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THE RELATIONSHIP BETWEEN IMPLIED

AND REALIZED VOLATILITY

Bachelor's Thesis

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TABLE OF CONTENTS

1 INTRODUCTION

Option markets have significantly expanded since 1973 when the Chicago Board Options Exchange (CBOE) began listing call options. The value of the OTC (over-the-counter) option trades in 2004 was 220 billion dollars and the value of exchange traded derivatives in 2004 was 49 billion dollars (Hull 2006:1-3). Today options are traded in all continents in amounts that exceed other exchange traded instruments, therefore pricing should be accurate and reliable. To price options it is needed to forecast the future volatility of the underlying stock. This forecast should be based on the best information available and it should reflect future expectations. The objective of this study is to evaluate how well these forecasts have succeeded by analyzing the relationship between implied and realized volatilities.

1.1 Background of the Study

OTC (over-the-counter) trading of derivatives began in 1980's in Finland. In Finland the first derivatives exchange was launched in 1987 and after some fusions the Helsinki Stock Exchange (HEX) was born in 1997. In the year 1999 HEX started to co-operate with the world biggest derivatives exchange Eurex[1].

Eurex is European futures and options exchange. It was founded in 1998 when Deutsche Terminbörse (DTB) and Swiss Options and Financial Futures (SOFFEX) merged. Eurex offers a wide variety of different derivatives. Equity segment contains options of most traded European companies from different countries. Eurex also trades U.S. equity options.

Call options is a contract that gives its holder the right to purchase an asset on a specified price. The opposite position is a put option which gives its holder the right to sell an asset on a specified price. Warrants are call options that are issued by a company. Warrants are typically call options that are added to bond issue to make

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[1] More information can be found from: http://www.eurexchange.com/

the bond more attractive to the investors. After bond issuance the call option part is separated and sold to secondary markets. When a warrant is exercised, the company releases new shares and this leads to a dilution of ownership of former owners. Warrants are typically used to commit the executives or the employees to the same goals that are beneficial for the stockholders. Executive and employee stock options (or warrants) are widely used in big publicly exchanged companies around the world.

Most of the options and warrants are traded in options exchanges. The average trade of warrants in Finnish exchange in May 2005 was 2,5 million euros and in year 2006 the value of trade had increased to 13,5 million euros (Arala: 2006) The worlds biggest option exchanges in the world are Chicago Board of Options Exchange (CBOE), Eurex, Euronext, and Amex. Domestic exchanges usually also trade with limited range of options. For example in Finland OMX Helsinki trades call options, put options and warrants. The main part of the options trading is done in option exchanges where the options are traded in standardized packages, which improves the liquidity of the paper and increases the variety of the products.

The option pricing is typically done using Black-Scholes pricing formula (Black, Scholes, 1973). Basic layout for the formula was presented in 1973 by Robert C. Merton by improving the work of Fischer Black and Myron Scholes. This achievement led to a Nobel price in Economics for Merton and Scholes in 1997. Nowadays the Black-Scholes pricing formula is one of the basic tools used in option and warrant pricing. Eurex uses binomial model to determine settlement prices for American options and for European options Eurex uses Black-Scholes 76 pricing formula.

The Black-Scholes formula can be used in reverse to find out certain factors of the formula when the price is already known. In this paper the formula is used to find out the volatility of the underlying stock when the price of the option is taken from the markets. This volatility is called implied volatility. Implied volatility is the only factor of the Black-Scholes model that is not known and therefore it is usually a guess based on a historical volatility and future expectations.

The implied volatility can be considered to be a forecast of the future volatility of the underlying stock for the remaining maturity of the option. If the investor thinks that the implied volatility is too low, the option can be considered cheap and if the implied

volatility is too high, the option can be considered expensive. Implied volatility represents the markets beliefs and assumptions of the future volatility of the underlying asset. Because it is hard to forecast future volatility and there are no correct answers when the implied volatility is estimated, it can be used also to overprice options and warrants to increase profits. Therefore it is important that implied volatility is on realistic level and it should be the best guess of the future volatility.

1.2 Objectives of the Study

The goal of this paper is to find out the nature of the connection between implied and realized (also called as historical) volatility and study if implied volatility has any forecasting power when compared to realized volatility. In theory the implied volatility should follow the same patterns as the realized volatility. It is also important to notice if the implied volatility is well beyond the realistic assumptions because it would be and indication of pricing errors.

The research questions to be discussed in this study are introduced as follows:

- Is the implied volatility a good forecast of realized volatility?

- Is the implied volatility used to cause pricing errors in options?

- Are there changes in implied volatility caused by some external factors?

- Does the pricing differ between different companies?

Call options were chosen from Eurex and one Helsinki Stock Exchange traded warrant is taken into study for comparison. It should be remembered that Eurex is one of the world's largest options exchanges and the pricing of traded securities should be on the best level achievable with current methods. Analyze was performed on time period from the beginning of 2003 to the end of 2006. The data is divided into two subperiods to achieve better statistical analysis.

1.3 Review of the Previous Research

Journal of Financial Economics has published a paper "The relation between implied and realized volatility" made by B.J Christensen and N.R Prabhala (1998) which deals with the same subject as this paper using index options. Christensen and Prabhala used S&P 100 index options when calculating monthly values of implied volatility for the index option. They used 139 months of data. One-month LIBOR was used as a risk-free rate and the implied volatility was calculated from the basic Black-Scholes pricing formula that doesn't take dividends into account. The ignorance of dividends does result to some degree of measurement error in calculated implied volatilities for the index option (S&P 100 index pays dividends). Therefore calculated implied volatilities were in some degree understated. Other source of measurement error was caused by the different timing of daily price records. Some of the one hundred stocks are less liquid than others and this leads to pricing errors. Also bid-ask spreads in option prices causes some degree of measurement error.

The study concluded that implied volatility outperforms past volatility in forecasting future volatility. This result is in contrast of previous researches that have found the implied volatility of S&P 100 index to be biased and inefficient in forecasting future volatility (OLS estimates were used). One explanation for less biased results is that earlier studies were made in advance of October 1987 crash which had in some degree changed the option pricing customs (Schwert: 1990)

Another study was made in 1993 by Linda Canina and Stephen Figlewski called "The informational content of implied volatility". They concentrated to find out if the implied volatility has any informational content when compared to the realized volatility. They found that the forecasting power of implied volatility was poor. They calculated implied volatilities of the S&P 100 index options and found that there was almost no correlation at all between implied and realized volatilities. The analyzed time period was from 1983 to 1987.

Mika Arala studied in his master's thesis in 2006 the pricing errors of Finnish warrants between 2003 and 2005 traded in Helsinki Stock Exchange. He concluded that there are significant arbitrage opportunities in warrants because of the pricing errors.

Significant part of warrants were traded below the basic value and in the last month of warrants maturity the average pricing error was -6,5 %. When the option prices were compared to Black-Scholes values, the average error in prices was -13 %. Similar errors are not found in warrants that are released by financial institutions or Finnish warrants traded in Eurex.

As a whole the relationship between implied and realized volatility has not been studied in a large scale. It seems that it is fairly easy in options market to intentionally overprice securities when compared to other securities. The pricing is less visible because implied volatilities are rarely easily available to the public.

1.4 Structure of the Study

This study consists of theoretical and empirical analysis. The theoretical part of the study is carried out by analyzing previous research papers. Section two introduces theoretical background of the study by going through the basic mathematical concepts. Section three explains the statistical methodology used in the study and reviews the data. Section three also goes through factors that cause measurement errors in the used data. Section four contains the empirical analysis of the study. Empirical analysis begins with the descriptive statistics of the data and continues with correlations and regression analysis. Section five sums up all the results and includes conclusions considering research hypothesis.

2 THEORETICAL BACKGROUND

2.1 Black-Scholes-Merton

Black-Scholes-Merton model is designed to value options which have underlying asset that pays dividends. The model can also be used to calculate implied volatilities of the underlying assets. Black-Scholes-Merton model is a modified version of the original equation. It is modified to take paid dividends into account. Average annual dividend yield is calculated by dividing the paid dividend per share with average share price in a particular year. Dividend yield is also transformed to continuously compounded form.

Calculation of call option value:

$$C = N(d_1)S^* - \frac{X}{e^{rt}}N(d_2) \tag{1}$$

Where S^* is the dividend-adjusted stock price; X is exercise price of the option or warrant; e is the neper ratio; r is the continuously compounded risk-free rate; t is the time to maturity in years; $N(x)$ are cumulative normal distribution function values.

The dividend adjusted stock price is calculated as follows:

$$S^* = \frac{S}{e^{qt}} \tag{2}$$

Where S is stock price; q is the average dividend return of the stock.

Value of d_1 is the hedge coefficient[2] and d_2 is the probability of the option to expire in-the-money.

Values are calculated as follows:

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[2] Hedge coefficient is the amount of underlying assets per one written call-option that is needed to make the position risk-free.

$$d_1 = \frac{\ln\left(\dfrac{S^*}{X}\right) + \left(r - q + \dfrac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \qquad (3)$$

$$d_2 = d_1 - \sigma\sqrt{t} = \frac{\ln\left(\dfrac{S^*}{X}\right) + \left(r - q - \dfrac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \qquad (4)$$

Where σ is the implied volatility of the underlying asset.

Assumptions of the Black-Scholes-Merton model: (Hull, 2006)

- The expected return and standard deviation of the underlying security are constant for the maturity of the option.

- Short selling is allowed.

- No transaction costs or taxes. All securities are perfectly divisible.

- Trading is constant

- Risk-free rate is constant for all maturities.

- There are no risk-free arbitrage opportunities.

2.2 Volatility

Volatility is the standard deviation of stock returns. Daily volatilities were calculated for the same period of time that was used to calculate implied volatilities. Volatilities were calculated for the remaining maturity of the option and results were annualized.

Volatility was calculated as follows:

$$\sigma_{day} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(r_i - \bar{r}_i)^2} \qquad (5)$$

Where n is the quantity of the data; r is the return.

8

$$r_i = \ln\left(\frac{P_i}{P_{i-1}}\right) \tag{6}$$

Where p is the price; p_{i-1} is the previous price.

To annualize daily volatility following equation was used:

$$\sigma_{annual} = \sigma_{day} \sqrt{h} \tag{7}$$

Where h is the number of trading days. In Europe it is typical to use 252 days.

2.3 OLS (Ordinary Least Squares)

Multiple linear regression model:

$$Y_i = \hat{\alpha} + \hat{\beta}_1 X_{i1} + \hat{\beta}_2 X_{i2} + ... + \hat{\beta}_k X_{ik} + \hat{\varepsilon}_i \tag{8}$$

Where Y is the dependent variable; α is the intercept; X is the independent variable; ε is the error term (residual)

Underlying assumptions:

1. $E(\varepsilon_t) = 0$
2. $Var(\varepsilon_t) = \sigma^2$
3. $Cov(\varepsilon_i, \varepsilon_j) = 0$
4. $Cov(\varepsilon_i, x_t) = 0$
5. $\varepsilon_t \sim N(0, \sigma^2)$
6. No perfect multicollinearity

Estimation of regression coefficients using ordinary least squares method can be done as follows:

$$\min_{\hat{\alpha}, \hat{\beta}_1 ... \hat{\beta}_k} \sum \hat{\varepsilon}_i^2 = \min_{\hat{\alpha}, \hat{\beta}_1 ... \hat{\beta}_k} \sum_{i=1}^{n} (Y_i - \hat{\alpha} - \hat{\beta}_1 X_{i1} - ... - \hat{\beta}_k \hat{X}_{ik})^2 \tag{9}$$

2.4 Correlation

Relationship between economic variables can be explored using correlation and co-variance. Correlation coefficients can have values between -1 and 1.

Correlation coefficient can be defined as follows.

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} \tag{10}$$

When X and Y are random variables, then covariance is:

$$\text{cov}(X,Y) = E\big[(X - E[X])(Y - E[Y])\big] \tag{11}$$

3 METHODOLOGY AND DATA

3.1 Methodology

Eurex continuous call series implied volatilities were imported using Thomson Data-stream. Datastream gives implied volatilities for continuous call series. All the series are at-the-money options. At-the-money prices are interpolated using two closest strikes available. Implied volatility for Helsinki Stock Exchange traded warrant was calculated using Black-Scholes-Merton model. Euribor (Euro Interbank Offered Rate) 12 months was used as risk-free rate (Figure 1.) and annual dividend yield was calculated by dividing dividend per share by average stock price of the examination year.

Figure 1. Euribor 12 months rate.

3.2 Statistical Methods

To find out if implied volatility has any forecasting power when compared to the realized volatility, regression analyses between the variables were performed. The following equation presents the regression model used to examine relation of the variables:

$$h_t = \alpha + \beta_1 i_t + \beta_2 h_{t-1} + \varepsilon_t \qquad (12)$$

Where h_t is the realized volatility of the stock return; i_t is the implied volatility of the call option or warrant; h_{t-1} is the realized volatility at the time t-1; ε_t is the residual. The aim of this regression is to find out if implied volatility has any statistical significance when explaining realized volatility and to test if it adds any predictive power when the volatility of previous trading day is taken into consideration. If implied volatility contains some information about the future volatility, β_1 should be nonzero and if implied volatility is an unbiased forecast of realized volatility, α should be zero and β should be one. In addition if implied volatility is efficient, the residuals should be white noise and uncorrelated. To further examine the relation between implied and realized volatility, correlations were tested. In addition the realized volatilities were lagged to find out if the correlation changes between variables of different time point.

11

3.3 Data and Sample Collection

The sample consists of ten different call options on two different time periods. The first period is from beginning of 2003 to the end of 2004 and the second period is from beginning of 2005 to the end of 2006. In addition one Finnish traded warrant is taken into study. Implied volatility of the warrant was calculated for year 2005. All the data was in daily format (5 days per week). Analyzed companies are listed either in Germany, Finland or in the United States. Stock prices and implied volatilities of Eurex options were collected from Thomson Datastream software. Implied volatilities of ten Eurex traded options were calculated from continuous call series using constant 30 days maturity. Implied volatility of the Finnish traded Nokia warrant was calculated manually by using Black-Scholes-Merton model. The illiquid trading of the Finnish warrants did not make it possible to select more warrants to be analyzed. Table 1 presents basic information of the analyzed data

Table 1. Analyzed Data
This table contains all the analyzed options/warrants.

Variable	Expiry Date	Option Exchange	Option ISIN*	Underlying Stock Exchange	Underlying ISIN*
Citigroup	Continuous	Eurex	US1729671016	NYSE	US1729671016
Daimler	Continuous	Eurex	DE0007100000	Deutsche Boerse	DE0007100000
Elisa Com.	Continuous	Eurex	FI0009007884	OMX Helsinki	FI0009007884
Microsoft	Continuous	Eurex	US5949181045	NASDAQ	US5949181045
Nokia	Continuous	Eurex	FI0009000681	OMX Helsinki	FI0009000681
SAP	Continuous	Eurex	DE0007164600	Deutsche Boerse	DE0007164600
Stora Enso	Continuous	Eurex	FI0009005961	OMX Helsinki	FI0009005961
Sun Micros.	Continuous	Eurex	US8668101046	NASDAQ	US8668102036
VW	Continuous	Eurex	DE0007664005	Deutsche Boerse	DE0007664005
Zurich	Continuous	Eurex	CH0011075394	Deutsche Boerse	CH0011075394
Nokia[1]	31.12.2007	OMX Helsinki	FI0009605513	OMX Helsinki	FI0009000681

[1]Warrant; * International Securities Identifying Number

Figure 2 shows the history of Citigroup's implied and realized volatility. It is interesting to notice that the implied volatility has been over the realized volatility from the beginning of 2003 to the beginning of 2004. The same phenomenon can be seen in several options analyzed in this study. In some reason the options of that time have been overvalued and therefore abnormal returns have been achieved by the traders.

The high volatility of the stocks in the beginning of the 21st century has significantly impacted implied volatilities and caused them to rise over realized volatility. The high volatility makes it more difficult to estimate future volatility and therefore the spread between realized and implied volatilities have widened in order to sell for sure at the premium.
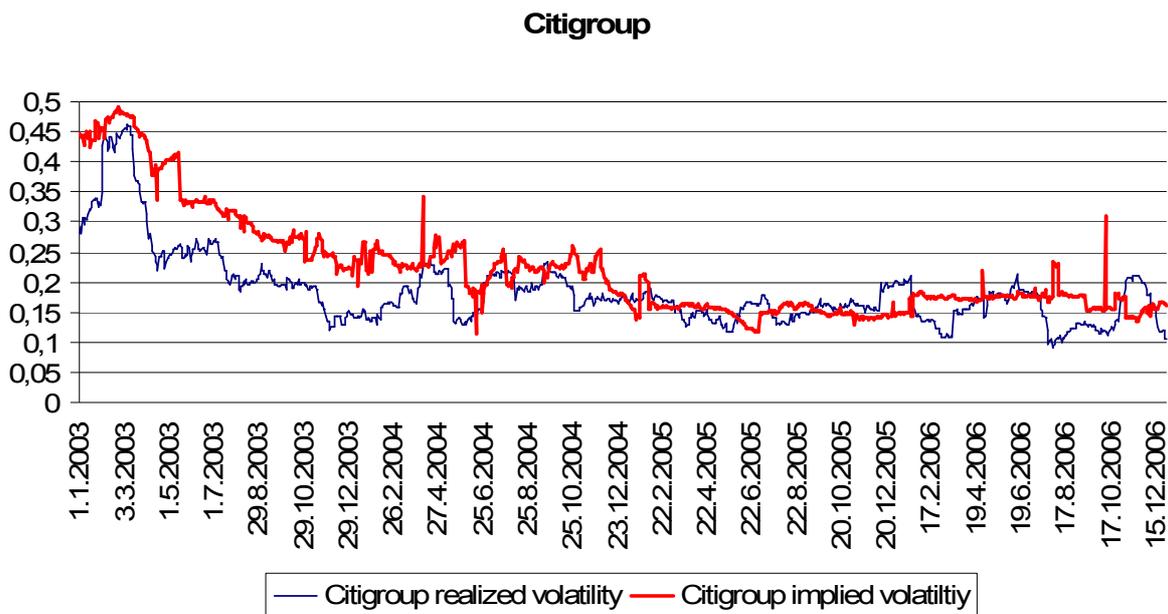
Citigroup

Figure 2. Citigroup realized and implied volatilities of the analyzed time period.

3.4 Measurement Errors

Some degree of measurement error in the warrants implied volatility is due to the fact that the Black-Scholes-Merton model is designed to work with so called European options, which means that the option can be exercised only in certain timeframe. All the analyzed Eurex options are American type and can be exercised any trading day during the lifetime of the option. Also the analyzed Finnish warrant is American style. Options are usually not exercised before the option expires because the option is worth more unexercised because of the time value it has left. This error should be constant in the whole measurement. Another possible source of measurement error is bid-ask spread in option prices which can vary between different days and there-fore is not constant in the measurement. The same error applies to the stock prices where the last trade of the day can be either bid or ask price.

Black-Scholes formula is also based on an assumption that the price of the underly-ing security follows a log-normal distribution. If the price of the underlying security changes value dramatically, the assumption is broken and the implied volatilities are not calculated correctly. In addition it is also possible that options and stocks do not have prices due a lack of trading (thin trading bias). Options that are deep-in-the-money or deep-out-of-the-money are very sensitive to volatility changes and there-fore implied volatilities of those options tend to be unreliable (Hull 2006: 300-301).

It is also notable that the analyzed warrant expires in the future of the time point when the implied volatilities were calculated, which means that the implied volatility of the warrant is for the whole maturity (to 31.12.2007) and the realized volatility is to date 11.10.2007. This should not make significant difference in the results.

4 EMPIRICAL RESULTS

4.1 Descriptive Statistics

This part discusses the descriptive statistics of different variables used in the empirical analyses. For each variable the minimum, maximum, mean, standard deviation, kurtosis, skewness and Jarque-Bera value are reported.

4.1.1 Realized Volatilities

Realized volatilities of the options are calculated for each day using constant 30 days maturity. Volatilities are calculated from continuously compounded returns and annualized. The realized volatility of the Nokia warrant is calculated to for each day using 12.10.2007 as a maturity date.

Table 2. Descriptive Statistics of the Realized Volatilities: Subperiod 1/2003 to 12/2004

This table contains descriptive statistics for analyzed options realized volatilities for period from 1.1.2003 to 31.12.2004. Volatilities are calculated for constant 30 days maturity. Every variable includes 523 observations.

Variable	Minimum	Maximum	Mean	Standard deviation	Kurtosis	Skewness	Jarque-Bera
Citigroup	0,121	0,462	0,217	0,074	2,644	1,656	385,83
Daimler	0,128	0,647	0,322	0,119	-0,145	0,754	49,77
Elisa Com.	0,166	0,549	0,358	0,113	-1,355	-0,226	44,32
Microsoft	0,125	0,434	0,242	0,080	-1,016	0,282	29,50
Nokia	0,161	0,654	0,373	0,133	-0,839	0,561	42,68
SAP	0,127	0,646	0,322	0,119	-0,145	0,754	49,77
Stora Enso	0,171	0,691	0,359	0,097	1,556	0,976	133,49
Sun Micros.	0,266	0,747	0,463	0,111	-0,548	0,585	36,3
VW	0,132	0,527	0,275	0,098	-0,073	0,916	72,85
Zurich	0,149	0,952	0,362	0,192	1,692	1,543	266,64

Realized volatilities of the period are in expected range. Zurich has the highest standard deviation. Zurich has also the widest range of the variables in this period. Kurtosis and skewness of all the variables differ from zero. Especially Zurich and Citigroup have high kurtosis and skewness.

The normality of the variables was examined using Jarque-Bera test. The null hypothesis was that the variable is normally distributed. The results of the normality test indicate that all the variables are not normally distributed at significance level of 95 percent. All the variables have high enough Jarque-Bera value to reject the null hypothesis. All the variables are not normally distributed. Variables Citigroup and Zurich have especially high Jarque-Bera values of the group.

Table 3. Descriptive Statistics of the Realized Volatilities: Subperiod 1/2005 to 12/2006
This table contains descriptive statistics for analyzed options realized volatilities for period from 3.1.2005 to 29.12.2006. Volatilities are calculated for constant 30 days maturity except Nokia[1] which is calculated to mature in 12.10.2007. Every variable includes 520 observations except Nokia warrant which includes 254 observations.

Variable	Minimum	Maximum	Mean	Standard deviation	Kurtosis	Skewness	Jarque-Bera
Citigroup	0,091	0,214	0,155	0,026	-0,644	-0,003	9,12[*]
Daimler	0,095	0,363	0,201	0,070	-0,208	0,852	63,59
Elisa Com.	0,109	0,469	0,247	0,074	1,592	0,949	130,72
Microsoft	0,132	0,394	0,175	0,057	6,808	2,604	1565,93
Nokia	0,132	0,388	0,226	0,066	-0,146	0,856	63,67
SAP	0,095	0,362	0,201	0,070	-0,208	0,852	63,59
Stora Enso	0,115	0,397	0,244	0,060	0,204	0,519	24,00
Sun Micros.	0,151	0,432	0,319	0,049	0,811	-0,726	59,06
VW	0,130	0,375	0,234	0,068	-1,207	0,350	42,11
Zurich	0,043	0,301	0,184	0,063	-0,665	-0,211	13,55
Nokia[1]	0,25	0,26	0,252	0,003	-0,659	0,033	4,81

*p > 0,01; [1]Period 2005 (maturity to 12.10.2007)

At the second subperiod Stora Enso has the greatest range in realized volatility. Elisa communication has the highest standard deviation. As a whole the standard devia-

tion of the second subperiod is on a much lower level than the standard deviation of the first subperiod. It is also notable that kurtosis of every variable differ from zero. Especially kurtosis of Microsoft is on a high level. Also skewness of most of the variables differs from zero. Citigroup is the only variable that is not skewed.

Variable Citigroup has the lowest significant Jarque-Bera value and therefore it is the most normally distributed variable of the group. It is also notable that the variable Microsoft was highly not normally distributed. As a whole the second period is much less normally distributed than the first period.

Table 4. Descriptive Statistics of Realized Volatilities: Full Period 1/2003 to 12/2006
This table contains descriptive statistics for analyzed options realized volatilities for full period from 1.1.2003 to 29.12.2006. Volatilities are calculated for constant 30 days maturity. Every variable includes 1043 observations.

Variable	Mini-mum	Maxi-mum	Mean	Standard deviation	Kurtosis	Skew-ness	Jarque-Bera
Citigroup	0,091	0,461	0,186	0,063	5,818	2,153	2257,5
Daimler	0,095	0,646	0,262	0,115	0,795	1,069	224,98
Elisa Com.	0,108	0,549	0,302	0,110	-0,972	0,475	80,311
Microsoft	0,114	0,434	0,208	0,077	-0,072	1,029	183,9
Nokia	0,132	0,65	0,299	0,128	0,367	1,068	203,24
SAP	0,095	0,646	0,262	0,115	0,795	1,069	224,98
Stora Enso	0,115	0,691	0,301	0,099	1,695	1,032	307,52
Sun Micros.	0,151	0,747	0,391	0,112	0,685	1,014	197,96
VW	0,130	0,527	0,254	0,087	0,601	0,985	183,37
Zurich	0,043	0,951	0,272	0,168	4,684	2,055	1674,1

The full period widens the range of every variable. Standard deviation of the full period is in between of standard deviation of individual subperiods. Variable Zurich has the highest standard deviation of the realized volatility. As a whole the kurtosis and skewness of the full period is on significantly higher level than on individual subperiods. Also the normality of the variables is on much lower level (higher Jarque-Bera values) on full period when compared to individual subperiods.

The standard deviation of the realized volatilities and implied volatilities on the full period are close to each other. The standard deviations of implied volatilities are little bit smaller which supports the notion that implied volatility is a smoothed expectation of realized volatility. Also the mean values of the implied and realized volatilities are close to each other which mean that the implied volatilities are on average on the correct level.

4.1.2 Implied Volatilities

Implied volatilities of the options are calculated for each day using constant 30 days maturity. Volatilities were calculated from continuously compounded returns and annualized. The implied volatility of the Nokia warrant was calculated to for each trading day using 31.12.2007 as a maturity date.

Table 5. Descriptive Statistics of the Implied Volatilities: Subperiod 1/2003 to 12/2004
This table contains descriptive statistics for analyzed option/warrant implied volatility series for the analyzed period from 1.1.2003 to 31.12.2004. Implied volatilities are calculated for constant 30 days maturity. Every variable includes 523 observations.

Variable	Minimum	Maxi-mum	Mean	Standard deviation	Kurtosis	Skew-ness	Jarque-Bera
Citigroup	0,115	0,490	0,284	0,084	0,023	1,063	97,88
Daimler	0,140	0,607	0,318	0,099	0,456	1,092	107,5
Elisa Com.	0,157	0,776	0,389	0,079	0,236	0,455	19,01
Microsoft	0,186	0,481	0,309	0,078	-1,180	0,408	44,78
Nokia	0,168	0,699	0,404	0,105	0,051	0,759	49,95
SAP	0,207	0,658	0,360	0,110	-0,168	0,875	67,02
Stora Enso	0,136	0,469	0,289	0,055	-0,363	0,124	4,32*
Sun Micros.	0,389	1,121	0,606	0,149	0,100	0,939	14986
VW	0,187	0,623	0,337	0,097	0,038	0,969	81,44
Zurich	0,166	0,789	0,341	0,142	0,735	1,240	144,49

* $p > 0,05$

In the first subperiod Sun Microsystems has the widest range with standard deviation of 0,149. Most of the standard deviations are on lower level than the standard deviations of realized volatilities on the same period. This is natural because implied volatility is typically a smoothed expectation of realized volatility. The range of all the variables is well in range of typical implied volatilities. Sun Microsystems is the only variable which implied volatility exceeds one hundred percents. Microsoft and Zurich have the highest kurtosis. Zurich, Citigroup and Daimler have the highest skewness of the group.

Stora Enso is the only variable that is normally distributed on significance level of 95 percent. It is notable that the Jarque-Bera value of Sun Microsystems is extremely high. Taking natural-logarithm of the values does not make a significant difference in the results.

Table 6. Descriptive Statistics of the Implied Volatilities: Subperiod 1/2005 to 12/2006
This table contains descriptive statistics for analyzed option/warrant implied volatility series for the analyzed period from 3.1.2005 to 29.12.2006. Implied volatilities are calculated for constant 30 days maturity except implied volatility of Nokia Warrant is calculated to 12.10.2007. Every variable except Nokia warrant include 520 observations. Nokia warrant includes 253 observations on time period 1.3.2005 to 30.12.2005.

Variable	Minimum	Maximum	Mean	Standard deviation	Kurtosis	Skewness	Jarque-Bera
Citigroup	0,116	0,311	0,161	0,019	9,342	1,491	2041,2
Daimler	0,139	0,379	0,227	0,044	0,047	0,289	7,22**
Elisa Com.	0,129	0,327	0,274	0,026	4,298	-1,504	585,58
Microsoft	0,075	0,289	0,188	0,032	1,218	0,451	48,46
Nokia	0,186	0,442	0,267	0,057	0,614	1,172	126,04
SAP	0,149	0,335	0,218	0,038	-0,032	0,796	54,66
Stora Enso	0,033	0,311	0,219	0,032	3,307	-0,344	241,00
Sun Micros.	0,230	0,499	0,362	0,046	0,227	-0,095	1,76*
VW	0,167	0,370	0,248	0,043	-0,669	0,294	17,30
Zurich	0,102	0,342	0,224	0,040	0,431	-0,087	4,39*
Nokia[1]	0,218	0,372	0,263	0,036	1,026	1,122	62,61

[1]warrant; * p > 0,05; ** p > 0,01

In the second subperiod variable Nokia has the greatest range and standard deviation. This period also is in line with the notion that implied volatility is a smoothed expectation of realized volatility and therefore implied volatilities are less variable than realized volatilities at the same period. Kurtosis of Citigroup, Elisa Communications and Stora Enso are at especially high level when compared to other variables. Skewness of implied volatilities of Nokia warrant, Nokia call option, Citigroup and Elisa Communications are on higher level than other variables.

Jarque-Bera normality test indicate that most of the variables are not normally distributed on significance level of 95 %. Variable Sun Microsystems has the smallest value of Jarque-Bera test with probability of 0,4149. The variable Zurich and Daimler are also fairly normally distributed with Jarque-Bera values 4,39 and 7,22 on probability 0,111 and 0,0269.

Table 7. Descriptive Statistics of the Implied Volatilities: Full Period 1/2003 to 12/2006
This table contains descriptive statistics for analyzed options implied volatilities for the analyzed period from 1.1.2003 to 29.12.2006. Implied volatilities are calculated for constant 30 days maturity. Every variable include 1043 observations.

Variable	Minimum	Maximum	Mean	Standard deviation	Kurtosis	Skewness	Jarque-Bera
Citigroup	0,115	0,490	0,223	0,087	1,493	1,457	462,97
Daimler	0,139	0,607	0,273	0,089	2,416	1,516	648,49
Elisa Com.	0,129	0,776	0,331	0,082	0,764	1,033	209,82
Microsoft	0,075	0,481	0,249	0,085	-0,135	0,924	148,81
Nokia	0,168	0,700	0,335	0,109	0,700	1,047	210,65
SAP	0,149	0,658	0,290	0,109	1,266	1,344	381,2
Stora Enso	0,033	0,469	0,254	0,057	0,216	0,584	61,03
Sun Micros.	0,230	1,121	0,484	0,164	0,872	1,171	270,09
VW	0,167	0,623	0,293	0,087	1,929	1,441	519,11
Zurich	0,102	0,789	0,283	0,120	3,886	1,981	1328,3

Skewness of the variables are on substantially lower level in the individual subperiods relative to the whole period. Kurtosis as a whole is highest on the second sub-

period. On subperiod one the kurtosis as a whole is on significantly lower level than on the second subperiod or on the full period.

Due to abnormally high Jarque-Bera value of Sun Microsystems on the first subperiod, the normality of the full period is on a better level than the normality of the first subperiod, although the normality of the second subperiod is on much lower level than the normality of the full period.

4.2 Hypothesis and Testing

The linear regression analyses were conducted by using ordinary least squares (OLS) method. The regression was extended to include two independent variables. Similar test layout was used by J. Christensen and N.R. Prabhala (1998). In multiple regression additional independent variable was added into regressions in order to observe whether the new independent variables bring additional explanation to the relations.

The research hypothesis tests whether there is association between realized volatility and implied volatility. Accordingly, the hypothesis (H_1) is introduced as:

>H_1: Options implied volatility is significantly related to the realized volatility of the underlying asset.

4.3 Analysis of the Results

4.3.1 Correlations

Table 8 present the results of the correlations between implied and realized volatility. Correlations were performed using Eviews 5.

Table 8. Correlation Matrix between implied volatility and realized volatility

Table presents the correlations between the implied volatilities and realized volatilities that were used in the study. The correlations were tested by using Pearsons's correlation coefficient. Correlations were calculated for the full period.

	Citigroup i_t	Daimler i_t	Elisa Com. i_t	Microsoft i_t	Nokia i_t	SAP i_t	Stora Enso i_t	Sun Micros. i_t	VW i_t	Zurich i_t	Average
Lag 0	0,823	0,698	0,603	0,610	0,646	0,772	0,718	0,668	0,712	0,681	**0,693**
Lag 10	0,836	0,750	0,612	0,613	0,682	0,822	0,725	0,690	0,783	0,734	**0,725**
Lag 30	0,823	0,732	0,629	0,664	0,615	0,779	0,763	0,688	0,806	0,744	**0,724**
Lag 60	0,768	0,537	0,622	0,680	0,666	0,733	0,686	0,675	0,653	0,633	**0,665**
Lag 90	0,725	0,486	0,595	0,601	0,507	0,666	0,639	0,676	0,556	0,439	**0,589**

Table 8 presents the correlations between implied volatilities and realized volatilities with different lags of realized volatility. It is notable that the correlation is on a highest level (0,725) when the realized volatility is lagged by 10 trading days. When the lag increases to 60 days, correlation decreases to 0,665. This finding tells that the realized volatility follow the implied volatility most strongly on ten day lag. The results is interesting because in theory the correlation should be at the highest when the lag is zero because then the maturity of the both variables is the same.

4.3.2 Regressions

Tables 9 - 14 present the results of the linear regressions. The linear regression analyses were conducted by using the ordinary least squares (OLS) method. The

SPSS 13.0 for Windows was utilized in the implementation of the empirical analysis. In simple regressions autocorrelation is very significant. Autocorrelation is probably caused by missing explanatory variables and therefore residuals reflect the effect of missing variables. The effect of autocorrelation is that OLS-estimates are not BLUE (Best Linear Unbiased Estimates) when compared to other linear estimators. Estimates of the standard errors are typically faulty and therefore t-statistics and F-statistics are not valid. Nevertheless estimates are linear and unbiased. Autocorrelation was found to not be a problem in multiple regressions on full period or on subperiod one. No evidence of heteroscedasticity was found.

Table 9. Results of the Simple Regressions: Subperiod 1/2003 to 12/2004

Table presents the results of the simple regressions, where dependent variable was realized volatility of stock returns and independent variable was implied volatility. The coefficients and t-statistics are presented for each variable used in the regressions. The t-statistics are in parentheses. In addition R^2, adjusted R^2 and F-statistics are reported for each simple regression.

Dependent variables	Intercept	i_t	DW	Adj. R^2	F-stat.	N
Citigroup h_t	0,002 (0,422)	0,756** (38,7)	0,127	0,741	1497	523
Daimler h_t	0,055** (4,361)	0,839** (22,0)	0,075	0,480	483	523
Elisa Communications h_t	0,088** (4,00)	0,695** (12,57)	0,127	0,231	158	523
Microsoft h_t	0,023* (2,205)	0,707** (21,36)	0,095	0,466	456	523
Nokia h_t	0,118** (5,87)	0,630** (13,07)	0,131	0,246	170	523
SAP h_t	0,018 (1,577)	0,843** (27,81)	0,083	0,597	773	523
Stora Enso h_t	0,033 (1,858)	1,130** (18,67)	0,234	0,400	348	523
Sun Microsystems h_t	0,456** (77,23)	0,009 (1,925)	0,073	0,005	3,705	523
VW h_t	-0,007 (-0,770)	0,838** (33,60)	0,162	0,684	1129	523
Zurich h_t	0,074** (4,335)	0,843** (18,29)	0,050	0,390	334	523

*$p < 0,05$; **$p < 0,01$

The findings in Table 9 indicate that in most regressions implied volatility is statistically significant variable when realized volatility is modelled. Sun Microsystems is the only variable with not significant implied volatility coefficient. The adjusted R^2 of the

regressions are highly varying. Due to high autocorrelation, adjusted R^2 and regression coefficients are probably overvalued. Log-volatilities were tested but the autocorrelation of the regressions didn't change satisfactorily. Regressions suffer from missing explanatory variables and therefore autocorrelation is on a very high level. The conclusion is that implied volatility alone is not sufficient factor to estimate realized volatilities and the results the results of the table 9 should be interpreted carefully.

Table 10. Results of the Multiple Regressions: Subperiod 1/2003 to 12/2004

Table presents the results of the multiple regressions, where dependent variable was realized volatility of stock returns and independent variables were implied volatility and realized volatility of previous trading day. The coefficients and t-statistics are presented for each variable used in the regressions. The t-statistics are in parentheses. In addition R^2, adjusted R^2 and F-statistics are reported for each multiple regression.

Dependent variables	Inter-cept	i_t	h_{t-1}	DW	Adj. R^2	F-stat.	N
Citigroup h_t	0,000 (0,318)	0,013 (1,489)	0,980** (97,68)	1,810	0,978	19234	523
Daimler h_t	0,002 (0,955)	0,005 (0,456)	0,987** (115,52)	1,941	0,980	13104	523
Elisa Communications h_t	0,002 (0,321)	0,028 (1,633)	0,963** (80,796)	1,817	0,943	4332	523
Microsoft h_t	0,000 (0,175)	0,022 (1,895)	0,969** (88,702)	1,901	0,965	7278	523
Nokia h_t	0,008 (1,426)	0,007 (0,478)	0,970** (81,623)	1,999	0,945	4509	523
SAP h_t	0,003 (1,345)	-0,003 (-0,27)	0,991** (101,152)	1,951	0,980	13100	523
Stora Enso h_t	-0,002 (-0,58)	0,037* (2,291)	0,975** (109,050)	1,761	0,975	10096	523
Sun Microsystems h_t	0,017** (3,014)	-0,001 (-0,42)	0,964** (83,623)	1,981	0,931	3523	523
VW h_t	0,003 (1,729)	-0,009 (-0,93)	0,997** (106,032)	2,195	0,986	18368	523
Zurich h_t	0,003 (0,974)	0,002 (0,137)	0,988** (117,097)	1,876	0,978	11429	523

*$p < 0,05$; **$p < 0,01$

Table 10 presents the results of the multiple regressions in the time period as the previous simple regressions table. The results differ significantly from simple regressions because the realized volatility of previous trading day is added to explanatory

variables. The results show that implied volatility looses all the statistically significant explanatory power when the realized volatility of the previous trading day is taken into regressions. The coefficients of the h_{t-1} are statistically very significant in every regression. Stora Enso is the only variable with statistically significant implied volatility (95% significance level). Autocorrelation problem encountered in simple regressions is not immanent in these multiple regressions. Durbin-Watson coefficients are close to two in every regression. The results show that in this time period the implied volatilities have not included statistically significant information of the future realized volatilities. In every regression the adjusted R^2's are on very high level due to h_{t-1} variables good explanatory power.

Table 11. Results of the Simple Regressions: Subperiod 1/2005 to 12/2006

Table presents the results of the simple regressions, where dependent variable was realized volatility of stock returns and independent variable was implied volatility. The coefficients and t-statistics are presented for each variable used in the regressions. The t-statistics are in parentheses. In addition R^2, adjusted R^2 and F-statistics are reported for each simple regression.

Dependent variables	Intercept	i_t	DW	Adj. R^2	F-stat.	N
Citigroup h_t	0,211** (21,230)	-0,350** (-5,70)	0,098	0,057	32,569	520
Daimler h_t	0,128** (8,069)	0,324** (4,728)	0,049	0,040	22,350	520
Elisa Communications h_t	0,141** (4,163)	0,388** (3,159)	0,050	0,017	9,980	520
Microsoft h_t	0,233** (15,568)	-0,307** (-3,92)	0,082	0,027	15,344	520
Nokia h_t	0,144** (10,647)	0,309** (6,248)	0,054	0,068	39,041	520
SAP h_t	0,115** (4,496)	0,398** (4,971)	0,047	0,044	24,711	520
Stora Enso h_t	0,091** (5,345)	0,697** (9,088)	0,099	0,136	82,590	520
Sun Microsystems h_t	0,376** (22,038)	-0,157** (-3,36)	0,091	0,019	11,299	520
VW h_t	0,093** (5,744)	0,567** (8,811)	0,048	0,129	77,641	520
Zurich h_t	0,192** (12,333)	-0,038 (-0,55)	0,039	-0,001	0,303	520
Nokia[1] h_t	0,264** (248,538)	-0,044** (-11,15)	0,110	0,329	124,363	253

[1]Warrant; *p < 0,05; **p < 0,01

Table 11 presents the simple regressions results of the second time period. The results are very similar with the results of the first time period. Implied volatilities are statistically significant in every regression except Zurich and the model suffers from very high levels of autocorrelation which makes the t-statistics and F-statistics overvalued. In comparison with the simple regression of period one, it is notable that the adjusted R^2 on the second period is on significantly lower level when compared to the simple regression of the first period.

Table 12. Results of the Multiple Regressions: Subperiod 1/2005 to 12/2006
Table presents the results of the multiple regressions, where dependent variable was realized volatility of stock returns and independent variables were implied volatility and realized volatility of previous trading day. The coefficients and t-statistics are presented for each variable used in the regressions. The t-statistics are in parentheses. In addition R^2, adjusted R^2 and F-statistics are reported for each multiple regression.

Dependent variables	Intercept	i_t	h_{t-1}	DW	Adj. R^2	F-stat.	N
Citigroup h_t	0,206** (21,878)	-0,472** (-7,813)	0,131** (7,624)	0,257	0,154	46	497
Daimler h_t	0,053** (4,120)	0,163** (2,956)	0,535** (15,354)	0,491	0,364	142	497
Elisa Communications h_t	0,077** (2,730)	0,068 (0,689)	0,615** (21,710)	0,691	0,487	236	497
Microsoft h_t	0,175** (12,526)	-0,449** (-6,391)	0,438** (13,046)	0,389	0,275	95	497
Nokia h_t	0,042** (3,407)	0,217** (5,300)	0,554** (17,343)	0,601	0,425	185	497
SAP h_t	0,032* (2,201)	0,282** (4,400)	0,520** (14,913)	0,479	0,365	143	497
Stora Enso h_t	0,028 (1,519)	0,688** (9,272)	0,297** (7,424)	0,232	0,225	73	497
Sun Microsystems h_t	0,271** (14,699)	-0,088 (-1,939)	0,284** (11,120)	0,276	0,206	65	497
VW h_t	0,041** (2,683)	0,395** (6,507)	0,444** (11,058)	0,309	0,305	110	497
Zurich h_t	0,123** (9,269)	-0,197** (-3,494)	0,521** (17,984)	0,506	0,394	161	497
Nokia[1] h_t	0,013* (2,565)	-0,003* (-2,123)	0,952** (49,836)	1,836	0,938	1919	253

[1]Warrant; *p < 0,05; **p < 0,01

Table 12 presents the results of the multiple regressions of the period two. The results differ greatly of the first period. The most significant difference is that the regressions suffer once again of autocorrelation problems. The autocorrelation is not as high as with simple regressions but it is still on very alarming level. The only variable that does not suffer of autocorrelation is the regression of realized volatility of the Nokia warrant. The results of the Nokia warrant shows that implied and realized volatilise are both statistically significant factors when the realized volatility is explained. This result shows that depending on the marketplace, the results differ. Unfortunately the lack of trading on Finnish markets did not make it possible to take more warrants under the study.

Table 13. Results of the Simple Regressions: Full period 1/2003 to 12/2006

Table presents the results of the simple regressions, where dependent variable was realized volatility of stock returns and independent variable was implied volatility. The coefficients and t-statistics are presented for each variable used in the regressions. The t-statistics are in parentheses. In addition R^2, adjusted R^2 and F-statistics are reported for each simple regression.

Dependent variables	Intercept	i_t	DW	Adj. R^2	F-stat.	N
Citigroup h_t	0,051** (16,413)	0,607** (46,780)	0,090	0,677	2188	1043
Daimler h_t	0,016 (1,905)	0,903** (31,464)	0,073	0,487	990	1043
Elisa Communications h_t	0,034** (2,970)	0,811** (24,396)	0,115	0,363	595	1043
Microsoft h_t	0,070** (11,884)	0,557** (24,848)	0,076	0,372	617	1043
Nokia h_t	0,044** (4,507)	0,761** (27,312)	0,127	0,417	746	1043
SAP h_t	0,025** (3,933)	0,817** (39,133)	0,068	0,595	1531	1043
Stora Enso h_t	-0,019 (-1,904)	1,262** (33,298)	0,245	0,515	1109	1043
Sun Microsystems h_t	0,170** (21,091)	0,457** (28,964)	0,089	0,446	839	1043
VW h_t	0,047** (7,059)	0,710** (32,754)	0,083	0,507	1073	1043
Zurich h_t	0,003 (0,260)	0,955** (29,994)	0,048	0,463	900	1043

*$p < 0,05$; **$p < 0,01$

Table 13 presents the results of simple regressions on full period ranging from the beginning of year 2003 to the end of year 2006. As with the previous simple regressions; the regression suffers from significant autocorrelation and therefore the esti-

mates of t-statistics and F-statistics are overvalued. Implied volatility alone is not sufficient factor to explain realized volatilities.

Table 14. Results of the Multiple Regressions: Full period 1/2003 to 12/2006

Table presents the results of the multiple regressions, where dependent variable was realized volatility of stock returns and independent variables were implied volatility and realized volatility of previous trading day. The coefficients and t-statistics are presented for each variable used in the regressions. The t-statistics are in parentheses. In addition R^2, adjusted R^2 and F-statistics are reported for each multiple regression.

Dependent variables	Intercept	i_t	h_{t-1}	DW	Adj. R^2	F-stat.	N
Citigroup h_t	0,001 (1,457)	0,011* (2,316)	0,980** (149,485)	1,866	0,986	35753	1043
Daimler h_t	0,002 (1,504)	0,002 (0,253)	0,989** (167,464)	1,919	0,982	27851	1043
Elisa Communications h_t	0,002 (0,670)	0,021* (2,012)	0,970** (125,036)	1,791	0,960	12583	1043
Microsoft h_t	0,002 (1,360)	0,020** (2,776)	0,966** (124,557)	1,938	0,960	12666	1043
Nokia h_t	0,004 (1,637)	0,009 (0,941)	0,976** (126,326)	1,982	0,964	14069	1043
SAP h_t	0,003 (1,914)	-0,001 (-0,074)	0,990** (148,060)	1,922	0,982	27850	1043
Stora Enso h_t	-0,001 (-0,291)	0,024* (2,354)	0,981** (167,543)	1,770	0,983	29537	1043
Sun Microsystems h_t	0,007** (2,823)	0,013* (2,163)	0,965** (112,428)	1,946	0,958	11832	1043
VW h_t	0,004** (2,829)	-0,010 (-1,635)	0,996** (169,170)	1,948	0,983	29592	1043
Zurich h_t	0,002 (0,899)	0,005 (0,600)	0,989** (177,397)	1,864	0,983	29782	1043

*p < 0,05; **p < 0,01

Table 14 shows the multiple regression results of the full period. The results are similar with the multiple regressions of the first subperiod. Regression coefficients of the explanatory variable h_{t-1} are statistically very significant in every regression. The most of the intercepts do not differ from zero. Also the variable i_t is statistically significant in several regressions. The most significant regression coefficient of the implied volatility is in Microsoft regression. Also regressions Citigroup, Elisa Communications, Stora Enso and Sun Microsystems have statistically significant implied volatilities. This means that implied volatility does have forecasting power over the realized vola-

tilities. The increase in amount of observations seems to have increased the statistical significance in implied volatilities. The adjusted R^2 also shows very high values. Regressions are also free of autocorrelation.

5 CONCLUSIONS

The empirical findings of this study, regarding the relation between implied and realized volatility, suggest a positive relation between these measures although the forecasting power of the implied volatility does not outperform realized volatility as in the study by B.J Christensen and N.R Prabhala (1998). This study supports the previous results by Canina and Stephen Figlewski (1993) where they found that the forecasting power of implied volatility was poor by studying S&P 100 index options. As a whole the conclusion is that the implied volatility does contain some data about future volatility and therefore the future volatility estimates are in some degree successful.

The implied volatilities of the studied options in the past years have achieved in average correct levels and therefore the options are in average correctly priced. It is notable that in times when volatility is high, the implied volatilities are on even higher level. This means that some of the options have been overpriced when the volatility of the underlying stock has been on a high level. It seems that options with high volatility underlying security are not as well priced as options with low volatility underlying stocks. The implied volatility easily overreacts to volatility changes of the underlying stock.

The results of regression analysis shows, that the implied volatility is not statistically significant explanatory factor when the time period is two years. When the time period is increased in four years the significance of the implied volatility increases. In this study half of the implied volatilities were found to be significant factors when the full four year time period was used in the regressions.

In single regressions the autocorrelation rose to be a problem and therefore the results are controversial. Implied volatilities were statistically significant but due to high autocorrelation the test statistics are not accurate and therefore the results are unreliable.

The results considering the analyzed Finnish warrant are that the implied volatility does contain some information of the realized volatility of the underlying stock. The implied volatility is in average on correct level.

Correlations between implied volatilities and lagged realized volatilities were also performed. The results were that the correlation is highest when the lag of realized volatility is around 10 trading days. This result is surprising because the correlation should be strongest when there are no lagged variables. In this context implied volatility has predictive power when it comes to realized volatility.

Some suggestions and possibilities for further research rise while processing this study. First of all to make the results more reliable, the sample size should be widened from four years to at least eight years. Also options could be selected from several different exchanges to get comparative data. In addition GLS (Generalized Least Squares) should be used in order to cope with high autocorrelation of the variables.

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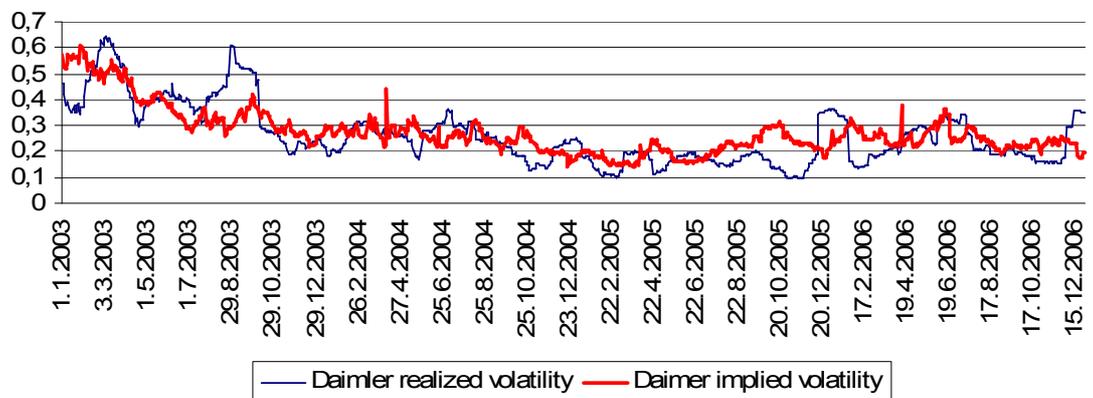Vaihekoski, M., 2004. *Rahoitusalan sovellukset ja Excel*. WSOY, Helsinki. 272-279.

APPENDICES

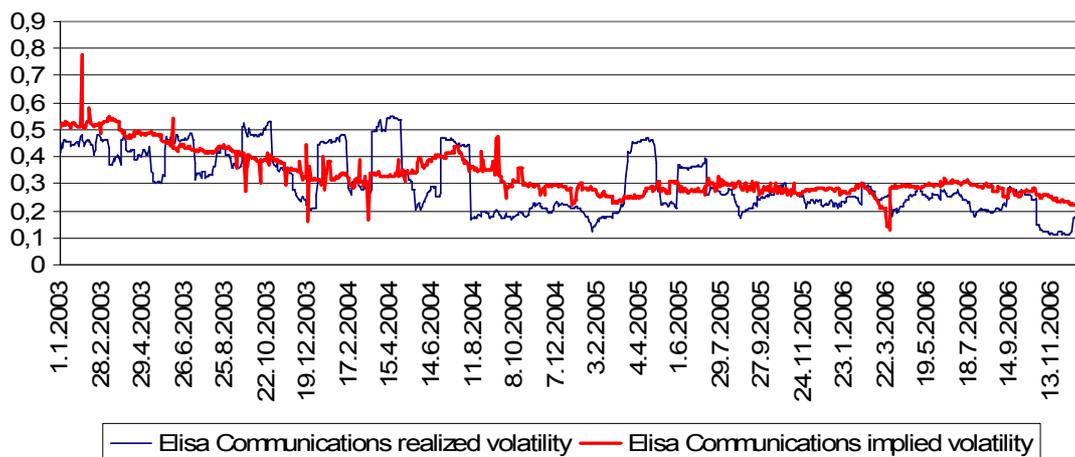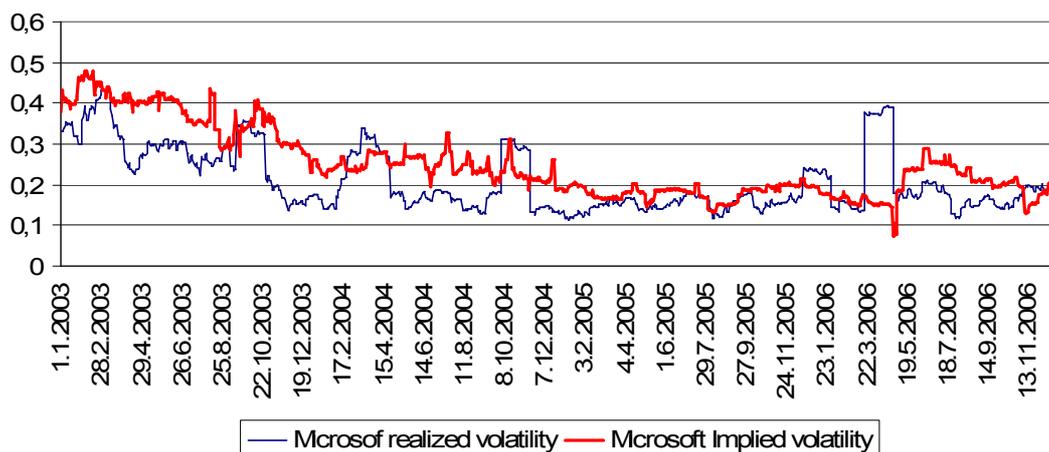Appendix 1 presents implied and realized volatilities of the studied data.

Citigroup

Citigroup realized volatility —— Citigroup implied volatiltiy

Daimler

Daimler realized volatility —— Daimer implied volatility

Elisa Communications

Elisa Communications realized volatility — Elisa Communications implied volatility

Microsoft

Mcrosof realized volatility — Mcrosoft Implied volatility

Nokia

Nokia realized volatiltiy — Nokia implied volatility

SAP

SAP realized volatility — SAP implied volatility

Stora Enso

Stora Enso realized volatility — Stora Enso implied volatility

Sun Microsystems

Sun Microsystems realized volatility — Sun Microsystems implied volatility

VW

Zurich

Nokia warrant