Hannu Sarén

ANALYSIS OF THE VOLTAGE SOURCE INVERTER WITH SMALL DC-LINK CAPACITOR

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In electric drives, frequency converters are used to generate for the electric motor the AC voltage with variable frequency and amplitude. When considering the annual sale of drives in values of money and units sold, the use of low-performance drives appears to be predominant. These drives have to be very cost effective to manufacture and use, while they are also expected to fulfill the harmonic distortion standards. One of the objectives has also been to extend the lifetime of the frequency converter.

In a traditional frequency converter, a relatively large electrolytic DC-link capacitor is used. Electrolytic capacitors are large, heavy and rather expensive components. In many cases, the lifetime of the electrolytic capacitor is the main factor limiting the lifetime of the frequency converter. To overcome the problem, the electrolytic capacitor is replaced with a metallized polypropylene film capacitor (MPPF). The MPPF has improved properties when compared to the electrolytic capacitor.

By replacing the electrolytic capacitor with a film capacitor the energy storage of the DC-link will be decreased. Thus, the instantaneous power supplied to the motor correlates with the instantaneous power taken from the network. This yields a continuous DC-link current fed by the diode rectifier bridge. As a consequence, the line current harmonics clearly decrease. Because of the decreased energy storage, the DC-link voltage fluctuates. This sets additional conditions to the
controllers of the frequency converter to compensate the fluctuation from the supplied motor phase voltages.

In this work three-phase and single-phase frequency converters with small DC-link capacitor are analyzed. The evaluation is obtained with simulations and laboratory measurements.

Keywords: frequency converter, voltage source inverter, electrolytic capacitor, film capacitor, pulse width modulation, overmodulation, space vector modulation, differential space vector modulation

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ACKNOWLEDGEMENTS

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I would like to express my gratitude to my supervisor, Professor Olli Pyrhönen for his valuable comments, guidance and very much needed encouragements. I would also like to thank Professor Juha Pyrhönen for maneuvering me at the beginning of my studies into the exciting fields of electrical engineering.

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Vantaa, 1st of October 2005            Hannu Sarén
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, C, E</td>
<td>system matrixes of the state-space model</td>
</tr>
<tr>
<td>a</td>
<td>weighting factor</td>
</tr>
<tr>
<td>b</td>
<td>Fourier series constant, weighting factor</td>
</tr>
<tr>
<td>C</td>
<td>capacitance</td>
</tr>
<tr>
<td>c</td>
<td>space vector scaling constant, cosine amplitude in DFT</td>
</tr>
<tr>
<td>c₀</td>
<td>scaling constant of the zero-sequence component of the pace vector</td>
</tr>
<tr>
<td>D</td>
<td>duty cycle</td>
</tr>
<tr>
<td>d</td>
<td>direct component in synchronously rotating dq reference frame</td>
</tr>
<tr>
<td>e</td>
<td>electro motive force</td>
</tr>
<tr>
<td>f</td>
<td>function, frequency</td>
</tr>
<tr>
<td>Gₙ</td>
<td>resonance peak amplitude gain</td>
</tr>
<tr>
<td>h</td>
<td>amplitude of a staircase waveform</td>
</tr>
<tr>
<td>i</td>
<td>current</td>
</tr>
<tr>
<td>Iₙ</td>
<td>nominal RMS-value of the phase current</td>
</tr>
<tr>
<td>J</td>
<td>inertia</td>
</tr>
<tr>
<td>K</td>
<td>torsional spring constant</td>
</tr>
<tr>
<td>k</td>
<td>sample number of the frequency</td>
</tr>
<tr>
<td>L</td>
<td>inductance</td>
</tr>
<tr>
<td>L₁, L₂, L₃</td>
<td>supply grid phases, line phases</td>
</tr>
<tr>
<td>m</td>
<td>sector index</td>
</tr>
<tr>
<td>M</td>
<td>motor</td>
</tr>
<tr>
<td>M</td>
<td>modulation index</td>
</tr>
<tr>
<td>N</td>
<td>neutral phase</td>
</tr>
<tr>
<td>N</td>
<td>number of samples</td>
</tr>
<tr>
<td>n</td>
<td>Fourier series harmonic number</td>
</tr>
<tr>
<td>p</td>
<td>power, motor pole pair</td>
</tr>
<tr>
<td>P</td>
<td>steady-state power</td>
</tr>
<tr>
<td>q</td>
<td>quadrature component in synchronously rotating dq reference frame</td>
</tr>
<tr>
<td>R</td>
<td>resistance</td>
</tr>
<tr>
<td>S</td>
<td>power switch control signal</td>
</tr>
<tr>
<td>s</td>
<td>sine amplitude in DFT, apparent power</td>
</tr>
<tr>
<td>sw</td>
<td>switching function</td>
</tr>
</tbody>
</table>
\[ t \] time
\[ t_e \] electric torque
\[ t_l \] load torque
\[ t_{sh} \] shaft torque
\[ T \] time period of the staircase waveform
\[ T_s \] switching period of the modulator
\[ u \] voltage
\[ U, V, W \] output voltage phases
\[ U_n \] nominal RMS-value line-to-line voltage
\[ U \] RMS-value line voltage
\[ V \] active voltage vector
\[ Z \] impedance
\[ x \] phase variable of the general three-phase system
\[ X \] set of complex numbers
\[ x \] real axis component in xy reference frame
\[ x_0 \] zero sequence component
\[ y \] filter output
\[ y \] imaginary axis component in xy reference frame
\[ \alpha_h \] hold-angle
\[ \psi \] flux linkage
\[ \omega \] angular frequency
\[ \Omega_1 \] mechanical system resonance frequency
\[ \Omega_2 \] mechanical system anti resonance frequency
\[ \Delta \] change, variation
\[ \Delta \psi \] the change of flux linkage
\[ \theta \] angle
\[ \phi \] phase shift between variables
**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>act</td>
<td>actual</td>
</tr>
<tr>
<td>B</td>
<td>base value</td>
</tr>
<tr>
<td>d</td>
<td>direct component in synchronously rotating dq reference frame</td>
</tr>
<tr>
<td>C</td>
<td>capacitor</td>
</tr>
<tr>
<td>CA</td>
<td>Constant Amplitude</td>
</tr>
<tr>
<td>calc</td>
<td>calculated</td>
</tr>
<tr>
<td>CV</td>
<td>closest vector</td>
</tr>
<tr>
<td>c2g</td>
<td>current to grid</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>estim</td>
<td>estimated</td>
</tr>
<tr>
<td>f</td>
<td>fundamental component</td>
</tr>
<tr>
<td>fil</td>
<td>filter</td>
</tr>
<tr>
<td>FP</td>
<td>Flower Power</td>
</tr>
<tr>
<td>grid</td>
<td>supply grid, line</td>
</tr>
<tr>
<td>gf</td>
<td>grid/filter system</td>
</tr>
<tr>
<td>i</td>
<td>current</td>
</tr>
<tr>
<td>i</td>
<td>index, harmonic number</td>
</tr>
<tr>
<td>in</td>
<td>input</td>
</tr>
<tr>
<td>initial</td>
<td>initial</td>
</tr>
<tr>
<td>inv</td>
<td>inverter</td>
</tr>
<tr>
<td>l</td>
<td>load</td>
</tr>
<tr>
<td>lim</td>
<td>limited quantity</td>
</tr>
<tr>
<td>L1, L2, L3</td>
<td>supply grid phases, line phases</td>
</tr>
<tr>
<td>m</td>
<td>motor</td>
</tr>
<tr>
<td>m</td>
<td>sector index</td>
</tr>
<tr>
<td>max</td>
<td>maximum</td>
</tr>
<tr>
<td>mean</td>
<td>mean value</td>
</tr>
<tr>
<td>meas</td>
<td>measured variable</td>
</tr>
<tr>
<td>mech</td>
<td>mechanical</td>
</tr>
<tr>
<td>mod</td>
<td>modulated</td>
</tr>
<tr>
<td>motor</td>
<td>motor</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>nom</td>
<td>nominal</td>
</tr>
<tr>
<td>$n$</td>
<td>Fourier series harmonic number</td>
</tr>
<tr>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>OM</td>
<td>over-modulation</td>
</tr>
<tr>
<td>OMI</td>
<td>over-modulation I</td>
</tr>
<tr>
<td>OMII</td>
<td>over-modulation II</td>
</tr>
<tr>
<td>pu</td>
<td>per-unit</td>
</tr>
<tr>
<td>$q$</td>
<td>quadrature component in synchronously rotating dq reference frame</td>
</tr>
<tr>
<td>rec</td>
<td>rectifier</td>
</tr>
<tr>
<td>ref</td>
<td>reference value</td>
</tr>
<tr>
<td>res</td>
<td>resonance</td>
</tr>
<tr>
<td>RMS</td>
<td>root mean square, RMS</td>
</tr>
<tr>
<td>$s$</td>
<td>stator</td>
</tr>
<tr>
<td>$sh$</td>
<td>shaft</td>
</tr>
<tr>
<td>six-step</td>
<td>six-step modulation</td>
</tr>
<tr>
<td>sw</td>
<td>switching</td>
</tr>
<tr>
<td>$u$</td>
<td>voltage</td>
</tr>
<tr>
<td>$U, V, W$</td>
<td>output voltage phases</td>
</tr>
<tr>
<td>wobble</td>
<td>wobble</td>
</tr>
<tr>
<td>$x$</td>
<td>real axis component in xy reference frame</td>
</tr>
<tr>
<td>$y$</td>
<td>imaginary axis component in xy reference frame</td>
</tr>
<tr>
<td>0</td>
<td>initial value, zero-sequence component</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>difference</td>
</tr>
</tbody>
</table>

**Other notations**

- $\hat{x}$: peak value of $x$
- $\bar{x}$: complex conjugate of space vector
- $|x|$: absolute value
- $|\hat{x}|$: vector length
<table>
<thead>
<tr>
<th>Acronyms</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A/D</td>
<td>Analog to Digital</td>
</tr>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>CA</td>
<td>Constant Amplitude</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DPFC</td>
<td>Dynamic Power Factor Control</td>
</tr>
<tr>
<td>DSVPWM</td>
<td>Differential Space Vector Pulse Width Modulation</td>
</tr>
<tr>
<td>DTC</td>
<td>Direct Torque Control</td>
</tr>
<tr>
<td>EMC</td>
<td>Electromagnetic Compatibility</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive Force</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>IGBT</td>
<td>Insulated Gate Bipolar Transistor</td>
</tr>
<tr>
<td>MPPF</td>
<td>metallized polypropylene film capacitor</td>
</tr>
<tr>
<td>OM</td>
<td>over-modulation</td>
</tr>
<tr>
<td>OM I</td>
<td>over-modulation I</td>
</tr>
<tr>
<td>OM II</td>
<td>over-modulation II</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional Integral Derivative</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SVPWM</td>
<td>Space Vector Pulse Width Modulation</td>
</tr>
<tr>
<td>THD</td>
<td>Total Harmonic Distortion</td>
</tr>
<tr>
<td>VSI</td>
<td>Voltage Source Inverter</td>
</tr>
<tr>
<td>TTL</td>
<td>Transistor-Transistor Logic</td>
</tr>
</tbody>
</table>

1 INTRODUCTION

Control of electric power is the main function of modern-day power converters. In most of the cases, adjustable voltage amplitude and frequency are required. One of the biggest application groups on which such demands are set are variable speed drives, where the rotor speed is controlled to match the need of the application. In this work, the variable speed drives are referred to as electric drives. In electric drives, frequency converters are used to generate the AC voltage with variable frequency and amplitude to the electric motor. The electric drives can be categorized by their dynamical output performance into low-performance and high-performance electric drives. High-performance applications are for example paper machines, elevators, rolling mills and various servo drives. The common feature of these applications is the fast torque response of the produced electric torque and the high precision demand of the speed. Low-performance electric drives are used in applications where inaccuracy in the produced speed is tolerable and where the dynamic performance is lower. Low-performance electric drive applications are for example fans and pumps.

1.1 Economical aspects of the electric drives

According to the market analysis made by a company named ARC Advisory Group (1998), the power ratings of the low-power, up to 200 kW, electric drives can be categorized into three groups. Micro drives belong to the category less than 4 kW, low-end drives are in the group from 4 kW to 40 kW and the midrange continues up to 200 kW. The micro drives sell in large quantities usually through the OEM markets. Low-end drives applications are used mainly in fans and pumps in building automation industry. Midrange drives are often sold through system integration or directly to the end-user. The ARC Advisory Group has gathered information on market shares of these power segments from worldwide statistics. Fig. 1.1 shows the reported units delivered in percent of the corresponding annual unit shipments. The micro drives are dominating the annual shipments in units. Fig. 1.2 illustrates the reported units delivered in percent of the corresponding annual sale in million US Dollars. In comparison with the delivered units in Fig. 1.1 the micro drives are the smallest group in annual sales in US Dollars. No big changes in monetary percentages are expected.
In the report by ARC Advisory Group (1998) the operation modes of the electric drives are categorized as volts/Hz, sensorless and flux/vector control mode. The volts/Hz mode, later referred to as scalar control, is used in low-performance drives. In high-performance electric drives a sensorless vector control or flux/vector control is required to maintain the speed control accuracy. The difference between sensorless vector control and flux/vector control mode is the absence of the speed or position sensors in the sensorless control mode. In both of the cases the controller adjusts the output with respect to the mechanical load. Although the sensorless vector controlled drives have rapidly improved, the flux/vector control defends its position in high-performance drives, where accurate speed control is required also near zero-speed. Sensorless drives are suffering from controller inaccuracies near zero-speed. In that sense, the sensorless induction motor drives can be categorized also as low-performance electric drives although the performance of the sensorless drives is greatly enhanced compared to the scalar control. From Fig. 1.3 it can be seen that the low performance drives, combining volts/Hz and sensorless drives, have an over 90 % share of the operation modes in annual market values. It is expected that the
share of sensorless vector controlled drives will be greatly increased at the expense of the scalar control.

Fig. 1.3: Reported operation mode of delivered AC-drives in the year 1997 as a percent of costs in US Dollars.

Low dynamic performance does not mean that the electric drive efficiency is not of interest. The amount of the electric drives is rapidly increasing. Already electric drives are the greatest electricity consumers in industry and the percentual proportion of the consumed electricity is increasing. Due to the high volume of the low-performance electric drives in global annual sales manufacturers have growing interest in low-performance electric drives.

A study by Haataja (2002) gathers information on energy saving potential by using high-efficiency electric drive technology. Although the efficiency of the motors may be improved, in many cases improving the efficiency of the mechanical parts of the electric drive has a greater impact on the total efficiency of the electric drive. As an example of a mechanical part decreasing the total efficiency of the electric drive, the gearbox may be mentioned.

Almeida (1997) estimates the consumption of electricity in the EU for the end-use in the industrial and tertiary sector. According to the study, the consumption of electricity can be divided into following categories: motor, lightning and other. Fig. 1.4 shows the consumption of electricity shared between these three categories. In industry, Fig. 1.4 a), the motors are the greatest group with an over 2/3 share. In the tertiary sector, Fig. 1.4 b), the motors are still representing 1/3 share. When for the motors the consumption of electricity is categorized according to the applications, the low-dynamic performance drives are found to represent the majority. Almeida (1997) divides up the share of the electricity consumption by motor drives in industry and in the tertiary sector according to the different motor applications. This is illustrated in Fig. 1.5. Although the statistics used are not the most updated, the information received is clear. It can be seen, that from the total consumption of electricity used by electric drives the share
of the low-dynamic performance electric drives varies from 60 % in industry to 80 % in the tertiary sector. The increase of the electricity used in motor applications per year is 2.2 % in the tertiary sector and 1.5 % in industry (Hanitsch 2002). In the tertiary sector the smaller motors are the biggest electricity consumers whereas in industry the consumption is rather equally distributed between the power ranges. The share of power consumption between the different power ranges is shown in Table 1.1.

Fig. 1.4: Estimated electricity consumption in the EU in 1997 for end-use in the a) industrial and b) the tertiary sector.

Fig. 1.5: Estimated electricity consumption of motor applications in the EU in 1997 for end-use in the a) industrial and b) tertiary sector.
Table 1.1: Estimated electricity consumption in 2010 of the AC motors by power range (Hanitsch 2002).

<table>
<thead>
<tr>
<th>Power range [kW]</th>
<th>Industrial consumption [TWh]</th>
<th>Tertiary consumption [TWh]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75 – 7.5</td>
<td>148</td>
<td>109</td>
</tr>
<tr>
<td>7.5 – 37</td>
<td>136</td>
<td>75</td>
</tr>
<tr>
<td>37 – 75</td>
<td>103</td>
<td>25</td>
</tr>
<tr>
<td>&gt; 75</td>
<td>258</td>
<td>18</td>
</tr>
<tr>
<td>Total</td>
<td>645</td>
<td>227</td>
</tr>
</tbody>
</table>

It can be concluded that the biggest proportion of the electric drives is low-performance drives. It is also concluded that low-performance drives are the biggest electricity consumer. The total electricity consumption is most affected by improvements in the category of low-performance electric drives.

The frequency converter is connected to a supply grid, which in this work is also referred to as a line. Interactions between the supply grid and the electric drive are of increasing interest. Due to their working principle power electronic converters cause disturbances in the supply grid. These disturbances cause the supply grid waveforms to differ from pure sinusoids. Along with the increasing amount of the electric drives the importance of limiting the disturbances caused by the electric drive in the supply grid is growing.

There exist different types of standards creating limits for electric device produced harmonic pollution and disturbances (IEC 61800-3, IEEE-519). These standards are for public electric grids. In non-public industry grids these standards are not mandatory. However the connection point between industry and public electric grid do have to meet the standards. Because of the increasing amount of devices creating electromagnetic disturbance into the grid, there is a growing tendency to tighten the limits of the standards. The electric drive connected to the supply grid has to pass the electromagnetic compatibility (EMC) test. To pass the EMC test the device must not create disturbances higher than allowed by the standards. This is to confirm that the device does not disturb other devices. The electric device has to be robust against disturbances created by other devices. This assures that the device remains functional in an environment where electromagnetic disturbances exist. For the electric drives this means that the line side filtering needs to be taken into account more carefully. A more complex type of the AC and DC filter needs to be included.
into the electric drive to fulfill the standards. Another method is to use an active rectifier bridge. In both cases the electric drive requires a more expensive hardware structure. Vienna rectifier is one very interesting method to reduce disturbances. In this kind of rectifier three extra switches as well as a second DC link capacitor are needed compared to the simple diode rectifier. More of the Vienna rectifier can be found for example from Kretschmar et al. (2001) and Kolar et al. (1994).

For low-performance electric drives the active rectifier bridge is not a cost-effective way to decrease the line disturbances. This is mainly because of the small demand of regenerative braking of the pumps and fans. One very comprehensive survey including complex filters and the active rectifier bridge is done by Pöllänen (2003). Recently, the use of frequency converter driven low-performance electric drive in home applications has become common. Usually, these kind of electric drives are single-phase fed systems. For these home appliances to appropriately function, improvement of the supply grid interactions without the use of expensive active or passive filtering is required. The dominating feature of electric drives made for home applications is the cost. Accurate speed control is in these applications usually less important. It can be concluded that in low-performance drives cost-effective and simple filtering for line side is needed.

The main types of frequency converter topologies are the current source inverter and the voltage source inverter (VSI). The popularity of the current source inverter is limited by the power electric configuration. The main drawbacks so far are the lack of proper switching devices, the bulky DC inductor and more complex controller structure. More information on the current source inverter topology can be found, for example, in Salo (2002). The most common topology is the voltage source inverter. In this work, voltage source inverter with diode rectifier is chosen to be the topology of the frequency converter.

1.2 Overview of the voltage source inverters

The main circuit of the voltage source inverter with three-phase motor load is shown in Fig. 1.6. In this work, the frequency converter refers to the system having a full-wave diode rectifier bridge, DC-link and inverter bridge. In the frequency converter, the supply grid AC-voltage is first rectified using a full-wave rectifier diode bridge. After rectifying, the inverter bridge is used to convert the DC-voltage into AC-voltage with variable frequency and amplitude. When being used as a rectifier, the active rectifier bridge, which is identical to the inverter bridge, can be used to enable regenerative operation of the electric drive. An intermediate circuit with DC-link capacitor is used to provide energy storage and to filter the DC-link voltage from the rectifier bridge and inverter bridge voltage spikes. Controlling of the voltage amplitude and frequency is done by using semiconductor switches, which are turned on and off at high frequency. The motor
control algorithms determine the reference to the motor variables. The modulator is used to convert the reference signal into instances for the six power switches.

\[ \text{motor variables} \]

**Fig. 1.6:** Main circuit of the voltage source frequency converter with three-phase input and three-phase motor. The line phases L1, L2 and L3 are connected to the full-wave diode rectifier bridge. The rectifier transforms AC-voltage into DC-voltage. The DC-link capacitor \( C \), located in the intermediate circuit, is used to smooth the DC-voltage. The inverter bridge is used to transform the DC-voltage into three-phase (U, V, W) AC-voltage with variable amplitude and frequency. The inverter bridge is controlled with the switch commands \( S_1, S_2, S_3 \).

### 1.2.1 Motor control

There exist different types of motor control algorithms. Field-oriented vector control method called current vector control is often applied to high-performance electric drives. The current vector control is well known and discussed in detail, for example, in Kazmierkowski et al. (2002) and Bose (1997). An alternative method for field oriented control is the direct torque control (DTC). The DTC was developed almost simultaneously by Depenbrock (1985) and Takahashi et al. (1986). The method was further developed by Tiitinen et al. (1995). As for the vector controlled drives, the current trend is to reduce the number of the external measurements as, for example, motor speed sensors.

The scalar control method is widely applied to low-performance electric drives. The scalar control adjusts the motor speed by varying the frequency converter output voltage amplitude and frequency. The induction motor speed is then defined by the loading conditions. Using a speed feedback, the speed accuracy can be improved. A system equipped with speed feedback is called closed-loop scalar control. In the scalar control the motor angular frequency is used to form the amplitude and frequency for the frequency converter output phase voltages. The general form for the variable frequency AC-motor drives electromotive force (EMF) vector \( \mathbf{e} \) in steady state is defined with the stator flux linkage vector \( \mathbf{\psi} \) and its angular frequency \( \omega \)

\[ \mathbf{e} = -j \omega \mathbf{\psi} . \]
The electromotive force can be defined with the stator voltage vector \( u_s \), resistance \( R_s \) and current vector \( i_s \):

\[
-e_s = u_s - R_s i_s.
\]  

(1.2)

A theoretical vector presentation of the loading state of the general rotating induction motor drive in a static xy-coordinate system is illustrated in Fig. 1.7. The coordinate system will be discussed in detail in Section 1.4.1.

![Fig. 1.7: Vector presentation of the general rotating induction motor drive. The electromotive force \( e_s \), stator voltage vector \( u_s \), resistance \( R_s \), current vector \( i_s \), and stator flux linkage vector \( \psi_s \) are shown in the static xy-coordinate system. In steady state the vectors are rotating in the xy-coordinate system with a constant angular frequency \( \omega \).](image)

The scalar control is based on the steady state operation. The mathematical formation of the scalar control is divided into the calculation of the instantaneous amplitude \( |u_{s,\text{ref}}| \) and angle \( \theta_{s,\text{ref}} \) for the stator voltage vector \( u_{s,\text{ref}} \) as

\[
|u_{s,\text{ref}}| = f_u(\omega) \bigg|_{\omega = \omega_{\text{ref}}}.
\]  

(1.3)

The vector presentation and definitions are discussed in detail later in Section 1.4.1. Different types of the voltage-frequency curves \( f_u \) between frequency reference \( \omega_{\text{ref}} \) and the amplitude exist. The traditional method to form the voltage curve for the AC-motor is to use the constant flux linkage. The simplest way is to form voltage amplitude modulus equal to be to the modulus of the electromotive force, that is \( |u_{s,\text{ref}}| = |e| \). From (1.1) it can be seen that this leads to a constant \( u/\omega \)
relation. Equation (1.2) shows that stator resistance $R_s$ causes a voltage drop. The effect of the stator resistance can be included to increase the voltage amplitude accuracy. This is called IR-compensation. From (1.1) and (1.2) it is possible to calculate the stator voltage vector modulus reference as

$$|u_{s, \text{ref}}| = |\omega_{s, \text{ref}} \psi_s + R_s i_s|.$$

(1.4)

Also a quadratic flux linkage frequency dependence can be used. The angle for the stator voltage vector reference $\theta_{s, \text{ref}}$ is calculated by integrating the frequency reference $\omega_{\text{ref}}$ as shown in

$$\theta_{s, \text{ref}} = \int \omega_{\text{ref}} dt + \theta_{s, \text{ref}0},$$

(1.5)

where the $\theta_{s, \text{ref}0}$ is the initial angle of the stator voltage vector. In many cases the zero initial value can be used.

Thus the output of the scalar control can be written in vector form as

$$u_{s, \text{ref}} = |u_{s, \text{ref}}| e^{j \theta_{s, \text{ref}}}. $$

(1.6)

### 1.2.2 Modulator

Holtz (1992) introduced a very good categorization of the different types of modulators. The two main methods are the feedforward and feedback schemes.

Feedback schemes generate the switching sequences in a closed control loop. The control loop can be based on the stator currents or stator flux linkage. As an example the current hysteresis control and direct torque control methods belong to this category.

Feedforward schemes generate the switched three-phase voltage such that the generated fundamental voltage vector equals the reference vector. The traditional analog pulse width modulation (PWM) method which is based on the triangular comparison method as well as the method called space vector pulse width modulation (SVPWM) belongs to this category. The SVPWM is suitable especially in digital implementation. The SVPWM method is presented, for example, in Van Der Broeck et al. (1986) and will be discussed in Section 2.1.1.

The inverter bridge has discrete circuit modes for each set of the switch states. To create a voltage vector with an arbitrary direction and length, the time averaging approach is used in the modulation. The modulator is a control circuit that converts the phase reference, or analogously the voltage vector reference, into switching commands to be fed to the power switches of the
The modulator does not restrict the applied motor control method. In the widely used Pulse Width Modulation (PWM) method the inverter fundamental output voltage waveform equals the reference value. The scalar control and modulator block diagram are shown in Fig. 1.8.

Fig. 1.8: Scalar control and modulator block diagram. The angular frequency reference is fed to the scalar controller. The scalar control transfers the angular frequency reference further to a reference stator voltage vector. The modulator is used to convert the voltage vector into switching commands for the inverter bridge.

When the sinusoidal output fundamental waveform is maintained, the modulation region is called a linear modulation region. If distortion in the output quantities is tolerable, overmodulation (OM) methods can be used to increase the output voltage. The behavior of the frequency converter greatly depends on the modulator type. Especially the overmodulation properties are dependent on the modulation method. Traditionally, the electric drive voltage output is designed to have its nominal value $u_{s,nom}$ when the nominal angular speed of the motor $\omega_{nom}$ is used. Overmodulation methods are used after this point to increase the stator voltage all the way to the maximum limit of $u_{s,max}$ when extra power is needed. This is illustrated in Fig. 1.9.

Fig. 1.9: Principle of the output voltage under linear and overmodulation region as a function of the angular rotating frequency. When the sinusoidal output quantities are maintained, the modulation region is named linear modulation region. Overmodulation can be used to increase the output voltage at the expense of additional distortion in output voltage.
1.3 Voltage source inverter with small DC-link capacitor

In the traditional frequency converter, shown in Fig. 1.6, a relatively large electrolytic DC-link capacitor is used. Many disadvantages are related to the use of this component. Electrolytic capacitors are large, heavy and rather expensive components. In many cases, the life-time of the electrolytic capacitor is the main factor limiting the life-time of the frequency converter (Military Handbook 217 F, Imam et al. (2005)). To overcome the problem, the electrolytic capacitor is removed. In practice, a small capacitor in the DC-link is necessary to by-pass switching harmonics of the inverter bridge. This capacitor, however, can be comparably small in capacitance, if it has sufficient RMS-current capability. Since the metallized polypropylene film capacitor (MPPF) does not have the same limitations as the electrolytic capacitor and because the capacitance of the MPPF is approximately one percent of the same volume electrolytic capacitor, the MPPF is applicable and here chosen to be used as DC-link capacitor. In this work small capacitor refers to a film capacitor. Technical details and comparison of different capacitor structures is done e.g. by El-Hussein et al. (2001), Bramoullé (1998) and Michalczyk et al. (2003). The driving force for this work is to replace the electrolytic capacitor, which works as the DC-link energy storage, by the MPPF capacitor. Although the changes required for the power electric hardware configuration remain rather small, the changes have a significant impact on the frequency converter dynamics and motor control algorithms. By changing the DC-link capacitance, the dynamic behavior of the DC-link voltage in the frequency converter changes dramatically. Also the line current harmonic content is improved.

Though replacing the electrolytic capacitor by an MPPF capacitor is a very attractive idea, not many publications dealing with the small DC-link capacitor frequency converters have been found by the author. The SED2, which is a product of the company SIEMENS, uses a small DC-link capacitor. One of the earliest papers on small DC-link capacitor drives was published by Takahashi et al. (1990). This paper reports the study of a small DC-link capacitor drive equipped with an active and a passive rectifier bridge. The compensation of the DC-link fluctuation is implemented in triangle-comparison analog PWM controllers. Bose et al. (1991) eliminates the DC-link electrolytic capacitor by introducing an active current-fed type high frequency filter in the DC-link. Minari et al. (1993) studied the frequency converter without DC-link capacitor. However, the authors used an AC-filter to smooth the rectifier and harmonic components caused by the inverter bridge. After the study by Minari et al. a few papers dealing with the minimization of the DC-link capacitor have been published (Kim et al 1993, Alaküla et al. 1994, Wen-Song et al. 1998, Jung et al. 1999, Namho et al. 2001 and Gu et al. 2002). The motivation for these studies
was to match the instantaneous input and output power with each other, thus attaining that no current flows through the DC-link capacitor. As a consequence, the DC-link capacitor may be minimized. In these papers mentioned above, an active rectifier bridge is required. Gu et al. (2005) presented the analytical form to find the minimum capacitance for a frequency converter having an active rectifier bridge. The matrix converter topology approach is used by Siyoung et al. (1998). However, the converter configuration presented by the authors is returned to the small DC-link capacitor VSI with the active rectifier bridge and an additional supply grid filter component. Kim et al. (1995) use the active rectifier bridge VSI with resonant DC-link without the electrolytic capacitor. The drawback of the method is that additional power electronic components are demanded for creating the resonant circuit into the DC-link. Klumpner et al. (2004) discuss the stability of the VSI equipped with an active rectifier bridge and the problems of the unbalanced supply grid voltages for small DC-link storage. Two studies on frequency converter with low-capacitance DC-link capacitor and active rectifier bridge with 120 degree leading angle have lately been published by Göpfrich et al. (2003) and by Piepenbreier et al. (2004). The emphasis in Piepenbreier’s et al. (2004) study is placed on the measuring of the efficiency of the small DC-link capacitor electric drive. Although the study deals with the frequency converter unit capable of operating in regenerative operation mode, the study can also be generalized to the motor drive system considered in this work. According to the results obtained, no considerable change in the efficiency between the small DC-link capacitor drive and the high-capacitance frequency converter electric drive can be found. Kretschmar et al. (1998) studied the DC-link fluctuation with simulations and analytic equations for permanent magnet synchronous motor driven by a frequency converter with small DC-link capacitor.

In this work, only the passive full-wave diode rectifier bridge is considered. The choice is justified with the small demand of regenerative braking of low-performance drives. No complex AC-filters are used. Takahashi et al. (1990) considered the same frequency converter structure as introduced in this work. Contrary to Takahashi’s study, in this work a modern digital control platform including modulation has been used. New digital modulation methods utilize the DC-link voltage much more efficiently. This is important especially in the overmodulation region. Improved measurements as well as control hardware capabilities enable faster controlling of the frequency converter dynamics. In this study, a DC-link capacitor as small as 2 µF is considered and its applicability to a frequency converter unit with a 10 A nominal current has been tested. The capacitance value of the capacitor is approximately 1 % of the original capacitor capacitance. The selected capacitor is thus even smaller than the capacitors used in the references mentioned
above. Kreschmar et al. (1998, 2005) have found that 10 \( \mu F \) capacitor is sufficient for 15 kW permanent magnet integral motor. This capacitor size leads approximately to same ratio of capacitance and electric drive power as in this research.

The above literature survey covers the three-phase fed VSI and does not take into consideration the single-phase supplied VSI. Only a few scientific studies dealing with the single-phase supplied small DC-link VSI have been published. Publications as by Takahashi et al. (2001, 2002), Haga et al. (2003) and Lamsahel et al. (2005) consider the single-phase fed small DC-link capacitor VSI with permanent magnet motor. The papers differ from each other by the applied motor control algorithms. One patent EP1396927 by Takahashi et al. (2004) has been registered. The power electronic configuration used in this patent is exactly the same as that used in this work. However, in this work different motor control algorithms are used. Lately, a study by the author (Sarén et al. 2005) presents the single-phase fed VSI with a small DC-link capacitor and DTC control for a three-phase induction motor. The fact that only few references for the single-phase fed small DC-link capacitor VSI were found by the author indicates that no extensive research and publications on this subject have yet been done.

1.3.1 Three-phase input

Replacing the electrolytic capacitor by a smaller MPPF capacitor will dramatically decrease the energy stored in DC-link. When a symmetrical load and sinusoidal output voltages and currents are assumed the three-phase instantaneous output power is constant at every instant. The three-phase full-wave diode rectifier bridge is capable of supplying a continuous power which is equal to the instantaneous output power. Thus, it is not necessary to have energy storage in DC-link in the case of a symmetrical three-phase system. The figures below illustrate the change in the DC-link voltage and the line currents comparing the high-capacitance frequency converter with the frequency converter with small DC-link capacitor. The inductor used as an AC-filter and the DC-link capacitor are scaled 10 \% and 1 \% from the original values being 4 mH and 200 \( \mu F \) respectively. In both cases, the simulated DC-link voltage, the supply grid phase voltages and currents are shown. And in both cases constant, equal power is taken from the DC-link. Fig. 1.10 shows the characteristic behavior of a high-capacitance three-phase frequency converter. The DC-link voltage \( u_{DC} \) is relatively stable and the line side currents \( i_{L1}, i_{L2} \) and \( i_{L3} \) include a high amount of harmonic current components at the frequencies 250 Hz and 350 Hz, causing problems to the supply grid. The corresponding behavior of a frequency converter with small DC-link capacitor is shown in Fig. 1.11. Replacing the electrolytic capacitor by a film capacitor will decrease the energy storage of the DC-link. Thus, the instantaneous power supplied into the motor correlates
with the instantaneous power taken from the network. This leads to a continuous DC-link current fed by the diode rectifier bridge. As a consequence, the line current harmonics clearly decrease. It can be seen that the DC-link voltage fluctuates. This causes additional conditions to the controllers of the frequency converter to compensate the fluctuation from the supplied motor phase voltages. The line current is significantly improved when the frequency content of the line current is taken into consideration. Thus, for the VSI with small DC-link capacitor only a small AC-side filter is needed to filter inverter bridge switching harmonics.

Fig. 1.10: Simulated characteristic behavior of a high-capacitance frequency converter. The DC-link voltage $u_{DC}$ is relatively smooth and the line side currents $i_{L1}$, $i_{L2}$ and $i_{L3}$ include a high amount of harmonic components at the frequencies 250 Hz and 350 Hz. The line side inductor and DC-link capacitor values are 4 mH and 200 µF.
Fig. 1.11: Simulated characteristic behavior of a frequency converter with line side inductor and DC-link capacitor. The values of the line side inductor and DC-link capacitor are scaled 10% and 1% from the original ones being 0.4 mH and 2 µF respectively. The DC-link voltage $u_{DC}$ fluctuates. The line current significantly improves when the frequency content of the current is taken into consideration.

The supply grid current waveform in Fig. 1.11 strongly defends the configuration of the small DC-link capacitor VSI. The line current waveform is a real benefit. Approximating the line current with a staircase waveform with amplitude $h$, shown in Fig. 1.12, the current can be written in terms of the Fourier series

$$f(t) = \sum_{n=1}^{\infty} b_n \sin \left( \frac{2\pi n}{T} t \right),$$

(1.7)
where \( n \) stands for the harmonic number, \( T \) is the period of the staircase waveform and the constants \( b_n \) can be calculated from

\[
b_n = \frac{2}{T} \int_0^T f(t) \sin\left(\frac{2\pi n t}{T}\right) dt \]

\[
= \frac{2}{T} \left[ \int_{5T/12}^{7T/12} h \cdot \sin\left(\frac{2\pi n t}{T}\right) dt + \int_{7T/12}^{11T/12} -h \cdot \sin\left(\frac{2\pi n t}{T}\right) dt \right]
\]

\[
= \frac{h}{\pi n} \left[ \cos\left(\frac{1}{6} \pi n\right) + \cos\left(\frac{11}{12} \pi n\right) - \cos\left(\frac{5}{6} \pi n\right) - \cos\left(\frac{7}{6} \pi n\right) \right].
\]

Now it is possible to approximate the amplitude of the different harmonic components of the supply grid current waveform shown in Fig. 1.11. The Fourier transform suggests that the amplitude of the 5th and the 7th harmonic compared to the fundamental component waveform magnitude are \( b_5/b_1 = 1/5 \) and \( b_7/b_1 = 1/7 \). From Fig. 1.11 it can be seen that this is not exactly the case. However, the current staircase waveform approximation is reasonably accurate.

![Fig. 1.12: Approximation of a small DC-link capacitor VSI line current waveform with the staircase waveform \( f(t) \) with the amplitude \( h \). The fundamental waveform amplitude is \( b_1 \).](image)

The DC-link voltage fluctuates periodically at a frequency which is 6-fold the supply grid frequency. In a 50 Hz supply grid the fluctuation frequency is thus 300 Hz. This indicates that the modulator should take into account the DC-link voltage level. Takahashi et al. (1990) describe an analog implementation of the PWM modulator that is capable of taking the 300 Hz DC-link...
fluctuation into account. The main idea proposed in the article is to use the amplitude of the carrier signal which fluctuates synchronously with the DC-link voltage at a frequency of 6-fold the supply grid frequency. In the space vector pulse width modulation, the DC-link voltage is observed in the calculation of the pulse width references.

### 1.3.2 Single-phase input

In the case of a single-phase input, dramatic changes will be observed in comparison with the three-phase system. The input power is not constant. The main diagram of the hardware configuration discussed is shown in Fig. 1.13.

![Diagram](image)

Fig. 1.13: Main circuit of the voltage source frequency converter with single-phase input and three-phase motor load.

The instantaneous power gained from the sinusoidal one phase system in steady state is

\[ P_{in} = u_{1}i_{1} \]

\[ = \hat{u} \sin(\omega_{\text{grid}}t) \cdot \hat{i} \sin(\omega_{\text{grid}}t - \phi), \]

where \( \hat{u} \) and \( \hat{i} \) indicate the peak values of the phase voltage and current respectively. The angle \( \phi \) is the phase shift between the voltage and current.

If the phases of the line voltage and current are identical, that is \( \phi = 0 \), the instantaneous power becomes

\[ P_{in} = \hat{u} \hat{i} \sin^{2}(\omega_{\text{grid}}t) \]

(1.10)

This indicates that the output power should also fluctuate in a similar way to gain a purely sinusoidal line current behavior and unity power factor. This also represents the maximum possible continuous power supply gained from the single-phase fed small capacitance frequency converter under sinusoidal line voltage and current. The situation is illustrated in Fig. 1.14.
using the small DC-link capacitor a cost-effective power electronic configuration can be achieved. This is important especially in the applications of electric drives in home appliances.

![Graph](image)

Fig. 1.14: The DC-link voltage $u_{\text{DC}}$, line voltage $u_{L1}$ and current $i_{L1}$ and input power $p_{\text{in}}$ waveforms with a small DC-link capacitor under resistive load. To gain a purely sinusoidal line current waveform the output power must match the waveform of the theoretical input power waveform.

### 1.4 Tools for analysis

Next, the most essential mathematical tools used in this work are defined and explained. The three-phase systems are simplified into a vector presentation by using the space vector presentation. Simulation is a very important method for verifying the ideas and methods before implementing them into practice. In this work a considerable number of simulation results are presented. The frequency domain analysis is used to study the behavior of the electric drive quantities. Per unit values are used to make comparison between different power scales easier.

#### 1.4.1 Space vector presentation

For the modeling of three-phase systems space vectors are proved to be a very useful means. The theory was invented by Kovacs and Racz (1959) and it was originally intended for the transient analysis of AC machines. In a general three-phase system, which has an angular frequency $\omega$, the instantaneous values of the phase quantities $x$ and phase the angle are expressed as

$$x(t) = \hat{x}(t) \cos(\omega t + \phi(t)) \quad (1.11)$$
\[ x_v(t) = \hat{x}_v(t) \cos \left( \theta(t) - \frac{2}{3} \pi + \phi_v(t) \right) \]  \hspace{1cm} (1.12)

\[ x_w(t) = \hat{x}_w(t) \cos \left( \theta(t) - \frac{4}{3} \pi + \phi_w(t) \right) \]  \hspace{1cm} (1.13)

\[ \theta(t) = \int_0^t \omega(t) \, dt \]  \hspace{1cm} (1.14)

where \( \hat{x} \) is the phase quantity peak value.

The system can be expressed with a complex space vector and real zero-sequence component \( x(t) \) and \( x_0(t) \) respectively as

\[
x(t) = c \left[ x_u(t) + x_v(t) e^{\frac{2\pi}{3}} + x_w(t) e^{\frac{4\pi}{3}} \right] = x(t) e^{j\phi(t)}
\]

\[ x_0(t) = c_0 [x_u(t) + x_v(t) + x_w(t)] . \]  \hspace{1cm} (1.16)

The underlined variables express the space vector in a stationary reference frame and the angle \( \theta \) is the space vector angle from the real axis of the stationary reference frame. The terms \( c \) and \( c_0 \) are used in the vector scaling. Peak value scaling is achieved when \( c = \frac{2}{3} \) and \( c_0 = \frac{1}{3} \) are selected whereas the power invariant form of the space vectors is achieved with the values \( c = \sqrt{2/3} \) and \( c_0 = 1/\sqrt{3} \). In this work peak value scaling is used.

The space vector presentation (1.15) can be expressed with a real and an imaginary component denoted with the subscripts x and y as

\[ \bar{x}(t) = x_x(t) + j x_y(t) . \]  \hspace{1cm} (1.17)

The real and imaginary components can be calculated from the following matrix relation as

\[
\begin{bmatrix}
x_x(t) \\ x_y(t) \\ x_0(t)
\end{bmatrix} =
\begin{bmatrix}
\frac{2}{3} & -1/3 & -1/3 \\
0 & 1/\sqrt{3} & -1/\sqrt{3} \\
1/3 & 1/3 & 1/3
\end{bmatrix}
\begin{bmatrix}
x_u(t) \\ x_v(t) \\ x_w(t)
\end{bmatrix},
\]  \hspace{1cm} (1.18)

where the \( x_0 \) is a zero-sequence component.
In a symmetric three-phase system the peak values and phase angles of the three-phase system are equal. The zero-sequence component \( x_0 \) thus becomes zero and can be omitted in the space vector analysis. The coordinate systems presented in this work are based on this real/imaginary coordinates. Later, the coordinates are shortly denoted as xy-coordinates.

The instantaneous value of the phase variables, when peak value scaling is used, can be obtained from

\[
\begin{align*}
    x_u(t) &= \text{Re}\left\{ x(t)e^{-j\frac{2\pi}{3}} \right\} + x_o(t), \\
    x_v(t) &= \text{Re}\left\{ x(t)e^{-j\frac{4\pi}{3}} \right\} + x_o(t), \\
    x_w(t) &= \text{Re}\left\{ x(t)e^{-j\frac{6\pi}{3}} \right\} + x_o(t)
\end{align*}
\]  

(1.19)

The transformation from the xy-components to the three-phase system is defined in the matrix form as

\[
\begin{bmatrix}
    x_u(t) \\ x_v(t) \\ x_w(t)
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \\ -1/2 & -\sqrt{3}/2 & 1
\end{bmatrix}
\begin{bmatrix}
    x_x(t) \\ x_y(t)
\end{bmatrix}.
\]  

(1.20)

The instantaneous power of the three-phase system \( p(t) \) is equal to the sum of each phase instantaneous power

\[
p(t) = u_u(t)x_u(t) + u_v(t)x_v(t) + u_w(t)x_w(t).
\]  

(1.21)

The instantaneous power can be expressed with the space vector presentation when the phase voltages and currents are expressed with equation (1.19) as

\[
p(t) = \frac{3}{2} \text{Re}\left\{ u^* \right\} + 3u_0i_0
\]

\[
= \frac{3}{2} \left( u_xi_x + u_yi_y \right) + 3u_0i_0
\]

(1.22)

In this work the most often used terms stator voltage and current vectors in the xy-coordinate system are denoted as \( u_x \) and \( i_x \). The vectors are defined by using the output phase U, V and W quantities of the voltage source inverter. The equations are written as
\[
\begin{equation}
\ddot{u}_j(t) = \frac{2}{3} \left[ u_{u_j}(t) + u_{v_j}(t) e^{\frac{2\pi}{3}} + u_{w_j}(t) e^{\frac{4\pi}{3}} \right] \\
= \left| u_j(t) \right| e^{j\theta_j(t)} \\
= u_{x_j} + j u_{y_j}
\end{equation}
\]

and

\[
\begin{equation}
\ddot{i}_j(t) = \frac{2}{3} \left[ i_{u_j}(t) + i_{v_j}(t) e^{\frac{2\pi}{3}} + i_{w_j}(t) e^{\frac{4\pi}{3}} \right] \\
= \left| i_j(t) \right| e^{j\phi_j(t)} \\
= i_{x_j} + j i_{y_j} .
\end{equation}
\]

### 1.4.2 Dynamic DC-link model

The dynamic model of the DC-link is needed to verify the behavior of the DC-link voltage under different load conditions and with different electrical parameters. The dynamic model includes a line side resistance \( R_{AC} \) and inductance \( L_{AC} \). The variables are defined without using the time variable for simplicity. The supply grid voltage vector \( u_{grid} \) is defined as

\[
\begin{equation}
\begin{aligned}
u_{grid} &= u_{L1} + u_{L2} e^{\frac{2\pi}{3}} + u_{L3} e^{\frac{4\pi}{3}} .
\end{aligned}
\end{equation}
\]

The line current vector \( i_{grid} \) on the AC-side can be calculated from

\[
\begin{equation}
L_{AC} \frac{di_{grid}}{dt} + R_{AC} i_{grid} = u_{grid} - u_{rec} ,
\end{equation}
\]

where \( u_{rec} \) is a rectifier voltage.

The functionality of the full-wave rectifier bridge can be modeled with the switching function \( sw_{rec} \). The function is defined as

\[
\begin{equation}
sw_{rec} = sw_{rec,L1} + sw_{rec,L2} e^{\frac{2\pi}{3}} + sw_{rec,L3} e^{\frac{4\pi}{3}} .
\end{equation}
\]

If a diode is modeled as an ideal switch the sign of the corresponding line current determines the on/off state of the diode. The function can be expressed as

\[
\begin{equation}
sw_{rec,i} = f\left(i_{grid,i}\right) ,
\end{equation}
\]

where the subscript \( i = L1, L2 \) and \( L3 \).
The state of $sw_{rec,i}$ is defined as

$$sw_{rec,i} = \begin{cases} 1, & \text{when } i_{\text{grid},i} > 0 \\ 0, & \text{when } i_{\text{grid},i} \leq 0 \end{cases}$$

(1.29)

The rectifier current $i_{rec}$ is expressed as a function of the switching function $sw_{rec}$ and the line current vector

$$i_{rec} = \frac{3}{2} \text{Re}\{sw_{rec} i_{*_{\text{grid}}^*}\}.$$  

(1.30)

The rectifier voltage vector $u_{rec}$ at the AC-side is also defined with the switching function.

$$u_{rec} = sw_{rec} u_{DC},$$  

(1.31)

where the DC-link voltage $u_{DC}$ depends on the rectifier current $i_{rec}$ and the inverter current $i_{inv}$ and is defined as

$$\frac{du_{DC}}{dt} = \frac{1}{C} (i_{rec} - i_{inv}).$$  

(1.32)

where $C$ is DC-link capacitance.

The approximate model for the DC-link voltage with no commutation and inverter current effect can be expressed as

$$u_{DC} = \max(\|u_{L1} - u_{L2}\|, \|u_{L2} - u_{L3}\|, \|u_{L3} - u_{L1}\|).$$  

(1.33)

The circuit diagram of the DC-link dynamic model is shown in Fig. 1.15.

![Fig. 1.15: Main circuit diagram of the dynamic DC-link model. The parameters are the line side resistance and the inductances $R_{AC}$ and $L_{AC}$ respectively and the DC-link capacitance $C$.](image_url)
1.4.3 Electric drive simulation tool

To verify the functionality of the whole electric drive a simulation model was built. For this work Matlab® Simulink® by the company MathWorks has been used as a simulation program. Simulink includes a power circuit simulator which, previously known as PowerSystem Blockset®, has been lately renamed SimPowerSystem®. The power circuit simulator allows the user to configure the power circuit diagram as well as the motor model by using ready-made blocks and simply initializing the electrical parameters of the blocks. All the control algorithms, including modulators, can be made by using standard Simulink blocks. The modulator using the PWM method needs a discrete time simulation model with a very short sample time or preferably a time continuous model. Due to the long simulation durations the main model used is a discretized simulation model. To preserve for the PWM a sufficient accuracy, sample time steps from 5 µs to 0.01 µs are used depending on the scope of the simulation case.

Fig. 1.16 shows the used electric drive simulation model in block diagram. The supply grid model includes the ideal three-phase 50 Hz sinusoidal voltage supply with a peak value of $230\sqrt{2}$ V. The line impedance is set to zero. The purely inductive inductor $L_{AC}$ is applied as the AC-filter. A universal bridge configured as full-wave diode bridge is used as the rectifier. A capacitor $C$ is used in the intermediate circuit. A universal bridge configured as an IGBT bridge is used as the inverter bridge. A three-phase asynchronous motor model with external load torque $t_l$ capability is used as the motor model. All these basic blocks are available in the SimPowerSystem® blockset. References for these blocks are given, for example, on MathWorks’ manual (Simulink, 2000).

The control circuits are built using discrete time blocks to emulate the digital behavior of the controllers. The scalar control has been described earlier in this work. The modulator equations will be presented in detail in the next chapter. Blanking time logic is used to make the control signals of the upper and lower IGBT arm switches with added blanking time between the switching instants. Blanking time is used to prevent short circuit between the same phase upper and lower IGBT arm. The parameters used in the simulations are listed in the appendices. Appendix A includes the parameters related to the three-phase fed simulation model. Appendix B includes the parameters related to the single-phase fed simulation model.
Fig. 1.16: Basic components of the electric drive simulation model. The ideal sinusoidal three-phase voltage supply is used as the supply grid model. A three-phase parallel inductor is used to describe the line side AC-filter. The universal bridge parameterized as the diode and the IGBT bridge are used to respectively model the full-wave rectifier and IGBT bridge. The three-phase asynchronous motor with pu units is used as the motor model. The load torque is adjustable. A fully working scalar control with modulator is included into the model to gain the IGBT bridge control signals. The blanking time logic is used to prevent short circuit between the DC-link positive and negative bus-bar.

### 1.4.4 Frequency domain analysis

In many cases, an analysis being capable of analyzing the frequencies that are contained in one signal offers considerable advantages. The Fourier analysis is used to decompose the signals into sinusoids. For the sampled signals, the discrete Fourier transform (DFT), also called finite Fourier transform, is employed. The DFT is widely used in signal processing and in related applications. The DFT can be quickly computed using a fast Fourier transform (FFT) algorithm. In the DFT sine and cosine waves are applied. The number of samples, indexed with $i$ in the time domain, is denoted as $N$. The time domain having $N$ points corresponds to $N$ points in the frequency domain. The output of the DFT is a set of complex numbers $X$ that represent the amplitudes of the cosine and sine waves $c$ and $s$ respectively for each frequency $f$ with index $k=\frac{fN}{N}$. The DFT output $X$ is

$$X[k] = c[k] + js[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N},$$

(1.34)
where the $i$ is the time domain index and $k \in \{0, 1, \ldots, N-1\}$ stands for the sample number of the frequency.

The inverse DFT assigns each of the amplitudes to the corresponding cosine and sine waves. The result is a set of scaled sine and cosine waves that can be added to form the time domain signal. In general, any $N$ point repetitive signal can be created by adding cosine and sine waves. Adding the scaled sine and cosine waves produces the time domain signal $x[i]$.

$$x[i] = \sum_{k=0}^{N-1} X[k] e^{j2\pi ik/N}$$  \hspace{1cm} (1.35)

More on the DFT can be found, for example, in Smith (1999). In this work, a ready-made Matlab® FFT function for studying sampled data in the frequency domain is used. Here, the frequency domain quantities are denoted shortly as FFT values.

### 1.4.5 Per-unit values

Per-unit values are used to scale different power scale values into a comparable form. The per-unit (pu) system is a dimensionless relative value system defined in terms of base values denoted with subscript B. A per-unit value is defined as

$$x_{pu} = \frac{x}{x_B},$$  \hspace{1cm} (1.36)

where $x$ is the absolute physical value of the quantity and $x_B$ is its base value.

The fundamental base values are defined as

- **voltage**: $u_B = \sqrt{\frac{2}{3}} U_n$

- **current**: $i_B = \sqrt{2} I_n$  \hspace{1cm} (1.37)

- **frequency**: $\omega_B = 2\pi f_{grid}$,

where $U_n$ is the nominal root mean square (RMS) value of the converter line-to-line voltage, $I_n$ is the nominal RMS-value of the converter phase current and $f_{grid}$ is the supply grid line voltage frequency.
From the fundamental base values other base values can be derived as follows:

\[
time: \tau_B = \frac{1}{\omega_B},
\]

impedance and resistance: \( Z_B = R_B = \frac{u_B}{i_B} \),

inductance: \( L_B = \frac{Z_B}{\omega_B} = \frac{u_B}{i_B(\omega_B i_B)} \),

\[
\text{capacitance: } C_B = \frac{1}{Z_B(\omega_B)} = \frac{i_B}{\omega_B u_B}.
\]

\[
\text{flux linkage: } \psi_B = u_B / \omega_B \text{ and}
\]

\[
\text{power: } s_B = \frac{3}{2} u_B i_B = \sqrt{3} U_B I_B.
\]

1.5 Experimental test setups

The presented ideas and theories can be verified by use of experimental devices. The commercial frequency converters that belong to the VACON NX™-family are used as power electronic hardware. With this hardware platform it is possible to feed the external switching commands \( S_1, S_2 \) and \( S_3 \). The ASIC integrated in a VACON NX™ frequency converter further converts these commands for each IGBT power switch with added hardware specific blanking time. The switching commands are obtained from the dSPACE™ real time control card. All the control algorithms including the modulator are included in the dSPACE™ real time control card. An example of dSPACE™ controller usage is given, for example, by Sarén et al. (2002a). For this study, slightly differing experimental settings for the three-phase and single-phase fed devices were built. During the research work a considerable number of measurements have been carried out.

1.5.1 Three-phase fed voltage source frequency converter

A standard 4 kW 3-phase four-pole induction motor is used as the controlled motor. A 5.5 kW induction motor with regenerative speed controlled VACON CX™ unit is used as the load. The motors are connected back to back by stiff shaft coupling. In appendix C the essential experimental setup parameters are listed. The theoretical diagram of the used experimental device is shown in Fig. 1.17. The original 165 \( \mu \)F electrolytic capacitor is replaced with a 2 \( \mu \)F MPPF capacitor. As the line filter, a 0.4 mH inductor is used instead of the original 3.4 mH. The phase current and DC-link voltage measurement cutoff frequency of the analog low pass filter is 100
kHz. The field programmable gate array circuit (FPGA) samples the voltage and current measurements and filters with sample weighting. Seven 12 bits samples from the phase currents and DC-link voltage are acquired within a period of 25 µs. The weighted samples \( x_i \) (with \( i = 1, 2, \ldots, 7 \)) are summed to form one sample \( y \) to the dSPACE™. The samples are weighted such that the three most recent samples are weighted with a factor \( a = 1/4 \) and the rest four with \( b = 1/16 \). The filter output \( y \) can be written using samples \( x_i \) as

\[
y = ax_1 + ax_2 + ax_3 + bx_4 + bx_5 + bx_6 + bx_7.
\]

The output of the FPGA is read from the digital I/O to the dSPACE™ at the interval of 40 kHz. No further filters are used in the measurements. A Nyquist frequency of 20 kHz is achieved.

Fig. 1.17: Basic components of the controlled three-phase fed experimental device. The dSPACE™ system is used as a controller which includes the scalar control and modulator algorithms. The Vacon NX™ commercial frequency converter unit is used as the power electric hardware. This type of converter is capable of receiving external IGBT command signals and may thus be controlled with the dSPACE™ system. The FPGA is used to sample the measurements and filter the data before transferring the measurements to the dSPACE™ digital I/O. The sampling interval for the dSPACE™ system is 40 kHz while the analog cutoff frequency for the low pass filter is at 100 kHz.

### 1.5.2 Single-phase fed voltage source frequency converter

The single-phase supplied VACON NXL™ frequency converter is used as the controlled frequency converter. The original 165 µF electrolytic capacitor is replaced with a 3µF MPPF capacitor. A 0.4 mH inductor is used as the line filter. In the measurements standard 3-phase induction motors are used to function as the controlled motor. For the verification measurements a 1.1 kW VEM induction motor is used whereas for the performance test a 1.3 kW Elektror induction motor is used. Different types of systems have been used as the load. A 4 kW induction
A machine with regenerative speed controlled VACON CX™ unit is used for the verification measurements. For the performance comparison measurements a fan with adjustable inflow is used as the load. The essential experimental setup parameters are listed in appendix D.

The theoretical diagram of the used experimental device with the used measurement points is shown in Fig. 1.18. The measurements of the motor phase currents and DC-link voltage are done by the use of analog sensors. The dSPACE™ analog inputs are used to read the measurement data. The sampling frequency for the dSPACE™ A/D conversion is set to 40 kHz.

![Fig. 1.18: Basic components of the single-phase fed experimental device. The analog current and voltage measurement points are directly connected to the dSPACE™ control board via the A/D measurement circuits.](image)

### 1.6 Outline of the thesis

By removing the electrolytic capacitor from the DC-link the supply grid harmonic content can be significantly improved without having to use complex line side filter structures. The supply grid inductive filter inductance can also be decreased. After removing the capacitor practically no energy storage in the DC-link exists. This causes the DC-link voltage to fluctuate.

The aim of this work is to study and verify a frequency converter in which the DC-link electrolytic capacitor is replaced with a metallized polypropylene film capacitor. The most common frequency converter is a low-performance voltage source inverter. For this reason such a configuration is used in this work. A diode rectifier bridge is used because of small demand for regenerative braking of low-performance drives. In this work a scalar control is used as the motor control principle. The choice of scalar control is justified by the generality of the low-performance drives. The three-phase fed as well as the exceptional single-phase fed frequency converter drive
with a small DC-link capacitor are studied. Solving the problem of the capacitor’s life time by replacing the electrolytic capacitor with the MPPF capacitor produces a new problem in the form of the fluctuating DC-link voltage. When a very small capacitor is used every switching action affects the DC-link voltage. By using only the scalar control it is guaranteed that motor control does not generate any additional phenomena in supplied output voltage. For example the current controllers in current vector control could cause additional fluctuation when combined with fluctuating DC-link voltage. Thereby finding the origin of different voltage and current frequency components and unexpected operation modes in VSI with small DC-link capacitor would be more complicated. With scalar control no torque controllers are utilized. Thus, the interactions between the modulation methods and the DC-link dynamics can be studied. This also justifies the use of scalar control. The objective for this work is to study and develop a modulation strategy that compensates the DC-link voltage fluctuation while the line side benefits are still preserved. In this work, only the steady state motor operation is considered. No wide attention has been paid to the transient operation. Although for the study induction motors have been used, the small DC-link capacitor VSI does not limit the choice of the motor type to be used. The induction motor type is chosen to be used for this study mainly because of its popularity in industrial applications.

In chapter 2 the “thoroughbred” three-phase fed frequency converter is studied. The fluctuating DC-link voltage sets additional conditions to the frequency converter. These are to be taken into consideration. While modulating the voltage to the motor, the DC-link fluctuation has to be recalled. To roughly compensate the DC-link voltage fluctuation the traditional SVPWM method has been employed. The SVPWM uses the DC-link voltage as feedforward information to form the switching pattern beforehand. The measurements and simulation results are given for the purposes of verifying the SVPWM method with the small DC-link capacitor VSI drive. The problem with the SVPWM is that with a small DC-link capacitor the DC-link voltage considerably changes even during the switching period. This is true especially when low switching frequencies are used. To compensate the voltage variations during the switching period a new method named Differential Space Vector PWM (DSVPWM) is introduced and studied. The main idea of the DSVPWM method is that the stator voltage vector reference is converted to the required change of the flux linkage reference during the switching period. During the switching period the DC-link voltage waveform must be constantly observed in order to rapidly calculate the actual achieved flux linkage change. The modulator thus uses active voltage vectors so as to accomplish the reference flux linkage change. Simulation results are given for verification of the DSVPWM.
Due to the DC-link fluctuation the linear modulation region ends earlier compared with a high-capacitance DC-link capacitor VSI. This indicates that the overmodulation methods are one of the main subjects for study. The total harmonic distortion for the VSI with small DC-link capacitor is calculated and compared to the high-capacitance DC-link capacitor VSI. The overmodulation methods for controlling the fundamental voltage starting from the linear modulation all the way to the maximum available fundamental voltage are introduced, studied and tested.

Extra attention needs to be paid to the protection of the DC-link from power fed back from the motor in case of braking. Under light loading condition or when the motor is braking the power flow direction might be changed. Due to the small DC-link capacitor the DC-link voltage slew rate is much higher compared to the traditional converter. Normally, the measured DC-link voltage is used as a feedback to point out the state of braking. With a small DC-link capacitor the DC-link voltage can raise intolerably high even during one switching period of the modulator circuit. Therefore, when there is no speed encoder installed in the shaft, restraining the scalar controlled motor drive from braking turns out to be a non-trivial task. To solve this problem, a new overvoltage protection method based on the dynamic power factor control is introduced, studied and tested. The introduced method is named as DPFC. Both the simulation and measurement results are given to verify the DPFC functionality.

The DC-link capacitor combined with the line side inductance creates a resonance circuit. Traditionally, the resonance frequency has been very low. Traditionally two decade lower than switching frequency. Now, with the use of small DC-link capacitors and line side inductors, the resonance frequency becomes significantly higher. The resonance frequency can be close to or even higher than the switching frequency of the modulator. The resonance phenomenon is studied. The study presents a great number of simulation and measurement results.

In chapter 3 the single-phase fed VSI is considered. The object of the work is to study a “very much affordable” single-phase fed frequency converter for high inertia applications. The assignment for this study was: Make a single-phase fed VSI that is very gentle to the supply grid and which “as a by-product also rotates the motor”. The gentle grid behavior is gained with the small DC-link capacitor. While loaded, the DC-link voltage follows the line voltage waveform. This leads to a sinusoidal line current waveform, which is desirable. The main problem is thus to make the motor rotate. The problem is approached using the traditional SVPWM combined with the simple scalar control. With the SVPWM the main considerations are focused on the stator voltage reference formation. The SVPWM variation, named as Flower Power by the author, is introduced and verified by simulations and measurements. Also the traditional SVPWM type
modulator with estimated DC-link voltage feedforward is studied. The performance measurements have been carried out using an adjustable fan load. It was presumed that due to the heavily fluctuating torque the efficiency of the single-phase fed small DC-link capacitor VSI could not be very satisfactory. The question remaining as to how good or bad this efficiency is can only be solved in a study of the results acquired from test measurements. The performance measurement results are compared with the results obtained with a commercial, unmodified frequency converter. The single-phase fed small DC-link capacitor VSI creates a high torque oscillation in the produced electric torque. The effect of the oscillating torque on the flexible shaft has been studied and tested. The oscillation with low stiffness shaft couplings may result in unintentional shaft deterioration due to the mechanical resonance.

All the simulation models included in the study are made by author. The measurement tests and the off-line processing are carried out entirely by the author. The control algorithms included in the simulations and the experimental devices are also produced by the author. Throughout the study, the simulation and measurement results obtained are presented next to the corresponding theoretical studies. The contributions of the study can be summarized as

The contributions
- Verification of the single and three-phase fed small DC-link capacitor VSI by simulations and measurements
- Invention of the DSVPWM modulator
- Overvoltage protection based on dynamic power factor control (DPFC)
- Study of overmodulation and overmodulation method
- Study of the unorthodox single-phase fed VSI with simple modulator structure

The scientific conference publications including the above subjects


**Patents or patent applications considering this topic**

- EP Application 1511168, FI115265, DSVPWM
- FI Application number 20040838, Flower Power
- FI Application number 20041454, DPFC
2 THREE-PHASE FED VOLTAGE SOURCE INVERTER

In industry the three-phase AC is the most commonly used electricity source. In this chapter the three-phase supplied frequency converter with a small DC-link capacitor is studied. Since the research work has been accomplished with the objective of studying the effects of the fluctuating DC-link voltage on the motor supply voltage, the scalar control has been used as the motor control method. A considerable part of the research deals with the subject of modulation methods.

In the first part of this chapter, the principles of three-phase AC modulation are discussed. A new differential space vector PWM (DSVPWM) method based on the idea of SVPWM is described and verified. To compensate the DC-link voltage fluctuation, feedforward of the DC-link voltage is necessary. In both the DSVPWM and SVPWM the DC-link voltage is used as the feedforward signal. Finally, a modulation strategy is proposed.

After the study of three-phase AC modulation, overvoltage protection is discussed. Since there exists, practically, no energy storage in the DC-link, the DC-link voltage may rise intolerably high even during the single switching period due to the returning energy from the motor. Typically, no external shaft pulse encoders are used. It is therefore necessary to find a method for identifying the braking situation and protecting the VSI. A new method for that purpose, named Dynamic Power Factor Control (DPFC), is introduced and verified by simulations and measurements.

The line side filter and DC-link capacitor together create a resonance circuit. Due to the small DC-link capacitor and decreased AC-filter inductor the resonance frequency becomes significantly high. The switching frequency of the inverter bridge works as a stimulus for the resonance. This resonance phenomenon is studied at the end of this chapter.

2.1 Principles of the three-phase AC modulation

2.1.1 Space Vector PWM

In a traditional space vector modulation the correct switching instants and combination are decided at the beginning of the switching period. The input for the space vector PWM is the stator voltage vector reference in a stationary two-axis real and imaginary reference frame. The reference voltage vector (1.6) can be written as a two-component vector with \( x \) and \( y \) as

\[
\mathbf{u}_{s,\text{ref}} = u_{s,\text{ref},x} + j u_{s,\text{ref},y}.
\]  

(2.1)

The duration of the active voltage vectors \( V_m \) and \( V_{m+1} \) in a current sector \( m \) during the switching period \( T_s \) is calculated from the reference voltage vector as shown in
where the voltage vectors $V_m$ and $V_{m+1}$ are defined as

$$
V_m = \frac{2}{3} u_{DC} \ e^{j \theta_m} \\
V_{m+1} = \frac{2}{3} u_{DC} \ e^{j \theta_{m+1}}.
$$

(2.3)

The times for the active voltage vector are limited as

$$
T_s \geq t_m + t_{m+1} \ .
$$

(2.4)

The reference voltage is achieved by using the active voltage vectors $V_m$ and $V_{m+1}$, with corresponding durations $t_m$ and $t_{m+1}$. The instantaneous length of the voltage vectors depend on the DC-link voltage $u_{DC}$ as $|V| = 2/3 u_{DC}$. It is worth to point out that in SVPWM the DC-link voltage is assumed to be constant during the switching period. In the low-speed region where no overmodulation is used, the maximum radius for the circular reference voltage vector is $|u_{s,ref}| = \sqrt{3}/2 |V| = u_{DC}/\sqrt{3}$. The SVPWM method is illustrated in Fig. 2.1, where the eight discrete voltage vectors $V_m$ and the circle describing the maximum available voltage vector are shown. Also the xy-coordinate system based on the real and imaginary components is shown. The voltage vectors are defined by a combination of the conducting and non-conducting switches S1, S2 and S3 in the inverter bridge. The conducting and non-conducting mode is denoted with logic 1 and 0 respectively. Six of the voltage vectors, subscripts $m=\{1, 2, \ldots, 6\}$, are active and two of them, subscripts $m=\{0, 7\}$, represent zero-vectors. The active voltage vectors are forming 6 sectors of equal size numbered as $m=\{1, 2, \ldots, 6\}$. The SVPWM locally averages the adjacent and zero-vectors to be equal to the reference vector over the switching period $T_s$. This is illustrated in Fig. 2.2.
Fig. 2.1: The eight discrete voltage vectors $V_m$ defined by the logic states of power switches. Six of the voltage vectors, subscripts $m=\{1, 2, \ldots, 6\}$, are active and two of them, subscripts $m=\{0, 7\}$, represent zero-vectors. The active voltage vectors are forming 6 sectors of equal size numbered as $m=\{1, 2, \ldots, 6\}$. The maximum voltage circle radius is $2/\sqrt{3} |V_m|$.

Fig. 2.2: The formulation of the average voltage vector reference $u_{s,ref}$ is done by combining in the current sector $m$ two active vectors with corresponding durations $t_m$ and $t_{m+1}$ during the switching period $T_s$. Zero-vectors are used the remaining time.

From (2.1) and (2.2) it is possible to calculate the durations $t_m$ and $t_{m+1}$ needed to fulfill the reference values for the components of the stator voltage vector.
\[
\begin{bmatrix}
    t_m \\
    t_{m+1}
\end{bmatrix} = \frac{\sqrt{3} T_s}{u_{DC}} \begin{bmatrix}
    \sin \left( \frac{\pi}{3} \frac{m}{m} \right) & -\cos \left( \frac{\pi}{3} \frac{m}{m} \right) \\
    -\sin \left( \frac{\pi}{3} (m-1) \right) & \cos \left( \frac{\pi}{3} (m-1) \right)
\end{bmatrix} \begin{bmatrix}
    u_{s,refx} \\
    u_{s,refy}
\end{bmatrix}.
\]

The remaining switching period is left for the zero-vectors \( V_0 \) and \( V_7 \). The duration for the zero-vectors is calculated from

\[
t_{\text{zero}} = t_0 + t_7 = T_s - (t_m + t_{m+1}).
\]

Furthermore, the durations of the active vectors can be transformed for each phase into switching instants. In Fig. 2.3 one switching sequence is shown. The active vectors are distributed symmetrically in time.

Fig. 2.3: Formulation of the switching pattern during the switching period \( T_s \). The switching sequence starts with zero-vectors \( V_0 \). After that, the first active vector \( V_m \) is selected and hold over the pre-calculated time length \( t_m/2 \). The second active vector \( V_{m+1} \) is selected after the first active voltage vector time is reached. The second zero-voltage vector \( V_7 \) is selected after the second active voltage vector pre-calculated time \( t_{m+1}/2 \) is reached. After the first half of the switching period is gone through, the switching combination is reversed.
The calculation for the active time for the particular phase U, V, W depends on the sector. In Table 2.1 the time instant for turning on the corresponding phase is shown.

Table 2.1: The time instant for turning on the corresponding phase as a function of the sector.

<table>
<thead>
<tr>
<th>Sector, m</th>
<th>$t_{U,\text{on}}$</th>
<th>$t_{V,\text{on}}$</th>
<th>$t_{W,\text{on}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$t_0/2$</td>
<td>$t_0/2 + t_m/2$</td>
<td>$t_0/2 + t_m/2 + t_{m+1}/2$</td>
</tr>
<tr>
<td>2</td>
<td>$t_0/2 + t_{m+1}/2$</td>
<td>$t_0/2$</td>
<td>$t_0/2 + t_m/2 + t_{m+1}/2$</td>
</tr>
<tr>
<td>3</td>
<td>$t_0/2 + t_m/2 + t_{m+1}/2$</td>
<td>$t_0/2$</td>
<td>$t_0/2 + t_m/2$</td>
</tr>
<tr>
<td>4</td>
<td>$t_0/2 + t_m/2 + t_{m+1}/2$</td>
<td>$t_0/2 + t_{m+1}/2$</td>
<td>$t_0/2$</td>
</tr>
<tr>
<td>5</td>
<td>$t_0/2 + t_m/2$</td>
<td>$t_0/2 + t_m/2 + t_{m+1}/2$</td>
<td>$t_0/2$</td>
</tr>
<tr>
<td>6</td>
<td>$t_0/2$</td>
<td>$t_0/2 + t_m/2 + t_{m+1}/2$</td>
<td>$t_0/2 + t_m/2$</td>
</tr>
</tbody>
</table>

The time to turn off the switch of the corresponding phase can be calculated from

$$t_{i,\text{off}} = T_s - t_{i,\text{on}},$$

(2.7)

where $i = U, V, W$.

In some cases, it is convenient to denote the phase references with duty cycles. The duty cycle indicates the amount of active time of the switch with respect to the switching period. The duty cycle $D$ for the corresponding phase can be calculated from

$$D_i = 1 - \frac{t_{i,\text{on}}}{T_s},$$

(2.8)

where the output phase label is denoted as $i = U, V, W$ and $D \in [0,1]$.

The distribution of the zero-time between the zero-vectors $t_0$ and $t_T$ can be freely chosen. The use of only one zero-vectors leads to a situation where one phase is clamped into one of the DC-buses through the whole switching sequence. According to the research done by Holmes (1996) an even distribution of the zero-time between two zero-vector, that is $t_0 = t_T$, results in a smaller motor phase current ripple. An even distribution of the zero-vectors results in a situation that is
comparable to the triangle-comparison with triple frequency component (Blasko, 1997). In this work even distribution of zero-vectors in SVPWM is used due to the aim of smaller motor phase current ripple (Holmes, 1996).

2.1.2 Overmodulation

The zero-vector time decreases as the stator voltage vector amplitude increases. The sinusoidal output quantities are retained when the zero-vectors are present in the space vector modulation. This modulation mode is called linear modulation. After this point, the overmodulation methods are needed. Overmodulation is used to increase the voltage output of the PWM controlled frequency converter. It is important to use the full frequency converter voltage because, in using the full frequency converter voltage, it is possible to both reduce the cost and increase the output power. Furthermore, an electric drive with high overmodulation performance is less sensitive to DC-link voltage disturbances.

A DC-link voltage sag may lead in an unintentional entering to the overmodulation region. The DC-link sag is a result of the line voltage sag or fault conditions of electric drive. When smaller DC-link capacitors are used, voltage sags caused by the full-wave rectifier bridge are present all the time. Under such conditions, a high performance overmodulation method could maintain the drive performance as much as possible. The inverter blanking time and minimum-pulse-width constraints could further reduce the linearity range. For reasons of simplicity, the following study of overmodulation does not take into consideration the blanking time and minimum-pulse-width limitations.

The modulation index $M$ is used to describe the voltage utilization of the modulator. Here, similar to the reference of Holtz (1993), the modulation index is defined with the magnitude of the fundamental voltages of the inverter output voltage $u_t$ and the theoretical maximum voltage under six-step operation $u_{six-step} = \frac{2}{\pi} u_{DC}$. For a given DC-link voltage $u_{DC}$, the modulation index $M$ is defined as

$$M = \frac{u_t}{u_{six-step}} = \frac{u_t}{\frac{2}{\pi} u_{DC}}.$$  \hfill (2.9)

In the linear modulation region the stator voltage reference does not exceed the maximum available voltage circle. When a big DC-link capacitor is used the DC-link voltage, essentially, remains constant. The constant DC-link voltage results in a hexagon-shaped maximum available
stator voltage vector. This is illustrated in Fig. 2.4. The maximum fundamental output voltage is thus \( u_f = \frac{u_{\text{DC}}}{\sqrt{3}} \approx 0.577u_{\text{DC}} \) and the modulation index \( M \leq \frac{\pi}{2\sqrt{3}} \approx 0.907 \). After this point, overmodulation methods are needed.

![Diagram showing modulation index](image_url)

Fig. 2.4: The maximum sinusoidal behavior retaining stator voltage vector is achieved with the modulation index \( M \leq 0.907 \). The hatched area represents the linear modulation region.

Overmodulation I (OM I) is a modulation region, where the stator voltage vector amplitude is increased all the way up to the maximum available stator voltage length. The stator voltage reference angle remains unchanged but the amplitude includes distortion. With a stiff DC-link the maximum amplitude follows the symmetrical time invariant hexagon. This is illustrated in Fig. 2.5. When the amplitude of the stator voltage vector is increased to the maximum, the resulting maximum output is limited to the hexagon. The fundamental output voltage magnitude becomes \( u_f = \frac{\sqrt{2/\pi}}{\sqrt{3}}u_{\text{DC}} \approx 0.606u_{\text{DC}} \). The modulation index becomes \( M \leq \frac{\sqrt{\pi/2}}{\sqrt{3}} \approx 0.952 \).

Bolognani et al. (1996) derived an analytical form for the amplitude of the fundamental component. According to the study, the maximum achievable modulation index under overmodulation I mode is \( M \leq \frac{\sqrt{3 \ln(3)}}{2} \approx 0.951 \). Whereas the linear modulation has a sinusoidal stator voltage reference, overmodulation I has a distorted continuous stator voltage reference. The original reference

\[
\begin{align*}
\mathbf{u}_{s,\text{ref}} &= \mathbf{u}_{s,\text{ref}}^0 + j\mathbf{u}_{s,\text{ref}}^\theta
\end{align*}
\]

[1.6]
is reformulated into

$$u_{\text{COMI}} = u_{\text{ref,lim}} e^{j\theta_{\text{ref}}},$$

(2.10)

where subscript lim stands for limited quantity.

![Diagram of voltage vectors](image)

Fig. 2.5: The maximum stator voltage vector under overmodulation I. When the modulator uses all the available voltage limiting only the amplitude to the voltage hexagon, a modulation index up to $M \leq 0.952$ is achieved. The modulated voltage is continuous but due to the amplitude limitations it includes distortion.

Under linear modulation the fundamental output voltage is expected to increase linearly as a function of the voltage vector reference amplitude. The limit is $u_{\text{DC}}/\sqrt{3}$. After this point overmodulation I is used and according to Bolognani et al. (1996) the fundamental voltage output $u_{\text{COMI}}$ can be calculated as a function of the DC-link voltage $u_{\text{DC}}$ and stator voltage vector amplitude $|u_{\text{ref}}|$. 

$$u_{\text{COMI}} = \frac{6}{\pi} \left( u_{\text{ref}} |\alpha_g + \frac{u_{\text{DC}}}{\sqrt{3}} \ln \left( \tan \left( \frac{\pi}{3} \frac{\alpha_g}{2} \right) \right) \right),$$

(2.11)

where
\[ \alpha_s \left( \frac{|u_{s,ref}|}{u_{DC}} \right) = \frac{\pi}{6} - \cos^{-1}\left( \frac{u_{DC}}{\sqrt{3}} \right) \left( \frac{u_{s,ref}}{u_{DC}} \right) \].

(2.12)

\[
\frac{u_{DC}}{\sqrt{3}} \leq \frac{|u_{s,ref}|}{u_{DC}} \leq \frac{2u_{DC}}{3}
\]

After overmodulation I has reached the upper limit, overmodulation II (OM II) becomes active. Under overmodulation II the essential feature is that the particular active voltage vector that is closest to the stator voltage reference vector is used gradually longer and longer time periods. In the final state the method called six-step is applied, which means that each state vector is used one sixth of the fundamental period. Whereas the linear modulation has a sinusoidal stator voltage reference, overmodulation I has a distorted continuous stator voltage reference. While overmodulation I has a distorted continuous stator voltage reference, overmodulation II has a distorted discontinuous stator voltage reference. The absolute maximum fundamental voltage available from the VSI is achieved under six-step operation.

Under overmodulation II the controller affects both the amplitude and angle of the stator voltage reference vector. The original stator voltage vector reference (1.6) is reformulated into

\[
\lim_{\text{ref}, s, \text{OMII}} = \left| \lim_{\text{ref}, s, \text{OMII}} \right| e^{j \theta_{\text{ref,lim}}}.
\]

(2.13)

In the overmodulation II method, shown in Holtz et al. (1993), the trajectory of the \( u_{s,ref} \) is limited to the hexagon while the angle is defined as a function of the hold-angle \( \alpha_h \). The hold-angle is a function of the modulation index, that is \( \alpha_h = f(M) \). The value of the hold-angle is limited between \([0 \ldots \pi/6]\). The value \( \alpha_h = \pi/6 \) equals the six-step mode. The controlled, limited stator voltage vector reference angle \( \theta_{\text{ref,lim}} \) can be calculated from

\[
\begin{align*}
\theta_{\text{ref,lim}} &= \begin{cases} 
0 & \text{for } 0 \leq \theta_{\text{ref}} \leq \alpha_h \\
\frac{\pi(\theta_{\text{ref}} - \alpha_h)}{(\pi - 6\alpha_h)} & \text{for } \alpha_h \leq \theta_{\text{ref}} \leq \frac{\pi}{3} - \alpha_h \\
\frac{\pi}{3} & \text{for } \frac{\pi}{3} - \alpha_h \leq \theta_{\text{ref}} \leq \frac{\pi}{3}.
\end{cases}
\end{align*}
\]

(2.14)

More about overmodulation methods can be found from literature. Holtz (1992, 1993, 1994) and Kerkman (1993, 1995, 1996), for example, are authors who have treated the subject in several...
studies that are widely referred to in the literature. One of the most comprehensive survey and research works on overmodulation is done by Hava (1998). In the study by Tarkiainen (2005) the modulation index and fundamental voltage produced by the DTC is introduced.

Fig. 2.6 shows the usage of the modulation methods and the index of the achievable modulation.

Fig. 2.6: Modulation methods and the index of the modulation. Under linear modulation the stator voltage vector reference does not need to be limited by the modulator in any way. The modulation index 0.907 can be achieved by using linear modulation. With overmodulation I the modulation index can be increased to 0.952, however at the expense that the limitations of the stator voltage vector amplitude will cause distortion. Overmodulation II is used to further increase the modulation index up to the theoretical maximum value of 1. The limitations in the stator voltage vector reference influence both the stator voltage vector amplitude and angle.

2.1.3 Analytical estimation of the DC-link voltage fluctuation

Kretschmar et al. (1998) have studied the DC-link fluctuation. They divide fluctuation into two parts depending on the fluctuation source. The first part is the fluctuation caused by the full-wave diode rectifier bridge. The mean value of the DC-link voltage $u_{\text{DC,mean}}$ gained from the rectifier bridge is

$$
u_{\text{DC,mean}} = \frac{3\sqrt{2}}{\pi}U_n$$

$$\approx 540 \text{ V}$$

$$\approx 1.65 \text{ pu,}$$

(2.15)

where $U_n$ is the nominal line-to-line RMS voltage. For the numerical evaluation the value of $U_n$ was set to 400 V.

The fluctuation caused by the rectifier $\Delta u_{\text{DC,rec}}$ can be calculated from
\[
\Delta u_{\text{DC}_{\text{inv}}} = \left( \sqrt{2} - \frac{3}{\sqrt{2}} \right) U_n
\]
\[
\approx 76 \text{ V}
\]
\[
\approx 0.23 \text{ pu}
\]  

(2.16)

According to Kretschmar et al. (1998) the DC-link voltage fluctuation caused by the inverter bridge \( \Delta u_{\text{DC}_{\text{inv}}} \) can be represented by means of the steady state power \( P \), the DC-link voltage mean value \( U_{\text{DC}} \) and the switching period \( T_s \) as

\[
\Delta u_{\text{DC}_{\text{inv}}} = \frac{4 M - 2}{8} \frac{P T_s}{C U_{\text{DC}}}
\]  

(2.17)

Hence, the modulation index used in Kretschmar et al. (1998) is scaled here to be equivalent to the modulation index based on the six-step unity modulation index, which is used in this work. The equation can thus be considered only up to the modulation index \( M = \pi/4 \), which is the upper limit of the analog realized PWM without an added third harmonic component in the carrier signal.

The total peak-to-peak worst case for DC-link voltage fluctuation \( \Delta u_{\text{DC}} \) can be estimated by summing the fluctuations due to the rectifier and inverter bridge as

\[
\Delta u_{\text{DC}} = \Delta u_{\text{DC}_{\text{inv}}} + 2|\Delta u_{\text{DC}_{\text{rec}}}|
\]  

(2.18)

where the fact that the inverter bridge fluctuation \( \Delta u_{\text{DC}_{\text{inv}}} \) can be both negative and positive is noticed. A more precise formulation and explanations of the equations are presented in the original reference (Kretschmar et al., 1998).

2.1.4 Verification of the small DC-link capacitor VSI

To verify the idea of the small DC-link capacitor VSI, simulations have been carried out. Scalar control and a SVPWM modulator with 100 \( \mu \)s switching period \( T_s \) yielding a 10 kHz switching frequency \( f_{\text{sw}} \) has been used. The nominal torque and half of the nominal mechanical speed has been used, that means the power \( P \) becomes 2 kW. A 2 \( \mu \)F capacitor has been used as the DC-link capacitor \( C \). The DC-link voltage has been used as the feedforward in the SVPWM. Fig. 2.7 shows simulation results of one line current \( i_{\text{grid}} \), the DC-link capacitor current \( i_C \) and the DC-link voltage \( u_{\text{DC}} \) waveforms with corresponding FFT-figures. For the purpose of simulation model verification, the same situation was created with an experimental device. In Fig. 2.8 a measured
line current $i_{\text{grid}}$, DC-link capacitor current $i_C$ and the DC-link voltage $u_{\text{DC}}$ are shown. The corresponding FFT-figures are also shown. Comparison between simulation results and experimental device measurement results clearly shows the accuracy of the simulation model. Another much desired feature, present in both the simulated and measured supply grid current, is the frequency content shown in the FFT. As can be seen, the supply grid current harmonic content is much friendlier to the line properties compared with traditional frequency converters. By substituting the values obtained from the identical simulations and measurements discussed above the estimated fluctuation due to the inverter bridge can be estimated analytically from (2.17). With half the nominal speed the modulation index becomes

$$M = \frac{1}{2} \frac{u_{\text{DC}}}{\frac{\sqrt{3}}{2} \frac{\pi}{u_{\text{DC}}}} \approx 0.45$$

(2.19)

and the fluctuation due to the inverter bridge can be evaluated as

$$\left| \Delta u_{\text{DC,inv}} \right| \approx 33 \text{ V}$$

$$\approx 0.1 \text{ pu}$$

(2.20)

The result is in good accordance with the simulated (Fig. 2.7) and measured (Fig. 2.8) DC-link voltage fluctuation. Hence, the fluctuation caused by the inverter bridge is distributed mainly to the switching frequency with a component of approximately 10 V and to the second harmonic component of the switching frequency with a component of approximately 20 V. The result of the fluctuation due to the rectifier bridge, given in (2.16), can be scaled to equal the FFT values shown in the figures. The FFT value represents the peak value of the corresponding sinusoidal signal. Thus, the fluctuation originating from the diode rectifier bridge, shown in (2.16), becomes

$$\Delta u_{\text{DC,rec}} = \frac{\Delta u_{\text{DC,rec}}}{2} = 38 \text{ V}$$

when scaled to correspond the result given in the figures. The fluctuation amplitudes $\Delta u_{\text{DC,rec}}$ and $\Delta u_{\text{DC,inv}}$ are in good accordance with the simulation and measurement results. The analytically evaluated DC-link voltage fluctuations described in Kretschmar et al. (1998) show the scalability of the analytic equations for different sizes of the DC-link capacitor. Thus, the equations (2.16) and (2.17) can be used to approximate the DC-link capacitor size required to limit the DC-link voltage fluctuation to an acceptable level.
Fig. 2.7: Simulated supply grid current, capacitor current and DC-link voltage waveforms and their FFT figures. The supply grid current frequency content shown in the corresponding FFT is as previously assumed. The 5th and 7th harmonic component are lower compared to the traditional case. The DC-link voltage waveform has a strong six-fold line frequency component of 50 V that is generated by the diode rectifier. The inverter bridge switching frequency is clearly visible in the DC-link and can be observed in the FFT figure as the 10 kHz and 20 kHz components. The sum of the 1st and 2nd component of the inverter bridge is approximately 30V which is in good accordance with the estimation.
Fig. 2.8: Measured supply grid current, capacitor current and DC-link voltage waveforms and their FFT figures. The measurements back up the simulation results. The 5th and 7th harmonic component as lower compared to the traditional case. The DC-link voltage waveform has a strong six-fold line frequency component of 40 V that is generated by the diode rectifier. The inverter bridge switching frequency is clearly visible in the DC-link and can be observed in the FFT figure as 10 kHz and 20 kHz components. The sum of the 1st and 2nd component of the inverter bridge is approximately 25V which is pretty close to the estimated fluctuation caused by the inverter bridge.

In Fig. 2.9 the measured motor phase currents $i_u$, $i_v$ and $i_w$ and the calculated stator current vector trajectory of the measurement case of Fig. 2.8 are shown. The electrical output frequency is approximately 27 Hz, which results in a 0.5 pu mechanical output frequency. There is no significant DC-link voltage fluctuation originated distortion in the motor phase currents and stator current vector trajectory. The traditional SVPWM is thus capable of compensating roughly the DC-link voltage fluctuation from the output voltage.
In order to illustrate the difference in the supply grid line current, the measurement results, pointing out the difference between the original and modified VSI, are shown in Fig. 2.10. In both cases, equal power is supplied to the motor. The switching frequency of the scalar controlled SVPWM is 10 kHz. The figure clearly shows that the traditional frequency converter with big DC-link capacitor causes significantly higher harmonic frequency components at the frequencies of 250 Hz and 350 Hz which correspond to the 5th and the 7th harmonic component.
Fig. 2.10: Measured supply grid current for a standard commercial high-capacitance frequency converter ($L_{AC}=3.4\, \text{mH}, C=165\, \mu\text{F}$) and for a frequency converter with small DC-link capacitor ($L_{AC}=0.4\, \text{mH}, C=2\, \mu\text{F}$). The corresponding FFT figures are shown. The supply grid current wave forms differ greatly from each other. The VSI with small DC-link capacitor generates smaller harmonic frequency components at the frequencies of 250 Hz and 350 Hz which correspond to the 5th and the 7th harmonic component.

2.2 Improvement of modulation in the small DC-link capacitor VSI

To compensate the DC-link fluctuation during the switching sequence, a new method called DSVPWM is introduced. Also changes in the overmodulation behavior are considered later in this chapter.

2.2.1 Differential SVPWM (DSVPWM)

DC-link voltage fluctuation naturally affects the maximum available instantaneous stator voltage vector (Fig. 2.11). Especially at low switching frequency the DC-link voltage differs significantly from the measured $u_{DC}$ during the switching period $T_s$. That is why the traditional space vector PWM cannot work accurately under heavily fluctuating DC-link voltage.
Fig. 2.11: The behavior of six discrete voltage vectors and the largest voltage circle under fluctuating DC-link voltage. The fluctuation of the DC-link voltage takes the shape of pulsating active voltage vectors. At a certain time instant the DC-link voltage level determines the length of the active vectors to $2/3 u_{DC}$. In the figure, four different lengths of active voltage vectors are shown with the corresponding values of DC-link voltage.

The new concept, named differential SVPWM (DSVPWM), is based on the needed change of the flux linkage $\Delta \psi$ during the switching period $T_s$. The change of the flux linkage reference can be derived from (2.2) by multiplying both sides of the equation by $T_s$. The reference value is updated once per switching cycle. The calculation of the reference for the flux linkage change is shown in

$$u_{s,ref} T_s = t_m V_m + t_{m+1} V_{m+1}$$

$$\Delta \psi_{s,ref} = \frac{u_{s,ref}}{T_s} = \Delta \psi_m + \Delta \psi_{m+1}$$

By substituting (2.3) into (2.21) we get the reference change for the flux linkage for both active vectors $V_m$ and $V_{m+1}$ in the current sector $m$

$$\Delta \psi_{s,ref} = t_m V_m + t_{m+1} V_{m+1}$$

$$= t_m \frac{2}{3} u_{DC} e^{j\frac{\pi}{3} (m-1)} + t_{m+1} \frac{2}{3} u_{DC} e^{j\frac{\pi}{3} m}.$$ (2.22)

Now the reference for the change of the flux linkage amplitude during the switching period in sector $m$ is defined as
The idea of the references of the flux linkage change in the direction of the active voltage vectors is illustrated in Fig. 2.12.

As the reference for the change of the flux linkage is known, it is necessary to make sure that the references are achieved despite the fluctuating DC-link voltage. To find out the time instant at which the reference flux linkage change is met during the switching period, the DC-link voltage $u_{\text{DC,meas}}$ is observed and integrated during the corresponding active voltage vector. The calculated flux linkage changes $\Delta\psi_{\text{m,calc}}$ and $\Delta\psi_{\text{m+1,calc}}$ can be defined as

$$\Delta\psi_{\text{m,calc}} = \int V_m \, dt = \int \left( \frac{2}{3} u_{\text{DC,meas}} e^{j\varphi_{\text{meas}}} \right) \, dt, \quad \text{when } V_m \text{ is active}$$

$$\Delta\psi_{\text{m+1,calc}} = \int V_{m+1} \, dt = \int \left( \frac{2}{3} u_{\text{DC,meas}} e^{j\varphi_{\text{meas}}} \right) \, dt, \quad \text{when } V_{m+1} \text{ is active}$$

The voltage vector is changed when the calculated change of the flux linkage reaches the corresponding reference value. Hence, the time instant at which the reference flux linkage change is met and the voltage vector is changed is not known in advance.

To preserve the symmetrical SVPWM switching pattern the DSVPWM switching behavior can be chosen such that the symmetrical behavior is maintained. The switching period can be divided
into two sections: Section 1 is from 0 to \(T_s/2\) and section 2 is from \(T_s/2\) to \(T_s\). During the first period the halved values of the flux linkage changes are fulfilled. The switching sequence goes as follows: The first active voltage vector \(V_m\) is chosen at the beginning of the switching period \(T_s\). The vector is kept until the limit of \(\Delta \psi_{m,ref}/2\) is achieved. After that, the second active voltage vector \(V_{m+1}\) is chosen and kept until the reference of \(\Delta \psi_{m+1,ref}/2\) is achieved. After both active vectors are used, the zero-vector \(V_7\) is kept until the switching period reaches halfway \(T_s/2\). After half of the switching period is accomplished, the sequence is reversed and the second active voltage vector \(V_{m+1}\) is chosen. The vector is kept until the second half of the flux linkage change \(\Delta \psi_{m+1,ref}/2\) is met. After that, the first active voltage vector \(V_m\) is used until the reference flux change \(\Delta \psi_{m,ref}/2\) is reached. After that, the zero-vector \(V_0\) is used until the end of the switching sequence. A theoretical drawing of the DSVPWM switching sequence is shown in Fig. 2.13.

![Switching sequence of the DSVPWM](image)

**Fig. 2.13:** The switching sequence of the DSVPWM. Differently from the SVPWM, the DSVPWM switching sequence begins with the active voltage vector. The first active voltage vector \(V_m\) is chosen at the beginning of the switching period \(T_s\). The vector is kept until the limit of \(\Delta \psi_{m,ref}/2\) is achieved. After that, the second active voltage vector \(V_{m+1}\) is chosen and kept until the reference of \(\Delta \psi_{m+1,ref}/2\) is achieved. After both active vectors are used, the zero-vector \(V_7\) is kept until the switching period reaches halfway \(T_s/2\). After half of switching period is accomplished, the sequence is reversed and the second active voltage vector \(V_{m+1}\) is chosen. The vector is kept until the second half of the change of flux linkage change \(\Delta \psi_{m+1,ref}/2\) is met. After that, the first active voltage vector \(V_m\) is used until the reference flux change \(\Delta \psi_{m,ref}/2\) is reached. After that, the zero-vector \(V_0\) is used until the end of the switching sequence.

The references for the flux linkage changes can be updated in the middle of the switching period \(T_s/2\). If the switching frequency is low, updating the references in the middle of the switching period improves the accuracy of the produced voltage.

During the switching period, both references \(\Delta \psi_{m,ref}\) and \(\Delta \psi_{m+1,ref}\) are being fulfilled. Thus, we achieve the voltage reference \(\Delta \psi_{m,ref}\) given by the upper controllers despite the fluctuating DC-link
voltage. Fig. 2.14 illustrates in principle the simulation results in a case where the voltage vector reference is in sector $m=3$. Two switching periods are shown. The switching sequence starts with the first active voltage vector $V_3$. The second active voltage vector is selected when
\[ \Delta \psi_{m,\text{calc}} \geq \frac{1}{2} \Delta \psi_{m,\text{ref}} \]
is reached. After that, $V_4$ is used until the condition
\[ \Delta \psi_{m+1,\text{calc}} \geq \frac{1}{2} \Delta \psi_{m+1,\text{ref}} \]
is true. The zero-vector $V_7$ is used during the remaining time of the halved switching period. The other side of the halved switching sequence is reversed and both active vectors are used until the corresponding reference flux changes are met. $V_0$ is used over the remaining switching period. During these two simulated switching periods the symmetrical switching pattern is preserved. Though symmetrical, it can be seen that the times used for each active voltage vector are not identical during the switching period. That is because of the DC-link voltage which is not constant. For simulation purposes, a 5 kHz switching frequency has been used and the integration time step in equation (2.24) was $1 \mu s$.

![Diagram](image-url)

Fig. 2.14: Simulated functionality of the switching sequence of the DSVPWM under heavily fluctuating DC-link voltage. Under low DC-link voltage the active time of the corresponding vector is increased and under high DC-link voltage the active time of the vector is decreased to ensure that the reference is achieved accurately.
To illustrate the superiority of the DSVPWM modulation method simulations have been carried out for the purpose of comparing the DSVPWM with the SVPWM. In the simulations, both modulation methods are exposed to DC-link voltage fluctuation. The reference vector amplitude is kept at 0.5 pu while the stator voltage vector angle reference is varied from 0 to \( \pi/3 \) at the steps of \( \pi/60 \). A switching frequency of 5 kHz is used. In each simulation, the reference voltage vector is kept in a constant position for multiple switching periods while the DC-link voltage is fluctuating. The error in the resulting modulated voltage vector over a switching period of 200 \( \mu s \) is calculated. The produced average voltage vector, denoted with subscript act, can be found by integrating the DC-link voltage during the corresponding active voltage vector as shown below

\[
\begin{align*}
    u_{m,act} &= \frac{1}{T_s} \int_{0}^{T_s/3} u_{dc} \, dt, \text{ when } V_m \text{ is active} \\
    u_{m+1,act} &= \frac{1}{T_s} \int_{0}^{T_s/3} u_{dc} \, dt, \text{ when } V_{m+1} \text{ is active}
\end{align*}
\]  

The produced voltage vector \( u_{s,act} \) in the stator reference frame is then calculated from

\[
\begin{bmatrix}
    u_{s,x,act} \\
    u_{s,y,act}
\end{bmatrix} =
\begin{bmatrix}
    \cos\left(\frac{(m-1)\pi}{3}\right) & \cos\left(\frac{m\pi}{3}\right) \\
    \sin\left(\frac{(m-1)\pi}{3}\right) & \sin\left(\frac{m\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
    u_{m,act} \\
    u_{m+1,act}
\end{bmatrix}
\]  

(2.26)

Fig. 2.15 and Fig. 2.16 demonstrate the error calculation. Three switching sequences are shown in both of the figures from which it is clear that the SVPWM modulator is not able to fulfill the references accurately during the switching period (Fig. 2.15). Small but still significant errors between the corresponding reference and actual values can be found. In the case of DSVPWM, the error is significantly smaller and the actual voltages match with the references (Fig. 2.16).
Fig. 2.15: Simulated realization of the modulated voltage vector with the SVPWM modulator under heavily fluctuating DC-link voltage. The SVPWM creates a significant error. The error can be seen at the end of the switching sequences at the time instants 200 µs, 400 µs and 600 µs. The produced voltage vectors $u_{m,act}$ and $u_{m+1,act}$ do not meet accurately their corresponding reference levels during the switching sequence.

Fig. 2.16: Simulated realization of the modulated voltage vector with the DSVPWM modulator under heavily fluctuating DC-link voltage. The DSVPWM compensates the DC-link fluctuation caused error. The produced voltage vectors $u_{m,act}$ and $u_{m+1,act}$ do meet accurately their corresponding reference levels during the switching sequences of 0 to 200 µs, 200 µs to 400 µs and 400µs to 600 µs.
Fig. 2.17 shows the variations of the resulting voltage vector time-averages produced by the SVPWM and the DSVPWM during multiple switching periods with different reference angles. With the SVPWM modulator the DC-link fluctuation causes error in the realization of the stator voltage vector whereas with the DSVPWM method the produced voltage follows the reference value with good accuracy.

So far, only the linear modulation region is covered with DSVPWM method. The use of the DSVPWM in the overmodulation region produces an error in the average voltage vector angle. The amplitude is limited to the hexagon. This is illustrated in the Fig. 2.18, where an overlong stator voltage vector reference $u_{s,ref}$ is used as an input for the DSVPWM modulator. In the figure, 20 simulated pairs of reference $u_{s,ref}$ and actual produced average voltage vector $u_{s,DSVPWM}$ with constant DC-link voltage are shown.
In overmodulation I the limitations in produced average voltage vector amplitude are allowed but the angle should remain unchanged as shown in (2.10). Overmodulation methods for covering the voltage supply region from the linear modulation to the maximum theoretical available voltage will be discussed next.

### 2.2.2 Overmodulation in the case of the small DC-link capacitor

Considering frequency converters that have constant DC-link voltage, overmodulation appears to be a challenging task. For frequency converters that have a small DC-link capacitor, overmodulation is an even more demanding task. Depending on the fundamental output frequency, there exists different amount of voltage sags in output voltage per single output cycle due to the rectifier bridge. The amount of sags can be found from
voltage sags during one period of the output cycle \[ \frac{6f_{\text{grid}}}{f_{\text{motor}}} \] (2.27)

Although the average value of the DC-link voltage is approximately equal to the version in conventional size, the voltage drops need to be taken into account. Usually, DC-link voltage drops due to the supply voltage failures are relatively slow. Now, as practically no energy storage exists, the dynamic behavior of the DC-link is considerably faster compared with the traditionally dimensioned frequency converter. Due to the voltage drops caused by the rectifier bridge, the entry into the overmodulation region occurs earlier compared with the traditional frequency converter.

Overmodulation

The DC-link voltage waveforms used in the theoretical overmodulation calculations for describing the VSI with small and big DC-link capacitor are shown in Fig. 2.19. The mean value is used to describe the stiff DC-link voltage when a big DC-link capacitor is used. The theoretical full-wave rectified voltage waveform is used to describe the DC-link voltage in the case of a small DC-link capacitor. No feedforward from the modulator output to the DC-link voltage is included. In reality, the modulator interacts with the DC-link voltage waveform and creates additional distortion. This phenomenon is neglected in the following study for reasons of simplicity.

![DC-link voltage waveforms](image.png)

Fig. 2.19: DC-link voltage waveforms used in the simulations. The ideal rectifier voltage is used as a DC-link voltage to model the DC-link without energy storage while the mean value of the ideal rectifier voltage is used as a stiff DC-link voltage.
The total harmonic distortion is used to illustrate the differences in output phase voltage comparing the high-capacitance and small DC-link capacitor. The THD is defined as the RMS value of the total harmonics of the signal, divided by the RMS value of its fundamental signal. The voltage vector can be transformed in the phase voltages with (1.20). In the case of the U-phase voltage, the THD is defined as

$$\text{THD}_U = \sqrt{\frac{\sum_{i=2}^{n} U_{U,i}^2}{U_{U,1}}},$$

(2.28)

where $U$ is the RMS value of the voltage. Subscript $U$ defines the phase and $i=\{2,3,\ldots,n\}$ defines the harmonic number. The fundamental value is identified with subscript 1.

The same overmodulation I methods discussed earlier can be utilized for overmodulation of the small DC-link capacitor VSI. Under the overmodulation I mode the voltage amplitude is supposed to track the fluctuating hexagon as shown in Fig. 2.11. Fig. 2.20 shows one electrical period of the stator voltage reference vector $u_{s,\text{ref}}$, the maximum vector $u_{s,\text{max}}$ and the actual modulated limited vector path $u_{s,\text{mod}}$. The frequency of the stator voltage vector reference in the case described in the figure is equal to the grid frequency. Due to the motor output frequency, 6 voltage drops are taking place during one revolution of the voltage vector. Also the phase of the line voltage and the motor voltage is chosen to be equal. Although this situation is possible, it is rare. However the case is used because it points out the maximum fluctuation of the maximum available voltage vector. The fluctuation is due to the combined dc-link and hexagon originated fluctuation. The combined effect of the fluctuating DC-link voltage and the hexagon shaped maximum voltage vector is represented in Fig. 2.21. The limits, which the combined effect of the hexagon and DC-link fluctuation produces, are shown. Four different local minimum and maximum points can be found. These are listed in Table 2.2. The absolute value depends on the line side properties that affect the minimum and maximum absolute values of the DC-link voltage.
Fig. 2.20: One electrical period of the stator voltage reference vector $u_{s,\text{ref}}$, the maximum vector $u_{s,\text{max}}$ and the actual modulated limited vector path $u_{s,\text{mod}}$. Here, the frequency of the stator voltage vector reference is equal to the grid frequency. When the voltage is insufficient the modulator limits the voltage vector amplitude to the maximum voltage vector amplitude available.

Table 2.2: Four local extreme values of the stator voltage vector value

<table>
<thead>
<tr>
<th>$u_{\text{DC}}$</th>
<th>Hexagon shape effect</th>
<th>Combined effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min, $\sim 1.5u_B$</td>
<td>Min, $u_{\text{DC}}/\sqrt{3}$</td>
<td>$\sim 0.866u_B$</td>
</tr>
<tr>
<td>Max, $\sim 1.73u_B$</td>
<td>Min, $u_{\text{DC}}/\sqrt{3}$</td>
<td>$\sim 1u_B$</td>
</tr>
<tr>
<td>Min, $\sim 1.5u_B$</td>
<td>Max, $(2/3)u_{\text{DC}}$</td>
<td>$\sim 1u_B$</td>
</tr>
<tr>
<td>Max, $\sim 1.73u_B$</td>
<td>Max, $(2/3)u_{\text{DC}}$</td>
<td>$\sim 1.153u_B$</td>
</tr>
</tbody>
</table>
Fig. 2.21: The combined effect of the DC-link voltage and hexagon shape on the instantaneous maximum available stator voltage vector trajectory. The limits set by the combined effect of the hexagon and the DC-link fluctuation produces are shown. Four different local minimum and maximum points can be found. When the minimum DC-link voltage level and the minimum hexagon radius are combined the maximum voltage radius is about 0.866 pu. When the maximum DC-link voltage level is combined with the minimum hexagon radius or the minimum DC-link voltage level with the maximum hexagon radius the output voltage radius is approximately 1 pu. When the maximum hexagon radius and the maximum DC-link voltage level are combined the output voltage vector in length of 1.153 pu is achieved.

Fluctuation due to the rectifier bridge occurs at a frequency of six times the line frequency, that is $6\omega_{\text{grid}}$. The fundamental frequency of the motor $\omega_{\text{ref}}$ and the DC-link fluctuation $6\omega_{\text{grid}}$ form a wobble phenomenon. This means that the instantaneous value of the fundamental voltage and its harmonics are fluctuating. The symmetry of the hexagon shaped maximum voltage vector trajectory creates a frequency which is six times the fundamental motor frequency. Because of the wobble, the overmodulation fundamental voltage and its harmonic content needs to be studied over the wobble period. Hence, the motor frequency is a parameter in the study. During the wobble period the fluctuation of the DC-link voltage and the hexagon shaped maximum instantaneous voltage together have combined all the amplitude limitation conditions in the different voltage vector directions. The time period of wobbling $t_{\text{wobble}}$ can be expressed as
At certain motor frequencies the idea of the wobble period is insufficient for the purpose of studying the variation levels of the total harmonic distortion (THD) and fundamental voltage. These frequencies are $\omega_{\text{ref}} = \{1, 1.2, 1.5, 2, 3, 6\}$ pu. These frequencies form a constant behavior with the hexagon and DC-link waveform. At these frequencies, the THD and fundamental voltage level in a certain phase are a function of the stator voltage vector initial angle versus the instantaneous DC-link voltage level. The stator voltage vector initial angle $\theta_{s,\text{ref},0}$, that is the initial phase of the output phase voltage, is included in the study as a parameter. The frequencies causing constant behavior of the phase voltage are illustrated in Fig. 2.22.

The simulations have been used to carry out the theoretical study of the effect of the small DC-link capacitor effect on the voltage supply of the VSI. The maximum and minimum THD level as a function of the initial stator voltage vector angle $\theta_{s,\text{ref},0}$ and angular frequency reference $\omega_{\text{ref}}$ are shown in Fig. 2.23. As the upper limit for the angular frequency reference, $\omega_{\text{ref}} = 2.6$ pu was chosen to cover most of the mechanical applications.
Fig. 2.23: Maximum and minimum THD levels during the wobble period as a function of the stator voltage vector initial reference angle and the electric frequency of the motor. The THD levels are depended on the electric frequency of the motor. The THD level is varying between the minimum and maximum level. The exceptions are the frequencies causing for the phase voltage waveform to have constant behavior. At these frequencies the hexagon shape and DC-link voltage instantaneous levels form the time invariant voltage supply waveform. At these frequencies the THD level is constant. However, the constant value level is a function of the initial phase of the phase voltage of the motor.

The corresponding fundamental voltage $u_f$ maximum and minimum RMS components as a function of the initial stator voltage vector angle and angular frequency are shown in Fig. 2.24.
Fig. 2.24: Maximum and minimum fundamental voltage levels during the wobble period as a function of the stator voltage vector initial reference angle and the electric frequency of the motor. The fundamental voltage levels are fluctuating in a similar way as the THD levels. In a similar way the fundamental voltage is affected by the frequencies causing for the phase voltage waveform to have constant behavior.

Projecting the Fig. 2.23 and Fig. 2.24 versus the angular frequency and corresponding maximum and minimum THD and fundamental voltage levels, the effect of the stator voltage vector initial angle can be omitted. The maximum and minimum THD levels and the corresponding fundamental voltage components for the big and small DC-link capacitor VSI are calculated and shown in Fig. 2.25. It can be seen, that the THD levels of the small DC-link capacitor VSI are higher compared to the corresponding levels of the stiff DC-link VSI. The average fundamental phase voltage component is however equal if the whole wobble period is considered. The
theoretical modulation index $M=0.952$ is thus achieved in both cases. The effect of the wobbling can be seen especially in the THD level variations.

Overmodulation II

The overmodulation II method described before is intended for constant DC-link voltage applications. However it could also be used with the small DC-link capacitor VSI. The most interesting alternative overmodulation method is presented by Bolognani et al. (1996). The method is called Continuous Control. The idea is simply to track the angle that fulfills the stator voltage vector reference amplitude. Under traditional overmodulation II, presented for example in Holtz (1993), both the amplitude and angle of the stator voltage vector are controlled. Now, only the angle is controlled. This results in simpler structure of the overmodulation control algorithms.
Bolognani et al. suggested that the Continuous Control can handle the whole region from the beginning of the overmodulation I mode to the sixstep mode. There is, however, no reason why the traditional overmodulation I mode could not be used first. On the contrary, by keeping the angle of the stator voltage vector unchanged, as in OMI, the torque oscillation remains smaller when compared to the Continuous Control. Torque oscillations are highly undesirable in many cases. In this work, the Continuous Control is used after the overmodulation I mode has reached the limit.

What makes this Continuous Control special in the case of a small DC-link capacitor is the inbuilt ability of the method to track the stator voltage vector angle that fulfills the requested stator voltage vector magnitude despite fluctuating DC-link voltage. Even Bolognani et al. do not discuss the ability of the method to respond to the change in DC-link voltage, this property can be seen from the equations. Since the method is used after the overmodulation I and because of its ability to track the angle fulfilling the voltage amplitude, the method is here renamed Constant Amplitude (CA) overmodulation. The Constant Amplitude is chosen to be used as the OM II method in this work.

The closest vector that satisfies the magnitude reference $u_{\text{ref}}$ can be found from the angle $\theta_{\text{CV}}$, which is defined from the middle of the sector and can be found from

$$\theta_{\text{CV}} = \cos^{-1}\left(\frac{\frac{u_{\text{DC}}}{\sqrt{3}}}{u_{\text{ref}}}\right).$$  \hspace{1cm} (2.30)

The values of the angle $\theta_{\text{CV}}$ are limited to $[0 \ldots \pi/6]$. Hence, the case of $\theta_{\text{CV}} = \pi/6$ corresponds to the sixstep operation mode and $\theta_{\text{CV}} = 0$ suggests that the stator voltage reference is achievable without any change in the angle. When $\theta_{\text{CV}} = 0$, the stator voltage vector is in linear modulation region. This is illustrated in Fig. 2.26. The linear modulation region is achievable in the hatched area. Depending on the stator voltage reference vector angle $\text{arg}(u_{\text{ref}}) = \theta_{\text{ref}}$, the term $\theta_{\text{CV}}$ is added to or subtracted from the final angle of the constant amplitude vector as
\[
\theta_{s,\text{ref}} \leq (m-1)\frac{\pi}{3} + \frac{\pi}{6}
\]

\[
\Rightarrow \theta_{\text{CA}} = \frac{\pi}{6} - \theta_{\text{CV}} + (m-1)\frac{\pi}{3}.
\]

\[
\theta_{s,\text{ref}} > (m-1)\frac{\pi}{3} + \frac{\pi}{6}
\]

\[
\Rightarrow \theta_{\text{CA}} = \frac{\pi}{6} + \theta_{\text{CV}} + (m-1)\frac{\pi}{3}.
\]

(2.31)

The original stator voltage reference vector (1.6) is then replaced by

\[
u_{s,\text{ref,CA}} = \nu_{s,\text{ref}} |e^{j\theta_{\text{CA}}}|	ext{.}
\]

(2.32)

The subscript CA stands for Constant Amplitude and describes the ability of the method to maintain the amplitude reference unchanged. The angle \(\theta_{\text{CA}}\) represents the angle where the stator voltage amplitude fulfills the reference amplitude.

\[|V| = \frac{2}{3}u_{\text{DC}}\]

\[M > 0.952\]

\[m = 1\]

Fig. 2.26: The Constant Amplitude overmodulation tracks the closest stator voltage vector that fulfills the magnitude of the reference stator voltage vector. In the case of a constant DC-link voltage level the reference stator voltage vector angle is limited to the closest angle where the amplitude of the voltage vector reference is located. The red line represents the reference voltage vector \(\nu_{s,\text{ref}}\). The green line represents the voltage vector reference limited by the CA overmodulation \(\nu_{s,\text{ref,CA}}\). The hatched area represents the angle references allowed by the CA overmodulation. The discontinuity between the hatched areas represents the stator voltage vector angle references that, in this case, do not fulfill the voltage vector amplitude reference.

To illustrate the functionality of the presented CA overmodulation method, two figures of the sector \(m=1\) are shown. In both of the figures the DC-link voltage is decreasing. Under the DC-link voltage level variations the angle is varied according to (2.30). In Fig. 2.27 the stator voltage vector angle reference is located in the first half of the sector, that is \(\theta_{s,\text{ref}} \leq (m-1)\frac{\pi}{3} + \frac{\pi}{6}\). Under
such conditions the stator voltage vector satisfying the amplitude reference gets closer and closer to the active voltage vector $V_1$ as shown in Fig. 2.27. When the angle of the stator voltage reference exceeds half of the sector $\theta_{\text{ref}} > (m-1)\frac{\pi}{3} + \frac{\pi}{6}$, the situation changes as the closest vector satisfying the amplitude is now located near to the active vector $V_2$. This is illustrated in Fig. 2.28.

**Fig. 2.27:** The closest stator voltage vector in the sector $m=1$ satisfying the amplitude of the stator voltage vector reference. Two possibilities for two DC-link voltage values are shown. The reference stator voltage vector is located in the first half of the sector. In the figure, the DC-link voltage decrease causes the CA OM to turn the voltage vector reference $u_{\text{ref,CA}}$ in the opposite direction than the original angular frequency suggests.

**Fig. 2.28:** The closest stator voltage vector satisfying the amplitude of the stator voltage vector reference. Two possibilities for two DC-link voltage values are shown. The reference stator voltage vector is located in the second half of the sector. In the figure, the DC-link voltage decrease causes the CA OM to turn the voltage vector reference $u_{\text{ref,CA}}$ further ahead of the stator voltage vector reference.
The constant amplitude fundamental voltage output \( u_{ca} \) can be found from

\[
u_{ca}(u_{ref}) = \frac{6}{\pi} \sqrt{\frac{3}{\pi}} \left( \alpha_g + \sin\left(\frac{\pi}{6} - \alpha_g\right) \right),
\]

(2.33)

where

\[
\alpha_g \left| u_{ref} \right| = \frac{\pi}{6} - \cos^{-1}\left( \frac{u_{DC}}{\sqrt{3} \left| u_{ref} \right|} \right).
\]

[2.12]

\[
\frac{u_{DC}}{\sqrt{3}} \leq \left| u_{ref} \right| \leq \frac{2u_{DC}}{3}
\]

A more precise mathematical formation of the fundamental output voltage equation can be found from Bolognani et al. (1996).

**Example measurement of the Constant Amplitude overmodulation**

The measured supply grid line current waveform and the FFT calculated under CA OM are shown in Fig. 2.29. As it can be seen, the line current has considerable distortion. However, the frequency content of the supply grid current, shown in the FFT graph, is still better than in the normal case of high-capacitance VSI shown in Figs. 1.10 and Fig. 2.10. The VSI with small DC-link capacitor generates smaller 5th and the 7th harmonic components even though the line current is strongly distorted because of the CA overmodulation. Fig. 2.30 shows the behavior of the angle limited by the CA OM algorithm during a single stator voltage vector revolution. The electric frequency is 80 Hz (1.6 pu).
Fig. 2.29: Measured waveform of the supply grid line current and calculated FFT. The line current waveform, having considerable distortion, has still satisfactory frequency content.

Fig. 2.30: Recorded behavior of the stator voltage angle reference $\theta_{s,\text{ref}}$ and the CA limited angle $\theta_{CA}$ during one stator voltage vector revolution. The stator voltage vector angle reference is controlled by the constant amplitude overmodulation method such that it turns into the closest direction fulfilling the amplitude reference.
Fig. 2.31 describes the realized voltage vector values - fulfilling the stator voltage amplitude reference - that are obtained under CA OM during a single revolution of the stator reference voltage vector. Only directions fulfilling the amplitude reference are used. The maximum instantaneous voltage vector trajectory in the direction of the stator voltage vector reference is also shown. For low voltage values, the active voltage vector directions are used. In cases when the DC-link voltage is high and the limit of the linear modulation voltage vector reference is exceeded, the linear modulation is used. The different stator voltage vector references of the OM I and the CA OM are due to the aim to supply equal fundamental voltage with different modulation methods. The direction of the realized CA OM voltage vector is alternating rapidly due to the heavily fluctuating DC-link voltage shown in Fig. 2.32.

![Diagram showing voltage vector references](image_url)

Fig. 2.31: Measurement values for the recorded voltage vector references under linear modulation and under CA OM. $u_{s,ref}$ is for the linear modulation. In two cases the DC-link voltage spikes reach so high that the linear modulation is used. Otherwise, CA OM is used. In the case of the low DC-link voltage, the voltage vector in the direction of the active voltage vector is chosen by the CA OM. This operation mode corresponds to the sixstep operation.
A lot of distortion in the DC-link voltage exists. Due to the extremely strong interaction between the modulator and the DC-link voltage waveform, no simplified DC-link voltage waveform approximation was found. Thus, the fundamental component of the output voltage and its THD level fluctuations are not possible to calculate in the same way as in the case of overmodulation I. The MPPF type capacitor can cope with the heavily fluctuating voltage without premature deterioration.

![Graph showing measured DC-link voltage during one stator voltage vector reference revolution. The DC-link voltage is heavily distorted. However, the MPPF type capacitors can take up to 1200 V DC-link voltage. The level is therefore no problem as service life-time is considered.](image)

The researcher familiar with field-oriented control may find it more familiar to study the phenomena of motor control through the produced stator flux linkage $\psi_s$. The produced stator flux linkage can be roughly estimated by integrating the recorded stator voltage vector references realized by the Constant Amplitude overmodulation $u_{s,CA}$. In Fig. 2.33 one revolution of the realized stator voltage vector trajectory and the estimated stator flux linkage are shown. The flux linkage path forms almost a hexagon.
The proposed modulation strategy for the VSI with small DC-link capacitor will be discussed next. It is suggested that the linear modulation region is to be covered with the DSVPWM method because of the more accurate voltage vector realization. Especially at low speeds, accurate voltage supply is needed. After the linear modulation region, the overmodulation I methods are suggested to be used with the traditional SVPWM. Overmodulation II is covered with the Constant Amplitude method, which results in a six-step operation when the maximum fundamental output voltage is needed.
The modulation strategy can be shown in accordance with the fundamental output voltage as a function of the scaled stator voltage reference vector length $r$. The vector length $r$ is scaled as

$$r = \frac{|u_{s,\text{ref}}|}{\frac{2}{3} u_{DC}}. \quad (2.34)$$

Fig. 2.35 shows the fundamental voltage output as a function of the active voltage vector length. The value of $r=1$ equals the maximum voltage in the direction of the active voltage vector $2u_{DC}/3$. After the linear modulation region has reached the end, that is $r \approx 0.866$, the overmodulation I method is used to increase the fundamental voltage. The fundamental voltage output is nonlinear. After the level of $r=1$ and $u_f \approx 0.606u_{DC}$, the fundamental voltage output is at its maximum level for the OM I. After this point, CA overmodulation is suggested to be used. The fundamental output of $u_f \approx 0.606u_{DC}$ can be achieved with both of the methods, that is $u_{f,\text{CA}} \approx (0.921 \cdot 2u_{DC}/3) = u_{f,\text{OMI}}(1 - 2u_{DC}/3)$. Using the CA overmodulation the fundamental voltage output can be increased from $0.606u_{DC}$ to the maximum theoretical level of $2u_{DC}/\pi$. 

Fig. 2.34: Proposed modulation strategy. The linear modulation is covered with the DSVPWM modulator to improve especially the slow speed modulation accuracy. Overmodulation I region is covered with traditional OM I methods and SVPWM modulator. CA is used to deliver the voltage supply from the OM I all the way to the sixstep operation.
Fig. 2.35: Fundamental voltage supply under proposed overmodulation combination. After the linear modulation region has reached the end, that is \( r \approx 0.866 \), the overmodulation I method is used to increase the fundamental voltage. The fundamental voltage output is strongly nonlinear. After the level of \( r = 1 \) and \( u_f = 0.606u_{DC} \), the fundamental voltage output is at its maximum level with OM I. After this point, CA overmodulation is used. The fundamental output of \( u_f = 0.606u_{DC} \) can be achieved with both of the method, that is \( u_{f,CA}(0.921 \cdot 2u_{DC}/3) = u_{f,OMI}(1 \cdot 2u_{DC}/3) \). In using the CA overmodulation the fundamental voltage output can be increased from \( 0.606u_{DC} \) to the maximum theoretical level of \( 2u_{DC}/\pi \).

### 2.3 Overvoltage protection

In this section a simple overvoltage protection method is explained and verified. The method is based on the control of the angle between the stator voltage and the current vectors. The cosine of this angle is commonly known as the power factor. The motor is braking when the direction of power flow changes from the motor towards the converter. In advanced frequency converter control systems braking may be easily prevented. In methods like current vector control and DTC, the electric torque \( t_e \) can be calculated and controlled through estimated fluxes (rotor or stator). As an example, the DTC torque calculation is formed by Takahashi et al. (1986) as

\[
\begin{align*}
\psi_s & = \int (u_s - R_i l_i) dt \\
I_e & = \frac{3}{2} \frac{\psi_s}{i_s} \times I_s,
\end{align*}
\]

where \( u_s, I_s \) and \( R_s \) are the stator voltage vector, current vector and resistance respectively. Motor pole pair is denoted with \( p \).
The vector diagram presentation in a stationary xy-coordinate system of the stator voltage $u_s$, current $i_s$ and flux linkage $\psi_s$ is shown in Fig. 2.36. The hatched area represents the absolute value of the produced electric torque. The angles $\theta_u$ and $\theta_i$ represent the angle of the stator voltage and current vector respectively from the x-axis. The angle $\theta_\Delta$ represents the angle difference between the stator voltage and current vectors. In scalar systems where no information of the fluxes is available the torque cannot be observed and controlled accurately.

Fig. 2.36: Vector diagram presentation of the stator voltage $u_s$, current $i_s$ and flux linkage $\psi_s$. The hatched area represents the magnitude of the electric torque $t_e$. The angles $\theta_u$ and $\theta_i$ represent the angle of the stator voltage and current vector respectively from the x-axis. The angle $\theta_\Delta$ represents the angle difference between the stator voltage and current vectors.

2.3.1 Traditional overvoltage protection

Traditionally, in single-quadrant scalar controlled drives the DC-link overvoltage protection has been carried out by measuring DC-link voltage $u_{DC}$ and controlling the stator voltage vector $u_{s,ref}$ such that the DC-link voltage remains under the reference limit. Traditionally, linear controllers such as PID-controllers are used. In scalar control the controlled value is the (angular) frequency that controls the stator voltage vector $u_{s,ref}$. The limitation control principle is shown in Fig. 2.37. The output of the controller is an additional correction angular frequency $\Delta \omega$ to be added to the angular frequency reference.
Fig. 2.37: Limiting the DC-link voltage in traditional scalar control by means of measured DC-link voltage. The measured DC-link voltage and DC-link voltage high limit is used as an input for the overvoltage controller. The overvoltage controller, traditionally a PID-type, affects the angular frequency with the term $\Delta \omega$.

Problems arise when a small DC-link capacitor is used. The DC-link voltage is by far not constant and fluctuates rapidly. To illustrate the problem an approximative equation for the change of DC-link voltage is

$$-i_{\text{inv}} = C \frac{d u_{\text{DC}}}{dt}$$

$$\Rightarrow \Delta u_{\text{DC}} = -\frac{1}{C} i_{\text{inv}} \Delta t_{\text{sw}}$$

(2.36)

For an example, let us consider a frequency converter unit having 10 A nominal current and a 2 $\mu$F DC-link capacitor. During a single switching period of $\Delta t_{\text{sw}} = 100$ $\mu$s and 1/5 of the returning nominal current $i_{\text{inv}} = -2$ A the DC-link voltage increases $\Delta u_{\text{DC}} = 100$ V. With half of the nominal current the increase of the DC-link voltage is already 250 V. Therefore, the DC-link voltage needs very fast protection. Fig. 2.38 shows the simulated responses of the electric torque $t_e$, load torque $t_l$, angular frequency reference $\omega_{\text{ref}}$ and its limited value $\omega_{\text{lim}}$ as well as the mechanical angular frequency $\omega_{\text{mech}}$ and correction angular frequency $\Delta \omega$. At the beginning of the simulation, the angular frequency reference is changed from 0 to 0.5 pu. The time instant being 0.25 s the angular frequency is changed again from 0.5 to 0.25 pu. The small 0.005 pu load is changed to –0.2 pu at the time instant 0.35 s. The negative load resulting in positive torque causes the motor to accelerate. During the acceleration the current limiter limits the stator current. In the figure, the torque causing acceleration of the motor is marked positive. In Fig. 2.39 the simulated DC-link voltage is shown. As it can be seen, the DC-link voltage $u_{\text{DC}}$ fluctuates. The limit for the DC-link voltage controller is set to 700 V. Despite the controller is trying to limit the DC-link voltage into the limit value of 700 V, peak values of the DC-link voltage go as high as 1300 V. This indicates
that it is not a trivial procedure to use the DC-link voltage as a feedback to protect the DC-link from overvoltage. More satisfactory results may be gained by tuning the controller parameters.

Fig. 2.38: Simulated responses of the torque, angular frequencies and correction \( \Delta \omega \) in per unit value under traditional overvoltage control. At the start, the angular frequency reference is changed from 0 to 0.5 pu. The time instant being 0.25 s the angular frequency reference is changed again from 0.5 to 0.25 pu. The small 0.005 pu load is changed to –0.2 pu load causing the motor to accelerate onwards from the 0.35 s time instant. During acceleration the current limiter limits the stator current. Due to the rapid changes in the DC-link the traditional overvoltage protection causes the angular frequency reference to fluctuate.
Fig. 2.39: Simulated DC-link voltage under traditional overvoltage control. A change in angular frequency reference from 0 to 0.5 pu was made at the beginning. The time instant being 0.25 s the angular frequency reference is changed again from 0.5 to 0.25 pu. Small load 0.005 pu was changed to –0.2 pu load at the time instant 0.35 s. The DC-link voltage gets extremely high.

In Fig. 2.40 the angle between stator voltage and current is observed during the same frequency step. During acceleration, the power flows from the inverter to motor. This can be observed from the angle. At the moment when the torque tends to get negative, the fast dynamic of the small DC-link capacitor is observed. The angle $\Delta \theta$ between stator voltage and current varies heavily when electric torque $t_e$ is close to zero-torque. This is because of the PID controller which reacts to the rapidly changing DC-link voltage every time the level of 700 V is exceeded.
2.3.2 Dynamic Power Factor Control (DPFC) for overvoltage protection

Because of the fast dynamics of the DC-link voltage a different aspect has to be adopted in the overvoltage protection. The ideal situation would be if the controller could identify the braking situation before it occurs. A new method named Dynamic power factor control (DPFC) is introduced and verified to fulfill the demand of overvoltage protection of the small DC-link capacitor VSI. The DPFC is intended to be used especially in scalar controlled electric drives where no torque estimation exists. The DPFC method is based on the control of the power factor, that is the cosine of the angle \( \theta_A \) locating between the stator voltage vector \( u_s \) and the current vector \( i_s \). The mathematical methods for finding the angle \( \theta_A \) can vary. One possible way is to align the d-axis of the synchronous rotating dq-reference frame with the stator current vector. The vector diagram is shown in Fig. 2.41.
The stator voltage reference vector $u_{s,\text{ref}}$ is transformed into the dq-frame by using equation

$$
\begin{bmatrix}
u_{s,d} \\
u_{s,q}
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_i) & \sin(\theta_i) \\
-\sin(\theta_i) & \cos(\theta_i)
\end{bmatrix} \begin{bmatrix}
u_{s,\text{ref,x}} \\
u_{s,\text{ref,y}}
\end{bmatrix}.
$$

The angle between the stator voltage and current vectors can now be calculated from

$$
\theta_{\Delta} = \tan^{-1}\left(\frac{u_{s,q}}{u_{s,d}}\right).
$$

The angle $\theta_{\Delta} = \theta_s - \theta_i$ between the stator voltage and current vectors works as an input for the controller. Despite the heavily fluctuating DC-link voltage, the angle $\theta_{\Delta}$ remains relatively calm in every situation.

Zero-torque is obtained in the instance in which the flux linkage vector $\psi_s$ and stator current vector $i_s$ are overlapping. The power starts to flow from the motor to the DC-link at the moment when the negative angle between flux linkage and current vector is created. During zero-torque the induction motor rotor rotates in synchronous angular frequency with the stator voltage vector. This leads to a zero-rotor current. The induction motor stator voltage space vector presentation, shown e.g. in Leonhard (1996), may thus be simplified during zero-torque as

$$
u_s = R_s i_s + j \omega L_s i_s,$n

where the $L_s$ is the stator inductance.
The angle between the stator voltage and current vector producing zero-torque can be found from

\[ \theta_{\Delta} = \tan^{-1} \left( \frac{\omega L_s}{R_s} \right) . \]  

(2.40)

Since the stator resistance is much smaller compared to the stator inductance, that is

\[ R_s << \omega L_s , \]

it can be approximated that zero-torque takes place at the angle \( \theta_{\Delta} \approx \pi / 2 \). This angle is used to indicate the negative torque compared to the direction of the rotation. The approximate zero-torque vector diagram is shown in Fig. 2.42.

![Vector diagram of the approximate zero-torque situation](image)

Fig. 2.42: Vector diagram of the approximate zero-torque situation

The DPFC should wake up at the moment when the \( |\theta_{\Delta}| \geq \pi / 2 \) is exceeded. Small margins for the controller to compensate the stator resistance and controller delays can be included in the reference limit angle. The output of the DPFC, \( \Delta \omega \), is summed in the angular frequency reference similarly as for the traditional overvoltage controller. A block diagram of the overvoltage control of the scalar control by means of the dynamic power factor control is shown in Fig. 2.43 and Fig. 2.44. As the controller in the DPFC, a traditional PID-controller is used. Integrative term is necessary in order to gain the offset for steady correction angular frequency \( \Delta \omega \). In steady state the condition \( \omega_{\text{ref}} + \Delta \omega = \omega_{\text{mech}} \) should be fulfilled. The derivative part should react to the speed at which the angle \( \theta_{\Delta} \) is increasing when exceeding the zero-torque indication limit of \( |\theta_{\Delta}| \geq \pi / 2 \).
Fig. 2.43: Block diagram of the overvoltage control of the scalar control by means of the dynamic power factor control. The power factor controller needs the stator current and voltage vectors as inputs. The output can be added to the angular frequency reference.

Fig. 2.44: Block diagram of the dynamic power factor control unit. The angle between the stator current vector and stator voltage vector $\theta_s$ is used as an input for the controller. As the controller a traditional PID controller can be used. The controller produces the overvoltage protection control signal $\Delta \omega$ to be summed in the angular frequency reference.

The same simulation sequence as used for the traditional overvoltage control has been used here to verify the new DPFC system. The simulated responses of the electric torque $t_e$ and the load torque $t_l$, angular frequency reference $\omega_{\text{ref}}$ and its limited value $\omega_{\text{lim}}$ as well as the mechanical angular frequency $\omega_{\text{mech}}$ and correction angular frequency $\Delta \omega$ during operation of the DPFC controller are shown in Fig. 2.45. The DPFC becomes active when the torque tends to get negative at the instant $0.2 \, \text{s}$. In Fig. 2.46 the DC-link voltage during operation of the DPFC is shown. No overvoltage situation occurs.
Fig. 2.45: Simulated responses of torque, angular frequencies and correction term \( \Delta \omega \) in per unit value under DPFC. The same sequence for this case is used as in the previous case.

Fig. 2.46: Simulated DC-link voltage under DPFC. No high voltage peaks occur.
In Fig. 2.47 the angle $\theta_\Delta$ is shown during a step change of the reference frequency. The angle is controlled such that the reference limit to the angle $|\theta_\Delta| = 90\% \cdot \frac{\pi}{2} = 0.45\pi$ is the upper limit. The value 90% gives a small margin for the controller and includes a small effect of the stator resistance shown in (2.40). This angle limit reference may result in a very small positive torque. Comparison to the case illustrated in Fig. 2.40, where traditional overvoltage control is used, the difference is clear. No fluctuation in the angle exists when the operating mode of the motor is close to zero-torque. The result is a more fluent behavior of the motor during retardation.

![Graph](image)

Fig. 2.47: The angle $\theta_\Delta$ between stator voltage and current during the frequency step during operation of the DPFC.

Verification has been done also with the experimental device. A measurement example of the DPFC is shown in Fig. 2.48. The load machine is controlled to rotate at a constant 0.3 pu angular frequency. The load motor nominal torque is much higher compared to the controlled test motor. The test motor was used with an angular speed reference sequence $\omega_{\text{ref}} = \{0.24, 0.32, 0.24\}$ pu. With the experimental device the angle limit reference is controlled up to the upper limit of $|\theta_\Delta| = 90\% \cdot \frac{\pi}{2} = 0.45\pi$. As can be seen, the DC-link can be protected by using DPFC. When the angular frequency reference is smaller than the load speed the DPFC controls the difference between load and test motor to be set equal. Thus the correction becomes $\Delta\omega = (0.3-0.24)\text{pu} = 0.06$ pu.
The DPFC is a very simple method for dealing with the problem of the overvoltage control. No additional measurements are needed. Only information from the reference stator voltage vector and measured stator currents are required.

2.4 Resonance between line inductor and DC-link capacitor

Due to the small DC-link capacitor the resonance frequency between the DC-link capacitance and line side inductance becomes significantly high. The resonance frequency can get close to or even exceed the switching frequency. In Phipps (1997) a transfer function approach is used to study the
harmonic filter design. Here, the method is used to study the interactions between the supply grid and the DC-link. The supply grid is modeled as the Thevenin equivalent impedance $Z_{\text{grid}}$. The line side AC-inductor $L_{\text{AC}}$ can be included in the supply grid impedance. The DC-link capacitor can be modeled as the impedance and referred to as a filter and denoted as $Z_{\text{fil}}$. This is illustrated in the Fig. 2.49. The harmonic source for the grid/filter impedance $H_{gf}$ system can be either the rectifier bridge $U_{\text{rec}}$ or the inverter bridge $U_{\text{inv}}$ voltage or both of them.

![Fig. 2.49: Representation of the line impedance and DC-link impedance connection. The rectifier bridge or inverter bridge is considered to be the input stimulus for the impedance system. $Z_{\text{grid}}$ is used to describe the combined supply grid and AC-filter impedance. The DC-link impedance is modeled with $Z_{\text{fil}}$.](image)

The grid/filter impedance transfer function presentation in Laplace’s $s$-domain is

$$H_{gf}(s) = \frac{U(s)}{I_{\text{grid}} + I_{\text{fil}}} = \frac{Z_{\text{grid}}(s)Z_{\text{fil}}(s)}{Z_{\text{grid}}(s) + Z_{\text{fil}}(s)}, \quad (2.42)$$

where $U$ is either the rectifier bridge $U_{\text{rec}}$ or the inverter bridge $U_{\text{inv}}$ voltage.

Assuming that the supply grid combined with the AC-inductor impedance includes an inductive and resistive part, the equation for $Z_{\text{grid}}$ becomes

$$Z_{\text{grid}}(s) = R_{\text{grid}} + sL_{\text{grid}}, \quad (2.43)$$

The DC-link inductor can be assumed to be a pure capacitor. The equation for $Z_{\text{fil}}$ is then

$$Z_{\text{fil}}(s) = \frac{1}{sC}, \quad (2.44)$$

By substituting (2.43) and (2.44) into (2.42) the grid/filter impedance equation becomes
Further substitution of \( s = j\omega \) in (2.45) yields the grid/filter impedance equation in the frequency domain. By solving the angular frequency when the real part of the denominator becomes zero, the resonance frequency \( \omega_{\text{res}} \) is obtained. The resonance angular frequency is

\[
\omega_{\text{res}} = \frac{1}{\sqrt{L_{\text{grid}} C}}. \quad (2.46)
\]

The ratio of the supply grid current and the injected current is defined as \( H_{c2g} \)

\[
H_{c2g}(s) = \frac{I_{\text{grid}}}{I_{\text{grid}} + I_{\text{filt}}} = \frac{H_{gf}(s)}{Z_{\text{grid}}(s)} = \frac{1}{L_{\text{grid}} Cs^2 + R_{\text{grid}} Cs + 1}. \quad (2.47)
\]

This is an important result since it presents the ratio of the supply grid current and the harmonic current injected by the harmonic voltage source. The magnitude of the grid/filter impedance and the magnitude of the supply grid current as a function of the injected current frequency in the frequency domain are shown in Fig. 2.50. It can be shown that the resonance peak amplitude gain \( G_g \) is

\[
G_g = |H_{c2g}(j\omega_{\text{res}})| = \frac{1}{R_{\text{grid}}} \sqrt{\frac{L_{\text{grid}}}{C}}. \quad (2.48)
\]

This gain is widely known as filter quality factor. If the harmonic voltage source injects harmonic current components close to the resonance frequency, the supply grid current includes a high amount of resonance frequency current components.
Fig. 2.50: The magnitude of the grid/filter impedance and the magnitude of the supply grid current in the injected current in the frequency domain are represented. Magnitude maxima are located in the resonance frequency. \( G_g \) represents the supply grid current magnification in correspondence to the injected harmonic current. The gain \( G_g \) can be found from \( H_c \) at the resonance frequency. Approximately the same gain can be found from the impedance transfer function at the resonance frequency when compared to either the inductance or capacitor curve.

The impedance transfer function approach is a rough estimation of the supply grid and voltage source frequency converter interactions. It does not include inductor nonlinearities and all the other un-idealities included in the overall power electric configuration. However, it gives a good hint of the system behavior. In Fig. 2.51 one possible formation of the line current resonance created in the supply grid is shown. The resonance frequency \( f_{res} \) can be calculated as

\[
f_{res} = \frac{1}{2\pi \sqrt{2L_{AC}C}}. \tag{2.49}
\]
Theoretically, when a purely inductive line is assumed and there exist no resistive components in the power circuit, the resonance frequency exists as its own frequency component. Resonance appears when the signal frequencies produced by the rectifier or inverter bridge come close enough. In practice, there exist resistive components in the power circuit. Consequently, the resonance frequency does not appear as its own frequency but it amplifies the switching frequency components when they approach the resonance frequency.

The problem of the resonance frequency can be solved based on the simulation and measurement results. In the following Fig. 2.52, Fig. 2.53 and Fig. 2.54 the simulated line current waveform examples and the calculated FFT graphs versus the switching frequency are shown. In the simulation, there exist no resistive components in the supply grid. The switching frequency and the resonance frequency are plotted by a dashed line. It is clearly illustrated how the resonance frequency $f_{res}$ component is amplified when switching frequency $f_{sw}$ approaches the frequency $f_{res}$. The approximate margin to avoid the resonance is $f_{sw} \geq 2.3f_{res}$. In practice, however, there exist resistive components in the power circuit. These resistive components successfully damp down the resonance frequency current components. In this case, the resistive supply grid components contribute to the content of the line current frequency. The resonance frequency is only excited when the switching frequency comes very close to the resonance frequency. This can be seen from the figures Fig. 2.55, Fig. 2.56 and Fig. 2.57. In these figures, the measured line current waveform examples and calculated FFT graphs are shown as a function of the switching frequency for the different resonance frequencies. The switching frequency and the estimated resonance frequency are marked by a dashed line. When the resistive components, modeling the iron losses of the line and line inductor, are included in the simulations as in parallel connection.
with the line side inductor the resonance component is damped down, as suggested by the measurement results.

![Waveform and FFT of the simulated line current as a parameter of the switching frequency. The DC-link capacitor is 2 \(\mu\)F and the line inductor 0.2 mH.](image)

Fig. 2.52: Waveform and FFT of the simulated line current as a parameter of the switching frequency. The DC-link capacitor is 2 \(\mu\)F and the line inductor 0.2 mH.
Fig. 2.53: Waveform and FFT of the simulated line current as a parameter of the switching frequency. The DC-link capacitor is 2 $\mu$F and the line inductor 0.4 mH.
Fig. 2.54: Waveform and FFT of the simulated line current as a parameter of the switching frequency. The cc-link capacitor is 2 μF and the line inductor 0.6 mH.
Fig. 2.55: Waveform and FFT of the measured line current as a parameter of the switching frequency. The cc-link capacitor is 2 µF and the line inductor 0.1 mH.
Fig. 2.56: Waveform and FFT of the measured line current as a parameter of the switching frequency. The DC-link capacitor is 2 $\mu$F and the line inductor 0.2 mH.
The resonance is not a problem with a sufficiently high switching frequency and highly resistive supply grids, such as domestic supply grids. The resistance effectively damps down the resonance effect. In supply grids having only a small resistive component, such as industrial networks, the resonance might become a problem. In such cases, implementation of some additional resistance in the line side filter for damping the resonance might be necessary (Göpfrich et al. (2003) and Piepenbreier et al (2004)). This, however, creates additional losses in the system.
3 SINGLE-PHASE FED VOLTAGE SOURCE INVERTER

The high-capacitance single-phase fed VSI without power factor circuit creates a significant amount of harmonic components in the line current. The simulated and measured behavior of the traditional, single-phase fed VSI is shown in Fig. 3.1 and Fig. 3.2, respectively. The motor phase currents are relatively sinusoidal and the DC-link voltage has only a moderate 100 Hz ripple. However, the line current FFT shows a significant amount of harmonic components in the line current waveform. A large DC-link capacitor causes high line current peaks during load of the DC-link capacitor. Traditionally, power factor correction circuits are used to correct the quality of the line current. To solve the problem of the line current quality and electrolytic capacitor, a small MPPF capacitor is used. This leads to a heavily fluctuating DC-link voltage and torque. Many industrial low performance applications, such as pumps and fans, are high inertia loads. Thus, even heavily fluctuating torque can be tolerated.

Fig. 3.1: Simulated behavior of the high-capacitance single-phase VSI motor phase currents, inverter line current and DC-link voltage with corresponding FFT figures. The motor phase currents waveforms are very sinusoidal. The line current waveform includes a high amount of harmonic components.
3.1 PWM modulation

When the DC-link voltage waveform follows the full-wave rectified line voltage waveform the line current remains sinusoidal. The situation is illustrated already in Fig. 1.14. This case correspond the situation where resistive load is connected to the grid instead of diode rectifier. Power factor correction circuits are thus not needed. To follow this idea, motor voltage modulation should be performed in a way that the absolute value of the stator voltage vector supplied to the motor is following the absolute value of the line voltage. In order to feed the scaled, line-voltage shaped voltage pulse to the motor, a SVPWM method is used. To gain a simple modulator structure with the line voltage amplitude tracking feature a method called Flower Power PWM is introduced. The scalar control is used as the motor control method.

3.1.1 Flower Power PWM

For a modulator, this name might sound weird. However, it will be justified later. When loaded, the DC-link voltage is approximately equal to the line voltage. Under resistive load the DC-link voltage equals the absolute value of the line voltage. As the amplitude of the stator voltage reference, $u_{\text{ref}}$, is wanted to follow the line voltage amplitude, the DC-link voltage instant value can be used as a scaling factor. Thus equation for the stator voltage vector reference for Flower Power method $u_{\text{ref,FP}}$ becomes
where the subscript FP stands for Flower Power.

Substituting this into the active time calculation equation (2.5), we notice from

\[
\begin{bmatrix}
    t_m \\
    t_{m+1}
\end{bmatrix} = \frac{2}{\sqrt{3}} T_s \begin{bmatrix}
    \sin\left(\frac{\pi}{3} m\right) & -\cos\left(\frac{\pi}{3} m\right) \\
    -\sin\left(\frac{\pi}{3} (m-1)\right) & \cos\left(\frac{\pi}{3} (m-1)\right)
\end{bmatrix}
\begin{bmatrix}
    u_{s_{\text{ref},x}} \\
    u_{s_{\text{ref},y}}
\end{bmatrix} \frac{2}{3} u_{\text{DC}},
\]

that no information of the DC-link voltage is needed. This leads to a more simple structure of the power circuit design. Also the modulator structure will be extremely simple.

This is the simplest way to make the motor rotate. To verify the idea and to show the basic features of the studied method a simulation and measurements performed at an output frequency of 25 Hz (0.5 pu) are given. The simulated and measured motor phase currents as well as the frequency converter line current with corresponding FFT-figures are shown in Fig. 3.3. As it can be seen, the motor phase currents are unorthodox but the line current waveform is very nice. Fig. 3.4 illustrates the simulated and measured xy-plot of the stator current vector trajectory \(i_s\) and the DC-link voltage vector trajectory waveform describing the maximum available stator voltage \(u_{s_{\text{max}}}\). Since a 25 Hz (0.5 pu) frequency has been chosen the modulator using Flower Power method carries out half of the available stator voltage \(u_{s_{\text{ref},FP}}\). The voltage vector trajectory plot in the xy-coordination system justifies the term Flower Power for the modulator. The maximum available voltage vector and also the stator voltage vector reference have the shape of a flower petal.
Fig. 3.3: a) Simulated and b) measured motor phase currents and inverter line current with corresponding FFT figures. The motor phase currents waveforms are very unorthodox. The line current waveform is close to the sinusoidal and thus very desirable.
Fig. 3.4: a) Simulated and b) measured motor stator current vector trajectory $i_s$ and the maximum stator voltage vector trajectory $u_s,max$ and the modulated stator voltage vector trajectory $u_{s,ref,FP}$. The term Flower Power comes from the shape of the voltage vector trajectory.
3.1.2  SVPWM with estimated DC-link voltage

The Flower Power PWM is a very rough method. To improve the behavior of the motor especially under low angular frequencies, the traditional scalar control with SVPWM modulator and overmodulation I is used. The amplitude reference for the stator voltage vector is used when sufficient DC-link voltage is available. When the DC-link voltage level is insufficient, the amplitude is limited according to the principle of overmodulation I. To accomplish compensation of the DC-link voltage variation, voltage feedforward from DC-link voltage is needed. Fig. 3.5 represents the simulated and measured xy-coordinate presentation of the voltage and current vector trajectories. The idea, shown in both figures, is to use compensation of the DC-link voltage according to the traditional scalar control and SVPWM with overmodulation I. This means that the stator voltage vector reference, marked by a red curve, is used unchanged as long as the level of the DC-link voltage remains sufficient. The level during which the DC-link voltage is insufficient, the overmodulation I method limits the amplitude of the modulated stator voltage vector, marked by a black curve, in order to reach the maximum available voltage vector. In both figures, the maximum available voltage vector $u_{s,\text{max}}$ trajectory, the stator voltage vector reference $u_{s,\text{ref}}$ trajectory, the modulated voltage vector $u_{s,\text{mod}}$ trajectory and the stator current vector $i_s$ trajectory, marked by a green curve, are shown. The line current waveform is expected to differ from sinusoidal. This is because the output power is not controlled according to the input power equation (1.10). The motor phase currents and line current waveform measurement results with corresponding FFT figures are shown in Fig. 3.6. From the figure it can be seen that the line current waveform is not as close to the sinusoidal as under the Flower Power method shown in Fig. 3.3. However, the waveform frequency content remains still very decent.
Fig. 3.5: a) Simulated and b) measured motor stator current vector trajectory ($\mathbf{i}_{s}$, green curve) and maximum stator voltage vector trajectory ($\mathbf{u}_{s,max}$, blue curve) and modulated stator voltage ($\mathbf{u}_{s,mod}$, black curve) vector trajectory. The stator voltage vector reference ($\mathbf{u}_{s,ref}$, red curve) is used unchanged during the period the DC-link voltage remains sufficient. When the DC-link voltage becomes insufficient the overmodulation I method limits the amplitude of the modulated stator voltage vector in order to reach the maximum available voltage vector.
Fig. 3.6: Measured motor phase currents and inverter line current with corresponding FFT figures under traditional DC-link voltage compensated scalar control with SVPWM and overmodulation I. The line current waveform is not as close to the sinusoidal as under the Flower Power method. However, the waveform frequency content remains still very decent.

When the aim is to achieve a very simple power electric hardware configuration, it should be tried to eliminate from the system every unnecessary high precision measurement. In this case, the aim is to replace the accurate DC-link voltage measurement with an estimation algorithm and a single bit line voltage sign measurement. In the case of resistive load, the DC-link voltage equals the absolute value of the line voltage. To estimate the DC-link voltage it is thus only necessary to know the line voltage phase. Only a single-bit measurement of the line voltage sign is needed to determine the phase. The peak value of the line voltage can be expected to be very close to its normal value. The equation for \( u_{DC,estim} \) is

\[
\bar{u}_{DC,estim} = \left| \hat{u}_{L1} \sin(\omega_{grid}t + \phi_{grid}) \right|,
\]

where \( \hat{u}_{L1} \) is the peak value for the line voltage, \( t \) is the time, \( \omega_{grid} \) is the angular frequency of the grid and the angle \( \phi_{grid} \) stands for the initial phase of the line voltage.

Due to the insufficient voltage, a negative torque is generated. Due to the negative torque the DC-link voltage rises and therefore differs from the absolute value of the line the voltage waveform.
This causes an error in the DC-link voltage estimation. The negative torque during the low DC-link voltage may be clearly seen from Fig. 3.7 (a) and (b). In the case of an ideal DC-link voltage measurement, the SVPWM modulator forms duty cycles as shown in Fig. 3.7 (a). At 0.01 s, there appears the effect of the energy returning from the motor during the negative torque. This negative torque is shown in Fig. 3.7 (b).

![Figure 3.7: a) Simulated effect of the measured DC-link voltage in the formulation of the three-phase duty cycles and b) simulated electric torque under SVPWM with measured DC-link feedback. The energy returning from the motor during the negative torque causes the DC-link voltage to arise. When measured DC-link voltage feedback in SVPWM modulator is used the pulse width reference reacts on the voltage level. This can be seen at the 0.01 s time instant.](image)

As a comparison to Fig. 3.7, the created three-phase duty cycles and the behavior of the torque when using the estimated DC-link voltage are shown in Fig. 3.8 (a) and Fig. 3.8 (b) respectively.
When the DC-link voltage gets lower than what the reference voltage modulated to the motor, the duty cycles are saturated to their maximum values. The estimation algorithm fails to estimate the DC-link rising during the returning energy. It can be seen, when comparing Fig. 3.7 and Fig. 3.8, that no substantial error occurs.

![Graph A](image1)

![Graph B](image2)

![Graph C](image3)

**Fig. 3.8:** (a) Simulated effect of the estimated DC-link voltage in the formulation of the three-phase duty cycles and (b) simulated electric torque under SVPWM with estimated DC-link feedforward. The energy returning from the motor during the negative torque causes the DC-link voltage to arise. When estimated DC-link voltage feedforward in SVPWM modulator is used the pulse width references do not react on the voltage level. This can be seen at the 0.01 s time instant.

The measured DC-link voltage feedforward has been tested with the experimental device. Due to the un-modeled features and other losses not included in the simulation model, the returning energy during the negative torque does not cause as high a DC-link voltage rise as seen in the
simulations shown in Fig. 3.7 and Fig. 3.8. Basically this means that in the case of the experimental device the estimated and measured DC-link voltage yield exactly the same three-phase duty cycle waveforms. The measured DC-link voltage and SVPWM generated three-phase duty cycles used are shown in Fig. 3.9.

![Graph of DC-link voltage and duty cycles](image)

Fig. 3.9: Measured DC-link voltage and the three-phase duty cycles used. Due to losses that are not included in the simulation model, the returning energy during the negative torque did not cause DC-link voltage to rise high. When measured DC-link voltage feedforward in SVPWM modulator is used the pulse width reference reacts to the voltage level. This can be seen at the 0.006 s time instant. In the case of the experimental device the estimated and measured DC-link voltage yielding exactly the same three-phase duty cycle waveforms.

### 3.2 Electromechanical resonance

In traditional electric drives the structures of the mechanical couplings should be chosen carefully to avoid mechanical failures. In demanding motor drive applications like rolling mill drives, fast torque transients have a serious drawback; they work as a stimulus for the mechanical vibrations. To avoid this, extremely stiff constructions have been used in shafts and mechanical switches. In many cases, lighter dimensioning would be advantageous, if mechanical vibrations could be actively damped. Some papers present active suppression using controlled electrical drives (Jun-Keun et al. 1995, Peter et al. 2001, Sarén et al. 2002b). Active suppression requires a dynamical model of the whole drive and smooth controllable electric torque. In the case of constantly fluctuating electric torque, the design of mechanical couplings is an even more crucial task. The electric torque cannot be controlled to be smooth as assumed in the papers above.
A theoretical drawing of the mechanical system is illustrated in Fig. 3.10. A model of two-mass system connected with the shaft having finite torsional stiffness is the simplest mechanical resonant system. While it is the simplest mechanical resonant system model, it is also accurate enough to produce a good model describing the wide range of different rotating systems connected with the shaft.

Fig. 3.10: Mechanical two-mass model. Inertia $J$, torsional spring constant $K$, torque $t$ and angular frequency $\omega$. The subscripts m, e, l and sh denotes the motor, electric, load and shaft respectively.

The mechanical two-mass system can be presented as a state-space model

$$
\dot{x}(t) = \begin{bmatrix} 0 & 0 & -\frac{1}{J_m} \\ 0 & 1 & \frac{1}{J_l} \\ K_{sh} & -K_{sh} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t_1(t),
$$

(3.4)

where the input $u(t) = t_e$, output $y(t) = \omega_m$, and the states $x(t) = [\omega_m \ \omega_1 \ t_{sh}]^T$. 
The mechanic system produces resonance $\Omega_1$ and anti resonance $\Omega_2$ frequency peaks

$$\Omega_1 = \frac{1}{2\pi} \sqrt{\frac{K_{sh}}{J_m} \left( \frac{1}{J_m} + \frac{1}{J_1} \right)},$$

$$\Omega_2 = \frac{1}{2\pi} \sqrt{\frac{K_{sh}}{J}}.$$  \hspace{1cm} (3.5)

A Bode-diagram, $\omega_m/t_e$, of the mechanical structure is shown in Fig. 3.11 and $t_{sh}/t_e$ in Fig. 3.12. The mechanic system produces resonance and anti-resonance frequency peaks as mentioned above. The simulated electric torque $t_e$ with its calculated FFT for a single-phase fed VSI with small DC-link capacitor is shown in Fig. 3.13. Due to the heavily fluctuating DC-link voltage, the electric torque also fluctuates.

Fig. 3.11: Bode-diagram of $\omega_m/t_e$ of the mechanical structure.
On comparing the frequency content of the electric torque shown in Fig. 3.13 and the Bode diagrams of the mechanical structure (Fig. 3.11 and Fig. 3.12) it is clearly seen that the main harmonic frequency of the electric torque, that is 100 Hz, is effectively damped in the electrical angular frequency whereas the resonance frequency, 200 Hz, is the dominating mechanical
harmonic frequency. On the shaft the resonance frequency is the dominant component. In Fig. 3.14(a) a simulated mechanical shaft torque and its FFT are shown. In Fig. 3.14(b) the corresponding measured shaft torque and its FFT are shown. The operating point is 0.8 pu electric frequency. Even tough the nominal torque of the controlled motor is approximately 7.5 Nm the shaft torque goes as high as the peak value of 40 Nm. The phenomenon of shaft torque is clear and can be tracked to the mechanical resonance. It can be concluded that extra attention should be given to the design of the mechanical couplings. Stiff couplings are needed to avoid resonance. Particularly traditional, low stiffness, torque measurement shafts are not applicable in the case of a single-phase supplied small DC-link capacitor VSI.

![Graphs showing shaft torque and FFT](image)

**Fig. 3.14:** (a) Simulated and (b) measured shaft torque.

### 3.3 Performance measurements

How good is this small capacitor, single-phase fed VSI? One very good criterion is the performance comparison with an unmodified commercial VACON NXL-device. Both modified and unmodified inverters are used with scalar motor control and 10 kHz switching frequency. The unmodified NXL is controlled with scalar control using the traditional SVPWM with DC-link voltage feedforward. With the MPPF DC-link capacitor modified NXL, named here YSK, both SVPWM methods, the Flower Power method and the traditional $u_{DC}$-feedforward compensated method, are tested. The basic diagram of the measurement setup is shown in Fig. 3.15. The input
power, which is the line voltage $u_{L1}$ and current $i_{L1}$, is measured. Both of the VSI configurations were used with equal output powers. The equal output power can be guaranteed by controlling the mechanical speed $\omega_m$ of the fan. The loading variation is gained by varying the air inflow of the fan.

![Measurement setup for constant output power using fan load.](image)

Fig. 3.15: Measurement setup for constant output power using fan load.

Three different inflow values with different rotating speeds of the fan are tested. The following figures Fig. 3.16, Fig. 3.17 and Fig. 3.18 show the input power of the inverter as a function of the mechanical angular speed of the fan for these three different inflow values.

![Graph showing input power vs. mechanical speed](image)

Fig. 3.16: Measured input power of the compared devices with minimal fan load. The difference in input power is insignificant below 75% of the nominal speed.
Fig. 3.17: Measured input power of the compared devices with medium fan load. The Flower Power can not take more than 50% of the nominal speed. The Udc compensated YSK remains competitive until 60% of the nominal speed.

Fig. 3.18: Measured input power of the compared devices with full fan load. The Udc compensated YSK remains competitive until 60% of the nominal speed.
To define the different electrical behavior of the traditional NXL and DC-link capacitor modified NXL device, named YSK, the following figures Fig. 3.19, Fig. 3.20 and Fig. 3.21 are shown. In each of the figures the waveforms of the supply grid current, DC-link voltage and motor phase currents are shown. The same mechanical output power could be ensured in observing the fan rotating speed. The mechanical frequency is 0.5 pu in all of the cases.

Fig. 3.19: Measured supply grid current, DC-link voltage and motor phase currents with full fan load for NXL-device.
Fig. 3.20: Measured supply grid current, DC-link voltage and motor phase currents with full fan load for YSK-device with DC-link feedforward modulation.
Based on the additional losses due to the increased magnetic hysteresis losses and iron losses it is suggested generally that the applications of the small capacitor single-phase fed VSI are among those applications that have a low annual operating time but demand for line current with low harmonic components. With Flower Power a very good grid behavior is gained. However the efficiency gets poor when loading gets higher. This indicates that the Flower Power is recommended for low dynamic applications which demand very good grid behavior, very low annual operating time and which are not heavily loaded. Thus domestic appliances are very tempting for this configuration. The single phase fed small DC-link capacitor VSI with traditional
DC-link voltage feedforward improves the loading capacity but the grid behavior is not as good as with Flower Power. However this method is still more tempting than traditional high-capacitance VSI when comparing the grid behavior. Manufacturing costs can be lowered by using the estimated DC-link voltage. High-capacitance VSI has better efficiency. This makes the choice of single phase fed small DC-link capacitor VSI with DC-link voltage feedforward and high-capacitance VSI an optimization task between annual energy consumption and the repayment period of the device. When smooth torque and when both annual operation time and the loading are high, the high-capacitance VSI should be chosen.

Because of the lack of big energy storage in the DC-link, the average output voltage is smaller compared with the traditional VSI with big DC-link capacitor. For this reason, a motor supplied with a single phase fed small DC-link capacitor VSI needs to be redesigned and adapted to the lower voltage level in order to gain enhanced performance. Since this kind of motor redesign has not been carried out in this work it is not clear how much the performance can be enhanced with proper motor design.
4 CONCLUSIONS

The need to improve the supply grid properties of the VSI is increasing. It is also sought for solutions to lengthen the expected life-time of the frequency converter. The life time of the frequency converter is expected to be increased by replacing its up to now “weakest link”, the electrolytic DC-link capacitor, with a film capacitor. Film capacitors have a lower energy density which leads to a smaller capacitance of the DC-link. This again leads to faster dynamics of the DC-link voltage. The physical changes in the power electric hardware configuration remain small when the electrolytic capacitor is replaced with a film capacitor of equal physical size having traditionally only about 1 % of the electrolytic capacitance. It has, however, a significant impact on the frequency converter dynamics and motor control algorithms. In order to compensate the DC-link voltage fluctuation out from the VSI output voltage, faster controllers are needed.

The major part of all electric drives applications are low performance applications, such as pumps and fans. In these applications the active rectifier bridge is hardly cost-effective. The supply grid current harmonic content can thus not be controlled actively with the controlled rectifier bridge. Traditionally, different types of passive filters are used. The physical size of the filters tends to be large. By replacing the electrolytic capacitor with a film capacitor, the energy storage of the DC-link decreases. Thus the instantaneous power supplied to the motor correlates with the power drawn from the supply grid. This leads to a continuous DC-link current fed by the diode rectifier bridge for which reason in the frequency converter with small DC-link capacitor the line current harmonics are clearly decreased. Only a small AC-side filter is needed in the VSI with small DC-link capacitor to filter the inverter bridge switching harmonics. When combining the reduced size of the DC-link capacitor and the line side filter a more compact VSI is obtained.

4.1 Three-phase fed VSI

The frequency converter with a small DC-link capacitor shows very good line current properties while the motor properties remain unchanged. No additional line side filter components are needed. This simplifies the structure of the frequency converter. The modulator needs a feedforward signal from the fluctuating DC-link voltage. The traditional SVPWM method includes the DC-link voltage feedforward and uses it in the formulation of the switching sequence.

When the capacitor is very small, every switching action of the power switches affects the DC-link voltage. Thus, the constant DC-link assumption during the switching period leads to an error in the produced voltage. The new modulating principle, named Differential Space Vector Pulse
Width Modulation (DSVPWM), being able to deal with the heavily fluctuating DC-link voltage, produces the correct voltage vector with high accuracy.

The voltage source inverter with a small DC-link capacitor needs efficient overmodulation methods. Since no energy storage exists in the DC-link, the DC-link voltage waveform follows the rectifier bridge waveform. The region of overmodulation I starts at an earlier point because of the voltage sags. The fundamental voltage fed to the motor can be increased smoothly from the overmodulation I all the way up to the sixstep mode by using Constant Amplitude overmodulation. The THD levels of the VSI with small DC-link capacitor are higher compared with the VSI having the high-capacitance DC-link capacitor.

Due to the small DC-link capacitor, the DC-link voltage slew rate is much higher compared with traditional high-capacitance converters. Traditionally, the measured DC-link voltage is used as a feedback to point out the state of braking in scalar controlled VSI. With a small DC-link capacitor the DC-link voltage can raise intolerably high even during a single switching period of the modulator circuit. Therefore, when there is no speed encoder installed on the shaft, restraining the scalar controlled motor drive from braking turns out to be a non-trivial task. To solve this problem, a new overvoltage protection method, named DPFC, based on dynamic power factor control has been introduced, studied and tested.

The DC-link capacitor combined with the line side inductance creates a resonance circuit. Normally, the resonance frequency remains very low. Now, with the use of the small DC-link capacitor and line side inductor, the resonance frequency significantly increases. The resonance frequency can be close to or even higher than the switching frequency of the modulator. The resonance phenomenon has been studied. A great number of simulation and measurement results have been presented. The resonance is not a problem in highly resistive supply grids, such as domestic supply grids. The resistance effectively damps down the resonance effect. In supply grids having only a small resistive component, such as industrial networks, the resonance might become a problem. In such cases, the implementation of additional resistance in the line side filter might be required for damping the resonance.

4.2 Single-phase fed VSI

A single-phase fed VSI with a high capacitance DC-link capacitor causes high line current peaks during DC-link capacitor load. To solve the problem of the line current quality and electrolytic capacitor, a relatively small MPPF capacitor is selected. If the DC-link capacitance is changed, also the dynamic behavior of the single-phase fed frequency converter changes drastically. The
DC-link voltage and electric torque include heavy fluctuation. Because many industrial low performance applications, such as pumps and fans, are high inertia loads, even heavily fluctuating torque may thus be tolerated. In home applications the accuracy of the speed controller is not of vital importance. A low line current harmonic content, however, is of crucial significance. By using the Flower Power modulation method the sinusoidal line current waveform can be attained. The performance of the single-phase fed VSI can be improved by using the DC-link voltage feedforward. To reduce the expenses the DC-link voltage can be estimated from the sign of the supply grid voltage. This means that the DC-link voltage feedforward can be achieved without having to implement higher resolution voltage measurement in the DC-link.

During the design of the mechanical couplings of the single-phase fed small DC-link capacitor VSI the constantly fluctuating electric torque has to be taken into account. The resonance frequency of the two-mass system with finite torsional stiffness shaft can come close to the fluctuating electric torque components. Simulation and measurement results illustrating the problem have been presented.

Based on the additional losses due to the increased magnetic hysteresis losses and iron losses it is suggested generally that the applications of the small capacitor single-phase fed VSI are among those applications that have a low annual operating time but demand for line current with low harmonic components. With Flower Power a very good grid behavior is gained. However the efficiency gets poor when loading gets higher. This indicates that the Flower Power is recommended for low dynamic applications which demand very good grid behavior, very low annual operating time and which are not heavily loaded. Thus domestic appliances are very tempting for this configuration. The single phase fed small DC-link capacitor VSI with traditional DC-link voltage feedforward improves the loading capacity but the grid behavior is not as good as with Flower Power. However this method is still more tempting than traditional high-capacitance VSI when comparing the grid behavior. Manufacturing costs can be lowered by using the estimated DC-link voltage. High-capacitance VSI has better efficiency. This makes the chose of single phase fed small DC-link capacitor VSI with DC-link voltage feedforward and high-capacitance VSI an optimization task between annual energy consumption and the repayment period of the device. When smooth torque and when both annual operation time and the loading are high, the traditional high-capacitance VSI should be chosen. These suggestions are confirmed by the performance measurements with fan-load.
4.3 The future scope of the research

The future scope of the work will, unquestionably, concentrate on implementing more sophisticated motor control algorithms, like current vector control, in the VSI with small DC-link capacitor. When the small DC-link capacitor VSI is to be considered in high performance electric drive applications the influence of the DC-link voltage fluctuation on the produced electric torque needs to be investigated. Theoretically, no extra fluctuation in the electric torque is expected until the overmodulation region starts. There is no known reason why the small DC-link capacitor configuration couldn’t be used in high-performance drives.

The DC-link fluctuation causes extra harmonic distortion in the output voltage. As a result, the efficiency decreases. As already mentioned, no drastic reduction of efficiency is expected and thus the subject does not concern the author. However, in order to be able to find or even design the optimal motor type for the small DC-link capacitor VSI, the additional distortion must to be taken into consideration.

Although the DPFC with linear PID-controller proved to work satisfactory, more attention needs to be paid to the controller. Finding a proper tuning method for the controller may be a subject for future work. Future research should also focus on the possibility of implementing alternative controller types into the DPFC method.

Traditionally, the impedance of the VSI filter has been the dominating part in the total line side inductance. Now as the AC-filter impedance is reduced along the DC-link capacitor capacitance, the supply grid impedance and AC-filter impedance values are now in the same decade. The influence of the supply grid impedance on the line current resonance needs to be investigated at different supply grid conditions.

Considering the single-phase supplied small DC-link capacitor VSI, attention may be paid to the possibility of implementing other motor types for the modulators presented in this work. As the reference publications suggest, the permanent magnet motor is the most interesting alternative compared with the induction motor. Furthermore, it may be of interest to see what can be done in designing a motor when the limitations in the supplied motor voltages of the single-phase supplied small DC-link capacitor VSI are known beforehand.
References


APPENDIX A, Simulation parameters for three-phase fed VSI

Nominal line-to-line RMS voltage $U_n = 400$ V

**Motor parameters:**

Nominal power: 4 kW
Nominal current: 8.7 A
Nominal frequency: 50 Hz
Stator resistance: 0.03 pu
Stator leakage inductance: 0.1 pu
Stator magnetization inductance: 1.7 pu
Rotor leakage inductance: 0.1 pu
Rotor resistance: 0.025 pu

**Frequency converter parameters:**

Line side filter inductance: varied, for example 0.4 mH, 0.0047 pu.
De-link capacitor: varied, for example 2 µF, 0.0167 pu
Blanking time of IGBT switches: 2 µs
Minimum pulse of IGBT switches: 2 µs
APPENDIX B, Simulation parameters for single-phase fed VSI

Nominal line-to-zero RMS voltage \( U_n = 230 \) V

Motor parameters:

Nominal power: 1.1 kW (VEM)
Nominal current: 4.55 A
Nominal frequency: 50 Hz
Stator resistance: 0.09 pu
Stator leakage inductance: 0.05 pu
Stator magnetization inductance: 1.77 pu
Rotor leakage inductance: 0.08 pu
Rotor resistance: 0.07 pu

Frequency converter parameters:

Line side filter inductance: varied, for example 0.4 mH, 0.0043 pu
De-link capacitor: varied, normally 3 \( \mu \)F, 0.0275 pu
Blanking time of IGBT switches: 2 \( \mu \)s
Minimum pulse of IGBT switches: 2 \( \mu \)s

Mechanical system parameters:

Motor inertia: 0.0035 kgm\(^2\) including motor inertia and half of the torque sensor inertia
Load inertia: 0.0146 kgm\(^2\) including load inertia and half of the torque sensor inertia
Shaft torsional spring constant: 6363 Nm/rad including the torque sensor spring constant
APPENDIX C, The parameters for three-phase fed test setup

The parameters of the motors are shown in Table 1.

Table 1: Controlled 4 kW motor and 5.5 kW load motor parameters for three-phase system.

<table>
<thead>
<tr>
<th>Manufacturer and serial number</th>
<th>Power [kW]</th>
<th>(\cos \phi)</th>
<th>Nominal speed [rpm]</th>
<th>Main voltage [V]</th>
<th>Nominal current [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strömberg, SN 26383 B</td>
<td>4</td>
<td>0.83</td>
<td>1435</td>
<td>380</td>
<td>8.7</td>
</tr>
<tr>
<td>Strömberg SN 0566 MP</td>
<td>5.5</td>
<td>0.84</td>
<td>1440</td>
<td>380</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Control board: dSPACE 1005
Control algorithms: Scalar control and SVPWM
Controller sample time: 25 \(\mu\)s
Switching frequency: varied, normally 10 kHz
Line side filter inductance: varied, normally 0.4 mH
Dc-link capacitor: varied, normally 2 \(\mu\)F
APPENDIX D, The parameters for single-phase fed test setup

The controlled motors are shown in Table 2. In verification measurements the 1.1 kW VEM induction motor was used. In performance test carried out with fan load, the 1.3 kW Elektror induction motor was used. The parameters for loading induction machine in verification measurements are shown in Table 3.

Table 2: Controlled motor and load motor parameters for single-phase fed system.

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>VEM</td>
<td>1.1</td>
<td>0.8</td>
<td>1410</td>
<td>230</td>
<td>4.55</td>
</tr>
<tr>
<td>ELEKTOR</td>
<td>1.3</td>
<td>0.7</td>
<td>2480</td>
<td>220 - 300</td>
<td>4.8 - 4.2</td>
</tr>
</tbody>
</table>

Table 3: Load induction motor for single-phase fed system.

<table>
<thead>
<tr>
<th>Manufacturer and serial number</th>
<th>Power [kW]</th>
<th>cosϕ</th>
<th>Nominal speed [rpm]</th>
<th>Main voltage [V]</th>
<th>Nominal current [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strömberg, SN 24569 B</td>
<td>4</td>
<td>0.83</td>
<td>1435</td>
<td>380</td>
<td>8.7</td>
</tr>
</tbody>
</table>

Control board: dSPACE 1103

Control algorithms: Scalar control and SVPWM and SVPWM variations

Controller sample time for measurements: 25 µs

Switching frequency: varied, normally 10 kHz

Line side filter inductance: varied, normally 0.4 mH

Dc-link capacitor: varied, normally 3 µF

The fan performance measurement

Power analyzer: YOKOGAWA PZ4000 with low pass filter for all channels in 20 kHz.

YSK FlowerPower & YSK Udc compensation: 0.4 mH and 3 µF

NX Scalar control and SVPWM: 0.4 mH and 165 µF
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