Katja Hynynen

BROADBAND EXCITATION IN THE SYSTEM IDENTIFICATION OF ACTIVE MAGNETIC BEARING ROTOR SYSTEMS

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Abstract

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One of the targets of the climate and energy package of the European Union is to increase the energy efficiency in order to achieve a 20 percent reduction in primary energy use compared with the projected level by 2020. The energy efficiency can be improved for example by increasing the rotational speed of large electrical drives, because this enables the elimination of gearboxes leading to a compact design with lower losses. The rotational speeds of traditional bearings, such as roller bearings, are limited by mechanical friction. Active magnetic bearings (AMBs), on the other hand, allow very high rotational speeds. Consequently, their use in large medium- and high-speed machines has rapidly increased.

An active magnetic bearing rotor system is an inherently unstable, nonlinear multiple-input, multiple-output system. Model-based controller design of AMBs requires an accurate system model. Finite element modeling (FEM) together with the experimental modal analysis provides a very accurate model for the rotor, and a linearized model of the magnetic actuators has proven to work well in normal conditions. However, the overall system may suffer from unmodeled dynamics, such as dynamics of foundation or shrink fits. This dynamics can be modeled by system identification. System identification can also be used for on-line
In this study, broadband excitation signals are adopted to the identification of an active magnetic bearing rotor system. The broadband excitation enables faster frequency response function measurements when compared with the widely used stepped sine and swept sine excitations. Different broadband excitations are reviewed, and the random phase multisine excitation is chosen for further study. The measurement times using the multisine excitation and the stepped sine excitation are compared. An excitation signal design with an analysis of the harmonics produced by the nonlinear system is presented. The suitability of different frequency response function estimators for an AMB rotor system are also compared. Additionally, analytical modeling of an AMB rotor system, obtaining a parametric model from the nonparametric frequency response functions, and model updating are discussed in brief, as they are key elements in the modeling for a control design.

Theoretical methods are tested with a laboratory test rig. The results conclude that an appropriately designed random phase multisine excitation is suitable for the identification of AMB rotor systems.

Keywords: active magnetic bearings, modeling, system identification, frequency domain identification, broadband excitation, multisine excitation, harmonics analysis

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Lappeenranta, October 9th, 2011

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Abstract

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Nomenclature

Roman Letters

A  system matrix
A(s)  matrix polynomial
A  bearing A
A  amplitude
A  cross-section, projected area  m²
A(s)  denominator polynomial of a transfer function
A'  position of sensor A
B  system matrix
B(s)  matrix polynomial
B  bearing B
B  flux density  T
B(s)  nominator polynomial of a transfer function
B'  position of sensor B
C  system matrix
C(jωk)  frequency response function of the feedback controller
D  damping matrix
D  system matrix
d  damping
d_A  distance of radial bearing A from the center of mass of the rotor  m
d_B  distance of radial bearing B from the center of mass of the rotor  m
d_s  distance of the sensor from the center of mass of the rotor  m
E  error
F+  force of the upper electromagnet  N
F−  force of the lower electromagnet  N
F  force vector  N
F  force  N
f  frequency  Hz
f₀  base frequency  Hz
f_c  clock frequency  Hz
F_g  gravitational force  N
G  gyroscopic matrix
g  standard gravity, g = 9.81 m/s²  m/s²
G₀  underlying linear system
$G_B$ bias or systematic error due to the nonlinear distortions
$G_{ff}$ feedforward gain of a current controller
$G(j\omega)$ frequency response function
$G_p$ proportional gain of a current controller
$G_R$ related linear dynamic system
$G(s)$ transfer function
$G_S$ stochastic nonlinear contribution
$H$ magnetic field $\text{A/m}$
$I$ identity matrix
$i$ current vector $\text{A}$
$i$ current $\text{A}$
$I_x$ transversal moment of inertia about the $x$ axis $\text{kg} \cdot \text{m}^2$
$I_y$ transversal moment of inertia about the $y$ axis $\text{kg} \cdot \text{m}^2$
$I_z$ rotational moment of inertia about the $z$ axis $\text{kg} \cdot \text{m}^2$
$J$ Jacobian matrix
$j$ imaginary unit, $j^2 = -1$
$K$ stiffness matrix
$k$ number of cylindrical elements in the FEM analysis
$k$ stiffness
$K_i$ current stiffness matrix $\text{N/A}$
$k_i$ current stiffness, force-current factor $\text{N/A}$
$K_s$ position stiffness matrix $\text{N/m}$
$k_s$ position stiffness, force-displacement factor $\text{N/m}$
$k_u$ velocity-induced voltage coefficient $\text{N/A}$
$\ell$ cost function $\text{H}$
$L$ inductance $\text{m}$
$l$ length $\text{m}$
$M$ mass matrix $\text{kg}$
$m$ mass $\text{kg}$
$M(\omega)$ discrete Fourier transform of the measurement noise $m(nT_s)$
$N$ shape function matrix
$N$ number of the samples of the time domain data
$N_b$ number of blocks used for averaging
$N_c$ number of coil windings
$N_t$ number of frequencies
$N_G$ error due to the output noise
$N(\omega)$ discrete Fourier transform of the noise $n(nT_s)$
$N_m$ number of modes
$N_u$ number of inputs
$N_y$ number of outputs
$P$ number of the nodes in the FEM analysis
$p$ pole $\text{m}$
$q$ displacement vector $\text{m}$
$R$ residual matrix $\text{\Omega}$
$r$ excitation signal in the time domain
$R(\omega_k)$ discrete Fourier transform of the reference signal
$s$ Laplace variable
$s$ air gap
$s$ local longitudinal coordinate
$S_i$ nodal location matrix of current stiffness
$S_s$ nodal location matrix of position stiffness
$S_{UU}$ autopower spectrum of input
$S_{YU}$ cross-spectrum of input and output
$S_{YY}$ autopower spectrum of output
$t$ time
$t_{sa}$ measurement time when using stepped sine excitation
$t_{T_1}$ transformation matrix
$t_{T_2}$ transformation matrix
$t_{bs}$ measurement time when using broadband excitation
$t_c$ clock period
$t_{rise}$ rise time
$T_s$ transformation matrix
$T_s$ sampling time
$T_w$ waiting time
$U$ input matrix
$u$ input signal in the time domain
$u$ voltage
$U(\omega_k)$ discrete Fourier transform of the input signal $u(nT_s)$
$V$ unmeasured disturbance
$V_a$ volume of the air gap
$m^3$
$W$ frequency-independent orthogonal matrix
$W$ measured disturbance
$W(j\omega_k)$ weighting function
$W_a$ field energy in the air gap
Ws
$x$ displacement vector in the $x$ direction
$m$
$x$ displacement in the $x$ direction
$m$
$Y$ output matrix
$y$ displacement in the $y$ direction
$m$
$y$ output signal in the time domain
$Y(\omega_k)$ discrete Fourier transform of the output signal $y(nT_s)$
$z$ zero

Greek Letters

$\alpha$ angle at which the magnetic force influences the rotor
$\beta$ tilting motion
$\Delta f$ frequency resolution
$\Phi$ magnetic flux
Wb
$\phi$ phase
$\Phi$ mode shape function matrix
$\phi$ eigenvector
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>harmonic coherence</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>magnetic permeability of a vacuum, $\mu_0 = 4\pi \times 10^{-7}$ Vs/Am</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>relative permeability Vs/Am</td>
</tr>
<tr>
<td>$\theta$</td>
<td>estimated parameter vector</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>moment Nm</td>
</tr>
<tr>
<td>$\theta$</td>
<td>estimated parameter</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time constant s</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>rotational speed rad/s</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency rad/s</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>eigenfrequency, natural frequency rad/s</td>
</tr>
<tr>
<td>$\omega_{BW}$</td>
<td>power bandwidth rad/s</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>damping ratio</td>
</tr>
</tbody>
</table>

**Superscripts**

- $^\hat{\cdot}$ estimate
- $^\sim$ in modal coordinates
- $^g$ in generalized coordinates
- $^H$ complex conjugate transpose
- $^T$ transpose

**Subscripts**

- $^0$ nominal
- $^a$ amplifier
- $^a$ actuator
- $^A$ bearing A
- $^a$ air gap
- $^b$ in bearing coordinates
- $^B$ bearing B
- $^b$ bias
- $^b$ blocks
- $^{bs}$ broadband signal
- $^{BW}$ bandwidth
- $^c$ in coordinates of the center of gravity
- $^c$ coil
- $^c$ control
- $^c$ cut-off
- $^{cc}$ current controller
- $^c{l}$ closed loop
- $^d$ direct current, dc link
- $^d$ disturbance
- $^{dyn}$ dynamic
- $^{ex}$ excitation
- $^{fe}$ ferromagnet
Abbreviations

ADC  analog-to-digital converter
AMB  active magnetic bearing
ARI  arithmetic mean
CDM  common-denominator model
CMIF complex mode indicator function
DFT  discrete Fourier transform
DIBS discrete interval binary sequence
DOF  degrees-of-freedom
EFRM empirical frequency response matrix
EIV errors-in-variables
EMA  experimental modal analysis
ERA  eigensystem realization algorithm
ETFE empirical transfer function estimate
EVD  eigenvalue decomposition
FDPI frequency-domain direct parameter identification
FEM  finite element method
FFT  fast Fourier transform
FPGA field programmable gate array
FRF  frequency response function
IGBT  insulated gate bipolar transistor
JIO   joint-input-output
LOG  logarithmic mean
LQG  linear quadratic Gaussian
LSFD least-squares frequency domain
LS   least-squares
MFD  matrix fraction description
MIMO multiple-input, multiple-output
ML   maximum likelihood
NLS  nonlinear least-squares
PEM  prediction-error method
PID  proportional-integral-derivative
PRBS pseudo random binary sequence
PWM  pulse-width-modulation
RLDS related linear dynamic system
SISO single-input, single-output
SNR  signal-to-noise ratio
SVD  singular value decomposition
Chapter 1

Introduction

This chapter presents the motivation, background, and scope of the thesis. An introduction to active magnetic bearing rotor systems, their operation principle, and characteristics is given. Analytical and experimental modeling are discussed, and earlier work related to the system identification of active magnetic bearing rotor systems is reviewed. Finally, the outline of the thesis and the main scientific contributions are provided.

1.1 Motivation and background

In 2008, the European Parliament and Council agreed upon a climate and energy package concerning all the union countries. The package includes obligations to reduce the greenhouse gas emissions by at least 20 percent below the 1990 level, a target to increase the use of renewable resources up to 20 percent of energy consumption, and to increase the energy efficiency in order to achieve a 20 percent reduction in the primary energy use compared with the projected level by 2020 (European Commission, 2008). The latter target can be achieved for instance by maximizing the energy efficiency of electrical devices. In the European Union, about 70% of the consumed electrical energy is used by industrial motor drives (de Almeida et al., 2001). Thus, remarkable climatic and energy economic benefits can be reached by improving their energy efficiency. In particular, the energy efficiency is improved by increasing the rotational speed of large electrical drives that are used to rotate pumps, blowers, and compressors, among others. The increase in the rotational speed enables the elimination of the gearbox, thus leading to a compact design with lower losses.
The bearing of large medium- and high-speed machines is increasingly implemented using actively controlled electromagnetic bearings, because the other bearing types, such as roller bearings, produce too much losses or they are otherwise inapplicable to demanding drives. In active magnetic bearings (AMBs), the rotor and the stator have no physical contact. The contact-free operation enables very high rotational speeds, where the only limiting factor for the speed is the strength of the rotor material. Additionally, no lubrication is needed, and there is no wear caused by friction. Hence, the AMBs can be used in cleanrooms and extreme conditions (such as vacuum, process gases, corrosive liquids, high temperatures) where traditional bearings are not applicable. Moreover, the absence of the contaminating mechanical wear and the need for lubricants reduce the need for maintenance and extend the lifetime of the system. This makes the AMBs a particularly interesting choice in turbomachinery applications. The other advantages of the active magnetic bearings include the condition monitoring during the operation and an opportunity to affect the rotordynamics. The both functions are provided by the digital control system of the bearings.

Active magnetic bearings are used for example in:

- **Turbomachinery**, which is their main application area. The AMBs are used for example in natural gas production, transportation, and treatment both offshore and onshore. The main advantages obtained when using AMBs come from the oil-free operation: There is no lubricant that should be separated from the process gases or fluids with seals, and the maintenance costs are lower compared with traditional bearings. Additionally, the AMBs provide an opportunity for vibration damping and diagnostics.

- **Energy production and storage**, e.g. flywheels and plant generators.

- **Machining.** AMBs are used in high-speed and high-precision milling and grinding.

- **Vacuum and cleanroom systems.** Turbomolecular vacuum pumps with AMBs are used in the semiconductor industry providing the ultrahigh vacuum needed in the chip manufacturing.

- **Medical devices.** An artificial heart pump is an example of a medical application of AMBs.

In spite of their indisputable advantages, the presence of AMBs in industrial applications is still rare because of their high investment costs. The design of active magnetic bearings requires knowledge of several engineering fields (mechanics, electromagnetism, electronics, control engineering, and software engineering). The design is application specific and always needs an analysis of the dynamics, mechanical, magnetic, and control design, and implementation for each application. This is very time consuming and thus raises the investment costs.

In commercial active magnetic bearings, lead-lag type compensators (e.g. PID controllers with filters) are commonly used. Because of the simple structure, their performance is limited, even if they are optimally tuned. When using the more developed controllers, such as...
model-based controllers, the flexible modes of the rotor, the known disturbances, and the rotational-speed-dependent dynamics can be taken into account. Furthermore, the modern tuning methods, such as robust loop shaping methods and genetic algorithms can be used effectively to improve the performance of the system.

The design of a controller requires an accurate system model. Previously, in the control design of the AMB rotor systems, the rigid body model of the rotor was commonly used, but the tendency towards higher rotational speeds and lower power consumption necessitates the identification of the flexible modes of the rotor. The modeling and control of the flexible modes require a significant effort. Traditionally, the flexible modes have been modeled by computers using the finite element method (FEM) and refining the obtained model by applying the experimental modal analysis (EMA) using hammer excitation. These methods provide a very accurate model of the rotor. After the installation of the rotor inside the stator, the system model is affected by the dynamics of the couplings of the magnetic bearings, shrink fits, and foundings, among others. The analytical modeling of these is difficult. However, they may have a substantial effect on the dynamics of the system, and thus, they should be considered in the control design.

System identification provides an opportunity to model the overall system experimentally. In identification, the system model is constructed using measured input and output signals. In an AMB rotor system, the control currents of the bearings are measured as inputs and the displacements of the rotor as outputs. The model obtained using system identification not only contains the dynamics of the rotor and the bearings, but also the dynamics of the couplings between the bearings, foundings, and so on. This unmodeled dynamics can be updated to the analytical model in order to obtain a more accurate system model. For the robust control design, system identification offers an option to verify the uncertainties of the model.

As the control of the AMBs requires continuous measurements of the rotor displacements, provides information about the control currents, and enables injection of the excitation signals, sufficient data for diagnostics and condition monitoring during the normal operation are readily available. Thus, the active magnetic bearings provide an option for diagnostics of the system using system identification without any additional instrumentation. In diagnostics, possible changes in the dynamics of the system are observed.

1.2 Objective and scope of the thesis

In this thesis, broadband excitation is adopted for the system identification of active magnetic bearing rotor systems. The broadband excitations provide faster measurement of frequency response functions (FRFs) compared with the stepped sine and swept sine excitations commonly used in the identification of AMBs. This is an advantage especially in the diagnostics of the system.
This doctoral thesis considers different broadband excitations and methods for calculating frequency response functions when using them with nonlinear, closed-loop, multiple-input, multiple-output (MIMO) AMB rotor systems. The design of the excitation signal, the time required for measurements, and the estimators used for calculating the FRFs are considered. In addition, the problem with the influence of the harmonics produced by a nonlinear system is treated. Because a parametric, physically meaningful model of the system for control design purposes is needed, the thesis also considers analytical modeling of the system, obtaining a parametric model from the frequency response functions, and updating of the model.

The thesis proves that accurate frequency response functions for an AMB rotor system can be measured by multisine excitation. The work also shows that the measurements can be carried out faster than when using a stepped sine excitation.

An advantage of the proposed method is that it enables fast frequency response function measurements for the entire frequency range of interest. The method can be used

- to update unmodeled dynamics to the analytical system model, as is done in this thesis,
- to verify the uncertainties of the system for robust control design,
- in the on-line identification for the diagnostics of the system. Fast FRF measurements provide knowledge of the system in the whole frequency range of interest.

Practical limitations:

- In this thesis, a nonrotating rotor is considered. However, the methods are directly applicable to a rotating rotor and on-line identification.
- The identification is only performed in the radial direction and the identification of the axial bearing is left out of the scope of the thesis.
- The proximity sensors used in the test setup are nonlinear, which may affect the results.

1.3 Active magnetic bearings

1.3.1 Operation principle of active magnetic bearings

The basic operation principle of electromagnetic levitation is shown in Fig. 1.1. When current is fed to the electrical magnet, it exerts a magnetic attraction force on a ferromagnetic ball. The ball remains in stable levitation, as the magnet pulls it upwards and gravity provides an equal counterforce. If the ball moves down from its equilibrium point and the current remains unchanged, the magnetic force decreases as the ball moves further away from it, and
the ball falls. If, on the other hand, the ball moves upwards, the magnetic force increases and pulls the ball toward the magnet. To avoid this unstable behavior, the magnetizing current has to be continuously controlled. The position of the ball is measured continuously using a displacement sensor, and the controller determines a suitable control current. A power amplifier provides the magnetizing current and feeds it to the magnet. With an appropriate control, the ball remains levitating in the reference position (Lösch, 2002; Schweitzer and Maslen, 2009).

In a practical active magnetic bearing, there are usually two counteracting magnets operating in a differential driving mode. A radial bearing consists of two pairs of electromagnets in the x and y directions, as shown in Fig. 1.2. In a typical AMB rotor system, there are two radial bearings such as presented in Fig. 1.2, and an additional axial bearing as shown in Fig. 1.3. Thus, there are five pairs of electromagnets and five position sensors constituting a five-degrees-of-freedom (5-DOF) system (Lösch, 2002; Schweitzer and Maslen, 2009). The controllers provide the reference currents, in this thesis called control currents $i_c$, according to the rotor displacements, and the power amplifiers provide the electromagnets with the magnetizing currents proportional to the references. In addition, there are safety bearings, or backup bearings, that hold the rotor when the power is turned off and during drop downs. The retainer bearings are typically bushing type or rolling element bearings, or combinations of the two types (Kärkkäinen, 2007). An AMB rotor system consists of electromagnetic actuators, a mechanical rotor, control electronics, and controllers with appropriate software. It is thus a typical mechatronic system, the design of which requires knowledge and cooperation of several engineering fields. Moreover, the design of the demanding software may substantially raise the investment costs of the system, which is typical of the mechatronics systems (Schweitzer and Maslen, 2009). The modeling of an active magnetic bearing rotor system is discussed in more detail in Chapter 2.
Fig. 1.2. Operation principle of a two-degrees-of-freedom AMB (Lösch, 2002).

Fig. 1.3. Functional structure of a five-degrees-of-freedom AMB. Adapted from Jastrzebski (2007); Lösch (2002).
1.3.2 Characteristics of active magnetic bearings

The contact-free operation provides numerous remarkable benefits (Schweitzer and Maslen, 2009):

- The contact-free operation allows high rotational speeds of the rotor. The only restriction for the speed is the strength of the rotor material. By using amorphous metals, even a circumferential speed of 350 m/s is achievable.
- Because of contact-free operation, there is no need for lubrication, and no contaminating mechanical wear will occur. Thus, the AMBs are applicable in vacuum, cleanrooms, with process gases and corrosive fluids, and extreme temperatures.
- Lower power losses compared with traditional (Schweitzer and Maslen, 2009) or fluid film bearings (Chen and Gunter, 2005).
- Diagnostics during the operation can be performed using the same control unit and sensors required for the normal operation of an AMB rotor system.
- An opportunity to interfere in the rotordynamics, and adjust the stiffness and damping of the system.
- Contact-free operation enables the unbalance compensation and force-free rotation.
- The maintenance costs are lower and the lifetime longer than for traditional bearings.

The disadvantages, on the other hand, are (Schweitzer and Maslen, 2009):

- The design of an AMB rotor system requires co-operation in several fields of engineering, such as mechanics, electromagnetics, electronics, control, and software engineering.
- Back-up bearings are required.
- Commissioning is not possible without skilled personnel.
- The investment costs are high compared with traditional bearings.

1.4 Analytical and experimental modeling

There are two ways to obtain a mathematical model for a physical system. In the first method, the system is divided into subsystems, the characteristics of which are well known based on physical laws (e.g. Maxwell, Newton). The overall model is obtained by combining the models of the subsystems. This method is called physical modeling. Another method is based
on experimental measurements. The inputs and outputs of the system are measured, and an experimental model is generated from them. This method is called system identification (Ljung, 1999).

In this study, the model constructed based on the physical laws is called an analytical model. An analytical model of an AMB rotor system typically consists of a model of the rotor determined by using the finite element method, a simple linearized model of the magnetic bearings, a power amplifier, and an approximated model of the sensors. The analytical modeling of an AMB rotor system is discussed in more detail in Chapter 2. The rest of this section deals with the basics of the system identification followed by a short survey of the system identification of AMB rotor systems.

### 1.4.1 System identification

As already explained, system identification is an experimental method for modeling physical systems. The model is constructed from the experimental measurements of the inputs and outputs of the system. The system identification procedure consists of four steps:

1. Collecting the data.
2. Selecting the model structure to represent the system in consideration.
3. Choosing the model parameters so that the model fits to the measurements as well as possible.
4. Validating the obtained model.

Consider a system of Fig. 1.4. The system contains an output signal \( y \), the measured signal that is of interest. It also contains an input signal \( u \) that can be manipulated in order to obtain the desired output. Additionally, there are disturbances affecting the system. The disturbances can be divided into measured disturbances \( w \), and unmeasured disturbances \( v \) depending on whether they can be measured directly or only their effects on the output can be observed (Ljung, 1999). When considering a 1-DOF AMB rotor system as shown in Fig. 1.5, the output signal is the displacement of the ball \( x \). Physically it would make sense to consider the bearing voltage as an input. However, when the power amplifier is included in the overall model, as is done in this study, the control current \( i_c \) is considered as an input. For an inherently unstable system, the identification is carried out in a closed loop. An excitation signal is fed to the control current. In this study, none of the disturbances are measured, and thus \( w = 0 \). Unmeasured disturbances \( v \) consist of sensor noise and disturbances caused by the process in the case of the on-line identification. Since a complete AMB rotor system includes two radial bearings and an axial bearing that is a 5-DOF system, it has five inputs and five outputs being thus a MIMO system.
1.4 Analytical and experimental modeling

Fig. 1.4. Identified system with the input $u$, output $y$, measured disturbance $w$, and unmeasured disturbance $v$.

Fig. 1.5. Identified 1-DOF AMB rotor system. As AMB systems are inherently unstable, the identification is carried out in a closed loop, and the excitation signal is fed to the control current $i_c$. Thus, the input of the system is the control current added by the excitation signal. The displacement $x$ is an output. The system is affected by unmeasured disturbances.
Usually, the experiments used for system identification are designed so that the system is excited with an excitation signal in the input. The type, amplitude, duration, and in MIMO systems also the combination of the excitations in different inputs are chosen appropriate for the experiment in consideration. The choices made may have a significant impact on the final result.

When considering the suitable model structure for the system to be identified, a parametric or a nonparametric, a linear or a nonlinear, or a black-box or a gray-box model can be chosen. For controller design purposes, a parametric model is often needed, but a nonparametric model is an easy way to obtain preliminary information of the system. A black-box model can be chosen, if it is only necessary to make the model fit the measured data, and the parameters do not have to be physically meaningful. Transfer function models and canonically parametrized state-space models are examples of black-box models. A gray-box, also known as a white-box model is needed, if the parameters have to have a physical interpretation. Continuous-time state-space models are typical examples of gray-box models. Nonparametric frequency domain methods provide a good insight into the suitable model structure and order.

After the model structure has been chosen, the parameters of the model are estimated using an appropriate identification method, for example prediction-error methods (PEM) such as least-squares (LS) or maximum-likelihood (ML) methods, or subspace methods, so that the model matches with the measurement data as well as possible.

Finally, the validity of the obtained model is assessed: Does the model describe the system well enough in the conditions where it will be used later? If the model is found to be valid, it can be used for further purposes. If it is not valid, it is necessary to change the model parameters or the model structure, or even collect new measurement data and start the identification procedure again as presented in Fig. 1.6 (Ljung, 1999; Pintelon and Schoukens, 2001b).

![Fig. 1.6. System identification procedure.](image-url)
The identified model does not necessarily coincide with the analytical model. This may be due to erroneous assumptions made when modeling the system analytically. The identified model may also contain completely unmodeled dynamics. The erroneous assumptions can be corrected and the unmodeled dynamics added to the analytical model by model updating.

In this study, nonparametric identification methods are used as they provide important information about the system. A parametric model is made based on that information. The nonparametric identification issues are discussed in more detail in Chapter 3. Methods to obtain a parametric model from the measured frequency response functions as well as model updating are discussed in Chapter 4.

1.4.2 System identification of active magnetic bearings

To the author’s knowledge, the research on the identification of AMBs started at the beginning of the 1990s (Herzog and Siegwart, 1993; Lee et al., 1994; Gäbler and Herzog, 1995). Gäbler (1998) used active magnetic bearings for rotordynamic measurements and a modal analysis of a rotating machine. Lösch (2002) invented an automated method for identification and controller design. Since then, these methods have been applied to obtain an experimental model for control purposes (Sawicki et al., 2007; Ahn et al., 2003b,a). The system identification has also been used for the verification of the applied model uncertainties (Jastrzebski et al., 2009; Sawicki and Maslen, 2008).

Maslen et al. (2002), Vázquez et al. (2003), and Wang and Maslen (2006) used system identification for updating the analytical model. An automated method for the updating was invented. Li et al. (2006) identified substructures separately and updated the analogical model accordingly.

The system identification can also be used for diagnostics as presented by Sawicki et al. (2008); Sawicki (2009). It is also possible to use AMBs as magnetic actuators for diagnosis purposes when some other types of bearings support the rotor.

In all the aforementioned publications, nonparametric frequency domain methods have been used. While a parametric model is usually required for control design, the FRF data must be converted into the parametric model. Gäbler et al. (1997) developed an algorithm to obtain a parametric model for a MIMO system considering it as several SISO systems with common poles. Ahn et al. (2003a) improved the method to consider the system as a MIMO over the whole procedure.

In the above-listed works, either a stepped or swept sine excitation has been used in the identification measurements. Hynynen and Jastrzebski (2009) and Hynynen et al. (2010) introduced a multisine excitation to be used in the identification of AMB rotor systems. The first publication showed that the quality of the FRFs is better when using an optimal set of excitations in each system input than having separate excitations in each input (Hynynen and Jastrzebski, 2009). Another paper compared the suitability of different FRF estimators for the identification of an AMB rotor systems (Hynynen et al., 2010).
1.5 Outline of the thesis

The thesis is divided into six chapters with the following outline:

Chapter 1 provides the motivation, background, and objectives of the thesis. The operation principle, characteristics, and applications of active magnetic bearings are introduced. The difference between the analytical and experimental modeling and the procedure of the system identification are discussed. A short survey of the system identification of the active magnetic bearings is presented.

Chapter 2 presents the experimental setup used in the study. An analytical modeling of an AMB rotor system is explained. The model contains both the rigid and flexible dynamics of the rotor obtained by finite element modeling and a linearized model of the electromagnets. The model is completed with a simple first-order transfer function model of the actuator dynamics. The chapter also describes the construction of an overall model of the system. The sensor dynamics, the controller, and the unmodeled dynamics affecting the system are also discussed.

Chapter 3 deals with the theory of nonparametric system identification for active magnetic bearings. Special issues concerning AMB rotor systems are addressed (nonlinearities, closed-loop identification, and multiple-input, multiple-output system identification). Different broadband excitation signals are introduced, and their suitability for the identification of AMB rotor systems is assessed. The excitation signals widely used in the AMB rotor system identification, namely the stepped sine and swept sine excitations, are also discussed. Moreover, the chapter presents the frequency response function estimators used to improve the signal-to-noise ratio when using the multisine excitation.

Chapter 4 introduces the methods to obtain a parametric model from the nonparametric frequency response functions. A brief literature review of the methods applied to the structural dynamics is provided. Least-squares methods for a common-denominator model and their suitability for AMB rotor systems are discussed. The selection of weighting functions is also addressed. Furthermore, the challenges of the real poles of the actuator of an AMB rotor system are considered. Finally, the model updating is dealt with.

Chapter 5 presents the experimental results of the system identification with a laboratory test setup. The excitation signals for the stepped sine and multisine excitation are designed, and the harmonics produced by a nonlinear system analyzed. A comparison of the FRFs obtained using the stepped sine and multisine excitation, a comparison of the multisine excitation with two different combinations of excitation signals in each input, and a comparison of the FRFs using different estimators are provided. Additionally, the fit of the parametric and nonparametric models is presented.

Chapter 6 summarizes the results and gives suggestions for future work.
1.6 Scientific contributions and publications

The doctoral thesis provides the following scientific contributions:

• Broadband excitation is adopted to the identification of AMB rotor systems.
• Some broadband excitations and their suitability for AMB rotor systems are studied.
• The suitability of different frequency response function estimators for an AMB rotor system identification is investigated.
• The harmonics produced by a nonlinear AMB rotor system are analyzed.
• Multisine excitation signals are designed in order to avoid the harmonics produced by a nonlinear system.

Some of the results presented in this thesis have also been published in the following conference papers:


The author has also published research results related to the control of AMB rotor systems that are not covered in this thesis:


Chapter 2

Analytical model of an active magnetic bearing rotor system

In this chapter, analytical modeling of an active magnetic bearing rotor system is provided. A linearized model for the control is required. First, the experimental setup used in the study is introduced. Then, the analytical modeling of each part of the setup is described and a state space model of the complete system is constructed. The real system includes components and disturbances that are difficult or even impossible to model analytically. These components are also discussed. Moreover, the position controller of the radial bearings is presented.

2.1 Experimental setup

An AMB rotor system consists of a rotor, magnetic actuators, power amplifiers, analog-to-digital converters (ADCs), sensors, and controllers. A schematic of the experimental setup of the active magnetic bearing rotor system under consideration is presented in Fig. 2.1 and a block diagram in Fig. 2.2.

The system consists of two radial eight-pole bearings and one axial active magnetic bearing as described in Fig. 1.3. The bearings not only support the system, but they can also be used to supply excitation signals to the system for the identification, see Chapter 3 and Section 5.1. Analytical modeling of the bearings is discussed in more detail in Section 2.2.1.

The mechanics of the AMB rotor system consists of a rotor, a stator, an axial disk of the axial bearing, couplings, and safety bearings. The rotor is a solid steel shaft (Fe52). Stacks of thin circular laminations of electrical steel M270-50A are added to the locations of the radial bearings to provide high magnetic permeability and prevent eddy current losses. At the locations of the position measurements, aluminum sleeves are added. The stators of the radial AMBs
Fig. 2.1. Schematic of the experimental setup.

Fig. 2.2. Block diagram of the system.
are similar to the stators used in the rotating electrical machines. The electromagnets are comprised of laminated poles of electrical steel M270-50A and coil windings. The windings are wound in such a way that the polarities of the stator poles vary in a sequence NSNS. For an axial bearing, a solid steel disk (Fe 52 C) is added to the rotor shaft. The stator of the axial bearing is constructed of C-shaped toroidal discs made of solid steel as seen in Fig. 1.3. The analytical modeling of the rotor is presented in Section 2.3 and the rotor used in this study in Section 2.3.3.

In this study, the bearings are operated with current control. This means that the position controller provides a reference current and the power amplifier then provides the electromagnets with a magnetizing current comparable with the reference. An inner controller of the power amplifier compares the measured magnetizing current with the reference current obtained from the position controller and adjusts the output current of the amplifier so that the required coil current is achieved. The power amplifier is an H-bridge switching amplifier that provides the coil currents. The switching is performed using a carrier-based pulse-width-modulator (PWM) with two carrier signals and asymmetric regular sampling. The modeling of the current controller and power amplifier is described in Section 2.2.2.

Each radial bearing has two and the axial bearing one eddy current proximity sensors. In radial bearing A, two differential (two-channel) DT3703 U3-A-C3 sensors, in the radial bearing B, three single-channel DT3701 U1-A-C3 sensors from MIKRO-EPSILON, and in the axial bearing, a single-channel CMSS 68 sensor from SKF are used. The magnetizing currents are measured using closed-loop Hall-effect LEM transducers (LA 25-NP). There are ten current sensors, two for each DOF. The measured analog signals of the radial displacements and the currents are sampled with an ADC board DS2001 that is part of the dSpace platform.

Position control is applied in the outer control loop. For the radial bearings, a centralized $H_{\infty}$ position controller and for the axial bearing, an individual $H_{\infty}$ position controller are used. The controllers provide control currents for each magnetic bearing, for the radial bearings both in the $x$ and $y$ directions. Thus, there are five control currents in total. Both the current and the position controllers are realized with a dSpace platform where the controllers are implemented in a graphical Simulink environment and compiled into a PowerPC processor. A DS4003 board from dSpace that contains its own PowerPC is used in the work. The position controllers of the radial bearings are presented in Section 2.7.

The parameters of the system are presented in Appendix A.
2.2 Magnetic actuators

Magnetic actuators consist of the electromagnets and the power amplifiers as shown in Fig. 2.2. This section deals with their analytical modeling.

2.2.1 Electromagnets

The modeling of the nonlinear electromagnets is based on the linear magnetic circuit theory with the following assumptions:

- The permeability of the ferromagnetic material is infinite.
- There is no hysteresis, nor magnetic saturation.
- The cross-section of the core is constant through the whole magnetic loop and equals the cross-section of the air gap.
- Flux flows in the air gap in the radial direction.
- There is no leakage flux.
- There is no eddy current losses.

The simplest representation of a magnetic bearing is to consider it as a U-shape magnet core as in Fig. 2.3 (a), where the current of the coil $i$ generates a magnetic field $H$ in the ferromagnetic core according to Ampere’s law,

$$ \oint H \cdot dl = N_c i, \quad (2.1) $$

where $l$ is the length of the magnetic path and $N_c$ is the number of coil windings. The magnetic field in the ferromagnetic core is $H_{fe}$ and in the air gap $H_a$. According to Fig. 2.3 (a), the length of the magnetic path is $l_{fe} + 2s$, where $l_{fe}$ is the mean length of the magnetic path in the magnetic core and $s$ is the air gap. Now Ampere’s law of Eq. (2.1) can be rewritten as

$$ l_{fe}H_{fe} + 2sH_a = N_c i. \quad (2.2) $$

Flux density $B$ is obtained from the magnetic field as follows

$$ B = \mu_0 \mu_r H, \quad (2.3) $$
2.2 Magnetic actuators

where \( \mu_0 \) is the magnetic permeability of a vacuum \( (\mu_0 = 4\pi \times 10^{-7}) \) and \( \mu_r \) is the relative permeability. For air, \( \mu_r \approx 1 \) and for the ferromagnetic materials \( \mu_r \gg 1 \). Assume that the magnetic flux \( \Phi \) flows entirely in the magnetic core without leakage flux, and the cross-section of the core \( A_{fe} \) is constant through the whole magnetic loop and equals the cross-section of the air gap \( A_a \),

\[
\Phi = B_{fe}A_{fe} = B_aA_a, \tag{2.4}
\]

\[
A_{fe} = A_a. \tag{2.5}
\]

From Eqs. (2.4) and (2.5), it follows that the flux density is constant and equal both in the core and the air gap,

\[
B_{fe} = B_a = B. \tag{2.6}
\]

Substitute Eqs. (2.3) and (2.6) to Eq. (2.2) and solve the flux density,
Because the relative permeability in the ferromagnetic materials is \( \mu_r \gg 1 \), the magnetization of the ferromagnet is often neglected and Eq. (2.7) simplifies to

\[
B = \mu_0 \frac{N_i}{\frac{1}{2} + 2s}.
\]  

(2.8)

The attraction force affecting on the ferromagnetic body is generated at the boundaries of the differing permeabilities \( \mu \). The force is determined based on the field energy \( W_a \) stored in the air gap,

\[
W_a = \frac{1}{2} B_a H_a V_a = \frac{1}{2} B_a H_a A_a (2s)
\]  

(2.9)

where \( V_a \) is the volume of the air gap. The magnetic force is obtained from the field energy as a partial derivative with respect to the air gap as follows

\[
F = \frac{\partial W_a}{\partial s} = B_a H_a A_a.
\]  

(2.10)

By substituting Eqs. (2.3) and (2.8) to Eq. (2.10), the magnetic force becomes

\[
F = \mu_0 A_a \left( \frac{N_i}{2s} \right)^2 = \frac{1}{4} \mu_0 \mu_c^2 A_a \frac{r^2}{s^2} = k \frac{r^2}{s^2}
\]  

(2.11)

with the stiffness \( k \) as

\[
k = \frac{1}{4} \mu_0 \mu_c^2 A_a.
\]  

(2.12)

In practical radial magnetic bearings, the magnetic forces influence the rotor in an angle \( \alpha \) as presented in Fig. 2.3 (b). For eight-pole bearings, \( \alpha = 22.5^\circ \). Thus, the magnetic force is

\[
F = \frac{1}{4} \mu_0 \mu_c^2 A_a \frac{r^2}{s^2} \cos \alpha = k \frac{r^2}{s^2} \cos \alpha.
\]  

(2.13)
In the majority of AMB applications, magnetic bearings consist of two counteracting magnets operating in the differential driving mode, see Fig. 2.4. The upper magnet is supplied by a current that is a sum of the bias current $i_b$ and the control current $i_c$, $i_b + i_c$, and the lower one by their difference $i_b - i_c$. This procedure improves the linearity of the force-current relationship, when neglecting the magnetization of the iron. The linearized force of a counteracting couple of magnets can be written as a sum of the both magnets according to Fig. 2.4 and Eq. (2.13) as

$$F_x = F_+ - F_- = k \left( \frac{(i_b + i_c, x)^2}{(s_0 - x)^2} - \frac{(i_b - i_c, x)^2}{(s_0 + x)^2} \right) \cos \alpha, \quad (2.14)$$

where $F_+$ and $F_-$ are the forces of the upper and lower magnets. $s_0$ represents the nominal air gap and $x$ the displacement from it in the x direction. The subscript $x$ refers to the $x$ direction. Simplifying Eq. (2.14) and linearizing it with respect to $x \ll s_0$ gives the relation

$$F_x = \frac{4k_i b}{s_0} \cos(\alpha)i_x + \frac{4k_i^2}{s_0^2} \cos(\alpha)x = k_i i_x + k_x x, \quad (2.15)$$

Fig. 2.4. Basic principle of an AMB. The bearing consists of two counteracting magnets that are operating in the differential driving mode (Schweitzer and Maslen, 2009).
where the current stiffness, also called force-current factor \( k_i \) and the position stiffness, also called a force-displacement factor, \( k_s \), are defined as

\[
k_i = \frac{4k_i b}{s_0^2} \quad (2.16)
\]

and

\[
k_s = \frac{-4k_i^2}{s_0^3}. \quad (2.17)
\]

Now the magnetic force of an active magnetic bearing, linearized to an operating point \((i_b, s_0)\), is

\[
F_x = k_i i_c + k_s x. \quad (2.18)
\]

Although Eq. (2.18) is only a linear approximation that holds true in the vicinity of the operational point, practical experience over the years has shown that it works extremely well in normal operating conditions in many applications. When considering special cases such as rotor-stator contact, flux saturation, and very low bias currents, more detailed, usually nonlinear models are required (Schweitzer and Maslen, 2009).

In this study, eight-pole radial magnetic bearings are used as described in Fig. 1.2. In the eight-pole bearings, there are two foregoing magnet pairs, both in the \(x\) and \(y\) directions. When considering a horizontal rotor, the gravitational force for the vertical plane is \(F_g = mg\) and for the horizontal plane \(F_g = 0\), where \(g\) is the standard gravity (\(g = 9.81 \text{ m/s}^2\)). The \(x\) and \(y\) planes are typically rotated by 45° with respect to the vertical and horizontal planes, as seen in Fig. 1.2, so that the gravitational force for both planes becomes \(F_g = 1/\sqrt{2}mg\). In large bearings, the number of poles can be increased in order to keep the outer diameter low with respect to the inner diameter, and in small bearings, also a three-pole configuration is used. An AMB rotor system consist of two radial bearings and an additional axial bearing as shown in Fig. 1.3.

### 2.2.2 Power amplifiers

In this study, a current controller with biased control currents is used. Fig. 2.5 shows a block diagram of the current controller together with the power amplifier. Both radial bearings contain four independent circuits of this kind and the axial bearing two, so that an AMB rotor system contains ten such current controllers and amplifier circuits in total. The input for the current controller is a reference current \(i_{\text{ref}}\) that is the control current \(i_c\) obtained from
2.2 Magnetic actuators

Fig. 2.5. Block diagram of the magnetic actuator consisting of the current controller, power amplifier, and electromagnets.

the displacement controller $\pm$ the bias current $i_b$ ($i_{ref} = i_c \pm i_b$). The reference current is compared with the magnetizing current $i_m$ measured from the coil of an electromagnet.

Alternative options for the current control are a voltage control and a flux control. Compared with the current control, the voltage control has a simpler power amplifier topology and a more accurate plant model, which leads to a higher overall system robustness. However, the voltage control requires more complex control algorithms whereas the current control can be stabilized using relatively simple PID type controllers (Schweitzer and Maslen, 2009). The advantages of the flux control over the current control are that the flux is more closely related to the force than the current is and the inner flux control loop does not destabilize the system as the current control loop does (Zingerli and Kolar, 2010). The flux can be measured using Hall sensors or field plates. Alternatively, the flux can be estimated from the bearing current and voltage. Most industrial AMBs for rotating machines have a current control (Schweitzer and Maslen, 2009).

A current controller consists of a feedback branch with a proportional controller gain $G_p$ and a feedforward branch with a controller gain $G_{ff}$. The fast current feedback compensates the inductive voltage drop and the variations in the inductances of the coils. The feedforward branch compensates the effect of a resistive voltage drop (Jastrzebski et al., 2006a,b). The current controller provides a control voltage $u_c$ for the power amplifier. The power amplifier is an H-bridge switching amplifier consisting of two IGBT switches and two diodes. PWM with a unipolar switching is chosen, where the both IGBT switches have their own control voltage $\pm u_c$, which is compared with the triangular carrier voltage $u_{tri}$ leading to the output voltages $+u_{dc}$, $0$, or $-u_{dc}$ depending on the switching combination of the IGBTs. $u_{dc}$ is the voltage of the dc link. With the chosen amplifier topology and PWM scheme, the current ripple is independent of the dc link voltage. Thus, an increase in the dc voltage do not increase the current harmonics as much as when using a full bridge topology (Zhang and Karrer, 1995).
According to Fig. 2.5, the voltage of the power amplifier can be written as

\[ u = L \frac{di}{dt} + Ri + k_u \frac{dx}{dt}, \]  

(2.19)

where \( L \) and \( R \) are the inductance and resistance of the coils. The inductance \( L \) varies according to the rotor position \( x \). In the linearized model, it is assumed to have a constant value of the operation point in \( x = 0 \). \( k_u \) is a velocity-induced voltage coefficient. According to the theory of electromechanical energy conversion, it can be shown that \( k_u = k_i \). When modeling the feedback loop of the current controller, the velocity-induced voltage \( k_u \frac{dx}{dt} \) is typically neglected as its magnitude is relatively low when compared with the voltage of the coil (Schweitzer and Maslen, 2009; Lantto, 1999). Also the resistance of the coil is typically small and can be neglected. Now, under these assumptions, a simple first-order model for the closed-loop dynamics can be written as

\[ G_{cc}(s) = \frac{i_{max}}{t_{ref}} \approx \frac{G_p}{sL + G_p} = \frac{1}{s\tau_{cl} + 1}, \]  

(2.20)

with \( \tau_{cl} \) as a closed-loop time constant. Another option for modeling the current feedback loop of the current controller according to Lantto (1999) is the transfer function

\[ G_{cc}(s) \approx \frac{\omega_{BW}}{s + \omega_{BW}} \]  

(2.21)

where the power bandwidth \( \omega_{BW} \) is approximated using the rise time \( t_{rise} \) of the coil current from zero to the maximum coil current \( i_{max} \) through a current and rotor position dependent dynamic inductance \( L_{dyn} \).

\[ \omega_{BW} = \frac{\ln(9)}{t_{rise}} \]  

(2.22)

with the rise time

\[ t_{rise} \approx \frac{1}{u_{dc}} \int_{0}^{i_{max}} L_{dyn}(i, x_0) di. \]  

(2.23)

When replacing the dynamic inductance by the nominal inductance \( L \), the power bandwidth is approximated as (Lantto, 1999; Jastrzebski, 2007)

\[ \omega_{BW} \approx \frac{\ln(9)u_{dc}}{Li_{max}}. \]  

(2.24)
2.3 Rotordynamics

The rotors can typically be divided into two types. The first rotor type has all the flexible eigenfrequencies beyond the bandwidth of the position controller and the maximum rotational speed. This is a rigid rotor. Another rotor type has flexible eigenfrequencies at low frequencies, they are crossed during the run-up and run-down, and they can be affected by the position controller. These are flexible rotors and they require modeling of the elastic behavior (Lösch, 2002). Machines operating at rotational speeds below the critical speeds are called undercritical or subcritical machines, and those operating at rotational speeds over the critical speeds are called overcritical or supercritical machines. Pure rigid rotors do not exist in reality, and in most cases the undercritical machines require the modeling of one or more flexible modes.

Modeling of the rigid and flexible modes are treated separately using a general, linearized equation of motion based on Newton’s II law of motion

$$M \ddot{q}(t) + (D + \Omega G)\dot{q}(t) + Kq(t) = F(t), \quad (2.25)$$

where $M$ is a mass matrix, $q$ is a displacement vector, $D$ is a damping matrix, $\Omega$ is a rotational speed, $G$ is a gyroscopic matrix, $K$ is a stiffness matrix, and $F$ is a force vector. The linearized equation of motion can be used if

- the rotor can be assumed to be axisymmetric (with the exception of small unbalances),
- the displacements from the reference points are small when compared with the rotor dimensions, and
- the rotational speed is constant.

Although the rotor can often be modeled using a linearized model, other related components such as bearings, dampers, or seals may be too nonlinear to be described using linear equations. Also a crack in a structural element cause nonlinear behavior (Genta, 2005).

2.3.1 Rigid rotor model

For the rigid rotor, there are two kinds of eccentric motions, cylindrical and conical whirling motion. The rigid rotors supported by AMBs are typically modeled in the radial direction as a 4-DOF model and in the axial direction as a simple mass assuming that the coupling between the radial and axial planes is negligible. Consider first a nonrotating rotor with no coupling between the $(x,z)$ and $(y,z)$ planes. Thus, it is sufficient to consider the rotor in the $x$ and $y$ planes as two equal 2-DOF systems. Now, consider a rotor bearing system of Fig. 2.6. A rotor, assumed to be a rigid body, is suspended with two radial active magnetic bearings in
positions A and B. The distances of the bearing positions A and B from the center of mass of the rotor are \(d_A\) and \(d_B\). The displacements of the rotor are measured with two sensors at positions A’ and B’ at distances \(d_{s,A}\) and \(d_{s,B}\) from the center of mass.

The internal damping of the rotor does not affect the behavior of the rigid modes, and thus the damping can be assumed zero, \(D = 0\). When considering the rotor only in one plane and thus assuming that there is no coupling between the planes \((x,z)\) and \((y,z)\), the gyroscopic matrix is also zero, \(G = 0\) (Genta, 2005). Now, the general equation of motion of (2.25) can be reduced to apply to a 2-DOF rigid rotor. The equation of motion in the coordinates of the center of gravity when the bearing forces of Eq. (2.18) are influencing the rotor at A and B is

\[
M_c \ddot{\mathbf{x}}_c = \mathbf{F}_c = T_1 \mathbf{K}_s \mathbf{x}_b + T_1 \mathbf{K}_i \mathbf{i}_c, \tag{2.26}
\]

where the mass matrix \(M_c\), the position stiffness matrix \(\mathbf{K}_s\), the current stiffness matrix \(\mathbf{K}_i\), and the control current vector \(\mathbf{i}_c\) are

\[
M_c = \begin{bmatrix} m & 0 \\ 0 & I_y \end{bmatrix}, \quad \mathbf{K}_s = \begin{bmatrix} k_s & 0 \\ 0 & k_s \end{bmatrix}, \quad \mathbf{K}_i = \begin{bmatrix} k_i & 0 \\ 0 & k_i \end{bmatrix}, \quad \mathbf{i}_c = \begin{bmatrix} i_{c,A,x} \\ i_{c,B,x} \end{bmatrix}, \tag{2.27}
\]

where \(I_y\) is the transversal moment of inertia about the y axis and \(i_{c,A,x}\) and \(i_{c,B,x}\) are the control currents of bearings A and B in the \(x\) direction. The vector \(\mathbf{x}_c\) represents the transversal motion of the rotor in the \(x\) direction and the tilting motion around the \(y\) axis \(\beta_y\), \(\mathbf{x}_c = [x_y \quad \beta_y]^T\). \(\mathbf{x}_b = [x_A \quad x_B]^T\) is a displacement vector with the displacements of the rotor at bearings A and B in the \(x\) direction. The subscripts c and b refer to the coordinates of the center of gravity and the bearing coordinates, respectively. The bearing forces influencing bearings A and B are transferred to the force vector in the coordinates of the center of gravity \(\mathbf{F}_c\) using a
transformation matrix $T_1$ as follows

$$T_1 = \begin{bmatrix} 1 & 1 \\ -d_A & d_B \end{bmatrix}. \quad (2.28)$$

For AMB rotor systems, it is more convenient to represent the equation of motion in the bearing coordinates than in the coordinates of the center of gravity. The equation of motion in bearing coordinates is

$$M_b \dot{x}_b = F_b = K_s x_b + K_i \dot{x}_b, \quad (2.29)$$

where $M_b$ and $F_b$ are the mass matrix and the force vector in the bearing coordinates. The transformation of the displacement vector $x_c$ and the mass matrix $M_c$ from the coordinates of the center of gravity to the bearing coordinates is made using transformation matrices $T_1$ and $T_2$ as follows

$$x_b = T_1^T x_c, \quad (2.30)$$
$$M_b = T_2^T M_c T_2, \quad (2.31)$$
$$T_2 = T_1^{-T}. \quad (2.32)$$

In AMB rotor systems, the magnetic actuators and the displacement sensors are typically not collocated, that is, the sensors are not exactly at the same locations with the actuators. Thus, the vector of the measured displacements is represented in sensor coordinates $x_s$, which are obtained from the coordinates of the center of gravity or bearing coordinates as follows

$$x_s = T_s^T x_c = T_s^T T_1^{-T} x_b, \quad (2.33)$$

with the transformation matrix $T_s$ determined as

$$T_s = \begin{bmatrix} 1 & 1 \\ -d_{s,A} & d_{s,B} \end{bmatrix}. \quad (2.34)$$

A 2-DOF AMB rigid rotor system has four real poles as shown in Fig. 2.7. The AMBs with current control move the poles of the free rotor from the origin to the real axis. The poles are placed symmetrically around the origin and two of them are unstable causing the unstable behavior of the AMB rotor system.
Now, consider a rotor bearing system with a rotating rotor with a considerable gyroscopic effect. The aforementioned 2-DOF models for the $x$ and $y$ planes are no longer decoupled. An equation of motion of a 4-DOF system in the coordinates of the center of gravity is

$$M_c \ddot{q}_c + \Omega G_c \dot{q}_c = F_c. \quad (2.35)$$

The vector $q_c$ represents the transversal motions of the rotor in the $x$ and $y$ directions, and the tilting motions around the $y$ and $x$ axes, $\beta_y$ and $\beta_x$, $q_c = [x \ y \ \beta_x \ \beta_y]^T$. The mass matrix $M_c$, gyroscopic matrix $G_c$, and force vector $F_c$ in the coordinates of the center of gravity are

$$M_c = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_x & 0 \\ 0 & 0 & 0 & I_y \end{bmatrix}, \quad G_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad F_c = \begin{bmatrix} F_x \\ F_y \\ \Theta_x \\ \Theta_y \end{bmatrix}, \quad (2.36)$$

where $I_x$ and $I_y$ are the transversal moments of inertia about the $x$ and $y$ axes, and $I_z$ is the rotational moment of inertia about the $z$ axis, respectively. $F_x$ and $F_y$ denote the forces acting in the $x$ and $y$ directions, and $\Theta_x$ and $\Theta_y$ denote the moments about the same axes. The rotor is assumed to be axisymmetric, and thus $I_x = I_y$. In the 2-DOF model, the $(x,z)$ and $(y,z)$ planes were assumed to be decoupled. Now, the gyroscopic matrix $G$ couples the planes. The 4-DOF model is transferred from the coordinates of the center of gravity to the bearing coordinates similarly to the 2-DOF system. The 4-DOF equation of motion in bearing
coordinates is

\[ M_b \ddot{q}_b + \Omega G_b \dot{q}_b = F_b = K_x q_b + K_i i_c, \quad (2.37) \]

where \( G_b \) is the gyroscopic matrix in bearing coordinates. \( q_b = [x_A \ y_A \ x_B \ y_B]^T \) is a displacement vector with the displacements of the rotor at bearings A and B in the x and y directions, and \( i_c = [i_{c,A,x} \ i_{c,A,y} \ i_{c,B,x} \ i_{c,B,y}]^T \) is a control current vector containing the control currents of the magnets in bearings A and B in the x and y directions, respectively. The mass matrix is transferred from the coordinates of the center of gravity to the bearing coordinates according to Eq. (2.31), and the displacement vector and the gyroscopic matrix are transformed as follows

\[ q_b = T_1^T q_c, \quad \quad \quad (2.38) \]
\[ G_b = T_2^T G_c T_2. \quad \quad \quad (2.39) \]

The displacements in the sensor coordinates are obtained similarly to Eq. (2.33) as

\[ q_s = T_1^T q_c = T_1^T T_1^{-1} q_b. \quad \quad \quad (2.40) \]

For the 4-DOF system, the transformation matrices \( T_1 \) and \( T_s \) are defined as

\[
T_1 = \begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
0 & -d_A & 0 & d_B \\
-d_A & 0 & d_B & 0
\end{bmatrix}, \quad T_s = \begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
0 & -d_{s,A} & 0 & d_{s,B} \\
-d_{s,A} & 0 & d_{s,B} & 0
\end{bmatrix}, \quad (2.41)
\]

and \( T_2 \) according to Eq. (2.32). The 4-DOF AMB rigid rotor system with an axisymmetric rotor has the same poles in both the x and y planes, and the distribution is identical to the 2-DOF system shown in Fig. 2.7 except that each pole occurs twice.

2.3.2 Flexible rotor model

Often rotors are so lightweight and slender that the rigid body model cannot describe them accurately enough but also the flexible modes have to be modeled. In this study, finite element modeling (FEM) is used to model the flexible rotor.
Finite element modeling

When applying the FEM in the elementary rotordynamics, the rotor under consideration is typically cut into \( k \) cylindrical beam elements that are thought to be bound together by nodes. Here, the Timoshenko beam model (Timoshenko, 1921, 1922) is used for a single beam element. An advantage of the model is that it also takes into account the shear deformation (Genta, 2005; Chen and Gunter, 2005). Fig. 2.8 shows one beam element and the nodes that bind it to the next beam elements. Each node has six degrees of freedom: displacements in the \( x \), \( y \), and \( s \) directions, and rotations around the same axes. \( s \) is a local longitudinal coordinate. Now, it is assumed that the radial and axial motions are uncoupled. Additionally, the rotor is supposed to be rotating. Therefore, the 4-DOF model is considered with the displacements in the \( x \) and \( y \) directions and the rotations around the \( x \) and \( y \) axes. An equation of motion is written for each node describing deformations of a single beam element.

![Fig. 2.8. Beam element and coordinates.](image)

The equation of motion for each node of each beam is

\[
\mathbf{M}_i \ddot{\mathbf{q}}_i + (\mathbf{D}_i + \Omega \mathbf{G}_i) \dot{\mathbf{q}}_i + \mathbf{K}_i \mathbf{q}_i = \mathbf{F}_i, \tag{2.42}
\]

where the displacement vector of the node \( \mathbf{q}_i = [x_i, y_i, \beta_{x,i}, \beta_{y,i}]^T \) consists of displacements in the \( x \) and \( y \) directions and rotations around the same axes for single nodes. The mass matrix \( \mathbf{M}_i \), gyroscopic matrix \( \mathbf{G}_i \), and force vector \( \mathbf{F}_i \) are defined analogously to the 4-DOF rigid rotor model as
2.3 Rotordynamics

\[
\mathbf{M}_i = \begin{bmatrix}
    m_i & 0 & 0 & 0 \\
    0 & m_i & 0 & 0 \\
    0 & 0 & I_{x,i} & 0 \\
    0 & 0 & 0 & I_{y,i}
\end{bmatrix}, \quad \mathbf{G}_i = \begin{bmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & I_{x,i} & 0 \\
    0 & 0 & 0 & -I_{y,i}
\end{bmatrix}, \quad \mathbf{F}_i = \begin{bmatrix}
    F_{x,i} \\
    F_{y,i} \\
    \Theta_{x,i} \\
    \Theta_{y,i}
\end{bmatrix}, \quad (2.43)
\]

A generalized displacement, that is, the displacement of the center point of a node on the s axis, is obtained using a shape function matrix \( \mathbf{N} \),

\[
\mathbf{q}_i^g = \begin{bmatrix}
    x(s,t) \\
    y(s,t) \\
    \beta_x(s,t) \\
    \beta_y(s,t)
\end{bmatrix} = \mathbf{N}(s)\mathbf{q}_i(t). \quad (2.44)
\]

The superscript \( g \) refers to global coordinates. The shape function matrix is constructed using the beam elasticity theory. The column vectors \( n_k \) of the shape function represent the static displacement modes for a single node when the coordinates of the other nodes are set zero (Chen and Gunter, 2005; Kärkkäinen, 2007). The complete system consists of the generalized displacements of all the nodes in global coordinates. Equation of motion of the complete system is

\[
\mathbf{M}_g^{\mathbf{q}^g} + (\mathbf{D}_g^{\mathbf{q}^g} + \Omega \mathbf{G}_g^{\mathbf{q}^g}) \mathbf{q}^g + \mathbf{K}_g^{\mathbf{q}^g} \mathbf{q}^g = \mathbf{F}_g^{\mathbf{q}^g}, \quad (2.45)
\]

where the global displacement vector is \( \mathbf{q}^g = [q_1^g \quad q_2^g \ldots q_P^g]^T \).

Modal reduction

The equation of motion of Eq. (2.45) is in physical coordinates, it has many DOFs, and the \((x,z)\) and \((y,z)\) planes are highly coupled. Thus, it is not a practical model for control design. The degrees of freedom can be reduced using modal reduction where the physical model of Eq. (2.45) is converted into a modal model using modal coordinate transformation and truncating the irrelevant high-frequency modes. The modal coordinate transformation is made using a mode shape function matrix \( \hat{\Phi} \) that is composed of the eigenvectors of the system as follows

\[
\hat{\Phi} = \begin{bmatrix}
    \phi_1 & \phi_2 & \ldots & \phi_M
\end{bmatrix}.
\]
Analytical model of an active magnetic bearing rotor system

$\tilde{\Phi}_1 - \tilde{\Phi}_4$ are the eigenvectors of the rigid body modes and $\tilde{\Phi}_5 - \tilde{\Phi}_M$ of the flexible body modes of the rotor. The eigenvectors $\tilde{\Phi}_k$ and eigenfrequencies or natural frequencies $\omega_{0,k}$ for an undamped, nonrotating rotor ($D = 0, \Omega = 0$) without external forces are obtained by solving a generalized eigenvalue problem

$$(K^g - \omega_{0,k}^2 M^g) \tilde{\Phi}_k = 0. \quad (2.47)$$

In the mode shape matrix $\Phi$, the eigenvectors of the rigid modes $\tilde{\Phi}_1 - \tilde{\Phi}_4$ correspond to the transversal motions in the $x$ and $y$ axes, and the tilting motions about the same axes, in that order. The eigenvectors of the flexible body modes are arranged in such a way that their eigenfrequencies are in an ascending order. Now, the modal reduction is done by truncating the irrelevant higher-frequency flexible modes. Usually, four to ten lower-frequency flexible modes are sufficient to model the flexible rotor accurately enough (Jastrzebski, 2007). In the control engineering, the columns of the mode shape function matrix are often scaled so that the modal mass matrix becomes an identity matrix, $\tilde{M} = \tilde{\Phi}^T M \tilde{\Phi} = I$.

Now, the relation of the physical nodal displacements $q^g$ and the displacements in the modal coordinates $\tilde{q}$ is

$$q^g = \tilde{\Phi} \tilde{q}, \quad (2.48)$$

and the equation of motion of a rotor with a constant rotation speed in the modal coordinates is

$$\ddot{\tilde{M}} \tilde{q} + (\tilde{D} + \Omega \tilde{G}) \dot{\tilde{q}} + \tilde{K} \tilde{q} = \tilde{F}. \quad (2.49)$$

The mass, damping, stiffness, and the gyroscopic matrices in modal coordinates are obtained as

$$\tilde{M} = \tilde{\Phi}^T M \tilde{\Phi}, \quad \tilde{D} = \tilde{\Phi}^T D \tilde{\Phi}, \quad (2.50)$$

$$\tilde{K} = \tilde{\Phi}^T K \tilde{\Phi}, \quad \tilde{G} = \tilde{\Phi}^T G \tilde{\Phi}, \quad (2.51)$$

$$\tilde{F} = \tilde{\Phi}^T F^g. \quad (2.52)$$

$\tilde{M}$, $\tilde{D}$ and $\tilde{K}$ are diagonal mass, damping, and stiffness matrices, and $\tilde{G}$ is a skew-symmetric gyroscopic matrix of the rotor in the modal coordinates.

Fig. 2.9 (a) shows the poles and zeros of a free flexible rotor at standstill. In the origin, there are eight rigid mode poles. The poles and the corresponding zeros of the three lowest-frequency flexible modes are placed close to the imaginary axis. In Fig. 2.9 (b), the AMBs
2.3 Rotordynamics

![Fig. 2.9. Typical pole-zero map of (a) a free flexible rotor (b) and a flexible rotor supported by AMBs with a current controller.](image)

with a current controller are added to the model. It can be seen that the AMBs, or more accurately, the current controllers move the rigid body poles from the origin to the real axis.

When the rotor is rotating, the $x$ and $y$ planes are coupled by the gyroscopic term $\Omega G$. At standstill, the flexible poles of both planes are equal as shown in Fig. 2.9. The rotating causes the eigenfrequencies to split into forward and backward eigenfrequencies. Usually, the frequencies of the forward modes increase and the frequencies of the backward modes decrease with the increasing rotational speed. The flexible poles of the system move from the positions of Fig. 2.9 along the imaginary axis towards increasing and decreasing frequencies. The Campbell diagram of Fig. 2.10 shows how the eigenmodes change as a function of rotational speed. The intersections of the eigenfrequencies and rotational speed are critical speeds.

2.3.3 Rotor of the test rig

In this study, the number of the nodes used in the FEM analysis of the rotor were $P = 32$, which corresponds to $4 \times P = 128$ DOFs. The frequency range of interest is $0$–$1200$ Hz and in the modal reduction, the number of modes was reduced to the first three flexible modes. The flexible frequencies of the modes can be seen in Table 2.1. The result of the FEM analysis was verified with an experimental modal analysis using Brüel’s & Kjær’s mode analyzer. The flexible frequencies and damping ratios obtained from the experimental modal analysis (EMA) can also be seen in Table 2.1.

It can be seen that the flexible frequencies obtained using the FEM are slightly higher than the flexible frequencies obtained using the EMA. As the FEM model does not include damping, the damping is updated to the model according to the experimental modal analysis.

The three lowest flexible mode shapes of the rotor are shown in Fig. 2.11. The blue and green
Fig. 2.10. Example of a Campbell diagram of a free flexible rotor. The frequencies of the forward modes increase and the frequencies of the backward modes decrease with the increasing rotational speed.
Table 2.1. Flexible frequencies of the studied rotor obtained using the FEM and experimental modal analysis, the differences between them in percents, and the damping ratios of each flexible mode obtained using the EMA (Jastrzebski, 2007).

<table>
<thead>
<tr>
<th>Mode k</th>
<th>Frequency [Hz], FEM model</th>
<th>Frequency [Hz], EMA model</th>
<th>Difference [%]</th>
<th>Damping ratio $\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>260.3</td>
<td>259.8</td>
<td>0.2</td>
<td>0.004118</td>
</tr>
<tr>
<td>2</td>
<td>539.0</td>
<td>526.7</td>
<td>2.3</td>
<td>0.002263</td>
</tr>
<tr>
<td>3</td>
<td>951.8</td>
<td>946.2</td>
<td>0.6</td>
<td>0.004345</td>
</tr>
</tbody>
</table>

Fig. 2.11. Schematic of the rotor model with the first three flexible mode shapes and frequencies. The locations of the bearings and the sensors are denoted by blue and green stars, respectively.

stars denote the locations of the actuators and the displacement sensors, respectively. It can be seen that the first flexible mode passes through bearing B, and the second flexible mode passes very close to bearing A and sensor B. Also the third flexible mode passes through sensor A. This causes difficulties in controlling the modes and also in the system identification, as will be seen in Chapter 5.

2.4 Sensors

In the test setup used in this thesis, the displacements of the rotor and the currents of the electromagnets are measured in radial bearings A and B in both the $x$ and $y$ directions, and in the axial bearing.

For the displacement measurements, eddy current sensors are chosen as they have high resolution, small physical size, small phase shift, and high dc stability. A disadvantage is their
52 Analytical model of an active magnetic bearing rotor system

high cost. The displacement sensors used in this study are nonlinear. The linearity of the sensors used in the end A is ±6% and of the sensors in the end B ±5%. Other alternatives for displacement sensors are optical, ultrasonic, Hall effect, capacitive, and inductive sensors. Their characteristics are described, for example, in Schweitzer and Maslen (2009).

The current sensors used are closed-loop Hall effect LEM transducers. They provide a fast response, a high linearity, and a low temperature drift. They are also relatively immune to electrical noise. These characteristics make the sensors suitable for AMB applications (Jastrzebski, 2007).

The sensors can be modeled using a simple first-order low-pass filter with a transfer function

\[ G_s(s) = \frac{\omega_c}{s + \omega_c}, \]  

(2.53)

with a cut-off frequency \( \omega_c \). The displacement sensors used in this study have a cut-off frequency of 10 kHz, and thus they have no influence in the frequency range of interest, 0–1200 Hz. Hence, the sensor model is not included in the overall model of the system.

2.5 Overall plant model

For the controller design purposes, the analytical model of the system is often described using state equations. First, combine the equation of motion of a rotor of Eq. (2.49) with the linear model of magnetic actuator dynamics of Eq. (2.18),

\[ M\ddot{\mathbf{q}} + (D + \Omega G)\dot{\mathbf{q}} + (\mathbf{K} + \tilde{\mathbf{K}}_s)\mathbf{q} = \tilde{\mathbf{K}}_i \mathbf{i}_m \]  

(2.54)

where the matrices of the force displacement factor \( \tilde{\mathbf{K}}_s \) and the force current factor \( \tilde{\mathbf{K}}_i \) in the modal coordinates are written as

\[ \tilde{\mathbf{K}}_s = \mathbf{\Phi}^T S_s (\mathbf{-K}_s) \mathbf{\Phi}, \]  

(2.55)

\[ \tilde{\mathbf{K}}_i = \mathbf{\Phi}^T S_i \mathbf{K}_i. \]  

(2.56)

\( S_s \) and \( S_i \) are the nodal location matrices of position and current stiffnesses, respectively. The state equations of the combined model of the rotor and the magnetic actuators are
\[ x_r = A_r x_r + B_r u_r \] (2.57)
\[ y_r = C_r x_r + D_r u_r \] (2.58)

with the system matrices \( A_r, B_r, C_r, \) and \( D_r, \) and the state vector \( x_r \) defined as

\[
A_r = \begin{bmatrix}
-M^{-1}(K + K_s) & I \\
-M^{-1}(D + \Omega G)
\end{bmatrix}, \quad B_r = \begin{bmatrix}
0 \\
M^{-1}K_s
\end{bmatrix},
\]

(2.59)

\[
C_r = [S_m \Phi \ 0], \quad D_r = [0], \quad x_r = \begin{bmatrix}
\dot{q} \\
\dot{\bar{q}}
\end{bmatrix}.
\] (2.60)

The input vector \( u_r \) is composed of the magnetizing currents of each plane in each bearing, \( u_r = [i_{m,x,A} \ i_{m,y,A} \ i_{m,x,B} \ i_{m,y,B}]^T \) and the output vector \( y_r \) contains the displacements of the rotor in the \( x \) and \( y \) directions both in bearings \( A \) and \( B, \) \( y_r = [x_A \ y_A \ x_B \ y_B]. \) \( S_m \) is a nodal location matrix of the measurement.

Another state equations are generated for the dynamics of the power amplifier with the current controller. A differential equation of the magnetizing current \( i_m \) can be written using the transfer function of Eq. (2.21),

\[ \dot{i}_m = -\omega BW i_m + \omega BW i_c. \] (2.61)

Now, the state equations of the power amplifier are written as

\[ x_a = A_a x_a + B_a u_a \] (2.62)
\[ y_a = C_a x_a + D_a u_a \] (2.63)

where the system matrices \( A_a, B_a, C_a, \) and \( D_a, \) are determined as

\[
A_a = -\text{diag}(\begin{bmatrix}
\omega_{BW} & \omega_{BW} & \omega_{BW} & \omega_{BW}
\end{bmatrix}), \quad B_a = \text{diag}(\begin{bmatrix}
\omega_{BW} & \omega_{BW} & \omega_{BW} & \omega_{BW}
\end{bmatrix}),
\]

(2.64)

(2.65)

\[
C_a = I_4, \quad D_a = 0_4.
\] (2.66)

The state vector \( x_a \) and the input vector \( u_a \) are composed of the magnetizing control currents \( i_m \) and the control currents \( i_c, \) and are defined as \( x_a = [i_{m,x,A} \ i_{m,y,A} \ i_{m,x,B} \ i_{m,y,B}]^T, \) and \( u_a = [i_{c,x,A} \ i_{c,y,A} \ i_{c,x,B} \ i_{c,y,B}]^T, \) respectively. The overall model of the active mag-
netic bearing rotor system containing rotor dynamics, the linearized model for the magnetic actuators, and the power amplifier dynamics is

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]

with the system matrices \(A\), \(B\), \(C\), and \(D\),

\[
A = \begin{bmatrix} A_a & 0 \\ B_r C_a & A_r \end{bmatrix}, \quad B = \begin{bmatrix} B_a \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & C_r \end{bmatrix}, \quad D = 0,
\]

and the state, input, and output vectors \(x\), \(u\), and \(y\), determined as

\[
x = \begin{bmatrix} x_a \\ x_r \end{bmatrix}, \quad u = \begin{bmatrix} u_a \\ u_r \end{bmatrix}, \quad y = \begin{bmatrix} y_a \\ y_r \end{bmatrix}.
\]

### 2.6 Unmodeled dynamics

Modeling of the rotor by the FEM and updating the model by the EMA gives an accurate model of the free rotor. The linearized model of the electromagnets has also proven sufficient to model the system in the normal operation conditions. However, in all the rotor bearing systems, there are usually some undermodeled or entirely unmodeled dynamics. For example, the foundations, shrink fits, and seals, influence the overall dynamics of the rotating system. When considering an AMB rotor system, there are also some couplings between the radial and axial bearings. The behavior of the dynamics of the above-mentioned components is considered frequency dependent and is problematic to model analytically. However, its influence on the systems can be determined using system identification (Wang and Maslen, 2006).

### 2.7 Position control

The position control is applied as an outer control loop for the current controlled AMB rotor system. The design of a position controller is out of the scope of this thesis. However, as it is an essential part of an AMB rotor system, it is discussed in brief.

The industrial AMB applications with a rigid rotor are commonly controlled using decentralized PID controllers. In the decentralized control, each bearing and each plane are controlled...
individually. However, because the magnetic actuators and the displacement sensors are typically not collocated in the AMB rotor systems, the decentralized PID controllers may lead to instability at certain rotational speeds. Nonconservative bearing forces, such as the internal damping of the shaft or the seal effect, may also cause problems (Schweitzer and Maslen, 2009). Another control scheme applied to AMB rotor systems is a state controller. The state control requires not only the displacements of the rotor but also the velocities. Because the velocities are not usually measured, a state controller with an estimator is required. Such an approach is the linear quadratic Gaussian (LQG) control (Jastrzebski, 2007; Zhuravlyov, 2000). The main drawback of the LQG control is its sensitivity to model errors and uncertainties (Doyle, 1978). The robustness properties have been improved using different $H_\infty$ approaches (Fujita et al., 1993; Arredondo and Jugo, 2007; Gosewski and Mystkowski, 2008; Jastrzebski et al., 2010) and $\mu$ synthesis (Nonami and Ito, 1996; Lösch et al., 1999; Lanzon and Tsiotras, 2005).

In this research, a centralized, signal-based $H_\infty$ controller is used. In the control design, a coupled plant model based on the FEM and corrected using measured FRFs of the AMB rotor system is applied. An uncertain model and the structure of the weights in the control design are also based on the measured system responses. The singular value plot of the controller $C(s)$ in the $\mathbf{s}$ plane can be seen in Fig. 2.12.

Fig. 2.13 shows the singular values of the MIMO open-loop $G_{ol}(s)$ system containing the rotor, the power amplifier determined according to Eq. (2.20), and the position controller. The bandwidth of a MIMO system can be determined at the frequency where the minimum singular value crosses 0 dB (Lewis and Syrmos, 1995; Skogestad and Postlethwaite, 2005). For the studied system the bandwidth is $f_{BW} = 74$ Hz ($\omega_{BW} = 465$ rad/s).
2.8 Conclusions

In this chapter, the experimental setup with the corresponding parts and their analytical modeling have been presented.

The magnetic actuators of the AMB rotor system consist of electromagnets and power amplifiers. The electromagnets are modeled using a simple linearized equation for the magnetizing force. The model has proven to work well in normal operating conditions. The power amplifier with a current controller is modeled with a simple first-order transfer function.

The modeling of the rotor dynamics has been presented for both 2-DOF and 4-DOF rigid rotors and for flexible rotors using modal coordinates. Finite element modeling and modal reduction have been explained in brief. The rotor of the test rig has also been presented.

The sensors are often modeled with a simple first-order low-pass filter. However, in this study, the cut-off frequency of the sensors is considerably high (10 kHz) when compared with the frequency range of interest, and thus, the sensor model is not included in the overall model of the system.

The state equations of the overall analytical model comprising the rotor and the magnetic actuators with the electromagnets and the power amplifier with a current control have been presented. The position controller has also been introduced as it is an essential part of the
2.8 Conclusions

operation of an unstable AMB rotor system. For the position control of the radial bearings, a centralized $H_{\infty}$ controller has been used.

In addition to the aforementioned dynamics, there are usually some unmodeled dynamics in the rotor bearing systems. In AMB rotor systems, the unmodeled dynamics may originate, for example, from the foundation, shrink fits, seals, or the coupling between the radial and axial bearings. The unmodeled dynamics can be modeled using system identification as discussed in Chapter 3, and further updated to the analytical model using model updating presented in Section 4.6.
Analytical model of an active magnetic bearing rotor system
Chapter 3

Nonparametric identification of an AMB rotor system

This chapter reviews some special issues concerning AMB rotor system identification, such as nonlinearities, closed-loop, and MIMO system identification. A few broadband excitation signals and their properties are presented and their suitability for the identification of an AMB rotor system is assessed. For comparison, also a stepped sine excitation is considered. Further, excitation signal combinations for MIMO system identification are discussed. Nonlinear systems produce additional harmonics that may distort the measured frequency response functions. Their analysis methods are presented and harmonics of an AMB rotor system are analyzed. Additionally, different frequency response function estimators required in improving the signal-to-noise ratio when using broadband excitations are introduced.

3.1 Special issues on AMB rotor system identification

In this study, nonparametric frequency domain methods of identification are applied to a non-rotating AMB rotor system. The nonparametric frequency response functions are used in the experimental modal analysis of mechanical structures, and they are also widely adopted to AMB rotor systems, for example (Gähler, 1998; Lösch, 2002; Maslen et al., 2002; Sawicki et al., 2007). The nonparametric methods present the system model in a large number of points, and they give valuable information about the possible model structure and order. Frequency domain identification also provides an option to detect the effects of possible nonlinearities in the system by investigating the harmonics in the system output (Evans et al., 1994). A restriction in the use of FRFs is that it should be possible to describe the system using a linear model. When considering the studied AMB rotor system, the main source of nonlinearities are the magnetic actuators. Further nonlinearities may result from the nonlinear sensors and rotordynamics. When modeling the system for the control purposes, a paramet-
Nonparametric identification of an AMB rotor system

Nonparametric methods are often used as an intermediate state when identifying parametric models. In the second step, a parametric model is constructed from the nonparametric model or directly from the measurement data (Pintelon and Schoukens, 2001b). For the diagnostics purposes, a nonparametric model is suitable.

This study considers only the identification of a nonrotating AMB rotor system. However, AMBs also enable the identification of a rotating system. Identification of a rotating AMB rotor system is considered, for example, in Gähler (1998); Lösch (2002).

In this section, nonparametric frequency domain methods for single-input, single-output (SISO) and multiple-input, multiple-output (MIMO) systems are presented and their applicability to nonlinear systems explained. Additionally, the identification in a closed loop is discussed.

3.1.1 Definitions

An active magnetic bearing rotor system is a continuous-time system. However, it is controlled by a discrete-time controller that operates with discrete-time signals. In addition to these time domain signals, frequency domain signals, spectra, and frequency response functions are also treated in this study. This section introduces different notations used for the time and frequency domain signals and functions.

A continuous-time signal \( y(t) \) is determined at every time instant \( t \). A sampled discrete-time signal \( y(nT_s) \), on the other hand, is only determined at discrete time instants, which are determined according to the sampling time \( T_s \), \( n = 0, 1, \ldots, N - 1 \). \( N \) is the number of the samples of the measured data. Fig. 3.1 (a) and (b) show an example of a continuous and discrete-time signals, respectively. The frequency spectrum of the discrete-time signal is determined by a discrete Fourier transform (DFT) \( Y(\omega_k) \), where \( \omega_k = 2\pi f_k \), \( k = 0, \ldots, N_f - 1 \). \( \omega_k \) and \( f_k \) are discrete sets of angular frequencies and frequencies in Hertz, respectively, and \( N_f \) is the number of frequencies. A frequency response function \( G(j\omega_k) \) describes the system from the input to the output at a discrete set of frequencies \( \omega_k \). Examples of a spectrum and frequency response function can be seen in Figs. 3.1 (c) and (d).

3.1.2 Nonparametric frequency domain identification

A simplest nonparametric identification method of the FRF for a SISO system is an empirical transfer function estimate (ETFE)

\[
\hat{G}_{\text{ETFE}}(j\omega_k) = \frac{Y(\omega_k)}{U(\omega_k)},
\]

(3.1)
3.1 Special issues on AMB rotor system identification

Fig. 3.1. Definitions. (a) Continuous-time signal \( y(t) \), (b) sampled discrete-time signal \( y(nT_s) \), (c) frequency spectrum of a discrete-time signal \( Y(\omega_k) \), and (d) frequency response function \( G(j\omega_k) \).
where $Y(\omega_k)$ and $U(\omega_k)$ are DFTs of one period of output and input samples $y(nT_s)$ and $u(nT_s)$, respectively,

$$Y(\omega_k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(nT_s) e^{-j2\pi f_k n/N},$$  

(3.2)

$$U(\omega_k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u(nT_s) e^{-j2\pi f_k n/N}.$$  

(3.3)

The ETFE gives a good performance for periodic input signals and noiseless measurements. In reality, however, the signals in the system contain noise as presented in Fig. 3.2. The input signal $U(\omega_k)$ includes generator noise $N_g(\omega_k)$ that distorts the original input signal $U_0(\omega_k)$, or the applied excitation. The output signal $Y(\omega_k)$ is affected by the process noise $N_p(\omega_k)$, and both the input and output measurements may contain measurement noise $M_U(\omega_k)$ and $M_Y(\omega_k)$, respectively (Pintelon and Schoukens, 2001b). For aperiodic signals and in the presence of disturbing noise, the ETFE is often smoothed by averaging over neighboring frequencies that are assumed to be asymptotically uncorrelated or by averaging over different data sets. Both smoothing methods lead to reduction of the variance (Ljung, 1999). Methods for averaging over different data sets are introduced in Section 3.3.

![Fig. 3.2. Identified plant $G_0(j\omega_k)$ with noise signals. $N_g(\omega_k)$ is generator noise and $N_p(\omega_k)$ is process noise. $M_U(\omega_k)$ and $M_Y(\omega_k)$ are input and output measurement noises, respectively.](image)

### 3.1.3 Identification of a MIMO system

Now, consider a multiple-input, multiple-output system of Fig. 3.3, with $N_u$ inputs and $N_y$ outputs, each suffering from generator noise, process noise, and measurement noise signals. The empirical transfer function estimate of Eq. (3.1) used for SISO systems can be extended to an empirical frequency response matrix (EFRM) estimate as

$$\hat{G}_{\text{EFRM}}(j\omega_k) = Y(\omega_k)U^{-1}(\omega_k),$$  

(3.4)
with the input and output matrices $U(\omega_k)$ and $Y(\omega_k)$ determined as

$$
U(\omega_k) = \begin{bmatrix}
U_1^{(1)}(\omega_k) & U_1^{(2)}(\omega_k) & \cdots & U_1^{(N_e)}(\omega_k) \\
U_2^{(1)}(\omega_k) & U_2^{(2)}(\omega_k) & \cdots & U_2^{(N_e)}(\omega_k) \\
\vdots & \vdots & \ddots & \vdots \\
U_{N_u}^{(1)}(\omega_k) & U_{N_u}^{(2)}(\omega_k) & \cdots & U_{N_u}^{(N_e)}(\omega_k)
\end{bmatrix},
$$

(3.5)

$$
Y(\omega_k) = \begin{bmatrix}
Y_1^{(1)}(\omega_k) & Y_1^{(2)}(\omega_k) & \cdots & Y_1^{(N_e)}(\omega_k) \\
Y_2^{(1)}(\omega_k) & Y_2^{(2)}(\omega_k) & \cdots & Y_2^{(N_e)}(\omega_k) \\
\vdots & \vdots & \ddots & \vdots \\
Y_{N_y}^{(1)}(\omega_k) & Y_{N_y}^{(2)}(\omega_k) & \cdots & Y_{N_y}^{(N_e)}(\omega_k)
\end{bmatrix}.
$$

(3.6)

The subscripts of the matrices $U$ and $Y$ refer to the inputs ($1, 2, \ldots N_u$) and outputs ($1, 2, \ldots N_y$) of the system, respectively, and the superscripts refer to the separate experiments. Thus, the column vectors $U^{(i)}(\omega_k)$ and $Y^{(i)}(\omega_k)$ contain the DFTs of all the input and output signals of the system from the separate experiments. The input matrix $U(\omega_k)$ of Eq. (3.5) must be invertible and as many distinct experiments with different sets of excitations must be made as there are system inputs.
Similar to the ETFE, the EFRM also gives correct FRFs for linear systems and periodic data only if the system has no noise. The practical estimators need to include some averaging methods, which are presented in Section 3.3.

A magnetic bearing rotor system, as a whole, is a 5-input, 5-output system containing control currents as inputs and displacements of the rotor as outputs in bearings A and B in both the x and y planes, and in the axial bearing, as presented in Chapter 2. In this study, a nonrotating rotor is considered and no coupling between the x, y, and z planes is assumed. Additionally, the x and y planes are assumed identical, and thus it is sufficient to consider the system only in one plain. The axial bearing is also not considered. Thus, a 2-input, 2-output system is studied.

3.1.4 FRF measurements of a nonlinear system

Nonlinear systems are often modeled using linear models by linearizing them in their respective operating points. This enables the application of the linear control theory. Also in this study, a linearized model for a magnetic bearing rotor system is used. Thus, the target in the identification is to obtain a linear approximation of the system. To this end, this research concentrates on linear identification methods. However, the system contains nonlinear components such as magnetic coils and nonlinear sensors, which have to be taken into account in the FRF measurements and estimators. The system does not contain seals or other nonlinear components, and the displacements of the rotor from the reference point are small. Thus, the nonlinearities of the mechanical parts are assumed to be minimal when compared with the magnetic bearings and sensors.

Often, the dynamics of a system can be adequately described by using a linear model even if there is a static nonlinearity affecting the input or output of the system. For example, a nonlinear actuator in saturation can be considered a static nonlinearity in the input and sensors with nonlinear characteristics as a static nonlinearity in the output. Fig. 3.4 shows different nonlinear models. A system with nonlinearity in the input can be described using a Hammerstein model (Narendra and Gallman, 1966), Fig. 3.4 (a), and a system with a nonlinearity in the output is described using a Wiener model, Fig. 3.4 (b). When the system has nonlinearities both in the input and the output, it can be modeled using a Wiener-Hammerstein model, Fig. 3.4 (c) (Ljung, 1999). If a nonlinearity can be considered to be in parallel with the linear part, an additive nonlinear model is used, Fig. 3.4 (d) (McCormack et al., 1994; Pintelon and Schoukens, 2001b). The additive nonlinear model can be used, for example, if a linear model suffers from nonlinear distortions.

Consider an additive nonlinear model of Fig. 3.4 (d) where the linear contribution is dominating. Now, the FRF of the underlying linear system can be determined by minimizing the influence of the nonlinear part. However, when considering the Hammerstein, Wiener, and Wiener-Hammerstein models of Fig. 3.4 (a)-(c), the nonlinearity is in series with the linear model and it is no more possible to measure the FRF of the underlying linear system. The same holds for the additive nonlinear model when the nonlinear contribution is high. Therefore, the objective in the FRF measurements is to find the best linear approximation to the
system including both the linear and nonlinear parts. If the nonlinearity of the system is too large, the described methods are not applicable but also the nonlinearity of the system has to be modeled (Pintelon and Schoukens, 2001b).

The output of a nonlinear system can be determined as

\[ Y(\omega_k) = G_R(j\omega_k)U(\omega_k) + G_S(j\omega_k)U(\omega_k) + N_G(\omega_k), \]  

(3.7)

where \( G_R(j\omega_k) \) is a related linear dynamic system (RLDS), \( G_S(j\omega_k) \) is a stochastic nonlinear contribution, and \( N_G(\omega_k) \) is an error caused by the output noise (the input noise is assumed to be minimal compared with the output noise). Thus, the measured FRF of the system from the input \( U(\omega_k) \) to the output \( Y(\omega_k) \) is

\[ G(j\omega_k) = G_R(j\omega_k) + G_S(j\omega_k). \]  

(3.8)

The RLDS can be further divided into two parts, an underlying linear system \( G_0(j\omega_k) \) and bias or systematic errors caused by nonlinear distortions \( G_B(j\omega_k) \),

\[ G_R(j\omega_k) = G_0(j\omega_k) + G_B(j\omega_k). \]  

(3.9)

For a nonlinear system, the RLDS is the best linear approximation of the system. When the influence of the stochastic nonlinear contribution \( G_S(j\omega_k) \) is weak, the expected value of the FRF \( G(j\omega_k) \) converges to the RLDS \( G_R(j\omega_k) \). Because of the stochastic nonlinear distortions, the FRF is not smooth but scatters around the expected value \( G_R(j\omega_k) \). The impact of the \( G_S(j\omega_k) \) can be minimized by a careful design of the excitation signal as explained later in Section 3.2, and by averaging the FRF over sufficiently many signal periods or blocks of
separate measurements. Different estimators for the averaging will be discussed in Section 3.3. The systematic error $G_B(j\omega_k)$ causes the RLDS to differ from the underlying linear system $G_0(j\omega_k)$. This is due to the harmonics produced by a nonlinear system, and its impact on the FRF cannot be affected by averaging over different periods or blocks (Pintelon and Schoukens, 2001b).

The dynamics of an AMB rotor system with the power amplifiers and the rotor is modeled using linear equations. Additionally, the magnetic coils are assumed to contain static nonlinearities in the input of the system, which leads to the Hammerstein model. When nonlinear sensors are used, as is the case in this study, the Wiener-Hammerstein model is used. The FRFs of the best linear approximation are measured. The applicability of the obtained model is determined by analyzing the harmonics possibly produced by the nonlinear system. The detection of the harmonics is described in Section 3.4.

### 3.1.5 Identification in a closed loop

So far, it is assumed that the frequency response function measurements can be made for an open-loop system, which is recommended if only possible. However, in some cases it is necessary to make the measurements for a closed-loop system if, for example, the system is inherently unstable as AMB rotor systems are, or it has to be controlled for the productional, economic, or safety reasons. A problem in the closed-loop identification is that it often contains less information about the open-loop system, because the feedback makes the closed-loop less sensitive to the changes in the open-loop. A more serious problem is that many identification methods, including the frequency domain nonparametric methods, may produce erroneous results when using closed-loop measurements (Ljung, 1999). The reason for the problem is that the open-loop identification methods assume that there is no correlation between the process noise and the system input, which is not true in the closed-loop identification. Consider Fig. 3.5 that shows an identified plant $G_0(j\omega_k)$ with a feedback controller $C_0(j\omega_k)$. $N_p(\omega_k)$ is the process noise, $N_c(\omega_k)$ is the controller noise, and $R(\omega_k)$ is the reference signal. It can be seen that the system input $U(\omega_k)$ not only consists of the reference signal but also of the feedback signal. Thus, the input signal is corrupted by the process noise (Pintelon and Schoukens, 2001b; Wernholt and Gunnarson, 2007).

Ljung (1999) presents three different approaches to the closed-loop identification:

- **Direct approach:** An open-loop model from the input $U(\omega_k)$ to the output $Y(\omega_k)$ is measured ignoring the feedback. An advantage of the approach is that no knowledge about the controller or the reference signal, nor special algorithms and software are required. A disadvantage of the method is that it may lead to biased estimates for the reasons explained above.

- **Indirect approach:** A closed-loop FRF $G_{cl}(j\omega_k)$ from the reference $R(\omega_k)$ to the output $Y(\omega_k)$ is estimated. While the output of the closed-loop system is

$$Y(\omega_k) = G_{cl}(j\omega_k)R(\omega_k) + N_{cl}(\omega_k)$$
3.1 Special issues on AMB rotor system identification

Fig. 3.5. Identified plant $G_0(j\omega_k)$ with a feedback controller $C_0(j\omega_k)$. $U(\omega_k)$ and $Y(\omega_k)$ are the measured input and output, and $R(\omega_k)$ is the reference signal. $N_p(\omega_k)$ is the process noise and $N_c(\omega_k)$ is the controller noise. $M_U(\omega_k)$ and $M_Y(\omega_k)$ are the input and output measurement noises, respectively. Adapted from Ljung (1999); Pintelon and Schoukens (2001b).

$$= \frac{G_0(j\omega_k)}{1 + G_0(j\omega_k)C_0(j\omega_k)}R(\omega_k) + \frac{1}{1 + G_0(j\omega_k)C_0(j\omega_k)}N_c(\omega_k),$$

(3.10)

the open-loop model $G_0 = G_0/(1 - G_0C_0)$ can be calculated when the controller is known. $N_c(\omega_k)$ is the noise of the closed-loop measurement. Problems arise if $C_0(j\omega_k)$ contains errors, such as deviation from a linear regulator, because of input saturation or anti-windup measures. In such a case, the open-loop model cannot be correctly resolved.

• Joint-input-output approach: The reference signal $R(\omega_k)$ is considered as a system input, and both the plant input $U(\omega_k)$ and the plant output $Y(\omega_k)$ are considered as the system outputs, which can be determined as

$$Y(\omega_k) = G_0(j\omega_k)S_0(j\omega_k)R(\omega_k) + S_0(j\omega_k)N_p(\omega_k) + G_0(j\omega_k)S_0(j\omega_k)N_c(\omega_k),$$

(3.11)

$$U(\omega_k) = S_0(j\omega_k)R(\omega_k) - C_0(j\omega_k)S_0(j\omega_k)N_p(\omega_k) + S_0(j\omega_k)N_c(\omega_k),$$

(3.12)

where $S_0 = 1/(1 - G_0C_0)$. A disadvantage of the joint-input-output approach is that it requires knowledge about the reference signal.

In this study, the direct and joint-input-output approaches are used. In Section 3.3, the suitability of the direct approach FRF estimators for the closed-loop data is discussed.
3.2 Selection of the excitation signal

When measuring the frequency response functions of a system that is assumed to be ideally linear and the measurements are noiseless, any persistent excitation is usually suitable. However, if the system is not ideally linear and there are disturbances affecting the measurements, the excitation signal has a considerable impact on the characteristics of the results. The influence of the nonlinearities can be minimized by choosing an appropriate excitation signal (Pintelon and Schoukens, 2001b).

3.2.1 Excitation signals

In the case of frequency domain identification methods, it is reasonable to choose a periodical excitation signal if only possible, and measure an integer number of periods. By doing so, the leakage problem in calculating the Fourier transform is avoided. The leakage problem can be reduced by windowing, but it cannot be totally avoided if the excitation is aperiodic or a noninteger number of periods are measured. In this research, the control platform enables the generation of almost arbitrary excitation signals, and thus, the focus of the further discussion is on the periodic excitations. A random phase multisine, periodic random noise, and a pseudorandom binary sequence (PRBS) as broadband signals have been taken into consideration. Also the stepped sine excitation is discussed as it is currently widely used in the identification of AMB rotor systems. More about different excitation signals can be found for example in Schoukens et al. (1988, 2004); Phillips and Allemang (2003). Schoukens et al. (1988) have investigated the suitability of different signals for the FFT (fast Fourier transform) based signal analyzers and compared their measurement times, accuracy, and sensitivity to nonlinear distortions. They have also investigated the option to modify the amplitude spectrum of the signals, that is, apply different amplitudes at different frequencies. Schoukens et al. (2004) have investigated the suitability of the random and multisine excitations when measuring the best linear approximation \( G_R(j\omega) \) in the presence of nonlinear distortions. Phillips and Allemang (2003) consider excitation signals suitable for the experimental modal analysis of the mechanical structures.

In AMB rotor system applications, the stepped sine excitation has been used, among others, by Ahn et al. (2003b); Gähler (1998); Lösch (2002) and the swept sine by Sawicki et al. (2007); Mushi et al. (2010). Hynynen and Jastrzebski (2009) and Hynynen et al. (2010) have used the multisine excitation.

Stepped sine excitation

A stepped sine excitation contains a series of measurements with single sine signals

\[
r_k(t) = A_k \cos(2\pi f_k t).
\]  

(3.13)
3.2 Selection of the excitation signal

The amplitudes $A_k$ and frequencies $f_k$ of the sine signals can be chosen arbitrarily. An advantage of the stepped sine excitation is that all the signal power is concentrated on one frequency at a time, and thus, the signal-to-noise ratio (SNR) of the measurement is maximized. A disadvantage is that in general, the measurement time is significantly longer when compared with a well-designed multisine excitation. The reason for this is that for each frequency of the stepped sine, at least one period of the sine has to be measured, and after each frequency step there is a waiting time until the transient is stabilized. The total measurement time $T_{ss}$, when using the stepped sine excitation and using one period of the sine signal at each frequency, is

$$T_{ss} = \left( \sum_{k=1}^{N_f} \frac{1}{f_k} + N_f T_w \right) N_e. \quad (3.14)$$

$N_f$ is the number of frequencies and $T_w$ is the waiting time required for the system to stabilize after the transients of the plant and the measurement system. The measurement time is multiplied by the number of experiments $N_e$. For a MIMO system, as many distinct experiments must be measured as there are system inputs. If the chosen frequencies are multiples of a base frequency $f_0$, $f_k = k f_0$, the measurement time can be rewritten as

$$T_{ss} = \left( \frac{1}{f_0} \sum_{k=1}^{N_f} \frac{1}{k} + N_f T_w \right) N_e. \quad (3.15)$$

The waiting time depends on the dynamics of the system, and for highly damped AMB rotor systems, the waiting time is relatively short. In the simplest approach, the waiting time is assumed to be a frequency-independent constant. However, if the system under consideration has well-separated poles, the waiting times can be selected separately for each frequency band (Schoukens et al., 2000).

An interesting property of a sine wave signal is that it allows the calculation of the Fourier transform without saving a long time domain measurement of the signal. This is an advantage if there is not a lot of memory available. Simplifying the equation of the DFT of Eq. (3.2) to the form

$$Y(\omega) = \sum_{n=0}^{N-1} y(n) e^{-j\omega n}, \quad (3.16)$$

substituting $e^{-j\omega n} = \cos(\omega n) - j \sin(\omega n)$, and assuming that the signal $y(n)$ is real, as a sine wave is, it follows that
Nonparametric identification of an AMB rotor system

\[ Y(\omega) = Y_R(\omega) + Y_I(\omega) = Y_R(\omega) + jY_I(\omega) = N^{-1} \sum_{n=0}^{N-1} [y(n) \cos(\omega n)] - j \sum_{n=0}^{N-1} [y(n) \sin(\omega n)]. \]  

(3.17)

Now, only the real and imaginary parts of the transform, \( Y_R(\omega) \) and \( Y_I(\omega) \), are saved and updated in real time during the measurement over the period \( n = 0 \ldots N - 1 \) (Proakis and Manolakis, 1996).

In the identification of AMB rotor systems, also a swept sine excitation has been used. A signal generator generates a sine wave signal for which the frequency sweeps very slowly through the frequency range of interest. When the sweep is slow enough, no observable transient occurs. In practice, the measurements are made at discrete intervals, and thus, the sine sweep excitation is equivalent to the stepped sine excitation with a very dense frequency resolution. In practice, the important characteristics of an AMB rotor system can be seen from an FRF measured using the resolution of 1 to 4 Hz.

**Random phase multisine**

A random phase multisine signal can be determined as

\[ r(t) = \sum_{k=1}^{N_f} A_k \cos(2\pi f_k t + \phi_k), \]

(3.18)

where \( A_k \) are the amplitudes of each frequency component and \( N_f \) is the number of frequencies \( f_k \) chosen from the grid \( \{ \frac{n\pi}{N_f}, n = 1, \ldots, N/2 - 1 \} \). Both the frequencies and amplitudes of the signal can be chosen arbitrarily. Random phases \( \phi_k \) are uniformly distributed to the interval \( [0, 2\pi) \) (Wernholt and Gunnarson, 2007). Another option for the determination of the phases is the Schroeder phases (Schroeder, 1970). Optimal phases have also been determined by minimizing the crest factor of the multisine signal; den Ouderaa et al. (1988b,a) have developed an iterative clipping algorithm for optimization and Guillaume et al. (1991) have used an algorithm based on an \( l_2p \) norm. By a careful design of the phases, it is possible to minimize the impact of the stochastic nonlinear contribution \( G_S(j\omega) \) on the measured FRF (Pintelon and Schoukens, 2001b). Fig. 3.6 shows an example of a multisine signal both in the frequency and time domains. The signal contains frequencies in the frequency range of 10 to 200 Hz with the frequency resolution of 10 Hz. The amplitudes are chosen as one at the frequencies 10–50 Hz, two at the frequencies 60–150 Hz, and three at the frequencies 160–200 Hz. The FRF is only estimated at the excited frequencies.

Because the broadband excitation contains all the required frequencies at the same time, the time required for the measurement is considerably shorter compared with the stepped sine excitation. If the SNR is high and only one measurement is required, the measurement time \( T_{bs} \), equals
3.2 Selection of the excitation signal

![Graph 1](image1.png)

![Graph 2](image2.png)

**Fig. 3.6.** Example of a multisine signal containing frequencies in the frequency range of 10 to 200 with the frequency resolution of 10 Hz. The signal is plotted both in the frequency and time domains.

$$T_{bs} = \left( \frac{1}{\Delta f} + T_w \right) N_e,$$

(3.19)

where $\Delta f$ is the frequency resolution. When the chosen frequencies are multiples of the base frequency $f_0$, similarly to the stepped sine excitation, the period length is determined according to the base frequency, and thus, the measurement time is

$$T_{bs} = \left( \frac{1}{f_0} + T_w \right) N_e.$$

(3.20)

Similar to the stepped sine excitation, the measurement times must be multiplied by the number of experiments $N_e$ when identifying MIMO systems. When comparing the measurement times using the stepped sine and multisine excitations, Eqs. (3.14) and (3.19), and assuming the same waiting time with both excitations, it can be seen that the measurements with the multisine signal are significantly faster.

The above measurement time of Eq. (3.20) for the FRF measurements with the broadband excitation holds for the measurements with a high SNR. However, when using the multisine or other broadband excitation, the signal distributes the power over $N_f$ frequencies. Thus, the amplitudes of each frequency component of the signal must be lower when compared with a single sine excitation, which degrades the SNR. The signal-to-noise ratio is improved
by averaging the measured FRF over several measurements, and the total measurement time depends on the blocks required for the averaging. If the SNR is very low, $N_f$ averages are needed in order to get the same signal-to-noise ratio as when applying the stepped sine excitation. Thus, the total measurement times would be the same. The measurement time using the multisine signal when several blocks are measured is

$$T_{bs} = \left( 1 / f_0 + N_b T_w \right) N_e, \quad (3.21)$$

where $N_b$ is the number of blocks used for the averaging in the case of nonsynchronous measurements. The measurements are synchronous, if a multiperiod measurement can be cut to blocks of the length of the period $1 / f_0$ that are used for the averaging. For the synchronous measurements $N_b = 1$ (Schoukens et al., 2000).

For a linear system, the averaging can be performed over different periods with the same excitation signal. For a nonlinear system, however, the averaging must done over periods or blocks with different realizations of the random phases $\phi_k$. Otherwise, the nonlinearities may distort the FRF estimate (Wernholt and Moberg, 2007; Dobrowiecki and Schoukens, 2007).

**Periodic random excitation**

A periodic random signal or periodic noise is a periodical noise signal that is repeated at least for one whole period after the transient. Because several frequencies are involved, the SNR deteriorates similarly to other broadband excitations. For the averaging, the measurement is repeated $N_b$ times using every time a new periodic random signal (Schoukens et al., 1988).

The frequencies in the periodic random signal can be chosen arbitrarily, but the amplitudes cannot be chosen. A disadvantage of the random amplitudes is that they require more averaging than, for example, the multisine signal that has fixed amplitudes in every block used for the averaging (Pintelon and Schoukens, 2001b).

**Pseudorandom binary sequence**

A pseudorandom binary sequence (PRBS) is a deterministic, periodic signal. The length of the sequence is $N$ and it varies between the levels $+a$ and $-a$ (or typically between $+1$ and $-1$) at multiples of a clock period $T_c$. The optimal choice for the clock frequency $f_c$ depends on the highest excited frequency $f_{\text{max}}$. This leads to the frequency resolution $\Delta f = f_c / N$. Thus, the desired frequencies can be chosen, but also other frequencies are excited. A more developed version of the PRBS is a discrete interval binary sequence (DIBS). The DIBS allows the signal power to be concentrated at the desired frequencies with the discrete grid $k f_0$. However, excitation of the intermediate frequencies cannot be avoided (Pintelon and Schoukens, 2001b).
Binary signals give an optimal spectrum for the excitation signal, but their disadvantage is that they do not provide an option for the detection of the possible nonlinearities generated in the system. The nonlinearities are detected by analyzing harmonics produced by a nonlinear system at nonexcited frequencies. Because the binary sequences excite several additional frequencies, the analysis of the harmonics is not possible (Ljung, 1999).

### 3.2.2 Excitation signals in MIMO identification

When exciting a MIMO system with random multisine signals, the excitation matrix \( \mathbf{R}_{N_u}(\omega_k) \) is written as

\[
\mathbf{R}_{N_u}(\omega_k) = \begin{bmatrix}
R_1^{(1)}(\omega_k) & R_1^{(2)}(\omega_k) & \ldots & R_1^{(N_e)}(\omega_k) \\
R_2^{(1)}(\omega_k) & R_2^{(2)}(\omega_k) & \ldots & R_2^{(N_e)}(\omega_k) \\
\vdots & \vdots & \ddots & \vdots \\
R_{N_u}^{(1)}(\omega_k) & R_{N_u}^{(2)}(\omega_k) & \ldots & R_{N_u}^{(N_e)}(\omega_k)
\end{bmatrix}.
\] (3.22)

The subscripts refer to the inputs \( u \) and the superscripts to the separate experiments \( m \). The individual excitation signals \( R_{um}^{(m)}(\omega_k) \) in each input and each experiment are independent of each other. With such a set of excitation signals, there is always the problem that the fluctuation of the excitations adds to the fluctuations caused by the stochastic nonlinear contribution \( G_N(j\omega_k) \), and thus the variance of the FRF increases.

To reduce the random fluctuations of the excitation signals, orthogonal random multisines are used for the excitation of MIMO systems. When using orthogonal random multisines, the excitation matrix is chosen as

\[
\mathbf{R}_{N_u}(\omega_k) = \begin{bmatrix}
w_{11}R_{11}^{(1)}(\omega_k) & w_{12}R_{12}^{(1)}(\omega_k) & \ldots & w_{1N_e}R_{11}^{(1)}(\omega_k) \\
w_{21}R_{21}^{(1)}(\omega_k) & w_{22}R_{22}^{(1)}(\omega_k) & \ldots & w_{2N_e}R_{21}^{(1)}(\omega_k) \\
\vdots & \vdots & \ddots & \vdots \\
w_{N_u1}R_{N_u1}^{(1)}(\omega_k) & w_{N_u1}R_{N_u1}^{(2)}(\omega_k) & \ldots & w_{N_u1}R_{N_u1}^{(N_e)}(\omega_k)
\end{bmatrix} = \mathbf{W} \cdot \begin{bmatrix}
R_1(\omega_k) \\
R_2(\omega_k) \\
\vdots \\
R_{N_u}(\omega_k)
\end{bmatrix}.
\] (3.23)

Instead of having an independent random excitation in every input and every experiment, there are only independent excitations for every input, which are scaled for each experiment with a frequency-independent orthogonal matrix \( \mathbf{W} \). The elements of the orthogonal matrix are determined as

\[
[W_{um}] = e^{-j\pi(u-1)(m-1)/N_u}.
\] (3.24)
When the number of system inputs is $N_u = 2^K$, the orthogonal matrix $W$ can be chosen even simpler as presented in Eq. (3.23) using a Hadamard matrix. For a two-input system with two separate experiments, the orthogonal random multisine excitation matrix simplifies to

$$R_2(\omega_k) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} R(\omega_k).$$ (3.25)

In practice, in the first experiment, both inputs are excited with the same random multisine excitation $r(t)$, whereas in the second experiment, the excitation signal of the second input is inverted. Similarly, the orthogonal random multisine excitation for a four-input system is written as

$$R_4(\omega_k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} R(\omega_k).$$ (3.26)

When improving the SNR of a nonlinear system by averaging over multiple blocks, a new excitation signal $r(t)$ with a different set of phases is used for every block (Dobrowiecki et al., 2005; Dobrowiecki and Schoukens, 2007).

The combination of Eq. (3.25) is suitable for the identification of the nonrotating, radial AMB rotor system. Hynynen and Jastrzebski (2009) show with simulations that this optimal set of excitations gives better results for the FRFs of a nonrotating AMB rotor system when compared with the random multisine signal of Eq. (3.22) and separate excitations in each input. When identifying a rotating rotor, a four-input, four-output system is considered and the excitation matrix is as presented in Eq. (3.26).

### 3.3 Frequency response function estimators

When using a multisine or other broadband excitation where the signal power is divided over several frequencies, the SNR is increased by averaging over multiple blocks or periods. These estimators have been studied for instance by Guillaume et al. (1992); Guillaume (1998); Wernholt and Gunnarson (2007).

Different frequency response function estimators differ in the requirements for the measurement setup and the signal-to-noise ratio (SNR). They also have different bias and variance properties. Some estimators also cause problems when applying to the closed-loop measurements.
For the closed-loop identification, a joint-input-output (JIO) estimator

\[ \hat{G}_{\text{JIO}}(j\omega_k) = \frac{1}{N_b} \sum_{i=1}^{N_b} Y^{(i)}(\omega_k) R^{(i)H}(\omega_k) \left( \frac{1}{N_b} \sum_{i=1}^{N_b} U^{(i)}(\omega_k) R^{(i)H}(\omega_k) \right)^{-1} \]  

(3.27)

gives asymptotically unbiased FRFs. \( N_b \) blocks are used for the averaging. Assume that synchronized measurements are used, which means that the blocks are single periods of a long measurement. Also assume that the same periodic excitation is applied in every block. Under these assumptions, the JIO estimator simplifies to an errors-in-variables (EIV) estimator (Wernholt and Moberg, 2007)

\[ \hat{G}_{\text{EIV}}(j\omega_k) = \left( \frac{1}{N_b} \sum_{i=1}^{N_b} Y^{(i)}(\omega_k) \right) \left( \frac{1}{N_b} \sum_{i=1}^{N_b} U^{(i)}(\omega_k) \right)^{-1} \]  

(3.28)

When the assumptions of the synchronized measurement and the same excitation signal in each block hold, the EIV estimator gives the asymptotically best linear approximation for the FRFs both for the open-loop and closed-loop measurements.

For random excitation signals, such as Gaussian noise or nonperiodic pseudorandom binary sequence (PRBS), an H1 estimator

\[ \hat{G}_{\text{H1}}(j\omega_k) = \frac{1}{N_b} \sum_{i=1}^{N_b} Y^{(i)}(\omega_k) U^{(i)(j)H}(\omega_k) \left( \frac{1}{N_b} \sum_{i=1}^{N_b} U^{(i)}(\omega_k) U^{(i)(j)H}(\omega_k) \right)^{-1} \]  

(3.29)

gives the asymptotically best linear approximation. \((\cdot)^H\) denotes complex conjugate transpose. The H1 estimator suits also for the nonsynchronous measurements with periodic excitation. A disadvantage of the H1 estimator is that it only considers disturbances in the output of the system. For noisy input measurements, it may give a large bias error. In the closed-loop measurements, the H1 estimator may give a biased estimate because the output disturbances get to the input through the feedback (Pintelon and Schoukens, 2001a).

Other suitable methods for the nonsynchronous MIMO measurements are an arithmetic mean (ARI) estimator and a logarithmic mean (LOG) estimator (Guillaume, 1998). The ARI estimator

\[ \hat{G}_{\text{ARI}}(j\omega_k) = \frac{1}{N_b} \sum_{i=1}^{N_b} \hat{G}_{\text{ERFM}}^{(i)}(j\omega_k) \]  

(3.30)
Nonparametric identification of an AMB rotor system

is sometimes called an empirical transfer function estimate, as it is obtained by averaging from Eq. (3.1) or Eq. (3.4) for MIMO systems. An advantage of the ARI estimator is that it has a smaller bias error in closed-loop measurements compared with the H1 estimator. Its disadvantage is that the estimate may deteriorate if some of the experiments have a low SNR (Wernholt and Gunnarson, 2007). The LOG estimator

\[
\hat{G}_{\text{LOG}}(j\omega_k) = \exp \left( \frac{1}{N_b} \sum_{i=1}^{N_b} \log \left( \hat{G}_{\text{ERFM}}^{(i)}(j\omega_k) \right) \right) \tag{3.31}
\]

uses nonlinear averaging. It is especially robust to outliers because the logarithmic function attenuates the influence of the occasional large errors. The classical estimators require a large number of averages in order to eliminate the effect of such errors (Wernholt and Gunnarson, 2007).

### 3.4 Detection of harmonics generated by a nonlinear system

Nonlinear systems or nonlinear disturbances influencing a linear system produce additional harmonics that degrade the measured FRF. Depending on the nonlinear system, it may produce one or more harmonics for a sine wave excitation \( r(t) = A \cos(2\pi f_{\text{ex}}t) \) according to the equation

\[
f_h = m f_{\text{ex}}, \tag{3.32}
\]

where \( m \) is a positive integer number and \( f_{\text{ex}} \) is the excitation frequency. The even harmonics are even multiples and odd harmonics are odd multiples of the frequency \( f_{\text{ex}} \). Fig. 3.7 shows the amplitude spectra of a linear, quadratic, and cubic systems. It can be seen that a linear system does not generate harmonics. The quadratic system generates harmonics at the second multiple of the base frequency and the cubic system both at the third multiple of the base frequency and at the base frequency itself. All the systems containing odd nonlinearities disturb the measurement, because they add signal power to the base frequency, similar to the cubic harmonic (Pintelon and Schoukens, 2001b).

If the system is suffering from significant disturbances at some frequencies, for example, the grid frequency 50 Hz, it may produce harmonics according to a mixer equation

\[
f_h = \pm m f_{\text{ex}} \pm nf_d \tag{3.33}
\]
3.4 Detection of harmonics generated by a nonlinear system

Fig. 3.7. Amplitude spectra of (a) a linear system, (b) a quadratic system \( y = u^2 \), and (c) a cubic system \( y = u^3 \) (Pintelon and Schoukens, 2001b).

When no significant disturbance frequencies are present, the existence of the harmonics generated (according to Eq. (3.32)) can be analyzed using a multisine excitation with the appropriately selected frequencies. One option is to use an odd-odd multisine that consists of every second odd multiples of the base frequency \( f_0 \), \( f_k = (4k + 1)f_0 \), \( k = 1, \ldots, N_f \). The even harmonics produced by a nonlinear system can be detected at the frequencies \((4k + 2)f_0, k = 1, 2, \ldots\) and odd harmonics at the frequencies \((4k + 3)f_0, k = 1, 2, \ldots\). If the system generates odd harmonics, the excited frequencies contain both the linear contribution and additional odd nonlinear distortions (odd harmonics). Different frequency combinations for the detection of the nonlinearities have been investigated in McCormack et al. (1994); Evans et al. (1994); Vanhoenacker and Schoukens (1999). The influence of the even harmonics can be eliminated by only exciting the odd multiples of the base frequency, but the disturbances caused by odd harmonics are almost impossible to avoid (Evans et al., 1994; McCormack et al., 1994; Pintelon and Schoukens, 2001b).

Often, nonlinear distortions and noise have approximately the same magnitude. McCormack et al. (1994) have proposed a coherence function method to separate the nonlinear distortions from the noise. The frequencies \((4k + 1)f_0\) are excited. The coherence function of the input and output of the system is calculated as

\[
\text{coherence} = \frac{\text{cross-spectral density}}{\text{power spectral density of the input} \times \text{power spectral density of the output}}
\]
for the nonexcited frequencies \((4k + 2)f_0\) and \((4k + 3)f_0\). The autospectra of input \(u(nT_s)\) and output \(y(nT_s)\) signals and their cross spectrum, \(S_{UU}(\omega_k)\), \(S_{YY}(\omega_k)\), and \(S_{UY}(\omega_k)\), are determined as

\[
S_{UU}(\omega_k) = \frac{1}{N_b} \sum_{i=1}^{N_b} U_i^*(\omega_k)U_i(\omega_k),
\]

\[
S_{YY}(\omega_k) = \frac{1}{N_b} \sum_{i=1}^{N_b} Y_i^*(\omega_k)Y_i(\omega_k),
\]

\[
S_{UY}(\omega_k) = \frac{1}{N_b} \sum_{i=1}^{N_b} U_i^*(\omega_k)Y_i(\omega_k).
\]

Where \(N_b\) blocks are measured and the auto and cross spectra are calculated by averaging over the blocks. For a purely linear system, the values of the coherence function equal unity at the excited frequencies. The nonexcited frequencies only consist of noise. In the presence of nonlinear distortions, the coherence function gives values less than unity at the excited frequencies. If the impact of the nonlinear distortions is significant, the coherence function gives values close to the unity at the nonexcited frequencies. If, on the other hand, the nonlinear distortions are under the noise level, the values at the nonexcited lines are low. A problem with the method is that for periodic inputs, it always gives values of unity at the excited frequencies and thus does not detect the harmonics at the nonexcited frequencies, even if the system is purely nonlinear (McCormack et al., 1994).

When using periodic excitation, the harmonics can be detected calculating the DFT of the output signal \(Y(\omega_k)\) averaged over sufficiently many blocks. The significance of the even and odd harmonics detected at the nonexcited frequencies \((4k + 2)f_0\) and \((4k + 3)f_0\) is determined by comparing their magnitudes both with the magnitudes at the excited frequencies \((4k + 1)f_0\) and with the standard deviation of the output

\[
\sigma_Y(\omega_k) = \sqrt{\frac{1}{N_b-1} \sum_{i=1}^{N_b} (Y_i(\omega_k) - Y(\omega_k))^2}
\]

where \(Y_i(\omega_k)\) is the DFT of the output of one experiment and \(Y(\omega_k)\) is the mean value of the output averaged over \(N_b\) measurements.
3.5 Conclusions

In this chapter, special issues concerning AMB rotor system identification have been addressed. An AMB rotor system is a nonlinear, unstable MIMO system. The identification of a MIMO system, identification in a closed loop, and the use of frequency response functions for a nonlinear system have all been discussed.

Selection of the excitation signal has been considered. A few broadband excitation signals have been discussed; a random phase multisine, periodic random excitation, and pseudorandom binary sequence. For comparison, also stepped sine and swept sine excitations have been studied. The times required for the frequency response function measurements using a stepped sine excitation and a random phase multisine excitation have been compared, and it can be seen that the measurements are faster when using multisine excitation. Only in the case of a very low signal-to-noise ratio, the measurement times are the same with both excitations. When considering the identification of a MIMO system, the combination of the excitation signals in each input has also been discussed.

Because a multisine signal contains \( N_f \) frequencies, the signal power is divided over all those frequencies thereby degrading the signal-to-noise ratio of the measurement. The SNR is improved by measuring several periods and estimating the FRFs by averaging over those periods. Different frequency response estimators suitable for MIMO systems have been presented.

Nonlinear systems produce additional harmonics that may degrade the measured frequency response functions. The multisine excitation provides an option to analyze the harmonics of the system when the frequencies of the signal are chosen appropriately. By using the odd-odd multisine, both the even and odd harmonics can be analyzed from the system output at the nonexcited frequencies, and their influence on the frequency response functions determined by combining them to the output at the excited frequencies, and to the standard deviation of the output.

As a conclusion, it can be stated that the multisine excitation enables faster FRF measurements than the stepped sine excitation when the SNR of the measurements is adequate so that there is no need for a large number of periods for averaging. Because a multisine signal contains several frequencies, a nonlinear system may produce additional harmonics that degrade the linear approximation of the FRF. The frequencies of the excitation signal can be selected so that the influence of the even harmonics is eliminated in the FRF, but the elimination of the effect of the possible odd harmonics is difficult. A means to reduce the influence of the odd harmonics is to use a lower amplitude for the excitation signal or to design several multisine excitations.
Nonparametric identification of an AMB rotor system
This chapter deals with constructing a parametric model from a nonparametric frequency response function model. The chapter starts with a short survey of the existing methods. Then, different parametric models are presented and a least-squares method for a common denominator model is explained. The choice of the weighting functions used to improve the quality of the parametrized model are also considered. The unstable rigid modes of a current-controlled AMB rotor system bring challenges to the parametrization of the nonparametric model, which is also discussed. Additionally, the model updating is addressed.

4.1 From a nonparametric to a parametric model

Chapter 3 focused on the nonparametric frequency domain methods for identification. They result in frequency response functions that are very accurate system models. However, for the control design, a parametric model is often required. A parametric model can be a transfer function model where the parameters are the coefficients of the nominator and denominator polynomials, or a state space model where the coefficients of the state matrices are defined according to the modal characteristics (eigenfrequencies, dampings, and mode shapes) of the structural dynamics of the system.

In this chapter, a short survey of making a parametric model is first provided in Section 4.2. Then, different parametric models are discussed in Section 4.3. Section 4.4 presents a least-squares approach to the parametrization of the FRFs using common-denominator models. Finally, obtaining a parametric model for the AMB rotor systems is considered in Section 4.5.
4.2 Overview of the identification of structural dynamics

Methods to describe reliable identification of structural dynamics using modal parameters have been developed for about the last 50 years. At the beginning, single-degree-of-freedom (SDOF) methods were used. The development of the FFT, multichannel measurements, and efficient storing and computing capacity have made the development of the methods easier, and nowadays the MIMO measurements and multiple-degrees-of-freedom (MDOF) approach are used.

The first SDOF methods assumed that the modes were real and that it was possible to describe dynamical behavior in a small frequency range using one single mode. In a peak amplitude method, the natural frequencies of the system are first determined according to the individual resonance peaks of the FRF. Then, the dampings of each mode are determined according to the sharpness of the resonance peaks. The mode shapes can be determined from the ratios of the peak amplitudes at the response locations. Another SDOF method, a circle fitting method was proposed by Kennedy and Pancu (1947). The SDOF methods often lead to erroneous results especially for the dampings because the methods deal with only one mode at a time and thus assume the modes to be decoupled. However, if the flexible frequencies are close to each other, they are very strongly coupled. Other disadvantages of the SDOF methods are that they are sensitive to noise and require a lot of attention of the operator (Verboven, 2002).

Since then, the SDOF methods have evolved first into time-domain MDOF methods and further into frequency-domain methods. In this study, only frequency-domain methods are discussed. A survey of the time-domain methods can be found, for example, in Verboven (2002). A least-squares frequency domain (LSFD) method directly solves parameters for the modal model that is non-linear-in-the-parameters, and thus, a nonlinear least-squares problem is to be solved using an optimization algorithm. A frequency domain eigensystem realization algorithm (ERA) forms a complex block matrix from the FRF data and estimates the modal parameters from that using a singular value decomposition (SVD) and eigenvalue decomposition (EVD) (Juang and Suzuki, 1988). A direct parameter identification algorithm (FDPI) identifies a low-order state space model from the FRF data. The poles and mode shapes can be further determined from that (Lembregts, 1988). A complex-mode indicator function (CMIF) uses the SVD of the FRFs. The modal parameters are determined according to the singular value plot (Phillips et al., 1998).

Several frequency-domain estimators are based on the least-squares (LS) minimization that uses a polynomial transfer function model. The principle of the estimators is to choose the polynomial coefficients so that an error between the measured FRFs and parametric transfer function model is minimized. For noisy data, maximum likelihood (ML) methods provide more accurate estimates (Schoukens and Pintelon, 1991; Guillaume, 1992). In the ML methods, the variance of the data is included in the minimized cost function, which thus increases the calculation time and effort.
4.3 Parametric models

In this chapter, models to describe vibrations of the structural mechanics are presented. A modal model gives the most transparent physical understanding of the system, but the model is highly nonlinear-in-the-parameters. That is why many parametrization methods use matrix fraction description (MFD) models and state space models. The modal model is further transformed from the MFD or state space model.

4.3.1 Matrix fraction model

For a rotating system, a transfer function matrix is obtained from the equation of motion of Eq. (2.49) or for an AMB rotor system with the rotor and the electromagnets from Eq. (2.54). In this study, a nonrotating rotor is considered, and thus the gyroscopic effect is ignored ($\Omega \mathbf{G} = 0$). The combined equation of motion in the Laplace domain is

$$[\mathbf{M}s^2 + \mathbf{D}s + (\mathbf{K} + \mathbf{K}_s)]\mathbf{Q}(s) = \mathbf{Ψ}^T\mathbf{I}_m(s).$$

(4.1)

$\mathbf{Q}(s)$ and $\mathbf{I}_m(s)$ are the displacement and magnetizing current vectors, respectively. The matrix $\mathbf{Φ}_s$ contains the locations of the sensors and transformation of the modal parameters into physical coordinates. The matrix $\mathbf{Ψ}^T = \mathbf{Κ}_s$ consists of the force-current factors and locations of the magnetic actuators. The matrices can be further written as

$$\mathbf{Φ}_s = \begin{bmatrix}
\mathbf{ϕ}_{s,r=1} & \mathbf{ϕ}_{s,r=2} & \mathbf{ϕ}_{s,f=1} & \cdots & \mathbf{ϕ}_{s,f=n}
\end{bmatrix},$$

(4.2)

$$\mathbf{Ψ} = \begin{bmatrix}
\mathbf{ψ}_{r=1} & \mathbf{ψ}_{r=2} & \mathbf{ψ}_{f=1} & \cdots & \mathbf{ψ}_{f=n}
\end{bmatrix}.$$ 

(4.3)

In AMBs, the sensors are usually not in the same locations as the magnetic actuators, and thus, $\mathbf{Φ}_s \neq \mathbf{Ψ}$ (Gähler et al., 1997). The transfer function matrix of an AMB rotor system from the magnetizing currents to the measured rotor displacements is written as

$$\mathbf{G}_r(s) = \mathbf{Φ}_s^{-1}[\mathbf{M}s^2 + \mathbf{D}s + (\mathbf{Κ} + \mathbf{Κ}_s)]^{-1}\mathbf{Ψ}^T.$$ 

(4.4)

The transfer function matrix of the overall system from the control currents to the measured rotor displacements also contain the dynamics of the power amplifier of Eq. (2.21) and is obtained as follows
\[ G(s) = G_r(s)G_{cc}(s). \]  

(4.5)

The transfer function matrices can be expressed using matrix fraction descriptions as presented by Kailath (1980). There are two kinds of MFDs, the left MFD

\[ G(s) = A(s)^{-1}B(s), \]  

(4.6)

where \( A(s) \) is a matrix polynomial of the size \( N_y \times N_y \) and \( B(s) \) is a matrix polynomial of the size \( N_y \times N_u \), and the right MFD,

\[ G(s) = B(s)A(s)^{-1}. \]  

(4.7)

A special case of the MFDs is a common-denominator model (CDM), which is also called a scalar matrix fraction model. A common-denominator model can be expressed as

\[
G(s) = \begin{bmatrix}
B_{1,1}(s) & \cdots & B_{1,N_u}(s) \\
\vdots & \ddots & \vdots \\
B_{N_y,1}(s) & \cdots & B_{N_y,N_u}(s)
\end{bmatrix}
\]

\[ A(s), \]  

(4.8)

where the denominator polynomial \( A(s) \) is common for all the input-output relations (Verboven, 2002; Cauberghe, 2004).

### 4.3.2 Modal model

According to Gähler et al. (1997), a modal model of an AMB rotor system can be written as a sum of the second-order systems

\[
G(s) = \sum_{r=1}^{2} \frac{R_r}{s^2 + 2d_r s - p_r^2} + \sum_{f=1}^{n} \frac{R_f}{s^2 + 2\zeta_f \omega_0 f s + \omega_0^2 f},
\]  

(4.9)

where \( p_r \) and \( d_r \) are the poles and dampings of the rigid body modes, and \( \zeta_f \) and \( \omega_0 f \) are the damping ratios and natural frequencies of the flexible modes, respectively. The dampings of the rigid body modes are small, and here they are assumed zero. The subscripts \( r \) and \( f \) refer to the rigid and flexible modes. When mass-normalized coordinates are used, the modal mass matrix is considered as a unity matrix, \( M = I \). The modal matrices of stiffness \( K + K_s \) and
damping $\mathbf{D}$ are obtained from the denominators of the modal model of Eq. (4.9) as follows

$$
\mathbf{K} + \mathbf{K}_s = \text{diag}\left(\begin{bmatrix}
-p_1^2 & -p_2^2 & \omega_{01}^2 & \ldots & \omega_{0n}^2
\end{bmatrix}\right),
$$

(4.10)

$$
\mathbf{D} = 2 \text{diag}\left(\begin{bmatrix}
d_1 & d_2 & \zeta_1 \omega_{01} & \ldots & \zeta_n \omega_{0n}
\end{bmatrix}\right).
$$

(4.11)

In the stiffness matrix $\mathbf{K} + \mathbf{K}_s$, the position stiffness matrix $\mathbf{K}_s$ is constructed from the rigid modes and the matrix $\mathbf{K}$ from the flexible modes. The residual matrices $\mathbf{R}_r$ and $\mathbf{R}_f$ are determined as dyadic products,

$$
\mathbf{R}_r = \phi_{s,r} \psi_{r}^T,
$$

(4.12)

$$
\mathbf{R}_f = \phi_{s,f} \psi_{f}^T.
$$

(4.13)

The dynamics of the power amplifier of Eq. (2.21) is added to the modal model of Eq. (4.9) as follows

$$
\mathbf{G}(s) = \left(\sum_{r=1}^{2} \frac{\mathbf{R}_r}{s^2 + 2d_r s - p_r^2} + \sum_{f=1}^{n} \frac{\mathbf{R}_f}{s^2 + 2\zeta_f \omega_{0f} s + \omega_{0f}^2}\right) \cdot \mathbf{G}_{cc}(s).
$$

(4.14)

### 4.3.3 State space model

The state space model is commonly written as

$$
\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu},
$$

(4.15)

$$
\mathbf{y} = \mathbf{Cx} + \mathbf{Du}.
$$

(4.16)

The conversion of the state space model into a transfer function model is

$$
\mathbf{G}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D}.
$$

(4.17)

Now, from the transfer function matrix of the rotor with the magnetic actuators of Eq. (4.4), the system matrices of the state equation of Eq. (2.58) are obtained as follows

$$
\mathbf{A}_r = \begin{bmatrix} 0 & \mathbf{1} \\ -\left(\mathbf{K} + \mathbf{K}_s\right) & -\mathbf{D} \end{bmatrix},
\mathbf{B}_r = \begin{bmatrix} 0 \\ \mathbf{\psi}^T \end{bmatrix},
\mathbf{C}_r = \begin{bmatrix} \mathbf{\Phi}_s & 0 \end{bmatrix},
\mathbf{D}_r = 0.
$$

(4.18)
The submatrices of the state matrix $A_r$ are obtained from Eqs. (4.10) and (4.11) and the submatrices of $B_r$ and $C_r$ according to Eqs. (4.2), (4.3), (4.12), and (4.13) (Gähler et al., 1997).

The system matrices for the state space model of the power amplifier are obtained directly from the pole of the power amplifier as presented in Eqs. (2.64)–(2.66).

### 4.4 Least-squares approach

The least-squares approach for the parametrization of the FRF data presented in this section is based on the studies of Verboven (2002) and Parloo (2003).

The basic idea in the cost-function-based methods is to estimate the parameters of the transfer function $\hat{G}_i(j\omega_k, \theta)$ such that it equals the initial nonparametric FRF $G_i(j\omega_k)$ as well as possible. Thus, an error $E_i(j\omega_k)$

$$E_i(j\omega_k) = \hat{G}_i(j\omega_k, \theta) - G_i(j\omega_k) = B_i(j\omega_k, \theta) - G_i(j\omega_k) \approx 0 \quad (4.19)$$

is to be minimized. Now, the common-denominator model of Eq. (4.8) is described using the frequency response functions with the nominator and denominator polynomials $B_i$ and $A_i$ written as

$$B_i(j\omega_k, \theta) = \sum_{j=0}^{2N_m} b_{ij}\omega_k^j, \quad (4.20)$$

$$A_i(j\omega_k, \theta) = \sum_{j=0}^{2N_m} a_{ij}\omega_k^j, \quad (4.21)$$

The coefficients $b_{ij}$ and $a_{ij}$ are the estimated, unknown parameters $\theta$. $k$ is a frequency sample index, $i$ is the number of the factors of the frequency response function matrix $i=1,2,\ldots,N_yN_u$, and $N_m$ is the number of the flexible modes (Verboven, 2002).

The solution of the error equation (4.19) is nonlinear in the parameters, but the linear least-squares approach requires model equations that are linear in the parameters. Levy (1959) presented a linear approximation for the problem. The method was first presented for a SISO system, but it was later extended to MIMO systems and is nowadays an often used approximation. A linear-in-the-parameters error equation is obtained from (4.19) by multiplying by the denominator polynomial $A_i(j\omega_k, \theta)$ as follows
4.4 Least-squares approach

\[ E_{i}^{LS}(j\omega_k) = B_i(j\omega_k, \theta) - A(j\omega_k, \theta)G_i(j\omega_k). \] (4.22)

A problem in estimating the parameters from Eq. (4.22) is that it overemphasizes the high-frequency measurements. A well-chosen weighting function \( W_i(j\omega_k) \) can be used to improve the quality of the estimate, and Eq. (4.22) is thus rewritten as

\[ E_{i}^{LS}(j\omega_k) = W_i(j\omega_k)(B_i(j\omega_k, \theta) - A(j\omega_k, \theta)G_i(j\omega_k)). \] (4.23)

The choice of the weighting function \( W_i(j\omega_k) \) is discussed in Section 4.4.3. Now, a weighted linear least-squares cost function \( \ell_{LS}(\theta) \) can be written as

\[ \ell_{LS}(\theta) = \sum_{i=1}^{N_yN_u} \sum_{k=1}^{N_f} |E_{i}^{LS}(j\omega_k)|^2. \] (4.24)

4.4.1 LS formulation based on a Jacobian matrix

The parameter estimates are found by minimizing the cost function of Eq. (4.24), that is, solving the equation

\[ E_{i}^{LS}(j\omega_k) = J\theta = 0 \] (4.25)

where \( J \) is a Jacobian matrix and \( \theta \) is a parameter vector containing the parameters of the nominator and denominator coefficients. When a common denominator model is used and Eqs. (4.23) and (4.25) are linear-in-the-parameters, the error equation can be reformulated as

\[ E_{i}^{LS}(j\omega_k) = \begin{bmatrix} \Gamma_1 & 0 & \cdots & 0 & \Phi_1 \\ 0 & \Gamma_2 & \cdots & 0 & \Phi_2 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \Gamma_{N_yN_u} & \Phi_{N_yN_u} \end{bmatrix} \begin{bmatrix} \theta_{B_1} \\ \theta_{B_2} \\ \vdots \\ \theta_{B_{N_yN_u}} \\ \theta_{A} \end{bmatrix} = 0 \] (4.26)

with the parameter vectors of the denominator \( \theta_{A} \) and the nominators \( \theta_{B_i} \) written as
\[ \boldsymbol{\theta}_A = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix}, \quad \boldsymbol{\theta}_{B_i} = \begin{bmatrix} b_{i0} \\ b_{i1} \\ \vdots \\ b_{i(m-1)} \\ b_{im} \end{bmatrix} \] (4.27)

The submatrices \( \boldsymbol{\Gamma}_i \) and \( \boldsymbol{\Phi}_i \) are determined as

\[ \boldsymbol{\Gamma}_i = \begin{bmatrix} \Gamma_i(j\omega_1) \\ \Gamma_i(j\omega_2) \\ \vdots \\ \Gamma_i(j\omega_{N_f}) \end{bmatrix}, \quad \boldsymbol{\Phi}_i = \begin{bmatrix} \Phi_i(\omega_1) \\ \Phi_i(\omega_2) \\ \vdots \\ \Phi_i(j\omega_{N_f}) \end{bmatrix} \] (4.28)

with \( \Gamma_i(j\omega_k) \) and \( \Phi_i(j\omega_k) \) written as

\[ \Gamma_i(j\omega_k) = \mathbf{W}_i(j\omega_k) \left[ (j\omega_k)^0 \quad (j\omega_k)^1 \quad \ldots \quad (j\omega_k)^{n-1} \quad (j\omega_k)^{2N_m} \right] , \] (4.29)

\[ \Phi_i(j\omega_k) = -\Gamma_i(j\omega_k) \mathbf{G}_i(j\omega_k) . \] (4.30)

If the parameters are determined directly using the error equations of Eqs. (4.25) or (4.26), a significant calculation effort is required while the Jacobian matrix has \( N_f N_u N_y \) rows and \( (n+1)(N_u N_y + 2N_y N_m + 1) \) columns with the number of the frequencies \( N_f \) much higher than the order of the system \( 2N_m \) \((N_f \gg 2N_m)\).

### 4.4.2 LS formulation based on a normal matrix

The modal parameter estimation problems are often solved using ‘normal equations’ instead of the Jacobian matrix approach in order to reduce the sizes of the matrices and the calculation time. The least squares cost function of Eq. (4.24) can be written as

\[ \ell_{LS}(\boldsymbol{\theta}) = \boldsymbol{\theta}^T (\mathbf{J}^T \mathbf{J}) \boldsymbol{\theta} \] (4.31)

where the square of the Jacobian matrix is determined as
4.4 Least-squares approach

\[ J^H J = \begin{bmatrix} \Gamma_1^H \Gamma_1 & 0 & \ldots & \Gamma_1^H \Phi_1 \\ 0 & \Gamma_2^H \Gamma_2 & \ldots & \Gamma_2^H \Phi_2 \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_1^H \Gamma_1 & \Phi_2^H \Gamma_2 & \ldots & \sum_{i=1}^{N_u N_y} \Phi_i^H \Phi_i \end{bmatrix}. \]  

(4.32)

Now, when the submatrices of the squared Jacobian matrix of Eq. (4.32) are defined as

\[ R_i = \Gamma_i^H \Gamma_i, \quad S_i = \Gamma_i^H \Phi_i, \quad T_i = \Phi_i^H \Phi_i, \]  

(4.33)

the normal equation can be written as

\[ \begin{bmatrix} R_1 & 0 & \ldots & S_1 \\ 0 & R_2 & \ldots & S_2 \\ \vdots & \vdots & \ddots & \vdots \\ S_1^H & S_2^H & \ldots & \sum_{i=1}^{N_u N_y} T_i \end{bmatrix} \begin{bmatrix} \theta_{Bi} \\ \theta_{B2} \\ \vdots \\ \theta_{B N_u N_y} \\ \theta_A \end{bmatrix} \approx 0. \]  

(4.34)

It can be seen that the size of the normal matrix of Eq. (4.34) is \((2N_m + 1)(N_u N_y + 1)\) and the number of frequencies \(N_f\) are eliminated when compared with the size of the Jacobian matrix in Eq. (4.26). This leads to a reduction in the computation time and memory requirements.

Now, we minimize the cost function of Eq. (4.34) with respect to the unknown parameters \(\theta_{Bi}\) and \(\theta_A\) as follows

\[ \frac{\partial \ell_{LS}(\theta)}{\partial \theta_{Bi}} = -2 (R_i \theta_{Bi} + S_i \theta_A), \quad i = 1, \ldots, N_u N_y, \]  

(4.35)

\[ \frac{\partial \ell_{LS}(\theta)}{\partial \theta_A} = 2 \left[ \sum_{i=1}^{N_u N_y} (S_i^T \theta_{Bi} + T_i \theta_A) \right] = 0. \]  

(4.36)

The coefficients of the nominator polynomials \(\theta_{Bi} = -R_i^{-1} S_i \theta_A\) can be solved from Eq. (4.35) and substituted into Eq. (4.37) as follows,

\[ \sum_{i=1}^{N_u N_y} (T_i - S_i^T R_i^{-1} S_i) \theta_A = 0. \]  

(4.37)
An LS solution for the coefficients of the denominator polynomial $\Theta_A$ is obtained, for example, by fixing the coefficient of the highest order $a_{N_m}$ to 1 and determining the remaining coefficients according to Eq. (4.37) as follows

$$\Theta_A = \left[-D(1 : N_m, 1 : N_m)^{-1} D(1 : N_m, N_m + 1)\right]. \tag{4.38}$$

### 4.4.3 Choice of a weighting function

The weighting function $W_i(j\omega_k)$ is used to improve the quality of the least squares estimate and to convert the initial nonlinear least squares (NLS) problem to a linear LS problem. The principal idea of using a weighting function was presented by Sanathanan and Koerner (1963). They introduced a weighting function

$$W_{SK,i}^2(j\omega_k) = \frac{1}{|A(j\omega_k, \theta_{m-1})|^2} \tag{4.39}$$

where $A(j\omega_k, \theta_{m-1})$ is a denominator coefficient of the previous estimation. The proposed weighting function leads to an iterative LS problem

$$\ell_{LS}(\theta) = \sum_{i=1}^{N_r} \sum_{k=1}^{N_f} \frac{|B_i(j\omega_k, \theta_m) - A(j\omega_k, \theta_m)G_i(j\omega_k)|^2}{|A(j\omega_k, \theta_{m-1})|^2} \tag{4.40}$$

which has an absolute error (Sanathanan and Koerner, 1963). When considering an AMB rotor system, the current controller causes a roll-off of 40 dB per decade. Thus, minimizing an absolute error leads to poor results at high frequencies (Gähler et al., 1997). For such systems, minimizing a relative error will lead to better estimates. The weighting function with a relative error is written as (Strobel, 1966)

$$W_{SR,i}^2(j\omega_k) = \frac{1}{|A(j\omega_k, \theta_{m-1})G_i(j\omega_k)|^2} \tag{4.41}$$

leading to a cost function

$$\ell_{LS}(\theta) = \sum_{i=1}^{N_r} \sum_{k=1}^{N_f} \frac{|B_i(j\omega_k, \theta_m) - A(j\omega_k, \theta_m)G_i(j\omega_k)|^2}{|A(j\omega_k, \theta_{m-1})G_i(j\omega_k)|^2}. \tag{4.42}$$

A survey of different weighting functions is presented in (Pintelon et al., 1994).
4.5 Parametrization of an AMB rotor system

The aforementioned methods for obtaining a parametric model from the FRF data are broadly used in the identification of the structural dynamics. The dynamics of the structural dynamics usually consists of a large number of flexible modes. Thus, their poles and zeros are complex conjugate pairs. The flexible modes of an AMB rotor system can also be described using such a system model. The dynamics of an AMB rotor system does not only contain the mechanics, but also the actuators with the current controllers. They cause the system to have real poles, and the modal model of an AMB rotor system is described using Eq. (4.9). The real poles are hard to estimate, and the methods designed for the structural dynamics fail in describing them (Gähler et al., 1997).

Gähler et al. (1997) invented an algorithm to obtain a parametric model for an AMB rotor system from the FRF data. The method first finds the poles of the system while for a MIMO system all the single FRFs have the same poles. Then, the zeros are estimated for each FRF considering them as separate SISO systems with common poles. The method uses a fictitious proportional gain to move the real poles first to the origin and further close to the imaginary axis. Ahn et al. (2003a) improved the method to consider the system as a MIMO using an MFD model. Fictitious gains were not needed, but the rigid and flexible modes were identified separately. This is possible when the flexible modes of the system are far from the rigid ones.

In this study, the rigid and flexible modes are treated separately. The method of Gähler et al. (1997) fails with the studied system, because the use of the fictitious gain damps the lowest flexible frequencies. According to the modal model of an AMB rotor system, Eq. (4.9), and assuming the dampings of the rigid body modes zero, the parametric model of the rigid body modes is written as

$$G_{\text{rigid}}(s) = \frac{(s^2 - z_1^2)}{(s^2 - p_1^2)(s^2 - p_2^2)}.$$  \hspace{1cm} (4.43)

A denominator of a transfer function of such a system has five parameters and the nominator has three parameters with the even number of parameters as zero. Thus, the above-mentioned LS method is applied to the rigid body modes by setting the even rows and columns in the matrices of Eqs. (4.32)–(4.34) zero.

The dynamics of the power amplifier cannot be identified from the FRF data of the overall system but it is identified separately.
4.6 Model updating

When modeling the system analytically, such assumptions has to be made in which the real system does not necessarily adjust. The model updating means that the possibly erroneous assumptions can be corrected using experimental, measured data (Friswell and Mottershead, 1995).

According to Mottershead and Friswell (1993), the erroneous assumptions can be divided into three types:

1. Structural errors, where the physical equation used cannot explain the behavior of the real system. Possible unmodeled dynamics may cause a structural error, and, for example, nonlinearities of a nonlinear system when trying to describe it by using linear equations.

2. Parametric errors may result from an erroneous use of the boundary values or erroneous assumptions when simplifying the model.

3. Errors in the model order originate typically from the discretizing of a complex system and lead to a deficient model order. The model order errors can be categorized as structural errors.

If it is assumed that the model structure is correct but there are errors in some parameters, the parameters are updated using the system identification. This method has been used to update the FEM model that lacks the damping terms by using the modal analysis data. If the parameters of the analytic model are assumed to be correct but the model is completely missing certain dynamics, a model updating should be used.

The model updating methods can be divided into direct and indirect methods. The direct methods lead to black box models, which are not physically meaningful. If a physically meaningful model is required, indirect updating methods should be used. Different methods for model updating can be found, for example, in Friswell and Mottershead (1995).

When considering an AMB rotor system, the indirect updating methods are used because the model is often used for the controller design, and a physically meaningful state space model is required. The model of the rotordynamics is assumed to be precise, but there may be some unmodeled dynamics influencing the system. The structural unmodeled dynamics, for example, the dynamics of the foundation and couplings between the radial and axial bearings, can be updated by enlargening of the model order, because they can be modeled as additional flexible modes.

Maslen et al. (2002), Vázquez et al. (2003), and Wang and Maslen (2006) have studied the model updating of an AMB rotor system. They suppose that the analytical model of the system has correct parameters, but the real system has some dynamics that is difficult or impossible to model analytically. The method keeps the state model of the analytical model
unchanged and adds the unmodeled dynamics as an additional disturbances influencing cer-
tain assumed locations. The parameters of the unmodeled dynamics are computed so that an
error between the updated analytical model and the identified plant is minimized. A limita-
tion of the method is that the disturbance sources must be known in order to determine the
locations where they enter the system (Maslen et al., 2002).

In this study, the purpose is to update only the structural unmodeled dynamics that can be
described as additional flexible modes, such as the dynamics of the foundation. This is done
simply by enlarging the order of the analytical model.

4.7 Conclusions

In this chapter, the methods to obtain a parametric model from the nonparametric frequency
response functions and model updating have been considered.

In the structural mechanics, the construction of the parametric model from the measured data
of an experimental modal analysis is widely investigated in the literature. These methods are
applicable to the flexible modes of active magnetic bearings. In this chapter, a short survey of
making the parametric model, starting from the simplest SISO methods and ending to MIMO
methods based on the least-squares and maximum likelihood estimation has been presented.

In this study, a linear least-squares approach to obtain a parametric common-denominator
model has been chosen and the method presented. The transformation of the polynomial
common-denominator model into a modal model and further into a state space model is pre-

tented.

As mentioned above, the common methods of the structural mechanics are suitable for the
flexible modes of the AMB rotor system. For the unstable rigid body modes of the AMB rotor
system, however, the parametric model is more challenging to obtain. In this study, the linear
least-squares method with two different weighting functions has been applied separately to
the rigid and flexible modes.

An analytical model of the system may contain structural or parametric errors because of
unmodeled dynamics or erroneous assumptions. The errors can be corrected according to
identified model using model or parametric updating. In this study, the purpose is to use
the model updating for the structural unmodeled dynamics that can be modeled as additional
flexible modes.
Parametric model and model updating
Chapter 5

Experimental results

In this chapter, the experimental results based on the theory presented in the previous chapters are provided and discussed. First, the test arrangements for identification of the system described in Section 2.1 are presented. Then, stepped sine and multisine excitation signals for the identification are designed and the harmonics of the AMB rotor system are analyzed. Comparisons of the FRFs calculated using different FRF estimators and different sets of excitations in each input with random phase multisine excitation, and comparison of the frequency response functions measured using both the stepped sine and multisine excitations are presented. Furthermore, a parametric model is composed from the nonparametric FRF data.

5.1 Description of the test arrangements

Frequency response function measurements are performed for a nonrotating rotor and assuming an axisymmetric rotor when the $x$ and $y$ planes can be considered as two equal 2-DOF systems. In this study, the identification is done in the $x$ plane. The excitation signals $r(nT_s)$ are fed to the control currents $i_c(nT_s)$ of both bearings in the $x$ direction. The measured input signals of the system are the control currents added by the excitation signals, $u(nT_s) = i_c(nT_s) + r(nT_s)$, and the measured outputs $y(nT_s)$ are the displacements of the rotor both in $A$ and $B$ bearings. The excitation signals are also measured separately. A block diagram of the test arrangements can be seen in Fig. 5.1. The experimental setup is described in more detail in Section 2.1 and the dimensions of the prototype are listed in Appendix A.

The excitation signals are generated using a dSpace platform that enables the generation of both the stepped sine and random phase multisine excitations. The stepped sine excitation is performed using a Python script under the dSpace that gives command of the new frequency of the sine excitation, and measures and saves the input, output, and reference signals. The
random multisine excitation signals are calculated in advance and implemented in Simulink using look-up tables. For the memory restrictions of the dSpace, the base frequency in the multisine excitation cannot be lower than 2 Hz.

5.2 Design of excitation signals and harmonics analysis

In the above sections, different broadband excitations and their design have been discussed. In this study, a multisine excitation is chosen because it provides an option to arbitrarily choose both the amplitudes and the frequencies. In the studied AMB rotor system, the current controller has a 40 dB roll-off per decade, which causes significant attenuation at the higher frequencies. Thus, it is advantageous if a higher amplitude of the excitation can be chosen at the frequencies with a higher attenuation. For the periodic random excitation, for example, it is not possible to choose arbitrary amplitudes. Additionally, because of the random amplitudes, more averages are required when compared with the multisine excitation. The PRBS would provide an ideal spectrum for the excitation; however, it is also not possible to change the amplitude. The PRBS excites additional frequencies in addition to the desired frequencies, which makes the analysis of the harmonics impossible. In this study, stepped sine excitation is also used and its results are compared with the results obtained using multisine excitation.

5.2.1 Design of the stepped sine excitation

The amplitude of the excitation signal is limited both by the linearity region of the rotor and the current slew rate of the magnetic actuator. Fig. 5.2 shows the force-displacement...
5.2 Design of excitation signals and harmonics analysis

characteristics of a radial magnetic bearing with the bias current \( i_b = 2.5 \) A determined using Eq. (2.15). It can be seen that the displacement of 0.15 mm of the rotor still keeps the operation in the linear region. However, the magnetic center of the system is far from the geometric center, and the allowable displacement of the rotor is only 0.035 mm.

The current slew rate of an AMB system is determined as

\[
\frac{di}{dt} = \frac{2s_0}{\mu_0 N_c A_a} [u_{dc} - (i_b + i_{ex} + i_c)(R_c + R_{IGBT})],
\]

where \( s_0 \) is the nominal air gap, \( N_c \) is the number of coil windings, \( A_a \) is the projected area of the pole face, \( u_{dc} \) is the dc link voltage, \( i_b \) is the bias current, \( i_{ex} \) is the amplitude of the excitation current, \( i_c \) is the control current from the position controller, and \( R_c \) and \( R_{IGBT} \) are the resistances of the coils and IGBTs, respectively (Ahn et al., 2003a). When the highest frequency of interest is 1200 Hz, the limiting factor of the amplitude of the excitation current is not the current slew rate but the linearity region of the rotor.

The amplitudes of the stepped sine excitation are chosen according to the magnitudes of the frequency response functions of the system so that the amplitude increases when the magnitudes of the FRFs decrease. Fig. 5.3 shows the magnitudes of the FRFs of the system measured from the control currents \( i_{c,Ax} \) and \( i_{c,Bx} \) to the rotor displacements \( x_A \) and \( x_B \) and Fig. 5.4 shows the frequency content of the stepped sine signal. The amplitudes can also be seen in Table 5.1. It must be noticed that in the FRF measurements with the stepped sine excitation, the frequencies are applied one at a time.
Fig. 5.3. Magnitudes of the frequency response functions of the studied AMB rotor system. $G_{AA}$ is the FRF from the control current $i_{c,Ax}$ to the displacement $x_A$. $G_{AB}$ is the FRF from $i_{c,Ax}$ to $x_B$. $G_{BA}$ is the FRF from $i_{c,Bx}$ to $x_A$, and $G_{BB}$ from $i_{c,Bx}$ to $x_B$. 
5.2 Design of excitation signals and harmonics analysis

Fig. 5.4. Frequency content of the stepped sine excitation signal. The frequencies are applied one at a time.
Table 5.1. Amplitudes of the stepped sine excitation at each frequency.

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>2–98 Hz</td>
<td>50 mA</td>
</tr>
<tr>
<td>102–166 Hz</td>
<td>300 mA</td>
</tr>
<tr>
<td>170–186 Hz</td>
<td>50 mA</td>
</tr>
<tr>
<td>190–250 Hz</td>
<td>300 mA</td>
</tr>
<tr>
<td>254–278 Hz</td>
<td>50 mA</td>
</tr>
<tr>
<td>282–510 Hz</td>
<td>300 mA</td>
</tr>
<tr>
<td>514–558 Hz</td>
<td>50 mA</td>
</tr>
<tr>
<td>562–734 Hz</td>
<td>300 mA</td>
</tr>
<tr>
<td>746–878 Hz</td>
<td>500 mA</td>
</tr>
<tr>
<td>882–898 Hz</td>
<td>500 mA</td>
</tr>
<tr>
<td>902–930 Hz</td>
<td>100 mA</td>
</tr>
<tr>
<td>934–970 Hz</td>
<td>50 mA</td>
</tr>
<tr>
<td>974–1190 Hz</td>
<td>500 mA</td>
</tr>
</tbody>
</table>

The frequency range of interest is 2–1200 Hz. The minimum frequency 2 Hz is determined according to the constraints that the dSpace platform sets to the base frequency \( f_0 \) of the multisine signal. The minimum resolution is 4 Hz which is explained further when designing the multisine excitation signal. The minimum frequency resolution 4 Hz is used at the low frequencies and in the vicinity of the poles and zeros of the system, that is, in the frequency ranges 2–330, 482–578, and 794–974 Hz. In the frequency range 330–482 Hz, the frequency resolution is 8 Hz and the resolution 12 Hz is used in the frequency ranges 578–794 and 974–1190 Hz.

5.2.2 Harmonics analysis

For the design of the multisine excitation signal, the possible harmonics produced by the system are analyzed. The harmonics analysis is performed using the stepped sine excitation designed in Section 5.2.1. The same combination of excitations in both system inputs is used as for the orthogonal random multisine (Eq. (3.25)) proposed for the multisine excitation. The harmonics analysis is performed for the displacements of the rotor in end B when the excitation \( \mathbf{r}(t) = [1 \quad -1]^T \mathbf{r}(t) \) is used. The same observations can be made from both ends and with both excitations \( \mathbf{r}(t) = [1 \quad 1]^T \mathbf{r}(t) \) and \( \mathbf{r}(t) = [1 \quad -1]^T \mathbf{r}(t) \).

Fig. 5.5 shows the DFTs of the displacements of the rotor in end B when the frequencies 2–50 Hz with the resolution of 4 Hz are exited. The displacements are analyzed with the frequency resolution of 2 Hz in order to display the possible even harmonics produced by the system. It can be seen in Fig. 5.5 that only the frequencies 30 and 50 Hz produce even harmonics \( 2f_{ex} \). The odd harmonics \( 3f_{ex} \), however, are produced by nearly all the considered frequencies.
The spectrum of the displacement $x_B$ when the frequencies 2–50 Hz are excited one at a time.
In Fig. 5.6, the same examination for the displacement of the rotor in end B can be seen when the frequencies 150–300 Hz with the resolution 4 Hz are excited one at a time. Now it can be seen that several frequencies produce the even harmonics \(2f_{ex}\) and \(4f_{ex}\). The odd harmonics, however, are not that significant. The same observation of the even harmonics can also be seen when the frequencies above 300 Hz are investigated.

The purpose of Figs. 5.5 and 5.6 was to investigate the possible even and odd harmonics that the system produces to the excitation signal \(f_{ex}\) according to Eq. (3.32). If there are no significant disturbance signals influencing the system, such harmonics can be analyzed using an odd-odd multisine signal as explained in Section 3.4. However, as can be seen in Fig. 5.7, the system suffers from harmonics that cannot be explained by Eq. (3.32). In Fig. 5.7, the DFTs of the displacements of the rotor in end B are shown when the frequencies 2–800 Hz are excited one at a time. It can be seen that several harmonics are produced especially at the low frequencies on the left side of the figure. The excitations at the frequencies 300–500 Hz produce harmonics at the frequencies 0–200 Hz according to the equation \(f_{ex} - 300\) Hz and excitations at the frequencies 100–300 Hz produce harmonics at the frequencies 0–200 Hz according to the equation \(300\) Hz – \(f_{ex}\). Additionally, the excitations at 600–800 Hz produce harmonics according to \(f_{ex} - 600\) Hz, the excitations at 100–200 Hz according to \(f_{ex} - 100\) Hz, and excitations at 200–300 Hz according to \(f_{ex} - 200\) Hz. The harmonics are produced according to the mixer equation, Eq. (3.33), with the grid frequency 50 Hz as the
5.2 Design of excitation signals and harmonics analysis

According to the harmonics analysis in Section 5.2.2, the system produces several harmonics, which has to be taken into consideration when designing the multisine excitation signal. The influence of the even harmonics produced by the frequencies above 100 Hz can be eliminated by using an odd-odd multisine signal that only contains the odd multiples of the base frequency \( f_0 = 2 \) Hz. Thus, the frequency resolution of the multisine signal is chosen to be 4 Hz. The influence of the other produced harmonics on the measured frequencies can only be eliminated by choosing several multisine excitations. A disadvantage of the separated excitations is that the total measurement time increases. However, it is possible that the required number of averaging over different blocks decreases when the number of excited frequencies in the multisine signals decreases.

![Harmonics analysis for the frequencies 2–800 Hz. The spectrum of the displacement \( x_B \) when the frequencies 2–800 Hz are excited one at a time.](image)

**Fig. 5.7.** Harmonics analysis for the frequencies 2–800 Hz. The spectrum of the displacement \( x_B \) when the frequencies 2–800 Hz are excited one at a time.

In addition to the aforementioned harmonics, several frequencies produce disturbances to the lower frequencies 10, 14, 18, 22, and 26 Hz, which can also be seen in Fig. 5.7.
Four separate multisine excitation signals are designed. The frequency resolution is the same as for the stepped sine excitation presented in Section 5.2.1; In the frequency ranges 2–330, 482–578, and 794–974 Hz the resolution is 4 Hz, in the range 330–482 Hz it is 8 Hz, and in the ranges 578–794 and 974–1190 Hz the resolution is 12 Hz. The amplitudes of each frequency component are chosen according to the magnitudes of the FRFs of the system in Fig. 5.3. At the frequencies where the FRFs have a higher magnitude, the excitations have lower amplitudes, and at the frequencies with a high attenuation in the FRFs, the excitations have higher amplitudes. Because a multisine signal contains several frequencies, an amplitude of each frequency component must be lower than for a single sine signal. However, while the phases of the sine components in the multisine are random, the amplitude of each component can still be higher than $i_{ex,ss}/N_1$ when compared with the single sine signal. The frequencies and the amplitudes of the signals can be seen in Table 5.2 and Fig. 5.8.

Table 5.2. Frequencies and amplitudes of four separate multisine excitation signals.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Frequencies</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation 1</td>
<td>2, 10, 14, 18, 22, 26, 90 Hz</td>
<td>15 mA</td>
</tr>
<tr>
<td></td>
<td>402–498 Hz</td>
<td>30 mA</td>
</tr>
<tr>
<td>Excitation 2</td>
<td>6, 30–86, 92–96 Hz</td>
<td>15 mA</td>
</tr>
<tr>
<td></td>
<td>402–498 Hz</td>
<td>30 mA</td>
</tr>
<tr>
<td>Excitation 3</td>
<td>102–146, 250–254 Hz</td>
<td>15 mA</td>
</tr>
<tr>
<td></td>
<td>258–262 Hz</td>
<td>7.5 mA</td>
</tr>
<tr>
<td></td>
<td>266–298 Hz</td>
<td>15 mA</td>
</tr>
<tr>
<td></td>
<td>502–518 Hz</td>
<td>30 mA</td>
</tr>
<tr>
<td></td>
<td>522–526 Hz</td>
<td>15 mA</td>
</tr>
<tr>
<td></td>
<td>530–538 Hz</td>
<td>7.5 mA</td>
</tr>
<tr>
<td></td>
<td>542–546, 602–698 Hz</td>
<td>30 mA</td>
</tr>
<tr>
<td></td>
<td>710–898 Hz</td>
<td>37.5 mA</td>
</tr>
<tr>
<td>Excitation 4</td>
<td>150–246, 302–362 Hz</td>
<td>15 mA</td>
</tr>
<tr>
<td></td>
<td>370–394, 550–590 Hz</td>
<td>30 mA</td>
</tr>
<tr>
<td></td>
<td>902–938 Hz</td>
<td>15 mA</td>
</tr>
<tr>
<td></td>
<td>942–958 Hz</td>
<td>7.5 mA</td>
</tr>
<tr>
<td></td>
<td>962–1190 Hz</td>
<td>30 mA</td>
</tr>
</tbody>
</table>
Fig. 5.8. Frequency content of four separate multisine excitations.
5.3 Frequency response function measurements

In this section, the results of the frequency response function measurements are reported. First, a comparison of different FRF estimators is presented. Then, a comparison is made of the FRFs measured using different combinations of excitations in each input, and finally, a comparison is provided of the FRFs measured using the stepped sine and random phase multisine excitations.

The frequency response function measurements with the multisine excitation are made in four sets exciting selected frequencies in order to avoid the influence of the produced harmonics on the measured FRFs. The excitation signals are presented in Section 5.2.

The measured signals in the frequency response function measurements are the control currents, rotor displacements, and excitation signals in each radial bearing in the $x$ plane.

5.3.1 Comparison of different FRF estimators

The frequency response functions of an AMB rotor system are measured in the $x$ plane using the orthogonal random multisine excitation. The FRFs are calculated using the EIV, JIO, H1, ARI, and LOG estimators averaged over different amount of blocks $N_b$. The FRFs using 50 blocks for the averaging are shown in Fig. 5.9. For comparison, the frequency response formed according to the results obtained with the FEM are included. $G_{AA}$ is the FRF from the control current $i_{c,A}$ to the displacement $x_A$, $G_{AB}$ the FRF from $i_{c,A}$ to $x_B$, $G_{BA}$ the FRF from $i_{c,B}$ to $x_A$, and $G_{BB}$ the FRF from $i_{c,B}$ to $x_B$.

Figure 5.9 shows that parts of the FRFs are not identifiable. For example, the third flexible mode is not visible in $G_{AA}(j\omega)$ and $G_{BA}(j\omega)$ because the third mode passes through sensor A as can be seen in Fig. 2.11.

When comparing the measured FRFs and the FEM-based frequency response plotted with the black dotted line, it can be seen that they are not fully comparable. There is some unmodeled dynamics influencing all four measured FRFs at the frequencies 170–190 Hz. Also the frequencies of the transmission zeros in $G_{BB}(j\omega)$ differ from the FEM-based model. Additionally, in $G_{AB}(j\omega)$ and $G_{BB}(j\omega)$ some unexpected dynamics or disturbances can be seen around the second flexible mode at the frequencies 400–530 Hz and in $G_{AB}(j\omega)$ also at the frequencies 540–700 Hz. The second mode passes through actuator A and sensor B, as is shown in Fig. 2.11 which may influence on the results of the $G_{AB}(j\omega)$. However, because several estimators give very same results for the FRFs at these frequencies, they are probably caused by unmodeled dynamics.

According to Fig. 5.9, the JIO, H1, and ARI estimators give very similar results when using such a large number of experiments. However, the H1 estimator gives biased estimate for $G_{BA}(j\omega)$ at the frequencies 128–325 Hz and for the zeros with a high attenuation in the $G_{AB}(j\omega)$. This was expected when applied for closed-loop measurements. The FRFs cal-
5.3 Frequency response function measurements

Fig. 5.9. Magnitudes of the frequency response functions of an AMB rotor system from the control currents $i_{c,Ax}$ and $i_{c,Bx}$ to the rotor displacements $x_A$ and $x_B$ using different FRF estimators and 50 blocks for the averaging.
Experimental results

culated using the ARI estimator have some fluctuation, which is because of the low SNR of some of the blocks used for the averaging. The LOG estimator gives a biased estimate at the frequencies 2–70 Hz and at the frequencies above 1020 Hz. Additionally, it cannot correctly estimate the transmission zero of the third flexible mode of the $G_{BB}(j\omega)$ at 854 Hz. The FRFs calculated with the EIV estimator have significant fluctuation, and it cannot estimate the zeros with a high attenuation in $G_{AB}(j\omega)$ and $G_{BB}(j\omega)$. This is because of the nonsynchronous measurements. For comparison, Fig. 5.10 shows the FRFs calculated using the JIO, H1, and ARI estimators, when only ten blocks are used. The FRFs calculated using the JIO estimator with 50 blocks are also shown. It can be seen that the variance is higher for all the estimators. Further, the H1 estimator gives a biased estimate at several frequencies.

Fig. 5.10. Magnitudes of the frequency response functions of an AMB rotor system using different FRF estimators and ten blocks for the averaging.

As there is no initial reference model to compare the results with, the FRFs calculated using the JIO estimator and 50 blocks are taken as one because it is especially intended for the closed-loop identification. The FRFs of the estimators with different numbers of blocks are compared with the reference FRFs. The difference is measured using the cost

$$c_{\text{log}}(G_{\text{JO}}, 50, G_{\text{est}, N_b}) = \left( \sum |\log G_{ij}^{\text{JO}, 50}(j\omega_k) - |\log G_{ij}^{\text{est}, N_b}(j\omega_k)|^2 \right)^{1/2}$$

(5.2)
where $i$ and $j$ indicate the row and column of the FRF matrix, and the superscript $\text{est}$ indicates the compared estimate with $N_b$ blocks (Wernholt and Moberg, 2007). The unidentifiable frequencies in $G_{AA}(j\omega)$, $G_{BA}(j\omega)$ above 662 Hz are not taken into consideration. The results are shown separately for all four excitation signals in Fig. 5.11. Without averaging, that is, with only one period or block, all the estimators give the same FRFs and thus with one block, they all start from the same difference.

![Graphs showing comparison of different FRF estimators](image)

**Fig. 5.11.** Comparison of different FRF estimators as a function of the number of blocks determined according to Eq. (5.2). The JIO estimator with 50 blocks is used as a reference.

It can be seen that the error of the EIV estimator does not converge with any of excitations 1–4. When using excitation 1, the error of the H1, ARI, and LOG estimators converge fast, and it is almost the same with only ten blocks as with 50 blocks. The error of the H1 estimator is the smallest when compared with the JIO estimator with 50 blocks, then comes the ARI estimator, and the error of the LOG estimator is the largest. When using the JIO estimator, the error is very low already with 35 blocks. With excitation 2, the H1 and ARI estimators converge already with 25 blocks at almost the same error they have with 50 blocks. The both converge to the same error. The error of the LOG estimator remains rather high. The error of the JIO estimator has almost the same error with 25 and 30 blocks as with 50 blocks. For some reason, the error increases with 35–45 blocks. This may be because of the more disturbed measurements of these blocks. With excitations 3 and 4, the estimators do not converge with 50 experiments and smoother results could be obtained with a higher number of measurement blocks. However, 50 blocks give sufficiently accurate FRF estimates for the
purposes of this study. The errors of the H1 and ARI estimators are approximately the same, and lower than the error of the LOG estimator when compared with the JIO estimator and 50 blocks.

According to the results, the JIO estimator gives the best estimate for this setup with the orthogonal random multisine excitation and nonsynchronous measurements. With excitations 1 and 2, the required numbers of blocks used for the averaging are 35 and 25, respectively. With excitations 3 and 4, 50 blocks are required in order to correctly estimate the zeros of the FRFs $G_{AB}(j\omega)$ and $G_{BB}(j\omega)$. If the reference signal cannot be measured, and consequently, the JIO estimator cannot be used, the ARI estimator should be used. The variance of the estimate can be smooth using the averaging over the neighbouring frequencies (Ljung, 1999).

5.3.2 Comparison of the orthogonal random multisine and separate multisine excitations in each input

The frequency response functions measured using the orthogonal random excitation signal and the separate multisine excitation in both inputs are compared. The FRFs calculated using the JIO estimator can be seen in Fig. 5.12. With excitations 1 and 2, 35 and 25 blocks are used for the averaging, respectively. With excitations 3 and 4, 50 blocks are used.

The results show that the FRFs measured using the orthogonal random multisine excitation determine the transmission zero of the third flexible mode of the rotor at 854 Hz in $G_{BB}(j\omega)$ more accurately than the FRFs measured with separate excitations in each input. The FRFs $G_{AB}(j\omega)$ also differ at the lower frequencies 2–50 Hz. Additionally, the FRFs measured using the orthogonal random multisine excitation are slightly smoother than the FRFs measured with separate excitations in both inputs in the $G_{BA}(j\omega)$ at the frequencies 290–380 Hz and $G_{AB}(j\omega)$ at the frequencies 350–850 Hz.

5.3.3 Comparison of the stepped sine and multisine excitations

The frequency response functions measured using the stepped sine excitation and the orthogonal random multisine excitation are compared. With the stepped sine excitation, the same combination of excitations in both system inputs is used as for the orthogonal random multisine (Eq. (3.25)). The frequencies and amplitudes used for the excitations are presented in Section 5.2.1. The FRFs measured using the multisine excitation are calculated using the JIO estimator. For the averaging, 35 and 25 blocks are used for excitations 1 and 2, and 50 blocks for excitations 3 and 4. The results of the comparison of the stepped and multisine excitations can be seen in Fig. 5.13.
Fig. 5.12. Frequency response functions of an AMB rotor system from the control currents $i_{c,Ax}$ and $i_{c,Bx}$ to the rotor displacements $x_A$ and $x_B$ measured using the orthogonal random multisine excitation and the separate multisine excitation in each input. The FRFs are calculated using the JIO estimator.
Fig. 5.13. Frequency response functions of an AMB rotor system from the control currents $i_{c,A}$ and $i_{c,B}$ to the rotor displacements $x_A$ and $x_B$ measured using the multisine excitation and the stepped sine excitation.
According to Fig. 5.13, both the stepped sine excitation and the multisine excitation give a very similar result for the FRFs at those frequencies where they are identifiable. Both give the same poles and transmission zeros for the first and second flexible modes of the rotor, and for the unmodeled dynamics at 170–190 Hz. When considering the third flexible mode of the rotor, the FRFs measured using the stepped sine excitation give 934 Hz for the frequency of the pole, and the FRFs measured with the multisine give 944 Hz. The third flexible frequency of the FEM-based model is 952 Hz, and according to the experimental modal analysis, 948 Hz. The third flexible mode of the rotor, which cannot be determined using the multisine excitation, can be roughly determined using the stepped sine excitation. With the multisine excitation, however, the transmission zero of the third flexible mode of the rotor \( G_{BB}(j\omega) \) at 854 Hz can be estimated more accurately. For the transmission zero of the first flexible mode of the rotor in \( G_{AB}(j\omega) \) at 370 Hz, the stepped sine excitation gives a lower magnitude than the multisine excitation.

The times required for the FRF measurements when using the stepped sine and multisine excitations can be calculated using Eqs. (3.15) and (3.21). With the both excitations, the waiting time for the system to stabilize is \( T_w = 0.5 \) s. When the number of total frequencies is \( N_f = 206 \), the number of experiments is \( N_e = 2 \), and the base frequency is \( f_0 = 2 \) Hz, the measurement time with the stepped sine excitation is \( T_{ss} = 3 \) min 32 s. Because the FRF measurements with the multisine excitation are made using four different excitation signals, the measurement time naturally increases. If 50 blocks are measured with each four excitation signal, the measurement time is \( T_{ms} = 3 \) min 24 s. That is, both the stepped sine and multisine measurements require about the same measurement time. When the blocks used for the averaging are 35 and 25 for excitations 1 and 2, and 50 for excitations 3 and 4, the measurement time is only \( T_{ms} = 2 \) min 44 s. The time saving in favor of the multisine excitation is about 23% of the time required for the stepped sine measurements. It must be noticed that only the waiting times for the system to stabilize and the effective measurement times are considered. The periods of transition from one excitation to another with the stepped sine excitation and the periods of transition from one experiment to another with the multisine excitation are ignored as well as the time required to calculate the DFTs and FRFs. These times depend on the implementation of the excitations in the platform used and the clock frequency and memory capacity of the computer or microprocessor.

### 5.4 Parametric model and model updating

In this section, a parametric model is constructed from the nonparametric FRF data. The model is made of the FRFs calculated using the JIO estimator. With excitations 1 and 2, 35 and 25 blocks are used for the averaging, respectively. With excitations 3 and 4, 50 blocks are used. The parametric model is obtained separately for the rigid and flexible modes of the AMB rotor system. Additionally, the model of the power amplifier is identified separately.

Fig. 5.14 shows the FRF of the power amplifier measured from the control current \( i_{c,x,A} \) to the magnetizing current \( i_{m,x,A} \). It is assumed that the power amplifiers of both ends A and B are equal. The parametric model of the power amplifier is made according to Eq. (2.21)
Experimental results

![Graph showing frequency response and phase](image)

Fig. 5.14. Measured FRF, parametric model, and two analytical models of the power amplifier. The analytical models are determined according to Eqs. (2.20) and (2.21).

as a simple first-order transfer function. The parametric model has a pole at -260 Hz, and thus, the power bandwidth $\omega_{BW} = 1634$ rad/s, and an additional gain of 0.97 when compared with Eq. (2.21). The frequency responses of the parametric model and the analytical model obtained both according to Eq. (2.20) and Eq. (2.21) can also be seen in Fig. 5.14. The analytical model of Eq. (2.20) has a pole at -286 Hz and of Eq. (2.21) at -100 Hz, and the power bandwidths are $\omega_{BW} = 1797$ rad/s and $\omega_{BW} = 628$ rad/s, respectively. Thus, the model of Eq. (2.20) describes the dynamics of the power amplifier more accurately. It can be seen from the phase of the measured FRF in Fig. 5.14 that the power amplifier has a delay. If a more accurate model for the power amplifier is required, the delay should be taken care of.

The parametric model of the rigid body modes is estimated in the frequency range of 2–100 Hz. Both the weighting functions presented by Sanathanan and Koerner, Eq. (4.39) and by Strobel, Eq. (4.41) are considered. The comparison of the measured FRFs and the estimated parametric models for the rigid modes is shown in Fig. 5.15.
Fig. 5.15. Comparison of the parametric models of the rigid body modes of an AMB rotor system estimated using the weighting functions of Sanathanan-Koerner and Strobel, and the measured frequency response functions.
It can be seen in Fig. 5.15 that the parametric models estimated using both the Sanathanan-Koerner and Strobel weighting functions estimate the FRF of $G_{AA}$ accurately. However, both fail with estimating the FRFs of $G_{AB}$, $G_{BA}$, and $G_{BB}$. Pole-zero maps of the rigid body model estimated using both weighting functions are shown in Fig. 5.16. It must be noticed that the pole-zero maps of Fig. 5.16 are made for individual elements of the transfer matrix. They contain rigid body zeros unlike the MIMO model presented in Figs. 2.7 and 2.9 (b). In Table 5.3, the poles of the parametric models are compared with the poles of the analytical model presented in Section 2.3.3. It can be seen that the poles of the identified models differ considerably from the poles of the analytical model.

![Pole-zero maps of the parametric models of the rigid body modes estimated both with the Sanathanan-Koerner weight and the Strobel weight.](image)

**Table 5.3.** Rigid body poles of the studied system calculated from the analytical model and the identified plant using two different weighting functions in the parametrization.

<table>
<thead>
<tr>
<th>Rigid mode</th>
<th>Pole [Hz], Analytical model</th>
<th>Pole [Hz], Measured, Strobel</th>
<th>Pole [Hz], Measured, SK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.8</td>
<td>11.5</td>
<td>13.7</td>
</tr>
<tr>
<td>2</td>
<td>38.0</td>
<td>33.2</td>
<td>33.0</td>
</tr>
</tbody>
</table>
Based on the significant differences between the estimated parametric models and the analytical model, it is suggested that the parameters of the analytical model are not updated according to the identified results.

The parametric model of the flexible modes of an AMB rotor system is estimated using the weighting functions presented by Sanathanan and Koerner (1963) and Strobel (1966). The frequency range of 122–404 Hz is considered. For the higher frequencies, it is not possible to make a parametric model. This is because the third mode passes through sensor A, for which the third flexible mode is not visible in $G_{AA}(j\omega)$ and $G_{BA}(j\omega)$. Additionally, there are some unmodeled dynamics visible only in $G_{AB}(j\omega)$ and $G_{BB}(j\omega)$ at the frequencies 400–530 Hz and in $G_{AB}(j\omega)$ also at the frequencies 540–700 Hz. This dynamics cannot be modeled as additional flexible modes by enlarging the model order, because it is not visible in all four FRFs. Therefore, the parametric model can only be obtained for the first flexible mode of the rotor and for the unmodeled dynamics at the frequencies 170–190 Hz. For the controller design this will be sufficient, because the bandwidth of the controller is about 74 Hz. The higher modes cannot be controlled but they have to be damped. The magnitudes of the frequency responses of the estimated parametric models and the nonparametric model are shown in Fig. 5.17 and the phases in Fig. 5.18.

![Comparison of the magnitudes of the parametric models of the flexible modes of an AMB rotor system estimated using the Sanathanan-Koerner and Strobel weighting functions. The measured FRFs and the model based on the FEM analysis are also shown.](image-url)
Fig. 5.18. Comparison of the phases of the parametric models of the flexible modes of an AMB rotor system estimated using the Sanathanan-Koerner and Strobel weighting functions, and measured frequency response functions.
5.5 Conclusions

There are two flexible modes in the frequency range under consideration, at 182 Hz and 270 Hz. Often, slight overmodeling is required, and also here the order of the modeled parametric models has been 11 instead of the minimum of four. It can be seen that both the parametric models with the Sanathanan-Koerner and Strobel weighting functions estimate quite well both the poles and zeros of all the four FRFs, except for the transmission zero of the first flexible mode of the rotor in the $G_{AB}$ at about 370 Hz, which cannot be estimated using the Sanathanan-Koerner weighting function. The model obtained using the Sanathanan-Koerner weight estimates better the transmission zero of the first flexible mode of the rotor at 262 Hz in $G_{BB}$. The phase of the parametric model with the Strobel weight has a phase shift of $\cdot 180^\circ$ at the frequencies 270–410 Hz in $G_{BB}$. The zero at 372 Hz in $G_{BB}$ is a transmission zero of the second flexible mode of the rotor or possibly some unmodeled dynamics and is not parametrized in this examination. The reason for the parametric estimate with the Sanathanan-Koerner weight to fail in describing the zero of $G_{AB}$ is that the weighting function leads to an absolute error. The Strobel weighting function suits better for AMB rotor systems, for which the current controller causes a roll-off of 40 dB per decade thereby leading to a high attenuation of the zeros at higher frequencies. The estimated parametric model with the Strobel weight required only five iterations to converge to the obtained result, whereas the model with the Sanathanan-Koerner required 100 iterations in order to correctly estimate the other poles and zeros except for the one in the FRF $G_{AB}$.

Table 5.4 shows the flexible frequencies of the FEM-based model and the parametric model obtained with the Strobel weight in the frequency range 122–400 Hz. The percentage difference of the flexible frequency of the first mode between the FEM-based model and the updated model is 3.6%. Additionally, in the identified model, the transmission zero of the first flexible mode in $G_{BB}$ precedes the pole, when in the FEM based model the zero comes just after the pole. The difference between the models is so significant, that the parameters of the analytical model related to the first flexible mode should be updated. The unmodeled dynamics is updated to the analytical model by increasing the order of the analytical model.

<table>
<thead>
<tr>
<th>Mode $k$</th>
<th>Frequency [Hz], FEM model</th>
<th>Frequency [Hz], Measured parametric model</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>182</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>260.3</td>
<td>270</td>
<td>3.6</td>
</tr>
</tbody>
</table>

5.5 Conclusions

In this chapter, the test arrangements for the frequency response function measurements of a nonrotating AMB rotor system have first been described. Then, the stepped sine and multisine signals for excitation have been designed, and the harmonics produced by a nonlinear AMB rotor system analyzed. A comparison of the FRFs calculated using different estimators, a comparison of the FRFs measured using different combinations of excitation in each inputs, and a comparison of the FRFs measured using stepped sine and multisine excitations have
Experimental results

been presented. Furthermore, a parametric model of the flexible frequencies obtained from
the nonparametric FRFs has been provided.

Excitation signals for the frequency response function measurements of the system under
consideration have been designed. The maximum amplitudes of a single sine signal have first
been determined. Then, a harmonics analysis using the designed stepped sine excitation has
been presented. The analysis has revealed that the system produces odd harmonics at low fre-
quencies 2–50 Hz and even harmonics mostly at the frequencies above 100 Hz. Additionally,
the system produces harmonics influenced by the grid frequency 50 Hz. Four separate multisine
signals have been designed in order to eliminate the influence of the produced harmonics
on the measured frequency response functions.

The comparison of different FRF estimators has shown that the JIO estimator that is espe-
cially intended for the closed-loop identification gives the best result for the nonsynchronous
measurements. A disadvantage of the JIO estimator is that it requires the reference signal to
be measured. If the measurement of the reference signal is not possible, the ARI estimator is
suggested to be used. The H1 estimator gives a slightly biased estimate at some frequencies
because of the closed-loop measurements. The LOG estimator gives biased estimates at the
frequencies 2–70 Hz and 1020–1190 Hz. Additionally, it cannot estimate all the zeros of the
system. The FRFs calculated using the EIV estimate have significant fluctuations, which are
caused by the nonsynchronous measurements.

The FRFs of the AMB rotor system have been measured using the orthogonal random multisine
excitation and separate multisine excitations in both inputs. The results show that when
using the orthogonal random multisine excitation, the FRFs have less fluctuation than when
using separate excitations in each input. Additionally, the orthogonal random multisine leads
to more accurate FRFs at lower frequencies 2–50 Hz and determines the transmission zero of
the third flexible mode in \(G_{BB}(j\omega)\) more accurately.

The comparison of the frequency response functions obtained using the stepped sine and
random phase multisine excitations has shown that very similar results can be obtained with
both excitations. With the multisine excitation, the transmission zero of the third mode in
\(G_{BB}(j\omega)\) is more accurately determined than with the stepped sine excitation. However,
the stepped sine determines better the third flexible mode in \(G_{AA}(j\omega)\) and \(G_{BA}(j\omega)\) that are
difficult to model because the third mode passes through sensor A. The results indicate that
the random phase excitation can be used in the FRF measurements of AMB rotor systems.
However, when using multisine excitation for a nonlinear system, the excitation signal must
be designed very carefully as several frequencies are considered at the same time. If the
excitation signal is designed negligently, the possibly generated harmonics may deteriorate
the measured frequency responses.

The measurement times of the FRFs using the stepped sine and multisine excitations have
been compared. If only the measurement times with the waiting times required for the sys-
tem to be stabilized are considered, and the time required to change from one frequency to
another is ignored, the measurement time of the FRFs for the studied system using the stepped
sine excitation is 3 min 32 s. When using the multisine excitation and nonsynchronous mea-
surements, the measurement time is 3 min 24 s when 50 blocks are used for each four FRF measurements with different excitations. When the number of the measured blocks with excitations 1 and 2 is reduced to 35 and 25, the measurement time is only 2 min 44 s. The time saving of 23% in favor of the multisine excitation is achieved.

The FRF of the power amplifier has been measured. The dynamics of the power amplifier is described with a simple first-order transfer function. The measured model has a pole at -260 Hz, and thus, the power bandwidth $\omega_{BW} = 1634 \text{ rad/s}$. The analytical model has a pole at -100 Hz ($\omega_{BW} = 628 \text{ rad/s}$) or -286 Hz ($\omega_{BW} = 1797 \text{ rad/s}$) depending on the equation used. The measured model also has a gain of 0.97 when compared with the analytical models. The parameters of the analytical model of the power amplifier are updated according to the result.

The parametric models of the rigid and flexible body modes of the AMB rotor system have been composed from the nonparametric FRF data using a linear least-squares approach for a common-denominator model. Different weighting functions have been used. For the rigid body modes, the parametric models estimated using both the Sanathanan-Koerner and Strobel weighting functions estimate one individual FRF of the MIMO model accurately. However, both estimates fail with describing the other three FRFs. The poles of the obtained parametric models differ significantly from the analytical model. The rigid body modes of the rotor are suggested to be identified using other methods, for example, the method presented by Lösch (2002). The parametric model of the flexible modes is made in the frequency range of 122–410 Hz containing the first flexible mode of the rotor at 270 Hz and an additional mode caused by unmodeled dynamics at 182 Hz. The parametric model with the Strobel weight can estimate all the poles and zeros in the examined frequency range fast and accurately. The model with the Sanathanan-Koerner weight fails to describe a zero with a high attenuation. The error in the first flexible mode between the FEM-based model and the parametric model identified using the Strobel weight is 3.6%. Additionally, in the identified model, the transmission zero of the first flexible mode in $G_{BB}$ precedes the pole, but in the FEM-based model the zero comes right after the pole. The differences are significant and the parameters of the first flexible mode are updated to the analytical model. Additionally, the unmodeled dynamics at the frequencies 170–190 Hz is updated to the analytical model by increasing the order of the model. It is not possible to obtain a parametric model for the second and third flexible body modes of the rotor because the third mode passes through sensor A and is thus not fully identifiable. Additionally, the system contains unmodeled dynamics at the frequencies 400–530 and 540–700 Hz in $G_{AB}(j\omega)$ and at 400–530 Hz in $G_{BB}(j\omega)$. Because this unmodeled dynamics is not visible in all four FRFs, it cannot be modeled as an additional flexible mode by enlarging of the model order. However, the obtained parametric model is sufficient for the controller design, because the controller bandwidth is about 74 Hz.
Chapter 6

Conclusions

6.1 Summary

In this doctoral thesis, broadband excitation was adopted to the identification of the AMB rotor systems. Broadband excitation contains all the desired frequencies in the frequency range of interest, and with an adequate signal-to-noise ratio, it enables faster frequency response function measurements when compared with the stepped sine and swept sine excitations widely used in AMB rotor identification.

In this study, special issues concerning system identification of an AMB rotor system were addressed. An AMB rotor system is a nonlinear, unstable multiple-input, multiple-output system. All these characteristics pose their own challenges to the identification. A few broadband excitations were presented, such as random phase multisine, periodic random excitation, and a pseudorandom binary signal. Random phase multisine excitation was chosen for further study as it provides the user with an opportunity to choose both the frequencies and amplitudes of the excitation signal.

The multisine signals used in the FRF measurements were designed according to the harmonics analysis. As the AMB rotor system is a nonlinear system, it produces additional harmonics that may degrade the measured FRFs if the multisine signal is not properly designed. The harmonics analysis proved that the system produces odd harmonics at low frequencies 2–50 Hz and even harmonics at the frequencies above 100 Hz. Additionally, significant harmonics were produced caused by the grid frequency 50 Hz. In order to avoid the influence of the harmonics on the measured FRFs, four separate multisine excitation signals were designed.

The comparison of the FRFs obtained using the stepped sine and multisine excitations showed that both excitations lead to very similar results. The result verifies the suitability of the random phase multisine excitation in the identification of AMB rotor systems.
Conclusions

In the broadband signals, the signal power is distributed to several frequencies, which degrades the signal-to-noise ratio of the measurement. The SNR is increased by measuring several blocks and averaging the FRFs over them. Different frequency response function estimators and their suitability for AMB rotor system identification were studied. It was shown that the joint-input-output estimator that is especially intended for closed-loop identification gives the best estimate for the FRF when the measurements are not synchronous. The JIO estimator requires the measurement of the excitation signal in addition to the measurements of the input and output signals. If the excitation signal cannot be measured, the ARI estimator should be used.

In the identification of MIMO systems, it is important to choose the combination of excitations in each input carefully so that the fluctuations of the excitation do not add to the fluctuations of the output noise. A comparison was made for the FRFs measured using the orthogonal random multisine excitation and separate multisine excitations in both inputs. The results showed that the FRFs measured using the orthogonal random multisine have less fluctuations than the FRFs measured using separate excitations in each input. Additionally, the orthogonal random multisine led to more accurate FRFs at low frequencies.

The measurement times of the frequency response functions using the stepped sine and random phase multisine excitations were compared. For the studied system, the measurement time using the stepped sine was 3 min 32 s, and with the random phase multisine 2 min 44 s. Thus, the measurements with the multisine excitation were 23% faster when compared with the time required for the stepped sine measurements, even if several measurements with different multisine excitations were performed in order to avoid the influence of the harmonics.

A parametric model was identified for the power amplifier, and the rigid and flexible modes of the system separately. For the rigid and flexible modes, least-squares methods for a common-denominator model were used. The analytical model of the system was updated with respect to the power amplifier and flexible modes.

It was shown that the random phase multisine excitation can be used in the identification of AMB rotor systems. The identification was performed for the controller design purposes and only for a nonrotating rotor.
6.2 Suggestions for future work

In this thesis, the purpose of the identification was to verify and, if required, update the analytical model. In the future, the suitability of the multisine excitation for diagnostics will be investigated. When used for diagnostic purposes, the identification is performed for the rotating rotor. The rotation of the rotor generates subharmonics and increases the level of the measurement noise. The subharmonics may influence the measurements if they combine with the excitation frequencies. The increase in the measurement noise level necessitates more blocks to be measured for averaging in order to increase the SNR. Therefore, the measurement times with the rotating rotor are assumed to become somewhat longer than with the nonrotating rotor.

However, in the diagnostics, the changes in frequencies of the flexible modes are of interest as they indicate possible faults in the mechanics of the system. Because an accurate linear model of the system is not the key factor in the diagnostics but the changes in the FRFs are of primary interest, the harmonics produced by the system would not be such a problem as they are in the identification for the controller design. Thus, one multisine excitation might be enough, which would enable faster FRF measurements. The continuous harmonics analysis could also be applied to the diagnostics.

In this study, the amplitudes of the frequency components of the random phase multisine excitation were chosen manually. In the future, the excitation signal design should be automated. The frequencies and amplitudes of each frequency component should be chosen according to the purpose of the identification. For the controller design, a higher-frequency resolution is required, and a continuous analysis of the harmonics is not necessary. In the diagnostics, the frequency resolution can be lower, and also a continuous harmonics analysis can be applied. The selection of the amplitudes of each frequency component can be automated according to the gains of the measured FRFs at different frequencies.

Obtaining a parametric model from the frequency response function data should be further investigated. The method applied in this thesis was not suitable for the identification of the rigid body modes, and therefore, other identification methods should be used for them. The parametrization of the flexible modes should also be automated.
Bibliography


Appendix A

Data of the laboratory test setup

A.1 Magnetic bearings and rotor

Table A.1. Dimensions and parameters of the radial magnetic bearings.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator diameter $\varnothing_s$</td>
<td>180 mm</td>
</tr>
<tr>
<td>Core length $d_{\text{core}}$</td>
<td>60 mm</td>
</tr>
<tr>
<td>Stator tooth width $b_{\text{dr}}$</td>
<td>18 mm</td>
</tr>
<tr>
<td>Slot to slot diameter</td>
<td>144 mm</td>
</tr>
<tr>
<td>Shaft diameter $\varnothing_{\text{shaft}}$</td>
<td>60 mm</td>
</tr>
<tr>
<td>Rotor outer diameter $\varnothing_r$</td>
<td>89.8 mm</td>
</tr>
<tr>
<td>Nominal air-gap $s_0$</td>
<td>0.6 mm</td>
</tr>
<tr>
<td>Mechanical air-gap $s_m$</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Lamination material</td>
<td>M270-50A</td>
</tr>
<tr>
<td>Lamination sheet thickness</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Current stiffness $k_i$</td>
<td>268 N/A</td>
</tr>
<tr>
<td>Force stiffness $k_s$</td>
<td>992 N/mm</td>
</tr>
<tr>
<td>Number of turns per magnet $N_c$</td>
<td>180</td>
</tr>
<tr>
<td>DC link voltage $U_{\text{dc}}$</td>
<td>120 V</td>
</tr>
<tr>
<td>Coil resistance $R_c$</td>
<td>0.43 $\Omega$</td>
</tr>
<tr>
<td>Inductance $L$</td>
<td>0.042 H</td>
</tr>
<tr>
<td>Bias current $i_b$</td>
<td>2.5 A</td>
</tr>
<tr>
<td>Maximum current $i_{\text{max}}$</td>
<td>10 A</td>
</tr>
</tbody>
</table>
Table A.2. Dimensions and parameters of the axial magnetic bearing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator outer radius</td>
<td>36 mm</td>
</tr>
<tr>
<td>Stator inner radius</td>
<td>27.5 mm</td>
</tr>
<tr>
<td>Nominal air-gap $s_0$</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Mechanical air-gap $s_m$</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Disc material</td>
<td>S355JO/EN 10025</td>
</tr>
<tr>
<td>Thickness of the disk</td>
<td>8.9 mm</td>
</tr>
<tr>
<td>Current stiffness $k_i$</td>
<td>213 N/A</td>
</tr>
<tr>
<td>Force stiffness $k_s$</td>
<td>1065 N/mm</td>
</tr>
<tr>
<td>Number of turns per magnet $N_c$</td>
<td>100</td>
</tr>
<tr>
<td>DC link voltage $U_{dc}$</td>
<td>120 V</td>
</tr>
<tr>
<td>Coil resistance $R_c$</td>
<td>0.52 Ω</td>
</tr>
<tr>
<td>Inductance $L$</td>
<td>0.045 H</td>
</tr>
<tr>
<td>Bias current $i_b$</td>
<td>2.5 A</td>
</tr>
<tr>
<td>Maximum current $i_{max}$</td>
<td>10 A</td>
</tr>
</tbody>
</table>

Table A.3. Parameters of the rotor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft mass $m_{shaft}$</td>
<td>36.5 kg</td>
</tr>
<tr>
<td>Rotor mass $m_r$</td>
<td>46.2 kg</td>
</tr>
<tr>
<td>Transversal moment of inertia</td>
<td>4.80 kg · m²</td>
</tr>
<tr>
<td>Polar moment of inertia</td>
<td>0.041 kg · m²</td>
</tr>
<tr>
<td>Location of the radial bearing A from the center of mass $d_A$</td>
<td>0.388 m</td>
</tr>
<tr>
<td>Location of the radial bearing B from the center of mass $d_B$</td>
<td>0.352 m</td>
</tr>
<tr>
<td>Location of the radial sensor A from the center of mass $d_{s,A}$</td>
<td>0.462 m</td>
</tr>
<tr>
<td>Location of the radial sensor B from the center of mass $d_{s,B}$</td>
<td>0.477 m</td>
</tr>
</tbody>
</table>

A.2 Sensors and control electronics

Table A.4. Specifications of the differential sensors DT3703 U3-A-C3 used in the radial bearing A and single-channel position sensors DT3701 U1-A-C3 used in the radial bearing B.

<table>
<thead>
<tr>
<th>Specification</th>
<th>DT3703 U3-A-C3</th>
<th>DT3701 U1-A-C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring range</td>
<td>0.1–1 mm</td>
<td>0.3–1.5 mm</td>
</tr>
<tr>
<td>Output range (from driver)</td>
<td>0–2.5 V</td>
<td>0–2.5 V</td>
</tr>
<tr>
<td>Dynamic resolution at 1 kHz</td>
<td>1.5 nm</td>
<td>2.25 nm</td>
</tr>
<tr>
<td>Linearity</td>
<td>± 6%</td>
<td>± 5%</td>
</tr>
<tr>
<td>Frequency bandwidth (-3 dB)</td>
<td>10 kHz</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Measurement target</td>
<td>Al</td>
<td>Al</td>
</tr>
</tbody>
</table>
A.2 Sensors and control electronics

Table A.5. Specifications of the single-channel position sensor CMSS 68 used in the axial bearing.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring range</td>
<td>0.2–2.5 mm</td>
</tr>
<tr>
<td>Output range (from driver)</td>
<td>0–18 V</td>
</tr>
<tr>
<td>Dynamic resolution at 1 kHz</td>
<td>–</td>
</tr>
<tr>
<td>Linearity</td>
<td>±1.1%</td>
</tr>
<tr>
<td>Frequency bandwidth (-3 dB)</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Measurement target</td>
<td>Fe</td>
</tr>
</tbody>
</table>

Table A.6. Specifications of the LEM current sensors LA 25-NP.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring range</td>
<td>0–36 A</td>
</tr>
<tr>
<td>Output range (at the input to ADC)</td>
<td>0–3.75 V</td>
</tr>
<tr>
<td>Accuracy at $i_{PN} = 25$ A</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Linearity</td>
<td>0.2%</td>
</tr>
<tr>
<td>Response time</td>
<td>1 µs</td>
</tr>
<tr>
<td>Frequency bandwidth (-1 dB)</td>
<td>0–150 kHz</td>
</tr>
</tbody>
</table>

Table A.7. Specifications of DS2001 ADC boards connected to the dSpace.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ADC channels per board</td>
<td>5</td>
</tr>
<tr>
<td>Resolution</td>
<td>16-bit</td>
</tr>
<tr>
<td>Conversion time</td>
<td>5.0 µs</td>
</tr>
<tr>
<td>Input range</td>
<td>±5 or ±10 V%</td>
</tr>
<tr>
<td>Organization of inputs</td>
<td>parallel</td>
</tr>
</tbody>
</table>

A custom-built power distribution board is run through a Xilinx Virtex-II Pro FPGA (field programmable gate array) that provides PWM for the gate drivers. A Memec development board with Xilinx’s Virtex-II Pro was chosen when finding a powerful and flexible FPGA-based platform to easily test and modify control algorithms (Jastrzebski et al., 2005; Jastrzebski, 2007). However, so far the dSpace platform has been used for the control. A custom-built Spartan-II board is used for communication between dSpace and Virtex-II Pro.
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