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Convertible Bonds: Arbitrage and hedging strategies in the U.S. markets

Master’s Thesis 2022

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ABSTRACT

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This thesis examines the risk-adjusted performances of several arbitrage and hedging strategies involving convertible bonds. Convertible arbitrage (CA) exploits the pricing inefficiencies of volatility or credit attributes embedded in the convertible bond by simultaneously taking a long position in the convertible bond and a short position in the underlying common stock. The methodology involves replicating various CA strategies set up on linear and non-linear attributes of convertible bonds. The sample consists of 159 U.S. market convertibles issued between 2013 and 2018. The returns of various CA strategies are first examined as individual trade returns and later aggregated to portfolios. Strategy returns are examined with the Sharpe ratio, skewness and kurtosis-adjusted Sharpe ratio (SKASR) and a linear risk-factor model incorporating equity and bond risk. The results present mixed news for investors interested in CA. Strategies involving dynamic hedging around convertible bonds generate statistically significant alpha on a risk-factor basis but lack robust evidence from a total risk perspective. Results indicate that arbitrageurs exploiting CA strategies might find it worthwhile to consider higher equity market risk when setting up hedges. All results are robust to modest transaction costs.

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Tämä tutkielma tarkastelee vaihtovelkakirjalainojen ympärille luotujen arbitraasi- ja suojausstrategioiden riskikorjattua suoriutumista. Vaihtovelkakirjalaina-arbitraasissa tavoitteena on hyötyä vaihtovelkakirjalainaan epätehokkaasti hinnoitellusta volatiliteetista tai luottoriskinhinnasta ostamalla vaihtovelkakirjalaina ja samanaikaisesti myymällä lyhyeksi kohde-etuutena olevaa osaketta. Tässä tutkielmassa replikoidaan useaa vaihtovelkakirjalaina-arbitraasistrategiaa hyödyntämällä 159:ää Yhdysvaltain markkinoilla vuosien 2013 ja 2018 välillä liikkeellelaskettua vaihtovelkakirjalainaa, joiden ympärille luodaan instrumenttien lineaaristen ja epälineaaristen ominaisuuksien mukaisesti erilaisia riskiarbitraasipositioita. Strategioiden tuottoja tarkastellaan ensin yksittäisten riskiarbitraasipositioiden kautta, ja myöhemmin tuotot aggregoidaan portfolioiksi. Strategioiden riskikorjattuja tuottoja tutkitaan Sharpen luvun, vinous- ja huipukkuus korjatun Sharpen luvun sekä lineaarisen osake -ja korkomuuttujia sisältävän riskifaktorimallin avulla. Tulokset osoittavat, että strategiat tuottavat tilastollisesti merkitseviä ylituottoja riskifaktorimallia vastaan, mutta kokonaisriskiin perustuvien mallien perusteella vahvoja todisteita strategioiden ylisuoriutumisesta ei ole. Tutkimustulokset indikoivat, että vaihtovelkakirjalaina-arbitraasistrategiaa harkitsevat saattaisivat hyötyä suuremman osakeriskin suosimisesta vaihtovelkakirjalainaposition suojaamisessa. Tutkimustuloksissa on huomioitu maltilliset kaupankäyntikustannukset.

Table of contents

List of figures

List of tables

List of Appendices

List of abbreviations

BSM = Black-Scholes-Merton

BPS = basis point

CA = convertible arbitrage

CB = convertible bond

CDS = credit default swap

HV= historical volatility

IG = investment grade

IV = implied volatility

OTC = over-the-counter

OTM = out of the money

SKASR = skewness and kurtosis-adjusted Sharpe ratio

YTM= yield to maturity

1. Introduction

1.1. Background

A convertible bond consists of a traditional bond with fixed payments and an embedded option on the equity. Investors owning the bond can earn a fixed return by receiving cashflows from the bond but have an option to convert the bond to common shares. Aggressive and skillful market entities such as hedge funds and proprietary trading desks use a vast range of offsetting positions around the convertibles and try to create attractive risk-return profiles. Convertible arbitrage belongs to the class of fixed income arbitrage where the aim is to spot and capture profits from the mispricing between the convertible bond and other instruments from the issuer's capital structure. In the turmoil of the financial crisis 2008, highly leveraged convertible arbitrage funds lost over 30 percent of their value and were among the worst-performing hedge fund strategies that year[1]. One of the oldest hedge fund strategies betting on the mispricing between a convertible bond and equity was no longer market neutral and profitable. Since the financial crisis 2008, investors have withdrawn approximately $ 30 billion from the convertible arbitrage funds.[2] When the COVID-19 pandemic hit the world economy in 2020 and the stock market plunged, convertible arbitrage funds raised their heads for the first time in years. Funds deriving return from the mispricing of the volatility in the convertibles show solid returns in 2020 despite the stock market crash, see Appendix 1.

Convertible arbitrage or any arbitrage is far away from textbook execution and can face a large amount of risk and uncertainty (Shleifer and Vishny,1997). The scientific evidence speaking for the strategy's superior risk and return characteristics is limited, controversial and lacks post 2012 coverage, see e.g. Fabozzi, Liu and Switner (2009), Hutchinson and Gallagher (2010), Agarwal, Fung, Loon and Naik (2011). In summary, the results indicate that on traditional risk exposure measures, the strategy generates abnormal excess returns. Prior papers have used

[1] Source: HFRI Convertible Arbitrage Index

[2] Source: BarclayHedge, 2021

mainly two approaches. The first and more popular approach has been to construct risk factors that incorporate, for example, a long-exposure and delta-hedged exposure to convertible bonds and use them to explain CA fund returns. In the second approach, CA portfolios are constructed from historical market data see, e.g. Fabozzi et al. (2009) or Hutchinson and Gallagher (2008). Hutchinson and Gallagher (2008) point out, there are issues related to historical hedge fund data e.g. survivorship bias and how to address proper risk factors. Following Hutchinson and Gallagher (2008, 2010) and Fabozzi et al. (2009), the approach in this thesis is to construct simulated convertible arbitrage trades and portfolios from real market and bond data. In addition to avoiding the possible biases in hedge fund data, this method allows full control of transaction costs and leverage throughout the time series.

Convertibles are often issued with a purpose to monetize the volatility i.e. obtain lower financing costs because investors are interested in a long-term call option on the equity and willing to pay for it. This volatility is often priced much lower than the volatility observed from the equity or options market would indicate. Sae-Sue, Sinthawat, and Srivisal (2020) show that implied volatility in the options embedded in convertible bonds is significantly mispriced in the U.S market during 2015-2016. This is an interesting observation as it is very closely related to convertible arbitrage and indicates the possible existence of arbitrage. From the volatility perspective, this gives the motivation to explore the strategy returns, again, as the strategy should derive some of its return from the mispricing of volatility.

To derive the proper hedging metrics, hedge funds and proprietary trading desks use a vast amount of models that are used to estimate convertible bonds' price sensitivity to the underlying stock, interest rate level, and so on. Fabozzi et al. (2009), Loncarski, Ter Horst, and Veld (2009) employ the Black-Scholes-Merton (1974) model to derive such metrics. To address the credit risk, to which the arbitrageur is also exposed, a binomial model incorporating the credit risk is used in this thesis. A binomial model with credit risk by Milanov, Kounchev, Fabozzi, Kim, and Rachev (2013) serves as the framework on hedging strategies which has not been used very often, if not ever, in the convertible arbitrage papers.

Arbitrageurs are exposed to market frictions such as direct and indirect costs that occur every time something is bought or sold. To enhance the robustness of results and to study the effect of transaction costs, a market impact model of the stock trading costs is employed. Bonds are traded OTC and transaction costs are rather difficult to estimate. A sensitivity analysis is performed to address the effect of bond bid-ask spread on the strategy's profitability. This again has not been done according to the author's knowledge, in convertible arbitrage context, and should bring value to the existing research pool.

This thesis should answer questions that have been left unanswered and gives a motivation to test the deviations from the law of one price in the convertible arbitrage context.

1.2. Hypothesis development

In this thesis, a variety of convertible bond arbitrage and hedging strategies are simulated and tested for the deviation from the law of one price. The sample consists of 159 convertible bonds issued between 2013 and 2018 in the U.S. markets. Strategies employed in this thesis are delta and gamma-based strategies that are set up between the convertible bond and the underlying stock. All trades are first studied separately and in the latter part, aggregated to portfolios that are examined with linear risk-factor and total-risk models. In this section, all hypotheses and explanations for them are presented.

Hypothesis 1: *Convertible arbitrage is a superior investment strategy on a risk-adjusted scale.*

A hedged position around convertible bond generates high risk-adjusted returns both from a systematic and total risk perspective. E.g. Hutchinson and Gallagher (2008) show annual returns of 8.47 % for equal-weighted simulated convertible arbitrage with an annual volatility of 6.04%. In terms of the Sharpe ratio, an investor received more return units per one risk-unit than investing in the Russell 3000 (return 6.99% with a volatility of 15.41% as p.a.) over the period from 1990 to 2002. Also, the HFRI Convertible Arbitrage Index returned on average 11.02% with a standard deviation of 3.37% during the same period. At least in history, the strategy has provided a high return to risk metrics both in the scientific and real world.

Hypothesis 2: *Convertible arbitrage is a market-neutral strategy.*

E.g. Gallagher, Hutchinson, O'Brien (2018) claim that convertible strategy has generated positive returns for a relatively long period with low volatility. The only exceptions are market shocks that have had a large negative effect on the returns and led to high volatility. However, they show that convertible arbitrage has relatively low exposure to common risk factors in a normal market regime.

Hypothesis 3: *The binomial model with credit risk is usable to calculate Greeks for convertible bonds and later to derive abnormal excess returns.*

The binomial model by Milanov et al. (2013) has not been used often, if not ever, in scientific articles that examine convertible arbitrage strategy. A regular binomial model result converges to the Black-Scholes-Merton result, when the number of steps is increased enough. Although the Milanov et al. (2013) model diverges from the basic Wiener Process approach, the result should be close to the regular Black-Scholes-Merton result as the tree construction parameters are close to the regular Cox, Ross and Rubinstein (1979) solution.

Hypothesis 4: *Hedge ratios calculated using implied volatilities lead to more precise hedging and generate higher risk-adjusted returns.*

Zeitsch (2017) challenged the use of historical volatility as a model calibration volatility in capital structure arbitrage strategies. Although these strategies were about trading mispriced CDS, the motivation to use 1-month 10-delta put implied volatilities was clear. Buying CDS protection inherently reminds of buying deep out of the money (OTM) put options as an insurance against financial distress. Market players start buying OTM put options as insurance, thereby driving the implied volatility up. This means that the CDS model should be calibrated with deep OTM put option volatility as these instruments are inherently for the same purpose, that is tail risk insurance. The same conclusion could be drawn from a convertible bond that has an embedded warrant on the equity, a call option-like feature. When the market expects the company's financials or other features to enhance, they start buying out-of-the-money calls, speculating on the increase in the stock price. So, convertibles that are issued OTM, should then possess the same features as OTM calls.

Hypothesis 5: *Modified-delta and gamma strategies outperform regular delta-hedging strategies.*

Hutchinson and Gallagher (2008), Calamos (2003) claim that daily delta-hedging is usually ignored by hedge funds due to its expensive nature. Ammann and Seiz (2006) and Batten, Khaw, and Young (2018) claim that deep OTM convertibles are less likely to be efficiently priced and therefore might face larger hedging errors. The trader might then enter into trades selling short too few or too many shares. This gives the motivation to study strategies that use different rebalancing and hedge ratio guidelines than a regular delta-hedge strategy.

1.3. Limitations of the study

On the limitations of this thesis, a few themes should be especially highlighted. Firstly, only the binomial model with credit risk component serves as a valuation model for the convertible bonds. Other notable models such as the model proposed by Ayache, Forsyth, and Vetzal (2003) or Tsiveriotis and Fernandes (1998) are excluded.

As many companies in the sample are smaller than for example companies that are part of the SP500 index, only one aggregate value addressing the implied volatilities for out-of-the-money calls is used in the model calibration. Smaller companies might not have enough liquid vanilla option quotes that could be employed in the model calibration.

The maximum holding period of a particular CA position is 14 months after opening the position. As the individual trades are aggregated to the portfolio level, there should be a clear limit when the position exits the portfolio, that is, either 14 months, call or default by the issuer. The 14 months were chosen for several reasons. Fabozzi et al. (2009) indicate that delta-hedged trades generate positive returns for the first 15 months from the issuance. Also, the liquidity aspect is considered. According to Batta, Chacko, and Dharan (2010), the issuer’s stock and the CB have the highest liquidity near the initial issuance. Marle and Verwijimeren (2017) claim that hedge funds are exposed to particular trade for approximately 1 year.

Other limitations consider the bond valuation and Greek letter derivation. Credit risk is incorporated in both models and there should be some educated guess where the credit spread should be for a particular company. Again, companies in the sample rarely have CDS quotes or liquid vanilla bond quotes so the credit spread is estimated with the Merton model (1974) framework.

1.4. Structure of the thesis

The thesis is structured as follows. Sections 2 and 3 focus on the theoretical framework of convertible bonds and convertible arbitrage as an investment strategy. The methodology employed is this thesis, and major studies concerning convertible arbitrage's abnormal performance and market efficiency are presented in the third section as well. Sections 4 and 5 consist of the data description, portfolio construction and results. Conclusions are presented thereafter.

2. Convertible Bonds: Valuation and Risk

Hybrid securities are between debt and equity. The most common instruments in this asset class are convertible bonds and preferred shares. Hybrid securities can possess characteristics such as long or perpetual maturity, convertible feature (convertible to equity or debt), lowest payment rank in a case of bankruptcy (subordinated debt), no voting right (preferred shares), and a possibility of a coupon or dividend deferral. The accounting treatment, whether treated as debt or equity, can vary between different countries. (De Spiegeleer, Van Hulle and Schoutens. 2014. 1-2)

2.1. Overview of convertible bonds

A convertible bond is a hybrid security that consists of a traditional bond and an embedded equity option. Like a regular bond, a convertible bond has a face value and investor receives coupons. The holder has a right to convert the security to a predetermined amount of the company's shares but has no obligation to do so. After the conversion has taken place, the holder foregoes the remaining coupons and the face value and receives the shares that the holder is entitled to. The payoff to the investor is either the pure fixed income return and/or the equity value when the bond position is converted to shares. It could be so that the equity trades deeply below the strike price and the investor has no incentive to convert but would rather receive cashflow from the coupons. Should the stock price rise enough, the holder converts the bond to shares.

Final Payoff

The convertible bond can be converted to equity during its life (American option) or at maturity (European option). At each time *t* during the life of the convertible, the conversion value is the value of an immediate conversion.

(1) $$Conversion\ Value = C_r S_t$$

Where C_r is the number of shares the convertible bond can be converted into or the conversion ratio and S_t is the stock price on trading day *t*.

If the bond is held until maturity, the final payoff to the convertible bondholder is either the debt value or conversion value, whichever is greater. Unless the holder decides to exercise his right to convert, the bond position exists.

(2) $$Payoff = max(DebtValue, Conversion\ Value)$$

or

(3) $$Payoff = max(FV + FV * C, C_r * S)$$

Where FV is the face value of the bond and C is the coupon rate.

Pricing and expressions

The bond floor is the pure debt component of the convertible bond. If the bond is not converted during its life, the return to the investor is the same as holding a regular fixed-income instrument. The return then equals the price change of the bond plus the coupon payments on the face value. The valuation of the fixed income leg is analogous to a regular fixed income valuation. The bond floor value is equal to the sum of discounted cashflows received by the bondholder.

(4) $$B_F = \sum_{i=1}^{N_c} C_{t_i}\, e^{-r_b t_i} + FV\, e^{-r_b T}$$

Where N_c is the number of coupons received during the life of the bond, C_t is the coupon paid at time t, r_b is the discount rate, t_i is the time of coupon arrival, T is the time to maturity and FV is the face value of the bond.

The convertible bond price is a sum of the pure debt component and the equity option value. The convertible bond is then economically the same as holding company's bond and a call option on the underlying equity. The conversion price, or strike price, is the stock price at which the equity conversion value is equal to the face value of the bond. In some occasions, the conversion price is changed during the life of the security for example should the company split its stock or issue new shares to the market. The precise valuation method for the option is explained in detail after this sub-section.

(5)
$$P_{CB} = P_{Call}C_r + B_F$$

Where P_{CB} is the price of the convertible bond, P_{Call} is the price of a call option, C_r is the conversion ratio and B_F is the bond floor.

(6)
$$C_r = \frac{FV}{C_p}$$

Where C_r is the conversion ratio and C_p is the conversion price.

The parity of a convertible is expressed as a percentage to the face value of the bond. For example, a parity of 120% means that the value of the conversion is 20 % higher than the face value of the bond.

(7)
$$Parity = \frac{S}{C_p}$$

or

$$Parity\ \% = \frac{C_r * S}{FV}$$

Where C_r is the conversion ratio, C_p is the conversion price and S is the stock price.

The investment premium is expressed as a percentage that describes the value of the equity option. The investment premium is calculated by taking the difference between the market value of the convertible bond and the fixed income value or bond floor and divided by the bond floor. The conversion premium describes the equity participation in the convertible, that is, if the conversion value is $75,000, the bond trades at par and has a face value of $100,000, the conversion premium would be 33%.

(8)
$$Investment\ Premium\ \% = \frac{(P_{CB} - B_F)}{B_F}$$

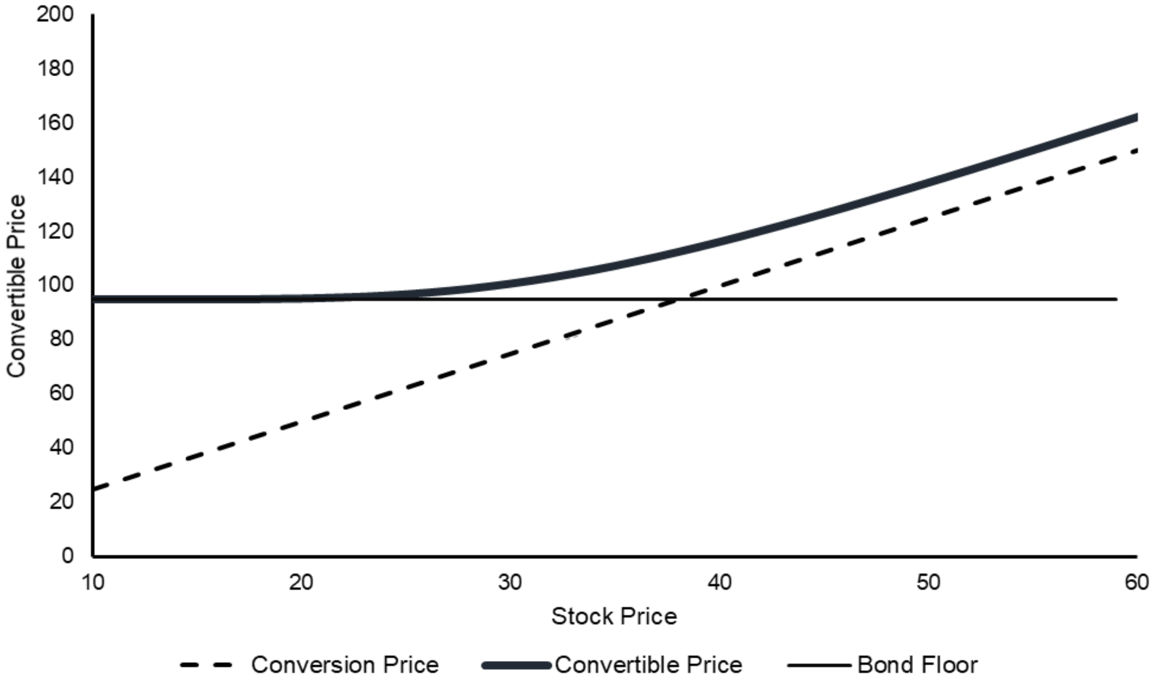
Where P_{CB} is the market price of the CB and B_F is the bond floor.

(9)
$$Conversion\ Premium\ \% = \frac{(P_{CB} - C_r S_t)}{C_r S_t}$$

Where P_{CB} is the market price of the bond and $C_r S_t$ is the conversion value.

When the CB is trading below the implied strike price, the security is more sensible to the changes in the level of interest rates and the credit spread. When the embedded option increases in value, or the delta increases, the CB becomes more equity-like and its sensitivity to traditional bond price drivers such as the credit spread and yield curve, decreases. Figure 1 shows the convertible price track with respect to the underlying stock price when the bond floor is kept as a constant. The minimum value of a convertible bond is equal to the bond floor. When the stock price increases, the convertible price increases and may become more than the value of the straight debt component.

Figure 1: Convertible bond's price sensitivity with respect to the underlying stock price



Greeks

Delta	$\frac{\partial P}{\partial S}$	Price sensitivity of the convertible bond to the underlying share. An increase in the underlying share price tends to increase the price of the CB.
Vega	$\frac{\partial P}{\partial \sigma}$	Price sensitivity of the convertible bond to the model volatility. An increase in volatility tends to increase the price of the CB.
Rho	$\frac{\partial P}{\partial r}$	Price sensitivity of the convertible bond to the overall level of interest rates. An increase in the level of interest rates tends to decrease the price of the CB.
Omicron	$\frac{\partial P}{\partial c}$	Price sensitivity of the convertible bond to the credit spread. An increase in the credit spread tends to decrease the price of the CB.
Phi	$\frac{\partial P}{\partial d}$	Price sensitivity of the convertible bond to the underlying dividend yield. An increase in the dividend yield tends to decrease the price of the CB.
Upsilon	$\frac{\partial P}{\partial rr}$	Price sensitivity of the convertible bond to the assumed recovery rate. A decrease in the bond's assumed recovery rate in case of default tends to decrease the price of the CB.
Theta	$\frac{\partial P}{\partial t}$	Price sensitivity of the convertible bond to the passage of time. A decrease in the CB's time to work-out or maturity tends to decrease the value of the embedded call option.

Other Features

Callable Feature

The issuer may call or redeem the convertible bond if it is specified so in the bond prospectus. The call feature reduces the price of the bond as the noteholder has an embedded short position in the bond's call option.

Hard Call Protection

If the convertible has hard call protection, the issuer may not call the bond before the maturity of the call protection.

Provisional Call Protection

If the bond has provisional call protection, the issuer may not call the bond unless it has traded at or over a certain price for a predetermined period.

Put Provision

If the bond has a put provision, the bondholder may redeem the bond at a specified price. Put provision tends to increase the price of the bond. A put option is usually included as a change-of-control covenant. The put option is triggered if the company is sold to another entity and the noteholders are entitled to the redemption of the notes at a specified price.

2.2. Valuation and Greeks

In this section, the convertible valuation method is presented. In previous literature, the BSM model is widely used due to its simplicity and easy implementation ability (see e.g. Fabozzi et al., 2009). The binomial model with credit risk by Milanov et al. (2013) was chosen for this study for several reasons. It offers a simple binomial tree framework and can be implemented with the data that is available for this thesis. The Milanov et al. (2013) model is mathematically close to the model proposed by Ayache et al. (2003) as both assume that stock prices follow a risk-neutral jump-diffusion process.

The binomial model with credit risk is a convertible valuation model derived by Milanov et al. (2013). The assumption is that the bond itself is subject to credit risk, hence the obligor can fail to fulfill its obligation to service debt. In this model, the obligor's default is associated with a drop in its equity price.

As the convertible bond is a hybrid security, it has features from both equity and debt. Assuming a European type convertible, the investor decides whether to exercise the equity option or receive face value and coupon at maturity. A rational investor exercises the option if the conversion value is higher than the present value of the fixed income cashflows. As the exercise decision depends on the underlying equity price, the equity price path is modelled through a stochastic model and affects the pricing of a convertible in a risk-neutral world.

Milanov et al. (2013) model for convertibles that incorporate credit risk is based on variable S, or the underlying stock price. The default by the issuer is associated with a drop in its equity price. A more efficient and traded market (equity) first obtains the information of financial distress. Clark and Weinstein (1983) show that equity price declines approximately 30% upon issuer default. The path followed by the stock price is a result of the Wiener process and Poisson process with a given intensity of λ , or a diffusion process and a jump process, respectively. The Poisson process can be expressed as a stochastic process, where the intensity is known but the occurrence is random. Usually, the default probability is known or at least an educated guess, whereas the timing of the default is unpredictable and random. For a non-dividend-paying stock, the stock price movement for a discrete timestep δt is described in Equation 10. The asset price grows at risk-free rate r (drift term) but is also subject to stochastic Wiener Process and Poisson process. From Ito's Lemma, it can be shown that the stock price distribution can be expressed as lognormal (both real and logarithmic stock prices follow geometric

Brownian motion). The change in stock price expressed as logarithmic value is presented in Equation 11.

(10) $$\delta S_t = (r + \lambda \text{ɲ}) S_t \delta t + \sigma S_t \delta W - \text{ɲ} S_t \delta q$$

Where the δS_t , δW and δq are small increments during an infinite timestep *t* in stock price, Wiener Process, and Poisson process, respectively. σ is the volatility of the stock and ɲ is the percentage by which the stock price drops upon default.

(11) $$ln(S_t) - ln(S_{t-1}) = (r + \lambda \text{ɲ} - \frac{\sigma^2}{2})\delta t + \sigma \delta W + ln(1 - \text{ɲ})\delta q$$

(12) $$ln(S_t) - ln(S_{t-1}) = ln(S_t^C) - ln(S_{t-1}) + ln(1 - \text{ɲ})$$

Where the $ln(S_t)$ is the logarithmic value of the process in one arrival (δq=1) and the $ln(S_t^C)$ is the value if the arrival is absent.

Milanov et al. (2013) propose when there is an arrival of the Poisson process, equal to one (1), the stock price drops by ɲ percent. If the value of the process is $ln(S_t)$ in case of exactly one arrival, then the right side of Equation should be decomposed into a process value of non-arrival i.e. $\delta q = 0$ hence $ln(S_t^C)$ plus $ln(1 - \text{ɲ})$ that makes the equality hold. Thus, when rearranging the terms in Equation 12, the $ln(S_{t-1})$' on both sides will cancel out and then a fall in stock price through default can be expressed as $S_t^C(1 - \text{ɲ})$.

(13) $$\frac{ln(S_t)}{ln(S_t^C)} - ln(1 - \text{ɲ})$$

Or,

$$S_t = S_t^C(1 - \text{ɲ})$$

Milanov et al. (2013) present that the expected stock price return after timestep δt is equal to the risk-free rate *r* as the model is by construction derived on the risk-neutral assumption. The variance of the stock price return is presented in Equation 16.

(14) $$\frac{\delta S_t}{S_t} = (r + \lambda \text{ɲ})\delta t + \sigma \delta W - \text{ɲ} \delta q$$

(15) $$\mathbb{E}\left(\frac{\delta S_t}{S_t}\right) = (r + \lambda \mathrm{n})\delta t - \lambda \mathrm{n}\delta t = r\delta t$$

(16) $$\mathbb{D}\left(\frac{\delta S_t}{S_t}\right) = \sigma^2 \delta t + \lambda \mathrm{n}^2 \delta t = (\sigma^2 + \lambda \mathrm{n}^2)\delta t$$

(17) $$\mathbb{E}\left(\frac{S_t}{S_{t-1}}\right) = \mathbb{E}\left(\frac{S_t}{S_{t-1}} - 1 + 1\right) = \mathbb{E}\left(\frac{\delta S}{S}\right) + 1$$

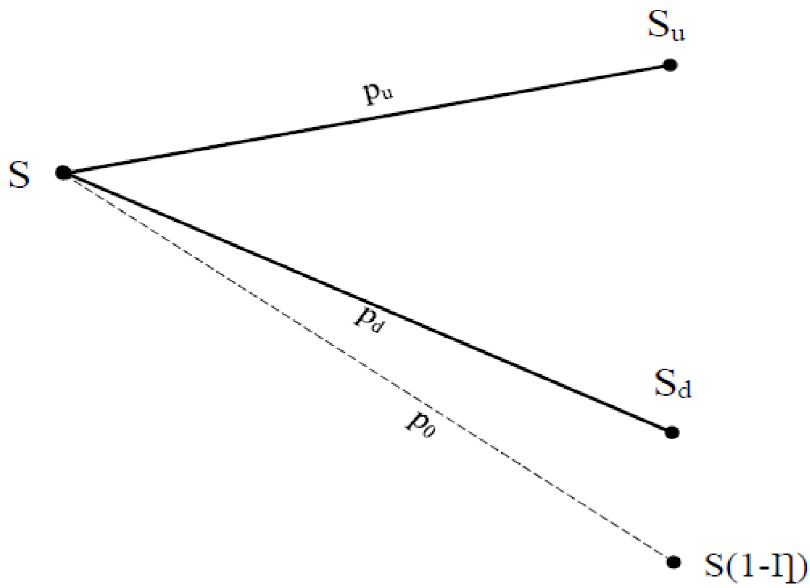
(18) $$\mathbb{D}\left(\frac{S_t}{S_{t-1}}\right) = \mathbb{D}\left(\frac{S_t}{S_{t-1}} - 1 + 1\right) = \mathbb{D}\left(\frac{\delta S}{S}\right)$$

(19) $$\mathbb{E}\left(\frac{S_t}{S_{t-1}}\right) = 1 + r\delta t$$

(20) $$\mathbb{D}\left(\frac{S_t}{S_{t-1}}\right) = (\sigma^2 + \lambda \mathrm{n}^2)\delta t$$

Since the $\mathbb{E}\left(\frac{S_t}{S_{t-1}}\right)$ can be also expressed as an expected change in stock price plus 1, it can be shown that the expected return multiplier in a stock tree is $1 + r\delta$. Given the dynamics of the stock price movement, the event of default during timestep δt means that $\{\delta q_t > 0\}$. The probability is then equal to 1- $\mathbb{P}(\{\delta q_t = 0\})$. Given that, the Poisson process has an intensity equal to λ, the probability of default during timestep δt is $1 - e^{-\lambda \delta t}$ or p_0. Authors assume that in a case of default triggered by the stock price fall, the stock never moves further hence the stochastic movement no longer exists. The possible stock price paths are presented in Figure 2. The stock may move up, down, and default. The default node is an imaginary node presenting the stock value after default hence, $S(1 - \mathrm{n})$. The node is imaginary because it is not seen in the tree as only up and downside movements are drawn.

Figure 2: Binomial-Tree with Credit Risk



The upside multiplier u and downside multiplier d for a non-dividend-paying stock are constructed like Cox, Ross, and Rubinstein (1979) propose. As mentioned earlier the default probability p_0 is known at this point. To address the proper upside movement probability p_u and downside movement probability p_d, probabilities in a traditional binomial tree are modified so that the default probability p_0 is deducted from these probabilities as presented in Equations 23 and 24. The sum of probabilities is then equal to 1.

Parameters for constructing the binomial tree assuming Ƞ=1

(21) $$u = e^{\sigma\sqrt{\delta t}}$$

(22) $$d = e^{-\sigma\sqrt{\delta t}}$$

(23) $$p_u = \frac{e^{r\delta t} - e^{-\lambda\delta t} d}{u - d}$$

(24) $$p_d = -\frac{e^{r\delta t} - e^{-\lambda\delta t} u}{u - d}$$

(25) $$p_0 = 1 - e^{-\lambda\delta t}$$

s.t. $$p_u + p_d + p_0 = 1$$

Where u is the coefficient for upside movement and d for downside movement, p_u and p_d are probabilities for these movements, respectively.

After the tree constructing parameters have been defined, the stock tree is constructed. The bond price tree is created using the stock tree working backward from the final nodes. Final node values are presented in Equation 26:

(26) $$Bond(i, Final) = max(C_r * S(i, Final), N * Coupon - \% + N)$$

(27) $$Bond\ Tree(i, j) = max(V, C_r * S\ )$$

Where the V is the European value of the convertible bond, $C_r * S$ is the intrinsic value of the convertible bond and $N\ is\ the\ notional\ value$.

(28) $$Bond(Default, j) = max(X, S(1 - \eta)\ *\ Conversion\ Ratio)$$

Where Ƞ is assumed 1, S is the stock price and X is the recovery value.

The assumption is in this thesis that the stock drops to zero upon the issuer default to avoid making ad hoc decisions about the proper percentage. In the final nodes, the pay-off to the investor is either conversion value or the face value plus the coupon. Note that Equation 28 holds only if the assumed stock price decline is under 100 percent, otherwise the max value is always the recovery value. By now, the final node values of the convertible bond have now been determined, the next step is to look at the derivation of possible portfolio values. During time t the portfolio may possess three different values specified in Equation 29. The diffusion or delta neutrality in the portfolio is achieved by finding the proper Δ or delta, that will ensure that the portfolio should have the same value, not depending on the direction of the stock price movement. Hence, the position is long in the convertible bond and short in the underlying stock. The short position offsets the loss on the convertible bond leg, should the equity price drop and vice versa.

(29) $$\Pi = \begin{matrix} V^+ - \Delta S u \\ V^- - \Delta S d \\ X - \Delta S(1 - \eta) \end{matrix}$$

Where V incorporates the convertible bond value, X incorporates the max(RN, $C_{R*}(1 - \eta)$S). However, in this case, as it is assumed that the stock defaults completely, the maximum value is always the recovery rate *R* multiplied with the bond's notional value *N*.

(30) $$V^- - \Delta S d = V^+ - \Delta S u$$

(31) $$\Delta = \frac{V^+ - V^-}{S(u - d)}$$

Now, when the hedge ratio Δ is used to eliminate the diffusion, Milanov et al. (2013) present portfolio values as 1) non-default state $\frac{V^{+}u - V^{-}d}{u-d}$ arriving at a probability of $e^{-\lambda\delta t}$ and 2) the default-state arriving at a probability of 1 minus $e^{-\lambda\delta t}$. The authors assume, however, that the default risk is diversifiable hence the portfolio value after timestep δt is equal to the risk-free rate. By arranging the terms in Equation 34, the solution is to discount probability-weighted portfolio values to get the convertible bond price, see Equation 35.

(32)
$$\mathbb{E}\left(\prod t + \delta t\right) = \frac{V^{-}u - V^{+}d}{u-d} e^{-\lambda\delta t} + \left(X - \frac{V^{+} - V^{-}}{u-d}(1-\text{ɲ})\right)(1 - e^{-\lambda\delta t})$$
$$= \frac{e^{-\lambda\delta t}u + (1-\text{ɲ})(1-e^{-\lambda\delta t})}{u-d} V^{-} - \frac{e^{-\lambda\delta t}d + (1-\text{ɲ})(1-e^{-\lambda\delta t})}{u-d} V^{+} + X(1 - e^{-\lambda\delta t})$$

(33)
$$\mathbb{E}\left(\prod t + \delta t\right) = \mathbb{E}\prod e^{r\delta t}$$

(34)
$$e^{r\delta t}\left(V - \frac{V^{+} - V^{-}}{u-d}\right) = e^{r\delta t} V$$
$$= \frac{e^{-\lambda\delta t}u + (1-\text{ɲ})(1-e^{-\lambda\delta t})}{u-d} V^{-} - \frac{e^{-\lambda\delta t}d + (1-\text{ɲ})(1-e^{-\lambda\delta t})}{u-d} V^{+} + X(1 - e^{-\lambda\delta t})$$

(35)
$$V = e^{-r\delta t}(p_u V^{+} + p_d V^{-} + p_0 X)$$

Note that, the previous derivation is for zero-coupon convertibles. To find the theoretical price for convertible paying a fixed coupon, the coupons must be added to the proper steps in the tree. Hence, now the possible portfolio values are presented in Equation 36. The coupon payment is only made in the absence of default as concluded in Milanov et al. (2013). After introducing the basic model, the modifications and assumptions for the model are presented in the next sub-section.

(36)
$$\Pi = \begin{matrix} V^{+} - \Delta S u + c_i e^{r(\mathrm{t}+\delta t - t_i^c)} \\ V^{-} - \Delta S d + c_i e^{r(\mathrm{t}+\delta t - t_i^c)} \\ X - \Delta S(1 - \pi) \end{matrix}$$

Where c_i is the coupon of the convertible bond and t_i^c is the moment of the coupon arrival.

(37)
$$V = e^{-r\delta t}(p_u V^{+} + p_d V^{-} + p_0 X) + c_i e^{-r(t_i^c - t) - \lambda \delta t}$$

2.3. Adjustments to the models: Yield curve and credit spread

The model visited in the previous sub-section assumes a flat yield curve. To enhance the model accuracy determining the CB price, a non-flat yield curve is applied. Instead of using just one Treasury rate, the curve is constructed from Treasury securities with tenors from 3 months to 10 years. The Treasury curve sample consists of 8 securities on the curve and missing datapoints are found by interpolating between the known values on the curve. E.g. if the coupon payment is due in 4,5 years, the appropriate riskless discount rate is found between the 3 and 5-year yield. The package containing the interpolation solution is a Python-based Scipy Library.

If a company has many outstanding debt securities that are quoted and traded by many market makers and/or the CDS market is effective on the particular name, the credit spread can be easily observed from the market prices. The spread is the probability of default during a certain period multiplied by the loss on given default. The loss given on default is widely assumed as 40% of the face value. If the bond's payment rank is 1st lien it might be sometimes more or if the bond is deeply subordinated the recovery rate could be zero. As many different trading strategies are being tested and the number of input parameters is relatively large, the credit risk component relies only on the Bloomberg-based (Bloomberg Credit Risk Function or DRSK) synthetic CDS-spread and default probability. The credit-risk component is estimated with a Merton (1974) - based model which takes market cap, debt, and volatility attributes (realized and implied) as inputs (Bloomberg, 2015). The Merton (1974) model is presented in this section to give an indicative explanation of the assumptions underlying the credit risk function. As mentioned in the limitations part, the credit risk assumption can be rather naïve and straightforward but there should be some educated guess where the spread should be given the capital structure and the volatility of the assets.

Like other debt instruments, convertible bonds are subject to credit risk i.e. where the obligor is unable to meet its obligation to service debt. The credit risk is the other major risk involved in the convertible bond alongside the equity risk although they are usually highly correlated. Maybe the most famous credit-risk model is Merton's (1974) model. In this structural model, there are two components, equity, and debt. The debt is assumed as a zero-coupon bond with a face value K due to time T. The firm's value V follows the Geometric Brownian Motion. From Ito's Lemma it can be shown that the log of V also follows Geometric Brownian motion as the Black-Scholes-Merton model proposes. The value of equity at time T, i.e. the maturity of the bond K, is the firm value V minus the bond's face value paid back to the noteholders, and should the firm value V be less than the face value of the bond K, the debtholders take over the firm. The probability of default is modelled by first calculating the present value of equity E_0 and using this extract the present value of the firm V. Once the debt and firm value are known, these are used as inputs in Equation 43 as proposed by the Black-Scholes-Merton model. The volatility of the assets can be derived by solving Equation 45. By using the cumulative probability distribution function, it is possible to get the probability of exercise as $\Phi(d2)$ and the probability of default as $1 - \Phi(d2)$ or as $\Phi(-d2)$. The credit spread required over the risk-free rate by a rational investor is a product of loss given default and the probability of default.

Merton's model (1974)

(38) $$dV_t = \mu V_t + \sigma V_t dW_t$$

Where the firm asset value V follows the Geometric Brownian motion

Or expressed as lognormal

(39) $$dlnV_t = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW_t$$

(40) $$V_t = V_0\left(\int_0^T dlnV_t\right)$$

$$V_t = V_0 exp\left(\sum_{i=1}^{M}\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma dW_t\right)$$

(41) $$E_t = E^P[max(V_t - K, 0)]$$

Where E_t is the equity value at the maturity of the zero-coupon bond K.

(42) $$E_0 = V_0\Phi(d1) - K^{-r(T-t)}\Phi(d2)$$

Where the E_0 is the value of equity today, the V_0 is the value of a firm's assets today and $K^{-r(T-t)}$ is the value of the zero-coupon bond K today.

(43) $$d1 = \frac{ln\left(\frac{V_0}{K}\right) + \left(\mu + \frac{\sigma_V^2}{2}\right)(T-t)}{\sigma_V\sqrt{(T-t)}}$$

$$d2 = d1 - \sigma_V\sqrt{(T-t)}$$

(44)

$$E_0\sigma_E = \frac{\partial E}{\partial V}\sigma_V V_0 = \Phi(d1)\sigma_V V_0$$

(45)

(46) $$PD = \Phi(-\mathrm{d}2)$$

(47) $$Spread\ to\ Treasury\ or\ Swap\ rate = p_D * \mathrm{LGD}$$

Where p_D is the probability of loss and LGD is the loss given default. E.g. if a 1-year zero-coupon bond has a default probability equal to 5% and the assumed recovery rate is 40%, the appropriate spread is 5%*(100%-40%) hence 3% or 300 bps.

To illustrate the pricing, I price two bonds from the sample using the first available market data after the issuance. I apply Tsiveriotis and Fernandes (1998) model as a benchmark for the comparison. Their model is a binomial tree for pricing convertibles with credit risk and is widely used as a reference in post-1998 scientific convertible pricing articles. The pricing model and code for the TF-model (1998) are available at Mathworks.com. The pricing parameters and results are presented in Table 1. The fair values of the example convertible bonds are relatively close to each other as the TF-model prices are 127.278 and 107.758 whereas binomial model prices are 127.420 and 107.850 of the face value, respectively. The prices of the option components are 38.967 and 19.857 reported as bond points, respectively. The price of the option component is the model CB price minus the bond floor. The bond floor is calculated using the yield curve on issue day *t* plus the synthetic spread. Yield to maturity is the yield as if the CB was a straight bond.

Table 1: Convertible Bond Pricing Example

The table presents the model inputs used to price the convertible bonds assuming a face value of $100,000 per bond. The implied hazard rate is derived from the synthetic 5-year CDS spread and the recovery value is assumed to be 40% of the face value of the CB. The call option implied volatility is the 30-day closest out-of-money call option volatility. The stock and CB price tree are 400-step trees. The first settlement price is the first available market price for the convertible bond after the issuance. The convertible bond implied volatility is the volatility figure that makes the convertible bond price equal to the market price when all other variables are kept as constant. The bond floor is the fair value of the fixed income leg, yield to maturity is the yield based on the bond floor and price of the call option is the Binomial model (2013) implied price minus the bond floor reported as bond points.

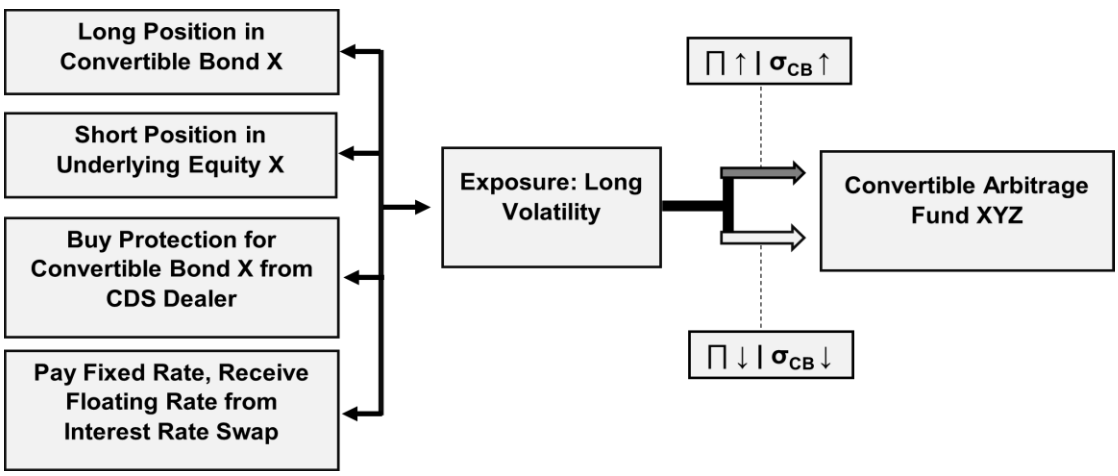
	COMPANY ABC	**COMPANY XYX**
Input parameters		
Minimum subscription size	$100,000	$100,000
Coupon -% p.a / Payment Frequency	1.25% / Semi-Annual	0.375% / Semi-Annual
Maturity in years	5	5
Option Type	American	American
Conversion Ratio	1880 shares	2358 shares
Call Protection Expires	Maturity	Maturity
Stock price on issue day	49.99	32.00
Call option implied volatility on issue day	51.17%	36.49%
Assumed recovery rate	40%	40%
Implied hazard rate on issue day	1.93%	1.03%
Number of steps in the binomial tree	400	400
Model Output and Components		
Binomial model (2013) theoretical price	127.420	107.850
TF-model (1998); Price on Issue Date	127.278	107.758
The first settlement price of the convertible bond	103.703	103.607
Convertible bond implied volatility on issue day	15.30%	30.00%
Bond Floor	88.453	87.993
Yield to maturity (based on the Bond Floor)	3.772 %	2.955 %
Price of the embedded call option (in bond points)	38.967	19.857

3. Convertible Arbitrage

3.1. Strategy description

Convertible arbitrage, one of the most popular market neural strategies among hedge funds, is based on finding mispricing between the equity or debt instrument and the convertible instrument (Loncarski et al., 2009). Maybe the most common and traditional way is to purchase the convertible bond and short sell the underlying stock i.e perform a delta-hedge. Other strategies are based on different risk metrics derived from the convertible bond. The main goal is to achieve attractive risk-return profiles that offer a positive return but have as little downside risk as possible. Arbitrageurs mainly capture the positive income from the coupon payments of the CB and short-sale proceeds. However, some market players look for undervalued convertibles to capture additional profits. By disassembling the bond to debt and option part, investors can spot mispricing of volatility or credit risk. Figure 3 represents an illustrative example of a convertible arbitrage trade set- up. A hedge fund seeking pure exposure to volatility would eliminate the credit risk and interest rate risk by entering to offsetting positions in the derivatives market and simultaneously gaining long volatility exposure by buying the convertible and shorting the underlying stock. The portfolio value $\prod$ increases when the volatility of the convertible's call option σ_{CB} increases and vice versa. Depending on the coupon rate and the underlying dividend, the fund may also gain net carry return from the coupon payment (positive) and underlying stock dividend (negative).

Figure 3: Convertible Arbitrage Trade Set-up, Long Volatility



3.2. Empirical evidence on convertible arbitrage returns and market efficiency

Fabozzi et al. (2009) perform a battery of tests on the law of one price by evaluating multiple trading strategies around convertibles. The full sample consists of 125 convertible bonds issued between 1990 and 2006 in the U.S. markets. Strategies examined include e.g. delta-hedge, gamma-hedge, implied volatility convergence hedge, and credit spread convergence hedge. Fabozzi et al. (2009) use the Black-Scholes-Merton model (1974) as an option valuation framework to determine the Greeks on which the position set up and rebalancing is based on.

The authors conclude that convertible arbitrage is profitable after accounting for transaction costs. Individual trades generated on average a significant 3.99 percent 12-month cumulative holding return at the 95 percent confidence level. Fabozzi et al. (2009) however demonstrate that cumulative returns started to diminish after the arbitrage position had been active for 30 consecutive months as the returns turned negative after 30 months for the complete sample. Although both sub-samples (until 2001 and after 2001) show positive and statistically significant returns, there is a slight indication that absolute returns are smaller for the post-2001- sample. Cumulative returns until 2001 and after 2001 samples were 5.77% and 1.12%, respectively. Only the prior sample indicated statistical significance for the 12-month cumulative return.

By construction, all convertible arbitrage positions are gamma positive, meaning that a larger tilt to any direction should increase the value of the portfolio. Fabozzi et al. (2009) claim that trades involving more gamma exposure and more infrequent delta-hedge rebalancing can lead to larger profits. As evidence, portfolios deriving returns from the larger equity exposure (bull gamma strategies) show 12-month cumulative returns of 4.79% for the complete sample at the highest confidence level.

Hutchinson and Gallagher (2010) examine the return and risk of convertible arbitrage using a sample that includes 503 convertible bonds listed in the U.S. markets between 1990 and 2002. Authors create simulated convertible arbitrage portfolios to study the risk and return in convertible arbitrage. In addition, they use hedge fund indices as a comparison to the simulated arbitrage portfolio. An individual arbitrage position was initially created by buying the convertible bond and shorting the underlying stock. They provide evidence of abnormal risk-adjusted returns that occur in individual hedge fund returns and simulated portfolios when applying the equal-weighted method.

The monthly excess return over the risk-free rate for HFRI Convertible Bond Arbitrage Index was 0.55% with a variance of 0.98 as the simulated portfolio generated an average of 0.33% per month with a variance of 3.1. The authors find that systematic stock risk embedded in the traditional market model (CAPM-based) is significantly positive, but the overall model lacks explanatory power. A traditional three-factor model (Fama and French, 1993) indicates that convertible arbitrage strategy derives return from its exposure to small and value stocks as coefficients were statistically significant and positive. One of the key issues in the argumentation for or against convertible arbitrage alpha is the liquidity risk. E.g. Batta et al. (2010) challenge the view on CA alphas and claim that it is just a product of bearing illiquidity risk. This is also an issue indicated and examined in Hutchinson and Gallagher (2010). The authors find, however, no evidence for the liquidity-based risk exposure. A linear model containing the liquidity factor (low minus high liquidity stocks) indicated coefficients ranging from -0.015 to 0.0079 with no statistical significance. According to Hutchinson and Gallagher (2010), convertible arbitrage strategy is affected most by the credit and term risk. Alpha was not significant for any of the linear models considering the simulated convertible arbitrage portfolio. However, hedge fund indices such as the HFRI Convertible Bond Arbitrage and the CSFB Tremont Convertible Bond Arbitrage indices captured almost the same risk factor loadings but also produced statistically significant alpha ranging from 36 to 50 bps monthly.

Gallagher, Hutchinson and O'Brien (2018) visit convertible arbitrage from hedge fund perspective. They summarize the CA strategy as a non-linear strategy to the risk factors from equity and debt markets. They argue that when equity markets are declining, CA funds tend to outperform indices incorporating common risk factors i.e. returning alpha. The case with bull markets seems to be different as the alpha is diminished. The linear model incorporating Fung and Hsieh (2004) factors reveals that portfolios formed from CA hedge funds produce statistically significant alpha ranging from 0.29 percent to 0.39 percent monthly. Also, a risk-factor model by Agarwal et al. (2011) which includes a delta-hedged portfolio and long-only portfolio of CBs as explanatory factors indicates superior returns produced by hedge-fund managers.

Agarwal et al. (2011) propose that convertible arbitrage hedge funds are acting as intermediaries by buying convertibles (financing the issuer) and using equity markets to hedge away their equity risk (delta-hedging). These CA hedge funds can assume a bigger role than regular mutual funds as they are allowed to use leverage and short-sell stocks which most of the mutual funds are not. Agarwal et al. (2011) construct an asset-based-style model which incorporates the dynamic features of convertible arbitrage. The buy-and-hedge factor is constructed from an issue-size weighted portfolio of convertibles and an offsetting portfolio of corresponding equities sold short to hedge away the equity risk. Authors use a trailing 30-day linear regression to estimate the delta for each CB and estimate the proper hedge ratio. They rebalance the short position daily, if needed. The buy and hedge - factor is includes on average 411 bonds with a current yield of 13% and parity of 69%. Parity is the conversion value expressed as a percentage to the nominal value of the bond. Authors use Vanguard CB mutual fund as a proxy for buy-and-hold style. The model explains 40 to 50 percent of the variation in CA hedge fund returns. Alpha is 0.4 % monthly for CA hedge funds and statistically significant. Agarwal et al. (2011) also specify another linear risk-factor model where the risk-factors incorporate duration and credit risk hedged delta-hedge strategy and the buy-and-hold strategy. The explanatory power of the modified risk factor model is within 30-40 percent range whereas monthly alpha is 0.3%.

The prior research has indicated the existence of excess returns or alphas in the convertible strategies. As mentioned earlier, Batta et al. (2010) challenge the traditional view of convertible alphas. If returns of CA strategy are benchmarked against risk factors such as term structure, equity risk, credit risk, and so on, the strategy has in many cases outperformed against its exposure. They show that if the liquidity factor is included as an explanatory factor, alphas are significantly reduced. This would indicate that alpha is nothing but a product of bearing the liquidity risk. However, they also claim that preceding would be true for off-the-run convertibles but convertibles that were issued recently were more liquid and arbitrage profits could be a result of volatility mispricing. There is also another implication made by Agarwal et al. (2011) considering the supply of convertible bonds. Adding the supply factor to the regression model changes the alpha sign to negative as the supply factor shows positive loading with the highest confidence level. This indicates that the overall CA hedge fund industry is relying on the issuance of convertibles bonds. A lesser amount of bonds to invest in reduces the opportunity space for convertible funds and contributes to the overall industry return.

For a sample consisting of convertibles issued between 1990 and 2007 Loncarski et al. (2009) present various explanations for diminishing returns in CA. First, popular explanations for diminishing returns are stable equity markets, rising interest rate level, withdrawals from arbitrage funds, and increased competition in the hedge fund industry. Although the CBs were under-priced in the timespan between 1990 and 2007, companies issuing convertible securities have started to purchase stocks around the issuance to stop losses to regular shareholders caused by hedge funds shorting the stock. A similar finding is presented also by Werner (2010) who finds that arbitrage-based short-selling has taken place around companies issuing convertible debt. Companies have started to combine convertible issuances with stock repurchases to lower the discounts of the issuance and reduce the short-sell pressure.

Batten et al. (2018) visit the convertible bond pricing efficiency theme in their study consisting of roughly 96 bonds from 2004 to 2011. They find that on average convertible bonds trade 6.31% lower than the model prices. Bonds with equity-like features such as high delta are found to be more efficiently priced. Deep out-of-the-money convertibles that are more sensitive to model inputs such as recovery rate and credit spread were found to be less efficiently priced. In addition, Batten et al. (2018) point out that liquidity affects mispricing significantly. Liquidity is measured by the issuance size and oversubscription at the issuance.

4. Data and Strategy Implementation

In this section, the used methodology is described in detail. This includes the data description alongside the trade and portfolio construction methodology.

4.1. Description of the data

The convertible bond deal data is from Thomson One M&A Database. As the scientific research coverage is mainly prior to 2012, the initial data search is performed on convertibles issued between 2013 and 2018. All convertibles used in this thesis are traditional convertible bonds exchangeable to common equity. All convertibles in the final sample are fixed coupon bonds with a maturity date. Zero-coupon convertibles are excluded as CA funds typically avoid zero-coupon CBs as the cashflows from coupon payments are an important part of the trade (Loncarski et al., 2009). Convertible bonds with special features such as perpetual maturity, call or put provisions, or mandatory conversions are excluded. The similarity of convertible bonds allows the usage of universal hedging methods over the entire sample. The minimum amount of proceeds plus overallotment sold to investors is set to $ 100M to ensure the liquidity and robustness of results. Common equity underlying the convertible bonds included in the sample had to trade at NASDAQ or NYSE marketplace during the sample and test period. The initial convertible bond sample consisted of 284 convertible bonds. All main terms such as the final maturity date, coupon rate, and conversion ratio were also collected from the Thomson One M&A Database. The term sheet was manually double-checked from either SEC Database or the issuer's Investor Relations website. Any issuances lacking or having contradictory information have been excluded from the sample. The final number of convertible bonds was 159 after some of the bonds had to be excluded due to missing deal or market data. In cases, where the stock had been split, the conversion ratio and conversion price were calculated using the conversion premium provided in the bond issuance prospectus.

Table 2 presents the bond sample statistics grouped by the GICS Sector. The average maturity for convertible bonds in this sample is 6.16 years with an average coupon of 2.65%. Convertible bonds had an average of $ 322M face value sold to investors and an average conversion premium of 30.92%. Early-stage or low valuation healthcare and technology companies are particularly active in the convertible bond market as they can obtain cheaper debt financing through convertible securities as they would from regular bond issuance or IPO due to higher risk of insolvency and business risk reasons. Although these companies are risky through either strong regulation (healthcare) or uncertainty related to technology yet left to monetize, a long-term call option may be valuable and attractive to investors as a speculative bet. Both sectors also possessed on average higher conversion premiums than the full sample average meaning that the implied strike price of the bond is higher than the current share price. All the equity market data is reported in Table 3. Columns include average stock price, dividend yield per annum, daily traded volume, and implied and historical volatilities, respectively. The daily traded volume is reported in millions of shares. The implied volatility is calculated from the 30-day closest out-of-the-money call option and the historical volatility is calculated as a trailing 252-day historical volatility using the daily stock price data. All volatility measures are reported as annualized values. The term structure of interest rates and synthetic spread statistics used to derive the fixed income value of the convertibles are reported in Table 4. The historical Treasury rates and the number of active positions in the simulated convertible arbitrage portfolio are presented in Figures 4 and 5, respectively. The most active period was 2013 to 2015 when the maximum number of positions in the simulated CA portfolio was almost 60 convertible bonds.

Table 2: CB Deal Statistics

The table provides the convertible bond data of the sample sorted by the GICS Sector. All reported values are averages excluding the number of the bonds.

	Number of Convertible Bonds	Average Maturity*	Average Conversion Premium -%	Average Coupon -%	Average Amount Issued in $ M
Sector					
Consumer Products and Services	11	5.97	28.49	2.13	236
Consumer Staples	2	5.61	28.75	1.63	329
Energy and Power	6	5.09	26.14	2.98	142
Financials	14	5.69	25.34	3.80	218
Healthcare	44	6.82	38.41	2.35	306
High Technology	38	6.54	32.20	1.38	426
Industrials	14	5.73	39.18	2.45	454
Materials	4	6.66	31.76	2.81	281
Media and Entertainment	3	5.74	45.00	2.46	600
Real Estate	16	5.32	17.48	4.39	253
Retail	4	6.32	27.48	1.69	331
Telecommunications	3	8.45	30.83	3.75	288
Total/ Average*	**159**	**6.16***	**30.92***	**2.65***	**322***

Table 3: Equity Market Data

The table provides the equity market data of the sample. The average traded volume is reported in millions of shares and market capitalization is reported in billions of dollars, respectively. The average IV is the 30-day closest out-of-the-money implied volatility and the average historical volatility is the 252-day trailing volatility calculated using the historical stock prices.

	Average Stock Price	Average Dividend Yield p.a.	Average Traded Daily Volume	Average IV	Average Historical Volatility	Average Market Cap
Sector						
Consumer Products and Services	40.24	0.07 %	0.78	45.61 %	41.53 %	9.60
Consumer Staples	100.92	2.94 %	0.16	60.92 %	44.10 %	1.61
Energy and Power	19.21	0.31 %	1.72	65.46 %	53.16 %	1.53
Financials	25.77	6.72 %	1.14	35.51 %	32.93 %	1.44
Healthcare	44.02	0.25 %	1.39	56.92 %	51.00 %	2.58
High Technology	42.38	0.43 %	4.74	44.18 %	43.67 %	6.41
Industrials	34.95	0.50 %	9.91	50.66 %	44.28 %	9.24
Materials	27.83	4.37 %	5.78	46.79 %	46.75 %	3.41
Media and Entertainment	34.80	0.62 %	1.82	37.90 %	31.54 %	11.10
Real Estate	13.73	6.58 %	1.01	30.70 %	26.49 %	1.35
Retail	46.88	0.00 %	1.09	56.10 %	54.29 %	2.74
Telecommunications	24.01	0.64 %	1.23	78.15 %	58.20 %	1.39
All-sector average	**37.90**	**1.95%**	**2.56**	**50.74%**	**43.99%**	**4.58**

Table 4: Rates and Spreads Statistics

The table provides the rate and spread statistics used in the pricing of the convertible bonds. Panel A contains the descriptive statistics of the U.S. Treasury rates sorted by tenor. Panel B contains the synthetic CDS spread descriptive statistics sorted by year. Panel B contains all the bonds in the sample.

Panel A: U.S. Treasury Rates

	3-month	6-month	1-year	2-year	3-year	5-year	7-year	10- year
Mean	0.80 %	0.89 %	0.98 %	1.18 %	1.38 %	1.76 %	2.06 %	2.31 %
Std	0.87 %	0.88 %	0.87 %	0.79 %	0.69 %	0.53 %	0.45 %	0.42 %
Min	-0.02 %	0.02 %	0.07 %	0.20 %	0.29 %	0.65 %	1.07 %	1.35 %
Max	2.46 %	2.56 %	2.74 %	2.97 %	3.04 %	3.09 %	3.18 %	3.24 %

Panel B: Synthetic CDS-Spreads

	2013	2014	2015	2016	2017	2018	2019	2020
Mean (bps)	172	157	201	298	231	229	231	256
Std (bps)	13	12	22	26	10	7	15	27
Min (bps)	147	142	180	264	220	221	215	214
Max (bps)	197	185	267	349	266	260	262	302

Figure 4: Historical Treasury Rates

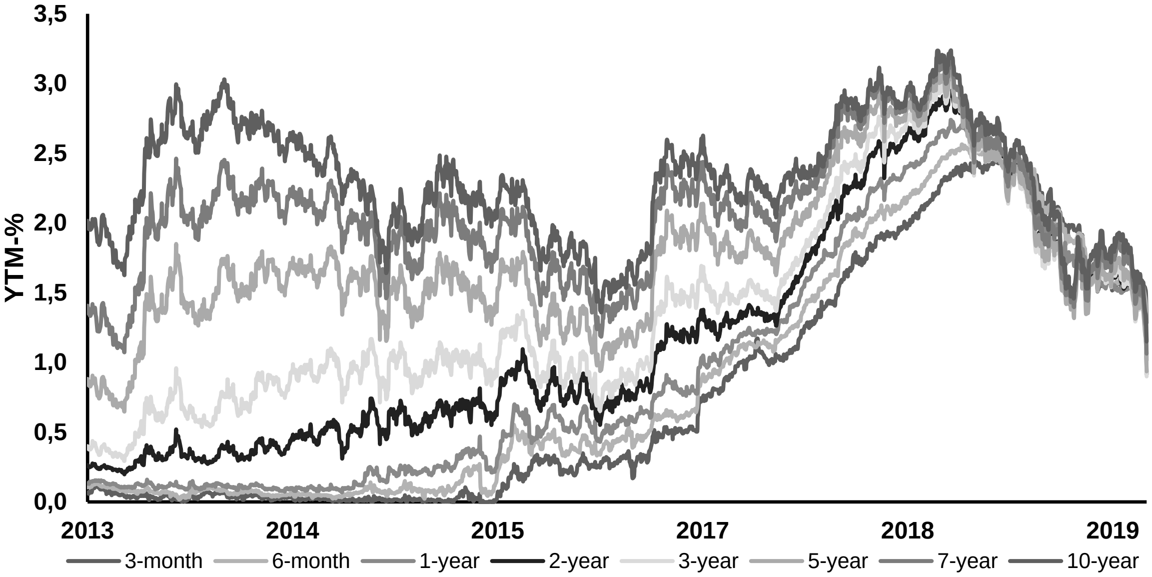
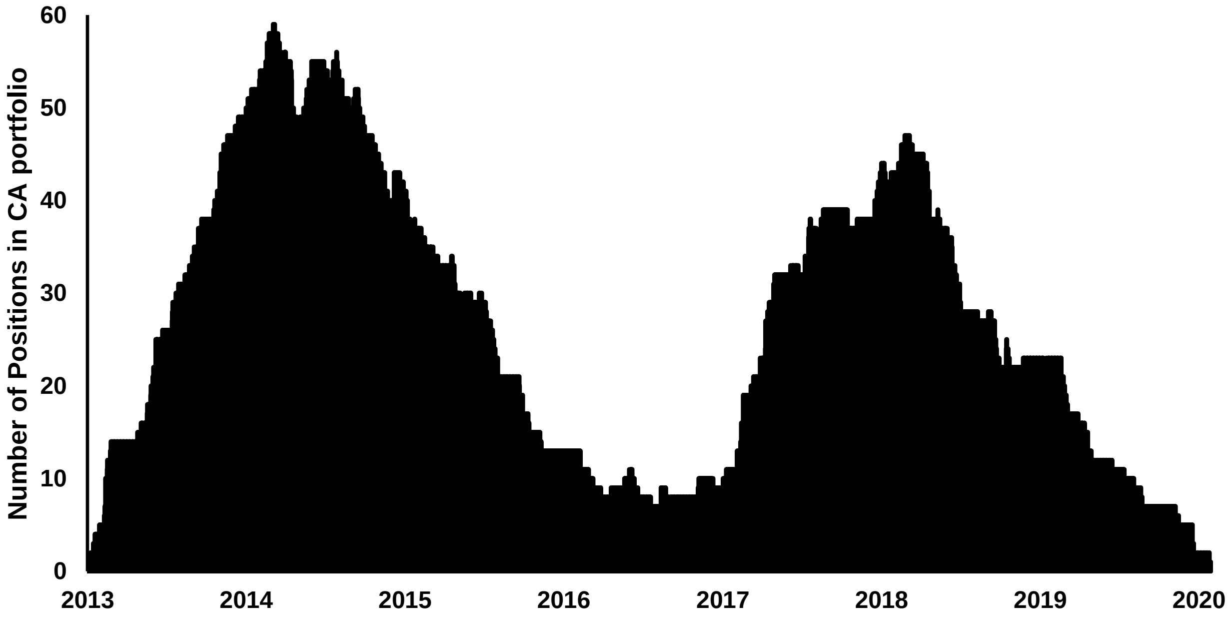


Figure 5: Number of Positions in CA portfolio



4.2. Convertible arbitrage trading methodology

Similar to e.g. Hutchinson and Gallagher (2008, 2010) and Fabozzi et al. (2009), the proposed methodology is to carry out simulations of convertible arbitrage trades from the first available market price up to 14 months. A long investment equal to $1,000,000 is applied to all convertibles at the first settlement price. The position is alive for 14 consecutive months, assuming no default-event occurs, and returns include price return of the CB and underlying stock and net carry interest from the coupon and dividend. The position is closed after 14 month holding period at the prevailing CB's market price. The binomial model by Milanov et al. (2013) serves as the framework for deriving proper hedging metrics for the convertible arbitrage trades. Investment strategies modelled and tested in this thesis are linear (delta) and non-linear (gamma) strategies set up between the CB and the underlying equity.

4.2.1. Delta-hedge

Delta-neutral portfolio is the proxy portfolio of CA in this thesis. The position consists of a long position in the CB and a short position in the company's equity. The idea of the delta-hedged CB position is to neutralize the position value from small changes in the underlying stock price while capturing income return from the coupons and non-income return from the long vega exposure.

The position is opened at the first available bond trading price in this study rather than assuming a bid in the issuance and eventually buying the bond at par. The amount invested in each CB position is $1,000,000. Simultaneous to the CB purchase, the underlying stock is sold short. The binomial tree used to derive the fair value of a convertible bond is applied in the delta estimation as depicted in Equation 48. To initiate a delta-hedged position for each convertible bond on the first trading day of the CB, the appropriate hedge ratio δ is determined by multiplying the conversion ratio C_r with the corresponding delta $\Delta_{Binomial,\mathrm{t}}$ (Hutchinson and Gallagher, 2008). This ratio determines how many shares are sold short against a particular CB on trading day t. On the following day, a new hedge ratio is estimated that is, if $\Delta_{Binomial,\mathrm{t+1}} > \Delta_{Binomial,\mathrm{t}}$ shares are sold or if, $\Delta_{Binomial,\mathrm{t+1}} < \Delta_{Binomial,\mathrm{t}}$ shares are purchased in order to maintain a delta-neutral hedge. The cash flow return is captured from the coupons minus the dividend. The CA position is closed after a maximum holding period of 14 months due to several reasons. First, Fabozzi et al. (2009) show positive returns for the first 15 months of the delta-neutral position. The second argument has to do with liquidity. Batta et al. (2010) claim that the closer the initial issuance, the higher the liquidity of both stock and bond. Similar arguments are presented also by Marle & Verwijimeren (2017) claiming that hedge funds keep arbitrage positions open for approximately 1 year.

Delta estimation

(48)
$$\Delta_{Binomial,t} = \frac{\partial CB}{\partial S} = \frac{CB_{1,1} - CB_{1,0}}{S_{1,1} - S_{1,0}}$$

Where Δ is the delta in the specified model framework on a trading day *t*.

4.2.2. Modified delta-hedge

As mentioned earlier in the hypothesis section, according to Hutchinson & Gallagher (2008) and Calamos (2003), the daily rebalancing of the CA trade is usually ignored by hedge funds due to transaction costs and also the possible inaccuracies in the delta-estimation. Also, as Ammann & Seiz (2006) conclude, thinly traded and deep OTM convertibles might not follow the underlying stock price very accurately. In a world with no market frictions in buying or selling assets, the position should be rebalanced at a daily frequency. If there are transaction costs that are paid, directly or indirectly, the position value decreases a small amount every time stocks are bought or sold to maintain the hedge. To observe the impact of larger delta-tolerance on total profits, I construct portfolios that use the same deltas as the regular delta strategy but are subject to 2, 5, and 10-unit delta tolerance rules. The short position is rebalanced only when the change in delta is larger than mentioned thresholds. In this thesis, three different rebalancing rules are applied.

(49)
$$Rebalance\ if\ abs(\Delta_t - \Delta_{t-rb}) > Threshold$$

Where threshold is $\begin{cases} 0.02 \\ 0.05 \\ 0.1 \end{cases}$, Δ_t is the delta on trading day *t* and Δ_{t-rb} is the delta of the last rebalance date.

4.2.3. Gamma capture hedge

As the delta-hedging focuses on the linear exposure elimination and is dynamic, gamma strategies are trying to derive alpha from positions set up on the non-linear exposures. A regular gamma-hedge means that non-linear securities are added to the portfolio so that if the underlying asset price changes, the delta stays unmoved. Inherently delta-neutral positions set up around convertibles have always positive gammas meaning that a tilt to either bearish or bullish direction should increase the value of the portfolio. Gamma capture hedges in this section take speculative positions meaning that the short position is left either lower or higher than in regular delta-hedge position.

A regular hedge fund manager would build a gamma position by combining the long position in the convertible and shorting the stock but initially taking a directional bet on the movement of the underlying stock. Compared to the regular delta-hedge, the short position would be adjusted so that the position would gain extra profits if the stock price declined (bearish gamma) or the stock price increased (bullish gamma). Following partly Fabozzi et al. (2009), gamma positions in this thesis are set up on the assumptions of the vanilla delta-hedge but the delta is 9 or 14 units lower for bullish gamma hedges and vice versa.The return profile of the gamma capture hedge to the change in underlying stock price is presented in Figure 6.

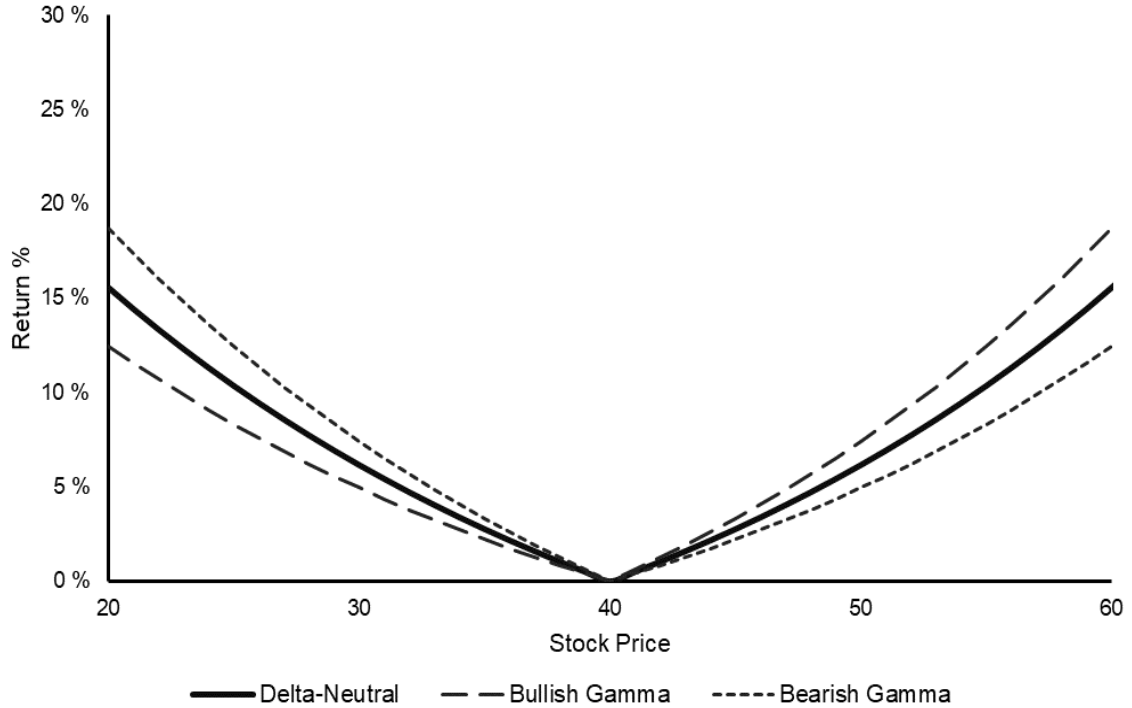
Gamma estimation

(50)

$$\Gamma_{Binomial} = \frac{\partial^2 CB}{\partial S^2} = \frac{\Delta_{uu-ud} - \Delta_{ud-dd}}{(S_{uu} - S_{dd}) * 0.5}$$

Where Γ is the gamma.

Figure 6: Return Profile of Gamma Capture Hedge



4.3. Return calculation and position mark-to-market

Following Fabozzi et al. (2009), long CB returns are calculated as the price change of the bond plus the accrued interest on a trading day *t*. Outflows are the cost of borrowing and dividends. A dividend on a given stock *S* is paid to the shareholder at a daily frequency. The dividend yield is an estimated 1-year dividend yield on a given stock *S*. The method used to calculate CA trade return is adapted from Hutchinson and Gallagher (2008). The return calculation method is presented in Equation 51. The position return is then equal to the price change of the bond and the stock plus the net carry return from the coupon and dividend payable. The short position is not marked as an invested capital, as it is assumed that the CA fund could use the CB as collateral.

(51)
$$R_{it} = \frac{P_{it}^{CB} - P_{it-1}^{CB} + C_{it} - \delta_{it-1}(P_{it}^{S} - P_{it-1}^{S} + D_{it}) + r_{t-1}S_{i,t-1}}{P_{it-1}^{CB}}$$

Where R_{it} is the return of the convertible arbitrage position on day *t*, P_{it}^{CB} is the closing price of the convertible on day *t*, C_{it} is the coupon received on day *t*, δ_{it-1} is the number of shares sold short on day t, D_{it} is the dividend payable on day t and $r_{t-1}S_{i,t-1}$ is the proceed from the short sale. The assumption is that the CA fund could use the bond as collateral on the short sale, therefore limiting the initial investment to the purchase of the bond.

(52)
$$C_{it} = \frac{C_i}{252} * FV$$

Where the C_{it} is the accrued interest during trading day t multiplied with face value FV.

(53)
$$D_{it} = \frac{D_i}{252} * \delta_{it-1}$$

Where the D_{it} is the dividend payable during trading day t multiplied with the number of shares sold short δ_{it-1} .

4.4. Transaction costs

To enhance the accuracy and robustness of the results, all trades are subject to transaction costs. Convertible bonds are traded OTC meaning that the regular way to estimate or gather transaction costs does not apply. To be able to obtain at least on some level precise transaction cost estimates, one would have to i.e. estimate the network size of the CB dealers and obtain reference prices from a major bond dealer (Hendershott, Li, Livdan and Schurhoff, 2020). This is beyond the scope of the study so I perform a sensitivity analysis on the CB leg of the trade and estimate when the returns converge to zero. E.g. Landschoot (2008) shows that bonds maturing in 5-7years with an average face value of $606M and a rating equal to BBB are on average quoted at 37 bps bid-ask spread. I assume this figure in the base case scenario for all the CA trades. Simply put, if the CB's mid-price is 100, the simulated CA fund buys the bond at 100.185 and sells it at 99.815, ceteris paribus.

I estimate the transaction costs for the equity leg using bid-ask spread, market impact, and cost of borrowing securities or cash. The market impact law says that less liquid stock, higher indirect transaction costs as the limited amount of sellers or buyers causes the execution price to slip to unfavourable direction. Frazzini, Israel and Moskowitz (2012) show that the price impact of short selling is not statistically different from the price impact of selling long-owned securities. E.g. Mitchell and Pulvino (2001) estimate indirect transaction costs with a similar method in the merger arbitrage context. The method in this thesis replicates the solution implied e.g. by Toth, Eisler, and Bouchaud (2016), see Equation 54.

(54)
$$\Delta p(Q) = \varepsilon(S_{BA}) + \varepsilon Y \sigma_D \sqrt{\frac{Q}{V_D}}$$

Where $\Delta p(Q)$ is the weighted average of the execution cost expressed as a percentage, $\varepsilon\ indicates\ buy(1)/sell(-1)\ order$ S_{BA} is the spread cost per trade, σ_D is the daily asset volatility, Y is a numerical constant of order unity, Q is the trade size and V_D is the total volume traded on trading day *t*.

In financial theory, short-selling is usually portrayed as costless and short-selling earns risk-free profit, see e.g. Fama (1965) and Ross (1976). However, the short-sellers face transaction costs directly and indirectly. Direct transaction costs are market-clearing loan fees and short interest on the market-clearing loan balance (D'Avolio, 2002).

CA funds often use leverage in their operations to enhance the returns. E.g. Agarwal et al. (2011) assume that the long position in the CB is financed by borrowing at the Fed Funds rate and the cash balance earns interest but at a lower rate, hence the Fed Funds rate minus a haircut of 50 bps. Funds may use the repo desks to enter trades without committing a large amount of their capital. However, as the convertible bond is subject to credit risk, the repo-dealer might require a relatively large haircut on the trade. According to FINRA (2020), the capital requirement for buying convertibles on margin is 15 to 25 percent of the market value if the bond is not close to default and trades close to the par value or higher. I do not apply borrowing on the convertible bond in this thesis but all trades are levered so that the initial investment is limited to the purchase of the bond. A cash collateral (or Treasury securities) in short position is usually around 102% of the market value of the short position (see e.g. D'Avolio, 2002) and settled daily. The number of shares shorted is in this thesis always less or equal to the conversion ratio but never greater. The market entity could always 1) sell the CB at the prevailing market price or 2) convert the CB to common shares and cover the short position.

D'Avolio (2002) claims that on average, a value-weighted proxy portfolio consisting of liquid stocks faces a 25-bps loan fee per annum in the U.S. markets. The cost of financing the short is subject to a net interest of 50 bps per annum in this thesis. The 50-bps assumption is rather conservative against the findings of D'Avolio (2002) but as most of the stocks in this thesis' sample are not included in the most liquid indexes (S&P 500, Nasdaq Composite), the 25 bps may not be valid. The effect of transaction costs on the position leg is carried out in the following way:

- Assume the underlying stock trades at $20.00 and the number of shares sold is 10,000 shares corresponding to a market value of $200,000. The intraday volatility of a particular stock is 1% whereas the total amount of traded volume is 100,000 shares.
- The fund pays stock-related transactions costs that are
  - 30 bps from the market impact. See Equation 54.
  - 30 bps is deducted from the selling price, that is the fund sells the shares on average at $19.94.
  - Assume that stock price stays unmoved during the life of the position, let's assume this a 1-year tenor, the fund pays 50bps on the financed amount, hence 50bps multiplied by $200,000 ≈ $1,000

4.5. Case study of Tesla

To gain a more comprehensive understanding of the strategy implementation and dynamics, a short case study of a convertible arbitrage trade set up on a Tesla's convertible is presented next. Tesla is an electric car manufacturer based in the U.S. The company issued a convertible bond in May 2013 carrying a fixed coupon of 1.5% with a 5-year maturity. Tesla's stock trades around $19.42 on the day the position is opened. In the first step, the delta is estimated with a 200-step IV-calibrated binomial model that yields a value around 0.75.

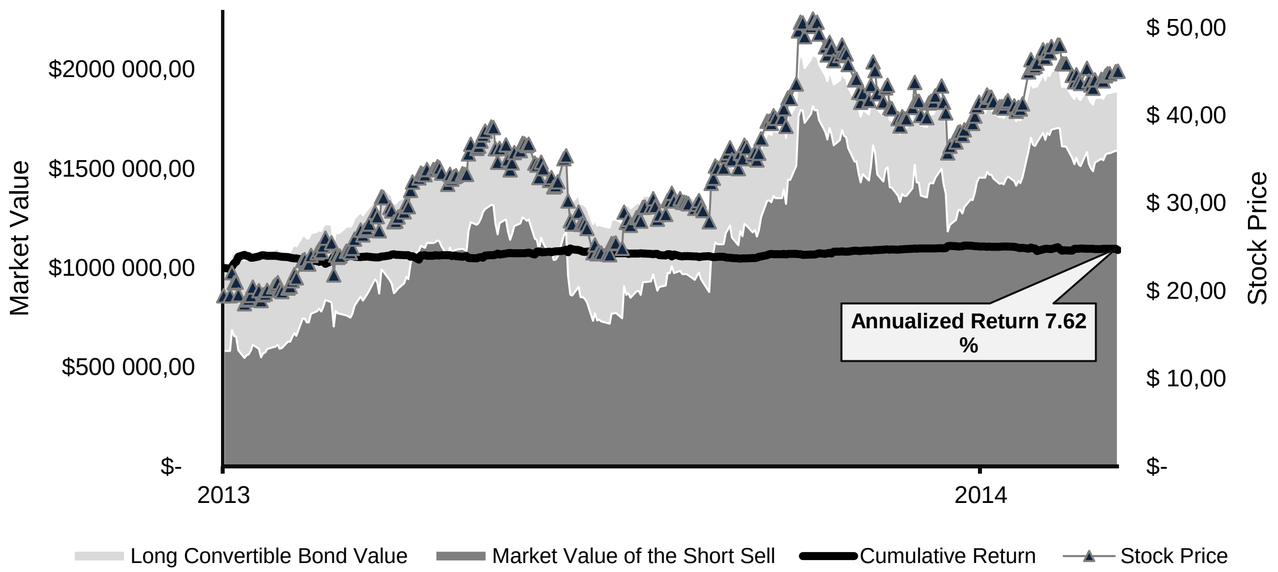
The fund invests $1,000,000 in the CB that is trading around 101% of the face value.[3] A notional value of $100,000 is convertible to approximately 4000.14 of the underlying shares[4]. To

[3] The fund is a simulated and fictional CA fund used as an example in this case study. Note that, this example is a simplified version, and some of the figures are rounded to nearest thousand to give a more straightforward illustration.

[4] Note that the stock price and the conversion ratio have been adjusted for the possible splits, that is, the conversion ratio reported in this example is not equal to the conversion ratio reported in the original Term Sheet. As the stock price is adjusted with respect to the possible splits, the conversion ratio, as depicted in Section 4, was calculated using the initial conversion premium. To be precise, the Tesla's stock price traded on 16th of May 2013 at $92.24 at NASDAQ marketplace as specified in the issuance prospectus. The conversion price specified in the issuance prospectus was $124.52 (35 % in terms of the conversion premium) and the conversion ratio was 8.306 per $1,000 of the notes. In the light of these facts, the conversion ratio indicated in this example is equivalent, as the split adjusted price in this thesis is 18.45 (price on 16th of May 2013) that multiplied with 4000.14 shares corresponds to around 74% of the $100,000 notional. This is equivalent to (8.0306x92.24)/1,000 ≈ 74% of the notional value as specified in the original Prospectus.

hedge the CB leg, the fund enters a short position equal to 29,700 shares. [5] To illustrate the rebalancing aspect, the following example is outlined. The stock price jumps to $22.067 on the following day after the position has been opened, the delta changes to 0.79 and the fund sells additional 1600 shares to rebalance the position. As the underlying stock performs rather well after the issuance until September 2013, the short position is more or less, increased. As a comparison, the market value of the short position is around $1.29M in September 2013, but as the delta declines, the fund buys more shares to adjust to the lower delta. In Figure 7, the return of the trade alongside the value of the long CB leg and short stock leg are depicted. The trade yields approximately 7.62 percent on annualized basis. The fund receives a gross carry return from coupons equal to approximately $17,325 throughout the trading period.[6]

Figure 7: Simulated CA trade using the delta-hedge approach with Tesla's convertible

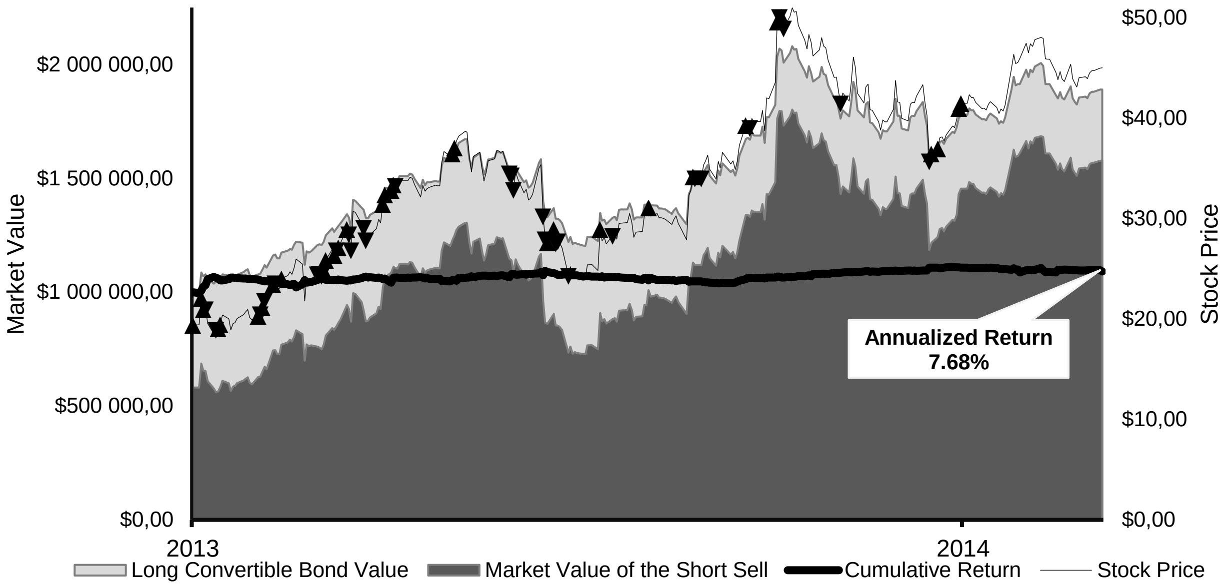


[5] ~39600 (number of shares included in the CB position) x ∆ of 0.75 ≈ 29700

[6] The CB carries a fixed coupon equal to 1.5% per annum. The fund has an exposure around $0.99M on notional terms, corresponding to $14,850 in terms of coupon cashflow per annum. The company does not distribute any dividends when the CA position is active. The fund holds the convertible bond for 14 consecutive months, hence the coupon cashflow is equal to 14/12 x1.5% x $990,000.00 ≈ $17,325. That is, the actual coupon cashflow paid in semi-annually and the accrued interest when the position is sold.

Let's consider the modified delta strategy. The hedging is carried out in a similar manner, but the short position is not balanced until the delta change is more than 2-points in absolute terms. The modified delta-strategy is depicted in Figure 8 using the 2-unit rule. The position is initially opened with a same long and short investment as in previous example in Figure 7. The rebalancing is marked with ▲ and ▼ that indicate increasing the short position and decreasing the short position, respectively. If the regular delta strategy would be depicted, these markers would appear daily, in this case, there is only limited number of rebalancing points when the delta has moved over the threshold of 2. The trade yields 6 bps over the regular rebalancing strategy.

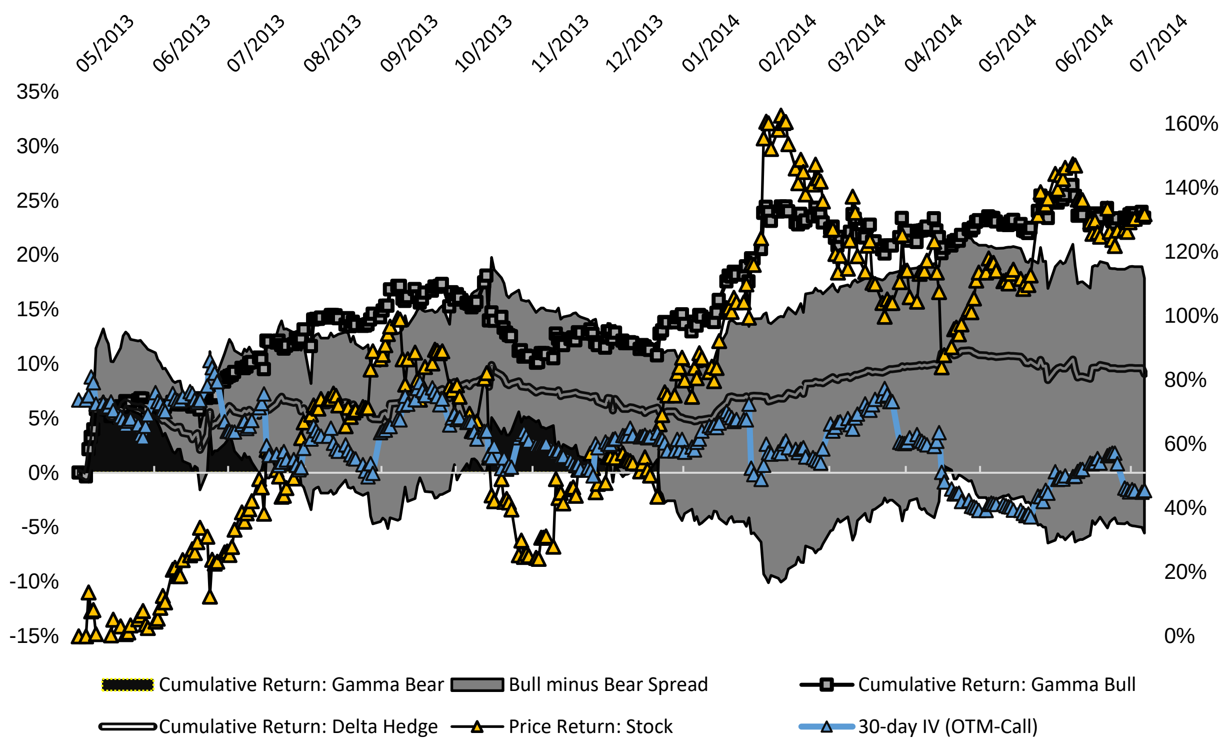
Figure 8: Modified Delta-Hedge Example



To illustrate the more aggressive strategies (bull and bear gamma), the returns of the 14-gamma strategies are depicted in Figure 9 alongside with the delta-hedge strategy return, underlying stock price return and the implied volatility. The position is opened with an investment equal to $1M. The hedge ratio diverges from the delta-hedge strategy as the number of shares shorted is ~24,150 and ~35,245 in bull gamma and bear gamma strategies, respectively. Considering the base case of the delta-hedging, the delta is now 14 units lower (higher) in bull (bear) gamma strategies The estimated delta is 0.75 and the directional gamma exposure is initiated by selling less (more) shares short in bull (bear) gamma strategies.

Positions are rebalanced at a daily frequency by either selling or buying more shares as depicted in the first example, but the hedge ratio is always lower (higher) in bullish (bearish) gamma position. E.g. the opening hedge ratio in the bullish gamma trade is 0.75 minus 0.14 multiplied with the conversion ratio. Both gamma strategies yield positive return for the first month as the stock price stays more or less unmoved. Tesla's share starts to gain value after 1-month holding period and the spread between gamma strategies starts to increase as the bullish gamma benefits from the larger equity exposure. The stock price declines from September 2013 until the end of November 2013 turning the bearish gamma strategy profitable for the next month. However, as the stock price continues to perform rather robustly, the bearish gamma strategy generates negative return after January 2014.

Figure 9: Strategies combined



5. Empirical Results

The empirical part is divided into two main sections. In the first part, the strategy returns are analysed at individual trade level. All individual trades are calculated on the basis of the long investment equal to $1,000,000 and presented as cumulative return on the long investment. Returns are then aggregated to monthly values and their statistical significance is reported. In the second part, the return and risk of the strategies are presented at portfolio level. To find out, how CA strategies behave and perform against equity and fixed income market factors, a regression analysis is performed. A sensitivity analysis is performed at the end of this section to find out when the possible excess returns converge to zero.

5.1. Individual trade analysis: Return and Risk

Following Fabozzi et al. (2009), the individual trade analysis starts with the regular delta-hedge strategy. Individual trade returns of the regular vanilla delta-hedge strategy are reported in Table 5. The table is sorted by the volatility used to calibrate the model. The average returns, t-statistics, and the number of positive and negative trades are reported for each trade type.

Table 5: Delta-Hedge Trade Returns

The table provides the average returns, t-values, and the number of positive and negative return trades, respectively. The returns are calculated as a cumulative return on the long investment of $1M per trade. Returns are not annualized. The table is sorted by the volatility used in the model calibration, 30-day implied volatility on the left side, and vice versa. Significance is marked as ***,**, and *, for confidence levels of 99%, 95%, and 90%, respectively.

	30D IV				252D Historical volatility			
FULL SAMPLE *Delta-hedge*	Returns	T-Value	Positive	Negative	Returns	T-Value	Positive	Negative
Months								
1	-0.03 %	-0.10	75	83	0.02 %	0.06	79	84
2	0.34 %	0.92	98	60	0.38 %	1.09	104	59
3	0.98 %	2.35**	105	53	1.03 %	2.52***	114	49
4	1.23 %	2.97***	109	49	1.38 %	3.38***	115	48
5	1.47 %	3.04***	107	51	1.61 %	3.33***	113	50
6	2.22 %	4.69***	111	47	2.38 %	4.92***	117	46
7	2.74 %	5.88***	117	41	2.96 %	6.36***	121	42
8	3.14 %	6.94***	118	40	3.42 %	7.42***	124	39
9	3.14 %	6.14***	110	48	3.47 %	6.76***	120	43
10	3.56 %	6.18***	115	43	3.96 %	7.03***	123	40
11	3.12 %	5.61***	110	48	3.53 %	6.32***	119	44
12	3.42 %	5.73***	116	42	3.85 %	6.51***	126	37
13	3.85 %	6.00***	116	42	4.38 %	6.89***	129	34
14	3.86 %	5.81***	118	40	4.10 %	6.27***	130	33

Results indicate that the delta-hedging strategy yields statistically significant positive returns after two months from opening the position. However, the first month is not significant either economically or statistically yielding approximately zero returns on average. The market impact of selling stocks short can be larger in this instance as the underlying stocks usually have rather a low trading volume compared to liquid names included e.g. in the SP500 index. The costs are largest at initiation as the CA fund has to sell a large number of shares compared to the small adjustments later as the delta changes. The 12-month return (IV) is approximately 57 bps below compared to Fabozzi et al. (2009) reporting a 12-month return as 3.99% at the 5% significance level. The 12-month historical volatility calibrated trades yield 3.85%.

The results of the modified delta strategy are presented in Table 6. The table is divided into two parts, the results of the implied volatility calibrated binomial model are on the left side and the historical volatility calibrated results are on the right side. Panel A depicts the 2-unit delta-tolerance portfolio, Panel B the 5-unit delta-tolerance portfolio and Panel C the 10-unit delta-tolerance portfolio, respectively. There is a minor return advantage in modified delta-portfolios over the regular strategy. When moving to portfolios assuming a larger equity exposure, the returns continue to increase. E.g. the 12-month cumulative return of the vanilla portfolio (IV) is 3.42 percent versus the 3.81 % return of the 10-delta strategy (IV). Fabozzi et al. (2009) perform a similar analysis and find that returns are increased by 1-2% percentage units compared to the vanilla strategy when the delta tolerance is increased. By construction, a convertible bond offers an asymmetric risk-return profile on its own without any short-selling required. Although the convertible bond price is driven partly by the stock price, it has fixed payments and it ranks better in a case of insolvency, assuming no subordination of the notes. When the position is rebalanced more infrequently, the trade benefits from the larger equity exposure as the upside equity potential is exploitable and the cumulative costs relating to hedging the CB leg are lower.

The results of bullish gamma strategy are reported in Table 7. The bullish gamma trades show statistically significant returns for a 12-month holding period for both 9 and 14 portfolios (IV) with returns of 4.46% and 5.02%, respectively. Compared against the regular and modified delta strategy returns, the bullish gamma hedge produces higher cumulative returns on 12 month holding period. The bullish gamma has a larger equity exposure than the delta-hedge portfolios usually have.

Table 6: Modified Delta-Strategy Individual Trade Returns

This table provides the average returns, t-values, and the number of positive and negative return trades, respectively. The returns are calculated as a cumulative return on the long investment of $1M per trade. Returns are not annualized. The table is sorted by the volatility used in the model calibration, 30-day implied volatility on the left side, and vice versa. Significance is marked as ***,**, and *, for confidence levels of 99%, 95%, and 90%, respectively.

	30D IV				**252D Historical volatility**			
FULL SAMPLE	Returns	T-Value	Positive	Negative	Returns	T-Value	Positive	Negative
Panel A: 2								
Months								
1	-0.05 %	-0.16	71	87	0.01 %	0.04	77	86
2	0.35 %	0.94	97	61	0.38 %	1.10	101	62
3	1.00 %	2.40**	106	52	1.02 %	2.53**	115	48
4	1.24 %	3.02***	108	50	1.37 %	3.40***	114	49
5	1.47 %	3.07***	105	53	1.60 %	3.35***	113	50
6	2.24 %	4.75***	111	47	2.38 %	5.00***	118	45
7	2.75 %	5.89***	116	42	2.96 %	6.46***	119	44
8	3.16 %	6.95***	118	40	3.41 %	7.51***	123	40
9	3.17 %	6.20***	109	49	3.46 %	6.82***	121	42
10	3.62 %	6.27***	114	44	3.95 %	7.10***	123	40
11	3.16 %	5.71***	108	50	3.53 %	6.39***	117	46
12	3.48 %	5.83***	114	44	3.85 %	6.56***	125	38
13	3.92 %	6.12***	119	39	4.37 %	6.96***	129	34
14	3.94 %	5.96***	119	39	4.11 %	6.34***	127	36
Panel B: 5								
1	0.00 %	-0.01	71	87	0.03 %	0.10	77	86
2	0.42 %	1.18	99	59	0.40 %	1.16	98	65
3	1.10 %	2.72***	105	53	1.11 %	2.77***	114	49
4	1.33 %	3.27***	111	47	1.41 %	3.50***	116	47
5	1.62 %	3.46***	110	48	1.66 %	3.53***	115	48
6	2.38 %	5.21***	110	48	2.37 %	5.11***	117	46
7	2.91 %	6.41***	118	40	2.95 %	6.52***	122	41
8	3.36 %	7.44***	120	38	3.45 %	7.50***	122	41
9	3.41 %	6.62***	111	47	3.57 %	6.80***	118	45
10	3.81 %	6.59***	116	42	4.07 %	7.14***	122	41
11	3.35 %	5.83***	115	43	3.61 %	6.31***	116	47
12	3.70 %	5.96***	119	39	3.97 %	6.51***	121	42
13	4.10 %	6.20***	121	37	4.49 %	6.86***	126	37
14	4.15 %	6.24***	122	36	4.23 %	6.47***	127	36
Panel C: 10								
1	0.03 %	0.09	73	85	0.01 %	0.06	75	88
2	0.50 %	1.39	103	55	0.38 %	1.06	99	64
3	1.23 %	2.99***	107	51	1.02 %	2.71***	115	48
4	1.50 %	3.74***	111	47	1.37 %	3.34***	113	50
5	1.80 %	3.77***	110	48	1.60 %	3.39***	114	49
6	2.58 %	5.51***	117	41	2.38 %	4.96***	117	46
7	3.16 %	6.60***	122	36	2.96 %	6.22***	121	42
8	3.65 %	7.42***	124	34	3.41 %	6.90***	122	41
9	3.63 %	6.66***	115	43	3.46 %	6.35***	117	46
10	4.02 %	6.80***	117	41	3.95 %	6.66***	122	41
11	3.42 %	6.07***	112	46	3.53 %	5.94***	117	46
12	3.81 %	6.32***	116	42	3.85 %	6.31***	121	42
13	4.32 %	6.55***	120	38	4.37 %	6.52***	119	44
14	4.30 %	6.61***	121	37	4.11 %	6.17***	127	36

As the proxy for the U.S. stock market (see Table 11) has generated excess return over risk-free rate on average 0.96 % monthly with a standard deviation of 3.45% or in terms of Sharpe ratio, 0.96 per annum, a larger equity exposure has been favourable to the bullish gamma portfolios. E.g Fabozzi et al. (2009) show that portfolios based on bullish gamma trade set up generate an average return of 4.79 % at the 99% confidence level which is higher than the regular delta-strategy that generated an average return of 3.99% after 12 month holding period. The difference is similar to the findings of this thesis. The regular delta-hedge trade generates an average 12-month return of 3.42% versus the 4.46% return of 9 bullish gamma trade.

Table 7: Bullish Gamma Trade Returns

This table provides the average returns, t-values, and the number of positive and negative return trades, respectively. The returns are calculated as a cumulative return on the long investment of $1M per trade. Returns are not annualized. The table is sorted by the volatility used in the model calibration, 30-day implied volatility on the left side, and vice versa. Significance is marked as ***,**, and *, for confidence levels of 99%, 95%, and 90%, respectively.

	30-D IV				**252D Historical volatility**			
FULL SAMPLE	Returns	T-Value	Positive	Negative	Returns	T-Value	Positive	Negative
Panel A: -9								
Months								
1	0.14 %	0.52	78	81	0.19 %	0.73	82	82
2	0.75 %	2.18**	106	53	0.79 %	2.41**	108	56
3	1.51 %	3.85***	113	46	1.55 %	3.99***	118	46
4	1.84 %	4.74***	121	38	1.97 %	5.03***	125	39
5	2.19 %	4.77***	120	39	2.31 %	4.96***	123	41
6	2.94 %	6.64***	124	35	3.08 %	6.74***	126	38
7	3.53 %	7.65***	123	36	3.73 %	8.03***	127	37
8	4.00 %	8.58***	127	32	4.26 %	8.99***	133	31
9	3.99 %	7.56***	119	40	4.30 %	8.12***	127	37
10	4.51 %	7.93***	121	38	4.87 %	8.66***	132	32
11	4.05 %	7.36***	120	39	4.43 %	7.87***	129	35
12	4.46 %	7.74***	122	37	4.85 %	8.37***	133	31
13	5.00 %	8.17***	123	36	5.48 %	8.90***	134	30
14	5.03 %	8.23***	126	33	5.22 %	8.49***	135	29
Panel B: -14								
1	0.23 %	0.88	85	74	0.28 %	1.12	91	73
2	0.98 %	2.87***	105	54	1.03 %	3.11***	114	50
3	1.80 %	4.58***	111	48	1.84 %	4.70***	118	46
4	2.18 %	5.54***	115	44	2.31 %	5.74***	122	42
5	2.59 %	5.52***	121	38	2.71 %	5.66***	127	37
6	3.34 %	7.40***	124	35	3.48 %	7.45***	122	42
7	3.98 %	8.17***	123	36	4.17 %	8.50***	128	36
8	4.49 %	8.88***	126	33	4.74 %	9.29***	134	30
9	4.46 %	7.84***	117	42	4.77 %	8.38***	130	34
10	5.04 %	8.40***	122	37	5.39 %	9.04***	131	33
11	4.56 %	7.84***	122	37	4.93 %	8.25***	127	37
12	5.02 %	8.37***	121	38	5.41 %	8.89***	131	33
13	5.62 %	8.87***	124	35	6.09 %	9.47***	133	31
14	5.66 %	9.09***	126	33	5.84 %	9.21***	137	27

The bearish gamma trade returns are presented in Table 8. The return is significantly lower than the delta or bullish gamma returns. The first two-month returns after the position initiation are negative although the returns are not statistically significant. The result is not surprising, as mentioned earlier, the position is already hedged because 1) bond floor is a partial hedge, 2) the stock market overall has performed well and e.g. Fabozzi et al. (2009) and Choi, Getmansky, and Tookes (2009) claim that stocks underlying the convertible tend to perform well after the convertible issuance, although there are negative short-term effects as the hedge funds tend to short the underlying stocks at issuance. The average stock price increased by ~15% during the first 252 trading days after issuance (see Appendix 4) so trades set up on heavier short exposure show lower returns. The 12-month returns are positive and statistically significant, the implied volatility calibrated strategy generated 2.43% and 1.89% returns for +9 and +14 portfolios, respectively. Less aggressive bearish gamma hedge strategy is statistically profitable after 5 (3) months in IV (HV) calibrated trades. Irrespective to the strategy, trades set up on historical volatility show slightly higher returns. With respect to the basics of call option delta, an increased volatility should tilt the delta higher. As concluded earlier, underlying stocks tend to perform well after the issuance. As the average implied volatility is higher for the total sample (see Table 3), one could assume that on average the estimated delta on a particular trading day *t* is higher with implied volatility calibrated model leading to higher short exposure. A larger short exposure should, all other things being equal, affect the CA returns negatively assuming upward trending stock price.

Table 8: Bearish Gamma Trade Returns

This table provides the average returns, t-values, and the number of positive and negative return trades, respectively. The returns are calculated as a cumulative return on the long investment of $1M per trade. Returns are not annualized. The table is sorted by the volatility used in the model calibration, 30-day Implied volatility on the left side, and vice versa. Significance is marked as ***,**, and *, for confidence levels of 99%, 95%, and 90%, respectively.

	30-D IV				**252D Historical volatility**			
FULL SAMPLE	Returns	T-Value	Positive	Negative	Returns	T-Value	Positive	Negative
Panel A: +9								
Months								
1	-0.20 %	-0.65	75	84	-0.15 %	-0.51	82	82
2	-0.09 %	-0.23	88	71	-0.04 %	-0.10	97	67
3	0.43 %	0.94	96	63	0.50 %	1.13	106	58
4	0.59 %	1.27	98	61	0.79 %	1.75*	104	60
5	0.73 %	1.35	95	64	0.92 %	1.74*	101	63
6	1.49 %	2.74***	103	56	1.69 %	3.16***	107	57
7	1.93 %	3.71***	106	53	2.19 %	4.32***	114	50
8	2.26 %	4.51***	105	54	2.57 %	5.12***	115	49
9	2.29 %	4.07***	104	55	2.64 %	4.74***	113	51
10	2.62 %	4.06***	107	52	3.04 %	4.90***	117	47
11	2.23 %	3.56***	101	58	2.67 %	4.37***	113	51
12	2.43 %	3.57***	107	52	2.90 %	4.40***	117	47
13	2.75 %	3.73***	105	54	3.32 %	4.65***	117	47
14	2.78 %	3.55***	105	54	3.05 %	4.05***	115	49
Panel B: +14								
1	-0.30 %	-0.91	72	87	-0.24 %	-0.77	79	85
2	-0.33 %	-0.78	78	81	-0.27 %	-0.67	92	72
3	0.15 %	0.30	94	65	0.23 %	0.49	100	64
4	0.27 %	0.55	92	67	0.48 %	1.02	97	67
5	0.36 %	0.63	88	71	0.57 %	1.02	102	62
6	1.14 %	1.97*	93	66	1.35 %	2.39**	98	66
7	1.53 %	2.74***	96	63	1.80 %	3.34***	103	61
8	1.82 %	3.34***	97	62	2.15 %	3.99***	107	57
9	1.88 %	3.07***	99	60	2.23 %	3.75***	107	57
10	2.14 %	3.09***	101	58	2.60 %	3.93***	107	57
11	1.75 %	2.59**	94	65	2.24 %	3.41***	106	58
12	1.89 %	2.56***	95	64	2.40 %	3.38***	108	56
13	2.16 %	2.72**	101	58	2.78 %	3.61***	108	56
14	2.19 %	2.57**	98	61	2.52 %	3.07***	106	58

5.2. Portfolio analysis: Return and risk

In this section, portfolio-level returns are examined. The position is opened in each convertible bond at the first available market price and subject to a long investment of $1,000,000. Each CB exits the portfolio when 1) it is either called by the issuer, 2) issuer defaults or 3) the CB has been in the portfolio 14 months, whichever comes first. The daily return of an individual bond position is equal to the method presented earlier. The equal-weighted method is applied to all portfolios, that is, every CA position receives equal weight in the portfolio. Following Agarwal et al. (2011), the simulated portfolios absorb new CBs arriving to the portfolio without a need to sell existing positions. It is assumed that a CA fund could manage this by employing leverage or capital infusion from the investors, or a combination of these. All portfolio returns are aggregated to monthly frequency from daily returns. The final time series consists of 85 months (February 2013 to February 2020) for which the CA returns have been calculated. CA returns are first examined with traditional Sharpe ratios to assess the risk-adjusted performance from total risk perspective.

A traditional Sharpe ratio is rather a straightforward and simple measure of the asset's risk and return ratio but has a shortcoming of not accounting for the risk embedded in return distributions that are non-normal.[7] To enhance the robustness of results, I use the skewness and kurtosis-adjusted Sharpe ratio (SKASR) introduced by Pätäri (2011) and assess the statistical significance. SKASR is a modified version of the Sharpe ratio where the third and fourth moments of the return distribution are captured. This method allows for enhanced Sharpe ratio comparison of portfolios showing different skewness and kurtosis measures (Pätäri, 2011). The adjusted Z-value Z_{CF} is estimated by employing the fourth-order Cornish and Fisher (1937) expansion that reveals the estimation of the true distribution. The skewness and kurtosis-adjusted deviation (SKAD) is estimated by multiplying Z_{CF}/Z_C by the standard deviation of excess returns (i.e. σ_i). SKAD is similar to σ_i but should the former be higher than the latter, the investor would encounter unfavorable distributional deviation from the normality.

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[7] Sharpe's ratio is presented in Sharpe (1966).

The SKASR is formed by replacing the standard deviation of excess returns σ_i with the SKAD. The modification of the resulting ratio is analogous to the refinement procedure proposed by Israelsen (2005) to deal with the negative excess return validity problem (Pätäri, 2011).

(55)
$$Z_{CF} = Z_C + \frac{1}{6}(Z_C^2 - 1)S + \frac{1}{24}(Z_C^3 - 3Z_C)K - \frac{1}{36}(2Z_C^3 - 5Z_C)S^2$$

Where Z_{CF} denotes the adjusted Z-value, Z_C is the critical value for the probability measure based on a standard normal distribution, whereas S and K denote skewness and kurtosis, respectively.

(56)
$$S = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_{it} - \overline{r_i}}{\sigma}\right)^3$$

Where the N denotes the number of outcomes and $\overline{r_i}$ is the average return.

(57)
$$K = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_{it} - \overline{r_i}}{\sigma}\right)^4 - 3$$

(58)
$$SKASR = \frac{r_p - r_f}{SKAD_p^{(ER/|ER|)}}$$

Where ER denotes the average excess returns of portfolio p.

The Sharpe ratio significance test follows the method first introduced by Jobson and Korkie (1981) and is implemented as Memmel (2003) suggests, see Equations 59 and 60. The test indicates whether the Sharpe ratios of two portfolios are statistically different. Pätäri (2021) proposes that the test format is also suitable in skewness and kurtosis-adjusted Sharpe ratio context. In this instance, the Sharpe ratios (i.e. $\hat{S}h_i, \hat{S}h_n$) are replaced with SKASR implied values.

$$Z = \frac{\hat{S}h_i - \hat{S}h_n}{\sqrt{\hat{V}}} \tag{59}$$

Where $\hat{S}h_i$ and $\hat{S}h_n$ are the Sharpe ratios of portfolios *i* and *n*, $\hat{V}$ is the asymptotic variance of Sharpe ratio difference.

$$\hat{V} = \frac{1}{T}[2 - 2\rho_{in} + \frac{1}{2}(Sh_i^2 + Sh_n^2 - 2Sh_iSh_n\rho_{in}^2)] \tag{60}$$

Where T is the number of periodic returns, ρ_{in} is the correlation between portfolio *i* and *n* returns.

Delta and gamma strategy returns are reported in Table 9. The regular delta-strategy produced on average a 0.43 % monthly return with a standard deviation of 1.29 percent. There is a gradual improvement in the monthly return of portfolios that rebalance more infrequently (i.e. modified delta). The average monthly return of the 10-delta portfolio (IV) is 0.45% with a lower risk (1.25%) compared to the daily delta-rebalancing strategy. Interestingly, the portfolio that is rebalanced only when the delta change is greater than 10 units shows the highest return but the lowest risk among delta-strategies. Comparing with Agarwal et al. (2011) return statistics are in line with this study. They show monthly returns of 0.50 % for equal-weighted CA portfolios. Unlike the findings of Agarwal et al. (2011), the arbitrage portfolio in this thesis show positive skewness indicating that returns are skewed more on the positive side of the mean.

When it comes to the counterintuitive return/risk ratio of the 10 rebalance, there is some prior evidence of similar results as e.g. Fabozzi et al. (2009) show that under daily rebalance the return and standard deviation are 2.99 % and 2.30 %, respectively, but for the 10 rebalance strategy they are 4.09 % and 1.98 %. The return is certainly higher, as the trade is allowed to drift more, allowing the position value to capture positive return more from the upward stock price movement. The initial assumption behind a CA volatility trade is that the equity and embedded option volatility will eventually converge. However, there are extra returns available if the trade captures capital gains from the stock price movement that are not swallowed by the excess amount of short-selling i.e. leaving the net exposure higher. Another point that was initially highlighted by Hutchinson and Gallagher (2008) and Calamos (2003) was that the daily rebalancing is expensive. The 10-delta rule does not require balancing as often and contributes to returns by reducing the amount of transaction costs.

Gamma 14 (IV) returns are on average 0.52 % monthly with a standard deviation of 1.18%. The average return and Sharpe ratios are lower for the bearish gamma portfolios. The number of shares sold short is much higher for the bearish gamma portfolio, hence the indirect transaction costs are higher and the yield advantage over the dividend yield is lower. Also, as indicated in Appendix 4, the underlying stock performs well after the issuance of the CB which is not beneficial for a heavy short position. E.g. Fabozzi et al. (2009) indicate that bullish gamma trade is by construction in a better position than bearish gamma because the stock underlying the convertible bond usually performs well approximately 3 years after issuance. The Kernel distribution of monthly returns is presented in Figure 10. As indicated in Table 9, none of the strategies exhibits large amount of skewness. The bearish gamma strategies possess the largest tail risk as the minimum monthly returns are lowest, whereas their standard deviation and kurtosis are among highest in the sample. One of the main risks embedded in the CA positions taking a directional bet on the stock price development is the rise of stock price. As the stock price by assumption and in practice has on average risen after the issuance, a significant appreciation causes the portfolio to lose value.

Due to the limitations in traditional Sharpe ratio measure, the statistical significance is carried out using the SKASR implied metrics. The SKASR results are presented in Table 10. The examined portfolios are more aggressive bull and bear portfolios and delta portfolios. These portfolios are included because 1) delta-strategy is a standard both in industry and theory, 2) most aggressive portfolios show different characteristics of the strategy as the hedging ratio is either very large or very small. As the results of the standard Sharpe ratio indicated, bullish portfolios show also highest risk-reward ratios in the terms of SKASR, 1.365 and 1.345 for IV and HV calibrated portfolios, respectively. None of the convertible arbitrage SKASRs is statistically different from the market indices equivalent. Considering the bullish IV gamma strategy, both the traditional Sharpe ratio and SKASR are however economically significant against the market indices. The results thus imply that an arbitrageur harvesting excess returns from CA related strategies, should assume more equity risk as more aggressive, bullish gamma strategies risk-reward ratio is statistically different against more conservative strategies such as the delta and bearish gamma. As indicated also in Fabozzi et al. (2009), bullish portfolios seem to dominate over conservative strategies.

Table 9: Portfolio Descriptive Statistics

This table provides the descriptive statistics of the CA portfolios. Values are reported as monthly values unless specified otherwise. The risk-free rate is the 1-month U.S. Treasury Rate.

Panel A: Delta-Strategies *Volatility: IV-OTM 30D* *Monthly Return and Risk*	Delta Hedge Vanilla	Rebalance 2	Rebalance 5	Rebalance 10
N	85	85	85	85
Mean	0.426 %	0.432 %	0.442 %	0.453 %
Standard Deviation	1.287 %	1.283 %	1.288 %	1.253 %
Sharpe Ratio p.a.	0.9875	1.006	1.031	1.092
Skewness	0.233	0.205	0.213	0.240
Kurtosis	3.347	3.381	3.876	3.159
Min	-3.749 %	-3.776 %	-4.051 %	-3.603 %
Max	5.434 %	5.412 %	5.596 %	5.302 %
Volatility: Historical 252D *Monthly Return and Risk*				
N	85	85	85	85
Mean	0.428 %	0.427 %	0.442 %	0.463 %
Standard Deviation	1.291 %	1.291 %	1.229 %	1.201 %
Sharpe Ratio p.a.	0.987	0.986	1.081	1.173
Skewness	0.214	0.270	0.112	0.261
Kurtosis	3.562	3.880	3.079	2.412
Min	-3.958 %	-3.959 %	-3.731 %	-2.907 %
Max	5.576 %	5.709 %	5.124 %	4.971 %
Panel B: Gamma Strategies *Volatility: IV-OTM 30D* *Monthly Return and Risk*	Bull Gamma -9	Bull Gamma -14	Bear Gamma +9	Bear Gamma +14
N	85	85	85	85
Mean	0.485 %	0.518 %	0.376 %	0.349 %
Standard Deviation	1.171 %	1.175 %	1.527 %	1.685 %
Sharpe Ratio p.a.	1.267	1.366	0.705	0.577
Skewness	0.164	0.158	0.320	0.385
Kurtosis	2.311	1.524	3.055	2.793
Min	-3.070 %	-2.778 %	-4.461 %	-4.886 %
Max	4.757 %	4.382 %	6.205 %	6.715 %
Volatility: Historical 252D *Monthly Return and Risk*				
N	85	85	85	85
Mean	0.486 %	0.520 %	0.381 %	0.357 %
Standard Deviation	1.186 %	1.199 %	1.513 %	1.661 %
Sharpe Ratio p.a.	1.255	1.342	0.724	0.602
Skewness	0.204	0.232	0.271	0.324
Kurtosis	2.443	1.843	3.398	3.055
Min	-3.237 %	-2.833 %	-4.670 %	-5.063 %
Max	4.832 %	4.423 %	6.333 %	6.761 %
Risk-free rate				
Mean p.a.	0.80%	0.80%	0.80%	0.80%

Table 10: SKASR results

This table provides the skewness and kurtosis-adjusted Sharpe ratios (SKASR) and the statistical significance against various market indices. SKASR results and their statistical significance level against market portfolios are presented in Panel A. In Panel B, the CA portfolio possessing the highest SKASR is set against other CA portfolios and the statistical significance is reported. MKTRF denotes the US Stock Market Total Return, CB Index denotes the Bloomberg U.S. Convertibles Liquid Bond Index Total Return and BBG IG denotes the Bloomberg Barclays US Investment Grade Index Total Return. The significance is in the parenthesis and is two-sided. All returns are in the excess of the risk-free return. The risk-free return is the 1-month T-bill. MKTRF is from Kenneth French Data Library (2021), CB index and BBG IG are from Bloomberg (2021). Note that the critical value $\boldsymbol{Z_c}$ applied in SKASR is -1.96.

Panel A: Strategy versus the market index

	Vanilla IV	Bull Gamma 14 IV	Bear Gamma 14 IV	Vanilla HV	Bull Gamma 14 HV	Bear Gamma 14 HV
Strategy SKASR	0.962	1.365	0.595	0.951	1.345	0.606
Vs.						
MKTRF SKASR	0.829	0.829	0.829	0.829	0.829	0.829
(Significance)	*(0.7853)*	*(0.2468)*	*(0.7460)*	*(0.7964)*	*(0.2473)*	*(0.7558)*
CB Index SKASR	0.993	0.993	0.993	0.993	0.993	0.993
(Significance)	*(0.9876)*	*(0.3905)*	*(0.5489)*	*(0.9706)*	*(0.4005)*	*(0.5566)*
BBG IG TR SKASR	0.903	0.903	0.903	0.903	0.903	0.903
(Significance)	*(0.9017)*	*(0.3238)*	*(0.5269)*	*(0.9214)*	*(0.3472)*	*(0.5406)*

Panel B: Highest SKASR versus other CA strategies

	Vanilla IV	Bear Gamma 14 IV	Vanilla HV	Bull Gamma 14 HV	Bear Gamma 14 HV
Bull Gamma 14 IV					
(Significance)	*(0.0663)*	*(0.0304)*	*(0.0491)*	*(0.7328)*	*(0.0291)*

Figure 10: Kernel Distribution of Monthly Portfolio Returns

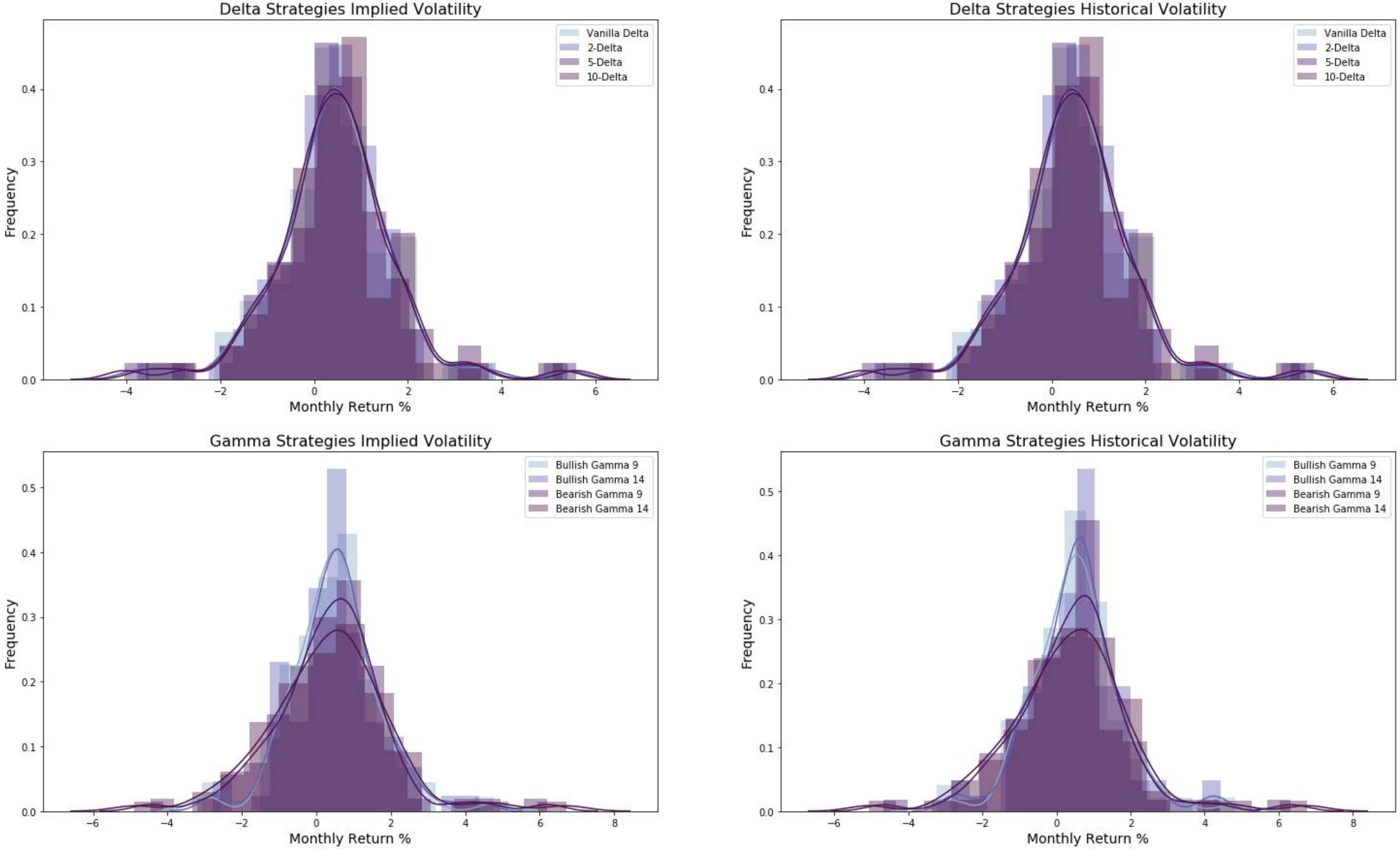
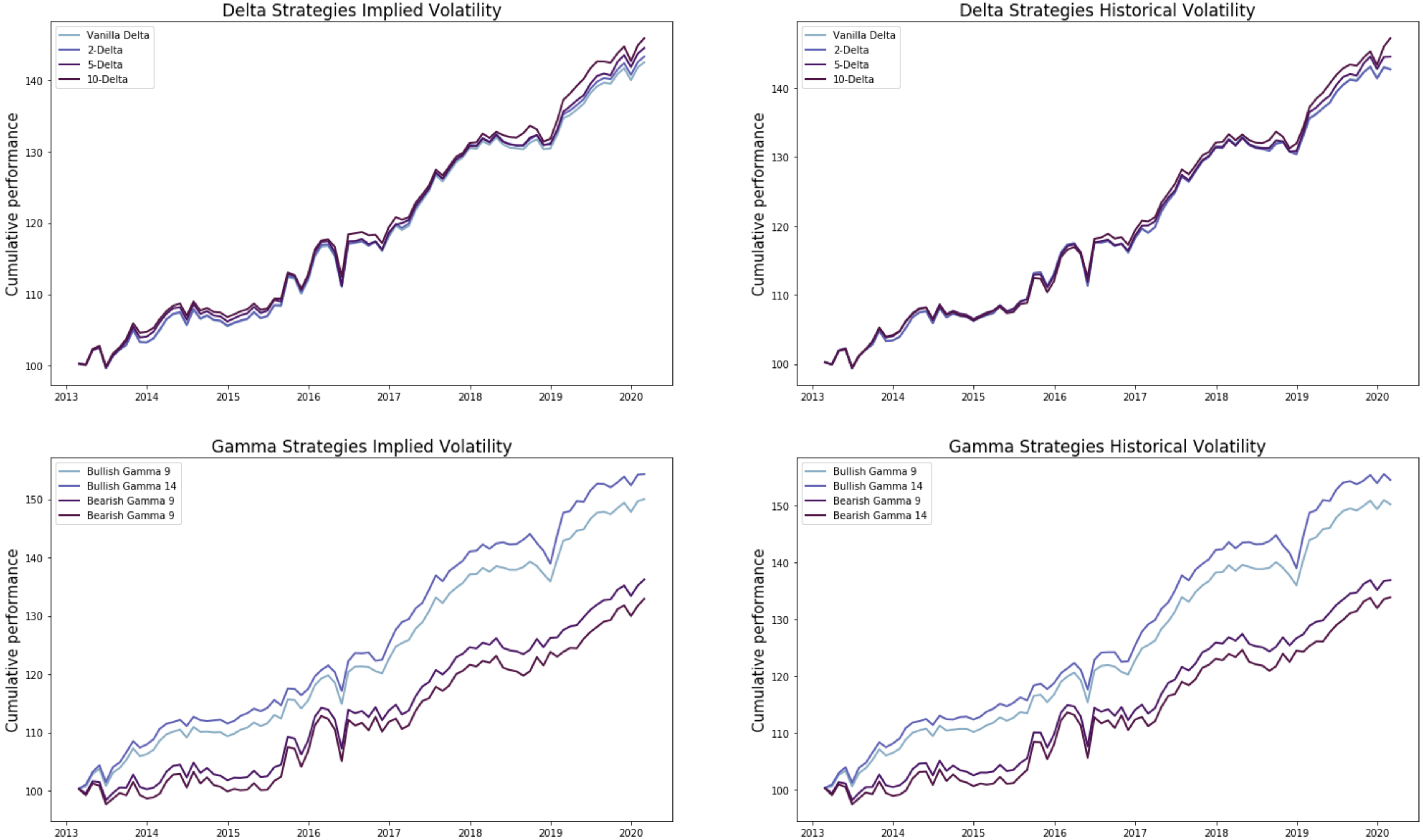


Figure 11: Cumulative Returns



5.3. Explaining strategy returns

The portfolios selected for the risk-factor approach are regular delta-hedge strategies and the most aggressive portfolios concerning hedging which are bull and bear gamma portfolios using the -/+ 14 rule when hedging the long convertible position. These portfolios are included because 1) delta-hedge is the industry and theory standard 2) most aggressive portfolios show different characteristics of the strategy as the hedging ratio is either very large or very small and should help to underline whether strategy provides excess return given its exposure to market risk factors.

Fung and Hsieh (1997a) propose that a portfolio of hedge funds can be presented as a linear combination of synthetic hedge fund strategies. In the spirit of Fung and Hsieh (2004) 7-factor model, the equity-related returns in this thesis are controlled by the Fama and French factors (1993) incorporating the market risk premia (MKTRF) and small minus large-cap portfolio (SMB). In addition, the Fama and French (1993) value minus growth stock portfolio (HML), Fama and French (2015) robust minus weak profitability (RMW) and conservative minus aggressive investing firm returns (CMA) are included in the linear model. The equity momentum is captured with the Carhart (1997) momentum-factor (UMD). The pure fixed income factor (FIX) is the Bloomberg Barclays US Corporate Investment Grade TR Index minus the risk-free return, analogous to what e.g. Fama and French (1993) and Fung and Hsieh (2004) propose as a fixed income factor[8]. Fung and Hsieh (2001, 2002, 2004) imply that hedge fund strategy payoffs are often similar to lookback straddle payoffs. The lookback straddle PTFSBD (Fung and Hsieh, 2001) incorporating the lookback straddle factor for bonds is also included the linear model. To address the hybrid capital component as a CB mutual fund return driver Ammann, Kind & Seiz (2010) use a CB index in the risk-factor model. However, the correlation between MKTRF and CB Index excess return (Bloomberg U.S. Convertibles Liquid Bond Index TR

[8] Fung and Hsieh (2004) propose that fixed income related returns can be express as term and default risk. Term risk-factor relates to the change in 10-year Treasury yield whereas the default risk relates to the change in the spread between Moody's Baa Yield minus the 10-year constant maturity yield. Fama and French (1993) propose these factors as the return of long-term government bonds minus the T-bill rate and the difference between the return of a portfolio of long-term corporate bonds and the return of long-term corporate bonds, respectively. The approach in this thesis combines the term and credit risk into one factor, that is the return of a portfolio of investment grade bonds minus the risk-free return.

Unhedged minus the risk-free return) is relatively large (see Table 11), one workaround is to use the CB index excess return minus the MKTRF as one risk-factor. This method is similar Agarwal et al. (2011) X-factor which is a portfolio of convertible bonds minus a portfolio consisting of underlying shares meant to represent a delta-hedge factor. The method applied in this thesis is rather simplistic but is by construction a self-financing portfolio and is long in the CB risk (vega) and short in the equity risk (delta). E.g. Hutchinson and Gallagher (2010), Agarwal et al. (2011) follow a similar design when constructing a linear combination of risk-factors that are used to explain CA returns. The risk-factors are shortly described at the end of this paragraph and the descriptive statistics of the risk-factors used in the linear regression are presented in Table 11.

Equity risk-factors

- **MKTRF**: The value-weight return in the excess of the risk-free return of all CRSP firms incorporated in the U.S.
- **SMB**: (Small Minus Big) is the return of small cap stock portfolios minus the return of the large cap stock portfolios
- **HML**: (High minus Low) is the return of value stock portfolios minus the return of the growth stock portfolios
- **RMW**: (Robust minus Weak) is the return of robust profitability stock portfolios minus the return of the weak profitability stock portfolios
- **CMA**: (Conservative minus Aggressive) is the return of conservative investment stock portfolio minus the return of the aggressive investment stock portfolio
- **UMD**: (Up minus Down) the return of the portfolio with high prior returns minus the return of the portfolios with low prior returns

Fixed income and option risk-factors

- **FIX**: Bloomberg Barclays US Corporate Investment Grade TR Index minus the risk-free return
- **PTFSBD (Fung and Hsieh, 2001)**[9]: Trend following factor for bonds.
- **CB**: The excess return of Bloomberg U.S. Convertibles Liquid Bond Index TR Unhedged. Represents a long investment in a portfolio of liquid U.S. convertibles.
- **CBMKTRF**: The excess return of Bloomberg U.S. Convertibles Liquid Bond Index TR Unhedged minus MKTRF. Represents a portfolio of liquid U.S. convertibles where the equity-risk is hedged.

[9] PTFSBD (Fung and Hsieh, 2001) is available at: http://faculty.fuqua.duke.edu/~dah7/DataLibrary/TF-Fac.xls

Table 11: Descriptive statistics of the independent variables

This table provides the descriptive statistics of the risk-factors used in the linear regression. Correlations are reported in Panel A and descriptive statistics in Panel B, respectively. MKTRF, SMB, HML, RMW and CMA are from Kenneth French Data Library (2021) whereas PTFSBD (Fung and Hsieh, 2001) is from David A. Hsieh's Data Library (2021). The risk-free rate is the return of 1-month T-bill. FIX and CB are from Bloomberg Terminal (2020). All values are in monthly frequency.

Panel A: Correlation Matrix

	MKTRF	SMB	HML	RMW	CMA	MOM	PTFSBD (Fung-Hsieh)	CBMK-TRF	FIX	CB
MKTRF	1.00	0.30	0.08	-0.06	-0.14	-0.30	-0.33	-0.70	0.10	0.88
SMB	0.30	1.00	0.23	-0.41	0.05	-0.23	0.01	-0.20	-0.19	0.27
HML	0.08	0.23	1.00	0.10	0.65	-0.55	-0.24	-0.33	-0.39	-0.11
RMW	-0.06	-0.41	0.10	1.00	0.24	-0.11	-0.04	-0.22	0.11	-0.23
CMA	-0.14	0.05	0.65	0.24	1.00	-0.37	0.03	-0.20	-0.18	-0.33
MOM	-0.30	-0.23	-0.55	-0.11	-0.37	1.00	0.32	0.26	0.13	-0.23
PTFSBD (Fung-Hsieh)	-0.33	0.01	-0.24	-0.04	0.03	0.32	1.00	0.27	0.19	-0.26
CBMKTRF	-0.70	-0.20	-0.33	-0.22	-0.20	0.26	0.27	1.00	0.22	-0.28
FIX	0.10	-0.19	-0.39	0.11	-0.18	0.13	0.19	0.22	1.00	0.28
CB	0.88	0.27	-0.11	-0.23	-0.33	-0.23	-0.26	-0.28	0.28	1.00

Panel B: Descriptive Statistics

	MKTRF	SMB	HML	RMW	CMA	MOM	PTFSBD (Fung-Hsieh)	CBMK-TRF	FIX	CB
N	85	85	85	85	85	85	85	85	85	85
Average	0.96 %	-0.20 %	-0.37 %	0.08 %	-0.23 %	0.33 %	-0.51 %	-0.17 %	0.32 %	0.79 %
Standard Deviation	3.45 %	2.43 %	2.49 %	1.45 %	1.49 %	3.34 %	16.57 %	1.72 %	1.18 %	2.55 %
Minimum	-9.55 %	-4.51 %	-6.27 %	-3.93 %	-3.35 %	-8.65 %	-22.60 %	-4.73 %	-2.80 %	-6.37 %
25 %	-0.11 %	-1.96 %	-1.89 %	-0.65 %	-1.26 %	-1.67 %	-12.15 %	-1.31 %	-0.26 %	-0.53 %
50 %	1.29 %	0.20 %	-0.42 %	0.10 %	-0.29 %	0.29 %	-4.06 %	-0.23 %	0.33 %	1.07 %
75 %	3.12 %	1.18 %	0.56 %	0.91 %	0.47 %	2.14 %	8.27 %	0.76 %	1.03 %	2.48 %
Maximum	8.41 %	6.80 %	8.22 %	3.53 %	3.78 %	10.29 %	60.88 %	5.08 %	2.99 %	7.77 %

The regression results are presented in Table 12. The results indicate that CA strategies exhibit statistically significant exposure to equity and bond related factors. The positive and significant loading of RMW implies that CA strategies respond to the variation of RMW. Considering the nature of convertible bond, it is a hybrid security between the debt and equity and allows the debt-investor to capture upside potential should the underlying share appreciate in price. The result is at first slightly counterintuitive as companies issuing CBs should rather be companies with non-robust financials and implicitly required to offer call option to keep the cost of debt at a decent level. Agarwal et al. (2011) indicate that small-cap stock premium is a positive and statistically significant variable in CA returns from 1993 to 2003. Also, Hutchinson & Gallagher (2010) present evidence that arbitrageurs might favour securities from small-cap issuers due to larger arbitrage opportunities. However, Gallagher et al. (2018) find that both equally and value-weighted portfolios of CA hedge funds have negative exposure to small-cap factors ranging from -0.12 to -0.03 with a sample spanning from 1994 to 2012. The profitability factor (RMW) remains unused in CA context according to the author’s knowledge. There is a modest negative correlation between the RMW and SMB indicating that smaller companies are less profitable, similar relationship is also presented in Fama and French (2015). This gives some support to the findings of this thesis as more conservative strategies, that are delta and bearish gamma, exhibit negative (positive) factor loadings in SMB (RMW). This may be caused by the employed threshold for deal value of $100M or higher. It would seem logical that hedge funds would prefer larger deals or companies considering the 1) better liquidity in a market distress 2) ability to employ leverage through e.g. repos and 3) better short-selling opportunities which might not be the case with smaller size CBs / companies.

The momentum factor is positive but statistically significant only in delta (IV) and bearish portfolios. The exposure is quite similar in all portfolios as the range is from 0.09 to 0.15. Similar exposure is also presented at least by Agarwal et al. (2011), Ammann et al. (2010), Hutchinson and Gallagher (2010) in convertible bond context. The positive momentum loading implies that winner stocks might play a role in CA strategies. The negative correlation between MKTRF or CB against UMD could indicate that the winner stocks tend to outperform when the overall stock market is in dire straits. Gallagher et al. (2018) present evidence that CA strategies have tendency to outperform during market distress.

All strategies exhibit significant negative exposure to the lookback straddle on bonds (PFTS-BSD). The result is intuitive and supported in previous literature, see Agarwal et al. (2011), Gallagher et al. (2018) who present evidence of a significant factor loading around -0.01. PTFSBD performs well when the long-term government bond yield expectations of various countries rise or lower aggressively, but poorly in a normal market regime, see Fung and Hsieh (2001, 2004). The fixed income risk-factor (FIX) is positive and significant in all examined portfolios. The result implies that CA strategies are exposed significantly to the duration and credit spread related factors. The result is somewhat analogous to prior evidence where CA hedge funds exhibit negative exposure to the change in 10-year Treasury yield and change in Moody's Baa spread which is virtually the same as a long investment in a portfolio of U.S. corporate bonds.

Finally, more aggressive strategies assuming more equity risk i.e. bullish gamma strategies, exhibit lower risk-factor loadings on CBMKTRF and FIX as their conservative counterparts do but show higher loadings on the CB index. E.g. Gallagher et al. (2018) find that the loadings of the risk-factors incorporating buy-and-hedge and buy-and-hold strategies in CBs are 0.39 and 0.10, respectively.[10]

The linear model explains 25.9% to 46.7% of the variation in CA returns. Putting this into comparison with Agarwal et al. (2011), Hutchinson and Gallagher (2008, 2010), the explanatory power is similar. Annual alphas range from 2.74% to 3.04% and are statistically significant at least at the 95% confidence level. However, Gallagher et al. (2018) show evidence that the outperformance ability of CA hedge funds is highly dependent on the current state of the market. Gallagher et al. (2018) conclude that CA funds post higher risk-adjusted returns when equity market return is below a threshold level.[11] Gallagher et al. (2018) that report an average return of the Fama and French (1993) MKTRF risk-factor as 6.27% p.a.[12] The reported figure

[10] These figures reported in Gallagher et al. (2018) are the factor loadings from a linear regression where the dependent variable is an equal-weight portfolio of CA hedge funds.

[11] Gallagher et al. (2018) use smooth transition regression (STR) to capture two risk regimes of the equity market. See Chan and Tong (1986), Teräsvirta and Anderson (1992) for STR methodology.

[12] The sample in Gallagher et al. (2018) spans from 1994 until 2012. MKTRF is the variable used to determine risk regimes in Gallager et al. (2018).

in Gallagher et al. (2018) is almost 6 %-units lower per annum than the same risk-factor in this thesis.[13] In summary, the results of this thesis do not provide evidence that CA strategies would perform badly despite the robust stock market from 2013 until the beginning of 2020. Arbitrageurs should maybe consider strategies that are robust to both total risk and risk-factor models, hence bullish CA strategies.

Table 12: Regression Results

This table provides an Ordinary Least Square (OLS) regression of the monthly excess return of convertible arbitrage portfolio against Fama and French (1993,2015) equity factors SMB, HML, RMW and CMA, Carhart (1997) UMD, Fung and Hsieh (2001) PTFSBD, FIX (U.S. investment grade excess return), CB (excess return of Bloomberg U.S. Convertibles Liquid Bond Index TR) and CBMKTRF. Coefficients and constants (alpha) are reported alongside the F-statistics, R-squared, and adjusted R-squared measures. The constant (alpha) is reported as an annualized value, that is, the monthly constant multiplied by 12. Newey-West (1987) heteroskedasticity and autocorrelation robust t-statistics are in the parenthesis and significance is marked with bold font and asterisk *. Significance is marked as ***,**, and *, for confidence levels of 99%, 95%, and 90%, respectively.

	Vanilla IV	Bull Gamma 14 IV	Bear Gamma 14 IV	Vanilla HV	Bull Gamma 14 HV	Bear Gamma 14 HV
Alpha	0.0285	0.0287	0.0304	0.0274	0.0274	0.0301
	(2.745)***	(2.862)***	(2.683)***	(2.386)**	(2.433)**	(2.511)**
SMB	-0.0186	0.0761	-0.1156	-0.0093	0.0839	-0.1067
	(-0.306)	(1.336)	(-1.735)*	(-0.16)	(1.554)	(-1.702)
HML	0.0381	0.0061	0.0822	0.0297	-0.0010	0.0793
	(0.618)	(0.107)	(1.123)	(0.494)	(-0.018)	(1.118)
RMW	0.3854	0.2928	0.4793	0.3959	0.2987	0.4851
	(3.009)***	(2.535)**	(3.169)***	(3.228)***	(2.758)***	(3.309)***
CMA	0.2454	0.1752	0.3110	0.2458	0.1717	0.3021
	(1.337)	(1.091)	(1.468)	(1.331)	(1.091)	(1.433)
MOM	0.1262	0.1010	0.1535	0.1203	0.0924	0.1473
	(1.772)*	(1.636)	(1.806)*	(1.653)	(1.473)	(1.717)*
PTFSBD (Fung-Hsieh)	-0.0185	-0.0155	-0.0214	-0.0185	-0.0155	-0.0213
	(-3.273)***	(-2.652)***	(-3.836)***	(-2.863)***	(-2.354)**	(-3.441)***
CBMKTRF	0.2809	0.1606	0.3987	0.2620	0.1392	0.3773
	(3.416)***	(2.341)**	(3.857)***	(3.182)***	(2.115)**	(3.648)***
FIX	0.1884	0.1827	0.2070	0.1837	0.1798	0.2066
	(2.153)**	(2.212)**	(1.996)**	(2.129)**	(2.241)**	(1.979)*
CB	0.1269	0.2279	0.0218	0.1396	0.2402	0.0303
	(1.848)*	(3.144)***	(0.314)	(2.052)**	(3.351)***	(0.444)
Observations	85	85	85	85	85	85
R2	0.385	0.339	0.524	0.368	0.35	0.504
Adj. R2	0.312	0.259	0.467	0.292	0.272	0.444
F-Statistic	19.79***	18.95***	17.21***	21.40***	22.43***	17.36***

[13] See Table 11. MKTRF (Fama and French, 1993) average is 0.96% monthly that is around 12.15 % (((1+0,0096)^12)-1) p.a.

5.4. Sensitivity analysis with respect to transaction costs

A convertible arbitrage trade includes buying and selling securities in both debt and equity markets of which the former can be highly illiquid, and pricing is determined by the market makers quoting the bond. To both check the robustness of results and to study the effect of the transaction costs on the trade profits, a sensitivity analysis is performed. The Saltelli (2002) method is used to construct 600 random points that represent the 1) trade size of the bond varying between $50,000 and $10,000,000 and 2) the bid-ask spread varying between zero and 2000 bps. After the random transaction costs and trade sizes had been created, the vanilla delta-hedge trades were simulated and aggregated to portfolios. To mention an example, one simulation consists of simulating 159 CA trades with an investment size of $2M per convertible bond and a bid-ask spread of 300 bps. The trade returns are then aggregated to portfolio level returns and corresponding T-statistic is reported. The results of the sensitivity analysis are presented in Table 13. The average return and the T-value are reported in the table sorted by the trade size and bid-ask spread. When moving to bid-ask spread over 200 bps, the average returns drop immediately and are no longer statistically different from zero. It is rather obvious that when transaction costs rise enough, they will turn any investment negative. However, this is an illustrative example of some limitations hedge funds might face when it comes to the market impact of the traded size plus the quoted bid-ask spreads of the CBs.

Table 13: Transaction Cost Sensitivity Analysis

This table provides the T-statistics and the average monthly returns against transaction costs and trade size. Trade size is indicated in millions of $ whereas the bid-ask spread is reported in basis points. T-statistics are reported in Panel A and the average monthly returns in Panel B.

Panel A: T-statistics

	Bid-Ask Spread									
Trade Size	**0-200**	**200-400**	**400-600**	**600-800**	**800-1000**	**1000-1200**	**1200-1400**	**1400-1600**	**1600-1800**	**1800-2000**
$0-1M	2.56	1.18	-0.26	-0.96	-1.66	-2.32	-2.83	-3.19	-3.59	-3.79
$1-2M	2.88	1.09	0.12	-0.74	-1.59	-2.25	-2.83	-3.16	-3.51	-3.91
$2-3M	2.64	1.36	-0.04	-0.99	-1.69	-2.22	-2.86	-3.34	-3.67	-3.93
$3-4M	2.54	1.06	0.21	-0.85	-1.82	-2.38	-2.81	-3.35	-3.68	-3.89
$4-5M	2.27	1.54	-0.03	-1.18	-1.83	-2.53	-2.77	-3.34	-3.74	-3.92
$5-6M	2.25	0.90	-0.04	-1.14	-1.70	-2.56	-3.06	-3.53	-3.76	-4.00
$6-7M	2.54	1.06	0.13	-0.82	-1.91	-2.41	-3.00	-3.38	-3.67	-3.97
$7-8M	2.64	1.04	-0.31	-1.26	-1.93	-2.49	-3.08	-3.47	-3.75	-3.99
$8-9M	2.28	1.23	-0.28	-1.16	-1.78	-2.52	-3.00	-3.35	-3.70	-4.04
$9-10M	1.99	1.39	-0.23	-1.34	-1.84	-2.63	-2.89	-3.43	-3.82	-4.05

Panel B: Average Monthly Returns

	Bid-Ask Spread									
Trade Size	**0-200**	**200-400**	**400-600**	**600-800**	**800-1000**	**1000-1200**	**1200-1400**	**1400-1600**	**1600-1800**	**1800-2000**
$0-1M	0.36 %	0.18 %	-0.04 %	-0.17 %	-0.33 %	-0.50 %	-0.67 %	-0.81 %	-1.00 %	-1.11 %
$1-2M	0.40 %	0.16 %	0.02 %	-0.13 %	-0.31 %	-0.48 %	-0.66 %	-0.79 %	-0.95 %	-1.17 %
$2-3M	0.37 %	0.20 %	-0.01 %	-0.18 %	-0.33 %	-0.47 %	-0.67 %	-0.87 %	-1.03 %	-1.17 %
$3-4M	0.36 %	0.16 %	0.03 %	-0.15 %	-0.36 %	-0.52 %	-0.65 %	-0.87 %	-1.03 %	-1.15 %
$4-5M	0.32 %	0.23 %	-0.01 %	-0.22 %	-0.36 %	-0.56 %	-0.63 %	-0.86 %	-1.06 %	-1.18 %
$5-6M	0.32 %	0.14 %	-0.01 %	-0.21 %	-0.33 %	-0.57 %	-0.74 %	-0.95 %	-1.07 %	-1.23 %
$6-7M	0.36 %	0.16 %	0.02 %	-0.14 %	-0.38 %	-0.52 %	-0.72 %	-0.87 %	-1.02 %	-1.24 %
$7-8M	0.37 %	0.16 %	-0.05 %	-0.23 %	-0.38 %	-0.54 %	-0.75 %	-0.91 %	-1.06 %	-1.17 %
$8-9M	0.33 %	0.18 %	-0.05 %	-0.21 %	-0.35 %	-0.55 %	-0.72 %	-0.86 %	-1.03 %	-1.21 %
$9-10M	0.29 %	0.21 %	-0.04 %	-0.25 %	-0.36 %	-0.58 %	-0.67 %	-0.89 %	-1.09 %	-1.22 %

6. Conclusion

In this thesis, I analyse the performance of convertible arbitrage strategies in the U.S. markets between 2013 and 2020 using 159 convertible bonds. Each simulated CA trade starts with a fixed amount of cash that is invested in the convertible bond and the underlying stock is simultaneously sold short using hedge ratios derived from the model proposed by Milanov et al. (2013). CA trade returns are aggregated to portfolios and their return and risk characteristics are examined with a linear set of risk-factors and total risk-models including the Sharpe ratio and SKASR. All results are robust to transaction costs.

Arbitrage and hedging strategies involving dynamic hedging around convertible bonds generate superior risk-adjusted returns against a set of equity and bond-related risk-factors. Gamma-based hedging strategy set up on bullish tilt generates higher absolute returns and exhibits statistically and economically higher skewness and kurtosis-adjusted Sharpe ratios than delta or bearish gamma strategies. In terms of Sharpe ratio and SKASR, the gamma portfolio exhibits appealing risk-return characteristics as these ratios are economically higher than market indices' equivalents. However, none of the SKASR values are statistically different from market indices' equivalent. On average, CA trades set up on implied volatility calibrated model earn slightly lower returns than historical volatility calibrated. However, as only one implied volatility measure is used, rather than a large spectrum of implied volatilities from market traded options, this result might not hold in cases where the hedge fund could employ liquid market traded options of longer maturities with various strikes.

CA strategies exhibit statistically significant exposure to multiple risk-factor, but the risk-adjusted return is positive and statistically and economically significant. The results imply that CA variation is explained mostly by convertible and corporate bond exposure and a non-linear strategy incorporating lookback straddle on bonds. These results and the explanatory power of the linear model are similar to e.g. Agarwal et al. (2011), Hutchinson and Gallagher (2008,2010). CA strategies are complex to some extent and returns are rarely explained well by risk-factor models, which implies that the alpha can also be a product of a misspecified set of risk-factors.

Arbitrage and hedging opportunities around CBs with relatively low market-factor exposure seem to exist in the U.S. markets. The results imply that there are meaningful opportunities to be pursued especially in more aggressive strategies that assume more equity risk.

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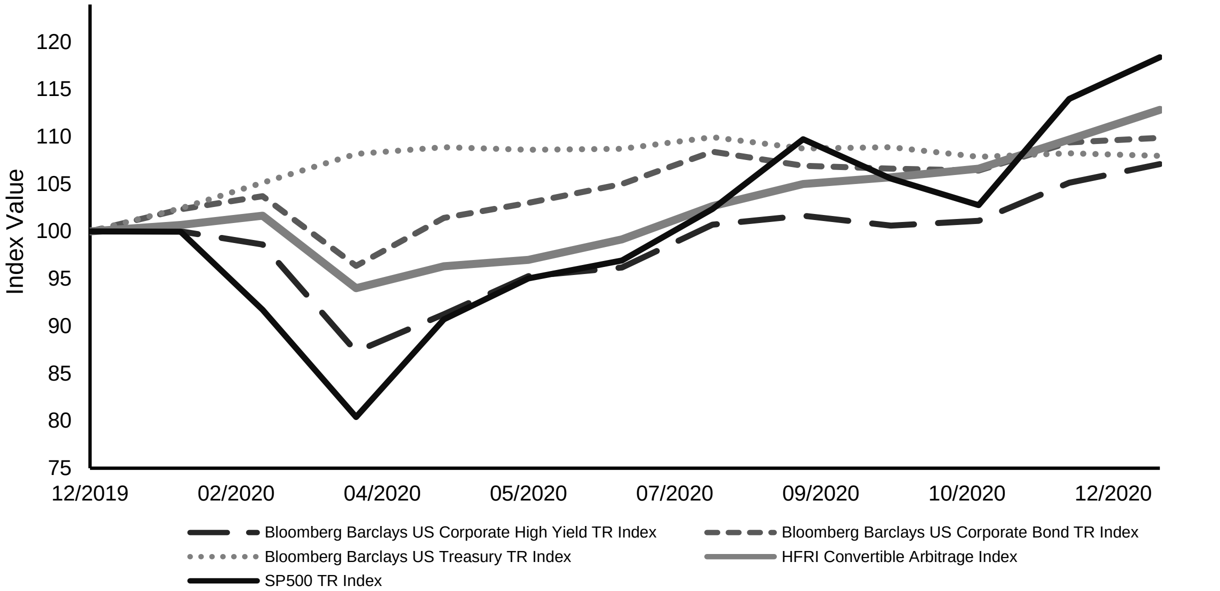
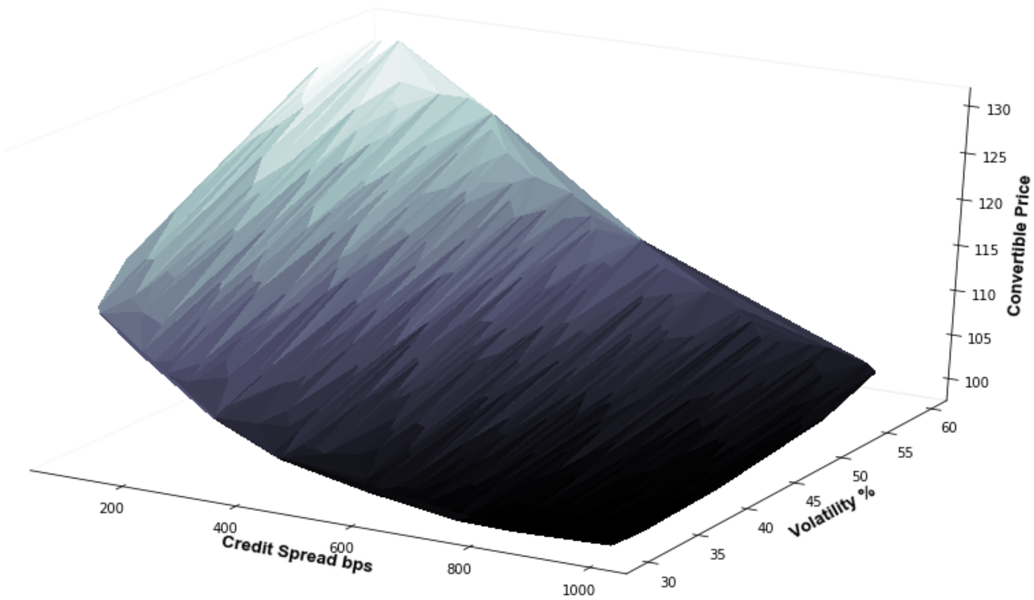
Appendices

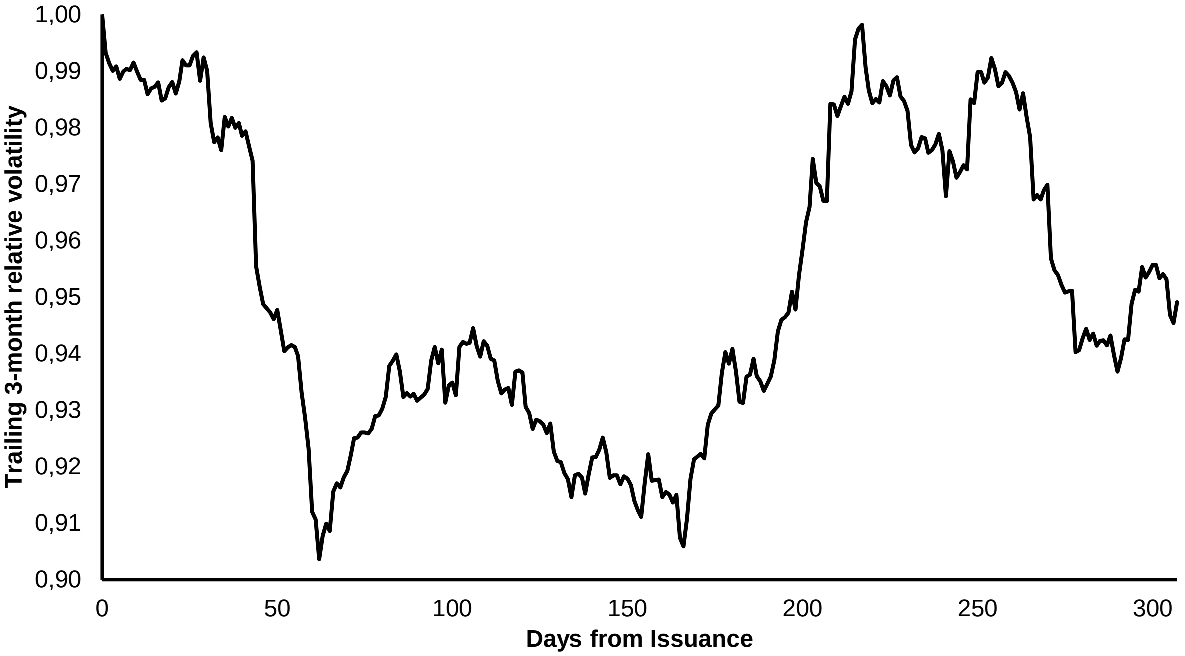
Appendix 1: Cumulative Returns of U.S. stock and bond market indices vs. HFRI Convertible Arbitrage Index



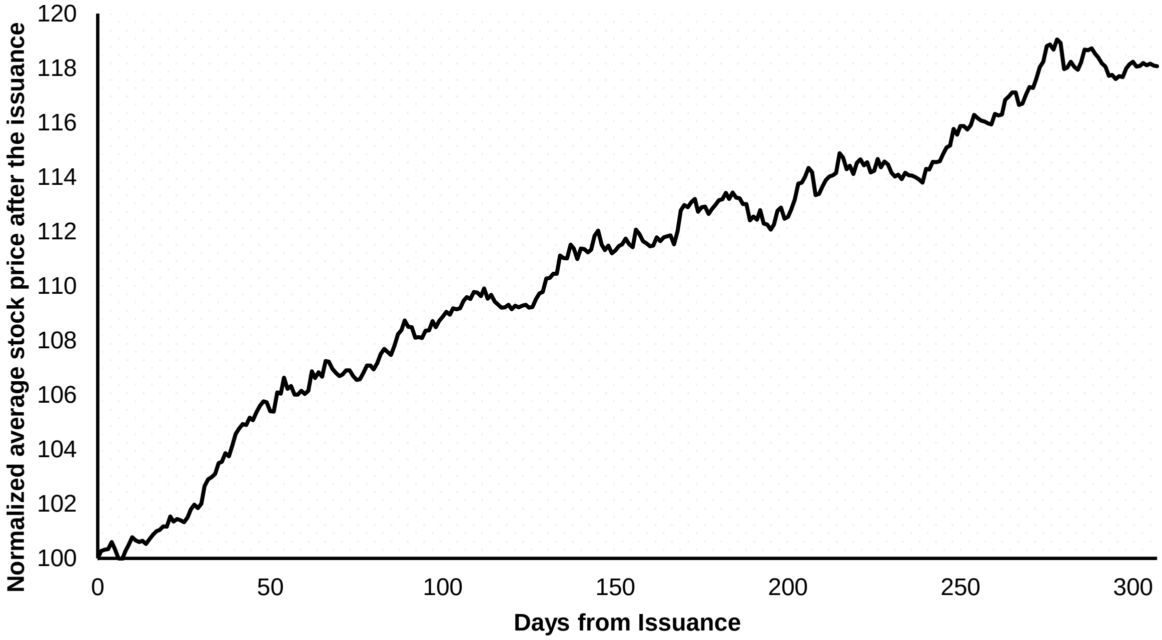
Appendix 2: Convertible bond price sensitivity to volatility and credit spread



Appendix 3: Trailing 3M relative historical volatility of the underlying stock after the issuance



Appendix 4: Normalized average stock price after the issuance of convertible bond



Historical volatility

I

$$\sigma_{252} = \sqrt{252}\sqrt{\frac{\sum_{i=1}^{i=n}(R_i - R_{mean})^2}{N-1}}$$

Where R_i is the daily return of portfolio *i* and σ_{252} denotes the annualized volatility.

Sharpe Ratio per annum

II

$$Sharpe\ Ratio = \frac{R_i - R_{rf}}{\sigma_i}\sqrt{12}$$

Where R_i, is the monthly return of portfolio *i*, and σ_i is the standard deviation of excess returns.