ASSESSING THE EFFECT OF FLASH CRASHES ON THE U.S. STOCK MARKET.

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ABSTRACT

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In recent years, different stock markets have observed increased numbers of sudden and sharp fluctuations in their trading systems. Remarkably, after the flash crash of May 06, 2010, the US stock market was the first to address this new phenomenon publicly. Since then, the US stock market has experienced multiple flash crashes that have led to the re-restructuring of the trading system and the adoption of new regulations.

This study examines the US stock market caps' reaction to flash crashers. The study centers on the flash crash of May 06, 2010, August 24, 2015, and February 05, 2018. The study provides an overview of the market cap volatility and distress observed after each flash crash. The study adopted the range-based volatility estimation model Yang-and-Zhang (2000) to estimate the daily historical volatility for the VIX index, the Large-Cap, the Mid-Cap, and the Small-Cap stock index on each evaluated year for a period of 252 trading dates.

After calculating the Yang-and-Zhang volatility estimation model, the study conducts different statistical analyses and hypothesis tests. The study finds that the US stock market cap indexes exhibit significant volatility fluctuations due to flash crashes. Similarly, the study observes that flash crashes cause considerable distress in the US stock market. Conclusively, the study finds that the Large-Cap stock index tends to be more susceptible to flash-crash than the Mid-Cap and the Small-Cap stock indexes.

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Many Thanks.

Gilbert, T.

"Satisfaction lies in the effort, not in the attainment; full effort is full victory."

- (Mahatma Gandhi, 1922)

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1. Introduction

The U.S. trading system has undergone a new form of market fluctuations characterized by sudden and sharp swings within short periods followed by rapid recoveries; a phenomenon categorized by the U.S. Commodity Futures Trading Commission and U.S. Securities & Exchange Commission (2010), Boulton, Braga-Alves and Kulchania (2014), and the Staff of the U.S. Securities and Exchange Commission (2020) as a Flash Crash. Flash Crashes are a phenomenon of great interest primordially because of the speed at which they occur and the danger it represents to the stock market mechanism and market participants. This phenomenon is responsible for triggering high levels of distress in the stock market, increasing volatility, and compelling the withdrawal of market participants across different sectors (Oriol and Veryzhenko, 2019).

One of the most critical concerns about the exiting relationship between stock markets and flash crashes comes from the market vulnerability as a function of its interconnectivity. Despite numerous research examining the effect of this phenomenon on the mechanism, and the dynamics of the U.S. stock market, a significant theory describing the cause, likelihood, and magnitude of such events remains far from being reached, Sornette (2003). Predominantly because a phenomenon of such scale can cause a deterioration in the market quality, increase the market volatility, and ultimately provoke a sharp decline in the value of the stock market securities (Black, 1988; AlShelahi and Saigal, 2018).

This thesis assesses the U.S. stock market's reaction to flash crashes. The subject is significant because, despite the existing literature, market participants still lack information describing the markets' reactions and behavior after a flash crash. Henceforth, the focus of the study centers on estimating the significance of the market volatility after a flash crash, assessing the market's sentiment, and ultimately evaluating the volatility among the Large-Cap, Mid-Cap, and Small-Cap stocks index.

1.1. Motivation

While currently, there is no single explanation behind flash crashes, there is a critical element that requires our attention: the assessment of the U.S. stock market reaction to Flash Crashes. According to The U.S. Commodity Futures Trading Commission (CFTC) (2010) and The U.S. Securities and Exchange Commission (SEC) (2010), a flash crash refers to a temporary downfall in the market followed by a swift recovery to levels before the shock. There is no concrete explanation among market participants and financial institutions about what might cause a temporary market crash. Therefore, factors such as high market volatility, speculative trading strategies, and market manipulation, among others, are linked to this phenomenon (Sushko and Turner, 2018).

The study of flash crashes is of great importance because, like conventional crashes in the stock market, the ramification of this new phenomenon can carry immeasurable outcomes. Thus, the more that we know about this phenomenon, the better prepared we will be for the future to respond and retaliate, Kathleen O'Toole (1999). This phenomenon's exact effects on the stock market and how markets and market participants react to this type of event is one of the many unanswered questions in today's trading markets. Hence, it is in this spirit that I wrote this thesis and sought to contribute to the existing literature.

This thesis evaluates the U.S. stock market's reaction to flash crashes. Remarkably, the study pays close attention to three recent years that experienced a flash crash: the flash crash of May 6th, 2010, August 24th, 2015, and February 5th, 2018. Therefore, to estimate the stock market's reaction to the flash crashes, this study adopted the price-range volatility model introduced by Yang and Zhang (2000); a model presented as an expansion of the earlier model introduced by (Garman and Klass, 1980).

Different conventional and less sophisticated volatility estimation models such as the Close-to-Close, ARCH, and the GARCH model account for the exiting volatility during a trading cycle. However, they have shown failure in estimating the actual value of volatility, particularly in the parcens of high drift and jump such as those observed during a flash crash. Contrastingly, the Yang and Zhang (2000) volatility estimation model provides a more accurate estimation of the actual value of volatility by considering drift motion, high price jumps, and historical volatility data based on High, Low, Open, and Close prices.

To conduct this study and examine the stock market volatility, the market sentiment, and the Market-Caps volatility, the studied variables are the SP500 index (Large-Cap), the SP600 index (Small-Cap), and the SP400 index (Mid-Cap), and the VIX index (Volatility Index). The SP500, SP400, and SP600 indexes are used for this research because each index encompasses a section of the U.S. Market Capitalization system. Moreover, following the existing literature, this study adopts the VIX and SP500 as a benchmark and yardstick to measure and assess the U.S. stock market sentiment and performance, respectively (S&P Global, 2021).

1.2. Objectives

The central objective of this thesis is to assess the U.S. stock market reaction to Flash Crashes. Predominantly, the research investigates three critical aspects: the U.S. stock market volatility, the significance of the level of distress in the market, and the volatility different across the stock market cap indexes. Therefore, the study adopts the YZ-Volatility estimation model to conduct this research. The model is broken down and performed in three analytical steps. Thus, corresponding to Ang (2015); first, the study estimates the close-to-Open volatility; second, the study calculates the weighted average of the Roger-Satchell volatility model; and third, the study calculates the Open-to-Close volatility. The Data and methodology chapter discusses the estimation of the YZ volatility model.

1.3. Market volatility

Financial and economics literature commonly defines market volatility as the degree to which prices in the market move, both up and down. Volatility is a phenomenon that is frequently measured mathematically by the standard deviation. Accordingly, a stock with high volatility is one where the price changes rapidly and with a bigger amplitude. Thus, the more volatile a stock security is, the riskier it is (Andersen and Bollerslev, 1997; Alizadeh, Brandt, and Diebold, 2002; Engle, 2004).

Debesh Bhowmik (2013) defines volatility as a measure of risk. Accordingly, the higher the standard deviation, the greater the dispersion and, consequently, the higher the risk. Bollerslev and Andersen (2018) describe volatility as the variability of a random variable or co-variability among two or more random variables. Bhowmik and Wang (2020) and Kissell (2021) define volatility as a function of uncertainty about price movements. Degiannakis and Floros (2015) describe volatility as a measure of the market's tendency to rise and fall within sharply periods. How to define volatility remains a challenge among market participants and researchers; thus, despite the intense research about this phenomenon, a universally accepted description remains abstract. Hence, it is this discrepancy that makes the analysis and interpretation of volatility a challenging task (Engle and Gallo, 2006).

The main characteristic of volatility is imperceptibility. Volatility is not observable—however, volatility is an estimate of different financial and econometric models. According to Engle and Gallo (2006), volatility shares the following characteristics: persistence, mean reversion, asymmetric impacts, and exogenous susceptibility. In historical financial data, large moves follow large movements (the contrary holds). Volatility oscillates whiting a normal range; that is, it does not fluctuate towards infinity. Volatility is usually high for specific periods and low for others, creating some volatility clustering. Volatility responds inversely to positive shocks as to negative ones, and volatility is vulnerable to information.

Expanding on this observation, early studies on financial data have identified several common factors across time series, such as volatility clustering, leptokurtosis, and leverage effect. Thus, early econometric models such as the ARCH and the GARCH address these findings. However, the models fail to capture the leverage effect. Therefore, to address this drawback, the ARCH and GARCH models have evolved into several others, such as the EGARCH, the GJR, and the APARCH model, among many others (Mandelbrot, 1963; Alberg, Shalit, and Yosef, 2008). Conversely, a major criticism of these models has been their failure to explain sudden changes in volatility as those generally found in stock prices (Kathleen O'Toole, 1999). This study contributes to the existing literature by analyzing the stock market after a sudden shock, such as a flash crash.

Table 1 Commonly Used Volatility Estimation Models:

Author	**Model**	**Description**
(Pearson, 1893)	Standard Deviation - Historical Volatility (HV)	It is a measure of dispersion that describes the degree of dispersal of the data around the mean or variance.
(Schwert, 1989)	Realized Volatility (RV)	It is the sum of the squared returns over a given period.
(Sharpe, 1964)	Capital Asset Pricing Model (CAPM)	The model computes the volatility for given security concerning a benchmark index, commonly the market index.
(Engle, 1982)	Autoregressive Conditional Heteroscedasticity (ARCH)	The model is commonly used to estimate the variance in time series. The model assumes variance as dependent on past information.
(Bollerslev, 1986)	Generalized Autoregressive Conditional Heteroskedasticity (GARCH)	The model is an expansion of the ARCH model. The modified model allows the estimation of stationary conditions autocorrelation and maximum likelihood.

Today volatility estimation has multiple approaches; for decades, volatility has been at the center of economic research; this has contributed to the development of several estimation models, such as those introduced by Garman and Klass (1980), Engle (1982), Bollerslev (1986) Rogers and Satchell (1991), and Yang and Zhang (2000). Moreover, traditional volatility estimation models for intraday historical financial data have relied for decades on Close-to-Close data; however, these models have failed to estimate the drifts and jumps commonly observed in economic data (Vințe, Ausloos, and Furtună, 2021).

Unlike traditional volatility estimation models, recently developed volatility estimation models are designed based on range-based and multifactor data. The models introduced by Rogers-and-Satchell (1991), Garman-and-Klass (1980), and Yang-and-Zhang (2000) are great examples of these models. This unique characteristic allows the models to provide more significant outcomes, such as high efficiency using publicly available data, ability to estimate volatility on high-frequency data, and robustness to microstructure effects. Hence, the range-based and multifactor models are equally efficient, if not better, than those models developed under the Close-to-Close prices (Fiszeder and Perczak, 2013).

There is not a single factor responsible for inciting volatility in the market. In finance, volatility is highly susceptible to all kinds of information. According to the existing literature, volatility is subjective to several factors, such as a political environment, economic conditions, monetary policies, investors sentiment, and sudden drops in the market, such as a flash crash, among other factors (Bhowmik and Wang, 2020).

For different practitioners and market participants, volatility is a yardstick used to understand price movements, compute trading costs, evaluate overall portfolio risk, and conduct asset allocation, among other applicability (Kissell, 2021). Thus, volatility is crucial to all market participants and researchers because volatility is a detrimental factor in the normal functioning of the stock market. Therefore, quantifying and measuring volatility can yield strongly valuable information imperative for understanding uncertainty and risk (Zheng et al., 2014).

Market distress is a phenomenon observed in historical financial data categorized as disruptions in the market scheme that deviate from standard performance. This phenomenon is commonly captured by examining and monitoring the trading movements of financial securities throughout time (Cboe Exchange Inc., 2022). The Fear Gauge index, i.e., the VIX index, was designed to capture and reflect expected market volatility; hence, the VIX index is used as a barometer to measure future levels of distress in the U.S. stock market.

Different Market Capitalizations (Market-Cap) refer to a security classification based on market value and growth. Market capitalization refers to the actual value of a company determined by multiplying the current value of a share by the total number of upstanding shares (U.S. Securities and Exchange Commission, 2022). Hence, based on this information, stocks are classified into four main groups: Large-Cap, Mid-Cap, Small-Cap, and Micro-Cap. Thus, market-cap indexes are constructed based on this classification.

A market-cap index tracks the performance of a specific set of stocks considered to represent a particular segment of the U.S. market capitalization. Therefore, we can interpret market-cap index volatility as the degree to which prices in the index move up and down. In contrast, the index with the highest volatility is one in which measured values change drastically with a high oscillation.

1.4. Research questions

This study analyses the U.S. stock market performance across three flash crashes based on the statistical analysis of the collected data. Hence, the events are studied individually and according to the year, they occurred. Flash crashes in the U.S. stock market were identified following the reports of the Staff of the U.S. Securities and Exchange Commission. (2020), and the U.S. Department of the Treasury (2021).

Flash crashes are different from non-highspeed shocks because, as their name implies, they occur within a short time interval. However, a common factor shared among both phenomena relates to their potential effect on a trading system and market participants. That is, market sentiment, volatility, and market confidence, among other factors. Thus, to further investigates such observations, this research studies the following questions and hypotheses:

Question 1: Does the market's volatility significantly fluctuates after flash crashes?

H_o: The US Market-Cap index exhibits significantly higher volatility after the flash crash.

The first question in this research seeks to assess and measure the market's volatility when a flash crash has occurred. This inquiry is significant for market participants and researchers because volatility can influence the market's behavior and future performance. Thus, the SP500 index was used as a benchmark for the U.S. stock market volatility to answer this question.

Question 2: Do flash crashes cause distress to the U.S. stock market?

Ho: The VIX index exhibits significantly higher volatility after the flash crashes.

The second question measures the market's fear level during flash crashes. These observations are critical information because the VIX index seeks to estimate distress in the U.S. stock market. Therefore, this research adopts the VIX index as a yardstick for the expected volatility in the U.S. stock market for 252 trading dates. Thus, the results provide crucial information to infer the forthcoming U.S. stock market.

Question 3: Do different Market-Caps exhibit the same response to flash crashes?

The third question of this research seeks to estimate and compare the volatility of three major U.S. stock market cap indexes during a flash crash. Thus, to answer this question, this research computed the Yang and Zhang (Y.Z.) volatility and evaluated the indexes' volatility when the flash crashes occurred. These obtained results greatly assisted in answering the underlying question that motivated this research, i.e., Are flash crashes critical to the U.S. stock market? However, this question is not part of the study since measuring criticality is a vague and extensive subject beyond the scope of this research.

1.5. Structure of the thesis

This study contains five central chapters. Following the introduction, Chapter two inspects the study background and the existing literature concerning flash crashes and the market reaction to such events. Chapter three presents the data and methodology. Chapter four examines the obtained results of the study. Chapter five concludes with a summary and an analysis of the obtained results, describe some of the study limitations, and ends with a suggestion for future research.

2. Background and literature review

This study estimates daily volatility to assess the U.S. stock market reaction to a flash crash. Although several papers have covered volatility, only a few are known for covering flash crashes. Some studies have used financial data from different trading sources, while others have used simulation models to replicate the phenomenon. This section introduces findings observed in previous research conducted about flash crashes, provides a table with some critical reach papers, and concludes with a summary of each studied flash crash.

2.1. Findings of the literature review

Madhavan (2012), McInish, Upson, and Wood (2014), Aldrich, Grundfest and Laughlin (2017), Kirilenko et al. (2017), Braun et al. (2018), and Paulin, Calinescu, and Wooldridge (2019) contribute to the literature, by providing an in-depth micro and macro analysis on the role and influence of government and policymakers, investors behavior, high-frequency trading systems, and assets' diversification.

Based on the currently available literature, it is evident that, analogous to a gradual decline in the stock market value; flash crashes have proven to frighten market participants; deteriorate the market quality and investors' confidence. Market congestion, high volatility, and a sharp decline in the market can persuade market participants to reframe from participating in the market (Black, 1988; Boulton, Braga-Alves and Kulchania, 2014; Virgilio, 2019).

Flash crashes have been unpredictable, and their effect range has been unmeasurable and powerful enough to disrupt an entire economy. Why market crashes occur is a question that remains unanswered; however, studying stocks and markets' historical data during and around these events is of pivotal importance to better understand this phenomenon and its impact on the market mechanism. Although historical data cannot accurately estimate market movements, it can help uncover past information encoded in the trading system and individual securities (Fama, 1965). Nonetheless, one thing remains evident; a market crash occurs because it has already been trading under an unstable environment where any disruption might have caused such a tremor (Sornette, 2003). Table 2 presents a summary of significant contributors to the existing literature.

Table 2 Flash Crashes - Critical Research List

Researchers	**Title**	**Research**
(Oriol and Veryzhenko, 2019)	Mechanistic origin of dragon-kings in a population of competing agents	Examines the behavior of extreme events on the stock market.
(Virgilio, 2019)	Understanding the Flash Crash – state of the art	Evaluates the origin and effect of flash crashes in the stock market
(Akansu, 2017)	The Flash Crash: a review	It reviews the flash crash and analyzes its origin and evolution.
(Kirilenko *et al.*, 2017)	The Flash Crash: High-Frequency Trading in an Electronic Market	Study the flash crash of May 06^{th,} 2010, before and during the event.

Johnson and Tivnan (2012) define flash crashes as extreme behavior in financial markets. The researchers argue that significant changes in the structure of a market are as predictable as small changes are. The researchers assess whether extreme behavior results from exogenous causes or endogenous causes. To investigate this phenomenon, the researchers adopted the 'El Farol' bar problem model, a generic-based model. The study found that an extreme event is an endogenous behavior that results from a vicious cycle created by market participants' trading strategies seeking to outperform the market. According to the study, the induction of a market intervention from market controllers can avoid this phenomenon. However, the feasibility of such an approach remains unexplored.

(Akansu (2017) ABC conducts a research summary of the flash crash observed in the US stock market on May 06th, 2010. The research provides an overview of the event and the market reaction; accordingly, this phenomenon was one of its kind in the trading system that highlighted the fragility of the US stock market to high-frequency trading. The research identifies critical aspects that might trigger flash crashes in the market. Based on the highlighted remarks, the study concludes by inferring that flash crashes share a substantial relationship with the size and frequency of the selling orders in the market, the current market volatility, and algorithmic trading, among other significant factors. According, this has motivated different regulatory and trading agencies to adopt new policies and regulations aimed at reducing the risk of suffering similar crashes in the future.

Virgilio (2019) conducts a methodological analysis of the flash crash of May 06th, 2010. The research summarizes the event and identifies some causes that might have driven this event. Accordingly, among the different hypothesis about flash crashes, the study finds that large selling order conducted by electronic program triggers sales waves into the market that might incentivize high volatility on the trading floor. Accordingly, the fear of violent and ultra-rapid trading strategies carried out by electronic trading systems is the most problematic to market participants about flash crashes.

Oriol and Veryzhenko (2019) examine the market reaction to flash crashes. Accordingly, the research infers that market crashes are not attributed to a single factor but rather several factors. The study investigates the structure of flash crashes caused by operational shocks involving several factors and market participants. Accordingly, the research simulates an artificial intraday selling shocks in the market, with and without multifactor agents. The study observes that flash crashes are outcomes or the resonance of different trading issues rather than outcomes of a single factor.

2.2. The flash crash of May 06, 2010

Since the flash crash of May 06th, 2010, there has been an increasing interest by market participants and academia in the study of flash crashes. The US Commodity Futures Trading Commission and the US Securities & Exchange Commission (2010) researched the flash crash of May 2010; according to their observations, during the event, approximately 8,000 trading securities were affected, and some trading securities lost around 60% of their original values. The Dow Jones Industrial Average (DJIA) lost about 3.20%, the VIX index rose to 31.70%, the SP500 declined about 20% in value, and the E-mini S&P 500 fell by approximately 50%.

Boulton, Braga-Alves, and Kulchania (2014) examined 29 of the most traded stocks on the New York Stock Exchange (NYSE) during the flash crash of May 06th, 2010, and the effect it had on stocks' returns. The study evaluated the stocks' return, the bid-ask spread, and market quality before, during, and after the event. According to the obtained results, the securities experienced a shareholder's wealth decline due to possible investors' reassessment of the stock's value and adverse shocks on liquidity around the event. Based on the obtained results, the study observed a wealth decline on the days around the event, wherein the average cumulative abnormal return around two days before the event was -1.77%. Figure 1 presents the adjusted price and log return movement for the SP500 during a 20-day trading period. Similarly, the graph displays the flash crash and the market's reaction.

Figure 1 SP500-2010 (Adj.Price & Daily Log-Return)

2.3. The flash crash of August 24th, 2015

The Staff of the Office of Analytics and Research and Division of Trading and Markets (2015) and Stafford and Mackenzie (2015) examined the market volatility on August 24th, 2015. According to their findings, several minutes after the opening of the NYSE on August 24th, 2015, the U.S. equity market experienced a flash crash where approximately 50% of the SP500 listed securities were affected, and about 471 exchange trading funds (ETFs) and stocks were placed on halt multiple times as volatility rose. During the first few minutes of trade, the Dow Jones Industrial Average, the SP500, and the Nasdaq Composite slumped nearly 1,100 points (6.60%), 77.68 points (3.90%), and 180 points (3.83%), respectively. Figure 2 presents the adjusted price and log return movement for the SP500 during a 20-day trading period. Similarly, the graph displays the flash crash and the market's reaction.

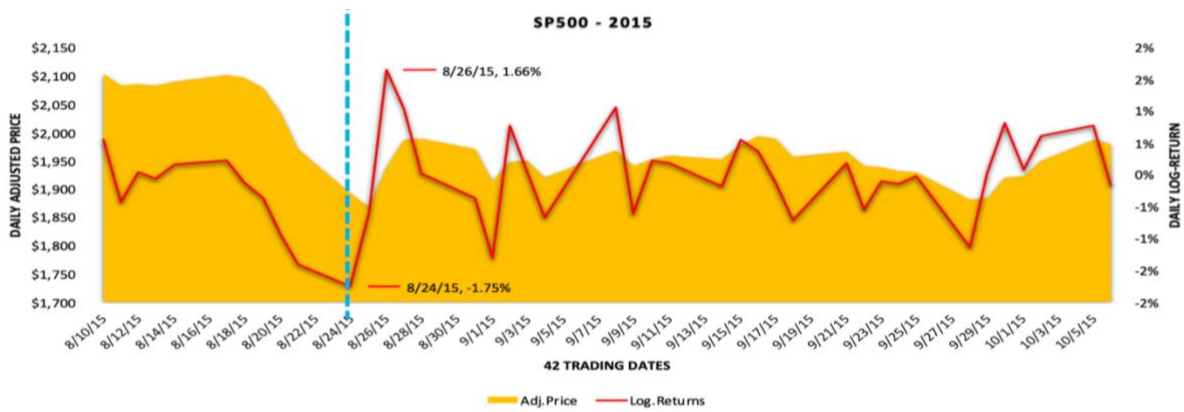


Figure 2 Figure 3 SP500 (Adj.Price & Log.Returns)

2.4. The flash crash of February 5th, 2018

Phillips and Hsu (2018), Sushko and Turner (2018), and Zacks (2018) studied the market flash crash of February 5th, 2018. Accordingly, this event lasted approximately 20 minutes. During the event, the US stock market lost on average $4 trillion as major indexes and securities underwent a flash crash. On February 5th, 2018, the Standard and Poor's 500, Nasdaq, and the Dow Jones Industrial Average fell by approximately 2%, 4%, and 4.6%, respectively. Figure 3 presents the adjusted price and log return movement for the SP500 during a 20-day trading period. Similarly, the graph displays the flash crash and the market's reaction.

Figure 3 Figure 3 SP500 (Adj.Price & Log.Returns)

3. Data and methodology

This chapter presents the research data and the adopted methodology. The chapter begins with an introduction to data research and collection. Then, it describes all the studied variables, and lastly, the study presents the development of the adopted methodology.

3.1. Data collection

Data collection and manipulation combine different analytical software and data repositories, such as Thomson Reuters, Yahoo! Finance, Microsoft Excel, RStudio, Elsevier, Wiley Online Library, and LUT-Primo. Utilizing as a benchmark the various research and reports conducted by the U.S. Commodity Futures Trading Commission and U.S. Securities & Exchange Commission (2010), Stafford and Mackenzie (2015), Phillips and Hsu (2018), Zacks (2018), and the U.S. Department of the Treasury (2021) the study identified the events and the stock securities of interest.

The data sets were collected from Yahoo Finance using R-Studio and the 'tidyquant' package. Therefore, this research downloaded three data sets encompassing three years of historical pricing data. Hence, the obtained values total 60 variables and 15,120 observations of historical pricing data. Moreover, the years used for data collection were January 1st, 2010, to January 1st, 2011; January 1st, 2015, to January 1st, 2016; January 1st, 2018, to January 1st, 2019. The retrieved data includes high, open, low, close, and adjusted prices for the VIX, the SP500, the SP400, and the SP600 index. The study uses the earlier introduced model for estimating and quantifying volatility; that is, the Yang and Zhang volatility estimation model, also known as the YZ range-based volatility estimation model. Table 3 presents the dates used to collect the data sets.

Table 3 Flash Crashes – Data Collection

Market Shock	Starting Point	Ending Pint	Trading Days
May 06th, 2010	*January 1st, 2010*	*January 1st 20211*	*252*
August 24th, 2015	*January 1st, 2015*	*January 1st 20216*	*252*
February 05th, 2018	*January 1st, 2018*	*January 1st 20219*	*252*

3.2. Market Indices

3.2.1. Large-Cap (SP500):

The Standard and Poor’s 500 (SP500) is a critical market index composed of eleven sectors and accounts for over 80% of the currently existing Market-Capitalization; thus, it is a large-cap index (S&P Global, 2022a). Larger-cap companies are companies with a high growth with a value of approximately $10 billion. Investors, financial institutions, and governments commonly use the SP500 as a benchmark and a yardstick to measure and assess the US stock market. Thus, it is considered a good representation of the US market’s performance (S&P Global, 2022).

3.2.2. Mid-Cap (SP400):

Parallel to the SP500 and the SP600, the Standard and Poor's 400 is a crucial market index that serves as a yardstick for measuring and assessing mid-sized stocks. Companies encompassed in the index tend to have a market cap between $3.7 billion and $14.6 billion (S&P Global, 2022a).

3.2.3. Small-Cap (SP600):

Comparable to the SP500 and the SP400, the Standard and Poor's 600 is an important index composed of eleven sectors. The SP600 accounts for 2.5% of the US equity market. Companies encompassed in this index have a market value of approximately $3 billion or less. The index measures a small segment of the US equity market; thus, it serves as a benchmark for assessing small-size companies (S&P Global, 2022).

3.2.4. Volatility Index (VIX)

Analogous to the SP500, the SP400, and the SP600, the Volatility Index (VIX) is an index of pivotal importance in the US economy. The index functions as a reflection of the market sentiment as it exposes market conditions. When markets are turbulent and facing extreme fluctuations, the index depicts high levels of volatility, while the opposite holds. During market tranquility, the VIX index displays a steady volatility level (S&P Global, 2022b). Market participants and academic researchers employ the VIX as a yardstick for measuring and assessing the existent level of volatility in the market at a given time. A fundamental feature of the VIX index is its negative correlation with the performance of major market indexes, such as the SP500.

3.3. Yang and Zhang (2000) – Volatility estimation model

The methodology applied in this study is quantitative. The methods implemented in this research were adapted and borrowed from the early models and analysis presented by Garman and Klass (1980), Rogers, Satchell, and Yoon (1994), Yang and Zhang (2000), Tsay (2005), Fiszeder and Perczak (2013) Petneházi and Gáll (2019), and Vințe, Ausloos, and Furtună (2021). The adopted model is known as the Drift-Independent Volatility Estimation model, introduced by Yang and Zhang (2000) as an extension to the Garman and Klass (1980) historical volatility estimator.

The Yang-Zhang (YZ) volatility estimator model allows for opening jumps and drifts; hence, the model handles and measures intraday price movements, such as extreme jumps and high fluctuations, commonly found in the stock market (Ang, 2015). The model estimation follows three steps; first, it estimates the sum of the Close-to-Open volatility (Overnight Volatility); second, it assesses the Open-to-Close volatility; and third, it calculates the weighted average of the Rogers and Satchell (RS) volatility model.

Contrastingly to other conventional volatility estimation models such as the classical estimator model, the Close-to-Close model, and the Open-to-Open model, among others, the Yan-Zhang volatility estimation model offers a further enhanced model through the use and application of various publicly available financial data such as Open (O), Close (C), High (H), and Low (L) trading prices (Petneházi and Gáll, 2019). Figure 4 shows the implemented steps for the estimation of the YZ volatility. The diagram simplifies the stages of this study. Briefly, the data were collected, classified according to the year of relevance, and inputted into the estimation model.

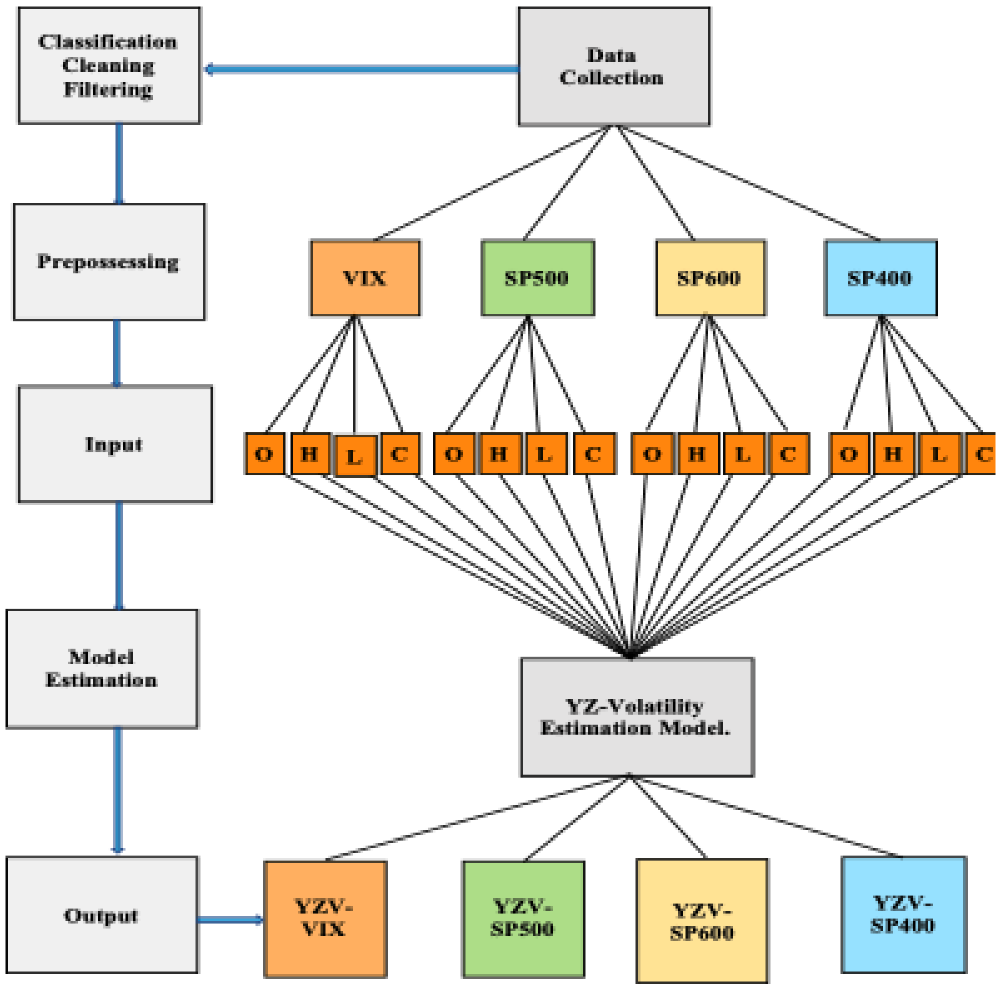


Figure 4 Data Processing Diagram

The Yang and Zhang (200) volatility estimation model was born after the expansion of several other models, specifically the Rogers-Satchell (1991) volatility estimator model and the Garman–Klass (1980) Volatility Estimator. Similarly, these models are an expansion and, in some cases, an extension of other conventional volatility estimation models such as the Parkinson (1980) Volatility Estimation model and the Unbiased Volatility Estimator model, among others (Vințe, Ausloos, and Furtună, 2021). Therefore, the following section briefly introduces those volatility estimation models that contributed to developing the Yang and Zhang volatility estimation model. The general notation adopted in this study follows the notation used by Yang and Zhang (2000):

Table 4 General Notation

Notation	Description:
n	Size of the time window (Days)
V	Unknown variance. It is expressed as the unknown volatility squared (σ^2)
C_0	Previous day closing price (at time 0)
O_1	Current day Opening price (at time f)
H_1	Current day's High during the trading interval
L_1	Current day's Low during the trading intervals
C_1	Closing price of the current day; at time 1
o	$\ln O_1 - \ln C_{0;}$ normalized Opening price
u	$\ln H_1 - \ln O_{1;}$ normalized High of the current period
d	$\ln L_1 - \ln O_{1;}$ normalized Low of the current period
c	$\ln C_1 - \ln O_{1;}$ normalized closing price of the current period
i	Represents the quantity of the i-th period
t	Denotes time

Unbiased Volatility Estimator – Classical Estimator (CC):

The model ignores other fundamentally available factors that might contribute to higher accuracy and efficiency, such as intraday price swings (Garman and Klass, 1980). Hence the following model is adopted from Petneházi and Gáll (2019). Moreover, following the convectional Close-to-Close price volatility estimation model, the following equation exhibits the computation of the variance or the drift of log returns for an n-period interval:

$$V_{CC} = \sqrt{\frac{\left(\sum_{i=1}^{n}\left(\ln\left(\frac{C_t}{C_{t-1}}\right) - \overline{\ln\left(\frac{C_t}{C_{t-1}}\right)}\right)^2\right)}{n-1}} \tag{1}$$

Unbiased Volatility Estimator - Open (O) And Close Prices (OC):

The model represents a better estimator than the previous one because, contrastingly to the classical Close-to-Close approach, this volatility estimator considers opening and closing prices in the calculation, which allows for better estimation, such as drifts and jumps (Garman and Klass, 1980). Adopting the literature from Vințe, Ausloos, and Furtună (2021), the estimation model is as follows:

$$V_{OC} = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}\left[(o_i + c_i) - \overline{(o + c)}\right]^2\right)} \tag{2}$$

Parkinson (P) Volatility Estimator:

In contrast to the conventional Close-to-Close estimation model, the Parkinson (1980) volatility estimation model is superior. However, the model does not consider opening jumps (Petneházi and Gáll, 2019). Hence, the model uses High and Low (HL) prices to estimate volatility. The following equation denotes the calculation of the Parkinson (P) model:

$$V_P = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{4\ln 2}(u_i - d_i)^2\right)} \tag{3}$$

Garman–Klass (GK) Volatility Estimator:

Garman and Klass (1980) introduce an extension of the previously presented model. The model is founded based on all commonly and publicly available stock prices, that is, Open (O), High (H), Low (L), and Close (C). Hence, contrastingly to the Close-to-Close model and other conventional models, the Garman–Klass (GK) volatility estimation model demonstrates a higher efficiency. However, the model fails to estimate price jumps and drifts (Yang and Zhang, 2000). The following equation expresses the GK model:

$$V_{GK} = \sqrt{\left(\frac{1}{n}\right)\sum_{i=1}^{n}\left[0.5\left(\ln\left(\frac{H_i}{L_i}\right)\right)^2 - (2\ln 2 - 1)\left(\ln\left(\frac{C_i}{O_i}\right)\right)^2\right]} \tag{4}$$

Rogers-Satchell (RS) volatility estimator model:

Rogers and Satchell (1991) present a volatility estimation model as an expansion of the GK model. Similarly, Rogers, Satchell, and Yoon (1994) introduce an addition to the previously submitted model by Rogers and Satchell (1991). Contrastingly to the GK model, the Rogers and Satchell (RS) model estimate volatility in the presence of drift. However, in the absence of drift, the model underperforms the GK model (Ang, 2015). The following equation denotes the RS volatility estimation model:

$$V_{RS} = \sqrt{\left[\left(\frac{1}{n}\right)\sum_{i=1}^{n}\left\{\ln\left(\frac{H_i}{O_i}\right)\ln\left(\frac{H_i}{C_i}\right) + \ln\left(\frac{L_i}{O_i}\right)\ln\left(\frac{L_i}{C_i}\right)\right\}\right]} \qquad (5)$$

Yang-Zhang (YZ) Volatility Estimator:

The Yang and Zhang (YZ) volatility estimation model applies multiple-period data sets. In contrast to other volatility estimation models, the YZ is unbiased, considers jumps in the opening prices, and is drift-independent (Yang and Zhang, 2000; Ang, 2015; Petneházi and Gáll, 2019). Hence, Yang and Zhang (2000) introduce an expansion to the GK and the RS models. The following equation denotes the mathematical estimation of the YZ model:

$$V_{YZ} = \sqrt{\left(\frac{\sum_{i=1}^{n}(o_i - \overline{o})^2}{(n-1)}\right) + \left(\frac{\sum_{i=1}^{n}(c_i - \overline{c})^2}{(n-1)}\right) + \left((1-k)\, V_{RS}\right)} \qquad (6)$$

Where "k" is the variance minimizer (Yang and Zhang, 2000). The following equation denotes the estimation of k:

$$k = \left(\frac{0.34}{1.34 + \left(\frac{n+1}{n-1}\right)}\right), \{k \mid k < 1 \text{ and } k > 0\} \qquad (7)$$

At this point, we can observe that the YZ volatility estimation model represents the evolution and expansion of several other conventional and less sophisticated models. This study applies the YZ volatility estimation model for its ability to estimate intraday volatility under the existence of high jumps and drifts in the stock prices.

3.4. Descriptive Statistics and Model of Fitness

Mean, $\bar{x}$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n-1} \tag{8}$$

Where the average values of the sample population are denoted by $\bar{x}$. The index returns at time i is represented by x_i, and n denoted the total number of observations. Hence, since the collected data is sample data, this study uses n – 1 degree of freedom to compute the descriptive statistics and ensure an unbiased model.

Variance, S^2

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \tag{9}$$

The study uses variance to measure the dispersion and spread of the estimated data points.

Std.Dev, (STD)

$$\text{STD} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} \tag{10}$$

Functions as a measure of risk. The higher the standard deviation, the greater the dispersion and, consequently, the higher the risk Debesh Bhowmik (2013).

Relative Standard Deviation, (RSD)

$$\text{RSD} = \frac{\sqrt{S^2}}{\bar{x}} \tag{11}$$

The relative standard deviation measures the degree of a variation of an observation point concerning the mean; that is, it measures how clustered or dispersed the data is from the mean (Vințe, Ausloos, and Furtună, 2021).

One-sample t-test, (t)

$$t = \frac{(\overline{x} - \mu)}{\left(\frac{s}{\sqrt{n}}\right)} \tag{12}$$

The one-sample t-test is a statistical test used to determine if the sample mean deviates significantly from the hypothesized mean, where μ denotes the hypothesized or theoretical value (Tsay, 2005).

Pearson correlation, (r)

$$r = \left(\frac{\left(\sum(X_i - \overline{X}) - (Y_i - \overline{Y}) \right)}{\left(\sum(X_i - \overline{X})^2 - (Y_i - \overline{Y})^2 \right)} \right) \tag{13}$$

Where r denotes the Pearson correlation coefficient, X_i denotes the estimated x values at time i, $\overline{X}$ is the mean, Y_i denotes the estimated y values at time i, and $\overline{Y}$ represents the mean values of the assessed variables at time i (Tsay, 2005).

Skewness, (SK)

$$SK = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(x-\overline{x})^3}{S}\right)} \tag{14}$$

Where SK = 0 indicates that the data is a normal distribution. Hence, a positive SK demonstrates that the data is right-skewed, and a negative SK suggests that the information is left-skewed (Kissell, 2021).

Kurtosis, (KU)

$$KU = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(x-\overline{x})^3}{S^2}\right)} \tag{15}$$

The model measures the steepness of the data. Accordingly, a negative kurtosis value infers a platykurtic distribution. Moreover, a positive kurtosis value indicates a leptokurtic distribution (Kissell, 2021).

Jarque–Bera Test, (JB)

$$\mathrm{JB} = \frac{n}{6}\left(\mathrm{SK}^2 + \frac{(k-3)^2}{4}\right) \sim \chi^2(2) \tag{16}$$

The Jarque–Bera test (JB) assesses whether the data follows a normal distribution. Accordingly, a normal distribution will have SK = 0 and a KU = 3, referred to as mesokurtic (MathWorks, 2022). Moreover, if JB = 1, the null hypothesis is rejected at the 0.05 level of significance – the data is not significant and normally distributed. Thus, the opposite holds; when the JB = 0, the test will fail to reject the null hypothesis.

Percentage change:

$$\mathrm{X} = \left\{\frac{\text{New Price} - \text{Old Price}}{\text{Old Price}}\right\} * 100 \tag{17}$$

The study uses the percentage change to estimate and compare observed value change in the indexes. Where New-Price denoted the cap index price at time t, and the Old-Price implies the cap index price at time t-1.

4. Results:

As expressed in Chapters 1 and 3, this study aims to estimate and evaluate the US stock market reaction to flash crashes. Therefore, the following four sub-sections of the study explore and describes the results of the Yang and Zhang (2000) volatility estimation model.

4.1. Raw data – Preliminary analysis

This section presents the first stage of the research study. Therefore, before further statical analysis or inference, the research starts with a preliminary investigation of the collected historical stock prices. Thus, for generalization, this subsection assessed the adjusted price for each studied index. Table A 2 (see appendix) presents the historical adjusted prices. Similarly, Figure 5 to Figure 7 illustrate the historical adjusted prices for each of the studied variables; the graphs offer the one-year historical development of the four indexes. Specifically, it presents the indexes' historical development during the years when a flash crash occurred. Hence, the graphs exhibit a horizontal blue line and a vertical red line demarking the mean of the historical adjusted price and the date of the flash crash, respectively. All three events were extreme enough to be visually identified upon examining the graphs.

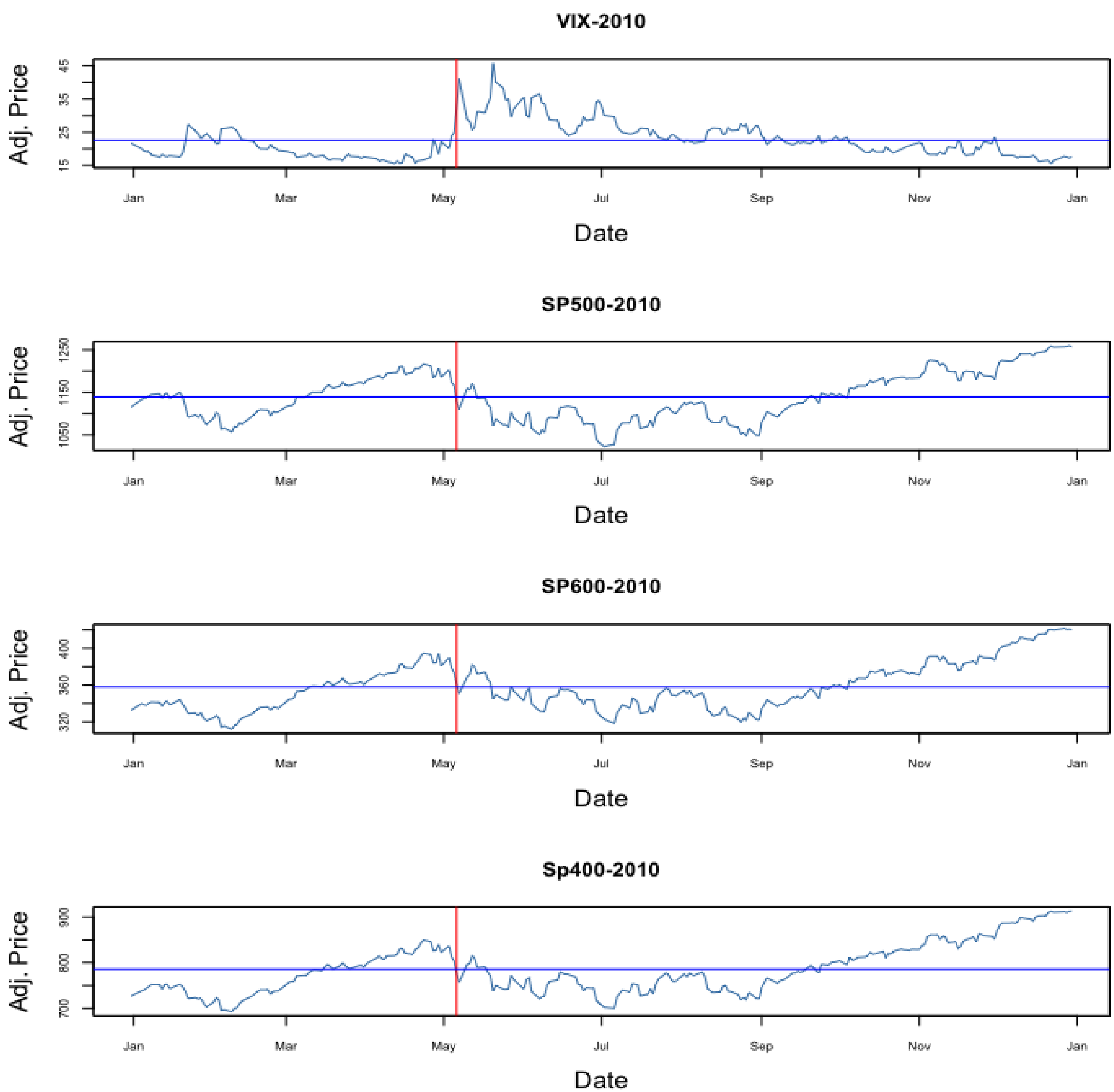


Figure 5 Adjusted Prices – 2010 Pace

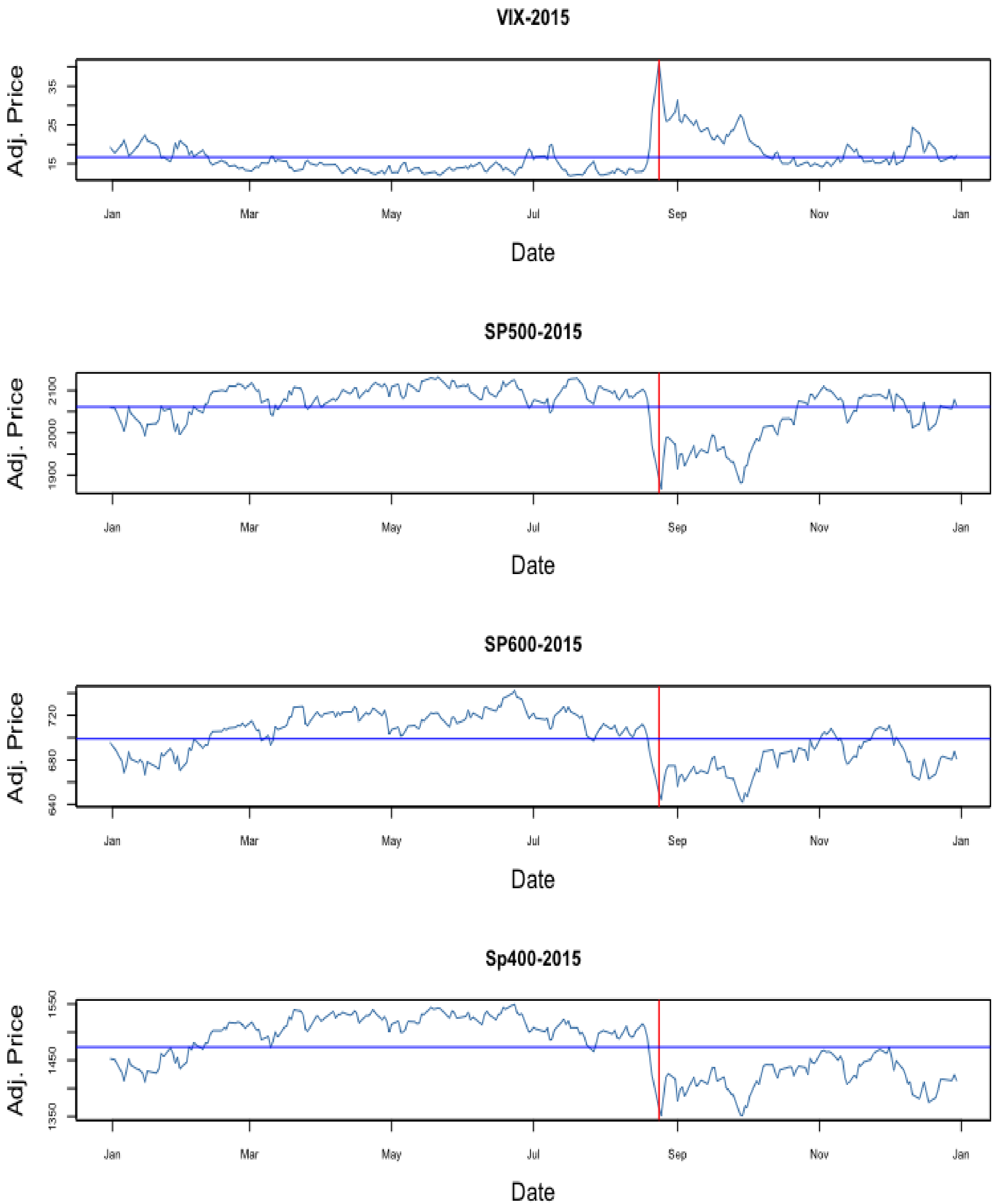


Figure 6 Adjusted Prices – 2015 Pace

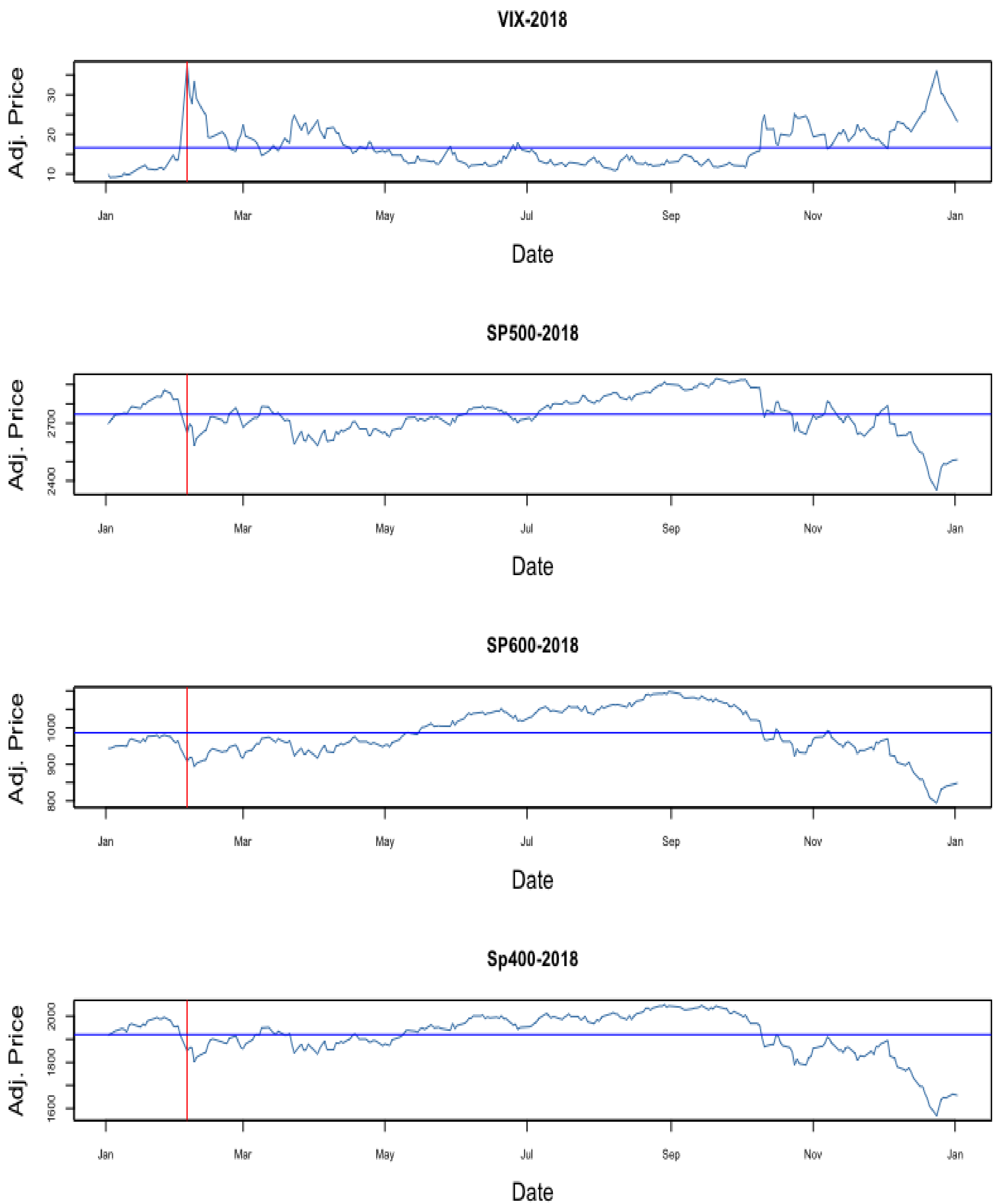
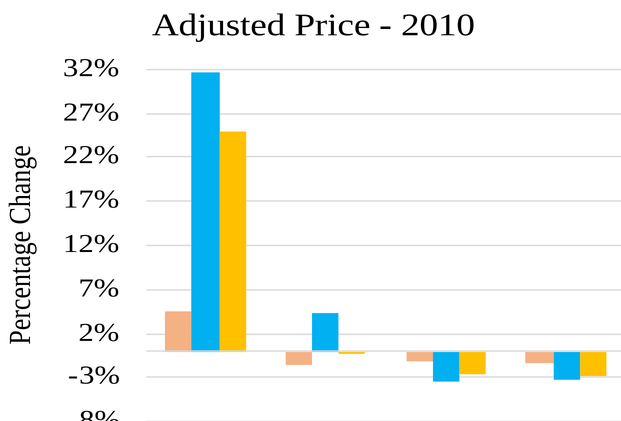


Figure 7 Adjusted Prices – 2018 Pace

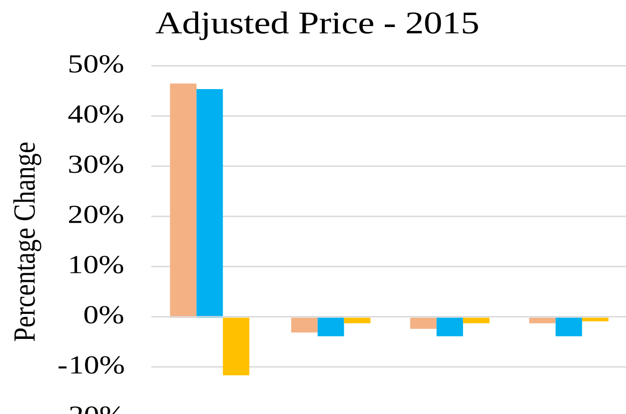
During the flash crash of May 06, 2010, the volatility index exhibited the highest percentage change in the adjusted prices. Hence, the index revealed a positive percentage change of 31.67%. Similarly, the Large-Cap, the Mid-Cap, and the Small-Cap stock index reported a percentage change of 4.40%, -3.36%, and -3.31%, respectively. Figure 8 illustrates these observations. Moreover, to calculate the adjusted price percentage change, the study follows equation 17.



	VIX	Large	Mid	Small
Before	4.49%	-1.53%	-1.10%	-1.34%
During	31.67%	4.40%	-3.36%	-3.31%
After	24.85%	-0.34%	-2.52%	-2.84%

Figure 8 Adjusted price - Percentage Change 2010

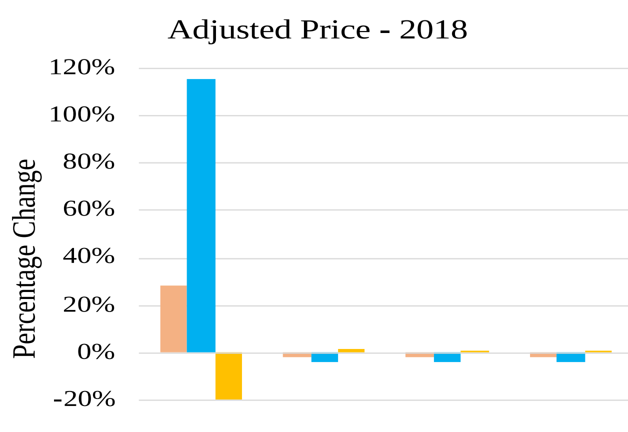
Similarly, during the flash crash of August 24, 2015, the volatility index exhibited the highest percentage change in the adjusted prices. Hence, the index revealed a positive percentage change of 46.45%. Moreover, the Large-Cap stock, the Mid-Cap stock, and the Small-Cap stock index reported a percentage change of -3.94%, -4.01%, and -3.82%, respectively. Figure 9 illustrates these observations. Moreover, to calculate the adjusted price percentage change, the study follows equation 17.



	VIX	Large	Mid	Small
Before	46.45%	-3.19%	-2.34%	-1.37%
During	45.34%	-3.94%	-4.01%	-3.82%
After	-11.59%	-1.35%	-1.08%	-1.04%

Figure 9 Adjusted Price - Percentage Change 2015

Moreover, analogous to 2010 and 2015, during the flash crash of February 05, 2018, the volatility index exhibited the highest percentage change in adjusted prices. Hence, the index revealed a positive percentage change of 115.60%. The Large-Cap, the Mid-Cap, and the Small-Cap stock index reported a negative percentage change of -4.10%, -3.56%, and -3.68%, respectively. Figure 10 illustrates these observations. Moreover, to calculate the adjusted price percentage change, the study follows equation 17.



	VIX	Large	Mid	Small
Before	28.51%	-2.12%	-2.02%	-2.12%
During	115.60%	-4.10%	-3.56%	-3.68%
After	-19.67%	1.74%	0.78%	1.06%

Figure 10 Adjusted Price - Percentage Change 2018

After estimating and examining the percentage change observed on each index, it is evident that flash crashes harm the U.S. stock market. Overall, the VIX index reported the highest percentage change among all the indexes. The large-cap, mid-cap, and small-cap indices reported a negative percentage change between 2010 and 2015. However, for the year 2018, the three market cap indexes exhibited a positive percentage a date after the flash crash, a behavior not found for the years 2010 and 2015. The research infers that the market recovery from a flash crash was much faster during the flash crash of 2018 than in 2010 and 2015.

4.1.1. Descriptive Statistics – Raw Data Analysis

The highest average adjusted price during 2010 was held by the large-cap stock index, followed by the mid-cap, the small-cap, and the VIX index. Respectively, the reported values were \$1,140, \$785, \$358, and \$22. Correspondingly, the highest dispersion as measured by the standard deviation was observed in the large-cap stock index, followed by the mid-cap, the small-cap, and the VIX index with respective values of approximately 56, 53, 25, and 5.3.

The reported values for the large-cap, the small-cap, and the mid-cap stock index had a negative kurtosis with values of (0.73), (0.36), and (0.32), respectively. Hence, the values were platykurtic distributed and positively skewed. The VIX index depicted a positive kurtosis with a value of 2.0 and a leptokurtic distribution with a positive skew. Figure 11 illustrates the indexes' density distribution plots. Hence, applying equation 16, i.e., The Jarque–Bera Test, at the 0.05 level of significance for the JB-test, the study rejects the null hypothesis for all variables and thus confirms that the dataset is not a normal distribution. Table A 5 to table A 7 (See Appendix) exhibit the reported statistical values.

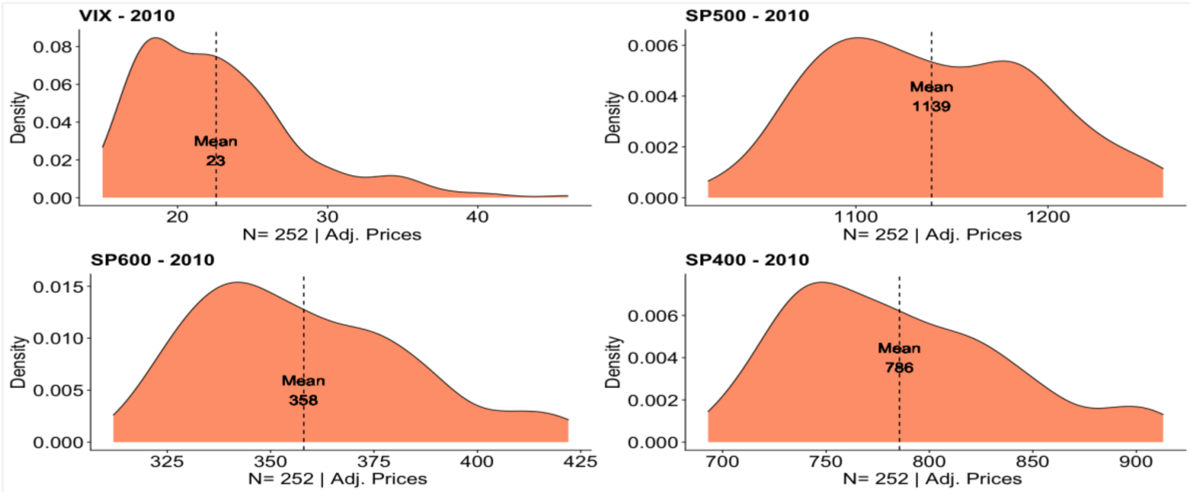


Figure 11 Distribution Density - Adjusted prices – 2010

Comparably to the previous analysis, the highest average adjusted price during 2015 was held by the large-cap stock index, followed by the mid-cap, the small-cap, and the VIX index. Respectively, the reported values were \$2,060, \$1,470, \$700, and \$17. Correspondingly, the highest dispersion as measured by the standard deviation was observed in the large-cap stock index, followed by the mid-cap, the small-cap, and the VIX index with respective values of approximately 55, 50, 21, and 4.30.

Figure 12 illustrates the indexes distribution density plots. The reported values for the mid-cap and the small-cap stock index had a negative kurtosis with (0.97) and (0.62), respectively—table A 9 to table A 12 (See Appendix) exhibit the reported statistical values. Thus, the values depicted a platykurtic distribution with negative skewness. The large-cap stock and the VIX index illustrated a positive kurtosis with a value of 2.20 and 5.30, respectively; hence, both variables depicted a leptokurtic distribution. However, the large-cap stock index showed a negative skewness, whereas the VIX displayed a positive skewness. Therefore, applying equation 16, i.e., The Jarque–Bera Test, at the 0.05 level of significance for the JB-test, the study rejects the null hypothesis for all variables and thus confirms that the dataset is not a normal distribution.

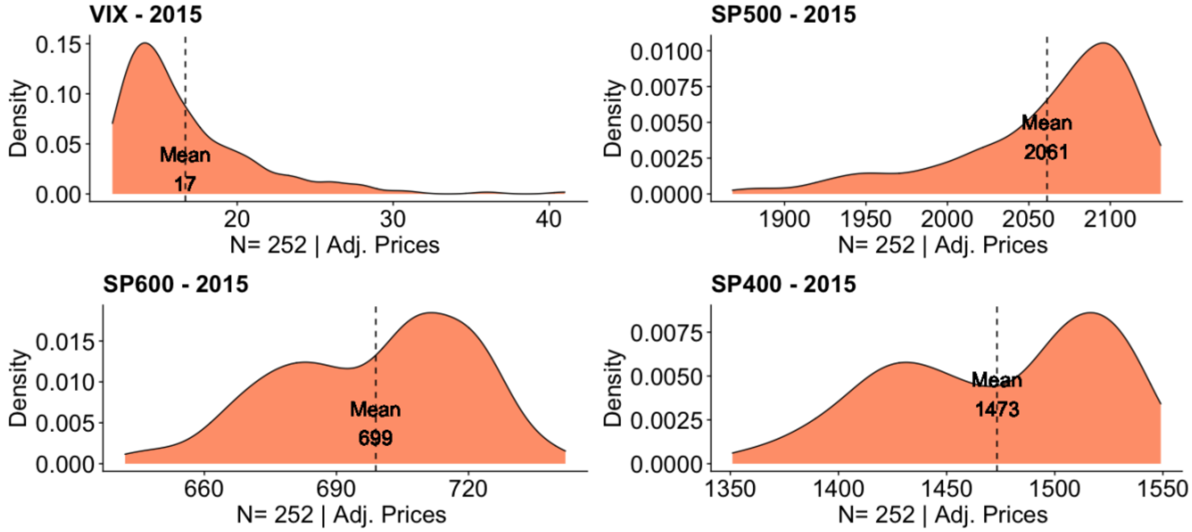


Figure 12 Distribution Density - Adjusted prices – 2015

Moreover, corresponding to the previous analysis, on average and in comparison, to the other three indexes, the large-cap stock index held the highest adjusted price during the year 2018, followed by the mid-cap, the small-cap, and the VIX index. Respectively, the reported values were \$2,740, \$1,920, \$980, and \$17. Correspondingly, the highest dispersion as measured by the standard deviation was observed in the large-cap stock index, followed by the mid-cap, the small-cap, and the VIX index with respective values of approximately 100, 90, 60, and 5.0.

The reported values for the large and the mid-cap stock index had a positive kurtosis with values of 0.85 and 1.90, respectively. Hence, the values were leptokurtic distributed and negatively skewed. Moreover, the small-cap stock index depicted a negative kurtosis with a value of (0.20). The values were platykurtic distributed and negatively skewed. The VIX index had a positive kurtosis with a value of 1.50. The values were leptokurtic distributed and appositively skewed. Figure 13 illustrates the indexes' density distribution plots. Moreover, applying equation 16, i.e., The Jarque–Bera Test, at the 0.05 level of significance for the JB-test, the study rejects the null hypothesis for all variables. It thus confirms that the dataset for the year 2015 is not a normal distribution —table A 13 to table A 16 (See Appendix) exhibit the reported statistical values.

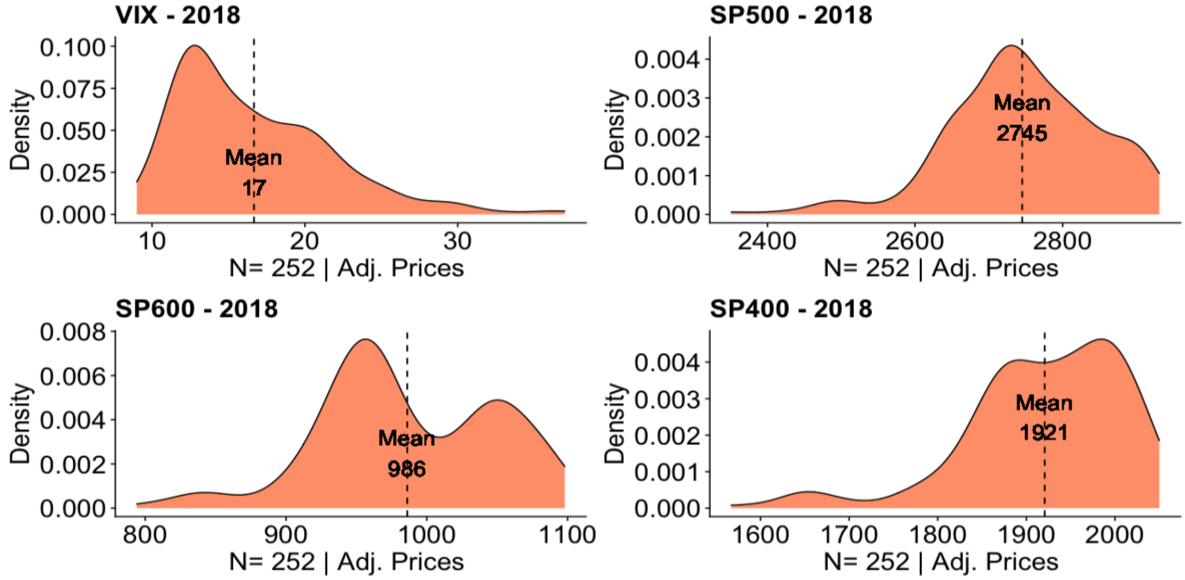


Figure 13 Distribution Density - Adjusted prices – 2018

4.2. YZ – Volatility

This section introduces and assesses the estimated volatility for the Large-Cap stock index, the Mid-Cap stock index, the Small-Cap stocks index, and the VIX index. The study estimated the daily volatility for 252 trading days to evaluate the development of the indexes —similarly, Figure 14 to Figure 16 exhibit the estimated daily volatility for all four indexes. The plots contain a horizontal blue line, a horizontal redline, and a vertical red lined demarketing the mean of the historical estimated volatility, the upper confidence interval, and the date on which the flash crash occurred, respectively. Upon visual examination, the model could capture the historical volatility pace, and the moment the flash crashes occurred.

Estimated Volatility:

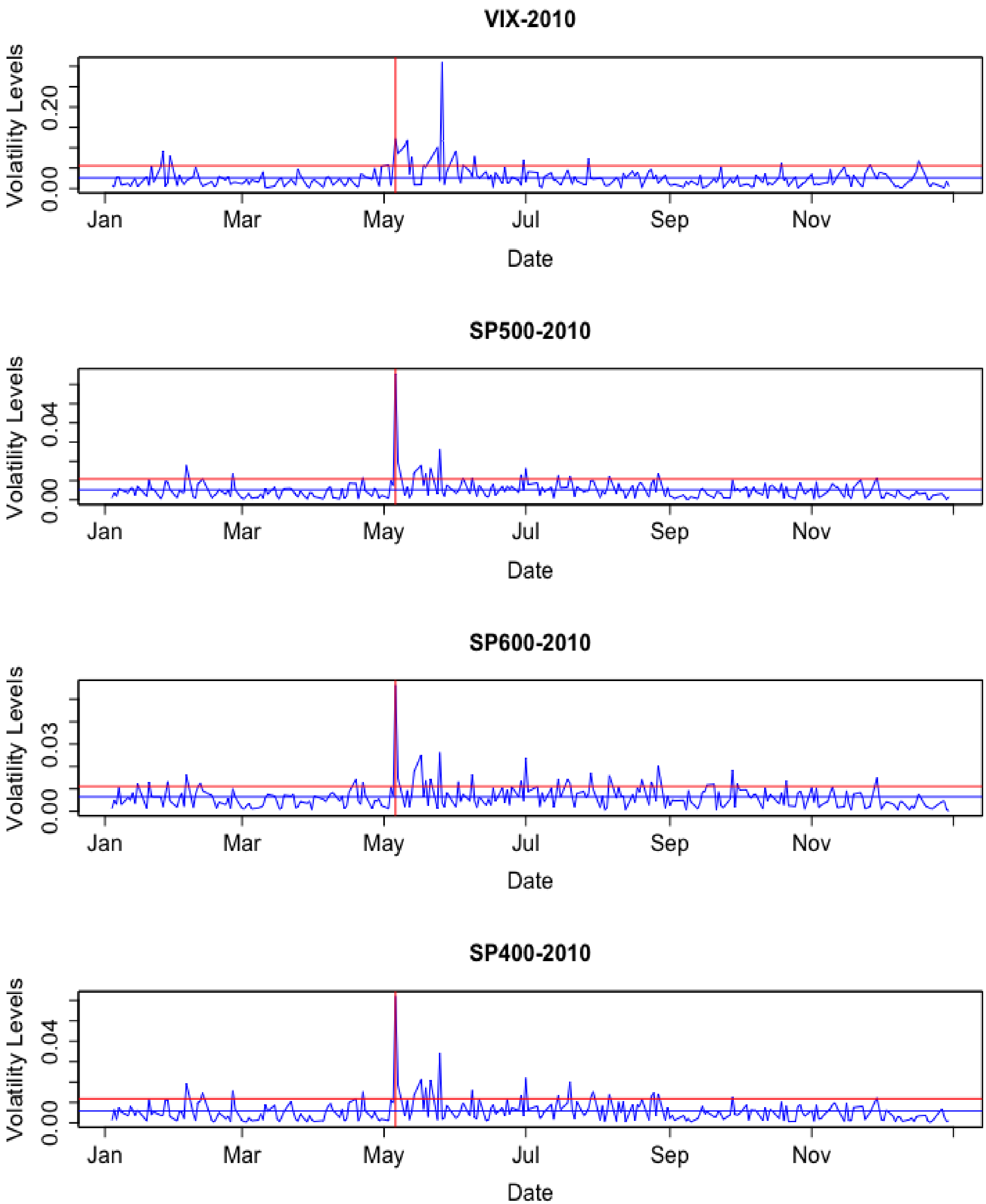


Figure 14 YZ - Volatility Estimates 2010

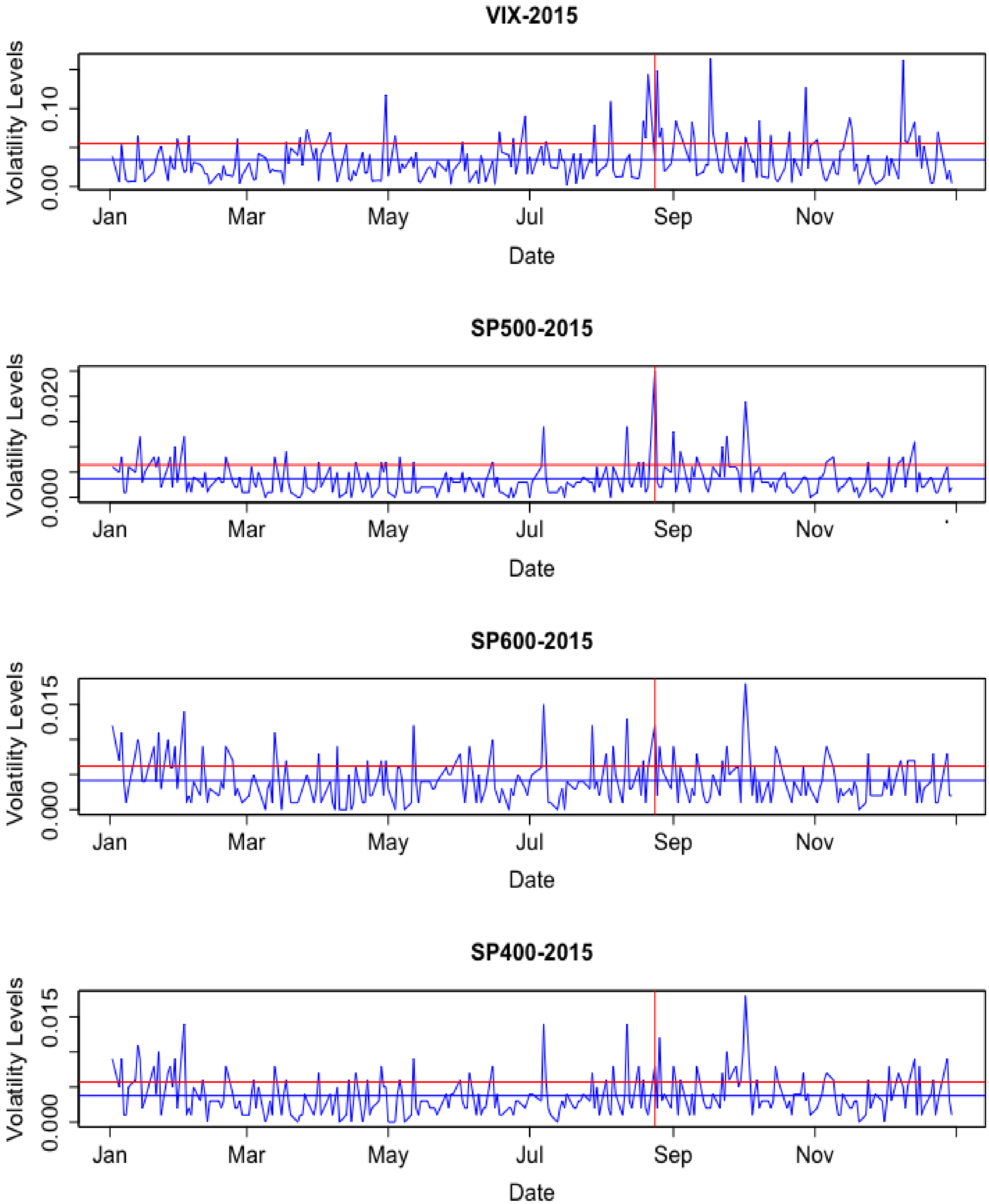


Figure 15 YZ - Volatility Estimates 2015

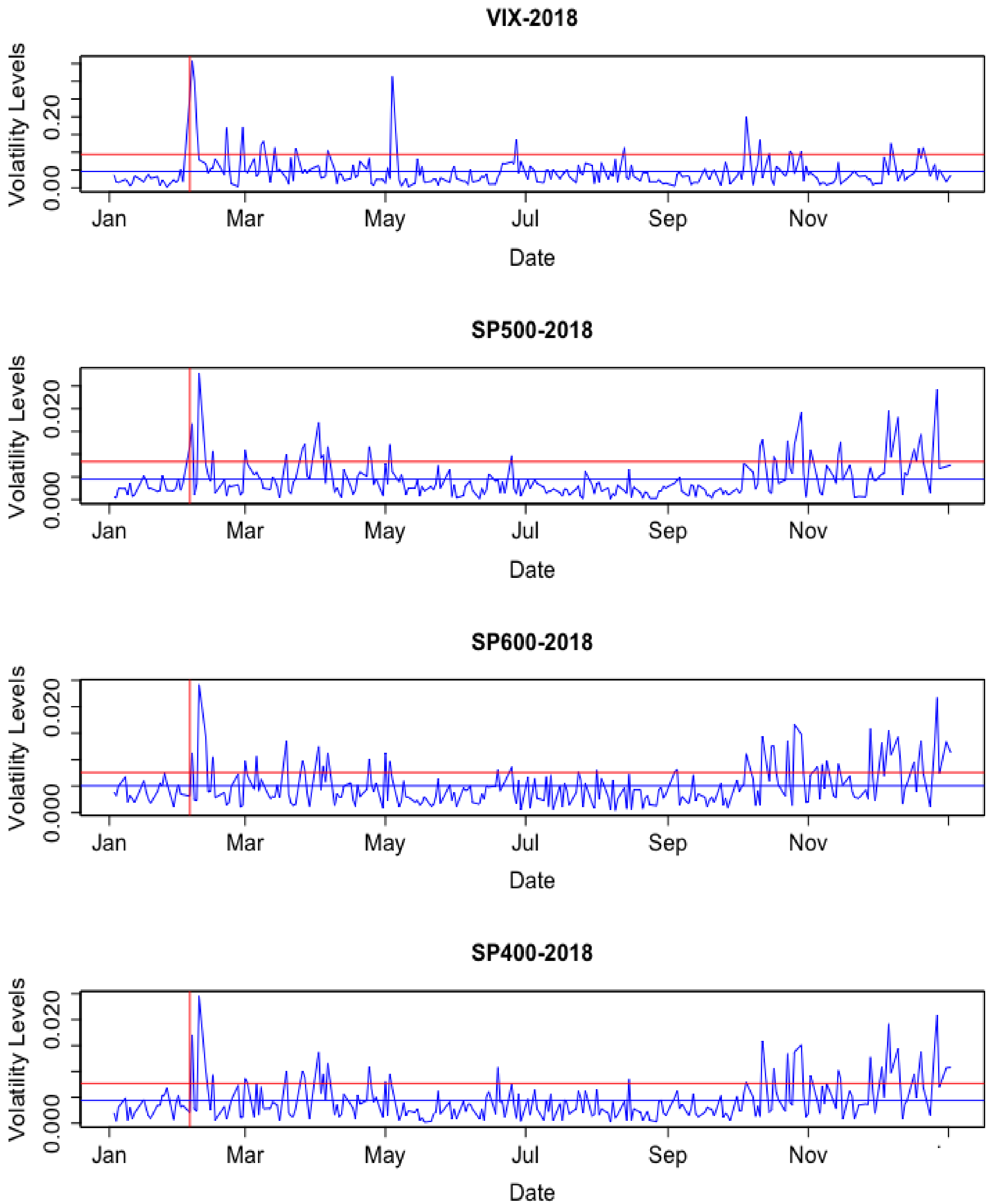


Figure 16 YZ - Volatility Estimates 2018

4.2.1. YZ estimated volatility

Table A 17 to table A 19 (See Appendix) shows the estimated daily volatility for all four indexes. The tables illustrate the estimated volatility for each index. The tables present the volatility estimates for 41 trading dates out of the 252 estimated dates. According to the obtained results, historically, during flash crashes, the VIX index tends to exhibit levels of fluctuation above those revealed by the large, mid, and small-cap stock index. Therefore, ranking the indexes from highest to lowest, based on the exhibited volatility levels during the flash crashes, the highest volatility was found in the VIX index, followed by the large, the small, and the mid-cap stock index. Table 5 presents the exhibited volatility values by each evaluated index during the episode of each flash crash.

Table 5 The Flash Crashes - Volatility Ranking

2010	YZV - Estimates	Rank	**2015**	YZV - Estimates	Rank	**2018**	YZV - Estimates	Rank
VIX	0.123	1	VIX	0.035	1	VIX	0.257	1
Large-Cap	0.066	2	Large-Cap	0.025	2	Large-Cap	0.012	2
Mid-Cap	0.062	3	Small-Cap	0.012	3	Small-Cap	0.003	3
Small-Cap	0.056	4	Mid-Cap	0.008	4	Mid-Cap	0.002	4

4.2.2. YZV – Descriptive Statistics – Analysis

Following the estimation of the daily volatility, the study conducted a descriptive statistical analysis of the obtained values—table

A *20* to table A 22 report the descriptive statistics. Respectively, the reported values were 0.0060, 0.0056, and 0.0055. However, among all the evaluated indexes, the volatility index displayed the highest standard deviation with a value of 0.0280. Moreover, based on the obtained values, for 2010, the stock index with the highest standard deviation was the mid-cap stock index, followed by the small and the large-cap stock index. Figure 17 illustrates the mean and standard deviation (STD).

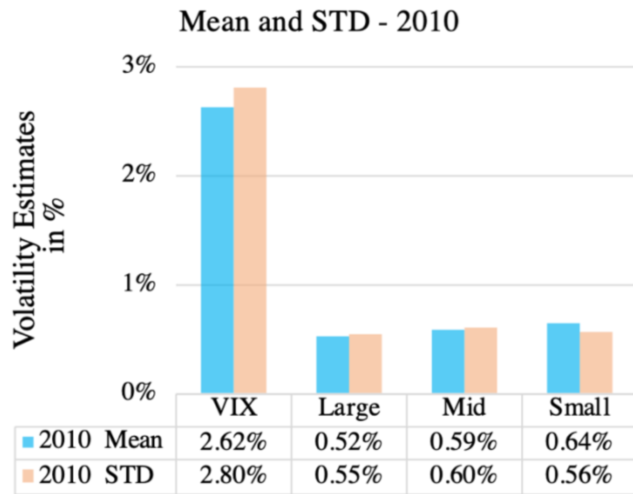


Figure 17 Mean & STD of the YZV Estimates – 2010

Moreover, the Volatility index, the Large-Cap, the Mid-Cap, and the Small-Cap index depicted a positive kurtosis with values of 42, 58, 24, and 30, respectively. Hence, the values were leptokurtic distributed and positively skewed. Figure 18 illustrates the density distribution. Therefore, this indicates that the estimated values were not a normal distribution. Furthermore, the study rejects the null hypothesis at the 0.05 level of significance for the JB-test.

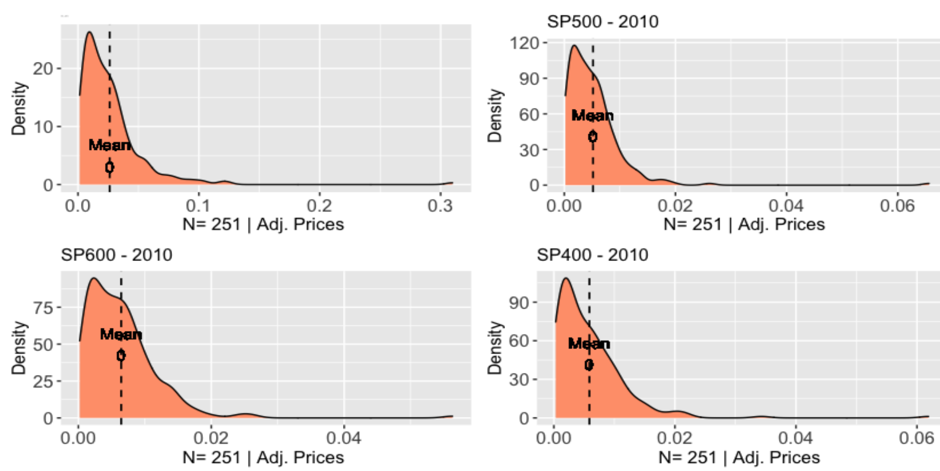
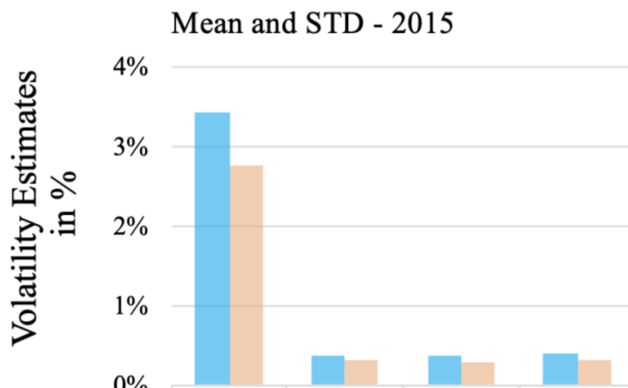


Figure 18 Density Distributions - YZV Estimates 2010

Similarly, based on the obtained values for the year 2015, the stock index with the highest standard deviation was held by the Large-Cap index, followed by the Small-Cap, and the Mid-Cap index with respective values of 0.0033, 0.0032, and 0.0029. However, among all the evaluated indexes, the Volatility index depicted the highest standard deviation among all other indexes, with a value of 0.0276. Figure 19 illustrates the mean and standard deviation (STD).



	VIX	Large	Mid	Small
2015 Mean	3.43%	0.37%	0.38%	0.42%
2015 STD	2.76%	0.33%	0.29%	0.32%

Figure 19 Mean & STD of the YZV Estimates – 2015

Furthermore, the Volatility index, the Large-Cap, the Mid-Cap, and the Small-Cap stock index depicted a positive kurtosis with values of approximately 4.90, 8.70, 1.70, and 2.80, respectively. Hence, the values were leptokurtic distributed and positively skewed. Figure 20 illustrates the density distribution plots. Furthermore, the study rejects the null hypothesis at the 0.05 level of significance for the JB-test. Hence, the estimated values were not a normal distribution.

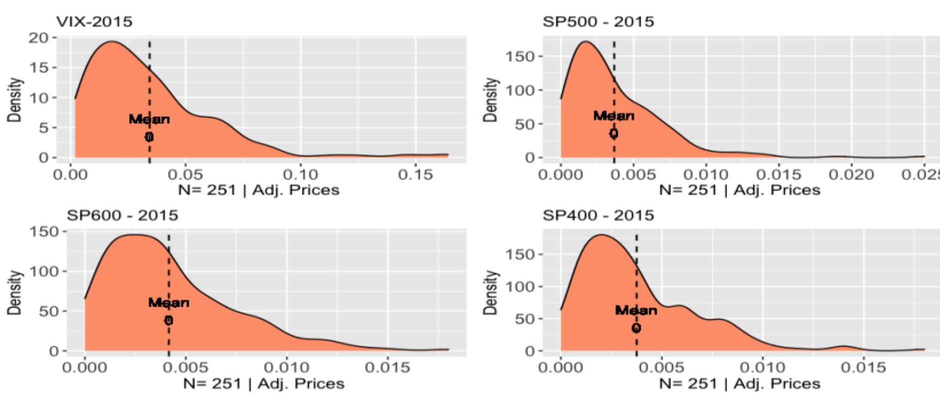


Figure 20 Density Distributions - YZV Estimates 2015

Similarly, based on the obtained values for the year 2018, the stock index with the highest standard deviation was the large-cap stock index, followed by the small-cap and the mid-cap stock index with respective values of 0.0043, 0.0039, and 0.0039. Moreover, among all the evaluated indexes, the Volatility index depicted the highest standard deviation among all other indexes, with a value of 0.0467. Figure 21 illustrates the mean and standard deviation (STD).

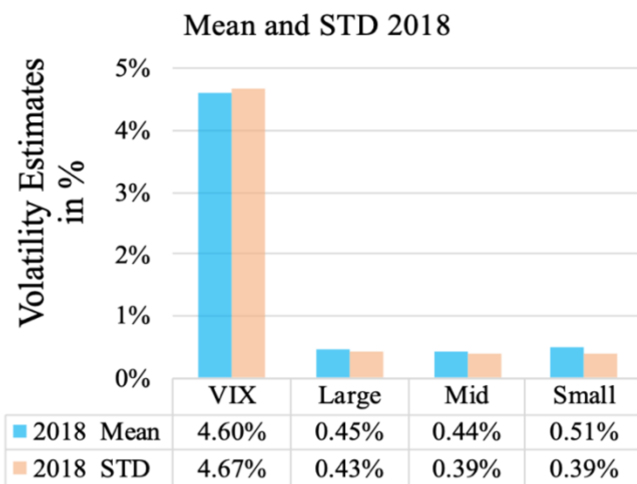


Figure 21 Mean & STD of Volatility Estimates – 2018

Furthermore, the Volatility index, the Large-Cap, the Mid-Cap, and the Small-Cap stock index depicted a positive kurtosis with values of approximately 16.30, 5.10, 3.50, and 4.60, respectively. Hence, the values were leptokurtic distributed and positively skewed. Figure 22 illustrates the density distribution plots. Moreover, the study rejects the null hypothesis at the 0.05 level of significance for the JB-test. Therefore, the estimated values were a normal distribution.

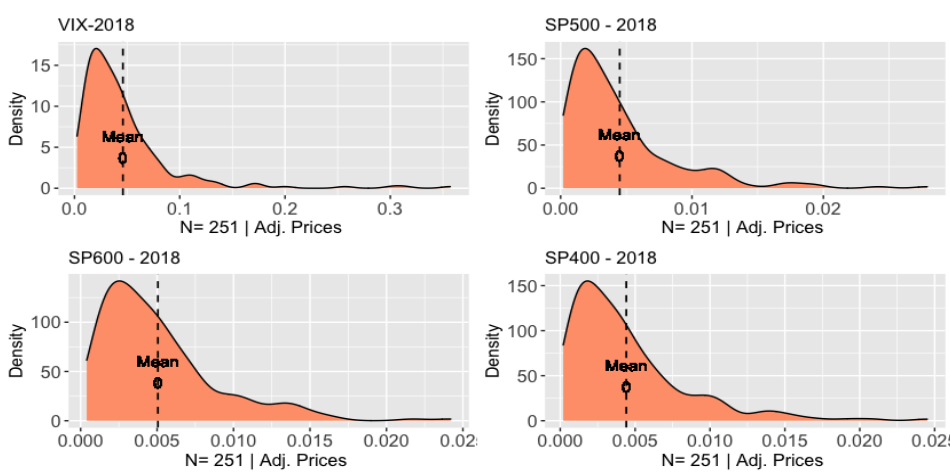


Figure 22 Density Distributions - YZV Estimates 2018

4.2.3. YZV Correlation

After estimating the historical volatility and conducting a descriptive statistics analysis, the study performs a Pearson's correlation test to analyze the possible relationship between the evaluated indexes. The "corrplot" package available in R-Studio contributed to the performance of the correlation test. Subsequently, for 2010, there appears to be a robust association between the three market-cap indexes. Hence, the mid-cap and the large-cap stock index exhibited the highest correlation with a value of r=.954 (N=251, p<.001). Table A 23 (See Appendix) shows the pair-wise correlation matrix, and Table 6 reports the Pearson's correlation hypotheses and tests for 2010.

Table 6 YZV- Reporting the correlation values, 2010

Reported values	***Ho**
SP400 and the SP500 (r=.954, N=251, p<.001)	FTR
SP600 and the SP400 (r=.892, N=251, p<.001)	FTR
SP500 and the SP600 (r=.878, N=251, p<.001)	FTR
*FTR = Fail to reject the Ho | *R= Reject the Ho | α:0.05	

Moreover, based on the obtained values for the year 2015, the study observes a robust and positive correlation between all three market-cap indexes. The mid-cap and the small-cap stock index exhibited the highest correlation, with a value of r=.881. Table A 24 (See Appendix) shows the pair-wise correlation matrix, and Table 7 reports the Pearson's correlation hypotheses and tests for 2015.

Table 7 Reporting the correlation values, 2015

Reported values	***Ho**
SP400 and the SP600 (r=.881, N=251, p<.001)	FTR
SP500 and the SP400 (r=.796, N=251, p<.001)	FTR
SP600 and the SP500 (r=.752, N=251, p<.001)	FTR
*FTR = Fail to reject the Ho | *R= Reject the Ho | α:0.05	

Furthermore, based on the obtained values for 2018 and analogous to the reported values over 2010 and 2015, the study observed a robust and positive correlation between all three market-cap indexes. The mid-cap and the small-cap stock index exhibited the highest correlation with a value of r=.926 (N=251, p<.001). Table A 25 (See Appendix) shows the pair-wise correlation matrix, and Table 8 reports the Pearson's correlation hypotheses and tests for 2018.

Table 8 Reporting the correlation values, 2018

Reported values	***Ho**
SP400 and the SP600 (r=.926, N=251, p<.001)	FTR
SP500 and the SP400 (r=.874, N=251, p<.001)	FTR
SP600 and the SP500 (r=.791, N=251, p<.001)	FTR

*FTR = Fail to reject the Ho | *R= Reject the Ho | α:0.05

4.3. Hypothesis testing

Hence, after the indexes' volatility was estimated through the application of the YZ volatility estimation model, the research seeks to answer questions 1 and 2, previously introduced in section 1.2, that is, Does the market's volatility significantly fluctuate after flash crashes? And Do flash crashes cause distress to the US stock market? Therefore, this study explores two hypotheses using R-Studio and the embedded "t-test" command. The following are the tested hypothesis: following the correlation analysis, this section describes the hypothesis testing and the observed results.

1. *H_o: The US Market-Cap index exhibits significantly higher volatility after the flash crash.*
2. *Ho: The VIX index exhibits significantly higher volatility after the flash crashes.*

Therefore, to establish the statistical significance of the observed fluctuation in the US stock market and the VIX index, a one-sample t-Test is conducted. Hence, researchers and market participants commonly use the SP500 and the VIX index to measure the US stock market volatility and distress, respectively. Accordingly, testing the above hypothesis during the flash crash of 2010, 2015, and 2018, this section uses the SP500 and the VIX index as a benchmark. Hence, the following subsection further illuminates the conducted hypothesis testing.

4.3.1. Market volatility

Examining the statistical significance of the US stock market volatility, the study test whether the mean of the estimated YZ historical volatility for the SP500 is significantly different from the hypothesized mean, i.e., the observed volatility values during the flash crash.

Hypothesis - 2010:

H_0: $\bar{x} \geq (0.0656)$

H_1: $\bar{x} < (0.0656)$

The average estimated volatility for 2010 was 0.0052 (0.52%), and the observed volatility for the SP500 during the flash crash was 0.0656 (6.56%). The obtained results for the t-test reported a T-Statistic greater than the critical values. Respectively, the obtained results were 173.80 and 1.70. Hence, the study rejects the null hypotheses at the five percent significance level (P-Value: 4.79E-263 < 0.05). There is insufficient evidence to infer that the mean of the estimated YZ historical volatility for 2010 was significantly higher than the hypothesized mean. Hence, on average, during this period, the US stock market volatility oscillated between the confidence intervals of 0.0059 (0.59%) and 0.0045 (0.45%). There is enough evidence to infer that the observed fluctuation for the SP500 during the flash crash of May 6th, 2010, was significantly higher than the estimated average volatility. Table A 26 (See Appendix) reports the descriptive statistics and the one-sample t-Test.

Hypothesis - 2015:

H_0: $\bar{x} \geq (0.025)$

H_1: $\bar{x} < (0.025)$

Similarly, the average estimated volatility for 2015 was 0.0037 (0.37%), and the observed volatility for the SP500 during the flash crash was 0.025 (2.5%). The obtained results for the t-test reported a T-Statistic greater than the critical values. Respectively, the obtained results were 103.20 and 1.70. Thus, the study rejects the null hypotheses at the five percent significance level (P-Value: 2.96E-207 < 0.05). There is insufficient evidence to infer that the mean of the estimated YZ historical volatility for 2015 was significantly higher than the hypothesized mean. During this period, the US stock market volatility oscillated between the confidence intervals of 0.0041 (0.41%) and 0.0033 (0.33%). There is enough evidence to infer that the observed fluctuation for the SP500 during the flash crash of August 24th, 2015, was significantly higher than the estimated average volatility. Table A 27 (See Appendix) reports the descriptive statistics and the one-sample t-Test.

Hypothesis - 2018:

H_0: $\bar{x} \geq (0.0118)$

H_1: $\bar{x} < (0.0118)$

The average estimated volatility for 2018 was 0.0045 (0.45%), and the observed volatility for the SP500 during the flash crash was 0.0118 (1.18%). Compared to the years 2010 and 2015, the obtained results for the t-test reported a T-Statistic greater than the critical values. Respectively, the obtained results were 27.10 and 1.70. Thus, the study rejected the null hypotheses at the five percent significance level (P-Value: 1.08E-76 < 0.05). There is insufficient evidence to infer that the mean of the estimated YZ historical volatility for 2018 was significantly higher than the hypothesized mean. During this period, the US stock market volatility oscillated between the confidence intervals of 0.0050 (0.50%) and 0.0040 (0.40%). There is enough evidence to infer that the observed fluctuation for the SP500 during the lash crash of February 05th, 2018, was significantly higher than the estimated average volatility. Table A 28 (See Appendix) reports the descriptive statistics and the one-sample t-Test.

Therefore, under these findings, the study infers that there is enough evidence to suggest that the US stock market exhibits significantly high volatility as a reaction to flash crashes.

4.3.2. Market distress

Following the same methodology as in the previous subsection, in this section, the study test whether the mean for the estimated YZ historical volatility (Distress) for the VIX index is significantly different from the hypothesized mean, i.e., the observed values for the VIX index during the flash crash.

Hypothesis:

$$H_0: \bar{x} \geq YZ_{\text{(VIX)}}$$

$$H_1: \bar{x} < YZ_{\text{(VIX)}}$$

The average estimated VIX volatility for 2010 was 0.0262 (2.62%), and the observed volatility for the VIX index during the flash crash was 0.123 (12.30%). The obtained results for the t-test reported a T-Statistic greater than the critical values. Respectively, the obtained results were 54.70 and 1.70. Thus, the study rejects the null hypotheses at the five percent significance level (P-value: 2.20E-141 < 0.05). Therefore, there is not enough evidence to infer that the estimated mean volatility for the VIX index for 2010 was significantly higher than the observed volatility during the flash crash of May 06th, 2010.

Similarly, the estimated average VIX volatility for 2015 was 0.0343 (3.43%), and the observed volatility for the VIX index during the flash crash was 0.035 (3.50%). The obtained results for the t-test reported a T-statics value higher than the critical value. Respectively, the obtained results were 38 and 1.70. The study rejects the null hypothesis at the five percent significance level (P-Value: 3.52E-01 <0.05). There is not enough evidence to infer that the estimated mean volatility for the VIX index for 2015 was significantly higher than the observed volatility during the flash crash of August 24th, 2015.

The obtained results for the t-test reported a T-Statistic greater than the critical values. The average estimated VIX volatility for 2018 was 0.0460 (4.60%), and the observed volatility for the VIX index during the flash crash was 0.2569 (25.69%). Respectively, the obtained results for the t-test reported, and the critical values were 71.50 and 1.70. Thus, the study rejects the null hypothesis at the five percent significance level (P-value: 1.04E-168 < 0.05). There is not enough evidence to infer that the estimated mean volatility for the VIX index for 2015 is significantly higher than the observed volatility during the flash crash of February 05th, 2018.

There is enough evidence to infer that the observed volatility for the VIX index during the flash crash of May 6th, 2010, August 24th, 2015, and February 05th, 2018, were significantly higher than the estimated average VIX volatility. Under these findings, the study infers that there is enough evidence to suggest that the VIX index exhibits extremely high volatility as a reaction to flash crashes. Consequently, the study concludes that the US stock market shows high levels of distress due to flash crashes. The one-sample t-test can be found in table A 29 to table A 31 (see appendix).

5. Conclusion and discussion

The study analyzed the US stock market caps and their reaction to Flash Crashes. Therefore, to conduct this research, the study adopted the Yang Zhang volatility estimation model and used the values to compute multiple statistical tests and analyses, such as the historical pace analysis, the descriptive statistics, the correlation analysis, and the hypothesis test. This section summarizes the obtained results.

5.1. Results Summary

The study began by estimating the percentage change for each index by applying equation 17 to understand the magnitude of the events and the indexes' responses. Figure 5 to Figure 7 exhibit the historical adjusted price for each index. The graphs depict the price movement for a period of 252 trading dates. The diagram exhibits the moment each flash crash occurred and the market's reaction; this observation provided a positive incite about the research. Hence, close attention was given to the flash crash dates.

After estimating the percentage change, the study found the VIX index as the most volatile across all three years for all the studied indexes. For 2015 and 2018, the VIX index reported a positive percentage change a date after the crash, inferring that the date after the crash, the expected market distress, and the incertitude for 2015 and 2018 were less dramatic than in 2010. Similarly, the study observed a negative percentage change in most cap stock indexes during all the flash crashes. Hence, 2018 exhibited the most significant decline with a percentage change of ≈ -4.10% held by the large-cap stock index. Furthermore, it is crucial to notice that contrastingly to the years 2010 and 2015, all market cap indexes exhibited a positive percentage change after the crash of February 2018, implying a market recovery.

The descriptive statistical analysis conducted on the raw data during 2010 showed all studied indexes as positively skewed and nonnormally distributed. However, for the years 2015 and 2018, the market-cap indexes were negatively skewed. Figure 11 to Figure 13 exhibit the distribution density plot. Furthermore, the study conducted a statistical analysis of the estimated YZ historical volatility. The estimated historical volatility across all three years was leptokurtic and positively skewed. Figure 11 to Figure 13 report the obtained descriptive statistics. The reported values exhibited that the market reaction to the flash crash of 2010 was the highest compared to the observed volatility estimates for 2015 and 2018. Hence, according to the values during the flash crash of May 2010, the study inferred that after the VIX index, the large-cap stock index held the highest volatility, followed by the mid-cap and the small-cap index. Table 5 reports the observed volatility for each event date.

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1.1. Answering the research questions

Does the market's volatility significantly fluctuate after flash crashes?

The study found significant evidence of market volatility indicating the US stock market's susceptibility to flash crashes. Therefore, the study strongly infers that volatility increases considerably during flash crashes. The study demonstrated the vulnerability of the US stock market to a flash crash; accordingly, the highest volatility in the market cap indexes was held by the large-cap stock index in the year 2010, followed by the mid-cap and the small-cap index. However, the reaction of the US stock market caps was less dramatic for the years 2015 and 2018 in comparison to the year 2010; hence, there seems to be a gradual decline in the market reaction to this type of phenomenon as time passes. Thus, the study infers that as the US stock market has learned new information about flash crashes, this has learned how to react better and mitigate this phenomenon; however, further research to corroborate such assumption is needed. Table 9 presents the YZ historical volatility estimates for the four assessed indexes.

Table 9 YZ- Average Volatility Estimates

Date	VIX	Large-Cap	Mid-Cap	Small-Cap
6-May-10	12.30%	6.56%	6.19%	5.63%
24-Aug-15	3.50%	2.50%	0.80%	1.20%
5-Feb-18	25.69%	1.18%	0.21%	0.31%
Ave.	***13.83%***	***3.41%***	***2.40%***	***2.38%***

Do flash crashes cause distress to the US stock market?

As mentioned in paragraph 3.2.4, the VIX index tracks the US stock market's future expected distress and market sentiment. Therefore, across the three different years assessed in this study, the observed results demonstrated a piece of strong and significant evidence in the VIX index indicating that flash crashes do cause market distress. Figure 14 to Figure 16 exhibit the index's dramatic oscillations for 2010, 2015, and 2018. The graphs locate the date of the flash crash and exhibit the volatility levels during that period. The one-sample t-Test can be observed in table A 29 to table A 31 (See Appendix).

Do different market-cap indexes exhibit the same response to flash crashes?

The study evaluated the estimated YZ volatility for each event date and compared the results to determine the difference in volatility. Historically, the highest volatility was observed on the VIX index, followed by the Large Cap, the Mid Cap, and the Small Cap stock index. The indexes exhibited average volatility of 13.80%, 3.40%, 2.40%, and 2.30%, respectively. The magnitude and intensity of the phenomenon differ from index to index. Therefore, based on the assessed historical data, the study infers that the market cap indexes do not respond similarly to flash crashes. Figure 23 displays a bar plot depicting each assessed index's observed YZ historical volatility estimates.

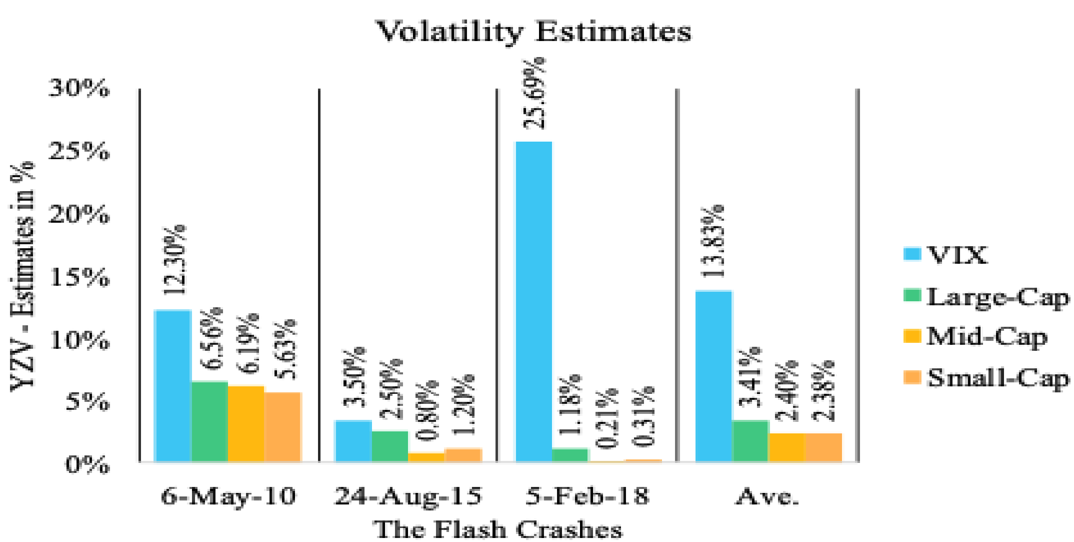

Figure 23 YZ- Volatility Estimates

One critical question from the research was, how long does the market indexes take to recover or dissipate the effect of a flash crash? After estimating the monthly average volatility for each index, the study infers that, on average, the market indexes take approximately one month to two months before returning oscillation values similar to those observed before the crash. This observation is a critical and helpful finding not only to make participants, hedge funds managers, and policymakers but also for those whose market activity relies on future market expectations. Based on this finding, in the presence of future flash crashes, market participants should expect to re-evaluate their position to mitigate unusual market fluctuations for the month to come. Table A 32 to A 34 in the appendix exhibit the obtained values for the estimated monthly average, and Figure 24 illustrates the estimated average monthly volatility for each studied flash crash.

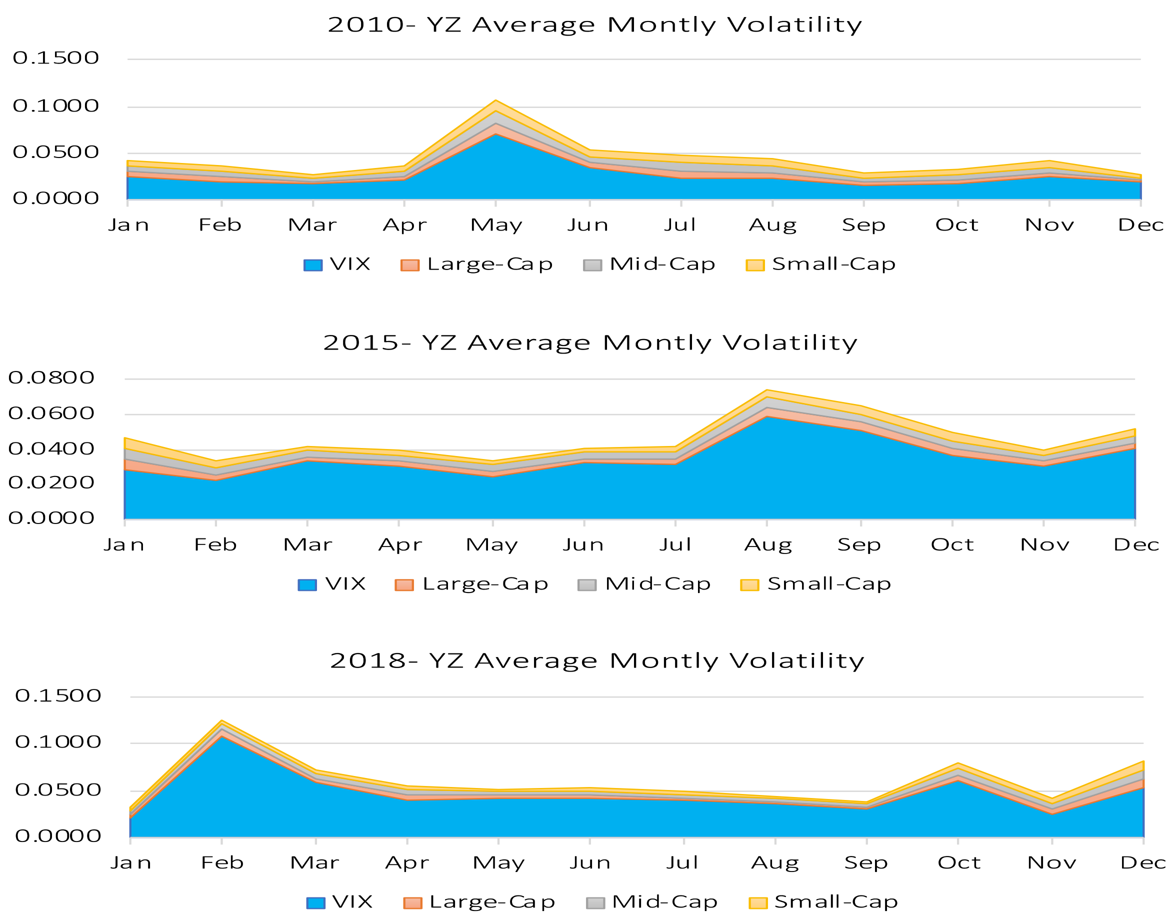

Figure 24 YZ - Monthly Average Volatility

1.2. Limitations

Estimating volatility is a challenge mainly because it is not observable. Hence, conventional volatility estimation models might not be appropriate when modeling financial data relevant to flash crashes because confounding events might affect the estimated values and can cause misestimation and thus provide false information. Although many different methodologies and techniques are available to calculate volatility, another major challenge arises from identifying the right model and the most suitable data type (Bauwens, Hafner, and Laurent, 2012). However, this is a challenge for most academic researchers because publicly available information is often available in daily values; therefore, isolating the data from cofounding events is often impossible. Consequently, it is challenging to assume how the market reacts to a particular event.

The limitations in this study were primarily associated with the methodology and the scale of the data. Accordingly, there is no individual procedure for estimating volatility; thus, it is crucial to identify the event or phenomenon of interest and the most efficient volatility model to calculate volatility for any given market security. This study analyzed and compared three independent events across three different points in time. However, due to the collected data's scale and the phenomenon's uniqueness, a significant challenge in this study arose from isolating the assessed values from other confounding events.

The stock market is a structured network of constant trading, collaboration, and information sharing among market participants. However, this structured atmosphere is restricted based on the existing degree of confidence and uncertainty among market participants. Because of this interconnection, understanding how each stock reacts to flash crashes is of great interest. However, in terms of data collection, because a flash crash is a shock in the market that occurs in a short period, it is suggested to consider high-frequency data to investigate this phenomenon on a grander scale. Thus, researchers could use high-frequency data such as tick-by-tick data. However, a significant challenge arises from the restricted availability of such information and the incongruence among different volatility estimation models. The finding can be of pivotal importance because it can provide in-depth evidence of the securities behaviors during flash crashes. Thus, a suggestion for future research could be a volatility analysis for a voluminous set of trading securities using tick-by-tick data rather than daily prices.

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Appendixes:

A 1 SP500 Adjusted Price Log-Returns

Obs.	**2010	**2015	**2018
-7	-2.34%	-0.06%	0.03%
-6	0.65%	0.17%	0.51%
-5	1.29%	0.23%	-0.29%
-4	-1.66%	-0.11%	-0.48%
-3	1.31%	-0.36%	0.02%
-2	-2.38%	-0.93%	-0.03%
-1	-0.66%	-1.41%	-0.93%
*0	**-3.24%**	**-1.75%**	**-1.82%**
1	-1.53%	-0.59%	0.75%
2	4.40%	1.66%	-0.22%
3	-0.34%	1.04%	-1.66%
4	1.37%	0.03%	0.64%
5	-1.21%	-0.37%	0.60%
6	-1.88%	-1.30%	0.11%
7	0.11%	0.79%	0.58%
8	-1.42%	0.05%	0.52%
9	-0.51%	-0.67%	0.02%
10	-3.90%	1.08%	-0.25%
11	1.50%	-0.61%	-0.24%
12	-1.29%	0.23%	0.04%
13	0.04%	0.19%	0.69%
14	-0.57%	-0.18%	0.51%
15	3.29%	0.55%	-0.56%
16	-1.24%	0.38%	-0.48%
17	-1.72%	-0.11%	-0.58%
18	2.58%	-0.71%	0.22%
19	0.41%	0.20%	0.48%
20	-3.44%	-0.54%	0.11%
21	-1.35%	-0.09%	-0.02%
22	1.10%	-0.15%	0.19%
23	-0.59%	-0.02%	0.75%
24	2.95%	-1.13%	-0.06%
25	0.44%	0.05%	-0.28%
26	-0.18%	0.82%	-0.25%
27	2.35%	0.09%	-0.03%
28	-0.06%	0.62%	0.07%
29	0.13%	0.79%	-0.62%
30	0.13%	-0.16%	0.06%
31	-0.39%	0.35%	-0.08%
32	-1.61%	0.38%	-1.11%
33	-0.30%	0.03%	-0.92%

** Obs. 0 denotes the observed volatility on the date when the FC occurred.*

*** Daily Adjusted prices for the SP500*

A 2 Trading days - Historical Adjusted Prices - 2010

Obs.	Date	SP500	SP400	SP600	VIX
1	April 26, 2010	$1,212	$842	$391	$16
2	April 27, 2010	$1,184	$850	$395	$17
3	April 28, 2010	$1,191	$846	$393	$17
4	April 29, 2010	$1,207	$826	$384	$23
5	April 30, 2010	$1,187	$827	$385	$21
6	May 3, 2010	$1,202	$840	$394	$18
7	May 4, 2010	$1,174	$823	$381	$22
8	May 5, 2010	$1,166	$836	$390	$20
9	May 6, 2010	$1,128	$812	$378	$24
10	May 7, 2010	$1,111	$803	$373	$25
***11**	**May 10, 2010**	**$1,160**	**$776**	**$361**	**$33**
12	May 11, 2010	$1,156	$757	$351	$41
13	May 12, 2010	$1,172	$796	$369	$29
14	May 13, 2010	$1,157	$798	$372	$28
15	May 14, 2010	$1,136	$816	$382	$26
16	May 17, 2010	$1,137	$809	$380	$27
17	May 18, 2010	$1,121	$789	$372	$31
18	May 19, 2010	$1,115	$791	$374	$31
19	May 20, 2010	$1,072	$779	$366	$34
20	May 21, 2010	$1,088	$771	$363	$35
21	May 24, 2010	$1,074	$738	$345	$46
22	May 25, 2010	$1,074	$750	$350	$40
23	May 26, 2010	$1,068	$742	$345	$38
24	May 27, 2010	$1,103	$741	$343	$35
25	May 28, 2010	$1,089	$743	$344	$35
26	June 1, 2010	$1,071	$771	$358	$30
27	June 2, 2010	$1,098	$763	$353	$32
28	June 3, 2010	$1,103	$743	$343	$36
29	June 4, 2010	$1,065	$762	$353	$30
30	June 7, 2010	$1,050	$768	$357	$29
31	June 8, 2010	$1,062	$736	$340	$35
32	June 9, 2010	$1,056	$721	$332	$37
33	June 10, 2010	$1,087	$725	$331	$34
34	June 11, 2010	$1,092	$727	$332	$34
35	June 14, 2010	$1,090	$751	$342	$31
36	June 15, 2010	$1,115	$759	$347	$29
37	June 16, 2010	$1,115	$762	$348	$29
38	June 17, 2010	$1,116	$780	$357	$26
39	June 18, 2010	$1,118	$777	$355	$26
40	June 21, 2010	$1,113	$775	$355	$25
41	June 22, 2010	$1,095	$774	$355	$24
42	June 23, 2010	$1,092	$769	$352	$25

- *Event Date. The date when the flash crash occurred*

A 3 Trading days - Historical Adjusted Prices - 2015

Obs.	date	SP500	SP400	SP600	VIX
1	August 10, 2015	$2,104	$1,510	$710	$12
2	August 11, 2015	$2,084	$1,496	$705	$14
3	August 12, 2015	$2,086	$1,492	$702	$14
4	August 13, 2015	$2,083	$1,492	$701	$13
5	August 14, 2015	$2,092	$1,502	$707	$13
6	August 17, 2015	$2,102	$1,514	$712	$13
7	August 18, 2015	$2,097	$1,507	$708	$14
8	August 19, 2015	$2,080	$1,491	$701	$15
9	August 20, 2015	$2,036	$1,457	$686	$19
10	August 21, 2015	$1,971	$1,423	$677	$28
***11**	**August 24, 2015**	**$1,893**	**$1,366**	**$651**	**$41**
12	August 25, 2015	$1,868	$1,351	$644	$36
13	August 26, 2015	$1,941	$1,386	$659	$30
14	August 27, 2015	$1,988	$1,420	$671	$26
15	August 28, 2015	$1,989	$1,426	$675	$26
16	August 31, 2015	$1,972	$1,417	$675	$28
17	September 1, 2015	$1,914	$1,377	$656	$31
18	September 2, 2015	$1,949	$1,396	$665	$26
19	September 3, 2015	$1,951	$1,403	$667	$26
20	September 4, 2015	$1,921	$1,386	$661	$28
21	September 8, 2015	$1,969	$1,420	$674	$25
22	September 9, 2015	$1,942	$1,404	$668	$26
23	September 10, 2015	$1,952	$1,407	$668	$24
24	September 11, 2015	$1,961	$1,414	$670	$23
25	September 14, 2015	$1,953	$1,408	$668	$24
26	September 15, 2015	$1,978	$1,424	$676	$23
27	September 16, 2015	$1,995	$1,437	$682	$21
28	September 17, 2015	$1,990	$1,436	$683	$21
29	September 18, 2015	$1,958	$1,413	$671	$22
30	September 21, 2015	$1,967	$1,420	$674	$20
31	September 22, 2015	$1,943	$1,401	$665	$22
32	September 23, 2015	$1,939	$1,397	$664	$22
33	September 24, 2015	$1,932	$1,391	$663	$23
34	September 25, 2015	$1,931	$1,388	$663	$24
35	September 28, 2015	$1,882	$1,353	$645	$28
36	September 29, 2015	$1,884	$1,352	$642	$27
37	September 30, 2015	$1,920	$1,369	$650	$25
38	October 1, 2015	$1,924	$1,366	$647	$23
39	October 2, 2015	$1,951	$1,386	$655	$21
40	October 5, 2015	$1,987	$1,413	$672	$20
41	October 6, 2015	$1,980	$1,407	$669	$19
42	October 7, 2015	$1,996	$1,425	$680	$18

- *Event Date. The date when the flash crash occurred*

A 4 Trading days - Historical Adjusted Prices - 2018

Obs.	date	SP500	SP400	SP600	VIX
1	January 2, 2018	$2,696	$1,917	$943	$10
2	January 3, 2018	$2,713	$1,923	$943	$9
3	January 4, 2018	$2,724	$1,928	$945	$9
4	January 5, 2018	$2,743	$1,936	$949	$9
5	January 8, 2018	$2,748	$1,946	$950	$10
6	January 9, 2018	$2,751	$1,943	$950	$10
7	January 10, 2018	$2,748	$1,933	$948	$10
8	January 11, 2018	$2,768	$1,961	$966	$10
9	January 12, 2018	$2,786	$1,966	$969	$10
10	January 16, 2018	$2,776	$1,952	$960	$12
11	January 17, 2018	$2,803	$1,966	$969	$12
12	January 18, 2018	$2,798	$1,958	$962	$12
13	January 19, 2018	$2,810	$1,979	$976	$11
14	January 22, 2018	$2,833	$1,990	$978	$11
15	January 23, 2018	$2,839	$1,994	$979	$11
16	January 24, 2018	$2,838	$1,988	$973	$11
17	January 25, 2018	$2,839	$1,987	$977	$12
18	January 26, 2018	$2,873	$1,995	$980	$11
19	January 29, 2018	$2,854	$1,979	$974	$14
20	January 30, 2018	$2,822	$1,957	$965	$15
21	January 31, 2018	$2,824	$1,954	$959	$14
22	February 1, 2018	$2,822	$1,957	$964	$13
23	February 2, 2018	$2,762	$1,918	$943	$17
24	**February 5, 2018**	**$2,649**	**$1,850**	**$909**	**$37**
25	February 6, 2018	$2,695	$1,864	$918	$30
26	February 7, 2018	$2,682	$1,863	$919	$28
27	February 8, 2018	$2,581	$1,801	$893	$33
28	February 9, 2018	$2,620	$1,821	$903	$29
29	February 12, 2018	$2,656	$1,839	$911	$26
30	February 13, 2018	$2,663	$1,844	$913	$25
31	February 14, 2018	$2,699	$1,878	$929	$19
32	February 15, 2018	$2,731	$1,897	$938	$19
33	February 16, 2018	$2,732	$1,901	$942	$19
34	February 20, 2018	$2,716	$1,888	$933	$21
35	February 21, 2018	$2,701	$1,884	$935	$20
36	February 22, 2018	$2,704	$1,881	$935	$19
37	February 23, 2018	$2,747	$1,904	$947	$16
38	February 26, 2018	$2,780	$1,915	$953	$16
39	February 27, 2018	$2,744	$1,887	$938	$19
40	February 28, 2018	$2,714	$1,865	$921	$20
41	March 1, 2018	$2,678	$1,858	$917	$22
42	March 2, 2018	$2,691	$1,879	$933	$20

- *Event Date. The date when the flash crash occurred*

Descriptive Statics – Raw Data – 2010

A 5 Descriptive Stat. VIX - Adj. 2010

VIX 2010	Open	High	Low	Close	Adjusted
Mean	22.735	23.705	21.700	22.564	22.564
Standard Error	0.333	0.366	0.290	0.332	0.332
Median	21.940	22.595	21.245	21.720	21.720
Minimum	15.440	16.000	15.230	15.450	15.450
Maximum	47.660	48.200	40.300	45.790	45.790
Standard Deviation	5.280	5.814	4.610	5.265	5.265
RSD	0.232	0.245	0.212	0.233	0.233
Sample Variance	27.883	33.808	21.248	27.719	27.719
Kurtosis	2.988	2.442	1.461	2.020	2.020
Skewness	1.410	1.403	1.109	1.283	1.283
Jarque-Berra	177.200	145.306	74.049	111.962	111.962
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	0.655	0.721	0.572	0.653	0.653

*Confidence Level (95.0%)

A 6 Descriptive Stat. SP500 - Adj. 2010

SP500 2010	Open	High	Low	Close	Adjusted
Mean	1,138.853	1,146.051	1,130.728	1,139.400	1,139.400
Standard Error	3.476	3.383	3.598	3.511	3.511
Median	1,136.395	1,144.095	1,127.910	1,136.730	1,136.730
Minimum	1,027.650	1,032.950	1,010.910	1,022.580	1,022.580
Maximum	1,259.440	1,262.600	1,258.780	1,259.780	1,259.780
Standard Deviation	55.178	53.700	57.111	55.728	55.728
RSD	0.048	0.047	0.051	0.049	0.049
Sample Variance	3,044.566	2,883.731	3,261.680	3,105.579	3,105.579
Kurtosis	(0.793)	(0.832)	(0.761)	(0.795)	(0.795)
Skewness	0.204	0.221	0.214	0.203	0.203
Jarque-Berra	8.347	9.318	8.018	8.370	8.370
P-Value	0.015	0.009	0.018	0.015	0.015
Count	252	252	252	252	252
*CL	6.846	6.662	7.085	6.914	6.914

*Confidence Level (95.0%)

A 7 Descriptive Stat. SP400 - Adj. 2010

SP400 2010	Open	High	Low	Close	Adjusted
Mean	784.776	791.125	778.169	785.504	785.504
Standard Error	3.322	3.281	3.399	3.354	3.354
Median	776.870	781.445	770.300	777.085	777.085
Minimum	692.500	697.490	681.910	692.520	692.520
Maximum	912.580	916.180	911.860	913.200	913.200
Standard Deviation	52.731	52.092	53.958	53.247	53.247
RSD	0.067	0.066	0.069	0.068	0.068
Sample Variance	2,780.567	2,713.587	2,911.489	2,835.200	2,835.200
Kurtosis	(0.355)	(0.378)	(0.292)	(0.357)	(0.357)
Skewness	0.583	0.581	0.616	0.585	0.585
Jarque-Berra	15.582	15.657	16.845	15.709	15.709
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	6.542	6.463	6.694	6.606	6.606

*Confidence Level (95.0%)

A 8 Descriptive Stat. SP600 - Adj. 2010

SP600 2010	Open	High	Low	Close	Adjusted
Mean	357.718	360.766	354.548	358.030	358.030
Standard Error	1.598	1.586	1.629	1.615	1.615
Median	353.475	357.255	350.085	354.820	354.820
Minimum	314.110	315.350	309.160	311.950	311.950
Maximum	421.990	421.990	420.310	421.560	421.560
Standard Deviation	25.370	25.172	25.852	25.631	25.631
RSD	0.071	0.070	0.073	0.072	0.072
Sample Variance	643.650	633.613	668.324	656.974	656.974
Kurtosis	(0.285)	(0.351)	(0.202)	(0.323)	(0.323)
Skewness	0.571	0.545	0.611	0.562	0.562
Jarque-Berra	14.568	13.766	16.130	14.345	14.345
P-Value	0.001	0.001	0.000	0.001	0.001
Count	252	252	252	252	252
*CL	3.148	3.123	3.207	3.180	3.180
*Confidence Level (95.0%)					

Descriptive Statics – Raw Data – 2015

A 9 Descriptive Stat. VIX - Adj. 2015

VIX 2015	Open	High	Low	Close	Adjusted
Mean	16.699	17.786	15.842	16.678	16.678
Standard Error	0.251	0.317	0.230	0.273	0.273
Median	15.550	16.285	14.865	15.315	15.315
Minimum	11.770	12.220	10.880	11.950	11.950
Maximum	31.910	53.290	29.910	40.740	40.740
Standard Deviation	3.991	5.033	3.647	4.337	4.337
RSD	0.239	0.283	0.230	0.260	0.260
Sample Variance	15.927	25.329	13.303	18.809	18.809
Kurtosis	2.061	10.722	2.049	5.323	5.323
Skewness	1.491	2.482	1.482	1.944	1.944
Jarque-Berra	137.986	1,465.832	136.250	456.235	456.235
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	0.495	0.624	0.453	0.538	0.538
*Confidence Level (95.0%)					

A 10 Descriptive Stat. SP500 - Adj. 2015

SP500 2015	Open	High	Low	Close	Adjusted
Mean	2,061.353	2,071.954	2,049.394	2,061.127	2,061.127
Standard Error	3.437	3.175	3.655	3.457	3.457
Median	2,079.515	2,086.965	2,067.020	2,079.395	2,079.395
Minimum	1,872.750	1,899.480	1,867.010	1,867.610	1,867.610
Maximum	2,130.360	2,134.720	2,126.060	2,130.820	2,130.820
Standard Deviation	54.563	50.394	58.020	54.875	54.875
RSD	0.026	0.024	0.028	0.027	0.027
Sample Variance	2,977.154	2,539.536	3,366.302	3,011.296	3,011.296
Kurtosis	1.147	0.922	1.104	1.199	1.199
Skewness	(1.281)	(1.257)	(1.265)	(1.287)	(1.287)
Jarque-Berra	82.745	75.302	79.996	84.625	84.625
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	6.769	6.252	7.198	6.808	6.808
*Confidence Level (95.0%)					

A 11 Descriptive Stat. SP600 - Adj. 2015

SP600 2015	Open	High	Low	Close	Adjusted
Mean	699.051	702.645	694.678	698.928	698.928
Standard Error	1.342	1.289	1.390	1.353	1.353
Median	703.115	707.260	699.195	703.010	703.010
Minimum	643.920	647.710	636.870	641.760	641.760
Maximum	741.690	742.230	738.540	742.130	742.130
Standard Deviation	21.301	20.466	22.060	21.482	21.482
RSD	0.030	0.029	0.032	0.031	0.031
Sample Variance	453.743	418.852	486.651	461.472	461.472
Kurtosis	(0.668)	(0.692)	(0.608)	(0.618)	(0.618)
Skewness	(0.381)	(0.361)	(0.396)	(0.408)	(0.408)
Jarque-Berra	10.790	10.520	10.457	10.990	10.990
P-Value	0.005	0.005	0.005	0.004	0.004
Count	252	252	252	252	252
*CL	2.643	2.539	2.737	2.665	2.665

*Confidence Level (95.0%)

A 12 Descriptive Stat. SP400 - Adj. 2015

SP400 2015	Open	High	Low	Close	Adjusted
Mean	1,473.628	1,480.349	1,465.304	1,473.309	1,473.309
Standard Error	3.122	3.010	3.232	3.151	3.151
Median	1,484.710	1,490.925	1,473.825	1,484.520	1,484.520
Minimum	1,352.810	1,361.380	1,344.800	1,351.290	1,351.290
Maximum	1,550.900	1,551.280	1,546.170	1,549.440	1,549.440
Standard Deviation	49.553	47.782	51.307	50.013	50.013
RSD	0.232	0.245	0.212	0.233	0.233
Sample Variance	2,455.547	2,283.164	2,632.431	2,501.302	2,501.302
Kurtosis	(1.036)	(1.066)	(0.971)	(0.965)	(0.965)
Skewness	(0.379)	(0.367)	(0.391)	(0.406)	(0.406)
Jarque-Berra	17.303	17.608	16.314	16.698	16.698
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	6.148	5.928	6.365	6.205	6.205

*Confidence Level (95.0%)

Descriptive Statics – Raw Data – 2018

A 13 Descriptive Stat. VIX - Adj. 2018

VIX 2018	Open	High	Low	Close	Adjusted
Mean	16.671	18.069	15.560	16.666	16.666
Standard Error	0.318	0.387	0.269	0.321	0.321
Median	15.425	16.735	14.610	15.545	15.545
Minimum	9.010	9.310	8.920	9.150	9.150
Maximum	37.320	50.300	29.660	37.320	37.320
Standard Deviation	5.048	6.140	4.265	5.096	5.096
RSD	0.303	0.340	0.274	0.306	0.306
Sample Variance	25.480	37.695	18.193	25.973	25.973
Kurtosis	1.485	3.568	0.410	1.522	1.522
Skewness	1.162	1.508	0.898	1.170	1.170
Jarque-Berra	79.892	229.198	35.657	81.789	81.789
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	0.626	0.762	0.529	0.632	0.632

*Confidence Level (95.0%)

A 14 Descriptive Stat. SP500 - Adj. 2018

SP500 2018	Open	High	Low	Close	Adjusted
Mean	2,746.836	2,761.680	2,729.241	2,745.277	2,745.277
Standard Error	6.324	5.936	6.788	6.382	6.382
Median	2,741.525	2,754.510	2,725.290	2,741.920	2,741.920
Minimum	2,363.120	2,410.340	2,346.580	2,351.100	2,351.100
Maximum	2,936.760	2,940.910	2,927.110	2,930.750	2,930.750
Standard Deviation	100.392	94.229	107.753	101.308	101.308
RSD	0.037	0.034	0.039	0.037	0.037
Sample Variance	10,078.482	8,879.093	11,610.764	10,263.223	10,263.223
Kurtosis	1.017	0.813	0.896	0.851	0.851
Skewness	(0.550)	(0.457)	(0.582)	(0.529)	(0.529)
Jarque-Berra	23.551	15.711	22.651	19.366	19.366
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	12.455	11.690	13.368	12.569	12.569

*Confidence Level (95.0%)

A 15 Descriptive Stat. SP600 - Adj. 2018

SP600 2018	Open	High	Low	Close	Adjusted
Mean	986.695	992.775	979.381	986.077	986.077
Standard Error	3.936	3.819	4.048	3.957	3.957
Median	973.570	977.800	966.260	972.430	972.430
Minimum	797.210	810.090	793.860	793.860	793.860
Maximum	1,096.530	1,100.580	1,092.280	1,098.360	1,098.360
Standard Deviation	62.478	60.632	64.259	62.817	62.817
RSD	0.063	0.061	0.066	0.064	0.064
Sample Variance	3,903.541	3,676.273	4,129.200	3,945.958	3,945.958
Kurtosis	(0.167)	(0.225)	(0.169)	(0.212)	(0.212)
Skewness	(0.231)	(0.191)	(0.255)	(0.231)	(0.231)
Jarque-Berra	2.532	2.065	3.036	2.707	2.707
P-Value	0.282	0.356	0.219	0.258	0.258
Count	252	252	252	252	252
*CL	7.751	7.522	7.972	7.793	7.793

*Confidence Level (95.0%)

A 16 Descriptive Stat. SP400 - Adj. 2018

SP400 2018	Open	High	Low	Close	Adjusted
Mean	1,922.082	1,932.631	1,909.383	1,920.659	1,920.659
Standard Error	5.634	5.380	5.928	5.695	5.695
Median	1,934.565	1,946.065	1,928.045	1,935.440	1,935.440
Minimum	1,574.720	1,602.650	1,565.980	1,567.400	1,567.400
Maximum	2,047.630	2,053.000	2,041.310	2,050.230	2,050.230
Standard Deviation	89.429	85.410	94.099	90.402	90.402
RSD	0.047	0.044	0.049	0.047	0.047
Sample Variance	7,997.608	7,294.892	8,854.576	8,172.497	8,172.497
Kurtosis	2.020	2.113	1.813	1.859	1.859
Skewness	(1.191)	(1.210)	(1.171)	(1.181)	(1.181)
Jarque-Berra	102.415	108.404	92.147	94.849	94.849
P-Value	0.000	0.000	0.000	0.000	0.000
Count	252	252	252	252	252
*CL	11.095	10.596	11.674	11.216	11.216

*Confidence Level (95.0%)

Daily YZ Volatility estimate – 2010

A 17. 41 Trading Days - YZ Volatility Estimate - 2010

Obs.	date	VIX	SP500	SP600	SP400
1	April 1, 2010	0.0213	0.0017	0.0068	0.0007
2	April 5, 2010	0.0035	0.0005	0.0053	0.0011
3	April 6, 2010	0.0253	0.0036	0.0044	0.0057
4	April 7, 2010	0.0287	0.0059	0.0060	0.0080
5	April 8, 2010	0.0229	0.0068	0.0078	0.0095
6	April 9, 2010	0.0084	0.0012	0.0084	0.0062
7	April 12, 2010	0.0236	0.0005	0.0045	0.0027
8	April 13, 2010	0.0152	0.0060	0.0068	0.0066
9	April 14, 2010	0.0064	0.0007	0.0011	0.0010
10	April 15, 2010	0.0119	0.0021	0.0025	0.0020
11	April 16, 2010	0.0261	0.0087	0.0086	0.0100
12	April 19, 2010	0.0068	0.0084	0.0143	0.0117
13	April 20, 2010	0.0042	0.0005	0.0046	0.0009
14	April 21, 2010	0.0353	0.0059	0.0030	0.0037
15	April 22, 2010	0.0332	0.0117	0.0131	0.0153
16	April 23, 2010	0.0283	0.0045	0.0070	0.0057
17	April 26, 2010	0.0227	0.0019	0.0011	0.0024
18	April 27, 2010	0.0494	0.0062	0.0048	0.0053
19	April 28, 2010	0.0230	0.0041	0.0046	0.0029
20	April 29, 2010	0.0062	0.0008	0.0054	0.0010
21	April 30, 2010	0.0544	0.0024	0.0025	0.0019
22	May 3, 2010	0.0581	0.0008	0.0012	0.0011
23	May 4, 2010	0.0074	0.0101	0.0107	0.0111
24	May 5, 2010	0.0604	0.0074	0.0068	0.0098
25	**May 6, 2010**	**0.1230**	**0.0656**	**0.0563**	**0.0619**
26	May 7, 2010	0.0860	0.0196	0.0148	0.0186
27	May 10, 2010	0.1043	0.0023	0.0021	0.0034
28	May 11, 2010	0.1193	0.0068	0.0093	0.0110
29	May 12, 2010	0.0336	0.0010	0.0019	0.0015
30	May 13, 2010	0.0770	0.0035	0.0077	0.0057
31	May 14, 2010	0.0090	0.0141	0.0178	0.0137
32	May 17, 2010	0.0097	0.0179	0.0252	0.0213
33	May 18, 2010	0.0569	0.0074	0.0065	0.0067
34	May 19, 2010	0.0516	0.0138	0.0139	0.0174
35	May 20, 2010	0.0634	0.0023	0.0026	0.0030
36	May 21, 2010	0.0718	0.0165	0.0145	0.0209
37	May 24, 2010	0.1011	0.0030	0.0011	0.0015
38	May 25, 2010	0.0181	0.0261	0.0262	0.0343
39	May 26, 2010	0.3098	0.0043	0.0037	0.0018
40	May 27, 2010	0.0100	0.0018	0.0019	0.0025
41	May 28, 2010	0.0411	0.0077	0.0085	0.0086

A 18. 41 Trading Days - YZ Volatility Estimate - 2015

Obs.	date	VIX	SP500	SP600	SP400
1	August 3, 2015	0.027	0.006	0.008	0.006
2	August 4, 2015	0.038	0.003	0.002	0.002
3	August 5, 2015	0.109	0.000	0.001	0.001
4	August 6, 2015	0.021	0.006	0.009	0.008
5	August 7, 2015	0.012	0.005	0.005	0.004
6	August 10, 2015	0.013	0.001	0.001	0.001
7	August 11, 2015	0.028	0.006	0.005	0.004
8	August 12, 2015	0.036	0.014	0.013	0.014
9	August 13, 2015	0.041	0.003	0.003	0.003
10	August 14, 2015	0.012	0.002	0.003	0.002
11	August 17, 2015	0.010	0.007	0.006	0.008
12	August 18, 2015	0.027	0.002	0.002	0.002
13	August 19, 2015	0.084	0.007	0.007	0.006
14	August 20, 2015	0.062	0.001	0.001	0.002
15	August 21, 2015	0.144	0.002	0.006	0.001
16	**August 24, 2015**	**0.035**	**0.025**	**0.012**	**0.008**
17	August 25, 2015	0.149	0.003	0.002	0.002
18	August 26, 2015	0.063	0.002	0.009	0.012
19	August 27, 2015	0.075	0.002	0.006	0.003
20	August 28, 2015	0.019	0.006	0.005	0.004
21	August 31, 2015	0.029	0.005	0.003	0.002
22	September 1, 2015	0.052	0.013	0.009	0.008
23	September 2, 2015	0.085	0.001	0.006	0.006
24	September 3, 2015	0.074	0.003	0.001	0.001
25	September 4, 2015	0.067	0.009	0.006	0.006
26	September 8, 2015	0.032	0.001	0.001	0.001
27	September 9, 2015	0.083	0.006	0.003	0.004
28	September 10, 2015	0.063	0.004	0.002	0.001
29	September 11, 2015	0.014	0.008	0.008	0.008
30	September 14, 2015	0.019	0.004	0.002	0.003
31	September 15, 2015	0.028	0.002	0.001	0.002
32	September 16, 2015	0.027	0.001	0.001	0.002
33	September 17, 2015	0.164	0.003	0.002	0.002
34	September 18, 2015	0.070	0.006	0.005	0.004
35	September 21, 2015	0.020	0.004	0.002	0.002
36	September 22, 2015	0.017	0.010	0.007	0.007
37	September 23, 2015	0.042	0.004	0.004	0.003
38	September 24, 2015	0.069	0.012	0.009	0.010
39	September 25, 2015	0.041	0.006	0.005	0.006
40	September 28, 2015	0.018	0.006	0.006	0.008
41	September 29, 2015	0.033	0.005	0.006	0.005

A 19. 41 Trading Days - YZ Volatility Estimate - 2018

Obs.	date	VIX	SP500	SP600	SP400
1	January 3, 2018	0.0366	0.0006	0.0039	0.0020
2	January 4, 2018	0.0170	0.0004	0.0032	0.0004
3	January 5, 2018	0.0153	0.0025	0.0050	0.0031
4	January 8, 2018	0.0237	0.0024	0.0068	0.0049
5	January 9, 2018	0.0170	0.0011	0.0020	0.0005
6	January 10, 2018	0.0052	0.0036	0.0032	0.0030
7	January 11, 2018	0.0130	0.0007	0.0026	0.0008
8	January 12, 2018	0.0336	0.0011	0.0019	0.0020
9	January 16, 2018	0.0169	0.0052	0.0060	0.0046
10	January 17, 2018	0.0293	0.0042	0.0039	0.0028
11	January 18, 2018	0.0377	0.0024	0.0025	0.0014
12	January 19, 2018	0.0262	0.0025	0.0018	0.0010
13	January 22, 2018	0.0317	0.0018	0.0040	0.0035
14	January 23, 2018	0.0063	0.0020	0.0055	0.0034
15	January 24, 2018	0.0218	0.0053	0.0051	0.0053
16	January 25, 2018	0.0138	0.0037	0.0075	0.0055
17	January 26, 2018	0.0024	0.0020	0.0056	0.0068
18	January 29, 2018	0.0225	0.0019	0.0011	0.0007
19	January 30, 2018	0.0151	0.0026	0.0036	0.0038
20	January 31, 2018	0.0224	0.0047	0.0053	0.0053
21	February 1, 2018	0.0527	0.0020	0.0035	0.0031
22	February 2, 2018	0.0172	0.0036	0.0034	0.0034
23	**February 5, 2018**	**0.2569**	**0.0118**	**0.0031**	**0.0021**
24	February 6, 2018	0.3569	0.0167	0.0112	0.0171
25	February 7, 2018	0.3015	0.0011	0.0024	0.0027
26	February 8, 2018	0.1742	0.0036	0.0023	0.0024
27	February 9, 2018	0.0793	0.0279	0.0242	0.0245
28	February 12, 2018	0.0672	0.0077	0.0142	0.0101
29	February 13, 2018	0.0412	0.0054	0.0039	0.0062
30	February 14, 2018	0.0532	0.0040	0.0041	0.0026
31	February 15, 2018	0.0563	0.0107	0.0105	0.0093
32	February 16, 2018	0.0823	0.0014	0.0028	0.0006
33	February 20, 2018	0.0375	0.0043	0.0038	0.0032
34	February 21, 2018	0.1688	0.0009	0.0014	0.0008
35	February 22, 2018	0.0629	0.0031	0.0019	0.0026
36	February 23, 2018	0.0119	0.0029	0.0049	0.0044
37	February 26, 2018	0.0037	0.0033	0.0073	0.0073
38	February 27, 2018	0.0771	0.0011	0.0011	0.0011
39	February 28, 2018	0.1715	0.0016	0.0014	0.0011
40	March 1, 2018	0.0500	0.0110	0.0098	0.0087
41	March 2, 2018	0.0417	0.0079	0.0070	0.0078

YZV – Descriptive Statistics – One year period

A 20 Descriptive Statics – YZV – 2010

YZV - Stats - 2010	VIX	SP500	SP600	SP400
Count	251	251	251	251
Mean	0.0262	0.0052	0.0064	0.0059
Minimum	0.0014	0.0002	0.0002	0.0003
Maximum	0.3098	0.0656	0.0563	0.0619
Standard Deviation (STD)	0.0280	0.0055	0.0056	0.0060
Sample Variance (s^2)	0.0008	0.0000	0.0000	0.0000
Standard Error (SE)	0.0018	0.0003	0.0004	0.0004
Relative Standard Deviation (RSD)	1.0710	1.0645	0.8740	1.0285
*CI	0.0035	0.0007	0.0007	0.0008
Upper Estimate	0.0297	0.0059	0.0071	0.0066
Lowe Estimate	0.0227	0.0045	0.0057	0.0051
Kurtosis	42.124	58.076	24.543	30.586
Skewness	4.906	5.835	3.455	4.060
Jarque-Berra	19564.404	36697.855	6799.288	10473.230
P-Value	0.00E+00	0.00E+00	0.00E+00	0.00E+00
*Confidence Level (95.0%)				

A 21 Descriptive Statics – YZV – 2015

YZV - Stats - 2015	VIX	SP500	SP600	SP400
Count	251	251	251	251
Mean	0.0343	0.0037	0.0042	0.0038
Minimum	0.002	0.0000	0.001	0.001
Maximum	0.164	0.025	0.018	0.018
Standard Deviation (STD)	0.0276	0.0033	0.0032	0.0029
Sample Variance (s^2)	0.0008	0.0000	0.0000	0.0000
Standard Error (SE)	0.0017	0.0002	0.0002	0.0002
Relative Std (RSD)	0.8048	0.8936	0.7584	0.7753
*CI	0.0034	0.0004	0.0004	0.0004
Upper Estimate	0.0378	0.0041	0.0046	0.0041
Lowe Estimate	0.0309	0.0033	0.0038	0.0034
Kurtosis	4.977	8.698	1.659	2.872
Skewness	1.84	2.212	1.202	1.426
Jarque-Berra	400.638	995.845	89.218	171.267
P-Value	1.01E-87	5.69E-217	4.23E-20	6.45E-38
*Confidence Level (95.0%)				

A 22 Descriptive Statics – YZV – 2018

YZV - Stats - 2018	VIX	SP500	SP600	SP400
Count	251	251	251	251
Mean	0.046	0.0045	0.0051	0.0044
Minimum	0.0024	0.0002	0.0004	0.0002
Maximum	0.3569	0.0279	0.0242	0.0245
Standard Deviation (STD)	0.0467	0.0043	0.0039	0.0039
Sample Variance (s^2)	0.0022	0.0000	0.0000	0.0000
Standard Error (SE)	0.003	0.0003	0.0002	0.0002
Relative Standard Deviation (RSD)	1.0162	0.949	0.7639	0.8864
*CI	0.0058	0.0005	0.0005	0.0005
Upper Estimate	0.0518	0.005	0.0055	0.0049
Lowe Estimate	0.0402	0.004	0.0046	0.0039
Kurtosis	16.294	6.063	3.492	4.551
Skewness	3.462	2.133	1.625	1.867
Jarque-Berra	3277.795	574.803	238.066	362.329
P-Value	0.00E+00	1.52E-125	2.02E-52	2.10E-79
*Confidence Level (95.0%)				

Pearson's Correlation

A 23 YZV Parsons Correlation - 2010

Corr - 2010	VIX	SP500	SP600	SP400
VIX	1			
SP500	0.176 (.005)	1		
SP600	0.085 (.177)	0.878 (<.001)	1	
SP400	0.128 (.043)	0.954 (<.001)	0.892 (<.001)	1

Computed correlation used pearson-method with listwise-deletion.

A 24 YZV Parsons Correlation - 2015

Corr - 2015	VIX	SP500	SP600	SP400
VIX	1			
SP500	0.053 (.403)	1		
SP600	-0.044 (.486)	0.752 (<.001)	1	
SP400	-0.04 (.527)	0.796 (<.001)	0.881 (<.001)	1

Computed correlation used Pearson-method with listwise-deletion.

A 25 YZV Parsons Correlation - 2018

Corr - 2018	VIX	SP500	SP600	SP400
VIX	1			
SP500	0.203 (.001)	1		
SP600	0.054 (.393)	0.791 (<.001)	1	
SP400	0.127 (.044)	0.874 (<.001)	0.926 (<.001)	1

Computed correlation used Pearson-method with listwise-deletion.

Market volatility – t-test

A 26 t-Test: One-Sample - Stock Market 2010

t-Test: One-Sample	SP500-2010
Mean	0.0052
Variance	3.03E-05
Observations	251
Hypothesized Mean	0.0656
df	250
t Stat	173.8083
P(T<=t) one-tail	4.79E-263
t Critical one-tail	1.6510
P(T<=t) two-tail	9.59E-263
t Critical two-tail	1.9695

A 27 t-Test: One-Sample - Stock Market 2015

t-Test: One-Sample	SP500-2015
Mean	0.0037
Variance	1.07E-05
Observations	251
Hypothesized Mean	0.025
df	250
t Stat	103.1982
P(T<=t) one-tail	2.96E-207
t Critical one-tail	1.6510
P(T<=t) two-tail	5.92E-207
t Critical two-tail	1.9695

A 28 t-Test: One-Sample - Stock Market 2015

t-Test: One-Sample	SP500-2018
Mean	0.0045
Variance	1.82E-05
Observations	251
Hypothesized Mean	0.0118
df	250
t Stat	27.1088
P(T<=t) one-tail	1.08E-76
t Critical one-tail	1.6510
P(T<=t) two-tail	2.16E-76
t Critical two-tail	1.9695

Market Distress – t-test

A 29 t-Test: One-Sample - Market Sentiment 2010

t-Test: One-Sample	**VIX-2010**
Mean	0.0262
Variance	7.87E-04
Observations	251
Hypothesized Mean	0.123
df	250
t Stat	54.6816
P(T<=t) one-tail	2.2E-141
t Critical one-tail	1.6510
P(T<=t) two-tail	4.39E-141
t Critical two-tail	1.9695

A 30 t-Test: One-Sample - Market Sentiment 2015

t-Test: One-Sample	**VIX-2015**
Mean	0.0343
Variance	7.64E-04
Observations	251
Hypothesized Mean	0.035
df	250
t Stat	37.9132
P(T<=t) one-tail	3.52E-01
t Critical one-tail	1.6510
P(T<=t) two-tail	7.05E-01
t Critical two-tail	1.9695

A 31 t-Test: One-Sample - Market Sentiment 2018

t-Test: One-Sample	**VIX-2018**
Mean	0.0460
Variance	2.19E-03
Observations	251
Hypothesized Mean	0.2569
df	250
t Stat	71.4771
P(T<=t) one-tail	1.0E-168
t Critical one-tail	1.6510
P(T<=t) two-tail	2.09E-168
t Critical two-tail	1.9695

Monthly Average YZ -Volatility

A 32 YZ-Average Monthly Volatility 2010

Months	VIX	Large-Cap	Mid-Cap	Small-Cap
Jan	0.0260	0.0049	0.0055	0.0063
Feb	0.0201	0.0054	0.0059	0.0063
Mar	0.0175	0.0029	0.0033	0.0035
Apr	0.0218	0.0040	0.0050	0.0058
May	0.0706	0.0116	0.0128	0.0116
Jun	0.0345	0.0059	0.0062	0.0068
Jul	0.0243	0.0070	0.0085	0.0091
Aug	0.0243	0.0057	0.0069	0.0073
Sep	0.0161	0.0030	0.0039	0.0061
Oct	0.0169	0.0047	0.0050	0.0064
Nov	0.0248	0.0052	0.0058	0.0059
Dec	0.0195	0.0022	0.0023	0.0026

A 33 YZ-Average Monthly Volatility 2015

Months	VIX	Large-Cap	Mid-Cap	Small-Cap
Jan	0.0288	0.0057	0.0067	0.0058
Feb	0.0224	0.0034	0.0039	0.0035
Mar	0.0333	0.0027	0.0033	0.0027
Apr	0.0300	0.0034	0.0033	0.0033
May	0.0247	0.0025	0.0041	0.0027
Jun	0.0325	0.0023	0.0035	0.0028
Jul	0.0315	0.0030	0.0041	0.0035
Aug	0.0592	0.0051	0.0052	0.0045
Sep	0.0509	0.0052	0.0041	0.0045
Oct	0.0365	0.0039	0.0045	0.0045
Nov	0.0303	0.0031	0.0034	0.0032
Dec	0.0402	0.0039	0.0040	0.0041

A 34 YZ-Average Monthly Volatility 2018

Months	VIX	Large-Cap	Mid-Cap	Small-Cap
Jan	0.0211	0.0028	0.0044	0.0034
Feb	0.1091	0.0060	0.0057	0.0055
Mar	0.0580	0.0053	0.0052	0.0045
Apr	0.0403	0.0055	0.0051	0.0051
May	0.0412	0.0040	0.0037	0.0028
Jun	0.0422	0.0028	0.0042	0.0035
Jul	0.0392	0.0024	0.0038	0.0033
Aug	0.0361	0.0020	0.0033	0.0024
Sep	0.0307	0.0021	0.0033	0.0028
Oct	0.0606	0.0065	0.0072	0.0059
Nov	0.0257	0.0051	0.0057	0.0051
Dec	0.0527	0.0101	0.0097	0.0092

RStudio - Research Packages

A 35 RStudio - Research Packages

Packages	Description
tidyquant	Tidy Quantitative Financial Analysis
quantmod	Quantitative Financial Modelling Framework
purrr	Apply Mapping Functions in Parallel using Futures
tidyverse	Easily Install and Load the 'Tidyverse'
ggplot2	Create Elegant Data Visualizations Using the Grammar of Graphics
magrittr	A Forward-Pipe Operator for R
broom	Convert Statistical Objects into Tidy Tibbles
rvest	Easily Harvest (Scrape) Web Pages
dplyr	Interface to 'Dygraphs' Interactive Time Series Charting Library
dygraphs	Interface to 'Dygraphs' Interactive Time Series Charting Library
PerformanceAnalytics	Econometric Tools for Performance and Risk Analysis
timetk	A Tool Kit for Working with Time Series in R
pacman	Package Management Tool
writexl	Export Data Frames to Excel 'xlsx' Format
corrplot	Visualization of a Correlation Matrix
readxl	Read Excel Files
pastecs	Package for Analysis of Space-Time Series