Lappeenranta-Lahti University of Technology LUT
School of Business and Management
Master’s Programme in Strategic Finance and Business Analytics

Henri Vainikka

ASYMMETRIC CORRELATIONS, VOLATILITY AND DOWNSIDE BETA

Examiners: Professor Eero Pätäri
Associate Professor Sheraz Ahmed

ABSTRACT

Lappeenranta-Lahti University of Technology LUT
School of Business and Management
Strategic Finance and Business Analytics

Henri Vainikka

Asymmetric correlations, volatility and downside beta

Master's Thesis
2020
89 pages, 8 figures, 11 tables and 2 appendices
Supervisor: Professor Eero Pätäri
2nd examiner: Associate Professor Sheraz Ahmed
Keywords: downside risk, asymmetric covariance, conditional volatility, conditional beta

Several studies have found that stocks react stronger to negative market movements. This effect is visible both in the downside covariance of the stocks and the market, and with stronger correlation with the volatility of the stocks. Downside covariance and conditional volatility impact the realised return distributions partly in different ways and this study examines the effect of asymmetries in systematic risk with conditional downside betas, both combined and separately with conditional volatility.

Downside beta and EGARCH model were used for portfolio-sorting of the weekly returns of stocks in the Finnish Stock Exchange during 2004-2020. The realised returns were observed through a rolling four-week holding period, with the realised comoments and lower-partial moment, to estimate the effects of the asymmetries both in the covariance and in the volatility. The results showed that the conditional volatility performed better as a single method, contributing to a gradually decreasing returns with a higher exposure to volatility. The outperformance of the lowest class was on a significant level. The downside beta showed a distinction between the highest and lowest classes, with a higher future returns for a higher exposure to downside risk but not on a significant level. After controlling for the effect of the volatility, downside beta better explained the realised return distribution. The best and the worst performing portfolios had the exact opposite sorting values. The observed effects were persistent throughout the study, excluding the highest volatility class. The downside beta contributed to future realised coskewness risk and conditional volatility to future cokurtosis. The results indicate that downside beta explains the returns better than the unconditional beta especially after controlling for the volatility. There were discounts in risk premiums for stocks with high upside asymmetry and with the effects of low returns to high volatility stocks this could be used for a risk-aversion purposes.

TIIVISTELMÄ

Lappeenrannan-Lahden teknillinen yliopisto LUT
School of Business and Management
Strategic Finance and Business Analytics

Henri Vainikka

Epäsymmetriset korrelaatiot, volatiliteetti ja laskumarkkinoiden beta-kerroin

Pro gradu-tutkielma
2020
89 sivua, 8 kuviota, 11 taulukkoa ja 2 liitettä
Ohjaaja: Professori Eero Pätäri
Toinen tarkastaja: Tutkijaopettaja Sheraz Ahmed
Hakusanat: negatiivinen riski, epäsymmetrinen kovarianssi, ehdollinen volatiliteetti

Useissa tutkimuksissa on havaittu, että osakkeet reagoivat voimakkaammin negatiivisiin markkinaliikkeisiin. Tämä ilmiö on havaittavissa sekä laskumarkkinoiden kovarianssissa että volatiliteetin voimakkaampana korrelaationa. Kovarianssi sekä volatiliteetti vaikuttavat toteutuneisiin tuottoihin osittain eri tavoin ja tämä työ tutkii epäsymmetrisyyksien vaikutusta systemaattiseen riskiin ehdollisen miinus-betan avulla, sekä yhdessä että erikseen ehdollisen volatiliteetin kanssa.

Miinus-betan ja EGARCH-mallin avulla Helsingin pörssin osakkeet lajiteltiin viikkotuottoja käyttäen vuosien 2004-2020 välillä. Tuotot laskettiin neljän viikon rullaavalla periodilla, yhdessä ehdollisten momenttien sekä tuottojakauman negatiivisen varianssin kanssa, joilla arvioitiin epäsymmetrian vaikutuksia sekä kovarianssissa sekä volatiliteetissa. Ehdollinen volatiliteetti suoriutui paremmin yksittäisenä lajittelumetodina, saaden aikaan asteittain laskevan tuottojakauman volatiliteetin kasvaessa. Alimman volatiliteettiluokan ylisuoriutuminen oli tilastollisesti merkittävää. Lajittelu Miinus-betalla aiheutti eroja suurimpien ja pienimpien luokkien välillä, tuottojen ollessa suurempia negatiivisen markkinariskin portfolioille mutta ilman tilastollista merkittävyyttä. Kontrolloimalla volatiliteetin vaikutusta, miinus-beta selitti paremmin toteutuneita tuottojakaumia. Parhaiten ja huonoiten suoriutuneet portfoliot olivat toistensa vastakohtia lajitteluarvoiltaan. Havaitut vaikutukset pysyivät samoina läpi tutkimuksen, pois lukien korkein volatiliteettiluokka. Miinus-beta ennakoi tulevaa ehdollisen vinouman riskiä ja volatiliteetti tulevaa ehdollista huipukkuutta. Tulokset viittaavat että, minus-beta selittää tuottojen jakaumaa normaalia betaa paremmin etenkin huomioitaessa volatiliteetin vaikutus. Riskipreemiot olivat pienempiä osakkeilla jotka korreloivat voimakkaammin nousevien markkinoiden kanssa ja yhdessä korkean volatiliteetin kanssa tätä voidaan käyttää riskien karttamiseen.

Acknowledgements

I would like to thank my supervisor professor Eero Pätäri for his guidance throughout the writing of this thesis. I am also thankful for Lappeeranta-Lahti university for the opportunity to study here.

Table of Contents

List of Figures

List of Tables

1.Introduction

Stock returns tend to correlate more strongly with the market when it goes down, than they do when the market is going up. This effect is often referred as asymmetric correlation and has been discussed on numerous studies which have found asymmetries in covariances, betas and volatilities of stock returns (Hong, Tu & Zhou 2007). While studies often focus on asymmetric volatility caused by leverage or volatility feedback effect, also known as time varying-betas and time-varying risk premium, there are also models for conditional beta which allow asymmetry. Cho & Engle (1999) find that betas are affected both by market shocks and by idiosyncratic shocks and it is evident that volatility and beta are interlinked in many ways, as variance and covariance used in beta estimation both in the denominator as market variance and in the numerator as a covariance might be affected by volatility. While high volatility often implies a high beta, it can also stem from high downside correlation with the market. This suggests that both the diversifiable part of risk, idiosyncratic risk, and volatility of assets returns over time are closely related to beta and observed covariances.

Traditionally, an exposure to risk and a compensation for bearing it in a form of higher risk premia, has been perhaps the most fundamental concept in economy since the introduction of the modern portfolio theory by Markowitz (1952). According to the modern portfolio theory, from the two categories of risk, systematic and idiosyncratic, only systematic risk is priced in equilibrium since idiosyncratic risk can be diversified by holding a portfolio. The coefficient for non-diversifiable risk, beta, measures both the upside and downside covariance of the stock and the market, which are assumed to be symmetric. Investors demand a higher return for holding a risky asset and it has been suggested since Roy (1952) and Markowitz (1959) that they also care more about the downside exposure to risk rather than the upside potential, and that the use of semivariance methods could be then a better measure for risk since it captures the actual covariance of an asset and market during downside movements. If this assumption of loss-aversion held then an asset with higher downside exposure should also have a higher risk premium.

Even if the beta is assumed to be linear or asymmetric, i.e. having different covariation depending the side of market movements, the relation is still straightforward. With higher systematic risk there needs to be higher risk premium for bearing the risk. However, the magnitude of this relation could vary a lot depending on the amount of asymmetry in conditional correlations with the market. The relation of volatility and beta is more complex; assuming that the correlation of market and assets stays constant, when volatility of a market increases (declines) more than asset volatility, based on definition of beta, the effect should then lower (rise) the value of beta-coefficient, which then would suggest a lower return for an asset. In most studies, the relation of volatility and returns is most often found to be negative (Black 1976), but there are many studies, for example French, Schwert and Stambaugh (1987), that find either an insignificant or a positive relation with volatility and expected return. This could be explained by the time-varying risk premiums and variation in beta as suggested by Cho & Engle (1999) Also, an anomaly of high returns and low volatility has been found by Baker, Bradley & Wurgler (2011) and Blitz and von Vliet (2007).

Volatility has also been used in measuring idiosyncratic risk and while the modern portfolio theory suggests that idiosyncratic risk is not priced when investors hold a diversified portfolio, many studies have found a positive relation with idiosyncratic risk and return, i.e. measuring residuals in CAPM or FF3-models, and some other suggest a negative relation with high idiosyncratic risk and returns. Merton (1987) and Malkiel & Xu (2002) find a positive relation with idiosyncratic risk and expected stock return. Ang, Hodrick, Xing and Zang (2006) find that stocks with high idiosyncratic risk, yield very low returns in the next month. Fu (2009), however, finds that the effect was mostly caused by the reversal of small subsample of stocks that had gained abnormally high returns the month before and finds further evidence of positive relation with idiosyncratic volatility and expected returns. Combining results of all above-mentioned studies, conclude to a fact that both the correlations and covariances of the stocks and the market work in many overlapping ways, and the amount of asymmetry on either the correlations or covariances of individual stocks could lead to a many different combinations of return structures. The purpose of this study is to find how the sign and magnitude of both the correlation and conditional volatility responses on asymmetric market movements together relate to future excess returns on aggregate level.

1.1 Objectives, scope and limitations of the study

This study mainly focuses on the joint effect of downside covariance and volatility, by measuring a different downside betas conditional on market down movements together with conditional volatility. Since volatility is also time-varying and asymmetric, reacting stronger to negative returns, this asymmetry could lead to higher downside betas on stocks with high volatility in respect to market. While the relation of both downside correlations in lower partial moments framework and conditional volatility to returns has been investigated in many studies separately, not much work can be found on their joint effect to future returns. Many studies mostly focus on finding the correct source of asymmetries in return distributions by trying to form a model that combines the beforementioned effects either through conditional covariances, time-varying betas or risk premiums. This thesis relies on the hypothesis that both the downside correlation and conditional volatility capture some unique risk measures that, while interlapping, are not completely the same. Controlling for volatility could then yield more uniform results in the risk estimate of a single stock than the use of the CAPM-beta, unconditional beta or volatility alone. The volatility estimated with EGARCH (1,1) is used to control for the effects of high idiosyncratic risk, which might affect returns on high levels, but also to compare how downside beta and volatility of an asset relate to each other and the returns. This will be done by various portfolio sorting methods that help to identify and capture the effects of these variables. The relation of downside covariance and conditional volatility can be further assessed by examining realised asymmetries on the return distributions and asymmetries found in the EGARCH parameters of the portfolios formed on basis of a different asymmetry or volatility criteria. While volatility is often estimated from daily returns, which might better capture some qualities of the volatility process itself through the parameters of the generalized autoregressive conditional heteroskedasticity models, this study uses weekly returns both in estimating the conditional volatility and downside betas. Lower frequency returns have been found robust for capturing the volatility clustering and persistence found in daily return series by Dannenburg and Jacobsen (2003), who find the estimates to be in line with the results obtained from higher frequency data. Gan, Nartea and Wu (2017) find similar results using weekly data to estimate volatility in Hong Kong stocks over the 35-year sample period. Since the purpose of this study is to find how volatility affects the returns within an investment window of four weeks, the use of weekly returns is chosen.

Volatility, it's modelling and asymmetries have been studied widely and the objective of this study will not be in finding the right or absolute source of the asymmetric correlations nor to estimate their time-varying properties or the fitness of the conditional volatility model, rather than to examine how the exposure to different levels of volatility and tendency to high downside correlation interlink and how it will affect the returns. Since the main source of time-variation and asymmetry in betas is caused by nonlinearities in the down- and up-movements of the market, focusing solely on the downside covariance could provide more information of the coeffects of volatility and covariance conditional on down-movements of the market. If there are empirical observations of stocks which have both a high downside correlation with the market and a low volatility, it could provide valuable information especially for portfolio diversification and for investment strategies by finding an investable spread of stocks based on their downside risk. The asymmetries in stock covariance can also be of much importance to investors with a strong disappointment aversion preference. The effects of conditional volatility and downside covariance have not been widely studied in the Finnish Stock Exchange, apart from the work of Koutmos and Knif (2002), who with the aid of bivariate GARCH, which allows asymmetric covariance response, find evidence of time-variation and asymmetry in betas between down and up markets. This thesis continues the study of asymmetries on Finnish stock market and because the market size, efficiency, number of stocks and liquidity it might provide different results than in more larger stock markets.

1.2 Research questions

1. Can historical conditional beta be used for predicting future excess returns?

2. Is there an additional risk premium for high b-minus assets?

3. How does conditional volatility affect the conditional beta's ability to explain returns? Does controlling for it yield better results for predicting future excess return with conditional beta?

4. Does conditional beta capture the spread of future returns better than conditional volatility?

5. Are asymmetric responses to market movements priced similarly in conditional betas and in conditional volatility, or do they capture different aspects of the realised return distributions?

1.3 Structure of the study

This thesis is structured in seven sections. The Second section introduces the theoretical framework of the study, downside risk, conditional volatility, possible reasons for asymmetries in the return distributions and all the main models used in this study. Section 3 presents a literature review on previous studies and summarizes their findings under subsections of downside covariance, volatility and higher statistical moments. Section 4 introduces the data with some descriptive statistics. The methodology will be presented in detail at section 5. Section 6 provides the empirical results of the study whereas section 7 concludes the empirical results and summarizes the study.

2. Theoretical framework

This section presents the different theories and standpoints in evaluating the reasons behind the asymmetric correlation of returns, ways of measuring their sign, magnitude, behaviour and impact. The first subsection demonstrates how higher risk premium for assets with high downside covariance may rise from risk-aversion and utility functions. The second subsection presents the traditional methods for assessing downside risk from semivariance and lower partial moments to downside covariance betas. The third subsection considers the conditional volatility, methods for capturing the time-variance and its effect on returns. Fourth subsection looks at the combination of downside covariance and conditional volatility and how their joint effect might alter the return distributions.

2.1 Semivariance and risk-aversion

The notion that investors are risk-averse and price down- and upside risk differently has been discussed since the first years of the modern portfolio theory by Roy (1952) and Markowitz (1959). Roy's Safety first Criterion states that rather than maximising their wealth in linear way, people and investor care more about their own economic survival and place more value in scenarios that reduce the possibility of individual economic disaster. Bawa (1978) employs Roy's Safety first rule in the von Neumann-Morgernstern expected utility paradigm, where under stochastic dominance rules, he shows that lower partial moment framework could be used as a more plausible method (instead of mean-variance) for determining risk and market equilibrium. In addition to Roy, also Markowitz considered the use of semi-variance instead of variance since it captures the actual downside risk, but limited computing capacities of the era made him to decide on using variance. Since then numerous studies about the utility of wealth have been conducted and for example Gordon, Paradis and Rorke (1972) in their experiments find the utility function of investors to be concave. Similar findings are reported in the prospect theory of Kahneman and Tversky (1979) in the field of behavioural economics. Kahneman and Tversky find that the value function is convex, thus making people react more strongly to losses than gains.

A more mathematical approach to how risk-aversion may lead to a higher risk premiums can be formed based on the work of Guls disappointment aversity function (1991), in which a coefficient for disappointment aversion determines how much more weight the investor puts to disappointing (negative) outcomes. Kyle and Xiong (2001) suggest that also wealth constraints may lead to risk-aversion. For a theoretical presentation of mean lower partial moment model considering the utility preference of the investors, a model following the work of Fishburn (1977) can be expressed as:

$$LPM = \frac{1}{n} \sum_{i=1}^{n} [max(0, \bar{R}_i - R_{it})]^{\alpha} \quad (1)$$

Where R_{it} are the return observations, $\bar{R}_i$ is their average, n is the number of observations below the mean. α is the risk-aversion coefficient of an investor with values above (below) one indicating risk-aversion (risk-seeking), enabling the different shaped utility functions of the investors. The mean return as a target rate can be replaced with a arbitrary value, thus the model estimates a part of the return distribution below the target rate.

2.2 Downside covariance

Since the downside and upside covariances with the market may exhibit different asymmetrical patterns, many models for capturing especially the downside risk has been developed around the CAPM framework. While studies such as Jensen, Black, Fischer and Scholes (1972) and Black (1972) proposed two-factor models with time-sifting alphas in order to adapt to asymmetries, the earliest actual downside beta models were based on the work of Hogan and Warren (1974), who laid the theoretical foundations to replace standard deviation with standard semideviation in the CAPM, which later models build on. Bawa and Lindenberg (1977) developed the CAPM with a lower-partial moment framework that has the same properties as the traditional mean-variance CAPM conditional on distribution staying either normal, stable or student-t. They conclude that the model is able to explain expected returns in cases of other distributions as well, thus performing better as the normal CAPM with fewer limiting assumptions. These first models chose the risk-free rate as a target rate for specifying the downside risk. The Bawa & Lindenberg downside beta, with risk-free rate replaced by average market excess return as expressed in Ang, Chen & Xing (2006):

$$\beta^{-} = \frac{cov\,(r_i, r_m \mid r_m < \mu_m)}{var(r_m \mid r_m < \mu_m)} \tag{2}$$

Where r_i denotes the excess return on security and r_m is the excess return on market. As a cut-off point for lower-partial moment is μ_m , the average market excess return.

Most of the earlier studies on the lower partial moments-framework focused on distributional assumptions of the models and on explaining asymmetries through third and fourth moments of the return distributions. The most variation in models comes through the selection of the cut-off point, which may lead to different properties in the way they capture the downside risk, especially in a case of heavily skewed distributions. (Post, van Vliet and Lansdorp, 2009)

2.3 Conditional volatility

Volatility measures the variability in a price of an asset over a time-period. In most simple case it can be expressed as the standard deviation of returns by assuming that the prices follow a geometric Brownian motion. However, in real world, the volatility is not constant (homoscedastic) rather than time-varying and heteroscedastic. The volatility process is autocorrelated, depending on previous volatility changes. Mandelbrot (1963) was among the first to notice that large changes in prices are followed by the next period large changes. Volatility clustering in periods of high and low volatility and time-variance can explain the properties found in return time-series around the world. First of these stylized facts is that the distribution of returns is not normal. Although it is approximately symmetric it has a higher kurtosis and fatter tails than a normal distribution. The distribution of the volatility process itself can be seen as a combination of normal distributions, with some having higher concentration around a mean and others a higher variance. Therefore, the mixed distribution has fatter tails than a normal distribution. Second stylized fact states that the correlation of daily returns is almost non-existent. The autocorrelation of returns, regardless of the time lags used, are close to zero. The third stylized fact is that the autocorrelation of absolute returns and their squared returns are positive for many lags. This demonstrates that the process is serially correlated, large returns (either positive or negative) are followed by large returns that can be observed in squared excess returns. (Taylor 2007, Campbell, Lo & MacKinlay 1997)

The first conditional volatility model, that captured the volatility clustering and the serial correlation of volatility was the Autoregressive Conditionally Heteroscedastic model by Engle (1982). ARCH (1) process is specified as:

$$\sigma_t^2 = \omega + \alpha(r_{t-1} - \mu)^2 \tag{3}$$

The model presents the variance of a return conditional on past returns with constant mean and a time-varying variance. Volatility parameter $\omega > 0$ and the volatility coefficient $\alpha \geq 0$. The past returns higher than the mean increases the future volatility when α is above 0. The ARCH model stays stationary when $\alpha < 1$.

The shortcomings of ARCH model are the limitation of parameters that cannot be negative. The Generalized Autoregressive Conditionally Heteroscedastic model by Bollerslev (1986) introduces a lagged variance term into an ARCH model, analogously to ARMA-models with GARCH (p,q) where p is the lag number of previous squared residuals and q is the lag number on previous conditional variance. With the aid of lagged variance, the GARCH model helps to capture the stylized fact of returns, volatility persistence and offers a good prediction accuracy, while estimation stays rather easy. GARCH (1,1) can be written as:

$$\sigma_t^2 = \omega + \alpha(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2 \tag{4}$$

The parameters ω, α and β are constrained to ≥ 0 to avoid negative variance. The process is stationary if $\alpha + \beta < 0$, then unconditional variance is finite, correlation between returns

r_t and r_{t-1} is zero when r > 0, and the correlation of the squared residuals is positive when r > 0. Furthermore α is measure of the effect of the past volatility shock to future volatility and together α + β give the rate of how long the effect lasts. (Taylor 2007)

The ARCH and generalized ARCH models treat both positive and negative shocks in the same way. However, it has been noted since 1976 by Black, that the effect of shocks to volatility is asymmetric, as negative innovations increase volatility more than positive ones. Nelson (1991) shows that when US stock markets fall, the effect on volatility is larger than effect from rise of the same magnitude. Therefore, the squared residuals used in GARCH are not able to capture this effect. The exponential GARCH introduces a residual function that gets different values depending on the sign of a function given by market up- or down movements. The EGARCH (1,1) with one lag for squared residuals and one lag of conditional variance can be estimated with following equations:

The conditional means of returns r_t are μ_t, residuals are:

$$e_t = r_t - \mu_t \tag{5}$$

Next they are standardized as:

$$z_t = e_t / \sqrt{\sigma_t^2} \tag{6}$$

The exponential GARCH model can be then expressed as:

$$\ln(\sigma_t^2) = \omega + \beta \, ln(\sigma_{t-1}^2) + \vartheta z_{t-1} + \gamma_i(|z_{t-1}| - E|z_{t-1}|) \tag{7}$$

Where the $\beta \, ln\left(\sigma_{t-j}^2\right)$ is the autoregressive parameter and the process stays stationary when -1 < β < 1. Parameter ω is the mean of the process $\ln(\sigma_t^2)$ and the asymmetric effects are generated together by the terms $\vartheta z_{t-1} + \gamma_i(|z_{t-1}| - E|z_{t-1}|)$ in the volatility residual function. The term $E|z_{t-1}|$ is the expectation of z_{t-1} for non-normal distributions, for normal distributions it can be replaced with $\sqrt{2/\pi}$, which generates a mean value of zero for the asymmetric function. When z is negative (markets fall), the function takes form $\vartheta - \gamma$, and when z is positive (markets rise), the form will be $\vartheta + \gamma$. Since the conditional volatility is an exponential of σ_t^2, there is no need for parametric restrictions to ensure non-negativity. The amount of asymmetry for EGARCH model can be estimated with an asymmetry ratio:

$$A = \frac{\gamma - \vartheta}{\gamma + \vartheta} \tag{8}$$

(Taylor 2007, Campbell, Lo & MacKinlay 1997)

Both the asymmetric response and the negative correlation of volatility and market movements has been explained mostly through the leverage effect and volatility feedback. The leverage effect was first introduced by Black (1976), who suggests that when the stock prices drop, it increases the leverage-ratio, which in turn increases the risk, resulting an increase in the volatility. Christie (1982) finds evidence supporting that an increase in firm's leverage increases volatility, causing a drop in stock value and making the remaining stock value more leveraged, thus increasing the required risk premium. He finds the leverage effect to be stronger with smaller firms which have a higher financial leverage. Duffee (1995) documents that the relation with returns and volatility is greater for small firms, finding a positive contemporaneous relation, and that the one-period-ahead relation stays positive at the daily frequency and turns negative at the monthly frequency. Cheung and Ng (1992) report similar findings that the volatility asymmetry is stronger for small firms in the U.S.

Another, in some ways similar, explanation is the volatility feedback, first suggested by Pindyck (1984), with a reverse causality compared to leverage effect. It bases on the assumption that volatility is priced as a risk measure. The feedback effect happens when the changes in volatility causes shocks that affect the market, thus increasing the future volatility, causing the risk premium required by investors to increase. Since the market reaction to positive and negative news is different in magnitude, the negative ones will further increase the original effect, causing the feedback. In case of positive news, the less strong magnitude and increase in volatility, balances the price rising effect of the positive news. In other words, when shocks or effects to returns are negative, the feedback effect will increase them further, and in case of positive ones, it will counterbalance the effect, thus causing the asymmetry in the way the volatility responds to the shocks of different sign. The volatility feedback effect then causes time-varying risk premiums. Bekaert and Wu (2000) investigate the sources of volatility asymmetry with multivariate GARCH using leveraged portfolios and find the volatility feedback effect to be more plausible cause of asymmetries, and especially when the reaction to market shocks is exhibiting strong asymmetries.

With various EGARCH specifications, Bollerslev and Mikkelsen (1996) find that the persistence of volatility in US market has a long memory and can be described to follow a mean-reverting fractionally integrated process, where effect of shocks to the conditional variance decays on slow hyperbolic rate. This is a necessary condition for volatility to be priced as a risk factor as noted by Poterba and Summers (1986). Under an assumption that the traditional CAPM holds, a volatility process can be priced through the following formula by Merton (1980):

$$E\left[r_{i,t}|\psi_{t-1}\right] = \lambda_t \, cov\left(r_{i,t} r_{m,t}\middle|\psi_{t-1}\right) \quad \forall\, i \tag{9}$$

Where r_i and r_m are excess returns on asset and market at time t. ψ_{t-1} is the information available at time t-1 and λ_t is a coefficient for the price of risk on the market, or risk aversion. The risk ratio λ as estimated by Dean and Faff (2004):

$$\lambda_t = \frac{E[r_{m.t}|\psi_{t-1}]}{E[(\sigma^2_{m,t})||\psi_{t-1}]} \tag{10}$$

As in the CAPM, the idiosyncratic risk is not priced, and the pricing mechanism would work only through the asset’s covariance with the market and the market price of risk, not including the variance of an asset itself. While the volatility of a market can be asymmetric, for a relatively stronger asymmetry to manifest on a firm level, there would then need to be a change in a leverage of a firm, resulting in an asymmetric covariance.

2.4 Time-varying beta and the joint effects of downside correlation and conditional volatility

Among the first to study the coeffects of betas and conditional volatility were Bollerslev, Engle and Wooldridge (1988), who use the GARCH-M process to model conditional covariance finding that the expected return and risk premia are significantly altered by the conditional variance. They also find indications that the risk premium is better defined through the covariance of an asset and the market than through its own variance. Harvey (1989) provides a method to test the CAPM under conditions that allowed time-variance in expected returns and conditional covariances. He finds that covariance changes through time and that even with this more relaxed definition, the CAPM is unable to capture the dynamics of the asset returns.

Since good and bad news are found to have different nonlinear effects to volatility, Braun, Nelson and Sunier (1995) study the volatility leverage effects and if the asymmetric response can be found in time-varying conditional betas as well. By following Hamada (1972) they reason, analogous to volatility leverage effect, that if a shock to value of firm rises the leverage that in turn rises the beta, therefore a negative correlation with unexpected shocks and beta should exist. The rise in beta would then again reflect to fall in price of an asset, since the higher risk needs to be compensated with a higher risk premium, assuming the market risk premia is not affected. By studying the conditional covariances of stock market with a bivariate EGARCH model that allows both the volatility and beta at market and portfolio-level to have an asymmetrical response to positive and negative returns of the market and portfolios, they find that the asymmetry mostly happens on the market level, with market volatility rising (falling) as a response to negative (positive) news. They find no evidence of asymmetries in conditional betas. Although they find that the volatility asymmetry shows in portfolios, but the effect comes from the covariance of the market and portfolios, rather than through beta or variance of the portfolio in question. This might suggest that also the downside beta could be a relatively stable predictor of future returns. However, studies such as Ball and Kothari (1989) and Chan (1988) find that betas are affected by shocks, but also that they react asymmetrically to good and bad news. Chan, studying contrarian strategies also finds that the losing stock betas and market risk premiums are positively correlated, and winning stock betas are negatively correlated with the market risk premium. With the double-beta model of Engle and Merzrich (1996) and EGARCH specification, Cho and Engle (1999) study if the betas increase with positive news and decrease with negative ones and find that the news have asymmetrical effect on an individual stock level. In more detail, the betas might be affected by two sources, the market shocks and idiosyncratic shocks to a firm, and individual stock betas could be affected by either both of them or one of them. They criticize the work of Braun et al. (1995) for that portfolio price aggregation together with monthly data, leads to a loss of much variation in the results. They conclude that asymmetric effects in betas could be partially explained by the change in expected returns due to changing beta-levels and that having two sources of variation in betas could be used for portfolio hedging, ie. combining stocks whose betas are dominated with market shocks and those who are mostly affected by their own idiosyncratic risk.

In light of the above studies, it appears that the downside correlation of a stock might not necessarily be constant, rather than that the asymmetry with the down- and upside covariation with the market could be a product of asymmetrical response, depending both on market and idiosyncratic shocks and their effect on individual stocks. However, when studying specifically the downside correlations of the market, Ang, Chen & Xing (2006) demonstrate the downside correlation and volatility relation with future returns by rewriting the downside beta:

$$\beta^{-} = \rho^{-} * \frac{\sigma_i^{-}}{\sigma_m^{-}} \tag{11}$$

Where, ρ^{-} is the downside correlation of an asset and the market, σ_i^{-} and σ_m^{-} are their downside volatilities, respectively. Thus, a rise in downside beta can then be produced either by rising downside correlation or increasing downside volatility of a stock in proportion to downside volatility of the market. Holding the downside correlation constant, the increase in stock volatility then increases the downside beta, which in turn should increase observed future returns. However, this relation does not seem to hold when the increase comes from the volatility as opposed to downside correlation. They find the downside beta to be poor predictor of future returns for high volatility stock, which is consistent with findings of Ang, Hodrick, Xing & Zhang (2006), where the authors find that stocks with very high levels of either total or idiosyncratic volatility have abnormally low future returns. In addition, Ang, Chen and Xing (2002) find that holding downside volatility of a stock constant, and sorting stocks by their downside correlation, results as 5% annual spread in the average monthly future returns between the tenth and the first decile portfolios. Ang and Chen (2002) notice that after controlling for size, stocks with low betas exhibit greater asymmetries in correlation with the US stock market. These findings suggest that the source of downside beta-values matters, with high downside correlation and high downside volatility working in contradictory ways, regarding future expected returns.

3. Literature review

The literature review section will be separated to three parts, first one focusing on downside variance, downside correlation with the market and studies centred around conditional beta models. Second section focuses on conditional volatility, volatility asymmetries and their co-effects to downside covariance, along with the observations about the relation of excess return and volatility. The effects of return distribution skewness and kurtosis and their co-moments with the market are review in the last section.

3.1 Downside beta

While many recent studies have approached return distribution asymmetry through conditional variances and time-varying risk premiums, some have focused on the beta asymmetry, especially in the form of downside covariance beta. First models date back as far as the semivariance suggested by Markowitz (1959), the actual downside beta-models of Hogan & Warren (1974) and Bawa & Lindenberg (1977) and the asymmetric response model of Harlow & Rao (1989). Starting from late 1970's and in 1980's much of the studies, including Price, Price & Nantell (1982) and Chow & Denning (1994), focused on the return distributions and higher moments in order to explain asymmetries in betas and in the distributions of the returns.

Fisburn (1977) shows that lower partial moments framework is consistent with stochastic dominance which is a requirement for utility functions under risk aversion. Continuing the work of Nantell and Price (1979), Nantell, Price and Price (1982) estimate lower partial-moment and regular betas for US stocks from 1935 to 1976, finding that positively skewed market causes the differences between LPM- and regular betas. When the normal distribution of returns holds, the analytical differences of the two models disappear and risk-free rate of interest separates the symmetrical down- and upside distributions for the LPM beta. Price et al. (1982) also conclude that in presence of the lognormal distributions, the asymmetries cause the regular beta to be lower than the LPM beta for stocks that exhibit high systematic risk and higher for stocks with low systematic risk. In other words, regular beta underestimates the risk of low beta stocks. However, Homaifar and Graddy (1990) come to an opposite conclusion. Both these works are later criticized by Chow and Denning (1994), who state that in equilibrium, these models should be equivalent and point out that misspecification in the Homaifar and Graddy-model leads to risk-free asset to have systematic risk. They address that even though lower partial moments can be considered to have less distributional limitations than mean-variance beta, both above-mentioned studies assume a lognormal distribution. Choosing an arbitrary target violates the equilibrium between mean-variance- and LPM-betas when the distribution is normal, and consistent with the models of Bawa & Lindenberg and Harlow & Rao, in equilibrium of the systematic risk measures of both have to be the same. Later work by Satchell (1996), using Fishburn utility functions, provides methods that can relax the distributional assumptions and ways of statistically test the difference of mean- and semi-variance betas.

Pettengill, Sundaram and Mathur (1995) perform a segmenting of the up- and downside markets to test the significance of the dual-beta model. They separate the periods of market returns with market risk premium being either positive or negative, finding a positive (negative) relation between beta and realised returns during upside (downside) market movements when market risk premium is positive (negative). Following the suggested methods of Pettengill et al. (1995), Isakov (1999) performs tests on Swiss stock market from 1983 to 1991. He notes the fact that realised and expected returns differ conditional on market movements and when markets are volatile and excess returns to the market negative, the relation of beta and realised returns reverses. When the realised market return is positive (negative) the relation with betas should be positive (negative). He finds that when dissociating the periods of positive and negative realised excess market returns, the relation with realised excess returns and beta is highly significant.

Some more recent studies include Ang, Chen & Xing (2006) who find that after controlling for other factors, such as coskewness and book-to-market ratio, there is still an additional risk premium for bearing downside risk, i.e. stocks with high conditional downside covariance have high unconditional returns, and sorting stocks based on their downside beta produces a significantly higher return spread between portfolios compared to sorting on regular beta. They also find downside beta to be a good predictor of future returns, excluding stocks with highest volatility in which case the relation becomes opposite. Post et al. (2009) compare the results of the semi-variance beta by Bawa and Lindenberg, the asymmetric response model of Harlow and Rao and the downside covariance beta of Ang et. al (2006) in US stock market from 1926 to 2007, finding that the semi-variance beta performs the best, generating an annual cross-sectional spread of 5,5 % compared to 3,7 % of a regular beta, while results of ARM being close to ones of regular beta, and downside covariance generating a 2,8 % annual cross-sectional spread. They note that replacing semi-variance with the conditional variance of Ang et al. (2006) leads to returns being estimated as a deviation of average return during downmarket and when mean return lowers during downmarket without affecting the downside variance this leads to higher downside betas. They conclude that when properly estimated, the downside betas explain stock returns and sorting by downside beta generates a high spread between returns, where stocks with high downside covariance are compensated with higher risk premium. Atilgan, Bali, Demirtas and Gunaydin (2018) perform a large international study and conclude that downside beta fails to explain returns in global setting. Both Ang et al. (2006) and Atilgan et al. (2018) use a conditional measure of variance when defining downside threshold as a value below a mean excess return on market and it is noted by Post et al. (2009) that since the mean returns decline during down markets, also the sensitivity to losses changes. Kaplanski (2004) finds that the combination of the conditional variance beta and regular beta outperforms both models when used separately but the differences come from market situations, in market down-swings the downside beta captures the risk most accurately. Estrada (2007) compares the performance of semi-variance and normal beta in both emerging and developed markets finding downside beta to be more plausible risk measure, especially in emerging markets. Tsai, Chen and Yang (2014) find results supporting the observations of Kaplanski (2004) that in depressed economy downside beta outperforms the traditional CAPM-beta. Their model compares the performance of the CAPM-beta and three downside beta specifications, including the models of Hogan & Warren (1974), Harlow & Rao (1989) and Estrada (2002), in 23 developed countries. In difference to other studies, they utilize the dynamic conditional correlation model of Engle (2002), in order to counter autocorrelation in return time series. In addition to dynamic conditional correlation, the DCC model also produces the time-varying conditional covariances and conditional variances that are used in the beta estimations on each model. Their results show that DCC modified betas of Hogan & Warren and Harlow & Rao exhibit both higher values than the CAPM beta, capture the downside risk better, and also outperform in explaining the expected stock market returns, concluding that a conditional time-varying beta outperforms the traditional constant beta. Huffman and

Moll (2011) study the differences of various risk measures using daily returns from 1988 to 2009. They find that although investors are compensated for a total risk, downside risk measures are better explaining the future returns and investors care more about the downside risk as compared to upside potential. However, they find the LPM measures to outperform other measures of downside risk and combined with a high risk-aversion coefficient, the model provides a best explanation for future returns. In addition, they note that there is no relation with skewness or kurtosis to future returns, and find traditional beta to be insignificant in explaining returns without additional explaining factors. Moreover, when controlling for downside risk, the traditional beta has a negative relation to future returns. For coskewness their results show that it has a negative coefficient regarding future returns when combined with other measures of risk, but as an independent risk measure, it fails to capture future returns.

3.2 Conditional volatility

The volatility approach for explaining asymmetric correlations has been studied more widely and with rather varying results. The autoregressive conditional heteroskedasticity ARCH by Engle (1982), the generalized ARCH by Bollerslev (1986), the exponential GARCH by Nelson (1991) and several other additional ARCH models have become very popular in modelling volatility, since they capture volatility clustering and time-variation of volatility. Ang and Chen (2002) find strong asymmetries in US stock market using various GARCH-models. The relation of beta and volatility has also been subject to some testing; Hong, Tu & Zhou (2007) create a test for asymmetric correlations and by constructing portfolios find evidence of asymmetries in conditional betas and covariances. Bekaert and Wu (2000) find significant asymmetries in Japanese stocks and market, and conclude that the sources for asymmetries vary, but are mostly caused by volatility feedback mechanisms and although they find asymmetry in conditional covariances, they don't find it in conditional betas. Dean and Faff (2004) investigate the conditional covariances of stock and market returns. Their findings support those of Bekaert and Wu (2000) as the covariance asymmetry could partly explain the volatility feedback and the mean reversion of stock prices, reported by De Bondt and Thaler (1989). They reject the leverage effect since the asymmetry comes from covariance process and the conditional covariance is not affected by time or cross-sectional aggregation, firm size or leverage. Dean and Faff (2004) further conclude that beta asymmetries are hard to estimate since the conditional covariance and conditional volatility both could show asymmetries. They find the covariance asymmetry as a more powerful and easier to detect than finding asymmetries in betas, therefore rejecting time-varying betas and suggesting time-varying risk premiums. As a conclusion they offer a covariance feedback as an option to volatility feedback, where the changes in covariance are more likely to cause the feedback effect. Koutmos and Knif (2002) create dynamic betas using a bivariate GARCH model and study the effects on size based equal-weighted portfolios on the Finnish Stock Exchange, finding that betas exhibit significant time-variation and asymmetries in down- and upside betas. Although the effect was not uniform throughout the size-based portfolios, with both the largest and smallest sizes exhibiting lower downside betas and the smallest portfolio having positive volatility asymmetry with volatility rising more with positive shocks. Furthermore, they find no evidence of asymmetric covariance. For a largest size portfolio, they estimate the negative volatility asymmetry to be 1.4 and increasing to two times higher for portfolio 4. The asymmetry ratio for various markets from previous studies has also been gathered by Taylor (2007). The ratio stays positive for the US stock market during different time periods, with value range of 2-7.2 and with different GARCH specification providing variability in results. For Japanese stock markets, according to studies by Engle and Ng (1993) and Bekaert and Wu (2000) the values range from 1.8 to 2.8. The UK stock market exhibit the least asymmetries with the ratio of 1.6 in study by Poon and Taylor (1992).

The work on studying correlation of volatility and expected returns has yielded quite various results, since the scale of models, frequencies and time-periods used varies greatly. French, Schwert and Stambaugh (1987), find a positive relation with volatility and expected return and negative in case of unexpected volatility changes, concluding that it suggests further evidence on the positive relation between risk premium and volatility. Glosten, Jagannathan and Runkle (1993) modify the GARCH-M model to allow positive and negative returns to have different impact on volatility and find a negative relation between conditional volatility and monthly returns. They also notice that negative residuals increase the variance, while positive ones cause a slight decrease. Baker et al. (2011) find a negative relation with volatility and future returns on U.S. markets, supporting a low-volatility anomaly. Similar findings have been reported by Ang, Hodrick, Xing and Zhan (2006), who while studying

U.S. stock portfolios find that stocks with high systematic volatility have low average returns, and also by using the Fama-French model discover that stocks with high idiosyncratic volatility have abnormally low returns. A finding that is inconsistent with the notion that idiosyncratic volatility should not be priced, since it can be diversified away. Fu (2009) criticizes the previous study on results concerning idiosyncratic volatility and shows how the reversal of previous period high return with high volatility stocks can lead to results of Ang, Hodrick, Xing and Zhan (2006). Also Blitz and van Vliet (2007) find a negative relation with volatility and expected returns, which seem to be consistent with most recent studies. In a relation to beta, Frazzini and Pedersen (2014) find that low beta stocks yield high returns and Hong et al. (2007) find strong asymmetries both in betas and covariances of their two best performing portfolios.

3.3 Conditional higher moments

The study on higher moments as a source of return asymmetries has also been extended to conditional skewness of the asset and market returns. Rubinstein (1973) first introduces risk-aversion to mean-variance valuation model, and the work of Kraus and Litzenberger (1976) extends the CAPM-framework and finds that investors have an aversion to variance and prefer a positive skewness in return distributions. The preference to coskewness is conditional on the skewness of the market, if the market return distribution is positively skewed and investors prefer positive skewness in their portfolios, then they would prefer positive coskewness with the market. And if the market returns are negatively skewed, they would be avoiding positive coskewness with the market. Friend & Westerfield (1980) find some evidence that supports the positive skewness preference of investors, and that they are willing to pay a premium for holding a positively skewed portfolio. Conditional on market skewness being positive, this could indicate a higher risk premium on stocks and portfolios with negative coskewness with the market.

Harvey and Siddique (1999) introduce a method for estimating time-varying coskewness with a model that allows for changing mean and variance. Their joint model extends the GARCH and EGARCH specifications and they find that two of the stylized facts found in conditional volatility models are affected by coskewness: the persistence of volatility, where high volatility is followed by more high volatility, and the asymmetric variance, where negative innovations have a greater impact on volatility. Including conditional skewness to GARCH-framework decreases both the persistence and asymmetric variance found in return time series. However, they also conclude that the relation on returns, variance and skewness is linked to seasonal variations, including January-effect. Also, the frequency of returns matters, daily and monthly returns have impact on the dynamics between second and third moments. Last, they note that the aggregation of stocks into a bigger portfolio has a great impact on the conditional variance.

Harvey and Siddique (2000) continue to explore the effects of conditional skewness in asset pricing context. Their model incorporates coskewness to the CAPM forming a three moment-pricing model. Their results suggest that conditional skewness helps to explain cross-sectional variation in stock returns that traditional mean-variance models fail to capture. They find that coskewness helps to capture the risk asymmetries, especially the downside risk. This suggests that the risk-averse investors, should prefer a right-skewed portfolio and since adding a left-skewed asset reduces the total skewness, then a higher expected return is needed for compensation. Therefore, a left-skewed asset can be perceived as containing more risk than an asset with identical risk-characteristics excluding coskewness. They estimate that the average risk premium for bearing systematic skewness to be 3,6 percent per year.

Ang, Chen and Xing (2006) find that, consistent with the findings of Harvey & Siddique (2000), stocks with high coskewness yield lower returns. They go on to separate the effect of coskewness from downside covariance and other factors by forming portfolios sorted based both on downside beta and coskewness. Based on the coskewness formula, the coskewness is either a covariance of assets return with the square of the market return or with volatility of the market. Thus, an asset with high negative coskewness has low returns when market volatility is high, which often are also periods of low market returns. The distinction between downside beta and coskewness emerges from the fact that the former especially measures the downside covariance. Their results suggest that for stocks with low coskewness with the market, the downside beta better captures the risk premium associated with downside risk.

Also, the fourth conditional moment of return distributions, cokurtosis, has been studied by Dittmar (2002). The author uses nonlinear pricing kernels to find cross-sectional variation in stock returns and finds a relation with positive cokurtosis and high returns. Scott &

Horvath (1980) link the preference to positive skewness to negative preference to kurtosis. Högholm, Knif, Koutmos and Pynnönen (2011) Study the combined factor loadings on third and fourth comoments across 49 US industry portfolios of Fama-French. They find that, across the conditional return distribution, the coskewness is not robust but has increasing negative effect on left tail of the distribution and positive on the right tail, similar in all 49 US industry portfolios. This is consistent with the findings of Galagedera (2003) that the impact of coskewness to excess returns varies across the conditional return distribution. For the effects of cokurtosis the results were more ambiguous and highly depending on the quintile of the distribution.

4. Methodology

The empirical part of this study consists of various portfolio sorting methods both in respect to volatility and downside correlation of the stocks and the market. The weekly excess returns for all the stocks in the portfolio sorting phase are computed using compounded simple returns and weekly risk-free rate. While many studies use logarithmic returns in financial time-series, when the part of the log-series of a single stock is placed in a different portfolio, in different periods, the results for the portfolio are not as accurate as when using simple returns. The annual three months Euribor rate is converted to weekly as follows:

$$r_{f weekly} = (1 + r_f)^{\frac{1}{52}} - 1 \tag{12}$$

The initial estimation period for the betas is three years (156 weeks) of historical weekly returns. This will provide enough observations for the β^- -calculation, since it is conditional on excess return on market being below zero. The estimation of β^- will be performed the following way:

$$\beta^- = \frac{cov(r_i, r_m | r_m < 0)}{var(r_m | < 0)} \tag{13}$$

where the cut-off point for downside beta is the zero value of the excess return on market when using the OMXH CAP as a proxy. The relative β^-, which will further separate the effect of actual downside risk from a regular systematic risk given by normal beta, will be then estimated by subtracting the regular beta of a given stock from conditional beta: $\beta^- - \beta$. If there is no observed beta-asymmetry for an asset, then the value of relative β^- will be zero.

For the estimation of volatility, this study will use the exponential generalized autoregressive heteroskedasticity (EGARCH) model by Nelson (1991), since it is suited for capturing the volatility asymmetry where variance reacts more strongly to negative shocks than to positive shocks of same magnitude. To ensure that there are enough observations when using weekly returns and to better capture the volatility clustering, the EGARCH process is run through the whole data, with the same data and starting date as for the initial beta estimation. (Taylor, 2007)

EGARCH (1,1) with one lag of squared residuals and one lag of conditional variance, will be estimated using R programming language and R-studio program with the rugarch-package that is specified in the following way:

$$\ln(\sigma_t^2) = \left(\omega + \sum_{j=1}^{m} \zeta_j \vartheta_{jt}\right) + \sum_{i=1}^{q} (\alpha_j z_{t-j} + \gamma_j(|z_{t-j}| - E|z_{t-j}|)) + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2) \quad (14)$$

Where α_j captures the sign and γ_j the size effect of asymmetric volatility. Parameters are estimated using the maximum likelihood estimation method based on the density function of standard distribution (Ghalanos 2020). Since the main purpose for the use of EGARCH in this study is to get estimations on volatility, and not to estimate model fitness or other qualities in the sorting phase, the residuals are simply assumed to be normally distributed since it won’t affect the actual volatility estimates that the model produces.

All stocks that are listed during the estimation period are included in a study after they have 40% of the 156 observations, resulting in stocks being included when they have return observations for the previous 63 weeks. Shorter time period could generate outlier values especially for the betas of a newly listed companies, which tend to stabilize after a longer observation period. Stocks are kept as part of the sample as long as they are listed in the Finnish Stock Exchange, or until their trading volume and weekly price changes are non-existent prior to delisting. After getting the estimations for both volatilities and betas, the stocks will then be sorted in portfolios based on their relative β^- and volatility values. The sorting will be done in two different ways. The first method (A) controls for high idiosyncratic risk and excludes the highest volatility quintile of stocks and sorts the remaining stocks into five quintile portfolios based on the relative β^-. The method A is partly based on the hypothesis that conditional downside betas are affected by high levels of volatility, as reported by Ang, Chen and Xing (2006). The second method (B), will include a double sorting of the stocks into portfolios, first in respect to their volatility (from high to low), forming four different volatility classes. Within each volatility class, the stocks will then be sorted again into three subportfolios, based on their relative downside betas. This sorting method also allows the construction of four pure volatility-based portfolios, based on the first ranking. All the portfolios formed on both methods are equally weighted and when the total amount of stock isn't dividable with either five, four or three, depending on the sorting, the remaining stock will be placed in the middle portfolio(s).

After the initial 156 weeks estimation period, the stocks are sorted on equal-weighted portfolios, based on their historical conditional volatility and relative downside beta. The portfolios are sorted again every fourth week, with a rolling estimation window of 156 weeks. The cumulative excess return for the whole range of the total valuation period and the geometrical average return of every four weeks holding period is estimated until the valuation period ends in the start of 2020. The average sorting phase values of conditional volatility and conditional betas can be then compared on portfolio-level, along with further test that are presented in the next two subsections. The realised annual future excess returns, descriptive statistics, regular beta, standard deviation and downside semidevation are estimated for every portfolio to examine the relation of conditional volatility and downside covariance in asymmetries of the realised return distributions.

4.1 Statistical estimates of portfolio distributions and volatility parameters

The EGARCH (1,1) process will be repeated on the portfolios and the residuals are then compared. The asymmetry given by the alpha and gamma-parameters of the EGARCH specification can then be compared with other portfolios. This could help to reveal more about relation of the downside beta and conditional volatility since the portfolios are initially sorted not only in respect to volatility but also with in respect to their relative downside betas. Engle and Ng (1993) present both the news impact curve and the sign bias test for estimating the impact of new information on volatility and the asymmetric response to it respectively, and the sign bias to test whether the model used explains these effects. The equations for news impact curve for EGARCH (1,1) as specified by Engle and Ng are as follows:

$$h_t = A * \exp\left[\frac{(\gamma+\alpha)}{\sigma} * \varepsilon_{t-1}\right], \text{ when } \varepsilon_{t-1} > 0, \text{ and} \tag{15}$$

$$h_t = A * exp\left[\frac{(\gamma-\alpha)}{\sigma} * \varepsilon_{t-1}\right], when\ \varepsilon_{t-1} < 0 \tag{16}$$

Where A is:

$$A = \sigma^{2\beta} * exp\left[\omega - \alpha * \sqrt{2/\pi}\right] \tag{17}$$

The EGARCH allows the news curve to have asymmetric slopes on either side of ε_{t-1} , thus demonstrating different reactions to positive and negative news. The sign bias test which determines whether variables, denoted z_{ot}, that are not being included in the EGARCH model can predict the squared residuals. The test consists of three regressions, sign bias, negative size bias and positive size bias and their joint effect, that are:

$$v_t^2 = a + b * S_{t-1}^{-} + \underline{\beta}' \underline{z}_{ot}^{*} + e_t \tag{18}$$

$$v_t^2 = a + b * S_{t-1e_t}^{-} + \underline{\beta}' \underline{z}_{ot}^{*} + e_t \tag{19}$$

$$v_t^2 = a + b * S_{t-1e_t}^{+} + \underline{\beta}' + \underline{z}_{ot}^{*} + e_t \tag{20}$$

where a and b are the constant coefficients, $\underline{\beta}$ is the constant coefficient vector and e_t the residual. For a sing bias test, the dummy S_{t-1}^{-} has a value one when the residual is negative and zero otherwise. For the positive size bias test, the variable $S_{t-1e_t}^{+}$ is one minus $S_{t-1e_t}^{-}$. The t-ratio of coefficient b is the test value in all regressions. The sign bias test determines whether the impact of negative and positive shocks not predicted by the model is significant.

The negative and positive size bias tests determine whether the different impact of small and large shocks, negative and positive respectively, not predicted by the model is significant.

To assess realised distributional asymmetries further, the conditional higher moments for each portfolio at post-sorting phase are computed using all stocks in this study as a proxy for the market portfolio. This can help to evaluate whether either the conditional volatility or tendency to downside correlation capture other higher moments used as risk-measures at an aggregate portfolio level. The portfolio-level coskewness of the realised four weeks excess returns is calculated with R and PerformanceAnalytics package in the following way:

$$CoSkew(r_{i,} r_m) = \sum((\, r_i - \overline{r_i})(r_m - \bar{r}_m\,)^2) \tag{21}$$

And for the cokurtosis:

$$CoKurt(r_i, r_m) = \sum((r_i - \overline{r_i})(r_m - \bar{r}_m)^3) \tag{22}$$

The relation of realised asymmetries with portfolios and market can be demonstrated more clearly by estimating systematic higher moment betas for coskewness and cokurtosis. Analogous to covariance beta, the value above (below) one indicates that the deviations from the proxy used, add (decrease) the systematic risk of the comoment. Martellini and Ziemann (2007) suggest them to be used as to measure the diversification potential of adding an asset to a portfolio, but in this study, they are used to measure the realised asymmetry measures in proportion to equal-weighted average of all stock in this study. Other advantage of using the Systematic comoments is that their interpretation is clear; the values above one indicate that, considering either coskewness or cokurtosis risk, the portfolio is riskier than the mean value of all stocks, or alternatively, that adding it to a portfolio of average market risk would raise the risk, considering the moment measured. Systematic coskewness is calculated as:

$$S_{r_i, r_m} = \frac{\sum((r_m - \bar{r}_m)(r_i - \overline{r_i})^2)}{\sum(r_m - \bar{r}_m)^3} \tag{23}$$

which in this study in simple terms is a conditional skewness of an asset and the market average divided with the skewness of the market. Systematic cokurtosis for estimating tail risks can be estimated in analogous way:

$$K_{r_i,r_m} = \frac{\sum((r_m - \bar{r}_m)(r_i - \bar{r}_i)^3)}{\sum(r_i - \bar{r}_i)^4} \tag{24}$$

When a systematic cokurtosis value is below one then there would be diversification benefits from adding an asset to a portfolio, and in this study value above one would indicate that in terms of realised returns of an portfolio, it is exposed to more extreme tail risks than an average, compared to an average of all stocks as a market portfolio. Because the interpretation of coskewness is conditional on market skewness, in R the result is multiplied with the sign of market skewness in order to compare the measure without considering the underlying market conditions. (Lestel 2019)

4.2 Testing the performance of the portfolios

For testing the performance of the portfolios regarding their volatility and realised returns, most simple method is the using of the Sharpe ratio presented by Sharpe (1966). It measures the risk-adjusted performance of a portfolio, while considering the volatility of excess returns:

$$Sharpe\ ratio = \frac{R_i - r_f}{\sigma_i} \tag{25}$$

Where, R_i denotes the mean return of portfolio and σ_i the standard deviation of excess returns to a portfolio. Dividing mean excess returns with the standard deviation causes the ratio to rise when volatility of the excess returns decline, making the total risk of the portfolio lower.

Since both positive and negative deviations from the mean return rise the standard deviation in Sharpe ratio in symmetric proportions, thus increasing the total risk, it is criticized for treating upside potential and downside risk the similar way. Pätäri (2011) proposes an extension to the Sharpe ratio, where standard deviation is replaced with a skewness- and kurtosis adjusted deviation (SKAD). It uses the adjusted Z-value that corresponds to the Z value of normal distribution by utilising the fourth order Cornish-Fisher expansion (Cornish and Fisher 1937):

$$Z_{CF} = Z_c + \frac{1}{6}(Z_C^2 - 1)S + \frac{1}{24}(Z_C^3 - 3Z_C)K - \frac{1}{36}(2Z_C^3 - 5Z_C)S^2 \tag{26}$$

where Z_c denotes the critical probability-value retrieved from standard normal distribution. Skewness (S) and kurtosis (K) are estimated as:

$$S = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_{it} - \bar{r}_i}{\sigma}\right)^3 \tag{27}$$

$$K = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_{it} - \bar{r}_i}{\sigma}\right)^4 - 3 \tag{28}$$

where $\bar{r}_i$ is the average return and N is the number of observations. Skewness and kurtosis adjusted deviation is then formed with multiplication of the standard deviation with the Z_{CF}/Z_c-ratio, and finally replacing the standard deviation in the Sharpe ration forms a skewness-and kurtosis adjusted Sharpe ratio:

$$SKASR = \frac{r_p - r_f}{SKAD_p^{(ER\ /\ |ER|)}} \quad (29)$$

As negative excess returns can cause the ratio to get a negative numerator, thus making a less risky portfolio to get a higher negative ratio, the SKASR is modified with the use of absolute values for deviation, as suggested by Israelsen (2005). Since this study focuses in assessing the asymmetrical effects of the return distributions it will use both Sharpe- and the SKASR-ratios to compare their differences to portfolio performance.

The performance of two portfolios can be compared further, Jobson and Korkie (1981) developed the Jobson-Korkie-test of equal Sharpe ratios which determines whether the Sharpe ratios of two portfolios are statistically different. The model is corrected and simplified as suggested by Memmel (2003) the following way:

$$Z = \frac{\hat{S}h_i - \hat{S}h_n}{\sqrt{\hat{V}}} \quad (30)$$

$\hat{S}h_i$ and $\hat{S}h_n$ represent the estimated Sharpe ratios of the portfolios under comparison and $\hat{V}$ denotes the estimated asymptotic variance:

$$\hat{V} = \frac{1}{T}[2 - 2p_{in} + \frac{1}{2}(Sh_i^2 + Sh_n^2 - 2Sh_i Sh_n p_{in}^2)] \quad (31)$$

where T is the quantity of return observations and p_{in} is the correlation of the portfolio returns:

$$p_{in} = \frac{\sigma_{in}}{\sigma_i \sigma_n} \quad (32)$$

The test is also fit for testing the significance of difference of the skewness- and kurtosis adjusted Sharpe ratios of two portfolios if standard Sharpe ratios are replaced with their skewness- and kurtosis adjusted counterparts.

The risk and the performance of the portfolios can also be measured with two additional measures, tracking error and information ratio. Tracking error measures how well the portfolio follows the movement of the market or a benchmark used, with lower values indicating less deviations from the proxy used, therefore it's also called as relative risk or active risk. In this study it can also identify if any of the portfolio sorting criteria causes high deviations from the market. The formula as represent by Bacon (2008):

$$TE = \sqrt{\frac{\sum_{i=1}^{i=n}(r_p - r_i)^2}{n}} \tag{33}$$

where r_p is the excess return on portfolio and r_i is the excess return on the market. Tracking error is therefore a standard deviation of the excess returns. With the aid of tracking error, the information ratio can be estimated as:

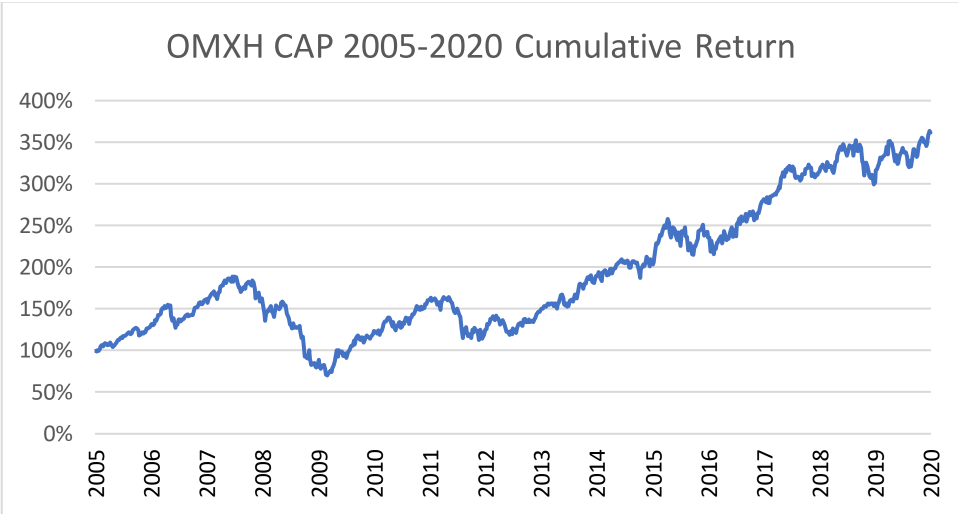
$$IR = \frac{r_p - r_i}{TE^{(ER\,/\,|ER|)}} \tag{34}$$

Information ratio then measures how much the tracking error contributes to the excess return deviations from the market. The information ratio is modified by adding the excess return divided by the absolute value of excess return as an exponent, as suggested by Israelsen (2005). In this study it can help to determine whether the realised deviations in different sorting criteriums are contributing positively or negatively to realised excess returns. (Bacon 2008)

5. Data

This study will be performed by using weekly returns of all Finnish stocks on the OMXH, from the first of December 2004 to the 8th of January 2020. The weekly return data is gathered from the Thomson Reuters Datastream using total return indices that are adjusted for dividends and stock splits. OMXH CAP Total Price index, used as a market portfolio, and monthly Euribor that is used as a risk-free rate are also retrieved from Datastream. The total amount of stocks that fit the criteria of having enough observations to be included in the study is 164, with an average of 122 stocks throughout the study, which will constitute to an average of 20,35 stocks per portfolio in sorting method A, where the top quintile of most volatile stocks is excluded. For the sorting method B, the first sorting phase has an average of 30,55 stocks, sorted in four portfolios based on their conditional volatility values and based on their downside beta, an average of 10,18 stocks in three subportfolios under the volatility classes. Overall, this totals at 99269 individual weekly price observations during the whole study, with maximum for a weekly return of single stock being 283 percent, minimum of minus 58 percent and weekly mean being 0,16 percent.

During this period, the stock market in Finland was largely impacted by global financial crisis of 2007-08, and moderately with the European debt crisis of 2010 followed by US stock market fall of late 2011. Also, the turmoil following US stock market selloff of 2015-16 affected the stock prices in Finland. Combined with a normal smaller scale market turbulence, with up- and downswings, this provides sufficient amount of data on how Finnish stocks behave throughout different market regimes. After the initial estimation period, the valuation period for portfolios starts in the midst of 2007-2008 crisis, contributing to relatively low returns cumulative and average returns in this study. Placing together the cumulative weekly returns on the OMXH CAP and their weekly price chances during this period demonstrates clearly the above market crisis and the asymmetric effect and persistence it has on the volatility on Finnish stocks, as can be seen on Figure 1:



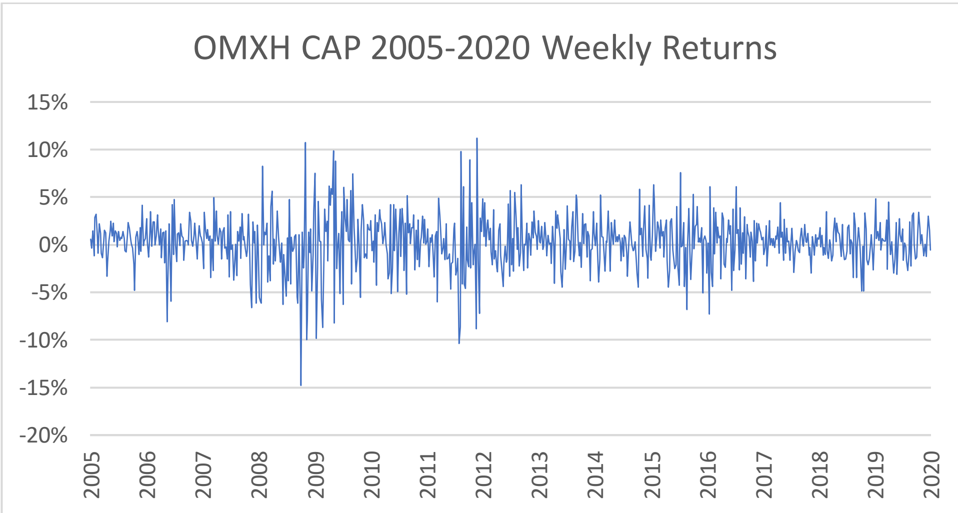


Figure 1: Comparison of the OMXH CAP cumulative and weekly returns from 2005 to 2020.

Since the estimation of downside beta used in this study is conditional on market excess return being below zero, there must be enough weeks that fulfil this criterion during every 156-week estimation period. Overall, the weekly excess return for the OMXH CAP was negative in 41 percent of the weeks during the whole study, the yearly distribution is listed at Table 1:

Table 1: Weeks with negative excess return to the OMXH CAP index from 2005 to 2020.

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
n	16	16	21	29	19	17	29	25	19	22	24	18	23	27	18
Ratio	31 %	31 %	40 %	55 %	37 %	33 %	56 %	48 %	37 %	42 %	46 %	35 %	44 %	52 %	35 %

The number of weeks with market going down is relatively high, but in some of the cases the excess return was just barely below zero, which technically makes them fit for estimating the downside beta, but it is possible that they don't capture the reaction of stocks covariance with the actual down-markets. For example, Fabozzi and Francis (1977) estimate downside betas for bear markets where the downside movement of the market is much stronger. However, most observation of downside market showed larger deviations from the zero and the total amount of observations for estimating downside beta is, on average, approximately 65 observations per one sorting period.

The asymmetric response to shocks in Finnish market during this period can be presented by the news impact curve of the OMX Helsinki CAP index:

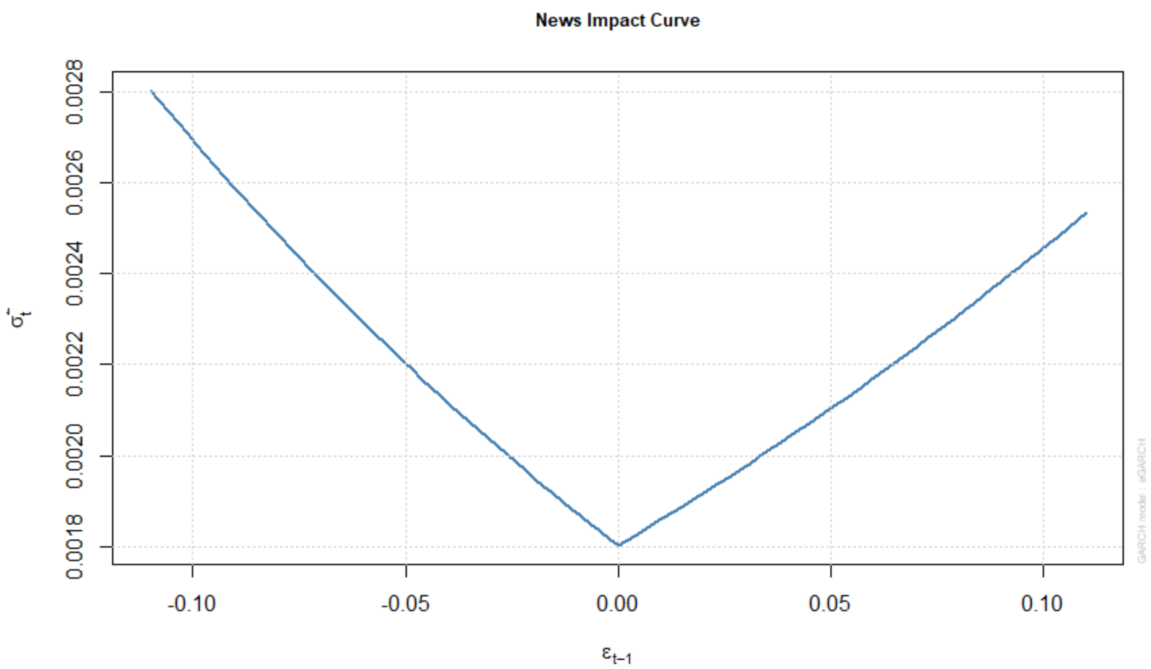


Figure 2: The effect of shocks on the Finnish Stock Exchange 2005-2020.

Figure 2 demonstrates how negative shocks have stronger impact on conditional volatility, with volatility rising more when the past periods residuals are negative. However, the positive shocks too seem to have had a notable impact on volatility, possibly due to the strong market turbulence during the estimation period. Section 6 will present how the sorting to portfolios has captured these effects.

6. Empirical results

The results are reported in three categories. Subsection one presents the overall performance of the portfolios and compares their risk-adjusted performance as measured by the Sharpe ratio and the SKASR-ratio and their significance along with other portfolio performance metrics. Second subsection presents the descriptive statistics of the portfolios and their ex-post higher moments from the realised returns on an aggregate portfolio-level. Subsection three looks into the asymmetric volatility effects and if they can explain deviations in the return distributions of the portfolios. The results are compared against the equal-weighted portfolio of all stocks used in empirical part instead of the OMXH CAP used for beta estimates. Since the CAP index is not completely equal-weighted, the weight-ratio of the smaller capitalization stocks differs quite largely from the equal-weighting used in this study. This can be seen with return difference, where the OMXH CAP has outperformed the average geometric returns of all stocks in this study by over two percentage points. The effect is also visible in unconditional beta estimates, which have a total average below one, caused by smaller capitalization stocks having larger deviations from a more capitalization-driven index. However, this only affects the scale of the beta estimates, not their relative order. Since even small deviations cause often larger impacts to estimates, especially when estimating comoments, the equal-weighted portfolio of all stocks is used. This provides a more accurate estimate of the relationship between the portfolios and their deviations from the mean.

Another aspect to consider is the turnover rates of the portfolios. It affects the cumulative performance of the portfolios in form of a transaction costs. Also in case of a high turnover rate for a specific portfolio it induces more randomness to statistical estimations presented in subsections two and three. To perfectly replicate this study as an investment strategy would require not just sorting the stocks every fourth week but also equal-weighting them. Since the latter might not be practical or necessary to adjust every time, the focus will be on looking how often, on average, a stock changes its class of portfolio. Overall, the changes in volatility classes were slow, with approximately 12 percent of stocks changing their class from period to next. These were mostly volatility jumps caused by idiosyncratic risk to a stock or stocks changing their class near the border of classes. The turnover-rate for downside beta sorting was under 10 percent, caused by the long estimation period used. The combination of the methods caused more turnover, with over 2 stocks changing per period in subclasses, resulting an approximately 23 percent turnover rate. However, most changes happened in the most volatile class and downside beta tail values of either sign reverted towards mean more slowly, thus the downside beta sorting and lowest volatility classes were quite stable and contributing less to an average turnover rate.

6.1 Portfolio performance

First sorting method based on excluding the most volatile quintile of stocks based on their conditional volatility estimated with EGARCH (1,1) and sorting the remaining stocks on five portfolios (from high to low) by their relative downside beta. The results are presented below:

Table 2: The performance of the portfolios sorted by their relative downside beta.

Annual geometric and cumulative returns from 2004 to 2020. The risk-adjusted performance and significance of the SKASR difference against equal-weighted average of all stocks.

Portfolio	R (Gm)	Cumul. R	St.dev.	Sharpe	SKASR	P-value
pf1	6,12 %	105,94 %	0,17	0,36	0,32	0,37
pf2	5,59 %	93,65 %	0,18	0,30	0,24	0,76
pf3	6,78 %	122,00 %	0,17	0,39	0,34	0,16
pf4	4,48 %	70,36 %	0,17	0,25	0,23	0,85
pf5	3,43 %	50,59 %	0,19	0,18	0,16	0,61
Hi-low	3,36 %	71,40 %	0,02	0,21	0,18	
Average	4,29 %	66,55 %	0,17	0,24	0,22	
OMXH CAP	6,63 %	118,17 %	0,20	0,32	0,28	

The returns are reported by their annual geometric mean and cumulative return for each portfolio from the fifth of December 2007 to the eight of January 2020, along with annualised standard deviation, the Sharpe- and the SKASR-ratios and the Jobson-Korkie significance test for the SKASR. The hi-low row notes the difference between the highest and the lowest values in each class. The results for the first sorting method show patterns with higher returns concentrating on the highest three portfolios, and the two lowest relative downside beta portfolios yielding also the lowest excess returns. However, there are no significant differences in the performance when compared to the equal-weighted portfolio of all stocks in study. Notably, the realised volatility of the portfolios increases only slightly when the downside beta decreases, revealing only small differences in correlation of ex-ante relative downside betas and realised volatility.

The second sorting method consisted of two phases and the main portfolios sorted on basis of their conditional volatility are presented next:

Table 3: The performance of the portfolios sorted in quartiles based on conditional volatility.

Annual geometric and cumulative returns from 2004 to 2020. The risk-adjusted performance and significance of the SKASR difference against equal-weighted average of all stocks.

Portfolio	R (Gm)	Cumul. R	St.dev.	Sharpe	SKASR	P-value
VOL1	1,41 %	18,56 %	0,23	0,06	0,06	0,16
VOL2	2,47 %	34,58 %	0,21	0,11	0,10	0,14
VOL3	5,47 %	91,00 %	0,18	0,30	0,26	0,64
VOL4	6,82 %	123,08 %	0,12	0,54	0,47	0,02
Hi-low	5,41 %	104,52 %	0,10	0,48	0,41	
Average	4,29 %	66,55 %	0,17	0,24	0,22	
OMXH CAP	6,63 %	118,17 %	0,20	0,32	0,28	

Sorting the stocks by solely based their conditional volatility does reveal many notable patterns. The relation of excess returns and volatility shows visible negative correlation, creating a large return spread between the highest and lowest quartiles. Also, the realised volatility declines steadily with portfolios with a lower ex-ante conditional volatility, suggesting that there is persistence in high conditional volatility that can measured from average realised volatility after a four-week holding period. The difference in performance against average is significant at 5% level for the lowest volatility quartile, with combination of high excess returns and low volatility generating high Sharpe and SKASR values. Notably the gap between the SKASRs and Sharpe ratios is constantly growing with lower volatility classes, indicating that with the lower volatility classes the combined effect of skewness and kurtosis has relatively more unfavourable deviations, although the total risk remains smaller.

The second phase of the method B goes on further by sorting stocks regarding their relative downside beta within the volatility quartiles is presented in Table 4 below:

Table 4: The performance of the downside beta subportfolios within volatility classes.

Annual geometric and cumulative returns from 2004 to 2020. The risk-adjusted performance and significance of the SKASR difference against equal-weighted average of all stocks.

Portfolio	R (Gm)	Cumul. R	St.dev.	Sharpe	SKASR	P-value
VOL1						
pf1	1,68 %	22,45 %	0,26	0,06	0,06	0,38
pf2	4,80 %	76,78 %	0,25	0,18	0,19	0,85
pf3	-4,69 %	-44,20 %	0,28	-0,16	-0,23	0,03
VOL2						
pf1	3,45 %	51,04 %	0,22	0,15	0,13	0,48
pf2	2,22 %	30,60 %	0,22	0,10	0,09	0,30
pf3	0,81 %	10,33 %	0,23	0,03	0,03	0,17
VOL3						
pf1	6,21 %	107,97 %	0,18	0,33	0,30	0,58
pf2	6,47 %	114,18 %	0,19	0,32	0,25	0,77
pf3	2,99 %	43,11 %	0,19	0,15	0,14	0,61
VOL4						
pf1	8,31 %	163,92 %	0,13	0,59	0,57	0,05
pf2	5,67 %	95,56 %	0,14	0,40	0,35	0,32
pf3	6,49 %	114,78 %	0,13	0,47	0,43	0,16
Hi-low	13,00 %	208,12 %	0,15	0,75	0,79	
Average	4,29 %	66,55 %	0,17	0,24	0,22	
OMXH CAP	6,63 %	118,17 %	0,20	0,32	0,28	

Sorting the volatility quartiles further with their relative downside betas, reveals some anomalous features. Portfolio three in the highest volatility class has yielded abysmally low returns, even when compared to other portfolios in the highest volatility class, which as total, performed the worst. The second notable feature is that the worst and best performing portfolios are the exact opposites in their two sorting criterions. While the highest volatility combined with lowest relative downside beta within the class has the lowest returns, the portfolio with the highest relative downside beta in lowest volatility class outperforms all the others. Similar pattern with the lowest downside beta portfolio underperforming seems to happen also within classes two and three with a decreasing magnitude. Once again, there seems to be little differences in realised volatility within classes. The underperformance of the VOL1 portfolio3 is significant at 5% level, and the outperformance of the VOL4 portfolio1 is just above the significant level when compared to average performance of all stocks. The combination of these two sorting methods generates quite a large spread both in returns and also in the performance metrics, suggesting that they could be used as a portfolio selection criterion and for risk-aversion. Overall, the performance measured with the SKASR-and the Sharpe-ratios were best in highest three downside beta portfolios of the sorting method A and with two of the lowest volatility classes in method B, where also the lower amount of volatility contributes to a better performance. Figure 3 plots the standard deviation of the portfolios against the annualised geometric excess returns to visualize the realised return-risk relation:

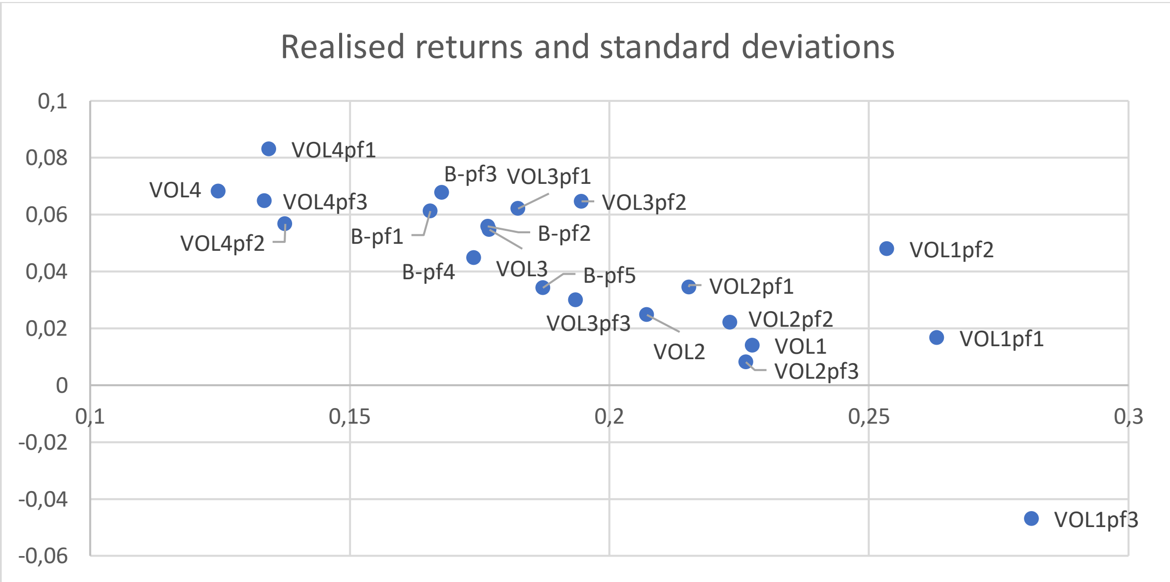


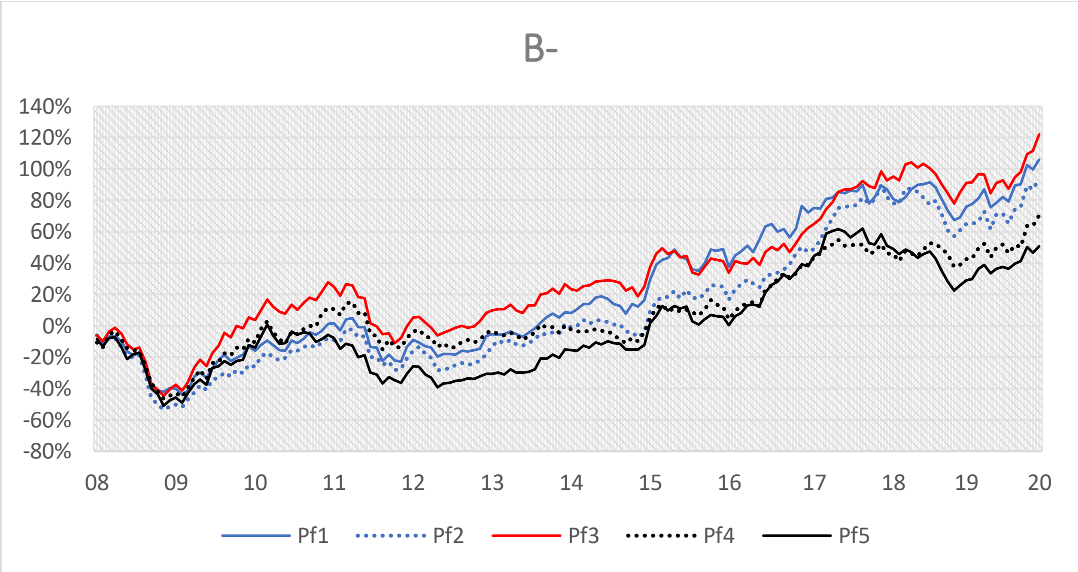
Figure 3: Annual geometric returns (y-axis) and standard deviations (x-axis) of the portfolios.

Figure 3 shows the geometric excess returns of the portfolios on the Y-axis and standard deviations on the X-axis. This demonstrates that when using weekly returns and a four-week holding period, the conditional volatility has a strong negative correlation with future returns dominating over the effect of downside beta. While the low volatility persists, they also outperform the higher volatility classes systematically. Next table will present the tracking error and the information ratio to see how the deviations of the portfolio returns differ in relation to the mean movements of the market:

Table 5: Tracking error and information ratio for all the portfolios.

Portfolio	Tracking Error	Annualised Tracking Error	Information Ratio
B-pf1	0,005	0,019	0,066
B-pf2	0,005	0,016	0,060
B-pf3	0,004	0,015	0,121
B-pf4	0,005	0,017	0,007
B-pf5	0,005	0,018	-0,008
VOL1	0,007	0,026	-0,036
VOL2	0,005	0,017	-0,015
VOL3	0,004	0,015	0,059
VOL4	0,006	0,021	0,074
VOL1pf1	0,012	0,044	-0,042
VOL1pf2	0,011	0,039	0,025
VOL1pf3	0,015	0,054	-0,258
VOL2pf1	0,007	0,026	-0,006
VOL2pf2	0,008	0,027	-0,025
VOL2pf3	0,008	0,029	-0,051
VOL3pf1	0,007	0,025	0,058
VOL3pf2	0,006	0,023	0,076
VOL3pf3	0,007	0,025	-0,017
VOL4pf1	0,008	0,029	0,093
VOL4pf2	0,006	0,022	0,035
VOL4pf3	0,007	0,025	0,054

The tracking error turns out to be more interesting since the information ratio outcome is mostly a function of realised excess returns to portfolios. In both the volatility and downside beta sorting, the tails seem to gather higher tracking error as to be expected, but this effect is moderately stronger for volatility than for downside beta and highest errors seem to be gathered around high volatility portfolios. The method A sorting has relatively lower values throughout all portfolios, likely due to excluding highest quintile of volatile stocks on the sorting phase. However, the tracking error does not consider the sign of an error and while a tracking error and information ratio do not seem to reveal much differences. A visual presentation of the cumulative returns allows an easy comparison how the returns are generated by each portfolio. First, the cumulative excess returns for downside beta and volatility classes are presented below in Figure 4:



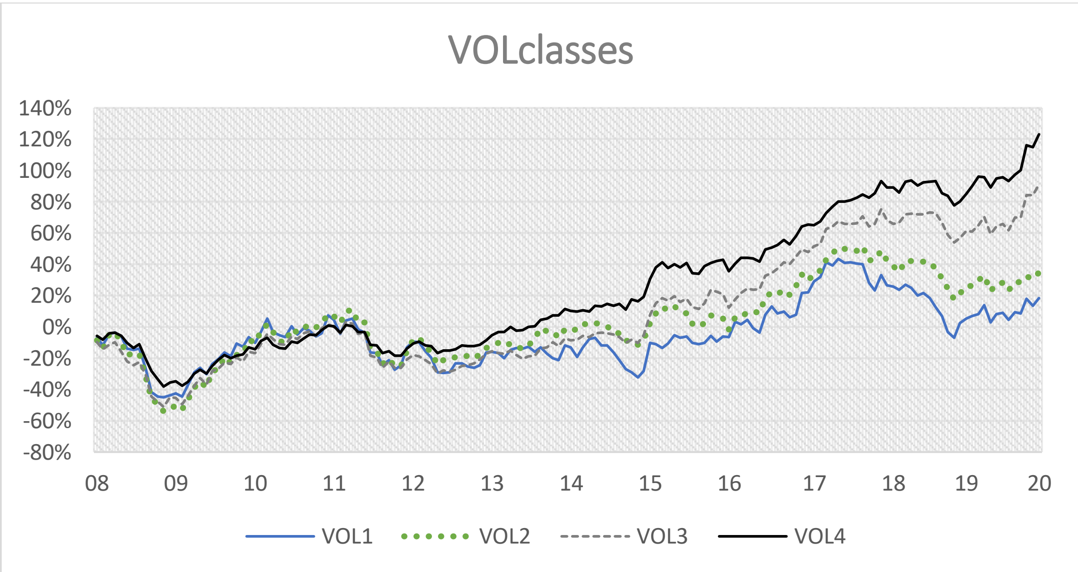


Figure 4: Cumulative excess returns of the portfolios sorted by downside beta and conditional volatility.

The differences in the downside beta sorting are less visible than with the volatility sorting where the gap between the highest and lowest classes is steadily increasing. But there also seems to be some consistency in the return generating process in the downside beta portfolios, where the lowest two portfolios stay below the others. However, this gap seems to get narrower when a market rises more strongly, and it is possible that in a special case of bullish market, the low downside beta stocks might outperform others. However, this is rare market condition and when the rise in the market moderates, the impact on the portfolios with low downside beta is visibly harder.

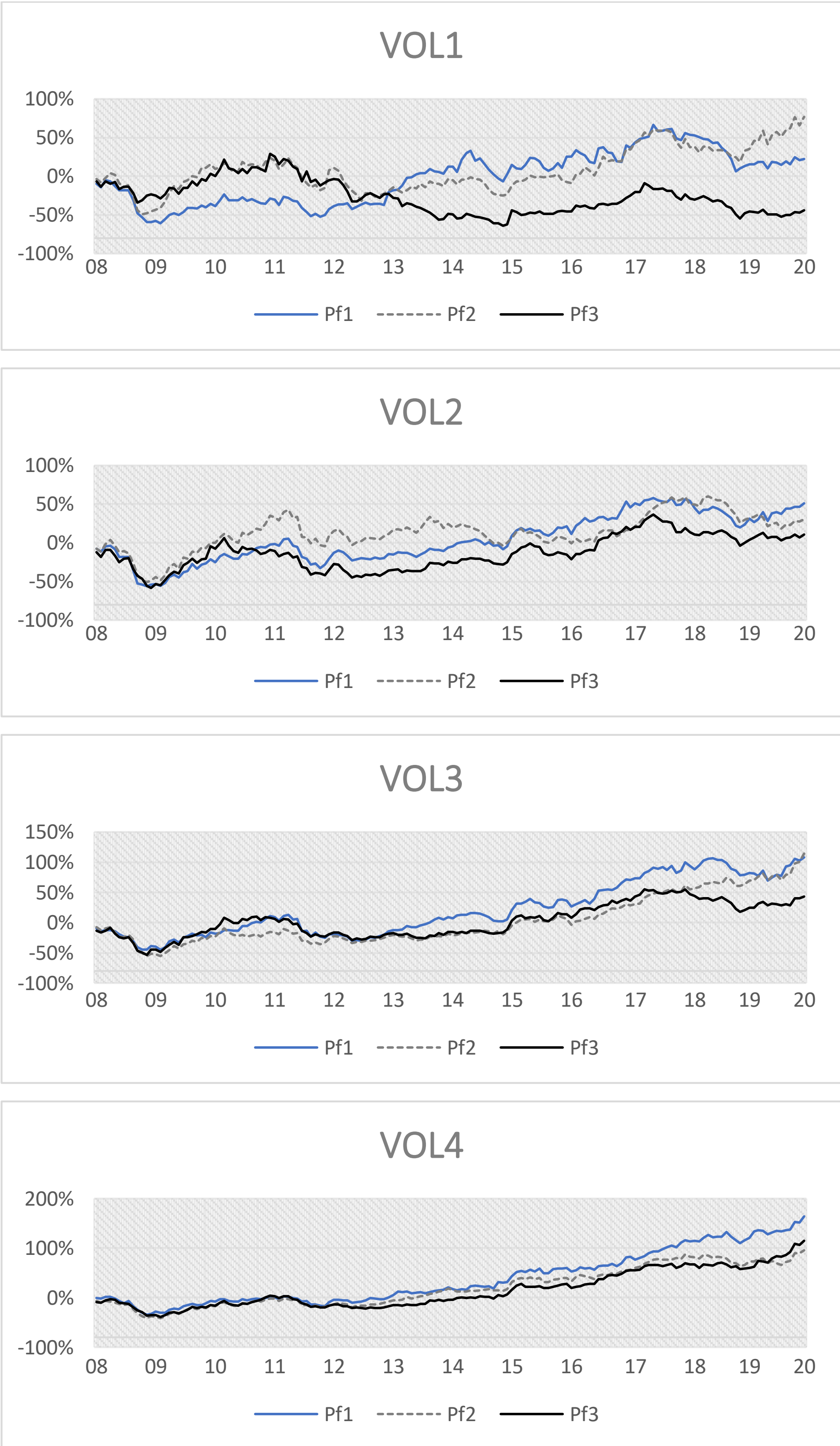


Figure 5: Cumulative excess returns of the downside beta portfolios in each volatility class.

Figure 5 shows that with the rising volatility the spread of the cumulative returns increases quite strongly. Most notable detail is that the black line, representing the lowest downside beta portfolio, stays under the others most of the times in every class. However, there seems to be sort of a paradigm shift in the highest volatility class starting around European debt crisis of 2012. This could be an indicator that the portfolio three gathers speculative stocks with high leverage, and the performance is highly dependable of the overall market situation. Also, the Figure 5 demonstrates that for an investment strategy, the two lowest volatility classes are quite stable, with rare changes in the mutual order of the portfolios. The high downside beta portfolios presented with the blue line are quite constantly above others in respect to their total cumulative excess return. The similar patterns are also visible in the two highest volatility classes, but with more randomness and also with a lower return to an investor. To summarize, the outperformance of the low volatility classes and low downside beta stocks is rather consistent throughout the total valuation period from the end of 2007 to the start of 2020.

To better estimate whether the sorting methods can be used for forming a profitable investment strategy in the real world, the average transaction costs of the portfolios should be considered. Pätäri and Vilska (2014) assume the average transaction cost of 0,1% per trade for institutional investors and 0,4% per trade for individual investors at the Finnish stock market from 1996 to 2012. Domowitz, Glen and Madhavan (2001) examine transaction costs in 42 countries from 1996 to 1998 and find the average one-way trading costs of 0,434% for the Finnish stocks. This study assumes the average transaction cost to be 0,3%. However, the realised transaction costs, especially between volatility classes, might differ depending on the liquidity of the stocks and the bid-ask spread. The average transaction costs are used for estimating the total transaction costs based on the turnover rate of the portfolios. This method considers only the stocks that switch their portfolio from period to next, not the equal-weighting of the portfolios. Perfectly balancing the weighting of the portfolios after every period would require a larger amount of transactions. However, the weighting differences from one period to next tend to be rather small. Table 6 will present performance rankings of the ten best performing portfolios after considering transaction costs:

Table 6: Turnover rate and the annual geometric returns after transaction costs for the ten best performing portfolios.

Portfolio	R (Gm)	Turnover rate	Transaction costs	Return after costs
VOL4pf1	8,31 %	22,10 %	1,74 %	6,57 %
pf3	6,78 %	9,10 %	0,71 %	6,07 %
VOL4	6,82 %	10,89 %	0,85 %	5,97 %
pf1	6,12 %	8,91 %	0,70 %	5,43 %
pf2	5,59 %	9,12 %	0,71 %	4,87 %
VOL3pf2	6,47 %	21,51 %	1,69 %	4,78 %
VOL4pf3	6,49 %	23,10 %	1,82 %	4,67 %
VOL3	5,47 %	11,01 %	0,86 %	4,61 %
VOL3pf1	6,21 %	22,30 %	1,75 %	4,46 %
VOL4pf2	5,67 %	22,20 %	1,75 %	3,93 %

Table 6 shows that the two-step sorting mechanism of the method B, contributing to a high turnover rate, affected the realised returns rather heavily. The average annual transaction costs are over 1,7% for the portfolios of the method B. However, the VOL4pf1 still outperforms others but the differences are much smaller. The performance of the Method A portfolios is relatively better; even though the method A did consist of two phases, exclusion of the most volatile quintile and sorting of the remaining stocks based on their downside beta, the turnover rate is relatively low. Compared to the average performance of all stocks in the study, top nine portfolios still yield higher returns. Although, after considering transaction costs, none of the portfolios outperformed the OMXH CAP index.

6.2 Beta asymmetries

This subsection investigates how the conditional beta values and conditional volatility estimates used as sorting criteria affect the post-sorting realised higher moments and comoments on aggregate portfolio level. This can further clarify the factors that generate the return distributions presented on previous subsection. To estimate how the performance of the downside betas as in predictor of the future returns differs from the unconditional beta, three different sorting phase beta values are reported. Table 7 will provides a more thorough analysis of the sorting phase conditional betas, normal beta, and conditional volatility along with the realised higher moments for all the portfolios:

Table 7: Conditional volatilities and betas of the sorting phase, with realised moments and geometric returns.

Portfolio	eGARCH	Relat. B-	Bminus	Beta	Skewness	Kurtosis	St.dev.	R (Gm)
B-pf1	0,047	0,47	1,09	0,62	-0,30	1,61	0,17	6,12 %
B-pf2	0,045	0,18	0,83	0,65	-0,85	2,92	0,18	5,59 %
B-pf3	0,044	0,05	0,70	0,65	-0,44	1,65	0,17	6,78 %
B-pf4	0,045	-0,08	0,64	0,71	-0,15	0,88	0,17	4,48 %
B-pf5	0,050	-0,35	0,42	0,77	-0,12	1,30	0,19	3,43 %
Hi-low	0,006	0,53	0,67	0,15	0,73	2,04	0,02	3,36 %
VOL1	0,119	0,04	0,80	0,75	0,32	1,43	0,23	1,41 %
VOL2	0,057	0,04	0,83	0,79	-0,26	1,62	0,21	2,47 %
VOL3	0,044	0,06	0,75	0,69	-0,41	2,22	0,18	5,47 %
VOL4	0,031	0,08	0,61	0,54	-0,54	1,16	0,12	6,82 %
Hi-low	0,088	-0,03	0,21	0,25	0,86	1,06	0,10	5,41 %
VOL1								
pf1	0,114	0,65	1,33	0,67	0,29	1,14	0,26	1,68 %
pf2	0,118	0,02	0,75	0,74	0,20	1,10	0,25	4,80 %
pf3	0,125	-0,54	0,31	0,85	1,65	8,68	0,28	-4,69 %
VOL2								
pf1	0,057	0,38	1,09	0,71	-0,61	2,92	0,22	3,45 %
pf2	0,057	0,04	0,86	0,82	-0,06	1,78	0,22	2,22 %
pf3	0,057	-0,30	0,53	0,83	0,03	1,03	0,23	0,81 %
VOL3								
pf1	0,044	0,34	0,98	0,63	-0,21	1,77	0,18	6,21 %
pf2	0,044	0,06	0,72	0,66	-0,84	3,42	0,19	6,47 %
pf3	0,044	-0,22	0,54	0,77	0,18	2,34	0,19	2,99 %
VOL4								
pf1	0,030	0,34	0,88	0,54	-0,14	0,62	0,13	8,31 %
pf2	0,031	0,06	0,59	0,53	-0,52	1,32	0,14	5,67 %
pf3	0,032	-0,18	0,37	0,55	-0,34	0,60	0,13	6,49 %
Hi-low	0,095	0,92	1,02	0,32	2,48	8,09	0,15	13,00 %
Average					-0,29	1,56		4,29 %
OMXH CAP					-0,52	1,23		6,63 %

The beta distribution in method A, at the top of Table 7 shows that the unconditional beta values have a negative correlation with both downside beta-measures. Moreover, inconsistent with the CAPM, the realised returns decline with rising unconditional betas of the sorting period. While the effects remain a bit scattered, the best performing portfolios are centred around higher downside beta values. The two downside betas used, while representing very synchronized values, measure different things. The non-relative downside beta is purely a measure of the downside covariance, without assuming any asymmetries it can react similarly to upside movements of the market. The relative downside beta measures the asymmetry in reactions to down- and up movements of the market, with values above zero indicating that the stock has a tendency to covary more strongly with the market when it goes down, thus it's beta being non-linear. However, it does not measure the magnitude of the impact, only the asymmetry in it, therefore it is a direct indicator if there are any additional risk premiums for stocks with asymmetrical reactions to market movements. The relation of the relative and non-relative downside beta has been studied by Ang, Chen and Xing (2006), who report that while the non-relative downside beta produces slightly wider positively correlating spread with excess returns, the t-test statistics for the relative downside beta are more significant. Using a relative downside beta then controls for the effects to the return distributions caused by the unconditional beta. Another finding by Ang, Chen and Xing (2006) shows that upside beta and its relative version both have significant negative correlation with returns, with latter producing a larger negative return spread. This effect is clearly visible in Table 7 above since the poorly performing lowest relative downside beta portfolios, with a negative sign, are therefore exhibiting an opposite upside asymmetry. Overall, the relative downside betas have more positive values in all portfolio classes, thus showing that an asymmetric stronger reaction to down-movements of the markets is present in each class. In portfolios sorted by the method B, both the unconditional and the non-relative downside beta seem to rise with the volatility, although the causality is opposite with the relative downside beta. And curiously, the highest and the lowest values for the relative and non-relative downside beta are in the highest volatility class portfolios 1 and 3. Thus, there is a link between a strong past asymmetric correlation of both signs and conditional volatility. However, the volatility does not consider the sign, and the measurement errors and the deviations in the distributions of beta measures are therefore largest in the most volatile class. And when considering that the class of most volatile stocks clearly underperforms when compared to others, it is then consistent with the findings of Ang, Chen and Xing (2006) that downside beta is a poor predictor of future returns for the most volatiles stocks. And when we look at the equation (11) and Table 7 where the distribution of downside betas is relatively flat across all volatility classes, and assuming that the past conditional volatility level are persistent, then for a different volatility levels to exhibit similar levels of downside betas would indicate that in the lowest volatility classes the downside correlation is stronger in order to produce similar downside betas. And with the higher volatility classes, the stocks variance or volatility dominates the process of producing high downside betas, conditional on the market down movements. This indicates that for the lowest classes, there would have to be more relative, not total, downside asymmetries. And within the volatility classes, the downside betas do seem to predict future returns quite well, an effect that is most visible in volatility class two, and excluding the lowest volatility class, portfolios with low downside betas yield noticeably lower returns. Outside the first volatility class, where also the highest downside beta-values are, there are clear patterns indicating that on aggregate level for stocks with higher downside exposure the risk is priced in form of a higher risk premium.

And looking at the Table 7 a different way, the unconditional beta seems to have a negative correlation with future returns, visible in all portfolios and sorting methods. While the sorting is not based on unconditional beta, and the distribution of unconditional betas that produces the mean values at Table 7 could be wide, there are still patterns of higher values in high volatility classes and in low downside beta classes. This is quite surprising and inconsistent with the CAPM since the average cumulative excess return from the beginning of the valuation period is positive, even though the amount of strong market down movements was relatively frequent. However, in this study the downside betas clearly outperform the unconditional beta in explaining future excess returns and share similar findings reported in section three, including Huffman and Moll (2011), who find that the downside risk is compensated more strongly than the total risk.

The weekly conditional volatility results given by EGARCH and their comparison to annualised standard deviation also confirms that the volatility persists quite well over the future valuation period and this is visible on every portfolio. Last columns on the Table 7 are the realised third and fourth moment after the valuation period. Again, most notable deviations from the average are in the highest volatility class, where the combining with low downside covariance seems to capture a special class of stocks that produce quite unique characteristics for portfolio three. There is also consistency with the sorting values and the realised skewness of the portfolios, which gathers more negative values both with the high downside beta loadings and low volatility. This is expected in case of the downside beta, but seems like the low volatility stocks also have this common realised characteristics. This could indicate that if the pricing mechanism is efficient in form of a higher risk premium for downside risk, then it could work as a balancing factor in regards to volatility. Overall, there seems to be no consistency between the realised kurtosis with neither downside betas or volatility, besides the single observation of VOL1 portfolio three, with high leptokurtic distribution and exposure to tail risks. Table 8 will further study how these moments relate to average market reaction, by using systematic comoments:

Table 8: Realised systematic comoments of the portfolios.

Portfolio	CoSkewness	CoKurtosis	CoVariance	R (Gm)
pf1	1,16	0,90	0,88	6,12 %
pf2	1,68	1,04	0,96	5,59 %
pf3	1,21	0,93	0,92	6,78 %
pf4	0,95	0,89	0,94	4,48 %
pf5	0,92	0,99	1,01	3,43 %
Hi-low	0,76	0,15	0,14	3,36 %
VOL1	0,47	1,20	1,21	1,41 %
VOL2	1,34	1,15	1,15	2,47 %
VOL3	1,26	1,01	0,97	5,47 %
VOL4	0,96	0,65	0,67	6,82 %
Hi-low	0,87	0,55	0,55	5,41 %
VOL1				
pf1	1,37	1,18	1,24	1,68 %
pf2	0,87	1,21	1,23	4,80 %
pf3	-0,82	1,21	1,18	-4,69 %
VOL2				
pf1	1,75	1,22	1,12	3,45 %
pf2	1,39	1,16	1,17	2,22 %
pf3	0,87	1,07	1,16	0,81 %
VOL3				
pf1	0,97	0,94	0,92	6,21 %
pf2	1,73	1,08	1,01	6,47 %
pf3	1,09	1,01	0,98	2,99 %
VOL4				
pf1	0,97	0,62	0,62	8,31 %
pf2	1,07	0,71	0,71	5,67 %
pf3	0,74	0,60	0,66	6,49 %
Hi-low	2,58	0,62	0,61	13,00 %

The systematic coskewness with the equal-weighted average of all stocks can be used to estimate which of the portfolios share either the right- or left-tailed movements of the market at the same time. To help the estimation, the systematic coskewness formula is modified so that the left-tailed coskewness values are positive, when market skewness is negative and vice versa. This way the values above one are always indicating more risk in case of every comoment and considering realised systematic coskewness, values above (below) one show that the portfolio has shared the negative left-tailed (positive right-tailed) deviations with the market. Based on values of the sorting method A, the relation with systematic coskewness and downside beta appears to be consistent with definitions of both these measurements. The downside beta of the sorting phase, which measures stocks tendency covary with market when it goes down, and systematic coskewness which is a realised measure of left-tailed co-distributions. Thus indicating that in fact the stocks with high downside correlation had a persistent tendency to undergo negative co-movements with the market during the valuation period. This effect, and its opposite with stocks with low downside beta, seems to be stronger with high volatility stocks and while the volatility decreases in lower volatility classes of method B, the effect is altered. This can be explained by looking at Equation (23), where when simplified, the denominator is basically a squared deviation of a market return and numerator is the portfolios covariance with the market. When the covariance with movements of either sign is high, it results to a large spread in coskewness of the portfolios with higher levels of volatility. Notably the portfolio 3 in highest volatility class has a negative signed systematic coskewness, another indication that it could gather high idiosyncratic risk stocks that do not follow market movements. Another interesting observation is that while the downside beta seems to indicate next period future coskewness, conditional volatility does the same for cokurtosis. Observed high conditional volatility and volatility clustering on asset and portfolio-level seems to contribute to higher extreme tail risks of the next period. The realised systematic covariance on the portfolio level, as to be expected, correlates strongly with the sorting period volatility and also seems to have rather ambiguous interaction with sorting period downside betas. To summarize, the downside beta does seem to reveal future coskewness risk, while the conditional volatility can measure future cokurtosis risk and volatility.

Yet another way to compare the differences in realised return distributions is the lower partial moments of the portfolios. While not a systematic risk measure, it captures the raw downside semivariance of the portfolios sorted based on their systematic risk measures and conditional volatility. The LPM calculates the variance below the mean excess returns of the market. In Table 9 below, the LPM is square rooted and then annualized in order to compare it to realised standard deviation of the portfolios:

Table 9: Comparison of the standard deviation and downside semideviation of the portfolios.

Portfolio	**St.dev**	$\sqrt{\mathbf{LPM}}$	**Diff.**	**R(Gm)**
B-pf1	0,166	0,173	-0,007	6,12 %
B-pf2	0,177	0,192	-0,016	5,59 %
B-pf3	0,168	0,174	-0,006	6,78 %
B-pf4	0,174	0,174	0,000	4,48 %
B-pf5	0,187	0,196	-0,008	3,43 %
VOL1	0,228	0,208	0,020	1,41 %
VOL2	0,207	0,208	-0,001	2,47 %
VOL3	0,177	0,179	-0,002	5,47 %
VOL4	0,125	0,135	-0,010	6,82 %
VOL1pf1	0,263	0,235	0,028	1,68 %
VOL1pf2	0,253	0,232	0,021	4,80 %
VOL1pf3	0,281	0,239	0,042	-4,69 %
VOL2pf1	0,215	0,225	-0,009	3,45 %
VOL2pf2	0,223	0,226	-0,003	2,22 %
VOL2pf3	0,226	0,222	0,005	0,81 %
VOL3pf1	0,182	0,185	-0,003	6,21 %
VOL3pf2	0,195	0,213	-0,019	6,47 %
VOL3pf3	0,194	0,187	0,006	2,99 %
VOL4pf1	0,134	0,135	-0,001	8,31 %
VOL4pf2	0,138	0,150	-0,012	5,67 %
VOL4pf3	0,134	0,138	-0,004	6,49 %

The difference column notes the difference of the standard deviation and downside semideviation, calculated as St.dev – $\sqrt{\mathrm{LPM}}$. First, looking at the downside beta portfolios, there is no clear correlation of the historical downside beta and difference between up- and downside variances of the portfolios, but the downside variance seems to be slightly higher in all portfolios. But in the volatility main classes, the difference is quite clear with the highest volatility class contributing more to the upside variance, and with a steady decline in lower classes. Interestingly, within the downside beta subportfolios, in two of the highest volatility classes (and partly in others too) , the downside beta captures the future asymmetry in size of the positive and negative variances, with high downside beta having relatively more downside variance. In volatility class 1, the upside variance dominates all subportfolios, but even there the lowest downside beta portfolio has the highest difference in upside asymmetry. This can be partly explained by the construction of relative downside beta, which in negative values results to a stock having a higher upside covariance with the market, and in every class the lowest downside beta portfolio has relatively more upside variance than the others. It is unclear why the method B subportfolios capture this but not the sorting in the method A, but this could be an indication that the downside volatility process in variance and covariance of the beta estimation dominates over the downside correlation, hence the differences come visible only after controlling for the volatility more thoroughly. This is also another indication that in the lowest volatility classes, the downside correlation could be stronger, thus contribution more to the downside beta values than in volatility dominated higher classes.

6.3 Volatility asymmetries

The previous section focused on properties of the conditional betas and their effect on the realised returns and higher moments. An alternative way is to repeat the EGARCH (1,1) process for the portfolio returns and study the parameters more closely. It should be noted that since the portfolios are sorted again every fourth week, the EGARCH process then captures the volatility parameters of a specific sorting criterium, rather than a time series of a stock or index with lesser structural changes. However, the stock seldomly change their portfolios, but especially in the case of the highest volatility class, due to idiosyncratic risk factors causing sudden high volatility surges on individual stocks during the previous four-week period, the portfolio class based on volatility might change rapidly, mostly affecting the high volatility classes. However, the downside beta is more stable in this relation, being more of an indicator of long-period downside risk with few periodical changes in the portfolios. First, to evaluate if the model has captured the conditional volatility in portfolios, various diagnostic tests for the goodness of fit of the EGARCH specifications used are presented in Appendix 1, but to summarize, the Ljung-Box and squared Ljung-Box tests show that there is no auto-correlation in residuals or squared residuals in any of the portfolios. The ARCH LM test shows similar results. The Nyblom stability test, estimates if the parameters in EGARCH process are constant or whether they change over time, indicating structural change. The null hypothesis for the joint statistic that there is no structural change is only rejected in case of the highest volatility class. The individual Nyblom test parameters are also significant at 5% level in every portfolio expect for the VOL1pf1 portfolio, where the combination of high volatility and downside covariance is at its peak. Engle and Ng's Sign bias test shows that the model does capture the sing and size effects quite well. The null hypothesis is that parameters do explain the asymmetries without additional parameters, and in joint test statistic it is not rejected in any portfolios. There are two individual tests for size bias and positive sign bias, for portfolios B-pf2 and VOL2, respectively, within a 10% confidence level. However, the overall performance of the model, in respect all these tests, is adequate. Table 10 presents the EGARCH parameters for portfolios of the sorting method A:

Table 10: EGARCH (1,1) model fit for the downside beta portfolios.

B-	mu	ar1	ma1	omega	alpha1	beta1	gamma1
pf1	1,005	-0,011	0,064	-0,541	-0,222	0,916	0,138
	(0,00)	(0,21)	(0,00)	(0,02)	(0,00)	(0,00)	(0,13)
pf2	0,997	0,817	-0,678	-0,203	-0,280	0,964	-0,181
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
pf3	1,005	0,648	-0,564	-0,379	-0,246	0,943	0,228
	(0,00)	(0,00)	(0,01)	(0,06)	(0,00)	(0,00)	(0,04)
pf4	1,003	0,648	-0,591	-0,156	-0,162	0,977	0,110
	(0,00)	(0,67)	(0,71)	(0,00)	(0,01)	(0,00)	(0,17)
pf5	1,004	0,743	-0,658	-0,152	-0,097	0,976	0,084
	(0,00)	(0,00)	(0,01)	(0,00)	(0,04)	(0,00)	(0,00)

The number in the row with the portfolio code corresponds to parameter value of that portfolio and p-value is presented in brackets below. The null hypothesis is that the parameter value is zero and thus p-value below 0,05 indicates that the parameter partly explains the conditional volatility of the portfolio in question. If a parameter would equal to zero, it does not indicate that the whole model fails to capture the conditional volatility, only that the specific parameter is redundant in this case. Especially three of the parameters in the Table 10 are most important in evaluating the effect of asymmetries in volatility and their significance is looked more closely. The parameter alpha1, together with gamma1 which is the leverage effect, capture the asymmetric properties of the volatility process. Beta1, the GARCH-parameter for lagged variance, captures the persistence effect. For every portfolio, the beta1 differs from zero with statistical significance, thus the persistence of volatility is present and captured for every portfolio. The alpha1 parameter, with a negative sign, is also significant in every portfolio. However, the gamma1 only captures the leverage-effect in portfolios two, four and five. Table 11 presents the model fit for portfolios sorted by the method B:

Table 11: EGARCH (1,.1) model fit for volatility classes and their subportfolios.

Portfolio	mu	ar1	ma1	omega	alpha1	beta1	gamma1
VOL1pf1	1,002	0,655	-0,619	-1,382	-0,304	0,740	0,012
	(0,00)	(0,01)	(0,02)	(0,02)	(0,00)	(0,00)	(0,94)
VOL1pf2	1,004	0,663	-0,648	-0,348	-0,211	0,936	0,047
	(0,00)	(0,22)	(0,25)	(0,00)	(0,00)	(0,00)	(0,13)
VOL1pf3	0,996	0,501	-0,577	-4,696	-0,497	0,085	-0,598
	(0,00)	(0,00)	(0,00)	(0,00)	(0,01)	(0,10)	(0,01)
VOL2pf1	1,004	0,952	-0,995	-0,973	-0,262	0,838	0,375
	(0,00)	(0,00)	(0,00)	(0,17)	(0,01)	(0,00)	(0,07)
VOL2pf2	1,000	0,657	-0,550	-0,065	-0,120	0,990	0,108
	(0,00)	(0,00)	(0,00)	(0,00)	(0,02)	(0,00)	(0,00)
VOL2pf3	1,001	0,568	-0,463	-0,100	-0,107	0,984	0,064
	(0,00)	(0,08)	(0,17)	(0,00)	(0,00)	(0,00)	(0,00)
VOL3pf1	1,006	0,760	-0,722	-0,701	-0,265	0,887	0,094
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,45)
VOL3pf2	1,008	0,917	-0,923	-0,441	-0,175	0,929	0,224
	(0,00)	(0,00)	(0,00)	(0,17)	(0,05)	(0,00)	(0,16)
VOL3pf3	0,993	0,858	-0,781	0,012	-0,204	0,999	-0,124
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
VOL4pf1	1,009	0,818	-0,903	-1,039	-0,184	0,849	0,258
	(0,00)	(0,00)	(0,00)	(0,00)	(0,01)	(0,00)	(0,04)
VOL4pf2	1,001	0,779	-0,661	-0,158	-0,270	0,975	-0,174
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
VOL4pf3	1,007	0,755	-0,691	-0,357	-0,072	0,948	0,109
	(0,00)	(0,00)	(0,00)	(0,00)	(0,16)	(0,00)	(0,15)
VOL1	0,997	-0,596	0,530	-0,166	-0,182	0,969	-0,171
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
VOL2	1,001	0,573	-0,479	-0,139	-0,171	0,978	0,103
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
VOL3	1,006	0,734	-0,694	-0,399	-0,206	0,938	0,180
	(0,00)	(0,00)	(0,00)	(0,07)	(0,00)	(0,00)	(0,05)
VOL4	1,007	0,629	-0,604	-0,512	-0,155	0,927	0,166
	(0,00)	(0,00)	(0,00)	(0,15)	(0,05)	(0,00)	(0,10)

Again, the alpha1 and beta1 are significant for every model, expect for the highest volatility class portfolio 3, where surprisingly the beta1 does not capture the volatility persistence. This is not the case with other portfolios in the highest volatility class, further indicating that combination of high volatility and low downside beta gathers stocks with quite erratic behaviour. The gamma1 is significant for three of the highest volatility classes, but not for the lowest class. On the subportfolio-level gamma1 is significant only for half of the portfolios. The news impact curves will further demonstrate the asymmetric reactions of the portfolios. They are presented for all method A portfolios and volatility classes in method B, along with the best and worst performing subportfolios, which have significant alphas and gammas together with robust results for the Nyblom stability test and the Sign bias test:

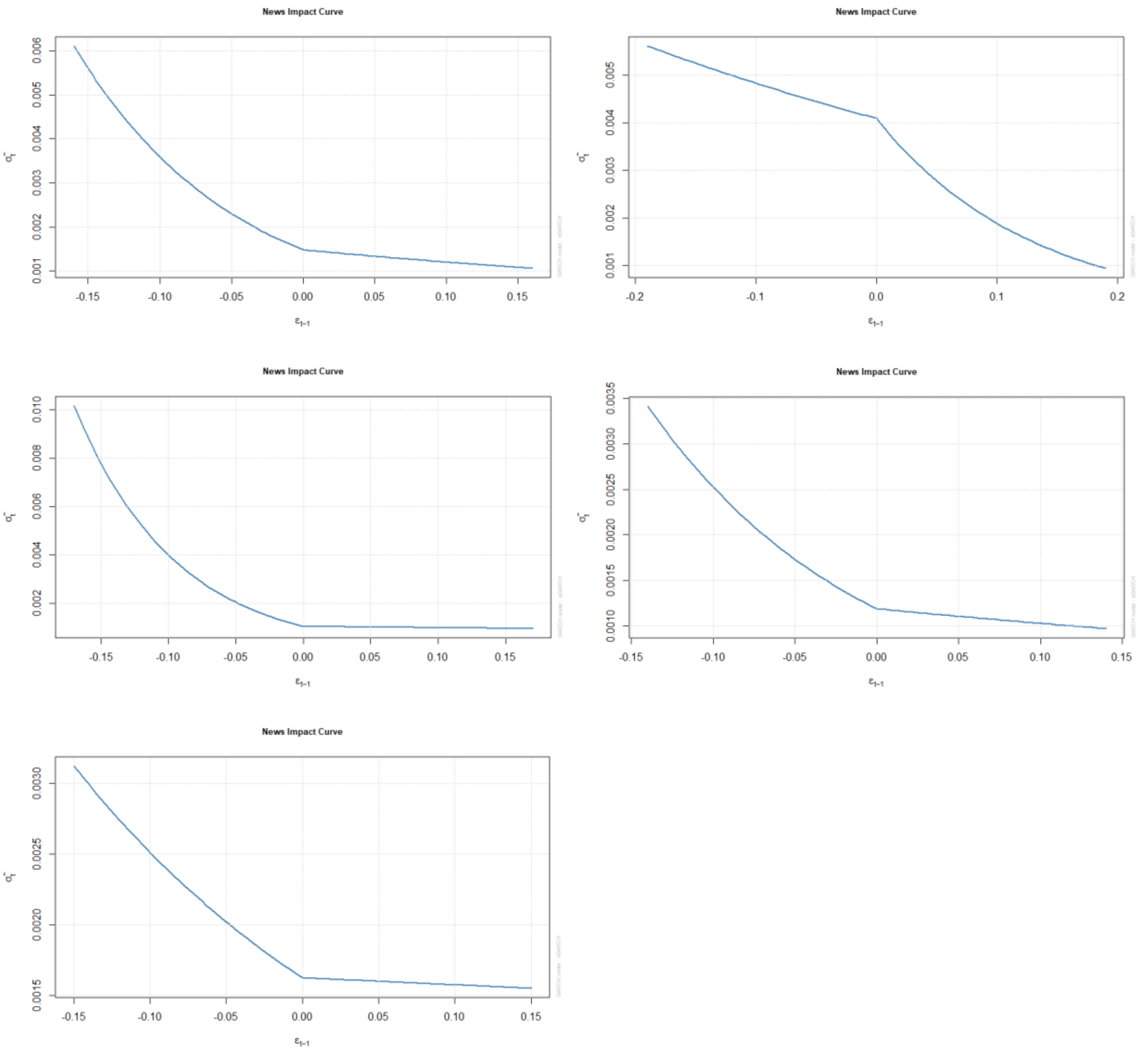


Figure 6: News impact curves of the downside beta portfolios.

Highest downside beta portfolios 1 and 2 are in the top row, from left to right. In the middle row, portfolios 3 and 4. Portfolio 5 is on the bottom row.

All the portfolios sorted by their downside beta exhibit rather similar asymmetric curves, with negative shocks or past residuals having stronger impact on volatility. However, it is worth noting that the scaling of the Y-axis changes with the portfolios and negative news have much larger impact on high downside beta portfolios and the effect is significantly lower at portfolios 4 and 5. This suggest that the news impact curve is picking up the past tendency of high b-minus stocks react stronger to negative news and to covary strongly with the market when it goes down. However, looking at Table 2, these portfolios also outperformed the ones with smaller magnitude reactions to negative shocks, suggesting that the asymmetric volatility is already priced as a high downside risk at portfolios 1, 2 and 3 of method A.

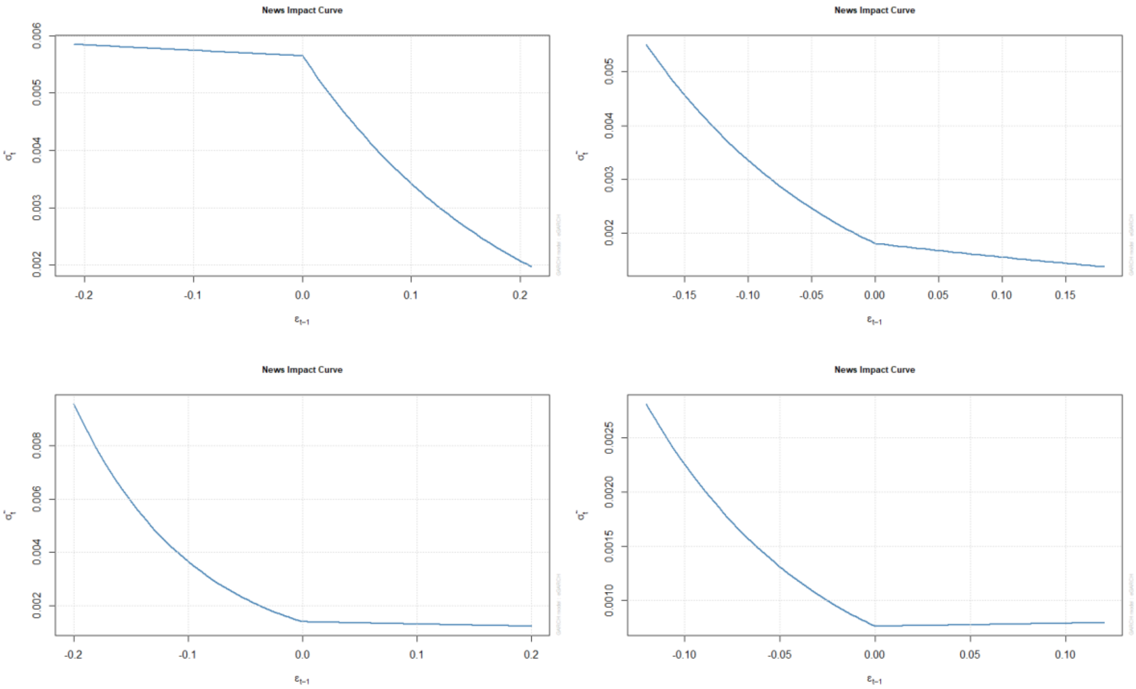


Figure 7: News impact curves of the main volatility classes.

VOL1 and VOL2 are on the top row from left to right. VOL3 and VOL4 are on the second row, from left to right.

In Figure 7, the news impact curves for the volatility classes 2, 3 and 4 show a rather similar shape, with the shocks total magnitude rising with higher volatility classes, while the lower classes show more relative negative asymmetries. However, the most volatile portfolio does have a different shaped curve, with negative shocks having only small effect and positive ones cause the volatility to decline stronger. This might sound counterintuitive but when looking at the volatility values at the zero point of the epsilon at X-axis, it shows that the volatility in highest class is already quite high, and only the incremental effect of past shocks to volatility is lesser, thus the high volatility is still increasing in case of negative shocks. The findings are also consistent with Glosten et al. (1993), who while using monthly returns, report the positive residuals causing a slight decrease in variance and negative ones causing an increase. Last, we compare the best and worst performing portfolios VOL4Pf1 and VOL4Pf3 under the volatility main classes:

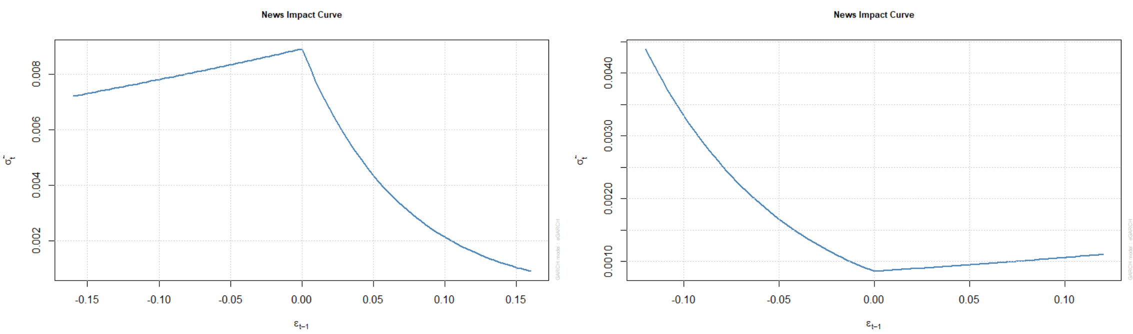


Figure 8: News impact curves of the worst (VOL1pf3) and best (VOL4pf1) performing portfolios. From left to right.

The news impact curve of the worst performing portfolio further confirms that the sorting mechanism captures a class of stocks with abnormal behaviour during the next period, even when compared to other portfolios in the highest volatility class which have more average reactions, presented in Appendix 2. When combining all results before, it indicates that the portfolio VOL1pf3 gathers high volatility stocks with low relative downside beta. A low relative downside beta stock, with a minus-sign, is by construction then also a high relative upside beta stock, covarying strongly with a market when they move up. This could lead to a similar effect of Ang, Hodrick, Xing and Zang (2006) with high idiosyncratic risk, yielding abnormally low returns in the next month. An effect which later concluded by Fu (2008), was mostly caused by the reversal of stocks that had gained abnormally high returns the month before. It might be possible that the portfolio in question generates an ongoing process of selecting stocks that have had positive over-reactions often a moment too late, resulting to a price drop in the next period when the prices stabilize. Also, lacking the downside covariance aspect, the stocks with only relative high upside covariance usually tend to be a group of speculative stocks, and in most cases, deemed to crash eventually. The best performing VOL4pf1 portfolio, has quite an opposite curve, with a low volatility but a strong asymmetrical reaction to shocks with also a volatility slightly increasing effect of positive shocks. Overall, negative asymmetries are stronger in lower classes, while their total impact might be lesser.

7. Conclusions

The impacts of downside correlation with the market showed some distinctive features when compared to conditional volatility. Both might be used as a risk measures, but their effects differed in magnitude and in the way they affect the realised return distributions. First when interpreting the results, a difference in how the used methodology worked in portfolio sorting phase should be noted. The conditional volatility reacts faster to changes, with the exponential GARCH model and lagged historical residuals, the effects of shocks in either idiosyncratic or systematic level affect the estimations more strongly and quickly. This causes the individual assets with relatively higher changes in volatility to jump into higher volatility classes in sorting phase rather quickly. Whereas with downside beta, which has no coefficient for past residuals and where the measurement is conditional on excess return being below zero, the adjustments to changes in underlying firm-level or market changes happen more slowly. The conditional volatility measures more rapid changes in a total risk and as a systematic risk measure, downside beta measures more slowly sifting downside correlation with the market. The underperformance of highest volatility classes could be explained with effects consistent with volatility feedback suggested by Pindyck (1984), relatively stronger changes in stock level volatility cause the prices to drop due to an increase in risk, and this also happens in case of positive shocks, when the rise in prices is moderated in the next period. When studying leverage effect, Duffee (1995) reported a positive contemporaneous relation with returns and volatility that reverses in monthly frequency. Looking at all the results presented in previous section, including the positive skewness values and the relatively lower downside deviation in the first class, the results strongly suggests that in many cases the highest volatility class stocks were the ones having positive price changes. This, combined with a holding period of four weeks, seems to capture a short period risk that has a negative correlation with realised excess returns. As a conditional volatility captures the rise in either idiosyncratic volatility or a relatively larger rise in total volatility compared to average, the downside beta will further capture the probable sign, or a tendency to covary strongly with the market conditional on its movements. Hence, as a two-step mechanism, conditional volatility increases and downside beta detects the probable cause. This could explain the return distributions in conditional beta portfolios under the highest three volatility classes, in which all the portfolio with a lowest downside beta underperformed others in the same class. And as a stock with a negative relative downside beta indicates that it has a strong positive asymmetry with a market, then there should be a discount in risk premium. This seems to happen in every class, excluding the lowest volatility class, where the differences and also the movements of stock were more sparse and the negative asymmetric volatility reaction to shocks was already stronger. However, more additional testing would be required to confirm this. The similar return distribution is visible in the first sorting method, where stocks were sorted solely on their downside beta values, excluding the most volatile quintile. While a downside beta stocks do seem to have higher realised returns or a higher risk premia, the opposite might be even clearer, the stocks with relatively higher upside beta in two bottom quintiles have lower than average realised excess returns with increasing negative correlation, thus indicating a lower risk premium.

As a single sorting method, the volatility clearly outperformed the downside beta, with increasing negative correlation to realised returns with higher levels of sorting-phase conditional volatility. However, the combination of both methods provided a largest return spread between portfolios, with significant performance differences. Also, the lowest volatility class significantly outperformed the average. Estimating transactions costs showed that the high turnover rate of the double sorting affected the returns severely. Thus, the method B might not work as an investments strategy without modifications that reduce the turnover rate. When compared to the OMXH CAP index, there were no significant performance differences, mostly due to the higher total return to the index during the estimation period. This raises a question did the equal-weighting of all stocks cause the small stocks to get a disproportionately large weight when compared to capital structure of the market. The effect of size and leverage to asymmetric volatility have been studied with various results. For example, Bekaert and Wu (2000) find no significant correlation with size and leverage when constructing leverage-based portfolios, but for the generalization of the result of this study, controlling for the size of the firms might offer more robust results.

The first research question was “can conditional betas be used for predicting future returns?”, and for a certain point and when not exposed to high levels of volatility, the downside beta does provide a specific structure of future returns. The differences were most clear when compared to portfolios with low downside betas and positively asymmetric covariance with the market, which proved to be the most underperforming class of stocks throughout all the sorting methods. With higher levels of downside betas in method A, the differences start to dissolve between highest portfolios, even after controlling for high volatility. This leads to second research question: “Is there an additional risk premium for high b-minus assets?”, and as presented at Table 7, the portfolios with the highest unconditional betas in the sorting phase yield low returns, while at the same time, having low downside beta values. This is inconsistent with the CAPM, and suggest that the downside risk is priced more strongly, in form of an additional risk premium, as compared to the total systematic risk. This might be the effect of the upside risk, that seems to strongly indicate lower future excess returns. This effect is present both in downside beta portfolios and subportfolios of the main volatility classes, excluding the highest class with highest return in the middle portfolio, where the combination of volatility and high downside risk might occasionally capture some stocks that are declining, unconditional of the market movements of the moment. However, generally the higher downside beta stocks command higher risk premiums, and compared to unconditional betas, the asymmetric betas, both down- and upside, explain the realised risk premiums better.

To answer the third research question, “How does conditional volatility affect the conditional beta’s ability to explain returns? Does controlling for it yield better results for predicting future excess return with conditional beta?”, it is best to look at the relative differences between the high and low downside beta portfolios in the different volatility classes. When exposed to more volatility, the conditional beta does capture the downside risk quite well, generating a large return spread withing the highest volatility classes. When at the same time, portfolios with highest volatility yield the lowest returns, this leads to a situation, where the effect of conditional volatility affects the realised returns in such a magnitude that sorting a portfolio solely based on downside beta would gather rather large deviations in individual asset return distributions. This could therefore explain why the sorting of the first method failed to find clear differences among the higher downside beta portfolios. When looking only at the realised returns, the volatility effect seem to dominate over downside beta. However, even in higher volatility classes, high downside beta portfolios outperform low downside beta portfolios in a volatility class one step lower, so that both measurements capture some aspects of risk better in comparison to the other. To conclude, controlling for volatility enhances the capacity of conditional beta to predict future excess return. This is shown most clearly with the highest return spread between the portfolios of exact opposite sorting values, the underperformance of the highest volatility and low downside beta portfolio compared to outperformance of the lowest volatility and high downside beta portfolio.

Yet another way to look at how the volatility interferes the downside beta is to look at the realised systematic comoments at the portfolio level and the amount of downside semideviation. The highest volatility class gathers both the highest realised portfolio-level covariance with the market as well as the highest difference in the coskewness withing subportfolios. At the same time, coskewness with the market and high downside beta have a clear positive correlation in highest volatility classes. This is consistent with the both the concepts of both the downside beta and coskewness since the average market skewness during the study was negative. Thus, showing that high volatility stocks that also have a strong reaction to down movements of the markets, continue to undergo negative co-movements with the market during the next four-week period. However, the coskewness effect deteriorates with lower volatility, as another indication that the differences with the downside beta and future returns are most clearly visible in higher volatility classes, and more importantly within the class. When combining downside betas with different levels of volatility, the combination causes opposite-sided reactions in future returns. This is visible when looking at the levels of realised total downside semideviation of the portfolios. While the single sorting with using downside beta fails to capture any differences, when first sorting stocks under volatility classes, the high downside beta portfolios have relatively higher downside semideviation. They also yield higher returns, thereby supporting the hypothesis that there is a higher risk premium for stocks that have more downside risk. Conditional volatility also contributed to future cokurtosis risks and acts as a more dominant risk-factor, but when controlling for it, the differences of different downside beta-levels become clearer. For estimating the exact coefficients and significances of these different risk measures, methods such as the Fama-MacBeth regression should be used, but these results indicate that the downside beta explains the returns after controlling for volatility.

The fourth question, "Does conditional beta capture the spread of future returns better than conditional volatility?" is rather clear considering the results. As a single sorting method, the conditional volatility produces a higher return spread and steadily decreasing return slope with more exposure to volatility. This finding in also consistent with most recent studies about the volatility-return relation, including Baker (2011) and especially with Glosten et al. (1993), who, by using GARH-M model allowing asymmetry, find a negative relation with monthly returns and conditional volatility. However, the findings about the mutual relation of downside beta and conditional volatility in this study might be to some extent altered with a longer valuation period. As noted before, the conditional volatility estimates react more faster to changes than downside betas, thus it might capture a more immediate risk of future turbulence and volatility clustering of the future four-week valuation period. Another difference is that when the downside beta is a measure of a downside systematic risk, the volatility in this study captures both the changes caused by idiosyncratic volatility and also the relatively stronger reactions of an asset to changes in total volatility. Considering this, the mutual relation of these risk metrics might change with a longer valuation period.

The final research question was "Are asymmetric responses to market movements priced similarly in conditional betas and in conditional volatility, or do they capture different aspects of the realised return distributions?". To answer this, the first thing is to find whether there are detectable asymmetries in betas or conditional volatilities. For beta, the average value of the relative downside beta had more positive values throughout all sorting methods, which clearly indicates that the responses in covariances are asymmetric, with stronger reactions to down movements of the market. And for volatility, the news impact curves of the EGARCH fit for the downside beta portfolios show that the negative asymmetric volatility effect is clear and strengthens with high downside beta values. This suggests that the two risk measures capture partly the same risk. For the volatility classes, the reactions are similar excluding the highest class that exhibits high volatility already on the base level. This is also visible on the downside beta portfolios of volatility classes, and by excluding the heavily underperforming VOL1pf3, the asymmetric reactions with different magnitudes are present in every portfolio. Hence, the asymmetric and stronger negative correlation is present for both the betas and for the volatility. For the downside beta, performance of the portfolios shows that it is also priced in the returns of the next period. Considering the volatility asymmetry, the shapes of the news impact curves in lowest three volatility classes are rather similar with slightly more steep reactions in lower classes. However, the effect is rather similar, but the low total volatility of the lower classes seems to persist in a way that even with a slightly stronger asymmetric reaction, the total volatility remains in a lower level. A similar reaction that can be seen when comparing semideviations of the portfolios. While the relative amount of downside deviations rises with a low volatility, the total downside deviations are higher in classes above. The lowest volatility portfolios and highest downside beta portfolios also shared some similar realised higher moments and coskewness and a tendency to have relatively more downside asymmetry. Also, there were high downside beta values present in the highest volatility classes, and the downside beta distribution between classes was rather flat, indicating that the sorting measures capture partly different things. However, there were indications that in the highest volatility classes the high downside beta were most likely produced by high variance, that also exhibited high covariance with the market. Whereas in the lower classes the downside correlation was stronger, combined with the relatively stronger negative asymmetric volatility reaction to market shocks. This shares similarities to covariance feedback effect of Dean and Faff (2004) and suggests that the conditional covariance contributes more to the behaviour of highest volatility classes. Estimating the price of the asymmetry in volatility would require further testing but in light of this study, in case of the volatility, the asymmetry is not priced as highly as with downside beta. Rather as the volatility in total has a negative correlation with returns, the downside beta can detect the asymmetric volatility increasing reaction, that is priced in all levels of volatility.

7.1 Summary

The aim of this study was to find how the stronger asymmetric reactions to negative market movements affect the risk premiums and ways to measure their effects with conditional downside betas and conditional volatility, both separately and combined. This was carried out with various portfolio sorting methods using all stocks in the Helsinki Stock Exchange from end of the year 2004 to the beginning of year 2020. The relative downside beta, that captures the asymmetry in systematic risk commanded a higher risk premium. However, this effect was clearest when among the portfolios with lowest downside betas. Sorting solely based on volatility produced a clear return pattern where the future excess returns of the four-week holding period decreased with higher levels of conditional volatility. Combining the methods by controlling for conditional volatility with the EGARCH model and sorting based on the downside betas withing the volatility classes proved to be most efficient method in finding out how both these measures affect the return distributions and their higher moments. In general, the downside beta combined with volatility generated recognizable patterns in future return distributions and could also be better suited to studies that examine the time-varying betas and risk premiums instead of the unconditional beta that does not capture the asymmetric nature of the covariance with the market. The portfolio sorts also revealed some variations that could be modified for an investment strategy or used as a risk control measures, as the combination of low volatility and high downside beta yielded highest return. Together, or even separately, high volatility and low downside beta contributed to lower future returns, producing a largest spread of future excess returns in exact opposite classes of the combined sorting methods. The effect of high upside potential stocks having a lower required risk premium, might have even contributed more to the results and realised return patters than the opposite relation of the downside risk. Outside the highest volatility class, these relations were quite persistent throughout the study, with stronger upside movement of the low downside (high upside) beta stocks, when the market is moving up. Although, the conditional volatility showed relatively stronger asymmetric reactions with lower levels of volatility, the total level of volatility impacted the returns more. The downside beta captured future realised negative coskewness with the market, whereas conditional volatility captured the cokurtosis and extreme tail risks. However, further research is needed to estimate the exact coefficients for the combinations of conditional betas and volatility. Also, the adding of controlling factors, most important being size and leverage, could further benefit this study.

References

Ang, A. and Chen, J. (2002) Asymmetric Correlations of Equity Portfolios. Journal of Financial Economics 63, 443-494.

Ang, A., Chen, J. and Xing, Y. (2002) Downside Correlation and Expected Stock Returns. NBER Working Paper 8643.

Ang, A., Chen, J. and Xing, Y. (2006) Downside Risk. The Review of Financial Studies, Vol. 19, Issue 4, pp. 1191-1239.

Ang, A., Hodrick, R.J., Xing, Y. and Zhang, X. (2006) The Cross-Section of Volatility and Expected Returns. Journal of Finance, 51, 259-299.

Atilgan, Y., Bali, T.G., Demirtas, K.O. and Gunaydin, A.D. (2018) Downside Beta and Equity Returns around the World. The Journal of Portfolio Management Summer 2018, 44 (7), 39-54.

Bacon, C. (2008) Practical Portfolio Performance Measurement and Attribution. John Wiley & Sons, New York.

Baker, M., Bradley, B., and Wurgler, J. (2011) Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly. Financial Analysts Journal, 67, (1), 40-54.

Ball, R. and Kothari, S.P. (1989) Nonstationary Expected Returns: Implications for Tests of Market Efficiency and Serial Correlation of Returns. Journal of Financial Economics. Vol. 25, P. 51-74.

Bawa, V. S., Lindenberg E.B. (1977) Capital Market Equilibrium in a Mean-Lower Partial Movement Framework. Journal of Financial Economics, 5, 189-200.

Bawa, V. S. (1978) Safety-First, Stochastic Dominance, and Optimal Portfolio Choice. The Journal of Financial and Quantitative Analysis, Vol. 13, No. 2 (June 1978), pp. 255-271

Bekaert, G. & Wu, G. (2000) Asymmetric Volatility and Risk in Equity Markets. The Review of Financial Studies. Vol 13, 1, 1-42.

Black, Fischer. (1972) Capital Market Equilibrium with Restricted Borrowing. Journal of Business, July 1972 444-455.

Black, F. (1976) Studies of Stock Price Volatility Changes. Proceedings of the 1976 Meeting of the Business and Economic Statistics Section, American Statistical Association, Washington DC, 177-181.

Blitz, D. & van Vliet, P. (2007) The Volatility Effect: Lower Risk without Lower Return. Journal of Portfolio Management, vol. 34, no. 1, 102–113.

Bollerslev, T. (1986) Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31, 307-328.

Bollerslev, T., Engle, R.F and Wooldridge, J.M. (1988) A Capital Asset Pricing Model with Time-Varying Covariances. Journal of Political Economy, 96 (1) p. 116-131.

Bollerslev, T. and Mikkelsen, H.O. (1996) Modelling and Pricing Long Memory in Stock Market Volatility. Journal of Econometrics, 73, p. 151-184.

Braun, A.P., Nelson, B.D. and Sunier, M.A. (1995) Good News, Bad News, Volatility and Betas. Journal of Finance. Vol. 50, No. 5, p. 1575-1603.

Campbell, J.Y., LO, A.W. and MacKinlay, A.C. (1997) The Econometrics of Financial Markets (2nd edition) Princeton University Press. Princeton, New Jersey.

Chan, K.C. (1988) On the Contrarian Investment Strategy. Journal of Business. 61, 147-163.

Cheun, Y-W. and Ng, L. (1982) Stock Price Dynamics and Firm Size: An Empirical Investigation. Journal of Finance, 47, p. 1985-1997.

Cho, H. Y. & Engle, F.R. (1999) Time-varying Betas and Asymmetric Effect of News: Empirical Analysis of Blue Chip Stocks. National Bureau of Economic Research. Working Paper.

Chow, K.V. and Denning, K.L. (1994) On Variance and Lower Partial Moment Betas and the Equivalence of Systematic Risk Measures. Journal of Business Finance and Accounting 21(2), 231-241.

Christie, A.A (1982) The Stochastic Behaviour of Common Stock Variances: Value, Leverage and Interest Rate Effects. Journal of Financial Economics, 10, p. 407-432.

Dannenburg, D. and Jacobsen, B. (2003) Volatility Clustering in Monthly Stock Returns. Journal of Empirical Finance, 10, p. 479-503.

Dean, G.W. and Faff, W.R. (2004) Asymmetric Covariance, Volatility and the Effect of News. Journal of Financial Research, 27 (3).

De Bondt, W.F. and Thaler, R.H. (1989) Anomalies: A Mean-reverting Walk down Wall Street. Journal of Economic Perspectives, 3, p. 189-202.

Dittmar, R. (2002) Nonlinear Pricing Kernels, Kurtosis Preference and Evidence from the Cross-section of Equity Returns. Journal of Finance. 57, p. 369-403.

Domowitz, I., Glen, J. and Madhavan, A. (2001) Liquidity, Volatility and Equity Trading Costs Across Countries and Over Time. International Finance 4 (2), p. 221-255.

Duffee, G.R. (1995) Stock Returns and Volatility: A Firm Level Analysis. Journal of Financial Economics, 37, p. 399-420.

Engle, F.R. (1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica. Vol 50, 4.

Engle, F.R. (2002) Dynamic Conditional Correlation – A Simple Class of Multivariate GARCH Models. Journal of Business and Economic Statistics, 20 (3), p. 339-350.

Engle F.R and Merzrich, J.J. (1996) Correlation with GARCH. Manuscript, UCDS and Salomon Bros.

Engle F.R. and Ng, K.V. (1993) Measuring and Testing the Impact of News on Volatility. Journal of Finance, Vol. 48, No 5.

Estrada, J. (2007) Mean-semivariance Behaviour: Downside Risk and Capital Asset Pricing. International Review of Economics and Finance, Vol. 16 (2), p. 169-185.

Fabozzi, F. and Francis, J. (1977) Stability Tests for Alphas and Betas Over Bull and Bear Market Conditions. Journal of Finance 32, 1093-1099.

Fishburn, P.C. (1977) Mean-risk Analysis with Risk Associated with Below-Target Returns. American Economic Review, 67(2). 116-126.

Frazzini, A. and Pedersen, L.H. (2014) Betting against Beta. Journal of Financial Economics, vol. 111(1), 1-25.

French, K. R., Schwert, G., and Stambaugh, R. F. (1987) Expected Stock Returns and Volatility. Journal of Financial Economics, 19 (1), 3-29.

Friend, I. and Westerfield, R. (1980) Co-skewness and Capital Asset Pricing. Journal of Finance, 35, 897-913.

Fu, F. (2009) Idiosyncratic risk and the Cross-Section of Expected Stock Returns. Journal of Financial Economics, 91, 1, 24-37.

Galagedera, D., Henry, D. and Silvapulle, P. (2003) Empirical Evidence on the Conditional Relation between Higher-order Systematic Co-moments and Security Returns. Quarterly Journal of Business & Economics, 42, p. 121-137.

Gan, C., Nartea, G. and Wu, J. (2017) Predictive Ability of Low-Frequency Volatility Measures: Evidence from the Hong Kong Stock Markets. Finance Research Letters.

Ghalanos, A. (2020) Introduction to rugarch package. [Internet document]. [referred 15.4.2020]. Available: https://cran.r-project.org/web/packages/rugarch/vignettes/Introduction_to_the_rugarch_package.pdf

Glosten, L.R, Jagannathan, R. and Runkle, D.E. (1993) On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. Journal of Finance, 48, p. 1779-1801.

Gordon, M.J., Paradis, G.E., and Rorke, C.H. (1972) Experimental Evidence on Alternative Portfolio Decision Rules. American Economic Review, Vol. 62, issue 1, 107-18

Gul, F. (1991) A Theory of Disappointment Aversion. Econometrica, 59, 667-686.

Hamada, R.S. (1972) The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks. Journal of Finance. Vol. 27, No 2. p. 435-452.

Harlow, W.V. and Rao, K.S. (1989) Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence. Journal of Financial and Quantitative Analysis 23 (3), 285-311.

Harvey, C.R. (1989) Time-Varying Conditional Covariances in Tests of Asset Pricing Models. Journal of Financial Economics, 24 (2) p. 289-317.

Harvey, C.R. and Siddique, A. (1999) Autoregressive Conditional Skewness. Journal of Financial and Quantitative Analysis, 34, p. 465-477.

Harvey, C.R. and Siddique, A. (2000) Conditional Skewness in Asset Pricing Tests. Journal of Finance, 55, 1263-1295.

Hogan, W, and Warren, J. (1974) Toward the Development of an Equilibrium Capital Market Model Based on Semivariance. Journal of Financial and Quantitative Analysis 9 (1). 1-11.

Homaifar, G. and Graddy, D.B. (1990) Variance and Lower Partial Moment Betas as Alternative Risk Measures in Cost of Capital Estimation: A Defence of the CAPM Beta. Journal of Business Finance & Accounting, Vol. 17, No. 5 (Winter 1990). 677-688.

Hong, Y., Tu, J. & Zhou, G. (2007) Asymmetries in Stock Returns: Statistical Tests and Economical Evaluation. The Review of Financial Studies. Vol. 20, 5.

Huffman S.P. and Moll C.R. (2011) The impact of Asymmetry on Expected Stock Returns: An Investigation of Alternative Risk Measures. Algorithmic Finance, 1, p. 79-93.

Högholm, K., Knif, J., Koutmos, G. and Pynnönen, S. (2011) Distributional Asymmetry of Loadings on Market Co-Moments. Journal of International Financial Markets, Institutions & Money, 21, p. 851-866.

Isakov, D. (1999) Is Beta Still Alive? Conclusive Evidence from the Swiss Stock Market. European Journal of Finance, 5, p. 202-212.

Israelsen, C.L. (2005) A Refinement to the Sharpe Ratio and Information Ratio. Journal of Asset Management 5 (6), 423-427.

Jensen, Michael C. and Black, Fischer and Scholes, Myron S., (1972) The Capital Asset Pricing Model: Some Empirical Tests. Michael C. Jensen, Studies in the Theory of Capital Markets, Praeger Publishers Inc.

Kahneman, D. and Tversky, A. (1979) Prospect theory: An Analysis of Decision under Risk. Econometrica 47, 263-291.

Kaplanski, G. (2004) Traditional Beta, Downside Risk Beta and Market Risk Premiums. The Quarterly Review of Economics and Finance, Vol. 44 (5), p. 636-653.

Koutmos, G. and Knif, J. (2002) Time Variation and Asymmetry in Systematic Risk: Evidence from the Finnish Stock Exchange. Journal of Multinational Financial Management. 12, p. 261-271.

Kraus, A. and Litzenberger, R. (1976) Skewness Preference and the Valuation of Risk Assets. Journal of Finance, 31, 1085-1100.

Kyle, A.W. and Xiong, W. (2001) Contagion as a Wealth Effect of Financial Intermediaries. Journal of Finance, 56, 1401-1440.

Lestel, M. (2020) Econometric Tools for Performance and Risk Analysis. [Internet document]. [referred 21.4.2020]. Available: https://cran.r-project.org/web/packages/PerformanceAnalytics/PerformanceAnalytics.pdf

Malkiel, B & Xu, Y. (2002) Idiosyncratic Risk and Security Returns. Working Paper.

Mandelbrot, B. (1963) The Variation of Certain Speculative Prices. Journal of Business, Vol. 36 (4), p. 394-419.

Markowitz, H. (1952) Portfolio Selection. The Journal of Finance 7, 1, 77-91.

Markowitz, H. (1959) Portfolio Selection: Efficient Diversification of Investment. John Wiley & Sons, New York.

Martellini, L., & Ziemann, V. (2010) Improved Estimates of Higher-order Comoments and Implications for Portfolio Selection. Review of Financial Studies, 23 (4), p. 1467-1502.

Memmel, C. (2003) Performance Hypothesis Testing with the Sharpe Ratio. Finance Letters 1, 21-23.

Merton, C.R. (1980) On Estimating the Expected Return on Market: An Exploratory Investigation, Journal of Financial Economics, 8, p. 326-361.

Merton, C.R. (1987) A Simple Model of Capital Market Equilibrium with Incomplete Information. Journal of Finance, 42, 483-510.

Nantell, T.J. and Price, B. (1979) Analytical Comparison of Variance and Semivariance Capital Market Theories. Journal of Financial and Quantitative Analysis, Vol. 14, p. 221-242.

Nantell T.J., Price K. and Price, B. (1982) Mean-Lower Partial Moment Asset Pricing Model: Some Empirical Evidence. Journal of Financial and Quantitative Analysis 17 (5), 764-782.

Nelson, D.B. (1991) Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica. Vol. 59, 347-370.

Pettengill, G., Sundaram, S., Mathur, I. (1995) The Conditional Relation between Beta and Returns. Journal of Financial and Quantitative analysis 30, 101-116.

Pindyck, R. (1984) Risk, Inflation and the Stock Market. American Economic Review. Vol. 74, issue 3, 335-51.

Poon, S-H, and Taylor, S.J. (1992) Stock Returns and Volatility: An Empirical Study of the U.K. Stock Market. Journal of Banking and Finance, 16, p. 37-59.

Post, T., van Vliet, P. and Lansdorp, S. (2009) Sorting out Downside Beta. ERIM Report Series Research in Management. Available: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1360708

Poterba, T.M. and Summers, L.H. (1986) The persistence of Volatility and Stock Market Fluctuations, American Economic Review, 76, 1142-1151.

Price, K., Price, B. and Nantell, T. (1982) Variance and Lower Partial Moment Measures of Systematic Risk: Some Analytical and Empirical Results. Journal of Finance, Vol. 37 (June 1982). 843-855.

Pätäri, E.J. (2011) Does the Risk-Adjustment Method Matter at All in Hedge Fund Rankings? International Research Journal of Finance and Economics, 75, 69-99.

Pätäri, E.J and Vilska, M. (2014) Performance of Moving Average Trading Strategies Over Varying Stock Market Conditions: The Finnish Evidence. Applied Economics, 46 (24), p. 2851-2872.

Roy, A. D., (1952) Safety First and the Holding of Assets. Econometrica, 20, 421-449.

Rubinstein, M. (1973) The Fundamental Theory of Parameter-Preference Security Validation. Journal of Financial and Quantitative Analysis, 8, p. 61-69.

Satchell, S. E. (1996) Lower Partial-Moment Capital Asset Pricing Models: A Re-examination. In Managing Downside Risk in Financial Markets: Theory, Practise and Implementation. Sortino, F. A and Satchell, S. E. (Editors). Reed Educational and Professional Publishing Ltd 2001. Great Britain. 156-168.

Sharpe, W. (1966) Mutual Fund Performance. The Journal of Business, 39(1), 119-138.

Scott, R.C. and Horvath, P.A. (1980) On the Direction of Preference for Moments of Higher Order than the Variance. Journal of Finance. 35, p. 915-919.

Taylor, S.J. (2007) Asset Price Dynamics, Volatility and Prediction. Princeton University Press. US.

Tsai, H-J., Chen, M-C, and Yang, C-Y. (2014) A Time-varying Perspective on the CAPM and Downside Betas. International Review of Economics and Finance, 29, p. 440-454.

APPENDIX I: EGARCH (1,1) diagnostics

VOL1 pf1

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.115	0.7346
Lag[2*(p+q)+(p+q)-1][5]	1.319	0.9996
Lag[4*(p+q)+(p+q)-1][9]	3.784	0.7405

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.01084	0.9171
Lag[2*(p+q)+(p+q)-1][5]	2.00398	0.6175
Lag[4*(p+q)+(p+q)-1][9]	3.05224	0.7503

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	1.082	0.500	2.000	0.2983
ARCH Lag[5]	2.400	1.440	1.667	0.3894
ARCH Lag[7]	2.659	2.315	1.543	0.5802

Nyblom stability test

Joint Statistic: 1.7483

Individual Statistics:

mu	0.31337
ar1	0.11391
ma1	0.12129
omega	0.75680
alpha1	0.06511
beta1	0.71176
gamma1	0.22832
shape	0.59372

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.7337	0.4642	
Negative Sign Bias	0.1977	0.8436	
Positive Sign Bias	0.3576	0.7211	
Joint Effect	0.5991	0.8966	

VOL1pf2

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.2093	0.6473
Lag[2*(p+q)+(p+q)-1][5]	2.6108	0.7167
Lag[4*(p+q)+(p+q)-1][9]	6.1338	0.2353

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.008661	0.9259
Lag[2*(p+q)+(p+q)-1][5]	1.357334	0.7749
Lag[4*(p+q)+(p+q)-1][9]	2.795943	0.7921

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.02604	0.500	2.000	0.8718
ARCH Lag[5]	0.06791	1.440	1.667	0.9924
ARCH Lag[7]	1.31700	2.315	1.543	0.8572

Nyblom stability test

Joint Statistic: 1.4249

Individual Statistics:

mu	0.07737
ar1	0.13834
ma1	0.14664
omega	0.16586
alpha1	0.07252
beta1	0.14875
gamma1	0.08990
shape	0.24515

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.2421	0.8090	
Negative Sign Bias	0.1976	0.8436	
Positive Sign Bias	0.4144	0.6792	
Joint Effect	0.2355	0.9717	

VOL1
pf3

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.06303	0.8018
Lag[2*(p+q)+(p+q)-1][5]	0.84763	1.0000
Lag[4*(p+q)+(p+q)-1][9]	1.35172	0.9985

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.03595	0.8496
Lag[2*(p+q)+(p+q)-1][5]	0.31108	0.9826
Lag[4*(p+q)+(p+q)-1][9]	0.70533	0.9953

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.3378	0.500	2.000	0.5611
ARCH Lag[5]	0.5222	1.440	1.667	0.8771
ARCH Lag[7]	0.7371	2.315	1.543	0.9522

Nyblom stability test

Joint Statistic: 1.1767

Individual Statistics:

mu 0.08029

ar1 0.12287

ma1 0.11874

omega 0.20567

alpha1 0.06600

beta1 0.21485

gamma1 0.09363

shape 0.25791

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	1.03638	0.3017	
Negative Sign Bias	0.17723	0.8596	
Positive Sign Bias	0.03957	0.9685	
Joint Effect	1.81407	0.6119	

VOL2pf1

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.0306	0.8611
Lag[2*(p+q)+(p+q)-1][5]	0.9517	1.0000
Lag[4*(p+q)+(p+q)-1][9]	2.2034	0.9742

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.6439	0.4223
Lag[2*(p+q)+(p+q)-1][5]	1.2261	0.8067
Lag[4*(p+q)+(p+q)-1][9]	2.3602	0.8579

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.04672	0.500	2.000	0.8289
ARCH Lag[5]	1.13296	1.440	1.667	0.6939
ARCH Lag[7]	1.85701	2.315	1.543	0.7476

Nyblom stability test

Joint Statistic: 0.8219

Individual Statistics:

mu 0.07868

ar1 0.06492

ma1 0.10193

omega 0.27150

alpha1 0.11115

beta1 0.24629

gamma1 0.06303

shape 0.02579

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.3139	0.7541	
Negative Sign Bias	0.5837	0.5603	
Positive Sign Bias	0.1955	0.8452	
Joint Effect	0.4281	0.9344	

VOL2
pf2

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.3399	0.5599
Lag[2*(p+q)+(p+q)-1][5]	1.2285	0.9998
Lag[4*(p+q)+(p+q)-1][9]	2.6893	0.9320

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	1.016	0.3135
Lag[2*(p+q)+(p+q)-1][5]	1.492	0.7418
Lag[4*(p+q)+(p+q)-1][9]	3.100	0.7424

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.2502	0.500	2.000	0.6169
ARCH Lag[5]	0.6731	1.440	1.667	0.8316
ARCH Lag[7]	2.2348	2.315	1.543	0.6677

Nyblom stability test

Joint Statistic: 1.1042

Individual Statistics:

mu 0.06143

ar1 0.05401

ma1 0.05685

omega 0.09477

alpha1 0.17651

beta1 0.08510

gamma1 0.13788

shape 0.09007

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.45562	0.6493	
Negative Sign Bias	0.04912	0.9609	
Positive Sign Bias	0.85423	0.3943	
Joint Effect	2.62736	0.4527	

VOL2pf3

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.04394	0.8340
Lag[2*(p+q)+(p+q)-1][5]	0.92186	1.0000
Lag[4*(p+q)+(p+q)-1][9]	1.84217	0.9902

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	3.555	0.05937
Lag[2*(p+q)+(p+q)-1][5]	4.380	0.21033
Lag[4*(p+q)+(p+q)-1][9]	7.075	0.19295

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.5822	0.500	2.000	0.4455
ARCH Lag[5]	0.6885	1.440	1.667	0.8269
ARCH Lag[7]	2.9813	2.315	1.543	0.5171

Nyblom stability test

Joint Statistic: 1.2399

Individual Statistics:

mu 0.06048

ar1 0.05247

ma1 0.05455

omega 0.08868

alpha1 0.29378

beta1 0.08025

gamma1 0.17711

shape 0.32749

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.4973	0.6197	
Negative Sign Bias	1.3125	0.1913	
Positive Sign Bias	1.2976	0.1964	
Joint Effect	4.0305	0.2582	

VOL3 pf1

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.006231	0.9371
Lag[2*(p+q)+(p+q)-1][5]	1.064209	1.0000
Lag[4*(p+q)+(p+q)-1][9]	3.469844	0.8074

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.4268	0.5136
Lag[2*(p+q)+(p+q)-1][5]	2.3248	0.5439
Lag[4*(p+q)+(p+q)-1][9]	2.8909	0.7768

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.5584	0.500	2.000	0.4549
ARCH Lag[5]	1.0618	1.440	1.667	0.7147
ARCH Lag[7]	1.1988	2.315	1.543	0.8792

Nyblom stability test

Joint Statistic: 1.0081

Individual Statistics:

mu	0.12459
ar1	0.15052
ma1	0.14735
omega	0.31651
alpha1	0.04241
beta1	0.31374
gamma1	0.07346
shape	0.12298

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	1.5453	0.1243	
Negative Sign Bias	0.1952	0.8455	
Positive Sign Bias	0.8002	0.4248	
Joint Effect	3.9483	0.2671	

VOL3pf2

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.151	0.6975
Lag[2*(p+q)+(p+q)-1][5]	1.026	1.0000
Lag[4*(p+q)+(p+q)-1][9]	2.001	0.9845

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.1634	0.6861
Lag[2*(p+q)+(p+q)-1][5]	1.2189	0.8085
Lag[4*(p+q)+(p+q)-1][9]	2.1435	0.8872

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	1.260	0.500	2.000	0.2616
ARCH Lag[5]	1.660	1.440	1.667	0.5512
ARCH Lag[7]	2.241	2.315	1.543	0.6663

Nyblom stability test

Joint Statistic: 1.4093

Individual Statistics:

mu	0.18755
ar1	0.04063
ma1	0.04280
omega	0.14757
alpha1	0.07010
beta1	0.12369
gamma1	0.23557
shape	0.03111

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.5999	0.5494	
Negative Sign Bias	0.1636	0.8703	
Positive Sign Bias	0.3036	0.7618	
Joint Effect	1.1929	0.7547	

VOL3
pf3

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.3916	0.5314
Lag[2*(p+q)+(p+q)-1][5]	0.7009	1.0000
Lag[4*(p+q)+(p+q)-1][9]	3.1845	0.8607

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.4015	0.5263
Lag[2*(p+q)+(p+q)-1][5]	1.0518	0.8478
Lag[4*(p+q)+(p+q)-1][9]	1.7441	0.9333

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.09491	0.500	2.000	0.7580
ARCH Lag[5]	0.65436	1.440	1.667	0.8373
ARCH Lag[7]	0.82185	2.315	1.543	0.9407

Nyblom stability test

Joint Statistic: NaN

Individual Statistics:

mu	NaN
ar1	0.006117
ma1	0.005251
omega	0.005392
alpha1	0.005431
beta1	NaN
gamma1	0.005469
shape	0.005359

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.6085	0.5437	
Negative Sign Bias	0.7894	0.4311	
Positive Sign Bias	0.6059	0.5455	
Joint Effect	1.0222	0.7959	

VOL4pf1

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.002889	0.9571
Lag[2*(p+q)+(p+q)-1][5]	0.322429	1.0000
Lag[4*(p+q)+(p+q)-1][9]	1.867631	0.9894

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.01032	0.9191
Lag[2*(p+q)+(p+q)-1][5]	0.76845	0.9095
Lag[4*(p+q)+(p+q)-1][9]	2.75003	0.7994

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.1565	0.500	2.000	0.6924
ARCH Lag[5]	0.2358	1.440	1.667	0.9568
ARCH Lag[7]	1.3765	2.315	1.543	0.8458

Nyblom stability test

Joint Statistic: 0.9612

Individual Statistics:

mu	0.20615
ar1	0.08977
ma1	0.08870
omega	0.11185
alpha1	0.02380
beta1	0.11031
gamma1	0.16498
shape	0.09659

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.007621	0.9939	
Negative Sign Bias	0.277867	0.7815	
Positive Sign Bias	0.558856	0.5771	
Joint Effect	0.502063	0.9184	

VOL4
pf2

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	3.286	0.06986
Lag[2*(p+q)+(p+q)-1][5]	4.541	0.01461
Lag[4*(p+q)+(p+q)-1][9]	7.530	0.08488

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.01789	0.8936
Lag[2*(p+q)+(p+q)-1][5]	0.70425	0.9221
Lag[4*(p+q)+(p+q)-1][9]	1.35770	0.9664

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.5820	0.500	2.000	0.4455
ARCH Lag[5]	0.9682	1.440	1.667	0.7424
ARCH Lag[7]	1.1490	2.315	1.543	0.8882

Nyblom stability test

Joint Statistic: 2.0666

Individual Statistics:

mu 0.01383

ar1 0.01128

ma1 0.01132

omega 0.01085

alpha1 0.01091

beta1 0.01768

gamma1 0.01098

shape 0.01085

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.5788	0.5636	
Negative Sign Bias	0.3987	0.6906	
Positive Sign Bias	0.7064	0.4810	
Joint Effect	0.6580	0.8830	

VOL4pf3

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.1041	0.7470
Lag[2*(p+q)+(p+q)-1][5]	1.2020	0.9999
Lag[4*(p+q)+(p+q)-1][9]	3.3537	0.8301

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.523	0.4695
Lag[2*(p+q)+(p+q)-1][5]	2.288	0.5520
Lag[4*(p+q)+(p+q)-1][9]	4.215	0.5528

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	2.025	0.500	2.000	0.1548
ARCH Lag[5]	2.837	1.440	1.667	0.3143
ARCH Lag[7]	4.256	2.315	1.543	0.3111

Nyblom stability test

Joint Statistic: 1.3198

Individual Statistics:

mu 0.12993

ar1 0.11441

ma1 0.13238

omega 0.08122

alpha1 0.28111

beta1 0.07375

gamma1 0.11778

shape 0.29627

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.002031	0.9984	
Negative Sign Bias	0.437765	0.6622	
Positive Sign Bias	1.027308	0.3059	
Joint Effect	1.506369	0.6808	

VOL1

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.7654	0.3817
Lag[2*(p+q)+(p+q)-1][5]	1.7746	0.9867
Lag[4*(p+q)+(p+q)-1][9]	5.0782	0.4361

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	1.408	0.2354
Lag[2*(p+q)+(p+q)-1][5]	3.247	0.3638
Lag[4*(p+q)+(p+q)-1][9]	3.918	0.6027

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	1.815	0.500	2.000	0.1779
ARCH Lag[5]	2.301	1.440	1.667	0.4085
ARCH Lag[7]	2.371	2.315	1.543	0.6392

Nyblom stability test

Joint Statistic: 20704.72

Individual Statistics:

mu	0.05860
ar1	0.05562
ma1	0.05774
omega	0.05776
alpha1	0.05663
beta1	0.02197
gamma1	0.03457
shape	0.05711

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.69205	0.4899	
Negative Sign Bias	0.04171	0.9668	
Positive Sign Bias	1.21323	0.2269	
Joint Effect	1.48237	0.6863	

VOL2

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.3172	0.5733
Lag[2*(p+q)+(p+q)-1][5]	1.5063	0.9979
Lag[4*(p+q)+(p+q)-1][9]	3.0015	0.8904

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.4565	0.4992
Lag[2*(p+q)+(p+q)-1][5]	0.7758	0.9080
Lag[4*(p+q)+(p+q)-1][9]	2.4618	0.8433

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.1961	0.500	2.000	0.6579
ARCH Lag[5]	0.7144	1.440	1.667	0.8191
ARCH Lag[7]	1.8610	2.315	1.543	0.7467

Nyblom stability test

Joint Statistic: 1.1304

Individual Statistics:

mu	0.05482
ar1	0.02357
ma1	0.02719
omega	0.09026
alpha1	0.19043
beta1	0.07709
gamma1	0.16172
shape	0.06915

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	1.4500	0.14908	
Negative Sign Bias	0.4234	0.67261	
Positive Sign Bias	1.9330	0.05508	*
Joint Effect	3.9422	0.26778	

VOL3

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.0005934	0.9806
Lag[2*(p+q)+(p+q)-1][5]	0.8764933	1.0000
Lag[4*(p+q)+(p+q)-1][9]	2.7204011	0.9284

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.06902	0.7928
Lag[2*(p+q)+(p+q)-1][5]	0.51944	0.9546
Lag[4*(p+q)+(p+q)-1][9]	1.06724	0.9833

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.3490	0.500	2.000	0.5547
ARCH Lag[5]	0.4895	1.440	1.667	0.8867
ARCH Lag[7]	0.5594	2.315	1.543	0.9727

Nyblom stability test

Joint Statistic: 1.1248

Individual Statistics:

mu	0.03988
ar1	0.05992
ma1	0.05926
omega	0.11187
alpha1	0.08869
beta1	0.09127
gamma1	0.16799
shape	0.22585

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.8881	0.3759	
Negative Sign Bias	0.4206	0.6746	
Positive Sign Bias	0.3962	0.6925	
Joint Effect	0.8111	0.8468	

VOL4

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.1528	0.6959
Lag[2*(p+q)+(p+q)-1][5]	1.4196	0.9990
Lag[4*(p+q)+(p+q)-1][9]	3.7776	0.7420

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.03663	0.8482
Lag[2*(p+q)+(p+q)-1][5]	0.28667	0.9851
Lag[4*(p+q)+(p+q)-1][9]	1.34533	0.9672

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.04156	0.500	2.000	0.8385
ARCH Lag[5]	0.33149	1.440	1.667	0.9318
ARCH Lag[7]	0.95880	2.315	1.543	0.9202

Nyblom stability test

Joint Statistic: 0.7827

Individual Statistics:

mu	0.03295
ar1	0.15794
ma1	0.15552
omega	0.12040
alpha1	0.11727
beta1	0.11253
gamma1	0.08766
shape	0.08103

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.3674	0.7138	
Negative Sign Bias	0.3631	0.7170	
Positive Sign Bias	1.0326	0.3034	
Joint Effect	1.2064	0.7515	

B- pf1

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.01388	0.9062
Lag[2*(p+q)+(p+q)-1][5]	0.68044	1.0000
Lag[4*(p+q)+(p+q)-1][9]	2.81565	0.9166

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.1792	0.6721
Lag[2*(p+q)+(p+q)-1][5]	1.1419	0.8268
Lag[4*(p+q)+(p+q)-1][9]	1.8751	0.9194

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.6075	0.500	2.000	0.4357
ARCH Lag[5]	1.5014	1.440	1.667	0.5919
ARCH Lag[7]	1.8620	2.315	1.543	0.7465

Nyblom stability test

Joint Statistic: 0.9811

Individual Statistics:

mu 0.09032

ar1 0.10851

ma1 0.11384

omega 0.17891

alpha1 0.21316

beta1 0.16696

gamma1 0.03931

shape 0.23379

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.1747	0.8615	
Negative Sign Bias	0.6504	0.5164	
Positive Sign Bias	0.5677	0.5711	
Joint Effect	0.8290	0.8425	

B- pf2

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.7793	0.3774
Lag[2*(p+q)+(p+q)-1][5]	3.1380	0.3878
Lag[4*(p+q)+(p+q)-1][9]	5.0760	0.4365

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.04007	0.8414
Lag[2*(p+q)+(p+q)-1][5]	1.42950	0.7572
Lag[4*(p+q)+(p+q)-1][9]	2.98263	0.7618

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.8816	0.500	2.000	0.3478
ARCH Lag[5]	2.5405	1.440	1.667	0.3637
ARCH Lag[7]	2.8284	2.315	1.543	0.5465

Nyblom stability test

Joint Statistic: 6.1697

Individual Statistics:

mu 0.02414

ar1 0.02789

ma1 0.02807

omega 0.02857

alpha1 0.02846

beta1 0.01396

gamma1 0.02808

shape 0.02902

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	1.755	0.08129	*
Negative Sign Bias	1.162	0.24709	
Positive Sign Bias	1.022	0.30826	
Joint Effect	3.216	0.35957	

B- pf3

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.554	0.4567
Lag[2*(p+q)+(p+q)-1][5]	1.486	0.9982
Lag[4*(p+q)+(p+q)-1][9]	3.512	0.7990

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.2293	0.6321
Lag[2*(p+q)+(p+q)-1][5]	1.1579	0.8230
Lag[4*(p+q)+(p+q)-1][9]	2.1944	0.8806

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.9321	0.500	2.000	0.3343
ARCH Lag[5]	1.5519	1.440	1.667	0.5788
ARCH Lag[7]	2.0502	2.315	1.543	0.7067

Nyblom stability test

Joint Statistic: 2.6084

Individual Statistics:

mu	0.02393
ar1	0.07184
ma1	0.07536
omega	0.16318
alpha1	0.04600
beta1	0.12907
gamma1	0.22740
shape	1.50399

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.79275	0.4291	
Negative Sign Bias	0.71077	0.4783	
Positive Sign Bias	0.07365	0.9414	
Joint Effect	0.92044	0.8205	

B- pf4

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.6412	0.4233
Lag[2*(p+q)+(p+q)-1][5]	3.2946	0.3011
Lag[4*(p+q)+(p+q)-1][9]	5.9402	0.2664

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	1.557	0.2121
Lag[2*(p+q)+(p+q)-1][5]	2.014	0.6152
Lag[4*(p+q)+(p+q)-1][9]	4.395	0.5233

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.06539	0.500	2.000	0.7982
ARCH Lag[5]	0.75859	1.440	1.667	0.8056
ARCH Lag[7]	3.20221	2.315	1.543	0.4762

Nyblom stability test

Joint Statistic: 1.2199

Individual Statistics:

mu	0.06601
ar1	0.12260
ma1	0.12369
omega	0.09957
alpha1	0.19941
beta1	0.08656
gamma1	0.26555
shape	0.04916

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.00656	0.9948	
Negative Sign Bias	0.29206	0.7706	
Positive Sign Bias	1.49780	0.1362	
Joint Effect	3.48480	0.3227	

B- pf5

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	1.656e-07	0.9997
Lag[2*(p+q)+(p+q)-1][5]	8.011e-01	1.0000
Lag[4*(p+q)+(p+q)-1][9]	2.061e+00	0.9818

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.4290	0.5125
Lag[2*(p+q)+(p+q)-1][5]	0.9849	0.8631
Lag[4*(p+q)+(p+q)-1][9]	4.0126	0.5867

d.o.f=2

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.0359	0.500	2.000	0.8497
ARCH Lag[5]	0.5762	1.440	1.667	0.8609
ARCH Lag[7]	3.4704	2.315	1.543	0.4294

Nyblom stability test

Joint Statistic: 1.3426

Individual Statistics:

mu 0.05797

ar1 0.03255

ma1 0.03849

omega 0.09618

alpha1 0.30127

beta1 0.08559

gamma1 0.17780

shape 0.28351

Asymptotic Critical Values (10% 5% 1%)

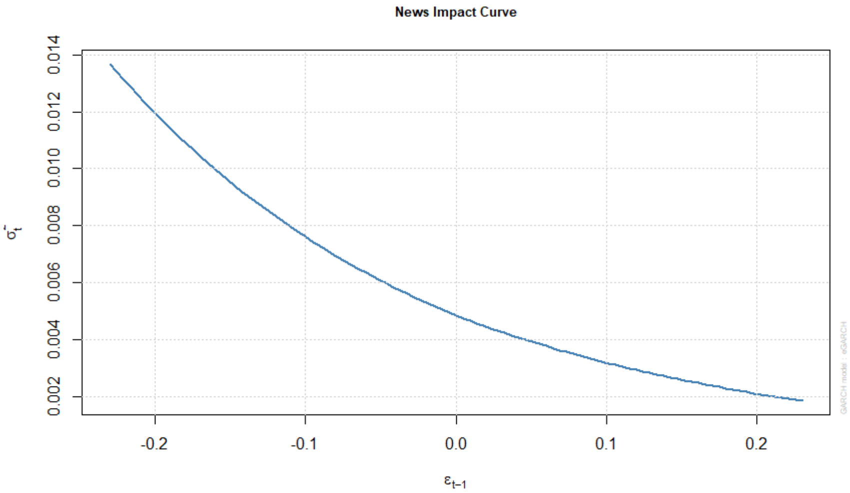
Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

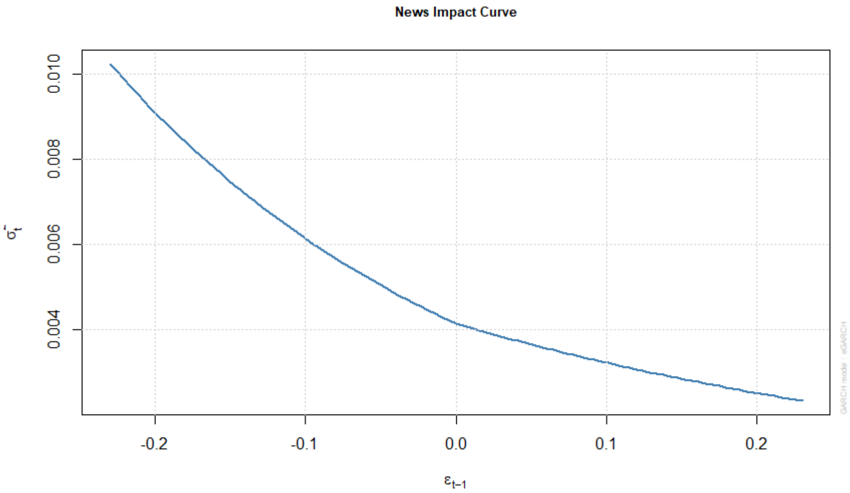
Sign Bias Test

	t-value	prob	sig
Sign Bias	1.5801	0.1161	
Negative Sign Bias	0.6554	0.5132	
Positive Sign Bias	1.5708	0.1183	
Joint Effect	3.3142	0.3457	

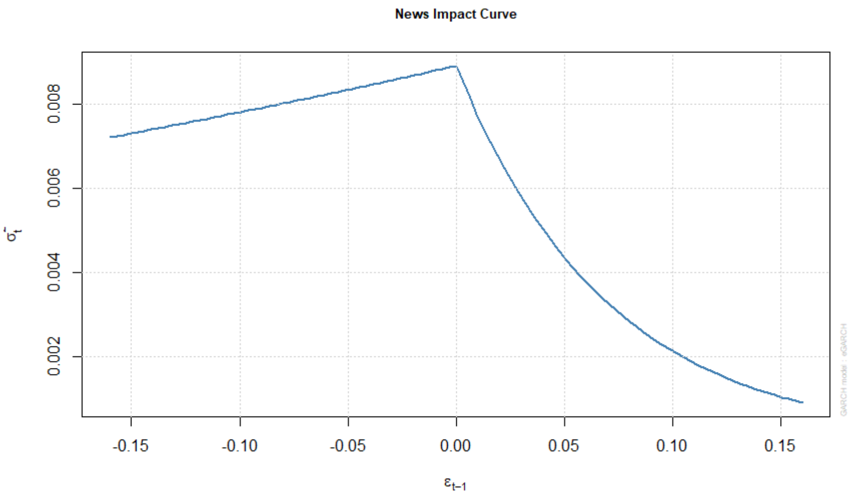
APPENDIX II: News impact curves of the portfolios in volatility classes



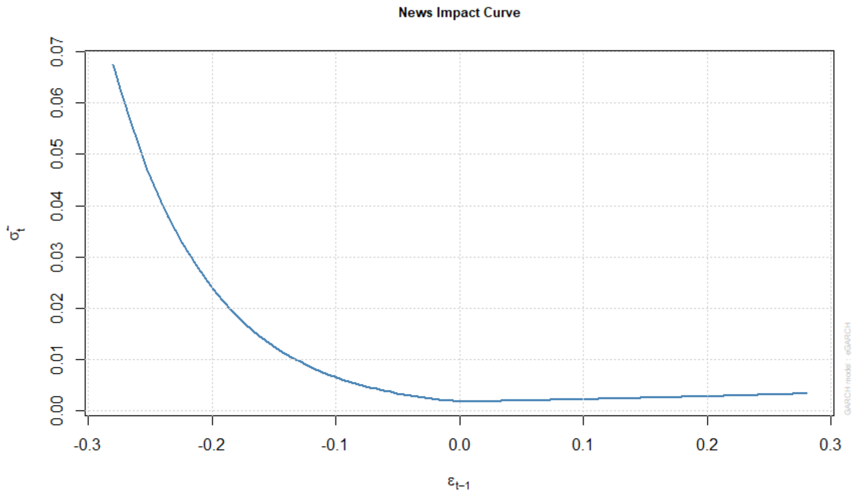
VOL1pf1



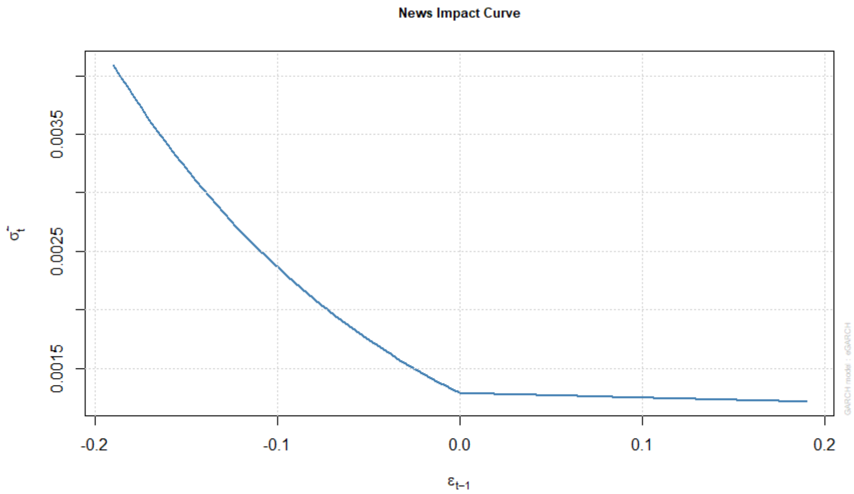
VOL1pf2



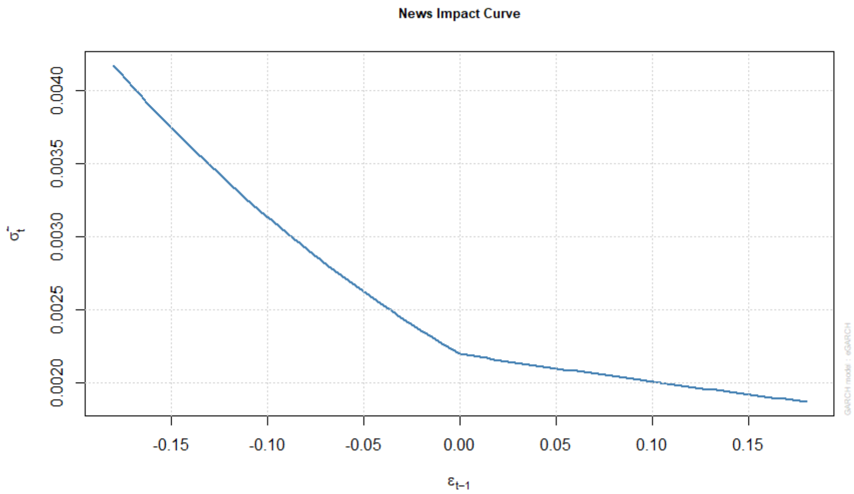
VOl1pf3



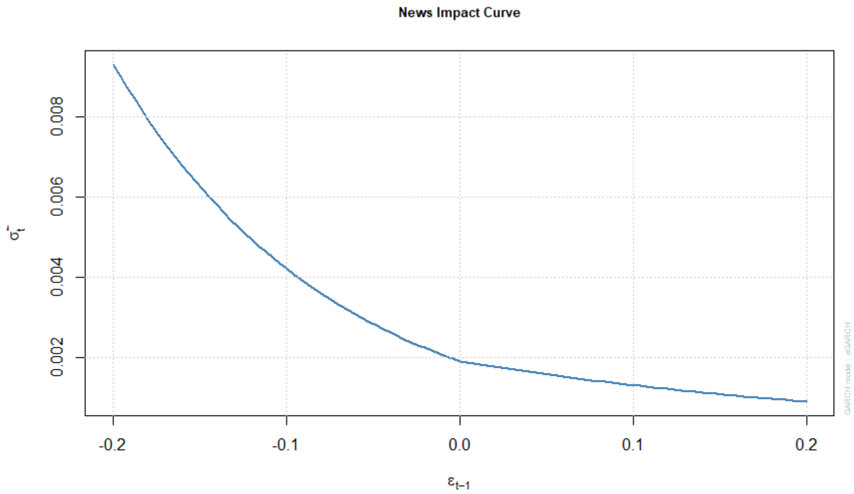
VOL2pf1



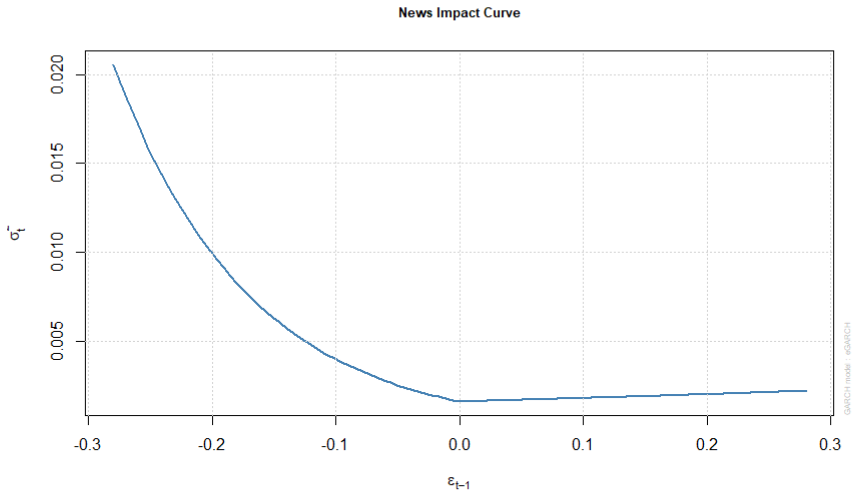
VOL2pf2



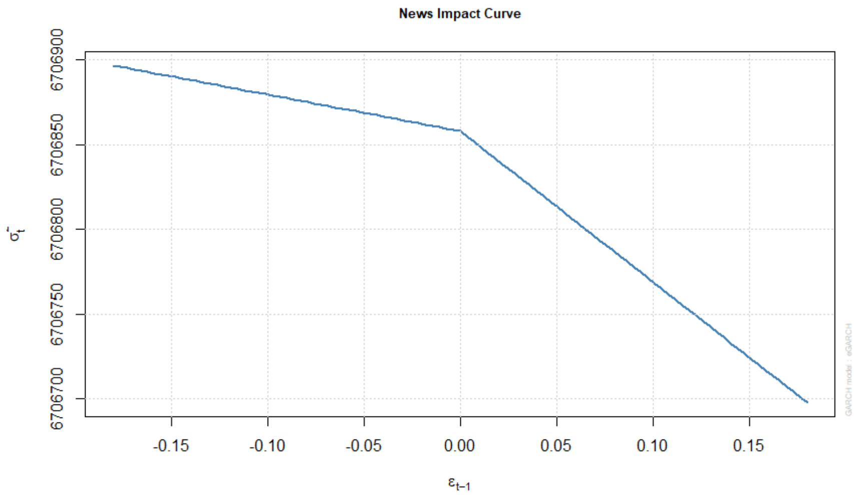
VOL2pf3



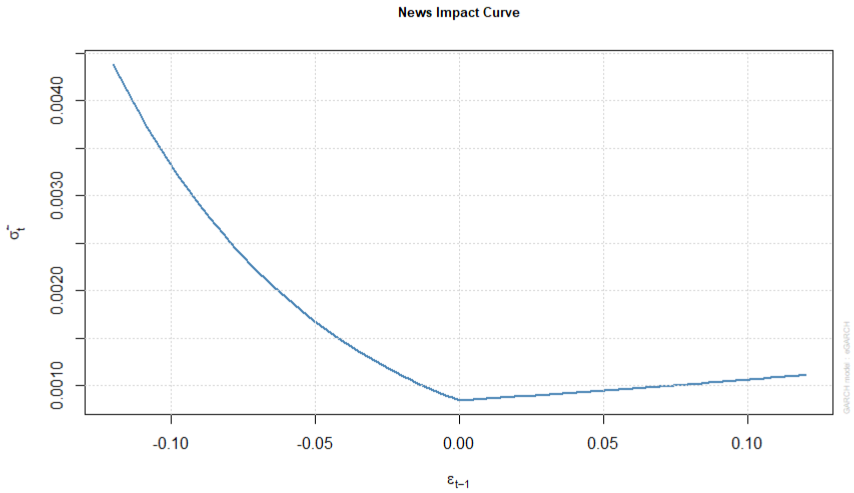
VOL3pf1



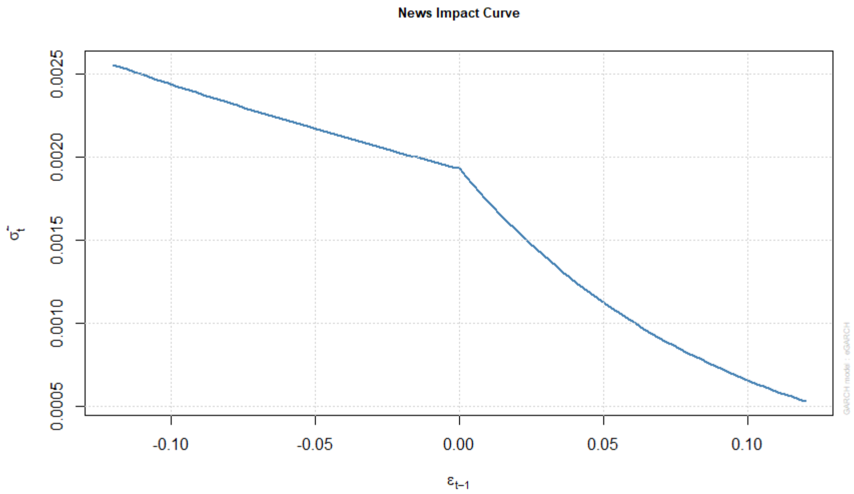
VOL3pf2



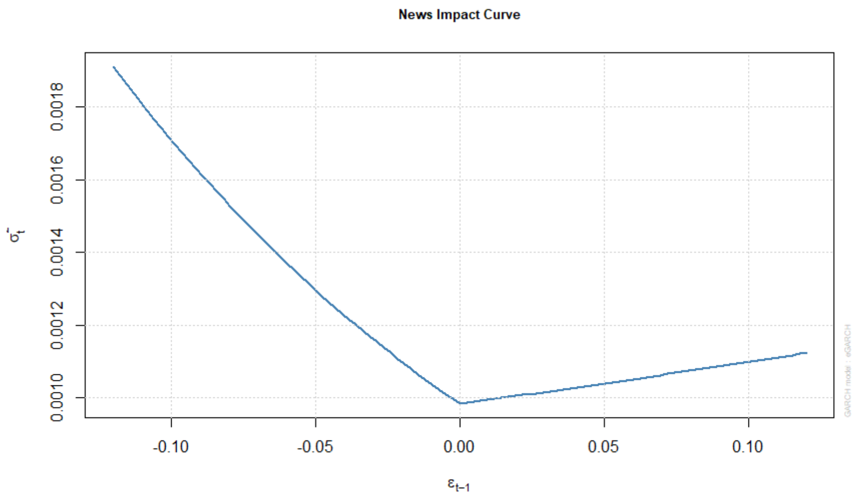
VOL3pf3



VOL4pf1



VOL4pf2



VOL4pf3