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Interlinkages between option-implied volatility and realized volatility: A case of S&P 500 and FTSE 100 index options

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TIIVISTELMÄ

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Tämän kandidaatintutkielman tavoitteena on selvittää tasaoptioiden implisiittisen volatiliteetin suhdetta toteutuneeseen volatiliteettiin. Tutkimuksessa käytetyt tasaoptiot ovat peräisin S&P 500 ja FTSE 100 indeksioptiomarkkinoilta ja ovat luonteeltaan osto-optioita. Tutkimus on luonteeltaan kvantitatiivinen ja se sisältää ei-päällekkäistä dataa ajanjaksolta 2016-2020. Tutkimuksessa käytetyt implisiittiset volatiliteetit ovat laskettu Black & Scholes optiohinnoittelumallista ja edustavat markkinoiden näkemystä kohde-etuuden tulevaisuuden volatiliteetista, kun taas toteutuneet volatiliteetit ovat laskettu kohde-etuuden markkinaliikkeistä.

Tulosten perusteella optio-implisiittinen volatiliteetti sisältää informaatiota tulevaisuuden volatiliteetista kaikissa regressiomalleissa. Kuitenkin perinteisen OLS-metodologian mukaan implisiittinen volatiliteetti on harhainen ja tehoton estimaattori tulevaisuuden volatiliteetille. Robustin regression, sekä kaksivaiheisen pienimmän neliösumman menetelmän tulokset osoittavat, että implisiittinen volatiliteetti muuttuu harhattomaksi estimaattoriksi pysyen silti tehottomana tarkasteluajanjakson aikana. Tutkimus myös osoittaa, että S&P500 indeksioptiomarkkinat ovat tehokkaammat verrattuna FTSE100 indeksioptiomarkkinoihin.

ABSTRACT

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This thesis re-evaluates the interlinkages between option-implied volatility derived from the call-options and realized volatility driven by underlying asset pricing movements in S&P 500 and FTSE 100 index options markets. The research is quantitative and consists of monthly nonoverlapping observations from the closest at the money call options during 2016 - 2020. The implied volatilities are calculated from the Black & Scholes option pricing model and represent the forecast of future volatility over the remaining life of the options contract, whereas the realized volatilities are calculated from the pricing movements of underlying asset.

The implied volatility subsumes information content of future volatility in all regression models based on the acquired results. The conventional OLS-methodology suggests that implied volatility is a biased and inefficient estimator for future volatility. However, the robust regression and 2SLS show that implied volatility turns to be an unbiased estimator for implied volatility, remaining still inefficient during the review period. The research also concludes, that the implied volatility is a better estimator for future volatility in terms of efficiency in S&P 500 index option markets compared to FTSE 100 index option markets.

Contents

List of Tables

List of Figures

1 Introduction

Through the ages, one of the biggest cornerstones of derivatives has been assessing the best value for the security of the derivatives. The volatility of the underlying assets is one of the critical components in the valuation of options securities. Implied volatility is the only variable not strictly measurable from the markets (Poon & Granger 2003). Consequently, market participants are said to be trading volatility, and often the one with the best prediction for the future volatility wins the battle.

Different approaches have been developed to forecast future returns and uncertainty. Most techniques use historical time series data as input variables and pass the information to the volatility forecast. These metrics are backward-looking, whereas the volatility implied by the options relies on real market awareness and is thus a forward-looking metric. Under the rational expectations principle, if the markets are information-efficient, implied volatility should be the best forecast for future volatility over an option contract's remaining lifetime.

Nonetheless, based on the previous studies, the consensus of the implied-realized volatility relation is not as unequivocal as it should be under the rational expectations principle. Many prior papers in this field of the study, such as the examination made by Canina & Figlewski (1993) reveal that implied volatility appears to be an inefficient and biased estimator for future volatility. In contrast, an examination made by Christensen & Prabhala (1998) with more robust methodology propound that implied volatility is an unbiased and efficient estimator for future realized volatility. A major part of the prior studies was published in the nineties and the start of twenties, hence new information about the relationship between implied and realized volatility has not been published as much as before. Different methodologies used in these and many other papers led to different results. Thus the consensus of the implied-realized volatility relation is still not clear.

1.1 Background of the Study

According to history, the foundations of derivatives stem from Mesopotamia's ancient culture, where farmers had an agreement that if the harvest season were dry, lenders would have to forego their debts to the farmers (Kummer & Pauletto 2012). Long after that, the Chicago Board Options Exchange started listing stock options in 1973, and the study for options pricing started. The usage of derivatives has substantially expanded during the 19th century, and it has been a popular subject in multiple academic papers. The primary purpose of derivatives, especially options, is to manage a portfolio's risk and ease speculation on specific instruments. (Hull 2003)

The pricing of options started in the same year, when the Chicago Board Options Exchange (CBOE) started listing call options, as Fischer Black and Myron Scholes invented the model, which derives the theoretical price of an option based on the information available from the markets (Black & Scholes 1973). This foundation was a breakthrough in mathematical finance and led to the Nobel Prize in 1997. Despite its theoretical assumptions, the Black & Scholes option pricing model is broadly used in finance, but later on, more sophisticated methods, which take the nature of the actual market conditions into account, are appropriating more and more recognition.

The index options turned into the most traded options contracts in their first trading year. (Evnine & Rudd 1985) In the 21 century, the popularity of index options grew exceedingly, and in June of 2019, approximately 1.5 million contracts of S&P 500 index options were traded on a daily basis (CBOE 2019). One thing raising the popularity of index options is its simplicity to hedge against multiple asset groups with just one instrument. The risk in index options is also smaller compared to single equity options due to the fact that the index fluctuations are generally more balanced than in single equity options. Index options also enjoy of the high liquidity of contracts and tighter bid-ask spreads than their equivalents. This thesis focuses purely on the index options since they are one of the most traded derivatives and most likely priced accurately.

1.2 Objectives of the Study

The main objective of this paper is to re-evaluate the relationship between implied and realized volatility. The common conception in academic finance is that the implied volatility presents the forecast of an underlying asset's future volatility (Canina & Figlewski 1993). This statement is testable with the regression analysis, which is introduced later in this paper. If the index options markets are information-efficient, implied volatility should be an unbiased and efficient estimator for forecasting an underlying asset's future volatility over the option's remaining lifetime (Day & Lewis 1992).

The research questions in this particular study are introduced as:

> *Is the implied volatility an efficient forecast of future volatility?*
>
> *Does the information content of implied volatility vary between different option markets?*

Call options of FTSE 100 traded in NYSE LIFFE, and S&P 500 index traded in Chicago Board Options Exchange (CBOE) are chosen to test the relationship later in this thesis. The analyzed period of the nonoverlapping samples starts from January 2016 to the end of September 2020.

1.3 Structure of the study

The study consists of a theoretical and empirical part. In the theoretical part, all the important theories related to this study are examined in turn. In the empirical part, the regression analysis is assembled to test the relationship between implied and realized volatility. Also, the conclusions are introduced based on the acquired results.

Chapter 2 will go through the theoretical framework of this thesis including theory of options, option pricing models and volatility estimating models. Subsequently, a brief literature review of the previous research is presented in Chapter 3. The structure of data and the sampling procedure is covered in Chapter 4. Chapters 5 and 6 focus on empirical research and conclusions.

2 Theoretical Framework

2.1 Options

An option is a derivative contract that grants the owner the right to purchase or sell the underlying instrument at a fixed price, known as the strike price. There are two types of options, put and call options. Put options grant its holder the right to sell the underlying at its expiry date with a given strike price, while the call option grants its holder the right to purchase the underlying at the given strike price. (Black & Scholes 1973, 637-638; Hull 2003, 10-14)

The exercising of the options relies on the exercise-style. Options can be either American or European styled. European style options can be exercised only on the expiry date, whereas American style options can be exercised at any time prior to the expiry date. Two participants per option contracts is always present. (Hull 2003, 10-14) One market participant is taking a long position, and the other one is taking a short position. The one that writes the contract, usually the bank, keeps the buyer's cash upfront but has potential liabilities later (Hull 2003, 10-14). What is noticeable in option contracts is that the writer's benefit or loss is always the reversed to the other market participant (Taleb 1997, 18).

The moneyness of an option is determined by the market price position linked to the exercise price. An option is out-of-the-money if the strike price is higher than the price of an underlying asset, at-the-money if the two variables are equal at that time, in-the-money if the strike price is lower than the price of an underlying asset. (Cox & Mark 1985, 9-22) The intrinsic value of options represents the prevailing value of options, to rephrase it, how much in-the-money the option is. For a call option, the intrinsic value is:

$$max[S - X, 0] \tag{1}$$

And for put option:

$$max[X - S, 0] \tag{2}$$

Where S represents the current stock price and X represents the Strike price of an option (Hull 2003, 107-110).

2.2 Trading strategies for Options

Traders typically try to find mispriced options. Mispricing in European styled options refers to the situation where the option price implies higher or lower volatility for the underlying asset (Barry & Taggart 2007). Since options are valued by using the return distributions of the underlying, the number of abnormal profits arises if the probabilities of price changes are incorrectly estimated (Hull 2005, 412-413). This chapter introduces some trading strategies for hedging and speculating with options.

The most straightforward strategy for speculation is called a naked position. This refers to the situation where one can buy a call or put option without actually holding the underlying asset. (Black 1975) Consequently, the position is not hedged against an adverse move in the underlying asset. The payoffs for writing naked positions are described below. Left picture represents the payoff for writing call options, whereas the right picture represents the payoff for writing put options

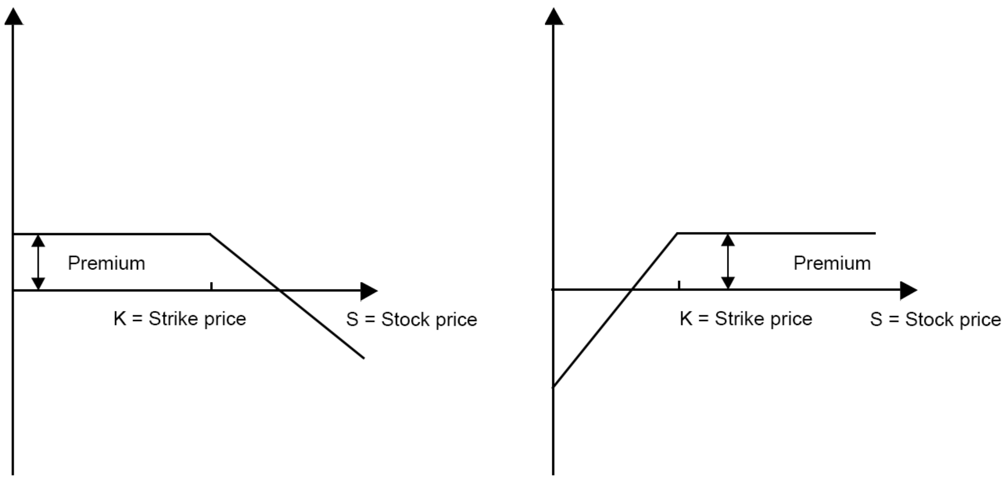

Figure 1: Call option writing payoffs

Options are also used to hedge against the price fluctuations in underlying assets. Another simple strategy, called covered position, refers to the situation where the position holder writes an option, and at the same time holds an equal and opposite position of underlying. Hull (2003, 181-185) states that the covered positions can lead to compellling losses and the results are not consistent. One general way of providing the hedging performance simultaneously for both sides of possible price movements is to use stangle and straddle strategies. A Strangle is an option strategy where the position holder purchases put- and call options with different exercise prices, but the same time to expiration date and underlying. The straddle is a similar strategy as the strangle, but the exercise prices are at the same level, whereas a strangle position holder uses different strike prices in put and call options. (Hull 2003, 14-15)

2.2.1 Delta hedging

The derivative of the option's value respect to the underlying price is noted as the Greek letter delta;

$$\Delta = \frac{\partial V}{\partial S}. \tag{3}$$

Delta is one of the risk measures of options. (Hull 2003, 186) Delta refers to the change rate between option and underlying asset. If the delta of a call option is 0.5, the price of an option changes around 50% of the amount that the underlying asset moves. (Hull 2003, 186-187) Delta hedging is a strategy where the trader constructs a delta neutral portfolio by offsetting negative and positive deltas close to the level of zero. The minimal variance (MV) delta hedge considers the effects of both the change in the underlying stock price and the expected change in volatility based on the change in the underlying asset's price. (Hull & White 2017) The portfolio remains hedged only for a small amount of time, and therefore it needs adjustments, which is known as re-balancing (Hull 2003, 188).

2.3 Black & Scholes Option Pricing model

When the Chicago Board Options Exchange started listing call options in 1973, later in that same year, Fischer Black and Myron Scholes made an immense breakthrough in options pricing as they developed the most famous and the most used option pricing model called Black & Scholes model. They managed to solve a differential equation which can be used to approximate a theoretical value of an European styled option based on the information available on the markets:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf, \tag{4}$$

where f stands for price of the option as a function of stock price S and time t. r is the risk-free rate and σ is a volatility of the stock (Black & Scholes 1973).

Later on, the Black & Scholes formula was refined by Robert Merton, to take the dividends into account (Merton 1973). Nowadays, the Black & Scholes model is widely used in the valuation of derivatives securities.

The Black & Scholes formula can be expressed as follows:

$$Call = S_0\Phi(d_1) - K\mathrm{e}^{-rT}\Phi(d_2) \tag{5}$$

$$Put = K\mathrm{e}^{-rT}\Phi(-d_2) - S_0\Phi(-d_1) \tag{6}$$

where:

$$d_1 = \frac{\log(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{7}$$

$$d_2 = \frac{\log(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma.\sqrt{T}. \tag{8}$$

S stands for the price of the underlying asset, K is the strike price, r is the risk-free rate σ is the expected volatility, T time-to-maturity, $d1$ and $d2$ are parameters to the phi ϕ in equations 5 and 6. ϕ and $N(x)$ stands for a cumulative probability distribution function for a stardardized normal variable, which can be expressed as:

$$N(X) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{x} exp(-\frac{z^2}{2})dz \qquad (9)$$

According to Hull (2003, 92-93), the assumptions of Black & Scholes option pricing model are listed as:

1) The stock price follows Geometric Brownian motion. Volatility and the expected rate of the underlying asset are constant over the lifetime of an option.
2) Short selling is allowed

3) No transaction costs or taxes

4) No dividends or other distributions are paid during the lifetime of the security

5) No arbitrage opportunities

6) Trading of security is continuous

7) The risk-free rate r is constant for all maturities

Some of the assumptions of Black & Scholes option pricing model do not reflect the real world qualities of stocks and options. Therefore, the Black & Scholes model has received plenty of criticism for its strict assumptions. Jackwerth & Rubinstein (1996) found in their study, about the distribution changes in the 1987 crash, pretty substantial evidence about the changes in the probability distributions between the test periods. This gives a basis for the fact, that the fluctuations of the underlying asset are infrequently log-normally distributed and do not adhere the normal distribution assumption of the Black & Scholes model. The model also presumes no transaction costs during the trading, which is rarely correct from a private investor's point of view.

2.4 Volatility

According to Poon & Granger (2003), the volatility is signified as a spread of all likely outcomes of an uncertain variable in a specified time horizon. However, in terms of statistics, the volatility-definition can be expressed as a standard deviation of a certain sample. Volatility has also been known to quantify the risk of an asset, since the negative return increments the financial leverage, constituting the object riskier and more volatile (Schwartz et al. 2010). However the volatility is not strictly the same as risk, since the risk is involved to the unwanted outcome in a defined time-frame, whereas volatility does not explicitly tell about the direction of the fluctuations and hence can be an outcome of a positive event. (Poon 2005, 1-2)

Volatility and stock prices tend to have an asymmetric relationship, which means that the volatility is usually higher in a declining market than in a mounting market. (Bekaert & Wu 2000) Multiple studies about the asymmetric relation between stock returns and volatilities have been published, and for example Andersen et al. (2001) made a study about the distributions of stock realized volatility and sidelined the topic. They tested the robustness of the previous studies, and in conclusion found out, that the asymmetric relation between stock returns and volatilities holds up its place.

Several different methods exist to estimate future volatility. Many of the volatility estimating models are created based on the realized volatility driven by the market fluctuations. The most common volatility estimating models, that are based on the realized volatility are listed as: Equally weighted average (EWMA), autoregressive conditionally heteroscedastic model (ARCH), stochastic volatility models (SV) and generalized autoregressive conditional heteroscedasticity model (GARCH). (Poon 2005, 32-33) Due to the limitations of the thesis, only two methods used in the empirical chapter are introduced over the next chapters.

2.4.1 Realized volatility

Realized volatility is expressed as a standard deviation over a fixed duration of asset fluctuations. Although more sophisticated time series models have been made to estimate volatility, the simple standard deviation offers a quick and reliable alternative to analyze the past volatility. According to Hull (2003, 121), the realized volatility can be expressed as:

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})}, \qquad (10)$$

where i represents the time interval, u_i is the continuously compunded returns and $\bar{u}$ is the mean of continously compounded returns.

Realized volatilities calculated in the empirical chapter represents the ex-post volatility. The ex-post return volatility is calculated over the remaining life of an option (Christensen & Prabhala 1998). The ex-post volatility is annualized and hence represents the measured annual value of the realized volatility from the chosen duration before the option's expiry.

2.4.2 Implied volatility

The implied volatility expresses an option's ex-ante outlook for the expected realized volatility over the remaining life of a contract, thus representing the forecast for the fluctuations in the underlying assets (Canina & Figlewski 1993). Implied volatilities in the empirical section are given as a annualized value. The inverted form of the Black & Scholes option pricing model does not provide any closed-form solution, so therefore numerical methods, such as Newton - Rhapsod or Bisection method is generally used to iterate the implied volatility over the Black & Scholes equation by using information available from the markets as input variables. (Brooks 2002, 420 - 421). According to Dumas et al. (1998), the mechanism of iterating the implied volatility can be illustrated as a mathematical formula

$$C(\sigma) - C_x, \tag{11}$$

where $C()$ is the option pricing equation, σ is the volatility parameter and C_x represents the theoretical value of option. The purposis of this equation is use an iterative algorithm to find the level of σ, where the substraction of this equation equals to zero.

2.4.3 Term structure and Volatility smile

One of the assumptions of Black & Scholes is that the fluctuations of underlying asset are log-normally distributed and volatility remains constant. However substantial evidence has been provided, that the implied volatility dynamics of the options, with the same strike and different time to maturity, changes over the time (Heynen et al. 1994). The presence of this activity in the options market is often referred as a term structure of implied volatility (Mixon 2007). One of its forms is a situation called volatility smile, where the plotted chart of options with same time-to-maturity and different strike prices is slighly skewed.

The existing literature provides different comphrehensions for the existence of volatility smile. For example Poon (2005) declares the two main theories related to the existence of volatility smile: Distributional assumption and Stochastic volatility. The fluctuations of stock returns are often "fat-tailed" meaning that the probability distribution occurs to have a large amount of skewness or kurtosis, consequently causing the higher price for the option and hence the implied volatility is higher (Poon 2005, 76-77). For example Aparicio & Estrada (2001) rejected the hypothesis of normally distributions in the European securities markets.

Another proven cause for the presence of volatility smiles is the situation, when underlying stock's volatility is stochastic. Stochastic volatility means that the underlying asset is instantaneously uncorrelated with the volatility (Hull & White 1987). Other theories apply to the market microstructure and measurement errors, but are not introduced due to the boundaries of this thesis. (Poon 2005, 76).

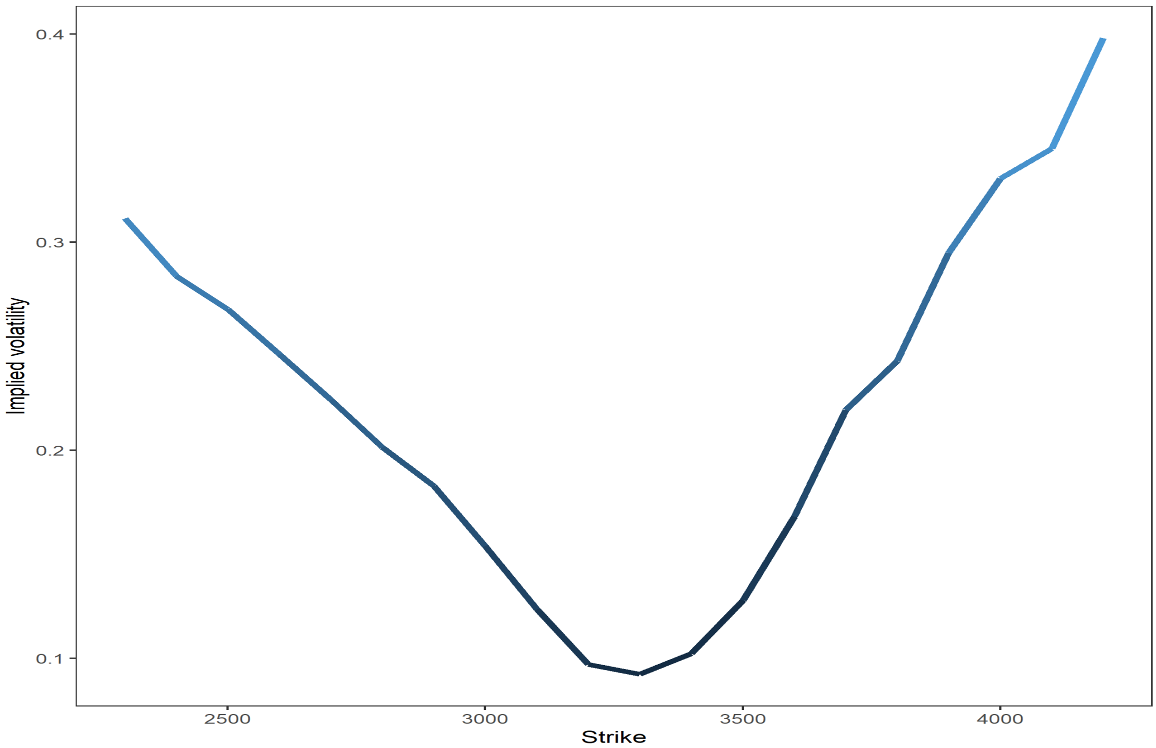

Figure 2: S&P 500 index option volatility smile

2.5 Itôs lemma

Kiyoshi Itô extended the theory of stochastic differential equations in his journal in 1951 (Itô 1951). Itô's proofs are extensively used in mathematical finance, and for instance, the Black & Scholes option pricing model stems from the Itô's lemma, which is one of the key components in Itô calculus (Mikosch 1998, 1-2). Itô's lemmas detailed proof is not relevant to the boundaries of the thesis, but however the important elements of Itô's lemma are shown as a mathematical formula next in this chapter. [1]

The price of option can be written in a function of underlying asset's price and time. In the other words, the price of an any derivative security can be written as a function of stochastic variables, time and the price of the underlying asset. (Hull 2003, 81-82)

According to Hull (2003, 86-87) Itô's lemma can be observed as follows.
Lets assume that the x follows the Itô process

$$dx = a(x,t)dt + b(x,t)dz, \tag{12}$$

where a and b are the functions of variable x and time t, dz represents the Wiener process. The parameter a is the drift rate of x. and the parameter a has a variance parameter of b^2. Itô's lemma proofs that G is a function of variable x and time t

$$dG = \left(\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \right) dt + \frac{\partial G}{\partial x} b\, dz, \tag{13}$$

where the drift rate of G is expressed as

[1] See Black & Scholes (1973) and Itô (1951) for more detailed proofs.

$$\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2, \tag{14}$$

and the variance

$$\left(\frac{\partial G}{\partial x}\right)^2 b^2. \tag{15}$$

3 Literature review

This chapter introduces the previous research and test results about the information content of implied volatility. In the recent decades, numerous studies about the informational content of implied volatility have been published. The results of many prior studies are completely different than others, and the differences are mostly caused by different methodologies used to test the relationship between the variables. However most of the previous studies show, that the implied volatility is a biased and inefficient estimator for future volatility, but it contains information with some specifications. Some relevant studies that are not introduced in this chapter are listed in the Appendix.

Christensen & Prabhala (1998) studied the information content of implied volatility with S&P 100 index options, during the timeframe of November 1983 to May 1995. The dataset in their study consisted of 139 monthly nonoverlapping observations, with 18 days time of maturity. Risk-free rate of 1-month LIBOR was used in calculations of implied volatility over the Black & Scholes option pricing model. They found out that the implied volatility outperforms historical volatility in predicting future volatility and it subsumes information of past realized volatility. By using the standard OLS-methodology, Christensen & Prabhala (1998) reported biased and inefficient relations between implied and realized volatility in OEX index options. By taking into account the nature of the EIV-problem, they found out that implied volatility turned out to be unbiased and efficient in estimating future volatility.

The empirical results reported by Christensen & Prabhala (1998) should be considered in the light of some limitations. The calculations of implied volatilities involve some degree of measurement error, because the S&P 100 index includes dividends and American-style in the S&P 100 options makes the early excercise possible. Other limitations comes from the pricing errors of options and the measurement errors in bid-ask spreads. Despite the measurement errors, this study reveals to be the most consistent in this field of study, because they took into account the telescoping overlapping problem, maturity mismatching problem and the EIV-problem in implied volatility calculations, which are the most common obstacles in prior studies.

The most perplexing results in this field of study were reported by Canina & Figlewski (1993) in their paper "The information content of implied volatility". Canina & Figlewski (1993) used also S&P 100 index options, but with different specifications as Christensen & Prabhala (1998). Despite the nature of the overlapping problem in options data, they used daily overlapping observations, and the data was divided into different maturities of option contracts. As a result, they reported no statistically significant correlation between implied and realized volatility. Nonetheless, studies using more sophisticated methodology, report the implied volatility to be an unbiased estimator the future volatility with some specifications.

The comparison between implied and realized volatility is also made for different asset classes of option underlyings. Jorion (1995) examined the relationship between implied- and realized volatility in foreign exchange market with currency futures. The data in Jorion's (1995) study covers the sample from January 1985 to February 1992, which is approximately seven years of daily observations. Jorion (1995) reports, that the implied volatility is efficient, but biased in estimating future volatility in foreign exchange markets. According to the study, one of two factors is causing the results; either the markets are inefficient or there is a misspecification in methodology.

4 Data

The quantitative data used in this paper is retrieved from the Refinitiv Eikon. S&P 500 index options were chosen, because they are the most traded and liquid contracts of index options. FTSE 100 index options, which are chosen for the second dataset, are also actively traded and liquid in terms of index options. They incorporate as a equilevant contracts to S&P 500 in European markets. Only few papers have re-evaluated this study in FTSE 100 index options market, hence this study fills the research gaps in European markets. Consequently, two major indices and their options in two different continents are chosen for the empirical tests.

The first dataset consists of call options of S&P 500 index from the timeframe of 1.1.2016 - 20.9.2020. The second dataset consists of call options of FTSE 100 index in the same time frame. Both datasets covers also the market plunging and V-shaped recovery in 2020. 1-month LIBOR rate, the inter-bank offered rate, is used in the calculations of implied volatilities, which is the same that Christensen & Prabhala (1998) used.

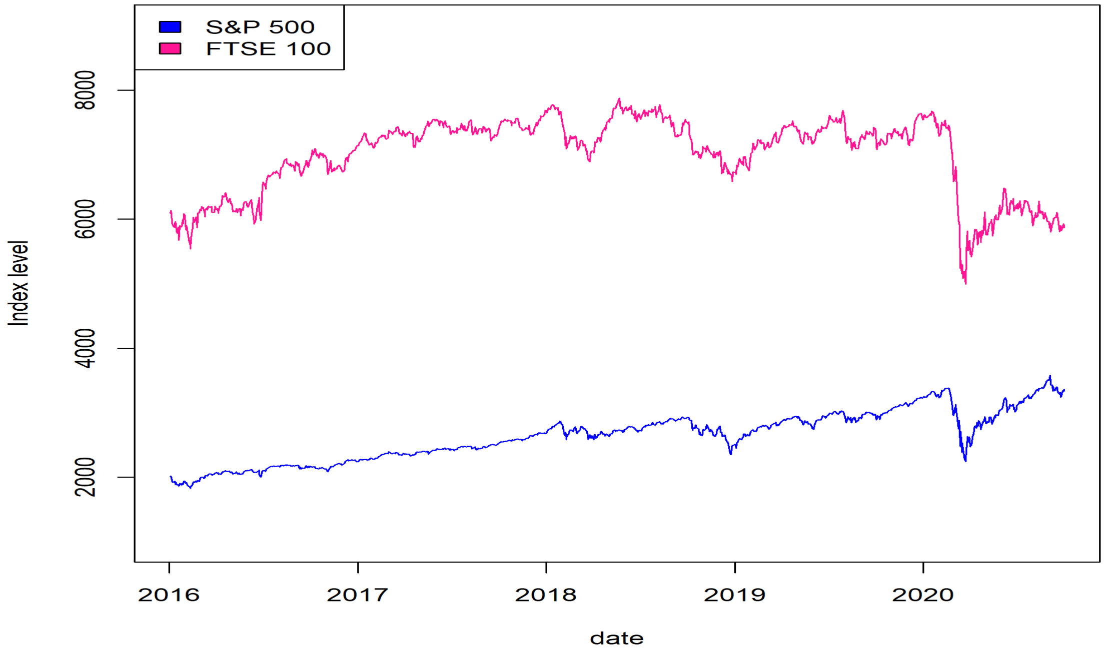

Figure 3: S&P500 and FTSE 100 indices during the sampling period

Every selected index call option is European styled and cash settled, and expires in the third Friday of the expiration month. The similarities in options contracts was a one factor for choosing the FTSE 100 and S&P 500 index options to this thesis. If the third Friday is not a trading day, the preceding day will be used. FTSE 100 and S&P 500 in the dataset are between 100 index points. Originally the strike prices of FTSE 100 and S&P 500 index options are quoted on a more frequent interval, but to reduce the amount of data only options between 100 index points are chosen. This scale allows the usage of the sampling procedure introduced over the next section.

4.1 Sampling procedure

The sampling procedure of this thesis follows the Christensen & Prabhala (1998) study. On each month, there are one observation of ex-ante implied volatility and ex-post realized volatility. The first observation is taken in January 2016 at Wednesday, immediatelly following the expiration. At that day, we take the observation of implied volatility for the closest at-the-money call option in the data set. This option expires on the third Friday of the following month $x+1$. This procedure is followed during the sampling preiod. The realized- and implied volatilites are expressed as an annualized terms.[2] The option contracts are rarely at-the-money, hence only options that have the moneyness rate between 0.95-1.05 are chosen to the analysis. At-the-money options with little time to expiration typically generates the most precise implied volatility estimates, and shrinks the possible detriments of the Black & Scholes model (Jorion 1995).

Following this sampling procedure, the maturities of all call options are virtually identical, three-to-four half weeks. This sampling procedure also enables to build nonoverlapping volatility series, because there will be only one observation of implied- and realized volatilities per sampling period (Shu & Zhang 2003). Nonoverlapping observations generates more reliable regression estimates than overlapping samples (Christensen & Prabhala 1998).

4.2 Descriptive statistics

Tables 1 and 2 show the descriptive statistics for the volatility series. The number of monthly nonoverlapping samples are 56 for S&P 500 and FTSE 100 index options respectively in a sampling period starting from January 2016 and ending in September

[2] All the calculations and plots are made with R-programming language, More information about R can be found directly from the CRAN R-website https://cran.r-project.org/

2020. Both of the indices display high level of kurtosis and skewness, which will lead to the situation where the volatility levels are not log-normally distributed. This can also be observed from the normal distribution plots shown in the appendix. Christensen & Prabhala (1998) also determined the same issue, and because of it they used log-volatility series to pacify the skewness and kurtosis levels in the regression analysis. The same thing is done in this paper, and we can see that the log-transformations shows smaller values of kurtosis and skewness. The volatility levels in 2020 are significantly higher than before, hence it may be argued that the condition of COVID-19 may give a rise to the charasteristics of these volatility series.

Table 1: FTSE 100 descriptive statistics

	RV	**IV**	**LOGRV**	**LOGIV**
Min	0.054	0.059	-2.91	-2.82
Max	0.60	0.43	-0.50	-0.82
Std	0.046	0.047	0.48	0.425
Mean	0.135	0.138	-2.12	-2.08
Var*100	0.800	0.550	23.2	18.06
Kurtosis	12.60	5.26	1.23	0.96
Skewness	3.05	2.20	0.93	0.97

Table 2: S&P 500 descriptive statistics

	RV	**IV**	**LOGRV**	**LOGIV**
Min	0.034	0.058	-3.364	-2.84
Max	0.961	0.612	-0.03	-0.489
Std	0.138	0.093	0.62	0.47
Mean	0.142	0.150	-2.18	-2.01
Var*100	1.906	0.880	39.2	22.1
Kurtosis	22.64	10.43	1.33	1.06
Skewness	4.23	2.80	0.85	0.925

The volatility plot below shows the volatility levels during the sampling period. The y-axis represents the volatility levels and x-axis represents the monthly nonoverlapping values. An important notice is that the volatility spikes are very similar in FTSE 100 and S&P 500 indices. The COVID-19 drop is displayed at x-axis level of 51, which represents the February 2020.

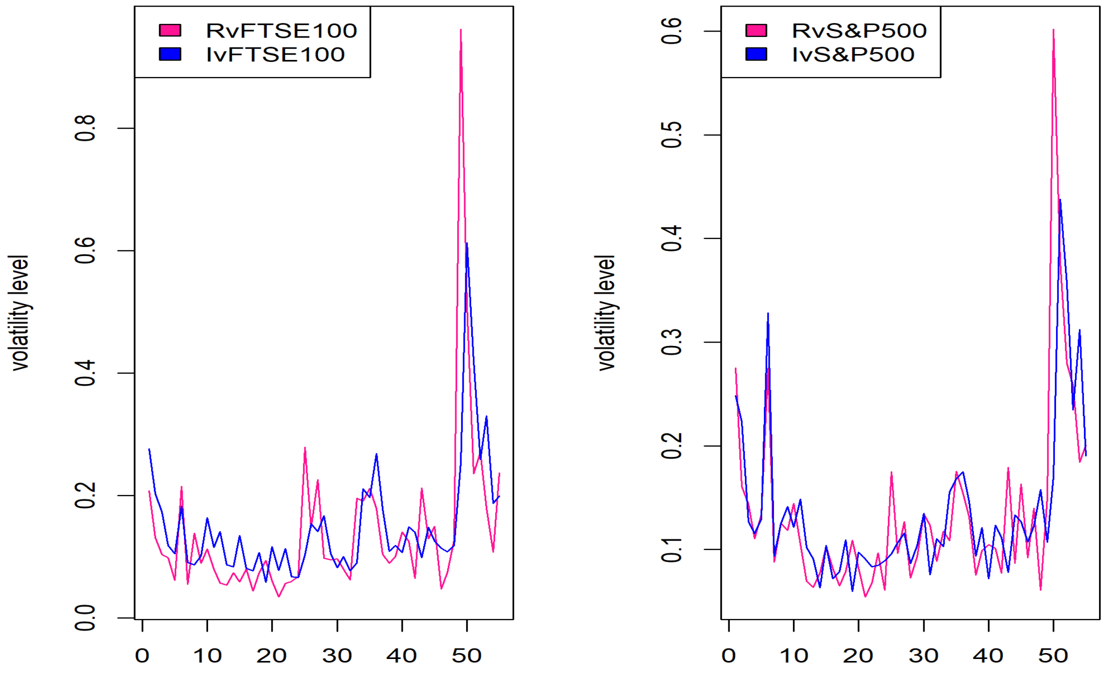

Figure 4: Volatility dynamics during the sampling period

4.3 Measurement errors

Some degree of measurement errors may occur in the estimations of volatilities, which should be taken into account before the results are shown. Both FTSE 100 and S&P 500 indices includes dividends, which are not taken into account in the implied volatility calculations. Consequently, dividends diminish the call-option values, and the implied volatilities obtained from the Black & Scholes model are underestimated (Hull 2005). The error by not including the dividends should be minuscule for the regression-estimates because the errors remains constant over the sampling period, and both FTSE 100 and S&P 500 dividends are relatively small. Nonetheless, measurement errors may occur in the estimations of implied volatilities, although the intercept terms in OLS-regressions should only be biased upwards if the implied volatility is used as an independent variable, and the slope coefficients are not affected by this problem (Christensen & Prabhala 1998). However as Christensen & Prabhala (1998) have shown, the errors-in-variables problem causes the effect where the slope coefficient of implied volatility is positive, but the intercept term is negative. However the EIV-problem can be revised with more sophisticated estimating methods.

Another possible measurement error that violates the assumptions of Black & Scholes is the stochastic volatility, which will lead to the jumps in underlying indices. The geometrian brownian motion assumption should be implied as a log-normal distribution for the underlying index fluctuations, which is not correct with this data. Table 2 and 3 shows a high degree of kurtosis and skewness and indicate that both indices' distributions are not log-normally distributed. This behaviour violates the B&S model's assumptions and can cause measurement errors in implied volatility calculations. Substantial evidence about the mispecifications of the B&S formula are shown by Cox & Ross (1976) and Heston (1993), hence the misspecifications and the possible EIV-problem are considered in the test results.

5 Empirical part

In this section, the obtained results are displayed alongside the used methodology. The focus remains on analyzing the coefficients of regressions and to provide answers for the research questions and hypothesis testing. The previous chapters discussed about the data and possible measurement errors, which are taken into account in order to obtain more robust answers for the research questions.

5.1 Methdodology

In the existing literature, the conventional regression form to evaluate the information content between implied and realized volatility is typically expressed in the following form

$$h_t = \alpha_0 + \alpha_i \, i_t + e_t, \tag{16}$$

where h_t indicates realized volatility in a period of t, and i_t indicates the implied volatility at the beginning of period t.

The option markets are efficient, if the implied volatility is found to be both efficient and unbiased forecast of future volatility (Christensen & Prabhala 1998). Both hypothesis are testable with the regression analysis. If the implied volatility contains some information of the future volatility, a_i should be different than zero. If the implied volatility is unbiased and efficient estimator, the joint hypothesis, which is $a_0 = 0$, and $a_i = 1$ can not be rejected. If the implied volatility is also found to be an efficient estimator for the future volatility, the residuals, which are notated as e_t should be white noise and uncorrelated with any variable. Bias refers to the gap between the estimated values and the actual values. Efficiency is determined by the fact that how close the slope coefficient of implied volatility to 1. Implied volatility achieves the perfect efficiency, when $a_i = 1$. (Christensen & Prabhala 1998; Shu & Zhang 2003)

Two alternative formulas are also introduced. Eq.(17) tests the predictive power of realized volatility without taking into account the values of implied volatility. Eq.(18) measures the predictive power of implied volatility with an additional control variable of past realized volatility.

$$h_t = a_0 + a_h h_{t-1} + e_t \tag{17}$$

$$h_t = a_0 + a_i i_t + a_h h_{t-1} + e_t, \tag{18}$$

where h_{t-1} denotes the past realized volatility. (Szakmary et al. 2003) Test results of the equations above are displayed in the next section.

5.2 Results of standard methodology

Tables 3 and 4 show the results of the OLS- regressions introduced in the methodology section. Descriptive statistics and the normal distribution plots exhibited a high level of kurtosis and skewness. Consequently, volatilities are transformed with natural logarithm to pacify linear regression's normality assumption. By observing the results of Eq.(16) from Tables 3 and 4, it can be concluded that implied volatility contains information about the future realized volatility with some specifications in both indices since the slope coefficients of implied volatilities are significantly different from zero at 1%. The implied volatility explains economically, almost 50 % (R^2) of the variation of the future realized volatility in both index options markets. Statistically, the intercepts in both markets differ from zero, signaling that the implied volatility is a biased estimator. The intercepts in both tables are also negative, which corresponds to the fact that the forecasting value of implied volatility is, on average, too low. The negative intercept is caused by the usage of log-volatilities and the EIV-problem (Christensen & Prabhala 1998). i_t, the slope coefficient, is less than unity at 1% significance, indicating that implied volatility explains future realized volatility inefficiently. F-statistics rejects the joint hypothesis of $\alpha_0 = 0$ and $\alpha_1 = 1$ in both markets, meaning that the implied volatility is both biased and inefficient in estimating future realized volatility.

Eq.(17) tests the predictive power of past realized volatility. The past realized volatility explains economically, around 20 % of the variation of future realized volatility. The R^2 values of past realized volatilities are also significantly lower than the corresponding values of implied volatilities in Eq(16), signaling that the prediction power of past realized volatility is lower than that of implied volatility. Past realized volatility is also a more biased estimator than implied volatility since the intercept terms are bigger in Eq.(17) than in Eq.(16). Over the decades, multiple papers have revealed that the predictive power of realized volatility is less than that of implied volatility, which is also noticed in this paper (Poon & Granger 2003).

Eq.(18) investigates whether or not implied volatility contains all the information content of past realized volatility. If the markets are informationally efficient, besides the aforementioned joint hypothesis, α_h should equal zero. However, the joint hypothesis is rejected in Tables 3 and 4 by the F-statistics at 1% significance level in both markets, indicating that implied volatility is a biased and inefficient predictor of future volatility, even when adding the control variable h_{t-1} to the equation. The slope coefficient of h_{t-1} drops dramatically in both markets when added to Eq.(18), but it remains statistically significant (p-value < 0.05), indicating violations in the efficient market hypothesis. The intercept value also reduces when adding the control variable to the formula in both indices, implicating less biased estimates when the possible autocorrelation is controlled with past realized volatility. Albeit the estimates in both markets' implicate that implied volatility predicts future realized volatility inefficiently, the slope coefficients (0.69 0.75) are statistically significant and economically good coefficients to explain the variations in future realized volatility. The slope coefficients of implied volatilities are also higher than the equilevant coefficients of past realized volatility, meaning the implied volatility is a better estimator for future volatility than the past realized volatility. Similar results are shown at this stage by E.g Christensen & Prabhala (1998), Shu & Zhang (2003), Li & Yang (2009)

The Durbin-Watson statistics shown in Tables 3 and 4, display no autocorrelation in the residuals. Figures 6 and 7 shown in Appendix display the normal distribution of residuals, which shows that the residuals are slightly skewed. Hence robust methods will provide more reliable estimates.

Table 3: FTSE 100 Index options log-regressions

	Intercept	i_t	h_{t-1}	$Adj.R^2$	DW	$F - Stat$
Eq.(16)	-0.44**	0.80 ***		0.4915	1.89	54.16***
Eq.(17)	-1.04***		0.50***	0.23	2.07	17.15***
Eq.(18)	-0.25**	0.69***	0.19**	0.506	2.55	28.73***

***<0.01 **<0.05 *<0.10

Table 4: S&P 500 Index options log-regressions

	Intercept	i_t	h_{t-1}	$Adj.R^2$	DW	$F - Stat$
Eq.(16)	-0.37**	0.89 ***		0.4432	1.92	43.99***
Eq.(17)	-1.16***		0.47***	0.21	2.07	15.16***
Eq.(18)	-0.04**	0.75***	0.28***	0.506	2.55	27.27***

***<0.01 **<0.05 *<0.10

5.3 Robustness

Since the standard distribution plot of residual indicates skewed and not normally distributed residuals, the White-adjusted robust regressions state more reliable estimates. White-adjusted estimates reduce the mean standard errors, thus making them more robust (Baltagi 2011, 100). Tables 5 and 6 below display the estimates obtained from the robust regression. Eq.(16) suggests, that the implied volatility remains biased and inefficient in both markets. The null hypothesis is rejected by the F-statistics at 1%

significance level in simple robust regression.

However, the White-adjusted regressions indicate different results than the OLS-regression, when the control variable h_{t-1} is added to the formula in Eq.(18). Because of the heteroscedasticity adjustments, the intercept terms and the slope coefficients of control variables in both markets become statistically insignificant from zero, thus the residuals are not autocorrelated anymore. Albeit the F-statistics endorse the rejection of the joint hypothesis, it can be inferred that the implied volatility turns out to be an unbiased, but inefficient, estimator for future realized volatility.

Table 5: FTSE 100 Index options white-adjusted regressions

	$Intercept$	i_t	h_{t-1}	$Adj.R^2$	$F-Stat$
Eq.(16)	-0.448**	0.80 ***		0.50	69.13***
Eq.(17	-1.04***		0.505***	0.245	9.73***
Eq.(18)	-0.25	0.69***	0.199	0.524	35.09***

***<0.01 **<0.05 *<0.10

Table 6: S&P 500 Index options white-adjusted regressions

	$Intercept$	i_t	h_{t-1}	$Adj.R^2$	$F-Stat$
Eq.(16)	-0.41**	0.88 ***		0.44	46.98***
Eq.(17)	-1.16***		0.47***	0.225	7.48***
Eq.(18)	-0.04	0.75***	0.28	0.516	35.09***

***<0.01 **<0.05 *<0.10

Due to the possible existence of the EIV-problem in implied volatilities, Christensen & Prabhala (1998) suggested running the regressions in instrumental variables framework to compensate the EIV-problem. Consequently, the implied volatilities used in the next regression are based on the fitted values of h_{t-1} and i_{t-1}. Therefore the implied volatility should be endogenously dependant on past volatility (Christensen & Prabhala 1998).[3]. The analysis with Two-Stage least squares regression (2SLS) can be considered a backtest for the White-adjusted regression because it is more powerful and robust in terms of endogeneity. The results of 2SLS are only compared to Eq.(18).

The results displayed in Tables 7 and 8 confirm the White-adjusted regressions' findings, as the implied volatility is an unbiased and inefficient estimator for the future realized volatility. The intercept terms and the control variables in both markets do not statistically differ from zero. However, the implied volatility remains inefficient for estimating future volatility, because the slope coefficients of i_{t-1} are statistically significant from unity at 1% in both markets.

The findings of White-adjusted regressions and 2SLS give answers for the research questions of this paper. Implied volatility remains an inefficient estimator for the future volatility in all regressions, although it turns out to be an unbiased estimator after the heteroscedasticity adjustments and the EIV-problem is taken into account. The implied volatility is also found to be less inefficient in S&P 500 index options compared to the FTSE 100 index options. The results also show that the implied volatility outperforms past realized volatility in prediction power, meaning that the implied volatility derived from the options markets gives roughly a good estimate for the future asset pricing movements.

[3]See Christensen and Prabhala (1998), for more information about the instrumental variables approach

Table 7: FTSE 100 Instrumental framework regression(2SLS)

	$Intercept$	i_{t-1}	h_{t-1}	$Durbin - Watson$
Eq.(18)	1.21	1.96***	-0.35	2.01

***<0.01 **<0.05 *<0.10

Table 8: S&P 500 Instrumental framework regression(2SLS)

	$Intercept$	i_{t-1}	h_{t-1}	$Durbin - Watson$
Eq.(18)	0.99	1.46***	0.10	1.96

***<0.01 **<0.05 *<0.10

5.4 Behavioral perspective

The previous section focused on the more technical part in the interlinkages between implied and realized volatility. This section tries to provide some pretexts of what caused these results. Firstly by observing the prediction power of past realized volatility, the conclusion was that the implied volatility predicts future realized volatility better than past realized volatility. This finding is in line with previous studies. One explanation for this may be that the market traders have already taken into account the past fluctuations in the underlying asset because it is one of the key elements in Black & Scholes option pricing model. Another interesting finding is that the mean values of implied volatilities are higher than that of realized volatility. This is indicates that the call options are on average overpriced. Behavioral finance states this as a compensation for the sell side of options contracts, which involves higher risks (Poon 2005).

While this paper concluded that implied volatility is not an efficient forecast of future realized volatility, it is an economically good indicator for the future flucticitions in underlying index. However, Christensen & Prabhala (1998) exhibited that the implied volatility turns out to be unbiased and efficient in instrumental variables framework.

This paper's results indicate that the implied volatility remains inefficient which is the opposite of what Christensen and Prabhala stated. One explanation of inefficiency is attributed to moneyness rate of options that are used in this thesis. The efficiency is most likely achieved when options are trading at the money, because the at the money options are most liquid contracts and hence at the money implied volatility is least prone to measurement errors (Poon 2005). However the option contracts are rarely at the money, moneyness rate of 0.95-1.05 was used to select the closest at the money call options. One possible explanation for the inefficiency is the COVID-19 dip, which occured after the February 2020.

The incorrectness of the Black & Scholes option pricing model is also a noticiable factor, that might have caused the inefficiency. This would mean that the implied volatility derived from the model does not signal the actual implied volatility of the markets. The final possible factor causing the inefficiency is the constraint of the dataset. (Christensen & Prabhala 1998) used over 100 nonoverlapping samples in their study, while the dataset of this particular study covers only 56 monthly nonoverlapping observations. Larger datasets typically provide more accurate predictions for regressions.

6 Conclusion and Discussion

The purpose of this thesis was to re-evaluate the interlinkages between option-implied and realized volatility. Two significant index options markets, FTSE 100 and S&P 500 were chosen to test the interlinkages between implied- and realized volatility. The empirical analysis results largely confirm previous studies by (Christensen & Prabhala 1998) and (Shu & Zhang 2003), suggesting that the implied volatility in OLS-regressions is biased and inefficient to explain variations in future realized volatility. Robust estimation approaches, which take the nature of the EIV-problem into account in estimations, indicate that the implied volatility is an unbiased estimator for future volatility, remaining inefficient, giving answer to the research question *Is the implied volatility an efficient forecast of future volatility.* The results of S&P500 index options remained better in terms of efficiency compared to FTSE100 index options indicating differences in market structures, giving answer to the research question *Does the information content of implied volatility vary between different option markets?.* Although the implied volatility in both markets is not an efficient estimator in terms of statistical testing, it had a significant connection to predicting the future realized volatility.

Index option traders may find this paper useful, as it tests the forecasting efficiency and biasedness of implied volatility. Estimates of future realized volatility that are not efficient may indicate mispricing in option contracts, which may generate arbitrage opportunities to the markets (Filis 2009). Because the implied volatility derived from the Black & Scholes option pricing model do not efficiently estimate future volatility, one can exploit the differences between the predicted values and actual values via delta-neutral portfolio. However, cross-market trading is typically costly and profitable only when the profits of the arbitrage opportunity exceed the transaction costs. (Fleming et al. 1998)

Nonetheless, answers for the research questions were found, and they are in-line with previous research in this field of study. Suggestions for future research would be to examine the interlinkages between implied and realized volatilities via more sophisticated option pricing models that take the markets' stochastic nature into account.

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A Appendix

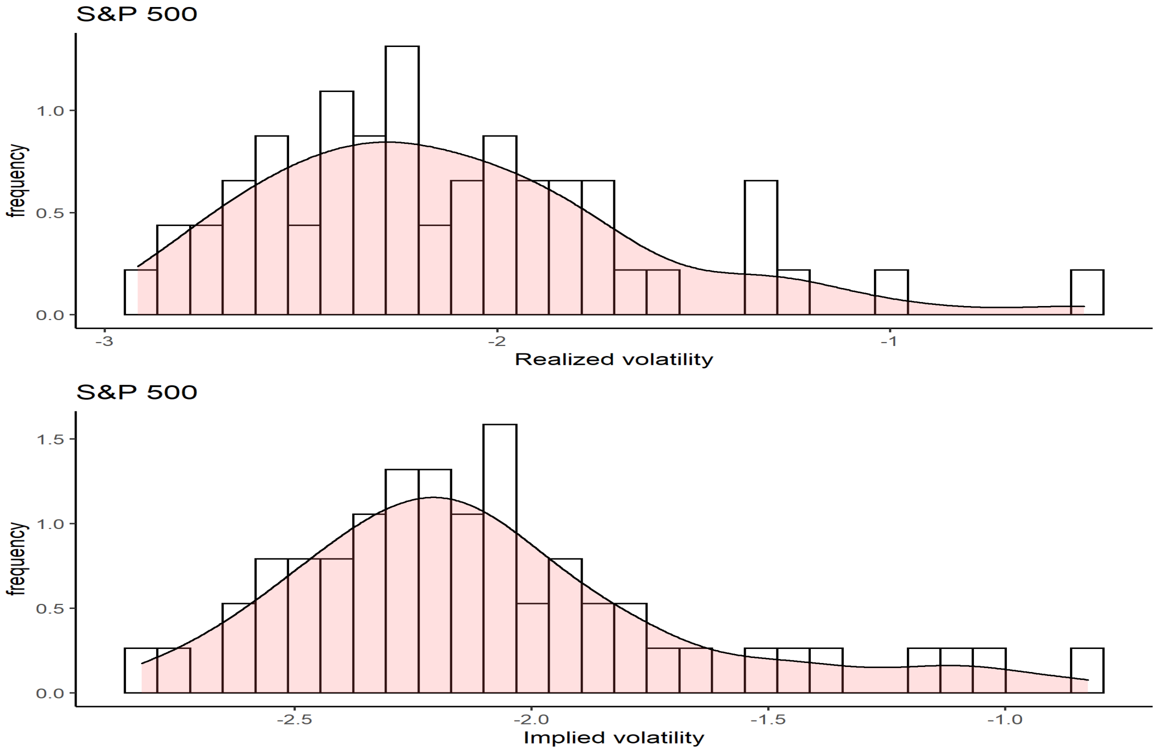

Figure 5: Normal distributions of logarithmic S&P 500 volatilities

Table 9: Relevant previous studies

	Authors	Data	OL/NOL	Results
	J Shu & JE Zhang	S&P 500 index options	NOL	IV contains information but remains biased
	CG Lamoureus& WD Lastapres	Ten selected stock-options	OL	IV is biased and inefficient forecast of future RV
	TE Day & CM Lewis	S&P 100 index options	OL	IV is biased and inefficient forecast of future RV
	JF Garvey & LA Gallagher	16 FTSE-100 Index stocks	NOL	IV contains predictive information of future RV. IV is optimal forecast-metric in medium term-horizons.

OL represents overlapping samples, whereas NOL represents non-overlapping samples.

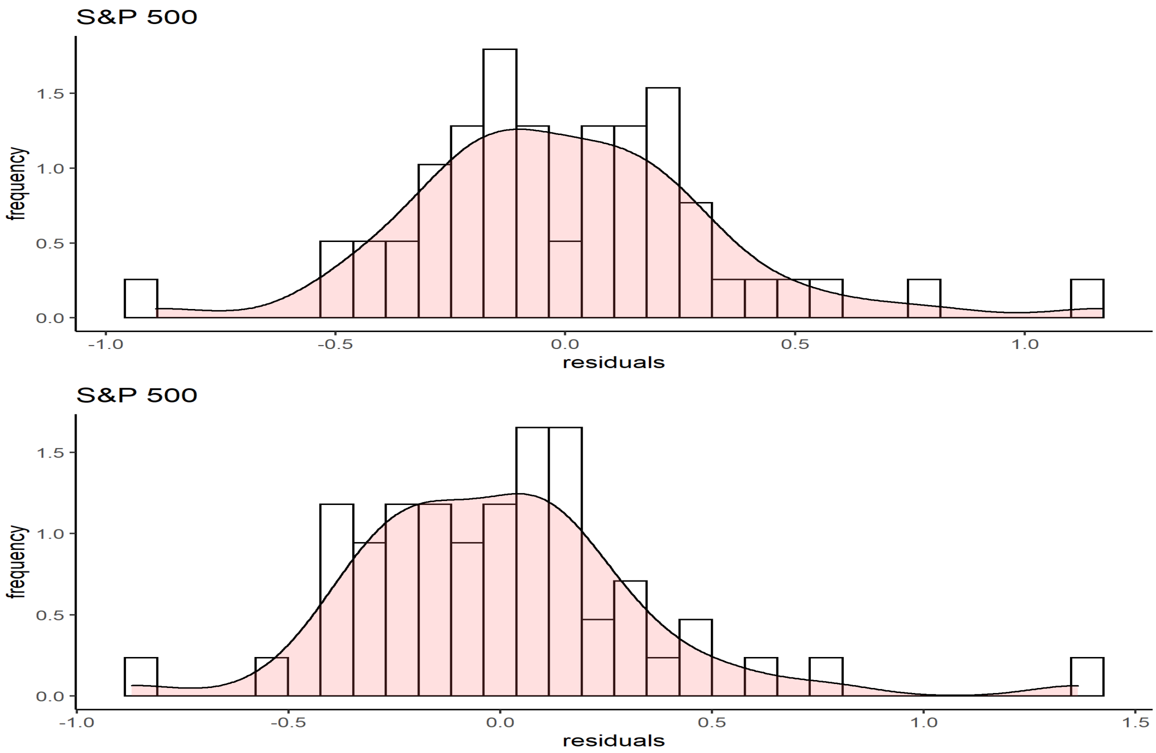

Figure 6: Normal distributions of residuals S&P 500 logarithmic regressions.

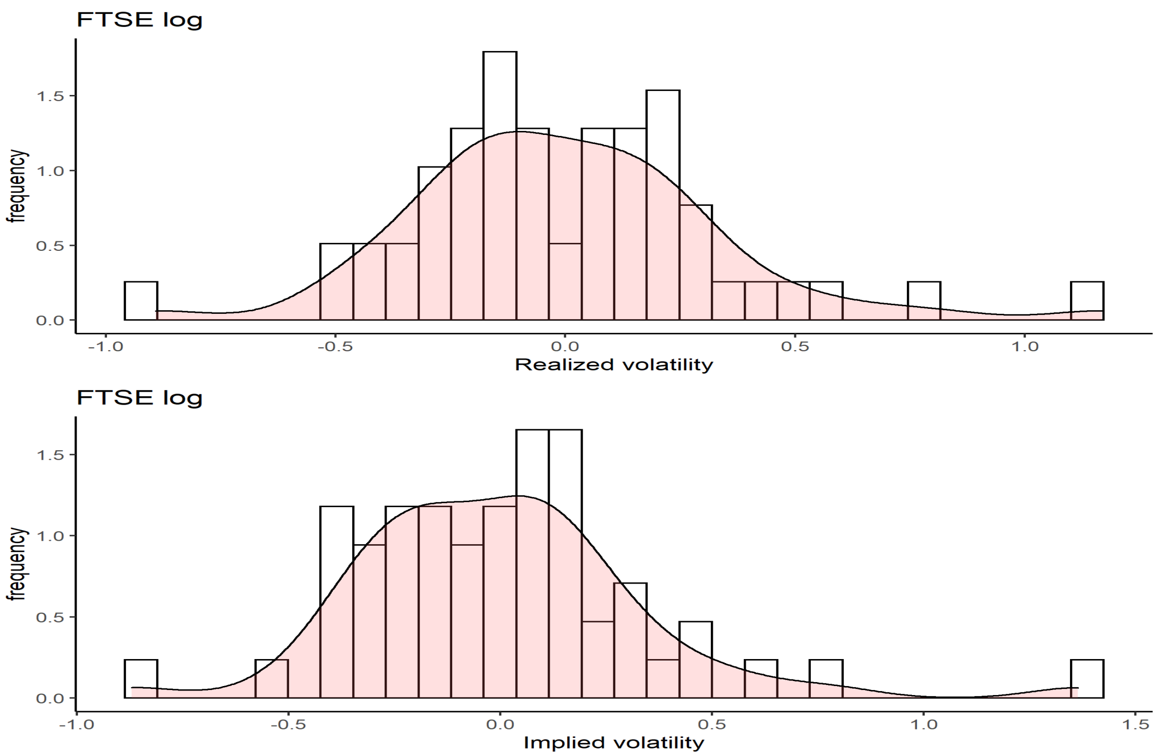

Figure 7: Normal distributions of logarithmic FTSE 100 volatilities

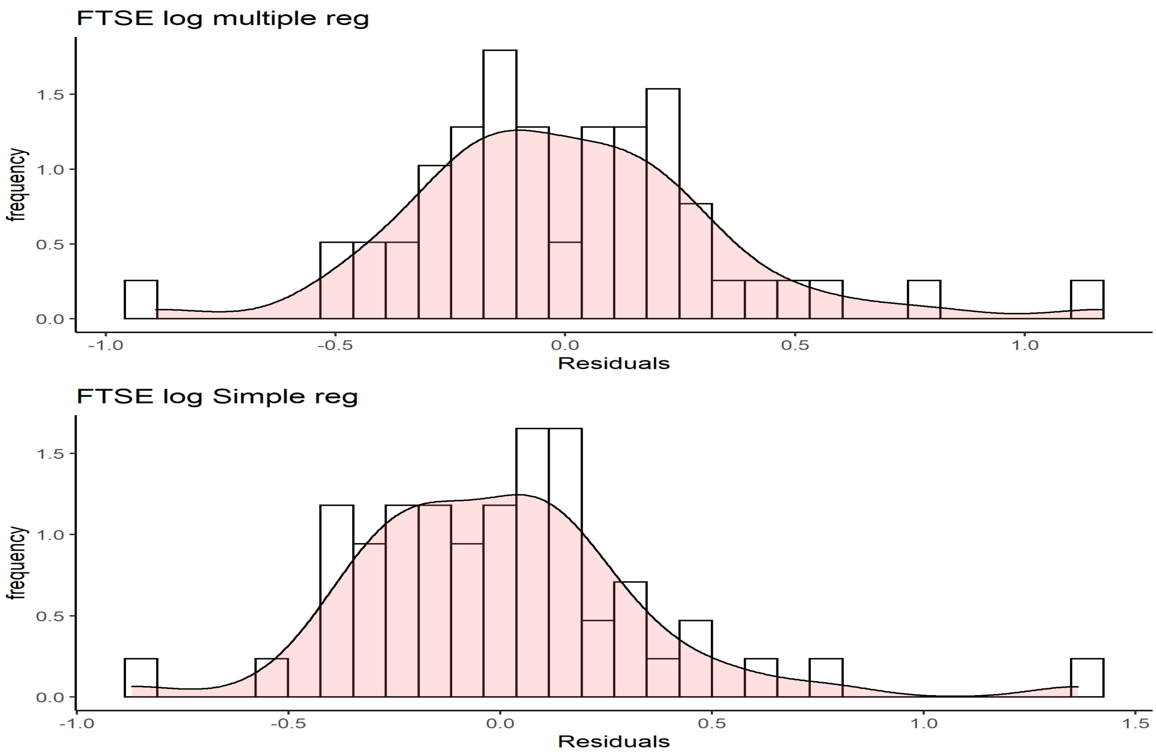

Figure 8: Normal distributions of residuals FTSE 100 logarithmic regressions.