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Convertible Bond Pricing: comparison of two pricing models against market prices on two German airline companies

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Introduction

Convertible bond issuances have increased significantly from $7.8 billion in 1992 to $50.2 billion in 2006. About 70 % to 80 % of these convertible bond issues are bought by convertible bond arbitrage hedge funds and they are very important investors both in primary and secondary markets. The trades caused by convertible arbitrage represent more than half of the secondary market trading in convertible securities, making the hedge funds the most important market participant. (Choi et al. 2008, Horst et al. 2004)

Relatively little research has been done about convertible arbitrage and convertible bond pricing, even though the markets are large. The underpricing of convertible bonds is one of the main sources of possible arbitrage in convertible arbitrage; different pricing models for convertible bonds have been developed in several papers Ingersoll (1977), Brennan and Schwartz (1995), Tsiveriotis and Fernandes (1986) and McConnell and Schwartz (1986). But only a little research has been done by the empirical side Ammann et al. (2003), Chan and Chen (2005), Horst et al. (2007) and Kang & Lee (1996). Agarwall et al. (2008), Choi et al. (2008) have researched the impacts of convertible bond arbitrage on liquidity. Agarwall et al. (2004/2007) and Horst et al. (2007) have examined the overall performance of convertible arbitrage strategy.

In this study the pricing of convertible bonds, of two airline companies Air Berlin and Deutsche Lufthansa, is compared against the markets. These companies were chosen because they represent the old and new business strategy in the airline industry and Deutsche Lufthansa is 8 times bigger in net sales than Air Berlin. The convertible bond valuation is done with two different models; binomial-tree model and component model. We find that binomial-tree model is more accurate model and the convertible bonds seem to be underpriced.

This paper begins with an overview of convertible bonds and recent studies of convertible bond pricing models and underpricing and convertible arbitrage issues. Then the pricing models used in this study are introduced. In the last part are data and results. There is a quite a lot about convertible arbitrage, but it is essential to understand the basics of convertible arbitrage in order to understand how the market works and what impacts it may have on the pricing and liquidity. Convertible arbitrage hedge funds are the main market participants on the convertible bond markets.

1. Resent research

1.1. Convertible bonds

Convertible bonds are corporate debt securities that give the holder the right to receive future coupon and/or principal payments and convert the bond into predetermined number of common stock of the issuing company or other company. Convertible bonds are complex securities that combine characteristics of both bonds and stocks. It is a hybrid security containing a straight bond and an option to call the underlying equity. Convertible bonds have limited loss potential, assuming that the issuer does not default on its bond obligation, and holder of the bond profits when stock prices rise. It is possible to hedge against default risk by purchasing credit default swap, if possible. Usually the issuer has the right to call the bond, but there are restrictions for calling, for example that the bond may not be called for during the first five years following the issuance date. (Ammann et al., 2003; Tsiveriotis & Fernandes, 1998)

There are a variety of different convertible bonds that can be issued and almost only imagination of man sets the limits to the development of different financial products, as we have seen. Some basic types of convertible bonds are described below.

- **Vanilla convertible bonds** are the most basic convertible bonds. These may be converted by the option owner into common stock of the issuer at a predetermined rate. These may not be redeemable by the issuer, depending on the contract.

- **Mandatory convertibles** are the most equity-like of convertible securities and they give only a little downside protection because they usually do not have fixed terminal value. They pay higher dividends than common stock and mandatorily convert into common stock at a specified date.
- **Exchangeables** are convertible bonds, which can be converted into other company's stocks than those of the issuer.
- **Mandatory exchangeabels** are mandatorily exchangeable upon maturity into a fixed amount of common stocks. They usually have higher yields than the underlying common stocks.
- **Convertible preferred stock** is almost like vanilla convertibles but its stocks have seniority in the capital structure. Each issues terms will define the seniority and dividends.
- **Contingent convertibles** allow the investor to convert the bond into the stock if the price of the stock exceeds the conversion price by a certain percentage.
- **OCEANEs** (Obligation Convertible En Actions Nouvelles ou Existance) may be converted into the equity of the issuer, but the issuer has the right to determine if it delivers old stocks or new stocks. Common in France.
- **LYON** (Liquid yield option note) is a zero-coupon, callable, redeemable convertible bond. Includes a call protection for the investor, because it may not be called during some predetermined period after the issuance or until the underlying stock rises above some predetermined level. It is also the most debt-like of convertibles. Created by Merrill Lynch in 1985. (McConnell & Schwartz, 1986)
- **Going public bonds** pay fixed interest and they may be converted or exchanged into shares of the issuing company if it later makes an initial public offering.
- **Hybrid bonds** are usually issued as a loan capital, but the issuer has the right to exchange or convert the bonds into convertible preference stocks with similar conversion income and rights.

1.2. Pricing of Convertible bonds

Pricing a convertible bond is more complex than pricing an option or a bond, because of periodic coupon payments and because it involves two options. Bondholder has the option to convert the bond into common stock of the issuing firm (or into other company's common stock in case of exchangeable convertible bond) and the firm has almost always the right to call the bond. Call option has usually restrictions e.g. that the bond may not be called during the first five years. Credit risk is the one of the most important factors when pricing a convertible bond, if it is ignored the coupons and principal payments will be overvalued thus leading to an unreliable value. (Hull, 2006; Kang & Lee 1996)

The first model for pricing of the convertible bond appeared in the 1960s. Idea was to set the price of the convertible bond equal to the maximum value of its ordinary bond or its maximum value in common stock after conversion and then discount the value to the present. These models have couple shortcomings, they don't say anything about which discount rate should be used and if the point of evaluation is before maturity then the duration of convertible privilege has been limited. And if only the maturity date is considered then the possible conversion is ignored. (Ingersoll, 1976)

Ingersoll (1977) has developed a convertible bond valuation model based on Merton's model. In short, the value of the convertible bond depends on the value of the underlying which is the stock of the company that has issued the bond adjusted with boundary conditions on optimal conversion and call strategy. Valuation is directly applicable to non-callable bonds and to callable bonds with slight modifications to the model. He divided assumptions into three categories: behavioural, general market and specific market assumptions. Behavioural assumptions stated that investors prefer more wealth to less and perfect substitutes are priced identically. The management shall at all times act so that it maximizes shareholder value. General market assumptions imply perfect markets, no dividends, constant conversion terms, the Modigliani-Miller theorem (no corporate taxes and the Modigliani-Miller proposition 1 obtains), no call notice and flat term structure. Specific market assumptions are Frictionless markets (no restrictions against borrowing or short selling and trading is continuous in time), The Ito dynamics (market value of company follows an Ito diffusion process) and such a capital structure in which the convertible bond is the only senior issue of the company and the only other claim is the common stock. It is also assumed that Brownian motion holds. Ingersoll also reports that the optimal call strategy for management is to call the bond when the bonds value equals to the call price.

Brennan and Schwartz (1980) developed a model similar to Ingersoll's model that assumed stochastic interest rates. But in 1980 they developed a new version of the previous model in which the interest rates are non-stochastic, thus making the model even more and extremely complex. The equilibrium value of the convertible bond is defined as a value that does not offer arbitrage opportunities for purchaser or to short seller. It is also assumed that both the bondholder and the issuer shall use optimal strategy when deciding about converting or calling the bond. Because of the complexity of the model it was interesting to test how these two models differed from each other. So they calculated the 10-year riskless interest rates corresponding to short-term riskless rates of 0, 10, 15 and 20 percent. Then they compared results when valuation was done with short-term riskless rates and when valuation was done with 10-year riskless rates. The results were that the differences were slight and therefore they suggested that in practice it might be preferable to use the simpler model for valuing convertible bonds.

Tsiveriotis and Fernandes (1998) suggest that in general convertible bonds can be accurately valued only by simultaneously pricing of the equity and the fixed-income part. Thus a convertible bond should be valued as a derivative of the underlying equity and interest rates. They argue that usually used two-factor Black-Scholes model is not valid approach for convertible bonds, because it accounts only for credit risk by using credit spread in discounting the derivative's value in time. But in convertible bonds only a part of its value is exposed to default risk and it is not known in advance. They assume that a rational bondholder will choose to receive future cash payments that are a derivative of the underlying equity and interest rates, thus easy to calculate. They extended the Black-Scholes model with credit risk, so that it involves one additional Black-Scholes equation for each credit class of payments involved in the convertible bond. As credit spread they used market-observed credit spreads for straight bonds. They also argue that it might be a good way to estimate credit spread by calculating the probability of conversion and use it to adjust the credit spread.

McConnell and Schwartz (1986) developed a model for pricing LYONs using contingent claims pricing techniques. While developing the model they wanted it to be commercially useable, unlike many other models that were far too complex to be used with an enhanced personal computer. It was the year 1986 so the personal computers where far away from today’s personal computers, but the approach was right. Again there are the assumptions that the issuers call policy is to minimize the value of the LYON and the investor has the opposite strategy to maximize the value of the LYON at each point of time and also the term-structure of the interest rates is flat and known in advance. In order to derive the LYON pricing model they assumed that the value of the LYON depends upon the issuer’s stock price, whose instantaneous changes follow a diffusion process with constant variance. They also created a simplified model for commercial use that appeared to mimic closely the reported market closing prices calculated with the more sophisticated model.

1.3. Convertible bond underpricing

A lot of research has found evidence that the initial public offerings of convertible bonds are underpriced. Henderson (2005) examined individual corporate issuances of convertible, equity and non-convertible bonds; the data included 483 convertible bonds. As a pricing model he used binomial-tree model with 50 periods in a calendar year. Evidence provided by empirical tests showed that new issues experience positive excess returns after the issuance, however the excess returns are restricted to the first two months following the issuance. The results suggest that profitable arbitrage opportunities arise during the first months following the issuance.

Henderson also found that the average firm-level volatility declines after the issuance, thus lowering the returns for convertible arbitrage strategy in the sample employed. Even tough previous research had pointed that because of market timing investors earn negative risk-adjusted returns, Henderson did not find evidence for this. (Henderson, 2005)

Ammann et al. (2003) studied French convertible bond markets and found that theoretical values for analyzed convertible bonds were on average more than 3 % higher than the observed market prices. They researched daily convertible bond prices as well as prices of the underlying securities from February 19, 1999, through September 5, 2000. They excluded illiquid issues from the sample by requiring three conditions from each issue: The market capitalization of convertibles should be above USD 75 million, minimum trading volume for the last two quarters at least USD 75 million and at least three of the top ten underwriters quote the prices with bid/ask-spread not larger than two percentage points. After satisfying these conditions there were 21 French convertible bonds including 7 exchangeable convertible bonds, as valuation methods they used the component model, the Margabe model and the binomial-tree model. From which the binomial-tree is most accurate and the component model the most inaccurate. The results were applicable for both standard convertibles and exchangeable bonds. They also found positive relationship between underpricing and maturity of the bonds. Convertible bonds with short maturity are priced more accurately than convertible bonds with longer maturity. The finding was in contrast to the previous research

Chan and Chen (2005) refer to the Moody's study of convertible bond issuers between 1970-2000. The study shows the recovery rates and default history of all convertible bonds that were rated by Moody's during that time period. There were no significant differences in default probability between convertible bonds and non-convertible bonds. But, there was a significant difference between recovery rates for all maturities. They suggest that the underpricing could be explained by the difference in recovery rates if the conditional default probabilities are substantial (25 percent or more). But, the same study also showed that the historical default probabilities were not high enough. So, they came to conclusion that the underpricing of convertible bonds should be apparent only during financial uncertainty of the issuing company (e.g. when company's bonds are downgraded or when they are first issued).

They studied convertible bond markets in the US, as the data they had all domestic convertible bonds with face value over US$100 million with Moody's initial rating of B2 or above. They researched the data at different data points in time from May 1999 and to 2003, in order to determine the survivorship of the used data. In the end they had 107 US convertible bonds that satisfied the requirements. As valuation method they used the same model as Ammann Kind and Wilde (2003), based on the theoretical model of Tsiveriotis and Fernandes (1998). There was a clear relationship between underpricing and credit rating: the lower the rating the higher the underpricing; a mean underpricing of -5.03% for companies with credit rating Aaa to A3, -8.23% (Baa1 to Baa3) and -11.80% (below Baa3). They also found that seasoned convertible bonds converge to their theoretical values during the first 500 weekdays, if the credit rating has not fallen. If credit rating falls then there was no evidence of convergence. Also if stock price fell significantly (fall of at least 50% within last ten trading days), the underpricing increased. After such an event there was an average pricing error of 9.77%. The final conclusion was that convertible arbitrage might not be risk-free and that the underpricing is not underpricing after all, it is caused by potential renegotiations on the covenant terms when the issuer faces financial difficulties. (Chan and Chen, 2005)

Kang & Lee (1996) studied U.S. markets with 91 convertible bond offerings from 1988 to 1992 and found an average excess return of 1.11%. They excluded convertible bonds that were not listed on the New York Exchange Bonds or American Exchange Bonds, they also issues for which closing prices were not in the Wall Street Journal for more than 30 days after issuing. From these 91 convertible bonds 56 were coupon bonds and 35 were zero-coupon bonds. The bonds ratings ranged from Aa to B. On average the issue amount of zero-coupon bonds were greater than the amounts of coupon bond issues. Median call protection period was 2.92 years and a coupon of 6.50%. Of the 91 issues, 87 were out-the-money, i.e. the conversion price exceeds the stock price at the time of offering.

As a pricing method to new convertible issues Kang and Lee first calculated the initial raw return (the return from buying at the offer price and selling at the closing price on the first day of public trading). To measure initial excess returns they used the Merrill Lynch Convertible Bond Index as a benchmark index for investment-grade and speculative-grade convertible bonds. The initial excess returns were calculated by subtracting the return on either the Merrill Lynch Investment-Grade or Speculative-Grade Convertible Bond Index from the initial raw return. The mean initial raw return for full sample was 1.31%, which was significantly different from zero at the 0.01 level. 68 issues showed positive returns on the first trading day. The mean and median convertible bond market-adjusted returns were 1.11% and 0.93%, that were also significant at the 0.01 level. The underpricing was higher for companies having equity-beta above 1 and an age less than the median 23 years. Unlike study by Ammann et al (2003) Kang and Lee found that buying at the offer price and holding for 125 or 250 trading days didn't seem to provide excess returns. (Kang & Lee, 1996)

Horst et al. (2007) studied convertible debt issues in the Canadian market between 1998 and 2004. In the end there were 72 new public issues denominated in Canadian Dollars, they excluded exchangeable and zero-coupon convertible bonds. As the valuation model they used binomial-tree formalized by Tsiveriotis and Fernandes. As a risk free rate they used Canadian government bonds of comparable maturity as the convertible bond. They found that more equity-like convertibles are more underpriced (25.2%) than debt-like convertibles (5.4%). The average underpricing in the whole sample was 10% at the issuance date. Companies that have below investment grade credit rating or don't have rating at all, issue more equity-like convertibles. They found a negative correlation between trading volume during the first three days following the issue and mis-/underpricing. Thus part of the mis-/underpricing is caused by lower liquidity of convertibles in the early trading, especially for the more equity-like convertible bonds. The underpricing decreases during the first 15 to 20 trading days and underpricing remains for a longer period of time.

1.4. Convertible arbitrage impacts on liquidity

Agarwal et al. (2007) studied U.S. and Japanese convertible bond and stock markets by directly modelling the trading strategies used by convertible arbitrageurs. They also found that discrete exogenous shocks on market liquidity such as Long Term Capital Management crisis may cause abnormal supply and demand situations on convertible bond markets. They argue that it is possible that demand creates its own supply in the convertible bond market, but more research is needed to prove it. Their conclusion was that arbitrageurs act as liquidity providers on the convertible market.

The research has shown that convertible arbitrage trades represent over half of the secondary market trading of convertible securities. Thus hedge funds are very important liquidity providers on the market. Choi et al. (2008) examined impact of convertible bond arbitrage on liquidity, as data they used all convertible bond issues by U.S. Publicly traded firms for the period July 1993 through May 2006. They examined changes in short interest near bond issuance date in order to estimate the equity positions taken by convertible arbitrageurs. They found that liquidity improves following issuance of convertible debt and the improvement was systematically related to their proxy (change in short interest intensity) for convertible bond arbitrage activity. Thus suggesting that there are positive liquidity spillovers on the equity markets that are caused by arbitrage activity, but they didn’t found systematically relationship between arbitrage activity and stock return volatility or efficiency. (Choi et al., 2008)

Horst et al. (2007) more equity-like convertible bonds have significantly lower trading volumes during first four days than more debt-like convertible bonds. 77.4 % of the total issue of the debt-like convertible bonds are traded at the issue date, but only 21.3 % of the equity-like convertibles are traded at the issue date. The results showed that at issuance date major of the convertible arbitrage activities took place.

2. Convertible arbitrage

2.1. Introduction to industry

Convertible arbitrage has been one of the most successful hedge fund strategies of the nineties and the beginning of 2000s. Horst et al. (2007) found that number of convertible arbitrage hedge funds grew from about 26 in 1994 to 145 in May 2003 and decreased to about 126 in November 2004. At the same time asset under management grew from $0.7 billion in January 1994 (about 2.2 % of the total assets under management in the hedge fund industry) to about $11.5 billion in May 2003 (about 2.8 % of the total assets under management) and even tough the assets under management grew to $13.9 billion the relative amount fell to 1.9 % of all assets under management.

The annual return for the period of 1994-2004 was 9.40 % and the annual standard deviation was 4.66 %. During the same time period annual return of S&P 500 was 11.68 % with an annual standard deviation of 15.24 % [Annual return of the OMX Helsinki index during the same period was 12.93 % with an annual standard deviation of 58.83 % (Data from Thomson OneBanker)]. Thus the risk-reward trade-off has been better to convertible arbitrage strategy than to pure equity strategy. (Horst et al 2007)

The history of convertible arbitrage goes back to second half of the ninetieth century when the first convertible securities were issued. Albeit the computation methods were not as sound as today, the principle was the same. (Choi et al. 2008, Horst et al. 2004)

Convertible arbitrage is a strategy, mainly used by hedge funds, that tries to exploit underpricing of convertible bonds. The strategy involves buying a portfolio of convertible securities, generally convertible bonds, and then hedging the equity risk by selling short the underlying stock (i.e. Delta-neutral hedging). The amount of underlying stock that should be sold short is a function of the conversion ratio, delta of the embedded call option and the gamma (the sensitivity of delta to changes in stock price). Some hedge fund managers may also try to hedge the interest rate risk and/or credit risk, for example. Studies also show that convertible bond arbitrageurs look for equity-like convertible bonds and that they prefer underlying stocks that pay no or low dividends, are undervalued, liquid and are easy to sell short. Zero coupon convertible bonds are usually avoided because they lack cash inflows (i.e. Coupon payments) even tough fund receives cash inflows also from the short interest credit on the short stock account. Non-or low dividend paying stocks are favoured since they don't cause so much cash outflows caused by the short position. (Agarwal et al., 2005; Horst et al., 2007) The strategy has been very successful at the end of the 1990s and beginning of the 2000s, with annual returns up to 20 % or more. By 2006 the returns have strongly decreased from the beginning of the 2000s and returns turned negative in 2005. Calamos has found out that similar results can be seen by warrant hedging, reverse hedging, capital structure arbitrage as well as with some other strategies. (Horst et al., 2007)

Convertible arbitrage as a term is not the best possible, because it is impossible to create risk-free returns with convertible arbitrage. This is because of corporate bonds that are risky and we should not forget the different options that are embedded in the convertible bond.

2.2. Convertible arbitrage risks and hedging

Agarwal et al. (2004) investigated Japanese convertible securities and they came out with two main findings. Firstly they found that Japanese convertible securities are predominantly associated with exposure to equity risk. But they didn't find significant exposure to interest rate risk, credit risk, volatility risk and currency risk, which can be thought as risk factors with such securities and investments. Secondly they found that a passive strategy with a long position in convertible bond and a short position in a portfolio of underlying stock explains from 7 % to 15 % of the return variations in the hedge fund indexes researched. The explanatory power is limited because hedge funds operate globally and not only in one specific country. They also found that Japanese hedge fund managers do not hedge against other risks such as interest rate risk or credit risk. When hedging equity risk by shorting stock results that at the same time also a portion of credit risk is hedged, this is because of the widening credit spread.

Agarwal et al. empirical results from 2007 also showed that active arbitrage strategy is more profitable than traditional buy-and-hold-strategy, because active strategy explains a large proportion of the return variation in convertible arbitrage hedge funds. Their other study from U.S. and Japanese markets from 2005 shows that following strategies; positive carry[1], credit arbitrage[2] and volatility arbitrage[3], explain a large portion of the return variations in convertible arbitrage hedge funds (Agarwal et al., 2005, 2007)

2.3. Convertible arbitrage: Delta-hedging

The classic convertible bond arbitrage strategy uses delta hedging, in which an arbitrageur buys the convertible bond and sells the underlying security at a specific delta. We know that if the stock price goes up, arbitrageur has to sell some of the

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[1] In positive carry strategy manager chooses bonds that have positive carry on daily basis. And they take delta-neutral position in the chosen bonds for the period for which they want to have positive carry. Carry=current income on delta-neutral position – financing cost on the convertible bond. Designed to minimize credit risk.

[2] In credit arbitrage strategy manager tries to capture value from the mispricing of the credit risk inherent in convertible bond. Designed to create a long credit spread position while minimizing interest rate and credit risk.

[3] In Volatility arbitrage strategy is similar to the two strategies above, but volatility arbitrage requires sizeable gamma. Gamma peaks when its equity option is at-the-money. The bond portfolio is equally weighted. It hedges against credit, interest rate and equity risk. Volatility arbitrage exploits the underpricing of embedded option in convertible bond by actively managing a delta-neutral, but at the same time having a long gamma position.

stock in order to keep the same delta hedge. And when price goes down an arbitrageur buys the stocks. The delta is defined as the change in the value of conversion right due to the change in the value of the underlying security. Actually it is derived from the option pricing model developed Black and Scholes, adjusted with continuous dividend payment model developed by Merton. (Horst et al., 2007; Choi et al. 2008)

$$\Delta = \frac{\partial C}{\partial S} \tag{1}$$

$$\Delta = e^{-\delta T} \cdot N\left[\frac{\ln\frac{S}{K} + (r - \delta + \frac{\sigma^2}{2}) \cdot T}{\sigma \cdot \sqrt{T}}\right] \tag{2}$$

where C is the value of conversion option, S is the current price of the underlying stock, K is the conversion price, δ is the continuously compounded dividend yield, r is the continuously compounded yield on a chosen risk-free bond, σ is the annualized stock return volatility, T is the maturity of the bond and N(.) is the cumulative standard normal probability distribution. The delta is always between 0 and 1, the closer to 1 the value is the more sensitive the convertible bond value is to underlying security (stock). Delta value close to 1 also predicts that the bond will be converted with high probability. The delta measure is never perfect measure for the sensitivity of the conversion option to changes in the stock price. There are several reasons for this. Firstly, conversion option is an option with a stochastic exercise price, because the underlying bond is used to pay the exercise price. Second, convertible bonds are almost always callable. Third, in some cases new shares are created when the option of conversion is exercised which leads to dilution among current shareholders positions. These are not taken into account with delta-measure, but as always there is no better measure available. (Horst et al. 2007; Choi et al. 2008)

3. Hypothesis and pricing models

3.1. Hypothesis

In this paper it is tested the theoretical prices of convertible bonds differ significantly from market prices, when calculated with the following pricing models.

H_0: theoretical price equals market price

H_1: theoretical price does not equal market price

3.2. Two pricing models for convertible bonds

Next we are going to introduce two different convertible bond-pricing models, which are used in this paper for valuation of the convertible bonds. First is the component model and second is the binomial-tree model, which is more accurate than the component model. While pricing convertible bonds and comparing results to other studies (even with same data) or to markets, we should remember that there are many models that are used. Thus it can be said that in this paper the analysis is more about testing these two models against the markets than testing the underpricing on the markets.

3.2.1. Component Model

In component model the convertible bond is divided into straight bond component, F_t, and an option component C_t on the conversion price $S_t n_t$ with strike price $X=F_t$. The fair value of both components can be calculated with basic formulas, thus the model is easy to use and it is also widely used in practice to price convertible bonds. (Ammann et al., 2003)

Component model has also its problems. Because the convertible bond is divided into a bond component and an option component there is a restrictive assumption, that there may not be any embedded options. Even tough convertible bonds are mostly callable and/or putable, these are not taken into account in the model. The value of the option is very simple to get, however it is good only for plain-vanilla options and a rough approximation for exotic options. (Ammann et al., 2003)

While call options have a strike price that is known when option is purchased, convertible bonds include an option with stochastic strike price. It is stochastic because the strike price is known only when the bond is converted into common stock. This is because of the price of the bond changes daily, but the conversion ratio stays the same. The strike value is known only if the bond is hold until to the maturity. Thus the future strike price depends on the future credit spread and the future interest rates. (Ammann et al., 2003) And the option component is valued as European option, but convertible bonds are usually American options that may be converted or called any time before maturity with some restrictions.

The fair value of bond component with face value N, a continuously compounded coupon rate c, and a credit spread ξ_t is calculated using the discount formula.

The notation for the component model:

t = current time
Ω_t = fair price of convertible bond
T = maturity of convertible bond
N = face value of the convertible bond
S_t = equity price (underlying) at time t
F_t = investment value (bond floor) at time t
$\sigma_{S,t}$ = volatility of the stock at time t
d_S = continuously compounded dividend yield
c = continuously compounded coupon rate
n_t = conversion ratio at time t
$r_{t,T}$ = continuously compounded risk-free rate from time t to time T
ξ_t = credit spread at time t
n_tS_t = conversion value at time t

κ = final redemption ratio at time T in percentage points of the face value

$$F_t = N \cdot \exp\left[-\left(r_{t,T} + \zeta - c\right)\left(T - t\right)\right] + N\left(\kappa - 1\right) \cdot \exp\left[-\left(r_{t,T} + \zeta - c\right)\left(T - t\right)\right] \quad (3)$$

κ is the repayment ratio, which shows the amount of cash (in percentage of the face value), which is paid out if the convertible bond is hold to the maturity.

The option part is valued using the Black & Scholes model, adjusted with dividend yield as suggested by Merton (1973), which is one of the most important models in finance. Black and Scholes (1973) model is derived using the following idea; there should not be arbitrage opportunities, by entering in long and short positions in options and their underlying stocks, if the options are priced correctly. It can be said that if the option is "in-the-money" (i.e. for call option, the exercise price below current stock price), the option is very likely going be exercised. For example, if the stock is price is higher than the exercise price of the option, then the value of the option should be stock price minus the price of a pure discount bond that has the same face value to the exercise price of the option and it expires the same day as the option. And if the stock price is much less than the exercise price of the option, then the option is very likely going to mature without exercise thus its value should be close to zero or zero. If the expiration date is very close in future then the option price should be close to stock price minus exercise price or zero, if the call option is "out-the-money" (i.e. the exercise price is above the stock price).

The assumption for the model:

- The short-term interest rate is known and constant through time.
- The stock price follows random walk, thus the distribution of possible stock prices at the end of any infinite interval is log-normal.
- The stock pays no dividends or other distributions. (In this paper dividend yield is taken into account.)
- The option is European, thus it can be exercised only at maturity.
- There are no transaction costs in buying or selling the stock or the option

- It is possible to borrow at the short term interest rate any fraction of the price of a security to buy or to hold it
- There are no penalties to short selling.

With these assumptions the value of the option is depended only on the price of the stock, time to maturity and the variables that are thought as constant.

Geometric Brownian motion under the risk-neutral measure for the stock price, with payoff $C_T = Max(S_T - X, 0)$, the fair price of the call option is

$$C_t = S_t N(d_1)\exp\left[-d_S(T-t)\right] - X \cdot \exp\left[-r_{r,T}(T-t)\right]N(d_2) \tag{4}$$

where

$$d_{1,2} = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r_{t,T} - d_S \pm \frac{\sigma_{S,t}^2}{2}\right)\cdot(T-t)}{\sigma_{S,t}\sqrt{T-t}} \tag{5}$$

Thus, the fair price of the convertible bond is $\Omega_t = C_t + F_t$ (6)

3.2.2. Binomial-Tree model

Binomial-tree model is more exact approach than the component model, because it takes embedded options and the possibility of early exercise into account. In this analysis binomial tree model created by Hull (2006) is used. Also here the value of the convertible bond is divided into a straight bond component and a stock component. These components have different kind of credit risk. First of all to the issuer the stock component is risk free, because company can always deliver its own stock. But, the bond component is risky because the company needs cash inflows in order to pay the coupon and principal payments. (Hull, 2006)

When calculating the value of convertible bond the life of the tree should be set equal to the life of the convertible bond. The value of the convertible at the final nodes of the tree is calculated based on conversion options that the holder has at that time and rolled back the tree. If the company has the option to call the bond, it is tested if it is optimal for it to do so and in turn re-test whether holders should convert if the bond is called. This is equivalent to setting the value at a node equal to

$$\text{Max}\ [\text{Min}(Q_1,Q_2),Q_3] \qquad (7)$$

Where Q_1 is the value of the rollback, before possible conversion or call, Q_2 is the call price and Q_3 is the value if the holder decides to convert.

Convertible bond has four possible final results: The convertible bond will not be called nor converted before maturity. The issuer will call the bond. The investor will convert the bond. The last possibility is forced conversion, where the issuer has called the bond and the investor has to convert the bond. The forced conversion is tempting to the issuer if interest rates decline significantly or the price of the underlying security is above the conversion price. (Ammann et al., 2003)

4. Data

The data contains of outstanding convertible bonds of Air Berlin and Deutsche Lufthansa from German markets from issuance date to 25th February 2009. Both Air Berlin and Lufthansa stocks are officially traded at XETRA (Air Berlin also at Frankfurt Securities Exchange) and the convertible bonds are traded at Luxembourg Stock Exchange, needed data is from Thomson DataStream. (Lufthansa.com a, Airberlin.com a)

Table 1.a. Specifications of the Convertible bonds (Lufthansa.com b,c; Airberlin.com, b,c)

Issuing Company	**Maturity**	**Coupon (%)**	**Initial conversion ratio**	**Maximum issue volume (millions of €)**
Air Berlin	2027	Fixed 1.5%	4550.38	220
Deutsche Lufthansa	2012	Fixed 1.25%	49.60	750

Table 1.b. Specifications of the Convertible bonds bonds (Lufthansa.com b,c; Airberlin.com, b,c)

Issuing Company	**Final redemption (%)**	**Call trigger ratio (%)**	**Call trigger basis**	**Call**	**Average stock volatility**
Air Berlin	100.00	150.00	Redemption	Yes	53.45%
Deutsche Lufthansa	100.00	130.00	Redemption	Yes	32.26%

In component model theoretical prices are calculated on each trading day. In binomial tree model theoretical prices are calculated on 7th of each month (or nearest trading day) from issuance date to 25th February 2009 and the tree has 100 steps.

The volatility of the underlying stock can be estimated in couple of ways. It can be calculated using historical data from changes in the stock prices. The second style is to estimate the implied volatility using the option pricing formulas to derive the implied volatility from at-the-money option prices. Ammann et al. (2003) suggests that it is better to estimate the volatility on historical basis, because most volatile options have shorter maturities than convertibles. Thus in this analysis the relevant volatility for each day is calculated as the standard deviation of the returns of the last 252 trading days[4]. In option pricing the volatility of an asset is expressed as volatility per year, the

[4] For Air Berlin, volatility is calculated from the initial public offering date 5/10/2006 (i.e. 274 trading days)

day volatility is transferred into volatility per year with following equation

$$\sigma_{year} = \sigma_{day}\sqrt{252} \tag{8}$$

In this analysis time period of 252 trading days (i.e. one year) is used, in order not to reduce the effect of the current financial crisis. This makes sense especially for convertibles maturing in three years or less, because future is very uncertain and the current volatile situation may hold for a longer period. But, volatility has an impact only to the option component of convertible bond.

As risk free interest rates for long maturities government bonds are used. Thus the risk free rate is assumed as Germany government bond with maturity closest to the maturity of the convertible bond at the current date.

Future dividend yields are assumed to be constant over time and past dividend yields are calculated from stock prices and dividend levels.

Credit spread is assumed as zero in both models, but for component model comparison values with credit spreads of 1%, 3% and 5% are calculated.

5. Results

The examined convertible bond prices of the two companies are compared with theoretical values received from binomial tree model and component model. The results are presented in table 2 and also in the appendices.

Table 2. Pricing overview for the models, and comparison values for component model with 1% and 3% credit spreads.

	Binomial-tree model	Component model	1%	3%	Data points[5]
Air Berlin	-14.36	14.98	25.82	42.06	456
Lufthansa	-1.54	11.53	16.80	26.29	1883

[5] *Data points* indicates the number days from which the model values are calculated

Table 2 shows the average overpricing percentages of convertible bonds. Positive value indicates that the convertible bond is overpriced and negative value that it is underpriced i.e. theoretical value is above the market price. The binomial-tree model shows in both cases the theoretical value is on average above the market price, thus market seems to be underpricing the convertible bonds. Convertible bond of Deutsche Lufthansa seems to be more accurately priced, average underpricing of 1.54%, than Air Berlins average underpricing of 14.36%. It may be because the convertible bond of Air Berlin is less liquid, the company is smaller, has larger credit spread and a different business strategy, which has been very challenging, many cheap airline companies have went bankrupt. The result is in line with previous study by Ammann et al. (2003) who found that convertible bonds with shorter maturities were more accurately priced than convertible bonds with longer maturities, in this study the Deutsche Lufthansa's convertible bond matures in 2012 and Air Berlin's in 2027 (table 1a). As seen in table 1b the stock of Air Berlin is more volatile and it is known that higher volatility should lead to higher option price that could explain some of the higher theoretical price. It may be that the market participants use different kind of volatility estimation procedures than in this study, especially in case where estimated volatility rises above 75% as it is the case with Air Berlin! This would lead to a lower option price, thus moving the value of the convertible bond closer to the market price. Also the theoretical price of the Air Berlin is more volatile than Deutsche Lufthansa's.

The price given by the component model is significantly below the market price, in both cases, and it decreases even more when credit spread is taken into account. According to the previous studies (Ammann et al., 2003) there should be underpricing or slight overpricing, but the amount of overpricing obtained from the component model is not rational[6]. Ammann et al. (2003), suggest that the larger underpricing is caused by the call feature, which is not taken into account in the component model. Callability has a negative impact on prices, because it reduces stocks upside potential. The component model also shows that convertible bond of Deutsche Lufthansa is more accurately priced than Air Berlin's, which is the same to

[6] The formula in Excel and input parameters were triple checked from errors, but none was found.

the binomial tree-model also here the shorter maturity of Deutsche Lufthansa's convertible bond may be one of the reasons along the others mentioned before. This is because component model gives small prices for both companies and the market price of the Air Berlin is lower for whole time period.

The prices given by the component model in Air Berlin case were above the market prices during the end of the researched time period. At the time the value of option component was zero and the value of the bond component was alone above the market value, this holds even with 3% credit spread. At the time there was a huge drop in the convertible bond price (from 72.14 to 23.32 in a single day). The drop could be have been a cause of change in credit rating, bad future forecasts etc., but no public announcement which could have had significance was found.

Also in the case of Deutsche Lufthansa the theoretical price wanders above the market price for a while, because of an increase in the stock price which increased the value of the option component. In the component model it mostly seems that the value of the convertible bond relies too much on the value of the bond and the option component plays only a little role, thus giving poor values to the whole convertible bond. The market price of Deutsche Lufthansa's stock went over the strike price couple of times and the conversion could have taken place. Air Berlin's stock wasn't close to the strike price anytime during the researched time period, so there were no opportunities for conversion.

From figures 1-6 we see that the theoretical prices obtained from component model replicate well the market prices, especially in case of Deutsche Lufthansa, in the beginning of the time period, even tough they are at lower level, but the drift is similar, thus the component model seems to be working at least to some extent. The binomial-tree model replicates the market price of Deutsche Lufthansa convertible bond surprisingly well (Figure 7), even tough it can't always follow the markets, and only in the last quarter of the time period the theoretical price differs from the market price significantly.

6. Conclusions

In this study it was tested if the theoretical prices of convertible bonds of Air Berlin and Deutsche Lufthansa differ from the market prices with two pricing models. We saw that convertible bonds seem to be underpriced when comparing market prices to theoretical prices, when using binomial tree-model. But the component model gave totally different results, suggesting that the market overprices the convertible bonds. The market prices of Deutsche Lufthansa convertible bond were closer to market values than Air Berlin convertible bonds, which were in line with previous studies. Overall the results from binomial tree-model were closer to the market prices and the results had smaller volatility than the results from component model.

The difference between pricing accuracy between the companies may be explained by credit spread, which was not taken into account in this study. Air Berlin as a newer and more unstable company has a higher credit spread than Deutsche Lufthansa, even though Air Berlin has a different business strategy. The illiquidity of Air Berlin makes the comparison harder, because the price of illiquidity is hard to estimate. The market price of Air Berlin stayed constant for longer periods, whereas the figure of market price of Deutsche Lufthansa is very vivid for the first two and half years before staying constant for a longer period.

The credit spread should be taken into account in future studies; the estimation could be done for example with credit-spread indices or using method proposed by Hull (2006), which were not available for this study. In this study the credit spread was assumed zero in binomial tree-model and comparison prices were calculated with 1% and 3% levels for component model. Also testing the statistical significance of the results could lead to dismissing some of the results.

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Appendix

Table 3. Comparison between Air Berlin and Lufthansa

Year 2008	Air Berlin	Deutsche Lufthansa
Revenue, €m	3,700	24,870
EBITDA, €m	476.8	2,421
Operating cash Flow, €m	0.68	2,473
Total Assets, €m	2,400	22,408
Earning per share	1.14	1.31
Passengers (thousands)	28,559	70,543

Table 4. Market price and binomial-tree model results compared

Company	**Average P_{market}-P_{Theory}**	**Std.dev. P_{market}-P_{Theory}**	**Average P_{market}**	**Average P_{Theory}**	**Std.dev. P_{market}**	**Std.dev P_{Theory}**
Air Berlin	-14.36	12.76	75.82	89.46	23.67	13.43
Deutsche Lufthansa	-1.54	3.52	98.41	99.998	3.18	2.13

Table 5. Market price and component model results compared

Company	**Average P_{market}-P_{Theory}**	**Std.dev. P_{market}-P_{Theory}**	**Average P_{market}**	**Average P_{Theory}**	**Std.dev. P_{market}**	**Std.dev P_{Theory}**
Air Berlin	14.98	28.8	75.82	61.19	23.67	6.04
Deutsche Lufthansa	11.53	9.84	98.41	86.88	3.18	9.80

Figure 1. Air Berlin's convertible bond prices with 3% credit spread (Component model and binomial-tree model)

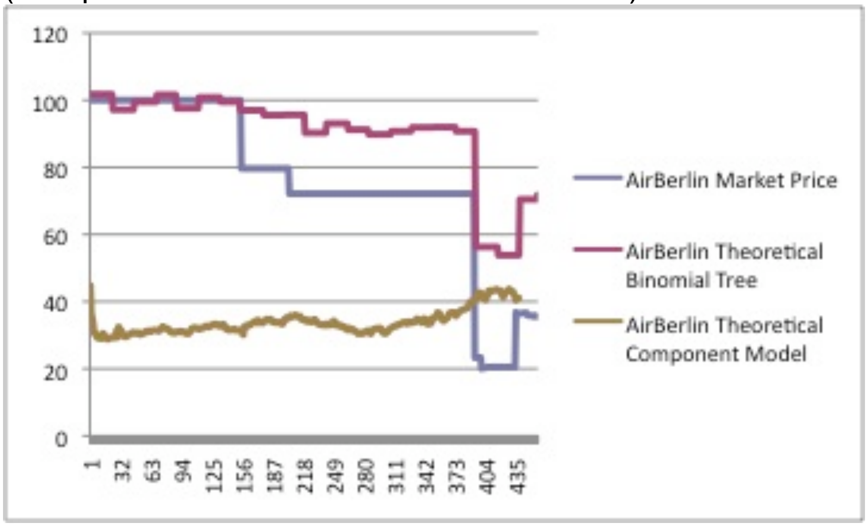

Figure 2. Air Berlin's convertible bond prices with 1% credit spread (Component model and binomial-tree model)

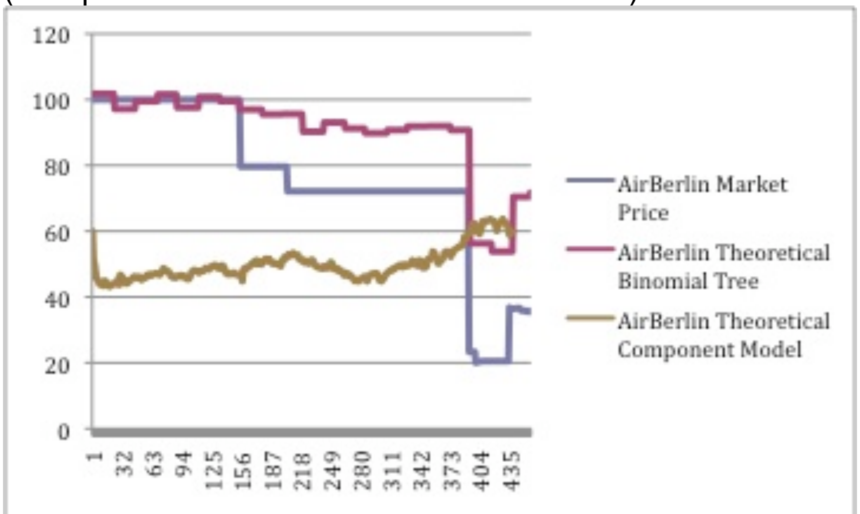

Figure 3. Air Berlin's convertible bond prices with 0% credit spread (Component model and binomial-tree model)

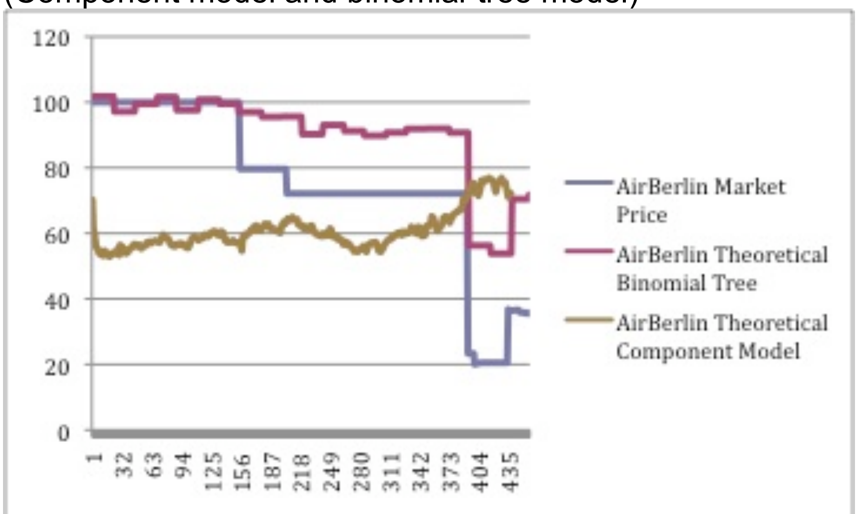

Figure 4. Deutsche Lufthansa's convertible bond prices with 3% credit spread (Component model)

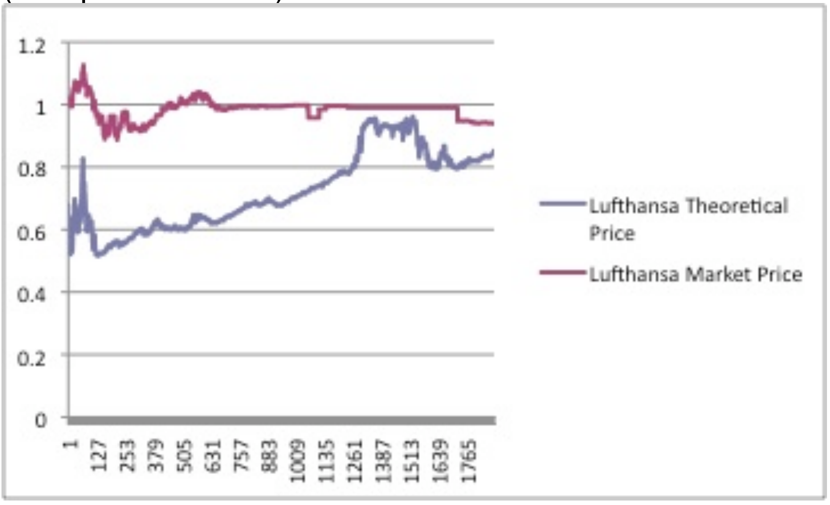

Figure 5. Deutsche Lufthansa's convertible bond prices with 1% credit spread (Component model)

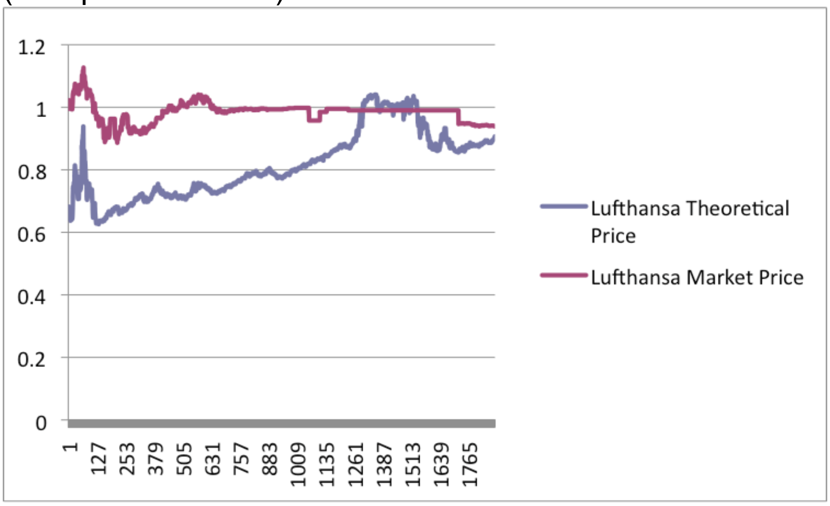

Figure 6. Deutsche Lufthansa's convertible bond prices with 0% credit spread (Component model)

Figure 7. Deutsche Lufthansa's convertible bond prices
(Binomial-tree model)