TIME-VARYING RISK PREMIUM AND CONDITIONAL VOLATILITY.
EMPIRICAL EVIDENCE FROM FINNISH STOCK MARKETS.
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1. INTRODUCTION

The expected risk premium and volatility has an important role in many financial applications. Volatility is generally used as a measurement of risk and uncertainty and in many applications relationship between required risk premium and volatility is assumed. Accurate estimates of future volatility and risk premium are seen as essential part for many financial applications. They are the key components for example to mean-variance portfolio theory and for different asset pricing models. From a theoretical perspective the relationship between stock returns and their own conditional volatility is in interest due the time-varying risk premium. Another question is that if markets are informationally efficient econometric volatility forecasting models should not be able to make such forecasts about future volatility based on past-realised volatility that can be turned to profitable trading rules.

A number of papers have studied relationship between stock returns and their own volatility. For the US market French et al. (1987) report a positive relationship between the expected volatility and the expected market risk premium and a negative relationship between the unexpected volatility and unexpected stock market returns. Nelson (1991) finds a negative relationship. For the Finnish stock market Stenius (1991) report a positive relation whereas Balaban and Bayar (2005) find a negative relationship between stock returns and both the expected and unexpected volatility. Nelson (1991) and Stenius (1991) have performed ex post analysis. French et al. (1987) report that their out-of-sample and in-sample results do not differ significantly from each other. Finally Balaban and Bayar (2005) have performed an ex ante analyse. These studies also contain information about the performance of different econometric models. Maukkonen (2002) has studied forecasting ability of different models of volatility on the Finnish stock markets. Day and Lewis (1992) have analysed forecasts derived from econometric models and from implied volatility on the S&P 100. The results of these studies show that both negative and positive relationships can be found, forecasting is a difficult task and the preferred forecasting model varies between studies.
However, there are only relative few previous studies using Finnish data to study time-varying risk premiums and their relationship to conditional volatility with GARCH (Generalized Autoregressive Conditional Heteroskedasticity) kind of models. Also the number of studies that compare relationship with unexpected volatility and stock market returns using Finnish data is limited. Therefore, it is of interest to analyse these issues from Finnish perspective.

This paper will analyse a relationship between stock returns and their own volatility to see whether there exists a time-varying risk premium in the sense of the increased expected rate of excess return required in response to an increase in the predictable volatility of the returns in Finnish stock markets. Volatility is modelled and forecasts are derived from the GARCH and EGARCH type of models. The relationship between unexpected volatility and returns is also analysed.

The rest of the paper is organized as follows. In Section 2 the theoretical background of time-varying risk premium and the main elements of the ARCH models are briefly presented. Applicability of the ARCH models is specially treated from the financial markets perspective. Section 3 contains data description, properties of the sample distribution and its implications for model building. Then in section 4, estimation and with-in and out-of-sample results of various return intervals and specifications of the GARCH kind of models is presented. Lastly, in section 5, the work is summarized.

2. THEORETICAL BACKGROUND

2.1 Time-varying risk premium

The Expected stock market risk premium can be defined as the expected return on a stock market portfolio minus the risk-free interest rate. It is the compensation required by risk averse economic agents for holding risky asset. Financial models like capital asset pricing model (CAPM) usually suggests that expected compensation should be positively related to the expected risk. According to Engle et al. (1987) the uncertainty in asset returns varies over time and so must also the risk premium vary. They argue that time series models of asset prices should therefore measure both risk and its movements over time and include it as a determinant of price.
Because we are interested about the expected risk premium and not realized, our estimation method should be forward looking. To illustrate ways for estimating risk premium we can select the conditional version of the CAPM and present it similar way as Vaihekoski (1998). If symbol $\Omega_{t-1}$ represents the information publicly available to agents at time $t-1$, the model for excess return can be presented as follows:

\[
E[r_i|\Omega_{t-1}] = \frac{\text{Cov}(r_i, r_{m|t-1})}{\text{Var}(r_{m|t-1})} E[r_{m|t-1}]
\]

where $E[r_{m|t-1}]$ and $E[r_i|\Omega_{t-1}]$ are the conditional expected excess returns for the market portfolio and for asset $i$ at time $t$. Similar $\text{Cov}(r_i, r_{m|t-1})$ and $\text{Var}(r_{m|t-1})$ are conditional covariance between asset $i$ and market portfolio and variance of market portfolio at time $t$. The risk-free rate is known at time $t-1$. Modifying equation (1) further by realising that covariance between markets is same as variance of market and replacing ratio $E[r_{m|t-1}]/\text{Var}(r_{m|t-1})$ by variable $\lambda_{mt}$, equation for market excess return can now be presented in a following form:

\[
E[r_{m|t-1}] = \lambda_{mt} \text{Var}(r_{m|t-1})
\]

where $\lambda_{mt}$ can be thought as the price of market risk. This variable measures the compensation representative agent must receive for unit increase in the market variance. This model can be estimated after proper model for conditional variance has been selected. In this paper GARCH and EGARCH type of models are chosen.

The CAPM theory suggests that the market portfolio should include all kinds of assets. In reality, it has to be placed by some market proxy. Elton (1999) points out that one big company can bias market proxy dramatically. In Finland this is probably true due to the impact of Nokia. When market proxy is used there comes also problem with the diversifiable and non-diversifiable risk because the true market portfolio does not have diversifiable risk. Stenius (1991) points out that theoretically it should be the covariance not the variance that enters into equation (2). If investors do not hold the market portfolio and they instead hold an incomplete set of assets due to taxes,
transaction cost or restrictions etc. has Levy (1978) shown that the risk measure of an asset is not its beta (covariance with the market portfolio divided by market variance), but instead its own variance. Because the purpose of this study is not to test CAPM, gives arguments above justifications for using variance and equity market proxy for our purposes.

In this paper time-varying risk premium is also analysed using standard deviation. Engle et al. (1987) argue that in the two-asset economy where only assets are risk-free and risky asset, in general one might expect that changes in variance are reflected less than proportionally in the mean. When interpreting results is now the compensation investors must receive for unit increase in the market standard deviation.

By adding intercept term to equation (2) conditional mean we are analysing is obtained. The conditional mean in interest, can be presented as follows:

\[
(3) \quad r_{mt} = \alpha + \lambda h_{mt}^s, \quad s = 1, 1/2
\]

where \( r_{mt} \) is the excess market return, \( h_{mt} \) is the conditional variance, \( \alpha \) is a constant that can be interpreted as an average risk premium for one step interval and \( \lambda \) is the market price of risk. If \( \lambda = 0 \), the expected risk premium is unrelated to the variability of stock returns. On the other hand if \( \alpha = 0 \) and \( \lambda > 0 \) then the expected risk premium is proportional to the conditional variance (s=1) or to the conditional standard deviation (s=1/2) of stock returns. If \( \lambda \neq 0 \) it is interpreted as direct evidence about time-varying risk premium that is induced by volatility. According to Nelson (1991) and French et al. (1987) there are more conditions, besides those mentioned before, that should be satisfied in order for theoretical model shown in equation (2) to hold. However, for purposes to analyse time-varying risk premium – changing volatility relationship, this model is enough valid.

In this paper both with-in- and out-of-sample analyses are carried out. The First is done because the best possible fit for model can be achieved and results can be compared with Stenius (1991) who found positive relation with Finnish data com-
posed (1949-1988). Out-of-sample analyse is done because as mentioned already we are interested about expected, not realised, so this is theoretically better alternative. French et al. (1987) argue that a positive relation between the expected risk premium and ex ante volatility will induce a negative one between the excess return and the unexpected change in volatility. This is tested with out-of-sample analyse and results can be compared with Balaban and Bayar (2005) findings with Finnish data.

2.2 ARCH models

Traditional econometric models are unable to explain number of typical features for financial data. Three of those features are treated here. First as Stenius (1991) points out evidence from stock markets usually indicate that returns have leptokurtic distributions rather than normal distribution. According to Watsham and Parramore (2002) one reason for this kind of distribution is example discontinuous trading which products periodic jumps in asset prices. Markets are not continuously open and information may arrive during this time, this may result a jump in asset prices, which in turn results larger negative or positive returns than one would expect if markets were continuously open. The result is a leptokurtic distribution with fat tails and excess peakedness. Second feature is volatility clustering. This means that large returns of either sign are expected to follow large returns and the same goes with small returns. Third features are leverage effects. As Watsham and Parramore (2002) mentions there is evidence that volatility raises more following a large price fall than after a price rise of same magnitude.

Engle (1982) introduced a model that can deal with the first and second issues mentioned above. This model is called autoregressive conditional heteroskedasticity (ARCH) model. Traditional models assume that variance of errors is constant, assumption about homoscedasticity. Situation where variance of errors is not constant is known as heteroscedasticity. Term autoregressive conditional heteroscedacticity is used about process where variance of the errors changes over time. The ARCH model allows the conditional variance of error term to change over time as a function of past errors leaving the unconditional variance constant. Bollerslev (1986) has generalised the ARCH model (GARCH) by allowing past conditional variances in the current conditional variance equation. Finally Engle et al. (1987) has extended the
ARCH to ARCH-in-mean (ARCH-M) model by allowing the conditional variance to enter into the conditional mean equation. Combining extensions together and presenting it in a similar form as Stenius (1991), the GARCH \((p, q)-\text{in-mean}\) model is given as:

\[
Y_t = X_t \beta + \lambda h_t + e_t, \quad e_t | \Omega_{t-1} \sim N(0, h_t)
\]

\[
h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i e^2_{t-i} + \sum_{i=1}^{p} \Phi_i h_{t-i}
\]

where equation (4) is for the conditional mean and equation (5) is for the conditional variance. \(X_t\) is a vector of explanatory variables and \(\beta\) is a vector of unknown parameters. As can be seen, the conditional variance \(h_t\) enters into conditional mean equation as an explanatory variable. Equation (5) is a function of a constant term \(\alpha_0\), the ARCH term \(e^2_{t-i}\) and the GARCH term \(h_{t-i}\). Orders of these terms are \(q\) and \(p\). The variance \(h_t\) of the error term \(e_t\) is obtained conditional on \(\Omega_{t-1}\), which represents the information set available at time \(t-1\). To ensure that variance is stationary and non-negative constrains are not violated should \(\alpha_0 \geq 0\), \(\alpha_i \geq 0\), \(\Phi_i \geq 0\) and \(\sum \alpha_i + \sum \Phi_i < 1\). The parameters \((\beta, \lambda, \alpha_0, \alpha_i\) and \(\Phi_i\)) of the model are estimated via maximum likelihood.

As Bollerslev (1986) points out by setting \(\beta=0, \lambda=0\) and \(p=q=0\) in equations (4) and (5) the process is simply white noise. If \(\lambda=0\) and \(p=q=0\) the standard linear model will be obtained. The original ARCH model can be obtained by setting \(p=0\) and \(\lambda=0\). Bollerslev (1986) argues that with ARCH \((q)\) model there is difficulties to set right lag structure and it will often lead to violation of the non-negativity constraints. The model introduced to overcome this problem, the GARCH \((p, q)\) model is obtained by setting only \(\lambda=0\). This model can be seen as infinite order ARCH model, which allows an infinite number of past squared errors to influence the current conditional variance. By letting \(\lambda\) to be non-zero GARCH \((p, q)\)-M specification as shown above is obtained. This model is capable to model time-varying risk premiums. Example equation (2) is well suited for this model. Day and Lewis (1992) argue that another advantage of this kind of models is that conditional variance is allowed to be a function of both ex-
ogenous and lagged dependent variables. This allows equation (5) to be further extended by adding regressors.

The Model above has still some drawbacks. First, it treats positive and negative volatility shocks symmetrically. This is because conditional variance in equation (5) is a function of squared lagged error terms and so signs of shocks are lost. Because of that, GARCH model is unable to account for possible leverage effects. Second problem with traditional GARCH are non-negativity constrains which are imposed to ensure that $h_t$ remains positive. This is because negative variance at any point would be meaningless. To overcome these two problems and third additional one, which concerns the interpretation of the persistence of shocks to conditional variance, has Nelson (1991) introduced the exponential GARCH model. The equation for conditional variance can be expressed as follows:

$$\ln(h_t) = \alpha_0 + \sum_{j=1}^{q} \beta_j \ln(h_{t-j}) + \sum_{i=1}^{p} \alpha_i \left| \frac{e_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{k=1}^{r} \gamma_k \frac{e_{t-k}}{\sqrt{h_{t-k}}}$$

where $\gamma_k$ is now the parameter and $r$ is the order for possible asymmetries. If $\gamma_k \neq 0$ the impact is asymmetric and the leverage effects can now be tested by the hypothesis that $\gamma_k < 0$. Because we are modelling $\ln(h_t)$ will conditional variance always be positive, even if parameters are negative. Artificial non-negativity constrains are thus not needed.

The Arguments above has demonstrated the ARCH models usefulness, flexibility and the special suitably for our purposes. In this paper we have chooses GARCH-M and EGARCH-M models to model time-varying risk premium. First model constitutes from equations (4) and (5). The latter model constitutes from equations (4) and (6). Finally, it is good to notice that Bollerslev et al. (1988) has extended the univariate model treated here to the multivariate generalization. This model can be used to model time-varying covariance/variance matrix. As Stenius (1991) points out this kind of

1 The Model specification presented here differs slightly from the original Nelsons specification. According to Eviews 5 user’s guide specifications will yield identical estimates, except for the intercept term, which will differ in a manner depending the order $p$ and the distributional assumption.
model makes example testing of CAPM model possible without having to use proxy variables as risk measures.

3. DATA AND ITS IMPLICATIONS FOR MODEL BUILDING

3.1 Data

The data used in this paper for market observations constitutes from daily, weekly and monthly values of the OMX Helsinki Cap –total return index (OMXHCAP).² The chosen index includes all shares listed in Helsinki and it is weight-restricted in a way that maximum weight of one company is limited to 10%. The OMXHCAP –total return index includes dividends. The sample period comprises 22.2.1995 to 28.2.2007. This period is chosen for two reasons. First, because current situation is in our interest, limiting our analyse to fairly fresh data should give us more detailed picture about current situation. Second, effects of recession in the early 1990s are smoothen with this choice. Monthly observations are based on the month’s last day’s value and weekly observations on every Wednesdays values. This procedure causes that first monthly and weekly observations do not start from the same day. Observations of Helsinki interbank offered rate (HELIBOR) are used as the risk-free interest rate for the period 22.2.1995 to 31.12.1998, after that Euro interbank offered rate (EURIBOR) comes available and it is then used. Excess returns are calculated as a difference between continuously compounded market returns and continuously compounded risk-free returns. True number of days between two observations is used when the risk-free return is calculated. The full excess return data set constitutes from 3136 daily, 628 weekly and 145 monthly observations.

A wide range of descriptive statistics for the used return series is presented in Table 1. These include mean, standard deviation, skewness, excess kurtosis, Bera-Jarque statistic and tests p-value, autocorrelation coefficients up to eight lags and the Ljung-Box statistic for eight lags of the autocorrelation function and tests p-value. Daily and weekly time-series exhibit significance negative skeweness, which indi-

² OMXHCAP-total return index and EURIBOR rate data are observed from DATASTREAM. The Lappeenranta University of Technology provides HELIBOR rate data.
icates that their distribution is more skewed to the left than normal distribution. All series show significantly positive excess kurtosis, series distributions are there for more peaked than normal distribution. The Bera-Jarque (BJ) statistic is distributed as chi-squared with 2 degrees of freedom. Test statistics are high for all series leading to the rejection of null hypothesis of normality in every sample. In fact only the monthly data with p-value 0.004 is even close to accept null hypothesis when 5% significance level is used. It can be concluded that series distributions are leptokurtic rather than normal. This is consistent with the previous findings of Maukonen (2002) on Finnish data.

Table 1. Descriptive statistics for the daily, weekly and monthly time series

Descriptive statistics for excess returns on the OMXHCAP index from 22.2.1995 through 31.12.1998. Sample sizes are 3136 daily, 628 weekly and 145 monthly observations T. a

<table>
<thead>
<tr>
<th>Return Interval</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>Bera-Jarque statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily</td>
<td>0.0004</td>
<td>0.0118</td>
<td>-0.352*</td>
<td>3.393*</td>
<td>1569.119*</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.0021</td>
<td>0.0284</td>
<td>-0.605*</td>
<td>2.782*</td>
<td>240.830*</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Monthly</td>
<td>0.0089</td>
<td>0.0598</td>
<td>-0.293</td>
<td>1.228*</td>
<td>11.186*</td>
<td>0.004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Return Interval</th>
<th>Autocorrelation at lag</th>
<th>Std. error</th>
<th>Q(8) b</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily</td>
<td>0.068* 0.023 0.022 0.016 -0.024 -0.042* -0.001 0.018 26.149*</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly</td>
<td>-0.002 0.028 0.099* -0.054 0.110* 0.086* -0.039 0.027 0.040 22.421*</td>
<td>0.004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly</td>
<td>0.197* 0.019 -0.012 -0.070 -0.046 0.010 0.024 0.046 0.083 7.305</td>
<td>0.504</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a The standard errors of the estimate for the sample statistics are in parentheses. Usual approximations for standard errors are used (For autocorrelation coefficients 1/√T, for skewness √(6/T) and for kurtosis √(24/T)).

b Q(8) is the Ljung-Box statistic for eight lags of the autocorrelation function

Ljung-Box statistic Q is employed to inspect the null hypothesis of independence in time domain. Q is distributed as chi-squared with the n (number of autocorrelations) degrees of freedom. The null is joint hypothesis that all first- to eight-order autocorrelation coefficients are zero. If 5% significance level is used only in the case of Monthly data the null is accepted, for weekly and daily data the null is rejected. Daily and monthly data exhibit strong positive first-lag autocorrelation. At later lags more
non-zero autocorrelation coefficients are found from daily and weekly data. As Akgi- ray (1989) points out, in the situation where returns are not normally distributed, may using approximations of standard errors, example \(1/\sqrt{T}\) for autocorrelation coefficients, be an understatement of the standard error. Despite this fact, values reported here are generally sufficiently large to give confidence for conclusions.

3.2 ARCH effects

Before estimating final GARCH-M and EGARCH-M models it is sensible first to make sure that chosen class of models is appropriate for the data. Testing method for ARCH effects is based on Engle (1982). To test the presence of ARCH effects in residuals, first OLS regression is run and resulting error terms are saved. First state is shown in equation (7).

\[
(7) \quad r_t = \alpha + \epsilon_t, \quad \epsilon \sim N(0, h)
\]

where \(r_t\) is excess return, \(\alpha\) is constant and error term \(\epsilon_t\) is thought to be normally distributed.

Second resulted estimated error terms are squared and regressed on \(q\) own lags. Second state is shown in equation (8).

\[
(8) \quad \epsilon_t^2 = \gamma_0 + \sum_{i=1}^{q} \gamma_i \epsilon_{t-i}^2 + v_t
\]

where \(\epsilon_t^2\) are squared estimated error terms resulted from equation (7), \(\gamma_0\) is constant, \(\gamma_i\) are autocorrelation coefficients and \(v_t\) is now models error term. Value of the Lagrange multiplier test statistic is given as \(TR^2\), where \(T\) is the number of observations and \(R^2\) is obtained coefficient of determination from equation (8). Statistic is distributed as \(\chi^2(q)\), where \(q\) is the number of tested lags. The null hypothesis is joint hypothesis that all first- to \(q^{th}\)-order autocorrelation coefficients are zero. Results from the test are shown in Table 2. Results show clear evidence of an ARCH effect in the daily and weakly excess returns. For them, ARCH types of models are clearly well suited. However, for monthly data the evidence of heteroscedasticity is weaker.
Table 2. ARCH effect test

<table>
<thead>
<tr>
<th>Q</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>193.409**</td>
<td>257.098**</td>
<td>293.018**</td>
<td>305.465**</td>
<td>317.042**</td>
<td>318.153**</td>
<td>328.214**</td>
</tr>
<tr>
<td></td>
<td>Weekly</td>
<td>6.513*</td>
<td>20.459**</td>
<td>22.139**</td>
<td>27.152**</td>
<td>27.712**</td>
<td>27.688**</td>
<td>52.233**</td>
</tr>
</tbody>
</table>

$\chi^2$ (q) 1%-critical values for 1…q are 6.653, 9.210, 11.345, 13.277, 15.086, 16.812, 18.475, and 20.090. $\chi^2$ (1) 5%-critical value is 3.841. ** and * denotes significance at least at the levels of 1% and 5%, respectively.

This leads us to expect that when final models are estimated, monthly data show insignificance ARCH and GARCH terms.

Overall, findings are consistent with the previous findings of Maukonen (2002) on Finnish data. As Stenius (1991) mentions, the lack of normality in the data may yield biased results. Again, values reported here are generally sufficiently large to give confidence for conclusions. To determine whether an asymmetric model like EGARCH is required for the given data is not front-tested. According to Brooks (2002) there exists a set of tests for this, known as sign and size bias tests, but in this paper justifying arguments rely on theory and earlier empirical studies.

3.2 Model building

Generally linear dependence in return series can be incorporated to various reasons. To give few examples, the presence of a common market factor, thin trading and calendar day effects, only mentioning few. Non-linear dependence may be explained by the changing variances. Variance changes, on the other hand, may be explained by the rate of information arrivals, level of trading activity and corporate leverage decisions, which tend to affect the level of stock price. (Akgiray (1989)) For example Lamoureux and Lastrapes (1990) used daily trading volume as a proxy for information arrival time and showed that it has significant explanatory power to the variance of daily returns. They also showed that this often leads ARCH effects to tend to disappear. This specific fact is overlooked here, but it is good to keep in mind that we use ARCH approach for modelling the conditional variance, so we are assuming that lagged squared residuals contribute information we need.
According to Akgiray (1989) any realistic daily model must recognise that time series of returns exhibit significant first-lag autocorrelation. As mentioned before our data is not an exception. To overcome this we can add an autoregressive (AR) term or moving average (MA) term into the mean equation. We have chose to use the AR (1) term. Because monthly data has also significant first-lag autocorrelation it is correct way to do the same modifications. Weekly data has no need for this treatment, so AR term is not included. Finally, according to Nelson (1991) there is no significant difference between the choices of MA or AR term for these purposes.

4. EMPIRICAL RESULTS

4.1 Model estimates

The general form of the conditional mean equation for the excess return to be estimated is:

\[ r_t = \delta_0 + \delta_1 r_{t-1} + \lambda h_t^s + e_t, \quad e_t | \Omega_{t-1} \sim N(0,h_t) \quad s = \frac{1}{2}, 1 \]

where \( r_t \) is excess return at time \( t \), \( \delta_0 \) is constant, \( r_{t-1} \) is first order autoregressive term, \( \delta_1 \) is AR (1) parameter (for weekly data this is set to zero), \( \lambda \) is parameter for market price of risk, \( h_t \) is the conditional variance, parameter \( s \) determines whether the conditional mean return is a function of the standard deviation or the variance. Error term \( e_t \) is thought to be conditionally normally distributed. The conditional variance \( h_t \) is given by equations (10) and (10’) depending model.

\[ h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 + \sum_{j=1}^{p} \beta_i h_{t-j} \]

\[ \ln(h_t) = \alpha_0 + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) + \sum_{i=1}^{q} \alpha_i \left| \frac{e_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{k=1}^{r} \gamma_k \frac{e_{t-k}}{\sqrt{h_{t-k}}} \]
where $\alpha_0$ is constant and $\alpha_i, \beta_j, \gamma_k$ are other model parameters. The parameters $\theta \equiv (\delta_0, \delta_1, \lambda, \alpha_0, \alpha_i, \beta_j, \gamma_k)$, depending model, are estimated simultaneously using maximum likelihood. Numerical maximization of the log-likelihood function $L(\theta)$ is done with Eview5 program using Berndt, Hall, Hall and Hausman (BHHH) algorithm. According to Brooks (2002) in the case when conditional normality assumption does not hold the parameter estimates will still be consistent if equations for mean and variance are correctly specified, however usual standard error estimates will be inappropriate. To make sure that possible non-normality does not make results biased, heteroskedasticity consistent covariance matrix estimator of Bollerslev-Woolridge is used for reported results. It should be noted that in the first stage, when all series are estimated, orders of models, GARCH-M (1,1) and (1,1,1) for EGARCH-M, was chosen in the basis of Maukkonen (2002) who finds (1,1) best for Finnish data and as Brooks (2002) mentions this is empirically found to be usually sufficient enough. Although, it would be possible to compare $L(\theta)$ and AIC values between different combinations, choice is done to keep results treatable. After diagnostic checking, most promising models are elaborated if needed. OLS regression results were used as starting coefficient values for BHHH.

### 4.2 With-in-sample results

In this section all observations are used to estimate models. Results from GARCH-M models are reported in Table 3 and results from EGARCH-M in Table 4. First, $L(\theta)$ and AIC values from standard model shown in equation (7) with or without AR(1) was obtained and concluded that models shown in Table 3 and 4 are superior in daily and weekly series, but on monthly data only slightly better values are obtained if conditional variance is allowed.

For the daily and weekly series all estimated GARCH-M model parameters are statistically significance at least at the level of 5%, except $\lambda$ in both and $\alpha_0$ in weekly data. Coefficient $\delta_1$ is significance for daily at 1%-level. This is just what was expected from AR (1) term in daily series. For daily series it should take account the positive first-order autocorrelation in the returns to portfolios of stocks induced by non-synchronous trading. For monthly series same interpretation is not so valid and
Table 3. GARCH-M (1,1) model results for the excess returns on the OMXHCAP index\textsuperscript{a}

Results from daily, weekly and monthly series. GARCH-M (1,1) model equations for conditional mean and for conditional variance are (9) \( r_t = \delta_0 + \delta_1 r_{t-1} + \lambda h_{t-1} + \epsilon_t \) and (10) \( h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta h_{t-1} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Daily</th>
<th>Daily\textsuperscript{c}</th>
<th>Weekly</th>
<th>Weekly\textsuperscript{c}</th>
<th>Monthly</th>
<th>Monthly\textsuperscript{c}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_0 )</td>
<td>0.002**</td>
<td>0.001**</td>
<td>0.007*</td>
<td>0.005**</td>
<td>0.064</td>
<td>0.035*</td>
</tr>
<tr>
<td></td>
<td>(2.741)</td>
<td>(4.099)</td>
<td>(2.290)</td>
<td>(3.264)</td>
<td>(1.695)</td>
<td>(2.045)</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>0.083**</td>
<td>0.084**</td>
<td>0.159</td>
<td>0.159</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(4.203)</td>
<td>(4.224)</td>
<td>(1.701)</td>
<td>(1.684)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda )</td>
<td>-0.071</td>
<td>-2.603</td>
<td>-0.147</td>
<td>-2.308</td>
<td>-0.967</td>
<td>-7.843</td>
</tr>
<tr>
<td></td>
<td>(-1.182)</td>
<td>(-1.066)</td>
<td>(-1.104)</td>
<td>(-0.998)</td>
<td>(-1.330)</td>
<td>(-1.267)</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>0.000**</td>
<td>0.000**</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(3.547)</td>
<td>(4.859)</td>
<td>(1.667)</td>
<td>(1.673)</td>
<td>(0.648)</td>
<td>(0.626)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.090**</td>
<td>0.090**</td>
<td>0.115**</td>
<td>0.115**</td>
<td>0.049</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(6.542)</td>
<td>(6.701)</td>
<td>(3.361)</td>
<td>(3.384)</td>
<td>(1.809)</td>
<td>(1.860)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.897**</td>
<td>0.897**</td>
<td>0.865**</td>
<td>0.865**</td>
<td>0.906**</td>
<td>0.908**</td>
</tr>
<tr>
<td></td>
<td>(63.648)</td>
<td>(69.914)</td>
<td>(22.489)</td>
<td>(22.729)</td>
<td>(12.418)</td>
<td>(12.898)</td>
</tr>
<tr>
<td>Log L</td>
<td>9871.635</td>
<td>9871.493</td>
<td>1401.980</td>
<td>1401.855</td>
<td>207.303</td>
<td>207.219</td>
</tr>
</tbody>
</table>

\textsuperscript{a} The z-statistics in parentheses reflect standard errors computed using robust inference procedures developed by Bollerslev and Wooldridge. ** and * denotes significance at least at the levels of 1% and 5%, respectively.

\textsuperscript{b} Return interval with index \( s \) on name stands for equation (9) with \( s=1 \) and without index with \( s=1/2 \).

reason for autocorrelation obtained earlier is probably some other phenomena that is now partly obtained by the rest of the model. For all series there is clearly strong ARCH effect in the return generating process, all \( \beta \) are significance at 1%-level and \( \alpha_1 \) at 1%-level, except monthly \( \alpha_1 \). This is in line with expectations that monthly data have no ARCH effects. Results show that the coefficient \( \lambda \) is not statistically significantly different from zero for any series. Further, all estimated values for \( \lambda \) are negative. Because \( \lambda =0 \) we have to conclude that although ARCH effects are present, time-varying risk premium induced by volatility is not, at least measured in this way. This contrast with the significant positive relation between returns and conditional variance found by Stenius (1991) using GARCH-M model and monthly Finnish data consist of years 1949-1988 and French \textit{et al.} (1987) with US data. Stationary conditions \( \alpha_1 + \beta <1 \) are satisfied for all series. The fact that all \( \alpha_1 + \beta \) are close to one indi-
cates that volatility shocks are highly persistent. $L(\theta)$ and AIC values between standard deviation ($s=1/2$) and variance ($s=1$) specifications are essentially same for all series, although standard deviation does slightly better for all series. Now conclusion can be drawn between them. One might suspect that the best-fit value for $s$ would be somewhere between these two.

Same diagnostic checking is done as Akgiray (1989). Because standard deviation specifications do slightly better testing values reported here is from these results, variance results are practically identical. If equations are correctly specified ARCH effects in standardized residuals should be disappeared. This is tested with Lagrange multiplier test. For weekly series no ARCH effects are found and for daily series there is still some in first lag and for monthly situation is same as before and no ARCH effects are present. Overall one can conclude that results indicate that ARCH effects are practically almost disappeared. Excess kurtosis in standardized residuals has reduced to 1.16724 and 1.87783 for daily and weekly series, when skewness has stayed virtually unchanged. Bera-Jargue statics are 242.0597 and 149.6487 for daily and weekly data. So although there are large improvements, leptokurticitiy is still present. For monthly data no improvements are obtained and together with results reported earlier, one can conclude that model does not fit for monthly data. One reason for this is probably low amount of observations.

Results from EGARCH-M model (Table 4.) show that for the daily series all estimated model parameters are statistically significance at least at the level of 1%, except $\lambda$ which is not significance at any tested level. Asymmetry coefficient $\gamma_1$ is significance and $\gamma_1<0$, which indicates that hypothesised leverage effects are present. In weekly data all parameters are statistically significance at least at the level of 5%, except $\lambda$ and $\gamma_1$. For the monthly data, not a single one parameter is significance. Even the conditional variance effect ($\beta$) is disappeared. This may indicate that autocorrelation shown earlier and significance $\beta$ in GARCH-M model is now taken account partly by asymmetry effects. Again, conclusion is that monthly data does not fit for tested model. For daily and weekly series there is clearly strong ARCH effect in the return generating process. As before, the coefficient $\lambda$ is not statistically significantly different from zero for any series and all estimated values for $\lambda$ are negative.
Table 4. EGARCH-M (1,1,1) model results for the excess returns on the
OMXHCAP index^a

Results from daily, weekly and monthly series. EGARCH-M (1,1,1) model equations for conditional
mean and for conditional variance are

\[
(9) \quad r_t = \delta_0 + \delta_1 r_{t-1} + \lambda h_t + e_t \quad \text{and} \quad (10') \quad \ln(h_t) = \alpha_0 + \beta \ln(h_{t-1}) + \alpha_1 \frac{e_{t-1}}{\sqrt{h_{t-1}}} + \gamma_1 \frac{e_{t-1}}{\sqrt{h_{t-1}}}
\]

Return interval^b

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Daily</th>
<th>Daily^2</th>
<th>Weekly</th>
<th>Weekly^2</th>
<th>Monthly</th>
<th>Monthly^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_0)</td>
<td>0.002** (2.718)</td>
<td>0.001** (3.513)</td>
<td>0.009** (2.679)</td>
<td>0.006** (3.352)</td>
<td>0.051 (0.856)</td>
<td>0.029 (1.191)</td>
</tr>
<tr>
<td>(\delta_1)</td>
<td>0.076** (3.811)</td>
<td>0.076** (3.786)</td>
<td>0.100 (0.546)</td>
<td>0.099 (0.581)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\lambda)</td>
<td>-0.101 (-1.647)</td>
<td>-4.148 (-1.557)</td>
<td>-0.240 (-1.707)</td>
<td>-4.097 (-1.620)</td>
<td>-0.712 (-0.653)</td>
<td>-5.315 (-0.677)</td>
</tr>
<tr>
<td>(\alpha_0)</td>
<td>-0.321** (-5.508)</td>
<td>-0.321** (-5.540)</td>
<td>-0.573* (-2.480)</td>
<td>-0.569* (-2.511)</td>
<td>-3.011 (-1.667)</td>
<td>-3.062 (-1.682)</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>0.175** (6.698)</td>
<td>0.175** (6.699)</td>
<td>0.239** (3.655)</td>
<td>0.238** (3.696)</td>
<td>0.226 (0.955)</td>
<td>0.235 (0.935)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.979** (180.474)</td>
<td>0.979** (182.274)</td>
<td>0.947** (35.121)</td>
<td>0.947** (35.833)</td>
<td>0.506 (1.554)</td>
<td>0.498 (1.525)</td>
</tr>
<tr>
<td>(\gamma_1)</td>
<td>-0.053** (-2.834)</td>
<td>-0.053** (-2.819)</td>
<td>-0.074 (-1.707)</td>
<td>-0.072 (-1.689)</td>
<td>-0.184 (-1.040)</td>
<td>-0.184 (-1.031)</td>
</tr>
<tr>
<td>Log L</td>
<td>9879.373</td>
<td>9879.179</td>
<td>1405.992</td>
<td>1405.870</td>
<td>207.737</td>
<td>207.675</td>
</tr>
</tbody>
</table>

^a The z-statistics in parentheses reflect standard errors computed using robust inference procedures
developed by Bollerslev and Wooldridge. ** and * denotes significance at least at the levels of 1% and
5%, respectively.
^b Return interval with index 2 on name stands for equation (9) with s=1 and without index with s=1/2.

The same conclusion has to be done that time-varying risk premium measured as \(\lambda\)
is not present. Our findings of negative and insignificant coefficients agree with findings of Nelson (1991) using same kind of model and US data. Now conclusion can be drawn between standard deviation and variance specification from \(L(\theta)\) and AIC
values. Again, standard deviation does slightly better for all series.

Diagnostic checking reported is again done with standard deviation specifications.
Variance specifications gave almost identical results. ARCH effects in standardized
residuals for weekly data has disappeared, however for daily data there is statistically
significance effects left in lags 1, 2 and 3. Excess kurtosis has reduced to 1.267835 and 1.83173 for daily and weekly series, respectively. Bera-Jargue statics are 263.2698 and 139.6741 for daily and weekly data. No improvements in skewness are obtained. Finally, no ARCH effects or improvements in third or fourth moments for Monthly data are appearing. Based on results obtained from monthly data, it is left out from the out-of-sample testing.

In the both models there are ARCH effects left in the daily series. Because of that, models are extended in away that this will be captured. Extension is done on the bases of $L(\theta)$, AIC values and diagnostic checking about ARCH effects. If only one term is added the best result is obtained with extra ARCH term in both cases. Models are now GARCH-M (1,2) and EGARCH-M (1,2,1) models. The ARCH effect in standardized residuals disappears with this treatment. Parameters added are tried to choose in away to keep them statistically significance if possible. Neither of the added extra parameters is statistically significance. Extra parameters has negative sign, but analyse of conditional variances show that non-negativity constrains are not violated To make sure that higher order model is statistically better than model estimated before is likelihood ratio (LR) test performed. LR test static is defined as $-2\{L(\theta_r) - L(\theta_u)\}$ and in our case it is distributed as $\chi_1^2$. $L(\theta_r)$ and $L(\theta_u)$ are the maximum log-likelihood function values of the restricted and unrestricted models. With LR static 5.668 and 3.908 for GARCH-M and GARCH-M^2 models and 5.444 and 5.324 for EGARCH-M and EGARCH-M^2 models, the null that additional set of regressors is not jointly significant can be rejected at 5%-level. It is concluded that (1,2) and (1,2,1) are better orders for daily series. Although estimated values for parameters change little, statistical conclusions are same as presented above, so statistical results are not treated further here. Statistical results from these models can be obtained from Table 5 from Appendix 1.

Performance of estimated GARCH-M (1,2) and EGARCH-M (1,2,1) models can be observed graphically in Figure 1 where daily models performance in risk premium modelling is shown in monthly level. As can be seen both models closely track each other’s. Models also show clear variability in risk premium if compared to simply
average. EGARCH-M model clearly captures some leverage effects, which can be seen steeper downward movements than GARCH-M models corresponding.

If actual observed risk premium is taken in the same figure will scale of y-axes change dramatically as shown in Figure 2. In this figure only GARCH-M models performance is shown because of scaling effect would make it hard to separate two models. As can be seen from Figure 2 actual risk premium measured as excess return on market is highly variable. Clearly GARCH kind of models are able to partly model this, but because observed magnitude of variability is so high they are unable to track so high variability. This may be a one reason for observed insignificant $\lambda$. Figure 2 also shows clearly that market risk premium is everything but constant over time.
Figure 2. Actual monthly risk premium and the estimates of the GARCH-M (1,2) and historical average computed from daily data over the whole sample period 1995-2007.

4.3 Out-of-sample results

We are analysing relation between expected returns and expected volatility so in reality we can only observe observations realised so far and use them for modelling purposes. In the previous section we used the whole sample and this is clearly unrealistic assumption. To get more realistic picture we have to analyse out-of-sample forecasts generated from models used in previous section. Eviews5 program can be used for forecasting purposes. There is two options to choose dynamic and static forecast. Unfortunately, neither of them does not re-estimate model parameters automatically after every step. We adapt a similar rolling GARCH estimation procedure as Day and Lewis (1992). Because it is necessary to re-estimate model after every step we limit our analyse to weekly series because of time economise reasons. Also because based on with-in-sample results, specifications with standard deviation in conditional mean are seem to be most promising the analyse is limited for those
Because it is clearly the latest information available, which is used to form expectations about variance our estimation procedure should give most realistic results. Time series of one-step ahead forecasts is generated using following methodology. We use a constant sample size of 428 weekly observations to estimate a conditional volatility forecast for each week \( t+1 \). In every step the week \( t-428 \) return is deleted and the week \( t \) return is added and GARCH-M (1,1) is re-estimated. After every re-estimation a step ahead forecast for week \( t+1 \) is generated. This is repeated until a sample of 200 weekly step-ahead forecasts is generated. It is good to notice that although this time series of weekly forecasts consist of partly same period that examined in previous section it is now based entirely on the history of returns prior to the forecast period.

French et al. (1987) argue that a positive relation between ex ante volatility and expected risk premium will induce a negative one between the unexpected change in volatility and the excess return. The argumentation they give is that for example in the situation where this weeks standard deviation is larger than expected the predicted standard deviations will be revised upward for future periods. Now if the risk premium is positively related to expected standard deviation will discount rates for future cash flows be higher. If the cash flows stay at the same level will higher discount rate reduce their present value and this will lead stock price to reduce. Therefore both relations are analysed here. Measure of the unexpected volatility is simply the difference between observed volatility and forecast volatility for each week. Observed variance \( \sigma^2 \) is defined as same way as Day and Lewis (1992). It is simply the square of the weekly return on the index and the observed volatility \( \sigma \) is its square root. To analyse these relationships same kind of regressions are run as French et al. (1987). First the relation between expected volatility and excess returns is investigated with following regression:

\[
(11) \quad r_t = \alpha + \beta_f \sigma_{f,t} + e_t
\]

where \( \sigma_{f,t} \) is one step-ahead forecast standard deviation produced by GARCH-M (1,1) model at time t-1 and \( r_t \) is excess return. Again, if \( \beta_f = 0 \) then risk premium is
unrelated to expected volatility and if $\beta_f > 0$ and $\alpha = 0$ then the risk premium is proportional to the predicted standard deviation. The relationship between unexpected volatility and excess returns is analysed as:

\[(12) \quad r_i = \alpha + \beta_u \sigma_{u,t} + e_i\]

where unexpected volatility $\sigma_{u,t}$ is simply $(\sigma_{a,t} - \sigma_{f,t})$ where $\sigma_{a,t}$ is the actual standard deviation. Finally combining both components together the following regression is run:

\[(13) \quad r_i = \alpha + \beta_f \sigma_{f,t} + \beta_u \sigma_{u,t} + e_w\]

According to French et al. (1987) this final regression has additional improvement because more of the excess returns are explained, will standard errors of the regression coefficients be smaller. Results from these regressions are reported in Table 5.

Table 5. Least squares regression of weekly OMXHCAP index excess returns against the predictable and unpredictable components of the standard deviations of market returns.\(^a\)

<table>
<thead>
<tr>
<th>Equation</th>
<th>$\alpha$</th>
<th>$\beta_f$</th>
<th>$\beta_u$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_i = \alpha + \beta_f \sigma_{f,t} + e_i$</td>
<td>0.005 (0.976)</td>
<td>-0.017 (-0.071)</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>$r_i = \alpha + \beta_u \sigma_{u,t} + e_i$</td>
<td>0.004 (1.519)</td>
<td>-0.060 (-0.217)</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>$r_i = \alpha + \beta_f \sigma_{f,t} + \beta_u \sigma_{u,t} + e_w$</td>
<td>0.005 (1.012)</td>
<td>-0.056 (-0.176)</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) The t-statistics in parentheses are computed using the Newey-West heteroscedasticity and autocorrelation consistent standard errors.

Neither one of the estimated coefficients is statistically significant. Further, the success of the regressions measured with R-squared statistic is poor. Multicollinearity is not a problem in the regression with two explanatory variables. Our findings of negative and insignificant coefficients partly agree with findings of Balaban and Bayar (2005) using same kind of model and Finnish data consist of years 1987-1997. They
find significant negative coefficient $\beta$, for expected volatility and significant intercept terms but observe poor fit for the model as we do.

### 4.4 Further interpretation of results

There are clearly ARCH effects in stock market excess return generating process. However, conditional variance or conditional standard deviation does not enter into conditional mean with significant effect. Hypothesised influence of changing stock market volatility to changing market risk premium has to be therefore abandoned and conclude that return-volatility relationship is not sufficiently good to describe time-varying risk-premium. As shown risk-premium is clearly not constant, but some other influences than that mentioned above is affecting its behaviour. Poterba and Summers (1986) has suggested that volatility shocks can have only a small impact on stock market prices. Their results lead them to doubt that volatility fluctuations and the movements induced by those fluctuations in equity risk premium could explain much of the variation in the stock market's level. After detailed analyse with Finnish stock market data we have to join to their opinion.

Stenius (1991) and Balaban and Bayar (2005) analyse in their papers market returns not market excess returns. This clearly can cause differences that were mainly observed if our results are compared to Stenius (1991) results. The second likely cause is that Stenius (1991) uses longer period of data, which starts as early as 1949. Because our results are mostly similar to those that Balaban and Bayar (2005) obtained using more recently data, can this be interpret as indirect evidence about resulted changes in variability and return generating process of Finnish stock market since Stenius (1991) study.
5. CONCLUSION

In this paper, I have studied the relationship between the Finnish stock market excess returns and its own conditional volatility to see if there exists a time-varying risk premium which variation in time is driven by the risk measured as its own conditional volatility. OMXHCAP-index is used as a market proxy for Finnish stock market and analysed data consists of years 1995-2007. Both variance and standard deviation specifications are analysed as conditional volatility and daily, weekly and monthly return intervals are all analysed. Models used to analyse relationship are generalised autoregressive conditional heteroskedasticity (GARCH)-in-mean model and exponential (GARCH)-in-mean model. With-in-sample analyse is conducted using all return intervals and different model specifications. Out-of-sample analyse is conducted with weekly returns and using standard deviation as volatility. Forecasts in this analyse are derived from GARCH-M model using rolling and re-estimating procedure to conduct one-step ahead forecasts. Forecasts from expected volatility are then regressed against observed excess returns. Unexpected changes in volatility are regressed in a same way and finally both components are regressed against returns. Used data is found to exhibit large excess kurtosis, negative skewness and autocorrelation in first lags for daily and monthly series. Clear ARCH effects are found from daily and weekly series. To eliminate non-synchronous trading effect first order autoregressive term is added in the conditional mean equation for daily series.

With-in-sample results show that although there are significant ARCH effects in excess return generating process, time-varying risk premium in the sense of the increased expected rate of excess return required in response to an increase in the predictable volatility of the returns is not present. Results from EGARCH-M model show that also leverage effects are present in Finnish stock markets. Result also show that ARCH models are better alternatives than standard linear model for modelling excess return generating process. For the daily and weekly series excess kurtosis in standardized residuals is reduced significantly and also ARCH effects are disappeared when these kinds of models are used. For the monthly series not much improvements are obtained. Out-of-sample results also show that there is no relationship between expected volatility and excess returns and the same is true with
unexpected changes in volatility. Giving these results hypothesised influence of changing stock market volatility to changing market risk premium has to be abandoned and conclude that return-volatility relationship is not sufficiently good to describe time-varying risk-premium in Finnish stock markets. Because this means that expected standard deviation or variance of returns is maybe not the appropriate measure of risk, developing some other risk measures like for example semi variance may be necessary.

Finally, it would be interesting to further investigate relations between risk and returns using some other definitions of risk and different econometric models. For example use of multivariate GARCH-M models would maybe give more accurate results. Use of longer time-series could also change results. Generating step ahead forecasts using rolling and re-estimating procedure from daily series would also be more detailed alternative for those weekly forecasts used in this paper in the out-of-sample section. Unfortunately Eviews5 programs forecasting procedures does not make it possible to re-estimate model after every step so for time economic reasons we had to chose weekly interval to our out-of-sample analyse.
REFERENCES


APPENDIX 1: Results from GARCH-M (1,2) and EGARCH-M (1,2,1) models.

Table 5. GARCH-M (1,2) and EGARCH-M (1,2,1) models for daily series

\[ r_t = \delta_0 + \delta_1 r_{t-1} + \lambda h^{s}_{t-1} + e_t \quad s = \frac{1}{2} \]  
\[ h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \beta h_{t-1} \]  
\[ \ln(h_t) = \alpha_0 + \beta \ln(h_{t-1}) + \alpha_1 \frac{e_{t-1}}{\sqrt{h_{t-1}}} + \alpha_2 \frac{e_{t-2}}{\sqrt{h_{t-2}}} + \gamma_1 \frac{e_{t-1}}{\sqrt{h_{t-1}}} \]  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(10)</th>
<th>(10)^2</th>
<th>(10)'</th>
<th>(10)'^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_0 )</td>
<td>0.002** (2.806)</td>
<td>0.001** (3.161)</td>
<td>0.002** (2.858)</td>
<td>0.001** (3.696)</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>0.082** (4.212)</td>
<td>0.084** (4.323)</td>
<td>0.076** (3.859)</td>
<td>0.077** (3.882)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>-0.072 (-1.208)</td>
<td>0.574 (0.234)</td>
<td>-0.102 (-1.683)</td>
<td>-4.335 (-1.636)</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>0.000** (3.226)</td>
<td>0.000** (3.491)</td>
<td>-0.284** (-5.288)</td>
<td>-0.285** (-5.327)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.131** (3.988)</td>
<td>0.129** (3.941)</td>
<td>0.242** (4.142)</td>
<td>0.241** (4.112)</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>-0.055 (-1.632)</td>
<td>-0.053 (-1.572)</td>
<td>-0.082 (-1.480)</td>
<td>-0.082 (-1.462)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.914** (67.610)</td>
<td>0.913** (68.503)</td>
<td>0.982** (198.489)</td>
<td>0.982** (199.864)</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>-0.049** (-2.773)</td>
<td>-0.049** (-2.777)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log L</td>
<td>9874.469</td>
<td>9873.447</td>
<td>9882.095</td>
<td>9881.841</td>
</tr>
<tr>
<td>AIC</td>
<td>-6.295</td>
<td>-6.294</td>
<td>-6.299</td>
<td>-6.299</td>
</tr>
</tbody>
</table>

\( a \) The z-statistics in parentheses reflect standard errors computed using robust inference procedures developed by Bollerslev and Wooldridge. ** and * denotes significance at least at the levels of 1% and 5%, respectively.

\( b \) Variance specification with index 2 on name stands for equation (9) with \( s=1 \) and without index with \( s=1/2 \).