DESIGN OF SHELL STRUCTURES FOR BUCKLING ACCORDING TO EN 1993-1-6 AND COMPARISON TO THE SECOND-GENERATION OF EUROCODE

Lappeenranta–Lahti University of Technology LUT

Master’s Programme in Mechanical Engineering, Master's thesis

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Markus Jääskeläinen

Examiners: Professor Timo Björk

Jimi Pulkka, M.Sc. (Tech.)

ABSTRACT

Lappeenranta–Lahti University of Technology LUT

LUT School of Energy Systems

Mechanical Engineering

Markus Jääskeläinen

Design of shell structures for buckling according to EN 1993-1-6 and comparison to the second-generation of Eurocode

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Keywords: buckling, steel structures, shell structures, EN 1993-1-6, EN 1993-1-14, Eurocode, LBA, GMNIA

The goal of the thesis was to create a way to calculate shell structure buckling strength according to the EN 1993-1-6 standard. The second goal was to compare the current standard to the upcoming second generation of Eurocode. Both hand calculations using Excel and Finite Element Method (FEM) using Ansys were used.

Both versions of the standard were compared. The primary changes to the second-generation standard were in the parameters that represent nonlinearity and imperfections. The goal of clarifying the Eurocode can be seen in the text of the new version.

Calculation model, with the same geometric values as the Excel calculations, was done in Ansys for FEM calculations. The results of FEM calculations were compared to the Excel calculation results.

The changes in the parameters in the second-generation Eurocode result in slightly lower utilization for medium length cylinders. Linear eigenvalue analysis using FEM results were in line with expectations. Geometrically and materially nonlinear analysis with imperfections resulted slightly lower utilization.

TIIVISTELMÄ

Lappeenrannan–Lahden teknillinen yliopisto LUT

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Konetekniikka

Markus Jääskeläinen

Kuorirakenteiden lommahdusmitoitus EN 1993-1-6 standardin mukaan ja vertailu toisen sukupolven Eurokoodiin

Konetekniikan diplomityö

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Diplomityön tavoitteena oli luoda tapa laskea kuorirakenteiden lommahduskestävyyttä EN 1993-1-6 standardin mukaan. Toisena tavoitteena on verrata nykyistä standardia tulevaan toisen sukupolven Eurokoodiin. Laskennassa käytettiin käsikaavoja Excelillä sekä elementtimenetelmä (Finite Element Method, FEM) analyysia Ansyksellä.

Molempia standardin versioita verrattiin. Suurimmat muutokset toisen sukupolven standardiin liittyivät parametreihin, joilla edustetaan epälineaarisuutta ja valmistusvirheitä. Tavoite selkeyttää Eurokoodin tekstiä on nähtävissä standardin uudessa versiossa.

Laskentamalli luotiin Ansykseen FEM laskentaa varten käyttämällä samoja geometrian arvoja kuin Excel laskennassa. FEM laskennan tuloksia verrattiin Excel laskennan tuloksiin.

Muutokset parametreihin toisen sukupolven Eurokoodissa johtavat lievästi alhaisempiin käyttöasteisiin keskipitkillä lieriöillä. FEM:llä suoritetun ominaisarvoanalyysin tulokset olivat odotuksien mukaiset. Geometrisesti and materiaalisesti epälineaarinen analyysi sisältäen valmistusvirheet johti hieman pienempiin käyttöasteisiin.

SYMBOLS AND ABBREVIATIONS

Roman characters

E	elastic modulus	[MPa]
f	strength	[MPa]
l	shell length	[mm]
l_g	gauge length	[mm]
r	shell mid surface radius	[mm]
t	shell thickness	[mm]

Greek characters

σ	stress	[MPa]
τ	shear stress	[MPa]

Dimensionless quantities

C	buckling strength coefficient
n_i	tolerance level multiplier
Q	fabrication tolerance quality parameter
r_{Rcr}	critical buckling resistance ratio
U_{n1}	dimple imperfection amplitude parameters
U_{n2}	dimple imperfection amplitude parameters
v	Poisson's ratio
$w_{0,eq}$	maximum deviation of imperfection
w_k	characteristic imperfection amplitude

α	elastic imperfection reduction factor
β	plastic range factor
γ	partial factor
η	interaction exponent
$\bar{\lambda}$	relative slenderness
$\bar{\lambda}_0$	squash limit relative slenderness
$\bar{\lambda}_p$	plastic limit relative slenderness
χ	buckling reduction factor
ω	shell length parameter

Subscripts

Ed	design value
M1	instability partial factor
Rcr	critical resistance
Rd	design resistance
Rk	characteristic resistance
yk	characteristic yield
x	axial
θ	circumferential
τ	shear

Abbreviations

BC	Boundary Condition
EC	Eurocode

FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
GMNA	Geometrically and Materially Nonlinear Analysis
GMNIA	Geometrically and Materially Nonlinear Analysis with Imperfections
LA	Linear Analysis
LBA	Linear Buckling Analysis
MNA	Materially Nonlinear Analysis

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1 Introduction

This master's thesis will cover stability analysis for structures according to Eurocode (EC). Specifically, buckling that occurs in shell structures, meaning that the EN 1993-1-6 standard will be the primary focus. The goal is to create a calculation tool that can be used in the buckling strength analysis of shells when designing according to EC. The result should allow quick and easy analysis to be performed for shell types that are supported according to the standard. The second main aspect of this thesis is to study the upcoming second generation of the Eurocode standard. The goal is to compare the upcoming changes to the standard and prepare a version of the calculation tool that is done according to the new version. This can then be used to compare how the results of the second generation of EC are different from the version that is currently in use. At the time of this thesis the approval process of the second generation of the standards is still on going, but preliminary versions of the standards are available. The EN 1993-1-6 standard includes basic hand calculations for simple structures. It also has methods and guidelines for the use of Finite Element Method (FEM). The calculation tool should therefore support the use of both. FEM will be applied in the verification of the results in this thesis. The second generation of EC includes the brand new EN 1993-1-14 standard. This standard offers general guidelines and working practises for Finite Element Analysis (FEA). The instructions given in EN 1993-1-14 will be studied and implemented during the FEM portion of this thesis. At the time of writing this standard is at a later stage of the approval process. Preliminary version of the standard was available and was used in the making of this thesis.

The calculation tool and the findings of the research will be created for Insinor Oy. The company specializes in design and strength analysis. Projects that involve the analysis of shell structures are among the most common types. Therefore, there is a great need to verify structures for buckling. Additionally, the study of the second-generation EC should prepare the company better for when these new standards become official. The tool will be created in Excel. It is a program that is familiar to most people and flexible enough to fit every need. The primary function of the tool should be, that after inputting the initial data, the result is the utilization percentage of the shell. In addition to being able to calculate the buckling resistance of simple shells through hand calculations, a standard is needed for the post processing of FEA results. Full nonlinear FEA can give a direct result, but these are often

difficult and very time consuming. Linear FEM is much easier to perform but it requires additional post processing for the result to be accurate. These additional calculations need a standard for a proper verification. Therefore, the calculation Excel includes a section where the results of a linear FEA can be input, and the buckling strength can be verified according to EN 1993-1-6. The FEM calculations will be performed using Ansys since it is the primary FEM program that is used by Insinor Oy. The choice of the program should not affect the results. The study of the new EN 1993-1-14 standard could also be used to aid in the general working practises in the company when using FEM.

The overall research problems of this thesis are to pinpoint the main aspects of linear buckling analysis according to EN 1993-1-6 and how can they be utilized in a calculation Excel. The second main problem of the research is the effect of the updates for EN-1993-1-6 standard. The main research questions are the following. *What are the main aspects needed for buckling analysis according to EN-1993-1-6? What are the differences in the second-generation of Eurocode for buckling analysis?*

The thesis will begin with a short introduction to the basic theory of buckling of shell structures. This will establish some of the working practices for shell stability analysis.

This is followed by a close look into the EN 1993-1-6 standard and the rules that it gives. Followed by a look into to some of the changes in the second-generation version of the standard.

After basics, definitions and limitations of the standard have been covered, the main equations needed for Excel are covered. This is followed by the FEM section which goes over the geometry, boundary conditions, meshing and loads in the Finite element (FE) model that is used in the test calculations. In this section, some of the guidelines given by EN 1993-1-14 are used.

The results section shows the results of both the Excel and Ansys calculations. This is then followed by the comparison and analysis of the results. The thesis will finish with conclusions that can be drawn from the results.

2 Methods of research

The primary methods of the research for this thesis are focused on studying the EN 1993-1-6 standard and supporting material. EN 1993-1-6 standard covers other failure modes, but the focus of this thesis will be on buckling. Literature that helps to open the information given in the standards will be utilized. This will help to understand the theory behind stability analysis. The actual equations used in the making to the calculation Excel must be directly from EN 1993-1-6.

2.1 Basis for buckling analysis

Buckling is challenging phenomenon to understand. The reason has to do with how sensitive a shell structure is to geometry, loading pattern, imperfections and boundary conditions during and after buckling (Rotter and Schmidt 2013, 13).

Initial imperfections are especially important due to how sensitive a shell is to them. This is because forces that would otherwise be carried by membrane forces can instead cause bending in the shell reducing the stiffness drastically. Experimental results show that the true buckling strength of a shell can be a fraction of what can be calculated with classical buckling theory. (Samuelson and Eggwertz 2005, 16-17.)

The geometric displacements during buckling can happen in a linear or nonlinear way. These are accounted for with small and large displacement theory respectively. Small displacement theory makes it possible to simplify the analysis at the cost of not being able to replicate the results of a large deflection analysis. Small displacement theory is able find the point where a structure loses stability but may mislead in some results. (Allen 1980, 14-15.)

The main cause for the difference between calculated buckling strength and experimental buckling strength is imperfections caused during manufacturing and fabrication. These imperfections make the shell geometrically nonlinear. Even small defects in manufacturing can cause a significant drop in the shells ability to maintain stability. The type of loading applied to the shell affects how sensitive the shell is to imperfections. Shell being subjected to axial loading is far more sensitive to imperfections than one that is subject to external

pressure. Determining imperfections in shell structure is challenging, because unlike beams and columns, the imperfection form that has the most serious effect on the strength is not simple to determine. (Rotter and Schmidt 2013, 18-19.)

The equations provided in the standards for buckling can be used to verify the design of a shell that has a defined geometry, material, loading and boundary conditions. The analysis can be done using hand calculation, mixed hand and computer calculation and full computer calculation. (Rotter and Schmidt 2013, 4.)

The method of analysis that is chosen depends on a variety of factors. These include the desired accuracy, available information, and time. Hand calculations are simple and quick to perform but are limited in the type of cases that can be solved. The limitations include boundary conditions as well as the geometry of the shell. Numerical analysis using FEM usually takes more time but is overall more flexible.

Loss of stability can occur in different ways. These can be seen in the different possible modes. Buckling can occur locally in specific areas of the shell. Bay buckling is a more global mode that still happens between stiffeners or the ends of the shell. General or global buckling happens through the whole surface of the shell. With long and small diameter shells, the buckling can happen as a column. Figure 1 shows typical buckling modes. (ABS 2022, 56.)

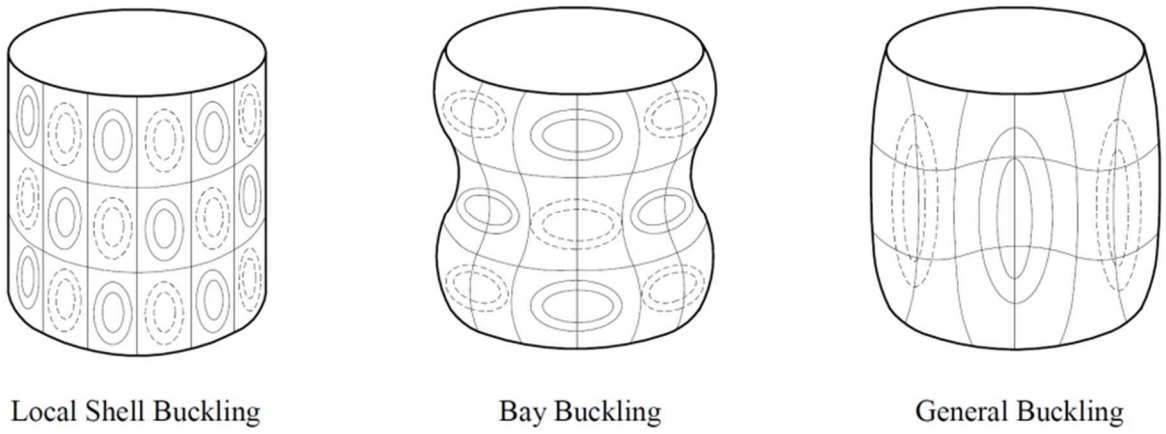


Figure 1. Typical buckling modes (ABS 2022, 57).

Simple hand calculations are mainly useful for assessing global buckling and can only account for local buckling using calculation parameters. This is why the use of FEM is important. Although the severity of local buckling differs based on the structure. Sometimes a small part of the structure will buckle but the overall structure will still maintain stability.

The elastic and plastic behaviour are represented by the relative slenderness. The relative slenderness is defined by the elastic critical buckling stress. The boundary conditions and fabrication quality are used to determine the imperfections and geometric nonlinearity in the shell which are used to acquire the characteristic buckling resistance. This is finally reduced by a partial factor to obtain the final design resistance of the shell. (Rotter and Schmidt 2013, 4.)

The verification for whether the shell will buckle is done by comparing design resistance against the stress resulting from the applied load. If the stress caused by the load is smaller than the design resistance, the design is acceptable. Utilization ratio can also be calculated by dividing the stress caused by the load with the design resistance. This can be used to access if how close the structure is to the limit. This is gives additional information over a binary verification check. Sometimes there may be a desire to leave extra margin. Especially if there is a potential for unexpected loads.

Using fully numerical analysis these steps can be skipped with exception of the partial factor. This is done by representing the imperfections in the geometry and using material and geometric nonlinearity in the analysis. (Rotter and Schmidt 2013, 5.)

Unintended features in the geometry are represented by the elastic imperfection reduction factor α. This factor is not specific to EN 1993-1-6 and the standard allows use of other sources. However, great care should be taken when choosing this factor. The value of α is typically based on experimental data. The imperfection reduction factor considers nonlinearity in the prebuckling path as well as geometric imperfections. The standard offers two primary methods for numerical analysis. First a numerical Linear elastic bifurcation analysis (LBA) which is followed by post processing using hand calculations. Second is a fully numerical analysis with geometric and material nonlinearity. (Rotter and Schmidt 2013, 20.)

2.2 EN 1993-1-6

As addressed previously, there is a discrepancy between buckling strength calculated using simple stability theory and true buckling strength. Since the accurate conditions of the structure that is being analysed is often difficult to determine accurately, calculation standards are needed for buckling analysis.

The current version of EN 1993-1-6 has been valid since 2007 and received a corrigendum in 2009 and an amendment in 2017. The latest addition added several fixes, clarifications and modifications to the standard. EN 1993-1-6 standard covers the strength and stability of shell structures. It therefore includes other failure modes but for this thesis the focus will be on the sections covering stability and the supporting Annexes.

> "The limit state of buckling should be taken as the condition in which all or part of the structure suddenly develops large displacements normal to the shell surface, caused by loss of stability under compressive membrane or shear membrane stresses in the shell wall, leading to inability to sustain any increase in the stress resultants, possibly causing total collapse of the structure." (SFS-EN 1993-1-6 2007, 20.)

The basic process for shell buckling calculation using hand calculations goes as follows. First the elastic critical buckling stress is determined. In EN 1993-1-6 this is determined using methods given in Annex D of the standard. These methods do not cover every case and are limited by the boundary conditions. Hand calculations can be replaced by a numerical LBA using FEM. The applied stress multiplied by the factor given by the eigenvalue analysis results in the elastic critical buckling stress. The imperfection factors are also determined in Annex D. Using these the elastic critical buckling stress is reduced based on the shape of the shell. The parameters account for imperfections and nonlinearities in the shell. These parameters are used to reduce the buckling resistance to better reflect reality.

When determining the strength against buckling, the quality of the construction is a major factor. Execution tolerances should be accounted for during strength assessment (SFS-EN 1993-1-6 2007, 22). Figure 2 shows the directions of the shell surface.

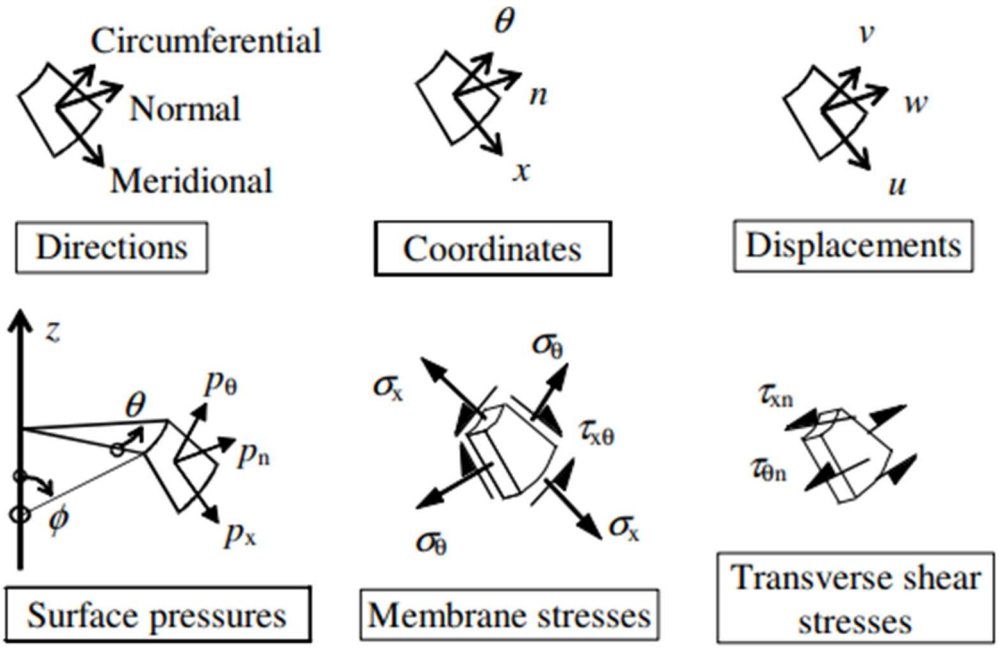


Figure 2. Directions and coordinates of the shell surface (SFS-EN 1993-1-6 2007, 13).

2.2.1 Material assumptions

The principal assumption is EN 1993-1-6 is that the material stress-strain relationship may be treated as ideally elastic-plastic. The hand calculations of the standard are simply based on three material parameters which are Poisson's ration v, Young's Modulus aka Elastic modulus E and yield stress f_{yk}. From these only later two have a major influence on the results. (Rotter and Schmidt 2013, 49.)

In a study conducted by Adam J. Sadowski, J. Michael Rotter, Thomas Reinke and Thomas Ummenhofer released in 2014, the estimated Elastic modulus of a material was shown to vary considerably from the assumed nominal value. The nominal value was 205 000 MPa, but the value of E could change could vary between 149 000 and 317 000 MPa. The coefficient of variation being much higher than what has traditionally been accepted in engineering. The study recognizes the difficulty in measuring the elastic modulus due to any variation in the chemical composition, structure of the crystals and the effects of heat treatment. The stiffness of the testing rig or errors in load application could also be a cause. The results are shown in Figure 3. (Sadowski, Rotter, Reinke and Ummenhofer 2014, 30.)

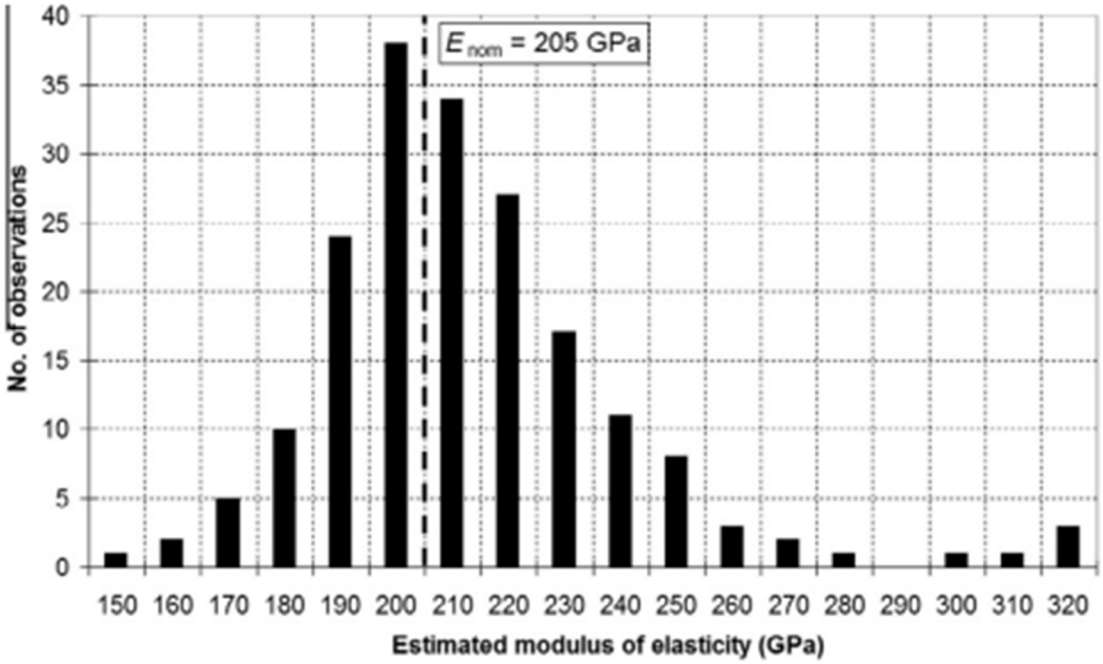


Figure 3. Estimated elastic modulus data set (Sadowski, Rotter, Reinke and Ummenhofer 2014, 30).

Based on the results of the study, great care should be taken when choosing *E* since the true value could be lower than expected. At the very least choosing the most conservative value appears to be advisable. EN 1993-1-6 standard itself does not give specific guidelines for the elastic modulus.

The material properties should be taken from the appropriate material standard (SFS-EN 1993-1-6 2007, 19). Shell structures are commonly operating at an elevated temperature which lowers the yield strength of the material. Materials used in the manufacturing of plate typically have reduced yield strength for thicker plates. According to the standard the material values are applicable up to 150°C.

2.2.2 Geometry

The geometry of the shell is limited by the *r/t* ratio or the ratio between the radius and the thickness. This ratio should be between 20 and 5000. The shells are modelled based on its midsurface, which is also how the loads are defined. The thickness of the shell is based on

the applicable standard but usually this means the nominal thickness of the shell minus any corrosion allowance.

The inclusion of imperfections in the geometry depends on the analysis type that is used. For analyses where imperfections are not included, they are represented through parameters in the calculations. Otherwise, the imperfections are considered according to chapter 8 of the standard.

The boundary conditions (BC) should be represented in a conservative manner. Table 1 shows the boundary conditions that are applicable for the standard.

Table 1. Boundary conditions for shells (SFS-EN 1993-1-6 2007, 25).

BC Code	Simple term	Description	Normal displacements	Meridional displacements	Meridional rotation
BC1r	Clamped	radially restrained meridionally restrained rotation restrained	$w = 0$	$u = 0$	$\beta_\phi = 0$
BC1f		radially restrained meridionally restrained rotation free	$w = 0$	$u = 0$	$\beta_\phi \neq 0$
BC2r		radially restrained meridionally free rotation restrained	$w = 0$	$u \neq 0$	$\beta_\phi = 0$
BC2f	Pinned	radially restrained meridionally free rotation free	$w = 0$	$u \neq 0$	$\beta_\phi \neq 0$
BC3	Free edge	radially free meridionally free rotation free	$w \neq 0$	$u \neq 0$	$\beta_\phi \neq 0$

Figure 4 shows some typical examples for boundary conditions.

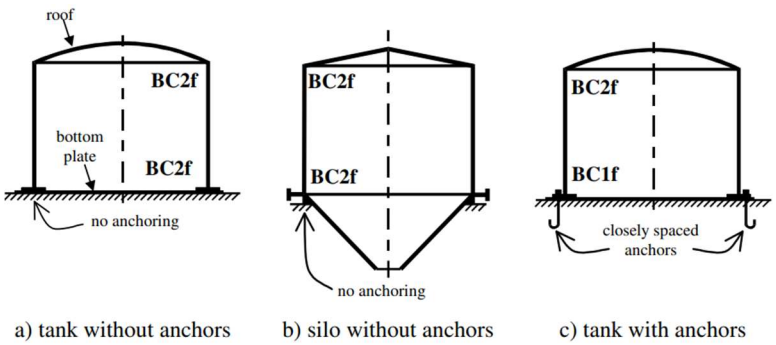


Figure 4. Examples of boundary conditions for buckling (SFS-EN 1993-1-6, 30).

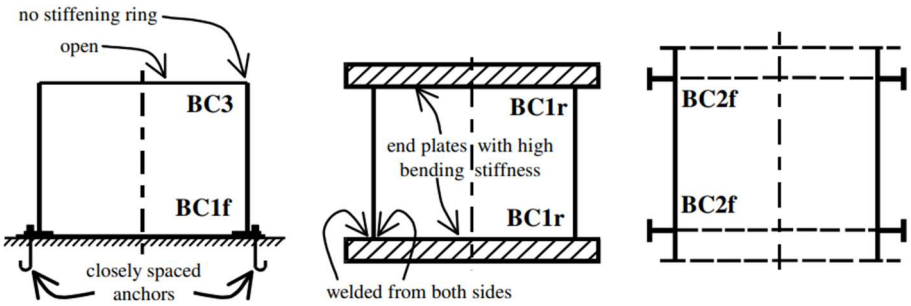


Figure 4 continues. Examples of boundary conditions for buckling (SFS-EN 1993-1-6, 30).

2.2.3 Types of analysis

When conducting buckling analysis, one or more of the methods of analysis defined in the standard should be used to determine the design stresses and stress resultants.

Linear elastic analysis (LA): Analysis that predicts the behaviour for an elastic structure using small deflections for displacements. Analysed with a perfect geometry and is the minimum requirement for stress analysis under general loading conditions. In EN 1993-1-6 this analysis can be done by using the equations presented in Annex D. (SFS-EN 1993-1-6 2007, 16, 20.)

Linear elastic bifurcation analysis (LBA): Evaluates the linear bifurcation eigenvalue on a shell using elastic shell bending theory with small deflections for displacements. Geometry is perfect. The eigenvalue is a point where the shell can buckle into a particular deformation mode. The analysis provides the elastic critical buckling resistance ratio r_{Rcr}. (SFS-EN 1993-1-6 2007, 16.)

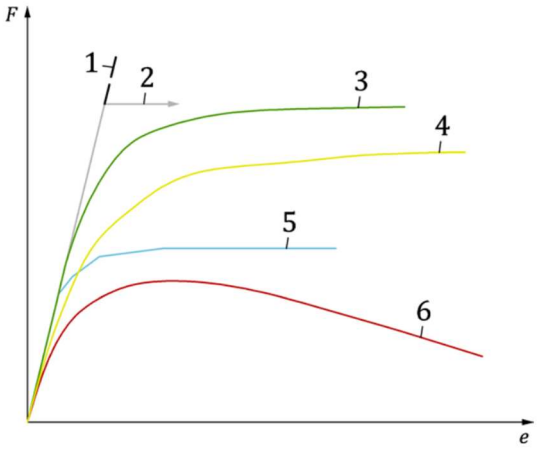
Materially nonlinear analysis (MNA): Analysis that uses shell bending theory with small deflections but includes a plastic (nonlinear) material model (SFS-EN 1993-1-6 2007, 10).

Geometrically and materially nonlinear analysis (GMNA): Gives the elasto-plastic buckling load for a perfect shell. Uses large deflection theory for displacements. Imperfections are not included. Used in combination with Geometrically and materially nonlinear analysis with imperfections (GMNIA) to identify the imperfection sensitivity of the shell. The analysis

can be used to check the loss in buckling capacity between a perfect and imperfect structure. The standard instructs that this analysis is used in cases compression and shear stresses are dominant. (SFS-EN 1993-1-6 2007, 16, 20.)

GMNIA: Fully nonlinear analysis that is done on a structure that includes unintended deviations from the ideal shape of the middle surface. The calculation should be compared with a perfect structure. Recommended for cases where compressive or shear stresses are dominant. (SFS-EN 1993-1-6 2007, 16, 20.)

Comparison for the different analysis methods can be seen in Figure 5. It shows the displacement when a force is applied. The figure shows how much the force capacity changes when nonlinearities and imperfections are included.



Key

1 Linear analysis: LA
2 Elastic bifurcation: LBA
3 Geometrically nonlinear elastic: GNA
4 Geometrically nonlinear elastic with imperfections: GNIA
5 Plastic collapse:MNA
6 Geometrically and materially nonlinear with imperfections: GMNIA

Figure 5. Comparison between different analysis types with force (F) as a function of displacement (e) (prEN 1993-1-14 2023, 32).

LA gives the result for the critical buckling stress. With LBA using FEM, the resulting eigenvalue is a load multiplier. The eigenvalue is used to multiply the stress caused by the applied load. The result is the critical buckling stress. Theoretically, the critical buckling stress could be determined directly from LA and LBA to access the strength of the structure.

However, this would be very unconservative. This is because, as mentioned above, these methods do not include geometric imperfections and linear material behaviour. The material and geometric imperfections in addition to the geometric imperfections are represented by parameters that are given in Annex D of the standard. The imperfection parameters used in EN 1993-1-6 are based on a large number of GMNIA calculations.

In theory GMNIA is the most accurate way to calculate the buckling strength since it considers features present in the real structure. This form of analysis is both resource and time intensive. Problem with GMNIA is that the accuracy of the analysis is highly dependent on accurate initial information and experience. The imperfections that the engineer chooses to include can dramatically change the result. The standard gives guidelines for evaluating the geometric imperfections. If the task is to analyse a structure that already exists, things like laser scanning can be used but this is not always possible.

2.2.4 Tolerances

The fabrication quality class is divided into Class A, Class B and Class C. The choice of the quality class affects the imperfection parameter in calculations that don't include them in the calculations. For GMNIA the fabrication tolerance class is used to define the imperfections on the shell. The imperfections include dimples on the surface and out-of-roundness of the shell.

The standard gives out of roundness tolerance for the shell cross section and the accidental eccentricity of the shell plates. Both of these tolerances are reflected in the quality classes. These tolerances are not taken into consideration in the linear calculations. When doing a GMNIA these imperfections should be modelled into the geometry based on the quality class that is chosen.

2.3 Second generation EC changes

The overarching objective of the second generation of Eurocode is simplify and improve the ease of use of the standards without taking anything important away from the previous version. These goals were addressed by listening to recommendations by experts from

different countries. During the development of the second generation of Eurocode, it was important to achieve consensus. This is partly done by voting on the different standards. This process is still ongoing at the time of writing this thesis. (Eurocodes 2023.)

The overall goal and strategy for the second generation EC3 is to not only incorporate new findings but to also clarify the standards. The overall structure of EC3 will stay the same. The attempt is to clarity and even reduce the overall volume of the standards. An example given is avoiding Informative Annexes. For EN 1993-1-6 specifically the mandate is the harmonisation and extension of rules for shells. (Kuhlmann and Knobloch 2023, 3-4.)

In the new version, a notable change has been made to the structures that the standard is applicable to. Previously the *r/t* ratio of the shell was within the range of 20 to 5000. This has been changed to 50-2000. This is a considerable change. For a shell with a 10 m radius this would mean a minimum wall thickness of 5 mm.

In general, the new version goes into further detail about calculations. Several notes are added to the newer standard to further explain the calculations and what assumptions are being made in them.

When addressing LBA, the new version is better at explaining what the result of the analysis is and how it should be used. The resulting critical buckling resistance is presented as a reference. This better reflects what the purpose of this value is. The new text more clearly explains that the result is a load multiplier that is applied to the input loads.

For GMNIA the new version wants additional LBA, MNA and GMNA analyses to be performed to verify the results. For the current version only GMNA was recommended. According to prEN 1993-1-6 (2023, 32-33) material values are defined as follows.

> "(1) The relevant material properties of carbon steels should be obtained from the relevant application standard.
>
> (2) For the mechanical properties of the structural carbon steels S235, S275, S355, S420 and S460 and also for weathering steel grades S235, S275 and S355, see EN 1993-1-1, except as defined in (3).
>
> (3) For all the steels covered by this document, the design value of Poisson's ratio should be taken as $\nu = 0{,}3$. The characteristic value of the elastic modulus for structural steel should be taken as $E = 200\,000$ N/mm^2, in accordance with the value defined for stability calculations in EN 1993-1-14.

> (4) For stainless steels covered by this document, the characteristic value of the elastic modulus should be taken as E = 191 000 N/mm^2, in accordance with the value defined for stability calculations in EN 1993-1-14."

The updated version of the standard has been created with the new EN 1993-1-14 standard in mind and is cited when FEM is covered. This includes following guidelines given in EN 1993-1-14 for making of the FEM model.

The main portion of the standard is mostly the same in terms of the equations and buckling strength verification. The new version offers more clarification and explains the methods in greater detail. The biggest changes to the calculations are in the Annexes which offer ways calculate specific buckling cases.

The new standard retains the same recommendation for the partial factor for resistance γ, which are shown in Figure 6. The new standard acknowledges that γ_{M1} which is used in buckling analysis depends heavily on the structural form, slenderness of the shell, load case and buckling mode. Due to a lack of reliable data on the subject the historical value for γ_{M1} has been retained. (prEN 1993-1-6 2023, 32.)

Resistance to failure mode	Relevant γ
Resistance of welded or bolted shell wall to plastic limit state	γ_{M0}
Resistance of shell to stability	γ_{M1}
Resistance of welded or bolted shell wall to rupture	γ_{M2}
Resistance of shell to cyclic plasticity	γ_{M4}
Resistance of shell to fatigue	γ_{Ff}

γ_{M0} = 1,00	γ_{M1} = 1,10	γ_{M2} = 1,25
γ_{M4} = 1,00		see EN 1993-1-9

Figure 6. Partial factor definitions and values (prEN 1993-1-6 2023, 32).

When calculating shear buckling with the current version of the standard, boundary conditions are not considered at all. This has changed in the newer version. Cases where

BC1 or BC2 are at both edges can be calculated. The boundary conditions only have an effect if the cylinder is short according to the length domain conditions. The C_τ factor used to calculate the critical stress is different for short cylinders in the new version of the standard.

2.4 Equations

The basis of linear buckling analysis is finding the design resistance σ_{Rd} and comparing it to the stress caused by the applied load. This basic concept applies for both hand calculations and LBA using FEM. The buckling strength is calculated with the following equation (SFS-EN 1993-1-6 2007, 38):

$$\sigma_{Rd}=\sigma_{Rk}/\gamma_{M1} \tag{1}$$

where γ_{M1} is partial factor for buckling and σ_{Rk} is the characteristic buckling stress

γ_{M1} should be taken from the National Annex if possible. If value has not been defined, then the standard recommends a value of no smaller than 1,1. The characteristic buckling stress is calculated from the following equation (SFS-EN 1993-1-6 2007, 38):

$$\sigma_{Rk}=\chi f_{yk} \tag{2}$$

where f_{yk} is the characteristic yield strength and χ is the buckling reduction factor.

The value of χ is defined by one of the following equations (SFS-EN 1993-1-6 2007, 38-39):

$$\begin{pmatrix} \bar{\lambda}_p \leq \bar{\lambda} \\ \chi=1 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} \bar{\lambda}_0<\bar{\lambda}<\bar{\lambda}_p \\ \chi=1-\beta\left(\frac{\bar{\lambda}-\bar{\lambda}_0}{\bar{\lambda}_p-\bar{\lambda}_0}\right)^{\eta} \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} \bar{\lambda}_p \leq \bar{\lambda} \\ \chi=\frac{\alpha}{\bar{\lambda}^2} \end{pmatrix} \tag{5}$$

where α is the elastic imperfection reduction factor, β is the plastic range factor, η is the interaction exponent, $\bar{\lambda}$ is the relative slenderness, $\bar{\lambda}_0$ is the squash limit relative slenderness and $\bar{\lambda}_p$ is the plastic limit relative slenderness.

$\bar{\lambda}_p$ is defined by the following equation (SFS-EN 1993-1-6 2007, 39):

$$\bar{\lambda}_p=\sqrt{\frac{\alpha}{1-\beta}} \tag{6}$$

$\bar{\lambda}$ is defined by the following equation (SFS-EN 1993-1-6 2007, 39):

$$\bar{\lambda}=\sqrt{f_{yk}/\sigma_{Rcr}} \tag{7}$$

where f_{yk} is the characteristic yield strength and σ_{Rcr} is the elastic critical buckling stress.

The elastic critical buckling stress can be defined in two ways. Either with FEM using LBA or with hand calculations. When using hand calculations, the value is defined by the equations in Annex D of the standard. There are three different definitions for each type of buckling. The equations in Annex D are applicable for only a limited number of cases.

In Annex D, for meridional (axial) buckling, the critical stress is defined by the following equation (SFS-EN 1993-1-6 2007, 74):

$$\sigma_{x,Rcr}=0{,}605EC_x\frac{t}{r} \tag{8}$$

where E is the elastic modulus, C_x is a parameter that depends on length parameter ω, t is the thickness of the shell and r is the radius of the shell.

Dimensionless length parameter ω is used to characterize the length of the shell segment (SFS-EN 1993-1-6 2007, 74):

$$\omega=\frac{l}{\sqrt{rt}} \tag{9}$$

where l is the length of the shell segment.

The shell can be characterized as either a short, medium or long cylinder based in the value of ω. These are shown in Table 2.

Table 2. C_x parameters for different cylinder lengths (SFS-EN 1993-1-6 2007, 74).

Length	**Definition**	$\boldsymbol{C_x}$
Short	$\omega \leq 1{,}7$	$1{,}36-\frac{1{,}83}{\omega}+\frac{2{,}07}{\omega^2}$
Medium	$1{,}7\leq\omega\leq 0{,}5\frac{r}{t}$	1
Long	$0{,}5\frac{r}{t}<\omega$	$C_{x,N}$

C_x is the larger of the following values (SFS-EN 1993-1-6 2007, 74):

$$C_{x,N}=1+\frac{0,2}{C_{xb}}\left[1\text{-}2\omega\frac{t}{r}\right] \tag{10}$$

$$C_{x,N}=0,60 \tag{11}$$

where C_{xb} is a parameter that represents the effects of the boundary conditions for a long cylinder being subject to meridional stress. These are shown in Table 3.

Table 3. C_{xb} parameter for long cylinders (SFS-EN 1993-1-6 2007, 74).

Case	**Boundary condition**	$\boldsymbol{C_{xb}}$
1	BC1, BC1	6
2	BC1, BC2	3
3	BC2, BC2	1

The axial elastic imperfection factor α_x is calculated with the following equation (SFS-EN 1993-1-6 2007, 75):

$$\alpha_x=\frac{0,62}{1+1,91\left(\frac{\Delta w_k}{t}\right)^{1,44}} \tag{12}$$

where Δw_k is the characteristic imperfection amplitude and is calculated as follows (SFS-EN 1993-1-6 2007, 75):

$$\Delta w_k=\frac{1}{Q}\sqrt{\frac{r}{t}}\,t \tag{13}$$

where Q is the fabrication tolerance quality parameter taken from Table 4. Quality class C is used in the calculations.

Table 4. Fabrication quality class (SFS-EN 1993-1-6 2007, 75).

Quality class	**Description**	***Q***
Class A	Excellent	40
Class B	High	25
Class C	Normal	16

The rest of the meridional buckling parameters are in Table 5.

Table 5. Meridional buckling parameters (SFS-EN 1993-1-6 2007, 75).

Parameter	**Symbol**	**Value**
squash limit slenderness	$\bar{\lambda}_{x0}$	0,20
plastic range factor	β	0,60
interaction exponent	η	1,0

The length domains for circumferential buckling caused by external pressure are shown in Table 6.

Table 6. Circumferential buckling length domain (SFS-EN 1993-1-6 2007, 76-77).

Length	**Definition**
Short	$\frac{\omega}{C_\theta} < 20$
Medium	$20 \leq \frac{\omega}{C_\theta} \leq 1{,}63\frac{r}{t}$
Long	$\frac{\omega}{C_\theta} > 1{,}63\frac{r}{t}$

For medium length cylinders the elastic critical circumferential buckling stress $\sigma_{\Theta,Rcr}$ is calculated with the following equation (SFS-EN 1993-1-6 2007, 76):

$$\sigma_{\Theta,Rcr} = 0{,}92E\left(\frac{C_\Theta}{\omega}\right)\left(\frac{t}{r}\right) \tag{14}$$

where C_Θ is circumferential buckling factor based on boundary conditions and is taken from Table 7.

Table 7. Factor C_Θ for medium and long cylinders at different boundary conditions (SFS-EN 1993-1-6 2007, 77).

Case	Cylinder end	Boundary conditions	C_Θ
1	end 1 end 2	BC 1 BC 1	1,5
2	end 1 end 2	BC 1 BC 2	1,25
3	end 1 end 2	BC 2 BC 2	1,0
4	end 1 end 2	BC 1 BC 3	0,6
5	end 1 end 2	BC 2 BC 3	0
6	end 1 end 2	BC 3 BC 3	0

For short length cylinders the elastic critical circumferential buckling stress $\sigma_{\Theta,Rcr}$ is calculated with the following equation (SFS-EN 1993-1-6 2007, 77):

$$\sigma_{\Theta,Rcr}=0{,}92E\left(\frac{C_{\Theta s}}{\omega}\right)\left(\frac{t}{r}\right) \tag{15}$$

where $C_{\Theta s}$ is circumferential buckling factor for short cylinders based on boundary conditions and is taken from the Table 8.

Table 8. Factor $C_{\Theta s}$ for short cylinders (SFS-EN 1993-1-6 2007, 77).

Case	Cylinder end	Boundary condition	$C_{\Theta s}$
1	end 1 end 2	BC 1 BC 1	$1{,}5+\frac{10}{\omega^2}-\frac{5}{\omega^3}$

Table 8 continues. Factor $C_{\Theta s}$ for short cylinders (SFS-EN 1993-1-6 2007, 77).

Case	Cylinder end	Boundary condition	**$C_{\Theta s}$**
2	end 1 end 2	BC 1 BC 2	$1{,}25+\frac{8}{\omega^2}-\frac{4}{\omega^3}$
3	end 1 end 2	BC 2 BC 2	$1{,}0+\frac{3}{\omega^{1{,}35}}$
4	end 1 end 2	BC 1 BC 3	$0{,}6+\frac{1}{\omega^2}-\frac{0{,}3}{\omega^3}$

$\sigma_{\Theta,Rcr}$ for long cylinders is calculated as follows (SFS-EN 1993-1-6 2007, 78):

$$\sigma_{\Theta,Rcr}=E\left(\frac{t}{r}\right)^2\left[0{,}275+2{,}03\left(\frac{C_\Theta}{\omega}\frac{r}{t}\right)^4\right] \tag{16}$$

The elastic imperfection reduction factor α_θ is based the fabrication quality in Table 9.

Table 9. Elastic imperfection reduction factor α_Θ for circumferential buckling (SFS-EN 1993-1-6 2007, 78).

Quality class	**Description**	**α_θ**
Class A	Excellent	0,75
Class B	High	0,65
Class C	Normal	0,50

The rest of the circumferential buckling parameters are shown in Table 10.

Table 10. Circumferential buckling parameters (SFS-EN 1993-1-6 2007, 78).

Parameter symbol	**Symbol**	**Value**
squash limit slenderness	$\bar{\lambda}_{\Theta 0}$	0,20
plastic range factor	β	0,60
interaction exponent	η	1,0

The elastic critical shear buckling stress $\tau_{x\Theta,Rcr}$ (SFS-EN 1993-1-6 2007, 79):

$$\tau_{x\Theta,Rcr}=0{,}75EC_{\tau}\sqrt{\frac{1}{\omega}}\left(\frac{t}{r}\right) \tag{17}$$

where C_τ is a shear buckling factor that is taken from Table 11.

Table 11. Shear buckling length domain and C_τ (SFS-EN 1993-1-6 2007, 79-80).

Length	**Length definition**	$\boldsymbol{C_\tau}$
Short	$\omega<10$	$\sqrt{1+\frac{42}{\omega^3}}$
Medium	$10\leq\omega\leq 8{,}7\frac{r}{t}$	1,0
Long	$\omega>8{,}7\frac{r}{t}$	$\frac{1}{3}\sqrt{\omega\frac{t}{r}}$

Table 12. Elastic imperfection reduction factor α_τ for circumferential buckling (SFS-EN 1993-1-6 2007, 79, 80).

Quality class	**Description**	$\boldsymbol{\alpha_\tau}$
Class A	Excellent	0,75
Class B	High	0,65
Class C	Normal	0,50

The rest of the shear buckling parameters are shown in Table 13.

Table 13. Shear buckling parameters (SFS-EN 1993-1-6 2007, 79-80).

Parameter symbol	**Symbol**	**Value**
squash limit slenderness	$\bar{\lambda}_{\tau 0}$	0,40
plastic range factor	β	0,60
interaction exponent	η	1,0

In Annex A of EN 1993-1-6 the standard gives methods for calculating the stresses in the shell. The following three equations and Figures 7, 8 and 9 show the most common load types (SFS-EN 1993-1-6 2007, 51):

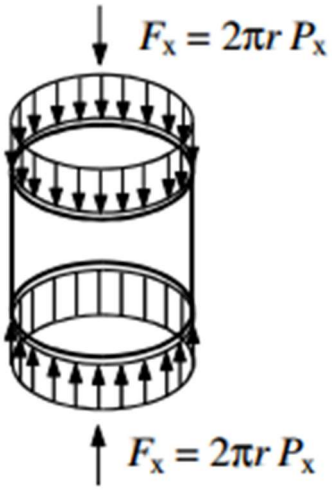


Figure 7. Uniform axial load (SFS-EN 1993-1-6 2007, 51).

$$\sigma_x = -\frac{F_x}{2\pi r t} \tag{18}$$

where F_x is the uniform axial force.

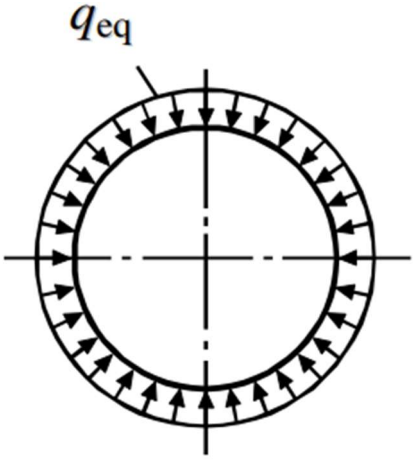


Figure 8. External pressure load (SFS-EN 1993-1-6 2007, 78).

$$\sigma_\theta = q\frac{r}{t} \tag{19}$$

where q is the uniform external pressure.

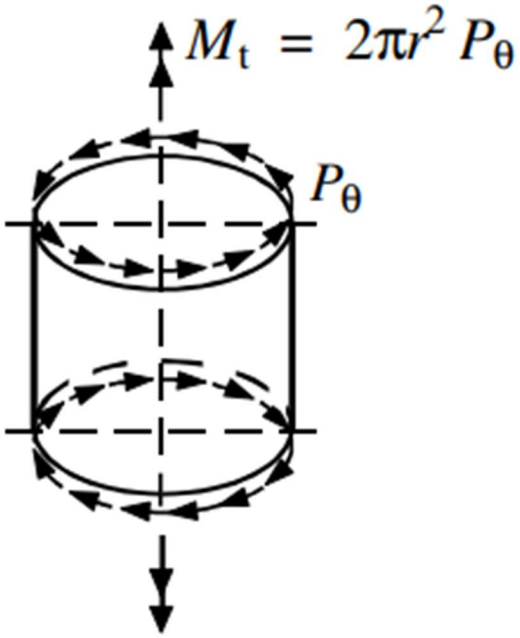


Figure 9. Uniform shear from torsion (SFS-EN 1993-1-6 2007, 51).

$$\tau = \frac{M_t}{2\pi r^2 t} \tag{20}$$

The resulting stress becomes the design stress σ_{Ed}. The buckling strength is verified with the following check for each relevant buckling type (SFS-EN 1993-1-6 2007, 39):

$$\sigma_{Ed} \leq \sigma_{Rd} \tag{21}$$

2.4.1 Second generation EC

The length domain definition for meridional (axial) buckling is defined in Table 14. Changes have been made to the definitions of medium and long cylinders. The multiplier used in the definition of long cylinders is 1,43 instead of 0,5 meaning that the definition for the medium cylinder is wider. According to the standard the 1,43 value is based on the assumption that the effective length of the member is 2. Long cylinders are considered to be subject to Euler buckling. Meaning that they are considered beams and should be checked for column buckling according to EN 1993-1-1. The length domains are shown in Table 14.

Table 14. Length domains (prEN 1993-1-6 2023, 120).

Length	**Definition**	C_x
Short	$\omega \leq 1{,}7$	$1{,}36-\frac{1{,}83}{\omega}+\frac{2{,}07}{\omega^2}$
Medium	$1{,}7 \leq \omega \leq 1{,}43\frac{r}{t}$	1
Long	$1{,}43\frac{r}{t} < \omega$	$C_{x,N}$

Most of the buckling parameters have seen changes. Axial squash limit relative slenderness $\bar{\lambda}_{x0}$ and axial elastic imperfection reduction factor α_x are obtained from the following equations (prEN 1993-1-6 2023, 121):

$$\bar{\lambda}_{x0}=0{,}1 \tag{22}$$

$$\alpha_x=\alpha_{xG}\alpha_{xI} \tag{23}$$

$$\alpha_{xG}=0{,}83 \tag{24}$$

$$\alpha_{xI}=\frac{1}{1+2{,}2(\delta_0/t)^{0{,}75}} \tag{25}$$

where imperfection amplitude δ_0 (prEN 1993-1-6 2023, 121):

$$\frac{\delta_0}{t}=\frac{1}{Q_x}\sqrt{\frac{r}{t}} \tag{26}$$

where fabrication quality class Q_x is taken from Table 4.

The plastic range factor β_x is calculated from (prEN 1993-1-6 2023, 121):

$$\beta_x=1-\frac{0{,}75}{1+1{,}1(\delta_0/t)} \tag{27}$$

The interaction exponent η_x depends on values η_{xo} and η_{xp} (prEN 1993-1-6 2023, 122):

$$\eta_{xo}=1{,}35-0{,}10(\delta_0/t) \tag{28}$$

$$\eta_{xp}=\frac{1}{0{,}45+0{,}72(\delta_0/t)} \tag{29}$$

$$\eta_x=\left[\frac{\bar{\lambda}\left(\eta_{xp}-\eta_{xo}\right)+\bar{\lambda}_{xp}\eta_{xo}-\bar{\lambda}_{xo}\eta_{xp}}{\bar{\lambda}_{xp}-\bar{\lambda}_{xo}}\right] \tag{30}$$

The hardening limit χ_x is (prEN 1993-1-6 2023, 122):

$$\chi_x=1{,}10 \tag{31}$$

The second-generation Eurocode introduces additional requirements for stainless steels. The parameters for different types are shown in Tables 15 and 16.

Table 15. Meridional buckling parameters χ_{xh} , $\bar{\lambda}_{xo}$, η_{x0}, η_{xh} for stainless steel types (prEN 1993-1-6 2023, 123).

Stainless steel type	**Hardening limit**	**Squash limit relative slenderness**	**Plastic interaction exponents**	
	χ_{xh}	$\bar{\lambda}_{xo}$	η_{x0}	η_{xh}
Austenitic	1,2	0,31	1,50	1,10
Duplex		0,36	1,08	0,60
Ferritic		0,30	1,125	0,70

Table 16. Buckling parameters β_x, η_{xo}, η_{xp} for different stainless-steel types (prEN 1993-1-6 2023, 123).

Stainless steel type	**Plastic range factor**	**Elastic-plastic interaction exponents**	
	β_x	η_{xo}	η_{xp}
Austenitic	$1-\frac{0,44}{1+0,955(\delta_0/t)^{0,96}}$	$0,85-0,51(\delta_0/t)^{0,71}$	$\frac{0,68}{1+0,56(\delta_0/t)}$
Duplex	$1-\frac{0,48}{1+1,35(\delta_0/t)^{0,95}}$	$1,1-0,631(\delta_0/t)^{0,62}$	$\frac{0,97}{1+1,06(\delta_0/t)}$
Ferritic	$1-\frac{0,66}{1+1,13(\delta_0/t)^{0,87}}$	$1,05-0,601(\delta_0/t)^{0,62}$	$\frac{1,04}{1+1,1(\delta_0/t)}$

Shells where $\bar{\lambda}_x<\bar{\lambda}_{x0}$ is true are considered stocky. With stainless steels the elastic-plastic buckling reduction factor should be calculated as (prEN 1993-1-6 2023, 123):

$$\chi_x=\chi_{xh}-(\chi_{xh}-1)(\bar{\lambda}_x/\bar{\lambda}_{x0})^{\mu_x} \tag{32}$$

The plastic interaction exponent μ_x is (prEN 1993-1-6 2023, 123):

$$\mu_x=\mu_{xh}+\left(\frac{\bar{\lambda}_x}{\bar{\lambda}_{x0}}\right)(\mu_{x0}-\mu_{xh}) \tag{33}$$

The circumferential buckling medium and long length domains are defined in the same way as in the current standard. The definition for short length cylinders is $\omega<\omega_s$, where ω_s is based on boundary conditions along with parameter $C_{\theta s}$. Both values are shown in Table 17.

With some of the boundary conditions the $C_{\theta s}$ value is applicable for both short and medium cylinders.

Table 17. $C_{\theta s}$ values for different boundary conditions (prEN 1993-1-6 2023, 125).

Boundary conditions at each end		$C_{\theta s}$	ω_s
End 1	**End 2**		
BC1r	BC1r	$1{,}50-\frac{1{,}64}{\omega}+\frac{8{,}7}{\omega^2}+\frac{24{,}2}{\omega^3}$	110
BC1r	BC1f	$1{,}50-\frac{1{,}9}{\omega}+\frac{8{,}9}{\omega^2}+\frac{0{,}9}{\omega^3}$	110
BC1f	BC1f	$1{,}50-\frac{2}{\omega}+\frac{5{,}1}{\omega^2}+\frac{2{,}76}{\omega^3}$	125
BC1r	BC2r	$1{,}25+\frac{0{,}86}{\omega}+\frac{2{,}6}{\omega^2}+\frac{27{,}8}{\omega^3}$	65
BC1r	BC2f	$1{,}25+\frac{5{,}8}{\omega^2}+\frac{2{,}8}{\omega^3}$	25
BC1f	BC2r	$1{,}25+\frac{0{,}82}{\omega}-\frac{0{,}84}{\omega^2}+\frac{18{,}3}{\omega^3}$	45
BC1f	BC2f	$1{,}25+\frac{1{,}9}{\omega^2}+\frac{2{,}9}{\omega^3}$	12
BC2r	BC2r	$1+\frac{2{,}6}{\omega}-\frac{1{,}6}{\omega^2}+\frac{30{,}4}{\omega^3}$	125
BC2r	BC2f	$1+\frac{1{,}8}{\omega}+\frac{0{,}1}{\omega^2}+\frac{9{,}3}{\omega^3}$	125
BC2f	BC2f	$1+\frac{1{,}3}{\omega}-\frac{0{,}8}{\omega^2}+\frac{6{,}9}{\omega^3}$	125
BC1r	BC3f	$0{,}6+\frac{0{,}77}{\omega^2}$	11
BC1f	BC3f	0,6	short and medium
BC2r	BC3f	$0{,}05+\frac{1{,}8}{\omega}-\frac{2{,}6}{\omega^2}+\frac{2{,}6}{\omega^3}$	short and medium
BC2f	BC3f	$\frac{0{,}34}{\omega}+\frac{0{,}27}{\omega^2}-\frac{0{,}25}{\omega^3}$ $-0{,}3\sqrt{\frac{t}{r}}\left(0{,}33-\omega\sqrt{\frac{t}{r}}\right)$	short and medium

The rest of the $C_{\theta s}$ values for medium length cylinders are taken from Table 18.

Table 18. C_θ for Medium length cylinders (prEN 1993-1-6 2023, 124).

Cylinder end	BC	$C_{\theta s}$
End 1 End 2	BC1 BC1	1,5
End 1 End 2	BC1 BC2	1,25
End 1 End 2	BC2 BC2	1,0
End 1 End 2	BC1 BC3	0,6
End 1 End 2	BC2 BC3	Table 14
End 1 End 2	BC3 BC3	0

The circumferential elastic imperfection reduction factor α_θ is now calculated from the following equations (prEN 1993-1-6 2023, 126):

$$\alpha_\theta = \alpha_{\theta G}\alpha_{\theta l} \tag{34}$$

$$\alpha_{\theta G} = 0{,}95 \tag{35}$$

$$\alpha_{\theta l} = \frac{1}{1+b(\delta_0/t)^{0{,}8}} \tag{36}$$

$$b = 9{,}8\left(\frac{r}{L}\right)^{0{,}75}\left(\frac{r}{t}\right)^{0{,}4} \tag{37}$$

$$\frac{\delta_0}{t} = \frac{1}{Q_\theta}\left(\frac{L}{r}\right)^{0{,}8}\left(\frac{r}{t}\right)^{0{,}5} \tag{38}$$

where Q_θ is the fabrication quality class for circumferential compression acquired from Table 19.

Table 19. Fabrication quality class for circumferential buckling (prEN 1993-1-6 2023, 126).

Quality class	Description	Q_θ
Class A	Excellent	75
Class B	High	40
Class C	Normal	20

Parameters β_θ and η_θ are the same as before. Parameter $\lambda_{\theta 0}$ squash limit slenderness has been updated (prEN 1993-1-6 2023, 126):

$$\lambda_{\theta 0}=0{,}4 \tag{39}$$

For critical shear buckling stress, parameter C_τ is calculated the same way for medium and long cylinders. For short cylinders the equation is (prEN 1993-1-6 2023, 128):

$$C_\tau=\sqrt{1+\frac{a_{\tau s}}{\omega^b}} \tag{40}$$

where $a_{\tau s}$ is a parameter that depends on in boundary conditions.

For BC1r or BC2r the equation is (prEN 1993-1-6 2023, 128):

$$a_{\tau s}=120-\frac{130}{1+0{,}015\frac{r}{t}} \tag{41}$$

For BC1f or BC2f the equation is (prEN 1993-1-6 2023, 128):

$$a_{\tau s}=70-\frac{75}{1+0{,}015\left(\frac{r}{t}\right)^{1{,}1}} \tag{42}$$

Parameter b is calculated with the following equation (prEN 1993-1-6 2023, 128):

$$b=3-\frac{5}{1+0,4\left(\frac{r}{t}\right)^{0,6}} \tag{43}$$

Elastic imperfection reduction factor α_τ is calculated with the following equations (prEN 1993-1-6 2023, 128):

$$\alpha_\tau=\alpha_{\tau G}\alpha_{\tau I} \tag{44}$$

$$\alpha_{\tau G}=0,96 \tag{45}$$

$$\alpha_{\tau I}=\frac{1}{1+0,5\left(\frac{\delta_0}{t}\right)} \tag{46}$$

$$\frac{\delta_0}{t}=\frac{1}{Q_\tau}\sqrt{\frac{r}{t}} \tag{47}$$

where Q_τ is the fabrication quality parameter and is taken from Table 4.

2.5 FE-analysis

This section will cover the building of an example FEM model that is used in combination with the calculation Excel. FEA is used to verify the results of the calculations that are based of the equations form the standard. The new EN 1993-1-14 standard will also be considered. This standard offers guidelines and instructions for making FEM calculations. The program used for this section is Ansys 2024 R1.

The primary use case for FEM analysis is with LBA analysis. The analysis is done by imputing the normal forces and boundary conditions into the FEM model. The analysis should result in an eigenvalue that is used as a multiplier for the value that was given at the start. The resulting critical buckling stress value can be input into the equations in the standard so that the final buckling strength can be determined.

The LBA is based on an eigenvalue problem and uses the perturbation method. This is a method that can solve a linear problem from a preloaded state. In Ansys, you must first solve a Static Structural analysis which eigenvalue analysis uses as the prestress state. The static analysis is connected to the Setup cell of the Eigenvalue Buckling system. After solving the problem, the results will show the load multipliers for each buckling mode. (Ansys Learning Hub 2023.)

Buckling analysis could be performed with a nonlinear static analysis. This should include imperfections, material nonlinearity and large deflections. The method is based on the calculation being unable to converge. This happens when buckling occurs in the structure due to a loss of stability. When stability is lost, the force stability is not achieved, and calculation won't converge. This method can be used to find both local and global buckling. (Ansys 2020, 11-12.)

Figure 10 shows the solution from the static structural analysis solution being connected to the eigenvalue buckling analysis.

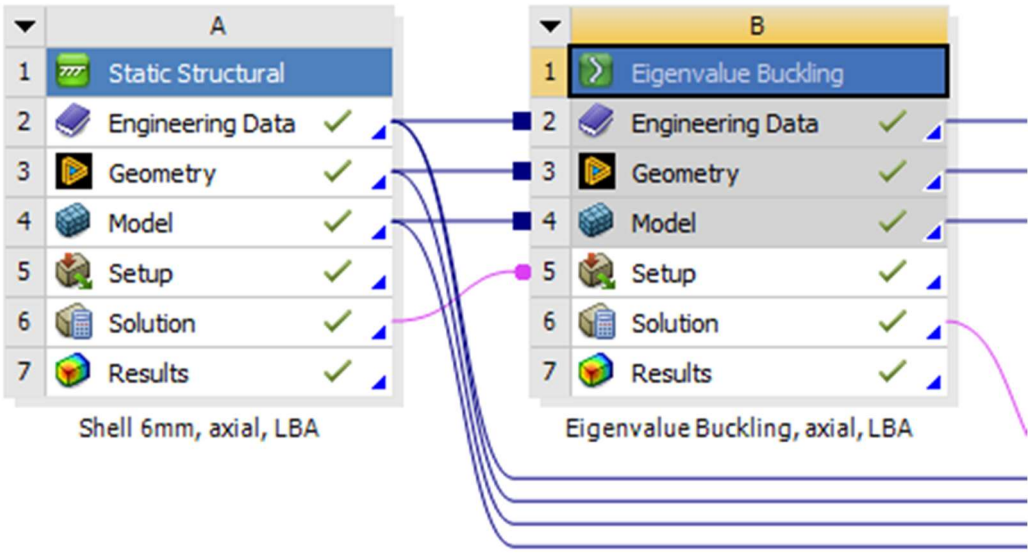


Figure 10. Ansys analysis cells

For GMNIA analysis FEM analysis is required. For this use case the calculation Excel does not serve much of purpose and the user mainly focus on following the instructions given in the standard.

2.5.1 EN 1993-1-14

At the time of writing this thesis, the EN 1993-1-14 standard has not been officially released yet. It is a brand-new addition to EC3. The copy used in this for this is a draft version which is still subject to change. The version used in this is dated 28.2.2024.

Overall, the standard gives general guidelines for FEM analysis. This includes broad instructions on the geometric model and mesh. The instructions are not in depth, but they do offer good working practices. In many cases the standard defers to the relevant part of EN 1993 for specific issues. The relevant part of EN 1993 should be primarily followed if the case that is being inspected is covered in one of the parts. EN 1993-1-14 should be relied on for additional guidelines.

The basis of design is EN 1990 and the relevant parts of EN 1993. The design is divided into two methods. Analysis that requires a subsequent design check and a direct resistance check analysis. The former should be performed with one of the following methods: LA, LBA, GNA, GNIA, MNA. For the latter the standard instructs the following methods: MNA, GMNA, GMNIA. Typically, when analysing buckling according to EN 1993-1-6, LBA and GMNIA are used for their respective cases.

Generally, LA and LBA should be expected to provide a higher load factor and therefore a less conservative result. As is typical with strength analysis, a more detailed inspection provides a more optimized result. However, it should be noted that with buckling this is not always the case since post-buckling behaviour can lead to higher loads before a complete failure occurs.

The methods outlined in the standard for a buckling resistance check using FEM are as follows.

1. LA (or MNA, GMNA) and LBA
2. GNIA combined with LBA

3. GNIA combined with cross section resistance
4. GMNIA

Out of these methods, 3 should not be used for shell structures.

2.5.2 FEM model

The results are to be cross referenced and verified with a simple FEM calculation using Ansys. The geometry of the shell for the test calculation is as follows.

Shell length: 8000 mm

Shell wall thickness: 6 mm

Shell radius: 4000 mm

The shell is supported at the bottom with a fixed support that fixes translations and rotations. At the top of the shell, a remote displacement is applied. The horizontal displacements are fixed while the vertical displacements and rotations are set free.

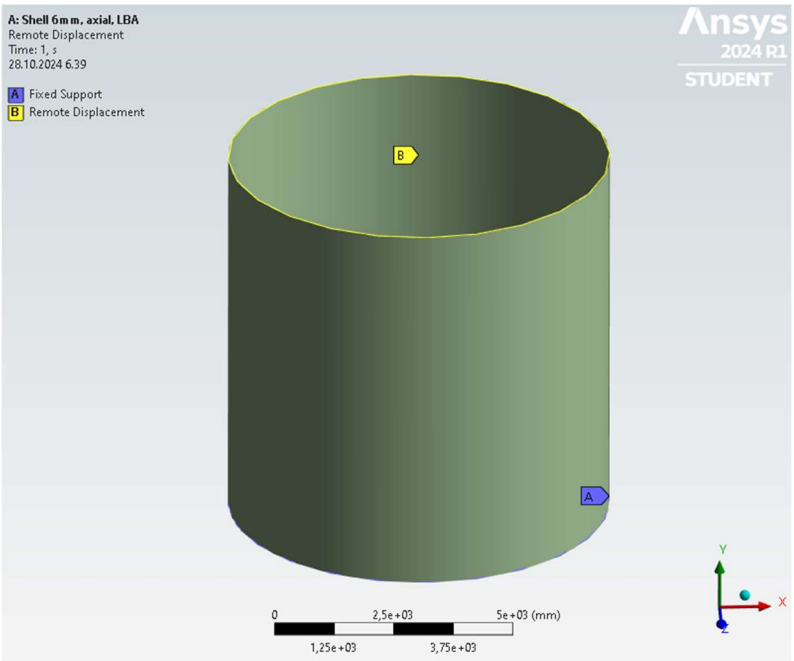


Figure 11. Shell supports

For the first load case a vertical 1000 kN load is applied to the top of the shell. The load is applied as nondeformable. For the second load case a 0,001 MPa pressure is applied to the outside surface of the shell. For the third load case a 1E+9 Nmm torsional moment is applied to the top edge of the shell. When solving an eigenvalue problem, the magnitude of the seed load does not matter as long as the load distribution is the same. The reason being that the actual buckling resistance is represented by the resulting eigenvalue. The eigenvalue or load multiplier is applied to the load that was originally used in the static structural analysis. (Ansys 2020, 8.)

Figures 12, 13 and 14 show the applied loads.

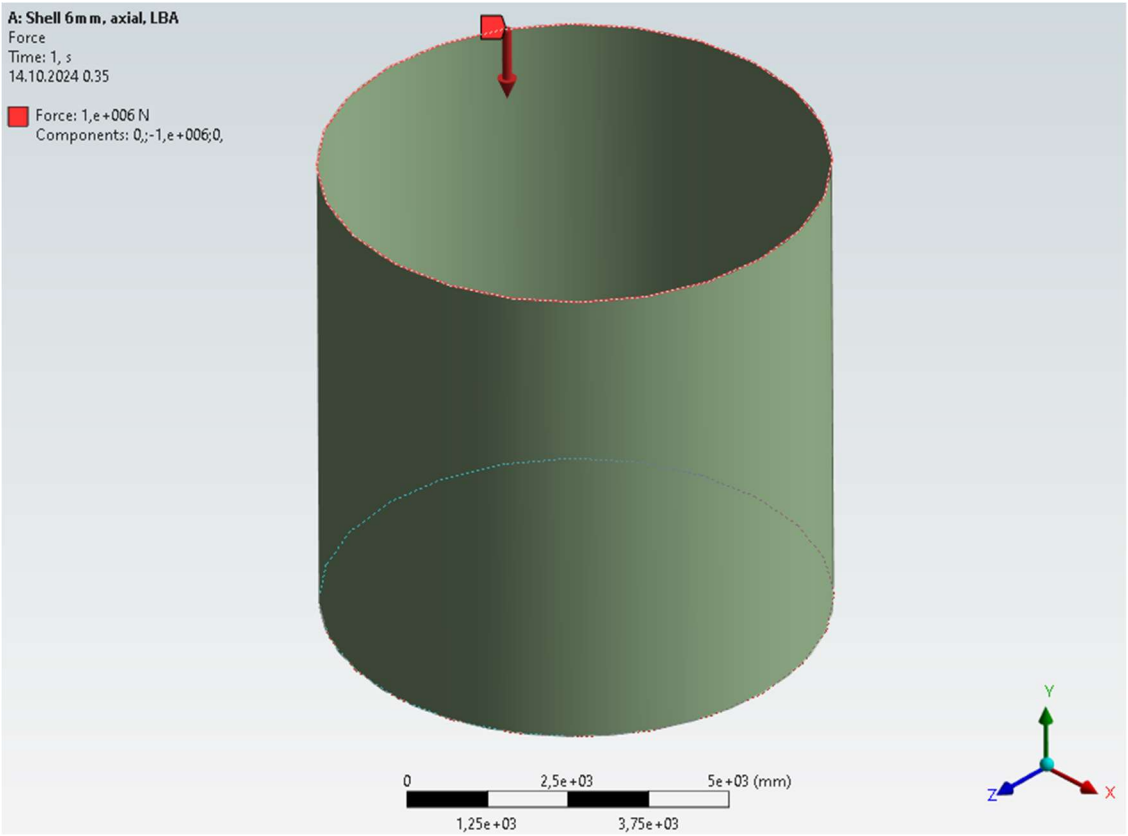


Figure 12. Ansys calculation axial load 1000kN

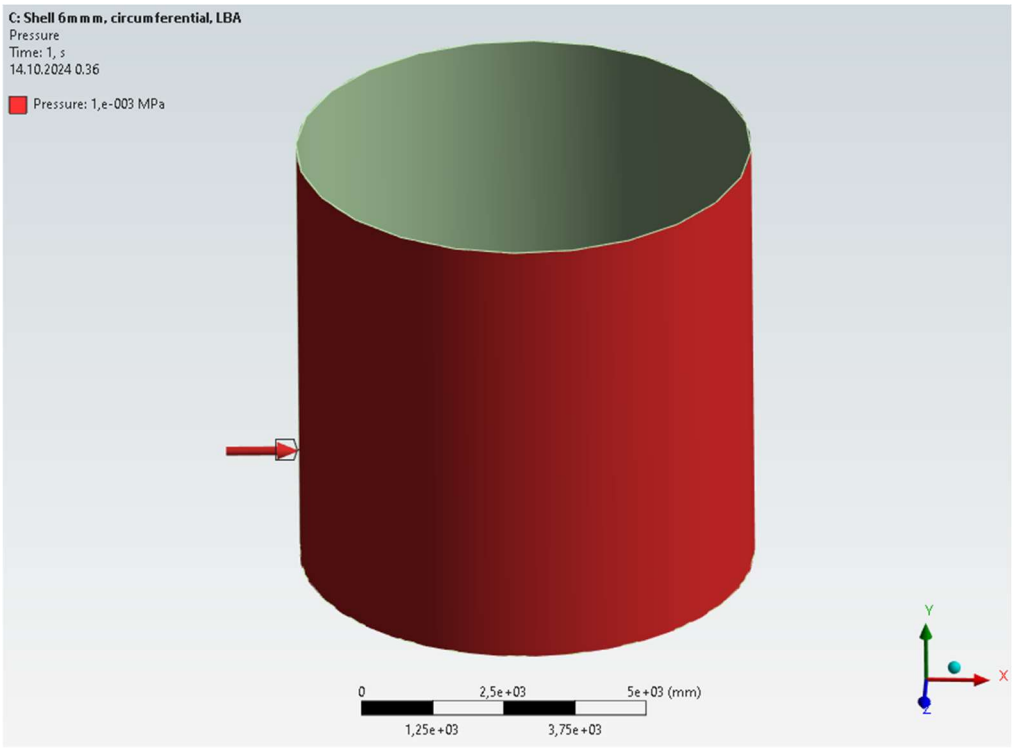


Figure 13. Ansys calculation pressure load 0,001 MPa

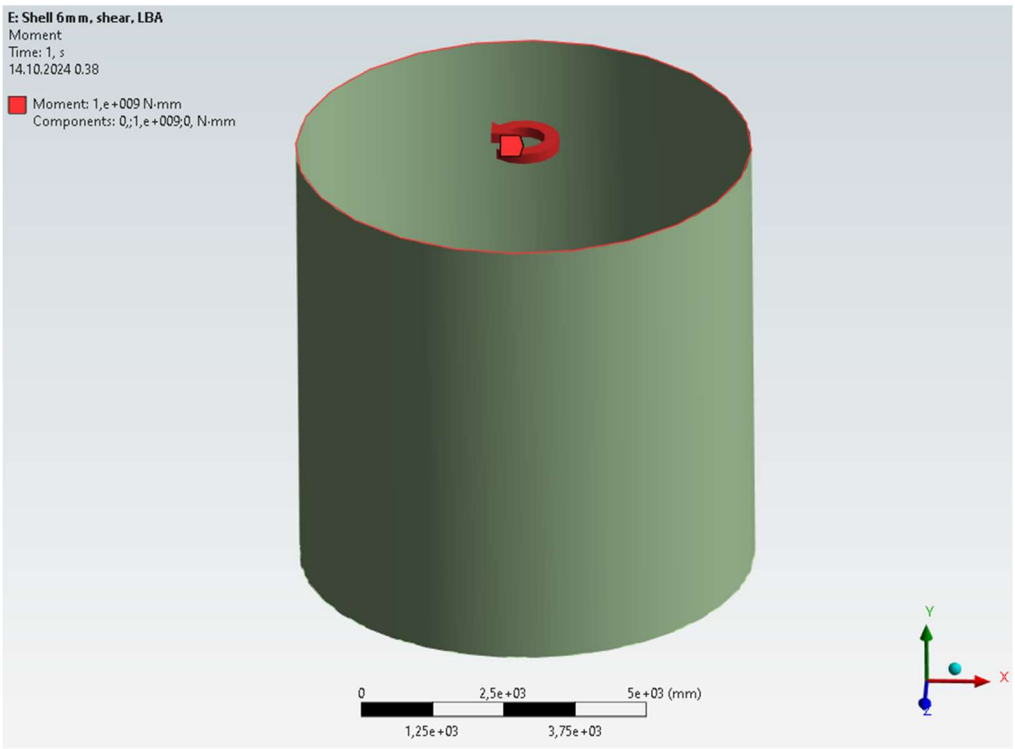


Figure 14. Ansys calculation shear moment 1E+9 MPa

Due to the simple geometry, a fine mesh can be used. The mesh uses linear hexahedral elements with an element size of 100 mm which is shown in Figure 15. The rest of the settings are left on default. Based on experience, for simple structures this is acceptable. Typically changes to the mesh settings need to be done when the program has trouble meshing complex structures.

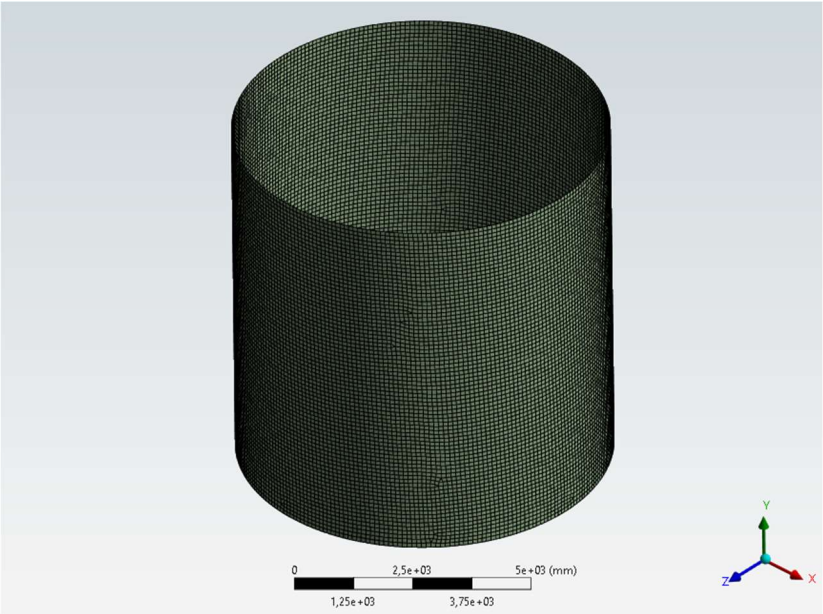


Figure 15. Ansys calculation mesh

2.5.3 GMNIA

When calculating linearly using hand calculations or LBA the critical buckling stress needed post processing to result in a realistic result. In GMNIA the result of the FEM analysis only needs to be divided by the partial factor, and this should result in the buckling strength of the structure.

For the GMNIA analysis a nonlinear material model needs to be added. For this analysis, the default nonlinear structural steel is used. For the purpose of this analysis the specific values of the material are not important. The most important thing is that the material considers nonlinear material behaviour. The second important difference for the GMNIA

analysis is to enable large deflections in the static analysis. This means gives the structure considers geometric nonlinearity.

In addition to the geometric and material nonlinearity, imperfections need to be included in the calculations. The geometry of the shell is perfect, also the loads are completely vertical or normal to the shell. This means that the result would be the same or very similar to the linear analysis. Small differences may be caused by for example the mesh not being completely even across the shell. Nonlinearity could be induced by applying a small load, for example in the horizontal direction. In the beginning of the section covering the buckling limit state, the EN 1993-1-6 standard offers tolerances for the different geometric features. These include the overall roundness of the shell and accidental eccentricities. For this analysis the dimple tolerances are the most relevant ones. The standard offers different dimple tolerances based on the fabrication quality class, this makes it possible to compare to the same quality class used in the hand calculation and LBA section.

The dimple imperfections can be done by using the buckling modes from the linear analysis. The LBA calculates a 1 mm displacement for the buckling mode. The resulting buckling mode is connected to the Model of a new static structural analysis in Ansys Workbench. When connecting to the Solution to the Model, the geometry mode can be scaled. Connecting the solution to a new calculation cell is shown in Figure 16.

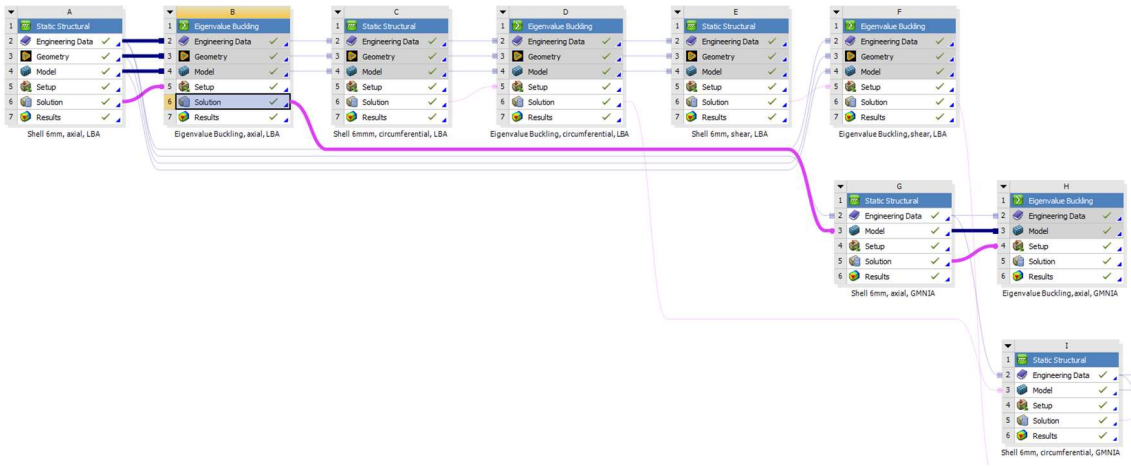


Figure 16. Connecting solution to a new static structural analysis

The scale factor is the depth of the dimple. The method for calculating the geometric tolerances is given in the section 8.7 Design by global numerical analysis GMNIA analysis. The depth of the dimple for a meridionally stressed is calculated with larger of the following two equations (SFS EN-1993-1-6 2007, 45):

$$\Delta w_{0,eq1}=l_g U_{n1} \tag{48}$$

$$\Delta w_{0,eq2}=n_i t U_{n2} \tag{49}$$

where l_g is the gauge length, n_i is a multiplier to achieve an appropriate tolerance level, t is the thickness of the shell, U_{n1} and U_{n2} are the dimple imperfection amplitude parameters.

The imperfection amplitude parameters are defined in the GMNIA section based on the fabrication tolerance class. The values are shown in Table 20.

Table 20. Imperfection amplitude parameters for different quality classes (SFS EN-1993-1-6 2007, 45).

Quality class	**Description**	U_{n1}	U_{n2}
A	Excellent	0,010	0,010
B	High	0,016	0,016
C	Normal	0,025	0,025

The gauge lengths l_g for axial, circumferential and shear stresses are calculated with the following equations (SFS EN-1993-1-6 2007, 35-36):

$$l_{gx}=4\sqrt{rt} \tag{50}$$

$$l_{g\theta}=2{,}3(l^2 rt)^{0{,}25} \tag{51}$$

where l is the length of the shell segment, r is the radius of the shell and t is the thickness of the shell.

The resulting load multiplier from the nonlinear analysis cannot be used directly. When doing a nonlinear eigenvalue analysis, Ansys allows additional loads to be applied. The other option is to copy the loads from the static structural analysis. This is the option that will be used here. The resulting load multiplier only considers the loads from the copied values used in the eigenvalue analysis. Meaning that the loads from the static structural analysis need to also be applied for the final load multiplier. When copying the loads, this means adding 1 to the load multiplier. For example, if the resulting multiplier is 3 than the actual value is 4.

The effect of the imperfections must be checked by comparing the load multiplier from an analysis without imperfections to one that includes them. In this case the result of the LBA can be used for the comparison since the stresses are likely to be well below the yield strength. The loads are applied evenly and the structure is completely symmetrical, which means that the large deflections should not change the result. According to the standard the ration should only be a little higher than the imperfection parameter α that is calculated with the hand equations.

$$\alpha_{GMNIA} = \frac{r_{GMNIA}}{r_{GMNA}} \tag{52}$$

The resulting load multiplier from the GMNIA analysis needs to be divided by the partial factor γ_{M1}. Resulting in the final load multiplier for buckling.

$$r_{Rd} = \frac{r_{Rk}}{\gamma_{M1}} \tag{53}$$

3 Results

This section will present the results from both the hand and FEM calculations. The results from the FEM calculations will then be input into the calculation Excel for post processing.

3.1 Hand calculation

This section will show the results for buckling under three different load types. The calculations are done for a cylinder classed as medium length. The results are shown for both the current EC and the second-generation EC. Input values are shown in Figure 17.

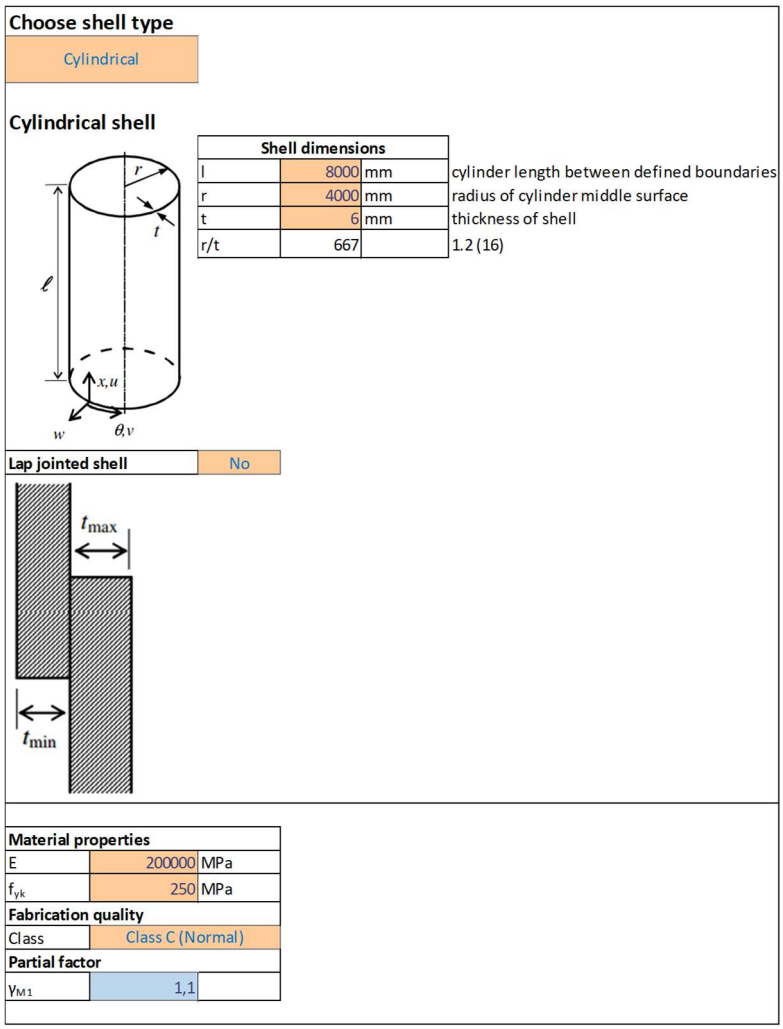


Figure 17. Geometric and material values

3.1.1 Meridional/axial buckling

Figure 18 shows the axial buckling calculation result. The axial load is 1000kN.

Stress design value		
$\sigma_{x,Ed}$	6,631	MPa

8.5.1 Design values of stresses

Hand calcualtions

Boundary conditions

End 1	BC1 (Clamped)
End 2	BC1 (Clamped)

8.5.2 Design resistance (buckling strength)			
λ_x	1,174		(8.17)
λ_{px}	0,768		(8.16)
χ_x	0,138		(8.13-8.15)
$\sigma_{x,Rk}$	34,614	MPa	(8.12)
$\sigma_{x,Rd}$	31,468	MPa	(8.11)

8.5.3 Stress limitation (buckling strength verification)

Utilization		
x	21 %	(8.18)

Figure 18. Axial buckling calculation

3.1.2 Circumferential buckling

Figure 19 shows the circumferential buckling calculation result. The external pressure is 0,001 MPa.

Boundary conditions	
End 1	BC1
End 2	BC2

8.5.2 Design resistance (buckling strength)			
λ_{Θ}	6,117		(8.17)
$\lambda_{p\Theta}$	1,118		(8.16)
χ_{Θ}	0,013		(8.13-8.15)
$\sigma_{\Theta,Rk}$	3,340	MPa	(8.12)
$\sigma_{\Theta,Rd}$	3,037	MPa	(8.11)

8.5.3 Stress limitation (buckling strength verification)

Utilization		
Θ	22 %	(8.18)

Figure 19. Circumferential buckling calculation

3.1.3 Shear buckling

Result for shear buckling is shown in Figure 20. Torsional moment at the top of the shell is 1E+9 Nmm.

Stress design value		
$\tau_{x\Theta,Ed}$	1,658	MPa

8.5.2 Design resistance (buckling strength)			
λ_{τ}	2,147		(8.17)
$\lambda_{p\tau}$	1,118		(8.16)
χ_{τ}	0,108		(8.13-8.15)
$\tau_{x\Theta,Rk}$	15,655	MPa	(8.12)
$\tau_{x\Theta,Rd}$	14,232	MPa	(8.11)

8.5.3 Stress limitation (buckling strength verification)

Utilization		
xΘ	12 %	(8.18)

Figure 20. Shear buckling result

3.1.4 Second generation EC

Comparison between the buckling parameters in the first- and second-generation standards is shown in Tables 21, 22 and 23.

Table 21. Axial buckling parameter comparison

Parameter	**1st Gen**	**2nd Gen**
λ_x	1,174	1,174
λ_{x0}	0,20	0,31
λ_{px}	0,768	1,069
α_x	0,191	0,200
β_x	0,676	0,825
η_x	0,641	0,388
χ_x	0,138	0,145
χ_{xh}	1,0	1,2

Table 22. Circumferential buckling parameters comparison

Parameter	**1st Gen**	**2nd Gen**
λ_θ	5,584	5,638
$\lambda_{\theta 0}$	0,4	0,4
$\lambda_{p\Theta}$	1,118	1,140
α_θ	0,5	0,520
β_θ	0,6	0,6
η_θ	1,0	1,0
χ_θ	0,016	0,016

Table 23. Shear buckling parameter comparison

Parameter	**1st Gen**	**2nd Gen**
λ_θ	2,147	2,147
$\lambda_{\tau 0}$	0,4	0,4
$\lambda_{p\tau}$	1,118	1,140
α_τ	0,5	0,96
β_τ	0,6	0,6
η_τ	1	1
χ_τ	0,108	0,115

Table 24 compares the utilization percentage of the different buckling types between the EC versions.

Table 24. Comparison between the first- and second-generation EC results

Buckling type	**1st Gen**	**2nd Gen**
Axial	21 %	20 %
Circumferential	22 %	21 %
Shear	12 %	11 %

3.2 FEM results

The first thing to check is the displacements in the static structural analysis. To verify that the loads and support conditions are working correctly. Figures 21, 22 and 23 show the total deformation for each load.

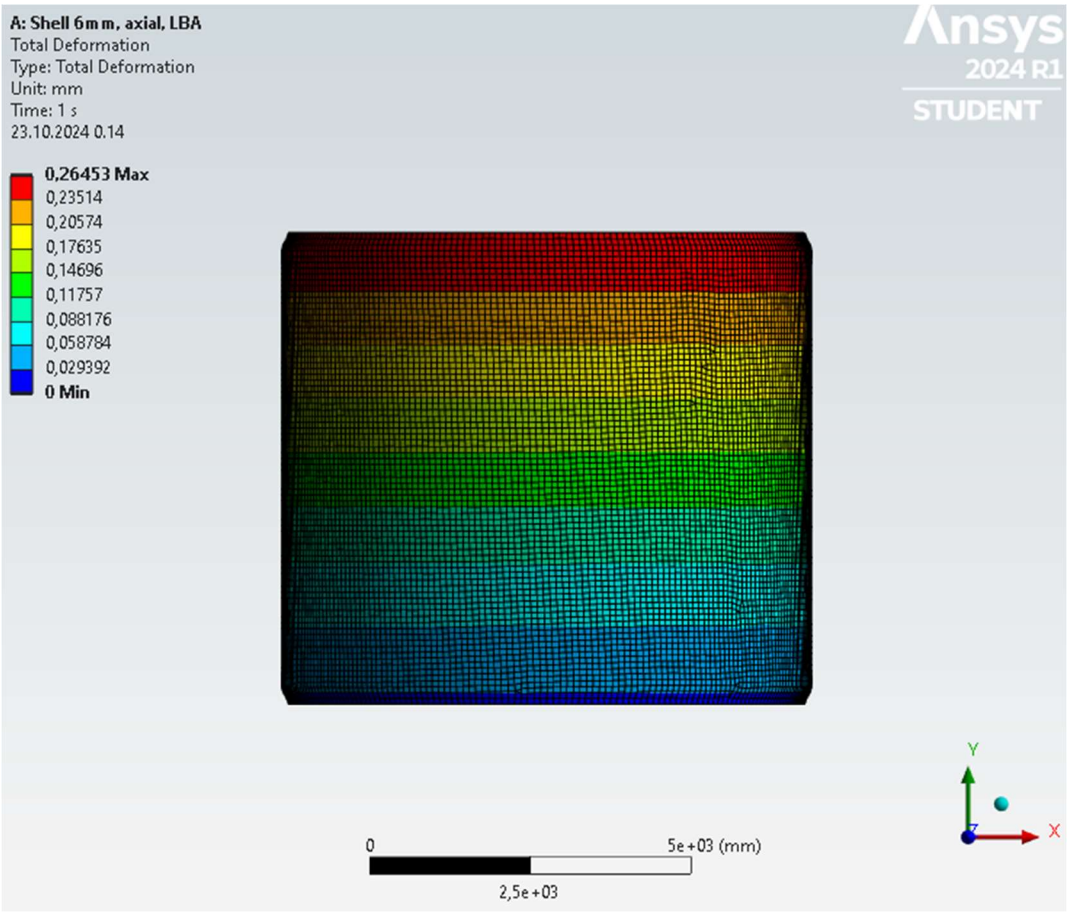


Figure 21. Total translation, axial load

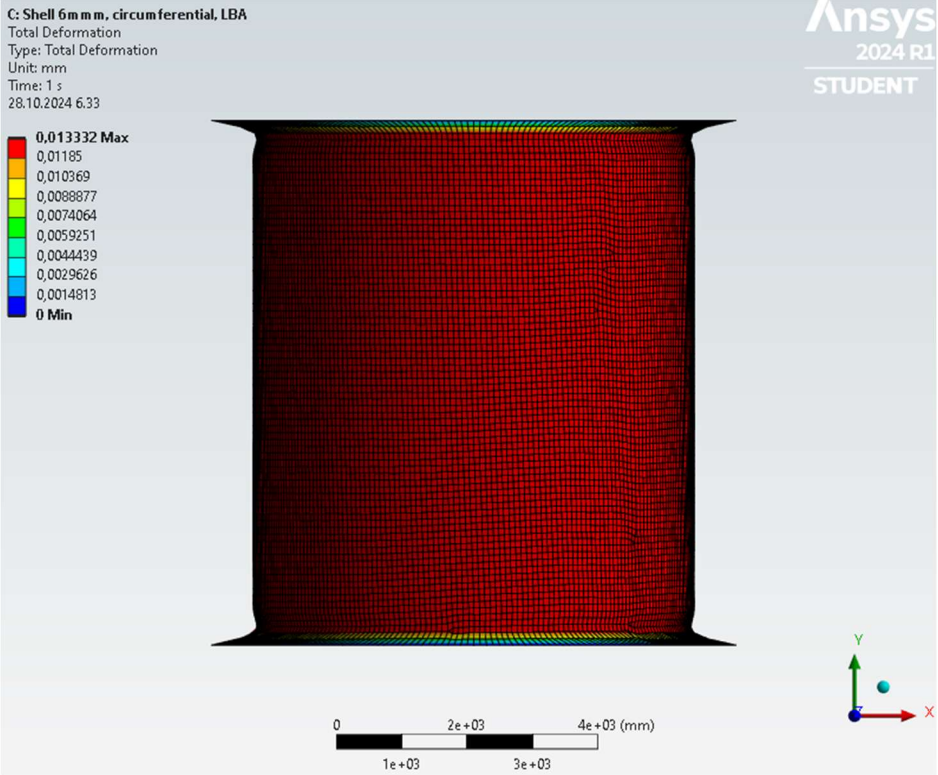


Figure 22. Total translation, external pressure

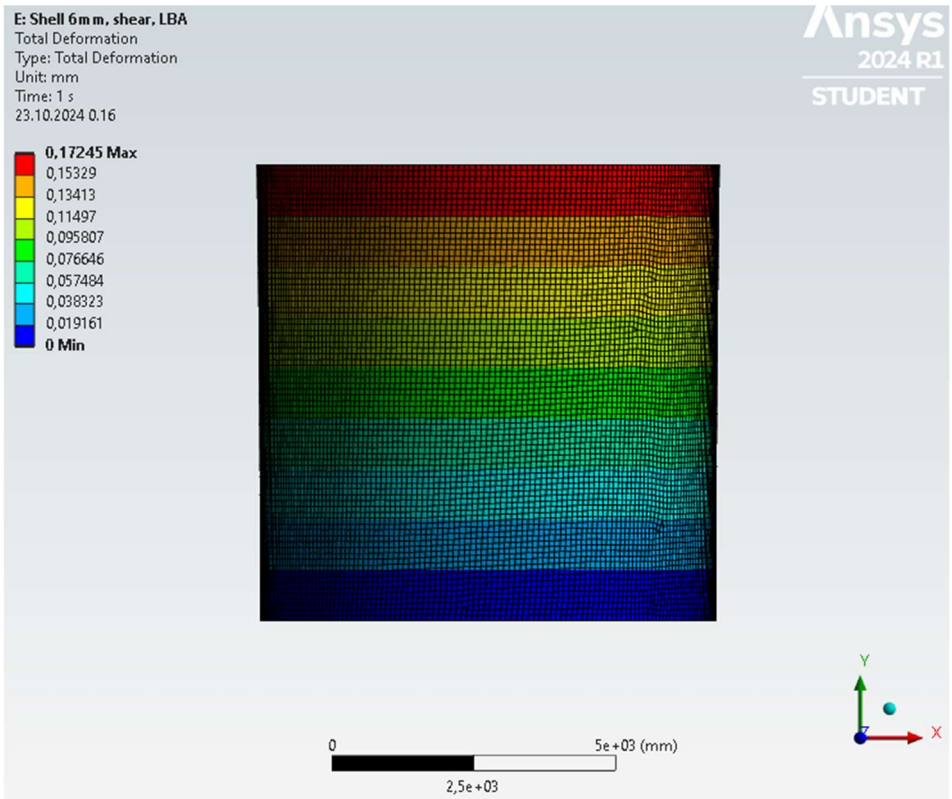


Figure 23. Total translation, moment

3.2.1 LBA

In the LBA section of the Excel tool, the eigenvalue (LBA multiplier) resulting from the FEM analysis is input into the appropriate cell along with the load that was used in the analysis. Figures 24-29 show the results of the LBA.

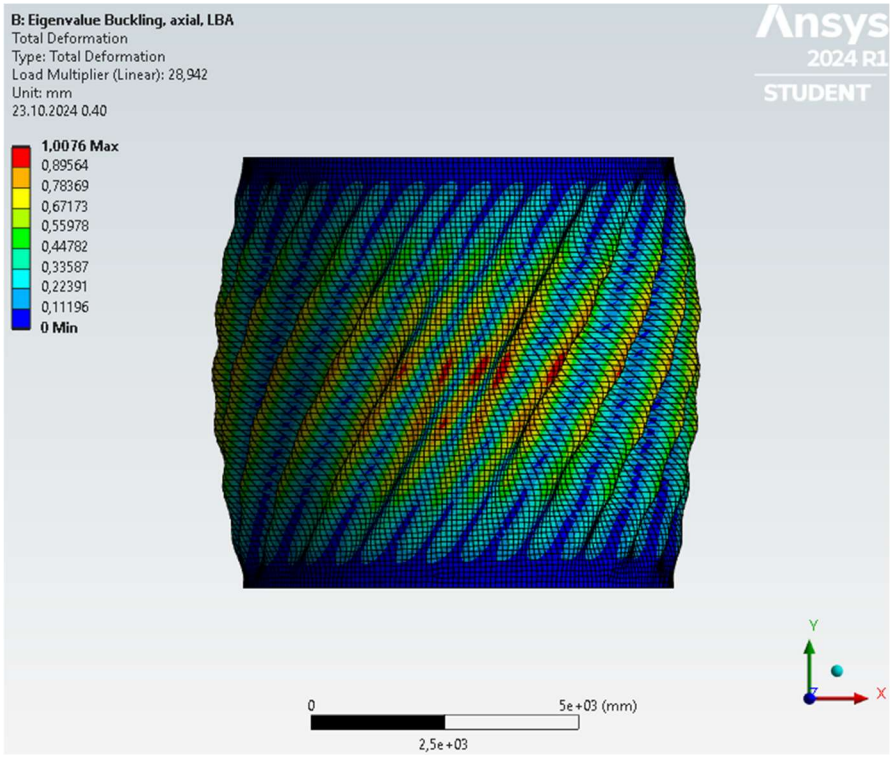


Figure 24. Axial buckling lowest eigenvalue, 28,942

LBA analysis values			
r_{Rcr}	28,942		Load multiplier
$\sigma_{x,Rcr}$	191,928	MPa	

8.5.2 Design resistance (buckling strength)			
λ_x	1,141		(8.17)
λ_{px}	0,768		(8.16)
χ_x	0,146		(8.13-8.15)
$\sigma_{x,Rk}$	36,603	MPa	(8.12)
$\sigma_{x,Rd}$	33,275	MPa	(8.11)

8.5.3 Stress limitation (buckling strength verification)

Utilization		
x	20 %	(8.18)

Figure 25. Axial buckling result using the critical buckling stress from LBA

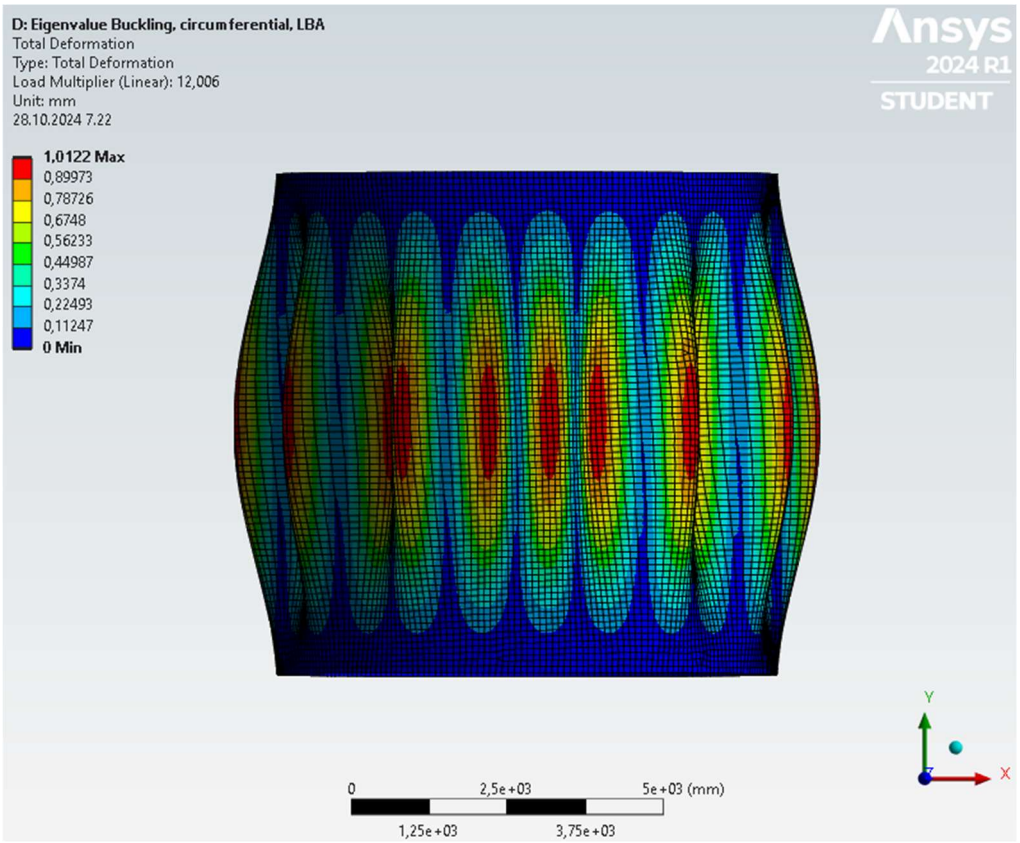


Figure 26. Circumferential buckling lowest eigenvalue, 12,006

LBA analysis values			
r_{Rcr}	12,006		Load multiplier
$\sigma_{\Theta,Rcr}$	8,004	MPa	

8.5.2 Design resistance (buckling strength)			
λ_{Θ}	5,589		(8.17)
$\lambda_{p\Theta}$	1,118		(8.16)
χ_{Θ}	0,016		(8.13-8.15)
$\sigma_{\Theta,Rk}$	4,002	MPa	(8.12)
$\sigma_{\Theta,Rd}$	3,638	MPa	(8.11)

8.5.3 Stress limitation (buckling strength verification)

Utilization		
Θ	18 %	(8.18)

Figure 27. Circumferential buckling result using the critical buckling stress from LBA

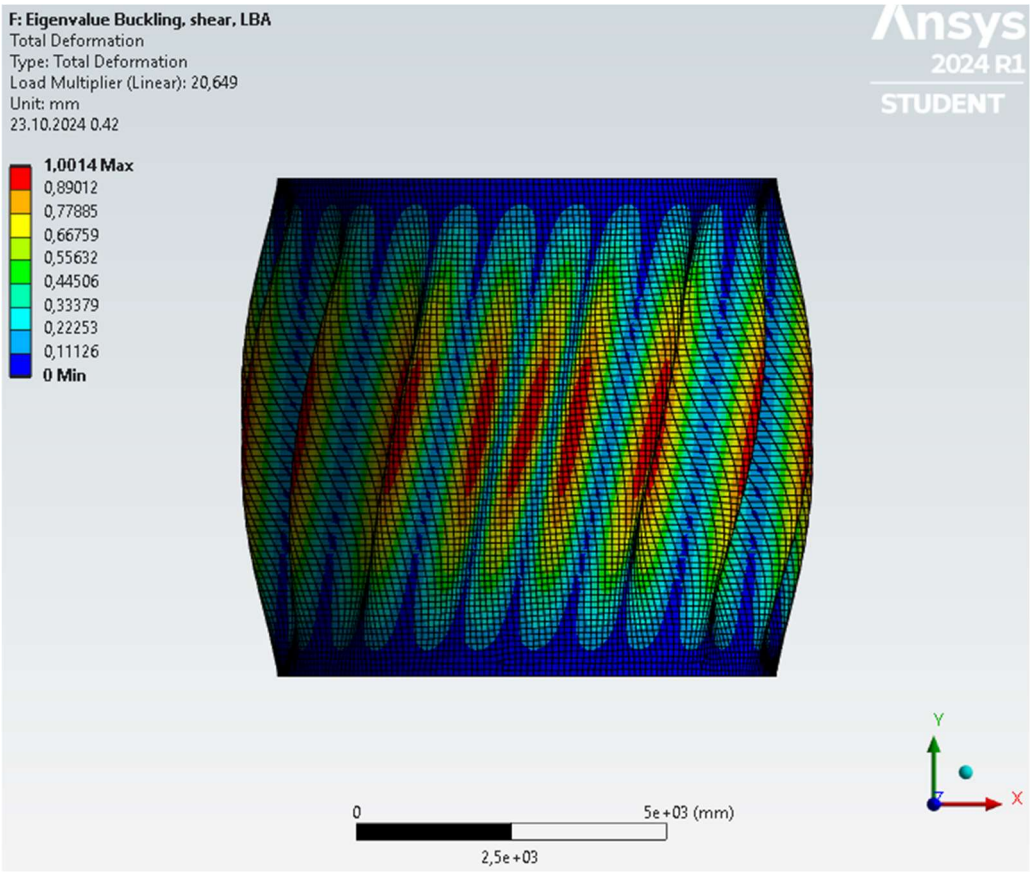


Figure 28. Shear buckling lowest eigenvalue, 20,649

LBA analysis values			
r_{Rcr}	20,649		Load multiplier
$\tau_{x\Theta,Rcr}$	34,233	MPa	

8.5.2 Design resistance (buckling strength)			
λ_{τ}	2,053		(8.17)
$\lambda_{p\tau}$	1,118		(8.16)
χ_{τ}	0,119		(8.13-8.15)
$\tau_{x\Theta,Rk}$	17,117	MPa	(8.12)
$\tau_{x\Theta,Rd}$	15,561	MPa	(8.11)

8.5.3 Stress limitation (buckling strength verification)

Utilization		
xΘ	11 %	(8.18)

Figure 29. Shear buckling result using the critical buckling stress from LBA

3.2.2 GMNIA

Figures 30, 31 and 32 show the results of the GMNIA for axial, circumferential and shear buckling and a comparison to the linear analysis. The imperfection multipliers were calculated with equations 48 and 49. The axial dimple multiplier is Δw_{0x}=15,5.

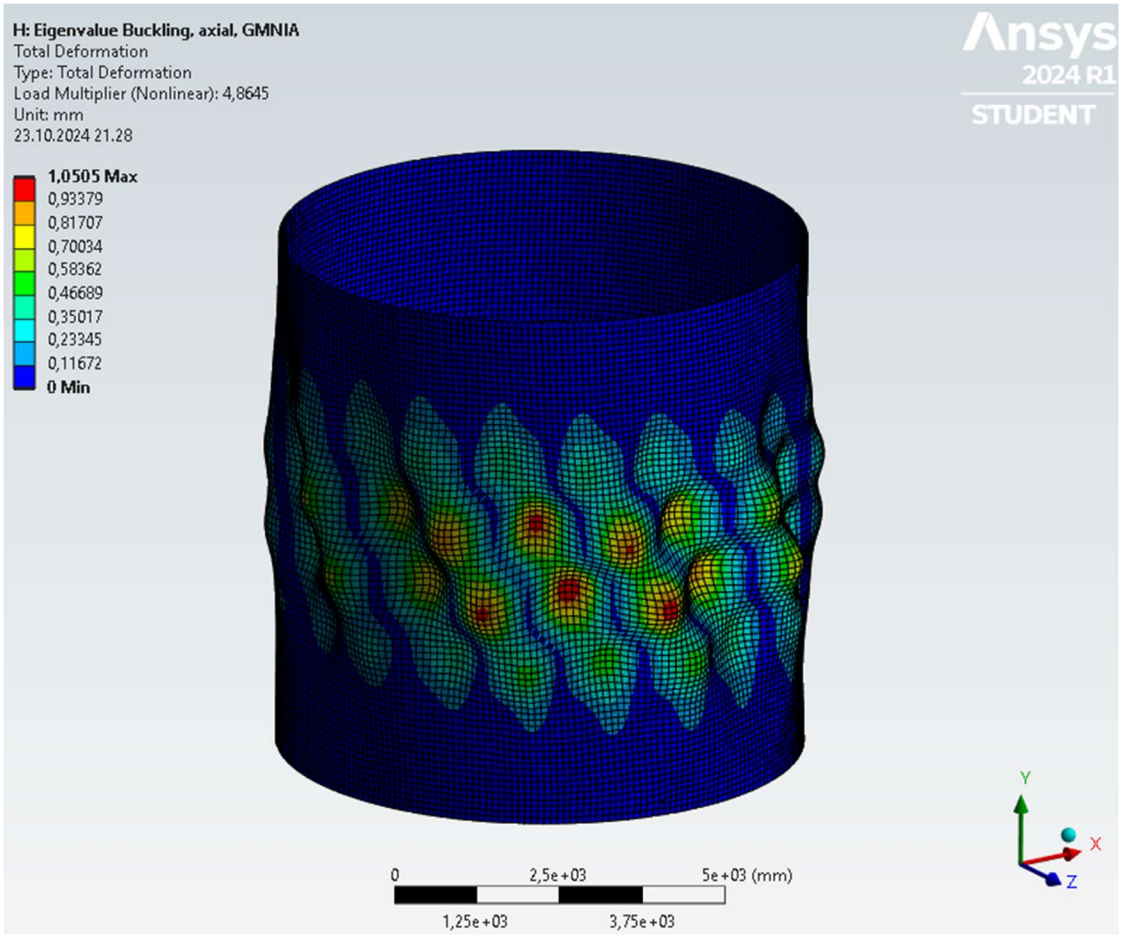


Figure 30. GMNIA axial buckling

The circumferential and shear dimple multiplier is $\Delta w_{0\theta\tau}$=64,0.

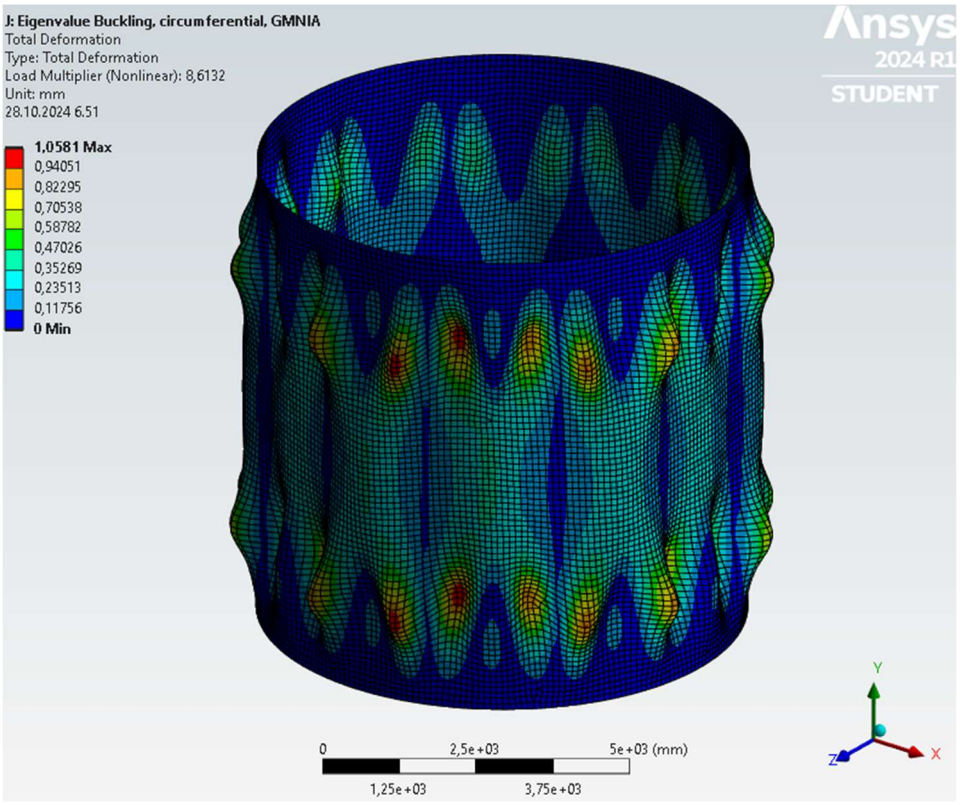


Figure 31. GMNIA circumferential buckling

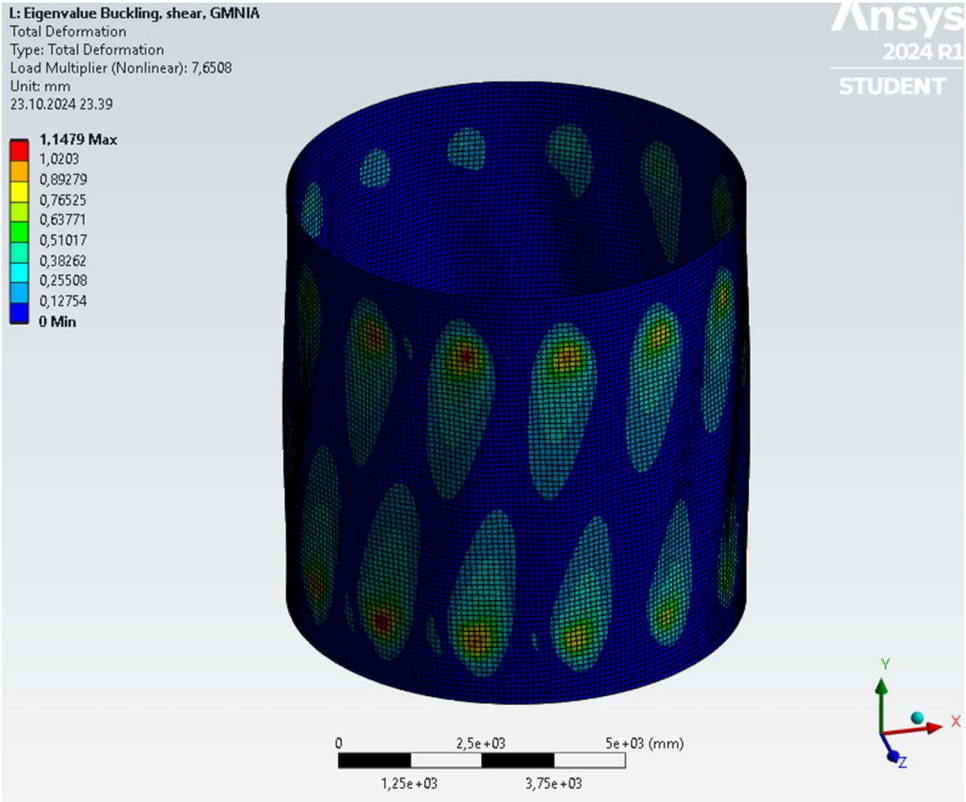


Figure 32. GMNIA shear buckling

The results of the GMNIA in Table 25 include the partial factor. The results for LA include the reduction factor from the standard.

Table 25. Comparison between LA and GMNIA

Buckling type	**LA**	**GMNIA**
Axial	21 %	18 %
Circumferential	22 %	11 %
Shear	12 %	13%

4 Conclusions

The goal to make the next generation of Eurocode clearer can be seen in the new EN 1993-1-6. An example of this is how LBA is explained. The new version clearly states that the result is a load multiplier that is applied to the input load and that this then becomes the critical buckling resistance. In the current version this aspect is less clear.

The change to the *r/t* ratio limit is presumably due to wall thickness. Based on previous experience, the strength of the shell would be very sensitive at lower wall thicknesses. It can be assumed that there is a large amount of unpredictability when you have a large shell with a small wall thickness. Perhaps due to this there was a desire to have these types of shells outside the scope of the standard. Presumably it was determined that the standard was simply not fit to analyse these structures. High *r/t* ratio makes the structure even more unstable and the load baring capacity will vary wildly based on the type of imperfections. Ultimately, linear buckling analysis involves a lot of guess work, and this means that a conservative approach should be taken.

For circumferential buckling the older standard appears to give more conservative results. This is most obviously seen in cylinders that are classified as medium length. This is due to the revised equations used in the calculations. The result is due to the revised parameters that represent the imperfections.

The results showing lower utilization of the structure when calculating according to the new EN 1993-1-6. This shows that the standard so far appears to have been conservative. This should be seen as a positive. Generally, in engineering, when there is a lack of information or uncertainty, the safer course of action should be taken. If the new version showed results that are worse, it could cause concerns whether structures built according to old version are sufficient. This could potentially lead to a need to strengthen existing structures. The move to a less conservative result also shows that the methods behind the analysis are in the right direction. The older version appears to be less refined. With the advance in technology and basic understanding, the equations can be refined to be more accurate and less conservative. If the result went in the opposite direction, it could imply that something wasn’t being considered. The parameters in EN 1993-1-6 are a result of many GMNIA calculations.

Presumably when conducting further analyses, the developers of the standard were able to optimize the results better.

The move into a less conservative direction is a positive from a design point of view. Based on experience from real projects at Insinor Oy, buckling has often been the limiting factor in shell structures by a wide margin. This has often led to a need to strengthen the structure from the customer's original plan. Shells with thin walls have especially been problematic.

One area where the second-generation EC can give more conservative values is with materials. The new standard both the new EN 1993-1-6 and EN 1993-1-14 instructs the use of an elastic modulus of 200 000 MPa for structural steels. In the current version the use of a higher elastic modulus is allowed. This change will likely have practical effects since the use of 210 000 MPa for the elastic modulus is common. The study referenced in 2.2.1 shows that there is good reason for lowering E. It shows that even the value given in the new standard may be too high in many cases.

The results of the LBA are closely in line with the linear hand calculations as they should be. In theory the results should be identical but things like the mesh and rounding will have small effects on the results.

The results for the GMNIA are more interesting. The method used to input the imperfections is rather simple and for a true buckling analysis multiple sets of imperfections would have to be calculated to get a reliable result. For axial and shear buckling the result is reasonably close to the result of the LBA which is a good sign. It helps validate the method. Perhaps somewhat surprising is the fact that the utilization was lower than in the linear analysis. Since the imperfection dimple shapes were quite large. The initial expectation was that using the mode shapes from a linear analysis would be on the conservative side since the shapes are global. The linear analysis likely has a considerable amount of safety. Generally, when using a more precise method, structures should work better.

The circumferential buckling showed considerably lower utilization when calculating with GMNIA. This may be a little surprising since once again the thought was that the imperfection modes that were used would result in unconservative results. Circumferential buckling is less sensitive to imperfections so this could explain the results. Another reason for the lower utilization could be the lack of a proper out-of-roundness imperfection.

One interesting note is that with the new standard in circumferential buckling the length domain is defined as short. In every other case the cylinder is considered to be medium length. This does not have a large effect on the result since the critical buckling stress in this case is similar. However, that may not always be the case.

Future work for the research done in this thesis would include further testing into different shell geometries. This could include a large amount of test results that are plotted out to see which areas of the geometry are most affected by the changes in the new standard. Further research could go to the mode shapes and their effect on the results of GMNIA. Running further analyses with different mode shapes would increase the accuracy of the results. Future work could also be done into the effects of the material strength. This should also include the additional requirements added to stainless steels in the second-generation standard.

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