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Intuitionistic Fuzzy Set Approach for Measuring Distances in Knowledge Management

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Abstract

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The main goal of this thesis is to present a way for measuring or estimating distances found in knowledge management. This is done by creating a new model for measuring distance in knowledge management setting. This "diamond model" is constructed according to the idea of Hamming distance, where binary strings are compared in a bit-wise manner. The chosen entities to be measured are considerd to be intuitionistic fuzzy sets in a common universe and a intuitionistic fuzzy Hamming distance is defined in this setting.

The secondary goal is to review the concept of distance found in knowledge management and to present its subdivision into different dimensions of distance. These different dimensions are intertwined and sometimes hard to distinguish from each other. Also, some ways to organize different dimensions of distance is presented. Cognitive distance is chosen for closer examination and the fact that it is possible describe cognitive processes using mathematical type terminology suggests that some kind of mathematical distance measure could be defined.

The mathematical side of the work presents the concepts of intuitionistic fuzzy sets and metric in order to build a solid base for creating a model for measuring distances. Together intuitionistic fuzzy approach and mathematical way of describing congnitive distance are combined and applied to the creation of the diamond model. Some examples of this are also given.

Tiivistelmä

Intuitionistic Fuzzy Set Approach for Measuring Distances in Knowledge Management

Tekijä: Tuure Välimaa
Hakusanat: Intuitionistiset sumeat joukot, intuitionistinen sumea etäisyys, kognitiivinen etäisyys, etäisyys tietojohtamisessa, läheisyys
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Tämän diplomityön tavoitteena on esitellä tapa mitata etäisyyksiä, joita esiintyy tietojohtamisessa. Tavoitteen saavuttamiseksi luodaan uusi malli etäisyyden mittaamiseen tietojohtamisen alalla. Tämä uusi ”timanttimalli” rakennetaan Hammingin etäisyydessä esitellyn idean, jossa binäärijonoja verrataan biteittäin, mukaisesti. Mitattavana olevat oliot tulkitaan intuitionistisiksi sumeiksi joukoiksi yhteisessä universumissa ja intuitionistinen sumea Hammingin etäisyys määritellään tässä ympäristössä.

Toissijaisena tavoitteena on esitellä tietojohtamisessa esiintyvä etäisyyden käsite sekä tämän etäisyyden jako useisiin eri ulottuvuuksiin. Nämä ulottuvuudet ovat kietoutuneet toisiinsa, joten usein niiden erottaminen toisistaan on vaikeaa. Lisäksi esitellään pari mallia, jolla pyritään järjestämään nämä eri etäisyyden ulottuvuudet. Kognitiivista etäisyyttä tarkastellaan lähemmin ja koska kognitiivisia prosesseja on mahdollista luonnehtia matemaattisin termein, tämä tukee ajatusta, että jonkinlainen matemaattinen etäisyyden mitta olisi mahdollista määritellä.

Työn matemaattinen osio esittelee intuitionistisen sumean joukon ja metriikan käsitteet, joiden perusteella on mahdollista rakentaa vankalle pohjalle malli etäisyyden mittaamista varten. Yhdessä intuitionistinen sumea lähestymistapa yhdistettynä kognitiivisen etäisyyden kuvaamiseen matemaattisin termein mahdollistaa timanttimallin määrittämisen. Tämän mallin käytöstä annetaan myös esimerkkejä.

Contents

Chapter 1

Introduction

In order to start a new business, a newly graduated barber tries to find an area in a city, where his/hers services are needed. Customers should be wealthy enough, have enough hair and work within couple minutes walking distance. At least ten customers are needed five days a week. Wealth is relatively easy to estimate, for example tax records are open for everyone and some parts of the city are ”better” than others. Similarily, distance can be physical or spatial and therefore measured in meters or in minutes and seconds. But how to decide whether a potential customer has enough hair or not. Is even currently completely bald character bald after two months anymore?

The previous example may seem naive, but it highlights the effect of the fuzzy approach to the subject. And more, there are some variables for which there are no knowledge at all. Some problems are hard to describe in an exact way. In recent years intuitionistic fuzzy sets have been used in decision theory and other fields to solve or describe these problems. In classical setting a customer either uses the service or not. In fuzzy setting there is some propability for using the service and obviously for not using the service - but no other options. When looking through intuitionistic fuzzy ”glasses” there is also room for not knowing - there exists some kind of estimates for positive result, negative result and the unknown.

Knowledge is a difficult thing to define and, furthermore, to manage. In the field of knowledge management, there is constant need to estimate, measure or predict the effects of current way of management and develop it to be more efficient. The first step is to decide what to measure and why. One interesting answer to the latter question is that knowledge is something which can be shared, transferred and stored - there is room for learning.

Several authors agree (see for example [Knoben and Oerlemans, 2006] and [Parjanen, 2014] for more comprehensive list) that distance or proximity is in critical role here - but how is distance defined. There is something more to it than yards and miles. Moreover, too much distance makes communication hard and too little distance means little or no new information at all. An enlightening example about this duality is a new couple in love living in different countries versus an older couple who have been married and living under same roof for decades. Sharing and transferring the knowledge is strongly connected to the field of innovation.

If the answers to the "why" have been found and the answers to the "what" have been identified, the next logical question is "how". In this thesis the working question is "how to measure distance between two (or more) actors in knowledge management setting?"

This thesis is arranged as follows. In chapter 2 the starting point is to present an agreeable definition of knowledge management and, after that, the main focus is in reviewing different concepts of distance used in knowledge management setting. The distance in general is considered as the main concept and it is divided possibly in several dimensions. Also, a couple of different ways to organize these concepts or dimensions is presented. An important note: it should be noted that when considering distance in knowledge management, it can be seen as a two-level concept - as a structural distance between firms or parts of firms, or, as an dyadic distance between two actors within an organizational setting. In this work the difference between those two interpretations is not explicitly presented. On the contrary, both organizational and dyadic ways of seeing distance are present in this chapter and the reason behind this solution is that when reviewing different dimensions of distance on a general level the main focus stays on the real subject. A thorough reader should be able to distinct the presence of structural or dyadic distance within the text.

Chapter 3 continues further discussion about one particular distance - cognitive distance. The choice of cognitive distance as the one to look closer is justified by its tight ties to the innovation processes. Bart Noteboom (see for example [Nooteboom, 2000, Nooteboom, 2012]) have done considerable work in developing the concept. Also, theories and philosophy behind the cognitive distance is also reviewed. Furthermore, absorptive capacity is a central concept when discussing about cognitive processes and cognitive distance. It was presented by Cohen and Levinthal [Cohen and Levinthal, 1990] and it has been a starting point for many later inquiries. Moreover, absorptive capacity plays a significant role when the ability to collaborate is considered. Also, cognitive function is considered in the last part of chapter 3. This and use of other mathematical type notions related to cognitive processes suggest that some kind of measure for cognitive distance could be found.

Chapter 4 mainly describes basic definitions of classical sets, fuzzy sets and intuitionistic fuzzy sets. The focus is on intuitionistic fuzzy sets, their presentation and some of their basic properties. Also two examples, both fuzzy and intuitionistic fuzzy approaches in knowledge management setting, are discussed. These examples are presented without any mathematical notions in order to enhance the readibility of these examples.

Chapter 5 begins with the discussion about metrics and norms. Then Hamming distance is discussed first in fuzzy setting and then in intuitionistic fuzzy setting. Finally, the 3-term Hamming distance measure for intuitionistic fuzzy sets is developed.

In chapter 6 the proposed diamond model is presented. First, it is considered theoretically and 3-term Hamming distance for intuitionistic sets is embedded to the model. Furthermore, some example of measuring or estimating the distances with it are given. It should be noted that the field of study of this thesis is industrial management and not mathematics. Intuitionistic fuzzy sets and Hamming measure defined in intuitionistic fuzzy setting are used as an tool or ruler within the proposed diamond model.

The final chapter 7 is conclusion. This thesis is reviewed in order to see what have been done and what kind of results have been obtained. Also the potential further lines of reseach will be suggested.

Chapter 2

Proximity and Distance in Knowledge Management

Knowledge management is a field of study which is somehow hard to define. As the name of it intuitively tells, it is management of knowledge within an organization or a network. There exist many definitions for knowledge management as Girard and Girard [Girard and Girard, 2015] present in their article. Often the definition is related to the field of operation of the considered organization. They suggest two possible definitions of general type for knowledge management:

Definition 2.1. *[Girard and Girard, 2015] Knowledge Management is the process of creating, sharing, using and managing the knowledge and information of an organization.*

Definition 2.2. *[Girard and Girard, 2015] Knowledge Management is the management process of creating, sharing and using organizational information and knowledge.*

The key words creating, sharing and using knowledge can be found from both of previous definitions, and these terms can be condensed under one term - knowledge transfer within organization. When considering the knowledge transfer in general the proximity of two actors involved play a general role whether the action of knowledge transfer is successful or not. Two major questions arise when one thinks about the effect of proximity to transferring knowledge within an organization:

Question 1. *How to define proximity or distance between two actors?*

And given that the definition is obtained somehow,

Question 2. *How is proximity or distance measured between two actors?*

Proximity and distance are two terms which are interpreted in many ways in the research literature. Quick first insight is usually ”normal” distance which is measured either in meters, kilometers, miles or, if travel time is considered, hours and minutes. This is natural, because many organizations have grown to have locations in several buildings, cities, countries or continents. The case is unfortunately not this simple and there exists many ways to define proximity.

Furthermore, different ways of defining the distance or the proximity lead to different dimensions of the concept. These different dimensions are used in a somehow confusing way and, in many cases, the scope of definitions of the distance is overlapping. Overall, this confusion is a result of the fact that it is hard to define the concept of distance (proximity) within a knowledge management setting.

The main goal of this this chapter is to review different ways of defining the distance between two actors or entities and what kind of dimensions are the reaults of these definitions. This should shed some light to the question number 1. The division and order of different proximities was chosen according to Parjanen [Parjanen, 2014, Parjanen and Hyypiä, 2018] and, at first, it is given in a slightly modified way, but without hierarchy or real classification of different concepts of distance. The question number 2 is a tricky one and one way to approach the solution is discussed later in this thesis in chapters 3, 5 and 6.

For this chapter, the secondary goal is to present some ways to organize the different dimensions of the proximity. A couple ways to create hierarchy between different types of distances will be presented.

2.1 Different concepts of distance and proximity

In their article Holmstrom et al [Holmström et al., 2006] present a case study in the field of global software development. They also find that there are several challenges to be overcome in this technologically and organizationally complex field of work. They identify three different dimensions of distance and also propose some solutions for crossing these.

Example 2.1. *[Holmström et al., 2006] In global software development actors from different national and organizational cultures take part in the software development process. When considering global software development teams, an organization can access larger labour pool and broader skill base. There exist also possibility to round the clock development and cost-effectiveness. In the optimal case, members of software development teams would have a possibility to collaborate in real-time, have face-to-face meetings regularly and, all in all, have rich interactions inside of the team. The members of the team together should also have a proper mix of required technical skills and relevant experience. Furthermore, common organizational culture promotes coordination and facilitates control.*

This kind of global way of working poses also challenges. These challenges include the effects of different kinds of distances. Truly, temporal, geographical and socio-cultural distances are seen to challenge processes related to development project such as communication, coordination and control.

This is one example of the effects of proximity or distance in a field of knowledge management. Moreover, there exist several dimensions of proximity and they are presented next.

The titles of distances are according to Parjanen [Parjanen, 2014] and and the order of the presentation is almost similar to hers. Originally, the distance was divided in eight dimensions: cognitive, communicative, organizational, functional, cultural, social, geographical and temporal. In this work, however, the division is slightly different. Here, the first subsection is dedicated to distances of cognitive type: cognitive, functional and communicative distances. Next, in individual subsections will be presented organizational, social and cultural distances. Geographical and temporal distances are discussed in the last subsection.

2.1.1 Distances of cognitive type

Cognitive proximity or distance is a concept which has been largely developed by Bart Nooteboom [Nooteboom, 2000, Nooteboom et al., 2007, Nooteboom, 2012, Nooteboom, 2013, Wuyts et al., 2005]. It is usually considered as the similarities in the way different actors perceive, interpret, understand and evaluate the world [Wuyts et al., 2005]. On the other hand, cognitive distance refers to differences in knowledge bases of actors. In the case of multi-disciplinary groups of individuals there is a possibility of knowledge transfer when the actors have similar frame of reference. This, in turn, can lead to positive results and innovative new way of thinking.

Cognitive distance

One definition for the cognitive distance is given by Nooteboom:

Definition 2.3. *[Nooteboom, 2013] Cognitive distance is both the difference in cognition in the sense of knowledge gathered during ones lifetime and the difference in perceptions and views of values, ethics and morality.*

The amount of cognitive distance between two actors affect to the ability to trust one another. Trust is both more needed and harder to gain when the two sides are in this sense further away from each other. In order to collaborate, the cognitive distance must be crossed somehow. The act of crossing this distance might require both trust and control to some extent and possibly a third party to be a mediator.

The concept of cognitive distance has dual implications, it presents both problem and opportunity. The problem is that larger cognitive distance makes collaboration harder, the actors involved understand each other less, find harder to see themselves in the place of other and have less empathy towards each other. There is, however, potential opportunity to learn, evolve and create new knowledge. The positive side of the cognitive distance is that there opens an opportunity for innovations.

In his article, Nooteboom [Nooteboom, 2013] presents the concept of optimal cognitive distance. It comes into play because the distance can be too small for new ideas and innovation or it can be too large to be crossed which prevents the utilization of new opportunities. The cognitive distance is optimal when both the novelty potential and the ability to collaborate together are as high as possible, or when the interaction of these two sides is in its peak.

The foundations of cognitive distance and cognitive distance in general is discussed futher in chapter 3.

Functional distance

Actors coming from different functional communities have different areas of expertise. Therefore, there is potentially difference in the way they interpret knowledge in a shared context. Functional distance refers to the difference in the actors' professional knowledge and expertise [Parjanen, 2014, Parjanen and Hyypiä, 2018].

Now, functional distance in this sense is included in the cognitive distance since the definition 2.3 states that the congnitive disctance takes into account the knowledge gathered during ones lifetime. This knowledge includes also the professional knowledge and expertise.

Communicative distance

When actors use common language and discuss problems, it is often silently assumed that they understand meanings of the terms used. Various concepts can have several meanings, or they are not understood in the same way by all the actors. Parjanen [Parjanen, 2014] refers to different meanings of concepts when actors from various fields of expertise are communicating with each other. This somehow binds the concept of communicative distance togerther with the concept of functional distance.

Again, in order to estimate the difference in perceptions, views of values, ethics and morality, there has to be some kind of communication involved. This, in turn, brings communicatice distance under the definition 2.3 and therefore, it is one part or dimension of the cognitive distance.

2.1.2 Organizational distance

In literature organizational proximity or distance is seen as an important factor when considering for example inter-organizational collaboration. If two or more organizations are working together, the processes and co-operation are more efficient and lead to better results when the organizational context is similar in every participating organization. Similarity in this sense makes mutual understanding easier and therefore short organizational distance facilitates ability to combine infomation and knowledge. Clearly, this similarity means proximity or a short distance between collaborating parties. Furthermore, this is seen beneficial for dyadic and collective learning and for creating new knowledge and innovation.
[Knoben and Oerlemans, 2006, Parjanen and Hyypiä, 2018]

Longer organizational distance means less ties and therefore less opportunities for interactive learning between independent actors. Now it should be noted that whether the actor is a individual person, a team working on a joint project or a independend production plant can differ from one scholar to another. The concept of organizational distance has been defined in many various ways in literature and different scholars give different definitions. This leads to an ambigious situation since some authors concentrate on the structural aspect and some others to the dyadic level of the relationship. In order to higlight this situation, some possible definitions will be presented here.

Definition 2.4. *[Boschma, 2005, Parjanen, 2014] Organizational distance is the extent to which relations are shared in organizational arrangements.*

The definition 2.4 does not explicitly identify actors involved in the arrangements or processes. The strong ties to innovation studies suggest that the organizational distance is seen as a possibility to make intellectual leaps and advancements in order to create new knowledge and innovation [Parjanen, 2014, Parjanen and Hyypiä, 2018].

Definition 2.5. *[Schamp et al., 2004] The distance between employees of a multi-plant firm who identify with each other as a result of belonging to the same firm and of their knowledge of firm specific routines is organizational distance.*

In the definition 2.5 Schamp et al look at specific relationships between members belonging to the organization. Here, proximity is considered in a dyadic way and the distance is defined by the the similarity of organizational context on which the actors are operating.

Now, it is possible to include both aspects, the dyadic and the structural level, in the definition of organizational distance. Indeed, this is seen in the next definition 2.6, where the dyadic level is included in the first part and the structural level in the second part.

Definition 2.6. *[Torre and Rallet, 2005] Organizational proximity is defined by actors whose interactions are facilitated by (explicit or implicit) rules and routines of behavior and that share a same system of representations, or set of beliefs.*

Now, if one aspect or another is missing from the definition, it can be argued that something is lost either accidently or on purpose. In the latter case the scholar have propably chosen to concentrate his or her analysis of the subject on the more specific case and potentially narrower area of research.

Some authors see organizational distance more general concept or upper concept compared to other dimensions of distance. This is discussed in the section 2.2 and subsection 2.2.2.

2.1.3 Social distance

Trust-based relations are greatly affected by the social distance of actors involved. Proximity in this sense, in turn, can potentially facilitate knowledge transfer [Boschma, 2005, Parjanen, 2014]. This is a significant fact when considering processes involved in knowledge management. Now, whether this makes cognitive distance an upper level concept or not depends on the view of the recearcher.

Social space

Social space is an environment for all social interactions and an actor in that environment can choose his position there freely. Beneficial social interaction will increase the proximity of two (or more) actors in this space. This concept of social space is developed by Akerlof [Akerlof, 1997] in his article and it is the environment or the universe where social distance is somehow measured.

Here, this means more economic trade than knowledge transfer, but the analogy carries further: Just as individual person has a reason to be connected to his family, relatives and current friends, firms and companies have a strong motivation to remain close to their current customers. And further, when actors transfer (tacit) knowledge in knowledge management setting from one to another, it happens because there exists close enough social relation and trust between actors. [Akerlof, 1997]

Social decisions

In his article Akerlof [Akerlof, 1997] pursues to develop a model for understanding social decisions. This model relies heavily on the concept of social distance, which is the underlying key concept.

Definition 2.7. *[Akerlof, 1997] One's location in the social space is partially inherited and partially result of social intercations. Social distance is the difference in this location in the social space.*

Social decisions are defined to be decisions which have social consequences. Therefore, the choice of groceries to buy in a local food market is not a social decision but a (micro)economical one. Whether one buys apples or bananas does not have an effect to ones social status or social situation. Instead, if one chooses to have children or no is clearly a social decision. People around the actor are affected by his attitudes and racial policies, possible marriages and divorces, educational and professional aspirations and so on. Any action which leads to involvement in some group of people or have some kind of effect to some group of people is at least partially social decision. In short, social decision is a decision made in social context.

According to Akerlof [Akerlof, 1997], every actor has an inherited position in social space. He defines social interaction to be a function of the difference of the actor's initial positions. In the sense of definition 2.7 this might seem to be a circular argument, but actually it just highlights the fact that social decisions are an ongoing process in the social space.

When it is a mutually beneficial trade, social interaction will increase the social proximity, or bring the actors socially closer. All this happens within the fore mentioned social space. Furthermore, as it is seen in definition 2.8, these kind of relations between people can also be used to define the concept of social distance:

Definition 2.8. *[Boschma, 2005] Social proximity is defined in terms of socially embedded relations between agents at the micro-level. Relations between actors are socially embedded when they involve trust based on friendship, kinship and experience.*

Now, according to Boschma [Boschma, 2005] social distance has effects to trust based relations. Proximity in this sense protects from the opportunistic behaviour to some extent, but it might also lead to the habit of doing things in the same way as usual, which diminishes the innovation and learning potential.

2.1.4 Cultural distance

Different ways of thinking, acting and reacting can be found both in any organization and its subunits. Beliefs, assumptions and values affect to these cultural ways of interacting between actors. There exist two major lines of defining the cultural distance, one is based on the work of Hofstede and the other on the analysis by Schwartz. (See for example [Redmond, 2000, Drogendijk and Slangen, 2006].) In addition to those two there is also a third way to approach the subject - individual level perceptual measures.

Cultural distance can be considered when examining factors contributing to culture shock. It is quite understandable that cultural distance is somehow proportional to the amount of social difficulties between native and host cultures. Similarly, cultural proximity results more accurate predictions and explanations when a newcomer tries to make sense of the new environment [Redmond, 2000]. Furthermore, when considering relationships between actors from two different cultures, greater cultural difference could result more problems in communication. This includes developing and maintaining relationships and meeting social needs, and could severely affect to the level of adapting to the different (new) culture.

Cultural distance can have also a direct effect to a multinational organization. In their article Drogendijk and Slangen propose the following hypothesis:

Hypothesis. *[Drogendijk and Slangen, 2006] The larger the cultural distance between the home country of the organization and the potential target country of expansion, the more likely the way of the expansion would be a greenfield investment than an acquisition.*

A greenfield investment is a type of foreign direct investment where the parent organization creates a new operation to a different country by building it from the ground up.

The four dimensions of cultural distance according to Hofstede

In his article Redmond [Redmond, 2000] reviews the concept of cultural distance. He uses the definition by Hofstede [Hofstede, 1983], which divides the concept of cultural distance in four dimensions for closer examination. These are presented in the figure 2.1.

Definition 2.9. *[Hofstede, 1983, Redmond, 2000] The cultural distance is a combination of power distance, uncertainty avoidance, individualism/collectivism and masculinity/femininity.*

Now, the dimensions from the definition 2.9 are described as follows. For members of certain culture, *power distance (1)* describes the ability to accept the institutions and organizations having power. *Uncertainty avoidance (2)* represents the amount of tolerance of the members of the culture towards ambiguity and uncertainty. The dimension of *individualism (3)* is intuitively understood and it describes how high emphasis is placed on individual goals and the wellbeing of immediate families compared to collective goals and the general good of society in general. The final dimension of *masculinity (4)* is used to represent the way to describe the culture when considering masculine and feminine values. For example, these values are as follows: on the masculine side, there is a preference for achievement, heroism and material success and, on the feminine side, preference for relationships, caring for the weak and quality of life. [Redmond, 2000]

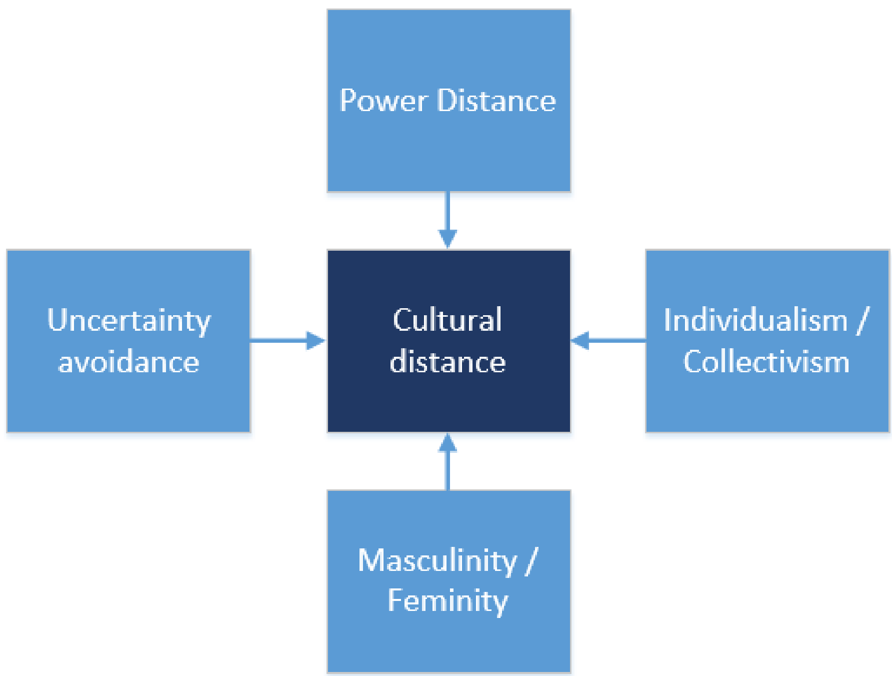

Figure 2.1: Four dimensions of cultural distance according to Hofstede [Hofstede, 1983].

These fore mentioned Hofstede dimensions are usually examined individually, without considering a full multidimensional analysis. According to Redmond [Redmond, 2000], the reason behind this is that the nature of the relations between the different dimensions is unclear. The dimensions are dimensions by the name, but there could exist overlaps between them. For example, there is evidence that the dimensions of power distance and individualism are not completely distinct.

According to Drogendijk and Slangen [Drogendijk and Slangen, 2006] many studies have verified the validity of Hofstede dimensions. Furthermore, it has been seen that they can reliably be used when considering different countries, their national cultures and cultural distances between them. Still, there have been several arguments concerning the Hofstede's study and the choice of dimensions.

Drogendijk and Slangen [Drogendijk and Slangen, 2006] reviewed the identified main points of concern in Hofstede dimensions, and they are as follows.

The research questions behind Hofstedes analysed data were not designed specially for identifying cultural dimension and therefore his analysis was not necessarily exhaustive. Next concern was that the sample of countries did not include all national cultures, which might have affected to the quality or the quantity of dimensions. Futher, the employees surveyed were working in IBM and since they were well educated technical and scientific personel they did not qualify as representatives of the general population of their home country.

Also, the data used in Hofstede's analysis was collected 1967-1973 and worldwide major cultural changes have occurred in last decades, so results or analysis based on them can be already outdated. Lastly, it was unclear if people from different cultures understood work-related values actually in the same way, so, according to the article by Drogendijk and Slangen [Drogendijk and Slangen, 2006], the conceptual equivalentness was questionable.

Schwartz's seven dimensions of cultural distance

In order to formulate his seven dimensions of cultural distance, Schwartz [Schwartz, 1999] conducted a thorough theoretical and empirical research. The initial set contained 56 different individual values recognized widely across cultures. These values explained inter-country cultural variation. Then, after carefully conducted surveys and further analysis the number of dimensions were reduced. The final set of cultural dimensions includes seven variables which form mutually distinct set of dimensions for distinguishing different national cultures.

Definition 2.10. *[Schwartz, 1999, Drogendijk and Slangen, 2006] The cultural distance is a combination of conservatism, intellectual autonomy, affective autonomy, hierarchy, egalitarian commitment, mastery and harmony.*

These selected seven dimensions given in the definition 2.10 are presented in the figure 2.2.

The dimension of *conservatism (1)* represents the amount of endeavor to maintain the status quo, propriety, and controlling actions and desires that could disturb the solidarity of the group. *Intellectual autonomy (2)* describes the level of freedom when considering pursuing one's own ideas and intellectual directions. Similarly, *affective autonomy (3)* refers to the extent to which one is able to follow own affective desires.

The dimension of *hierarchy (4)* is used to measure which extent it is legitimate to distribute power, roles and resources unequally. Somehow opposite, the concept of *egalitarian commitment (5)* describes the amount of personal resources people are willing to use for promoting the welfare of others instead of pursuing some other more selfish goals. *Mastery (6)* represents the importance of individual advancement, by being determined to advance one's own interests. And lastly, *harmony (7)* represents how important it is to fit into the cultural environment in an harmonious way. [Schwartz, 1999, Drogendijk and Slangen, 2006]

Individual level perceptual measures

The leaderboard or managers of an organization make majority of the strategic decisions based on their perceptions. Therefore some authors (See [Drogendijk and Slangen, 2006, p. 364] for refences.) suggest that individual level perceptual measures should be used to estimate the cultural differences and to assess cultures in general. These measures are usually based on some theoretical measure and applied to a organization specific way. For example, it is possible to create a managerial questionnaire with culture specific questions and use 7-point Likert scale for grouping the answers. This line of research could be interesting to pursue forward, but it would require organization specific data and research questions.

Figure 2.2: Seven dimensions of cultural distance according to Schwartz [Schwartz, 1999].

2.1.5 Geographical and temporal distances

When the concept of distance is considered, the normal first idea is usually the geographical distance. Some may also think about the time needed to cross that distance or even different time zones when thinking about distant loactions. These are different manifestations of physical distance.

Geographical distance

Intuitively, geographical distance is the spatial or physical distance between actors. It is measured either in hours and minutes or kilometers and meters (miles and yards in selected countries). It can also be seen as the amount of effort needed for crossing certain physical distance. This is also the definition Holmstrom et al have ended up in their article:

Definition 2.11. *[Holmström et al., 2006] A measure of the effort required one actor to visit another is called geographical distance.*

Knoben and Oerlemans [Knoben and Oerlemans, 2006, p. 72] are also in the same page with the previous definition 2.11, stating that the importance of geographical proximity lies in the fact that close proximity in this sense facilitates face-to-face interactions and therefore also knowledge transfer and innovation.

Furthermore, increased geographical distance can reduce the intensity of communication. This happens especially when it is hard to replace face-to-face interaction with or via some other media. However, geographical distance is more the just plain kilometers or miles between two locations. It should be considered by the ease of relocating. Different means of transportation could have a dramatic effect to the ease and time of travel and therefore also to the geographical distance between two locations.

In his article, Akerlof [Akerlof, 1997] argues that geographical distance is just one dimension of social distance and therefor social distance would be a generalization of geographical distance. Other way to express the same idea is to note that short geographical distance increases the propability of social interaction and therefore trust building [Boschma, 2005, Parjanen, 2014]. Furthermore, it can be seen that the transfer of tacit knowledge gets easier with shorter distances.

All in all, geographical proximity is strongly tied with social proximity. It is, depending on the point of view, either prerequisite for social proximity or one dimension of social proximity.

Temporal distance

How temporal distance is understood and how it is defined depends on the field of research. Anyway, it has obviously something to do with time. More common way of understanding the temporal distance is seeing it as a dislocation or shift in time needed to accomplish cooperation or communication between two (or more) actors.

Definition 2.12. *[Ågerfalk et al., 2005] The measured amount of the dislocation in time experienced by two actors trying to interact is called the temporal distance.*

Working in different time zones or different shift patterns can reduce possibilities for real-time collaboration and be the cause of temporal distance. Anyhow, if these are taken into account when organizing work patterns within a organization, it is also possible to decrease the temporal distance and create more overlapping hours between two (or more) different locations. [Holmström et al., 2006]

While seeing temporal distance as a somekind of difference in either real or experienced time, there exist also another way of interpreting the concept. Namely, it can refer to the ability to imagine different potential versions of the future. In the field of innovation studies this is the working definition:

Definition 2.13. *[Parjanen, 2014] The differences in the ability to imagine different possible versions of the future and their potential outcomes is called temporal distance.*

One can handle future oriented information in a reactive or proactive way. This naturally depends on the way one sees the information about the possible outcome of the future events. If they are seen as a negative development, it could lead to proactive or protective measures. On the other hand, if the view of the possible future is brighter, the measures taken can be reactive or the predicted future is included in the planning processes.

2.2 Some ways to order different proximities

Different definitions of distances and various ways of organizing mutual hierachy of the dimensions highlight the fact that there does not exist unanimous system or classification for proximities, distances or their dimensions. From the definitions it is quite easy to see that the concepts are intertwined in a such manner that the hierarchy or mutual order of different dimensions of distance can be organized in several ways.

2.2.1 The socio-cultural distance by Holmström et al.

Different actors have different cultural backgrounds and different values. In their article Holmström et al [Holmström et al., 2006] couple the concepts of social distance and cultural distance in one complex multidimensional distance, socio-cultural distance. It is a measure for understanding other actor's values and normative practices. When considering this kind of measure, one must take organizational culture, national culture, language, politics and one's motivations and work ethics into account. This subdividision of concepts is pictured in the figure 2.3.

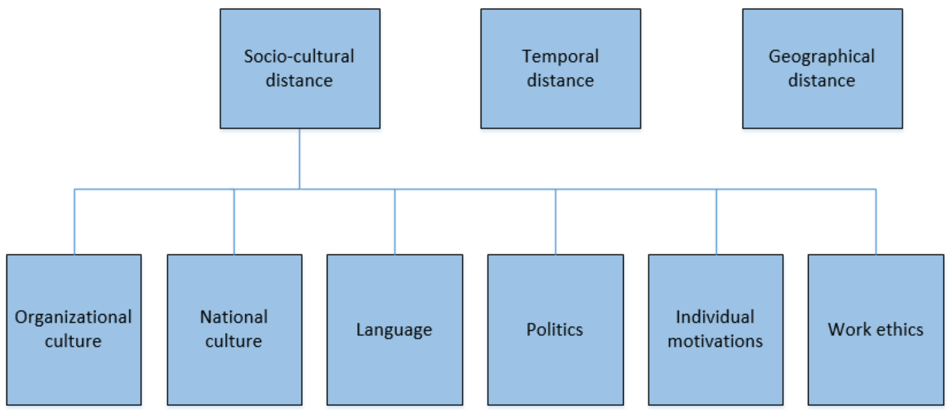

Figure 2.3: Distances and their dimensions discussed by Holmström et al [Holmström et al., 2006].

Considering the earlier sections in this chapter and the definitions given, it is quite straightforward to see that in addition to social and cultural distances the concept of socio-cultural distance includes at least bits and pieces from organizational, cognitive and communicative distances. And yet, there exists also somehow unclear connection to geographical distance, because greater geographical distance can imply also greater socio-cultural distance.

2.2.2 Dyadic level hierarchy by Knoben and Oerlemans

In their article Knoben and Oerlemans [Knoben and Oerlemans, 2006] proposed that three major dimensions of distance are relevant, when considering inter-organizational collaboration. They are organizational, technological and geographic proximities. Furhermore, they also discuss on other dimensions of distance and place most of them under the organizational distance or proximity in their hierarchy, which is shown here in the figure 2.4.

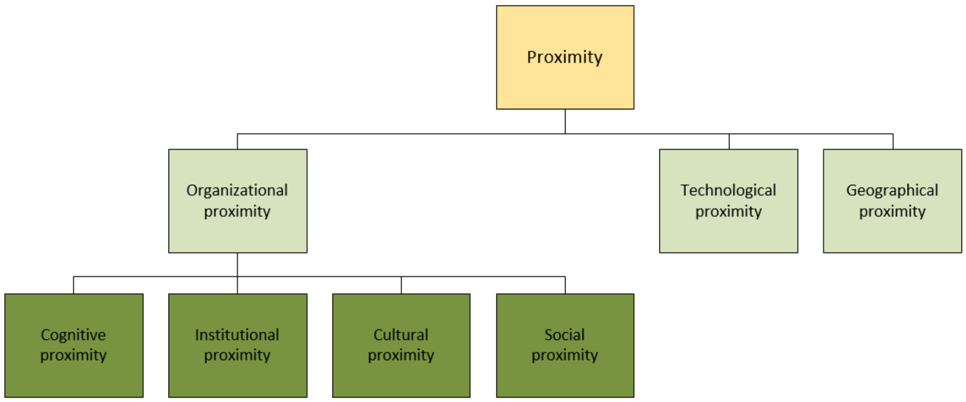

Figure 2.4: Dyadic proximity categories according to Knoben and Oerlemans [Knoben and Oerlemans, 2006].

It should be noted that institutional proximity and technological proximity have been omitted from this chapter. The reason behind this is twofold. First, the subdivision of the previous section is based on work of Parjanen [Parjanen, 2014], and both of those proximities are included in the other dimensions in that subdivision. Moreover, according to Knoben and Oerlemans [Knoben and Oerlemans, 2006], institutional proximity is identical to cultural proximity, at least in the context of inter-organizational collaboration.

Technological proximity, in turn, deals with aquisition and development of technological knowledge. The central concepts are absorptive capacity in general level and relative absorptive capasity in dyadic level. Both of them describe the firm's ability learn by assimilating new external knowledge which is recoqnized somehow valuable, but the difference is in the initial assumption whether the capacity to learn depends only on the firm itself (general level) or does the source of the knowlegde have also some effect to it (dyadic level). In any case, absorptive capacity is discussed also with cognitive distance in next chapter 3. Furthermore, cognitive distance is seen as a upper concept in this work, since it includes most of the concepts of technological distance.

2.2.3 Proximities in this work

Different dimensions of proximity or distance have been presented in the current chapter 2. These dimensions are pictured in the figure 2.5. The different ways of subdividing and organizing the distances in various ways have been justified by different research settings and different ways to approach the subject. The qualities included under each dimension also define the hierarchy and subdivision of distances just according to the scholar who has made the initial definitions for the distances in his or her original work.

In any case, not depending on the choise of definitions, proximity or distance between two or more actors have a significant effect to knowledge related processes. This will be considered further in chapter 3 and section 4.3 of chapter 4.

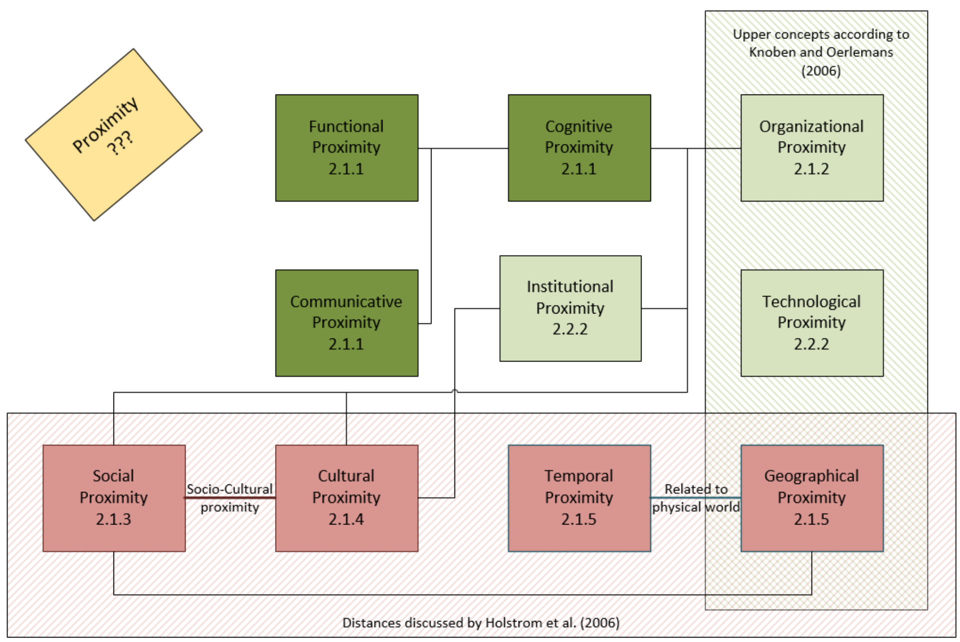

Figure 2.5: Distances or proximities discussed in this chapter and where to find them.

Chapter 3

Cognitive Distance

If one would like to propose one general model for measuring or at least making educated guesses of distances between different entities, the underlying assumptions would have to combine measuring time and physical distance as well as the distance of different levels of expertise. Common measure for measurable quantities (for example time or distance) and intuitively estimated quantities (professional skill or trust between actors) would be either very vague or extremely complicated, and possibly unusable in many cases.

However, it is possible to measure or at least estimate one dimension at a time. Therefore it makes sense to concentrate on one dimension and progress further in fairly straightforward way.

The concept of cognitive proximity presented by Bart Nooteboom (see for example [Nooteboom, 2000, Nooteboom et al., 2007, Nooteboom, 2012, Nooteboom, 2013, Wuyts et al., 2005]) is a facinating one. When one reads his articles it becomes obvious that his theory has matured and thought processes behind it have changed and refined in the course of time. What makes it especially interesting when looked through mathematical glasses, is the fact that Nooteboom himself has brought forward some mathematical notions and describes some of the processes involved by using terms like function, domain, range and mapping [Nooteboom, 2000].

3.1 Cognitive distance and innovation

In his article concerning innovation and cognitive distance, Nooteboom presents the concept of *optimal cognitive distance* [Nooteboom, 2013]. He, however, suggests that is not calculated in any way, but approximated by trial and error.

Trust and control are both complement and substitutes for each other. Innovation requires more trust than control because of uncertainty related to innovation processes and the nature of those processes. Trust is related to the ability to understand each other and therefore also to the cognitive structures developed by the different actors during their life cycle. (Here the term actor can refer to an individual human being as well as a complete firm or a section of such.) Hence the cognitive distance comes into play.

Cognitive distance affects to the innovation potential, but in order to collaborate it must be crossed. When considering innovation processes, Nooteboom [Nooteboom, 2013] discusses about bilateral relationships, the role of third parties in relationships and networks and the different factors emerging in different situations.

3.1.1 Innovation

Two major concepts related to field of innovation studies are exploration and exploitation. Nooteboom [Nooteboom, 2013] uses the definitions given by March as a starting point of his discussion:

Definition 3.1. *[March, 1991] Exploration includes things captured by terms such as search, variation, risk taking, experimentation, flexibility, discovery, and innovation. Exploitation includes such things as refinement, choice, production, efficiency, selection, implementation, and execution.*

Now, exploration is considered as a radical innovation and exploitation as an incremental innovation. It is seen that exploitation is more conservative side of innovation processes and it it usually related to short-term success. On the other hand, exploration means more risk taking and requires out-of-the-box thinking, but obtaining any economical benefits is a long-term process.

The main question is how to utilize both aspects of innovation, so that the firm can be successful both in short and long time periods. Some firms are more exploration oriented and others concentrate more on the exploitation side of the business. One obvious solution to the problem in hand is collaboration between two different companies, one exploitation-oriented and the other exploration-oriented. [Nooteboom, 2013]

The concepts of exploitation and exploration have different definitions. In their article Li et al survey several articles in order to find more general definition for those terms:

Definition 3.2. *[Li et al., 2008] Firms exploit by searching for knowledge within the organizational boundary and knowledge that is local to their existing knowledge base and explore by searching distant knowledge that is unfamiliar.*

The analysis here is based on sequence of the value chain which they divide in the scientific, technological and product-market levels. This correspond to the product developments early stages of scientific research, middle stages of technology development and the final stage of commercialization.

They define two function domains, science vs. technology and technology vs. product market knowledge where the exploitation acts as a function from the first domain to the second one. They also define a three dimensional knowledge distance domain where they claim that knowledge search can be executed and the knowledge distance can be measured along the cognitive dimension, temporal dimension and spatial dimension. They do not give any explicit way to execute the measurement, so in this sense they operate on a conceptual level only. Moreover, according to their analysis exploitation is approximated by local knowledge search and exploitation is approximated by distant knowledge search. [Li et al., 2008]

3.1.2 Trust and uncertainty

If conditions, procedures and the final outcomes of a certain action are known, trust would not be an issue. Trust is needed under uncertainty and in innovation processes uncertainty is usually high.

There exists a paradox of information concerning trust. Trust is often based on some information about either observed or reported chain of events, which act as an accepted estimate for possible outcomes in the future. However, there has to be lack of information, since the concept of trust includes also component of vulnerability. One is often dependent on the actions of others and the final outcome is often not known. [Nooteboom, 2013]

Nooteboom [Nooteboom, 2013] divides uncertainty of conditions, conduct or outcomes in two categories: calculable and incalculable. The first one is usually referred to as risk, and there exists methods for estimating it. The second one, incalculable uncertainty, is called radical uncertainty. Furthermore, the probability and the size of possible loss should be estimated and, in the case of radical uncertainty, probability of possible loss as well as the size of possible loss are not known.

3.1.3 Cognitive distance

People develop cognitive structures during their lifetime. Different people might experience similar events differently and their thought patterns can differ greatly. This lead to the cognitive distance. Now, definition 2.3 given by Nooteboom should be recalled here.

Definition 3.3. *[Nooteboom, 2013]* (Definition 2.3.) *Cognitive distance is both the difference in cognition in the sense of knowledge gathered during ones lifetime and the difference in perceptions and views of values, ethics and morality.*

Also the main points discussed in the section 2.1.1 should be considered again. The proximity in this sense between two actors have effects to the ability to trust one another - increased distance both increases the need for trust and makes it harder to gain. There exist dual implications when considering cognitive distance. Larger cognitive distance makes understanding each other harder in a broad sense and therefore also collaboration becomes harder. But, if the distance can be crossed, there is potential to learn and create new knowledge. The act of crossing this distance might require both trust and control to some extent and possibly third party to be a mediator.

The previous situation is modeled by Nooteboom [Nooteboom, 2013] in his article. He begins with a downward straight line which describes the decline in ability to collaborate when cognitive distance grows. The novelty potential increases as the cognitive distance gets bigger and that is pictured with another upward sloping straight line. Now the mathematical product of these two lines becomes an inverted U-shape, a parabola. The maximum point on that parabolic line is defined to be the optimal cognitive distance is a by Nooteboom. This corroborates the fact that cognitive distance is optimal when both the novelty potential and the ability to collaborate together are as high as possible. Now, the optimal cognitive distance is pictured in figure 3.1.

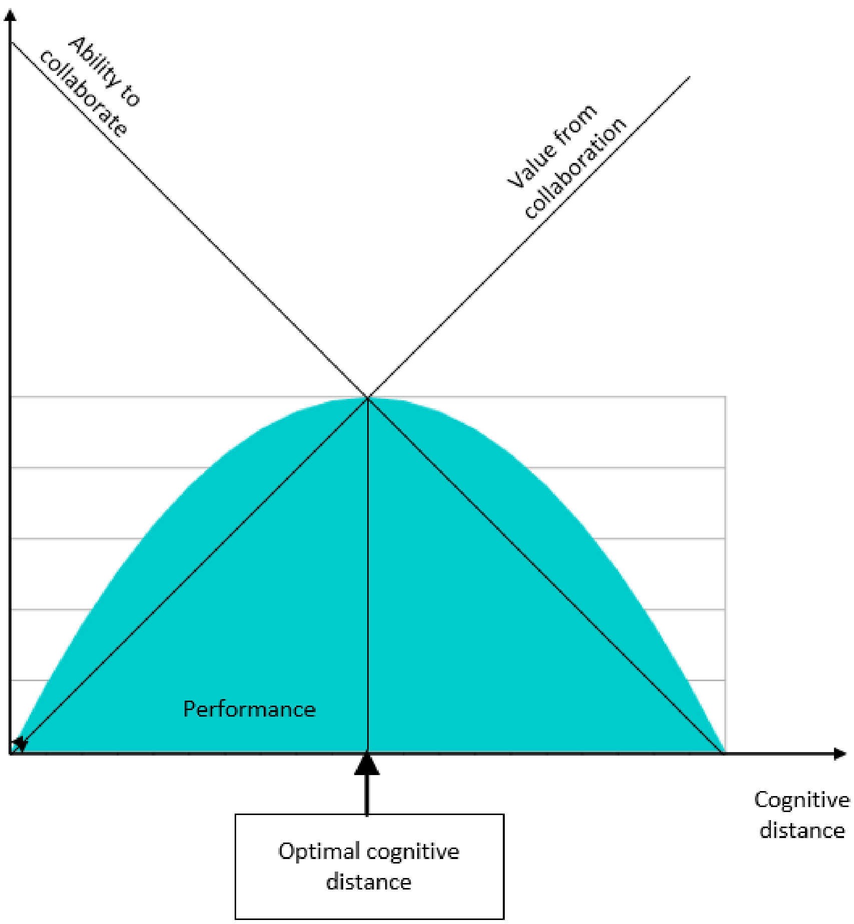

Figure 3.1: Optimal cognitive distance presented by Nooteboom [Nooteboom, 2013].

Now, if the innovation involved in this case is radical, the slope of novelty value is steeper than it would be with the incremental type of innovation. Moreover, if the ability to collaborate increases over time of mutual commitment, it is modeled with a shift of the downward sloping line. In this case, it is suggested that the model is more conceptual and the optimum cannot be calculated. According to Nooteboom [Nooteboom, 2013] it can only be approximated by trial and error.

3.2 Some philosophy and theory behind the concept

In his earlier work Nooteboom [Nooteboom, 2000] has chosen his starting point to be a constructivist, interactionist theory of knowledge for the value and processes of knowledge exchange. The development psychological ideas of Piaget and Vygotsky complemented up to a point his view of knowledge and learning. This line of thought is linked to his earlier attempt to bring forth mathematical and logical concepts when considering cognitive distance and related concepts.

In his later work Nooteboom [Nooteboom, 2012] has chosen a different starting point, namely pragmatist view of philosophy, which have changed his view of cognition and related processes. In this way of thinking, he includes development psychological ideas of Piaget more fully in his analysis of absorptive and expressive capacities and cognitive distance.

3.2.1 Constructivist theory of knowledge

Nooteboom [Nooteboom, 2000] uses the term knowledge in a broad sense. It includes perception, understanding and value judgements. People develop forms of thought in interaction with the physical and social environment. These mental models guide the way people perceive, interpret and evaluate the surrounding world. Two people see things differently and both interpret and evaluate happenings and situations differently.

The current knowledge has been affected by the past experiences and natural environments of an individual. These all together determine ones absorptive capacity. Nooteboom linked his approach to the work of Piaget and Vygotsky in developmental psychology [Nooteboom, 2000, p. 71]. He states that intelligence is internalized action and speech. Moreover, both knowledge and meaning are context-dependent.

Different people have always some differences in their ways of thinking. In order to work together for a specific joint goal successfully these different mental models of different people involved must be co-ordinated to some extent. The ability to learn or accept information, absorptive capacity, is determined by the environment and past experiences. This is discussed further in the section 3.3.

According to Nooteboom [Nooteboom, 2000], his theory of knowledge leads to an issue in governance, heavily related to the cognitive distance. In order to create new information or innovation, there exists a trade-off to be made between cognitive distance and cognitive proximity. Information is useless if it cannot be understood, but the same is true, if the information has lost its novelty value.

Now, this theory of knowledge leads to notions of mathematical type. These are quite fascinating and will be discussed in the section 3.4.

3.2.2 Pragmatist theory of innovation

In his later work, according to Nooteboom [Nooteboom, 2012], the concept of cognitive distance is derived from the embodied cognition school of thought in cognitive science. The line of thought behind his reasoning comes from the pragmatist philosophy.

Embodied cognition

Previous work from the predecessors state that cognition is a wide concept [Nooteboom, 2012, section 2 on Embodied cognition]. Embodied cognition roots cognition in the body and mind. Furthermore, mind and body are not separate and bodily processes of perception, feelings and emotions build up to form the collective called cognition. The theoretical base developed by Nooteboom [Nooteboom, 2012] connects both neural science and social psychology.

In social psychology there are connections with both mental framing and decision heuristics where rational evaluation is mixed together with emotions and unconscious psychological mechanisms. The decision heuristics may seem irrational in this frame of thought. However, the fact that primitive decision situations require fast response and interpretation of the perceived surroundings and developments of events makes this kind of reasoning adaptive and in that sense it is a rational method of making decisions.

The concept of mental framing combine both cognition and action. Mental frames are mental constructions which are developed in advance and applied (possibly unconsciously) in a situation to make evaluations of different choices in hand. These mental frames consist of forms of perception and dispositions of interpretation, judgement and action. This creates preprogrammed behavior and may enhance prejudice but makes also possible to respond quickly to different situations and utilize previous experience while doing that. [Nooteboom, 2012]

Pragmatism

Nooteboom [Nooteboom, 2012, section 3 on Pragmatism] binds his work to the American tradition of pragmatic philosophy. He mentions for example James, Pierce, Dewey, Mead and Hans Joas, who either belong to that tradition directly or are related to the tradition of pragmatic philosophy. His view of philosophical pragmatism is that cognition is based on mental dispositions and categories which both are developed in interaction with the physical and social environment. Here cognition is taken in a wide sense including normative judgement and goals. Moreover, intelligence is seen as an internal practice.

Nooteboom derives inspiration from the work of developmental psychologist Jean Piaget [Nooteboom, 2000, Nooteboom, 2012]. He describes the creation of knowledge in similar terms as the development of intelligence in children is described. The main concepts related to this line of reasoning are assimilation, accommodation, generalization, differentiation and reciprocation [Nooteboom, 2012].

New experience is assimilated into already existing cognitive structures, which, in turn, are accommodating the new experience and transform during the process. Now, the gathered knowledge and skills are generalized in new situations in order to exceed the limitations of the current know-how. If this does not work, the different, but already known, potential options will be tried. However, if this differentiation fails, one has to try reciprocation. It means that practices related to the newly faced situation,which seem to lead potential success, are tried. This does not change the existing cognitive structures, it adds temporarily new elements for playing around with the newly acquired methods or practices. The temporary hybrid structures of new and old might lead to permanent architectural change in cognitive structures if the benefits of new practices overcome the ones of old know-how. This permanent change is called accommodation. According to Nooteboom [Nooteboom, 2012], this process is similar to the pragmatic principle of exploring while being engaged to application.

That pragmatic way of reason states that goals, means and actions are bind together. They do not exist separately but interact with each other. Pre-established goals and preferences do exist, but they are re-evaluated when new opportunities or problems arise. This happens as results of actions and results of discovery of (possible) new means.

According to Nooteboom [Nooteboom, 2012], both embodied cognition and pragmatism leads to the fact that collaboration is important for creativity and innovation. This will lead to the concepts of absorptive capacity and, which is important in this work, cognitive distance.

3.3 Collaborative capacity

It is quite clear that collaboration requires (at least) two sides, receiver and sender. Absorptive capacity refers to the cognitive processes related to the receiver side of the communication. Moreover, absorptive capacity is wider notion than cognition, and it includes both competence and governance sides of thought processes.

In order to get a message through, the sender side has its own requirements. A clear message which triggers understanding has a potential to have an effect to receivers actions, and a good expressive capacity of the sender can have a positive influence on the mutual collaboration.

3.3.1 Absorptive capacity

In his earlier work, Nooteboom gave a definition of absortive capacity as a domain in cognition, which is a suitable starting point for the current work:

Definition 3.4. *[Nooteboom, 2000] Absorptive capacity can be interpreted as the domain of cognition: the phenomena one can make sense of, i.e. which one can perceive, interpret and evaluate.*

At the same time expressive capacity was not explicitly discussed, but it was placed under the title range for cognitive function, which also included linguistic expressions. This line of reasoning is discussed further in the next section 3.4.

The notion of absorptive capacity was elaborated further in Nootebooms later work [Nooteboom, 2012]. It includes not only substantive understanding but also moral views, motives, insights, styles of thought and empathy. Now, here substantive understanding refers to the competence side of thought processes and moral views, motives et cetera refer to the governance side of thought processes. Nooteboom [Nooteboom, 2000] takes the definition by Cohen and Levinthal as his starting point:

Definition 3.5. *[Cohen and Levinthal, 1990] Absorptive capacity is the ability of a firm to recognize the value of new, external information, assimilate it, and apply it to commercial ends.*

Now, in dyadic level interaction between people can be seen as the main source of new knowledge. Moreover, recalling concepts borrowed from developmental psychology discussed in previous section 3.2, absorption of knowledge is similar to assimilation. Furthermore, there exist counterparts also to generalization, differentiation and reciprocation.

When one actor tries to fit knowledge into absorptive capacity of the other, it can be seen as generalization. If the message does not go through, in the case of misunderstanding, the situation leads to the need for an alternative approach, or, one has to differentiate. Lastly, reciprocation comes into play, when one actor have an opportunity and need to fit elements of knowledge coming from other source into one's own cognitive framework.

Nooteboom [Nooteboom, 2012] has discussed also about impacts of the concept of absorptive capacity on organizational level. In his analysis he cites his own earlier work and gives the definition on organizational level.

Definition 3.6. *[Nooteboom, 2012] Absorptive capacity on organizational level includes ways of communication and knowledge sharing, organizational memory, and cultural features concerning views and attitudes towards the outside world, in organizational 'cognitive focus'.*

When compared to the earlier definition 3.5 by Cohen and Levinthal, the focus is more on the knowledge related processes. The connection to the psychology and neuroscience can be seen as an organization is considered to be an learning entity itself.

3.3.2 Expressive capacity and collaboration

Absorption of knowledge does not happen on its own. The source of knowledge has also a significant role in the process. In addition to the receivers ability to get the message, the sender side has to be able to express itself as understandable way as possible. This leads to the next definition 3.7.

Definition 3.7. *[Nooteboom, 2012] Expressive capacity means to ability to be clear, to give examples and use metaphors that trigger understanding.*

Now, both absorptive capacity and expressive capacity together define a larger concept of collaborative capacity.

From earlier discussion in section 3.1.3 should be remembered that each individual develops different cognitive structures during different paths of life. This includes different educational backgrounds, experiences and social environments. Moreover, all this affects to the absorptive capacity and expressive capacity of an individual. And, since the cognitive structures of two individuals are hardly ever identical, there exists difference in their cognitive stucture. This difference is called the cognitive distance between those two individuals or possibly entities of larger scale. Therefore, in order to be able to collaborate, this cognitive distance must be somehow crossed.

3.4 Cognitive distance and cognitive function

Absorptive capacity and cognitive distance are somehow hard concepts to define exactly. Intuitively both are quite understandable and related to everyone's own experiences, but how to define general concepts which are individually understood within ones own known world and collection of experiences.

Cognitive function

In order to be more precise, Nooteboom [Nooteboom, 2000] introduces the notion of *cognitive function*. The starting point here is a cognitive domain which consists of observed phenomena. It includes ones own observations, what happens in observable surroundings, other people's actions and linguistic expressions.

A cognitive range consists of conclusions and categorization and the cognitive function is considered to be a mapping from a cognitive domain to a cognitive range.

Definition 3.8. *[Nooteboom, 2000] Cognitive function is a mapping from a cognitive domain to a cognitive range.*

Further, both cognitive domain and cognitive range and the mapping between them is pictured in the figure 3.2. This mapping is done by applying actors own mental forms of thought, perception, interpretation and evaluation. Or, to use common vocabulary, this is thinking and the forms of thought build up actors cognitive repertoire. To keep his analysis simple enough Nooteboom restricts himself for the use of one cognitive function with its range including also both verbal and non-verbal expressions, linguistic expressions.

Here, the concept of absorptive capacity can be stated to be the phenomena which one can perceive, interpret and evaluate- all in all, the phenomena one can make sense of. Nooteboom proposes that absorptive capacity could be interpreted as the domain of cognition.

Now, learning in organizational context can be divided in two levels. Fiol and Lyles [Fiol and Lyles, 1985] discuss about lower- and higher-level learning, while Nooteboom [Nooteboom, 2000] uses the terms first and second order learning, respectively. Here, the first order learning is often result of repetition of some kind and it takes place within prevailing organizational structure and rules. Any changes caused by the first order learning are basicly minor corrections in the existing structure. Second order learning, however, is a cognitive process which changes cognitive frameworks and gives entirely new environment where to make decisions. This kind of learning might need some kind of unlearning in order to make room for new structures. Moreover, second order learning has profound impacts on the whole system.

According to Nooteboom [Nooteboom, 2000], learning induces an extension of cognitive function. He gives interpretation for first order learning as an extension of domain or range for given forms of thought. Then, second order learning is seen as change of the forms of thought, that is the change of the way the domain is mapped into the range. Further, the change in the cognitive function, forms of thought, would probably induce a change of domain and range. The change of domain and range does not necessarily affect to the mapping between them, in other words,

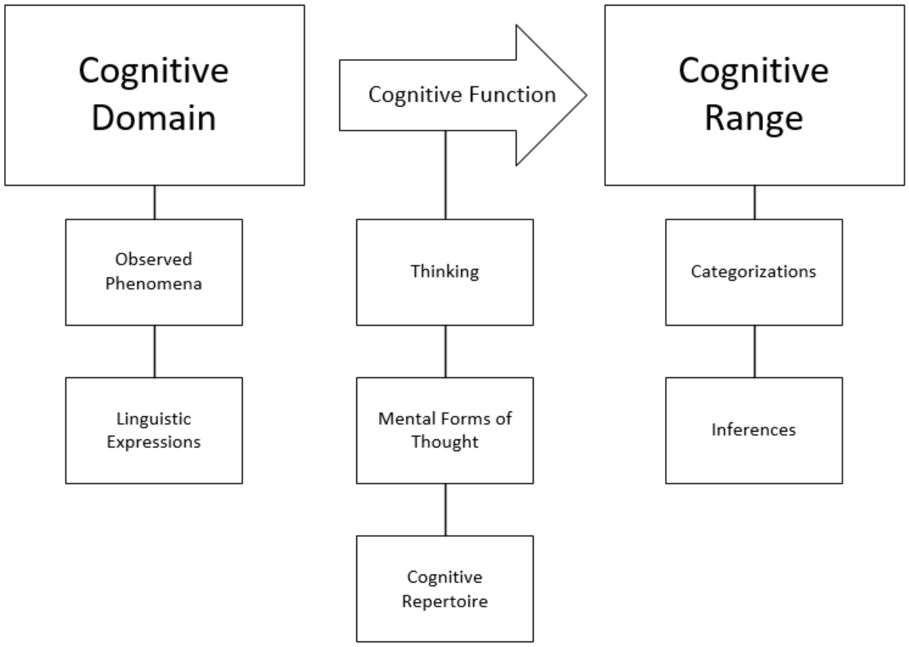

Figure 3.2: Cognitive domain, cognitive function and cognitive range according to Nooteboom [Nooteboom, 2000].

the change of domain and range does not have to have an effect to the forms of thought.

It should be noted that any part, domain, range and the mapping between them can be overlapping.

Cognitive distance

In his article Nooteboom defines cognitive distance by using the cognitive function:

Definition 3.9. *[Nooteboom, 2000] Cognitive distance is difference in cognitive function.*

Here, the difference can be in any component of the function: domain, range or the mapping itself. This is easy to understand intuitively, since in similar situation people tend to think differently, that is, they make sense of the same phenomena, but do it differently.

Actually, in this case they share domain, but have differences in their mapping function. There are two possible outcomes from this situation and they correspond to the fact whether the two actors have shared range or not. If their ranges are not shared, the outcome of the mapping is different because of differences in both mental forms of thought and mental categorizations. To put this in other words, their interpretation of the current phenomena is different and the final outcome of the thought process is different. It is also possible that with the shared domain and range, the final outcome of the thought process is the same - even with different ways of thinking!

In the figure 3.3 cognitive domain, function and range of two actors are pictured. The starting point is the cognitive domain and it could be shared or different. Here shared means exactly same and different includes even slightly different ones. When considering observed phenomena, they either make sense of it similarily or not, leading either to the path marked CD1 (cognitive domain 1) or paths CD1 and CD2. Now, their cognitive functions can also be identical or differ in some point of reasoning. This is pictured as paths labeled CF1 (cognitive function 1) and CF2. The target of the mapping is cognitive range and similarily, because of the potential differences the target could be any of pictured CR's - 1,2,3 or 4 (CR standing for cognitive range). Now, there are four different cognitive ranges listed while there are only two actors. The reason behind this is that only one or two of these are real and the rest either are identical to the real ones or they do not exist at all. The "green line" consisting of CD1, CF1 and CR1 is the first actors cognitive domain, function and range. In the case where two actors share cognitive function and range but not the cognitive domain, the other actors line would be CD2, CF1 and CR3, where CD3 equals CD1.

Cognitive distance will eventually get shorter if the actors involved work closely for longer periods and get to know each other well enough. This is not necessarily optimal case, when considering novelty value of the work and innovation potential. The shorter cognitive distance gives less opportunities for learning.

Therefore, in the long run reducing cognitive distance is inferior solution to bridging it. Bridging can be interpreted as a mapping from one actor's cognitive range to another actor's cognitive domain. This is essentially communication, someone can make sense of the phenomena in

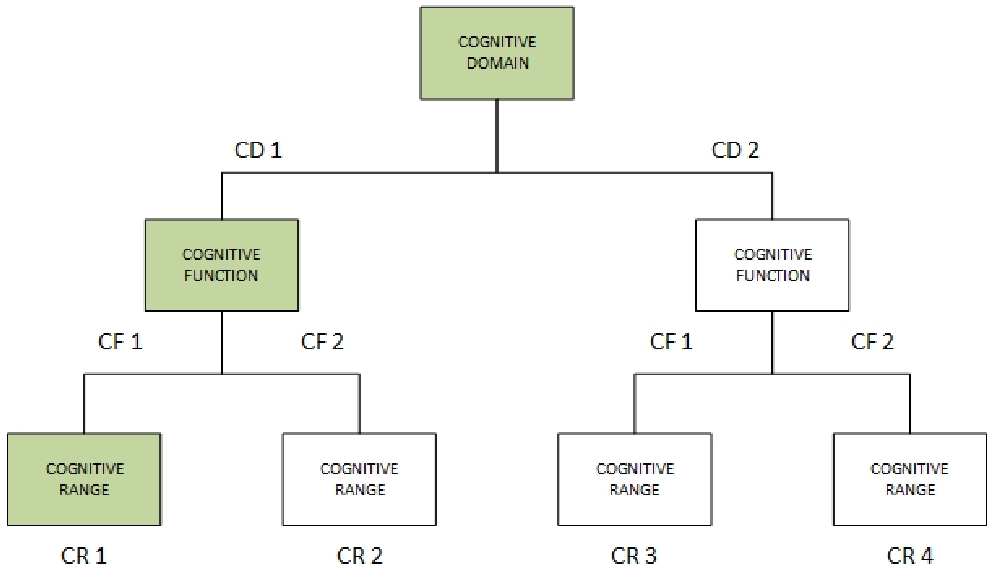

Figure 3.3: One way to visualize the source of cognitive distance between two actors.

hand and is able to help another one to make sense of the interpretation of that specific phenomena. This can also eventually lead to the reducing the cognitive distance, but it is not necessarily the case. Communication is a mapping which uses language as a tool. At best it could result overlapping between ranges, domains and forms of thought.

Nooteboom [Nooteboom, 2000] breaks down communication, or the aforementioned mapping, in his analysis and develops his idea further. Key terms in his analysis are making sense, understanding and explanation. According to his analysis, the zeroth step is for an single actor to make sense of the phenomena in hand. This act of thinking is made by earlier mentioned cognitive function by mapping from one's cognitive domain to one's cognitive range.

First step for having an inter actor mapping is to make sense: two different actors make enough sense of each other in order to make a mapping from one's domain to another's range possible. This is the actual sense for which Nooteboom uses the term communication [Nooteboom, 2000].

Further, the second step in this analysis is understanding. In ordinary language, it can refer to one person to make sense how another person thinks. This is essentially the sense how the term is used. To use mathematical-like terminology, this can be considered as a mapping from one actor's forms of thought to another's domain. This means that the latter actor understands how the former actor thinks, or what kind of cognitive function he has. To be precise, it should be noted that the understanding described here is usually only partial. It is related to the current phenomena, which are under the looking glass for sense making. Secondly, the understanding described happens over cognitive distance, one actor can make sense of other's cognitive function even though it differs from his own.

The third term used here is explanation. In everyday situations people explain their views to each other in order to help other people understanding. Or, to reason according to Nooteboom, one thinks certain way and this way of thinking is explained to another. This actually done by mapping one's forms of thought to another's cognitive domain.

Now there exists four mappings considered here: thought, communication, understanding and explanation. These are pictured in figure 3.4, where actor 1 is the active player, or the focus is on the mappings in point of his/her view. To summarize these mappings, actor 1 thinks by making sense of the observed phenomena and placing the result into his/her own cognitive categorizations. Further, actor 1 communicates with actor 2 and helps him/her to observe the sense made of the current phenomena. When actor 1 makes sense of how actor 2 thinks, he/she makes sense of the other actors mapping from cognitive domain to cognitive range, which means that actor 1 understands actor 2. And lastly, when actor 1 shares his/her way of thinking to actor 2, he/she is mapping the mental forms of thought to other actors domain, that is explaining.

This has consequences to collaborative possibilities. Agreement between two actors is not guaranteed even though they could understand each other perfectly. This, however, does not prevent collaboration - it is still possible. Collaborating becomes harder without understanding and virtually impossible without making sense to each other. In this context, making sense means communication.

According to Nooteboom [Nooteboom, 2000], in order to achieve optimal learning by, interaction, one should try to optimize the overlap in both cognitive domain and cognitive range. This means limited but sufficient overlap in domain while the overlap in range is still sufficient. And fur-

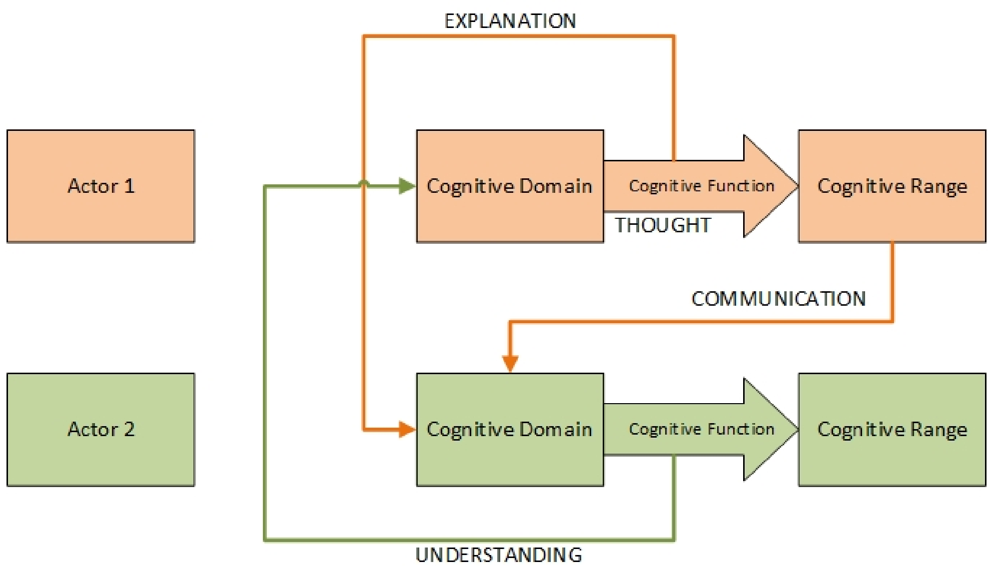

Figure 3.4: Four mappings: thought, communication, understanding and explanation. The actor 1 thinks, communicates, understands and explains.

ther, it leads to an interesting question: how does one measure the needed overlap in cognitive domains, functions and ranges?

The fore mentioned overlap can be interpreted as the proximity or distance between cognitive domains, functions and ranges. The main goal of this work is to provide one possible tool for finding an answer to this question.

Chapter 4

Intuitionistic Fuzzy Sets And Some Properties

In this chapter basic properties and definitions of sets, fuzzy sets and intuitionistic fuzzy sets are reviewed. In classical (crisp) sets, an object either belongs to a set or not, while in fuzzy sets an object belongs to a set with a degree. This is generalized further in intuitionistic fuzzy sets, where an object belongs to a set with a degree and does not belong to the same set with an another degree. The consequenses of this fact are reviewed later in this chapter. The nested structure containing crisp, fuzzy and intutionistic fuzzy sets is pictured in the figure 4.1.

The main focus here is in intuitionistic fuzzy sets, how they are defind and to give some examples. Moreover, in the last section 4.3 examples of both fuzzy and intuitionstic fuzzy approach used in knowledge management setting is presented. It should be noted that while both normal mathematical symbols and notations are used in this chapter, they are omitted in the last section in order to make text more readable.

4.1 Some preliminaries

This section is devoted to classical sets and fuzzy sets. Therefore, basic definitions and operations between sets are reviewed in both settings. Also, in order to highlight the differences of classial and fuzzy sets, theorem about the relation between equality and subset-relation is presented with a simple proof (theorems 4.1 and 4.2). In this section 4.1 plain unaccented

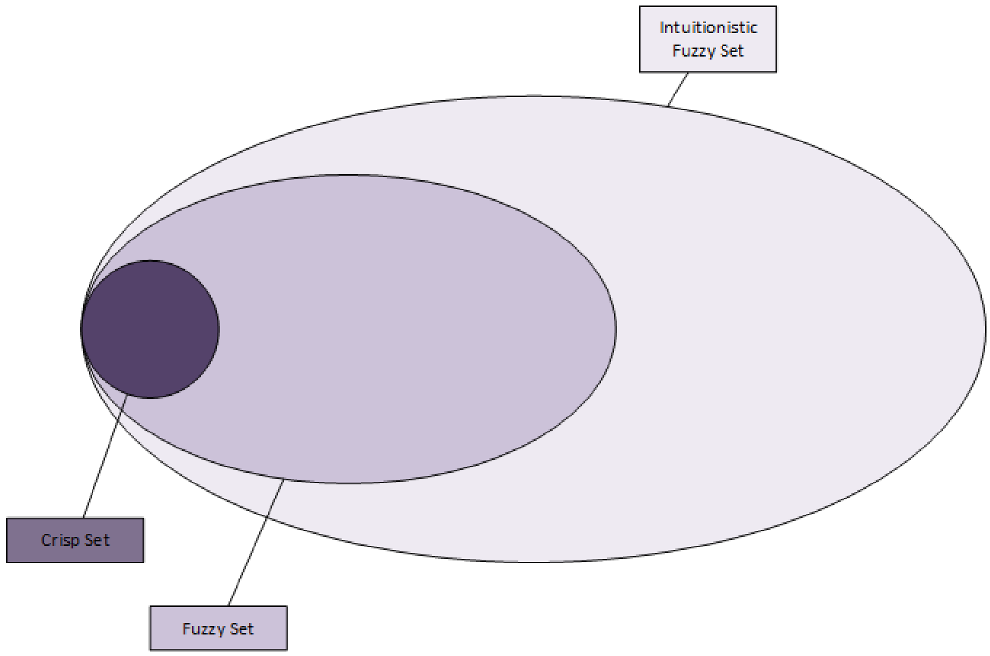

Figure 4.1: Intuitionistic fuzzy sets includes fuzzy sets and crisp sets as a special case.

letters $(A, B \ldots)$ are used as the notation related to classical sets and letters with a prime $(A', B' \ldots)$ are used related to fuzzy sets. Capital letters denote sets and the universe and small letters denote elements of sets.

4.1.1 Classical sets

Classical (crisp) set is one of the basic concepts in mathematics. Common sense understanding is that it is a collection of objects from the same ″universe″ X which have some kind of common properties. An element from that universe X either belongs to the set A or not - there does not exist other option. In conventional logic this called the principle of the excluded middle.

Here everything is considered within the universe X. So the expression ″$\exists x$″ should be interpreted as ″$\exists x : x \in X$″. Classical definition of a set is as follows.

Definition 4.1. *[Suppes, 1972, p. 19] A is a set $\iff \exists x : x \in A \vee A = \varnothing$.*

Initially, a set either includes at least one element or it is empty. In the latter case, it is a empty collection of elements. Now, a set can be part of another set:

Definition 4.2. *[Suppes, 1972, p. 22] Let A and B be sets in the universe X. A is a subset of B,*

$$A \subset B \iff \forall x : x \in A \implies x \in B.$$

A subset is included completely in the set it is part of. Other way to say this is, that there do not exist an element from the set A which is outside of the set B. Obviously a set is its own subset: $\forall x : x \in A \implies x \in A$.

Theorem 4.1. *[Suppes, 1972, p. 22] Let A and B be sets in the universe X.*

$$A = B \iff A \subset B \wedge B \subset A.$$

First, let $A = B$. From the fact that each set is its own subset follows that $A \subset B \wedge B \subset A$. Next, let $A \subset B \wedge B \subset A$ hold. By the definition 4.2 every element belonging to the set A belongs to the set B and vice versa. Therefore, $\forall x \in X : x \in A \iff x \in B$. Now, it is clear that if both of them have all the same elements and nothing more they are the exactly same set.

Next, the basic operations between sets are defined.

Definition 4.3. *[Suppes, 1972, pp. 22,30] Let A and B be sets in the universe X. The following basic operations are defined:*

1. *the complement* $A^C = \{x \in X : x \notin A\}$,
2. *the intersection* $A \cap B = \{x \in X : x \in A \wedge x \in B\}$,
3. *the union* $A \cup B = \{x \in X : x \in A \vee x \in B\}$, *and*
4. *the differcence* $A - B = \{x \in X : x \in A \wedge x \notin B\}$.

In every case, any result of previous basic operations presented in the definition 4.3 is also a set in the universe X. Now, the complement A^C is a set including everything which is not included in the set A. The intersection $A \cap B$ which includes only such elements of sets A and B which are included in both of them. The union $A \cup B$ includes every element which belongs to at least one of the sets A or B. Lastly, the difference $A - B$ includes all elements from the set A which do not belong to the set B.

Example 4.1. *Let* $X = \{1, 2, \ldots, 6\}$ *be the the set of the numbers on the dice,* $E = \{2, 4, 6\}$ *the set of even numbers and* $L = \{1, 2, 3\}$ *the set of low numbers. Now the results of operations of the definition 4.3 are as follows:*

1. $E^C = \{1, 3, 5\}$,
2. $E \cap L = \{2\}$,
3. $E \cup L = \{1, 2, 3, 4, 6\}$, *and*
4. $E - L = \{4, 6\}$.

It shoud be noted that the difference between two sets A and B can be written also using both the intersection $\cap$ and the complement B^C: $A - B = A \cap (B^C)$. This also express the set of elements belonging to the set A and not in the set B.

Definition 4.4. *[Szmidt, 2014, p. 8] For a set* X *and a subset* A *of* X $(A \subseteq X)$ *it is defined that*

$$\varphi(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

is the characteristic function of the set A *in* X.

This characteristic function $\varphi(x)$ describes the set A in X and using that, a classical set A is given by

$$A = \{< x, \varphi_A(x) > \mid \forall x \in X\}.$$

Example 4.2. *If numbers on the sides of a normal six-sided dice are* $1, 2, \ldots, 6$*, how to describe the set of even numbers on the sides of a dice?*

Let $X = \{1, 2, \ldots, 6\}$ *be the the set of the numbers on the dice. The set* E *of even numbers is obviously given by* $E = \{2, 4, 6\}$. *The characteristic function* $\varphi_E(x)$ *can be defined as*

$$\varphi_E(x) = \begin{cases} 1, & \text{if } x \in \{2, 4, 6\}, \text{ and} \\ 0, & \text{otherwise.} \end{cases}$$

Other possibility for defining the characteristic function $\varphi_E(x)$ *is by using* mod 2-*operator, the remainder of division by* 2*:*

$$\varphi_E(x) = |x(\text{mod } 2) - 1|.$$

Therefore, presented in a tabular form:

x	$x(\text{mod } 2)$	$\varphi_E(x)$	$< x, \varphi_E(x) >$
1	1	0	$< 1, 0 >$
2	0	1	$< 2, 1 >$
3	1	0	$< 3, 0 >$
4	0	1	$< 4, 1 >$
5	1	0	$< 5, 0 >$
6	0	1	$< 6, 1 >$

and now it is easy to see that a set E given by

$$E = \{< x, \varphi_E(x) > | \ \forall x \in X\} = \{2, 4, 6\},$$

is the set of even alternatives from a regular dice.

4.1.2 Fuzzy sets

While in a crisp setting an element either belongs to a set or not, this is not the case when considering fuzzy sets. A fuzzy set is a generalization of a classical set and elements of a fuzzy set belong to that set in a degree. That degree is presented by a membership function μ which can have any real numerical value from 0 to 1. For an element in a crisp set, the membership function μ coincides with the characteristic function φ given in the definition 4.4.

Fuzzy sets were introduced by Lotfi Zadeh in 1965:

Definition 4.5. *[Zadeh, 1965, p. 339] A fuzzy set A' in the universe X' is given by*

$$A' = \{< x', \mu_{A'}(x') > | x' \in X'\},$$

where $\mu_{A'} : X' \to [0, 1]$ is the membership function of the fuzzy set A'. For every element $x' \in X'$, $\mu_{A'}$ describes its extent of membership to fuzzy set A'.

Obviously, $1 - \mu$ is the degree of non-membership and it is given also for every element. Now, while the membership is defined in a fuzzy way, it still means that there exist no room for not knowing - the memberhship and non-membership are the only options and that is all there exist.

Example 4.3. *Since the elements of a fuzzy set are characterized by the value of the membership function and it can get any real numerical value from 0 to 1, it* *can be said that a person with a height of 160 cm belong to the set of tall people with a value of* 0.36*. Normally it would be said that 160 cm tall person is quite short.*

Height is, however, a quantity that is estimated and compared to the normal average height of the surrounding community or people in it - in medieval times 160 cm tall person would have been above average or even tall!

Let the universe X' *include all living people, TALL be the set of tall people and* h' *be the height of an idividual in centimeters. The mapping*

$$\mu_{TALL} : X' \to [0, 1]$$

is the membership function for the set TALL and it can be defined for example in the following way:

$$\mu_{TALL}(h') = \begin{cases} 1, & \text{if } h' \geq 200 \text{ cm}, \\ (\frac{h'-100}{100})^2, & \text{if } 100 \text{ cm} < h' < 200 \text{ cm}, \\ 0, & \text{if } h' \leq 100 \text{ cm}. \end{cases}$$

Then the ultimate limit for being tall is set to 200cm and the results would look like following:

Height h' *in cm*	$\mu_{TALL}(h')$
< 100	0
100	0
110	0.01
120	0.04
130	0,09
140	0.16
150	0.25
160	0.36
170	0.49
180	0.64
190	0.81
200	1
> 200	1

Again, with this definition of mebership function, person with the height of 160 cm belongs to the set of TALL with a value 0.36*.*

However, when considering a basket ball player from a professional NBA team, say 190 cm tall male, most of us consider him to be a tall person. The average *height of his team could be 203 cm and therefore he is propably not considered as a tall basket ball player within his team. When these arguments are taken into consideration, maybe it would be fair to say that basket ball player with the height of 190 cm belongs to TALL by the value of* 0.81.

Definition 4.6. *[Zadeh, 1965, p. 340] Let* $A' = \{< x', \mu_{A'}(x') > | x' \in X'\}$ *be a fuzzy set in the universe* X'. *Now,* A' *is a empty set,*

$$A' = \varnothing \Longleftrightarrow \forall x' \in X' : \mu_{A'}(x') = 0.$$

This definition is similar to the definition of empty set in the classical sets would be, if the definition 4.1 would have been given using the characteristic function φ given in the definition 4.4. In this case the membership function $\mu_{A'}$ equals 0 for all elements of the universe X' and the fuzzy set A' does not contain any elements.

Definition 4.7. *[Zadeh, 1965, p. 340] Let* A' *and* B' *be fuzzy sets in the universe* X', *and* $\mu_{A'}$ *and* $\mu_{B'}$ *be their corresponding membership functions,*

$$A' = \{< x', \mu_{A'}(x') > | x' \in X'\} \text{ and } B' = \{< x', \mu_{B'}(x') > | x' \in X'\}.$$

A' *equals* B',

$$A' = B' \Longleftrightarrow \forall x' \in X' : \mu_{A'}(x') = \mu_{B'}(x').$$

Now, the membership function μ defines the fuzzy set and if the two sets are equal, also their membership functions are equal.

Definition 4.8. *[Zadeh, 1965, p. 340] Let* A' *and* B' *be fuzzy sets in the universe* X', *and* $\mu_{A'}$ *and* $\mu_{B'}$ *be their corresponding membership functions,*

$$A' = \{< x', \mu_{A'}(x') > | x' \in X'\} \text{ and } B' = \{< x', \mu_{B'}(x') > | x' \in X'\}.$$

A' *is a subset of* B',

$$A' \subset B' \Longleftrightarrow \forall x' \in X' : \mu_{A'}(x') \leq \mu_{B'}(x').$$

Now, since for any fuzzy set A' holds $\forall x' : \mu_{A'}(x') \leq \mu_{A'}(x')$, any fuzzy set is clearly its own subset. Moreover, if a fuzzy set A' is a subset of a fuzzy set B', it means that every element x' is "more" member in the fuzzy set B' than in the fuzzy set A'.

Theorem 4.2. *Let A' and B' be fuzzy sets in the universe X', and $\mu_{A'}$ and $\mu_{B'}$ be their corresponding membership functions,*

$$A' = \{< x', \mu_{A'}(x') > | x' \in X'\} \text{ and } B' = \{< x', \mu_{B'}(x') > | x' \in X'\}.$$

Fuzzy sets A' and B' are equal,

$$A' = B' \iff A' \subset B' \wedge B' \subset A'.$$

First, let $A' = B'$. From the definition 4.8 and from fact that each set is its own subset follows that $A' \subset B' \wedge B' \subset A'$ is true. Next, let $A' \subset B' \wedge B' \subset A'$ hold. By the definition 4.8 both $\forall x' \in X' : \mu_{A'}(x') \leq \mu_{B'}(x')$ and $\forall x' \in X' : \mu_{B'}(x') \leq \mu_{A'}(x')$ hold. Therefore $\forall x' \in X' : \mu_{A'}(x') = \mu_{B'}(x')$ and $A' = B'$ and the theorem 4.2 holds.

Next, the basic operations between fuzzy sets are defined. Furthermore, fuzzy sets are defined using the membership function μ, so it is used also to define following operations.

Definition 4.9. *[Zadeh, 1965, p. 340] Let A' and B' be fuzzy sets in the universe X', and $\mu_{A'}$ and $\mu_{B'}$ be their corresponding membership functions,*

$$A' = \{< x', \mu_{A'}(x') > | x' \in X'\} \text{ and } B' = \{< x', \mu_{B'}(x') > | x' \in X'\}.$$

The following basic operations are defined:

1. *the complement* $A'^C = \{< x', 1 - \mu_{A'}(x') > | x' \in X'\}$,
2. *the intersection* $A' \cap B' = \{< x', \mathit{min}\,(\mu_{A'}(x'), \mu_{B'}(x')) > | x' \in X'\}$,
3. *the union* $A' \cup B' = \{< x', \mathit{max}\,(\mu_{A'}(x'), \mu_{B'}(x')) > | x' \in X'\}$, *and*
4. *the differcence* $A' - B' = \{< x', \mathit{min}\,(\mu_{A'}(x'), 1 - \mu_{B'}(x')) > | x' \in X'\}$.

Now, the complement reverses the values of membership and non-membership. This fact is also used to define the difference between two fuzzy sets, where classical type approach would have been unnecessarily complicated, which is seen if definitions 4.3 and 4.9 are compared. Now the definition is based on the fact that $A' - B' = A' \cap (B'^C)$.

Both the intersection and the union follow the logic inherited from the classical setting reformulated after the definition 4.5. It also makes sense when considering the subset relation ($\subset$) given in the definition 4.8. Namely, the intersection ($A' \cap B'$) of two sets, A' and B', is always a subset of both original sets and in fuzzy setting this means that $\forall x' \in X' : \mu_{A' \cap B'}(x') \leq \mu_{A'}(x') \wedge \mu_{A' \cap B'}(x') \leq \mu_{B'}(x')$,from which follows that $\mu_{A' \cap B'}(x') = \min\,(\mu_{A'}(x'), \mu_{B'}(x'))$ has to hold. Also, the union $A' \cup B$ can be considered similarly.

Example 4.4. *Let* $X' = \{x'_1, x'_2, x'_3, x'_4\}$ *be the finite universe and* A' *and* B' *be fuzzy sets over* X' *defined as follows:*

$$\begin{array}{ll} A = & \{< x'_1, 0.6 >, < x'_2, 0.5 >, < x'_3, 0.0 >, < x'_4, 1.0 >\}, \\ B = & \{< x'_1, 0.3 >, < x'_2, 1.0, >, < x'_3, 0.2 >, < x'_4, 0.3 >\}. \end{array}$$

Now, the operations from the definition 4.9 give following results:

1. *the complement*
   $A'^C = \{< x'_1, 0.4 >, < x'_2, 0.5 >, < x'_3, 1.0 >, < x'_4, 0.0 >\},$
2. *the intersection*
   $A' \cap B' = \{< x'_1, 0.3 >, < x'_2, 0.5 >, < x'_3, 0.0 >, < x'_4, 0.3 >\},$
3. *the union*
   $A' \cup B' = \{< x'_1, 0.6 >, < x'_2, 01.0, < x'_3, 0.2 >, < x'_4, 1.0 >\},$
4. *the differcence*
   $A' - B' = \{< x'_1, 0.6 >, < x'_2, 0.0 >, < x'_3, 0.0 >, < x'_4, 0.7 >\}.$

There are, of course, a multitude of other operators defined over fuzzy sets (see for example [Zadeh, 1965, Atanassov, 2012]). Here are only basic operations presented in order to give some insight to the different "families" of sets.

4.2 Intuitionistic fuzzy sets

Classical and fuzzy sets and some basic properties related to them were presented previous section 4.1. While fuzzy sets and their elements are fuzzy in the sense of membership, there are no room for unknown or undefined. To change this, to make room for not-knowing, definitions have to be adjusted somehow. In intuitionistic fuzzy sets there exist separate ways to express membership and non-membership and that results also the fact that there is also room for unknown.

It should be noted that in order to keep notation as simple as possible in the current section 4.2, plain unaccented letters $(A, B \ldots$ and $x, y, \ldots)$ are used as the notation for anything related to *intuitionistic fuzzy sets* and letters with a prime $(A', B' \ldots)$ are stil used related to fuzzy sets. Capital letters denote sets and the universe sets are in and small letters denote the elements of sets. If the meaning is something else it will be clearly pointed out in the each related context.

4.2.1 Presentation of intuitionistic fuzzy sets

Intuitionistic fuzzy sets are characterized by two different functions - membership μ and non-membership ν. Together the sum of these two functions is equal or greater than 0 and equal or less than 1. Moreover, if their sum is less than 1, there is something unknown involved.

Definition 4.10. *[Atanassov, 2012, pp. 1-2] An intuitionistic fuzzy set A in the universe X is given by (Atanassov viitteet)*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\},$$

where

$$\begin{aligned} \mu_A : \quad & X \to [0,1] \\ \nu_A : \quad & X \to [0,1] \end{aligned}$$

with the condition

$$0 \leq \mu_A(x) + \nu_A \leq 1, \ \forall x \in X.$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the degrees of membership and non-membership, respectively, of the element $x \in X$ to the set A.

Now, in the following definition a description of the unknown part is given.

Definition 4.11. *[Atanassov, 2012, p. 2],[Szmidt, 2014, p. 9] For an intuitionistic fuzzy set A*

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is called the intuitionistic fuzzy index *(hesitation margin) of the element x in the set A.*

The hesitation margin $\pi_A(x)$ expresses the lack of knowledge on whether x belongs to A or not. Now, if the hesitation margin vanishes completely (i.e. $\forall x \in X : \pi_A(x) = 0$) and the non-membership function $\nu_A(x)$ do not do that (i.e. $\exists x \in X : \nu_A(x) \neq 0$) the set is actually a fuzzy set. If both hesitation margin and non-membership function equal zero for all elements in the universe, the set is a classical set. This is presented in the example 4.5.

Because every element in the universe has been assigned three numbers, namely the degree of membership, the degree of non-membership and the hesitation margin ($\mu_A(x)$, $\nu_A(x)$ and $\pi_A(x)$), any intuitionistic fuzzy sets can be described by presenting every element of the set with theese characteristic numbers. This is called the *three-term presentation of intuitionistic fuzzy sets*.

Definition 4.12. *Let X be the universe and $A \subset X$ an intuitionistic fuzzy set with membership function $\mu_A(x)$, non-membership function $\nu_A(x)$ and hesitation margin $\pi_A(x)$. Then A can be written as*

$$A_{x \in X} = \left\{ \frac{(\mu_A(x_1), \nu_A(x_1), \pi_A(x_1))}{x_1}, \frac{(\mu_A(x_2), \nu_A(x_2), \pi_A(x_2))}{x_2}, \dots \right\}$$

However, it should be noted that since two numbers, $\mu_A(x)$ and $\nu_A(x)$, describe the element x fully, these two connected to the element x are usually used in the *two-term* notation of intuitionistic fuzzy sets [Atanassov, 2012, Li, 2014, Szmidt, 2014].

Now, if the universe X happens to be a finite space with n elements, it naturally suffices to determine the values of the three characteristic functions for every n elements.

Example 4.5. *Let $X = \{1, 2, 3, 4\}$ and let intuitionistic fuzzy sets A_i, $i = 1, 2, 3$, be defined as follows:*

$$A_1 = \left\{ \frac{(0,1,0)}{1}, \frac{(1,0,0)}{2}, \frac{(0,1,0)}{3}, \frac{(1,0,0)}{4} \right\},$$

$$A_2 = \left\{ \frac{(0.2,0.8,0)}{1}, \frac{(1,0,0)}{2}, \frac{(0.5,0.5,0)}{3}, \frac{(1,0,0)}{4} \right\}, \text{ and}$$

$$A_3 = \left\{ \frac{(1,0,0)}{1}, \frac{(0.6,0.2,0.2)}{2}, \frac{(0.1,0.4,0.5)}{3}, \frac{(0.8,0.1,0.1)}{4} \right\}.$$

Now $A_1 = \{2, 4\}$ is actually a classical set with two elements, $A_2 = \left\{ \frac{(0.2,0.8)}{1}, 2, \frac{(0.5,0.5)}{3}, 4 \right\}$ is really a fuzzy set and $A_3 = \left\{ 1, \frac{(0.6,0.2,0.2)}{2}, \frac{(0.1,0.4,0.5)}{3}, \frac{(0.8,0.1,0.1)}{4} \right\}$ is only truly intuitionistic fuzzy set. These three sets are presented in the figure 4.2.

It can be noticed also from the previous example 4.5 that the hesitation margin π is calculated directly and uniquely by substracting both membership μ and non-membership ν from 1, it would be enough to present intuitionistic fuzzy sets and elements of intuitionistic fuzzy sets by using just two functions, the membership μ and the non-membership ν. However, this two-term presentation would lead slightly different distance compared to the three-term presentation. This is discussed in sections 5.2 and 5.3.

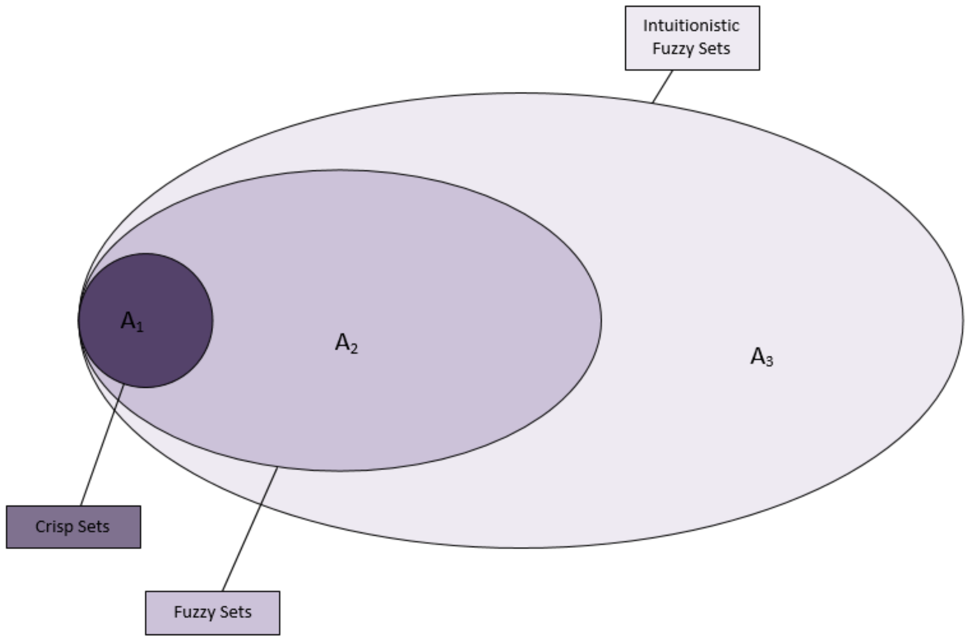

Figure 4.2: In the example 4.5 sets are presented in intuitionistic fuzzy way.

4.2.2 Some properties of intuitionistic fuzzy sets

The presentation of these properties is similar to the presentation in previous section 4.1 in classical and fuzzy setting.

Definition 4.13. *Let* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ *be an intuitionistic fuzzy set in the universe* X*. Now,* A *is a empty set,*

$$A = \varnothing \iff \forall x \in X : \mu_A(x) = 0 \wedge \nu_A(x) = 1.$$

Intuitively an empty set does not contain any elements and that should be a fact where is no room for uncertainty. If a set is propably empty it does not qualify as an empty set. In this case both the membership function μ_A equals 0 and the non-membership function equals 1 for all elements of the universe X, which means that the intuitionistic fuzzy set A in definition 4.13 does not contain any elements. Furthermore, the hesitation margin $\pi_A(x)$ equals 0 for all elements of the universe X, so it sure that in the whole universe X there do not exist a single element which could be so much as partly a member in the empty set. This definition of the intuitionstic fuzzy empty set is actually exactly same as the fuzzy empty set [Zadeh, 1965, p. 340].

Definition 4.14. *[Atanassov, 2012, p. 17] Let A and B be intuitionistic fuzzy sets in the universe X, μ_A and μ_B be their corresponding membership functions, and ν_A and ν_B be their corresponding non-membership functions,*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \text{ and } B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}.$$

A equals B,

$$A = B \iff \forall x \in X : \mu_A(x) = \mu_B(x) \wedge \nu_A(x) = \nu_B(x).$$

Now, intuitionistic fuzzy sets are defined by their membership and non-membership functions and if these two functions are equal, so must be also the case with both intuitionistic fuzzy sets and vice versa.

Definition 4.15. *[Atanassov, 2012, p. 17] Let A and B be intuitionistic fuzzy sets in the universe X, μ_A and μ_B be their corresponding membership functions, and ν_A and ν_B be their corresponding non-membership functions,*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \text{ and } B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}.$$

A is a subset of B,

$$A \subset B \iff \forall x \in X : \mu_A(x) \leq \mu_B(x) \wedge \nu_A(x) \geq \nu_B(x).$$

Furthermore, if in the previous definition 4.15 the intuitionistic fuzzy set B is replaced with A, it is easy to see that the partial inequalities still hold and, therefore, any intuitionistic fuzzy set is its own subset. Moreover, if an intuitionistic fuzzy set A is a subset of an intuitionistic fuzzy set B, it means that every element x is included "more" in the intuitionistic fuzzy set B than in the intuitionistic fuzzy set A. This notion of "more" is considered in a such way, that an element is both as much or more member and as much or less non-member.

Theorem 4.3. *Let A and B be intuitionistic fuzzy sets in the universe X, μ_A and μ_B be their corresponding membership functions, and ν_A and ν_B be their corresponding non-membership functions,*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \text{ and } B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}.$$

Intuitionistic fuzzy sets A and B are equal,

$$A = B \iff A \subset B \wedge B \subset A.$$

First, let $A = B$. Now, from the definition 4.15 and from fact that each set is its own subset follows that $A \subset B \wedge B \subset A$ is true. Next, let $A \subset B \wedge B \subset A$ hold. By the definition 4.15 both $\forall x \in X : \mu_A(x) \leq \mu_B(x) \wedge \nu_A(x) \geq \nu_B(x)$ and $\forall x \in X : \mu_B(x) \leq \mu_A(x) \wedge \nu_B(x) \geq \nu_A(x)$ and therefore $\forall x \in X : \mu_A(x) = \mu_B(x) \wedge \nu_A(x) = \nu_B(x)$, which by the definition 4.14 mean that $A = B$ and the theorem 4.3 holds.

The basic operations between intuitionistic fuzzy sets are defined in similar manner than operations over fuzzy sets. However, the difference is that also the non-membership function plays a role in the definition.

Definition 4.16. *[Atanassov, 2012, p. 17],[Li, 2014, pp. 9-10],[Szmidt, 2014, pp. 17-18] Let A and B be intuitionistic fuzzy sets in the universe X, μ_A and μ_B be their corresponding membership functions, and ν_A and ν_B be their corresponding non-membership functions,*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \textit{ and } B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}.$$

Let $\lambda > 0$ be an arbitrary real number. The following basic operations are defined:

1. *the complement*
   $A^C = \{< x, \nu_A(x), \mu_A(x) > | x \in X\}$,
2. *the intersection*
   $A \cap B = \{< x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)) > | x \in X\}$,
3. *the union*
   $A \cup B = \{< x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x)) > | x \in X\}$,
4. *the differcence*
   $A - B = \{< x, min(\mu_A(x), \nu_B(x)), max(\nu_A(x), \mu_B(x)) > | x \in X\}$,
5. *the addition*
   $A + B = \{< x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x) > | x \in X\}$,
6. *the multiplication*
   $AB = \{< x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) > | x \in X\}$,
7. *the product of an intuitionistic fuzzy set and a real number*
   $\lambda A = \{< x, 1 - (1 - \mu_A(x))^\lambda, (\nu_A(x))^\lambda > | x \in X\}$, *and*
8. *the power*
   $A^\lambda = \{< x, (\mu_A(x))^\lambda, 1 - (1 - \nu_A(x))^\lambda > | x \in X\}$.

Here it is important to notice that the complement reverses the values of membership and non-membership. Furthermore, in the classical setting an element is included either in the set or its complement. Now, in the intuitionistic fuzzy setting it feels reasonable, that if an element belongs to an intuitionistic fuzzy set with some value, it does not belong to the complement of the original set with the same exact value and also vice versa. Therefore, the reversal of the membership and non-membership functions make sense. The complement is also used to define the difference between intuitionistic two fuzzy sets and the definition is based on the fact that $A - B = A \cap (B^C)$.

The definition of both the intersection and the union follow the fuzzy definition, only the non-membership function is the difference. Again, it makes sense when considering the subset relation ($\subset$) given in the definition 4.15. Namely, the intersection ($A \cap B$) of two intuitionistic fuzzy sets, A and B, is always a subset of both original sets and in the intuitionistic fuzzy setting this means that $\forall x \in X : \mu_{A\cap B}(x) \leq \mu_A(x) \wedge \nu_{A\cap B}(x) \geq \nu_A(x)$ and $\forall x \in X : \mu_{A\cap B}(x) \leq \mu_B(x) \wedge \nu_{A\cap B}(x) \geq \nu_B(x)$. This, in turn, means that $\mu_{A\cap B}(x) = min(\mu_A(x), \mu_B(x))$ and $\nu_{A\cap B}(x) = max(\nu_A(x), \nu_B(x))$. Further, if the element x from the intersection $A \cap B$ would have smaller membership than $min(\mu_A(x), \mu_B(x))$, it would mean that there would exist an intuitionistic fuzzy set C such that $C \subset A, C \subset B, A \cap B \subset C$ and $A \cap B \neq C$, which is a contradiction.

Example 4.6. *Let $X = \{x_1, x_2, x_3, x_4\}$ be the finite universe, $\lambda = 2$, and A and B be intuitionistic fuzzy sets over X defined as follows:*

$$\begin{aligned} A = \quad & \{< x_1, 0.6, 0.2 >, < x_2, 0.5, 0.3 >, < x_3, 0.0, 0.0 >, < x_4, 0.0, 1.0 >\}, \\ B = \quad & \{< x_1, 0.3, 0.4 >, < x_2, 1.0, 0.0 >, < x_3, 0.2, 0.5 >, < x_4, 0.3, 0.4 >\}. \end{aligned}$$

Now, the operations from the definition 4.16 give following results:

1. *the complement*
$A^C = \{< x_1, 0.2, 0.6 >, < x_2, 0.3, 0.5 >, < x_3, 0.0, 0.0 >, < x_4, 1.0, 0.0 >\}$,
2. *the intersection*
$A \cap B = \{< x_1, 0.3, 0.4 >, < x_2, 0.5, 0.3 >, < x_3, 0.0, 0.5 >, < x_4, 0.0, 1.0 >\}$,
3. *the union*
$A \cup B = \{< x_1, 0.6, 0.2 >, < x_2, 1.0, 0.0 >, < x_3, 0.2, 0.0 >, < x_4, 0.3, 0.4 >\}$,

4. *the differcence*
$A - B = \{< x_1, 0.4, 0.3 >, < x_2, 0.0, 1.0 >, < x_3, 0.0, 0.2 >, < x_4, 0.0, 1.0 >\}$,
5. *the addition*
$A + B = \{< x_1, 0.72, 0.08 >, < x_2, 1.0, 0.0 >, < x_3, 0.2, 0.0 >, < x_4, 0.3, 0.4 >\}$,
6. *the multiplication*
$AB = \{< x_1, 0.18, 0.52 >, < x_2, 0.5, 0.3 >, < x_3, 0.0, 0.5 >, < x_4, 0.0, 1.0 >\}$,
7. *the product of an intuitionistic fuzzy set and a real number*
$2A = \{< x_1, 0.84, 0.04 >, < x_2, 0.75, 0.09 >, < x_3, 0.0, 0.0 >, < x_4, 0.0, 1.0 >\}$,
8. *the power*
$A^2 = \{< x_1, 0.36, 0.36 >, < x_2, 0.25, 0.51 >, < x_3, 0.0, 0.0 >, < x_4, 0.0, 1.0 >\}$.

It should be noted, that all operations presented here involve only membership and non-membership functions, so therefore the two-term presentation was used in the previous example 4.6.

4.3 Examples of fuzzy and intuitionistic fuzzy approaches in knowledge management

In knowledge management refining the current methods or processes requires some kind of evaluation of the current situation. Many of the variables included in the discussion are often hard to define and measure. Here are two different approaches suggested for tackling this kind of problems.

4.3.1 A fuzzy approach to develop metrics by Liebowitz

Liebowitz [Liebowitz, 2005] states that knowledge management in general is seen as an enigma among management. One part of the problem is that measuring the success of the knowledge management initiatives has been difficult or based mainly on observed results after certain amount of time.

The main challenge is that knowledge management deals with variables which are somehow vaguely defined and hard to quantify. Because of the complexity of the task at hand, there exists a tendency to use soft measures instead of defined metrics. For example, this includes using anecdotes and organizational narratives to describe the usefulness and performance of the current knowledge management system.

Liebowitz [Liebowitz, 2005] proposes that since knowledge management is a fuzzy area and its success is measured using anecdotal evidence, methods from fuzzy logic could be useful. Furthermore, some concepts from fuzzy logic could be applied to generated set of metrics in order to measure the success of a knowledge management system.

While the fuzziness brings important way to handle variables which cannot be measured exactly, it does not take in to account the fact that there are also unknown factors. The fuzzy approach states that some part of the variable satisfies the criteria and rest does not. Usually this is not the case. The examined variable can satisfy the criteria partly, be partly outside of the criteria and there could exist an area or part for which it is unknown whether it satisfies the criteria or not.

The last fact suggest that the unknown should be taken into account when trying to find measures suitable for knowledge management purposes. The intuitionistic fuzzy approach includes also the unknown, so it can be the right way to proceed forward.

4.3.2 An intuitionistic fuzzy approach to multi-person multi-attribute decision making by Xu

Intuitionistic fuzzy numbers are defined by two functions, membership and non-membership. According to Xu [Xu, 2007] they are a useful tool when trying to describe the information in the process of decision making. They include both the positive information (membership) and negative information (non-membership) while still leaving room for the unknown or uncertain.

Decision making is a common activity in various environments ranging from making grocery list to large scale organizational partnership decisions. In a multi-person multi-attribute decision making a group of decision makers participate in the process of ordering a set of potential alternatives in order to find the most beneficial or desirable alternative. There exist a set of attributes which are either given before the decision process or determined during it. Every decision maker orders the attributes, or in other words, provides own preference information of the attributes by giving them weights.

Because of the complexity socio-economic environment and the subjective nature of human thinking, the information about the values assigned to different attributes is vague or, at best, uncertain. In his article Xu [Xu, 2007] proposed that, the use of the intuitionistic fuzzy number is highly useful in handling fuzziness and uncertainty. Here, the intuitionistic fuzzy number is the basic element of an intuitionistic fuzzy set.

There exists several ways to approach intuitionistic fuzzy numbers and sets and applying them in the research literature. Most of them deal with different decision, communication and even game theoretical problems. (See for example [Atanassov, 2012, Li, 2014, Szmidt, 2014, Xu, 2007].) Taking this and the advanced mathematics required to deal with these topics, going further in these lines of research is out of the scope of the current work in hand.

In his article, Xu [Xu, 2007] proposes an approach to multi-person multi-attribute intuitionistic fuzzy decision making under intuitionistic fuzzy environment. This proposed method involves gathering all individual data and presenting it in a numerical matrix form. Then the individual intuitionistic decision matrices are fused into a collective intuitionistic decision matrix by applying a specific intuitionistic fuzzy hybrid geometric operator defined by Xu in [Xu, 2007].

Next phase is to find optimal weight vectors for different alternatives and after that to construct the weight matrix and to apply another operator by Xu, intuitionistic fuzzy weighted geometric operator, to get the overall values of the alternatives.

In the last phase of the decision process according to Xu, the alternatives are ranked by the scores of the overall values. These scores are calculated by using the score function defined by Chen and Tan [Xu, 2007, p. 224]. If two scores happen to be equal, their mutual order will be decided by the accuracy degrees of those two overall values which scores are equal.

In short, Xu's approach to multi-person multi-attribute intuitionistic fuzzy decision making under intuitionistic fuzzy environment consists of gathering the intuitionistic fuzzy data, finding the weights and ordering the alternatives by calculated scores and possible accuracy degrees.

Chapter 5

Distance In Intuitionistic Fuzzy Sets

The main goal of this chapter is to present a distance measure for intuitionistic fuzzy sets. In order to be useful and applicable, the measure should be simple enought and computable. Therefore the chosen distance measure, which is also defined in this chapter, is a Hamming distance.

The basic defitions of the metric and the norm are presented in the first section of this chapter. In the second section the definition of a Hamming distance for fuzzy sets is given and developed further into intuitionistic fuzzy setting using the two-term presentation of both the fuzzy and intuitionistic fuzzy sets.

Finally, the last step of the development in the current chapter is to define the three-term version of the intuitionistic fuzzy Hamming distance.

5.1 Basic definitions - metrics and norms

When people consider travelling from one location to another, usually two things are considered - the travel time and distance. Now, if the one- way roads are omitted from this imaginary example and the distance is considered to be the lenght of the shortest route between locations, any system of roads can be considered as a metric space. Measuring and estimating distance is a normal, maybe even daily, activity for most people.

Mathematically speaking, nothing more is assumed about the universe X that it is a set. Now, a metric is a function which introduces the concept of distance between the elements of the set it is paired with. This distance can be defined in various ways and it can be even simply just either 1 or 0!

Definition 5.1. *[Szmidt, 2014, p. 39],[Väisälä, 1999, p. 20] A distance on a set X is a positive function d from pairs of elements of X to the set $\mathbb{R}^+$ of non-negative real numbers with the following properties $\forall x_1, x_2, x_3 \in X$:*

1. $d(x_1, x_1) = 0$ *(reflexivity);*
2. $d(x_1, x_2) = 0 \Longleftrightarrow x_1 = x_2$ *(separability);*
3. $d(x_1, x_2) = d(x_2, x_1)$ *(symmetry);*
4. $d(x_1, x_3) \leq d(x_1, x_2) + d(x_2, x_3)$ *(triangle inequality).*

The postiive function $d : X^2 \rightarrow \mathbb{R}^+$ is called metric and the pair (X, d) is called metric space.

If there exists a function which relates a positive number for every member of a set, it is not necessarily a metric. Functions which fullfill weaker conditions are named as follows [Szmidt, 2014, 40]. A pseudometric is a measure which fulfills requirements 1, 3 and 4 of the previous definition (separability does not hold). A semimetric fulfills requirements 1, 2 and 3 of the previous definition (triangle inequality does not hold). Lastly, a semi-pseudometric satisfies requirements 1 and 3 only.

Vector space is a concept from linear algebra. Here it suffices to note that a vector space is a set X and its elements $\vec{x} \in X$ are called vectors. These fulfill the axioms of vector space which can be reviewed for example from [Väisälä, 1999, p. 13]. The most usual example of vector space is $\Re^n$, where a vector's dimension in each coordinate is given as a n-tuplet of numbers. Now, a norm of a vector can be thought as the lenght or the magnitude of the current vector. [Szmidt, 2014, 40]

Definition 5.2. *[Väisälä, 1999, p. 16] A norm of a vector is a real positive number $\|\vec{x}\|$ is assinged to the vector $\vec{x}$ $(\in \mathbb{R}^n)$. In order to be a norm, the number $\|\vec{x}\|$ must satisfy the following axioms:*

1. $\|\vec{x}\| \geq 0$ *for every $\vec{x}$;*
2. $\|\vec{x}\| = 0 \Longleftrightarrow \vec{x} = 0$*;*
3. $\|\alpha\vec{x}\| = |\alpha|\|\vec{x}\|$ *for every $\vec{x}$ and every real number α;*
4. $\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$ *for every $\vec{x}$ and $\vec{y}$.*

Here, it should be noted that the *norm* in the previous definition 5.2 is defined in a real vector space $\mathbb{R}^n$.

Definition 5.3. *[Szmidt, 2014, p. 41],[Väisälä, 1999, p. 16] If X is a vector space and $\|\cdot\|$ is a norm in X, the pair $(X, \|\cdot\|)$ is called normed vector space.*

However, norms can be defined in many different ways. Let then $\vec{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ be a real vector (of n dimensions). The most used vector norms are:

1. $\|\vec{x}\| = \|\vec{x}\|_2 = \sqrt{\sum_i^n x_i^2}$ (Euclidean norm);
2. $\|\vec{x}\| = \|\vec{x}\|_\infty = \max_{1 \le i \le n} |x_i|$ (Supremum or Uniform norm);
3. $\|\vec{x}\| = \|\vec{x}\|_1 = \sum_i^n |x_i|$ (Sum norm).

Now, these previous three norms are actually special cases of more general norms, for which definitions 5.4 and 5.5 will be given next.

Definition 5.4. *[Szmidt, 2014, p. 40] Let $r \ge 1$ be a real number. Then the l_r-norm of a vector $\vec{x} = (x_1, x_2, \ldots, n_n)^T \in \mathbb{R}^n$ is defined as follows:*

$$l_r(\vec{x}) = \|\vec{x}\|_r = (\sum_i^n |x_i|^r)^{\frac{1}{r}}.$$

Definition 5.5. *[Szmidt, 2014, p. 41] Let $r \ge 1$ be a real number. Now the l^r-norm of a vector $\vec{x} = (x_1, x_2, \ldots, n_n)^T \in \mathbb{R}^n$, i.e. the i-th power of l_r-norm is defined by*

$$l^r(\vec{x}) = \|\vec{x}\|^r = \sum_i^n |x_i|^r.$$

Moreover, the Euclidean norm is a special case of the l_r-norm, where $r = 2$, and the sum norm is a special case of both l_r- and l^r-norms, namely if $r = 1$, the defitions 5.4 and 5.5 can be both written as follows:

$$l_1(\vec{x}) = \|\vec{x}\|_1 = (\sum_i^n |x_i|^1)^{\frac{1}{1}} = \sum_i^n |x_i|, \text{ and}$$
$$l^1(\vec{x}) = \|\vec{x}\|^1 = \sum_i^n |x_i|^1 = \sum_i^n |x_i|.$$

There exists a correspondence between norms and metrics on vector spaces. Every norm on a vector space determines a metric and sometimes vice versa. In a given normed vector space $(X, \|\cdot\|)$, a metric on X can be defined by $d(x, y) = \|x - y\|)$. Then it is said that the norm $\|\cdot\|$ induces the metric d. [Väisälä, 1999, pp. 20-21]

Let $\vec{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ and $\vec{y} = (y_1, y_2, \ldots, y_n)^T \in \Re^n$ be a real vectors (of n dimensions). The following three metrics are among the most used in different applications.

1. Manhattan distance: $d(x, y) = \sum_{i=1}^{n} |x_i - y_i|$
2. Euclidean distance: $d(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$
3. Minkowski distance: $d(x, y) = (\sum_{i=1}^{n} |x_i - y_i|^p)^{\frac{1}{p}}$

Now, when previous l_r-norm is considered, it clearly induces the Minkowski distance. Furthermore, as its special cases, for $r = 1$ it becomes Manhattan distance and for $r = 2$ the Euclidean distance.

5.2 From the fuzzy distance to the intuitionistic fuzzy distance - the two-term version

There is an obvious generalization of defining distance from fuzzy set to intuitionistic fuzzy sets. In fuzzy setting the distance is characterized by the membership function $\mu_A(x)$. Because in an common universe a fuzzy set is a special case of an intuitionistic fuzzy set, it can be written in an intuitionistic fuzzy way. This procedure can be applied also for distances.

Now, it is recalled that intuitionistic fuzzy sets are constructed in such a way that the norm does not have to be related to elements of the universe in any way. This means that it is not a norm in a classical sense, but some kind of "pseudo-norm", which relates a positive nomber to every element of the universe [Atanassov, 2012, p. 133], [Szmidt, 2014, 43]. Further, it means that instead of

$$\begin{aligned} \|x\| = 0 \quad &\Longleftrightarrow x = 0, \text{and} \\ \|x\| = \|y\| \quad &\Longleftrightarrow x = y, \end{aligned}$$

which are not valid here, the following condition will hold:

$$\|x\| = \|y\| \Longleftrightarrow \mu_A(x) = \mu_A(y) \wedge \nu_A(x) = \nu_A(y).$$

Next, it should be noted that for any element $x' \in X'$ and for any fuzzy set $A' \subset X'$ the value of membership function $\mu_{A'}(x')$ can be used as a pseudo-norm. In a fuzzy setting the metric $m'_A(x', y') = |\mu_{A'}(x') - \mu_{A'}(y')|$ is used to define distances, Hamming distance included.

For fuzzy sets A' and B' in a space X' the usual definition for Hamming distance is as follows:

Definition 5.6. *[Atanassov, 2012, p. 138], [Szmidt, 2014, p. 49] For a given finite universe X' and fuzzy sets $A' \subset X'$ and $B' \subset X'$, the Hamming distance is*

$$d_{FS}(A', B') = \sum_{x' \in X'} |\mu_{A'}(x') - \mu_{B'}(x')|$$

and the corresponding normalized Hamming distace is

$$l_{FS}(A', B') = \frac{1}{n} \sum_{x' \in X'} (|\mu_{A'}(x') - \mu_{B'}(y')|).$$

Let X' be a finite universe with cardinality n and let $A' \subset X$ and $B' \subset X$ be fuzzy sets. Since any fuzzy set is also an intuitionistic fuzzy set (a special case of one, to be exact) both A' and B' can be expressed as an intuitionistic fuzzy set. Therefore the following can be written:

$$\begin{aligned} A' &= \{< x', \mu_{A'}(x') > | x' \in X'\}_{FS} \\ &= \{< x', \mu_{A'}(x'), 1 - \mu_{A'}(x') > | x' \in X'\}_{IFS} \\ &= \{< x', \mu_{A'}(x'), \nu_{A'}(x') > | x' \in X'\}_{IFS}, \end{aligned}$$

and

$$\begin{aligned} B' &= \{< x', \mu_{B'}(x') > | x' \in X'\}_{FS} \\ &= \{< x', \mu_{B'}(x'), 1 - \mu_{B'}(x') > | x' \in X'\}_{IFS} \\ &= \{< x, \mu_{B'}(x'), \nu_{B'}(x') > | x' \in X'\}_{IFS}. \end{aligned}$$

Because of the fact that A' is a fuzzy set, the hesitation margin $\pi_{A'}$ equals zero:

$$\pi_{A'}(x') = 1 - \mu_{A'}(x') - \nu_{A'}(x') = 1 - \mu_{A'}(x') - (1 - \mu_{A'}(x')) = 0.$$

And since the hesitation margin π equals zero for all fuzzy sets, it can be omitted in the intuitionistic type presentation of the fuzzy Hamming distance, which can be expressed as follows.

$$\begin{aligned} d_{IFS}(A', B') &= \textstyle\sum_{x' \in X'} |\mu_{A'}(x') - \mu_{B'}(x')| + |\nu_{A'}(x') - \nu_{B'}(x')| \\ &= \textstyle\sum_{x' \in X'} |\mu_{A'}(x') - \mu_{B'}(x')| + |1 - \mu_{A'}(x') - (1 - \mu_{B'}(x'))| \\ &= \textstyle\sum_{x' \in X'} |\mu_{A'}(x') - \mu_{B'}(x')| + |\mu_{A'}(x') - \mu_{B'}(x')| \\ &= 2 \textstyle\sum_{x' \in X'} |\mu_{A'}(x') - \mu_{B'}(x')| \\ &= 2\, d_{FS}(A', B'). \end{aligned}$$

Similarily, the normalized Hamming distance $l_{IFS}(A', B')$ is the Hamming distance $d_{IFS}(A', B')$ divided by cardinality n of the universe X' and

$$l_{IFS}(A', B') = \frac{1}{n}\, d_{IFS}(A', B') = \frac{2}{n}\, d_{FS}(A', B') = \frac{2}{n} \sum_{x' \in X'} |\mu_{A'}(x') - \mu_{B'}(x')|.$$

Therefore, it is convenient to give following definitions for 2-term expression of Hamming metric and Hamming distance for *intuitionistic fuzzy sets*.

Definition 5.7. *[Atanassov, 2012, p. 137] For a intuitionistic fuzzy set A in a finite universe X, the 2-term Hamming metric is defined by*

$$h_A(x, y) = \frac{1}{2}(|\mu_A(x) - \mu_A(y) + \nu_A(x) - \nu_A(y)|).$$

Now, in the sense of the discussion in the beginning of the current section, this 2-term Hamming metric is actually a pseudo-metric of a sort. The corresponding two-term Hamming distancefor intuitionistic fuzzy sets is defined as follows:

Definition 5.8. *[Atanassov, 2012, p. 139], [Szmidt, 2014, p. 50] For a given finite universe X and intuitionistic fuzzy sets $A \subset X$ and $B \subset X$, the 2-term Hamming distance is*

$$d_{IFS(2)}(A, B) = \frac{1}{2} \sum_{x \in X} (|\mu_A(x) - \mu_B(y)| + |\nu_A(x) - \nu_B(y)|),$$

and the normalized 2-term Hamming distace is

$$l_{IFS(2)}(A, B) = \frac{1}{2n} \sum_{x \in X} (|\mu_A(x) - \mu_B(y)| + |\nu_A(x) - \nu_B(y)|).$$

5.3 The Hamming distance for intuitionistic fuzzy sets - the three-term version

In previous section both membership and non-membership functions were taken into consideration. This, however, left out the hesitation margin from the discussion. Next, it will be taken into account when considering the concept of the distance further. Also, somehow it would also be nice to be free from the the problematic term "pseudo-metric". In his book, Li gives the following definion:

Definition 5.9. *[Li, 2014, p. 19] For a given finite universe X, let $d : f(X) \times f(X) \to [0,1]$ be a mapping such that for any intuitionistic fuzzy sets A, B and C, the following four properties (1. - 4.) are satisfied:*

$$\begin{array}{ll} 1. & 0 \leq d(A,B) \leq 1, \\ 2. & d(A,B) = 0 \Longleftrightarrow A = B \\ 3. & d(A,B) = d(B,A) \\ 4. & d(A,B) \leq d(A,C) + d(C,B) \end{array}$$

Then $d(A,B)$ is called the normalized distance between intuitionistic fuzzy sets A and B.

The previous definition 5.9 is a nice way to go around the "pseudo-metric problem", it gives a definition to the normalized distance between intuitionistic fuzzy sets. Moreover, the pseudo-metric is silently embedded into the concept of distance. Anyway, the concept here is intuitively understood and (1.) the normalized distance is a positive number between 0 and 1. Also, (2.) the distance from a set to itself is zero and if the distance of two sets equals zero they are both the same exact set. (3.) The distance can be measured starting from either end of the "route" and (4.) the detour route via other point is always same or longer than the original route.

Now, the hesitation margin should be introduced explicitly to the discussion about distance. Let X be a finite universe with cardinality n and let $A \subset X$ be an intuitionistic fuzzy set. Now, recalling the definitions 4.10 and 4.11,

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\},$$

and the corresponding hesitation margin π is

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore

$$\begin{aligned} |\pi_A(x) - \pi_A(y)| &= |1 - \mu_A(x) - \nu_A(x) - (1 - \mu_A(y) - \nu_A(y))| \\ &= |-\mu_A(x) - \nu_A(x) + \mu_A(y) + \nu_A(y)| \\ &= |\mu_A(y) - \mu_A(x) + \nu_A(y) - \nu_A(x)| \\ &\leq |\mu_A(y) - \mu_A(x)| + |\nu_A(y) - \nu_A(x)|, \end{aligned}$$

and

$$|\mu_A(x) - \mu_A(y)| + |\nu_A(x) - \nu_A(y)| \geq |\pi_A(x) - \pi_A(y)| \geq 0.$$

Now both $|\mu_A(x) - \mu_A(y)| \geq 0$ and $|\nu_A(x) - \nu_A(y)| \geq 0$ and therefore the value of the difference in hesitation margins $|\pi_A(x) - \pi_A(y)|$ is somewhere between a positive number and zero, $r \geq |\pi_A(x) - \pi_A(y)| \geq 0 \wedge r > 0$, and cannot be omitted when considering the distance between intuitionistic fuzzy sets.

Next phase is to define Hamming metric and Hamming distance for intuitionistic fuzzy sets. It is done as follows.

Definition 5.10. *[Atanassov, 2012, p. 138] For a intuitionistic fuzzy set A in a finite universe X, the 3-term Hamming metric is defined by*

$$h_A(x, y) = \frac{1}{2}(|\mu_A(x) - \mu_A(y)| + |\nu_A(x) - \nu_A(y)| + |\pi_A(x) - \pi_A(y)|).$$

Now, the 3-term Hamming metric from the previous definition 5.10 is used to determine the distance between two elements x and y in the intuitionistic fuzzy set A. Further, the 3-term Hamming distance is a measure for the distance or difference between two intuitionistic fuzzy sets A and B, in all their elements.

It should be noted that both sources, Szmidt [Szmidt, 2014] and Szmidt and Kacprzyk [Szmidt and Kacprzyk, 2000], do not explicitly give the definition for the 3-term Hamming metric. Atanassov [Atanassov, 2012], however, presents the definition but with a slight error: In his presentation the multiplier in front of the expression is $\frac{1}{3}$, while according to other two authors line of reason it should be $\frac{1}{2}$. The same difference applies also to the next definition 5.11.

Definition 5.11. *[Szmidt, 2014, pp. 60-61],[Szmidt and Kacprzyk, 2000, p. 514] For a given finite universe X and intuitionistic fuzzy sets $A \subset X$ and $B \subset X$, the 3-term Hamming distance is*

$$d_{IFS(3)}(A, B) = \frac{1}{2}\sum_{x \in X}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|),$$

and the normalized 3-term Hamming distace is

$$l_{IFS(3)}(A, B) = \frac{1}{2n}\sum_{x \in X}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|).$$

It is easy to see that when the cardinality of the universe X is n, the normalized 3-term Hamming distance is the 3-term Hamming distance divided by the cardinality of universe.

$$\begin{aligned} & l_{IFS(3)}(A,B) \\ = & \tfrac{1}{2n}\textstyle\sum_{x\in X}(|\mu_A(x)-\mu_B(x)|+|\nu_A(x)-\nu_B(x)|+|\pi_A(x)-\pi_B(x)|) \\ = & \tfrac{1}{n}\cdot\tfrac{1}{2}\textstyle\sum_{x\in X}(|\mu_A(x)-\mu_B(x)|+|\nu_A(x)-\nu_B(x)|+|\pi_A(x)-\pi_B(x)|) \\ = & \tfrac{1}{n}d_{IFS(3)}(A,B). \end{aligned}$$

Moreover, both 3-term Hamming distance and normalized 3-term Hamming distance from the definition 5.11 are both pseudo-metrics in a sense of the discussion in the beginning of the current section. Now, according to the definition 5.9 the normalized 3-term Hamming distance is actually a normalized distance between intuitionistic fuzzy sets:

Theorem 5.1. *[Li, 2014, pp. 20-21] The normalized 3-term Hamming distance $l_{IFS(3)}(A,B)$ is a normalized distance in the sense of the definition 5.9.*

Now, in order to prove the theorem 5.1 it is enough to show that properties (1. - 4.) of the definition 5.9 are satisfied. Let A, B and C be intuitionistic fuzzy sets in the universe X.
1. Since

$$l_{IFS(3)}(A,B)=\frac{1}{2n}\sum_{x\in X}(|\mu_A(x)-\mu_B(x)|+|\nu_A(x)-\nu_B(x)|+|\pi_A(x)-\pi_B(x)|),$$

and

$$\forall x\in X: 0\leq|\mu_A(x)-\mu_B(x)|+|\nu_A(x)-\nu_B(x)|\leq 1\wedge|\pi_A(x)-\pi_B(x)|\leq 1,$$

then (remembering that the cardinality of X, $Card(X)=n$)

$$0\leq l_{IFS(3)}(A,B)\leq\frac{1}{2n}\sum_{x\in X}2=\frac{2n}{2n}=1.$$

2. Clearly, if $A=B$, then

$$\forall x\in X: |\mu_A(x)-\mu_B(x)|=0\wedge|\nu_A(x)-\nu_B(x)|=0\wedge|\pi_A(x)-\pi_B(x)|=0,$$

and $l_{IFS(3)}(A,B)=0$. On the other way round, if $l_{IFS(3)}(A,B)=0$, then

$$\forall x\in X: |\mu_A(x)-\mu_B(x)|=0\wedge|\nu_A(x)-\nu_B(x)|=0\wedge|\pi_A(x)-\pi_B(x)|=0$$

and therefore

$$\forall x \in X : \mu_A(x) = \mu_B(x) \wedge \nu_A(x) = \nu_B(x) \wedge \pi_A(x) = \pi_B(x),$$

which means that $A = B$.
3. Now the equation can be written as follows:

$$\begin{aligned}
& l_{IFS(3)}(A, B) \\
= & \tfrac{1}{2n} \textstyle\sum_{x \in X} (|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|) \\
= & \tfrac{1}{2n} \textstyle\sum_{x \in X} (| - (\mu_B(x) - \mu_A(x))| + | - (\nu_B(x) - \nu_A(x))| \\
& + | - (\pi_B(x) - \pi_A(x))|) \\
= & \tfrac{1}{2n} \textstyle\sum_{x \in X} (|\mu_B(x) - \mu_A(x)| + |\nu_B(x) - \nu_A(x)| + |\pi_B(x) - \pi_A(x)|) \\
= & l_{IFS(3)}(B, A).
\end{aligned}$$

4. Now, from the definition 4.10 of intuitionistic fuzzy sets and the triangle inequity follows that

$$\begin{aligned}
& l_{IFS(3)}(A, B) \\
= & \tfrac{1}{2n} \textstyle\sum_{x \in X} (|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|) \\
= & \tfrac{1}{2n} \textstyle\sum_{x \in X} (|\mu_A(x) - \mu_C(x) + \mu_C(x) - \mu_B(x)| \\
& + |\nu_A(x) - \nu_A(x) + \nu_B(x) - \nu_B(x)| \\
& + |\pi_A(x) - \pi_A(x) + \pi_B(x) - \pi_B(x)|) \\
\leq & \tfrac{1}{2n} \textstyle\sum_{x \in X} (|\mu_A(x) - \mu_C(x)| + |\mu_C(x) - \mu_B(x)| \\
& + |\nu_A(x) - \nu_A(x)| + |\nu_B(x) + \nu_B(x)| \\
& + |\pi_A(x) - \pi_A(x)| + |\pi_B(x) + \pi_B(x)|) \\
= & \tfrac{1}{2n} \textstyle\sum_{x \in X} (|\mu_A(x) - \mu_C(x)| + |\nu_A(x) - \nu_C(x)| + |\pi_A(x) - \pi_C(x)|) \\
& + \tfrac{1}{2n} \textstyle\sum_{x \in X} (|\mu_C(x) - \mu_B(x)| + |\nu_C(x) - \nu_B(x)| + |\pi_C(x) - \pi_B(x)|) \\
= & l_{IFS(3)}(A, C) + l_{IFS(3)}(C, B),
\end{aligned}$$

and the proof of theorem 5.1 is complete.

Now, the three-term intuitionistic fuzzy Hamming distance is a valid tool for measuring distances between intuitionistic fuzzy sets. The example 6.3 how to calculate distance between two intuitionistic fuzzy sets is given in the next chapter 6 along further discussion of applying it to proposed Diamond model.

Chapter 6

Proposed "Diamond Model"

The proposed model for measuring or estimating proximity in knowledge management setting is presented in this chapter. One everyday situation where this kind of approach can be used is presented in the example 6.1, where different transportation methods are considered. The discussion on different categories of distances and measuring distance between intuitionistic fuzzy sets in previous chapters should be kept in mind during the presentation in the current chapter. It should be also noted that the notation of the terms is slightly different compared to previous chapters.

The current chapter is arranged as follows. The proposed model for measuring distance is discussed in general or theoretical level in the first section 6.1 of this chapter. In the second section 6.2 the theory discussed in earlier chapters is embedded into the theoretical model. Lastly, some examples are given in the final section 6.3.

Example 6.1. *How to compare two (or more) different routes between two locations? Let's say that a person is looking for different options for travelling from Tampere to Rovaniemi. The reason behind his choice could be as follows:*

	Aeroplane	*Train*	*Bus*	*Car*
1. Cost	−	+	+	+
2. Travel time	+	−	?	−
3. Schedule	?	+	?	+
4. Comfort	+	+	−	−
5. Services	+	+	−	?
6. Ability to work	?	+	±	−
7. Environment friendliness	−	+	±	−

It seems that the train would be a clear winner in this competition, but is it necessarily so? Are all factors taken into consideration of equal value? Or, for example, is the cost of the transportation just little too much or just under the tolerable level?

6.1 The model - theoretical version

Depending on the situation there are basically two ways to estimate distance between two entities. The first one is to simply measure distance from one to another, given that there exist a measure for that. It is a straightforward way to measure the distance.

If the case is more complicated, the second way is to measure the distance to an ideal entity, and then compare all measured distances between different entities and the ideal entity. This is an intuitive way to make a decision between different options in a certain choice situation. This can be the case, for example, if the ”distance” has to be evaluated for multiple entities and a certain decision has to be made based on that result.

Many approaches require a lot of data gathered from a long period of time and the statistical methods are complicated. Furthemore, the analysis of results require also an expert in order to understand what is going on. Therefore there exist a need for a model, which is easy to understand and use at least to approximate facts or to be used as a basis of analysis. Now, the proposed model is pictured in figure 6.1. The main model is quite simple, but depending on the application statistical methods may be still required.

The diamond model

The model is named to be the diamond model, based on the shape of the visualization of it. Now, the model pictured figure 6.1 is given in general level and the notations used in it are as follows: X and Y are two entities which are compared to each other, $C_i,\ i \in \{1, 2, 3, 4, 5\}$ are the criteria used in comparison and $D_i(X, Y)$ is the distance according to ith criteria. It shoud be noted that the amount of criteria is not set to be exactly 5, it could be anything from 1 to n, where n is a finite natural number.

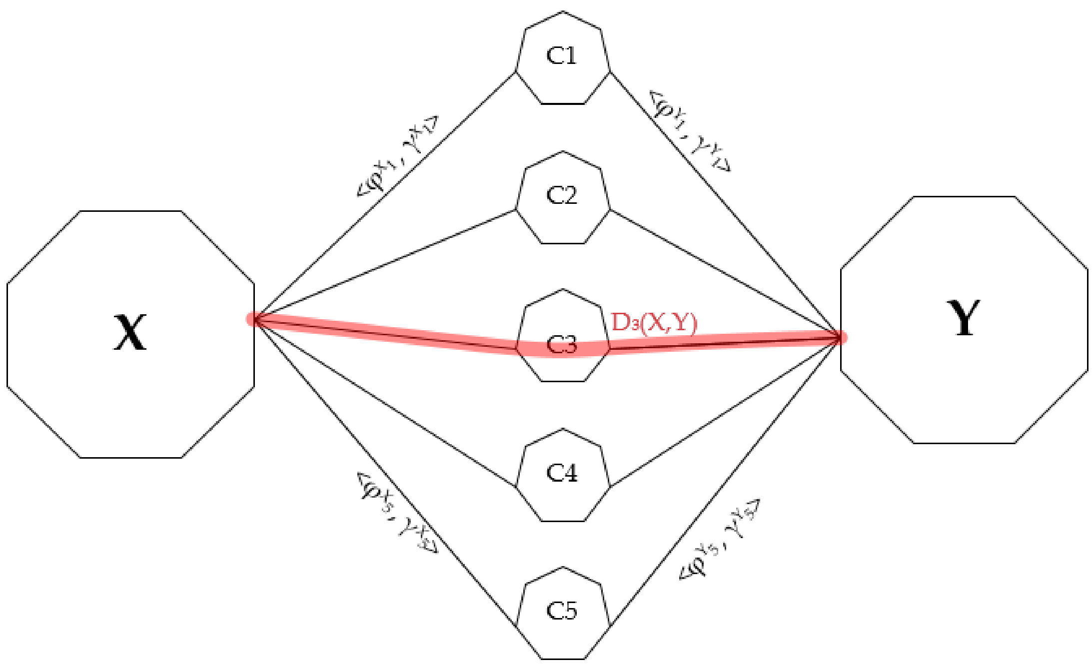

Figure 6.1: The proposed diamond model for measuring (cognitive) distance between two entities.

The validity of this model depends on several things. First, the entities which are to be compared have to be somehow similar. They have to belong in the same class, which could be basicly anything. In the examples 6.1 and 6.3 it is means of transportation, but the class can be firms, plants from different countries, subcontractors, research teams or even persons competing for a position within the firm.

Next, how to choose right criteria? This is a choice for the experts, the quality which is to be measured must be identified and right variables chosen. When considering person to hire, the criteria could include education, work experience, professional skills, ability to team work, langue skills , etc. Anyway, the right dimensions for the measured distance must be chosen.

Lastly, the right ruler must be chosen. The main question is how to measure the distance between the entities X and Y using the criteria $C_i,\ i \in \{1, 2, \dots, n\}$. The aim of this work is to present one solution to that question. The easiest solution is the *discrete metric*, where the distance is 0, if the compared elements are diffrent, or 1, if they are same. The choice of the distance measure is discussed in the next section 6.2. If there exists several criteria, each one can be measured individually and then results can be added up and, finally, the result can be normalized if needed.

6.2 Embedding intuitionistic fuzzy measure

One way to measure distances within the diamond model is discussed in the current section.

6.2.1 The hamming distance

Originally the Hamming distance was related to binary data strings. It is a metric which tells how many bits are different in two data strings of same length. Now, the original example of distance in binary strings is reviewed in example 6.2.

Example 6.2. *[Hamming, 1950] The following binary strings are at the distance of* 2 *from each other:*

$$\begin{matrix} 0 & 0 & 1, \\ 0 & 1 & 0, \\ 1 & 0 & 0, \\ 1 & 1 & 1. \end{matrix}$$

It is clearly seen, that there is only one common binary number between any two strings of the previous example 6.2 and the two other numbers are different, which gives the distance of 2 between any two of them.

Further, the criteria in the diamond model can be seen as a string of control variables $C_i,\ i \in \{1, 2, \dots, n\}$ and the corresponding variables $x_i,\ i \in \{1, 2, \dots, n\}$ and $y_i,\ i \in \{1, 2, \dots, n\}$ are found from the entities X and Y, $\forall i \in \{1, 2, \dots, n\} : x_i \in X \wedge y_i \in Y$. Therefore, it is possible to order variables in both entities according to criteria and compare them one by one:

$$\begin{array}{ccccc} C_1 & C_2 & C_3 & \dots & C_n \\ \hline x_1 & x_2 & x_3 & \dots & x_n \\ | & | & | & \dots & | \\ y_1 & y_2 & y_3 & \dots & y_n \end{array}$$

Now, if the metric used is the original Hamming distance given by Hamming [Hamming, 1950], the distance would be the number of different pairs in places $i \in \{1, 2, \dots, n\}$, where $x_i \neq y_i$. This would correspond to using the discrete metrics to measure distance in individual positions and adding the results together. In some cases this approach would be appropriate, but when considering more complex cases, where for example unknown or undetermined is involved, something more refined is needed.

6.2.2 Intuitionistic fuzzy sets

When considering intuitionistic fuzzy sets in the current setting of the diamond model, one should keep in mind the discussion in the section 4.2, where basic properties of intuitionistic fuzzy sets were reviewed.

Definition 6.1. *The universe U_C is formed as a set of all criteria,*

$$U_C = \{C_i | i \in \{1, 2, \dots, n\} \wedge n \in \mathbb{N}\}.$$

Given that the well defined criteria has been obtained, according to this definition 6.1, the set of all criteria is considered to be the universe U_C. This is a critical phase, since here actually will be defined what is going to be measured. There have to be solid arguments behind the choice of this set.

Definition 6.2. *The entities, which distance is measured in the diamond model are considered to be intuitionistic fuzzy sets over the universe U_C. These intuitionistic fuzzy sets are denoted with capital letters and the elements included are denoted with corresponding small letters.*

Now, the definition 6.2 tells that every entity, which will be measured, is considered to be an intuitionistic fuzzy set. It means that for every entity to be measured is assigned a membership function and a non-membership function, which both are unique to every single entity. These functions define the degree of membership and non-membership of every element (criteria C_i) from the universe U_C to the intuitionistic fuzzy set. Recalling the definition 4.10, an entity A can be written as follows:

$$A = \{< C_i, \mu_A(C_i), \nu_A(C_i) > | i \in \{1, 2, \ldots, n\}, C_i \in U_C\}$$

Again, finding the membership and non-memership functions can be difficult. Here possible solutions are again experts evaluation depending on the situation ([Szmidt, 2014, pp. 30-31] and [Atanassov, 2012, pp. 12-16]) or statistical methods as discussed in [Szmidt, 2014, pp. 32-38].

6.2.3 The diamond model and distance

The last piece of the puzzle here is distances between intuitionistic fuzzy sets discussed in the chapter 5. Since the many variables which have to be measures can be defined by people and often the definition is vague and made somehow intuitively. The goal here is to provide some tool in order to tackle this vagueness and the unknown part of variables and therefore to include both fuzziness and unknown to the proposed diamond model.

Therefore, in this setting the definition 5.11 needs to be written again, only difference here in the definition 6.3 is just slightly changed notation.

Definition 6.3. *For a given finite universe U_C, its elements $\forall i \in \{1, 2, \ldots, n\}$: $C_i \in U_C$ and intuitionistic fuzzy sets $A \subset U_C$ and $B \subset U_C$, the 3-term Hamming distance is*

$$d_\diamond(A, B) = \frac{1}{2} \sum_{i=1}^{n} (|\mu_A(C_i) - \mu_B(C_i)| + |\nu_A(C_i) - \nu_B(C_i)| + |\pi_A(C_i) - \pi_B(C_i)|),$$

and the normalized 3-term Hamming distace is

$$l_\diamond(A, B) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(C_i) - \mu_B(C_i)| + |\nu_A(C_i) - \nu_B(C_i)| + |\pi_A(C_i) - \pi_B(C_i)|).$$

Now, it should be noted, that the cardinality of U_C is n, there are n predefined criteria and the distance is measured according to every criteria.

6.3 Examples

The consideration of different means of transport from the example 6.1 is continued in the following example 6.3. Here, the values of membership function, non-membership function and hesitation margin are simply invented and in the real situation these values could be almost anything between 0 and 1. In the following example, the theoretical optimal way of transportation would have the value 1 in every component of membership function. Therefore the transportation of choise is the one with shortest distance to the theoretical one.

Example 6.3. *In example 6.1 different options of transportation were considered. Firstly, the universe X contains all components of consideration, or the facts taken in to account when making the decision,*

$$X = \left\{ \begin{array}{l} \textit{Cost} \\ \textit{Travel time} \\ \textit{Schedule} \\ \textit{Comfort} \\ \textit{Services} \\ \textit{Ability to work} \\ \textit{Environmental friendliness} \end{array} \right\}.$$

Since there is seven elements in the universe X, the cardinality of X,

$$Card(X) = 7.$$

The means of travel are in fact intuitionistic fuzzy subsets of this universe X, they just happen to be named as "Aeroplane", "Train", "Bus", "Car" and "Ideal transport method". For example,

$$\textit{Aeroplane} = \left\{ \begin{array}{l} \frac{\textit{Cost}}{(0.3;0.5;0.2)}, \frac{\textit{Travel time}}{(0.7;0.1;0.2)}, \frac{\textit{Schedule}}{(0.3;0.3;0.4)}, \\ \frac{\textit{Comfort}}{(0.6;0.3;0.1)}, \frac{\textit{Services}}{(0.5;0.3;0.2)}, \\ \frac{\textit{Ability to work}}{(0.4;0.2;0.4)}, \frac{\textit{Environmental friendliness}}{(0.1;0.8;0.1)} \end{array} \right\} \supset X.$$

The reason behind the final decision could be as follows:

	Aeroplane	*Train*	*Bus*	*Car*
1. Cost	$(0.3; 0.5; 0.2)$	$(0.8; 0.2; 0)$	$(0.9; 0; 0.1)$	$(0.5; 0.1; 0.4)$
2. Travel time	$(0.7; 0.1; 0.2)$	$(0.3; 0.5; 0.2)$	$(0.2; 0.3; 0.5)$	$(0.6; 0.1; 0.3)$
3. Schedule	$(0.3; 0.3; 0.4)$	$(0.7; 0.1; 0.2)$	$(0.3; 0.3; 0.4)$	$(0.7; 0.2; 0.1)$
4. Comfort	$(0.6; 0.3; 0.1)$	$(0.7; 0.1; 0.2)$	$(0.4; 0.3; 0.3)$	$(0.3; 0.6; 0.1)$
5. Services	$(0.5; 0.3; 0.2)$	$(0.6; 0.2; 0.2)$	$(0.2; 0.6; 0.2)$	$(0.3; 0.3; 0.4)$
6. Ability to work	$(0.4; 0.2; 0.4)$	$(0.7; 0; 0.3)$	$(0.3; 0.3; 0.4)$	$(0.1; 0.8; 0.1)$
7. Env. friendl.	$(0.1; 0.8; 0.1)$	$(0.6; 0.2; 0.2)$	$(0.4; 0.4; 0.2)$	$(0.2; 0.7; 0.1)$

Now, it is quite straight forward procedure to calculate both 3-term Hamming distance and normalized 3-term Hamming distance using the values from the decision table.

The ideal transportation method have membership function value 1 *in every component: cost, travel time, schedule, comfort, services, possibility of working and environment friendliness. Therefore, the values can be written* $(1.0; 0.0; 0.0)$ *for every component. The calculation for aeroplane is done as follows. First the 3-term Hamming distance between the ideal situation and travelling by aeroplane is*

$$\begin{aligned}
& d_{IFS(3)}(Ideal, Aeroplane) \\
= & \tfrac{1}{2}\sum_{x\in X}(|\mu_{Id}(x)-\mu_{Ap}(y)|+|\nu_{Id}(x)-\nu_{Ap}(y)|+|\pi_{Id}(x)-\pi_{Ap}(y)|) \\
= & \tfrac{1}{2}(|1-0.3|+|0-0.5|+|0-0.2|+|1-0.7|+|0-0.1|+|0-0.2| \\
& +|1-0.3|+|0-0.3|+|0-0.4|+|1-0.6|+|0-0.3|+|0-0.1| \\
& +|1-0.5|+|0-0.3|+|0-0.2|+|1-0.4|+|0-0.2|+|0-0.4| \\
& +|1-0.1|+|0-0.8|+|0-0.1|) \\
= & \tfrac{1}{2}(1.4+0.6+1.4+0.8+1.0+1.2+1.8)=4.1
\end{aligned}$$

and because the cardinality of the universe is 7*, the normalized 3-term Hamming distance is*

$$l_{IFS(3)}(Ideal, Aeroplane)=\frac{1}{7}d_{IFS(3)}(Ideal, Aeroplane)=0.586.$$

Next, all values for the other means of transportation are calculated:

	Aeroplane	*Train*	*Bus*	*Car*
1. Cost	1.4	0.4	0.2	1.0
2. Travel time	0.6	1.4	1.6	0.8
3. Schedule	1.4	0.6	1.4	0.6
4. Comfort	0.8	0.6	1.2	1.4
5. Services	1.0	0.8	1.6	1.4
6. Ability to work	1.2	0.6	1.4	1.8
7. Env. friendl.	1.8	0.6	1.2	1.6
8. 3-t Hamming	4.1	2.6	4.3	4.3
9. 3-t H. norm.	0.586	0.371	0.614	0.614

When considering all components chosen for this decision, it is easy to see that the "Train" is the transportation of choice in the universe of this example. It is also noteworthy that different values in components can still lead to the same final result, as is the case with the "Bus" and the "Car" in this example.

The row number 8*, the 3-term Hamming distance, equals the sum of previous rows from* 1 *to* 7 *divided by* 2*. And further, the row number* 9*, the normalized 3-term Hamming distance, can be calculated by dividing the value from the previous row* 8 *by* 7*, the cardinality of the universe.*

After the previous example 6.3 and its predecessor, example 6.1, the method and processes of the proposed model should be quite clear. Next phase is to jump into unknown. The main purpose of the next example 6.4 is to give some kind of insight of the possibilities of the approach used in this work. Before the actual example, the figures 3.2, **??** and 3.4 together with discussion related to them should be reviewed.

Example 6.4. *Let's say that in a research team exists an open postion for a junior researcher. Somehow there are only two potential applicants and one of them will get a position or at least a chance to have one. So, the potential problem with different entities is already solved, there are only applicants* A *and* B*.*

Next, the criteria has to be defined. Of course, in the real life situation there are former studies and education, recommendations and personal network, etc, which have some kind of effect to the final result. Here, for the sake of an argument, these are assumed to be even and are therefore omitted from this consideration. If nothing else is known, the cognitive function can be considered as a starting point. Furthermore, in a knowledge intensive field of work, superior cognitive abilities would be an advantage.

Potential sources of differences in cognitive function can be found in the domain, the function itself and the range. One way of defining the criteria here is to use Bloom's taxonomy presented in their article by Kasilingam with his coauthors [Kasilingam et al., 2014]. It is system of different categories of learning behavior developed originally in 1950's. Further, the main goal was to assist in the design and assesment of educational learning. Now, the criteria is as follows.

Criteria	Name	Description
C_1	Knowledge	Retrieve and recall knowledge
C_2	Comprehension	Understand, translate, explain
C_3	Application	Use knowledge and skills to solve problems
C_4	Analysis	Similarities, differences and relationships
C_5	Synthesis	Merge, combine, integrate
C_6	Evaluation	Prove, evaluate, conlude, criticize

From the previous criteria C_1 and C_2 are related to the cognitive domain, C_2, C_3 and C_5 are related to the cognitive function, and C_4, C_5 and C_6 are related to the cognitive range. The overlapping of terms is almost unavoidable and here it does not interfere the main goal of the example. The universe $U_C = \{C_i | i \in \{1, 2, \ldots, n\}\}$.

Next, in the sense of the diamond model, both applicants are considered to be intuitionistic fuzzy sets. Therefore they can be written as follows:

$$A = \{< C_i, \mu_A(C_i), \nu_A(C_i) > | i \in \{1, 2, \ldots, n\}, C_i \in U_C\}, \text{and}$$
$$B = \{< C_i, \mu_B(C_i), \nu_B(C_i) > | i \in \{1, 2, \ldots, n\}, C_i \in U_C\}.$$

Now, how are the membership (μ_A and μ_B) and non-membership (ν_A and ν_B) functions defined? Here, it is assumed that a board of experts have evaluated the applicants in a work-like situation and they have decided the values of the two characteristic function in every point of criteria. The hesitation margins (π_A and π_B) are also calculated here.

Criteria	μ_A	ν_A	π_A	μ_B	ν_B	π_B
C_1	0.6	0.2	0.2	0.7	0.2	0.1
C_2	0.5	0.2	0.3	0.5	0.3	0.2
C_3	0.5	0.3	0.2	0.6	0.4	0.0
C_4	0.4	0.4	0.2	0.4	0.3	0.3
C_5	0.7	0.1	0.2	0.6	0.1	0.3
C_6	0.4	0.4	0.2	0.4	0.2	0.4

Here it is appropriate to consider the ideal applicant, who would have membership function value of 1 and non-membership function value of 0 in every point of criteria. Furthermore, absolute values of calculated distances from the ideal are as follows:

	$\|\mu_I - \mu_A\|$	$\|\nu_I - \nu_A\|$	$\|\pi_I - \pi_A\|$	$\|\mu_I - \mu_B\|$	$\|\nu_I - \nu_B\|$	$\|\pi_I - \pi_B\|$
C_1	0.4	0.2	0.2	0.3	0.2	0.1
C_2	0.5	0.2	0.3	0.5	0.3	0.2
C_3	0.5	0.3	0.2	0.4	0.4	0.0
C_4	0.6	0.4	0.2	0.6	0.3	0.3
C_5	0.3	0.1	0.2	0.4	0.1	0.3
C_6	0.6	0.4	0.2	0.6	0.2	0.4
$\|Sum\|$	2.9	1.6	1.3	2.8	1.5	1.3

And finally, the normalized 3-term Hamming distances are $l_\diamond(A, Ideal) = 0.483$ and $l_\diamond(B, Ideal) = 0.467$, which means that is the required criteria is otherwise fullfilled, the applicant B gets the job. Now, it is possible that the board of experts can decide that the maximum acceptabe difference from the ideal is for example 0.333, in which case no one will be hired.

Now, what happens if it is acknowledged that there are some thing which are not known about the ideal applicant. For all criteria, the ideal membership function could be 0.7, non-membership 0.1 and therefore hesitation margin 0.2. Now, the results would look as follows:

	$\|\mu_I - \mu_A\|$	$\|\nu_I - \nu_A\|$	$\|\pi_I - \pi_A\|$	$\|\mu_I - \mu_B\|$	$\|\nu_I - \nu_B\|$	$\|\pi_I - \pi_B\|$
C_1	0.1	0.1	0.0	0.0	0.1	0.1
C_2	0.2	0.1	0.1	0.2	0.2	0.0
C_3	0.2	0.2	0.0	0.1	0.3	0.2
C_4	0.3	0.3	0.0	0.3	0.2	0.1
C_5	0.0	0.0	0.0	0.1	0.0	0.1
C_6	0.3	0.3	0.0	0.3	0.1	0.2
$\|Sum\|$	1.1	1.0	0.1	1.0	10.9	0.1

Furthermore, the normalized 3-term Hamming distances are $l_\diamond(A, Ideal) = 0.183$ and $l_\diamond(B, Ideal) = 0.167$, and the applicant B gets the job again.

Chapter 7

Conclusion

Distance is intuitively an easy concept to understand, since people have to relocate themselves regularly in the every day life. However, this concept is not as simple as it might seem at the first sight. When considering communication and especially knowledge transfer, it turns out that the concept is really multifaceted and divided in several dimensions. The main theme through the work is distance. What does the concept of distance or proximity mean in Knowledge management? How is this distance defined? How it is measured? Can it be measured in a such way that intuitively and somehow vaguely defined dimensions of distance can be translated into easily understandable values? And, if the answer is ”yes”, what kind of tool is needed in order to accomplish this task?

There exists a multitude of different processes which have to be estimated or measured in knowledge management . Many of those processes are related to research or innovation. Since it is an accepted truth that distance between different actors involved in these processes have a profound effect to the knowledge transfer, learning and innovation potential, the concept of distance or proximity has been studied and characterized widely in reseach literature. In the chapter 2 these different dimensions presented in the literature were reviewed and some possible classifications were given. The definitions of different dimensions of distance are often vague and the exact limits are often hard to find and, furthermore, many definitions are overlapping.

The role of cognition is significant in different dimensions of distance. In some classification cognitive distance is located within organizational distance or it is embedded into socio-cultural distance. Since cognitive distance is in central role when considering knowledge transfer and innovation processes, it has been discussed more thoroughly in the chapter 3. The ability of express the ideas and the ability of absorb information are invaluable. Also, the cognitive processes of learning and knowledge transfer can be considered in mathematical type terms. Thinking can be seen as a function from the cognitive domain to the cognitive range. Here, the cognitive domain can be translated as the observed phenomena and the cognitive range as the conclusions or categorizations. The use of this kind of terminology suggests that some kind of mathematical approach could be used in measuring or estimating the cognitive distance.

Since cognitive processes including common everyday logic are hard to measure and based on individual estimates on observed phenomena, the unknown or undefined is always present. In order to present this mathematically, intuitionistic fuzzy sets were introduced in the chapter 4. Intutionistic fuzzy sets are characterized by the membership and the non-membership functions, which define the relation of every element of a intuitionistic fuzzy set to the universe. In a finite universe this is done by defining the membership and non-membership within the intuitionistic fuzzy set for every element of the universe. As a prerequisite both classical sets and fuzzy sets were reviewed in order to build solid enough foundation for the reader to approach the subject. Main qualities of intuitionistic fuzzy sets are that it includes fuzziness in the sense that membership can be partial and it includes the unknown, since memership and non-membership are defined and dealt with separately. Therefore, there exists room for the undefined or unknown.

Intutionistic fuzzy approach has been used, for example, in decision making where multiple actors are making decisions based on multiple attributes or variables. The challenge is that the distance has to be defined for intuitionistic fuzzy sets or entities interpreted as such. In mathematics the term "metric" is used to describe distance measure. At this point, some theory involving metrics and norms were presentes in the chapter 5 and in intuitionistic fuzzy setting the 3-term Hamming distance were introduced. This measure takes every aspect of intuitionistic fuzzy set into account: the membeship function, the non-membership function and the hesitation margin, which describes the unknown in the set. Now, distance between intuitionistic fuzzy sets can be measured by using this tool.

The original Hamming distance were used to measure difference of binary strings and it was applied bit-by-bit. The similar idea is presented in the diamond model, which is an way of measuring vague notions emerged in the field of knowledge management. The universe is constructed by selecting the criteria by which the measurement of distance is done. The entities to be measured are considered to be intuitionistic fuzzy sets. Furthermore, the membership and non-membership functions are defined and the distance between the entities is measured by using the 3-term Hamming distance. The actual process of defining the two characteristic functions related to an intuitionistic fuzzy set can be complicated and require estimates from experts. This proposed diamond model and some examples how it can be used were presented in the chapter 6.

The main goal was to present a way to measure distance in the knowledge management setting. The diamond model is that tool and it is general enough in order to be defined or applied for several different scenarios. The selection of criteria, or, construction of the universe is difficult and requires thorough expert analysis of the subject. The same applies to the consturcting membership and non-membership functions. It can be done from the statistical data, but the process would be complicated [Szmidt, 2014, pp. 32-38] and out of the scope of the current work. It is quite propable that other ways to import numerical values to this model could be considered with success. Both the statistical methods and other way to import data to this model could be interesting subjects for further studies. The previous could also result interesting case studies in various knowledge management or innovation settings. Also, in the field of intuitionistic fuzzy sets theory, there are open questions related to different applications. All in all, the actual idea behind the diamond model is sophisticated and makes sense intuitively.

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