MASTER’S THESIS

Investigations of GaAs based heterostructures for spintronics

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ABSTRACT

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In the present work are reported investigations of structural, magnetic and electronic properties of GaAs/Ga$_{1-x}$In$_x$As/GaAs quantum wells (QW) having a 0.5 - 1.8 monolayer thick Mn layer, separated from the quantum well by a 3 nm thick spacer. The structure of the samples is analyzed in details by photoluminescence and high-resolution X-ray diffractometry and reflectometry, confirming that Mn atoms are practically absent from the QW. Transport properties and crystal structure are analyzed for the first time for this type of QW structures with so high mobility. Observed conductivity and the Hall effect in quantizing magnetic fields in wide temperature range, defined by transport of holes in the quantum well, demonstrate properties inherent to ferromagnetic systems with spin polarization of charge carriers in the QW. Investigation of the Shubnikov – de Haas and the Hall effects gave the possibility to estimate the energy band parameters such as cyclotron mass and Fermi level and calculate concentrations and mobilities of holes and show the high–quality of structures. Magnetic ordering is confirmed by the existence of the anomalous Hall effect.
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Roman Kochetov
Table of symbols

Roman letters

- $c$: light velocity
- $d$: thickness of quantum well
- $e$: electron charge
- $g$: electron (or hole) $g$ – factor
- $h$: Plank constant
- $k$: electron’s (or hole’s) wave vector
- $k_B$: Boltzmann constant
- $m_e$: cyclotron mass
- $m_0$: mass of free electron
- $n$: concentration of charge carriers
- $p_{SdH}$: concentration of holes which take part in SdH oscillations
- $r_{xx}$: magnetoresistance
- $w$: width of the sample
- $A$: amplitude of oscillation
- $B$: magnetic field (inductance)
- $E$: energy
- $\tilde{E}$: electric field
- $E_f$: Fermi level
- $F$: ferromagnetic material
- $G$: conductance
- $H$: intensity of magnetic field
- $I$: electric current
- $\tilde{J}$: electric current density
- $M$: magnetization
- $N$: quantum number
- $P$: polarization
- $P_{SdH}$: period of SdH oscillations
- $R_{Hall}$: the Hall coefficient
- $R_{xy}$: the Hall resistance
\( S_m \)  extremal cross-sectional area of the Fermi surface
\( T \)  temperature
\( T_C \)  Curie temperature
\( T_D \)  Dingle temperature
\( U \)  voltage

Greek letters
\( \lambda \)  free path
\( \mu \)  hole mobility
\( \rho \)  specific resistance
\( \rho(E) \)  charge carrier’s density of states
\( \tau \)  relaxation time
\( \omega_c \)  cyclotron frequency

Acronyms
\textit{AHE} anomalous Hall effect
\textit{CIP} current in plane
\textit{CPP} current perpendicular to the plane
\textit{dHvA} de Haas – van Alphen
\textit{DMS} diluted magnetic semiconductors
\textit{GMR} giant magnetoresistive
\textit{LL} Landau level
\textit{MBE} molecular beam epitaxy
\textit{ML} monolayer
\textit{MRAM} magnetic random access memory
\textit{MTJ} magnetic tunnel junction
\textit{PHL} photoluminescence
\textit{QHE} quantum Hall effect
\textit{QW} quantum well
\textit{SdH} Shubnikov – de Haas
\textit{SFET} spin field-effect transistor
\textit{TMR} tunneling magnetoresistance
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1. Introduction

Investigations of the diluted magnetic semiconductors (DMS) as potential materials for spintronic devices became one of the fastest growing directions of nowadays solid state physics [1-3]. DMS are semiconductors containing considerable amount (up to 10%) of magnetic impurities. Exists a number of experimental data about ferromagnetism in DMS and its influence to transport phenomena in DMS based on p-type A\text{III}B\text{V} materials doped with Mn at concentrations up to $10^{21}$ cm$^{-3}$ [3,4]. Microscopic mechanism of the magnetic ordering in these materials is still under discussion but it is generally accepted that the ferromagnetism is mediated by free and/or localized holes in the impurity band. There exist big amount of publications dedicated to GaAs based DMS, but the number of structures with a two-dimensional (2D) conductivity channel is relatively small [5-9].

In the present work correlation of structural, transport and magnetic properties of ferromagnetic GaAs/$\delta$-Mn/GaAs/In$_x$Ga$_{1-x}$As/GaAs quantum well (QW) structures are investigated. Most investigations of 2D DMS structures concern the influence of ferromagnetic ordering on transport properties [1-8]. Only recently attention has been paid to the role of structural details and inherent disorder on physical properties of DMS [10]. In the majority of previous investigations of 2D DMS structures, showing the anomalous Hall effect (AHE), the Mn layer was applied directly on the 2D electron channel [5, 6]. Some amount of Mn ions probably penetrated into the channel as indicated by the very low mobility of the charge carriers (2 - 5 cm$^2$/Vs) even in the case of spacer between Mn layer and QW. Under such circumstances it is difficult to evaluate the influence of the quality of the QW or the roughness of its borders on the magnetic properties of the structure.

One of the main questions is the right thickness of the spacer layer, i.e. the distance between our source of the localized magnetic moments and spins of the carriers in QW.

This work is an attempt to answer some complicated questions concerning possible prospective materials and structures for spintronics.
2. Spintronics – materials and devices (literature review)

Spintronics refers to the role played by electron (and more generally nuclear) spin in solid state physics, and possible devices that exploit spin properties instead of or in addition to charge degrees of freedom [11]. Spintronics is sometimes also referred to as spinelectronics or 'magneto-electronics' although we prefer the 'spintronics' terminology because magnetic field or the presence of magnetic material is not necessarily essential for manipulating of spins. Spintronics is an emerging field [12] of active control of spin dynamics and transport in electronic materials. This is particularly, but not necessarily, limited to semiconductors. The term spin stands for either the spin $s$ of a single electron, which can be detected by its magnetic moment $2g\mu_B s$, or the average spin of a group of electrons, manifested by magnetization. In the equation for magnetic moment $\mu_B$ is the Bohr magneton and $g$ is the electron $g$ - factor, in a solid generally different from the free-electron value of $g = 2.0023$. Control of spin is a control of either the population and the phase of the spin of an ensemble of particles, or a coherent spin manipulation of a single or a few-spin system. The goal of spintronics is to understand the interaction between the particle spin and its solid-state environments and to make useful devices using the acquired knowledge. Fundamental investigations are spin transport, spin dynamics and spin relaxation in electronic materials. Typical questions are (a) what is an effective way to polarize a spin system? (b) how long is the system is able to remember its spin orientation? and (c) how can spin be detected?

Generation of spin polarization usually means creating a nonequilibrium spin population. This can be reached by several methods. Traditionally spin has been oriented using optical techniques in which circularly polarized photons transmit their angular momentum to electrons. For device applications electrical spin injection is more desirable. In electrical spin injection a magnetic electrode is connected to the sample. When the current drives spin-polarized electrons from the electrode to the sample, nonequilibrium spin accumulates there. The value of spin accumulation depends on spin relaxation, the process of bringing the accumulated spin population back to equilibrium. There are several mechanisms of spin relaxation, most involving spin-orbit coupling to provide the spin-dependent potential, in combination with
momentum scattering to provide a randomizing force. Typical time scale for spin relaxation in electronic systems is nanoseconds, while the range is from picoseconds to microseconds. Spin detection and part of a generic spintronic scheme relies on sensing the changes in the signals caused by the nonequilibrium spin in the system. Common aim in many spintronic devices is to maximize the spin detection sensitivity to the level that it detects not the spin itself, but changes in the spin states.

For example, spin relaxation and spin transport in metals and semiconductors are fundamental research interest for basic solid state physics issues and also for the potential these phenomena have in electric technology [11, 13-17]. The prototype device that is already in use in industry as a read head and a memory – storage cell is a giant magnetoresistive (GMR) sandwich structure [11] which consists of alternating ferromagnetic and nonferromagnetic metal layers. Depending on the relative orientation of the magnetizations in the magnetic layers, the device resistance changes from small (parallel magnetizations) to large (antiparallel magnetizations). This change in resistance, also called magnetoresistance, is used to sense changes in magnetic fields. Recent efforts in GMR technology have also involved magnetic tunnel junction devices where the tunneling current depends on spin orientations of the electrodes.

Existing technologies such as GMR-based memory devices and spin valves are elementary spintronic applications. Here the role of spin is passive in limiting the value of resistance or tunneling current. Spin direction in this case is controlled by local magnetic fields. Spintronics go beyond passive spin devices, and introduce applications, and possibly whole new technologies, based on the active control of spin dynamics. Such active control of spin dynamics is predicted to lead to novel quantum-mechanical enabling technologies. The future devices are spin transistors, spin filters and modulators, new memory devices, and perhaps eventually quantum information processing and quantum computation. The possibility to integrate magnetic, optical, and electronic applications on a single device, where magnetic field and polarized light control spin dynamics, is an exciting new spintronic prospect for creating novel magneto-electro-optical technology. Two important physical principles in spintronics are the quantum mechanical nature of spin as a dynamical variable and the long relaxation or coherence time associated with spin states. The first one is leading to the possibility of novel spintronic quantum devices not feasible within the present-day
charge-based electronics. It is important for developing spintronics applications that carrier spin in semiconductors can be manipulated by using local magnetic fields, by applying external electric fields through controlled gates, and even by shining polarized light.

Let illustrate the generic spintronic scheme on a prototypical device, the Datta-Das spin field-effect transistor [18], shown in Fig. 2.1.

![Scheme of the Datta-Das spin field-effect transistor (SFET) [19].](image)

**Fig. 2.1.** Scheme of the Datta-Das spin field-effect transistor (SFET) [19].

White circles describe behavior of electron spin under the different transport conditions (ON is open case and OFF is closed case). Dashed lines show the direction of precession of the electron magnetic moment.

The scheme shows the structure of the usual FET, with a drain, a source, a narrow channel, and a gate for controlling the current. The gate either allows the current to flow (ON) or does not (OFF). Also spin transistor control the charge current through the narrow channel. The difference is in the physical realization of the current control. In the Datta-Das SFET the source and the drain are ferromagnets acting as the injector and detector of the electron spin. The source injects electrons with spins parallel to the transport direction. The electrons are transported ballistically through the channel. When they arrive at the drain, their spin is detected. In a simplified picture, the electron can enter the drain (ON) if its spin points in the same direction as the spin of the drain. Otherwise it is scattered away (OFF). The role of the gate is to generate an
effective magnetic field in the direction of Ω in Fig. 2.1. This is arising from the spin-orbit coupling in the substrate material, from the confinement geometry of the transport channel, and the electrostatic potential of the gate. This effective magnetic field causes the electron spins to precess. By modifying the gate voltage, one can turn the precession either parallel or antiparallel (or anything between) to electron spin at the drain. Therefore the gate is effectively controlling the current.

Even though the name spintronics is rather new, research in spintronics relies closely on a long tradition of results obtained in different branches of physics (magnetism, semiconductor physics, superconductivity, optics) and establishes new connections between its different subfields [20, 21]. The recent researches, often described as magnetoelectronics, typically covers paramagnetic and ferromagnetic metals and insulators, which utilize magnetoresistive effects. It can be realized, for example, as magnetic read heads in computer hard drives, nonvolatile magnetic random access memory (MRAM), and circuit isolators [22]. These more established aspects of spintronics have also been addressed in several books [23-27]. Spintronics also benefits from a large class of emerging materials, such as ferromagnetic semiconductors [28, 29], organic semiconductors [30], organic ferromagnets [31-32], high-temperature superconductors [33], and carbon nanotubes [34, 35], which can bring novel functionalities to the traditional devices. There is a continuing need for fundamental studies before the potential of spintronic applications can be fully realized.
2.1. Mechanisms of ferromagnetism

Fig. 2.2 shows some of the operative mechanisms for magnetic ordering in dilute magnetic semiconductor (DMS) materials.

Fig. 2.2 (A, B). Semiconductor matrix with high concentrations of randomly distributed magnetic impurities (i.e. Mn) (triangles in the figure) can be insulator (A) for group II–VI materials where divalent Mn ions occupy group II sites. At high concentrations, Mn ions are antiferromagnetically coupled, but at dilute limits, atomic distances between magnetic ions are large, and antiferromagnetic coupling is weak. For the cases where there is high concentrations
of carriers (B) (i.e. (Ga, Mn)As where Mn ions behave as acceptors and provide magnetic moment as Mn occupy trivalent Ga sites), the carriers are thought to mediate ferromagnetic coupling between magnetic ions (“carrier mediated ferromagnetism”). Red and blue balls are atoms (Ga and As) of semiconductor matrix.

Fig. 2.2 (C, D). When the concentration is low, hole carriers are localized near the magnetic impurity. This happens near the metal – insulator transition, in the insulator side. Below certain temperature, a percolation network (C) is formed. Percolation limit it is a “collective” effects in magnetic
ordering. Clusters of holes are delocalized and hop from site to site. Energetically favors maintaining the carriers’ spin orientation during the process, an effective mechanism for aligning Mn moments within the cluster network. Alternatively, at percolation limits, localized hole near the magnetic impurity is polarized, and the energy of the system is lowered when the polarization of the localized holes are parallel (D) [36]. Gimlets in the figure mean magnetic moments of free carriers (holes).

Two basic approaches to understand the magnetic properties of dilute magnetic semiconductors have emerged. The first class is based on mean-field theory which originates in the original model of Zener [37]. In the mean-field approach it is supposed that field in every point of the sample is equal to some mean-field value. In these theories is assumed that the dilute magnetic semiconductor is a more-or-less random alloy, e.g. (Ga,Mn)N, in which Mn substitutes for one of the lattice constituents. Within these theories, there are differences in how the free carriers are assumed to interact, as shown in Fig. 2.3.

**Fig. 2.3.** Schematic of role of carriers (holes) in the various theories for carrier-induced ferromagnetism in dilute magnetic III–V semiconductors [36].

The second class of approaches suggests that the magnetic atoms form small, a few atoms, clusters that make the observed ferromagnetism [38]. It is experimentally difficult to verify the mechanism responsible for the observed magnetic properties because depending on the growth conditions. It is possible to produce samples of all
kinds, meaning single-phase random alloys, nanoclusters of the magnetic atoms, precipitates and formation of second phase. Therefore, it is necessary to decide on a case-by-case basis which mechanism is applicable. This can only be achieved by careful correlation of the measured magnetic properties with materials analysis methods that are capable of detecting other phases or precipitates. If, for example, the magnetic behavior of the DMS is similar to a known ferromagnetic phase (such as MnGa or Mn₄N in (Ga,Mn)N), then the mean-field models are not applicable. Most experimental reports concerning room temperature ferromagnetism in DMS use X-ray diffraction, selected-area diffraction patterns (SADP), transmission electron microscopy (TEM), and photoemission or X-ray absorption to determine whether the magnetic atoms are substituting one lattice constituent to form an alloy. Given the concentration of the magnetic atoms, it is often very difficult to categorically determine the origin of the ferromagnetism. Magnetic measurements may be insufficient to exclude any ferromagnetic intermetallic compounds as the source of magnetic signals. Even the presence of so called anomalous Hall effect (AHE) may be insufficient to characterize a DMS material. It is also possible that magnetically-active clusters or second phases could be present in a pseudorandom alloy and therefore several different mechanisms could contribute to the observed magnetic behavior. There is major opportunity for the application of new, element- and lattice position-specific analysis techniques, such as the various scanning tunneling microcopies and Z-contrast scanning transmission electron microscopy (Z-contrast STEM) of ferromagnetism in the new DMS materials.

In the mean-field approach the ferromagnetic interaction between the local moments of the Mn atoms is mediated by free holes in the material. The spin–spin coupling (interaction between electrons’ spins) is assumed to be a long-range interaction, allowing use of a mean-field approximation [39-41]. In its basic form, this model employs a virtual-crystal approximation to calculate the effective spin-density due to the Mn ion distribution. The direct Mn–Mn interaction is antiferromagnetic in the absence of free carriers but may be ferromagnetic in their presence. The Curie temperature, $T_C$, for a given material with a specific Mn concentration and hole density, is determined by a competition between the ferromagnetic and antiferromagnetic interactions. In the presence of carriers, $T_C$ is given by the expression [39, 41]
\[ T_C = \left[ \frac{N_0 X_{\text{eff}} S(S + 1)\beta^2 A_F P_S (T_C)}{12 k_B} \right] - T_{AF}, \] (2.1)

where \( N_0 X_{\text{eff}} \) is the effective spin concentration, \( S \) the localized spin state, \( \beta \) the p–d exchange integral, \( A_F \) the Fermi liquid parameter, \( P_S \) the total density of states, \( k_B \) is Boltzmann’s constant and \( T_{AF} \) describes the contribution of antiferromagnetic interactions. Numerous refinements of this approach have appeared recently, taking into account the effects of positional disorder [43, 44], indirect exchange interactions [45], spatial inhomogeneities and free-carrier spin polarization [46, 47].

A further issue that needs additional exploration in the theories is the role of electrons, rather than holes, in stabilizing the ferromagnetism in DMS materials. All reports of ferromagnetism in (Ga, Mn)N, for example, occur for material that is actually n-type. Since the material must be grown at relatively low temperatures, to avoid Mn precipitation, only molecular beam epitaxy (MBE) can be used. Therefore there is always a possibility of unintentional n-type doping from nitrogen vacancies, residual lattice defects or impurities such as oxygen. In this case stoichiometric effects, crystal defects or unintentional impurities may control the final conductivity, rather than Mn or the intentionally-introduced acceptor dopants. Once again, this is much less of an issue in materials such as GaAs, whose low temperature growth is relatively well understood and controlled.

2.2. Spin-polarized transport and magnetoresistive effects

In a pioneering work, Mott [48, 49] provided a basis for our understanding of spin-polarized transport. Mott sought an explanation for an unusual behavior of resistance in ferromagnetic metals. He realized that at sufficiently low temperatures, where magnon scattering becomes vanishingly small, electrons of majority and minority spin, with magnetic moment parallel and antiparallel to the magnetization of a ferromagnet, respectively, do not mix in the scattering processes. Magnons are waves (oscillations) of localized spins in crystal caused by phonons. The conductivity can then be expressed as a sum of two independent and unequal parts. This way the current in
ferromagnets is spin polarized. This is also known as the two-current model and has been extended by Campbell [50] and Fert and Campbell [51]. In its modifications is provided an explanation for various magnetoresistive phenomena [52]. Tunneling measurements played a key role in early experimental work on spin-polarized transport. Studying N/F/N junctions, where N was a nonmagnetic metal and F was an Eu-based ferromagnetic semiconductor [53, 54], revealed that I-V curves could be modified by an applied magnetic field [55] and now show potential for developing a solid-state spin filter. When unpolarized current is passed across a ferromagnetic semiconductor, the current becomes spin-polarized [56, 57].

A series of experiments [58-60] in ferromagnetic / insulator / superconductor (F/I/S) junctions has unambiguously proved that the tunneling current remains spin polarized even outside of the ferromagnetic region [61]. The Zeeman-split quasiparticle density of states in a superconductor [59, 62] was used as a detector of spin polarization of conduction electrons in various magnetic materials. Julliére [63] measured tunneling conductance of F/I/F junctions, where I was an amorphous Ge. By adopting the Tedrow and Meservey [58, 59] analysis of the tunneling conductance from F/I/S to the F/I/F junctions, Julliére [63] formulated a model for a change of conductance between the parallel (↑↑) and antiparallel (↑↓) magnetization in the two ferromagnetic regions F1 and F2, as described in Fig. 2.4.
**Fig. 2.4.** Schematic illustration of electron tunneling in ferromagnet / insulator / ferromagnet (F/I/F) tunnel junctions: (a) Parallel and (b) antiparallel orientation of magnetizations. Dashed lines depict spinconserved tunneling [19].

The corresponding tunneling magnetoresistance (TMR) in an F/I/F magnetic tunnel junction (MTJ) is defined as

\[
TMR = \frac{\Delta R}{R_{\uparrow\uparrow}} = \frac{R_{\downarrow\downarrow} - R_{\uparrow\uparrow}}{R_{\uparrow\uparrow}} = \frac{G_{\uparrow\uparrow} - G_{\downarrow\downarrow}}{G_{\downarrow\downarrow}},
\]

(2.2)

where conductance \( G \) and resistance \( R = 1/G \) are labeled by the relative orientations of the magnetizations in F1 and F2. It is possible to change the relative orientations, between \( \uparrow\uparrow \) and \( \downarrow\downarrow \), even at small applied magnetic fields ~ 1 mT. TMR is a particular manifestation of a magnetoresistance that yields a change of electrical resistance in the presence of an external magnetic field [64, 65]. Historically, the anisotropic magnetoresistance in bulk ferromagnets such as Fe and Ni was discovered first, dating back to the experiments of Lord Kelvin [66]. Due to spin-orbit interaction, electrical resistivity changes with the relative direction of the charge current (for example, parallel or perpendicular) with respect to the direction of magnetization.
Within Jullie`re’s model [63], which assumes constant tunneling matrix elements and that electrons tunnel without spin flip, Eq. 2.2 yields

\[ TMR = \frac{2P_1P_2}{1-P_1P_2}, \]  

(2.3)

where the polarization \( P_i = (N_{M_i} - N_{m_i})/(N_{M_i} + N_{m_i}) \) is expressed in terms of the spin-resolved density of states \( N_{M_i} \) and \( N_{m_i} \), for majority and minority spin in the region of F1 and F2, respectively (see Fig. 2.4). Conductance in Eq. 2.2 can then be expressed as \([67] G_{\uparrow\uparrow} \approx (N_{M_1}N_{M_2} + N_{m_1}N_{m_2}) \) and \( G_{\uparrow\downarrow} \approx (N_{M_1}N_{m_2} + N_{m_1}N_{M_2}) \) to give Eq. 2.3.

While the early results of Jullie`re [63] were not confirmed, TMR at 4.2 K was observed using NiO as a tunnel barrier by Maekawa and Gäfvert [67].

The prediction of Jullie`re’s model illustrates the spinvalve effect: the resistance of a device can be changed by manipulating the relative orientation of the magnetizations \( M_1 \) and \( M_2 \), in F1 and F2, respectively. Such orientation can be preserved even in the absence of a power supply, and the spin-valve effect, later discovered in multilayer structures displaying the giant magnetoresistance (GMR) effect [68, 69] can be used for nonvolatile memory applications [23, 24, 70]. GMR structures are often classified according to whether the current flows parallel (CIP, current in plane) or perpendicular (CPP, current perpendicular to the plane) to the interfaces between the different layers, as shown in Fig. 2.5.

![Fig. 2.5. Schematic illustration of (a) the current in plane (CIP), (b) the current perpendicular to the plane (CPP) of giant magnetoresistance geometry [19].](image-url)
Most of the GMR applications use the CIP geometry, while the CPP version, first realized by Pratt [71], is easier to analyze theoretically [72, 25] and relates to the physics of the tunneling magnetoresistance effect [73]. The size of magnetoresistance in the GMR structures can be expressed analogously to Eq. 2.2, where parallel and antiparallel orientations of the magnetizations in the two ferromagnetic regions are often denoted by “P” and “AP,” respectively (instead of ↑↑ and ↑↓). Realization of a large roomtemperature GMR [74, 75] enabled a fast transition from basic physics to commercial applications in magnetic recording [76].

One of the keys to the success of magnetoresistance based applications is their ability to control [70, 77, 78] the relative orientation of $M_1$ and $M_2$. An interesting realization of such control was proposed independently by Berger [79] and Slonczewski [80]. While in GMR or TMR structures the relative orientation of magnetizations affect the flow of spin-polarized current, they predicted a reverse effect. The flow of spin-polarized current can transfer angular momentum from carriers to ferromagnet and alter the orientation of the corresponding magnetization, even in the absence of an applied magnetic field. This phenomenon, known as spin-transfer torque, has since been extensively studied both theoretically and experimentally [81-84], and current-induced magnetization reversal has been demonstrated at room temperature [85]. It was also shown that the magnetic field generated by passing the current through a CPP giant magnetoresonance device could produce roomtemperature magnetization reversal [86]. In the context of ferromagnetic semiconductors additional control of magnetization was demonstrated optically, by shining light [87, 88] and electrically, by applying gate voltage [89, 90] to perform switching between the ferromagnetic and paramagnetic states.

Jullie’s model also justifies the continued quest for highly spin-polarized materials. They would provide large magnetoresistive effects, desirable for device applications. In an extreme case, spins would be completely polarized even in the absence of magnetic field. Numerical support for the existence of such materials, so-called half-metallic ferromagnets, was provided by de Groot, Janner, and Mueller, and these materials were reviewed by Pickett and Moodera [91]. Half-metallic ferromagnets behave near the Fermi level as metals only for one spin, the density of states vanishes completely for the other spin. In addition to ferromagnets, such as CrO$_2$ [92]; and
manganite perovskites [93], there is evidence for high spin polarization in III-V ferromagnetic semiconductors like (Ga,Mn)As [94]. The challenge remains to preserve such spin polarization above room temperature and in contacts with other materials, since the surface (interface) and bulk magnetic properties can be significantly different [95, 96]. While many existing spintronic applications [23, 24] are based on the GMR effects, the discovery of large room-temperature TMR [97] has renewed interest in the study of magnetic tunnel junctions, which are now the basis for several magnetic random-access memory prototypes [98, 99]. Future generations of magnetic read heads are expected to use magnetic tunnel junctions (MTJ) instead of CIP giant magnetoresonance. To improve the switching performance of related devices it is important to reduce the junction resistance, which determines the RC time constant \( \tau = \frac{1}{RC} \), where \( R \) is resistance, \( C \) is capacitance) of the MTJ cell. Consequently, semiconductors, which would provide a tunneling barrier lower than the usually employed oxides, are being investigated both as the nonferromagnetic region in MTJ's and as the basis for an all-semiconductor junction that would demonstrate large TMR at low temperatures [100, 101]. Another desirable property of semiconductors has been demonstrated by the extraordinary large room-temperature magnetoresistance in hybrid structures with metals, reaching 750 000% at a magnetic field of 4 T [102], which could lead to improved magnetic read heads [103]. Magnetoresistance effects of similar magnitude have been found also in hybrid metal / semiconductor granular films [104]. Another approach to obtaining large room-temperature magnetoresistance (>100% at B ~ 0.01 T) is to fabricate ferromagnetic regions separated by a nanosize contact. For simplicity, such a structure could be thought of as the limiting case of the CPP giant magnetoresonance scheme in Fig. 2.5(b). This behavior, also known as ballistic magnetoresistance, has already been studied in a large number of materials and geometries [105-107].
2.3. Spin injection and optical orientation

Many materials in their ferromagnetic state can have a substantial degree of equilibrium carrier spin polarization. However, as illustrated in Fig. 2.1, this alone is usually not sufficient for spintronic applications, which typically require current flow and/or manipulation of the nonequilibrium spin (polarization). The importance of generating nonequilibrium spin is not limited to device applications; it can also be used as a sensitive spectroscopic tool to study wide variety of fundamental properties ranging from spin-orbit and hyperfine interactions [109] to the pairing symmetry of high-temperature superconductors [110, 111] and the creation of spin-polarized beams to measure parity violation in high-energy physics [112].

Nonequilibrium spin is the result of some source of pumping arising from transport, optical, or resonance methods. Once the pumping is turned off the spin will return to its equilibrium value. While for most applications it is desirable to have long spin relaxation times, it has been demonstrated that short spin relaxation times are useful in the implementation of fast switching [113].

Electrical spin injection, an example of a transport method for generating nonequilibrium spin, has already been realized experimentally by Clark and Feher [114], who drove a direct current through a sample of InSb in the presence of a constant applied magnetic field. The principle was based on the Feher effect [115], in which the hyperfine coupling between the electron and nuclear spins, together with different temperatures representing electron velocity and electron spin populations, is responsible for the dynamical nuclear polarization [116]. Motivated by the work of Clark and Feher [114] and Tedrow and Meservey [58, 59] and the principle of optical orientation [109], Aronov [117], and Aronov and Pikus [118] established several key concepts in electrical spin injection from ferromagnets into metals, semiconductors, and superconductors. Aronov [117] predicted that, when a charge current flowed across the F/N junction (Fig. 2.6), spin-polarized carriers in a ferromagnet would contribute to the net current of magnetization entering the nonmagnetic region. Carriers would lead to nonequilibrium magnetization \( \delta M \), shown in Fig. 2.6(b), with the spatial extent given by the spin diffusion length [117, 118].
Fig. 2.6. Illustration of the concept of electrical spin injection from a ferromagnet (F) into a normal metal (N). Electrons flow from F to N: (a) schematic device geometry; (b) magnetization $M$ as a function of position. Nonequilibrium magnetization $\delta M$ (spin accumulation) is injected into a normal metal; (c) contribution of different spin-resolved densities of states to both charge and spin transport across the F/N interface. Unequally filled levels in the density of states describe spin-resolved electrochemical potentials different from the equilibrium value $\mu_0$ [19].

Such a $\delta M$, which is also equivalent to a nonequilibrium spin accumulation, was first measured in metals by Johnson and Silsbee [119, 120]. In the steady state $\delta M$ is realized as the balance between spins added by the magnetization current and spins removed by spin relaxation.

Generation of nonequilibrium spin polarization and spin accumulation is possible also by optical methods known as optical orientation or optical pumping. In optical orientation, the angular momentum of absorbed circularly polarized light is transferred to the medium. Electron orbital moment is directly oriented by light and through spin-orbit interaction electron spins become polarized. In a pioneering work Lampel [121] demonstrated that spins in silicon can be optically oriented (polarized). This technique is derived from the optical pumping proposed by Kastler [122] in which optical irradiation changes the relative populations within the Zeeman and hyperfine levels of the ground states of atoms. There are similarities with previous studies of free atoms.
but optical orientation in semiconductors has important differences related to the strong coupling between the electron and nuclear spin and the macroscopic number of particles [125, 109]. Polarized nuclei can exert large magnetic fields (~5 T) on electrons. In bulk III-V semiconductors, such as GaAs, optical orientation can lead to 50% polarization of electron density. This could be further enhanced in quantum structures of reduced dimensionality or by applying a stress. A simple reversal of the polarization of the illuminating light (from positive to negative helicity) also reverses the sign of the electron density polarization. Such properties of optical orientation in semiconductors allow getting a negative electron affinity. In this case photoemission of spindipolarized electrons can use as a powerful detection technique in high-energy physics and for investigating surface magnetism [112].

2.4. Materials considerations

Nominally highly spin-polarized materials could provide both effective spin injection into nonmagnetic materials and large magnetoresistance effects, important for nonvolatile applications. Examples include half-metallic oxides such as CrO$_2$, Fe$_3$O$_4$, CMR materials, and double perovskites [91, 126]. Ferromagnetic semiconductors [53], known since CrBr$_3$ [127], have been demonstrated to be highly spin polarized. However, more recent interest in ferromagnetic semiconductors was spurred by the fabrication of (III,Mn)V compounds because of the good technological opportunities for manufacturing of high grade devices on their base. After the initial discovery of (In,Mn)As [128, 129], most of the research has focused on (Ga,Mn)As [130, 131]. In contrast to (In,Mn)As and (Ga,Mn)As with high carrier density (~$10^{20}$ cm$^{-3}$), a much lower carrier density in (Zn,Cr)Te [132], a II-VI ferromagnetic semiconductor with Curie temperature $T_C$ near room temperature [133], suggests that transport properties can be effectively controlled by carrier doping. Most of the currently studied ferromagnetic semiconductors are p-doped with holes as spin-polarized carriers, which typically leads to lower mobilities and shorter spin relaxation times than in n-doped materials. It is possible to use selective doping to substantially increase $T_C$, as compared to the uniformly doped bulk ferromagnetic semiconductors [134].
Early work on (Ga,Mn)As [135] showed low solubility of Mn and the formation of magnetic nanoclusters characteristic of many subsequent compounds and different magnetic impurities. Presence of such nanoclusters often complicates accurate determination of $T_C$ as well as of whether the compound is actually in a single phase. Consequently, the reported room-temperature ferromagnetism in an increasing number of compounds [28] is not universally accepted. Conclusive evidence for intrinsic ferromagnetism in semiconductors is highly nontrivial. For example, early work reporting ferromagnetism even at nearly 900 K in La-doped CaBa$_6$ [136], was later revisited suggesting extrinsic effect [137].

High $T_C$ and almost complete spin polarization in bulk samples are not enough for successful applications. Spintronic devices typically rely on inhomogeneous doping, structures of reduced dimensionality, and/or structures containing different materials. Interfacial properties, as discussed in the previous sections, can significantly influence the magnitude of magnetoresistive effects and the efficiency of spin injection. Doping properties and the possibility of fabricating wide range of structures allow spintronic applications beyond magnetoresistance effects, for example, spin transistors, spin lasers, and spin-based quantum computers. Materials properties of hybrid F/Sm heterostructures, relevant to device applications, were reviewed by Samarth et al. [138].

Different type of photoinduced magnetization was measured in ferromagnetic (Ga,Mn)As. In Faraday geometry, by changing the polarization of a circularly polarized light, one can modulate the Hall resistance and thus the induced magnetization by up to 15% of the saturation value [88]. Additional experiments on photoinduced magnetization rotation [139] are realized by generating an effective magnetic field through the p-d exchange interaction, rather than by spin-transfer torque [140]. In GaAs-Fe composite films an observation of room-temperature photoenhanced magnetization was used to demonstrate that a magnetic force can be changed by light illumination [141].

Subsequent work by Park et al. [142] showed that ferromagnetism can be manipulated in MnGe at higher temperature and at significantly lower gate voltage (at $\sim$50 K and $\sim$1 V). The combination of light and electric-field control of ferromagnetism was used
in modulation-doped p-type (Cd,Mn)Te quantum wells [90]. It was demonstrated that illumination by light in p-i-n diodes would enhance the spontaneous magnetization, while illumination in p-i-p structures would destroy ferromagnetism.

In semiconductors $g$ factors can be very different from the free-electron value. This $g$ factor determine the spin splitting of carrier bands and consequently influence the spin dynamics and spin resonance. With strong spinorbit coupling in narrow-band III-V’s $g$ factors are $\approx -50$ for InSb and $\approx -15$ for InAs, while doping with magnetic impurities can give even $|g|\sim 500$. Manipulation of the $g$ factor in a GaAs / AlGaAs quantum well relies on the results for a bulk Al$_x$Ga$_{1-x}$As; the variation of Al concentration changes the $g$ factor [143, 144] to $g = -0.44$ for $x = 0$ and $g = 0.40$ for $x = 0.3$. Related experiments on modulation-doped GaAs/Al$_{0.3}$Ga$_{0.7}$As have shown that by applying gate voltage $V_G$ one can shift the electron wave function in the quantum well and produce $\sim 1\%$ change in the $g$ factor [145]. Subsequently, in an optimized Al$_x$Ga$_{1-x}$As quantum well, where $x$ varied gradually across the structure, much larger changes were measured. When $V_G$ is changed, the electron wave function efficiently senses different regions with different $g$ factors [146].

In spite of the great current interest in the basic principles and concepts of spintronics a large number of obstacles need to be overcome before one can manufacture spintronics applications. For example, a basic spintronics transport requirement is to produce and sustain large spin-polarized currents in electronic materials (semiconductors) for long times. This has not yet been accomplished. In fact, it has turned out to be problematic to introduce spin-polarized carriers in any significant amount into semiconductor materials. Similarly, for quantum computation one requires significant and precisely controllable spin entanglement as well as single spin (i. e., single Bohr magneton) manipulation using local magnetic fields. Currently there is no good idea about how to accomplish this. It is clear that a great deal of basic fundamental physics research is needed before spintronics applications become a reality.
3. Galvanomagnetic effects in the range of classical magnetic fields

3.1. Magnetoresistance

Magnetoresistance (magnetoresistive effect) is the change of electrical resistivity of a material in a magnetic field. In semiconductors the relative change of resistivity is bigger than in metals and can achieve hundreds of percents.

Let the current \( \vec{J} \) flows in the sample along axis \( x \). In the absence of magnetic field the charge carriers are moving along straight lines and between two collisions they pass a distance defined as free path \( \lambda \).

In the external magnetic field \( B \) their trajectories represent a part of cycloid with the length \( \lambda \) in an infinite large specimen. On the free path along the electric field \( E \) the particle will move the way shorter than \( \lambda \), namely

\[
\lambda_s \approx \lambda \cos \phi \approx \lambda \left( 1 - \frac{\mu^2 B^2}{2} \right),
\]

(3.1)

where \( B \) is magnetic field, \( \mu \) is electron mobility.

For the time of free path \( \tau \) the particle moves along the shorter way parallel to the electric field \( E \) and it is equivalent to the decreasing of drift velocity (or mobility); by other words the resistivity will increase. Taking into account statistic dispersion of free path times (and lengths),

\[
\frac{\Delta \rho}{\rho} = \mu^2 B^2.
\]

(3.2)

In a finite sample the Hall field compensates the influence of magnetic field and as a result, the charge carriers move along the straight lines, therefore magnetoresistance should not exist. However velocities of electrons and holes are different and magnetic field influences stronger to the fast particles than Hall field, i.e. slow particles deviate
under the influence of the Hall field. Consequence of dispersion in particles’ velocities leads to decreasing of the contribution to the conductivity from fast carriers, and this leads to the increasing of resistivity.

If magnetic field is directed along \( \mathbf{J} \), changing in resistivity can not take place. But in some materials the magnetoresistance is observed, which can be explained by the complex shape of Fermi surfaces in these compounds.

### 3.2. The Hall effect

If an electric current flows along the \( x \) direction and a magnetic field is applied in the \( z \) direction, then an electric field is produced in the \( y \) direction. This phenomenon is called the Hall effect.

The Hall effect is due to the nature of the current flow in conductor in magnetic field. Current consists of charge-carrying particles (typically electrons), which experience a force (called the Lorentz force) in the presence of a magnetic field. When perpendicular magnetic field is absent, there is no Lorentz force and electron follows an approximate 'line of sight' or free path. When perpendicular magnetic field is applied, the free path is curved perpendicular to the magnetic field due to the Lorentz force. The result is an asymmetric charge distribution across the conductor perpendicular to the direction of electric current. This charge distribution would not take place in the absence of the magnetic field. As a result, it is generated an electric field, which compensates the influence of the Lorentz force.

Let us denote by \( \mathbf{J} \) the current density, by \( \mathbf{B} \) the inductance of magnetic field, by \( \mathbf{E}_{\text{Hall}} \) the strength of the electric field (see Fig. 3.1) and by \( R_{\text{Hall}} \) the Hall constant (characteristic of the material of the conductor). In this case

\[
\mathbf{E}_{\text{Hall}} = R_{\text{Hall}} \cdot \left[ \mathbf{J} \times \mathbf{B} \right],
\]

(3.3)

where \( R_{\text{Hall}} \) is a characteristic of the material of the sample.
Since the angle between $\vec{J}$ and $\vec{B}$ is 90°, therefore $\left[\vec{J} \times \vec{B}\right] = J_x \cdot B_z$, and

$$E_{\text{Hall}} = \frac{U_{\text{Hall}}}{w} = R_{\text{Hall}} \cdot \frac{I_x}{wd} \cdot B_z = \Rightarrow U_{\text{Hall}} = R_{\text{Hall}} \cdot \frac{I_x}{d} \cdot B_z,$$

(3.4)

where $U_{\text{Hall}} = E_{\text{Hall}} \cdot w$ is the Hall voltage, $I = J \cdot w \cdot d$ is electric current through the sample, $w$ is width of the sample and $d$ is thickness of the sample. It can be seen that the Hall resistance increases linearly with magnetic field.

The Hall coefficient can be estimated as

$$R_{\text{Hall}} = \frac{U_{\text{Hall}} \cdot d}{I_x B_z}.$$

(3.5)
If the current in a conductor represents the charge carrier stream, then Hall constant is equal to \( R_{\text{Hall}} = \frac{1}{q n_{\text{Hall}}} \), where \( n_{\text{Hall}} \) is the density of free charge carriers and \( q \) is their charge.

The anomalous Hall effect (AHE) may be observed in the samples with the magnetic ordering. Classical Hall resistance \( R_{xy} \) is proportional to the external magnetic field which is equal to the internal one in the absence of magnetic impurity (\( R_{xy} \sim B \)). When sample exhibits the pronounced magnetic properties the AHE takes place and \( R_{xy} \) is proportional to the internal magnetic field, which is the sum of the external magnetic field and magnetization of the sample.

\[
R_{xy}^a \sim (B + M), \tag{3.6}
\]

where \( M \) is magnetization and \( R_{xy}^a \) is a constant for anomalous Hall effect.

Since \( M \) has hysteresis, the anomalous Hall resistance also would exhibit the same behavior.
4. Galvanomagnetic effects in the quantizing magnetic fields

4.1. The Shubnikov – de Haas effect

Energy spectrum of charge carriers in the solid state materials is quantized when applying of strong magnetic field and density of states of charge carriers demonstrates oscillating behavior vs. energy when changing the magnetic field.

Shubnikov – de Haas (SdH) oscillations are periodical changing of magnetoresistance due to the changing charge carrier’s density of states on the Fermi level if the intensity of magnetic field is changed.

Landau described this phenomenon in the frames of quantum mechanics [147]. Nature of quantization for charge carrier’s density of states is presented within the bounds of isotropic quadratic law of dispersion for free electrons. Analysis shows that in magnetic field the energy of charge carriers can be described by expression:

\[ \varepsilon_{N_mk_z} = (N_m + 1/2)\hbar \omega_c + \frac{\hbar^2 k_z}{2m}, \]  

(4.1)

where electron frequency \( \omega_c = eB/m_e \); \( m_e \) is cyclotron mass of electron (or hole), \( N_m \) is integer and \( k_z \) is electron’s (or hole’s) wave vector component along the axis \( \hat{Z} \) with magnetic field parallel to axis \( \vec{Z} \). The first item in Eq. 4.1 is discrete variable of energy of electron (hole) motion in the plane perpendicular to magnetic field direction. Second term is the energy of continuous electron (hole) motion along \( \vec{Z} \) – axis. Thus, three-dimensional zone in \( k \) – space with quasicontinuous energy levels distribution splits to a number of one dimensional magnetic sub bands, co called Landau levels. This is a result of energy quantization of charge carrier’s orbital motion in the plane perpendicular to the magnetic field direction. The distance between energy sub bands is equal to cyclotron energy \( \hbar \omega_c \). The level with \( N_m = 0 \) is situated \( \frac{\hbar \omega_c}{2} \) above the conduction band without magnetic field (see Fig. 4.1).
Fig. 4.1. Energy sub bands of electron in magnetic field $B = B_Z$.

Distribution of charge carrier’s density of states $\rho(E)$ in quantizing magnetic field starts to depend on the magnetic field:

$$
\rho(E) = \frac{m^{3/2}}{\sqrt{2\pi^2h^3}} \frac{\hbar\omega_c}{2} \sum_{N_m=0}^{N_{\text{max}}} \left[ E - (N_m + 1/2)\hbar\omega_c \right]^{-3/2}.
$$

Discontinuous character of function $\rho(E)$ close to points $E = (N_m + 1/2)\hbar\omega_c$ leads to the non monotonic peculiarities of the transport properties (in particular of magnetoresistance) because density of states is infinite in the vicinity of the bottom of each Landau sub bands (Fig. 4.2).
Fig. 4.2. Density of states for electron $\rho(E)$ in magnetic field. Hatch shows density of states without magnetic field.

Fermi energy $E_F$ changes in magnetic field and is connected to Fermi energy without magnetic field

$$E_{F_0} = \left(\frac{\hbar^2}{2m}\right)(3\pi^2n)^{2/3} \quad (4.3)$$

by

$$\frac{2}{3} \left(\frac{E_{F_0}}{\hbar\omega_c}\right) = \sum_{N_m=0}^{N_{max}} [E - (N_m + 1/2)h\omega_c]^{1/2}. \quad (4.4)$$

Dependence $E_F(B)$ should be taken into account only for small values of $N_m$ ($N_m \leq 3$), where ratio $E/\hbar\omega_c$ is small (see Eq. 4.3).

Observation of SdH oscillations is possible under the following conditions:

$$\omega_c \tau >> 1, \quad (4.5)$$

$$\hbar\omega_c >> k_B T, \quad (4.6)$$

$$E_{F_0} > \hbar\omega_c. \quad (4.7)$$

Condition (4.5) means that the distance between Landau levels must be bigger than broadening of each level $\hbar/\tau$ (or $\mu B >> 1$). From (4.6) follows that distance between
Landau levels must be bigger than their thermal broadening at the temperature of experiment. Condition (4.7) shows the highest limit of magnetic field when the oscillations disappear. For observation of SdH oscillations degeneration of electron (hole) gas is necessary ($E_F \gg k_B T$, see (4.6), (4.7)).

Expression for longitudinal conductivity $G_{zz}$, showing SdH oscillations, if charge carriers dissipate on the acoustical phonons [148] at $\vec{B} (0,0, B_Z), \vec{J} (0,0, J_Z)$

$$\frac{G_{zz}}{G_0} = 1 - x \left( \frac{\hbar \omega_c}{2E_F} \right)^{1/2} \sum_{m=1}^{\infty} \frac{(-1)^m M^{1/2}}{sh(Mx)} \exp \left( - \frac{2\pi M}{\omega_c \tau'} \right) \cos \left( \frac{2\pi ME_F}{\hbar \omega_c} - \frac{\pi}{4} \right), \quad (4.8)$$

where $x = 2\pi^2 k_B T / \hbar \omega_c$, $\tau'$ is relaxation time, which characterizes no thermal broadening of the Landau levels and $sh$ is a function of hyperbolic sine.

Non thermal broadening of the Landau levels can be caused by some non homogeneity of investigated samples and by dispersion of charge carriers on the defects of crystal lattice. In some cases it is convenient instead of relaxation time to use effective temperature $T_D = \hbar / \pi k_B \tau'$, named Dingle temperature.

Calculation of transversal conductivity $G_{xx} = G_{cl} + G_1 + G_2$ was done in [149], where $G_{cl} = e^2 \hbar / m \tau^2 \omega_c^2$ is conductivity in the limit of classical magnetic field. Expressions in the case of dispersion on the acoustical phonons for the finite temperature and taking into account broadening levels are

$$\frac{G_1}{G_{cl}} = \frac{5x}{\sqrt{2}} \left( \frac{\hbar \omega_c}{E_F} \right)^{1/2} \sum_{m=1}^{\infty} \frac{(-1)^m M^{1/2}}{sh(Mx)} \exp \left( - \frac{2\pi M}{\omega_c \tau'} \right) \cos \left( \frac{2\pi ME_F}{\hbar \omega_c} - \frac{\pi}{4} \right), \quad (4.9)$$

$$\frac{G_2}{G_{cl}} = \frac{3\pi x}{8} \left( \frac{\hbar \omega_c}{E_F} \right) \sum_{m=1}^{\infty} \frac{(-1)^m M}{sh(Mx)} \exp \left( - \frac{2\pi M}{\omega_c \tau'} \right) \cos \left( \frac{2\pi ME_F}{\hbar \omega_c} - \frac{\pi}{2} \right). \quad (4.10)$$
Observations of the oscillations of the Hall coefficient $R_{\text{Hall}}$ are theoretically described by the contribution of $G_{XX}$ to $\rho_{XY}$, where $\rho_{XY} = \frac{U_{\text{Hall}} \cdot d}{I}$. Amplitudes of Hall coefficient oscillations should be small since they appear in theory only in the second order on dispersion.

As it is shown in [150], period of SdH oscillations $P_{\text{SdH}}$ is inversely proportional to extremal cross section $S_m$ of Fermi surface by the plane perpendicular to the direction of magnetic field and is equal to

$$P_{\text{SdH}} = \frac{2e}{\hbar S_m}. \quad (4.11)$$

That’s why it is possible to study topology of the Fermi surface of charge careers by investigation of the anisotropy of SdH oscillations’ period. For the isotropic quadratic dispersion law

$$S_m = \pi \cdot (3\pi^2 n)^{2/3}, \quad (4.12)$$

and period of SdH oscillations

$$P_{\text{SdH}} = \frac{2e}{\hbar} \cdot \frac{1}{(3\pi^2 n)^{2/3}} \quad (4.13)$$

shows that period of SdH oscillations depends only on the concentration of charge careers $n$.

For anisotropic quadratic dispersion Eq. 4.13 may be converted to

$$P_{\text{SdH}} = \frac{e\hbar}{cm_c E_F}, \quad (4.14)$$

where $m_c$ is cyclotron mass of charge carriers.
According to [151], in the case of isotropic dispersion law, positions of maximums of SdH oscillations are described by equation

\[
\frac{1}{B_{\text{MAX}}} = \frac{2e}{h(3\pi^2 n)^{2/3}} \left[ \left( M + \frac{1}{2} \right)^{2/3} - \left( \frac{1}{2} \right)^{2/3} \right]^{2/3},
\]

(4.15)

where \( M \) is natural number. Taking the ratio \((V/B_{\text{MAX}}) / P_{\text{SdH}}\) for different \( M \) one gets not the numbers 1,5; 2,5; 3,5; etc., but the consecution 1,33; 2,36; 3,38; etc. because of the dependence of \( E_F(B) \).

\( g \) – factor of free carriers can be found from the spin split of maximums or by using the equation for zero plus – maximum, which is given in [149]

\[
B_0' = \frac{\hbar}{e} \left[ \frac{4\pi^2 n^2}{|g|} \cdot \frac{m_0}{m_e} \right]^{1/2}.
\]

(4.16)

Estimation of cyclotron mass charge of carries, which take part in SdH oscillations, is done using Eq. 4.17 from the temperature dependence of amplitude of SdH oscillations and under the assumption of independence of \( T_D \) from temperature [152].

The term for the ratio of amplitudes of SdH oscillations at two temperatures \( T_1 \) and \( T_2 \) corresponds to the equation

\[
\frac{A_{T_1}}{A_{T_2}} = \frac{x_{1/sh} \cdot x_1}{x_{2/sh} \cdot x_2},
\]

(4.17)

where \( A_{T_i} \) are amplitudes of SdH oscillations at temperature \( T_i \) and

\[
x_i = \frac{2\pi^2 k_B}{\hbar e} \cdot \frac{m_e}{m_0} \cdot \frac{T_i}{B_i}.
\]

(4.18)
After solving Eq. 4.17 the cyclotron mass of charge carriers, which take part in SdH oscillations is found.

Amplitude of SdH oscillations is described by

\[ A \approx x \cdot \exp(-2\pi^2 k_B T_D \gamma \cdot \hbar \omega_c) \frac{(BE_F)^{1/2} \cdot \text{sh}(x)}{(BE_F)^{1/2} \cdot \text{sh}(x)}. \]  

(4.19)

Therefore, it is possible to find Dingle temperature from the inclination of the dependence \[ \ln\left(A(BE_F)^{1/2} \cdot \text{sh}(x)/x\right) \] vs. \(1/B\). Comparison of the Dingle temperature value calculated from Hall mobility

\[ T_{DH} = \frac{\hbar e}{\pi k_B m_e \mu}, \]  

(4.20)

and \(T_D\) shows whether or not the dispersion is a dominant reason in the broadening of Landau levels.

Finally it is possible to make a conclusion that experimental investigation of Shubnikov – de Haas effect is an efficient method for analysis of energy band structure of semiconductors. This method allows to study the topology of Fermi surface in the material and to estimate important parameters of energy band structure as tensor components of effective mass, \(g\) – factor, effective mass of density of states and, moreover, to evaluate the perfection degree of the grown crystal.

4.2. Quantum wells

A quantum well (QW) is a well of electric potential that confines particles. Particles are originally free to move in three dimensions, but in case when thickness of structure is much smaller than two others dimensions, these conditions force particles to occupy a planar region. The effects of quantum confinement take place when the QW thickness becomes comparable at the de Broglie wavelength of the carriers. This leads
to appearance of energy levels called “energy subbands”, i.e., the carriers can only have discrete energy values. Quantum wells are formed in semiconductor structures by growing a narrow-gap material (gallium arsenide, see Fig. 4.3) sandwiched between two layers of a material with a wider bandgap (aluminium gallium arsenide). In this case all carriers will be confined in the narrow-gap semiconductor.

An electron in the GaAs layer, represented in the Fig 4.3 by its wavefunction $\Psi$, can be considered to be partially confined in a quantum well of barrier height $\Delta E$. $\Delta E$ is the energy difference between the bottoms of the conducting band $E_C$ for the two layers materials (GaAs and Al$_x$Ga$_{1-x}$As). Situation is similar to one-dimensional particle-in-a-box problem of elementary quantum mechanics [153, 154]. In the limit as $\Delta E \to \infty$ electron energy states are quantized

$$E_n = \frac{n^2 \hbar^2}{8m^*L^2}, \quad (4.21)$$

where $\hbar$ is Planck’s constant, $m^*$ is the effective mass of the electron, $n$ is a quantum number, and $L$ is the width of the well. It is seen that $E_n$ is proportional to $L^{-2}$, and thus it is possible to change the energy states by varying the width of the well [155]. For the infinite-deep well, an electron has infinite set of discrete energy states.

Second important difference is the energy dependence of the electron density of states [154]. For a three-dimensional free electron solid, the density of states is proportional to $E^{3/2}$, but for QW it does not depend on energy.

Confinement of one motional degree of free carriers’ freedom is very interesting phenomenon. That is why quantum wires and quantum dots are becoming increasingly important since they yield two-dimensional and three-dimensional quantum confinement, respectively. Three-dimensional quantum- confinement effects in semiconductor micro-crystallites occur when the particle size approaches the bulk exciton Bohr radius [155].
Fig 4.3. GaAs layer between two $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layers shows quantum well behavior. $E_C$ is the energy of the bottom of the conduction band, $E_V$ is the energy of the top of the valance band, and $E_g$ is the band gap energy. The barrier height $\Delta E$ is the difference between the values of $E_C$ for $\text{Al}_x\text{Ga}_{1-x}\text{As}$ and GaAs [155].

4.3. The quantum Hall effect

At low temperatures in a two-dimensional metal or semiconductor the observed Hall effect demonstrates series of steps that appear in the Hall resistance instead of the monotonic increase as a function of magnetic field. Moreover, these steps occur at incredibly precise values of resistance and it does not matter what sample is investigated. The resistance is quantized in units of $\frac{\hbar}{e^2}$ divided by an integer. This phenomenon got a name of the quantum Hall effect (QHE).
In Fig. 4.4 is shown the integer quantum Hall effect (QHE) in a GaAs-Al_xGa_{1-x}As heterojunction at 30 mK. The QHE can be seen at liquid helium temperatures, but in the millikelvin temperatures the plateaus are much wider. The diagonal component of resistivity is also presented, which shows regions of zero resistance corresponding to each QHE plateau. In this figure the plateau index is, from top right to left down, 1, 2, 3, 4, 6, 8,... Odd integers correspond to the Fermi energy being in a spin gap and even integers to an orbital Landau level (LL) gap. As the spin splitting is small compared to LL gaps, the odd integer plateaus are only seen in the highest magnetic fields.

The zeroes and plateaus in the two components of the resistivity tensor are intimately connected and both can be understood in terms of the Landau levels formed in a magnetic field.
In the absence of magnetic field, the density of states in the 2D object is constant as a function of energy. In a magnetic field the available states clump into Landau levels separated by the cyclotron energy, with the regions of energy between the LLs where states are not allowed. When the magnetic field increases the LLs move relatively to the Fermi energy.

When the Fermi energy lays in a gap between LLs, electrons can not move to new states and so there is no scattering. Thus the transport has no dissipation and the resistance falls to zero.

Classical Hall resistance was given by $\frac{B}{ne}$. In QHE the number of current carrying states in each Landau level is $\frac{eB}{\hbar}$, so when there are $i$ Landau levels at energies below the Fermi energy, filled with $\frac{ieB}{\hbar}$ electrons, the Hall resistance is $\frac{\hbar}{ie^2}$. Integer filling factor is exactly the same as in the classical case.

The difference in the QHE is that the Hall resistance can not be changed from the quantized value when the Fermi energy is in a gap, i.e. between the fields (a) and (b) in the diagram (see Fig. 4.5), and so a plateau results. Only when case (c) occurs and the
Fermi energy is inside the Landau level, the Hall voltage can be changed and a finite value of resistance appear.

This picture has assumed a fixed Fermi energy, i.e. fixed carrier density, and a changing magnetic field. The QHE can also be observed by fixing the magnetic field and varying the carrier density, for instance by sweeping a surface gate.

4.4. Investigation of complex Fermi surfaces by means of the de Haas – van Alphen effect

De Haas and van Alphen measured in 1930 magnetization $M$ of bismuth as a function of magnetic field in strong fields at 14.2 K, and found oscillations of $M/H$.

The oscillations display a remarkable regularity, if the susceptibility is plotted not versus magnetic field, but as a function of inverse magnetic field. It then becomes clear that susceptibility $\chi$ has a periodic dependence on $(1/H)$, though frequently two or more periods are superposed.

Similar oscillatory behavior has been observed not also in the conductivity (Shubnikov - de Haas effect), the magnetostriction (dependence of sample size on magnetic field strength), and, when measured with sufficient care, in almost all other quantities. Oscillations of this kind have been observed in the high-field Hall resistivity, a clear indication that the effect must lie in a failure of the semiclassical model.

Even before the theory of de Haas – van Alphen (dHvA) effect for Bloch electrons was pointed out by Onsager. Landau [156] was able to calculate the oscillations in free electron theory, as a direct consequence of the quantization of closed electron orbits in a magnetic field, giving direct manifestation of a purely quantum phenomenon. This phenomenon became of even greater interest and importance when Onsager [157] pointed out that the change in $1/H$ through a single period of oscillations, $\Delta\left(\frac{1}{H}\right)$, was determined by the remarkably simple relation:
\[
\Delta \left( \frac{1}{H} \right) = \frac{2ne}{hc} \frac{1}{S_m},
\]

(4.22)

where \( S_m \) is any extremal cross-sectional area of the Fermi surface in a plane normal to the magnetic field.

Some Fermi surface cross sections are illustrated in Fig. 4.6. If the \( z \)-axis is taken along the magnetic field, then the area of a Fermi surface cross section at height \( k_z \) is \( S(k_z) \), and the external areas \( S_m \) are the values of \( S(k_z) \) at the \( k_z \) where \( dS/dk_z = 0 \). Thus maximum and minimum cross sections are among the external ones.

![Illustration of various extremal Fermi orbits](image)

**Fig. 4.6.** Illustration of various extremal Fermi orbits. For \( \vec{H} \) along the \( k_1 \)-axis, (1) and (2) are maximum extremal orbits and (3) is a minimum extremal orbit. When the field along the \( k_2 \)-axis, only one extremal orbit, (4), is allowed [158].

Since altering the magnetic field direction brings different extremal areas into play, all extremal areas of the Fermi surface can be mapped out. This frequently provides enough information to reconstruct the actual shape of the Fermi surface. In practice, this may be a complex task, because if more than one extremal orbit is allowed in certain directions, or if more then one band is partially field, then several periods will
be superposed. Rather than directly disentangling the geometric information from the data, it is often easier to guess what the surface is (using, for example, an approximate calculations of the band structure), and later refining the guess by testing it against the data.

4.4.1. Origin of the oscillatory phenomena

Underlying the de Haas – van Alphen and related oscillations is a sharp oscillatory structure in the electronic density of levels imposed by the quantization condition (4.22).

\[
S(E_{\nu}(k_z),k_z) = (\nu + \lambda)\Delta S, \tag{4.23}
\]

where \( S(E_{\nu}(k_z)) \) is the k-space area enclosed by the orbit and \( \nu \) is a quantum number.

The difference between levels will be

\[
\Delta S = S(E_{\nu+1}) - S(E_{\nu}) = \frac{2\pi eH}{hc}. \tag{4.24}
\]

The level density will have a sharp peak whenever \( E \) is equal to energy of an extremal orbit satisfying the quantization condition. The reason for this is shown in Fig. 4.7. Fig. 4.7a depicts a set of all orbits satisfying Eq. 4.23 for a given \( \nu \). These form a tubular structure (of cross-sectional area \((\nu + \lambda)\Delta S\)) in k-space. The contribution to \( g(E)dE \) from the LLs associated with orbits on the vth such tube will be the number of such levels with energies between \( E \) and \( (E + dE) \). This, in turn, is proportional to the area of the portion of tube contained between the constant energy surfaces of energies \( E \) and \( (E + dE) \). Fig. 4.7b shows this portion of tube when the orbits of energy \( E \) on the tube are not extremal, and Fig. 4.7c shows the portion of tube when there is an extremal orbit of energy \( E \) on the tube. Evidently the area of the portion of tube is enormously enhanced in the latter case, as a result of the very slow energy variation of levels along the tube near the given orbit.
Fig. 4.7. (a) Landau tube. Its cross sections by planes perpendicular to $H$ have the same area - $(ν + λ)ΔS$ for the $ν$th tube, and are bounded by curves of constant energy $E_ν(k_ν)$ at height $k_ν$. (b) The portion of the tube containing orbits in the energy range from $E$ to $(E + dE)$ when none of the orbits in that range occupy extremal positions on their constant-energy surfaces. (c) Same construction as in (b), except that $E$ is now the energy of an extremal orbit [158].

Using the value (see Eq. 4.24) for $ΔS$, it follows that $g(E_F)$ will be singular at regularly spaced intervals in $1/H$ given by

$$Δ\left(\frac{1}{H}\right) = \frac{2πe}{hc} \cdot \frac{1}{S_m(E_F)}.$$  

(4.25)

Thus oscillatory behavior as a function of $1/H$ with period (see Eq. 4.25) should appear in any quantity that depends on the level density at $E_F$, which, at zero temperature, includes almost all characteristics metallic properties.

At nonzero temperatures typical metallic properties are determined by averages over a range of energies within $k_BT$ of $E_F$. If this range is so broad that for any value of $H$ extremal orbits satisfying (see Eq. 4.23) contribute appreciably to the average, then the oscillatory structure in $1/H$ will be washed out. This will happen when $k_BT$ is greater
than the typical energy separation between adjoining tubes of Landau levels. We estimate this energy separation by its free electron value, $\hbar\omega_c$. Therefore one must use fields of the order of 1T and temperatures of a few Kelvin to avoid the thermal obliteration of the oscillations.

5. Topic of investigation

We investigated the influence of magnetic ordering on the galvanomagnetic properties of GaAs / δ-Mn / GaAs / In$_x$Ga$_{1-x}$As / GaAs heterostructures. Purpose was to estimate the application perspectives of these structures for spintronics.

The investigated samples were grown by MOS–hydride epitaxy and were characterized by means of photoluminescence and high–resolution X–ray diffractometry and reflectometry.

Transport properties of the samples were measured at temperatures up to 1.6 K and in different magnetic fields up to 12 T.
6. Sample preparation and characterization

Two samples containing a 10 nm wide In\textsubscript{x}Ga\textsubscript{1-x}As quantum well (QW) layer inside GaAs matrix were grown by MOS-hydride epitaxy in the form of a sandwich structure. A Mn quasi-δ-layer separated from the QW by a 3 nm thick spacer was prepared by laser ablation. The phrase “quasi-δ-layer” is used here because sometimes its thickness exceeds 1 monolayer. The buffer layer and the spacers were grown at the temperature of 600 °C, while the Mn layer was deposited at 450 °C. The cross-section of the samples and their physical parameters are given in Figs. 6.1, 6.5 and in Table 6.1, correspondingly.

![Sandwich structure of the investigated samples.](image)

Existence of the optimal spacer thickness of ≈ 3 nm is connected, most probably, with nonzero thickness of the Mn δ-layer. These observations suggest that penetration of the Mn atoms into the QW layer takes place if the spacer is thinner than 3 nm, leading to changes of the energy of the Mn acceptor levels inside the QW. Samples with Mn atoms that penetrated into the 2D conductivity channel were investigated in [159] where magnetic order in the 2D DMS QW was confirmed by presence of AHE. As supported by the structural investigations, the high value of the Hall mobility give evidence that in our samples the Mn ions are located outside the 2D channel.
Table 6.1. Physical parameters of the samples. PHL stands for photoluminescence measurements. $x$ is concentration of In in In$_x$Ga$_{1-x}$As.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$x$</th>
<th>$d_{Mn}$</th>
<th>$\mu_{eff}$, cm$^2$/Vs</th>
<th>$p_x \cdot 10^{12}$, cm$^2$</th>
<th>$R_{ss}$, $\Omega$/□</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PHL</td>
<td>X-ray</td>
<td>ML</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>0.25</td>
<td>0.21</td>
<td>0.5</td>
<td>176</td>
<td>10</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>0.21</td>
<td>0.16</td>
<td>1.8</td>
<td>158</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Increasing distance of the Mn $\delta$-layer from the QW leads to decreasing of the AHE i.e., to weakening of the magnetic ordering because the exchange interaction in such systems is based on carrier mediated ferromagnetism. Existence of the optimal Mn concentration in the $\delta$-layer is due to formation of MnAs clusters when the concentration of Mn exceeds an optimal value. In bulk samples this concentration is about 6 at.% [160, 161]. The clusters give considerable contribution to the magnetization of the whole structure, but not to the AHE because of the energy barrier between the free charge carriers and clusters [162]. This conjecture is confirmed by comparison of the data obtained from magnetization and Hall measurements of samples with different Mn contents in the $\delta$-layer [10]. When the Mn layer is thicker than 1 monolayer, the saturation magnetization of the sample is about $10^{-4}$ emu, but when the thickness is 0.5 monolayer or smaller the magnetization is reduced by two orders of magnitude. On the other hand, the AHE could be observed only in the structures with Mn content less than 1 monolayer.

Formation of ferromagnetic MnAs granules is not the only process, which changes the properties of GaMnAs films when the concentration of Mn is high. At low concentration the Mn atoms substitute Ga acting as acceptors. At concentrations higher than 5 at.%, as it was noticed for bulk samples, a considerable part of Mn atoms are in interstitial sights playing a role of double donors [163, 164]. With random distribution of Mn ions this leads to appearance of an intensive Coulomb potential [165]. At high Mn concentration the conductivity of the 2D channel at low temperature is much smaller than in the samples with low Mn concentration.
The main channel of conduction is the QW. This is confirmed by the hole mobility, which increases more than one order of magnitude when the sample is cooled from room temperature down to 77 K. Finally, the experimental value of the activation energy of conductivity agrees with the value calculated for transport of holes along the QW at moderate Mn concentrations [10].

Spots for contacts on the samples were prepared by photolithography and gold wires were soldered by indium.

Fig. 6.2. Configuration of the measuring contacts on the sample with

\[ l = 3.11 \text{ mm}; w = 0.78 \text{ mm}; d = (90-100) \text{ Å}. \]

X-ray characterization of the samples was made with a two-crystal X-ray spectrometer using CuKα1 radiation, a Ge (400) crystal monochromator in the diffractometry and a Si (111) crystal in reflectometry measurements. With this spectrometer decrease of the diffuse scattering was obtained in registration of oscillations on the far “tails” of the reflection curves (DRC) at the level of about \(10^{-6}\) times the background from the GaAs substrate [166 - 168].
Fig. 6.3. Experimental X-ray reflectometry curves of GaAs/$\delta$-Mn/In$_x$Ga$_{1-x}$As/GaAs heterostructures (vertical dashes) and theoretical curves (solid lines) calculated in the frames of the four layer resulting model ($\chi^2 = 1.39$ and 5.22 for A and B samples, correspondingly).

It is well visible from the Table 6.1 and Figs. 6.3, 6.4 that the QW in the samples is not definite concerning the In concentration and its interfaces on the buffer side is much sharper than on the opposite side. This fact may be connected to the well-known phenomenon of the partial segregation of In atoms on GaAs surface. Besides that, the In content in the QWs is lower than ideal. This is apparently connected with the washing away of the QW due to its high doping level. It should be noted 3 nm thickness of the Mn $\delta$-layer in the samples A and B are reliably determined by X-ray measurements.
Table 6.2. Parameters of the layers in heterostructures from the DRC resulting models. Parameters of Mn $\delta$-layer and quantum well is shown by the bold characters. $\chi^2$ is calculation parameter, $l_j$ is the thickness of layer, $\Delta a_{\perp}$ is the normal component of deformation of the layer (lattice parameter), $f_j$ is the parameter of amorphism, $j$ is a number of layer.

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi^2$</td>
<td>1.04</td>
</tr>
<tr>
<td>$j$</td>
<td>$l_j$, nm</td>
<td>$\Delta a_{\perp}$, %</td>
</tr>
<tr>
<td>1</td>
<td>3.6(2)</td>
<td>0.1(1)</td>
</tr>
<tr>
<td>2</td>
<td>38.0(4)</td>
<td>0.012(1)</td>
</tr>
<tr>
<td>3</td>
<td>3.3(8)</td>
<td>0.42(8)</td>
</tr>
<tr>
<td>4</td>
<td>3.8(6)</td>
<td>0.21(7)</td>
</tr>
<tr>
<td>5</td>
<td>4.4(3)</td>
<td>1.35(6)</td>
</tr>
<tr>
<td>6</td>
<td>5.4(2)</td>
<td>2.99(6)</td>
</tr>
<tr>
<td>7</td>
<td>3.7(3)</td>
<td>0.48(7)</td>
</tr>
<tr>
<td>8</td>
<td>449(1)</td>
<td>-0.02(1)</td>
</tr>
<tr>
<td>9</td>
<td>12.1(4)</td>
<td>-0.070(3)</td>
</tr>
<tr>
<td>Substrate</td>
<td>0.82(2)</td>
<td>Substrate</td>
</tr>
</tbody>
</table>

The data (Table 6.2) give evidence for random distribution of the Mn atoms in these layers. The most interesting result is the fact that in sample A the GaAs spacer between the QW and the Mn $\delta$-layer is relatively well formed while in the sample B the GaAs spacer with practically same technological parameters was strongly doped with Mn, or with In atoms from the QW (Table 6.2 and Fig. 6.3). This obstacle influences the transport properties of the samples. At the same time results of X-ray reflectometry allow to conclude that the Mn atoms do not penetrate into the QW of the samples A and B.
Fig. 6.4. Experimental X-ray diffraction curves observed from (004) planes of GaAs/δ-Mn/ InₓGa₁₋ₓAs/ GaAs (vertical dashes) and theoretical curves (solid lines) calculated for the four-layer model ($\chi^2 = 1.45$ and 1.04 for A, B samples, correspondingly). Dotted lines show contribution of the non-coherent diffusion substrate.
Fig. 6.5. Distribution profiles of the relative changes of the lattice parameter $a$ vs. the depth of heterostructures.

The In content in the quantum well was determined by means of photoluminescence measurements. In the spectrum shown in Fig. 6.6 the right hand maximum is caused by transitions of GaAs and the left hand maximum by transitions in the QW and its position was used for estimation of $x$. 
As can be observed from Table 6.1 there is some difference between the $x$ values of obtained from photoluminescence and from X-ray measurements. Because this difference is very similar be caused by changes of the energy gap because of the exchange interaction. And may for all samples and independent on the Mn content such effect cannot be explained by fluctuation of the potential due to nonrandom distribution of the acceptor atoms in the doping layer. Such potential leads to formation of tails of states in the forbidden gap shifting the photoluminescence peak to smaller energies and increasing the measured value of $x$. 

Fig. 6.6. Photoluminescent spectra of GaAs/$\delta$-Mn/In$_x$Ga$_{1-x}$As/GaAs for $A$ and $B$ samples. Right hand side maximum at 1.5 eV originates from GaAs, left hand side are related to the QWs in $A$ and $B$ samples.
7. Experimental setup and main theoretical formulas

Measurements of galvanomagnetic properties of GaAs/δ-Mn/GaAs/In_xGa_{1-x}As/GaAs – heterostructures were done by means of computer-assisted set up at low temperatures in magnetic fields up to 12 T. Measuring equipment (see block scheme in Fig. 7.1) consists of:

- 4 digital voltmeters,
- sample’s current source,
- Hall-sensor and its current source,
- thermocontroller,
- switching unit,
- cryostat (with anticryostat),
- superconducting solenoid with thermal switch and its power supply,
- gas-outlet system with valves and manometers,
- forevacuum mechanical pump,
- mercurial McLeod manometer,
- Allen Bradley resistor,
- Ar - laser,
- differential thermocouple.
Fig 7.1. Block scheme of the measuring set up
Signals from the sample are registered by two voltmeters (measuring $r_{xx}$ and $R_{xy}$). Third voltmeter is used for measuring the temperature of specimen (or magnetic field). Forth one controls helium level with leading-out wires attached to the Allen Bradley resistor, which is most sensitive at the temperatures around the boiling point of helium.

Fig. 7.2. Experimental setup for measurements of transport properties in semiconductors.

Contacts from the sample are connected to switching unit together with the voltmeters and power supplies. Mechanical pump is used for pumping out the helium vapor, the lower helium vapor is the lower liquid helium boiling temperature is created. McLeod mercury manometer measures the helium vapor pressure to determine the real
temperature of the sample. Thermocontroller is needed for stabilizing of the required sample temperature above 5 K. The temperature of specimen is measured by Cu-Cu:Fe1.5% differential thermocouple and is regulated by means of heater, which is connected to thermocontroller. Liquid helium penetrates from external cryostat into the internal one via the capillary installed into the cryostat bottom.

Electric current to the sample was provided by the computer controlled dc power supply and the polarity of the dc current were changed with the desirable frequency.

Parameters of superconducting solenoid:
- highest possible magnetic induction 12T;
- current 11.2 A / T;
- current of thermal switch 35 mA.

Fig. 7.3. The cryomagnetic unit.
Experimental data is processed by the LabVIEW based computer program, which drives sample’s current source, voltmeters and writes down the data from voltmeters in real-time mode.

Obtained experimental data is used for estimation of the Hall coefficient and specific resistivity of the samples in different magnetic fields and at different temperatures. The Hall coefficient is calculated using equation:

$$R_{\text{Hall}} = \frac{R_{xy} d}{B},$$  \hspace{1cm} (7.1)

where $R_{xy} = \frac{U_{xy}}{I}$ is the Hall resistance, $d$ is thickness of quantum well.

In our experiments the dependencies $r_{xx}(B)$ and $R_{xy}(B)$ are studied at positive and negative direction of applied magnetic field. This was done to reduce the error, which is connected with nonequipotentiality of the Hall contacts. Nonequipotentiality appears when the Hall contacts are not exactly opposite to each other. In this case the voltage $U$ registered by voltmeter from the Hall contacts is

$$U^+_B = U_{r_{xx}} + U_{xy},$$  \hspace{1cm} (7.2)

$$U^-_B = -U_{r_{xx}} + U_{xy},$$  \hspace{1cm} (7.3)

where $U^+_B$ and $U^-_B$ are signals at positive and negative direction of applied magnetic field correspondingly, $U_{r_{xx}}$ is magnetoresistance signal and $U_{xy}$ is the Hall signal. Summarizing these equations and dividing the sum by 2 will give $U_{r_{xx}}$ and subtracting these equations and dividing the sum by 2 will give $U_{r_{xy}}$. 
The current is applied to the specimen in positive and negative directions to remove inaccuracy due to thermoelectromotive force, where \( U_i^+ \) and \( U_i^- \) are signals with different directions of dc current

\[
U_i^+ = (U_{R_{xy}} + U_{r_x}) + U_a,
\]

\[
U_i^- = -(U_{R_{xy}} + U_{r_x}) + U_a,
\]

where \( (U_{R_{xy}} + U_{r_x}) \) is the sum of magnetoresistance and Hall signals, and \( U_a \) is inaccuracy due to thermoelectromotive force. Subtracting from one equation the second one and dividing the sum by 2 will give \( (U_{R_{xy}} + U_{r_x}) \).

All this averaging was taken into account during analyzing of the results obtained.

The correct value of the dc current through the sample, which should be applied, was estimated from the part of the I-V characteristic where resistivity of the sample was linear (or well far from the non linear range). The example of such approach is presented in Fig. 7.4.

Fig. 7.4. Current-voltage characteristic of sample B at 4.2 K.
Period of SdH oscillations \( (P_{SdH}) \) is determined from \( \frac{1}{B_{\text{MAX}}} = f(N) \) dependence by the linear fit, where \( N \) is a quantum number of the certain maximum.

\( P_{SdH} \) is calculated from the following equation:

\[
\frac{1}{B_{\text{MAX}}} = P_{SdH} \left( N + \frac{1}{2} \right),
\]

The real quantum numbers may be found by dividing the equation for the maximum with quantum number \( N \) to the equation for the maximum with quantum number \( (N+1) \)

\[
\frac{B_{\text{MAX}}}{B_{\text{MAX}+1}} = \left( \frac{N + 1}{N + \frac{1}{2}} \right). \tag{7.7}
\]

Period of oscillations in our case depends on concentration of free charge carriers only because in In\(_x\)Ga\(_{1-x}\)As the Fermi surface of holes is spherical (see Eq. 4.13).

From the Hall coefficient in the infinite magnetic field, \( R_{\text{Hall}} \), one can estimate concentration of all holes, which take part in the conductivity, using Eq. 7.8.

\[
R_{\text{Hall}} = -\frac{1}{p_{\text{tot}} \cdot e}, \tag{7.8}
\]

where \( p_{\text{tot}} \) is total concentration of free holes.

In our case \( p_{\text{Hall}} > p_{SdH} \) (see Tables 8.2 and 8.4). Therefore exist free holes that do not take part in SdH oscillations. Most probably, this is due to the existence of the second valence subband, where holes with bigger cyclotron mass are located. This is illustrated in Fig. 7.5.
We can find their concentration:

\[ R_{Hall} = \frac{1}{(p_1 + p_2) \cdot e} \]  \hspace{1cm} (7.9)

where \( p_1 = p_{sH} \) and \( p_2 = \frac{(1-R_{Hall} \cdot e \rho)}{eR_{Hall} \cdot c} \) is holes from the second valence subband.

![Energy-band structure model](image)

**Fig. 7.5.** The model of energy-band structure for the investigated sample.

Using equations 7.10 and 7.11 it is possible to calculate mobility of both types of holes if their concentration is known.

\[ G_0 = e(p_1 \mu_1 + p_2 \mu_2) = \frac{1}{\rho_0} = \frac{l}{r_{xx} \cdot w \cdot d}, \]  \hspace{1cm} (7.10)

\[ R_{Hall0} = \frac{1}{e} \cdot \left( \frac{p_1 \mu_1^2 + p_2 \mu_2^2}{(p_1 \mu_1 + p_2 \mu_2)^2} \right), \]  \hspace{1cm} (7.11)
where $G_0$ is conductivity, $\rho_0$ is specific resistance, $l, d, w$ are dimensions of specimen, $r_{xx0}$ is resistance in the absence of magnetic field and $R_{Hall0}$ is the Hall coefficient in the absence of magnetic field.

The ratio of amplitudes of Shubnikov-de Haas oscillations at temperatures $T_1$ and $T_2$ is represented by the Eq. 4.17 vs. $m_c$.

Logarithmic decay of amplitude of SdH oscillations characterizes non thermal widening of the Landau levels and may be reflected by the Dingle temperature.

The studying of SdH oscillations gives an opportunity to calculate the Fermi level. If Fermi energy is independent from magnetic field, we can write

$$E_F = \hbar \omega_c \left( N + \frac{1}{2} \right) = \frac{\hbar B}{m_c} \left( N + \frac{1}{2} \right).$$  (7.12)
8. Results and discussion

We investigated temperature and magnetic field dependencies of resistivity and the Hall coefficient for QW in the samples A and B. Temperature dependencies of resistivity in the sample A and B were measured in the temperature range 4.2 – 90 K at two directions of dc current through the sample and is presented in Fig. 8.1.

![Graph showing temperature dependences of resistivity of samples A and B.](image)

**Fig. 8.1.** The temperature dependences of resistivity of samples A and B.

Behavior of resistivity with temperature have similar character for both specimens, but the conductivity value in the sample A is smaller for approximately one order of magnitude than in the sample B (see Fig. 8.1.). The maximums on the dependencies are attributed to the magnetic ordering of spins in the QW. Since Mn is ferromagnetic, it is influenced on holes in the conducting channel. For sample A maximum happened to be at about 32 K, for sample B at 27 K.
Temperature dependencies of resistivity is plotted in the coordinates \( \lg(r_{xx}) \) vs. inverse temperature in Fig 8.2 and 8.3. It is visible that \( \lg(r_{xx}) \) vs. \((1/T)\) contain two linear parts for both samples. It means that \( r_{xx}(T) \) dependencies have activation character and may described by Eq. 8.1.

\[
G(T) = G_0 \exp\left(-\frac{E_i}{k_B T}\right), \tag{8.1}
\]

where \( G \) is conductivity, \( E_i \) is activation energy of the impurity level, \( k_B \) is Boltzmann constant and \( T \) is temperature.

The activation energy was calculated for both samples using Eq. 8.1 (see Figs. 8.2, 8.3).

**Fig. 8.2.** Estimation of activation energy for the sample A.
Temperature dependencies of the Hall coefficient in the samples A and B were investigated in the temperature range for 4.2 – 80 K. These temperature dependencies exhibit changes of their character at temperatures, which correlate with the temperatures of maximums of resistivity for the certain sample. As it is visible in Fig. 8.4, where curves $R_{\text{Hall}}(T)$ is presented, the Hall coefficient for the sample A is smaller than for sample B meaning that concentration of holes in the sample A is higher.

Fig. 8.3. Estimation of activation energy for the sample B.
Magnetic field dependencies of resistivity of samples A and B were investigated in the steady magnetic fields up to 12 T and in the temperature range 1.6 – 60 K. Well pronounced quantum oscillations of SdH origin were detected in both samples. In Fig. 8.5 is presented the magnetic field dependencies of resistivity of sample A at T = 4.2; 11 and 17 K. Positions of SdH oscillations are practically the same for all temperatures but amplitudes of oscillations exhibit strong temperature dependence.
Fig. 8.5. The magnetic field dependence of the resistivity of sample A at T = 4.2; 11 and 17 K. Numerical values on the picture (1, 2, 3) are the quantum numbers of the corresponding maximums.

Cyclotron mass of holes taking part in SdH oscillations were calculated from the ratio of amplitudes at two different temperatures using Eq. 4.17 and the estimated values is presented in Table 8.1. To increasing the accuracy of calculations not only one pair of temperatures was used. Calculations include the whole set of temperatures at which SdH oscillations is observed, namely 1.6; 2.8 and 4.2 K. Calculations were performed for different temperatures for maximums with different quantum numbers. We obtained the average value of the cyclotron mass is $m_c = (0.14 \pm 0.02) m_o$. 
Table 8.1. Values of the cyclotron effective masses of extremums of SdH oscillations at different temperatures (sample A).

<table>
<thead>
<tr>
<th>$N$, quantum number</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{1,6}/A_{2,8}$</td>
<td>-</td>
<td>0.1649 $m_o$</td>
<td>0.1631 $m_o$</td>
<td>0.1633 $m_o$</td>
<td>0.1721 $m_o$</td>
</tr>
<tr>
<td>$A_{2,8}/A_{4,2}$</td>
<td>0.1268 $m_o$</td>
<td>0.1268 $m_o$</td>
<td>0.1275 $m_o$</td>
<td>0.1289 $m_o$</td>
<td>0.1251 $m_o$</td>
</tr>
<tr>
<td>$A_{1,6}/A_{4,2}$</td>
<td>-</td>
<td>0.1413 $m_o$</td>
<td>0.142 $m_o$</td>
<td>0.1435 $m_o$</td>
<td>0.1453 $m_o$</td>
</tr>
</tbody>
</table>

Average value $m_c = (0.14 \pm 0.02)m_o$

Decreasing of temperature down to $1.6$ K intensifies SdH oscillations and allows to distinguish oscillations with quantum numbers $N$ up to 7 (see Fig. 8.6).

![Fig. 8.6.](image-url) The magnetic field dependence of the resistivity of sample A at $T = 1.6; 2.8$ and $4.2$ K.
Increasing of temperature higher than 30 K destroys SdH oscillations. This correlates with the position of maximums on the temperature dependencies of resistivity (see Fig. 8.7) and with the values of activation energy estimated from the $r_{xx}(T)$ dependencies: SdH oscillations disappear when the level of degeneration of the hole gas decreases.

**Fig. 8.7.** The magnetic field dependence of the resistivity of sample A at T = 32, 45 and 60 K.

Magnetic field dependencies of the Hall coefficient in the sample A were investigated in the same temperature range 1.6 – 60 K in steady magnetic fields up to 12 T. The Hall resistance $R_{xy}$ demonstrates oscillatory behavior, which is presented in Fig. 8.8 – 8.10 for sample A in the form of calculated Hall constant ($R_{\text{Hall}}$) using $d = 100$ Å for the QW thickness (see Eq. 7.1).

Oscillations of the Hall coefficient are rather intensive and can not be attributed only to the quantum SdH oscillations of $r_{xx}$. Moreover, oscillations of the Hall coefficient have opposite phase to the oscillations of magnetoresistance. This gives evidence for the
existence of at least one more group of holes. In Figs. 8.8 and 8.9 the magnetic field 
dependences of the Hall coefficient in the sample A at T = 1.6; 4.2; 11 and 17K is 
presented.

![Graph showing magnetic field dependence of Hall coefficient](image)

**Fig. 8.8** The magnetic field dependence of the Hall coefficient in sample A at T = 4.2; 11 and 17K.

Small shift in the Hall coefficient oscillations is observed. It can be explained by 
increasing of the Fermi surface (and concentration of charge carriers) and therefore 
decreasing of the Hall coefficient itself and increasing of the period of oscillations as 
well (see Eq. 7.9).

On the magnetic field dependencies of the Hall coefficient and magnetoresistivity is 
observed some peculiarity close to 9 T at 4.2 K. This maximum may be attributed to 
the splitting of the first SdH maximum or to the influence of heavy holes subband.
Fig. 8.9. The magnetic field dependence of the Hall coefficient in sample A at $T = 1.6$ and $4.2$ K.

Oscillations of the Hall coefficient in sample A at $T = 17$ and $32$ K are still visible but their amplitude drastically decreases with the temperature (see Fig. 8.10).
Fig. 8.10. The magnetic field dependence of the Hall coefficient in the sample A at $T = 17$ and $32 \, \text{K}$.

Period of SdH and Hall coefficient oscillations in the inverse magnetic field was calculated for all temperatures where oscillations were observed. One of the plots of position of maximums in the inverse magnetic field vs. quantum number of the maximum is presented in the Fig. 8.11. It is visible that position of maximums follows periodic dependence in the inverse magnetic field and slope of linear fit gives an opportunity to estimate the period of SdH oscillations in every case.
Using Eq. 7.9 we can estimate concentration of holes that take part in SdH oscillations at different temperatures (see Table 8.2 for sample A and Table 8.4 for sample B). The Hall concentration of holes (Eq. 7.9) is bigger than SdH concentration (Table 8.2). This gives evidence of the existing of the second subband in the valence band of our QW. By solving Eq. 7.10 – 7.11 it is possible to estimate the hole mobility in both subbands (see Table 8.3).
Table 8.2. Concentrations of holes, calculated from SdH and Hall effect (sample A).

<table>
<thead>
<tr>
<th>T, K</th>
<th>$P_{SdH}$, T$^{-1}$</th>
<th>$p_{SdH} \times 10^{17}$, cm$^{-3}$</th>
<th>$R_{Hall}$, cm$^3$/C</th>
<th>$R_{Hall&lt;}$, cm$^3$/C</th>
<th>$p_{Hall} \times 10^{17}$, cm$^{-3}$</th>
<th>$p_{Hall}/p_{SdH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>0.072</td>
<td>2.92</td>
<td>8.62</td>
<td>8.83</td>
<td>7.24</td>
<td>2.48</td>
</tr>
<tr>
<td>11</td>
<td>0.072</td>
<td>2.92</td>
<td>8.53</td>
<td>8.77</td>
<td>7.315</td>
<td>2.51</td>
</tr>
<tr>
<td>17</td>
<td>0.072</td>
<td>2.9</td>
<td>8.3</td>
<td>8.8</td>
<td>7.52</td>
<td>2.59</td>
</tr>
<tr>
<td>32</td>
<td>-</td>
<td>-</td>
<td>8.28</td>
<td>8.72</td>
<td>7.536</td>
<td>-</td>
</tr>
<tr>
<td>45</td>
<td>-</td>
<td>-</td>
<td>7</td>
<td>6.85</td>
<td>8.914</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>-</td>
<td>-</td>
<td>5.43</td>
<td>5.19</td>
<td>11.49</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 8.3. Results of calculations using two – band model for concentrations and mobilities in different subbands (sample A).

<table>
<thead>
<tr>
<th>T, K</th>
<th>$p_1 \times 10^{17}$, cm$^{-3}$</th>
<th>$p_2 \times 10^{17}$, cm$^{-3}$</th>
<th>$\mu_1$, cm$^2$/Vs</th>
<th>$\mu_2$, cm$^2$/Vs</th>
<th>$T_D$, K</th>
<th>$T_{D\mu}$, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>2.92</td>
<td>4.16</td>
<td>1424</td>
<td>831</td>
<td>16.2</td>
<td>194</td>
</tr>
<tr>
<td>11</td>
<td>2.92</td>
<td>4.21</td>
<td>1175</td>
<td>1000</td>
<td>25.7</td>
<td>236</td>
</tr>
<tr>
<td>17</td>
<td>2.9</td>
<td>4.2</td>
<td>1354</td>
<td>872</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Using SdH oscillations the Dingle temperature $T_D$ was calculated. Dingle temperature is a parameter that characterizes non thermal broadening of SdH maximums due to non homogeneity in the sample. The value of this parameter allows us to estimate whether the thermal scattering of the carriers in our object is the dominant one. Dingle temperature was calculated from the inverse magnetic field dependence of logarithm of amplitudes ($\ln A = f(1/B)$) using Eq. 4.19. One of the plots of logarithm of amplitudes vs. inverse magnetic field is presented on the Fig. 8.12.

Points in the Fig. 8.12 are the reciprocal values of logarithm of magnetoresistance at the magnetic field of certain maximum at temperature 4.2 K. $T_D$ in this case is the slope of the linear fit (see Table 8.3).
Fig. 8.12. Plot of $\ln(r_{xx})$ vs. $1/B_{max}$ at $T = 4.2$ K. The line is a linear fit.

In Fig. 8.13 – 8.14 is presented magnetic field dependencies of resistivity of sample B at different temperatures. SdH oscillations are observed only for temperatures below 10 K on the magnetic field dependencies of the resistivity and the Hall coefficient of the sample B.
Fig. 8.13. The magnetic field dependence of the resistivity of sample B at different temperatures.

Fig. 8.14. The magnetic field dependence of the resistivity of sample B at $T = 1.6$ and 4.2 K.
In Fig. 8.15 – 8.17 is presented the magnetic field dependencies of the Hall coefficient of sample B at different temperatures. It is observed only the first maximums in the strong magnetic fields on the Hall coefficient oscillations (see Fig. 8.15).

**Fig. 8.15.** The magnetic field dependence of the Hall coefficient in sample B at T = 4.2 and 10K.
**Fig. 8.16.** The magnetic field dependence of the Hall coefficient in sample at $T = 1.6$ and 4.2 K.

**Fig. 8.17.** The magnetic field dependence of the Hall coefficient in sample B at different temperatures.
**Table 8.4.** Concentrations of holes, calculated from SdH and Hall effect (sample B).

<table>
<thead>
<tr>
<th>T, K</th>
<th>p_{SdH}, T^{-1}</th>
<th>p_{SdH}*10^{17}, cm^{-3}</th>
<th>R_{Hall}°, cm^3/C</th>
<th>R_{Hall&lt;5}, cm^3/C</th>
<th>p_{Hall}*10^{17}, cm^{-3}</th>
<th>p_{Hall}/p_{SdH}</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>0.18</td>
<td>0.74</td>
<td>20.6</td>
<td>21.4</td>
<td>3.03</td>
<td>4.09</td>
</tr>
<tr>
<td>10</td>
<td>0.18</td>
<td>0.74</td>
<td>19</td>
<td>20.1</td>
<td>3.284</td>
<td>4.44</td>
</tr>
</tbody>
</table>

**Table 8.5.** Results of calculations in the frames of two–band model for concentrations and mobilities in different subbands (sample B).

<table>
<thead>
<tr>
<th>T, K</th>
<th>p_1*10^{17}, cm^{-3}</th>
<th>p_2*10^{17}, cm^{-3}</th>
<th>\mu_1, cm^2/Vs</th>
<th>\mu_2, cm^2/Vs</th>
<th>T_{Dp}, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>0.74</td>
<td>2.18</td>
<td>74.3</td>
<td>173.9</td>
<td>384</td>
</tr>
<tr>
<td>10</td>
<td>0.74</td>
<td>2.37</td>
<td>65.9</td>
<td>6.3</td>
<td>-</td>
</tr>
</tbody>
</table>

The Fermi energy for both samples was calculated using Eq. 7.12 and the obtained values are 72 meV for sample A and 18 meV for sample B.

In present experiments the anomalous Hall effect was observed (see Fig. 8.18). Although there should be no Mn atoms in the volume of the quantum well in our samples the presence of AHE and the maximum observed in the temperature dependence of the resistivity component \( r_{xx} \) at 32K give evidence for existence of spin polarized charge carriers in the quantum well of the samples.

On the other hand, our X-ray data shows that the distribution of Mn atoms is non random neither in the growth direction of the structure nor in its plane. This gives evidence that the sample consists of a distribution of ferromagnetic and paramagnetic regions. Such phase separation can be understood by nonrandom distribution of the local magnetic moments and fluctuations of the exchange interaction in the samples having semiconductor type of conductivity. In addition, fluctuations of the Coulomb potential may also have a role in the separation of the sample to different magnetic phases.
Fig. 8.18. Magnetic field dependencies of the normal Hall resistivity (upper panels) and of the anomalous Hall resistivity (lower panels) for the samples A (T = 17 K) and B (T = 55 K). The value of the anomalous component was obtained by extracting of the normal Hall component estimated in magnetic fields up to 12 T from the total Hall response.
Since for such objects as QW it is accepted existence of the size quantizing it is needed to analyze our results in the frames of the 2D model. This case is described by following equations:

$$P_{\text{SdH}} = \frac{2e}{\epsilon_p \cdot \epsilon_{\text{p}}} = \frac{4.381 \cdot 10^{10} \left( T^{-1} \text{cm}^{-2} \right)}{\epsilon_p \left( \text{cm}^{-2} \right)} \Rightarrow \epsilon_p \left( \text{cm}^{-2} \right) = \frac{4.381 \cdot 10^{10} \left( T^{-1} \text{cm}^{-2} \right)}{P_{\text{SdH}}}, \quad (8.2)$$

where $\epsilon_p = p_{\text{Hall}}$ is surface concentration of holes. Period of SdH oscillations was estimated earlier in Fig. 8.11, therefore it is possible to calculate $\epsilon_p$.

Using surface concentration of holes ($\epsilon_p$) and cyclotron mass ($m_c$) we can estimate Fermi level for both samples:

$$E_F = \frac{\pi \cdot h \cdot \epsilon_p}{m^*} = \frac{3.15 \cdot 10^{-12} \cdot \epsilon_p}{m_c/m_0} \text{(meV)}. \quad (8.3)$$

The concentration of charge carriers estimated by the Hall effect in 2D case:

$$p_{sx} = \frac{1}{e \cdot R_{\text{Hall}}^{2D}} \left( \text{cm}^{-2} \right), \quad (8.4)$$

where

$$R_{\text{Hall}}^{2D} = \frac{R_{\text{Hall}} \left( \Omega \right)}{B(T)} \cdot 10^4 \left( \frac{\text{cm}^{-2}}{A \cdot \text{s}} \right). \quad (8.5)$$
Table 8.6. Concentrations of holes, calculated from SdH and Hall effect, 2D model (sample A).

<table>
<thead>
<tr>
<th>T, K</th>
<th>$P_{SdH}, T^{-1}$</th>
<th>$p_{SdH} \times 10^{11}, \text{cm}^{-2}$</th>
<th>$p_{Hall} \times 10^{11}, \text{cm}^{-2}$</th>
<th>$p_{Hall}/p_{SdH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.072</td>
<td>6.7</td>
<td>7.45</td>
<td>1.11</td>
</tr>
<tr>
<td>2.8</td>
<td>0.072</td>
<td>6.7</td>
<td>7.83</td>
<td>1.17</td>
</tr>
<tr>
<td>4.2</td>
<td>0.072</td>
<td>6.7</td>
<td>7.26</td>
<td>1.08</td>
</tr>
<tr>
<td>11</td>
<td>0.072</td>
<td>6.7</td>
<td>7.25</td>
<td>1.08</td>
</tr>
<tr>
<td>17</td>
<td>0.072</td>
<td>6.7</td>
<td>7.22</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 8.7. Concentrations of holes, calculated from SdH and Hall effect, 2D model (sample B).

<table>
<thead>
<tr>
<th>T, K</th>
<th>$P_{SdH}, T^{-1}$</th>
<th>$p_{SdH} \times 10^{11}, \text{cm}^{-2}$</th>
<th>$p_{Hall} \times 10^{11}, \text{cm}^{-2}$</th>
<th>$p_{Hall}/p_{SdH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>0.134</td>
<td>3.6</td>
<td>3.23</td>
<td>0.9</td>
</tr>
<tr>
<td>11</td>
<td>0.134</td>
<td>3.6</td>
<td>3.36</td>
<td>0.93</td>
</tr>
<tr>
<td>17</td>
<td>0.134</td>
<td>3.6</td>
<td>3.16</td>
<td>0.88</td>
</tr>
<tr>
<td>27</td>
<td>0.134</td>
<td>3.6</td>
<td>3.06</td>
<td>0.85</td>
</tr>
</tbody>
</table>

The Fermi energy for both samples was calculated using Eq. 8.3 and the obtained values are 14.9 meV for sample A and 8 meV for sample B.

Calculations of the QW energy band parameters performed in the frames of 2D model confirm 2D character of transport properties of the investigated QW because detected phenomena may be attributed to QHE.
9. Conclusions

During the experiments it was shown that manganese in the investigated structures penetrates to some extent into the QW. This is confirmed by the different concentration of free charge carriers in the QWs in spite of the fact that both samples were grown simultaneously. This difference is well visible from the values of resistivities, the Hall coefficients and periods of SdH oscillations.

Analyzes of the obtained results in the frames of 3D and 2D models show existence of the quantum Hall effect at low temperatures and indicate existence of at least two groups of holes in the valance band of our QW and gives estimation of their concentrations and mobilities.

Calculations of the Dingle temperatures show that thermal scattering of holes in the QW is not the dominant scattering mechanism.

Cyclotron mass was calculated by using temperature dependencies of the amplitudes of SdH oscillations. The mass of holes participating in the quantum effect appeared to be \((0.14 \pm 0.02) \, m_o\) what is higher that the expected one. This fact may be attributed to the influence of the thermal stretch and strain of our samples together with the influence of the magnetic ordering in the QW.

Magnetic ordering in the QW is confirmed by the existence of the well pronounced maximums on the \(r_{xx}(T)\) dependencies for both specimens and by the existence of AHE in the sample with a higher concentration of holes. Such mechanism of ferromagnetic ordering may be attributed to the “carrier mediated ferromagnetism”.
10. References

2. B.P. Zakharchenya, V.L. Korenev, UFN, No 6, 175 (2005), 629
53. E.L. Nagaev, Physics of magnetic semiconductors (Mir, Moscow, 1983)
106. N. Garcia, M. Munoz, and Y.-W. Zhao, “Magnetoresistance in excess of 200% in ballistic Ni nanostructures at room temperatures and 100 Oe”, Phys. Rev. Lett. 82 (1999), 2923-2926


139. H. Munekata, A. Oiwa, Y. Mitsumori, R. Moriya, and T. Slupinski, ”Rotation of ferromagnetically coupled Mn spins in (Ga,Mn)As by hole spins”, J. Supercond. 16 (2003), 411-414


144. C. Weisbuch, and C. Hermann, “Optical detection of conduction-electron spin resonance in GaAs, Ga$_{1-x}$In$_x$As, and Ga$_{1-x}$Al$_x$As alloys”, Phys. Rev. B 15 (1977), 816-822


150. I.M. Lifshitz, A.M. Kosevich, “To the theory of magnetic susceptibility of metals at low temperatures”, ZETF, v.29, №6 (1955), 730-742


156. L.D. Landau, Z. Phys. 64 (1930), 629

157. L. Onsager, Phil. Mag. 43 (1952), 1006


161. V.V. Rylkov, B.A. Aronzon, Yu.A. Danilov, Yu.N. Drozdov, V.P. Lesnikov, K.I. Maslakov, and V.V. Podol’ski, JETP 100 (2005), 742


