STOCKS PORTFOLIO OPTIMIZATION BASED ON FORECASTING THROUGH THE INTEGRATION OF THE MVO METHOD AND LSTM MODEL

Lappeenranta–Lahti University of Technology LUT

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Post-doctoral researcher Mahinda Mailagaha Kumbure

ABSTRACT

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This thesis explores optimal securities portfolio forecasting in the financial market using Long-Short Term Memory (LSTM) models and Mean-Variance Optimization (MVO). LSTM models have proven effective in stock market prediction, while MVO is widely used for portfolio construction. The article "9 of the Best Stocks for a Starter Portfolio" inspired the research. The dataset covers the adjusted closing prices of various stocks from 2010 to April 30, 2023. The LSTM model is trained on 80% of the dataset, with the remaining 20% for testing. The LSTM model consists of two hidden layers with 200 and 100 neurons, a 20% dropout rate, a batch size of 32, and 100 epochs. Daily stock returns are calculated and used for portfolio simulation through MVO. The forecasting performance is evaluated using MAPE, where a value below 2.5% indicates the accuracy of models. The Maximum Sharpe Ratio portfolio is constructed by constructing and analyzing a hypothetical investment portfolio based on different combinations of assets and selecting the one that outperforms the equally-weighted portfolio and the market index (S&P 500) in terms of Sharpe Ratio and

Annualized return. Although there is a difference in value, the Maximum Sharpe Ratio portfolio based on predicted data exhibits similar trends to the one based on actual historical data. Asset allocation within the portfolio varies across rebalancing periods, but portfolios constructed with actual and forecasted data are the same. These findings give some insight into combining LSTM models and MVO for optimal portfolio management. The thesis contributes to understanding portfolio optimization by integrating predictive modeling and quantitative techniques.

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The learning journey at LUT is coming to an end. Thanks for the meaningful and wonderful two years of studying in Finland at the age of 30s. Someone essential to me once said, "The hard line to walk is the rising road." Although this journey is difficult, sometimes it makes me stand down, but it also makes me feel happy.

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Con muốn bày tỏ lòng biết ơn vô hạn đến gia đình vì đã luôn ở bên con.

Con vô cùng biết ơn dì Loan và dì Bé, nếu không có sự hỗ trợ từ hai dì, hành trình du học của con đã không thể bắt đầu.

Cảm ơn các chị em XFX luôn khiến mình vui vẻ.

Thank each old or new friend for sharing and accompanying me on this journey.

The new life is just about ready to roll.

Lappeenranta, July 2023

Linh Pham

SYMBOLS AND ABBREVIATIONS

Constants

Abbreviations

MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MSE	Mean Squared Error
MSR	Maximum Sharpe Ratio
MVO	Mean-Variance Optimization
LSTM	Long-Short Term Memory
RMSE	Root Mean Squared Error

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1 Introduction

One common goal of optimizing portfolio is a critical task for investors who want to make as much money as possible while taking on as little risk as possible (Markowitz 1952). In the last few years, cutting-edge strategies like deep learning models have become valuable tools for improving investment portfolios. The Long-Short-Term Memory (LSTM) model is one of them, and it has sparked a lot of attention due to its ability to manage time series data and capture long-term relationships (Greff et al. 2016). There has been a lot of success in using LSTM models to optimize portfolios and make predictions about the returns of assets.

Stocks and stock portfolios are highly sought-after investment options that have attracted significant attention from investors worldwide. Investing in individual stocks provides the opportunity for capital appreciation and potential dividend income. Furthermore, constructing diversified stock portfolios allows investors to spread their risk across different companies, sectors, or regions, reducing the impact of any single stock's performance on the overall portfolio. In addition, diversification enhances stability and long-term growth potential. Investment goals, risk tolerance, and time horizon influence portfolio construction. Regular monitoring and rebalancing are crucial for maintaining the desired portfolio composition.

This study looks at how LSTM models can predict the prices of stocks and improve stock portfolios in the real world based on those predictions. As a starter investor, the author faces considerable challenges in selecting stocks to include in the portfolio due to the vast array of options available. Consequently, this study relies on a stock search engine to aid the portfolio selection process and leverages internet recommendations as part of this research. This study focuses on the stocks recommended in a April 20, 2023 article titled " 9 of the Best Stocks for a Starter Portfolio" on https://money.usnews.com/ because the article has an attractive title and content that it considers useful advice for "beginner" investors. This post included A. O. Smith Corp. (AOS), Johnson & Johnson (JNJ), and other stocks. For the LSTM model to accurately forecast future returns, the historical data of these stocks will be needed to train it. After that, these forecasts will be utilized in the process of portfolio optimization.

The findings of this experiment are encouraging. First, the study shows that the performance of the MVO portfolio outperforms the market index (S&P 500 index) and equally-weighted portfolio. This study then shows that an MVO portfolio based on quarterly adjusted LSTM predictions achieves much better results than an equally weighted portfolio or portfolio that is not adjusted based on actual historical data. As a result of these results, investors who want to improve their investment strategies can use LSTM models and the MVO method for portfolio optimization to get helpful information. This thesis offers a growing database of research on portfolio optimization through building deep learning models and using them. It demonstrates how these models may be applied in the real world with stocks. In addition, this thesis contributes to the expanding database of studies on artificial intelligence, which has been done previously.

1.1 Motivation

The general public's interest in personal finance and money management has significantly increased over the past few decades, as evidenced by the rise in private finance education, media coverage, bestselling books, online communities, and increased investment activity. As the internet and information have become more common, investors have easier access to financial news, opinions, and advice. Nevertheless, to the ease with which this information can be obtained, it is more difficult for investors to sift through the many types of irrelevant information and identify trustworthy data sources. As a result, despite needing more skills and knowledge to analyze the financial market, a significant number of inexperienced investors tend to rely on guidance or recommendations provided by the media (Taleb et al. 2021). Even when they receive information from reputable and influential sources, investors face the challenge of effectively utilizing it. Because of this, those who want to invest their money need to have the information and abilities necessary to assess the financial markets and then base their investment choices on what they discover. Using advanced scientific methods like LSTM models can potentially improve the performance of investments by effectively managing time series data and capturing long-term relationships.

LSTM models are a type of useful deep learning algorithms for making predictions based on time series data (Liu et al. 2019). As a consequence of this, it has the prospective to become a useful tool that may be included in investment portfolios. But only a few studies have looked into how LSTM models can be used in this framework. Based on personal needs, starting with this issue want to use recent scientific advances to make smart investment decisions, and LSTM models may help improve the performance of investments.

This thesis investigates the use of forecasting results from LSTM models in building investment portfolios using the MVO method. The research will also suggest way that integration of the LSTM models and MVO method can be used to improve the performance of investments. Suppose the reader looks into how sophisticated scientific methods are used in portfolio investing. In that case, they will better understand what might be good about using these methods in their investments.

The author of the thesis believes that this study's findings will help establish a community of individual investors who are more educated, knowledgeable, and capable of applying advanced scientific methodologies when making financial portfolio decisions. Individual investors in this community will be able to make decisions based on the findings of this study since it will assist them in becoming more educated, aware, and competent. This concept provides a beneficial resource for investors looking to enhance their assets' performance and help them achieve their financial goals and feel more confident about their money.

1.2 Objective

This project aims to build an stocks ratio allocation tool based on mathematical portfolio optimization algorithms employed in strategic and tactical portfolio development allocation techniques. When making investments with a long-term horizon, it is important to ascertain the investment rate per stock that confers the most benefit to the investor. The following research question has been established in order to ensure the success of this thesis:

> *" Does the utilization of Long Short-Term Memory (LSTM) models for stock forecasting, followed by Mean-Variance Optimization (MVO) on the forecasted data, prove to be effective in portfolio management?"*

Along with the primary research topic, the following follow-up questions might be posed:

1) How accurate is the forecast stock value using the LSTM model?
2) How does an optimized portfolio based on historical data using the MVO method compare to an equality-proportioned portfolio of stocks and the market benchmark?

3) Is portfolio management with an MVO portfolio approach incorporating predictive data more effective than the other portfolio (as an equally weighted portfolio)?
4) During the portfolio management process, does the portfolio allocation need to be changed to ensure the effectiveness of the portfolio optimization?

This investigation has been broken up into four parts with the goal mentioned above in mind. The first section primarily describes using Mean-Variance Optimization to build an optimal portfolio. The second section discusses how the LSTM model has been used to predict stocks' values and returns. The third section presents a comparative analysis of potential outcomes when individual investors manage their portfolios using different choices based on actual historical data and forecasted data. Subsequently, the fourth section showcases the results of optimal portfolio allocation in the investment management process.

1.3 Thesis Structure

The investigation's many facets will be discussed in the following parts. The second portion, the literature review, comes immediately after the first part, the introduction. A survey of the relevant theories and formulations connected to this subject might assist in finding appropriate theories and formulations. The academic part will then discuss all the essential theories and equations that affect portfolio optimization as a part of the theoretical framework. The data and techniques used in this study are used to produce the fourth portion of the report. This section covers the data, the prediction model, and the optimization model. After the data and method are analyzed and judged, the thesis results are measured. In the end, I conclude the study in the final part, where we also talk about the way an investor may put those findings to use. Following the sections devoted to the literature, there will also be appendices and references included in the report.

2 Literature Review

Portfolio optimization is an essential step in investment management that tries to maximize returns while avoiding risk. The significance of portfolio optimization stems from investors striving to achieve a trade-off between maximizing potential returns and managing risk effectively. While the objective is to maximize returns, investors aim to strike a balance that aligns with their risk tolerance and investment goals, acknowledging financial markets' volatile and unpredictable nature. Machine learning approaches are increasingly used in portfolio optimization to solve this difficulty (Ban, El Karoui & Lim, 2018). The Long Short-Term Memory (LSTM) model is one of the potential machine learning models for forecasting financial time series data. LSTM is a sort of recurrent neural network that can learn and predict patterns in long-term dependent sequential data. Through leveraging the predictive capabilities of LSTM models, the accuracy and reliability of forecast results can enhance the performance of a portfolio.

2.1 Financial time-series or stock prediction with deep learning and LSTM model

Deep learning techniques, especially Long-Short-Term Memory (LSTM) networks, have recently gained considerable attention in financial market predictions. The study of Thomas and Fischer (2017) demonstrates the practical significance of the LSTM network in predicting stock movements with high accuracy, outperforming other non-memory classifiers. Findings from the study provide insight into the development of profitable trading strategies. In addition, future research directions suggested by the authors include exploring the use of LSTM networks to predict other financial time series and investigating the integration of alternative data sources such as newspapers and social media.

Similarly, Dingli and Fournier (2017) explore using Convolutional Neural Networks (CNNs) for financial time series forecasting. Their research accurately predicted the price direction for the next month and week. Zhang et al. (2019) propose an attention-based LSTM model (AT-LSTM), which efficiently selects relevant feature sequences by the attention mechanism. This model has practical implications for financial and investment decision-making, enhances predictive interpretability, and extends its application to other areas.

Incorporating deep learning models, Mehtab et al. (2020-2021-2022) propose models based on CNN and LSTM to forecast stock prices accurately. Their works provide practical implications for investors and traders to make informed decisions and develop trading strategies. Similarly, Yan and Aasma et al. (2020) introduce the hybrid prediction model CEMMD-PCA-LSTM, which outperforms the standard models regarding prediction accuracy and profitability. This model brings practical benefits to investors in the stock market, allowing for more accurate predictions and improved profitability.

Using sentiment analysis and external factors is another avenue explored in stock price prediction. Jin and associates (2020) propose a deep learning-based model that combines investor sentiment and empirical method decomposition (EMD) to improve prediction accuracy and reduce latency. Ma (2020) compares ARIMA, ANN, and LSTM models for stock price prediction, highlighting the potential of the LSTM model in terms of superior predictability, even though the model is sensitive to data processing. In addition, the combination of time series and extrinsic factors are suggested for further study.

Furthermore, the impact of news and text information on stock price predictions is investigated. Hong (2020) presents a model that uses stock data and news information to predict stock market behavior, assisting investors and traders in their decision-making. However, it is essential to consider the complexity and many factors that affect the stock market, acknowledging that predictions may not always be accurate.

And then, Thu et al. (2021) propose a neighborhood deep neural network model that combines LSTMs with historical data about stocks and their nearest neighbors. Empirical results demonstrate the model's effectiveness in predicting stock prices, outperforming other competing methods. However, the limitation of this method lies in the fact that it relies only on historical data without considering external factors that may affect the stock price.

The above studies highlight the practical significance of using deep learning techniques, especially LSTM networks, to predict stock prices. These models exhibit high accuracy and profitability, assisting investors and traders in decision-making. Combining sentiment analysis, external factors, and textual information further enhances predictability. However, it is crucial to consider the limitations of the proposed methods, such as their sensitivity to data processing and the exclusion of critical external factors influencing the stock market. Studies suggest that in the future, it is possible to explore the combination of time series and external factors to improve prediction accuracy and investigate the applicability of these models in various fields in different areas.

2.2 Prediction-based portfolio optimization using forecasting model and Mean-Variance Optimization method

Portfolio optimization is crucial for investors aiming to maximize their returns while managing risks. Traditional approaches, such as the mean-variance model, have been widely used for portfolio optimization. However, these models often fail to capture short-term investment opportunities and may not perform optimally in dynamic market conditions. Researchers have explored integrating forecasting models and mean-variance optimization methods in portfolio management to address these limitations.

The article by Freitas et al. (2009) introduces a prediction-based portfolio optimization model that utilizes neural network predictors to capture short-term investment opportunities. The results demonstrate that the model outperforms the mean-variance model and achieves returns above the market index by leveraging short-term opportunities. Similarly, Arik et al. (2014) propose a framework for supervised classification-based stock prediction and portfolio optimization, which leverages machine learning techniques to automate stock picking and potentially improve investment decisions.

Luo et al. (2015) compare the performance of different models (OGARCH, MSM, and EWMA) in hedge fund portfolio optimization using the mean-variance method. Their findings reveal that the OGARCH model produces the best-performing optimal portfolio with the highest Sharpe ratio and the lowest risk. However, it is essential to note the limitations of this study, such as the limited number of indices considered and the lack of consideration for transaction costs and practical constraints.

Kulali (2016) tests the effectiveness of the Markowitz mean-variance approach on the Istanbul Stock Exchange and highlights the significance of diversification in minimizing risk and maximizing returns. While the study provides valuable insights, it only analyzes data from a single year and does not consider market fluctuations or transaction costs. Hoe and Siew (2016) examine portfolio optimization using the mean-variance model and find that investors can achieve the desired level of return with minimum risk through an optimal mean-variance portfolio. However, this study is limited to a specific market, and the assumptions of normal distribution and the absence of practical constraints may limit its generalizability.

In another approach, using stock clustering and a variety of investing methods, Goudarzi et al. (2017) offer a hybrid model for portfolio optimization. The model considers investors' risk-taking behavior and utilizes clustering and ranking algorithms to establish portfolios for different risk levels. Their results show that the proposed model outperforms general and industry indices in the Tehran Stock Exchange.

Soeryana et al. (2017) present two papers that discuss mean-variance portfolio optimization using time series approaches and non-constant mean and volatility. The studies consider logarithmic utility and negative exponential utility functions, respectively, and employ ARMA and GARCH models to analyze non-constant mean and volatility. These approaches provide a more realistic perspective on portfolio optimization. However, limitations such as focusing on specific markets and lacking practical constraints should be acknowledged.

An et al. (2019) propose a model for portfolio optimization based on minimizing period value at risk (PVaR) while satisfying expected return requirements. Their findings suggest that PVaR can provide a conservative investment strategy for risk-averse investors compared to the value at risk (VaR). Additionally, Ta et al. (2020) proposed a long short-term memory (LSTM) network for stock prediction and demonstrated its effectiveness in constructing portfolios. Their research demonstrates the potential of integrating deep learning techniques in quantitative trading and portfolio management. Their LSTM prediction model achieved high accuracy and outperformed linear regression and support vector machine models. Additionally, the constructed portfolios using optimization techniques showed significant return and Sharpe ratio improvements, surpassing the benchmark S&P 500 index.

Cao et al. (2020) presented an efficient portfolio optimization algorithm using deep neural networks (DNNs). Their AA+GRU model outperformed other methods in terms of the Sharpe ratio and exhibited promising results in the Vietnamese stock market and other countries. The use of deep learning methodologies enhanced the accuracy of portfolio optimization. However, limitations such as dataset generalizability, resource requirements, and considerations for transaction costs were noted.

Ma et al. (2020) focused on prediction-based portfolio optimization models for the Chinese stock market. They demonstrated that the use of DNNs improved the performance of these models. Among the models tested, the DMLP+MSAD model showed the best results and high desired portfolio returns further enhanced its performance. The authors also suggested future research directions, including trading simulation with transaction fees and incorporating additional input features and risk metrics.

Chen et al. (2021) proposed a hybrid model that combined machine learning for stock prediction and the mean-variance (MV) model for portfolio selection. Their method, which employed eXtreme Gradient Boosting (XGBoost) and an improved firefly algorithm (IFA), outperformed traditional approaches and benchmarks regarding returns and risks. The study used the Shanghai Stock Exchange as the sample, highlighting the superiority of the proposed method.

Gusliana and Salih (2022) explored a mean-variance investment portfolio optimization model without risk-free assets, specifically focusing on the JII70 stock list. They demonstrated that the model could determine the optimal asset weight allocation and expand the Mean-Variance framework. However, limitations such as the applicability to other stock lists or markets, homogeneity of risk preferences among investors, and the exclusion of transaction costs and external factors were identified.

The above articles have contributed to the literature on prediction-based portfolio optimization using forecasting models and mean-variance optimization. They have demonstrated the effectiveness of deep learning models, such as LSTM and DNNs, in predicting stock movements and constructing portfolios that outperform traditional methods. Additionally, optimization techniques have shown significant improvements in portfolio performance.

3 Theoretical Framework

This chapter should look at the basic ideas and theories behind deep learning (with LSTM models as the primary focus) and portfolio optimization. It should also explain how these ideas and theories relate to the research question and hypothesis.

3.1 Long-Short Term Memory Model

3.1.1 Deep learning

Deep learning, a subset of machine learning, has garnered significant attention for its ability to extract data representations through multiple layers of non-linear transformations. This approach is based on artificial neural networks, which aim to mimic the construction and operation of the human brain. Recent years have seen a surge in academic interest surrounding deep learning, resulting in numerous articles exploring its diverse applications. These articles cover various domains, such as computer vision, natural language processing, and speech recognition, demonstrating the successful implementation of deep learning techniques. One can consult academic databases and reputable journals to access an extensive collection of research papers on the advancements and achievements in deep learning.

A deep learning model is made up of numerous linked layers of neurons that examine input and create predictions collectively to build the model. Each layer is in responsible of extracting and manipulating different elements of the data, with higher-level layers building on what lower-level levels have learnt. This hierarchical structure enables deep learning models to understand complicated patterns and correlations in data and generate accurate predictions. Based on the anthology "Deep learning application Volume 3" by Wani M.A. (2022), some types of deep learning models can be brief description as follows:

CNN, or Convolutional Neural Network, is a well-known deep learning model that is frequently used for computer vision tasks such as detecting photos and videos, locating and separating objects, and breaking up images. CNNs employ convolutional layers to extract features from the input they are provided and filter the data to detect patterns and forms.

Recurrent Neural Networks, or RNNs, are another type of deep learning model that is frequently used for natural language processing tasks such as language translation and speech identification. RNNs are also known as recurrent neural architectures. RNNs are designed to handle sequential data by utilizing loops that enable information to be transmitted from one phase of the network to the next, allowing the network to learn temporal relationships in the input.

Generative Adversarial Networks, or GANs for short, are a type of deep learning model that can produce new data by building on top of previously obtained data. GANs are made up of two networks, a generator and a discriminator, which compete to create realistic pictures or other sorts of data.

Deep learning has several advantages over traditional machine learning algorithms, including the ability to learn from massive amounts of data, the ability to extract complex representations of the data, and the ability to provide accurate predictions over a wide range of domains. However, deep learning has certain drawbacks, such as the requirement for massive quantities of data, the risk of overfitting, and the inability of the models to be comprehended.

To conclude, deep learning is an effective machine learning subfield that can potentially change many other subfields, including computer vision, natural language processing, voice recognition, and many more. Through an anthology of articles researching the application of deep learning, deep learning models have been proven to be effective in detecting complex patterns and relationships in data, and researchers are constantly working to overcome their shortcomings and limitations so that they may be utilized more effectively. Deep learning models have been proven successful in learning complicated patterns and correlations in data.

3.1.2 Long-Short Term Memory model

In financial analysis, the Long Short-Term Memory (LSTM) model has emerged as a powerful neural network architecture for capturing long-term dependencies in sequential data. Proposed by Sepp Hochreiter and Jurgen Schmidhuber in their 1997 paper titled "Long-Short-term Memory," LSTM addressed the limitations of basic Recurrent Neural Networks

(RNNs). These limitations encompassed difficulties in retaining long-term dependencies, the issue of gradient explosion or vanishing, the absence of explicit memory control, and the challenge of learning long-term dependencies between distant time steps in a sequence. LSTM introduces a novel architecture that includes memory cells and gating mechanisms to selectively retain or forget information from previous time steps. This effectively manages long-term dependencies and overcomes the vanishing gradient problem commonly encountered in traditional RNNs.

The core components of an LSTM cell are the cell state and the hidden state. The cell state serves as a long-term memory, carrying information throughout the sequence, while the hidden state represents the current output and is passed to the subsequent LSTM cell. The memory cells within the LSTM are responsible for remembering information and undergo modifications through three primary processes known as gates: the input gate, the forget gate, and the output gate. (Fischer & Krauss 2018)

A diagram of an LSTM cell is presented as a schematic in Figure 1.

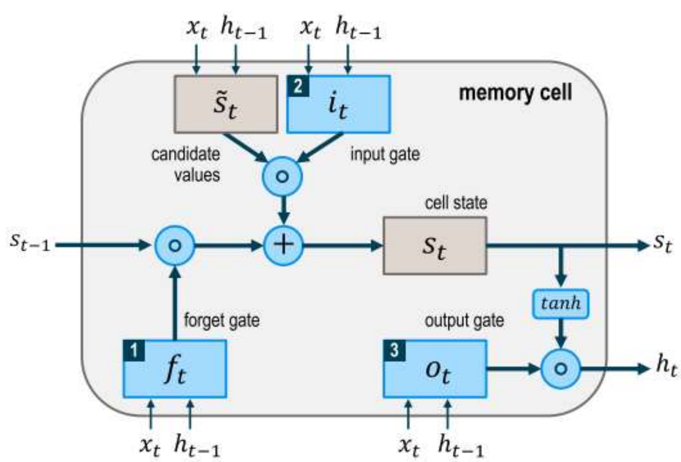


Figure 1. The structure of Long-Short Term Memory (LSTM) cell – (Fischer & Krauss 2018)

In the study of Fischer and Krauss (2018), the work of the memory cell of LSTM is discussed in more detail. As illustrated in Figure 1, a LSTM cell includes forget gate (f_t), input gate (i_t), and output gate (o_t) and these gates are used to adjust a cell state (s_t).

During each timestep t, the three gates receive the input x_t, which represents a single element from the input sequence, as well as the output h_{t-1} from the memory cells at the previous timestep t - 1. The equations provided below are expressed in a vectorized form and outline the process of updating the memory cells in the LSTM layer at each timestep t. In these equations, the following notations are utilized (*Fischer & Krauss 2018)*:

- x_t is the input vector at timestep t.
- $W_{f,x}$, $W_{f,h}$, $W_{\tilde{s},x}$, $W_{\tilde{s},h}$, $W_{i,x}$, $W_{i,h}$, $W_{o,x}$, and $W_{o,h}$ are weight matrices.
- b_f, $b_{\tilde{s}}$, b_i, and b_o are bias vectors.
- f_t, i_t, and o_t are vectors for the activation values of the respective gates.
- s_t and $\tilde{s}_t$ are vectors for the cell states and candidate values.
- h_t is a vector for the output of the LSTM layer.

Starting on the left side of Figure 1 is the cell state, denoted as s_{t-1}. The initial step involves the forget gate, which utilizes activation values f_t derived from the input x_t at timestep t and the outputs h_{t-1} from the previous timestep t-1. Both values are subjected to a sigmoid function in equation (1), which scales the activation values between 0 (indicating complete forgetting) and 1 (indicating complete retention) in the previous cell state s_{t-1}:

$$f_t = sigmoid(W_{f,x}x_t + W_{f,h}h_{t-1} + b_f) \qquad (1)$$

Moving to the middle part of Figure 1, the second step involves the input gate. The LSTM layer determines the relevant information to be incorporated into the network's cell states (s_t). This process comprises two operations. First, candidate values $\tilde{s}_t$, representing potential additions to the cell states, are calculated using the hyperbolic tangent (tanh) function in equation (2) (Fischer & Krauss, 2018):

$$\tilde{s}_t = tanh(W_{\tilde{s},x}x_t + W_{s,h}h_{t-1} + b_{\tilde{s}}) \qquad (2)$$

Second, the activation values i_t of the input gates are computed using equation (3) (Fischer & Krauss, 2018):

$$i_t = sigmoid(W_{i,x}x_t + W_{i,h}h_{t-1} + b_i) \qquad (3)$$

In the third step, the previous calculations are employed to update the new cell state s_t using equation (4). The new cell states s_t are determined based on the outcomes of the previous two steps, with the symbol ∘ denoting the Hadamard (elementwise) product:

$$s_t = f_t \circ s_{t-1} + i_t \circ \tilde{s}_t \quad (4)$$

Here, the activation values f_t denote which values are forgotten, the activation values it signify the information to be updated and the degree of updating, and $\tilde{s}_t$ represents the candidate values (Fischer & Krauss, 2018).

In the final step, the output of the memory cell h_t is derived using the following two equations:

$$o_t = sigmoid(W_{o,x}x_t + W_{o,h}h_{t-1} + b_o) \quad (5)$$

$$h_t = o_t \circ tanh(s_t) \quad (6)$$

Equation (6) is utilized to determine which components of the cell state s_t will be outputted. These values are then multiplied by the cell state s_t, which is initially scaled between -1 and 1 using the tanh function (Fischer & Krauss, 2018).

When processing an input sequence, the features are presented to the LSTM network timestep by timestep. Each time step corresponds to a standardized return, and the network processes it according to the abovementioned equations. Once the final element of the sequence has been processed, the overall output for the entire sequence is obtained. Like traditional feed-forward networks during training, the weights and bias terms are adjusted to minimize the loss of the specified objective function across the training samples.

Utilizing these gating mechanisms, the LSTM model can selectively retain important information in the cell state, forget irrelevant information, and output relevant information to the hidden state. Consequently, LSTM effectively captures long-term dependencies, accommodates sequences with varying time lags, and overcomes the vanishing gradient problem encountered in traditional RNNs.

The advantages of using LSTM in financial applications are noteworthy. LSTM models excel at modeling complex nonlinear relationships between input features and target variables, making them well-suited for a wide range of financial tasks, including risk management and trading strategy development (Troiano et al., 2018). Moreover, LSTM models demonstrate superior robustness against overfitting compared to traditional statistical models, making them suitable for processing noisy and incomplete financial data.

Despite the numerous benefits of LSTM, there are some limitations to consider when applying it to financial data. The complexity of LSTM requires additional computational resources and longer training times compared to traditional statistical models. Interpreting the inner workings of LSTM can take time, hindering the understanding of how the model arrives at its predictions or recommendations. Furthermore, LSTM models typically demand a large amount of training data to perform well, which can be a challenge in financial applications where data availability may be limited or acquiring data can be expensive.

In summary, LSTM is an advanced neural network architecture that effectively models long-term dependencies in financial data. The unique combination of memory cells and gating mechanisms enables LSTM to capture complex relationships and patterns in financial time-series data. However, the complexity, interpretability, and data requirements associated with LSTM should be carefully considered before employing it for financial analysis. By thoroughly assessing these pros and cons, practitioners can leverage LSTM to enhance their financial decision-making processes.

3.2 Portfolio Optimization

The last and most important step of the technique for the development of a portfolio that is based on Modern Portfolio Theory is called portfolio optimization. And when building a portfolio, it is essential to keep in mind the following three crucial inputs: the expected return, the volatility of asset returns, and the correlation (or covariance) of asset returns.

3.2.1 Modern Portfolio Theory

Modern portfolio theory (MPT) is a widely accepted investment framework that was first proposed by Harry Markowitz in 1952. The theory suggests that investors can construct a diversified portfolio of assets that balances risk and return. The main idea behind MPT is that an asset portfolio should not be constructed based on the performance of individual assets but rather on the performance of the portfolio as a whole. Therefore, when looking at a portfolio, one should consider how its internal risk-return dynamics work.

In the conventional mathematical framework of Modern Portfolio Theory (MPT), the measure of risk is represented by the variance of returns, whereas the expected return is a metric used to assess projected gains. The mean-variance (MV) model was introduced by Markowitz as a means to assess the anticipated return and risk of a portfolio by using historical asset prices to determine the mean and variance. The model being offered posits that investors have the ability to mitigate their risk exposure by diversifying their investment portfolio over a diverse array of assets that possess differing features in terms of risk and return.

The concept of Modern Portfolio Theory, first introduced by Markowitz, conceptualizes the temporal aspect of investment as a singular period during which the parameters of the probability distribution of asset returns are established with complete confidence and remain constant. This implies that the future is seen as a contiguous timeframe, beginning in the present but concluding at an indeterminate point in time. Nevertheless, this assumption has garnered significant attention in the theoretical literature, although there has been little development in terms of practical advancements accessible to financial practitioners.

The "single-period" paradigm for portfolio optimization is justified by the assumption that any frictions that affect the process of portfolio building and rebalancing may be disregarded. According to the study conducted by Francis and Kim in 2013, If the cost associated with rebalancing is negligible, the single-period assumption yields the most optimum strategy. The single-period assumption in portfolio optimization refers to the idea that the investment horizon is limited to a single period, typically a fixed duration such as a quarter or a year. It assumes that investors can freely adjust their portfolio weights without incurring any costs or frictions during the rebalancing process. Under this assumption, portfolio optimization models aim to maximize returns or minimize risk based on the investor's preferences and available information. Since no rebalancing costs are associated, the optimal solution can be determined solely based on return and risk considerations. However, portfolio rebalancing is often costly for almost all real-world investors. In the context of taxable investors who own illiquid assets like private equity or real estate, the significance of transaction costs in relation to return and risk concerns tends to be prominent, particularly when holding durations do not extend beyond several decades. Therefore, investors should consider transaction costs when rebalancing their portfolios.

In conclusion, MPT provides investors with a framework to construct portfolios that balance risk and return by diversifying across a range of assets. However, the assumptions of the single-period framework have been criticized for not reflecting the real-world situation of investing. Modern portfolio theory gives investors an excellent way to make investment decisions that match their goals, willingness to take risks, and time horizon. But, investors should also consider transaction costs when rebalancing their portfolios, especially for illiquid assets held by taxable investors.

3.2.2 Portfolio Construction

Portfolio construction is a three-step procedure according to modern portfolio theory. Asset allocation is the first stage in the construction of a portfolio (Prigent 2007). This means dividing the investment portfolio into different asset classes, like stocks, bonds, and cash, to get the best balance of risk and return based on the investor's investment goals, risk tolerance, and time horizon. The second step in building a diversified portfolio is picking specific securities from each asset class. This is called "security selection." The goal is to choose securities with different risk and return levels so that gains in one area will not make up for losses in another. Portfolio optimization is the third and concluding stage of the portfolio construction procedure. In this step, mathematical models and statistical analysis are used to find the combination of securities with the highest expected return for a given level of risk or the lowest expected return for a given level of risk. By following these three steps, investors can build a diversified portfolio that aims to get the most out of expected returns while taking the least risk. (Prigent 2007, 66)

Asset allocation is an essential part of managing investments. It means dividing the money between different types of assets based on the investor's goals, risk tolerance, and time horizon. Asset allocation aims to balance the predicted risk and return of an investment portfolio so that it can make the most money while minimizing risk. There are variety of investment opportunities available, including stocks, bonds, real estate, commodities, and cash. Because each asset class has its own risk and return, spreading investments across different asset classes is crucial to reducing risk and maximizing a return. Asset allocation techniques seek to diversify the portfolio by investing in several asset classes, thereby lowering the overall risk and volatility of the portfolio.

Asset allocation options to think about include strategic asset allocation, tactical asset allocation, dynamic asset allocation, constant-weight asset allocation, insured asset allocation, and integrated asset allocation. Strategic asset allocation is the most common strategy. It involves putting together a portfolio with a fixed mix of assets based on how risky an investor is willing to be and their investment goals. Tactical asset allocation is a more active method that involves changing the portfolio's asset allocation based on short-term changes in the market and predictions. Dynamic asset allocation is a more complicated method using quantitative and qualitative data to make tactical decisions about where to invest. In constant-weight asset allocation, the asset allocation is kept the same by rebalancing the portfolio often. On the other hand, insured asset allocation uses derivatives like options and futures to protect the portfolio from downside risk. Integrated asset allocation is a way to set up an investment portfolio that considers both financial and non-financial factors, such as the investor's beliefs about society and the environment.

Portfolio creation may be divided into two types: active and passive. Active portfolio building is actively managing an investment portfolio to outperform the market and create more significant returns, while passive portfolio construction entails investing in a diversified portfolio that replicates the performance of a specific market index, such as the S&P500. Active portfolio management often has higher costs and necessitates more resources and skill, while passive portfolio management is typically less expensive and requires less work.

Security selection is an essential part of managing a portfolio. It involves finding and choosing individual securities within a particular asset class to build a portfolio. After figuring out the asset allocation strategy, choosing the suitable securities to reach risk and return goals is crucial. Security selection aims to find and choose individual securities with the best-projected return for a given degree of risk. It involves looking at the fundamentals of the securities, such as their financial statements, industry trends, and economic forecasts, to figure out how much they are worth and how they might change in the future. Short-term trading opportunities may also be identified using technical analysis methods like charting and trend analysis. Security selection, on the other hand, requires forecasting and prediction, which introduces uncertainty and risk into the process. This risk can be lessened by doing thorough fundamental and technical research and spreading investments across a wide range of assets and securities. (Prigent 2007)

As previously said, three critical input of returns must be considered while constructing a portfolio: the expected return, the volatility of asset returns, and the correlation (or covariance) of asset returns. The expected return is the expected return on a portfolio given the asset allocation and security selection strategy. The degree of change in the price of a security that impacts the portfolio's overall risk is referred to as the volatility of asset returns. Asset return correlation is the degree to which two assets move in lockstep and may impact the benefits of diversity in a portfolio. Effective security selection is critical to meeting the investment portfolio's risk and return goals. A detailed grasp of the investor's investing goals, risk tolerance, and investment horizon is required to guide the securities selection process. Also, a strict investment approach that incorporates thorough fundamental and technical analysis, as well as effective risk management techniques, may potentially mitigate risk and enhance returns.

3.2.3 Portfolio Optimization and Performance Analysis

Portfolio optimization is a critical area of financial research that aims to identify the ideal asset allocation within an investment portfolio to achieve the best possible risk-return tradeoff. Portfolio optimization is a process that involves selecting a combination of assets from a set of available assets in order to create a diversified portfolio that is optimized for maximum return with minimum risk. This is a complex task because the selection of assets is influenced by various factors, such as market conditions, investor preferences, and risk appetite.

Mean-variance optimization (MVO) stands as a widely adapted technique within the field of portfolio optimization. The mean-variance optimization approach was introduced by Harry Markowitz in 1952 and is based on the principle that investors seek to maximize their expected return while minimizing their risk. The Markowitz model, which is based on mean-variance optimization, provides a framework for selecting a portfolio that maximizes returns for a given level of risk.

The objective of MVO is to find the set of portfolio weights that minimizes the portfolio's variance while achieving a target expected return. This is achieved by considering the covariance matrix of asset returns, which measures the relationship between different assets in terms of their price movements. By incorporating the expected returns and covariance matrix, MVO aims to construct an efficient frontier, which represents the set of optimal portfolios with the highest return for a given level of risk.

However, mean-variance optimization (MVO) faces certain challenges that need to be acknowledged. One prominent challenge pertains to the estimation of expected returns and the covariance matrix, as these parameters are inherently subject to estimation errors and can be influenced by the selection of historical data. It is crucial to recognize that the accuracy of these estimations can directly impact the effectiveness of the resulting portfolio allocations. Additionally, MVO relies on the assumption of a normal distribution of asset returns, which may not hold in practice, especially during periods characterized by market turbulence or extreme events. Although this assumption has not been explicitly mentioned earlier, it is an underlying assumption of the MVO framework, and its validity can affect the reliability of the optimization outcomes.

Mean-variance optimization (MVO) employs statistical measures such as variance and covariance to determine optimal portfolio allocations. Variance quantifies the extent of the deviation of an asset's returns from its expected value, while covariance measures the relationship between the returns of two assets. By considering the covariance between assets, MVO aims to achieve diversification benefits. This is accomplished by incorporating assets with lower or negative correlations, which reduces the overall portfolio risk without sacrificing potential returns. MVO aims to strike a balance between risk and return by minimizing the portfolio's variance. This entails constructing portfolios that maximize expected returns while reducing the risk level, thus achieving an optimal trade-off between these two factors.

Another popular technique for portfolio optimization is the Black-Litterman model, which was introduced in 1990 by Fischer Black and Robert Litterman. The Black-Litterman model uses Bayesian inference to incorporate the views and beliefs of investors into the portfolio optimization process. This model provides a way to adjust the optimal portfolio allocation to account for market trends or any other relevant information that may not be fully captured by traditional portfolio optimization techniques (Da Silva et al. 2009). Moreover, Monte Carlo simulation emerges as an alternative method that can be employed in portfolio optimization. While it does not directly perform optimization, Monte Carlo simulation generates numerous simulations of portfolio returns based on diverse asset allocation strategies and market scenarios. These simulations estimate the probability distribution of portfolio returns, enabling investors to make informed decisions regarding portfolio allocation. In addition to the above techniques, an equal-weighted portfolio (EQ) is a simple approach that involves allocating equal weights to each asset in the portfolio. This technique is straightforward and easy to implement but may not be the most efficient in terms of risk-return trade-off.

Performance analysis is a critical component of portfolio optimization, as it allows investors to evaluate the effectiveness of their investment strategies and make informed decisions about portfolio allocation. Performance analysis involves assessing the returns of a portfolio relative to a benchmark or a set of benchmarks. One widely used performance measure is the Sharpe ratio, which measures the excess return earned per unit of risk taken by the investor. Other performance measures include the Treynor ratio, the Jensen's alpha, and the Information Ratio (Prigent 2007, 133 - 150).

4 Data and Methodology

In order to achieve predictions and optimize investment strategies, this chapter on data and methodology provides a comprehensive description of the critical techniques used in this study, namely the LSTM model and the MVO method, and discusses their applications and significance. This chapter also provides information and processing of raw data for preparing the LSTM model's input and determining the optimal portfolio.

Figure 2 depicts how the empirical portion of the study will be conducted. First, unprocessed data and its statistics are introduced and described. The Exploratory Data Analysis (EDA) step is used to uncover the raw data of the stock's daily adjusting price and then analyze the stock's return. The implementation of the LSTM model for forecasting based on the actual historical adjusted closing price of stocks is then discussed. Next, by using the actual historical returns and the predicted returns (calculated from the actual historical data and the forecasted data), the MVO method is applied to find the optimal portfolio. In conclusion, the outcomes of optimization models and their comparisons are discussed. Using the Python programming language (Jupyter - Anaconda Navigator (anaconda3),) on an Intel(R) Core(TM) i7-9750H CPU @ 2.6GHz, 2592 Mhz, 6 Cores with 16GB RAM, all techniques are executed.

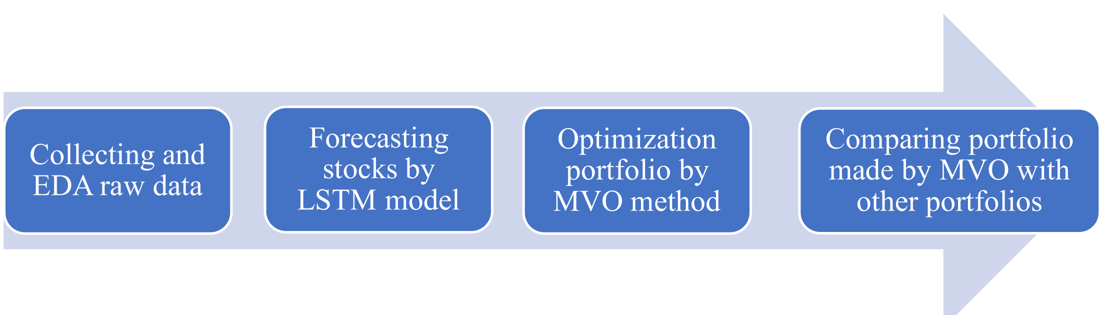


Figure 2. Process of study

4.1 Description of the data

This research looks at the daily adjusted close prices of 9 stocks from 01/10/2010 to 30/04/2023. The data of all stocks are time series data and available on Yahoo! Finance site (https://finance.yahoo.com). As mentioned in the introduction chapter, the study focuses on the 9 stock are recommended in article " 9 of the Best Stocks for a Starter Portfolio". The article's author justifies the selection of the following nine stocks based on the operating history of companies with a consistent record of dividend growth spanning 25 consecutive years. This criterion demonstrates the ability of these companies to thrive under various economic conditions, making them suitable candidates for an initial stock portfolio. The selected companies are as follows:

- Albemarle Corp. (ALB): Albemarle is an excellent growth holding that offers a balanced component to a starter portfolio. With a long-established presence in the materials industry and a record of over 25 years of dividend growth, Albemarle is well-positioned to capitalize on the emerging electric vehicle revolution by focusing on lithium. Its global operations in America, Chile, and Australia further strengthen its prospects. Notably, Albemarle stands out from many green energy companies by already being highly profitable and trading at an attractive valuation.
- A. O. Smith Corp. (AOS): A. O. Smith's dominant market position in hot water heaters and boilers exemplifies the recurring theme of dividend stocks excelling in stable, slow-moving industries. As demand for new housing rises, the company benefits from increased sales. Furthermore, with the millennial generation reaching the home-buying stage, A. O. Smith stands to benefit from a demographic tailwind. The company's long-term growth prospects remain intact despite near-term concerns surrounding a housing market slowdown and inflationary pressures.
- Brown-Forman Corp. (BF-B): Brown-Forman's success can be attributed to its tremendous brand value, with Jack Daniels being the world's most famous whiskey. The company's management has demonstrated an ability to expand its market share globally, particularly in Latin America, Europe, and East Asia. Noteworthy acquisitions like the Herradura tequila franchise have further fueled the company's growth. Additionally, Brown-Forman's resilience as an alcohol producer positions it favorably, regardless of economic conditions.
- Hormel Foods Corp. (HRL): Hormel, a leading packaged foods company specializing in protein products, has diversified its product portfolio beyond the well-known Spam brand. The company has capitalized on millennial-friendly products

such as almond and peanut butters, snack nuts, pepperoni, bacon, turkey, deli meats, salsa, and guacamole. Moreover, Hormel's unique ownership structure, controlled by the Hormel Foundation, incentivizes management to prioritize long-term growth over short-term fluctuations. Recent price dips associated with near-term inflation pressures present an attractive entry point for long-term investors.

- Johnson & Johnson (JNJ): Johnson & Johnson's consistent profitability and dividend growth spanning more than 50 years make it a staple in many investors' portfolios. The company's diversification across the health care industry, encompassing pharmaceutical drugs, medical devices, and consumer health and wellness products, has contributed to its success. The upcoming spin-off of its consumer health business into a separate entity named Kenvue presents an additional opportunity for investors. With a current undervaluation, JNJ stock offers a compelling investment proposition.
- McCormick & Co. Inc. (MKC): McCormick's strong presence in the consumer staples industry, particularly in spices and seasonings, makes it a suitable core holding for new investors. The company's leadership in institutional flavors, catering to world-renowned chain restaurants, provides a unique competitive advantage. McCormick's recent expansion into the hot sauce market through the acquisitions of Frank's Red Hot and Cholula positions it well to capitalize on younger consumers' preferences for diverse cuisines and recipes. Limited competition in the spice market ensures robust profit margins for McCormick.
- Roper Technologies Inc. (ROP): Roper Technologies stands out as a technology company with a track record of annual dividend increases spanning over 25 years. While the tech industry is known for its cyclicality, Roper's business model mitigates this risk by owning niche software applications in various industries, such as life insurance, K-12 school management, power plants, and graphic design. As software continues to permeate every sector, Roper has the potential to expand its software assets, resulting in sustained long-term cash flows.
- Sherwin-Williams Co. (SHW): Sherwin-Williams has demonstrated remarkable growth over the past three decades, owing to its focus on professional clients in the paints and coatings industry. The company's strong relationships with contractors and industry professionals create a stable recurring revenue stream. Despite recent

market concerns related to the housing industry, SHW stock presents an appealing long-term investment opportunity, given its historical performance and market position.

- Stanley Black & Decker Inc. (SWK): As a leading power tools and appliances manufacturer, Stanley Black & Decker boasts a history of consistent growth, as reflected in its dividend track record. Limited competition within the space and the strength of its brands have been instrumental in the company's ongoing success. While recent sales slowdowns and the durability of power tools may affect short-term performance, the significant decline in SWK stock since 2020 presents an attractive entry point for long-term investors.
- At the same time, this study choose the S&P 500 index (consists of the 500 large companies which are listed in the United States markets) as the market index as a basis for comparing the portfolio's performance to the market. According to the statistics from the financial markets, global indexes such as the S&P 500 and EAFE most of the time outperform the performance of the median manager (Prigent 2007. 103)

4.1.1 Data description of stock's original data

The dataset includes descriptive statistics regarding the adjusted closing prices of a selection of equities. As stated previously, the data sample was collected between 01/01/2010 and 30/04/2023, and after removing null values, the dataset contains 3,353 observations. Table 1 displays descriptive statistics regarding the dataset. The table lists each stock's mean, minimum value, maximum value, variance, standard deviation, skewness, and kurtosis.

Table 1. Descriptive statistics of adjusted close price of all stocks

Stocks	mean	min	max	variance	std	skewness	kurtosis
ALB	92.0480	28.8813	324.2554	4116.7625	64.1620	1.6166	1.5324
AOS	35.6576	5.6829	83.4770	414.5227	20.3598	0.0778	-1.1711
BF-B	40.3626	10.4912	78.6080	385.2935	19.6289	0.3080	-1.1889
HRL	28.5894	7.3223	53.5175	175.3634	13.2425	-0.0744	-1.3418
JNJ	99.4943	39.4091	181.1088	1697.8499	41.2050	0.1971	-1.1426
MKC	48.7285	13.7930	102.3292	660.9972	25.7099	0.4387	-1.2296
ROP	227.6453	46.3289	494.0063	17950.7828	133.9805	0.4415	-1.2126
SHW	117.8675	16.6254	347.8784	6911.7941	83.1372	0.7442	-0.5279
SWK	99.2484	37.2280	209.5333	1741.7981	41.7349	0.5326	-0.6744

The minimal and maximum values for the adjusted close prices vary among equities. For example, the range for ALB stock is between 28.8813 and 324.2554, whereas the range for ROP stock is between 46.3289 and 494.0063. For securities with a maturity of more than 12 years, the maximum price value can be five to ten times the minimum (depending on the security).

Skewness quantifies the asymmetry of the data distribution. Positive skewness indicates that the distribution has an extended right tail and that the majority of the data is concentrated on the left. In contrast, a negative skewness indicates that the left tail of the distribution is lengthier and that most of the data is concentrated on the right. Observing the skewness values in the table, we can see that ALB, AOS, BF-B, JNJ, MKC, ROP, SHW, and SWK have positive values, indicating an extended right tail. HRL, on the other hand, has a negative skewness value, which indicates an extended left tail.

In comparison to a normal distribution, kurtosis assesses the peakedness or flattening of the data distribution. A distribution with positive kurtosis has heavier tails and a higher peak than one with a normal distribution. As a result, the data is more concentrated around the mean and the distribution contains more extreme values, or outliers. As opposed to a normal distribution, a distribution with negative kurtosis has lighter tails and a flatter shape. As a result, the distribution of the data is greater and there are fewer extreme values or outliers. The table's kurtosis values indicate that all of the equities exhibit negative kurtosis, indicating that their distributions have thinner and a flattened shape compared to a normal distribution. This suggests that the data for those equities is less extreme and more spread out compared to a normal distribution.

When comparing the securities according to their mean, minimum, and maximum value, ROP has the highest mean at 227.6453, while HRL has the lowest mean at 28.5894. ALB has the lowest minimum value at 28.8813, while SHW has the highest at 16.6254. ROP has the highest maximal value at 494.0063, while ALB has the lowest at 324.2554.

Variance and standard deviation are measurements of the data's dispersion or distribution. The variance is the average of the squared differences between each data point and the mean, whereas the standard deviation is the variance's square root. They provide insight into the distribution of data points around the mean. The variance and standard deviation values for each stock are provided. For instance, the variance of ALB is 4116.7625, and its standard deviation is 64.1620. These values indicate that the adjusted close prices for ALB are more dispersed from the mean than those of other stocks in the dataset.

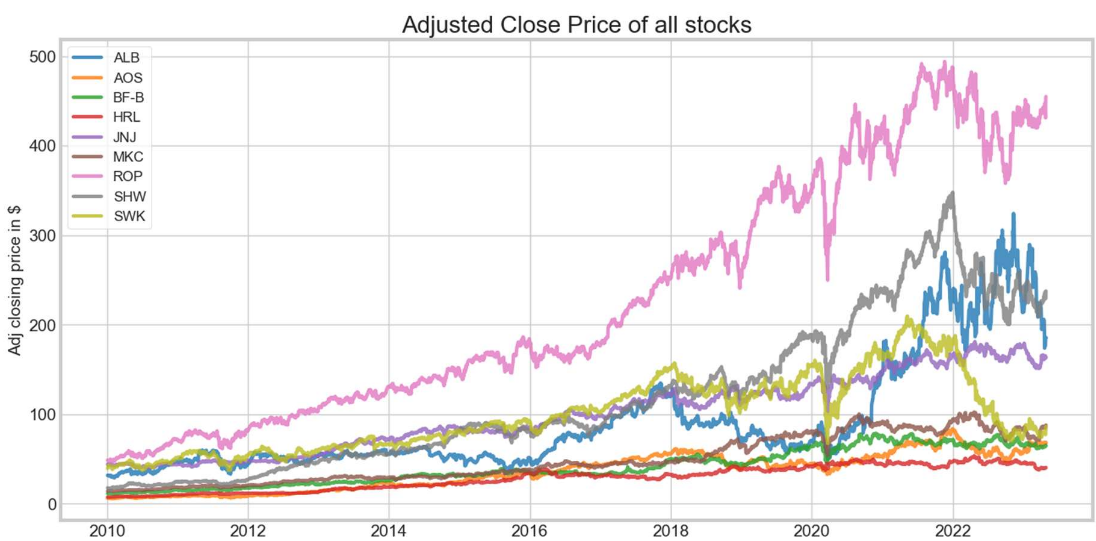


Figure 3. Adjusted close price development of 9 stocks during 01.01.2010 – 30.04.2023

4.1.2 Data description of stock's return data

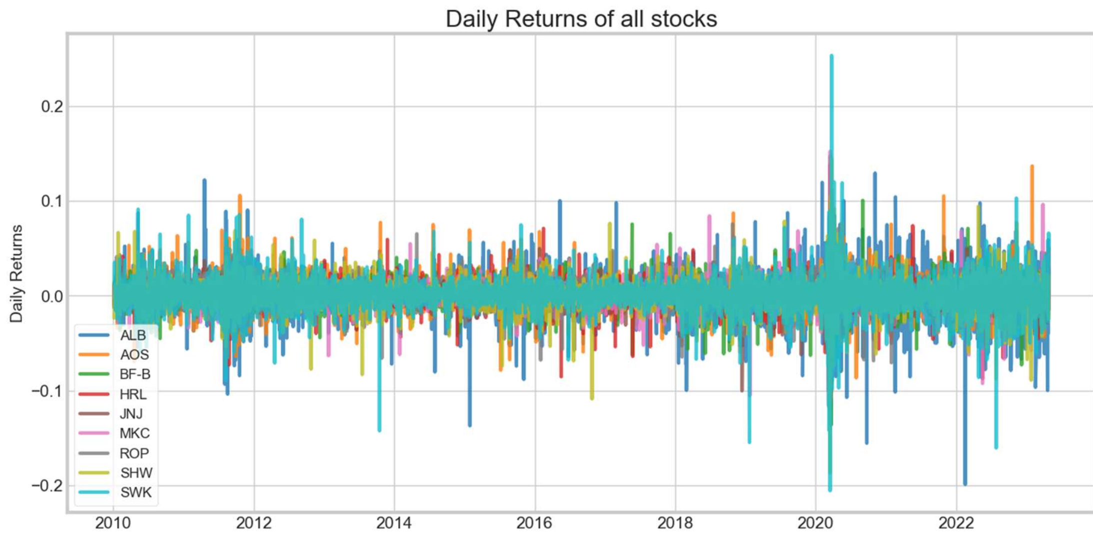


Figure 4. Fluctuation of daily returns

Similarly, Table 2 presents the descriptive statistics of daily returns for a dataset consisting of several stocks. Each stock's daily return is represented by the corresponding statistical measures given for each stock, including mean, minimum, maximum, variance, standard deviation, skewness, and kurtosis.

Table 2. Descriptive statistics of daily returns of all stocks

Stocks	mean	min	max	variance	std	skewness	kurtosis
ALB	0.0008	-0.1991	0.1374	0.0006	0.0241	-0.3882	5.6490
AOS	0.0009	-0.0867	0.1367	0.0003	0.0175	0.2903	3.9076
BF-B	0.0006	-0.1356	0.1473	0.0002	0.0142	-0.1142	10.0453
HRL	0.0006	-0.0876	0.1384	0.0002	0.0127	-0.0629	8.7513
JNJ	0.0004	-0.1004	0.0800	0.0001	0.0106	-0.1870	9.3732
MKC	0.0006	-0.1092	0.1523	0.0002	0.0130	-0.1594	16.6688
ROP	0.0008	-0.1019	0.1124	0.0002	0.0146	-0.2680	6.8789
SHW	0.0009	-0.1868	0.1445	0.0003	0.0159	-0.3435	12.2614
SWK	0.0004	-0.2058	0.2532	0.0004	0.0201	-0.1930	17.3566

The range between the minimum and maximum values provides an understanding of the spread of daily returns for each stock. Based on the minimum and maximum values, SWK appears to have the most vital fluctuations in daily returns compared to the other stocks in the dataset. SWK has a minimum value of -0.2058, indicating a relatively large negative return, and a maximum value of 0.2532, representing a relatively large positive return. This wide range between the minimum and maximum values suggests significant volatility in SWK's daily returns. Other stocks in the dataset may have smaller ranges between their minimum and maximum values, indicating comparatively less fluctuation. We can view the presence of fluctuations in daily returns in Figure 4 above.

The skewness values of the returns of stocks range from -0.3882 to 0.2903. All of the stocks' returns have negative skewness, indicating that the majority of the data is concentrated on the right side of the distribution, with a lengthier left tail. This suggests a higher likelihood of negative returns or outliers on the lower end of the distribution.

Regarding kurtosis, which measures the shape of the distribution, all stocks demonstrate positive kurtosis values. This indicates that the distributions have heavier tails and a sharper peak, implying a higher likelihood of extreme returns or outliers in the data. The presence of heavier tails and extreme values in the data can have an impact on the predictive modeling process. The traditional linear models may not be suitable for capturing the non-linearity and potential outliers present in the data. Therefore, it may be necessary to explore alternative modeling techniques that can better handle skewed and heavy-tailed distributions, such as robust regression methods, machine learning or deep learning algorithms.

Specific observations can be made when comparing the stocks based on their mean, minimum, maximum, variance, and standard deviation. For instance, the stocks with the highest mean daily return are AOS and SHW, with a mean of 0.0009. On the other hand, JNJ and SWK have the lowest mean return of 0.0004. When considering the variance and standard deviation, these measures provide insights into the volatility or dispersion of the daily returns. A lower variance and standard deviation indicate lower volatility, while higher values suggest greater volatility. JNJ has the most minor variance and standard deviation in this dataset, indicating relatively less volatility than other stocks. Conversely, ALB has an enormous variance and standard deviation, suggesting a higher level of volatility.

This study needs to consider the correlations between the returns of stocks; because they play a crucial role in optimal portfolio construction using MVO by influencing risk and diversification. Understanding the correlation structure is essential for both modeling and constructing efficient portfolios. The Figure 5 below shows the correlation between the returns of 9 stocks

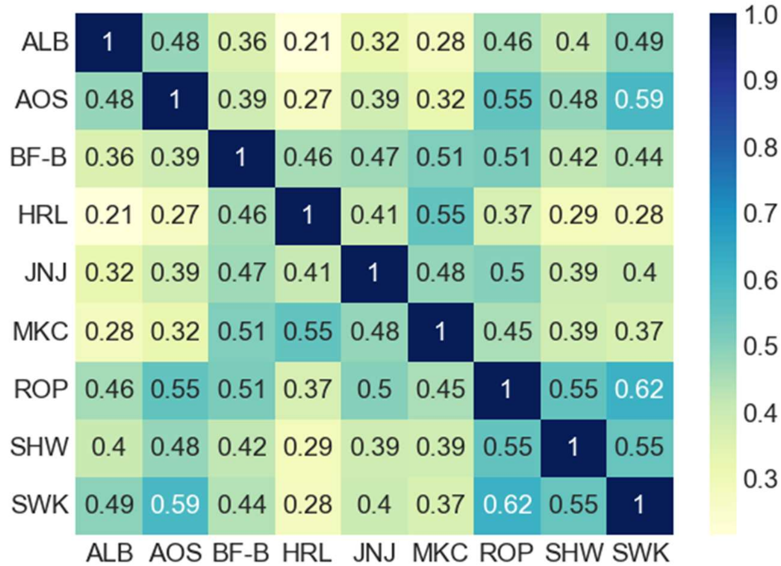


Figure 5. Correlation between daily returns of 9 stocks

Analyzing the correlation values, we can observe varying degrees of correlation among the stocks. Some notable observations include:

- Strong Positive Correlation: AOS and SWK have a correlation coefficient of 0.593342, indicating a relatively strong positive correlation between their returns. Similarly, other pairs such as ROP and SWK, AOS and ROP, and BF-B and MKC also exhibit relatively strong positive correlations.

- Moderate Positive Correlation: Several pairs of stocks, such as ALB and SWK, ALB and ROP, and AOS and SHW, display moderate positive correlations ranging from approximately 0.4 to 0.5.

- Weak to Moderate Positive Correlation: The remaining pairs exhibit weaker positive correlations, ranging from approximately 0.21 to 0.39.

4.2 Methodology

4.2.1 Prediction Model – LSTM Model implementation

The present study utilizes the LSTM model, building upon the research conducted by Chaweewanchon and Chaysiri (2022), to predict stock prices using historical data's actual adjusted closing price. An integral aspect of machine learning success lies in the training and testing process. The LSTM model is trained on a portion of the data and validated on a separate part to ensure that it is not overfitting to the training data.

Consistent with the study by Chaweewanchon and Chaysiri (2022), this research divides the adjusted closing price of each stock into training and testing sets, following an 80:20 ratio. This ratio is also applied in research by Ta et al. (2020). Consequently, the training process employs the initial 2683 days of data, while the remaining 670 days comprise the testing set. Additionally, a 100-day time step is utilized for retrospective analysis.

Before the training phase, standardization is applied to all datasets. This standardization transforms the data into a shared scale, enhancing the learning process's efficiency. Specifically, the input data is standardized by calculating the means and standard deviations of the variables, subtracting the mean (μ) from returns (R_t) and dividing the difference by standard deviation (σ) as follows (Fischer & Krauss, 2018):

$$\tilde{R}_t = \frac{R_t - \mu}{\sigma} \qquad (7)$$

Regarding the default activation function in the LSTM layers of Keras (open-source deep learning framework written in Python supports the process of building and training neural networks for developing deep learning models), the recurrent unit (s_t) uses the tanh activation function by default, while the gates (f_t, i_t, and o_t) use the sigmoid activation function by default. The final Dense layer following the LSTM layers does not specify an activation function, which means it will use the default linear activation function. The default linear activation function is a simple identity function that returns the input values as is, without any transformation.

The number of hidden layers and neurons represents critical hyperparameters significantly impacting the neural network's effectiveness. Identifying the optimal hyperparameters remains a major challenge in deep learning (Chaweewanchon & Chaysiri, 2022). In this study, the hyperparameters were manually set through trial and error, selecting the best parameters from experiments aligned with the researcher's operating system capabilities. The subsequent section provides a detailed description of the hyperparameters and their corresponding values.

The present study constructs a two-layer LSTM model. The first layer comprises 200 neurons, while the second layer contains 100 neurons. The output layer consists of a dense solitary unit. A 20% dropout rate is applied after each layer to mitigate overfitting, as observed in prior studies (Jia et al., 2019; Muhammad et al., 2021). According to Chung et al. (2014), the Adam Optimizer is well-suited for deep learning problems involving large datasets and proves effective for LSTM-based networks. As for the LSTM model, a batch size of 32, representing the number of training examples processed together during each iteration, and 100 epochs, indicating one complete round of training, are considered suitable default values (Kandel & Castelli (2020); Xayasouk et al. 2020).

The following information is provided to provide an overview of the LSTM model architecture:

```
Model: "sequential"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 lstm (LSTM)                 (None, 100, 200)          161600

 dropout (Dropout)           (None, 100, 200)          0

 lstm_1 (LSTM)               (None, 100)               120400

 dropout_1 (Dropout)         (None, 100)               0

 dense (Dense)               (None, 1)                 101

=================================================================
Total params: 282,101
Trainable params: 282,101
Non-trainable params: 0
```

Figure 6. LSTM model architecture

The first LSTM layer takes the input data and produces an output of shape (None, 100, 200). This means it processes the input data in sequences of 100 timesteps (100 days look back), each with a feature dimension of 200. The number of units (neurons) in this layer is 200.

This layer has 161,600 trainable parameters. In the model architecture, the "None" dimension in the output shapes represents the batch size, but it appears as "None" because it is a symbolic representation. After the first LSTM layer, a Dropout layer is added to drop 20% of units (dropout rate of 0.2) to help prevent overfitting.

The second LSTM layer takes the output of the first LSTM layer as input and produces an output of shape (None, 100). It processes the data in sequences of 100 timesteps but reduces the feature dimension to 100. The number of units (neurons) in this layer is 100. It has 120,400 trainable parameters. Another Dropout layer is added after the second LSTM layer with the same purpose first dropout layer.

The final layer is a Dense layer, which is a fully connected layer. It takes the output of the second LSTM layer and produces a single output value (None, 1), representing the predicted stock price. The number of units in this layer is 1.

During training, the model takes the historical data as input, processes it through the layers, and compares the predicted output with the actual output. The difference between the predicted and actual values is used to compute a loss function, quantifying the model's performance. Using techniques like backpropagation, the optimizer then adjusts the model's parameters to minimize this loss. Once the model is trained, it can predict new, unseen data. The input data is fed into the model, and the output is generated by propagating it through the layers. The final output represents the predicted stock price based on the model's learned patterns and relationships from the training data.

The study does not employ a single LSTM model trained on one stock and subsequently applies it to the remaining securities. Instead, it adopts a training and testing approach specific to each stock's data. Consequently, the study develops nine distinct LSTM models, each sharing identical hyperparameters, to accommodate the nine types of securities under investigation. Because each stock has its unique characteristics and patterns, by building separate LSTM models for each stock, the models can capture and learn the specific dynamics and trends relevant to that particular stock. This could lead to more accurate and tailored predictions for each stock.

4.2.2 Prediction Evaluation

Mean Absolute Error (MAE), Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE), and Root Mean Squared Error (RMSE) are frequently used metrics for evaluating models (Chicco et al. (2021)):

- Mean Absolute Error (MAE) quantifies the average absolute difference between predicted and observed values. It measures the average magnitude of defects without taking their direction into account.
- Mean Squared Error (MSE) is a popular metric for evaluating regression models. It determines the average squared deviation between the predicted and actual values. MSE is computed by averaging the squared differences between predicted and actual values.
- Mean Absolute Percentage Error (MAPE) measures the average percentage difference between predicted and observed values. MAPE is frequently employed in forecasting duties and provides a relative measure of prediction accuracy.
- Root Mean Squared Error (RMSE) measures the square root of the average squared difference between predicted and actual values. Like MAE, RMSE measures the average magnitude of errors but gives greater weight to more significant errors. Due to its sensitivity to outliers and large errors, RMSE is applicable when such errors must be addressed.

The following formulas are used to calculate the metrics (Chicco et al. (2021):

$$MAE = \frac{1}{m}\sum_{i=1}^{m}|X_i - Y_i| \tag{8}$$

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(X_i - Y_i)^2 \tag{9}$$

$$MAPE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{X_i - Y_i}{Y_i}\right| \tag{10}$$

$$RMSE = \sqrt{MSE} \quad (11)$$

Where X_i is the predicted i^{th} value, and the Y_i element is the actual i^{th} value

This study incorporates several evaluation metrics, namely RMSE, MAE, and particularly MAPE, to assess the performance of the LSTM prediction model. MAPE, in particular, possesses an intuitive interpretation concerning the relative error. Due to its definition, MAPE is particularly suitable for tasks where sensitivity to relative variations is more significant than absolute variations (De Myttenaere et al. 2016). The calculation of MAPE is derived from equation 10 mentioned earlier in this study.

4.2.3 Portfolio Optimization

Portfolio optimization aims to construct an optimal portfolio by considering the trade-off between expected returns and risk. The mean-variance optimization (MVO) method, pioneered by Markowitz (1952), has been widely adopted in financial literature due to its ability to generate efficient portfolios. The explanation of the MVO method can understand that the optimization will select the weights of stocks in the portfolio which gives the variance of the portfolio as minimum OR the Sharpe Ratio of the portfolio as maximum. This idea can be used for ranking the stocks to reduce the standard deviation of the day's return and get a better sharpe value.

The mean and covariance of returns primarily determine the return and risk of a portfolio. The mean and variance of the portfolio's fluctuations were used to model the portfolio's return and risk, respectively. Based on the post-processing data described in the previous section, the return or volatility of each stock is determined by calculating the percentage change between the stock's daily adjusted closing price value.

The annualized portfolio return, assuming that there was T = 252 trading days in a year, is calculated as follows (Pai 2018):

$$r_{Ann} = \left(\sum_{i=1}^{N} W_i . r_i \right) . T \quad (12)$$

where N stocks comprising the portfolio, with weights W_1, W_2, … W_N as their individula weights and r_1, r_2, … r_N as the daily returns of the stocks.

Similarly, the annualized portfolio risk in percentage is the standard deviation of portfolio returns is given by (Pai (2018):

$$\sigma_{Ann} = \sqrt{\left(\sum_{i}\sum_{j} W_i \cdot W_j \cdot \sigma_{ij}\right) \cdot T} \qquad (13)$$

Where σ_{ij} is the covariance between daily returns (%) of stocks i and j of the portfolio, also referred to as the variance-covariance matrix of returns

The mean-variance optimization model is employed to determine the optimal portfolio weights by combining estimated returns, risks, and the covariance matrix. This study generates a large number of random portfolios, namely 100,000 , in order to determine the optimal portfolio. The model's primary goal is to identify the portfolio allocation that maximizes expected return r_{Ann} for a given level of risk or minimizes risk σ_{Ann} for a give level of expected return.

In order to accomplish mean-variance optimization model, the optimization problem entails locating the portfolio weights that reside on the efficient frontier. The efficient frontier is constructed by repeatedly solving the mean-variance optimization model for varying target returns or risk levels. The resulting portfolios are then plotted on a graph, visually representing the risk-return trade-off. An efficient frontier is established by graphically portraying the efficient set derived through mean-variance optimization. This graph illustrating the risk-return trade-off shows a variety of optimal portfolios that provide the greatest anticipated return for a given level of risk or the lowest risk for a given level of expected return. The efficient frontier describes the ideal portfolio structure that produces the best-anticipated return for a certain degree of risk, or vice versa. Portfolio optimization aims to construct portfolios that reside on or above the efficient frontier, as portfolios below it are deemed suboptimal (Francis & Kim 2013). Commonly, the Markowitz efficient set refers to the set of all optimal portfolios that reside on the efficient frontier and generate the greatest return for a specified level of risk or vice versa.

Figure 7 illustrates the efficient frontier, which extends from point B on the left to point C on the top right. This region represents a subset of portfolios that offer the maximum return (y-axis) while minimizing variance (x-axis). Portfolios located between points A and B or inside the curve X are deemed inefficient as they do not maximize return relative to their risk. (Grasse et al. 2016)

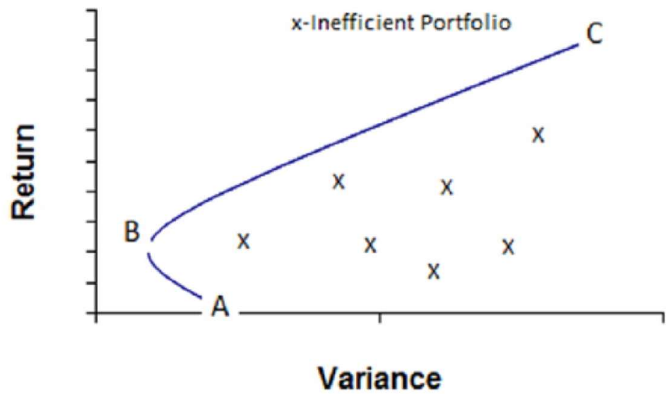


Figure 7. Efficient Frontier (Grasse et al. 2016, 829)

4.2.4 Portfolio Evaluation and Management

Nobel Laureate William F. Sharpe introduced the Sharpe Ratio, also known as the "reward-to-variability ratio" in 1966. The Sharpe Ratio is a measure used to calculate risk-adjusted returns. This ratio facilitates investors in assessing the performance of their investments concerning the risks involved. By comparing the expected returns of two investment opportunities, the Sharpe Ratio quantifies the additional return per unit of risk that an investor could obtain by selecting one opportunity over another. (Francis & Kim 2013)

To compute the Sharpe Ratio, the difference in returns between two investments is examined, and the average difference is compared to the standard deviation of this difference, which serves as a measure of risk. A higher Sharpe Ratio implies that the potential reward is more remarkable for a given level of risk. A specific investment opportunity is typically compared against a benchmark representing an entire investment category.

The Sharpe Ratio provides valuable insights to investors by evaluating the risk-reward trade-off of different investment options. This measure enables informed decision-making regarding portfolio allocation, incorporating both the expected returns and associated risks. By considering risk-adjusted returns, the Sharpe Ratio offers a comprehensive perspective on investment performance beyond solely focusing on returns. (Francis & Kim 2013)

Sharpe's methodology involves subtracting the estimated riskless interest rate (r_f) from each asset's average rate of the portfolio ($\overline{r}_p$). This difference, known as the risk premium, represents the portion of the average holding period return (HPR) that exceeds the riskless interest rate. The risk premium, alternatively referred to as an excess return, quantifies the additional return investors earn when investing in assets with more than zero risk. To determine the risk-adjusted performance of each portfolio, Sharpe divides the risk premium by the standard deviation (σ_p) of the portfolio's returns. This calculation yields the ratio of risk premium per unit of risk, denoted as S_p for portfolio p. (Francis & Kim 2013)

$$Sharpe\ Ratio\ (S_p) = \frac{\bar{r}_P - r_f}{\sigma_P} \tag{14}$$

Sharpe's desirability index was developed to compare and rank investment portfolios across different risk classes, considering their varying average rates of return. By combining both risk and return statistics into a single index number, denoted as S_p, Sharpe's ratio facilitates the ranking of portfolios. (Francis & Kim 2013)

The risk-free rate chosen for this study is based on the 52-week Treasury bill rates obtained from the U.S. Department of the Treasury. Notably, the value of the Treasury bill rate experiences significant fluctuations between 2010 and 2023. The increase from 0.1% at the start of 2021 to 4% by the conclusion of 2022 is particularly noteworthy. This selection aligns intending to match the annualized return and risk. Two Treasury bill rates are utilized in the analysis. The first-rate, 4.63%, corresponds to the 52-week Treasury bill rate at the beginning of May 2023. This rate is applied to calculate the Sharpe Ratio for the portfolio constructed on May 1st, representing the conclusion of the dataset timeframe. The second rate, 1.5%, represents the average Treasury bill rate from 2021 to April 2023. This rate is employed during the rebalancing periods to construct the portfolio from 2021 to 2023.

This study employs the MVO (Mean-Variance Optimization) method to select the portfolio with the highest Sharpe index. Once an investment portfolio is chosen, portfolio management becomes crucial for investors. The study assumes a holding period of approximately two years (from 28/01/2021 to 28/04/2023) for the selected portfolio, which encompasses various stocks. The initial asset allocation is based on the portfolio outcomes on January 28, 2021 with the highest Sharpe index.

However, it is important to note that portfolio management is an ongoing process due to market fluctuations. If left unattended, a portfolio can deviate over time, resulting in increased exposure to risk that may not align with the investor's risk appetite and investment preferences. Therefore, portfolio rebalancing becomes necessary. This involves buying and selling components of the portfolio to realign the portfolio weights, aiming to meet the original goals or establish a new asset allocation that adjusts each asset's weights and brings the portfolio's risk-return characteristics back in line with the desired parameters. (Pai 2018)

In the context of this academic thesis, the portfolio rebalancing process involves re-implementing the approach to identify the highest Sharpe index, considering the return, risk, and covariance matrix of the selected securities. It is worth noting that the study does not consider transaction costs associated with buying and selling securities. The frequency of portfolio rebalancing varies based on the preferences and techniques employed by individual investors. However, in this study, portfolio rebalancing occurs quarterly, from 28/01/2021 to 28/04/2023. This predetermined time frame ensures consistency in the rebalancing process and facilitates evaluating the portfolio's performance over the specified period.

5 Findings

This chapter will introduce the study's results. First, the study performs portfolio optimization based on all historical data using the MVO method; and compares the performance of the found two portfolios from MVO method with the market index and equally weighted portfolio. Next, the study sets up the LSTM prediction model and analyzes the accuracy of the prediction. To test the effectiveness of portfolio management by forecasting results, the study re-implements the optimization of the portfolio on January 28, 2021 (referred to as point P0). And then, use the remaining period's data (from 28/01/2021 to 28/04/2023) to analyze and compare the portfolios based on forecast data and historical data in the same next interval of time to find out the difference and change of the portfolio.

5.1 Portfolio Optimization based on whole historical data

The presented information in the Table 3 and pie charts (Figure 9) outline the portfolio allocations and performance metrics of two distinct investment strategies for whole historical data: the Maximum Sharpe Ratio Portfolio and the Minimum Volatility Portfolio. These strategies are both parts of the MVO method to optimize the risk-return tradeoff of investment portfolios.

Table 3. Two portfolios based on whole historical data

Index	Maximum Sharpe Ratio Portfolio Allocation	Minimum Volatility Portfolio Allocation
Annualized Return	19.6752%	14.3587%
Annualized Volatility	17.7592%	14.7242%
Sharpe Ratio	0.847179	0.660728

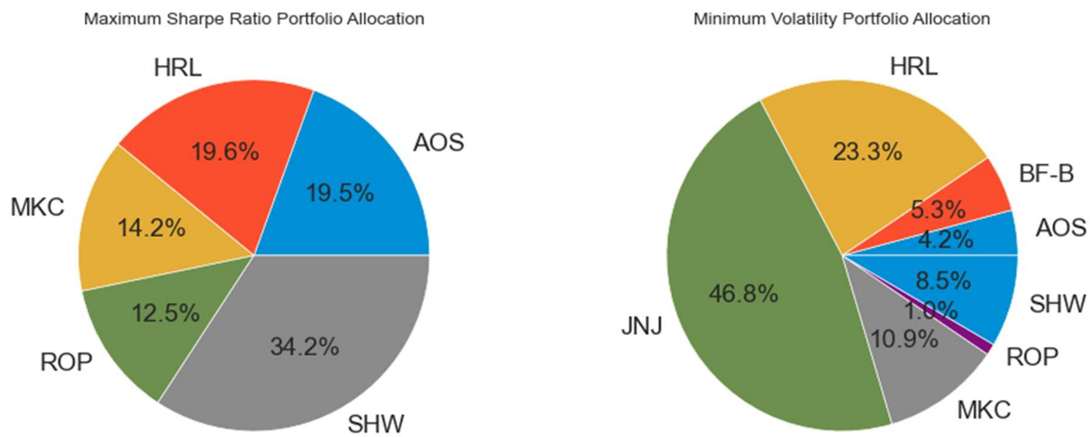


Figure 8. Portfolio Allocation based on whole historical data

The Maximum Sharpe Ratio Portfolio demonstrates an annualized return of 19.6752% and an annualized volatility of 17.7592%. The Sharpe Ratio, a measure of risk-adjusted return, is calculated as 0.847179. This portfolio comprises five assets: AOS, HRL, MKC, ROP, and SHW, with respective allocations of 19.4598%, 19.6262%, 14.1612%, 12.5435%, and 34.2093%.

Conversely, the Minimum Volatility Portfolio exhibits an annualized return of 14.3587%, with an annualized volatility of 14.7242%. The corresponding Sharpe ratio is determined to be 0.660728. The portfolio allocation for this strategy encompasses seven assets: AOS, BF-B, HRL, JNJ, MKC, ROP, and SHW, with allocations of 4.169%, 5.2627%, 23.3077%, 46.8431%, 10.9187%, 1.0466%, and 8.4522%, respectively.

These findings highlight the two examined investment strategies' performance characteristics and asset allocations. The Maximum Sharpe Ratio Portfolio prioritizes achieving a higher risk-adjusted return by allocating a more significant portion of the portfolio to SHW and a relatively smaller proportion to AOS, HRL, MKC, and ROP. Conversely, the Minimum Volatility Portfolio aims to minimize volatility, with a relatively higher allocation to JNJ and significant diversification across the remaining assets.

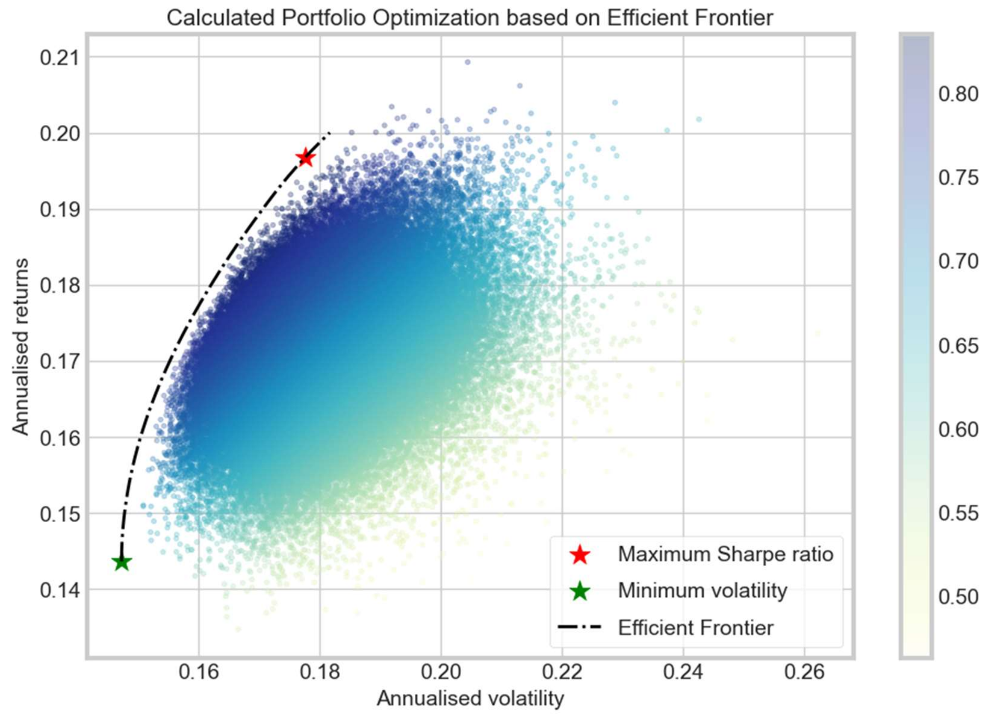


Figure 9. Calculated Portfolio Optimization with historical data

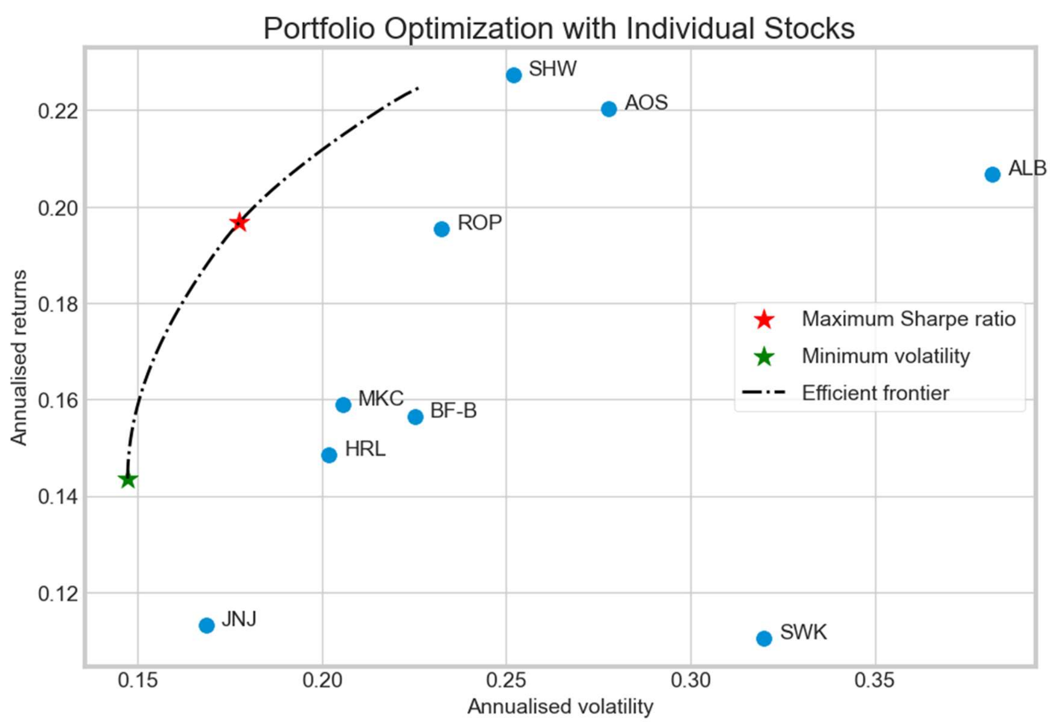


Figure 10. Portfolio Optimization and performance of individual stock (historical data)

In addition to portfolio findings based on the MVO method, another straightforward approach is the equal weighting method, which involves equally dividing the ratio of the nine existing securities in a portfolio. Based on the graphical results presented below (Figure 11), it is evident that the equal-weighted portfolio, with an annualized return (17.0767%) and a Sharpe ratio (0.701689), despite not achieving the same volume as the Maximum Sharpe Ratio Portfolio, but outperforms both the Minimum Volatility Portfolio and market benchmarks in terms of yield and risk-adjusted-performance.

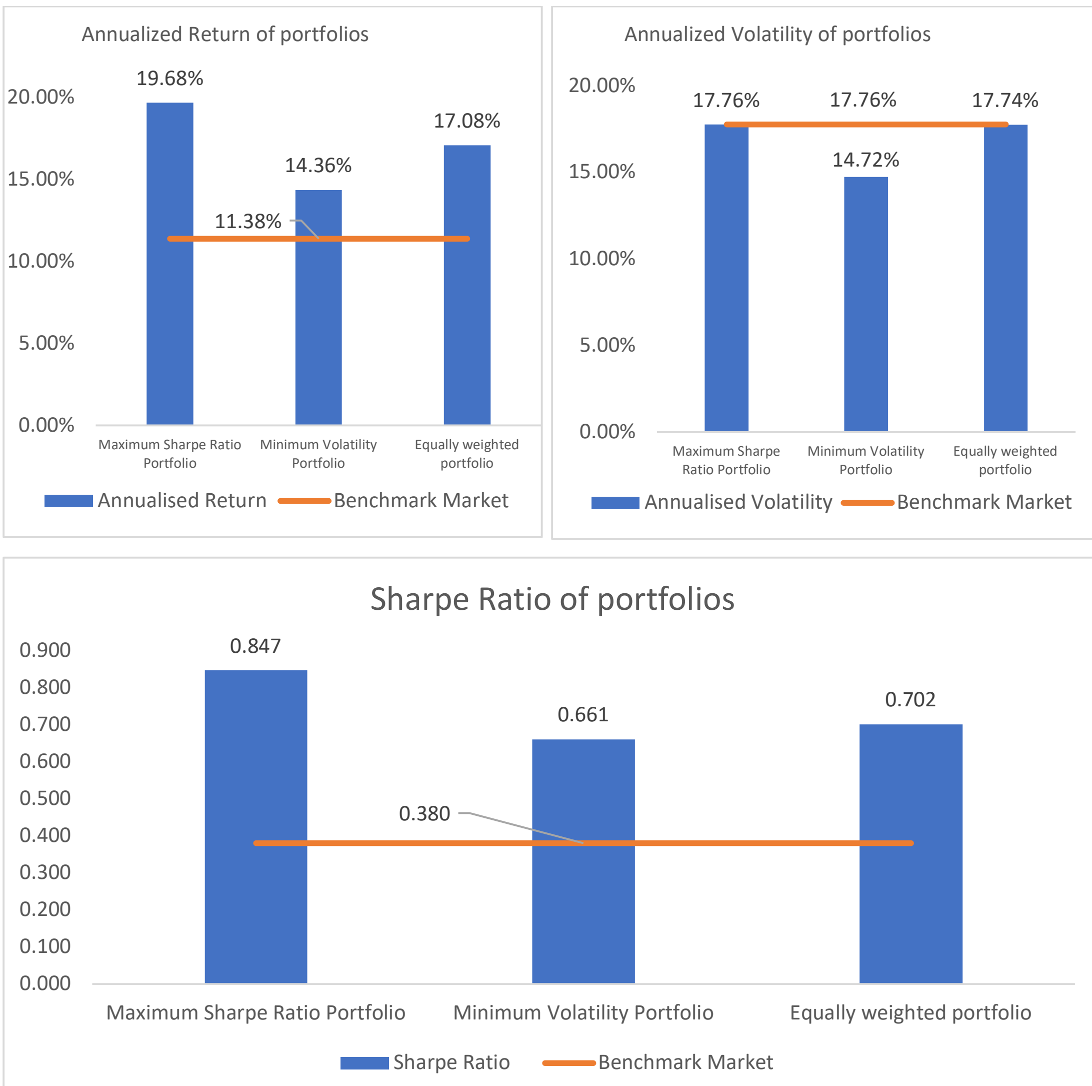


Figure 11. Comparing Portfolios based on historical data with benchmark market

5.2 Stocks Prediction Results

The model is constructed through training and testing using individual data sets corresponding to each security. This procedure creates nine distinct models, each associated with a specific stock (nine stocks considered for portfolio construction). Consistency in the architectural configuration of the LSTM models is ensured, with parameters such as the number of layers, number of neurons, batch size, and epochs being maintained at identical values across all models employed for different stocks.

The provided data tables below present the performance metrics of an LSTM (Long Short-Term Memory) model applied to different stocks in both the training and test phases (2683 days of observations for training and 670 days of observations for testing). As mentioned in the methodology chapter, the metrics used for evaluation include Mean Absolute Error (MAE), Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE), and Root Mean Squared Error (RMSE). Analyzing these metrics allows for a comprehensive assessment of the model's predictive accuracy and precision.

During the training phase, the LSTM model exhibited diverse performance levels across the different stocks. Notably, "HRL" demonstrated the lowest MAE value of 0.3809, indicating a relatively small average absolute deviation between the predicted and actual values. Similarly, “MKC”, "BF-B" and "AOS" displayed favorable MAE values of 0.4385, 0.4182 and 0.4910, respectively, suggesting accurate predictions for these stocks.

Table 4. Performance of LSTM model – train part

Stocks	MAE	MSE	MAPE	RMSE
ALB	1.0879	2.4648	1.7467	1.5700
AOS	0.4910	0.4277	2.2933	0.6540
BF-B	0.4182	0.3933	1.3163	0.6271
HRL	0.3809	0.3011	1.4146	0.5487
JNJ	0.8523	1.5130	1.1394	1.2300
MKC	0.4385	0.5125	1.0938	0.7159
ROP	2.1456	11.7537	1.2915	3.4284
SHW	1.1272	3.3779	1.4757	1.8379
SWK	1.2285	3.6315	1.4887	1.9056

When considering the RMSE values, "HRL" showcased the best performance with an RMSE of 0.5487, indicating a minimal overall deviation between the predicted and actual values. “MKC”, “BF-B” and “AOS” also exhibited commendable RMSE values of 0.7159, 0.6271 and 0.6540, respectively.

The MAPE values provide insight into the average percentage deviation between the predicted and actual values. Most stocks' LSTM models achieved low MAPE values during the training phase. Noteworthy stocks with low MAPE values include “MKC” with a value of 1.0938 and "JNJ" with a value of 1.1394. These values suggest that the model's predictions deviated minimally in percentage terms from the actual values for these stocks.

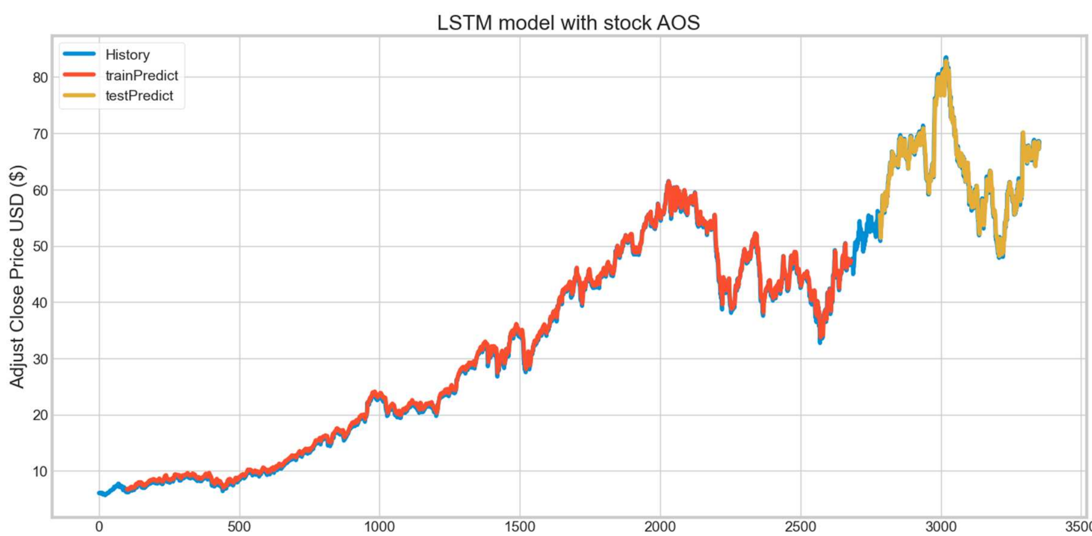
Table 5. Performance of LSTM model – test part

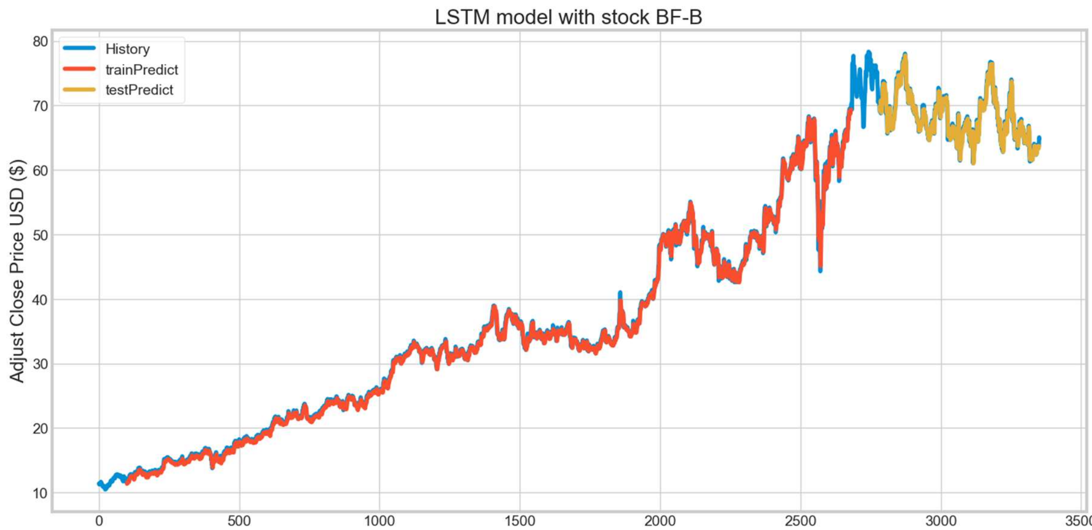
Stocks	MAE	MSE	MAPE	RMSE
ALB	5.2354	49.8031	2.3815	7.0571
AOS	0.9320	1.5204	1.4826	1.2331
BF-B	0.7142	0.9725	1.0491	0.9861
HRL	0.8414	0.9701	1.8419	0.9849
JNJ	1.3161	2.9652	0.8092	1.7220
MKC	0.9452	1.7703	1.1116	1.3305
ROP	5.0762	44.0746	1.1831	6.6389
SHW	3.7168	24.4308	1.4742	4.9428
SWK	2.1191	7.8583	1.7170	2.8033

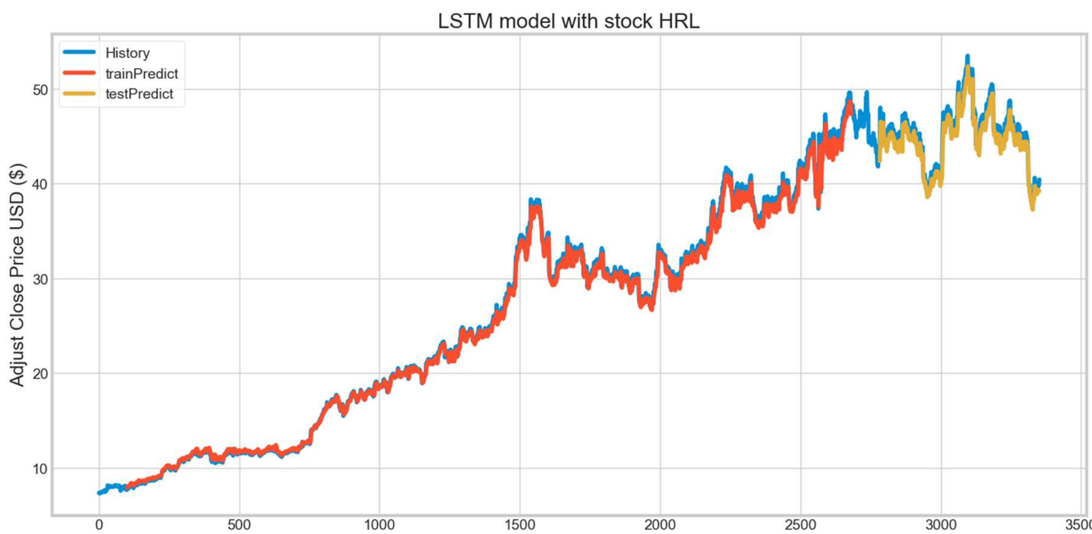
In the test phase, the MAPE values also reveal the small average percentage difference between the predicted and actual values for the LSTM model applied to various equities. Notably, "BF-B," and "JNJ" equities exhibited comparatively low MAPE values of 1.0491, and 0.8092, respectively. These values indicate that, on average, the LSTM model's forecasts for these equities deviated from the actual values by a small percentage. During the test phase, specific equities exhibited higher MAPE values. For instance, "ALB" exhibited a greater MAPE of 2.3815, indicating a greater average percentage deviation between predicted and actual values for this stock. Similarly, "MKC," "ROP," and "SWK" displayed elevated MAPE values.

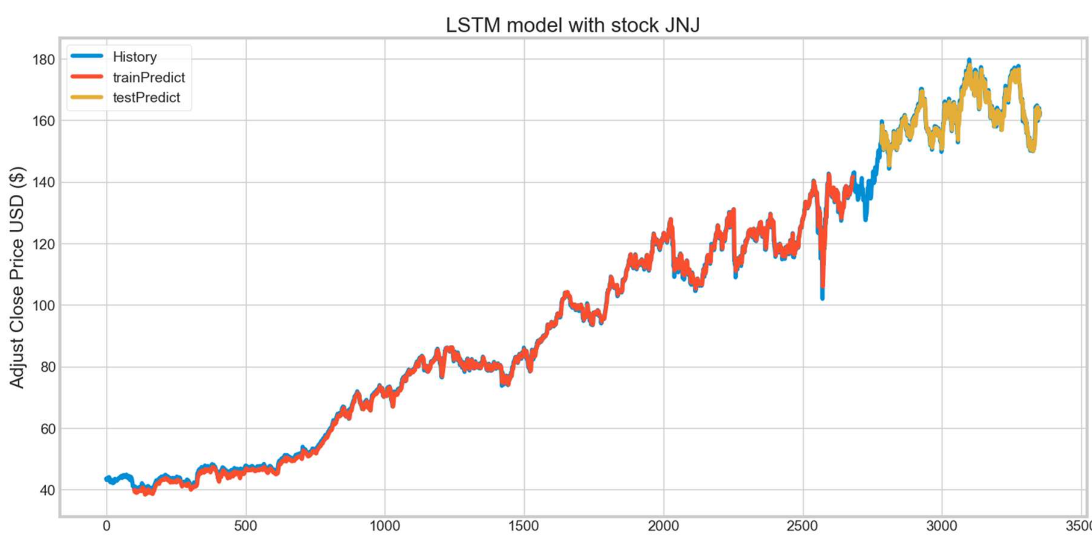
Some stocks exhibited relatively low MAPE values during the test phase, indicating accurate predictions with minimal percentage deviations, whereas other stocks exhibited higher MAPE values.

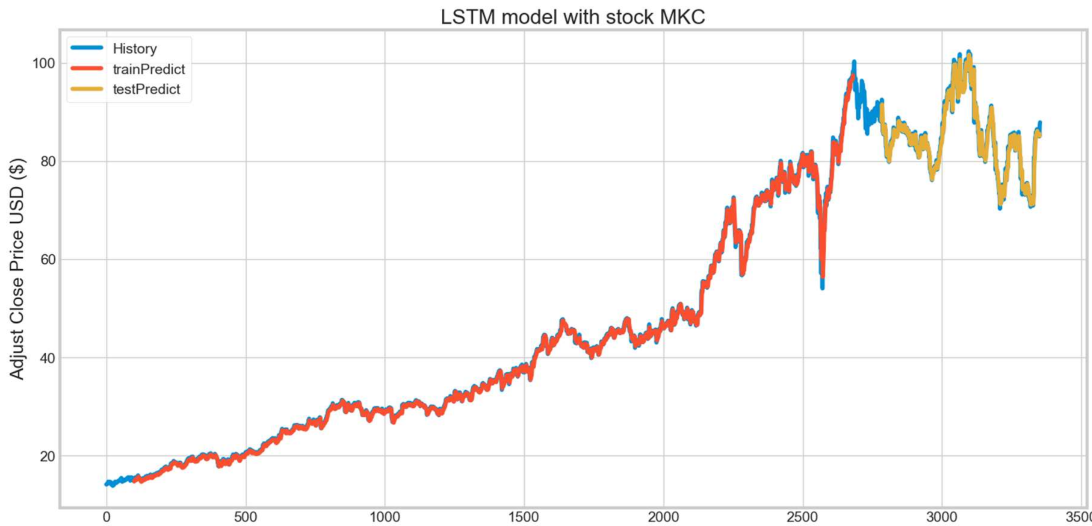
The subsequent sequence of charts will depict the functioning of the LSTM model in relation to each individual stock symbol:




LSTM model with stock BF-B
History
trainPredict
testPredict
Adjust Close Price USD ($)
80
70
60
50
40
30
20
10
0
500
1000
1500
2000
2500
3000
3500


LSTM model with stock HRL
History
trainPredict
testPredict
Adjust Close Price USD ($)
50
40
30
20
10
0
500
1000
1500
2000
2500
3000
3500


LSTM model with stock JNJ
History
trainPredict
testPredict
Adjust Close Price USD ($)
180
160
140
120
100
80
60
40
0
500
1000
1500
2000
2500
3000
3500


LSTM model with stock MKC
History
trainPredict
testPredict
Adjust Close Price USD ($)
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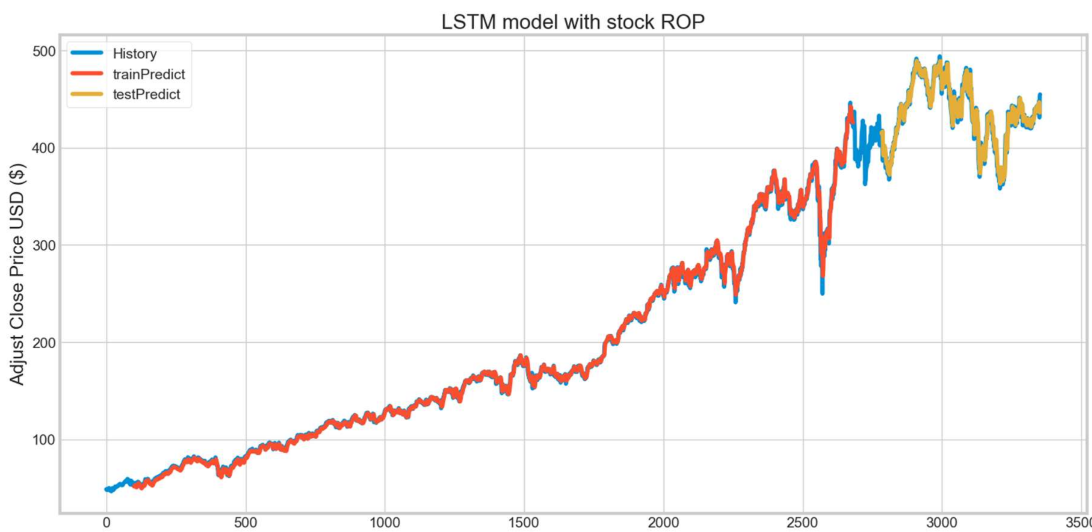
40
60
80
100
0
500
1000
1500
2000
2500
3000
3500


LSTM model with stock ROP
History
trainPredict
testPredict
Adjust Close Price USD ($)
100
200
300
400
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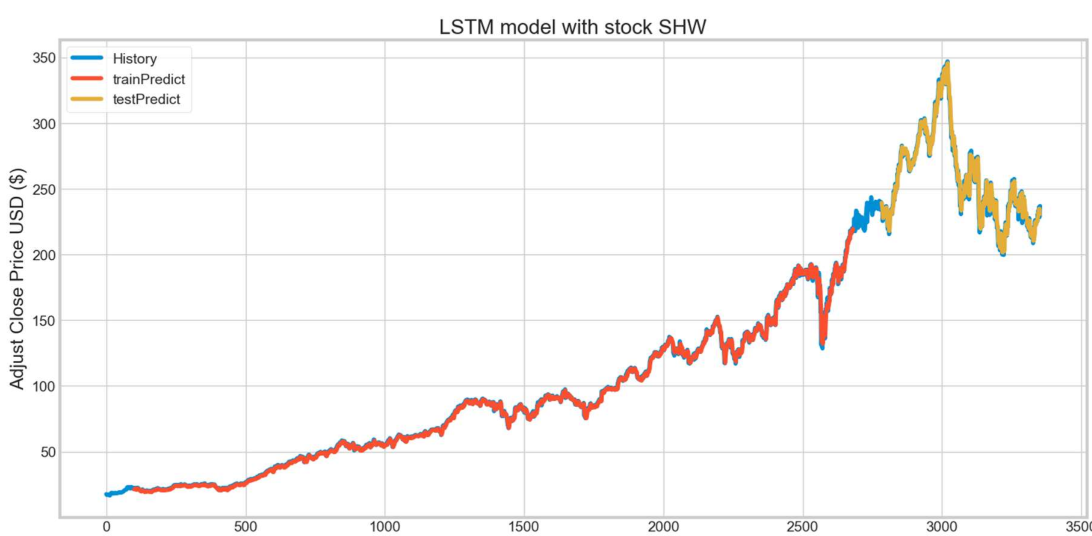
3500


LSTM model with stock SHW
History
trainPredict
testPredict
Adjust Close Price USD ($)
50
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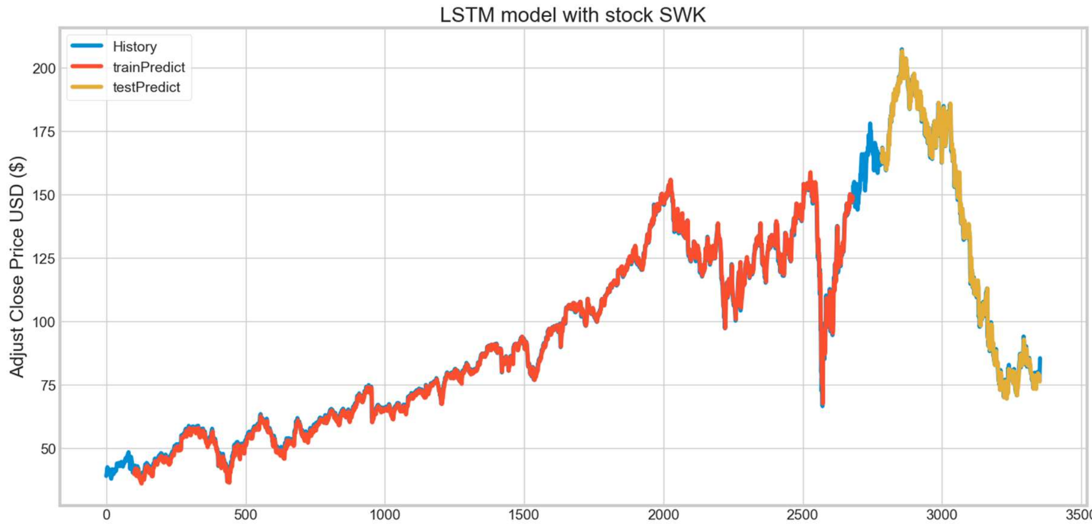

Figure 12. LSTM model chart series with nine stocks

5.3 Portfolio management with predicted data

In order to test the effectiveness of long-term portfolio management by forecasting results, this study assumes that investors start building a portfolio on January 28, 2021 (referred to as starting point P0). The next period (January 28, 2021 to April 30, 2023) is a research time frame that looks at portfolio management on both historical and predict data, and is divided into 9 rebalancing periods. Each period is separated by 63 trading days which equates to a portfolio management that will be rebalanced quarterly. The rebalancing period is a necessary activity in long-term portfolio management to refers to the practice of periodically adjusting the asset allocation of a portfolio back to its original target or desired allocation (Yesim et al. 2007). The use of rebalancing periods in the establishment of the six portfolios also helps to highlight the results of the comparison.

Table 6. Rebalancing periods

Rebalancing period	Date	Days
P0	28/01/2021	-
P1	to 28/04/2021	63
P2	to 28/07/2021	126
P3	to 26/10/2021	189
P4	to 26/01/2022	252
P5	to 27/04/2022	315
P6	to 28/07/2022	378
P7	to 26/10/2022	441
P8	to 27/01/2023	504
P9	to 28/04/2023	567

5.3.1 Portfolio optimization based on historical data (on 28.01.2021)

The MVO method is once again applied, following a similar approach as presented in section 5.1, but utilizing data from January 1, 2010, to January 27, 2021. Figure 13 below provides an overview of the performance of the identified portfolios that surpass the market index in terms of information content. Among these portfolios, the one with the highest Sharpe Ratio and Annualized Return, denoted as the Sharpe Ratio maximum portfolio, continues to outperform the other two portfolios and the benchmark market. It achieves a Sharpe Ratio of 1.229 and an Annualized Return of 22.16%. The Sharpe Ratio of the Minimum Volatility Portfolio and equally-weighted portfolio are quite similar, with values of 1.060 and 1.049, respectively. However, the equally-weighted portfolio exhibits a higher annualized return of 20.06%, compared to the Minimum Volatility Portfolio, which achieves an annualized return of 17.20%.

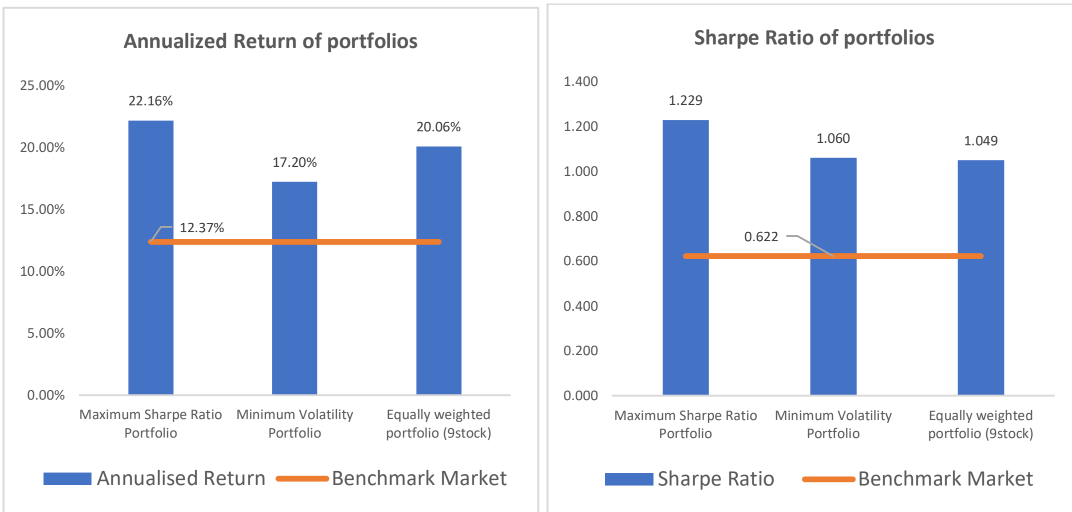

Figure 13. Comparing MVO Portfolios with benchmark market at time P0

The presented information in Figure 14 reveals the asset allocation for two portfolios: the Maximum Sharpe Ratio Portfolio and the Minimum Volatility Portfolio. The Maximum Sharpe Ratio Portfolio encompasses the assets AOS, BF-B, HRL, MKC, ROP, and SHW, assigned allocation weights of 10.7511%, 4.0183%, 27.4649%, 15.8581%, 7.7328%, and 34.1749% respectively. In contrast, the Minimum Volatility Portfolio comprises the assets AOS, BF-B, HRL, JNJ, MKC, and SHW, with allocation weights of 4.589%, 5.2034%, 22.4605%, 44.907%, 13.1675%, and 9.6726% respectively.

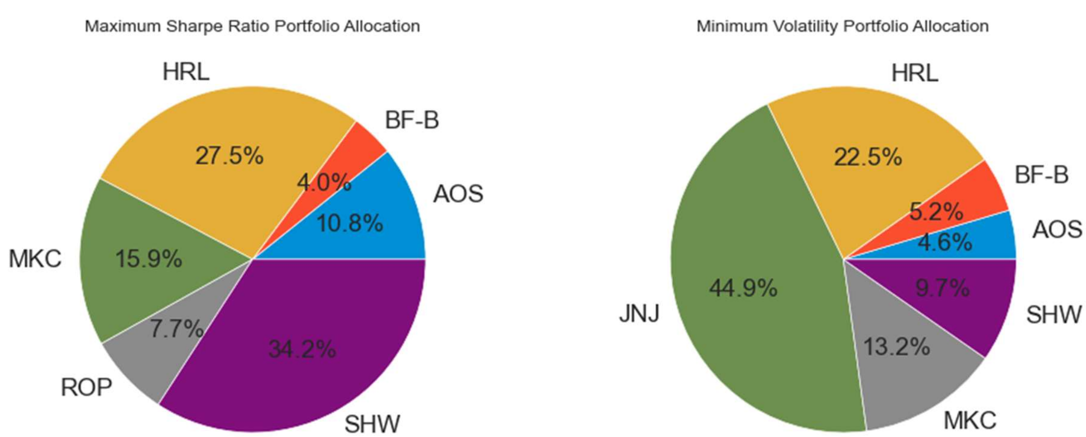

Figure 14. Portfolio Allocation at starting point P0

5.3.2 Portfolio managements with predicted data

As the result of section 5.3.1 can see the Maximum Sharpe Ratio portfolio is the outstanding performance portpolio both in Sharpe Ratio and Annualized returns at starting point P0. The study proposes selecting the investment weight based on the Maximum Sharpe Ratio Portfolio from the investors' perspective at P0. For the subsequent timeframe (corresponding to the rebalancing period from January 28, 2021, to April 28, 2023), the study considers three investment options for implementation. The first option is to maintain the investment without altering the portfolio weight at time P0, referred to as the "Unchanged allocation at P0_historical" portfolio. The second option is a balanced weight investment with six stock, denoted as the "Equally_historical" portfolio. The third option is to continue using the Maximum Sharpe Ratio portfolio but with weight adjustments during each rebalancing period, known as the "Max_Sharpe_historical" portfolio. These three portfolio options are constructed based on actual historical data. To evaluate the effectiveness of portfolio management through result preditction, the study uses the Maximum Sharpe Ratio portfolio with ratio adjustments for each rebalancing period using forecast data from the same period. This portfolio is referred to as the "Max_Sharpe_predict" portfolio. In addition, the study uses forecast data to find the results for the investment option assuming that maintaining the investment without changing the portfolio weight at time P0, is called portfolio "Allocation unchanged at P0_predict"; and a balanced weighted investment of six stocks, denoted "Equally_predict" portfolio.

The study also investigated the performance of the Minimum Volatility portfolio. However, the findings indicate that this portfolio did not demonstrate any advantages in terms of both Sharpe Ratio and Annualized Return, irrespective of whether the data sets used were actual historical or predicted. Specific metric information presented graphically can be found in the Appendix.

The following Figure 15 presents the values of the Sharpe Ratio for six different portfolios mentioned above for each portfolio across nine specific rebalancing periods, denoted as P1 to P9. Each period (P1 to P9) represents a distinct time interval during which the portfolios are rebalanced according to maximum Sharpe ratio strategies.

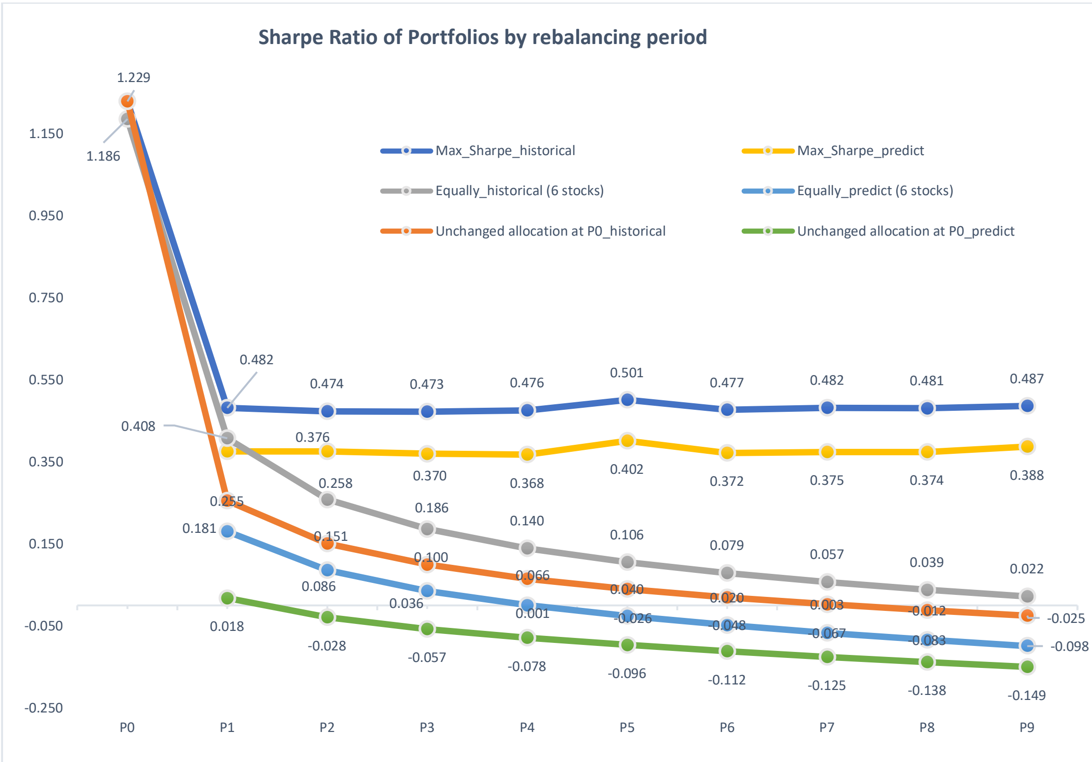

Figure 15. Sharpe Ratio of Portfolios by rebalancing periods

Based on the analysis depicted in Figure 15, it is notable that the Max_Sharpe_historical portfolio consistently exhibits superior Sharpe Ratio values throughout all rebalancing periods. This phenomenon is characterized by a sustained risk-adjusted outperformance, as supported by empirical historical data. In contrast, when using historical data, the Equally_historical portfolio, which employs an equal weighting strategy, and the "Unchanged allocation at P0_historical" portfolio exhibit lower Sharpe Ratio values than the Max_Sharpe_historical and a downward trend.

The Max_Sharpe_predict portfolio, created by combining predictive data and the MVO method, yields a Sharpe ratio value lower than the Max_Sharpe_historical portfolio, but higher than both the 'Equally_historical' and 'Unchanged allocation at P0_historical' portfolios. At the same time, it also gives better Sharpe ratios than the "Equally_predict" portfolio and the "Unchanged allocation at P0_predict" portfolio.

Based on the value of the Sharpe index, it is shown that, when forecasting data is found, the use of a portfolio built by the MVO method and adjusted for the rebalancing periods on the forecast data will perform better than maintaining a fixed allocation constant (equal or proportional at time P0). Moreover, the "Allocation unchanged at P0_predict" portfolio exhibits the lowest performance, as evidenced by its Sharpe ratio. This observation implies that utilizing forecast data in combination with the MVO model without performing phased rebalancing carries inherent risks.

Examining the graph (Figure 16) depicting the difference in Sharpe ratio between the other portfolios and the Maximum Sharpe Ratio portfolio based on historical data (Max_Sharpe_historical). Each legend in the graph illustrates the disparity in the Sharpe Ratio values among portfolios. These portfolios are denoted as "Max_Sharpe_historical" (referred to as MS_hist), "Equally_historical (6 stocks)" (denoted as Equal), "Max_Sharpe_predict" (denoted as MS_pred), and "Unchanged allocation at P0_historical" (referred to as Unchange). The "MS_hist-Equal" legend depicts the difference between the Sharpe Ratio value of the "Max_Sharpe_historical" portfolio and the "Equally_historical (6 stocks)" portfolio. On the other hand, the "MS_hist-MS_pred" legend illustrates the difference in Sharpe Ratio between the "Max_Sharpe_historical" portfolio and the "Max_Sharpe_predict" portfolio. Lastly, the "MS_hist-Unchange" legend showcases the discrepancy in Sharpe Ratio between the "Max_Sharpe_historical" portfolio and the "Unchanged allocation at P0_historical" portfolio.

It is evident that the application of predicted data to the Maximum Sharpe Ratio portfolio results in a consistently smaller disparity in Sharpe Ratio values compared to the other two portfolios. This disparity is effectively managed through regular rebalancing intervals, ranging from approximately 0.08 to 0.11 units. In contrast, the difference between the "Max_Sharpe_historical" portfolio and the "Equally_historical (6 stocks)" portfolio shows a gradual increase from 0.07 to 0.46, and the difference between the "Max_Sharpe_historical" portfolio and the "Unchanged allocation at P0_historical" portfolio also gradually increases from 0.227 to 0.507.

The "Max_Sharpe_historical" portfolio undoubtedly exhibits the highest Sharpe ratio across all rebalancing periods. However, the consistent and well-managed spreads between the "Max_Sharpe_historical" and "Max_Sharpe_predict" portfolios suggest that predicted data can lead to similar trends as observed with actual historical data. This observation implies that employing predicted data in constructing a Maximum Sharpe Ratio portfolio can offer advantages over maintaining a balanced portfolio or adhering to an unchanged allocation strategy throughout the rebalancing periods.

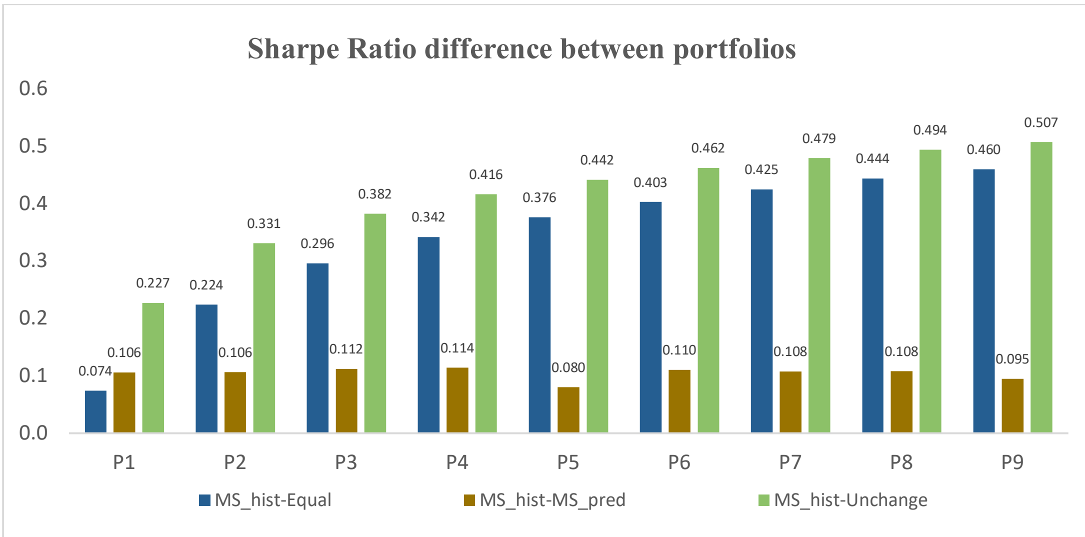

Figure 16. Sharpe Ratio's difference between portfolios compared with Max_Sharp_historical portfolio

Continuing from the analysis of the Sharpe Ratio, the subsequent Figure 17 presents the annualized returns for the six portfolios. Examining the annualized returns across the nine rebalancing periods (P1 to P9), this study observed that the Equally_historical portfolio has a higher annualized returns compared to all other portfolios in rebalancing period P1, this advantage is short-lived and quickly diminishes starting from rebalancing period P2. And after that, the Equally_historical portfolio underperformed compared to the other two Maximum Sharpe Ratio portfolios in most of the other rebalancing periods.

The Maximum Sharpe Ratio portfolios tend to exhibit higher annualized returns than the equal weight portfolio and Unchange allocation at P0 portfolio across most rebalancing periods. Commencing from period P2 and onwards, the "Max_Sharpe_historical" portfolio maintains its lead, demonstrating annual yields within the range of 12.63% to 14.54%. Subsequently, the "Max_Sharpe_predict" portfolio follows with annual returns ranging from 7.94% to as high as 9.51%. The chart shows that the movements in these two portfolios are relatively insignificant, exhibiting similar sideways trends, suggesting that the application of forecast data in portfolio construction Max_Sharpe_predict can help predict the trend of annualized returns to be similar to those obtained from historical data.

On the contrary, the remaining four portfolios all exhibit a decreasing trend across the rebalancing periods. Specifically, the "Allocation unchanged at P0_predict" portfolio displays the lowest annualized return, declining from 1.97% in period P1 to a mere 0.22% in period P9. Once again, this outcome underscores the inefficiency of utilizing forecasting data without making adjustments to the stock allocation ratio in the portfolio throughout the rebalancing periods.

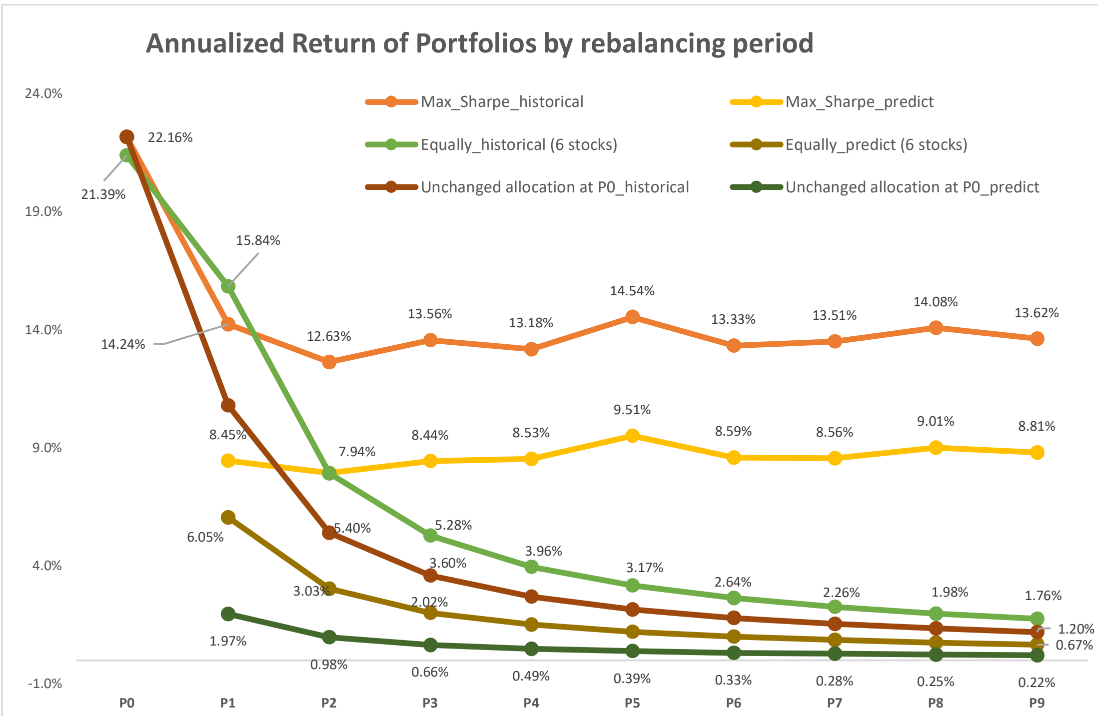

Figure 17. Annualized Return of Portfolios by rebalancing periods

5.3.3 Maximum Sharpe Ratio Portfolio Allocation by Rebalancing Periods

In the portfolio management progress, the study considers the allocation of stocks within the forecast value-based portfolios and compares it to the actual historical value-based portfolios, focusing on the Maximum Sharpe ratio portfolio. Notably, the findings reveal no difference in stock allocation between portfolios using actual historical data and portfolios using forecast data. This suggests that the predicted data accurately captures the underlying dynamics of the market and enables the construction of a portfolio with a similar asset allocation to the actual data-based portfolio.

Although there is no difference in the portfolio distribution between the forecast and the actual, it is essential to note that the allocation of stocks within the portfolio fluctuates across the rebalancing periods, as shown in the following Table 7. The table provides a detailed overview of the stock distribution within the portfolio across different rebalancing periods, offering valuable insights into the changing composition of the portfolio over time.

Table 7. Portfolio Allocation by Rebalancing Period

Rebalancing Period	AOS	BF-B	HRL	MKC	ROP	SHW
P0	10.8%	4.0%	27.5%	15.9%	7.7%	34.2%
P1	19.4%	15.1%	11.2%	16.5%	22.2%	15.2%
P2	13.2%	1.3%	37.1%	1.6%	19.9%	26.7%
P3	33.4%	22.9%	15.2%	16.5%	9.9%	2.2%
P4	15.4%	9.9%	25.7%	7.6%	27.0%	14.3%
P5	12.7%	27.3%	21.2%	24.6%	14.1%	0.1%
P6	5.7%	4.3%	26.8%	19.8%	18.4%	25.0%
P7	18.7%	15.5%	12.9%	28.1%	24.8%	0.1%
P8	18.7%	14.4%	15.6%	23.2%	22.6%	5.5%
P9	2.8%	29.1%	13.9%	25.4%	19.7%	9.1%

Each period exhibits varying weightings for the individual stocks, reflecting market conditions changes and the specific securities' performance. This information emphasizes the dynamic nature of portfolio management and the need for periodic rebalancing to maintain the desired asset allocation.

Throughout the various rebalancing periods, a consistent pattern emerges regarding the number of securities within the portfolio with an allocation greater than 16.67%. This proportion represents the expected allocation for each stock in an equal-weight portfolio. Notably, in specific rebalancing periods, except P1 and P8, there is an occurrence of securities that account for more than 25% of the portfolio's allocation. This implies a more concentrated distribution where fewer stocks hold a substantial proportion of the portfolio. These stocks surpassing the 25% threshold indicate periods of potentially higher concentration or overweighting in specific stocks. Such concentration can arise from various factors, including market dynamics or performance trends.

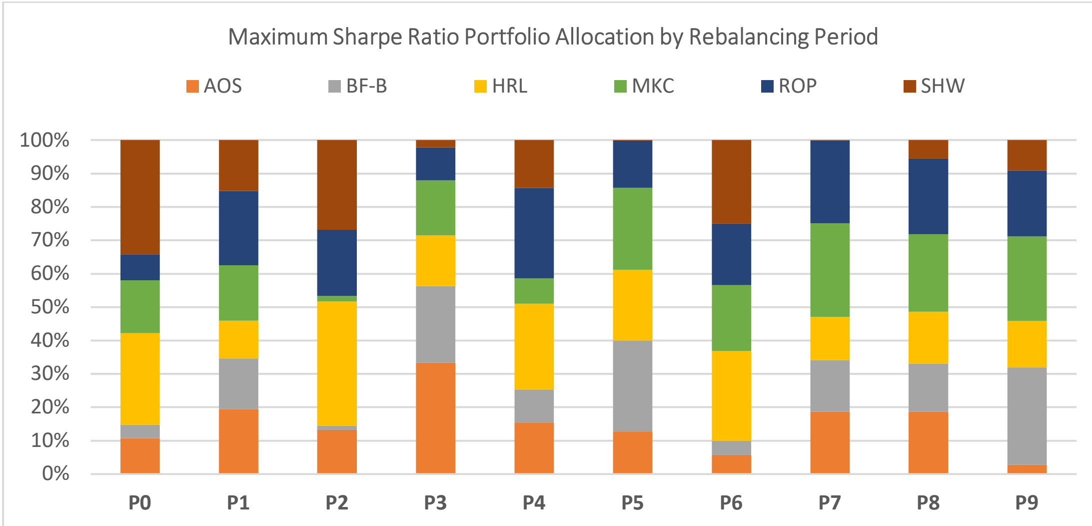

Figure 18. Maximum Sharpe Ratio Portfolio Allocation by Rebalancing Periods

Understanding these fluctuations in the number of securities and their corresponding allocations provides insights into the portfolio's composition and the potential impact of certain stocks on their performance. It emphasizes the need for careful monitoring and rebalancing to maintain a desired level of diversification and mitigate potential risks associated with a concentration in a few stocks. However, it is necessary to reiterate that this study does not consider transaction costs in buying-selling to adjust the proportion of securities allocated in the portfolio.

6 Conclusions and Discussions

This study was motivated by a curiosity to predict an optimal portfolio through scientific analysis. The methodology employed in this study involved an integration of the Long Short-Term Memory (LSTM) model for forecasting and the Mean-Variability Optimization (MVO) approach for constructing an optimal portfolio based on the forecast outcomes. Considering the author's inexperience with financial investments, the author concentrated on the nine securities in the article "9 of the Best Stocks for a Starter Portfolio." To ensure a comprehensive analysis, a dataset encompassing a collection period of more than twelve years was utilized to facilitate this research.

6.1 Answer the research questions

The study successfully develops performance LSTM models for nine securities, achieving MAPE values below 2.5% during both the training and testing phases. These low MAPE values indicate the models' accurate forecasting ability, aligning closely with the market's direction. Investors can confidently rely on these LSTM models to make well-informed investment decisions and identify market trends for optimizing their portfolios. This strategic approach improves investment returns by utilizing accurate forecasts and staying ahead of market movements to seize emerging opportunities, ultimately enhancing overall investment performance.

Upon comparing an optimized portfolio constructed using the MVO method with both an equally weighted stock portfolio and market benchmarks, it becomes evident that the Maximum Sharpe Ratio (MSR) Portfolio outperforms in terms of both Sharpe Ratio and Annualized Return, based on historical and predictive data. This outcome highlights the appeal of utilizing the Maximum Sharpe Ratio portfolio as an attractive investment option for investors. Furthermore, it indicates that the MVO method effectively identifies an allocation that offers superior risk-adjusted returns when compared to an equally weighted stock portfolio and market benchmarks. It is important to note that these portfolios have distinct objectives and, consequently, exhibit different performance characteristics aligned with their specific goals and expectations. Each portfolio is designed to cater to the preferences and requirements of individual investors, leading to variations in their risk-return profiles and overall performance. Therefore, when evaluating the performance of these portfolios, investors should consider their specific investment goals and preferences. In the context of this study, which utilizes the Sharpe Ratio as a measure of portfolio performance, it is advisable for investors to consider the MSR Portfolio as it aims to strike an optimal balance between risk and return, potentially enhancing overall investment performance.

Although the individual results of stock price forecasting by the LSTM model and portfolio optimization by the MVO method demonstrate positive outcomes, their combination in this study does not exhibit effective portfolio management as initially anticipated. Several insights emerge from comparing portfolios reliant on predicted and actual data. The study examines six portfolios: three based on historical data, namely the Maximum Sharpe Ratio portfolio, the Equally weighted portfolio, and a portfolio with an unchanged allocation (P0); and three analogous portfolios constructed using predicted data. Admittedly, the performance of portfolios using predicted data yields inferior results compared to those using historical data, both in terms of the Sharpe ratio and annualized returns. Depending on individual investor strategies, this outcome may represent a potential risk when utilizing this result to make investment decisions. For instance, if an investor sets an expectation for the Sharpe ratio and the annual return of their portfolio, but the result obtained from the portfolio using forecasted data is lower, the investor may consider abandoning their current portfolio, resulting in the loss of their actual productive portfolio.

However, the study reveals the practical value of managing a portfolio with allocation adjustments across rebalancing periods. When applying predicted data to the Maximum Sharpe Ratio (MSR) portfolio, the discrepancy in Sharpe Ratio values compared to the MSR portfolio based on actual historical data consistently remains smaller than in the other portfolios. This disparity is effectively managed through periodic rebalancing, highlighting the potential of utilizing predicted data to achieve comparable performance trends as observed with actual historical data in portfolio management.

Moreover, the Maximum Sharpe Ratio portfolios consistently demonstrate higher annualized returns than the Equally Weighted and unchanged allocation (P0) portfolios throughout the rebalancing period. This suggests that investors can derive benefits from incorporating forecasted data to construct more effective portfolios, surpassing the performance of maintaining an unchanged allocation or investing in an equally weighted portfolio.

Additionally, the study highlights the portfolio's performance with the unchanged allocation (P0) when using forecasted data, revealing its inferiority in both the Sharpe ratio and annualized returns, with a downward trend over time during which the portfolio remains unchanged. In light of these findings, the study proposes that investors consider using the value obtained from this portfolio as a performance threshold. Such an approach would enable them to make appropriate adjustments in the portfolio management process based on this benchmark.

The study's finding reveals no significant difference in stock allocation between portfolios constructed based on forecasted and actual historical data, specifically in the Maximum Sharpe Ratio Portfolio Allocation. This implies that investors can confidently utilize the portfolio with stock allocation derived from predicted data in real-world conditions. However, it is important to note that the allocation of the Maximum Sharpe Ratio portfolio undergoes substantial variations and fluctuations over different rebalancing periods. This highlights the dynamic nature of portfolio management, where adjustments are necessary to maintain diversification and effectively manage the risks associated with concentrated holdings. It is imperative for investors to closely monitor the composition of the portfolio across various periods, carefully examining the factors that drive its performance. This will enable them to make well-informed decisions regarding portfolio management and ensure alignment with their investment objectives.

The final aspect addressed in this study pertains to the number of securities utilized in constructing the portfolio. While the original idea proposed incorporating 9 different types of stocks, applying the MVO method to achieve an optimal portfolio has resulted in a subset of securities with fewer stocks. Specifically, the Maximum Sharpe Ratio portfolio includes only 5 securities (based on April 30, 2023) or 6 securities (based on January 1, 2021). Similarly, the Minimum Volatility portfolio consists of only 7 securities (based on April 30, 2023) or 6 securities (based on January 1, 2021). This finding highlights the importance of employing analytical techniques to determine the appropriate number of stocks in a portfolio, whether the goal is to maximize the Sharpe ratio or minimize volatility (risk). It emphasizes the need to move beyond a discretionary approach of investing in all 9 securities as suggested in the article. From there, investors can enhance the effectiveness and efficiency of their investment strategies.

6.2 Limitations and Discussions

Despite extensive efforts to conduct a comprehensive study on stock portfolio forecasting, this research has limitations that necessitate further exploration and development in future investigations. Firstly, it is essential to acknowledge that the forecasting model employed in this study solely relies on historical data of securities, neglecting other micro and macro factors specific to the stock market and the global economic environment. The portfolio construction procedure also does not account for various influential factors, including transaction costs, liquidity fees, and other relevant costs. These costs directly impact investors' returns, consequently influencing investment decisions and portfolio rebalancing.

Additionally, the selection criteria for this study were primarily based on recommendations from an article highlighting nine stocks deemed suitable for beginners. While this approach provided a foundation for portfolio selection, it inadvertently introduced an element of unfairness when comparing these portfolios with the S&P 500 index. Furthermore, the nine selected stocks were portrayed as having stable growth over time, potentially influencing the outcomes of the forecast model due to their consistent uptrend. The findings might need to be interpreted more cautiously, but they contribute to understanding beginner-oriented portfolio performance and investment choices for novice investors. This choice's strengths and weaknesses ensure a balanced and informative analysis for academia and practical decision-making.

This study is constructing the predictive model that divides the data into two distinct parts: training and testing sets. Although this partitioning approach was adopted in the two mentioned studies (as discussed in the previous section), there is a compelling argument in favor of adopting a three-part division comprising training, validation, and testing sets. Embracing this tripartite division could offer improved model validation and enhanced accuracy in future research endeavors concerning this subject. Therefore, considering the inclusion of a validation set may prove beneficial for ensuring the robustness and generalizability of the predictive model if further investigations are pursued.

Utilizing the Long Short-Term Memory (LSTM) model in this study while producing favorable predictive outcomes presents implementation challenges and limitations regarding universal applicability. Deep learning approaches, including the LSTM model, require significant computational resources with substantial processing capabilities. As highlighted previously, constructing the LSTM model necessitates two critical factors: determining the optimal batch size and the number of epochs, which may exceed the computing power available within the researcher's current system. This study adopted hyperparameters previously identified as effective in academic studies, but further research should explore optimal hyperparameter search techniques for the LSTM model.

Furthermore, the testing and modeling phases involve a significant time investment. On average, each iteration of testing the LSTM model for a specific stock, combined with trend analysis, requires approximately 40 minutes to identify the best-performing approach. To solve this problem, Hua and Zhao, and their colleagues (2019) proposed the novel model RCLSTM, which shortens the LSTM model's execution time. This alternative warrants consideration for future research in this domain.

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APPENDICES

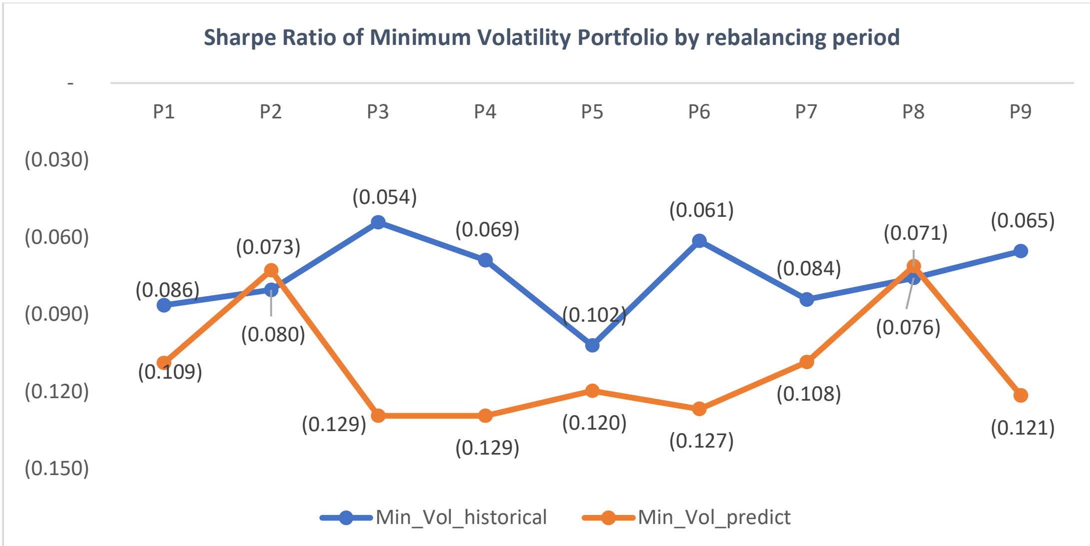
Appendix 1. Minimum Volatility Portfolio Allocation based on actual historical data

Rebalancing Period	AOS	BF-B	HRL	MKC	ROP	SHW
P1	3.7%	19.6%	40.3%	7.4%	24.3%	4.6%
P2	3.5%	14.7%	42.6%	6.9%	26.3%	6.0%
P3	6.4%	14.7%	40.2%	11.6%	24.9%	2.1%
P4	3.5%	12.6%	40.9%	11.1%	28.7%	3.2%
P5	2.5%	17.6%	41.7%	10.5%	26.0%	7.4%
P6	5.8%	12.8%	42.1%	10.6%	25.5%	3.3%
P7	5.3%	17.2%	41.3%	12.9%	23.1%	0.1%
P8	3.6%	15.9%	40.3%	10.9%	28.9%	0.4%
P9	5.6%	17.0%	40.7%	8.7%	25.7%	2.2%

Appendix 2. Minimum Volatility Portfolio Allocation based on predicted data

Rebalancing Period	AOS	BF-B	HRL	MKC	ROP	SHW
P1	1.7%	8.5%	29.5%	15.0%	44.6%	0.7%
P2	2.5%	2.6%	27.1%	18.2%	49.0%	0.5%
P3	0.0%	6.6%	31.8%	14.8%	44.5%	2.3%
P4	1.4%	2.4%	33.2%	18.1%	38.5%	6.5%
P5	0.1%	3.8%	28.4%	21.4%	45.0%	1.3%
P6	0.1%	6.0%	28.7%	18.8%	42.7%	3.7%
P7	1.2%	4.6%	33.3%	13.0%	47.3%	0.7%
P8	3.7%	3.8%	28.3%	15.7%	46.1%	2.5%
P9	0.0%	7.1%	32.7%	10.5%	46.7%	3.0%

Appendix 3. Sharpe Ratio of Minimum Volatility Portfolio by rebalancing periods



Appendix 4. Annulized Returns of Minimum Volatility Portfolio by rebalancing periods