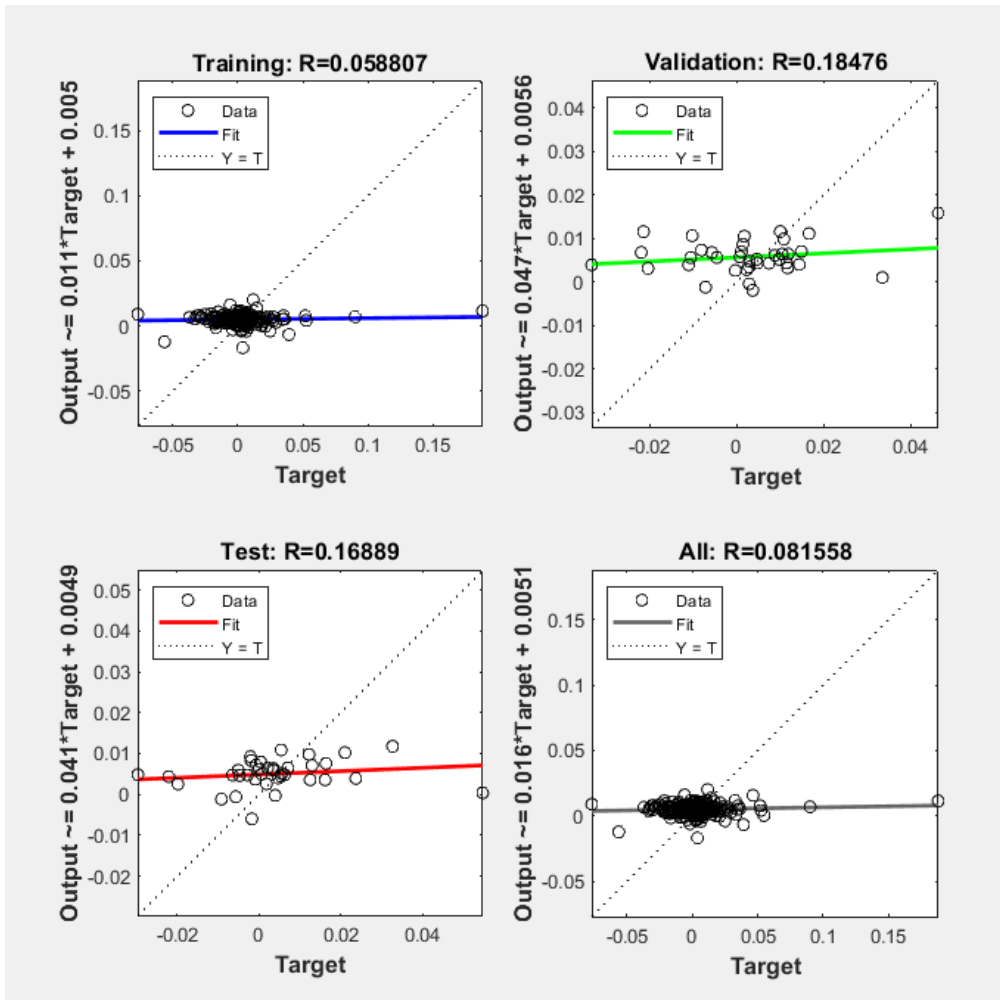


Error histogram:





Regression:



Error autocorrelation:

