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EMPIRICAL TESTS OF ASSET PRICING MODELS IN FINNISH STOCK MARKET

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ABSTRACT

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This study investigates the relationship between different sorts of risk and return on six Finnish value-weighted portfolios from the year 1987 to 2004. Furthermore, we investigate if there is a large equity premium in Finnish markets. Our models are the CAPM, APT and CCAPM. For the CCAPM we concentrate on the parameters of the coefficient of the relative risk-aversion and the marginal rate of intertemporal substitution of consumption, whereas for the CAPM we estimate the market beta and for the APT we will select some macroeconomic factors a priori.

The main contribution of this study is the use of General Method of Moments (GMM). We implement it to all of our models. We conclude that the CAPM is still a robust model, but we find also support for the APT. In contradiction to majority of studies, we are able to get theoretically sound values for the CCAPM's parameters. The risk-aversion parameters stay below two and the marginal rate of intertemporal substitution of consumption is close to one. The market beta is still the most dominant risk factor, but the CAPM and APT are as good in terms of explanatory power.

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Tämän tutkielman tavoitteena on selvittää mitkä riskitekijät vaikuttavat osakkeiden tuottoihin. Arvopapereina käytetään kuutta portfoliota, jotka ovat jaoteltu markkina-arvon mukaan. Aikaperiodi on vuoden 1987 alusta vuoden 2004 loppuun. Malleina käytetään pääomamarkkinoiden hinnoittelumallia, arbitraasihinnoitteluteoriaa sekä kulutuspohjaista pääomamarkkinoiden hinnoittelumallia. Riskifaktoreina kahteen ensimmäiseen malliin käytetään markkinariskiä sekä makrotalou-dellisia riskitekijöitä. Kulutuspohjaiseen pääomamarkkinoiden hinnoi-noittelumallissa keskitytään estimoimaan kuluttajien riskitottumuksia sekä diskonttaustekijää, jolla kuluttaja arvostavat tulevaisuuden kulutusta.

Tämä työ esittelee momenttiteorian, jolla pystymme estimoimaan lineaarisia sekä epälineaarisia yhtälöitä. Käytämme tätä menetelmää testaamissamme malleissa. Yhteenvetona tuloksista voidaan sanoa, että markkinabeeta on edelleen tärkein riskitekijä, mutta löydämme myös tukea makrotaloudellisille riskitekijöille. Kulutuspohjainen mallimme toimii melko hyvin antaen teoreettisesti hyväksyttäviä arvoja.

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Now my academic career is ending, I am happy to continue with my life. But, on the other hand it is quite hard to give up the academic world's challenges. I will surely return one day. The idea for this thesis came when I was in Gothenburg as an exchange student for one year – one of the best times of my life. With this thesis I jumped to unknown world without knowing what the results will be. I did not know anything about the method I was supposed to use but I knew that with dedicated work and endless hours of studying, I could do something special.

I want to thank the best parents and sister in the world for supporting and carrying me through all of the good and also the bad times. I want to thank all my friends without whom I could have not done this. Regarding this thesis and my studies I want to thank Professors Mika Vaihekoski and Eero Pätäri for making finance so intriguing for me. I also want to thank M.Sc. (Econ) Simo Kalatie for helping me with Matlab.

This thesis is dedicated to my beloved friend, Mr. Jari Simola, who died accidentally in the summer of 2006 doing what he loved.

Helsinki, 30.10.2007

Mauri Paavola

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1 INTRODUCTION

1.1 Background

The economic theory of capital asset pricing relies heavily on the principles of present value calculations and the hypothesis of efficient capital markets. The former tells us that the price of an asset is a function of the expected future yields discounted to the current date. This should apply to all assets, such as stocks, land, houses and durables, since they are alternative investment objects. (Takala & Pere, 1991) In particular, modern financial theory is founded on three central assumptions: markets are highly efficient, investors exploit arbitrage opportunities and investors are rational. (Dimson & Mussavian, 1999)

An important body of research in financial economics has been the behaviour of asset returns and especially the forces that determine the prices of risky assets. There are also a number of competing theories of asset pricing. These include the original capital asset pricing models (hereafter CAPM) of Sharpe (1964), Lintner (1965) and Black (1972), the intertemporal models of Merton (1973), Long (1974), Rubinstein (1976) and Cox et al. (1985), the consumption-based asset pricing theory (hereafter CCAPM) of Breeden (1979); Lucas (1978) and the arbitrage pricing theory (hereafter APT) of Ross (1976).[1]

Breeden (1979) and Lucas (1978) took a different approach in defining equilibrium in capital markets. They are able to show, under certain assumptions, that return on assets should be linearly related to the growth rate in aggregate consumption if the parameters of the linear relationship can be assumed to be constant over time (Elton et al., 2007). Breeden (1979) and Lucas (1978) models are so called “representative” agent

[1] For an excellent history review of these models, see Dimson & Mussavian (1999).

models of asset returns in which per capita consumption is perfectly correlated with the consumption stream of a typical investor. In this type of models, a security's risk can be measured using the covariance of its return with per capita consumption (Kocherlakota, 1996).

The CAPM is by far the most famous asset pricing model. It is widely used and examined both in literature and in practice. However, the CAPM is only a description of the reality. By this we mean that the CAPM does not help us understand what the ground factors are and how they affect the risky returns. If we want to go deeper and try to understand what the affecting forces are, how the investors define the returns for the risky assets, we have to start from the basic utility theory and try to find the solution from there.[2] The intuitive model to examine is the CCAPM, where the return is given with the covariance of investor's marginal utility. Moreover, in contrast to the CAPM, intertemporal general-equilibrium models identify clearly the economic forces that influence the risk-free real interest rate and the compensation that investors earn by accepting risk.[3] (Carmichael, 1998).

1.2 Objectives and methodology

The purpose of this thesis is to find out what the affecting forces behind the stock returns are, and which of these risk factors are significant. We will test the traditional market beta of the CAPM and some other macroeconomic risk factors employed in the APT. For the CAPM and APT our focus is on the risk factors (betas), whereas for the CCAPM we will focus on the risk-aversion and discount factor parameters, γ and β. Our purpose

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[2] Kocherlakota (1996) writes: "The representative agent model of asset pricing is not as widely used as the CAPM in the "real world" applications. However, from an academic economist's perspective, the representative agent model of asset pricing is more important than the CAPM."
[3] The CAPM deals with the question of how asset prices and yields are determined, under the hypothesis that the risk-free interest rate and market return are variables determined outside the model (exogenous).

is to compare all of these models. We will also try to find reasonable values for the CCAPM's parameters that have failed numerous times in empirical studies.[4] Our focus will be on the CCAPM, because it is the least known from these asset pricing models. The CAPM and APT serve more as a benchmark models, although we are going to present them in detail. The results of testing the different models are quite inconclusive. CAPM is widely used and its functions are well documented. On the other hand, there is a big group of researchers[5] that say that the CCAPM and its consumption beta should be preferable on theoretical grounds, although its empirical testing has failed numerous times.

We will employ Lucas (1978) study for testing the CCAPM with the standard Constant Relative Risk-aversion (hereafter CRRA) power utility function. We will examine a developed stock market of Finland and try to explain the differences in the returns of value-weighted portfolios. The other purpose of this thesis is to examine the equity-premium puzzle emerged from the fact that the consumption tends to be too smooth.[6] Mehra & Prescott (1985) presented the equity-premium puzzle and this rock solid evidence against the CCAPM is still unsolved.[7]

These differences in our value-weighted portfolio returns should be explained only by the different risk factors and the sensitivities of returns to the risk factors according to the theory. In the CAPM the expected equity premium (excess return) is proportional to market beta. The APT relates the expected rate of return on a sequence of primitive securities to their factor sensitivities, suggesting that factor risk is of critical importance in asset pricing. In comparison, the standard CCAPM measures the risk of a security by the covariance of its return with per capita consumption. (Elton et al., 2007)

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[4] See, e.g., Fama (1991) for an extensive review of these tests.
[5] See, e.g., Mankiw & Shapiro (1986) and Kocherlakota (1996).
[6] Thus, it is a well known fact that nondurable consumption growth barely moves over the business cycle and it is poorly correlated with stock returns. (Cochrane, 1996).
[7] Although numerous authors have presented "the solution" for the equity-premium puzzle, in our opinion it is still unsolved, because these solutions are not robust.

Most of the empirical tests of these models have been conducted for developed markets, e.g., U.S. and Germany. This study will examine the stock market of Finland. To the best of our knowledge, the Finnish stock market has not been examined this way. There are tests of the CCAPM and of course of the CAPM and APT, but no comparisons of these models in the same data set. We will compare the realized asset returns within these models to see which model provides the best results of explaining the time-series variation of value-weighted stock portfolios.

One of the main contributions of this thesis is the use of the Generalized Method of Moments (hereafter GMM) method. Again, to the best of our knowledge, this method has not been used in this way for the Finnish data, i.e., comparing these asset pricing models in the same data set. We will employ the GMM to all of our empirical tests and make comprehensive conclusions of the asset pricing models' ability to explain the portfolio returns. The GMM is a general statistical method for obtaining estimates of parameters of statistical models and it is widely used in the finance literature. All the empirical tests are done with Matlab.

1.3 Limitations and motivations

This study is performed from the European investor's point of view so that the currency used in this study is euro and also risk-free rate is quarterly[8] Euribor. In selecting the factors for our different models we will choose them a priori, as in Chen et al. (1986). The data used in this study is gathered from ETLA, Research Institute of the Finnish Economy, and Datastream. The research period will be from the beginning of the year 1987 to the end of year 2004.

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[8] "Quarterly", because the consumption data is reported quarterly.

We will test the asset pricing models in their purest form. This means that, e.g., the CCAPM is tested as it was presented in Lucas (1978). Thus, we will not use any other implications of the CCAPM that are, e.g., the habit formation of Constantinides (1990), the "non-expected utility" preferences of Epstein and Zin (1989) or the investment-based asset pricing model of Cochrane (1996), to name a few. However, these studies are important because they had had some success in solving the problems of the CCAPM and the key results are presented in this thesis. We are well aware that doing this thesis in this way, without any further assumptions or modifications of the CCAPM, may lead to a rejection of the CCAPM. This is probably the case, because we know that Finnish stock market has a relatively high equity premium, especially in our research period.[9] However, we also know that other models that we presented above have not had success in solving the equity premium puzzle, at least not in different markets, data sets, etc. Thus, right now we do not have an explicit solution to the equity premium puzzle, which we will show in further chapters.

1.4 Structure

The thesis is structured as follows. The next chapter introduces the basics of the utility theory and the concept of Stochastic Discount Factors (SDF). The third and fourth chapter presents the different asset pricing models in great detail and also one of the biggest debate issues in earlier studies, the determination of relevant factors, is represented. We will separate the discussion between the CAPM/APT and the CCAPM, because our main focus in on the less known CCAPM. Furthermore, the CAPM and APT are quite alike models, but the CCAPM comes from totally different grounds. In the fifth chapter we will go through some of the basic problems associated with the asset pricing models and go through an extensive amount of previous empirical studies. The sixth chapter describes the data for our

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[9] See Nyberg & Vaihekoski (2005).

purposes. We will especially concentrate on the research methodology, because we have found out that there are a lot of different methods to choose from. Furthermore, the GMM is explained in a difficult way and also quite irrationally in many parts of the literature, although it is a quite simple and effective method. The seventh chapter reports the empirical results and findings. The final chapter is for conclusions and suggestions for further research.

2 THEORETICAL BACKGROUND

2.1 The basics of the utility theory

The CCAPM is an equilibrium model that can be derived from the choices of a utility-maximizing investor. The utility theory is the base for a significant part of the finance theory. Utility theory is a potentially powerful tool for portfolio decision making. Utility is a measure of relative happiness or satisfaction gained by consuming different bundles of goods and services. We can explain economic behaviour in terms of attempts to increase one's utility. It is convenient in practice to represent this "happiness" or "satisfaction" as utility functions. All utility functions have a constant that serves to specify the trade-off between risk and return. This constant is called the coefficient of risk-aversion. However, over the past twenty years, the use of the utility theory as a practical tool has received a lot of criticism. (Elton et al., 2007)

Kahra (2003) gives a good introduction to the problems that a typical consumer with a flow of income has. Firstly, the consumer has to decide how to allocate present and future consumption among goods and services. Consumption decisions are also saving decisions from which the funds available for investment arise. This is known as the consumption-savings problem of the consumer. Secondly, he or she has to decide how to invest among various assets. The desire that consumers might have to smooth fluctuations in consumption over time determines the demand for appropriate assets. This is called the portfolio selection problem. The two problems are interrelated, since the consumer can affect his or her consumption path by transferring wealth between different time periods through portfolio selection. Both of the problems involve making decisions under uncertainty, considering simultaneously the probable portions of risk and expected return. Consumer's attitudes to saving, risk bearing and uncertainty are essential building blocks in explaining the behaviour of financial markets and asset prices. In the following we will explain the basic decisions that consumers face and how they handle the situations according to the utility theory.

With an initial (safe) level of wealth W_0, a utility function has the property as follows:

$$\frac{U(W)}{U(W_0)} = F\left(\frac{W}{W_0}\right), \tag{1}$$

From the Equation (1) we see that utility reacts to the relative difference in wealth. (Cuthbertson & Nitzsche, 2004) The utility function that we will use throughout this thesis will be the momentary utility function for a representative agent that takes the following isoelastic form:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{2}$$

where the parameter γ is the Arrow-Pratt relative risk-aversion coefficient. With this utility function, investors' CRRA is constant over time and is given by γ. The utility function has several interesting properties. First, it is compatible with risk neutrality (i.e., $\gamma = 0$), and it also includes, when γ tends towards unity, the case where preferences are logarithmic. Second, with this functional form, the risk premiums predicted by the model are resistant to changes in wealth levels and in the size of the economy. Third, to the extent that economic agents share the same utility function, we can aggregate individual choices, even if agents have different levels of wealth. This property offers some theoretical support for using aggregate consumption, rather than individual consumption, in econometric studies on the determination of returns. Fourth, with isoelastic utility function, the parameter γ determines simultaneously the relative risk-aversion coefficient and the elasticity of intertemporal substitution, ρ. In fact, with this functional form, the elasticity of intertemporal substitution is the reciprocal of the relative risk-aversion coefficient. This means that if consumers are highly risk-averse, they must have low intertemporal substitution as well. Hall (1998) also points out that this property is not necessarily desirable. In theory, there should be no such rigid link between these two distinct preference aspects. Risk-aversion influences the rate at which the agent is prepared to exchange units of consumption between different states of nature, whereas the elasticity of intertemporal substitution reflects the agent's willingness to exchange units of consumption between periods. Risk-aversion is a notion that can exist only in the presence of uncertainty, and it does not have to have temporal dimension. (Carmichael, 1998)

The fifth property is that the utility function is order preserving. This means that if we measure the utility of x as greater than the utility of y, it means that x is preferred to $U(x>y)$. Finally, expected utility can be used to rank combinations of risky alternatives. Copeland et al., (2005) show mathematically that the correct ranking function for risky alternatives is expected utility. This means that it is the expected utility that represents a linear combination of the utilities of outcomes. The expected utility of wealth can be presented as:

$$E[U(W)] = \sum_i p_i U(W_i), \tag{3}$$

The utility from any wealth outcome in Equation (3) is the outcome of W_i (wealth), denoted $U(W_i)$.

We can conclude that with rational investor behaviour and the assumption that all investors always prefer more wealth to less, investors will always seek to maximize their expected utility of wealth. Investors seem to calculate the expected utility of wealth for all possible alternative choices and then choose the outcome that maximizes their expected utility of wealth.[10]

[10] We will need the axioms of cardinal utility to provide the minimum set of conditions for consistent and rational behaviour of an investor. These axioms are: comparability, transitivity, strong independence, measurability and ranking. See, e.g., Copeland et al. (2005, p. 46–47).

Thus, the first restriction placed on utility functions is that more is always preferred to less so that $U'(W)>0$. (Copeland et al., 2005)

It is also assumed that individuals are risk-averse. This means that if the consumption level is high and the wealth (or consumption)[11] rises x units, it has a lower rise in the utility of an investor when the same happens if the consumption level is low. Thus, risk-aversion implies that the second derivative of the utility function is negative $U''(W)<0$. In other words, a risk-averse investor is said to have diminishing marginal utility of wealth: each additional unit of wealth adds less to her utility, the higher the initial level of wealth. [12] (Ibid)

2.2 The theory of the stochastic discount factors

The use of the stochastic discount factor (hereafter SDF)[13] method for econometric evaluation of asset pricing models has become common in the recent empirical finance literature. An SDF has the following property: the value of a financial asset equals the expected value of the product of the payoff on the asset and the SDF. An asset pricing model identifies a particular SDF that is a function of observable variables and model parameters. Hence, an SDF relates the future cash flows of any financial instrument to their respective present market values. For example, a linear factor pricing model (such as the CAPM) identifies a specific linear function of the factors as an SDF. The SDF method involves estimating the asset pricing model using its SDF representation and the generalized method of moments (GMM). This SDF method, which is the combination of the SDF representation and the GMM, provides a convenient and gen-

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[11] Traditionally, in this context, wealth and consumption are the same things in economics, because we get utility from consuming different goods and services.

[12] Also, as Kocherlakota (1996) writes: “The agent’s momentary utility is a decreasing function of last period’s consumption: it takes more consumption today to make him happy if he consumed more yesterday.”

[13] E.g., Cochrane (2005) bases his celebrated book solely on this framework and modifies it in different occasions.

eral framework for analyzing linear and nonlinear asset pricing models. (Jagannathan & Wang, 2002)

Lucas (1978) made a basic pricing equation which applies to wide class of intertemporal asset pricing models. The basic equation of asset pricing can be written as follows:

$$P_{it} = E_t\left[M_{t+1} X_{i,t+1}\right], \tag{4}$$

where P_{it} is the price of an asset i at time t ("today"), E_t is the conditional expectations operator conditioning on today's information, $X_{i,t+1}$ is the random payoff on asset i at time $t+1$ ("tomorrow"), and M_{t+1} is the stochastic discount factor. It generalizes the familiar notion of a discount factor to a world of uncertainty. If there is no uncertainty, or if investors are risk-neutral, the SDF is just a constant that converts expected payoffs of tomorrow into value today. A key element of SDF is that M_{t+1} is the same for all assets.[14] (Campbell, 2000)

If there is no uncertainty, we can express prices via the standard present value formula:

$$P_t = \frac{1}{R_f} x_{t+1}, \tag{5}$$

where R_f is the gross risk-free rate and $1/\mathrm{R}_f$ is the discount factor. Since gross interest rates are typically greater than one, the payoff x_{t+1} sells at a discount. When we turn into risky assets, their prices are usually lower than equivalent risk-free assets, so they are often valued by using risk-adjusted discount factors:

[14] Note that Equation (4) applies to any individual asset, and does not require any special assumption about utility. Only when we come to implement the model do we then use some of these additional assumptions (e.g., power utility function to the CCAPM). (Cuthbertson & Nitzsche, 2004)

$$P_t^i = \frac{1}{R_i} E_t(x_{t+1}^i), \tag{6}$$

If we compare Equations (5) and (6), we can see that in Equation (6) we have added the i superscript to emphasize that each risky asset i must be discounted by an asset-specific risk-adjusted discount factor $1/R_i$. Of course, finding all the asset-specific discount factors is an enormous project to apply. However, if we look at Equation (4), we can see that it suggests that one can incorporate all risk corrections by defining a single stochastic discount factor – which is the same one for each asset – and putting it inside the expectations. This M_{t+1} is stochastic or random because it is not known with certainty at time t. The correlation between the random components of the common discount factor M and the asset-specific payoff x_i are used to generate asset-specific risk corrections. (Cochrane, 2005).

Equation (6) has two important properties. First, in discrete-state setting, the asset price can be written as a state-price-weighted average of the payoffs in each state of nature. Equivalently, it can be written as a probability-weighted average of the payoffs, multiplied by the ratio of the state price to probability to each state. The conditional expectation in Equation (6) is just that probability-weighted average. The absence of arbitrage opportunities ensures that a set of positive state prices exists and hence that a positive SDF exists. If markets are complete, then state prices and the SDF are unique. (Campbell, 2000 & Cochrane, 2005)

Second, turning our focus to the optimization problem of the utility, we will introduce an agent k with time-separable[15] utility function $U(C_{kt})+\delta U(C_{k,t+1})$. If the agent is able to freely trade asset i, then the first-order condition[16] is:

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[15] This does not have to be "time-separable", but here we are going to follow a good introduction to SDF framework by Campbell (2000).

[16] First-order conditions are necessary to implement CCAPM to practical use.

$$U'(C_{kt})P_{it} = \delta E_t\left[U'(C_{k,t+1})X_{i,t+1}\right], \tag{7}$$

which equates the marginal cost of an extra unit of asset i, purchased today, to the expected marginal benefit of the extra payoff received tomorrow. Equation (7) is consistent with Equation (4) for $M_{t+1}= \delta U'(C_{k,t+1})/U'(C_{kt})$, the discounted ratio of marginal utility tomorrow to marginal utility today. This marginal utility ratio can always be used as the SDF for that set of assets. (Campbell, 2000)

Equation (4) allows for the existence of assets – or investment strategies – with zero cost today. However, if P_{it} is nonzero one can divide through by P_{it} (which is known at time t and thus can be passed through the conditional expectations operator to obtain):

$$1 = E_t\left[M_{t+1}(1 + R_{i,t+1})\right], \tag{8}$$

where $1+(R_{i,t+1})=X_{i,t+1}/P_{it}$. This form is more commonly used in empirical work. The origins of this presentation lie in the Arrow & Deubreu (1954) model of general equilibrium.[17] (Campbell, 2000).

2.2.1 The use of the stochastic discount factors

How are these equations used in empirical work? As Cochrane (2005) points out, the SDF method is sufficiently general that it can be used for analysis of linear as well as nonlinear asset pricing models, including pricing model for derivative securities. A first possibility is to impose minimal theoretical structure, using data on asset returns alone and to draw implications for the SDF. If we work in this style, we get information that simply documents stylized facts about means, variances and predictability of as-

[17] This is the so called "state preference theory" that is developed by Arrow & Debreu (1954). This model describes how resources are optimally allocated under uncertainty. See, e.g., Copeland et al. (2005).

set returns. A second possibility is to build a time-series model of the SDF that fits data on both asset payoffs and prices. This is done by including research on the term structure of interest rates. A third approach is micro-economic. One can use Equation (7), along with assumed preferences for an investor, the investor's temporal budget constraint, and a process for asset returns or equivalently for the SDF, to find the investor's optimal consumption and portfolio rules. Fourth, one can assume that Equation (7) applies to representative investor who consumes aggregate consumption; in this case C_{kt} is replaced by aggregate consumption C_t. Equation (7) restricts asset prices in relation to consumption data.[18] Finally, one can try to explain equilibrium asset prices as arising from the interactions of heterogeneous agents. (Campbell, 2000)

The SDF framework has gained a lot of popularity due to its desired properties. First of all, the Equation (4) with the price of the asset is so general that it places almost no restrictions to the data. Remember, that the discount factor relates to the returns only by covariance that is purely a mathematical expression. On the other hand, this generality places a lot of pressure on the researcher, because the researcher has to express the "correct" M_{t+1}.[19] If we compare this theory with the basic linear regression, we know that OLS method places a lot of restrictions on the data, e.g., the assumptions for the error term.

The other desired property of the SDF is that there is a relation between its necessary volatility to price the return of the Sharpe ratio of an excess return. Thus, we can calculate the lower bound for the discount factor that can be compared with the observed Sharpe ratio. Because the correlation between the SDF and the excess return must be greater than minus one,

[18] For the purpose of this thesis, this use of the SDF is the one we are going to use in our empirical section for the CCAPM.

[19] We will use different symbols for the SDF in this thesis. It can be M_{t+1}, θ or in the empirical section β. This might seem distracting, but this is deliberate choice and it will make sense when we approach the empirical section.

the negative covariance must be less than the product of the standard deviations of the excess return and the SDF. Thus, we have:

$$\frac{\sigma_t(M_{t+1})}{E_t M_{t+1}} \geq \frac{E_t(R_{i,t+1} - R_{f,t+1})}{\sigma_t(R_{i,t+1} - R_{f,t+1})}, \tag{9}$$

or taking the form of M_{t+1}:

$$\frac{\sigma_t\left[(c_{t+1}/c_t)^{-\alpha}\right]}{E_t\left[(c_{t+1}/c_t)^{-\alpha}\right]} \geq \frac{E_t(R_{i,t+1} - R_{f,t+1})}{\sigma_t(R_{i,t+1} - R_{f,t+1})}, \tag{10}$$

In words, the Sharpe ratio for asset i–the asset's risk premium divided by its standard deviation–puts a lower bound on the volatility of the SDF. The tightest lower bound is achieved by finding the risky asset, or portfolio of assets, with the highest Sharpe ratio. Equation (9) uses the form of M_{t+1} to link this framework to the CCAPM. It says that Sharpe ratio is higher if the economy is risky, i.e., when the volatility of consumption is high. The Sharpe ratio is also high when investors are risk-averse, i.e. the risk-aversion coefficient is high. This bound was first stated by Shiller (1982). Hansen & Jagannathan (1991) have extended it to a setting with many risky assets but no riskless asset, showing how to construct a frontier relating the lower bound on the volatility of the SDF to the mean of the SDF.[20] This frontier contains the same information as the familiar mean-variance efficient frontier relating the lower bound on the variance of a portfolio return to the mean portfolio return. The lower bound is achieved by an SDF that is a linear combination of a hypothetical riskless asset and the risky assets. (Campbell, 2000) The flexibility of this approach is that the bound can be calculated for a wide variety of possible utility functions and for various values of the risk-aversion parameter. For example, this enables us to see what alternative values of the risk-aversion parameter are consistent with a given utility function. (Cuthbertson & Nitzsche, 2004).

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[20] See also Cochrane & Hansen (1992).

We will need a very volatile discount factor to derive stock returns. The aggregate U.S. stock market is the best-known example. In post-war quarterly U.S. data summarized by Campbell et al. (1997) the annualized Sharpe ratio for a value-weighted stock index is about one-half, implying a minimum annualized standard deviation of 50 percent for the SDF.[21] Kasa (1997) explains further that there is a trade-off between the volatility of a discount factor and its correlation with asset returns. The less correlated a discount factor is with asset returns, the more volatile it must be to account for the observed volatility in asset returns. Thus, because the contemporaneous correlation between stock returns and consumption is relatively small, using them as discount factor proxies leads to an extremely volatile discount rate. Unfortunately, such volatility seems to be impossible to any attempt to link stock returns to underlying macroeconomic variables. It is difficult to find macroeconomic variables (as opposed to financial market variables, such as the market return) that are highly correlated with stock returns.

In spite of its wide use, little is known about the estimation efficiency of the SDF method relative to the classical beta method, e.g., the ordinary least squares (OLS) linear regression method. This method estimates the parameters in a linear factor pricing model using its beta representation, in which the expected return on an asset is a linear function of its factor betas (e.g, the CAPM and APT).[22] A question that arises is whether the generality of the SDF framework comes at the costs of estimation efficiency

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[21] This is a very large value for a random variable whose mean must be close to one and whose lover bound is zero. With $E(M) = 1/R_f \approx 0.99$, then the Sharpe ratio of around 0.5 implies that: $\sigma(M) \approx \gamma\sigma(\Delta c) > 50$ % p.a. and with $\sigma(\Delta c) \approx 1\%$ p.a., this implies $\gamma = 50$. The correlation between aggregate consumption and the return on the S&P500 is around 0.2 between 1959-1982, and this implies $\gamma = 250$, which is far from reasonable values. (Mankiw & Shapiro, 1986 and Cuthbertson & Nitzsche, 2004)

[22] As Kan & Zhou (1999) states, this implication has been tested extensively in the finance literature by the so-called "traditional methodologies". In the traditional methodologies, a data-generating process is first proposed for the returns, and then the restrictions imposed by an asset pricing model are tested as parametric constraints on the return-generating process. The approach taken by the traditional methodologies has a potential problem, which is that when the proposed return-generating process is misspecified the test results could be misleading. Therefore, in applying the traditional methodologies, researchers typically have to justify that the proposed data-generating process provides a good description of the returns.

for risk premiums and testing power for model specification. When returns and factors are jointly normally distributed and independent over time, the classical beta method provides the most efficient unbiased estimator of factor risk premiums in linear models. If the SDF method turns out to be inefficient relative to the classical beta method for linear models under these assumptions, some variation of the beta method may well dominate the SDF method for nonlinear models as well in terms of estimation efficiency. On the other hand, if the SDF method is as efficient as the beta method, it would become the preferred method because of its generality. (Jagannathan & Wang, 2002).

This is exactly what Jagannathan & Wang (2002) proved. They wanted to show that Kan & Zhou (1999) made some serious mistakes in their paper. The conclusion of Kan & Zhou (1999) is the following: "When asset returns are generated by a linear factor model, there are two potential problems associated with the use of the SDF methodology: (1) the accuracy of the estimated risk premium can be very poor and (2) its over identification test has very little power in detecting misspecified models." In addition they found out that the standard error of the estimated risk premium is often 30 times greater than that of the traditional methodologies, which should make one extra cautious when applying the SDF methodology. However, Jagannathan & Wang (2002) stated that Kan & Zhou (1999) comparison, as well as their conclusion about the relative inefficiency of the SDF method, is inappropriate for two reasons. First, it is incorrect to ignore the fact that the risk premium measures in the two methods are not identical, even though they are equal at certain parameter values. Second, the assumption that the factor can be standardized to have zero mean and unit variance is equivalent to the assumption that the factor mean and variance are known or predetermined by the econometricians. By making that assumption, they give an informational advantage to the beta method but not the SDF method. Overall, Jagannathan & Wang (2002) conclude that in spite of the SDF method's generality, it has the asymptotic precision as efficient as the beta method for estimating risk premiums in linear factor-pricing models. Therefore, the SDF method should be preferable to the traditional methodologies because of its generality.

2.2.2 Stochastic discount factors and consumption

In an intertemporal utility space, a number of economic models of individual behaviour assume that investors obtain utility solely from consumption goods. At any point in time, utility depends positively on consumption and exhibits diminishing marginal utility. The utility function, therefore, has the same slope as the "risk-averter". Thus, it is concave. The only other issue is how we deal with consumption that accrues at different points in time. The most general form of such an intertemporal lifetime utility function is:

$$U_N = U(C_t, C_{t+1}, C_{t+2}..., C_{t+N}), \quad (11)$$

However, to imply this framework to stochastic discount factor concept, some restrictions are usually placed on the form of U, the most common being additive separability with a constant subjective rate of discount, $0<\theta<1$:

$$U_N = U(C_t) + \theta U(C_{t+1}) + \theta^2 U(C_{t+2}) + ... + \theta^N U(C_{t+N}), \quad (12)$$

The discount rate used in Equation (12) depends on the "tastes" of the individual between present and future consumption. If we define $\theta=1/(1+d)$, then d is known as the subjective rate of time preference. Thus, it is the rate at which the individual will swap utility at time $t+j$ for utility at time $t+j+1$ and still keep lifetime utility constant. As we can see from Equation (12), the discount factor plays a key role and when we are defining the utility of an individual. Within this concept, we can apply the SDF-framework with some mathematical definitions for the variables, especially for utility function and to the θ. (Cuthbertson & Nitzsche, 2004)

Comparing Equation (8) for a risky asset and for the riskless asset, we have Equation (13):

$$0 = E_t\left[M_{t+1}(R_{i,t+1} - R_{f,t+1})\right] = E_t M_{t+1} E_t(R_{i,t+1} - R_{f,t+1}) + Cov_t(M_{t+1}, R_{i,t+1} - R_{f,t+1})$$

Rearranging Equation (13), the expected excess return on any asset satisfies:

$$E_t(R_{i,t+1} - R_{f,t+1}) = -\frac{Cov_t(M_{t+1}, R_{i,t+1} - R_{f,t+1})}{E_t M_{t+1}}, \quad (14)$$

The expected excess return is determined by risk, as measured by the negative covariance with the SDF, divided by the expected SDF or equivalently the price of a riskless asset. An asset whose covariance with the SDF is large and negative tends to have low returns when the SDF is high, that is, when marginal utility is high. In equilibrium such an asset must have a high excess return to compensate for its tendency to do poorly in states of the world where wealth is particularly valuable to investors, e.g., in recessions. (Campbell, 2000). Thus, investors who want to maintain a flat consumption path over time will attempt to "smooth out" transitory movements in their asset wealth arising from time variation in expected returns – when excess returns are expected to be higher in the future, forward-looking investors will react by increasing consumption out of current asset wealth and labour income, allowing consumption to rise above its common trend with those variables and vice versa (Lettau & Ludvigson, 2001a).

3 CLASSIC ASSET PRICING MODELS

A fundamental principle in finance is the trade-off between risk and return. This means that one portfolio can be expected to outperform another portfolio only if the former is riskier in some appropriate sense. Currently, the two most famous theories that provide a rigorous foundation for computing trade-off between risk and return are the CAPM and the APT.

3.1 The capital asset pricing model

Over the last two decades researches have spent a great deal of time in evaluating the performance of the CAPM by testing how well the model fits the data. The empirical evidence on the validity of the CAPM is mixed. While some studies have concluded that the model is misspecified, others have found support for the predictions of the model. However, many of these studies have encountered serious and difficult problems in their efforts to provide the "best" empirical tests of the model.[23] (Akdeniz, 2000)

Early work on the Sharpe–Lintner CAPM tended to be broadly accepted. The classic studies of Black, Jensen & Scholes (1972) and and Fama & MacBeth (1973), for example, found that high-beta stocks tended to have higher average returns than low-beta stocks and the relation was roughly linear. This line of work follows the mean-variance approach to asset demand, according to which the demand for an asset is fully described by two moments of the return distribution, i.e. the expected return and variance. The purpose of the CAPM is to show how expected returns of portfolios are also functions of the riskiness of assets. The riskiness of an asset is defined as the variance of the distribution of the return. The investor's choice of a portfolio is based on risk adjusted expected returns. The

[23] Contradictory to the predictions of the CAPM, factors other than beta have been found to explain the cross-section of expected stock returns. This empirical evidence has led researchers to deduce that the pure theoretical form of the CAPM does not agree well with reality. (Akdeniz, 2000)

core of portfolio theories is that the risk of an asset depends on the return of other assets as well. In this way the CAPM relies on the concept of efficient markets. Thus, in an efficient market, the return on an asset includes a premium for risk that cannot be diversified away. More importantly, the risk premium is proportionate to the covariance between asset and market returns, not to the variance itself. This risk premium should be fully explained by the covariance between asset returns and market return. (Takala & Pere, 1991)

The CAPM is based on the idea that all investors choose mean-variance efficient portfolios. Market portfolio is defined to be the linear combination of all assets (including e.g. human capital) that satisfy the efficient mean-variance condition. The expected return of an asset i is:

$$E(R_{it}) = R_{f,t-1} + \beta_i (E(R_{mt}) - R_{f,t-1}) , \tag{15}$$

where β_i is the beta coefficient that measures the risk of security by the security's covariance with the stock market return $\beta_i = \text{cov}(R_{it}, R_{mt})/\text{var}(R_{mt})$.[24] Due to its mathematically simple form of relation between risk and return, the CAPM has been widely used in the financial field, e.g., in a firm's capital budgeting, portfolio construction, project evaluations and also to measure the effect of policy change on risk. (Chen, 2003)

To be consisted in our work of implementing the SDF framework, we will represent the CAPM also in the SDF form. The CAPM ties the discount factor M_{t+1} to the return on the "wealth portfolio". The function is linear:

[24] Thus, the CAPM predicts that investors in security *i* will be rewarded only for market related risk. Also, portfolio theory emphasizes the interdependence between the risk of particular asset and the risk of other available assets. The riskiness of an asset depends ex post on how much the actual return differs from the expected return. For a riskier asset the required rate of return is higher because of the risk premium. But over the efficient market risk, adjusted returns should be equal. Therefore different observed nominal returns simply reflect different expectations of risk. (Takala & Pere, 1991).

$$M_{t+1} = a_{t+1} + bR_{t+1}^{W}, \tag{16}$$

where a and b are free parameters. There is one factor that is the covariance of the security with the market return. One can find theoretical values for the parameters a and b by requiring the discount factor M_{t+1} to price any two assets, such as the wealth portfolio return and risk-free rate, $1=E(mR^{w})$ and $1=E(m)R^{f}$. The problem here is of course the definition of the wealth portfolio. We do not have good data on that or even a good empirical definition for it. That is why in empirical testing we will use a broad-based stock portfolio such as the value-weighed NYSE, S&P500 etc. (Cochrane, 2005)

3.2 The arbitrage pricing model

Understanding the empirical linkages between macroeconomic variables and financial markets has long been a goal of financial economics. One reason for the interest in these linkages is that expected excess returns on common stocks appear to vary with the business cycle. This evidence suggests that stock returns should be forecastable by business cycle variables at cyclical frequencies. Thus, macroeconomic variables may play an important role in understanding the financial markets and asset prices. (Lettau & Ludvigson, 2001b)

The APT formulated by Ross (1976) rests on the hypothesis that the equity price is influenced by limited and non-correlated common factors and by a specific factor totally independent from the other factors. By using this arbitrage reasoning it can be shown that in an efficient market, the expected return is linear combination of each factor's beta. (Morel, 2001) The core idea of the APT is that only a small number of systematic influences affect the long term average returns of assets. Each measure captures sensitivity of the asset to the corresponding pervasive factor. This may seem similar to the multiple betas in Merton (1973) Intertemporal

Capital Asset Pricing Model (ICAPM), but the resemblance to the CAPM ends here for, as Ross (1976) states, the APT is "much more an arbitrage relation than an equilibrium condition". If the factor model holds exactly and assets do not have specific risk, then the law of one price implies that the expected return of any asset is just a linear function of the other assets' expected return. If this was not the case, arbitrageurs would be able to create long-short trading strategy that would have no initial cost, but would give positive profits for sure.[25] (Dimson & Mussavian, 1999).

The risk associated with holding a particular security comes from two sources. The first source of risk is the macroeconomic factors that affect all securities. Their influence pervades the whole asset market and can not be diversified away. The second source of risk is the idiosyncratic element. This element is unique to each security and, according to the APT, in a broadly diversified portfolio it can be diversified away. Thus, an efficient market will only reward the risks associated with the systematic (macroeconomic) factors. (Watsham & Parramore, 1997)

The APT model assumes that the return to the ith security, R_{it}, is generated by a multi-index model:

$$R_{it} = a_{it} + b_{i1}F_{1t} + ... + b_{iJ}F_{Jt} + e_{it} \quad i = 1,2,...,N \text{ ,} \tag{17}$$

where the F_{jt} are factors (j=1,2,…,J); the b_{ij} are factor loadings or sensitivities and e_i is a random variable with $E(e_i)=0$, $E(e_i^2) = \sigma_i^2$, $E(e_ie_k)=0$, $i \neq k$, and $\text{cov}(e_i,F_j)=0$ for all i and j. There are N assets. The CAPM can be linked to the APT in the following way: the CAPM can be thought simply as a special case of the APT where $J=1$ and $F_1 = (E(R_m\text{-}R_f)$ (Groenewold & Fraser, 1997)

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[25] This arbitrage-free assumption is a fundamental principle in the whole financial theory. Long-short strategy means the total position in an asset, i.e. long position means buying the asset and short position means selling the asset.

The focus of the APT is on the expected rate of return $E(R_{it})$. Assuming that: a) there are no arbitrage possibilities, and b) the law of large numbers,[26] the model implies the following relationship between the expected return to asset and the factor sensitivities:

$$E(R_{it}) = \lambda_0 + \lambda_1 b_{i1} + ... + \lambda_J b_{iJ} + \varepsilon_{it} , \tag{18}$$

where λ_0 usually equals the risk-free rate of return and λ_j has the interpretation of the expected return to a portfolio (price of risk) with unit sensitivity to factor j and zero sensitivity to all other factors. (Cheng, 1996)

It can be seen from Equation (17) that each security i has a unique sensitivity to each F_{jt} but that any F_{jt} has a value that is the same for all securities. Any F_{jt} affects more than one security. Finally, from Equation (18) we see that λ_j is the extra expected return required because of a security's sensitivity to the jth attribute of the security. The problem here is that, whereas for the CAPM the correct F_{jt} is defined, for the multifactor model and the APT the set of F_{jt}'s is not defined by the theory. (Elton et al., 2007)

When implying the SDF framework to the APT, the statement that the discount factor M_{t+1} is a linear function of factors, is equivalent to the conventional statements of factor pricing models in terms of betas and factor risk premia. (Cochrane, 1996). This means that if all investors are single-period mean-variance optimizers, then the market portfolio is mean-variance efficient, which implies a beta pricing relation between all assets and the market portfolio.[27] Within the SDF framework, these conclusions can be reached directly from the assumption that the SDF is a linear combination of K common factors $F_{k,t+1}$, $k = 1...K$. The multifactor model can be written as follows:

[26] In a statistical context, the law of large numbers implies that the average of a random sample from a large population is likely to be close to the mean of the whole population.
[27] See, e.g., Sharpe (1964) and Lintner (1965), and also Roll (1977) critique.

$$M_{t+1} = a_t - \sum_{k=1}^{K} b_{kt} F_{k,t+1} \ , \quad (19)$$

then the negative of the covariance[28] of any excess return with the SDF can be written as:

$$-Cov_t(M_t R_{i,t+1} - R_{f,t+1}) = \sum_{k=1}^{K} b_{kt}\sigma_{ikt} = \sum_{k=1}^{K} (b_{kt}\sigma_{kt}^2)\left(\frac{\sigma_{ikt}}{\sigma_{kt}^2}\right) = \sum_{k=1}^{K} \lambda_{kt}\beta_{ikt} \ , \quad (20)$$

Here σ_{ikt} is the conditional covariance of asset return i with the k^{th} factor, σ_{kt}^2 is the conditional variance of the k^{th} factor, $\lambda_{kt} = b_{kt}\sigma_{kt}^2$ is "the price of risk" of the k^{th} factor, and $\beta_{ikt} = \sigma_{ikt}/\sigma_{kt}^2$ is the beta or regression coefficient of asset return i on that factor. This equation implies that the risk premium on any asset can be written as a sum of the asset's betas with common factors times the risk prices of those factors. (Campbell, 2000 and Jagannathan & Wang, 2002).[29]

3.3 General discussion of the CAPM and APT

The classic two-step cross-sectional regression approach proposed by Fama & MacBeth (1973) is widely used in finance literature. For example, Fama & French (1992) use this approach to show that there is a relationship between the expected stock return and beta, and Chen et al. (1986) use this approach to study a linear multifactor asset pricing model. A shortcoming of the cross-sectional regression approach is that it ignores the sampling errors associated with the estimated betas (errors-in-variables problem). Shanken (1982) shows that the Fama & MacBeth (1973) method overstates the precision of the estimated parameters when

[28] This expression is important, because remember, that the SDF relates to the returns only by covariance that is purely a mathematical expression.
[29] See also Cochrane (2005, p. 584–586). He does the same factor structure implementation to SDF framework, but with more proofing and rigorous style.

returns and factors are conditionally homoskedastic and temporally independent.

The question is that is it really the CAPM that is misspecified or is it the empirical tests of the CAPM that are performed in a wrong manner?[30] The CAPM is a two-period, linear model expressed in terms of the expected return and expected risk. Since these expectations cannot be measured, empirical studies use observed data to test for this linear relationship. However, e.g., Akdeniz (2000) is able to calculate both expected return and the true beta at a given period in time; thus, he is able to theoretically test the CAPM in its ex ante form. He finds that there is a linear relationship between beta and the expected return at any given period in time, as predicted by the "pure" CAPM.

Turning to the APT, the main problem in testing the APT is that it has been silent about which events and factors are likely to influence all assets. There is a rather embarrassing gap between theoretically exclusive importance of systematic "state variables" and our complete ignorance of their identity. The co-movements of asset prices suggest the presence of underlying exogenous influences, but no one has yet determined which economic variables, if any, are responsible. (Chen et al., 1986) Thus, there is no formal theoretical guidance in choosing the appropriate group of economic factors to be included in the APT model (Azeez & Yonoezawa, 2003).

This is both the APT's strength and its weakness. It is strength in empirical work since it permits the researcher to select whatever factors provide the

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[30] For this question Fama (1991) states that "I argue that the SLB (Sharpe-Lintner-Black model, that is, the CAPM) does the job expected of a good model. In rejecting it, repeadetly, our understanding of asset pricing is enhanced. Some of the most striking empirical regularities discovered in the last 20 years are "anomalies" from the tests of the SLB. These anomalies are now stylized facts to be explained by other asset pricing models."

best explanation for the particular sample at hand;[31] it is weakness in practical applications because, in contrast to the CAPM, it can not explain variation in asset returns in terms of limited and easily identifiable factors, such as equity's beta. (Groenewold & Fraser, 1997)

Since the nature and number of the priced factors are unspecified by the APT, two approaches have been used to empirically implement the theory.[32] The most widely used approach, originally proposed by Gehr (1978) and subsequently extended by Roll & Ross (1980), relies on factor analysis techniques to simultaneously estimate the common factors and factor loadings of security returns. The second approach is in contrast to the factor analysis approach. Chen et al. (1986) attempt to use macroeconomic variables to explain asset returns in the APT context. The macroeconomic variables are treated as factors in the APT return generating process.

The most common test of the APT is a two-step test; the first step involves the use of time-series data to estimate a set of factor loadings for each asset; the second step then regresses the (sample) mean returns on the factor loadings in a cross-section regression. The tests are not straightforward, however, for the model gives no guidance as to the number neither of factors nor to the identity of factors. Most empirical work is based on the use of factor analysis or principal components analysis to both identify the factors and provide estimates of the factor loadings. The estimated factor loadings are then used in a cross-section regression to explain mean asset returns. (van Rensburg, 1999) Factor and principal components analysis are based on a notion that observations on real, observ-

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[31] Fama (1991) states that: "The multifactor models are an empiricist's dream. They are off-the-shelf theories that can accommodate tests for cross-sectional relations between expected returns and the loadings of security returns on any set of factors that are correlated with returns."

[32] Connor (1995) tested three methods' explanatory power for U.S. equity returns. These methods were macroeconomic, fundamental and statistical factor analysis. He concluded that although statistical and fundamental methods outperformed macroeconomic model in terms of explanatory power, by other important criteria such as intuitive appeal and theoretical consistency, using a macroeconomic factor model is probably the strongest of the three approaches.

able, economic or other variables can be viewed as being a weighted sum of unobservable variables called “principal components” or “factors”. (Herbst, 2002) Principal component analysis is somewhat analogous to factor analysis – the goal is to determine a specific set of F_j’s and b_{ij}’s such that the covariance of residual returns is as small as possible (Elton et al., 2007).

An alternative to the use of artificially structured factors and their corresponding sensitivities is to identify factors *a priori*. In most empirical work, factors are artificially generated using factor analysis or principal components analysis and therefore have no “real-world” interpretation at all. From this perspective, the work initiated by Chen et al. (1986) is worth attention for they seek to identify the factors in the APT with macroeconomic variables that they feel should influence asset returns.

Finally, we want to make a comparison between these classic asset pricing models. Many textbooks and articles repeat two common complaints about the CAPM:

1) The empirical evidence that it takes more than one factor to explain the shared, or systematic, risk in securities discredits the CAPM.
2) In demonstrating that the risk premium of an asset depends only on its systematic factor loadings, the APT provides investors with a result of great practical value that the CAPM does not provide. (Treynor, 1993)

The APT is commonly put forward as a superior alternative to the criticized but widely-used CAPM. The alleged weakness of the CAPM, its baggage of “unrealistic assumptions” and its empirical shortcoming, are well known. Test of the CAPM typically display poor explanatory power as well as overestimating the risk-free rate and underestimating the market risk premium. The main criticism is particularly the use of betas to predict an asset's return – returns on high-beta stocks will tend to be overestimated and vice versa for low-beta stocks. (Groenewold & Fraser, 1997)

The advances of the APT over the CAPM are widely discussed in the literature and we will sum up a few of the main notes that we have found. First, in favour of the APT is that it does not make assumptions about the empirical distribution of asset returns. Second, the strong assumptions made about utility theory in deriving the CAPM are not necessary. The APT also admits several risk sources and therefore can be more operational and has a better forecasting ability than the CAPM. There is no special role for the market portfolio in the APT, whereas the CAPM requires that the market portfolio is efficient. The APT is also easily extended to a multi-period framework. (Elton et al., 2007; Morel, 2001)

There has to be made several rigorous assumptions when deriving the CAPM. When deriving the APT there has to be made only three assumptions:

1) There are no market frictions, e.g., short selling is unrestricted, investors can borrow and lend at risk-free rate and there are no taxes
2) There are numerous securities so that idiosyncratic risk can be diversified away
3) Investors are risk-averse and seek to maximize their wealth. (Lofthouse, 2001)

4 CONSUMPTION-BASED ASSET PRICING MODEL

4.1 Theoretical background

The CCAPM was developed independently by Lucas (1978) and Breeden (1979). Since those studies appeared, the model has been a target for an extensive amount of research and testing. The interesting part of the model is that prices and yields of financial assets are linked, in a general-equilibrium context, to investors' decisions about consumption and savings. The yield structure predicted by the model is therefore intimately tied to the nature of investors' preferences and, in particular, to the parameters of risk-aversion and intertemporal substitutions. Moreover, in contrast to the CAPM, intertemporal general-equilibrium models (such as the CCAPM) identify clearly the underlying economic forces that influence the risk-free real interest rate and the compensation that investors gain by accepting risk. (Carmichael, 1998)

In the CCAPM the consumer reduces consumption in period t by an infinitesimal amount and uses the resulting savings into an asset to raise consumption in period $t+1$. The only aspect of riskiness that matters to the consumer's decision of whether hold more of an asset is the relation between the asset's payoff and the utility of consumption. If an asset's payoff is highly correlated with consumption, its price must go down where its expected return is high enough for the consumers to hold it. The main element is the covariance between an asset's return and consumption. (Kahra, 2003).

The CCAPM is a more general asset pricing framework than the standard mean-variance CAPM. In this model, investors do not base their behaviour on the one-period mean and standard deviation of returns as in the CAPM, but the model is intertemporal in a sense that investors are assumed to maximise expected intertemporal utility of current and future consumption. In the CCAPM, financial assets allow the consumer to smooth her consumption pattern over time, selling assets to finance consumption in "bad" times and saving in "good" times, i.e. securities are held to transfer purchasing power from one period to another. Assets whose returns have a high negative conditional covariance with consumption will be willingly held even though they have low expected returns. This is because they can be sold at a time when they are most needed, namely when consumption is low, and, therefore, extra consumption yields high marginal utility. This model associates an asset's systematic risk with the state of the economy (i.e., consumption). (Cuthbertson & Nitzsche, 2004) The standard approach to test the CCAPM is that one uses the aggregate consumption data in conjunction with an assumed parameterization of a representative consumer's utility function to model the marginal rate of substitution across dates and states (Kasa, 1997).

Mankiw & Shapiro (1986) have argued that the consumption beta should offer a better measure of systematic risk, because it is a preferable on theoretical grounds.[33] First, it incorporates the intertemporal nature of portfolio decisions, as in Merton (1973) and Breeden (1979). Second, it incorporates other forms of wealth beyond stock market wealth that are relevant for measuring systematic risk in general.[34] However, the existing literature reports that the basic CCAPM cannot explain the asset returns in the U.S. nor in international context.[35]

[33] Furthermore, Kocherlakota (1996) states: "Representative agent models that imbed the Lucas-Breeden paradigm for explaining asset return differentials are an integral part of modern macroeconomics and international economics. Thus, any empirical defects in the representative agent model of asset returns represent holes in our understanding of these important subfields."

[34] Other point is that because it comprises about two-thirds of GDP, consumption spending has a large impact on the economy. Optimistic consumers are more willing to spend and borrow money for expensive goods, such as houses, cars, new clothes, and furniture. Therefore, the performance of consumer cyclical industries will be affected by changes in consumer sentiment and by consumers' willingness and ability to borrow and spend money. (Reilly & Brown, 2002)

[35] See, e.g., Hansen & Singleton (1982); Kocherlakota (1996) in the U.S. and Cumby (1990) in international context.

Lucas (1978) described the economy with a representative agent[36] having a standard utility function, and maximizing the expected present discounted value of lifetime utility. He was the first to provide a complete theoretical examination of the stochastic behaviour of the equilibrium asset prices resulting from a pure exchange economy in which a stochastic technological process describes the availability of a single perishable good and identical agents allocate among themselves a set of contingent claims written on the outcome of the random production process. Lucas (1978) shows that the equilibrium of this stylized economy implies that a stochastic factor M_t exists such that its expected product with any asset real return is equal to one:

$$E_t\left[M_{t+j}\left(1+R_{i,t+j}\right)\right]=1, \tag{21}$$

Because Equation (21) aggregates all investors into a single representative agent, who derives his utility from the aggregate consumption of the underlying economy, Lucas (1978) shows that M_t is simply the intertemporal marginal rate of substitution between current and future consumption for that representative agent. This is known as the marginal rate of intertemporal substitution of consumption:

$$M_{t+1}=\beta\frac{U'(C_{t+1})}{U'(C_t)}, \tag{22}$$

The variable M_{t+1} plays the role of the SDF. In a consumption model, the SDF is in fact assimilated into the consumers' marginal rate of intertemporal substitution. From the Equation (22) we can see that if the SDF is above one, investors prefer the marginal utility of the future, which is an irrational assumption.[37] (Carmichael, 1998) By employing the necessary

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[36] Since preferences are homothetic, if they are common to all consumers and if consumers are similar in all other respects with the possible exception of differing wealth, then the representative agent assumption can be justified in usual way. (Epstein & Zin, 1989)
[37] This theorem is usually compared with the "time value of money"-theory.

first-order Euler conditions[38] for the model, asset prices can be generated explicitly by using the exogenous consumption process and the stochastic discount factor. (Chen, 2003; Pasquariello, 2000)

The representative agent faces the following maximization problem:

$$\max E_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}U(C_t)\right], \tag{23}$$

where C_t is consumption in period t, $U(\cdot)$ is strictly concave utility function, β is a constant discount factor and $E_t[\cdot]$ denotes conditional expectation. In any period t the agent can choose to buy consumption goods or purchase an asset which yields payoff R_{t+1} in the next period. The agent is assumed to receive income from assets purchased in the previous period and labour income W_t. All prices are denominated in terms of the consumption good. Therefore the agent's budget constraint is:

$$C_t + \sum_{i=0}^{N} P_{it}Q_{it} \le \sum_{i=0}^{N} P_{it}Q_{it-1} + W_t\,, \tag{24}$$

where N is the number of assets in the economy, P_{it} is the value of asset i in period t (price plus any cash payout), Q_{it} is the amount of asset i owned by the individual at the end of period t, and W_t is real labor income at date t. (Hansen & Singleton, 1982; Hall, 1993)

The maximization of Equation (23) subject to Equation (24) gives the necessary first order condition (FOC) (Hansen & Singleton, 1982):

$$P_{it}U'(C_t) = \beta^{M_i}E_t\left[R_{it+M_i}U'(C_{t+M_i})\right], \tag{25}$$

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38 Euler equation is the first-order conditions for optimal consumption and portfolio choices of the representative agent that can be used to link asset returns and consumption. (Campbell, 2000)

Equation (25) states that the representative agent will forego a dollar of consumption today and invest it in asset i so long as the expected additional discounted marginal utility that it provides is greater than today's marginal utility (Smoluk & Neveu, 2000).[39] The FOC for maximising the expected utility has the agent equating the utility loss from a reduction in current consumption, with the additional expected gain in (discounted) consumption next period (Cuthbertson & Nitzsche, 2004).

In the theory of finance, the expected excess return on any risky asset over the risk-free rate is explained by the quantity of risk times the price of risk. In the CCAPM the quantity of equity market risk is measured by the covariance of the excess stock return with consumption growth, whereas the price of risk is measured by the coefficient of relative risk-aversion of a representative investor. (Kahra, 2003) In order to get testable implications we must specify a utility function for the representative investor. As in most studies we use the constant relative risk-aversion (CRRA) utility function given by:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{26}$$

$$U'(C_t) = C^{-\gamma},$$

where γ is the coefficient of relative risk-aversion that is a constant (γ>0). Higher values of γ correspond to a more risk-averse agent. Power utility is a natural choice because of its desirable theoretical properties.[40] The log utility, when γ approaches one, has also attracted some theoretical sup-

[39] From the Equation (25) above, we can show a simplified example. If, e.g., the ith asset is a default-free, zero coupon bond with term to maturity M_i, then R_{it+M_i} equals the real par value of the bond at date $t+M_i$. Alternatively, if Q_{it} denotes the quantity of shares of stock a firm held at date t, D_{it} denotes the dividend per share of stock i at date t, and M_i=1, then $R_{it+1} = (P_{it} + D_{it+1})$, then Equation (25) becomes: $P_{it}U'(C_t) = \beta E_t[(P_{it}+D_{it+1})U'(C_{t+1})]$.

[40] Mehra (2003) states that the CRRA-function is used because it is not depend on the amount of capital. This means the returns are stationary. With the CRRA-function we can also assume that there is a representative agent that we can use in the empirical studies.

port.[41] Since γ is estimated, its distance from one can be determined. (Brown & Gibbons, 1985) Finally, Arrow (1951), among others, has emphasized that absolute risk-aversion should be decreasing, and power utility displays this characteristic.

When employing the Euler equation and using the utility function specified in Equation (26), we get the following formula:

$$E_t\left[\beta^i\left(\frac{C_{t+j}}{C_t}\right)^{-\gamma}\left(1+R_{i,t+j}\right)\right]=1\,, \tag{27}$$

Equation (27) represents the building block for the CCAPM: asset i's expected excess return results to be inversely correlated to the covariance between the asset's return and the stochastic discount factor M_t. This equation represents equilibrium, as it equates the expected marginal rate of substitution between current and future consumption with expected rate of transformation between current and future wealth. Our main interested points are the parameters γ and β. (Lund & Engsted, 1996; Pasquariello, 2000) Because β is the rate of time preference, higher values of β imply that consumers place a higher value on future consumption, meaning that they would prefer to save today and to consume tomorrow. An economically plausible β should be less than one indicating that consumers discount future consumption. According to Mehra & Prescott (1985), economically plausible estimates for γ are greater than zero and less than or equal to 10. Coefficients of relative risk-aversion that are around 10 or more suggest extremely risk-averse consumers who prefer low consumption growth (less volatile consumption over time). Coefficients of less than zero contradict Equation (27) by indicating nonconvex preferences (increasing marginal utility) and the non-existence of an intertemporal equilibrium for asset prices. (Smoluk & Neveu, 2000)

[41] See, e.g., Rubinstein (1976).

Thus, an asset is considered to be risky if its excess return has a negative covariance with the marginal rate of interemporal substitution of consumption (the SDF). A negative covariance means that the asset tends to offer a higher excess return than expected when the marginal utility of consumption is weaker than expected, i.e., when consumption is stronger than expected. The risk is systematic in the sense that it is linked to the rate of growth of the marginal utility of aggregate consumption. (Carmichael, 1998)

The direct applications of the CCAPM require an ex post consumption-based model which is assumed to adequately describe how realized returns (R_{it}) are generated in terms of the growth in per capita real aggregate consumption, C_t:

$$R_{it} = \alpha_{ci} + \beta_{ci} C_t + \varepsilon_{it}, \qquad (28)$$

While the β_{ci} estimates measure the sensitivity of the asset's return to the growth rate in real aggregate per capita consumption, they are unconstrained estimates relative to the CCAPM. Thus, the theoretical restrictions implied by the CCAPM can be identified and applied in the estimation process to produce (constrained) CCAPM consumption beta estimates. (Faff & Oliver, 1998).

To employ the SDF framework, the CCAPM is a SDF model in which, under power utility, the discount rate is determined by consumption growth. Consumption, the risk-free rate, the expected equilibrium return on risky assets and assets prices are all simultaneously determined in the SDF model. In the SDF model, M_{t+1} may be time-varying applying a time-varying risk premium. (Cuthbertson & Nitzsche, 2004)

Lucas (1978) showed how the equilibrium SDF implied by an exogenous consumption process and utility specification can be derived. He assumed that the economy can be described by a representative investor, with a standard utility function, who consumes aggregate consumption. The first-order conditions of this investor determine the SDF. Asset prices can be determined explicitly by modelling payoffs jointly with the SDF.[42] (Campbell, 2000)

4.2 The puzzles and properties of the CCAPM

4.2.1 Equity premium and risk-free rate puzzles

It is often claimed by financial advisors and academics that over longer periods of time equity investments should generate on average a higher return than bond investments in order to compensate the investor for taking the higher risk of equity returns. Understanding these characteristics of equity returns, bond returns and the equity premium is important for the investors' long term asset allocation decision as well as for academic research on securities markets. Despite some empirical evidence it still remains an unsolved question, whether stocks outperform bonds on a risk-adjusted basis by an adequate margin. Many empirical studies of this issue are controversially debated.[43] (Bessler, 1999) The simplest solution for the higher average return of stocks could be that stock returns covary more with consumption growth than do the bond returns and therefore investors see stocks as a poorer hedge against consumption risk, and so stocks must earn a higher average return (Kocherlakota, 1996).

The whole equity premium puzzle can be written as follows: the theory predicts that the expected excess return, on any risky asset over the risk-free asset, is explained as the quantity of risk times the price of risk. Thus:

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[42] Lucas (1978) modelled the aggregate stock market as paying a dividend equal to aggregate consumption; this is equivalent to the traditional assumption in finance that the stock market is "market portfolio" of all wealth.

[43] As Cochrane (1996) concludes: "The volatility of year to year stock returns is so great that it's very hard to measure average returns with any sort of statistical certainty. The best that statistics can do is to say that we are 95% certain that the true average excess returns are between 3% and 13%.

$$E(R_t^e) = Corr(R_t^e, \Delta c_t)\sigma(R_t^e)\sigma(\Delta c_t)\gamma, \quad (29)$$

The quantity of stock market risk is measured by the covariance of excess return with consumption growth. The coefficient of relative risk-aversion of a representative investor captures the price of risk. The well-known fact is that consumption is smooth and that makes the covariance to be small. Hence, the premium can be explained by allowing a very high coefficient of risk-aversion.[44] (Kahra, 2003 & Söderlind, 2006)

In the equilibrium models, such as the CCAPM, the expected return of an asset is a result of equilibrium of supply and demand for the asset. In an equilibrium dominated by very risk-averse investors, we should see risky assets like stock providing a higher expected return in order to attract cautious investors. In an equilibrium in which the average investor is only mildly risk-averse, the spread in expected return between stocks and less risky assets should be much smaller. What you do not expect to find is a high expected return relative to less risky assets and mildly risk-averse investors. However, this is exactly what Mehra & Prescott (1985) discover in their equilibrium analysis of the U.S. capital markets over the period 1889 to 1978. (Elton et al., 2007)

Many empirical studies for the U.S. and other countries have found that for longer investment horizons the equity premium is usually positive and substantial. Moreover, the premium is generally larger than standard economic models would predict, given plausible assumptions about investor's risk-aversion. This paradox is commonly referred to as the "equity premium puzzle". (Bessler, 1999) The standard model of the CCAPM predicts

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[44] Söderlind (2006) gives a descriptive example for the Mehra & Prescott (1985) data:

$$\underset{0.06}{E(R_t^e)} = \underset{0.33}{Corr(R_t^e, \Delta c_t)}\,\underset{0.14}{\sigma(R_t^e)}\,\underset{0.01}{\sigma(\Delta c_t)}\,\gamma$$

The puzzle can be seen from here that the volatility of consumption growth is very smooth. Therefore, even if the correlation between the excess return and consumption growth was perfect (one), then we would need a relative risk-aversion of 43 to make this equation hold.

an equity premium of less than half a percentage point per year. Over the long run, buying and holding the market portfolio of stocks has been a much better investment strategy than investing in government bonds. Since 1889 in the U.S., the real return on America's Cowles Commission and S&P Composite indices has averaged 7% percent per year, while the average real return on short-term government dept was less than 1% per year, so that the historical equity premium of US is around 6%.[45] This difference between the CCAPM prediction and empirical findings is called the "equity premium". In the CCAPM, the equity premium is the result of the representative investor's aversion to risk. Since average consumption is stable (it is smooth, because its coefficient of variation is about 3 percent p.a.), risk-aversion produces a small equity premium. The difference between the large historical equity premium and the small predicted value of the CCAPM is the equity premium puzzle. (Grant & Quinn, 2005).

Calibration exercises conducted by Mehra & Prescott (1985) show that it is very difficult to generate a significant premium (more than 2 percent) with isoelastic preferences when the risk-aversion coefficient is kept below 10. Stated differently, the consumption growth appears to be too smooth to justify the mean equity premium (Constantinides, 1990). There are a lot of papers that try to justify the use of the CCAPM or solve the equity premium puzzle.[46] They seek to break down the assumptions employed by Mehra & Prescott (1985) in hopes of explaining this puzzle. This kind of literature is usually categorized into three areas: consumer preferences, the representative agent and market frictions. Papers focusing on the con-

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[45] Nyberg & Vaihekoski (2005) find that the equity premium for Finnish equity market during the period 1920-2004 is 5.33%. They argue that the Finnish equity premium appears to be on the same level as in the U.S. supporting the equity premium puzzle also in Finland.

[46] However, the whole equity premium puzzle has had a bit of controversy. An interesting point was made by Fama (1991). He states that: "The habit formation argument has a ring of truth, but I also think that a large equity premium is not necessarily a puzzle; high risk-aversion (or low intertemporal elasticity of substitution for consumption) may be a fact. Roughly speaking, a large premium says that consumers are extremely averse to small negative consumption shocks. This is in line with the perception that consumers live in morbid fear of recessions (and economists devote enormous energy to studying them) even though, at least in the post-war period, recessions are associated with small changes in per capita consumption."

sumer preference assumption seek to generalize the very convenient time-separable power utility function employed by Mehra & Prescott (1985).[47] Papers focusing on the representative agent assumption examine the implications using aggregate consumption data to test the CCAPM. And finally, papers studying the market frictions examine transaction costs, the implication of incomplete markets, and economically constrained[48] consumers.

The academic literature has shown that the CCAPM requires that the risk-aversion coefficient is somewhere between 10 and 250 that the model can produce the large equity premium observed in real life. Mehra & Prescott (1985) calculated that the coefficient of risk-aversion required to generate to generate the historical spread between stock returns and riskless bond returns would have to be between 30 and 40. What does this number mean? Suppose an individual with a risk-aversion of 50 faced a 50-50 gamble of doubling or halving her savings. With this level of risk-aversion, they would pay 49% of their savings to avoid the loss of 50%. This kind of behaviour is difficult to rationalize. This individual would forego a 50% chance of doubling her money, and accept a certain loss of 49% to avoid losing an additional 1% more.[49] (Elton et al., 2007)

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[47] E.g., Constantinides (1990) concludes that the large equity premium is consistent with models in which utility depends on past consumption (habit formation).

[48] The theory of the economically constrained consumers is intuitively interesting, because it makes sense that the consumption patterns of lower income consumers, who live from paycheck-to-paycheck, should bear a weaker relation to real asset returns that higher income consumers since they lack the ability to save and manipulate consumption. However, e.g., Smoluk & Neveu (2002) tested this theory and they conclude that "the test fail to find any discernible patterns across income groups that are consistent with the idea that liquidity constraints bind lower income consumers." Due to these results, our discussions concentrate on the use of utility functions and the use of aggregate consumption data.

[49] Cochrane (2005) shows also an interesting example, where the different values of the risk-aversion coefficients are used. The losses that highly risk-averse investors are willing to take are irrational. Let us assume that an investor has a risk-aversion coefficient of 50. What does this imply for the real interest rates? If one earns $50.000 p.a., and one normally spends 5% ($2500) on an annual vacation, then one will voluntarily skip this year's vacation only if interest rates rise to about: $R_{ft} = (52.500/47.500)^{50}-1 = 14.800\%$ p.a. Thus, with time-separable power or logarithmic or exponential utility functions, the equity premium puzzle arises because the CCAPM cannot explain both the observed high average return for risky assets together with relatively low real interest rates that also exhibit low volatility (Cuthbertson & Nitzsche, 2004).

This is also the conclusion of Mankiw & Zeldes (1991). They state that an individual with a CRRA above ten would be willing to pay unrealistically large amounts to avoid bets. However, they consider only extremely large bets. In contrast, Kandel & Stambaugh (1991) show that even values of the CRRA as high as 30 imply quite reasonable behaviour when the bet involves a maximal potential loss of around one percent of the gambler's wealth.

Kocherlakota (1996) studies this whole problem and he concludes that the equity premium is still a puzzle, no matter what the researchers try to do with that. The right values of the risk-aversion coefficient, however, have been a target of a lot of criticism. In the first paper of Mehra & Prescott (1985) they state that the value should be between zero and three. Friend & Blume (1975) state that the CRRA should be around two or a bit more. A lot of other studies have shown that the CRRA value should be below five.[50] Of course, there are a lot of researchers[51] that believe that there is not at all equity premium puzzle: individuals just are more risk-averse than we thought, and this high degree of risk-aversion is reflected in the spread between stock and bonds.

Weil (1989) shows that the equity premium puzzle presents a second anomaly. According to standard models of individual preferences, when individuals want consumption to be smooth over states (they dislike risk), they also desire smoothness of consumption over time (they dislike growth). Given that large equity premium implies that investors are highly risk-averse, the standard models of preferences would in turn imply that they do not like growth very much. Yet, although bond returns offer only a low rate of return, individuals defer consumption at a sufficiently fast rate to generate average per capita consumption growth around two percent p.a. This is known as the risk-free rate puzzle: although individuals like consumption to be very smooth, and although the risk-free rate is very

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[50] See, e.g., Kocherlakota (1996).
[51] See, e.g., Cecchetti et al. (1990) and Hansen et al. (1994).

low, they still save enough that per capita consumption grows rapidly. (Kocherlakota, 1996)

The risk-free rate puzzle comes from the equity premium puzzle, because we need a high value of the CRRA to explain the equity premium. Thus, our reciprocal of the CRRA, which is the elasticity of intertemporal substitution (EIS), becomes smaller. Then extremely small values for the EIS imply that investors have an overpowering preference for a flat consumption path. Given the historical upward drift in consumption, this implies an extremely strong desire to borrow from the future. Unless this is offset by a low or even negative rate of time preference, the result is a counterfactually high real interest rate. (Campbell et al., 1997)

The data period of Weil (1989) was the same as in Mehra & Prescott (1985). Over that period, the average annual growth rate of consumption was 0.018, with a variance of 0.0013. Yet, unless the relative risk-aversion coefficient is very weak and the elasticity of intertemporal substitution is consequently very high, the annual average real interest rate is several times higher than 0.80 percent, the level observed by Mehra & Prescott (1985). E.g., even if the value of the CRRA is as low as two, the predict value of a real interest rate is higher than the time preference rate of 3.34 percent. Hence, unless the CRRA is negative, the model is incapable of predicting the observed the real interest rate. (Carmichael, 1998). To conclude the risk-free rate has to be a lot larger than it is in the reality that there is a demand for bonds and investor will be interested in investing in them.

4.2.2 Non-expected utility, habit formation and other factors

As we saw in the previous chapter, the CCAPM produces a couple of interesting puzzles. These puzzles and relaxing them has contributed to an extensive amount of research in the finance field. Here we will go through the most important papers that appear often in the finance literature. The conclusion is that we can get rid of the risk-free rate puzzle, but the equity premium puzzle stays, i.e., we can not solve both of them at the same time.

In the face of these empirical problems, some authors have suggested modifications to the consumption model to make it more general. Essentially, consumers in the CCAPM use financial assets above all as a means of smoothing out the marginally utility of consumption over time. In principle, there is no constraint that would require marginal utility for the period t to be dependent solely on consumption for period t. It is reasonable to assume that the marginal utility of consumption may also be influenced by other variables, such as the consumption of leisure or the level of consumption attained in the recent past. In this case, the covariance of excess returns with these other variables will also influence risk premiums, and may even help alleviate some of the empirical difficulties noted earlier. (Carmichael, 1998)

As we stated in the previous chapter, the risk-free rate puzzle arises from the fact that the CRRA and the EIS are reciprocals of one another. The natural way to solve this problem is to separate them. Eipstein & Zin (1989 & 1991) were the first ones to empirically test this kind of model. Their model consists of three appealing features: (1) intertemporal substitution and risk-aversion are separated; (2) they integrate atemporal non-expected utility theories into a temporal framework and (3) they generate implications for the temporal behaviour of consumption and asset returns.

In their model a risk-averse consumer may still be highly willing to substitute consumption intertemporally. The other important property of their model was that they introduced “non-expected utility” preferences into the model. This means that the marginal utility of consumption in good times is not independent of the level of consumption in bad times. Their model solved the risk-free rate puzzle by allowing for high levels of risk-aversion and intertemporal substitution.[52] However, it did not solve the equity-premium puzzle.[53]

Sundaresan (1989) and Constantides (1990) have argued in favour of the habit formation, a positive effect of today's consumption on tomorrow's marginal utility of consumption. They try to solve the puzzles by introducing non-separability of preferences over time into the model. This is done by means of mechanism for consumption habit-forming. The basic idea of this model is that the utility of current consumption is not independent of consumption levels attained in past periods. The utility function in equation (26) is replaced with the following function:

$$U(C-X) = C - X^{1-\eta}, \qquad (30)$$

where X represents the accustomed level of consumption and η influences the preferences curve.[54] The accustomed level of consumption is modelled as a variable that adjusts gradually to variations in consumption. With these preferences, the marginal utility of consumption rises as consumption approaches its accustomed level.[55] For this reason, the risk-aversion coefficient varies with the business cycle, according to this relationship:

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[52] Weil (1989) tested also the non-expected utility hypothesis. He manages to reproduce exactly the observed levels of the equity premium and the real interest rate, by setting the risk-aversion coefficient to 45 and the EIS to 0.10. As we see from these numbers, at least the risk-aversion coefficient is extremely large. Another excellent study is from Lee (1997). He studies five international stock markets: Canada, Hong Kong, Japan, UK and US. He founds evidence to doubt the validity of the conventional expected utility model. He concludes that "As long as timing of resolution matters, both consumption growth and world market portfolio return are important in characterizing international stock market returns."

[53] See also Eichenbaum et al. (1988) and Jagannathan & Wang (1996). They also show that nonseparable utility over consumption and leisure does not help explain mean asset returns.

[54] As always, there is also controversy of the functional form of the utility function. E.g., Abel (1990) proposes that it should be a power function of the ratio C_t/X_t.

[55] This property states that the effect is only the agent's *own* decisions on future levels of habit. There are also external habit models, such as Abel (1990), where habit depends on aggregate consumption that is unaffected by any agent's decisions. Abel (1990) calls this "catching up with the Joneses".

$$\gamma_t = \eta \frac{C_t}{C_t - X_t}, \tag{31}$$

Equation (31) shows that a consumer's risk-aversion rises as his level of consumption approaches the accustomed level. Thus, the preferences curve is no longer the only determinant of risk-aversion. This model can reproduce the observed level of the equity premium by allowing the risk-aversion premium to be high. There is no news here, because other models do this also. However, this model offers a new explanation of the risk-free rate puzzle. With isoelastic preferences, an increase in the risk-aversion coefficient, leads to an increase in the level and the variability of the real interest rate. A model with consumption habits gets around this problem by adding a precautionary or "rainy day" savings component that will counter the upward pressure on the real interest rate. Thus, consumers who are aware of their consumption habits will become more averse to risk when their current consumption drops relative to the accustomed level. (Carmichael, 1998).

Li & Zhong (2005) tested the CCAPM with habit formation to study the predictability and cross-section of returns from the international equity markets. They found that the predictability of returns from many developed countries' equity markets is explained in party by changing prices of risks associated with consumption relative to habit at the world as well as local levels. They also found out that the model performs mildly better than the traditional CCAPM, the unconditional and conditional world CAPMs and a three-factor international asset pricing model. Watcher (2002) uses the habit formation model to predict returns on bonds and stocks. He is able to construct a model that produces positive excess returns on long-term bonds, an upward sloping average yield curve, and allows for realistic levels of time-variation in the mean of consumption growth.

One possibility is to introduce additional state variables like leisure or wealth in the utility function. Bakshi & Chen (1997) provide an intertemporal model, where utility also depends on a social wealth index $S_t = W_t/V_t$, where W_t = individual's wealth, V_t = social wealth index (e.g., being middle class). They call this the "spirit of capitalism" model since utility depends not only on consumption but also independently on your "wealth status", relative to that of others. The utility function then can be written as follows:

$$U(C_t, W_t, V_t) = \frac{C_t^{1-\gamma}}{(1-\gamma)} \left(\frac{W_t}{V_t} \right)^{-\gamma}, \tag{32}$$

The wealth portfolio they use is the aggregate stock market index and the returns are different size-based portfolios. This model is able to solve the equity premium puzzle in a sense, but still the other puzzle of the risk-free rate remains, so we are still left with no answer with solving the puzzles at the same time.

One particularly interesting paper is made by Piazzesi et al. (2006). The argument is that consumers concern residential real estate in their intertemporal decision. The paper is inspired by the fact when Cochrane (1996) found that real estate investment growth has impact on cross-section of stock returns. They introduce a new risk factor, which is called "composition risk". That risk factor relates changes in asset prices also to changes in expenditure shares as a second factor. Thus, changes in expenditure share on housing emerge as a second factor that drives asset prices. Their model does well compared to the traditional CCAPM: the model implies higher equity and housing premia, higher stock return volatility, a lower risk-free rate (which is not volatile), and lower bond premia.

Lettau & Ludvigson (2001a) introduces the role of fluctuations in the lagged aggregate consumption-wealth ratio for predicting stock returns. Using U.S. post-war quarterly stock market data, they find that these fluctuations in the consumption wealth-ratio are strong predictors of both real stock returns and excess returns over a Treasury bill rate. They note that aggregate consumption, asset holdings, and labour income share a common long-term trend, but they deviate substantially from one another in the short run. The consumption-wealth ratio that they construct is the following:

$$c_t - w_t = c_t - wa_t - (1-w)y_t\,, \qquad (33)$$

where c_t, a_t and y_t denote consumption, asset holding and labour income, respectively, and w_t is nonhuman wealth share. They call their variable *cay.* Their model performs well: the CCAPM fits the data well and it outperforms the CAPM in being able to explain the value- and size-premia. Ioannidis et al. (2006) tests the same methodology as Lettau & Ludvigson (2001a) for Australia, Canada and the United Kingdom. They conclude that lagged cointegrating variable (*cay*) is an important addition to the set of variables, which can forecast future stock returns.

Couple of other interesting papers worth mentioning are Gregoriou & Ioannidis (2003) and Smoluk & Neveu (2002). In the former the CCAPM under transaction costs is tested in the UK stock market. Overall the statistical tests are unable to reject the bid-ask spread as an independent explanatory variable in the CCAPM. Thus, this leads to the conclusion that transaction costs should be included in asset pricing models. The latter research of Smoluk & Neveu (2002) tests intuitively interesting hypothesis that the using of aggregate data is likely to lead to a specification error since a significant portion of consumers live from paycheck-to-paycheck and are therefore constrained in their ability to intertemporally allocate consumption. He uses consumption expenditures grouped by consumer income, but he concludes that “Overall, the tests fail to find any discernible patterns across income groups.”

To conclude this chapter, we can say that attempts to solve the equity premium puzzle using the SDF model have met with some but not complete success, most notably by introducing additional state variables (e.g., wealth, habit consumption) into the utility function. The real problem is that there is not a widely agreed (both theoretically and empirically) model that can simultaneously explain the risk-free rate and the equity premium puzzles.

4.2.3 Measuring the consumption

The theoretical models that try to bridge consumption and finance have found an inconsistent relationship between consumption data and asset returns. The inconsistency stems from the finding that equity risk, as measured by the covariance between stock returns and consumption growth within the context of the CCAPM, is too small to warrant the large average risk premium on stocks. In other words, the CCAPM shows that aggregate investors, or the representative single agent, must be extremely averse to consumption (showing a large coefficient of risk-aversion) to demand such a large equity premium. (Smoluk & Neveu, 2002)

A large portion of this problem is usually referred to the fact that the aggregate consumption is too “smooth”, i.e., its volatility is too small. This means the asset return covariances with consumption growth are small. Hence, the equilibrium model can fit the equity premium only if the price of risk, e.g., the coefficient of relative risk-aversion, is very large. Breeden et al. (1989) have recognized and attempted to solve four types of problems that arise in measuring the rate of growth in per capita consumption. They examine four econometric problems associated with measured consumption: 1) the durables problem, 2) the problem of measured consumption as an integral of spot consumption rates, 3) the problem that consumption data are reported infrequently, and 4) the problem of pure sampling error in consumption measures.

The durables problem means that the consumption data is reported as expenditures, rather than consumption. The CCAPM prices assets with respect to changes in aggregate consumption between two points in time.

In contrast, the available data on aggregate "consumption" provide total expenditures on goods and services over a period of time. This problem is usually dealt with an assumption that expenditures on nondurable goods plus services act as a good proxy for consumption. (Breeden et al., 1989)

The second problem means that the reported ("interval") consumption rate for a quarter is the integral of the instantaneous ("spot") consumption rates during the quarter. Hence, the variance of interval consumption changes is shown to have only two thirds the variance of spot consumption changes, while the autocorrelation of interval consumption is 0.25 due to the integration of the spot rates. The averaging caused by the integration leads to the lower variance for reported consumption. Furthermore, the covariance is also smaller[56] and that leads to the fact that betas measured relative to reported quarterly consumption changes are ¾ times the corresponding betas with spot consumption. This analysis is used to adjust the growth in consumption so that betas will be the appropriate size. (Breeden et al., 1989)

The third problem is the unavailability of monthly data. The authors construct a portfolio that has a maximum correlation with the appropriate consumption series. By doing this, Breeden et al. (1989) are able to test the CCAPM using monthly observations on this portfolio. The correlation between this portfolio for the period of study, years 1929–1982, with the CRPS value-weighted index is 0.67. For the last problem of pure sampling in consumption measures, the authors conclude that if errors in measuring consumption are random and uncorrelated with economic variables, the estimate of the price of risk will be upward biased but their test of the significance of the model will not be biased.

[56] Precisely, "The population covariance of an asset's quarterly return with reported (interval) consumption is half the population covariance of the asset's return with spot consumption changes". Breeden et al. (1989)

5 EMPIRICAL RESULTS FROM PREVIOUS STUDIES

5.1 Tests of the CAPM

The basic tests of the CAPM have always been based on the natural implications that arise from the theory: higher returns should be expected from stocks that have higher beta, and the relationship between expected return and beta should be linear. In this case of the Treynor-Sharpe-Lintner-Mossin CAPM, the slope of this line should be equal to the market risk premium and the intercept should be equal to the risk-free rate. For the zero-beta CAPM, the slope should be less than the market risk premium, while the intercept should be greater than the risk-free rate. Moreover, there should be no systematic reward for bearing non-market risk and any deviations in realised returns from the CAPM should not be predictable. (Dimson & Mussavian, 1999)

These implications provided the testable hypotheses for the early empirical studies of the CAPM. Black et al. (1972) performed the earliest rigorous tests of the CAPM. They found that relation between mean excess return and beta was linear. They also found that the intercept and the slope of the cross-sectional relation varied with different sub periods and were not consistent with the traditional form of the CAPM. The Black-Jensen-Scholes (1972) (hereafter BJS) and Fama & MacBeth (1973) studies were methodological breakthroughs. Many following tests of the CAPM employ techniques derived from these works. In principle, the common way is to regress security or portfolio returns (the dependent variable) on their betas (the independent variable). Of course this beta is not known and can only be estimated with error, and this violates the assumptions underpinning regression. To solve this problem, BJS and Fama & MacBeth (1973) introduced the two-pass methodology. Although this methodology has limitations[57], this is called a breakthrough because it is so widely used in empirical work. In the first pass they run a time series regression of portfolio returns on the market return, which gives estimates of portfolio betas. At the second pass, BJS regressed the average returns on the estimated betas from the first pass. This cross-sectional regression provides a test for the traditional CAPM. However, BJS note that this does not give "any direct tests aimed at explaining the existence of the beta factor", i.e., the zero-beta CAPM. To overcome this limitation, Fama & MacBeth (1973) modify the second pass by performing cross-sectional regressions on a month-by-month basis and then taking the time-series average of the estimated risk premium. This, it turns out, allows them to test directly for the validity of the zero-beta CAPM. (Dimson & Mussavian, 1999)

However, as Fama (1991) states: "Because the CAPM is just a model, we should have known better because surely the model is false." The first attack is Roll (1977) criticism that the early tests are not much evidence for the CAPM model because the proxies used for the market portfolio (like the equally weighted NYSE portfolio) do not come close to the portfolio of invested wealth called for by the model. The market index must include bonds, property, foreign assets, human capital and anything else, tangible or intangible that adds to the wealth of mankind. Roll (1977) points out that "the portfolio used by BJS was certainly not the true portfolio" and if this "true portfolio" can not be found, the CAPM can never be tested.[58]

The CAPM has had its most criticism because of the assumption that market risk only describes the cross-section of expected returns. There is mounting evidence that other risk factors also affect stock returns. These include the price/earnings ratio (Basu, 1983) company size (Banz, 1981),

[57] Probably the most severe is the errors-in-variables problem, that arises because the second-pass independent variables (i.e., the betas) are themselves estimates (with error) from the first-pass regression.

[58] Stambaugh (1982) evidence that tests of the CAPM are not sensitive to the proxy used for the market return, suggests that Roll (1977) criticism is too strong, but this issue can never be entirely solved. (Fama, 1991) Also, because labor income is about two-thirds of U.S. GDP and capital income is only one-third of GDP, it is clearly important to model human capital as a component of wealth (Campbell et al., 1997).

book-to-market equity and momentum effect (Fama & French, 1992, 1996, respectively), to name a few. They all found that these factors affect stock returns and therefore the CAPM is rejected.[59] The empirical evidence provided further motivation for research into other models of asset pricing that might more successfully explain returns, or at least indicate why in practice the CAPM did not seem to hold. One direction is to obtain single-period models with multiple risk factors that could better explain returns and risks.

5.2 Tests of the APT

Around that time that the Roll (1977) published his critiques against the CAPM, Ross (1976) developed the APT as an alternative model that could potentially overcome the CAPM's problems while still retaining the underlying message of the latter. The core idea of the APT is that only a small number of systematic influences affect the long term average returns. These systematic influences are called factors in the APT framework. From the invention of the APT the number of factors, the choice of the factors and the interpretation has been the most debated issue. There is usually two ways to start testing the APT: the factor analysis and choosing the factors a priori. [60]

The first test of the APT is conducted by Gehr (1978) who applies factor analysis to U.S. stock returns. This approach is further developed by Roll

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[59] Empirically, these anomalies can be described parsimoniously using multi-factor models in which the factors are chosen atheoratically to fit the empirical evidence. One can also ask that if everyone knows these so called anomalies, are they really "anomalies" or are they just true facts from the empirical research? To go through all of these anomalies and test them is beyond the scope of this thesis. The interested ones are again referred to excellent reviews of Fama (1991), Dimson & Mussawian (1999) and Capmbell (2000).

[60] Connor (1995) tested three methods' explanatory power for U.S. equity returns. These methods were macroeconomic, fundamental and statistical factor analysis. He concluded that although statistical and fundamental methods outperformed macroeconomic model in terms of explanatory power, by other important criteria such as intuitive appeal and theoretical consistency, using a macroeconomic factor model is probably the strongest of the three approaches.

and Ross (1980) who report a five-factor structure of which two are priced after cross-sectional testing. There have also been a lot of tests in different economies[61], but the main criticism of Dhrymes et al. (1984) made about factor analysis approach still holds fast: significance tests of individual risk premium in the context of factor analysis with orthogonal rotation are not valid and in addition, the number of significant factors increases as the number of assets is increased in the sample. In their samples the number of significant factors increased from 3 for groups of 15 securities to 7 for groups of 60 securities.

In general, the number of factors that influence equity returns extracted from factor analysis has been a source of much contention.[62] For example, Trzinka (1986) finds five dominant factors within returns for a sample of US firms. Cho (1984) uses inter-battery factor analysis on a range of US industries and documents that the number of factors ranges from between two and five. Cho et al. (1986) perform a similar analysis at the international level for eleven industrial economies and report between one and five factors. Groenewold & Fraser (1997) found three factors for Australian share market. Cheng (1996) examined the UK market and found out, that "the market factor alone appears to incorporate most of the information contained in the underlying multiple factors". The number of factors ranges from zero to almost ten in our papers examined and because of this lack of guidance in choosing the factors to the APT from factor or principal component analysis, we will continue to the theory that selects factors *a priori*.

While there is no formal guidance choosing the right macroeconomic variables to the APT model, Chen et al. (1986) suggest a discounted cash flow approach to be used for the selection. They also argue that because

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[61] See, e.g., Roll & Ross (1980), Chen (1983), Chamberlain & Rothschild (1983), Lehmann & Modest (1988), Kryzanowwski et al. (1994).
[62] The advantages and disadvantages of using factor analysis to identify the influences are well documented in the literature: See, e.g., Brown & Weinstein (1983) and Gibbons (1982).

current beliefs about these variables are incorporated in price, it is only innovations or unexpected changes that can affect returns. On this basis they select five variables for their study: (1) the unanticipated change in inflation rate; (2) the change in expected inflation; (3) the unanticipated change in term structure; (4) the unanticipated change in risk premium; and (5) the unanticipated change in the growth rate of industrial production. They found out that variables (1), (4) and (5) are significant determinants of U.S. equity returns. Almost all published studies of testing the APT through selected macroeconomic variables have used these macroeconomic variables, or else very close related to these (Chen et al., 1997). Papers that have implemented this macro-economic APT for other countries find that the same types of variables as those used by Chen et al. (1986) are priced as well as other more country-specific variables.[63] However, this approach has been stated to be controversial, because the results are entirely spurious, the result of "data snooping" that has found accidental patterns in historical data (Lo & MacKinlay (1990).[64]

One of the most interesting tests of the APT for the purpose of this thesis is Cochrane (1996), for he uses the GMM framework to test the APT. He uses 10 portfolios of NYSE stocks sorted by market capitalization. He founds out that in the unconditional estimates, only one of the Chen et al. (1986) is individually significant, though they are jointly marginally significant with a 6.1 percent p-value. However, in the more efficient conditional estimate, two factors are individually significant, and the factors together

[63] E.g., the growth rate of money supply, oil and gold prices and exchange rates with various countries. See van Rensburg (1999) for South Africa, Groenewold & Fraser (1997) for Australia and Antoniou et al. (1998) for the United Kingdom. Sadorsky (1999) studied the relationship of oil prices changes and stock return for the U.S. and found out that oil price changes and oil price volatility play important roles in affecting equity returns. Others worth mentioning here are: Berry & Burmeister (1988) for the U.S., Beenstock & Chan (1988), Poon & Taylor (1991) and Clare & Thomas (1994) for the United Kingdom.

[64] Furhtermore, Cheng (1996) states that the method of Chen et al. (1986) is very sensitive to the number of independent variables included in the regression. Cheng (1996) also notes that when a researcher is testing the APT, a factor may be significant in one multivariate analysis and then will not be significant when testing in a univariate model. The multicollinearity among economic variables presents another drawback of this approach.

are jointly significant with 3.6 percent p-value.[65] Cochrane (1996) also states that although there is empirical evidence that expected returns are related to covariances of returns with macroeconomic variables, there is as yet no accepted economic explanation for this evidence.

The most disappointing feature of the APT is that it does not identify the common factors (nor even their number). It is not also supported by the theoretical foundations of the CAPM that describes the investors' behaviour (Morel, 2001). Gilles & LeRoy (1990) state that the APT contains no useful information about prices, because they think that the APT does not include any clear restrictions and it can be thought as a too general asset pricing model. They also state that many economists have all along been sceptical about the content of the APT, because they believe that the APT should depend on the validity of assumed restrictions on preferences and technology.

5.3 Tests of the CCAPM

The comparison between different studies of the CCAPM that have been tested earlier is difficult because of the vast number of different methods that have been used. Here we will present a couple of studies in which researchers have tried to estimate empirical values for the parameters γ and the β, and for the whole validity of the CCAPM. Surprisingly, the amount of these kinds of studies is quite limited and the extensions, that arise because of the poor performance of the CCAPM empirically, are dominating the literature.

[65] Cochrane (1996) tests a new investment-based asset pricing model that turns out to be at least as good as the other asset pricing models. He concludes that: "The model (investment-based asset pricing model) performs about as well as two standard finance models, the CAPM and the APT. The investment return model performs substantially better than the standard consumption-based asset pricing model and an ad hoc consumption growth factor model."

Probably the most cited article of the empirical tests of the CCAPM is Breeden et al. (1989). They test the CCAPM using betas based on both consumption and the portfolio having the maximum correlation with consumption. They use monthly returns from 1926 to 1982. They use the basic linear regression and maximum-likelihood techniques to answer two questions: does expected return increase as the risk increases and is the relation linear? They answer to these questions by stating that “the positive sign for the market price risk is verified for all periods and the relation between expected returns and betas is linear – while the CCAPM is by no means a perfect description of the data, we found the fit better than we anticipated.”

Most of the empirical studies are done for the U.S. data. However, as Kocherlakota (1996) says, the puzzles of the equity premium and the risk-free rate are not particular for the U.S. data. E.g., Kocherlakota (1994) shows that adding ten more years of data to the Mehra & Prescott (1985) series does not eliminate these puzzles. Hansen & Singleton (1982) and Aiayagari (1993) find that similar phenomena appear post-World War II monthly data in the U.S. Roy (1994) documents the existence of the two puzzles in post-World War II quarterly data in Germany and Japan. Thus, the equity premium and risk-free rate puzzles appear to be a general feature of organized asset markets.

Lund & Engsted (1996) are one the few that make a joint test of the CCAPM outside the U.S.[66] Using the GMM procedure they test the CCAPM with constant relative risk-aversion using monthly time series data from Danish, German, Swedish and UK stock markets. The empirical results for the Denmark (1922–90), Germany (1885–1913 and 1952–90), Sweden (1918–90) and UK (1919–87) show that first of all the risk-aversion parameter γ is estimated with great uncertainty, and only one out of 20 estimates is significantly different from zero. Furthermore, nine out of

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[66] Also couple of others exist, e.g. Sauer & Murphy (1992) for the German market; Rubio (1989) for the Spanish market; Hamori (1992) for the Japanese market.

20 cases imply a negative sign for the γ that is by no means consistent with the theory. They also conclude that the discount factor can be estimated somewhat more precisely, but many of the estimates are either unrealistically low or high (they referrer to realistic values as 0.95–0.98). Indeed, the most fitting values are for Sweden (between 0.92–0.94) but even those are quite low. For the Germany and UK the values of the SDF are quite irrational, ranging from 0.47 to 1.11.

Kasa (1997) tests two discount rate models to explain the cross-sectional and time-series variation of stock returns in the U.S., Japan, the UK, Germany and Canada. The data consists of quarterly data MSCI indices for the period from January 1972 to March 1993. The two models are the CCAPM and the production-based CAPM that links the discount rate to the intertemporal marginal rate of transformation in production. The main result is that a production-based CAPM performs better than the CCAPM in explaining the national stock market returns. However, neither model works very well unless allowance is made for time variation in the price of risk. He also finds that both factor models lead to extremely volatile discount rate processes. He argues that such volatility is inevitable unless we can find (and rationalize the use of) macroeconomic variables that are much more highly correlated with stock returns than investment.

Faff & Oliver (1997) compare the CCAPM and the traditional CAPM for 23 Australian industry portfolios, using quarterly Australian data from January 1974 to December 1992. The results reveal that there is very little sensitivity of returns to consumption growth where consumption data is measured contemporaneously. However, when a lagged relationship is examined, the consumption sensitivities become much stronger. The writers acknowledge the problem that given highly competitive capital markets in which information is rapidly impounded in prices, it would be expected that macroeconomic variables, such as aggregate consumption, are led by equity prices. They lag the aggregate consumption for two-months, which seems to be the best fit for their sample period. They also conclude that this reinforces the view that noncontemporaneous relationships must be examined in studies which seek to uncover links between stock markets and macroeconomic variables. They found out that consumption beta is significant for 15 portfolios, but when it is lagged two months, it is significant in 21 portfolios out of 23. However, the R^2-values are extremely small – only of them exceeds 10%.

Cuthbertson et al. (1999) uses the VAR approach to test the CCAPM in sub-sectors of the UK stock market on quarterly data from 1965 to 1992. Their conclusions are relatively strong: "We can draw three conclusions from this analysis. First, we find no support whatsoever for the CCAPM. Second, the standard CAPM, also finds little support. Third, there is some significant support for the own variance model for the sector portfolios."

One of the few successful empirical tests of the CCAPM is Pasquariello (2000), who applies the iterated GMM procedure of Hansen & Singleton (1982) to estimate the Lucas (1978) equilibrium model with different specifications for the assets' sets, the instrumental variables and the representative investor's preferences. The sample period is from the year 1959 to 1998 for the U.S. data. The empirical results seem to confirm the weak evidence for the Lucas (1978) model. Estimates of the relative risk-aversion coefficient appear to be generally low, usually not higher than 4. The time discount factor beta is generally higher than 0.99 but lower than one. He also concludes that a set of selected macroeconomic variables (real GPD per capita, real federal expenses per capita, personal savings rate and the real M3 money growth rate) seems to fare better than lagged return variables in explaining the cross-sectional variability of asset returns and the inter-temporal consumption profile of the representative American investor.

Cuthbertson & Nitzsche (2004) discuss about these problems and they try to rationalize why the CCAPM performs so poorly empirically although it is the starting point for a lot of asset pricing models. They state that the possible reasons for these problems are:

a) Power utility is correct, but we need to add non-separabilities where the marginal utility depends for example on wealth or past consumption
b) The basic SDF is correct but the power utility function is wrong, so we have to try other functional forms
c) The last 50 years of US stock returns have been good luck rather than a payment for risk, so the whole theory of the equity premium puzzle is questionable
d) Individual consumption is poorly measured by aggregate per capita consumption.

A more recent test of the CCAPM is made by Chen (2003). He evaluates the CCAPM based on their exact pricing performance across seven different industry (or financial) sub-sectors in the Taiwan stock market over the period July 1991 to March 2000. He concludes that "the empirical performance of the CCAPM across seven industry sub-sectors in the Taiwan stock market is disappointing although the CCAPM and consumption beta should offer a better measure of systematic risk theoretically. The CCAPM could not even outperform the simple random walk model."

5.4 Comparison of the different risk factors

The classic asset pricing models, the APT and CAPM, treat asset prices as being determined by the portfolio choices of investors who have preferences defined wealth over one period in the future, i.e., these models assume that investors consume all their wealth after one period, or at least that wealth uniquely determines consumption so that preferences defined over consumption are equivalent to preferences defined over wealth. This simplification is ultimately unsatisfactory. In the real world investors consider many periods in making their portfolio decisions, and in this intertemporal setting one must model consumption and portfolio choices simultaneously. These intertemporal equilibrium models have the potential to answer two questions that the classic asset pricing models can not. First, what forces *determine* the riskless interest rate and the rewards that investors demand for bearing risk? These variables are endogenous parameters in CCAPM; the model tries to explain where these variables come from. (Campbell et al., 1997)

It is quite surprising that we could not find a study, where all of these asset pricing models are compared in the same data set. Cochrane (1996) is probably the closest one, but his study does not compare these three basic models, because he tests the investment-based asset pricing model in comparison of these three models presented in this study. However, individual studies exist. As we saw in the previous chapter, the usual opinion is that the standard CCAPM with power utility does not simultaneously explain the high equity premium and the risk-free rate puzzle. E.g., Mankiw & Shapiro (1986) test the CAPM and CCAPM using cross-section data on 464 U.S. companies over the period 1959–1982. They found that the CAPM clearly outperforms the CCAPM. Breeden et al. (1989) find similar results for industry and bond portfolios, while Cochrane (1996) finds that the CCAPM performs worse than the CAPM using a cross-section of size-sorted portfolios. Lund & Engsted (1996) test the CCAPM on Danish, German, Swedish and UK stock markets and conclude that their results tend to support the findings in the U.S. studies: "The inability to explain time-varying discount rates in terms of simple and theoretically consistent consumption based models continues to pose a major challenge to the profession." Faff & Oliver (1997) also compare the CAPM and CCAPM. They conclude that the consumption betas remain considerably less significant than their market beta counterparts.

Cochrane (1996) tests the investment-based asset pricing model on quarterly U.S. data, which outperforms all of these asset pricing models presented in this study. The APT and the CAPM perform in his data set about as well, when the CCAPM performs worse. He concludes that "the fact that any model whose factors are related to economic theory and are based solely on quantity data is even in a position to challenge the empirical success of traditional finance models may be regarded as an encouraging initial success." Kasa (1997) argues that probably the reason for the success of his model is that investment growth was nearly eight times more volatile than consumption growth. This translates into a stock return/investment growth covariance that is eight times greater than the covariance between consumption growth and stock returns. This is important because standard asset pricing theories relate expected returns to covariances, so the weak covariance between consumption and stock returns implies an extreme degree of risk-aversion.

Finally, Chen (2003) compares the CAPM and CCAPM on the emerging Taiwan stock market, as we presented in the previous section. He concludes that the empirical performance of the CAPM is encouraging. The market beta is statistically significant at the 1% level in all seven market financial sectors. The relationship between stock returns and beta is statistically significant and the coefficient of determination of the regression high across all of seven industry sub-sectors. In comparison, the CCAPM fails to explain the Taiwan stock market.

Studies comparing the APT and the CAPM have used both factor or principal component analysis and selecting macroeconomic variables *a priori*. Connor & Korajczyk (1986) used principal components analysis and found five factors that could explain the size and January effect better than the CAPM. Berry et al. (1988) conclude that the APT model is better explaining equities returns than the CAPM and that at the 0.01 significance level the CAPM model can be rejected in favour of the APT model. Josev et al. (2001) conclude for Australian industry equity portfolios that "the results show that there is strong evidence in favour of the APT model". In a recent study for Indian stock markets Dhankar & Esq (2005) conclude that "APT with multiple factors provides a better indication of asset risk and estimates of required rate of return than the CAPM which uses beta as the single market of risk." Elton et al. (2007) state that the APT remains the newest and most promising explanation of relative returns. The APT promises to supply us with a more complete description of returns than the CAPM.

6 DATA AND METHODOLOGY

6.1 Methodology of the GMM

Since its introduction by Hansen & Singleton (1982), GMM has had a huge impact on econometrics and financial analyses. It provides a unifying framework for the analysis of many familiar estimators and includes least squares, instrumental variables and maximum likelihood as special cases. It also offers a convenient method of estimation in certain models that were before computationally very burdensome to estimate using traditional methods. The GMM approach is especially convenient when it comes to testing the dynamic properties of a discount rate model, i.e., when assessing a model's ability to capture variation over time in expected rates of return. In this case, all we have to do is scale the time $t+1$ returns by any variables that are presumed observable as of time t. Unfortunately, the selection of instruments is a problem that has made it more difficult to implement the GMM. In principle, any time t observable variable is a valid instrument. However, if we are to obtain efficient estimates of the parameters and powerful tests of the over-identifying restrictions of the model, we want to choose only those instruments that have some ability to predict returns and conditional covariances between returns and factors. (Kasa, 1997 & Cochrane, 2005) In this thesis we will follow the usual ad hoc procedure of picking out variables that have been known to forecast returns.

We were fortunate enough to find a simple comprehensive introduction to GMM from Cliff (2003) and this chapter relies heavily on his article.[67] Also

[67] This article was the one that made this methodology possible to understand and implement it to our analysis. For more detailed approach readers are referred to Cochrane (2005), Hall (1993) and Ogaki (1993).

the CAPM and APT algorithms are from his webpage.[68] All the models in this thesis are estimated with Hansen & Singleton (1982) GMM procedure.

The basic idea underlying Hansen & Singleton (1982) estimating strategy is presented next. The dynamic optimization problems of economic agents typically imply a set of stochastic Euler equations that must be satisfied in equilibrium. These Euler equations in turn imply a set of population orthogonality conditions that depend in a nonlinear way on variables observed by an econometrician. The estimation strategy permits to identify and estimate the parameters of the representative agent's dynamic objective function, as well as to test the over-identified restrictions imposed by the theoretical model, while at the same time avoiding any distributional assumption regarding the stochastic technology and the resulting equilibrium. The main idea is that economic models based on agents' expectations about the future, hence by definition not observable, can still be estimated if those agents form their expectations rationally, i.e., if the error they make in forecasting is uncorrelated with the information available to them at the time of the forecast. As long as it is possible to observe a subset of the information agents actually use, then this rational-expectation hypothesis suggests a set of orthogonality conditions that we can adapt in a GMM setting to estimate the unknown parameters of the CCAPM. (Pasquariello, 2000 and Hansen & Singleton, 1982)

Due to the generality of the GMM estimation principle, many theoretical treatments of the estimator are at an advance level. While this generality is very desirable for practical purposes, a researcher can easily find it difficult to understand the estimation procedure due to the large amount of different theoretical explanations.

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[68] http://www.mgmt.purdue.edu/faculty/mcliff/progs.html. Also a Statistics & Operation Research centre's article http://support.sas.com/rnd/app/examples/ets/harvey/index.htm provided great help.

The basic idea in the GMM approach is relatively straightforward. The asset pricing model predicts:

$$E(P_t) = E\left[m(data_{t+1}, parameters)x_{t+1}\right], \tag{34}$$

The most natural way to check this prediction is to examine sample averages, i.e., to calculate:

$$\frac{1}{T}\sum_{t=1}^{T} p_t \text{ and } \frac{1}{T}\sum_{t=1}^{T}\left[m(data_{t+1}, parameters)x_{t+1}\right], \tag{35}$$

GMM estimates the parameters by making the sample averages in Equation (35) as close to each other as possible. It seems natural, before evaluating a model, to pick parameters that give it its best chance. GMM then works out a distribution theory for the estimates. Then it suggest that we evaluate the model by looking at how close the sample averages of price and discounted payoff are to each other, or equivalently looking at the pricing errors. GMM gives a statistical test of the hypothesis that the underlying population means are in fact zero. This is a natural way to proceed since the fundamental asset pricing equation in Equation (4) is in the form of orthogonality conditions. More importantly, it permits general forms of conditional heteroskedasticity that is quite common among macroeconomic variables, such as consumption. Hansen & Singleton (1982) show that if our goal is to obtain the most efficient parameter estimates we should minimize a quadratic form in the otrhogonality conditions with a weighting matrix equal to the variance-covariance matrix of the orthogonality conditions. Cohrane, 2005)

Hall (1993) provides a simple example: let us consider an estimator that we get by minimizing a quadratic form in a vector of sample moments which are functions of the parameters β and the data. To estimate β all that is required is the population moments[69] whose expectation is zero when evaluated at β_0. The GMM estimate is obtained by minimizing a quadratic form in the analogous sample moments. A weighting matrix determines the relative importance of matching each moment (Cliff, 2003).

To present this in mathematical form, GMM chooses the parameters which minimize the quadratic as follows:

$$J_t = m(\theta)' W m(\theta)\,, \tag{36}$$

where θ is a k-vector of parameters, $m(\theta)$ is a L-vector of orthogonality conditions, and W is an $L \times L$ positive definite weighting matrix. The objective function has a least-squares flavour. (Ibid)

Most common estimation procedures can be couched in this framework, including ordinary least squares, instrumental variables estimators, two-stage least squares, and in some cases maximum likelihood. It is also important to realize the generality of GMM. Thus, saying that "I estimate the parameters via GMM" is about as informative as saying "My estimates are based on econometrics". A key advantage to GMM over other estimation procedures is that the statistical assumptions required for hypothesis testing are quite weak. Of course, there is a caveat. The cost is a loss of efficiency over methods such as maximum likelihood. However, the trouble with maximum likelihood is that the errors may not follow a known distribution (such as the normal distribution). Thus, GMM offers a compromise between the efficiency of maximum likelihood and robustness to deviations from normality. (Ibid)

[69] There must be at least as many moment conditions as parameters.

6.1.1 Orthogonality conditions

The moment, or orthogonality, conditions $m(\theta)$ set means of functions of the data and parameters to zero. One simple restriction estimates the mean μ of data y_t:

$$E[y_t] = \mu\,, \tag{37}$$

giving the *population* orthogonality condition:

$$E[y_t - \mu] = 0\,, \tag{38}$$

and *sample* counterpart:

$$m(\theta) = \frac{1}{T}\sum_{t=1}^{T} y_t - \mu\,, \tag{39}$$

Another restriction, on the variance (σ^2), is:

$$E\left[(y_t - \mu)^2\right] = \sigma^2 \quad \text{giving the system} \quad E\begin{bmatrix} y_t - \mu \\ (y_t - \mu)^2 - \sigma^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{40}$$

Note that the moment condition for the mean is needed to estimate the covariance. Similarly, a covariance restriction would be:

$$E\left[(x_t - \mu_x)(y_t - \mu_y)\right] = \sigma_{x,y} \text{ giving } E\begin{bmatrix} x_t - \mu_x \\ y_t - \mu_y \\ (x_t - \mu_x)(y_t - \mu_y) - \sigma_{x,y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{41}$$

The terms μ_x, μ_y, σ_x, σ_y and $\sigma_{x,y}$ are parameters we wish to estimate, whereas x_t and y_t are data. A key ingredient to GMM is the specification of the moment, or orthogonality, conditions $m(\theta)$. The moment conditions used, though somewhat arbitrary, are often guided by economic principles and the model of interest. E.g., the return on an asset this period is generally modelled as unpredictable by (orthogonal to) information in prior periods, so moment conditions often incorporate past returns, interest rates, etc. Note that there must be at least as many moment conditions as there are parameters to achieve identification. If we have too few restrictions, we can create more by using instruments, Z_t. (Cliff, 2003)

For more rigorous approach, we will follow the work by Ogaki (1993). Let $\{X_t : t = 1, 2, ...\}$ be a collection of random vectors, X_t, β_0 be a p-dimensional vector of the parameters to be estimated, and $f(X_t, \beta)$ be a q-dimensional vector of functions. Assume that X_t is stationary. We refer to $u_t = f(X_t, \beta)$ as the disturbance of GMM. Consider the (unconditional) moment restrictions:

$$E(f(X_t, \beta_0)) = 0\,, \tag{42}$$

Suppose that a law of large numbers can be applied to $f(X_t, \beta)$ for all admissible β, so that the sample mean of $f(X_t, \beta)$ converges to its population mean:

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} f(X_t, \beta) = E(f(X_t, \beta))\,, \tag{43}$$

with probability one. The basic idea of GMM estimation is to mimic the moment restrictions in equation $E(f(X_t, \beta)) = 0$ by minimizing a quadratic form of the sample means:

$$J_t(\beta) = \left[\frac{1}{T}\sum_{t=1}^{T} f(X_t, \beta)\right]' W_t \left[\frac{1}{T}\sum_{t=1}^{T} f(X_t, \beta)\right]\,, \tag{44}$$

with respect to β; where W_t is a positive semidefinite matrix. If there are as many moment conditions as parameters to be estimated, the moments will all be perfectly matched and the objective function in Equation (44) will have a value of zero. This is referred to as the “just-identified” case. In the situation where there are more moment conditions than parameters ("over-identified", usually the case in financial models) not all the moment restrictions will be satisfied so a weighting matrix W_t determines the relative importance of the various moment conditions (Cliff, 2003).

Now we will turn to a more practical use of the GMM and implement it to the CCAPM. The GMM uses the first-order condition in Equation (7) to estimate the unknown parameters $\theta = (\beta, \gamma)$ and to test the underlying equilibrium model. In the following x_t denotes a 2 x 1 vector consisting of the asset return R_t and the consumption growth rate C_t/C_{t-1} while θ_0 denotes the true (but unknown) value of the parameter vector. The basic idea behind the GMM is that the excess return is the following:

$$h(x_{t+1}, \theta_0) = \beta R_{t+1}(C_{t+1} / C_t)^{\gamma} - 1\,, \tag{45}$$

This excess return should be unpredictable given information at time t. Therefore, if we have instruments Z_t containing variables known at time t, we have that:

$$E(h(x_{t+1}, \theta_0) z_t) = 0\,, \tag{46}$$

Corresponding to the vector *population* orthogonality conditions we can define a *sample* counterpart (Lund & Engsted, 1996):

$$m(\theta) = \frac{1}{T}\sum_{t=1}^{T} h(x_{t+1}, \theta) Z_t\ , \tag{47}$$

In practice, there is no shortage of candidates of Z_t. One can use C_{t-1}, R_{t-1}, P_{t-1}, $i \geq 0$, any function of these variables or any other macroeconomic variables, such as lagged money supply, which are known to the agent at time t. This abundance can create a problem because the parameter estimates will vary with the choice of Z_t.[70] (Hall, 1993) This issue has been investigated by Tauchen (1986). It could be intriguing to use as many instruments as one can imagine and then let GMM just put less weight on the irrelevant instruments. Tauchen (1986) found that the estimator performed reasonably well but appeared to be sensitive to the choice and number of Z_t. However, when the number of instruments increased the estimator's variance decreased but there was an increase in bias. He tested the small sample properties of the GMM estimator with a different number of instruments. His conclusion was that the best performance of the GMM is obtained with a small number of instruments.

The great advantage of the GMM is that (1) it does not require distributional assumptions, like normality, (2) it can allow for heteroskedasticity of unknown form and (3) it can estimate parameters even if the model cannot be solved analytically from the first order conditions. (Verbeek, 2000)

6.1.2 Weighting matrix and hypothesis testing

As we stated before, usually in econometric analysis we have more moment conditions than parameters to be estimated. Thus, the weighting matrix W determines the relative importance of the specified moment conditions. The most GMM efficient estimator is obtained if W is equal to the inverse of the asymptotic covariance matrix ($W = S^{-1}$). It is optimal in a sense that it yields θ with the smallest asymptotic variance. Intuitively, more weight is given to the moment conditions with less uncertainty.[71] In general, an optimal weighting matrix requires an estimate of the parameter vector, yet at the same time, estimating the parameters requires a weighting matrix. To solve this dependency, common practice is to set the initial

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[70] It has been empirically tested in Pasquariello (2000) that the including and excluding of the instrumental variables affect the results significantly (Pasquariello, 2000).
[71] There are many approaches for estimating S which can account for various forms of heteroskedasticity and/or serial correlation. We will use a Matlab add-in of Kyriakoulis (2004), which allows us to use Martingale differences, Barlett and Parzen kernels. The add-in is available at: http://www4.ncsu.edu/~kkyriak/gmmgui.html.

weighting matrix to the identity matrix and then calculate the parameter estimates. (Cliff, 2003)

Thus, this presents a problem that we do not encounter in the linear models because in this more general setting such an estimator will inevitably depend on θ^* because the definition of S. The solution is to adopt a multi-step estimation procedure. For example one could estimate θ in two steps. In the first step we use a suboptimal choice of W to obtain $\hat{\theta}$. Although $\hat{\theta}$ is inefficient it is still consistent and so we can use $\hat{\theta}$ to construct $\hat{S}$. In the second step $\hat{\theta}^*$ is obtained using $W_n = \hat{S}^{-1}$. While this two-step estimator is asymptotically efficient, one need not stop here. The second step estimator of θ can be used to form a new estimator of S and the model re-estimated to produce a new estimate of θ which in turn yields a new estimator of S. This iterative procedure is continued until convergence and the resulting estimator has the same asymptotic properties as the two-step estimator. Furthermore, it has been stated in Kocherlakota (1990) and Ferson & Foerster (1984) that an iterated version gives better small sample properties.[72] (Hall, 1993) The iterated version of the GMM is also the method in our empirical sections.

Of course, we are interested how significant our model is. To test this, we need to know the standard errors and the distribution of our parameter estimates. The asymptotic covariance matrix of the GMM estimator $\hat{\theta}$ is given by:

$$\operatorname{cov}(\theta) = \frac{1}{T}\left[\frac{\partial m(\hat{\theta})'}{\partial \theta} S^{-1} \frac{\partial m(\hat{\theta})}{\partial \theta'}\right]^{-1} = \frac{1}{T}(D' S^{-1} D)^{-1}, \quad (48)$$

[72] We were also a bit worried that our sample size might be too small. However, Ferson & Foerster (1991) test the GMM with different lengths of asset sample sizes. In their study GMM exhibits relatively small biases even in samples of size 60. Thus, the sample size can be seen as adequate to estimate with GMM.

If and only if an equation is overidentified, we may test whether the excluded instruments are appropriately independent of the error process. This test of the overidentifying restrictions is known as the J-test. If the p-value is smaller than, e.g. 0.05, we reject the null hypothesis that all the instruments are uncorrelated with the error term. In a same time it is a test of the validity of the whole model, i.e. the ability of the moment conditions and instruments to satisfy the underlying GMM. This J-test gives us a value that we can compare with χ^2-distributed with (moment conditions – parameters) degrees of freedom value. The parameter vector θ can be compared between the models estimated returns and observed returns. The model can be rejected if the difference between estimation and observed values is statistically significant. (Cochrane, 2005 & Verbeek, 2000)

There are three classic tests for the GMM: Likelihood Ratio-test (LRT), Lagrange Multiplier (LM) and Wald-test (W). For the purpose of this thesis we will use LRT- and W-statistics. Asymptotically these tests measure the same difference between the unrestricted and restricted versions of the estimated models (Ogaki, 1993). The tests are based on nesting, meaning that the null hypothesis is a special case of the alternative. For example, in the model:

$$y_t = \alpha + \beta_1 x_{1,t} + \varepsilon_t \,, \tag{49}$$

the null hypothesis $\beta_1 = 0$ can be tested against the alternative $\beta_1 \neq 0$.

The Wald-test is calculated as follows:

$$W = (R\hat{b} - r)'(R\hat{C}R')^{-1}(R\hat{b} - r) \,, \tag{50}$$

where R and r are $(N \times M)$ and $(N \times 1)$ restriction matrices, $\hat{b}$ is a $(M \times 1)$ vector of coefficient estimates, $\hat{C}$ is a $(M \times M)$ Newey-West covariance matrix estimator, N is the number of test assets (or restrictions), and M is the number of assets times the number of parameters estimated for each equation (alpha and betas). In the tests, R is a matrix of zeros, except of those parameters that we want to pick for the tests and r is a vector of zeros. (Vaihekoski, 2000)

The likelihood ratio type test statistic is:

$$LRT = T\left(J_T\left(\beta_T^r\right) - J_t\left(\beta_T^u\right)\right), \tag{51}$$

which is T times the difference between the minimized value of the objective function when the parameters are restricted (β_T^r) and the minimized value of the objective function when the parameters are unrestricted (β_T^u). Thus, the Likelihood Ratio-test estimates the model in both restricted and unrestricted form. The essence of the test is to see how much the loss function (J_T) increases when the restrictions are imposed. It is also important to estimate the restricted model using the same weighting matrix as the unrestricted model. (Ogaki, 1993)

The W-statistics is almost like the t-test. This test estimates the unrestricted version of the model only and we see how many standard errors our restrictions are from zero. In a linear model with one restriction, the Wald-test is simply a squared t-test. However, we have to be cautious when using the Wald-test for nonlinear models. We can get different inferences by simply changing the way we write the model, so the Wald-test should not be used as the only description of reality. (Cliff, 2000)

These two tests are asymptotically distributed chi-square with degrees of freedom equal to the number of restrictions. (Cliff, 2000) Thus, the idea is to run the GMM-procedure two times. In the first time the GMM-procedure estimates the parameters of interest that are *unrestricted*. In the second time we will put some restrictions on the parameters. For example, for the CCAPM we can put restrictions on the risk-aversion coefficient γ and on β.

6.2 Test assets and risk factors

The data for this study is collected mainly from the Datastream system and from ETLA[73]. The value-weighted portfolio returns are from Vaihekoski (2004), updated until the end of 2004. From ETLA's database we are able to get quarterly[74] reported consumption data for non-durable goods (ND) and services (S). Individual consumption is measured by aggregate per capita consumption, so we have to divide these data series with the Finnish population numbers that are also available in ETLA's database. All time-series of equities and indices are in total return.[75] Because the CCAPM requires real prices, we have to deflate our series by the implicit consumption deflator. The empirical study is based on logarithmic excess returns, but we will also use normal nominal returns in some cases – mainly when we are estimating the parameters of the CCAPM and calculating the equity premium.

The analysis will be done on value-weighted portfolios.[76] Value-weighted portfolios highlight the role of size (and investability) and the value-weighted approach has been the predominant choice for research purposes (Vaihekoski, 2004). It is also interesting from our point of view to test whether the size of the company matter to a utility-maximizing investor. Intuitively speaking it is possible, that risk-averse investors try to find bigger, more stable and more liquid companies to smooth their consumption between different time periods and furthermore the risk-aversion pa-

[73] ETLA collects data from different sources. Our consumption, population and inflation data is originally from Statistics Finland, http://www.tilastokeskus.fi/index_en.html, but from ETLA the data is easily accessible.

[74] Breeden et al. (1989) suggest that the use of quarterly consumption data should provide more accurate estimates of the parameters of interest. However, this also implies that we can use a smaller amount of data for the GMM estimation itself and makes the selection of the proper instruments a more critical issue.

[75] Total Return Indices measure the market performance, including price performance and income from dividend payments.

[76] These portfolios are constructed according to Vaihekoski (2004). We are greatly indebted to Professor Mika Vaihekoski for providing this data. Furthermore, we are not going to go deep in explaining the Finnish financial markets, because Nyberg & Vaihekoski (2005) and Nyberg (2003) have done this in a brilliant way and we would not bring anything new to the table. However, we will present the results that have to be known to calculate the equity premium.

rameter could be larger in portfolios with larger companies. The research period is from the beginning of the year 1987 to the end of 2004.[77] Thus, we will have 72 quarterly observations.

First of all the use of our consumption series, NDS, has to be explained. This set is for nondurable goods and services, denoted by NDS.[78] The use of this set is motivated by the work of Blinder & Deaton (1985), who argue that this measure of consumption is closer to the theoretical ideal of a flow of services. The consumption is defined as the personal consumption expenditures less durables by summing up consumption expenditures on nondurable goods and services. But why not use the whole consumption that investors spend? The use of NDS expenditure categories is justified on the grounds that the theory applies to the *flow* of the consumption; expenditures on *durable* goods are not part of this flow because they represent replacements and additions to stock, rather than a service flow from the existing stock (Alan et al., 2005).

The equity premium and risk-free rate puzzles concern the co-movements of three variables: the real return to the market index, the real return to short term nominally risk-free rate, and the growth rate of per capita real consumption, which is defined as NDS.

The stock market index for our research period will not produce a problem. From January 1970, a total return equity marked index is available for Finland. Before 1990, the Department of Finance and Statistics at the Swedish School of Economics and Business Administration has calculated so-called WI-index.[79] After 1990, we use the HEX index calculated by the Helsinki Stock Exchange. Both indices are value-weighted return indices that include all the stocks in the Helsinki's main list. These indices account

[77] As Vaihekoski (2004) states: "This period is chosen because the open and transparent market for certificates of deposit began 1987 in Finland. The birth of the money market provided the market participants a fair and equal chance for (virtually) risk-free investments and thus, gives the researcher a change to calculate excess returns."
[78] This definition of consumption is the most dominant one in the empirical studies.
[79] For more information, see Berglund et al. (1983).

for e.g., dividends, splits and new issues. In the case of the CCAPM, these market returns are subtracted by the inflation series that we will get from the ETLA's database.

All quantities in the Lucas (1978) model are assumed to be "real", as the agents are supposed to interact in a pure exchange economy, with no money or wealth-preserving tools for the perishable goods.[80] Thus, we need real returns for the CCAPM. As a starting point, we use the CPI Index P_t in January 1987 as a proxy for the nominal price of the bundle of goods that we assume being included in the consumption series. The log returns for the inflation are I_t. As we use logarithmic returns, Cochrane (2005) shows that subtracting inflation from nominal returns is exactly true because:

$$\ln(A/B) = \ln A - \ln B\,, \tag{52}$$

and we have:

$$\ln R_{t+1}^{real} = \ln R_{t+1}^{nomial} - \ln I_{t+1}\,, \tag{53}$$

The risk-free rate that we will use from the beginning of the year 1987 to the end of the 1998 will be the so-called Helibor-rates (Helsinki Interbank Offered Rate). They are averages of the bid rates for certificates of deposits (CD). Rates are calculated for one, two, three, six, nine and twelve months. Starting 1999, the Finnish currency, Markka, was tied to the Euro and Helibor interest rates were replaced by the Euribor interest rates. (Nyberg & Vaihekoski, 2005) The risk-free rate is calculated from the three month annual interest rates and transformed to quarterly continuously compounded interest rate according to Vaihekoski (2007).

[80] This procedure is based on Pasquariello (2000).

Our portfolios will be six value-weighted weighted portfolios constructed as in Vaihekoski (2004). These size portfolios are constructed by ranking companies into six portfolios on the basis of their total market values as of time *t*. The number of companies varies between 50 and 106. The portfolio construction process has three important stages. The first stage calculates the company market values from the asset class specific market values. The second stage calculates the weights to be used in the portfolios. The third stage calculates the monthly value-weighted returns for size portfolios. These stock returns are calculated for all stocks quoted on the Main list of the Helsinki Stock Exchange (HEX).[81] Stock returns are measured as the continuously compounded return on stock price indices, i.e., the logarithmic relative of the price. (Vaihekoski, 2004) We decided to choose the whole index for our study. We did not choose, e.g., some restricted weighting index for the market index. It is possible that large companies in Finland, e.g., Nokia that had sometimes over 50% of the total market capitalization, determine the growth of the market. However, Nyberg (2003) did not find this relationship to be very severe. Overall, the great performance and the turbulence of the techno-bubble era of the Finnish stock market are presented in Figure 1.

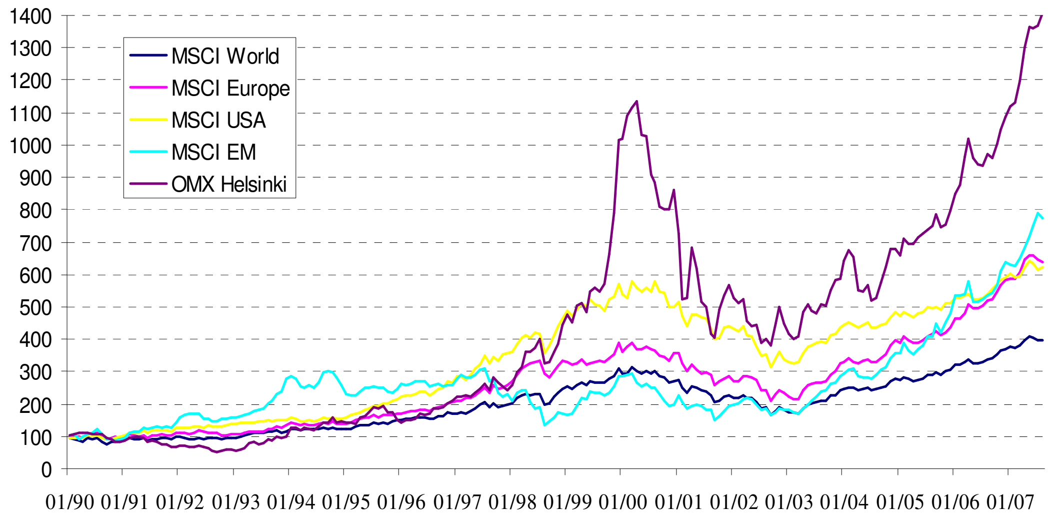

Figure 1. Good performance of Finnish stock market relative to others. The OMX Helsinki index compared to MSCI World, Europe, USA and Emerging market indices. The indices have been scaled to start from 100.

[81] Nowadays, OMX Nordic Exchange Helsinki.

For our purposes, we will accumulate the three months that are calculated in our data series. In this manner we will get quarterly returns. This is an advantage of logarithmic returns, because we can accumulate returns over the desired period – multi-period returns are just sums of period returns (Vaihekoski, 2004). However, our quarterly portfolio returns are calculated from monthly returns. This is a minor drawback, because the portfolio returns are calculated from the companies' market values. Therefore, if a company is only, e.g., one month in the highest market value portfolio and then is transferred to another portfolio, our accumulation of the returns is a bit false. However, this sort of timing factor is not considered to be a major drawback.

When it comes to the lagging of the instruments, we will introduce additional instrumental variables, denoted by Z_t. An instrument that is typically used in the estimation of the CCAPM is the lagged rate of growth of consumption. In the presence of measurement error such an instrument would be inappropriate. As it has become standard practice, one should lag such an instrument twice, to guarantee that it is not correlated with the error term. However, it should also be said that once lagged interest rates are valid instruments: there is no reason to believe that they are correlated with measurement error in consumption. (Alan et al., 2005) Furthermore, our lagged instrument variables will be one and two lags of the consumption growth and portfolio returns along with the risk-free rate and macroeconomic factors. These instruments usually include a constant, because we want to restrict the model errors to have mean zero.

The timing of the data implicitly assumes that consumers make economic decisions that coincide with the sampling interval of the data. Consumers make their consumption decisions at the beginning of the current quarter, based on knowledge of their investment returns for that quarter. The 3-month Euribor-returns are from the last day of the previous quarter (as consumers earn this rate during the quarter if they invest into this risk-free asset) and total stock returns are based on the current quarter ending of the market index. Consumers are assumed not to intertemporally store purchased items. (Smoluk & Neveu, 2002)

As it is, according to theory, the unanticipated change that affects the returns, we use AR(1) procedure to get the residuals from the factors and use these residuals as the unanticipated component in our analysis. The AR(1) model is specified as:

$$y_t = x_t'\beta + u_t \text{ and } u_t = \rho u_{t-1} + \varepsilon_t, \tag{54}$$

The parameter ρ is the first-order serial correlation coefficient. In effect, the AR(1) model incorporates the residual from the past observation into the regression model for the current observation. (Eviews 5 user's guide, 2004)

7 EMPIRICAL RESULTS

7.1 Descriptive statistics

Our quarterly data consists of six value-weighted portfolios' continuously compounded excess returns (ER_{P1}–ER_{P6}) (largest companies–smallest companies), consumption growth (C_t) for real[82] returns of nondurables plus services (C_{NDSt}), risk-free rate (R_{ft}); the factors for the CAPM and APT: market return M_{rt}, inflation I_t, the term structure of interest rates TS_t, risk premium RP_t, and industrial production IP_t. We will use the same risk factors as did Chen et al. (1986) in their paper for testing the APT.

The following risk-factor returns are logarithmic and retrieved from Datastream and ETLA. For the inflation factor we will simply use the return of inflation. The term structure of interest rates is calculated by subtracting 12-months Euribor interest rate from one-month interest rate. These series are also calculated to match the quarterly frequency. For the risk premium we calculate a series that is US Corporate Bond Moody's BAA subtracted by US Corporate Bond Moody's AAA returns. This is a measure for the market's reaction to risk. For the fourth and the last factor we will use industrial production's return. Table 1 presents the descriptive statistics for quarterly data consisting of the period of years 1987–2004.

[82] The CCAPM needs real returns so we subtract inflation from nominal returns that are logarithmic. We put *R* in the end of the series if it is measured in real terms. The rule of thumb is that when we speak of the CCAPM we will present the returns in real terms, otherwise they are continuously compounded returns.

Table 1. Descriptive statistics 1987-2004. Portfolios and market return are excess returns, marked with *E*. In the table are shown mean, standard deviation, skewness, excess kurtosis and series' normality for our data series. Mean and standard deviation are annualised. C_{NDSt} is in real terms. Jarque-Bera tests the series' normality. H_0 is normal distribution. The compared chi-square value is 5.991 with two degrees of freedom. ** indicates significance at 0.01 level and * at 0.05 level and N = 72.

Variable	Mean	Min.	Max.	Std. Dev.	Skew-ness	Kurto-sis	Jarque-Bera
ER_{P1}	1.70%	-49.92%	64.20%	37.80%	0.10	1.15	4.08
ER_{P2}	-0.27%	-46.71%	23.73%	28.35%	-0.76	0.39	7.46*
ER_{P3}	-1.60%	-39.63%	39.66%	28.62%	-0.34	0.23	1.51
ER_{P4}	-0.13%	-44.99%	33.32%	29.16%	-0.48	0.49	3.47
ER_{P5}	-2.04%	-34.68%	23.77%	26.53%	-0.13	-0.38	0.64
ER_{P6}	-3.56%	-38.10%	26.26%	26.08%	-0.13	-0.16	0.29
EM_{rt}	5.20%	-44.22%	62.74%	35.36%	0.22	1.25	5.26
C_{NDStR}	1.87%	-8.75%	7.51%	8.19%	-0.77	-0.64	8.31*
R_{ft}	1.68%	0.49%	3.96%	2.13%	0.73	-0.90	8.84*
I_t	2.36%	-0.40%	2.47%	1.25%	0.86	0.41	9.44*
TS_t	0.94%	-3.62%	1.89%	0.86%	-1.89	7.35	204.67**
RP_t	0.28%	-3.57%	6.88%	3.80%	0.58	1.25	8.68*
IP_t	3.77%	-5.16%	3.63%	3.63%	-1.12	1.75	24.24**

Probably the most surprising result from these descriptive statistics is the poor performance of our portfolios. However, these portfolios are constructed relative to their market value so direct comparison between them and the market index is misleading. The market return is clearly the best performer when comparing it with the other portfolios. The portfolios have performed badly and still the standard deviations are quite large. Only market and the largest companies' portfolio have positive skewness. This implies that these series have had many large positive returns, because the mean is larger than median or mode, and distribution's tail is long to right. Besides ER_{P2} every one of them are normally distributed. Not surprisingly, all the macroeconomic variables are not normally distributed.

The most interesting series is the consumption series, C_{NDSt}. This series with nondurables plus services has a standard deviation of 8.19% that is quite small. This is actually the whole problem of consumption based asset pricing models – the standard deviation of consumption is too small to warrant for the large equity premium. From the Table 1 we can see that the risk factor used in the CAPM estimation, the market return, has a standard deviation of 35.36% – clearly larger than that of consumption growth's. One more thing to point out is the large percent changes in interest rates. This is apparent from the empirical evidence that Finland has had large deviations in the 1980's and 1990's when the 3-month Helibor rate was about 15–17%, so the excess returns in the beginning of our sample period are low. We also calculated the real market percentage return that was 17.90% p.a. It is a large return, but we have to remember that our sample period is exceptionally good for Finnish stock markets.[83] The real 3-months risk-free rate is 4.51% and real consumption growth is 1.87% p.a. We will need these values in order to calculate the equity premium for Finnish markets.

7.2 Correlation matrix and the equity premium

We are specifically interested in the correlations between our macroeconomic factors and the returns. The correlations matrix is presented in Appendix 1. We see that the correlation between our portfolio returns and the market return is high, as we expected. Portfolio ER_{P1} has the highest correlation coefficient with the market return, which is not surprising because it has the largest market capitalization, thus the largest weight in the index. The correlation diminishes when we look at the smaller portfolios. When comparing the real market return with the continuously compounded market return we find out that taking the way the inflation factor from the continuously compounded market return does not affect the correlation almost at all because the correlation coefficient is 0.999. As other studies have

[83] Nyberg & Vaihekoski (2005) calculated the real percentage returns for a period of 1920-2004. The market return was 7.141 % p.a. for the whole sample. However, when they calculated it to a sample period of 1990-2004 the return was 20.025 % p.a., almost the same as our value. The real risk-free rate for our period was 4.51% compared to their period that was 4.30%.

shown, we also see a negative relationship between inflation and nominal stock returns.[84] It is also widely accepted that current stock levels are positively related to future levels of real activity, as measured by industrial production and that is also our finding because all the returns are significantly and positively correlated with industrial production. The consumption parameter is positively correlated with our portfolio returns. On the contrary, it is negatively correlated with the risk-free rate, as it should according to the theory.

One important thing to notice is also the relatively small correlation between consumption and portfolio returns compared to market return and portfolio returns. Remember that assets whose returns have a high negative conditional covariance with consumption will be willingly held even though they have low expected returns. Kocherlakota (1996) states that there is a simple explanation for the higher average return of stocks: stock returns covary more with consumption growth than do the risk-free rate. Thus, investors see stocks as a poorer hedge against consumption risk, and so stocks must earn a higher average returns. This is what our analysis also suggests: the correlation between stocks and consumption is positive and negative between the risk-free rate and consumption. Thus, the investors are willing to invest in the risk-free rate because the risk-free rate offers a better hedge against consumption fluctuations. Furthermore, our other variables are in line with the theory: the risk parameter that is the difference between BAA and AAA rated bonds is positively correlated with stock market returns – when the risk increases, returns increase as well. The term structure is negatively correlated with stock market returns and almost perfectly correlated with the risk-free rate.

We calculate the equity premium using the same methodology as Mehra & Prescott (1985), although their sample period is clearly different. To do so, we need arithmetic averages (real percentage returns) that we calculated

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[84]See, e.g., Gultekin (1983).

in the previous section. There is also a possibility to use government bonds or some “risk-free” returns, but we are interested in the absolute premium that stockholders require that they will invest in risky assets. Thus, our equity premium is the real percentage return of the stock market subtracted with the real percentage risk-free rate: 16.62% – 4.51% = 12.11%.[85] We can see that also Finland has a large equity premium and that CCAPM might be rejected because of this.

In the theory part we discussed about the fact that we need an unrealistically high risk-aversion parameter to account for the equity premium. We know now that the Finnish equity premium is large, so intuitively speaking it is possible that we need an unrealistic value of the risk-aversion coefficient to account for the equity premium. Thus, the equation that we presented was:

$$E(R_t^e) = Corr(R_t^e, \Delta c_t)\sigma(R_t^e)\sigma(\Delta c_t)\gamma\,, \tag{55}$$

We will plug in our own values to Equation (55):

$$\underset{0.1211}{E(R_t^e)} = \underset{0.19}{Corr(R_t^e, \Delta c_t)}\,\underset{0.35}{\sigma(R_t^e)}\,\underset{0.0819}{\sigma(\Delta c_t)}\,\gamma$$

When we calculate the value of γ, we get that our risk-aversion coefficient needed to account for the equity premium of our sample 1987–2004 is around 22. The overall conclusion from our theoretical part was that the relative risk-aversion coefficient cannot be more than five. If the correlation between the market return and the consumption growth would be perfect, we would still need a γ of around 8. However, for the Finnish stock market these values are not as unreasonable as for some countries where γ-value of 250 is needed. This is largely due to the fact that although the equity

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[85] Nyberg (2003) calculates for a period of 1990-2001 that the equity premium is 21.78% using the same definitions as we said. Nyberg & Vaihekoski (2005) calculates for a period of 1920-2004 that the equity premium is 5.33%, but when they use a sample of 1990-2004 the equity premium is 15.73%, quite close to our value.

premium in Finnish stock market is large, the standard deviation of the real consumption growth is relatively large. This is due to the fact that our sample period has had some dramatic changes during the good times in the beginning of the 1990's, then a bit weaker time in the depression, and the explosion in the 21st century, so the consumption has fluctuated relatively lot because the rapid wealth decreases and increases of Finnish consumers. The consumption of nondurables and services has doubled in our sample period, whereas the consumption of services has more then tripled.

7.3 Asset pricing models

7.3.1 The CAPM and the APT

Table 2 contains the results from fitting the models as described in our theory section. We compare the results of the GMM-procedure with the OLS using White (1980) standard errors. The small difference in the standard errors, and thus in t-values, arises because the White version does not include the degrees of freedom correction.

A natural question to ask is how well the model fits the data. If the model is "just-identified" there is one parameter for each restriction so the restrictions can be satisfied exactly. This we will be the case in our CAPM and APT estimations, for the CAPM we have one asset, the portfolio, and two parameters to be estimated (the alpha and the market beta). Thus, we have two moment conditions. In the CAPM we will use the market portfolio returns and the constant also as an instrument. Thus, we have as many moment conditions as parameters, so our model is just-identified. The same applies for the APT, but we have seven parameters (a constant and six risk factors) to be estimated and we have seven moment conditions. Thus, the *J*-test has no meaning in the unrestricted version, because it is zero, but when we add the restriction and take away the constant, the J-test will give us indication whether the constant is significant or not.

The risk factors are constructed as the theory suggests – we use the unanticipated component of the factors. We get this component by running the AR(1)-procedure, where we run this estimation for all of our factors and take the “noises” as the explanatory parameters. In this way we got a solid estimator for the unanticipated component. As we know, the theory suggests that if we know a future event, say in the oil price, this event is already priced in the market. Thus, it is the unanticipated component that affects the returns.

The constant and macroeconomic factors are used as instruments. “Just-identification” also implies that the GMM estimator will not depend on our W, the weighting matrix. The GMM estimators are compared with the OLS using White (1980)[86] correction for standard errors. We will also include a constant in our analyses to account for the alpha. According to null hypothesis, the constant should be zero and it should not be significant. The special density matrix that we will use is the Barlett kernel that was used in Newey & West (1987). The parameters are estimated from running the GMM and OLS-procedures one portfolio at a time. Thus, after running a one procedure, we change the dependent variable (i.e., the portfolio return series).

[86] White (1980) has derived a heteroskedasticity consistent covariance matrix estimator which provides correct estimates of the coefficient covariances in the presence of heteroskedasticity of unknown form.

Table 2. Results from the APT and CAPM estimation with the GMM. In the table are different factors' statistical significances. ** indicates significance at 0.01 level and * at 0.05 level. In the table are also Adjusted R^2, Durbin-Watson value to test the first-order autocorrelation and the overall F-test for the validity of the OLS (White). J-test, LRT and Wald-test are the test-values for the imposed restrictions' significance. Numbers in parentheses ( ) are t-values and numbers in square brackets [ ] are p-values. The sample period is 1987–2004 and N = 71.

	APT										CAPM				
GMM	$\alpha \times 100$	EM_{rt}	C_{NDStR}	I_t	TS_t	RP_t	IP_t	**J-test**	**LRT**	**Wald**	$\alpha \times 100$	β_{rm}	**J-test**	**LRT**	**Wald**
Portfolio 1	-0.390	0.978**	0.169	-0.247	0.064	0.434	-0.865	0.054	0.052	0.055	-0.805	0.940**	0.554	0.554	0.546
	(-0.22)	(12.16)	(1.17)	(-0.14)	(0.18)	(0.92)	(-1.07)	[0.815]	[0.818]	[0.813]	(-1.10)	(13.36)	[0.456]	[0.456]	[0.459]
Portfolio 2	3.061	0.529**	0.369	0.189	-0.616	1.001	-0.845	1.498	1.477	1.505	-0.929	0.529**	0.532	0.531	0.525
	(1.10)	(6.29)	(1.68)	(0.09)	(-1.45)	(1.71)	-(1.18)	[0.220]	[0.224]	[0.219]	(-0.73)	(6.03)	[0.465]	[0.465]	[0.468]
Portfolio 3	2.245	0.486**	0.151	3.264	-0.854*	0.608	-0.501	0.757	0.755	0.749	-11.676	0.499**	0.764	0.764	0.753
	(0.71)	(4.83)	(0.51)	(1.42)	(-2.03)	(0.95)	(-0.61)	[0.384]	[0.384]	[0.386]	(-0.87)	(5.19)	[0.382]	[0.381]	[0.385]
Portfolio 4	4.013	0.495**	-0.242	5.955*	-1.381**	-0.080	-0.305	1.932	1.842	2.177	-0.985	0.536**	0.560	0.559	0.552
	(1.36)	(5.61)	(-1.09)	(2.17)	(-2.75)	(-0.15)	(-0.49)	[0.164]	[0.174]	[0.140]	(-0.75)	(6.28)	[0.454]	[0.454]	[0.457]
Portfolio 5	1.841	0.409**	-0.023	3.252	-0.866**	0.422	-0.028	0.483	0.484	0.491	-1.332	0.447**	1.140	1.140	1.124
	(0.61)	(6.09)	(-0.09)	(1.43)	(-2.15)	(0.68)	(-0.05)	[0.486]	[0.486]	[0.483]	(-1.09)	(6.53)	[0.285]	[0.285]	[0.288]
Portfolio 6	2.079	0.343	0.375	2.053	-0.804	0.293	-0.262	0.435	0.437	0.454	-1.444	0.355**	1.137	1.137	1.121
	(0.60)	(4.26)	(1.29)	(0.65)	(-1.40)	(0.42)	(-0.32)	[0.509]	[0.508]	[0.500]	(-0.94)	(3.86)	[0.286]	[0.286]	[0.289]
OLS (White)	$\alpha \times 100$	EM_{rt}	C_{NDStR}	I_t	TS_t	RP_t	IP_t	**Adj. R^2**	**D-W**	**F-test**	$\alpha \times 100$	β_{rm}	**Adj. R^2**	**D-W**	**F-test**
Portfolio 1	-0.390	0.978**	0.169	-0.247	0.064	0.434	-0.865	0.762	2.367	[0.008]	-0.805	0.940**	0.770	2.428	[0.008]
	(-0.25)	(15.32)	(0.88)	(-0.08)	(0.18)	(0.71)	(-1.21)				(-0.75)	(16.19)			
Portfolio 2	3.061	0.529**	0.369	0.189	-0.616	1.001	-0.845	0.446	1.906	[0.011]	-0.929	0.529**	0.435	1.940	[0.011]
	(1.29)	(6.43)	(1.54)	(0.07)	(-1.54)	(1.48)	(-1.22)				(-0.74)	(6.75)			
Portfolio 3	2.245	0.486**	0.151	3.264	-0.854	0.608	-0.501	0.368	1.996	[0.013]	-11.676	0.449**	0.375	2.104	[0.012]
	(0.92)	(4.94)	(0.51)	(1.23)	(-1.86)	(0.90)	(-0.65)				(-0.88)	(5.54)			
Portfolio 4	4.013	0.495**	-0.242	5.955*	-1.381**	-0.080	-0.305	0.473	1.898	[0.010]	-0.985	0.536**	0.429	1.988	[0.011]
	(1.55)	(5.93)	(-0.88)	(2.07)	(-2.95)	(-0.15)	(-0.43)				(-0.75)	(6.28)			
Portfolio 5	1.841	0.409**	-0.023	3.252	-0.866**	0.422	-0.028	0.354	1.890	[0.011]	-1.332	0.447**	0.358	1.949	[0.011]
	(0.74)	(6.15)	(-0.09)	(1.11)	(-2.18)	(0.66)	(-0.05)				(-1.08)	(6.66)			
Portfolio 6	2.079	0.343**	0.375	2.053	-0.804	0.293	-0.262	0.213	1.572	[0.013]	-1.444	0.355**	0.221	1.649	[0.013]
	(0.71)	(4.28)	(1.13)	(0.69)	(-1.63)	(0.42)	(-0.32)				(-1.07)	(4.05)			

We can see from the Table 2 that, opposite to our expectations, the APT is able to explain the variations in portfolios' excess returns quite well. For the APT, none of the alphas are significant, which is supported by the theory. Furthermore, all of our models are significant, so we can reject the null hypothesis that all of our risk factors are zero. However, the market beta is still the most dominant factor.[87] Interestingly, the market beta is almost one in the largest capitalization portfolio that makes sense – these companies are the ones that determine where the market index is moving. The beta diminishes as we go to the smaller portfolios, leading to the 6th portfolio that has a market beta of 0.34. However, the other factors do not seem to add much explanatory power to our analysis. Consumption, risk premium and industrial production are never significant in our 0.05 significance level. However, some of the *t*-values are quite close to be significant. The Durbin-Watson statistics tests the serial correlation in our residuals. The critical values for our sample are for the lower one 1.43 and the upper one 1.80 at 0.05 level of significance. Only for Portfolio 6 the test is inconclusive, with other portfolios there is no evidence of serial correlation in our residuals.

The inflation factor is significant in one case, for Portfolio 4. Surprisingly, it is positive. We can conclude that these unexpected inflation changes can affect the stock returns positively. In a short run, we can think that if inflation stays in its current level, it does not affect the stock returns. However, if there is an unanticipated rise, it means that consumption has also risen and this raises the demand for stocks. In the long run inflation pressures make the central banks nervous and if they raise the rates, this will affect stock returns negatively.

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[87] We could have used only the macroeconomic factors in our analysis, without the market beta. However, when we tested this, the results were in line with the model presented here. Furthermore, a lot of the tests of the APT are done this way. Furthermore, the comparison between the APT and the CAPM is more reasonable, because now we can compare the adjusted R^2's.

This can also be seen from the term structure factor that is negatively significant for three portfolios. This means that if the 12-months Euribor rises more than the 1-month Euribor, making the spread wider, we can expect rises in the shorter-term risk-free rates that in turn make the discount rate for the stocks to be higher affecting negatively to stock prices. Monetary Portfolio Theory suggests that changes in money supply alters the equilibrium position of money, thereby altering the composition and price of assets in an investor's portfolio. In addition, changes in money supply may impact on real economic variables, thereby having a lagged influence on stock returns. Both of these mechanisms suggest a positive relationship between changes in money supply and equity returns. Thus, when the interest rates are expected to rise, the money supply decreases and stock prices decline.

We then put a restriction that we take away the constant α from the APT and CAPM models. J-, Wald- and Likelihood Ratio-tests do not reject the null hypothesis that the restriction is not statistically significant. For the Wald-test our restriction was simply that the alpha has to be zero, i.e. for the CAPM: R = [1 0] and r = 0. The loss function increase in the LRT is not statistically significant when we impose the restriction. These three tests' values are almost equal, because we are measuring the same error. It is also interesting to see that when the alpha is quite close of being statistically significant, i.e., in Portfolio 4 for the APT and Portfolio 5 for the CAPM, these test values approach 20 % significance levels.

Just to give an example how the J- and Likelihood Ratio-tests work, we took away the beta from the Portfolio 1 when testing the CAPM. We got J-test value of 14.9168 with probability 0.0002 and LRT-value 14.9168 with probability 0.0002, the same test values for both tests. Here we can see that, of course, taking away the only statistically significant explanatory parameter, the market beta, test statistics imply strongly that this parameter was significant and the estimated models differ significantly between the unrestricted and restricted versions.

The adjusted R^2's are almost the same for our portfolios, which might be due to the inclusion of the market risk factor to our APT. However, the other macroeconomic risk factors have some explanatory power as well, because the adjusted R^2 takes to account the adding of multiple explanatory parameters. Portfolio 1 has the highest adjusted R^2, because of the market beta. It declines as we move forward to smaller portfolios and in Portfolio 6 the value is only about 0.22.

From the Table 2 we can also see that the APT and the CAPM perform almost as well. Adding of the macroeconomic variables does not weaken the results which can be thought as a quite promising result, because now we get a lot more information about the risk factors that affect the stock returns. Especially the inflation and the term structure factor results were quite intriguing. It is also worth noticing the exactly same parameter estimates from the GMM and OLS (White) procedures.

7.3.2 The CCAPM and the estimation of the parameters

The results for the CCAPM can be found in below Tables 3–7. This model is "over-identified" and now we can use the J-test for testing the validity of the whole model, because it will not be possible to set every moment to zero. The question is how far from zero are we. The answer is provided by the "test of over-identifying restrictions", often denoted TJ_T. This test statistic is distributed χ^2_{L-k} under the null. (Cliff, 2000) TJ_T can be seen as the total pricing error that the model gives to the parameter estimation. The model can be rejected if the pricing errors are large. To give an intuitive meaning for the model we can state that the purpose of the model is to give significance for the vector θ in Equation (48) so that the model's estimated returns are in line with the observed returns in a most precise way. The result from this is that we can reject the model if the difference between the estimation and observed returns is statistically significant. (Nyberg, 2003)

For each of the portfolios we will use five different sets of instrumental variables, referred to as INSVAR1-INSVAR5. In INSVAR1 and INSVAR2 the instruments are a constant as well as lagged values of returns of the portfolios and a C_{NDStR}, the real consumption growth rate of nondurables plus services (one and two lags, respectively). Thus, INSVAR1 has three instruments and INSVAR 2 has five instruments. In INSVAR3 and INSVAR4 the instruments are a constant as well as lagged returns of the portfolios, the consumption growth rate and the risk-free rate (one and two lags, respectively). Thus, INSVAR 3 has four instruments and INSVAR 4 has seven instruments. Constantinides (1991) has argued that it is preferable to use other variables as instruments since measurement errors may lead to biased parameter estimates and spurious rejection of the overidentifying restrictions.[88] Therefore, in INSVAR5 we use a constant as well as our macroeconomic factors studied in the APT.

The overall impression from our Tables 3–7 is that the GMM is able to estimate the CCAPM's parameters with more theoretic intuition that we were prepared for. However, the risk-aversion parameter γ is estimated with large uncertainty that we can see from the confidence intervals. If we want to use null hypothesis H_0: $\gamma=0$, we can not reject that in any of the cases with 95 % confidence interval. However, this null hypothesis is quite misleading because the assumption of risk-neutrality that $\gamma=0$ is not so intriguing. Furthermore, if we want to use same null hypothesis Brown & Gibbons (1985) that H_0: $\gamma=1$, we cannot reject that either. This null hypothesis means that the data cannot reject log utility as the appropriate specification for the aggregate utility function. This means that the income effect and the substitution effect on savings exactly offset.

Nevertheless, in our opinion the confidence interval gives more information than the exact hypothesis testing in this case. Furthermore, the esti-

[88] Furthermore, Pasquariello (2000) used different macroeconomic factors in his analysis and found out that the macroeconomic factors perform well in the GMM-framework. However, he also said that one has to be quite imaginative to link these two sets of data to a reasonable conclusion.

mates of γ vary considerably over different definitions of the instrumental variables, and while this can be explained by the high standard errors, it could also be an indication of misspesification of the utility function. However, the risk-aversion parameters are generally really low and they make some economic intuition– excluding the negative sign of the Portfolio 6 that are the companies with the smallest market capitalization. One interesting pattern also reveals from the risk-aversion coefficient estimates. As we can see, the risk-aversion coefficient is the highest in Portfolio 1 and diminishes when we look at the smaller portfolios. This could mean that more risk-averse investors seek to find more stable, liquid and financially stronger companies with large market capitalizations to give them returns also during the recession and distressed times, when smaller companies usually do worse or even default. The β's are estimated with greater certainty. However, we estimate 30 β-values and only nine of them are below one that was the theoretically correct upper value for β.

For the CCAPM we use the three test statistics: LRT, Wald-, and J-test. It is important to notice now that the J-test is not measuring the same property as it did in the APT and CAPM-section above. The J-test measures now the whole validity of the models. We can see from the tables that the null hypothesis of the validity of the instruments, i.e., over-identifying restrictions, is not rejected for INSVAR1-INSVAR4. These instrumental variables are the common ones used in the GMM estimation. Thus, despite the uncertainty in our risk-aversion coefficient estimators, we do not reject the models. However, for the INSVAR5, the results are quite the opposite. As we can see, the confidence intervals widen a lot when we are using the macroeconomic factors for our instruments. This leads to the conclusion that J-test rejects the validity of the instruments. Thus, the validity of the whole model is rejected. We can draw a conclusion that the macroeconomic factors alone do not determine the consumption patterns and preferences of an individual.

Tables 3–4. The CCAPM test using INSVAR1 and INSVAR2. There are parameter estimates and confidence intervals for γ and β. Wald- and Likelihood Ratio-values are also reported. J-test is the test of the "over-identifying" restrictions. Numbers in parentheses ( ) are p-values and N = 69.

	CCAPM						
GMM (INSVAR1)	γ	β	**Confidence interval (γ)**	**Confidence interval (β)**	**Wald-test**	**Likelihood Ratio**	**J-Test (Prob.)**
Portfolio 1	1.113	0.999	[-0.874:3.101]	[0.953:1.045]	1.205 (0.272)	1.237 (0.266)	0.242 (0.622)
Portfolio 2	0.268	1.008	[-1.054:1.591]	[0.974:1.043]	0.158 (0.690)	0.159 (0.689)	0.015 (0.901)
Portfolio 3	0.441	1.008	[-0.963:1.845]	[0.975:1.041]	0.378 (0.827)	0.382 (0.826)	0.189 (0.663)
Portfolio 4	0.281	1.003	[-1.102:1.666]	[0.968:1.038]	0.159 (0.923)	0.159 (0.923)	0.645 (0.422)
Portfolio 5	0.111	1.013	[-1.153:1.376]	[0.979:1.047]	0.029 (0.985)	0.033 (0.983)	0.152 (0.696)
Portfolio 6	-0.505	1.017	[-1.833:0.822]	[0.982:1.052]	0.557 (0.756)	0.548 (0.760)	1.265 (0.261)

	CCAPM						
GMM (INSVAR2)	γ	β	**Confidence interval (γ)**	**Confidence interval (β)**	**Wald-test**	**Likelihood Ratio**	**J-Test (Prob.)**
Portfolio 1	1.172	1.000	[-0.561:2.905]	[0.954:1.045]	1.757 (0.185)	1.838 (0.175)	0.373 (0.946)
Portfolio 2	0.307	1.007	[-0.886:1.501]	[0.974:1.040]	0.2546 (0.613)	0.257 (0.612)	0.136 (0.987)
Portfolio 3	0.666	0.993	[-2.915:4.249]	[0.986:1.000]	0.592 (0.743)	0.651 (0.722)	1.748 (0.626)
Portfolio 4	0.555	0.999	[-0.648:1.760]	[0.966:1.033]	0.817 (0.664)	0.827 (0.661)	1.620 (0.655)
Portfolio 5	0.257	1.009	[-0.903:1.418]	[0.981:1.038]	0.189 (0.909)	0.190 (0.909)	0.717 (0.869)
Portfolio 6	-0.354	1.016	[-1.571:0.860]	[0.982:1.050]	0.328 (0.848)	0.327 (0.848)	2.348 (0.503)

Tables 5–6. The CCAPM test using INSVAR3 and INSVAR4. There are parameter estimates and confidence intervals for γ and β. Wald- and Likelihood Ratio-values are also reported. J-test is the test of the "over-identifying" restrictions. Numbers in parentheses ( ) are p-values and N = 69.

	CCAPM						
GMM (INSVAR3)	γ	β	**Confidence interval (γ)**	**Confidence interval (β)**	**Wald-test**	**Likelihood Ratio**	**J-Test (Prob.)**
Portfolio 1	0.791	1.010	[-1.210:2.793]	[0.964:1.055]	0.600 (0.438)	0.626 (0.428)	3.810 (0.149)
Portfolio 2	0.329	1.002	[-0.966:1.624]	[0.968:1.036]	0.248 (0.618)	0.266 (0.606)	4.328 (0.115)
Portfolio 3	0.735	1.001	[-0.583:2.055]	[0.970:1.032]	1.193 (0.550)	1.377 (0.502)	2.658 (0.625)
Portfolio 4	0.248	1.000	[-1.131:1.628]	[0.965:1.035]	0.124 (0.939)	0.129 (0.937)	3.045 (0.218)
Portfolio 5	0.054	1.014	[-1.215:1.324]	[0.980:1.047]	0.007 (0.996)	0.007 (0.996)	3.877 (0.144)
Portfolio 6	-0.506	1.015	[-1.829:0.818]	[0.981:1.050]	0.560 (0.755)	0.553 (0.758)	2.300 (0.317)

	CCAPM						
GMM (INSVAR4)	γ	β	**Confidence interval (γ)**	**Confidence interval (β)**	**Wald-test**	**Likelihood Ratio**	**J-Test (Prob.)**
Portfolio 1	0.838	1.010	[-0.858:2.536]	[0.964:1.055]	0.937 (0.332)	0.983 (0.321)	4.033 (0.545)
Portfolio 2	0.440	0.991	[-0.701:1.583]	[0.961:1.020]	0.572 (0.449)	0.612 (0.433)	6.438 (0.266)
Portfolio 3	0.696	1.002	[-0.497:1.890]	[0.971:1.032]	1.305 (0.520)	1.442 (0.486)	3.047 (0.693)
Portfolio 4	0.395	0.993	[-0.757:1.547]	[0.961:1.025]	0.451 (0.797)	0.465 (0.792)	4.223 (0.518)
Portfolio 5	0.087	1.008	[-1.043:1.218]	[0.979:1.037]	0.023 (0.988)	0.023 (0.988)	4.224 (0.518)
Portfolio 6	-0.343	1.012	[-1.529:0.844]	[0.979:1.046]	0.319 (0.852)	0.319 (0.852)	3.901 (0.564)

Table 7. The CCAPM test using INSVAR5. There are parameter estimates and confidence intervals for γ and β. Wald- and Likelihood Ratio-values are also reported. J-test is the test of the "over-identifying" restrictions. Numbers in parentheses ( ) are p-values and N = 69.

	CCAPM						
GMM (INSVAR5)	γ	β	**Confidence interval (γ)**	**Confidence interval (β)**	**Wald-test**	**Likelihood Ratio**	**J-Test (Prob.)**
Portfolio 1	1.703	0.986	[-1.850:5.256]	[0.944:1.027]	0.882 (0.347)	1.025 (0.311)	13.47 (0.009)
Portfolio 2	0.752	0.980	[-2.737:4.243]	[0.951:1.009]	0.178 (0.672)	0.246 (0.619)	14.13 (0.007)
Portfolio 3	-0.636	0.992	[-4.075:2.812]	[0.956:1.029]	0.129 (0.937)	0.154 (0.925)	10.58 (0.032)
Portfolio 4	-0.248	0.994	[-4.058:3.562]	[0.958:1.030]	0.016 (0.991)	0.016 (0.991)	12.92 (0.012)
Portfolio 5	-0.188	1.003	[-3.280:2.902]	[0.970:1.036]	0.014 (0.992)	0.018 (0.990)	13.46 (0.009)
Portfolio 6	-0.766	1.009	[-3.951:2.419]	[0.975:1.043]	0.222 (0.894)	0.231 (0.890)	7.393 (0.117)

The Wald- and Likelihood Ratio-tests differ also from the APT and CAPM-section. Now we put some restrictions on the parameters γ and β. We allow the risk-aversion parameter vary from zero to ten and β from 0.9 to 1.1. These values are quite in line with the theory, except the fact that the "rate of time preference" is allowed to vary above one. This was done because we do not want to restrict the tests too harshly. As we can see from the tables, none of the test statistics reject the null hypothesis that the restrictions do not affect the estimation significantly – excluding the macroeconomic model that we already abandoned. Actually, the restrictions are quite well accepted by the test values, because the values of tests are quite small. Thus, the restrictions do not affect the model performance significantly. This is also expected, because the restrictions are in line with the unrestricted versions of the model, when our risk-aversion coefficients were around zero to two and the β was close to one.

We stated before that it could be intriguing to use as many instruments as one can imagine and then let GMM just put less weight on the irrelevant instruments. We can see that this happens also in practice: The confidence intervals go smaller and smaller – our estimators' variances decrease – as we add more instrumental variables. However, the J-test values also rise and that is due to the fact the there is an increase in the bias of the whole model.

7.4 Summary

From the descriptive statistics we saw that the consumption series are quite smooth. However, they are relatively volatile compared to other countries, e.g., in Mehra & Prescott (1985) it was around three percent p.a compared to our 8.19% p.a. over the period of 1987–2004. We also saw that the Finnish stock market has performed well compared to bigger areas, such as U.S. and Europe. The portfolio returns were normally distributed except for Portfolio 2. The macroeconomic factors were not normally distributed due to the heteroskedasticity that is quite common among economic series.

The most interesting result from the correlation matrix was that the correlation between stocks and consumption was positive and negative between the risk-free rate and consumption. Kocherlakota (1996) states that there is a simple explanation for the higher average return of stocks: stock returns covary more with consumption growth than do the risk-free rate, which was also our conclusion from the correlation matrix.

We calculated the equity premium using the same methodology as Mehra & Prescott (1985) for their U.S. sample. Our estimated value for the equity premium was 12.11% for the Finnish stock market, which was quite similar to, e.g., Nyberg & Vaihekoski (2005). This equity premium is remarkable and we calculated that we needed risk-aversion coefficient of 22 to account for this equity premium. Even if the correlation between the market return and consumption growth rate would be a perfect one, we would still need a risk-aversion coefficient of eight. That is still larger than the hypothesized value found in previous studies were the coefficient was calculated to be less than five.

The CAPM and APT were tested with the GMM-procedure and OLS with White (1980) standard errors. The CAPM and the APT performed almost as well if we are looking the adjusted R^2's. Adjusted R^2 was used because it takes to account for the number of explanatory variables used in a model. For the largest portfolios we found the adjusted R^2 to be around 76–80% which can be interpreted as approximately 76–80 % in the dependent variable can be explained by the independent variables. This number is quite large for a finance study. This number went down to around 20% when we tested the CAPM and APT with the smallest capitalization portfolio. This makes a lot of sense because the corporations with largest capitalizations determine the direction of the whole index. All of the models were significant, so there is a linear relationship between risk and return in our tested models. Furthermore, we tested that if the constant, alpha, is significant for the model. We did not have to reject any model based on the F-, J-, LR- and Wald-tests. We were also able to see another relationship between the market beta and stock returns – portfolios with higher betas had higher returns, as it should be according to theory.

The CCAPM was able to explain the real returns better than expected and with theoretically sound parameter estimates. Risk-aversion coefficients were between zero and one, without couple of exceptions where the estimated value was above one and below zero. However, the estimated risk-aversion coefficients were volatile and we can not make any robust conclusions of the real coefficient values, because the risk-aversion coefficients were not significantly different from zero. However, we also stated that it is more intuitive to look at the confidence intervals, because although the risk-aversion coefficients were also below zero in this confidence interval scale, the estimated parameters were above zero in 22 out of the 30 cases. An interesting pattern was also found that the more risk-averse investors invested in the larger portfolios. This might mean that more risk-averse investors tend to seek large and stable companies that can go through distressed times without defaults and any unexpected events.

We tested the CCAPM with the same setting of tests than we did in the CAPM and APT-section. Now the J-test was a test of the validity of the whole model and we did not reject the model in the basic setting of where the instruments were lagged returns, consumption growth and risk-free interest rates. However, when we used the macroeconomic factors as instruments, five of the six portfolios were rejected at 0.05 significance level. This led to the conclusion that the macroeconomic factors do not explain the consumption patterns of an individual and they should not be used in our sample. Wald- and LR-tests were also used to test the restrictions in for our models. For the LRT we used two restrictions: the risk-aversion coefficient should be between zero and 10, and the SDF between 0.9 and 1.1. None of the tests rejected these restrictions, except for the macro-economic factor version that we rejected already, so we can conclude that these restrictions are robust for the model.

8 CONCLUSIONS AND DISCUSSION

This study examines three models' power to predict the risk-return relationship for six value-weighted portfolios on the Finnish stock market for the years 1987–2004. These three models are the CAPM, APT and CCAPM. We also tested if there is a significant equity premium between the stock returns and risk-free rates. One of the main contributions of this thesis is the use of Stochastic Discount Factor-methodology along with the GMM-framework. We tested the CAPM and APT with linear GMM-procedure along with OLS-method. The CCAPM equation is non-linear by its nature, so the GMM was used to estimate two important parameters: the risk-aversion coefficient and the rate of time preference that tells us by which value consumers discount future consumption.

The introduction of the GMM is perhaps the most important methodological innovation of the decade in the field. In fact, the SDF and GMM are related concepts that make the whole asset pricing theory viewed from a different angle. As Cochrane (2005) states: "Where once there were three apparently different theories for stocks, bonds and options, now we see each as special cases of the same theory". Hence, an SDF relates the future cash flows of any financial instrument to their respective present market values.

The development of consumption-based asset pricing theory is also one of the major advances in the finance field. This model provides a simple framework in which a simple relation between consumption and asset returns captures the implications of complex dynamic intertemporal multifactor asset pricing model. One of main puzzles in the CCAPM-framework has been the fact that variation in consumption is too smooth. This means that we need an unrealistically high risk-aversion coefficient to account for the difference between the large historical equity premium and the small predicted value of the CCAPM. This is called the equity premium puzzle.

We estimated that the equity premium for our sample period is 12.11% and we needed a risk-aversion coefficient of 22 to account for that premium.

In the comparison between the CAPM and APT the result was quite inconclusive. For the APT we selected macroeconomic factors a priori that were proved to influence stock returns. Both of the models were able to explain relatively high portion of the variation of the dependent variables, our six value-weighted portfolios. We did not reject any of these models, and in fact, we got interesting information of the APT's ability to explain some variation in the portfolio returns with inflation- and term structure-factors. However, the market beta was clearly the most dominant risk factor, which is in favour for the CAPM.

We estimated two parameters of the CCAPM with the GMM, the coefficient of relative risk-aversion (γ) and the marginal rate of intertemporal substitution of consumption (β). Risk-aversion influences the rate at which the agent is prepared to exchange units of consumption between different states of nature, whereas the elasticity of intertemporal substitution reflects the agent's willingness to exchange units of consumption between periods.

We were able to get reasonable values for both of the parameters, although they were quite volatile in our sample, especially the risk-aversion coefficient. The risk-aversion parameter was the highest in our portfolio with largest capitalization. Thus, investors who are more risk-averse want to invest in corporations that are stable and whose stocks are more liquid. The marginal rate of intertemporal substitution of consumption parameters were estimated with less volatility, but they were below one in only nine cases out of 30. Furthermore, we did not find any proof for the CCAPM when macroeconomic factors were used as instruments. For the suggested way of lagging consumption growth, portfolio returns and risk-free rate we did not reject any of these models. Therefore, the results are quite inconclusive, because the tests statistics do not reject the models but we know that the parameters are measured with lot of deviation. All in all, the CCAPM proved to be quite sound model for the Finnish stock market and we got information of consumers' habits when they are making decisions in their consumption patterns. Our results were in line with, e.g., Pasquariello (2000) who had totally the same research setting but he tested the CCAPM on U.S. data. For the CCAPM we can conclude that the original specification of the CCAPM has shown poor success in explaining the equity premium and there are not enough empirical evaluations of alternative models based on estimation and testing.

What ever the reason is, there has not been a lot of empirical success for the purest CCAPM. There are a number of well known stylized facts or puzzles which have offered challenges for financial economist and econometricians for almost three decades. Two possible explanations to these puzzles can be given either: (1) the models are wrong in come crucial way, or (2) investors have been extremely lucky in the past. However, as Kocherlakota (1996) argues that because there is a lack of compelling explanation for the large equity premium, are we just wasting our time? The empirical literature seems sometimes as a litany of failure for the CCAPM. We think that this is misleading in many ways. We have learned much from the equity premium puzzle literature about the properties of asset pricing models, about methods of estimating and testing asset pricing models, and about methods of solving for the implications of asset pricing models – all these contributions are very significant.

There are several suggestions for further research. We could test if the marginal utility of consumption depends on other variables other than consumption. We could introduce additional state variables like leisure or wealth into the utility function. One interesting addition could be that we could introduce habit-persistence effects, which means that past values as well as current values of consumption appear in the utility function. In the pure CCAPM the consumption does not depend on consumption in some other state, but we could modify utility functions so that marginal utility of consumption in one state is affected by the level of consumption in another state. We also used constant relative risk-aversion that does not change over time, but a time-varying risk premium could also be used. It is also possible to test the CCAPM without consumption data.

However, these are the traditional ways of looking at the problem and we learned from our theory section that they have met with some success, but these methods have not been able to solve the two puzzles at the same time. One particularly interesting paper was introduced by Piazzesi et al. (2006). They found that real estate investment growth has impact on cross-section of stock returns. Thus, changes in expenditure share on housing emerge as a second factor that drives asset prices and they were able to solve the puzzles. Another interesting research was made by Lettau & Ludvigson (2001a). They note that aggregate consumption, asset holdings, and labour income share a common long-term trend, but they deviate substantially from one another in the short run. They name their variable *cay*, which is more volatile than the usual consumption growth factor. Their model outperforms the CAPM in explaining the size- and value-premia. These kinds of studies could be easily implemented also in Finnish markets.

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APPENDICES

Appendix 1. The Pearson correlation coefficients between all our variables. The test is 1-tailed and ** represents statistical significance at 0.05 level and * at 0.10 level and N = 72.

Variable	ER_{P1}	ER_{P2}	ER_{P3}	ER_{P4}	ER_{P5}	ER_{P6}	EM_{rt}	$EM_{rt\ real}$	C_{NDSt}	R_{ft}	I_t	TS_t	RP_t
ER_{P1}	1.000												
ER_{P2}	0.725**	1.000											
ER_{P3}	0.664**	0.878**	1.000										
ER_{P4}	0.656**	0.831**	0.837**	1.000									
ER_{P5}	0.645**	0.817**	0.849**	0.823**	1.000								
ER_{P6}	0.517**	0.699**	0.729**	0.744**	0.752**	1.000							
EM_{rt}	0.880**	0.666**	0.622**	0.660**	0.606**	0.485**	1.000						
EM_{Rrt}	0.879**	0.668**	0.620**	0.656**	0.604**	0.486**	0.999**	1.000					
C_{NDSRt}	0.203*	0.243*	0.204*	0.301*	0.281**	0.207*	0.194	0.190	1.000				
R_{ft}	-0.207*	-0.299**	-0.276**	-0.324*	-0.280**	-0.278**	-0.253*	-0.272*	-0.078	1.000			
I_t	-0.113	-0.147	-0.052	-0.009	-0.045	-0.085	-0.138	-0.172	0.095	0.591**	1.000		
TS_t	-0.198*	-0.284**	-0.266**	-0.323**	-0.277**	-0.274**	-0.254*	-0.273*	-0.088	0.997**	0.586**	1.000	
RP_t	0.173	0.209*	0.165	0.094	0.141	0.096	0.155	0.155	0.375	-0.066	-0.037	-0.072	1.000
IP_t	0.288*	0.227*	0.243*	0.294**	0.287**	0.223*	0.406**	0.406**	0.068	-0.340**	-0.167	-0.359**	0.043