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Matylda Jabłońska

FROM FLUID DYNAMICS TO HUMAN PSYCHOLOGY. WHAT DRIVES FINANCIAL MARKETS TOWARDS EXTREME EVENTS

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Supervisor Docent, PhD Tuomo Kauranne
Faculty of Technology
Department of Mathematics and Physics
Lappeenranta University of Technology
Finland

Reviewers Prof. Vincenzo Capasso
Department of Mathematics
University of Milano
Italy

PhD Marta Posada
Escuela S. T. Ingenieros Industriales
University of Valladolid
Spain

Opponent Prof. Vincenzo Capasso
Department of Mathematics
University of Milano
Italy

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Abstract

Matylda Jabłońska
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For decades researchers have been trying to build models that would help understand price performance in financial markets and, therefore, to be able to forecast future prices. However, any econometric approaches have notoriously failed in predicting extreme events in markets. At the end of 20th century, market specialists started to admit that the reasons for economy meltdowns may originate as much in rational actions of traders as in human psychology. The latter forces have been described as trading biases, also known as animal spirits.

This study aims at expressing in mathematical form some of the basic trading biases as well as the idea of market momentum and, therefore, reconstructing the dynamics of prices in financial markets. It is proposed through a novel family of models originating in population and fluid dynamics, applied to an electricity spot price time series. The main goal of this work is to investigate via numerical solutions how well the equations succeed in reproducing the real market time series properties, especially those that seemingly contradict standard assumptions of neoclassical economic theory, in particular the Efficient Market Hypothesis.

The results show that the proposed model is able to generate price realizations that closely reproduce the behaviour and statistics of the original electricity spot price. That is achieved in all price levels, from small and medium-range variations to price spikes. The latter were generated from price dynamics and market momentum, without superimposing jump processes in the model. In the light of the presented results, it seems that the latest assumptions about human psychology and market momentum ruling market dynamics may be true. Therefore, other commodity markets should be analyzed with this model as well.

Keywords: electricity spot price, animal spirits, ensemble models, population dynamics

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To my beloved parents,
on their 30th wedding anniversary.

Moim Kochanym Rodzicom,
w ich 30-stą rocznicę ślubu.

Preface

This work was carried out in the Department of Mathematics and Physics in Lappeenranta University of Technology, Finland, between 2009-2011. I highly acknowledge all the institutions that provided financial support for this work, that is, the Department of Mathematics and Physics of LUT, the Research Foundation of LUT (LTY:n Tukisäätiö), and the Finnish Academy of Science and Letters (Suomalainen Tiedeakatemia). I also thank the Fortum Foundation (Fortumin Säätiö) for my Master thesis grant, as that study was the cradle of our present research. Moreover, this work would not have been possible without the extensive data sets that our research group acquired. Therefore, I would like to acknowledge Nord Pool Spot and the New Zealand Power Market for providing all the information free of charge.

Throughout this study a number of people have influenced the directions and progress of my work. In the first place, I thank my supervisor Tuomo Kauranne for all the scientific guidance, as well as for showing to us, students, how much there is left to learn about the world. Your knowledge and numerous ideas "to try out" have shaped the essence of this work and have inspired and encouraged me to challenge my own skills. You have been a mentor to me during my stay in Lappeenranta and I believe this will continue for long years to come. I would also like to thank you for being such a warm and caring person with our heart wide open, always ready to help not only with scientific, but personal matters as well.

I would also like to thank the reviewers of this work, Vincenzo Capasso and Marta Posada, for their insightful comments to help make this work as good as possible. Valuable remarks and questions from prof. Capasso made me realize some of the biggest mathematical challenges in this field of research and have been very inspiring in terms of future work considerations. I have also received a lot of help from a number of electricity market specialists. Therefore, my gratitude goes to the LUT Energy team lead by Jarmo Partanen and Satu Viljainen, for our long fruitful discussions that helped me understand in details the character of power trading. On the same grounds I would like to thank Karri Mäkelä, Pasi Kuokkanen and Jan Fredrik Foyn from Nord Pool Spot, Marko Pollari from Lappeenrannan Energia, and Brian Bull from the New Zealand Electricity Market, for their patience in answering my numerous questions. Also, this work would not have been possible without coauthors of my articles and all the LUT Master's students that completed their theses in the related topics. This dissertation refers to wide range of their results; therefore, I am grateful they were willing to join and contribute to our semi-finite research group.

I was able to work efficiently only because my friends created a perfect resting environment in my spare time. First, I would like to thank Piotr and Olga Ptak. It was you who in the first weeks of my stay in Finland made sure that I would feel like home. Also, our conversations with Piotr in the lab filled perfectly the breaks from the scientific work. Nothing could boost working energy more than a decent dose of Polish humour. In no smaller scope do I want to thank Ania Lewandowska from Bank Zachodni WBK. As your "professional child" I have gained a lot of self-confidence and learned how to sell my own skills. My dear friends Luna and Hasifa, you left some unforgettable memories from your stays in

Lappeenranta; thank you for all the laughters and serious talks that we had, and will still have together. I would also like to thank Ania B., Tanya, Ivan, Rustam and Andrey, for being my friends in good and bad times (special thanks to Ivan for answering all my LaTeX- and Matlab-related questions), as well as to all the co-workers from the Department, fellow students and other friends and acquaintances in and outside of Finland.

Last but not least, I would like to thank my closest family. My parents Elżbieta and Mirosław, to whom I dedicate this thesis, have always believed in me, not only in successful pursuit of my doctoral degree, but in any other challenges that I was undertaking. It is thanks for my mom's time and patience devoted in my earliest school years, that I was able to come this far in the field of mathematics. I have always wanted to make the two of you proud and, even if sometimes some human weakness stands on my way, I will try my best to reciprocate for your love, care and help throughout my life and studies. I also thank my big brother, Patryk, for being there for me to answer my IT- and non-IT-related questions. Finally, the biggest happiness in my life appeared with my wonderful daughter, Patricia. It is thanks to her peaceful sleep on day and night time, as well as my joyful time spent with her, together with love and great support from my fiancé, Stewart, that I found strength and time to write this dissertation. Thank you. I love you all.

Lappeenranta, November 2011

Matylda Jabłońska

List of the original articles and the author's contribution

This thesis consists of an introductory part and five original refereed articles. Two of them were presented on international conferences. Two have been published in scientific journals. The fifth one has also been submitted for review to a European scientific journal. The articles and the author's contributions in them are summarized below.

- **I** **Ptak, P., Jabłońska, M., Habimana, D., and Kauranne, T.**, (2008) Reliability of ARMA and GARCH Models of Electricity Spot Market Prices. In: *Proceedings of European Symposium on Time Series Prediction, Porvoo, Finland*
- **II** **Jabłońska, M., Mayrhofer, A., and Gleeson, J.**: Stochastic simulation of the Uplift process for the Irish Electricity Market. *Mathematics-in-Industry Case Studies*. **2** 86–110 (2010)
- **III** **Jabłońska, M., Nampala, H., and Kauranne, T.**: Multiple mean reversion jump diffusion model for Nordic electricity spot prices. *The Journal of Energy Markets*. **4**(2) Summer 2011.
- **IV** **Jabłońska, M., Viljainen, S., Partanen, J., and Kauranne, T.** (2010) The Impact of Emissions Trading on Electricity Spot Market Price Behavior. Submitted to *International Journal of Energy Sector Management*.
- **V** **Jabłońska, M. and Kauranne, T.** (2011) Multi-agent stochastic simulation for the electricity spot market price. *Lecture Notes in Economics and Mathematical Systems, vol. 652. Emergent results on Artificial Economics.* Springer

M. Jabłońska is the principal author of four of the listed articles, and a coauthor or the remaining one. In publication **I**, she carried out a part of the analysis related to basic GARCH modeling and wrote the respective portion of the paper. In publication, **II** she proposed the first of the two presented algorithms. The main numerical scheme in article **III** has been prepared by the author together with H. Nampala. In the remaining two publications, M. Jabłońska is the sole author of the practical analyzes. Moreover, she wrote majority of the articles' text and has been the corresponding author in the revision process of publications **II**-**V**.

ABBREVIATIONS

ACF	Autocorrelation Function
AIC	Akaike Information Criterion
ARCH	Autoregressive Conditional Heteroscedasticity
ARMA	Autoregressive Moving Average
cdf	cumulative distribution function
coef	coefficient
cooc	co-occurrence
EMH	Efficient Market Hypothesis
EUA	European Emission Allowances
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GlGARCH	Generalized long-memory GARCH
HMM	hidden Markov models
MCMC	Markov chain Monte Carlo
MLE	Maximum Likelihood Estimation
MWh	megawatt hour
NEPool	New England Pool
NYSE	New York Stock Exchange
orig	original
OU	Ornstein-Uhlenbeck
PACF	Partial Autocorrelation Function
pdf	probability density function
R&D	Research and Development
rev	reversion
RSS	Residual Sum of Squares
SARIMA	Seasonal Autoregressive
SBIC	Schwarz-Bayesian Information Criterion
SDE	Stochastic Differential Equation
sim	simulated
SLEIC	Schwarz-Bayesian Ljung-Box Engle Information Criterion

SNAR	Smoothed Nonparametric Autoregressive
SNARX	Smoothed Nonparametric Autoregressive with Exogenous Input
SSQ	Sum of Squares
std	standard deviation
TSO	transmission system operator
W	Wiener process
WMAE	Weekly-weighted Mean Absolute Error

PART I: OVERVIEW OF THE THESIS

CHAPTER I

Introduction

Financial and commodity markets have been subjects of research for decades, even though they seem to form the least tangible branch of science. In the same time, they also remain challenging and somewhat mysterious, not easy at all to be closed in the rigorous frames of mathematical or physical theories. When trading, participants of financial markets usually have either of the two main goals, depending on their character and position: spend least or gain most. One can expect that it is easier to achieve these goals if it is possible to predict market behavior. But a very specific characteristic of financial markets are extreme events, such as stock exchange crashes. These always come unexpected and cannot be statistically predicted as easily as, for instance, failure of a part in a mechanical system.

Different types of financial markets produce different families of time series. Therefore, researchers work on building models that would be able to explain market dynamics and forecast future prices. A very common assumption is the Efficient Market Hypothesis (EMH) saying that having all the information available, traders cannot permanently benefit from the market. In other words, knowledge of past stock performance should not be any indicator for its future results. But this has been recently questioned by a study proving existence of so called market momentum, that is the fact that markets navigate towards higher prices (Dimson et al., 2008). Also, EMH assumes that traders' decisions are based only on quantifiable economic facts, whereas it does not have to be the case.

A very distinct type of commodity markets is an electricity spot market. Prices in any electricity spot market are characterized as being highly volatile. What contributes to the high volatility is the large variations in the demand and supply of electricity, which are very uncertain in deregulated markets. The main difference of electricity markets from the other markets is that the commodity, that is electricity, cannot be stored on a bigger scale and, therefore, has to be consumed at the instant it is produced. As a consequence, extreme events in the form of price spikes are sudden and prominent.

Researchers working with electricity spot time series can be divided in two main groups. One is formed by those who model the prices' regular behavior, that is the strong intraday and weekly periodicity, and use the models for short term forecasting of the regular price evolution. The second group gathers researchers aiming at modeling price high volatility and spikes. A number of econometric models have been used to model spikes behavior, but none of them had a power to accurately predict their occurrence. Most suggested models base on a combination of mean reverting processes with jump components. In this work, the author argues that the spikes form directly from price dynamics as a result of market momentum as well as traders' psychology influencing their actions. These forces are commonly referred to as *animal spirits*, as first suggested by Keynes (1936).

This study aims at expressing in mathematical form some of the basic trading biases as well as the idea of market momentum and, therefore, reconstructing the dynamics of electricity spot price. It is proposed through a novel family of models originating in population and fluid dynamics. Here, traders in the market are treated as a population of individuals that interact in three scales through a system of stochastic differential equations. These scales are included in a model proposed by Morale et al. (2005). The macroscale drives the direction of the whole population. The microscale deals with each individual separately. Finally, the mesoscale allows interaction with its closest neighborhood. Another novelty in the presented work is that the global interaction is formulated in terms of a momentum component in analogy to Burgers' equation for fluid dynamics. Due to the fact that the topology and the dimensions of the domain are not known in this study, the work does not provide any mathematical analysis of the well-posedness of the proposed system of equations. The focus is set on how well the system is able to reproduce the real price dynamics.

The analysis is performed on spot price data from which all periodicities as well as influences of known deterministic factors have been removed. That series is referred to as *pure trading* series and is claimed to be reflecting the real market dynamics. The results of this dissertation show that the proposed model is able to reproduce most of the statistical features of the electricity spot prices. That includes not only mean reversion, but also price spikes. What differentiates this work from others is that the spikes are generated from pure price dynamics, not through any jump component. Moreover, the model accounts for basic animal spirits such as short-term thinking and herding. Results of those are magnified by market momentum into price spikes.

This work is organized as follows. Chapter 2 reviews the theory and literature related to deregulated electricity markets. That includes the history and performance of deregulation in different countries, as well as its consequences. Also, the Nord Pool Power Exchange is introduced, as the main source of data for this study. Finally, the past research in modeling electricity spot prices in presented. Chapter 3 goes in detail through a number of recent studies using the classical time series approaches and stochastic processes. Moreover, the deterministic factors driving electricity spot prices are discussed and the pure trading price series is constructed. This chapter refers to publications **I-IV**, as well as a number of other studies which form the base for the main contribution of this work. Chapter 4 suggests the reasons for failure of econometric models in predicting price spikes. In Chapter 5 the heart of this work is presented. It introduces the population dynamics models utilized in this study, refers to analogy between prices and fluids, and, finally, presents a number of models and their simulation results. That includes models proposed in publication **V** and their improvement. Chapter 6 provides discussion and future prospects.

CHAPTER II

Deregulated electricity markets – literature review

The history of energy market deregulation does not reach any further in the past than the last two decades of 20th century. Section 2.1 of this chapter reviews the reasons behind implementation of market deregulation in different countries around the world as well as its consequences. Section 2.3 presents the functioning and character of the Nordic electricity market, Nord Pool, which provides most of the empirical data used in this study. Then, Section 2.4 reviews the literature about a wide range of different modeling approaches used for analysis and forecasting of electricity spot prices of different markets by numerous researchers worldwide.

2.1 Reasons behind deregulation and its consequences

Before market deregulation, traditional energy contracts were based on a well-understood optimization problem and were fairly risk-free. Fees and prices were covering variable costs of the producers. If the distributors found them too high, they could forward those to the end-users as fixed or variable costs (Makkonen and Lahdelma, 2001). The main aim of deregulating electricity markets was to evolve market competitiveness and abandon local or national monopolies (Nakajima and Hamori, 2010). In such an environment, prices were expected to get lower and customer service was supposed to improve (Cunningham, 2001; Kinnard and Beron, Winter 1999/2000; Everything to play for in deregulated markets., 2003). The decrease of price level was expected to have a positive effect on industries as well. For example, in Singapore researchers found that cost reduction should be at least 8%. Also, they hoped that a higher number of market players should smoothen the reaction of electricity price to outside price shocks (Chang and Tay, 2006). However, it appeared that many of these goals have not been attained in many countries.

South America was the first continent to implement privatization and deregulation, as Chile created an energy market in the early 1980s. An originally successful solution eventually ended as being dominated by a few big market participants, with whom the smaller ones were not able to compete, i.e. an oligopoly emerged. Following in Chile's footsteps, Argentina implemented specific precautions against this during their deregulation in the form of strict limits that were imposed on market concentration and the right structure of reserve units.

The first Act about US market deregulation appeared in 1992 (Nakajima and Hamori, 2010). Over the next two decades the process was followed by different states at different pace and often with limited success. California, the first one to be deregulated, failed in all key goals of deregulation. Deregulation did not reduce costs, did not improve customer service, and did not end up having any higher competition (Cho and Kim, 2007; Ritschel and Smestad, 2003). Some producers fell into financial problems as they were not able to sell electricity at a cost-covering price. Over 90% of generators have withdrawn from the market, as the state simply appeared to be too expensive to compete in, and the savings projections they originally believed in never came true (Schmid and Leong, Dec 1999). Moreover, deregulation led to additional energy subsidies, whereas there was no change noticed in price elasticity.

Chilean, Brazilian or Californian style market breakdown was not seen often in other industrialized countries. The Nordic market proved to be able to function very well. Through wide studies and simulations, researchers such as Bye et al. (2008) showed that there are no problems with a deregulated market. The key features that drive market response are: demand flexibility, patterns and handling of inflow shortages, storage capacities, opportunities for trading between different regions that have different production technologies, and, finally, market general design and level of concentration. The only dramatic event in the case of Nord Pool was the winter 2002/03, a time of heavy hydrological storage speculation, which gave a harsh lesson about the importance of focusing on security of supply issues and market failures. More details about the events of that period can be found in publication **IV**.

Nevertheless, market operators and participants realized how important investment in new capacity is. Some researchers discussed the difficulties in this matter. It is expensive to maintain excess capacity in electricity markets. Such storage has to be kept mainly in the form of energy sources, not electricity itself. Also, introducing new capacity is possible only with significant delays, as it implies constructing new facilities. Simply, the industry cannot react quickly to supply shortage, which leads spot prices to skyrocket (Kocan, 2008). Finally, in a competitive environment, where prices are not regulated but set by the market, it is generally a lot more difficult to make investment decisions. According to some specialists, deregulation is failing in providing dynamic efficiency. That is, oversupply of base plant production (hydro, nuclear) may suppress the prices to a level at which they do not signal an entry of a peak power plant. It means that a more expensive generation has to be used suddenly without any early warning. And this leads to price spike emergence (Simshauser, 2006).

In short term, deregulation seems to lower the prices. For instance, in Scandinavia the spot price dropped dramatically after market deregulation, even below variable costs of most production plants (Makkonen and Lahdelma, 2001). From policy perspective, deregulation is encouraging (Linden and Peltola-Ojala, 2010), as some countries seek opportunities to have their local energy prices adjusted to the prices of their neighbors (Bojnec, 2010). Indeed, power pools are useful for different reasons to different market participants: producers, consumers and distributors (Makkonen and Lahdelma, 2001). However, one has to remember that deregulation imposes a serious uncertainty to supply and demand (Burger et al., 2004) and if specific markets fail in electricity R&D investments, that will have a negative impact on economies and environmental wellbeing (Dooley, 1998).

2.2 Nord Pool structure

2.2.1 Elspot – day-ahead spot market

Elspot trades daily contracts for physical delivery of energy in every hour of the following day. The offers can be placed for the whole of the following week. They have to be sent by 12 o'clock of the given day. None of the participants knows the others' bids. The auction is closed at noon and all data is processed. Price settlement is based on finding a balance between the total demand and total supply curve. At 14:00 Nord Pool publishes the prices to the participants who have half an hour to place any complaints. When everything is settled, at 14:30 the prices go public. At that moment the participants are informed about the contracts, that is the amounts and prices qualified for trading. The financial transfers take place between the Pool and participants. The invoices are sent to the traders every Monday and concern contracts from the whole past week. The total market is currently divided into 11 bidding areas: 5 in Norway, two in Denmark, and three other countries, each being a separate area: Sweden, Finland, and Estonia, as well as a part of Germany. These may become separate price areas if the calculated flow of power between bid areas for a given hour exceeds the capacity allocated for Elspot contracts by the transmission system operators. Figure 2.1 presents the current geographic structure of Nord Pool Spot market with repartition into possible price areas created when grid congestions occur.

Figure 2.1: Nord Pool Spot price areas (source: www.nordpoolspot.com).

2.2.2 The balancing power market

The role of balancing power market is to provide a real time balance in the grid between total generation and consumption. It is assured by the transmission system operator (TSO).

There are active and passive participants in the balancing market. Most of the active participants are producers. However, if a consumer is able to regulate his generation or consumption on TSO's request, he can join the balancing market as well. One of special requirements for the participants is a lower limit on the bidding volume and time restrictions for responds. On the passive side there are all companies connected to the central grid. The total consumption and production is measured for the grid and the difference between planned and measured generation and production is settled according to the prices established in the real time balancing.

The power sold to customers by retailers is estimated in terms of expected consumption. It creates a base for their bids in the auctions before the delivery. But it can be the case that the customers use less or more power than the retailer has bought. Then, respectively, the retailer will buy the missing amount from the transmission system operator, or will sell it back to TSO. In both cases the goal is to make the retailer's net purchase and consumption be balanced and, therefore, these trades form the balancing market.

2.2.3 Elbas – cross-border intraday market

Elbas is a continuous cross border intra-day market. It covers all the Nord Pool bidding areas. There, the adjustments are made until one hour prior to delivery. Trading at Elbas starts at 14:00, that is when the day-ahead market (Elspot) auction is closed.

The roles and advantages of the Elbas market are:

- Ensure instant access to all bidding areas and maximally utilize the cross border capacities.
- Reduce the risk of the prices in balancing market.
- Create optimal profit potential.
- Allow trading every day until one hour before delivery.
- Provide a user-friendly and effective web based trading system.

The cross border capacities are updated after each trade is executed. The reporting in each area is done only to the local TSO. Elbas is an alternative solution to the balancing market which, indeed, can have very high volatility in prices than are known only after the delivery. Trading at Elbas allows to know the price one hour prior to delivery which reduces economical risk.

2.2.4 The financial market

The role of financial market is to create hedging opportunities for both supplier and retailers through term contracts. The participants take out a mutual insurance. Each contract applies to one specific day, week, month, quarter or year with a specific amount and execution (strike, hedging) price. The contracts are purely financial and they provide hedging for physical power deliveries settled in the spot market.

If the average system price in a given trading period is higher than the hedging price, the supplier will compensate the retailer with the difference. On the other side, if the price is lower, the retailer is obliged to compensate the supplier. There money is then transferred between the parties. A futures contract is therefore not only a mutual insurance. It is also a mutual obligation.

2.3 Electricity spot prices in Nord Pool Spot

The deregulated Nordic electricity market is characterized as an *energy-only* market with a single, uniform market clearing price. Geographically, the market is composed of five dominating countries, that is Norway, Sweden, Denmark, Finland, Estonia and a part of Germany. However, currently there are participants from over 20 countries trading there. Marginal pricing is applied in the price formation on the Nordic electricity spot market. The market clearing price is found at the intersection of the supply and demand curves that are formulated in the day-ahead spot markets for each hour of the following day, based on the supply offers of electricity generators and the demand bids of electricity retailers and large electricity users. As depicted in Figure 2.2, the system price is the one for which the total demand and supply curves meet, and the amount of electricity at which they cross forms the turnover for a given trading period. Generators' offers reflect the marginal costs of producing electricity, whereas the demand bids indicate the buyers' willingness to pay. The spot market is organized by the power exchange Nord Pool Spot. The trading cycle is characterized as one undisclosed auction being closed at noon every day.

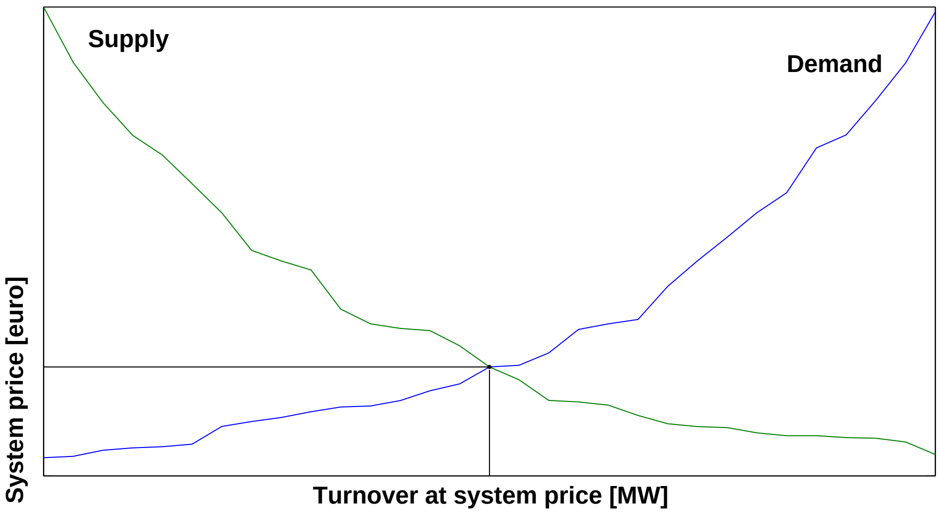

Figure 2.2: System price formation in Nord Pool.

The power exchange contributes to balancing the supply and demand in short-, mid- and long-term planning horizons. It provides motivation and regulations for using the power plants in the right merit order when it comes to production costs (fixed and variable) and enables the efficient use of the generation plants located across the market area, especially if market concentration is well monitored. The market price formed at the power exchange also acts as a reference price in bilateral electricity trading that takes place outside the power exchange.

A uniform market clearing price applied in the Nordic electricity market means that the market is, in principle, cleared with a single price that is applied to all electricity trades that take place in the electricity spot market. However, in case of transmission constraints, the market is divided into predefined price areas that get separated by congested transmission lines. Within the price areas, congestions are not expected to occur. The shape of those areas was presented in Figure 2.1. This system differs from nodal pricing approach employed in some countries (Russia, New Zealand, etc.) where heavy grid density and huge number of generators and consumers makes it more efficient to establish different electricity prices at every grid entry or exit point. However, character and variability of nodal prices does not differ in any degree from the classical system and area spot prices.

Prices in the Nordic electricity market are characterized as being highly volatile. This follows partly from the fact that prices are allowed to peak when the market is short, unlike on markets where prices are capped. Another thing that contributes to the high volatility is the large variations in the demand and supply of electricity, which are very uncertain in deregulated markets (Burger et al., 2004). For instance, temperature strongly affects the demand; in total, the demand varies between 50-100%. Thus, as some say, forecasting demand is almost equivalent to forecasting weather (Podraza, Fall 2006). Next to any climatic factors, hydrological balance, demand and base load supply (Vehviläinen and Pyykkönen, 2005) can be considered with equal importance as the key spot price drivers.

With respect to the logic standing behind the marginal pricing, the generator with the highest marginal costs needed to satisfy the demand for a given trading period defines the market clearing price. All the employed generators are then paid the same market price. Generators that are called to operate are always guaranteed to receive enough money to cover their variable costs. For the generator at the margin, the compensation will be exactly equal to its variable costs. For the other generators, the obtained revenues also cover some of their fixed costs. The principles of price formation are illustrated in Figure 2.3.

In addition to spot market revenues, generators may also earn money by operating in the regulating power markets. In the Nordic electricity market, the regulating power markets are organized for reliability reasons by the national transmission system operators. Demand resources may also participate in the regulating power markets.

Nordic price formation differs from, for instance, the Irish electricity market. There, the market operator calculates first the *shadow price* as an average price per MWh found based on offers of all generators chosen for supply within a given trading period. Then, if that mean price does not cover all costs of the generators, a price called *uplift* is established for each trading period and added on top of the shadow price. In case of Ireland, uplift is the most interesting type of price in terms of modeling, as it represents consequences of variability in non-base demand. More details regarding uplift formation can be found in publication **II**.

The fact that the Nordic electricity market is an energy-only market means that the revenues earned by the generators in the electricity spot market suffice to cover the short-term marginal costs as well as the long-term, 'going-forward' costs of the electricity generation plants. Generators' offers are not subject to offer capping. In shortage situations, prices

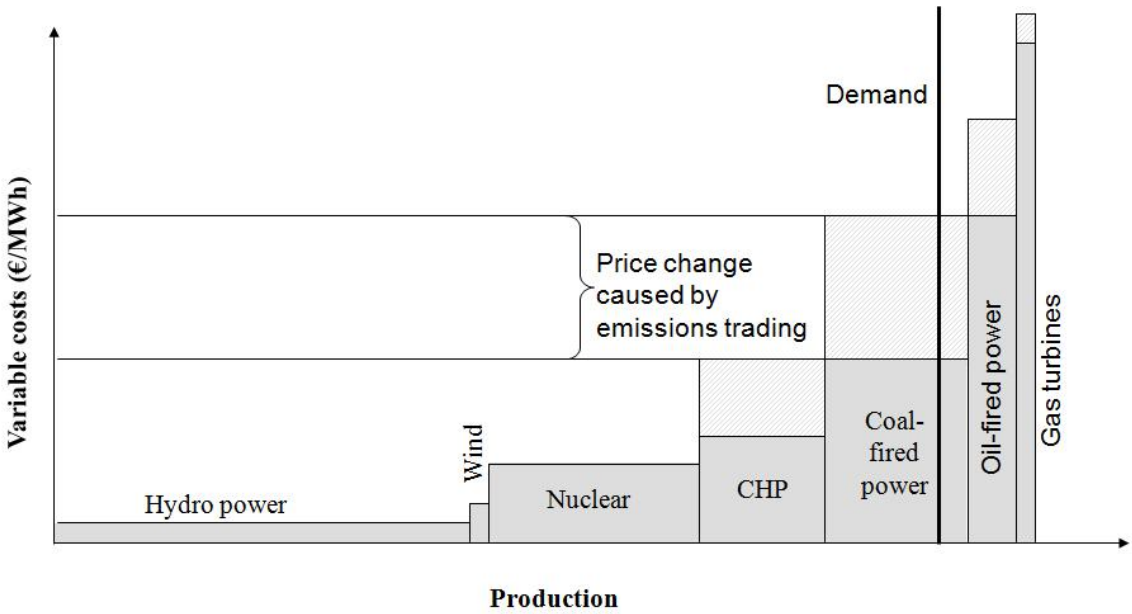

Figure 2.3: Principles of marginal price formation in the Nordic electricity market (source: publication **IV**).

are allowed to peak and the demand's willingness to pay for electricity settles the market price. During these shortage hours, generators are able to earn profits on their fixed costs. Separate capacity markets are not considered necessary as the energy market alone, by default, provides the generators with adequate revenues that facilitate new entry and enable maintaining the existing power plants in operation.

2.4 Modeling electricity spot prices

The emergence of spot prices is the main consequence of electricity market deregulation. Studies reveal that even though in some markets it was possible to lower the spot price levels through market deregulation, the competitiveness on the market increased price volatility. And it is that variability and prominent spikes that are the most difficult phenomena to model and predict. Some studies seek their origin in the uniform auction type implemented in spot markets and propose a discriminatory price auction as an alternative that would lead to eliminating spike occurrence (Mount, 2001).

Nevertheless, an ability to forecast spot and forward prices is of high importance and to have any predictive skill one needs a proper model. Researches show that spot and forward prices are strongly related, though forward prices tend to be higher than spot prices (Botterud et al., 2010). The relationship between them can be explained by the deterministic factors such as hydrological water storage and demand. Most recent studies focus on seeking the best approaches for day-ahead price forecasting, as the spot price's high volatility and prominent spikes are the basic risk factors for market participants. Their main cause is the non-storability of electricity, but also the competitive character of the deregulated markets. A big number of traders can significantly lower the mean price level (as proven for

Nord Pool case by Makkonen and Lahdelma (2001)), but it will also make it more volatile at the same time (Ruibal and Mazumdar, 2008).

Most recently proposed approaches are based on background deterministic variables known to be influencing electricity prices, such as demand (Vucetic et al., 2001) together with its slope, curvature and volatility (Karakatsani and Bunn, 2008), production type (Batlle and Barquin, 2005; Hreinsson, 2009), temperatures (Ruibal and Mazumdar, 2008), and other different climatic factors (Laitinen et al., 2000). To reduce electricity price forecasting errors, one can also account for known types of spot price periodicity. Among those, seasonal weather influence (Zhou and Chan, 2009), as well as weekday effects (Mandal et al., 2007) have been considered. More discussion on research in this field is provided in Section 3.2.1. In this area stochastic factor models were found reasonable for mid-term price estimation (Vehviläinen and Pyykkönen, 2005). Moreover, research has demonstrated that electricity production type can have a significant influence on the prices, especially in markets with a high share of renewable energy sources (Sensfuß et al., 2008). Other factors that have an impact on spot prices are of a more economic, technical, strategic or risk character, and their role can be dynamic over time (Karakatsani and Bunn, 2008).

One spot price feature that has received a lot of attention is mean-reversion. That is, even if the price spikes by a ten-fold increase overnight, it will eventually relax back to the previous level. The most common base for modeling this feature is a mean-reverting Ornstein-Uhlenbeck (OU) process (which will be presented mathematically in Section 3.4.4). Of course, it can capture only one of many spot price characteristics. Thus it is often combined with other processes. An example can be an OU process with a compound Poisson process to capture the spikes. The model parameters are modulated through a hidden Markov chain (Erlwein et al., 2010). This group of models is called hidden Markov models (HMM). The regimes can, for instance, be switching between a univariate process of the regular price and a bivariate process of the spiky regime (Haldrup and Nielsen, 2010). Erlwein et al. (2010) proposed to apply a model on deseasonalized Nord Pool prices and proved sufficient in capturing basic spot price characteristics, that is mean reversion and spikes. The jump diffusion models are claimed to perform better than the regime switching approaches (Weron et al., 2004).

When comparing basic stochastic models in the Ornstein-Uhlenbeck form with regime-switching proposals, the latter outperform the former (Higgs and Worthington, 2008). Also, mean reversion is not the same for spiky and non-spiky observations in the price series. Moreover, the variation of spikes seems to be very strong in different trading periods and they are often related to extreme weather events. For instance, jumps are more likely in warm and cold months as the demand grows then due to the use of air-conditioning use or heating, respectively. Some researchers use ARMA to describe the price base behaviour (non-spiky regime) and then employ simple probabilistic models for spike generation (Cuaresma et al., 2004). In other works it can be found that in terms of model fit, more elaborate approaches like k-factor GIGARCH outperform the traditional ones, e.g. SARIMA-GARCH (Diongue et al., 2009; Swider and Weber, 2007).

Another common spot price model categorization is dividing them into parametric and semiparametric models. A wide group of those was compared for two data sets, Californian and Nord Pool electricity prices, and it was found that the latter outperform the former (Weron and Misiorek, 2008), having SNAR/SNARX models in the lead. The results were robust for both point and interval prediction, which were verified through Weekly-weighted Mean Absolute Error (WMAE). It was also concluded that electricity consumption is a lot more accurate as an explanatory variable than air temperature, even in markets highly dependent on weather conditions, like Nord Pool.

Within last decades the discipline of evolutionary computation has been developed and used in countless applications. Recently it has been proposed to use evolutionary strategies for forecasting electricity spot prices (Unsihuay-Vila et al., 2010). It has been shown that this approach works a lot better than the classical ARIMA models or artificial neural networks. The results were confirmed for three different data sets. However, the classical time series models are still useful for simple comparison of data sets from different markets (Park et al., 2006). Another novel method proposed for electricity spot prices is a Takagi-Sugeno-Kang (TSK) fuzzy inference system in forecasting the one-day-ahead real-time peak price, which beats the classical time series approaches as well as neural network models (Arciniegas and Rueda, 2008). Also, wavelet transform has been found useful for data price series preprocessing, before model fitting (Schlueter, 2010).

A lot of efforts have focused on investigating spot price interdependencies. For instance, the New Zealand spot prices can be divided into five intraday groups: overnight off-peak, morning peak, day-time off-peak, evening peak, and evening off-peak. Then it appears that prices within these groups are a lot more correlated than between these groups along different trading periods (Guthriea and Videbeck, 2007). The authors also showed that spikes in the peak hours are significantly larger but less persistent when compared with off-peak hours. Another paper analyzes a group of models classified as Markov regime-switching (MRS) (Janczura and Weron, 2010). The authors focus on the performance of different models in terms of statistical goodness-of-fit and find that the best one is an independent spike 3-regime model with time-varying transition probabilities, heteroscedastic diffusion-type base regime dynamics and shifted spike regime distributions.

Many spot market price series reveal not only high but also non-constant variance. This can be modeled with the use of generalized autoregressive conditional heteroscedastic family of models (GARCH). In some countries there is a number of electricity markets. Through a GARCH approach one can find the non-constant variance estimates and compare them for different markets. In the case of Australia, it appears that information on what is happening on some of the markets can rarely be used to predict other markets' behaviour (Higgs, 2009). Moreover, spikes occurring in markets individually are usually more persistent than those coming in all the markets simultaneously (Worthington et al., 2005).

As no perfect model for short term spot price forecasting has been found so far, it is crucial from the risk-management point of view to know at least confidence intervals of the computed predictions (Zhao et al., 2008). Some have proposed price distribution forecasting through combination of Markov Chain Monte Carlo methods with multivariate skew t distribution (Panagiotelis and Smith, 2008), which is supposed to account for price skewness. Also, being able to model long-term price trajectories is equally important. The latter has been proposed through, for example, a price duration curve approach (Valenzuela and Mazumdar, 2005). A common measure of risk in financial markets in general is value-at-risk. This has been suggested as most efficient for electricity spot prices when based on extreme value theory (Chan and Gray, 2006). Along with forecasting efforts, one should be aware of any possible economic impacts of electricity market price forecasting errors

(Zareipour et al., 2010). Each new model always has to be revised in an on-going fashion because, as it is later discussed in publication **IV** and Section 3.2.2, the influence of price driving factors, as well as new economic situation and policies, can significantly change model parameters and, hence, its forecasting performance.

CHAPTER III

Classical approaches in modeling electricity spot prices

This chapter reviews some common spot price models and their application to different electricity spot price time series. In particular, Section 3.2 discusses deterministic factors that influence electricity spot prices and proposes a multiple regression model to remove those effects from price series. Section 3.3 presents details and application of ARMA and GARCH models. In Section 3.4 the mean reverting Ornstein-Uhlenbeck model is introduced, with both white and coloured noise. A multiple mean reversion variation of this model is presented in Section 3.5. Section 3.6 discusses analysis of deterministic indicators for possible two-regime models.

3.1 Basic statistical features of electricity spot prices

This section introduces the Nord Pool electricity spot price time series, which is used in further analyses. Basic statistical features of the data are presented.

3.1.1 Prices and price log-returns

Starting with visual investigation of the data, Figure 3.1 illustrates the Nord Pool system spot price over the period from 1 Jan 1999 until 28 Feb 2009. The series is clearly non-stationary, that is, its mean value does not remain constant over time. Globally, the data seems to have an upward trend, but there are also distinctive local trends in different periods. These, especially in the first few years, are highly related to seasons, with prices reaching higher levels in winter and lower in summer.

The series is also non-stationary with respect to variance. It may not be immediately seen from the prices, but their transformation to logarithmic returns reveals the high volatility as plotted in Figure 3.2. Variance in the series is not constant. This feature is called *heteroscedasticity*. Also, there are visible periods of low and high variance, which is also referred to as *variance clustering*. Moreover, both prices and price returns show prominent spikes. When the series jumps, it always comes back to the previous mean level in a short time.

Next, basic statistics are computed for distributions of both price and return series. These are collected in Table 3.1. The fact that the time series have spikes is reflected in the high

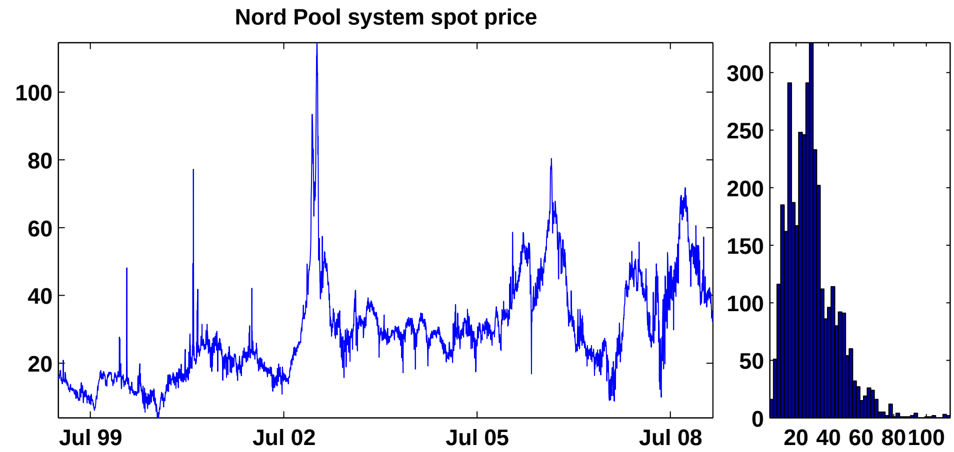

Figure 3.1: Nord Pool Spot daily system price.

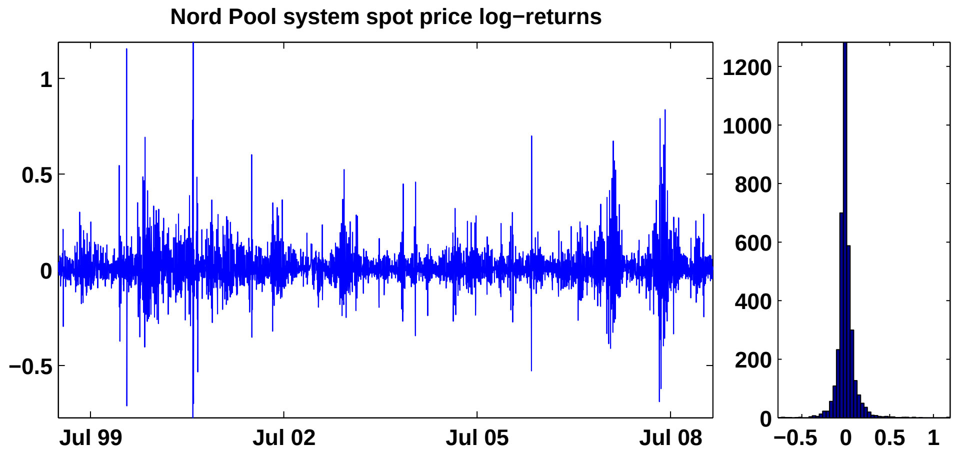

Figure 3.2: Nord Pool Spot daily system price log-returns.

values of kurtosis for both prices and returns. From Figure 3.2, it is visible how strongly leptokurtic the return distribution is. Also, both series are positively skewed. The skewness in the case of prices causes the histogram to have a shape close to log-normal.

Table 3.1: Basic statistics of Nord Pool Spot system price and price logarithmic returns.

	Prices	Price returns
Mean	29.4141	0.0002
St. dev.	14.7107	0.1017
Skewness	1.2176	1.5770
Kurtosis	5.6114	24.0171

One can also verify the interdependencies in the price series. As presented in Figure 3.3, the data is strongly autocorrelated. These shapes of autocorrelation function (ACF) and partial autocorrelation function (PACF) confirm the fact that prices are not stationary.

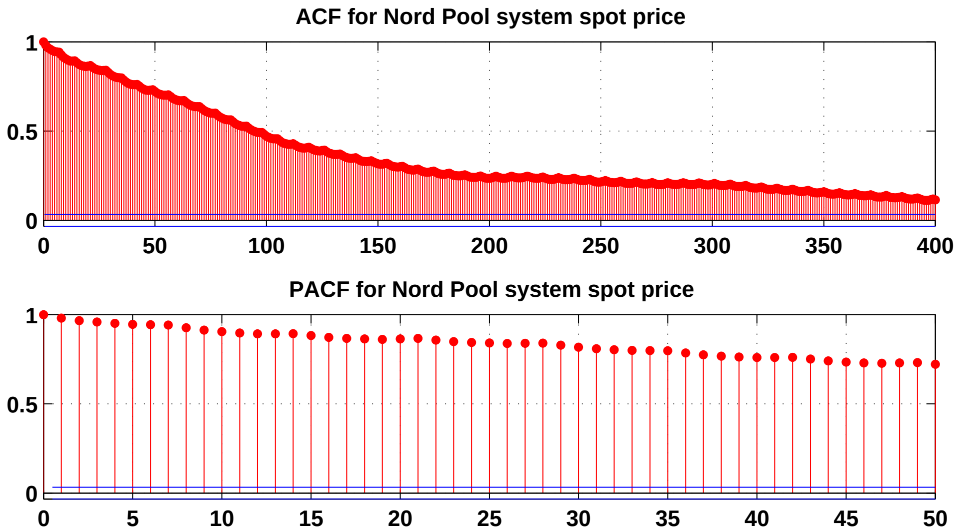

Figure 3.3: Autocorrelation and partial autocorrelation function for Nord Pool Spot daily system price.

When the prices are transformed to logarithmic returns, one can find a strong weekly periodicity in the data. This is revealed by the significant spikes in ACF and PACF at every 7th lag, as visible in Figure 3.4. There is also a slight annual dependence.

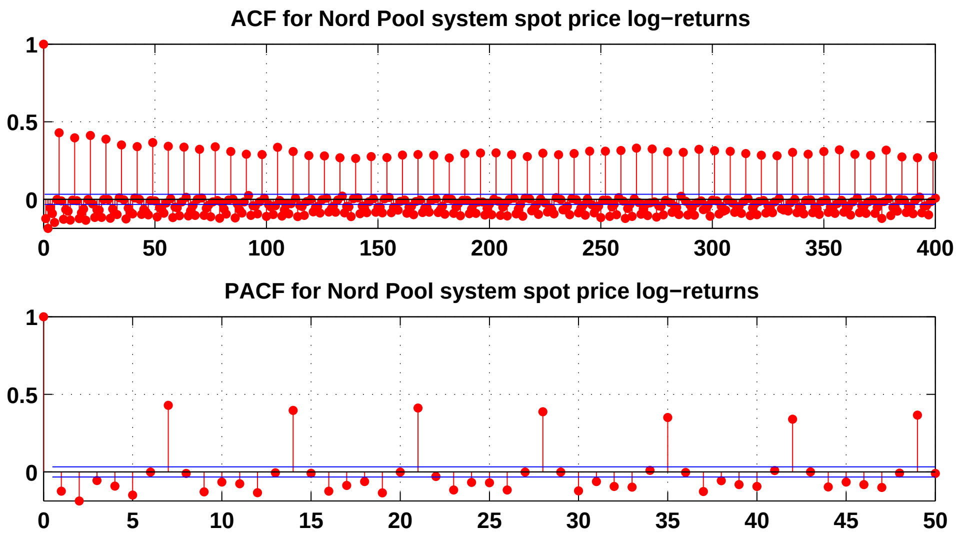

Figure 3.4: Autocorrelation and partial autocorrelation function for Nord Pool Spot daily system price log-returns.

3.1.2 Price spikes

As already mentioned, both prices and price returns have prominent spikes. And it is those spikes that form the main focus of this thesis. Here, their basic features are presented. A spike is understood as an observation that exceeds the mean value of its neighborhood by more than twice its standard deviation. The neighborhood is understood as a range of observations before and after the spike. For instance, a window $w = 30$ means the horizon of approximately a month around the observation, that is, half a month before and half a month after the given time point. Then the spike value is calculated as the difference between the spiky observation and the window mean value.

Figure 3.5 shows results of such spikes extraction. Clearly, spikes are not uniformly distributed in time. They seem to cluster on a non-regular basis. When the analysis window is changed to two months ($w = 60$) the number of spikes decreases, as illustrated in Figure 3.6. However, the clustering is still visible.

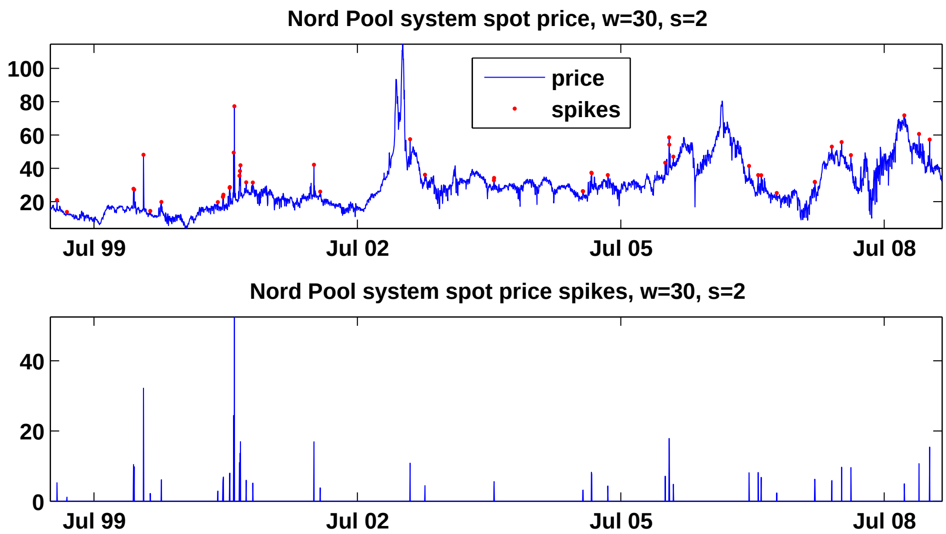

Figure 3.5: Spikes in Nord Pool Spot daily system price with analysis window $w = 30$ and standard deviation threshold $s = 2$.

For both cases, the spike distributions are constructed, as plotted in Figure 3.7. It seems that the size of jumps, especially the most prominent ones, could be approximated by an exponential distribution.

Finally, we take a closer look at the spike microstructure. Figure 3.8 illustrates the four most prominent spikes in the Nord Pool daily system price. It is clearly visible that the spikes are not symmetric. That is, they rise within one day and need from two to four days to relax.

3.2 Multiple regression models – pure trading dynamics

Regression methods were first proposed by Legendre (1805) and Gauss (1809). They thus introduced the *method of least squares*. The main idea behind this approach is to express

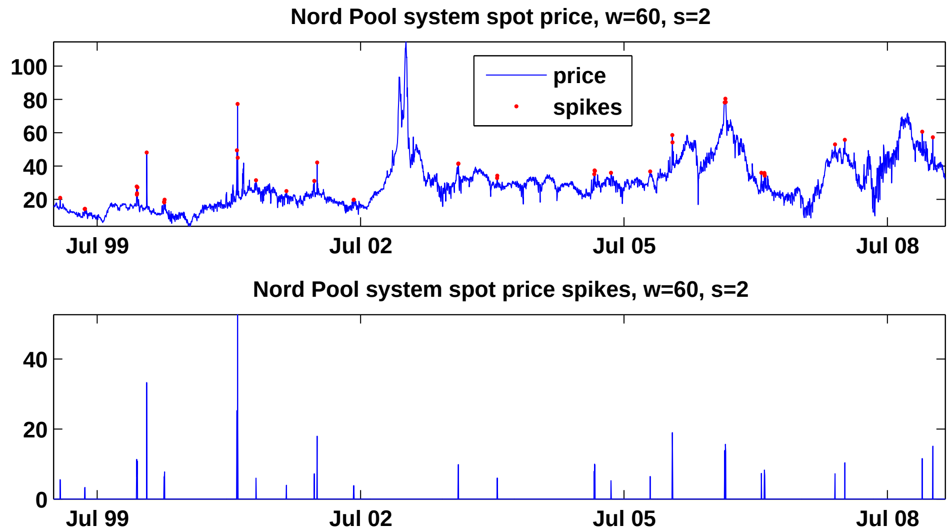

Figure 3.6: Spikes in Nord Pool Spot daily system price with analysis window $w = 60$ and standard deviation threshold $s = 2$.

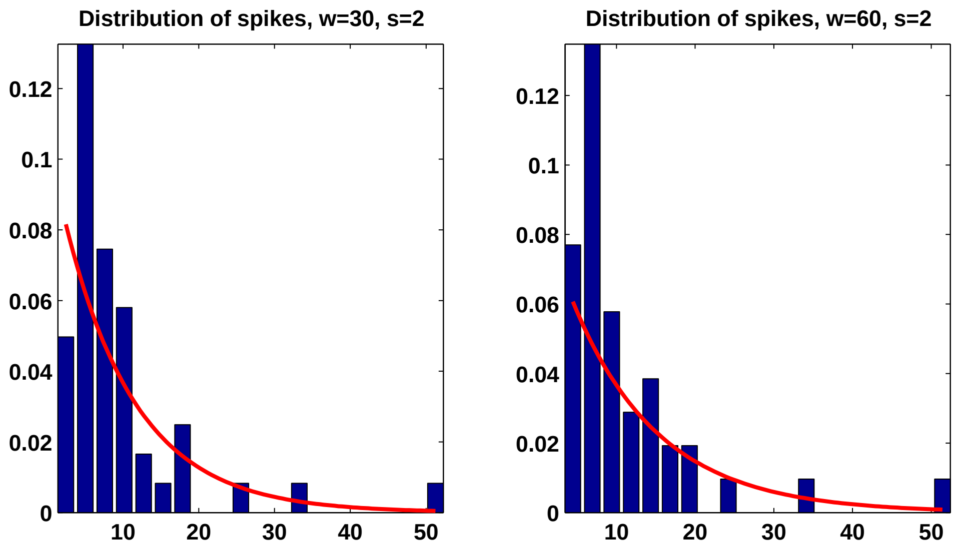

Figure 3.7: Distribution of spikes in Nord Pool Spot daily system price with analysis windows $w = 30$ and $w = 60$, and standard deviation threshold $s = 2$.

the relation between a dependent variable and one or more independent variables. A model that includes more than one variable is called a multiple regression model. This technique allows us to understand how the dependent variable changes when all but one of the independent variables are fixed.

The three sets of variables involved in a multiple regression model are:

- the dependent variable Y – to be explained by the model,

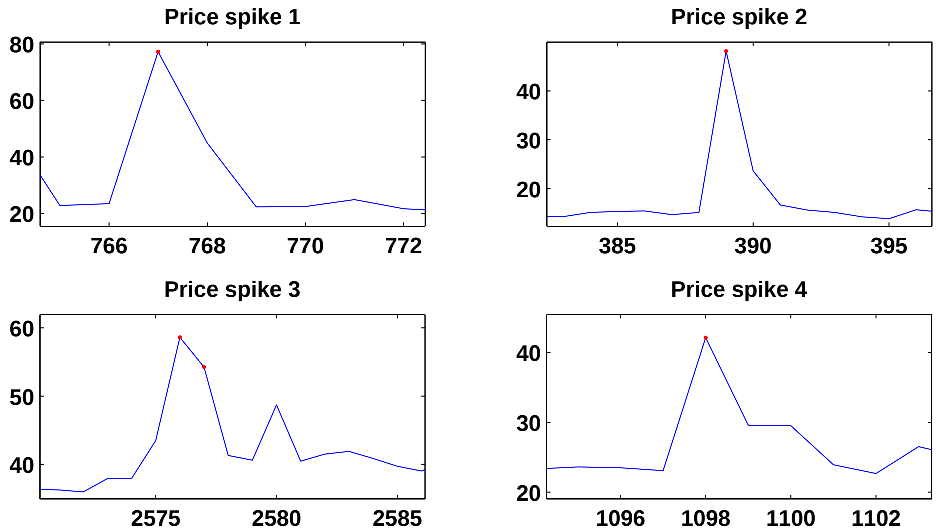

Figure 3.8: Four most prominent spikes in Nord Pool Spot daily system price.

- independent variables $\mathbf{X}$,
- unknown parameters β – to be estimated.

A regression model relates Y to $\mathbf{X}$ and β through a function f, that is $Y \approx f(\mathbf{X}, \beta)$. More formally, the point is to estimate the value of the dependent variable as a conditional expectation when the independent variables are fixed to given values. The model is derived from a set of observation values for Y and $\mathbf{X}$. Depending on the number of parameters β to be estimated, and the available number of observations, the system can be undetermined, exact or overdetermined. The last one is the most common case, where the method of least squares is applied to find the best values of parameters β.

Now, consider the linear time series regression model

$$Y_t = \beta_0 + \beta_1 X_{1t} + \ldots + \beta_k X_{kt} + \epsilon_t = X_t'\beta + \epsilon_t, \quad t = 1, \ldots, T \tag{3.1}$$

where $X_t = (1, X_{1t}, \ldots, X_{kt})'$ of size $(k+1) \times 1$ is the vector of explanatory variables, $\beta = (\beta_0, \beta_1, \ldots, \beta_k)'$ of size $(k+1) \times 1$ is the vector of coefficients to be estimated, and ϵ_t is a random error term. Note that the dimension $k+1$ comes from the fact that besides differently valued explanatory variables, we also allow a constant term in the model. In matrix form the model is expressed as

$$\mathbf{Y} = \mathbf{X}\beta + \epsilon \tag{3.2}$$

where $\mathbf{Y}$ and ϵ are $(T \times 1)$ vectors and

$$\mathbf{X} = \begin{bmatrix} 1 & X_{11} & X_{12} & \ldots & X_{1k} \\ 1 & X_{21} & X_{22} & \ldots & X_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{T1} & X_{T2} & \ldots & X_{Tk} \end{bmatrix}.$$

The standard assumptions of the time series regression model are:

i. the linear model (Equation (3.1)) is correctly specified,

ii. Y_t, X_t are jointly stationary and ergodic,

iii. the regressors x_t are predetermined: $E[X_{is}\epsilon_t] = 0$ for all $s \leq t$ and $i = 1, \ldots, k$,

iv. $E[X_t X_t'] = \Sigma_{XX}$ is of full rank $k + 1$, and

v. $X_t \epsilon_t$ is an uncorrelated process with a finite $(k+1) \times (k+1)$ covariance matrix $E[\epsilon_t^2 X_t X_t'] = S = \sigma^2 \Sigma_{XX}$.

The second assumption rules out trending regressors, the third rules out endogenous regressors but allows lagged dependent variables, the fourth avoids redundant regressors or exact multicollinearity, and the fifth implies that the error term is a serially uncorrelated process with constant unconditional variance σ^2. In the time series regression model, the regressors x_t are random and the error term ϵ_t is not assumed to be normally distributed.

3.2.1 Deterministic factors driving spot markets

There are many factors known to be influencing electricity spot prices. On the supply side, the variations are caused mainly by changes in fuel prices, the hydro situation and the prices of emission allowances. Historically, as Nord Pool is a hydropower-dominated market, deviations of water levels from their normal level have been reflected in Nord Pool electricity spot prices. Also, the introduction of emission trading of the EU changed the dynamics of the market, as depicted in Figure 3.9, and studied statistically in publication **IV**. The "hydro situation" here is understood as the level of hydrological storage reservoirs. Their deviation from normal level means that in a given week the level stays below or above the expected mean value for that specific week.

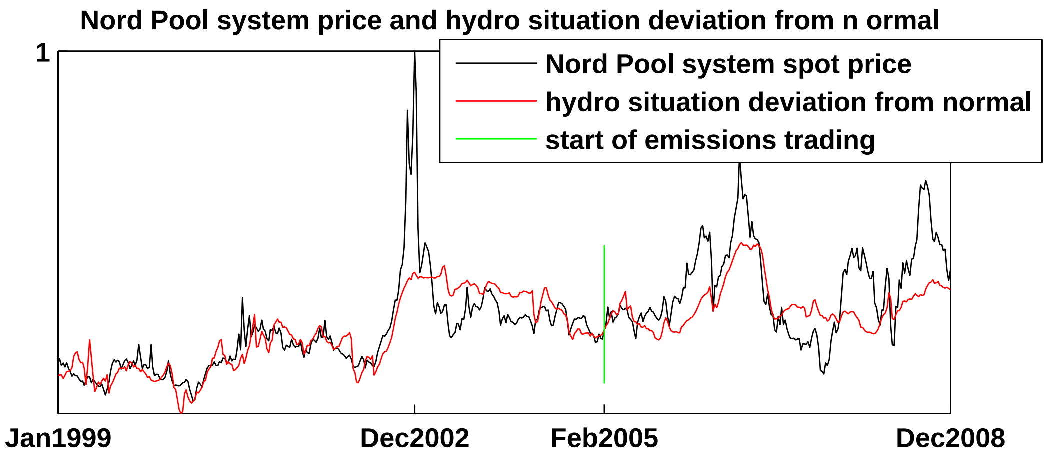

Figure 3.9: Normalized Nord Pool system price with respect to deviation of hydrological situation from normal (source: publication **IV**).

In a good hydro year, the electricity spot price, on average, is slightly below the marginal cost of a coal-fired power plant, including the cost of emissions. In a bad hydro year, on the other hand, the electricity spot price is little over the marginal cost of a coal-fired power plant, including the cost of emissions.

3.2.2 Pure spot market dynamics

As discussed in past sections, many different electricity spot price models have been applied to simulate spot market behavior. The purpose of this dissertation is to introduce a novel family of models that pinpoints to reflect the true market dynamics, independent of any known deterministic factors driving electricity prices. Therefore, any further modeling is done on Nord Pool electricity price time series which is detrended and deseasonalized with respect to any available background variables. The results of such decomposition can be found from Kirabo (2010) and publication **IV**.

The idea is based on classical time series theories (Box et al., 1994) as well as a novel approach with a moving regression component. This approach was introduced, because it can be seen that the effect of different factors on prices varies a lot over the years. Any single regression model describes poorly a price series that covers as much as 10 years of daily, let alone hourly, observations (Baya et al., 2009). With a six-monthly moving window for regression we can see clearly how hydrological storage information overruled the temperature variable in the winter time of 2002/03. Please, see Figure 3.10. This period faced a shortage of hydrological storage and a supposed market speculation.

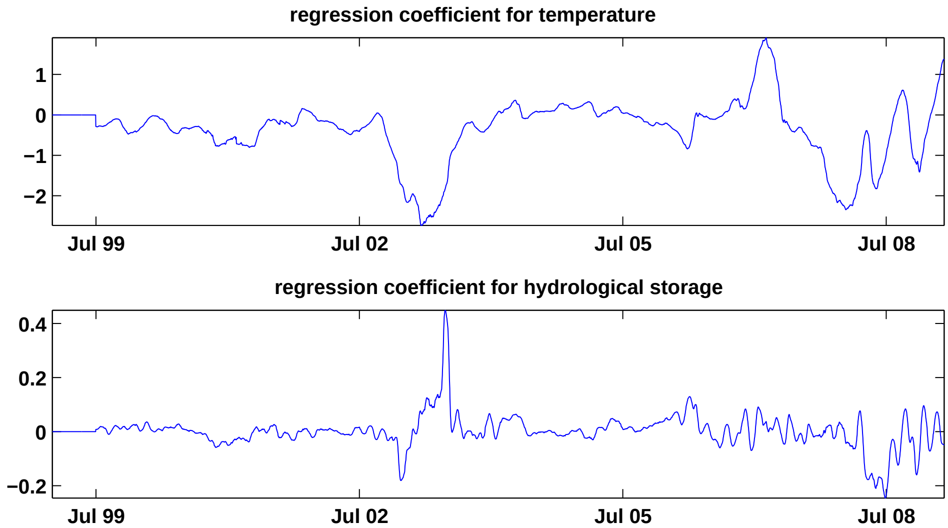

Figure 3.10: Moving regression coefficients for explanatory variables (source: publication **IV**).

The results of such a moving regression model applied to Nordic system price evolution are plotted in Figure 3.11. Depending on data availability, such a model can be produced for any other spot market, too. The residual series presented can be interpreted as reflecting the true character of electricity spot market dynamics and it is the one used in most of the further analyses in this thesis. One can see that background variables are not sufficient for modeling and predicting spot prices, as the resulting difference between the data and the fit still reveals non-constant mean level and significant price spikes in both upward and downward directions.

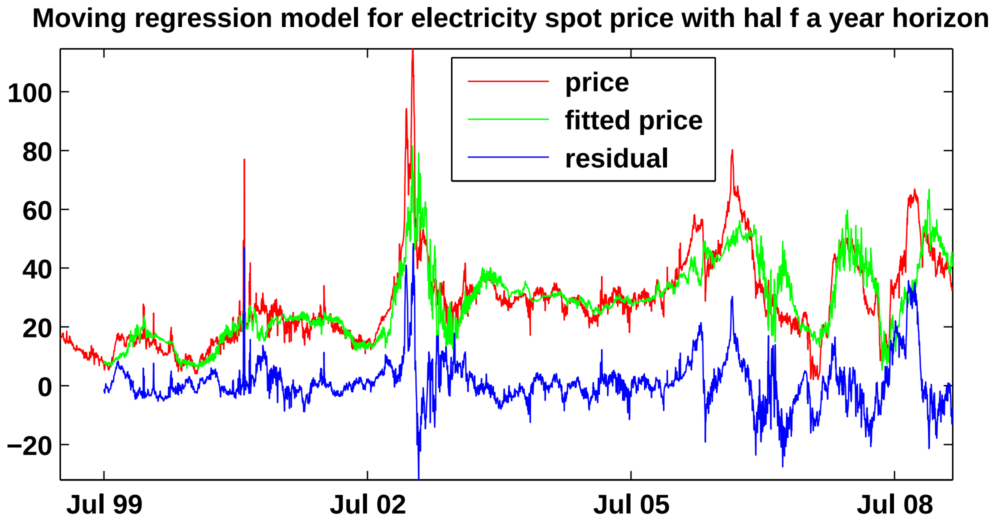

Figure 3.11: Moving regression fit for Nord Pool system price with half-a-year horizon window (source: Kirabo (2010)).

The residual series will from now be referred to as the *pure trading price* and will be used for calibrating the target model in this thesis.

3.2.3 Influence of CO_2 emissions trading on electricity spot price behaviour

As presented in Figure 3.9 of Section 3.2.1, one of the factors that has a profound influence on electricity spot market after emissions trading had been introduced. This study is done on the pure trading series, since this series should not contain any deterministic information any more. It is visible in Figure 3.11 that the performance of regression fit is not equal along the whole time horizon. There are periods with a distinctly poor fit. One of these is the fall-winter time of 2002–2003, and it is due to market speculation concerning water reservoirs level in Nord Pool. A second such period starts in the beginning of the year 2005 and continues throughout the remaining series part.

Indeed, February 2005 was the time when European Emission Allowances (EUA) trading was introduced to Nord Pool trading. Therefore, the time series can be split in two parts, one before and the other one after that date, and analyzed statistically. As publication **V** discusses, the difference between statistical features of the prices before (period 1) and after (period 2) February 2005 is significant. It is visible from Figures 3.12 and 3.13 presenting residual time series distributions for period 1 and period 2, respectively. The distribution of pure prices in period 1 is very close to normal, whereas the histogram representing period 2 is a lot more irregular, skewed and leptokurtic (having kurtosis higher than the normal value of 3).

Finally, it appears that price residual irregularity remains in the series even after EUA prices have been added as an additional variable in the regression model. The residual series after February 2005 still remain significantly different and a lot more irregularly distributed than the one before, as presented in Figure 3.14.

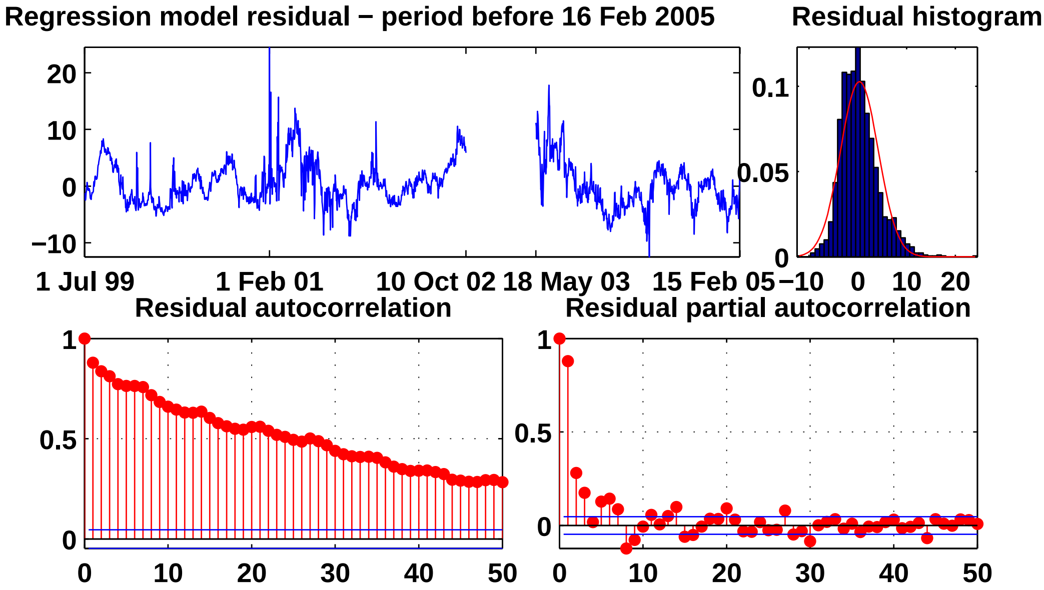

Figure 3.12: Residual price series for the period from 1 Jul 1999 to 15 Feb 2005, with accompanying statistics (source: publication **V**).

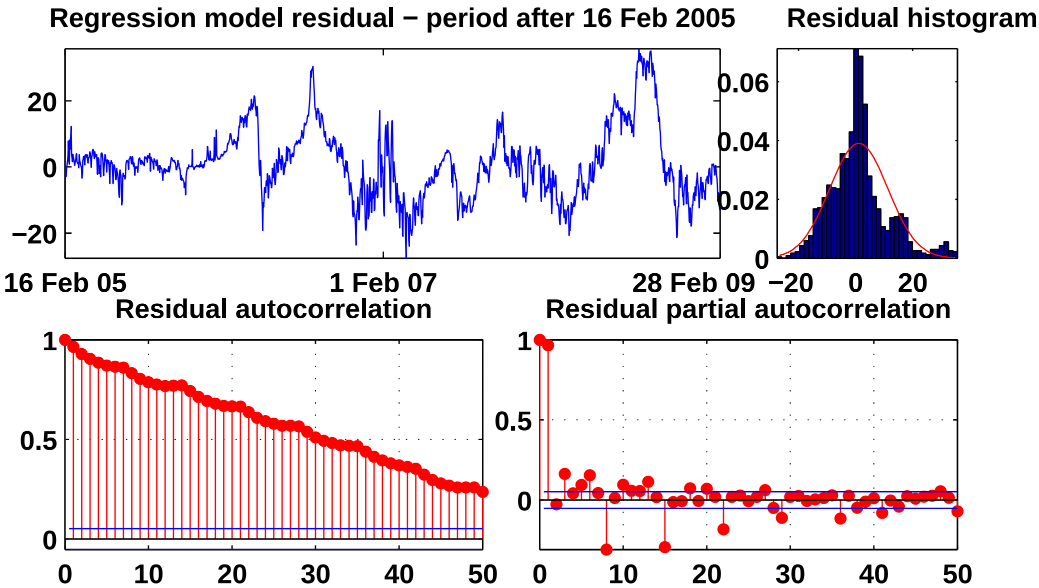

Figure 3.13: Residual price series for the period from 16 Feb 2005 to 28 Feb 2009, with accompanying statistics (source: publication **V**).

Clearly, a regression model defined uniformly for the whole 1999–2008 period loses its fitting skills from the beginning of February 2005. This leads to the conclusion that spot markets have adopted distinctly different dynamics since emissions trading started, and the influence of EUAs is deeper than a simple regression relation. Perhaps it has more of

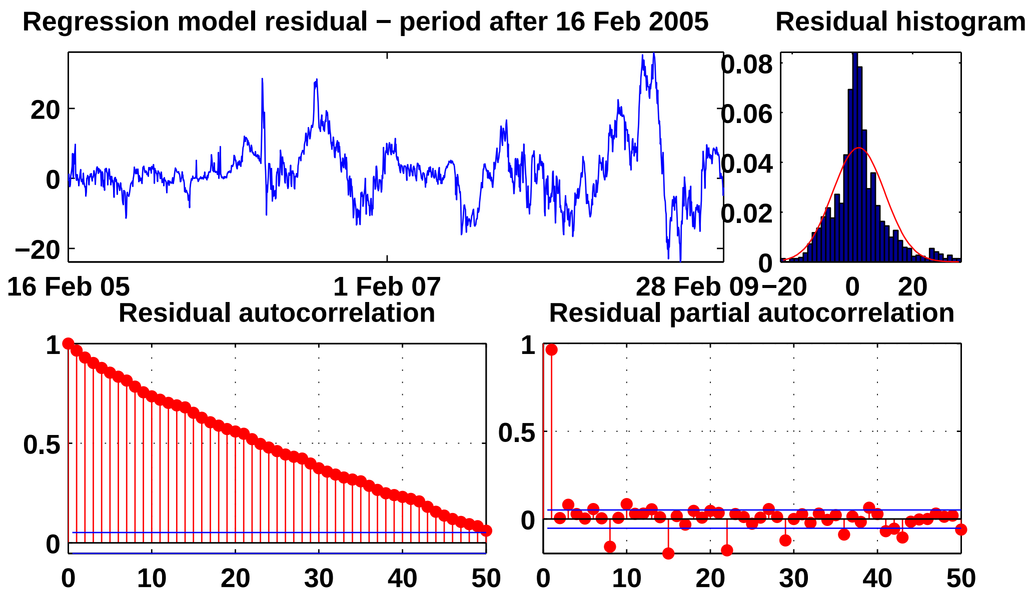

Figure 3.14: Residual price series for the period from 16 Feb 2005 to 28 Feb 2009, with its accompanying statistics, with emission allowance prices included in the regression model (source: publication **V**).

speculation character, similar to the period of fall-winter time of 2002–2003.

3.3 The classical time series models – ARMA and GARCH

3.3.1 Basic models - ARMA

A time series is a collection of observations at regular intervals, *e.g.* hourly, daily, monthly, annually, *etc*. Classical Box-Jenkins type time series analysis (see Box et al. (1994)) considers fitting Autoregressive (AR) and Moving Average (MA) models to such a time series. Basically, their main goal is to analyze the data in order to find dependencies between current and historical observations. These models can also be extended by associated heteroscedastic models when the data variance is not constant over time. These are discussed in more detail in Section 3.3.3.

An Autoregressive model represents a current observation in terms of lagged past realizations of a given process. An autoregressive model of order r, *i.e.* AR(r), is introduced by the following definition

- $x_t = C + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_n x_{t-r} + u_t$
- $u_t \sim N(0, \sigma^2)$ – white noise

A Moving Average model, on the other hand, states that a given observation is not related to the previous process realizations but to the historical values of process noise. A moving average model of order m, *i.e.* MA(m), is introduced by the following definition

- $x_t = C + \psi_1 u_{t-1} + \psi_2 u_{t-2} + \ldots + \psi_n u_{t-m} + u_t$
- $u_t \sim N(0, \sigma^2)$

The AR and MA models may also be combined together to create the Autoregressive Moving Average models (ARMA(r, m)), which join the properties of previously presented ones.

- $x_t = C + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_n x_{t-r} + \psi_1 u_{t-1} + \psi_2 u_{t-2} + \ldots + \psi_n u_{t-m} + u_t$
- $u_t \sim N(0, \sigma^2)$

The main assumption for this approach is that the residuals of models mentioned above are white noise – normally distributed random numbers. Therefore, the r lags of series observations and m lags of white noise are to be introduced so that the remaining residuals are purely random. Moreover, both AR(r) and MA(m) are special cases of ARMA(r, m) model, *i.e.* ARMA($r, 0$) and ARMA($0, m$) applied simultaneously.

3.3.2 Preparing Box-Jenkins models

Each attempt to fit an ARMA model to a given series consists of a full set of pre-analysis and fitting steps. There are certain requirements concerning the data, such that they make it possible to find a well fitting ARMA model.

The first prerequisite is that the series is stationary, *i.e.* the mean value and standard deviation remain constant in the series over time. There are certain statistical tests making it possible to verify hypotheses whether a series is stationary or has a unit root. If data appear to be non-stationary, the easiest way is to create an integrated series (a series of differences). Basically, the matter is to eliminate trend from the data. There also happens to exist strong seasonality in the observations, which is why seasonal differencing might be necessary. When it comes to financial time series, the differences are usually made on the logarithm of the series to create a series of the so-called logarithmic returns.

If the series is stationary, the next step is to analyze the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the series. Based on that, a decision is made to choose proper orders of the ARMA(r, m) model. Then the process moves on to parameter estimation for the chosen model. Finally, a forecast is prepared. However, ARMA models need to be monitored in an on-going manner so that amendments can be done to parameter values, if necessary.

3.3.3 ARCH/GARCH modeling

Not all time series can be explained by ARMA models. Sometimes they reveal some non-stationarity in terms of volatility, *i.e.* the series variance is not constant and it depends on its historical values. One of the first approaches aimed at modeling heteroscedasticity are Autoregressive Conditional Heteroscedasticity models, also known as ARCH (see R.F. (1982)) and Generalized Autoregressive Conditional Heteroscedasticity, or GARCH (see Bollerslev (1986)) models. A good survey of modern variations of these models can be found in Tsay (2005).

An ARCH model (see R.F. (1982)) represents the variance of the current error term as a function of the variances of previous time period error terms. ARCH simply describes the error variance by the square of a previous period's error. These types of models are widely used for time series that display so-called variance clustering, *i.e.* noticeable periods of higher-lower variance in the series. In general, an ARCH(q) model is represented as follows

- $u_t = \sigma_t z_t$
- $\sigma_t^2 = K + \alpha_1 u_{t-1}^2 + \ldots + \alpha_q u_{t-q}^2$,

where u_t is the corresponding ARMA(r, m) model residual series, $z_t \sim N(0, 1)$ and σ_t^2 are the variance estimates for time points t.

A model is a GARCH model (see Bollerslev (1986)), if second Autoregressive Moving Average model (ARMA-type model) is adopted to represent error variance. In that case, the GARCH(p, q) model (where p stands for the order of the GARCH terms σ_t^2 and q stands for the order of the ARCH terms u_t) is given by:

- $u_t = \sigma_t z_t$
- $\sigma_t^2 = K + \alpha_1 u_{t-1}^2 + \ldots + \alpha_q u_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_q \sigma_{t-p}^2$

The models presented above are the most popular ones for explaining heteroscedasticity in time series. Usually, GARCH$(1, 1)$ is sufficient as a compromise between simplicity of a model and its satisfactory fit to the empirical data. One of the best arguments supporting this choice is Albert Einstein's statement that the model should be "as simple as possible – but not more simple than that". However, any specific model order can be determined based on appropriate information criteria.

3.3.4 Markov Chain Monte Carlo methods

Monte Carlo is a general name for statistical methods that concern sampling random numbers in order to investigate a given problem. They are used for problems with complicated analytical representation (Hubbard, 2007) or in order to complement theoretical derivations. Also, Monte Carlo approaches are recommended for modeling phenomena that are characterized by high uncertainty of their inputs. The initial step of the approach is to define a domain for such inputs. Then the input values are generated randomly from a probability distribution that is known or assumed *a priori*. These values are used in model computations based on a deterministic model of the problem. Finally, the results are aggregated and analyzed.

Monte Carlo methods are used in different fields like physical sciences, engineering, computational biology, applied statistics, games, finance and business, and many more. In mathematics, they are most commonly used in integration, optimization, inverse problems and computational mathematics. Monte Carlo simulations are often combined with the theory of *Markov chains*, and thus construct the so-called Markov Chain Monte Carlo (MCMC) methods. These aim at creating specific types of Markov chains that can represent the posterior distributions of the model parameters (Solonen, 2006). A Markov chain is a stochastic process whose next state depends only on the current state and not on any other past values. That creates a definition of the Markov property (3.3).

$$P(X^{(t+1)} = s_{t+1}|X^{(0)} = s_0, X^{(1)} = s_1, \dots, X^{(t)} = s_t) = P(X^{(t+1)} = s_{t+1}|X^{(t)} = s_t). \tag{3.3}$$

MCMC runs create chains based on different algorithms. One of the most common ones, and used in one of the further studies, is presented here – the Random Walk Metropolis Hastings algorithm (Solonen, 2006):

Step 1: Initialization

- Choose θ_0, then set $\theta_{old} = \theta_0$
- Choose the covariance matrix for proposal distribution C
- Choose the number of MCMC runs, that is the length of the chain M, and set $i = 1$

Step 2: Acceptance step (Metropolis step)

- Choose sample θ_{old} from $N(\theta_{old}, C)$ and u from $\mathrm{U}[0, 1]$
- Calculate $\mathrm{SSQ}_{\theta old}$ and $\mathrm{SSQ}_{\theta new}$
- If $\mathrm{SSQ}_{\theta new} < \mathrm{SSQ}_{\theta old}$ or $u < e^{-\frac{1}{2\sigma^2}(\mathrm{SSQ}_{\theta new} - \mathrm{SSQ}_{\theta old})}$, set $\theta_i = \theta_{new}$. Else set $\theta_i = \theta_{old}$
- if $i < M$, set $i = i + 1$ and go to step 1. Else, stop the algorithm.

Where

- θ_0 is a vector of initial parameter values of the model,
- θ_{old} is a vector of the previous sampled parameter values,
- θ_{new} is a vector of the new sampled parameter values,
- M is the length of the chain,
- i is the number of iterations,
- u is the random value,
- $SSQ_{\theta_{old}}$ is the total sum of squares of previous sampled parameter values,
- $SSQ_{\theta_{new}}$ is the total sum of squares of new sampled parameter values.

3.3.5 Reliability of ARMA-GARCH models for electricity spot prices

The choice of appropriate ARMA-GARCH models is very difficult for most real-life data sets. The choice is done either based on the ACF and PACF functions, or on information criteria. The two most commonly used ones for ARMA models are Akaike Information Criterion (AIC) and Schwarz Criterion (also known as Schwarz-Bayesian Criterion, $SBIC$). The former is represented by the following formula

$$AIC = 2k + n\left[\ln\left(\frac{RSS}{n}\right)\right] \tag{3.4}$$

where k is the number of free parameters, n is the total number of observations and RSS stands for the residual sum of squares. It rewards goodness of fit, while penalizing an overly large number of free parameters. However, the penalty in AIC discourages overfitting less than the following Schwarz Criterion does

$$SBIC = \ln(\sigma_e^2) + \frac{k}{n}\ln(n) \tag{3.5}$$

where k and n have the previous meaning and σ_e^2 is the error variance.

For both approaches, the aim is to choose the model order that provides minimal values of AIC and $SBIC$. Of course, the suggested models may be different from the two methods and it is up to the analyst which one to favor.

When it comes to combined ARMA-GARCH models, the choice of model order is even more challenging. One can analyze specific statistical tests on the model residuals, such as Engle's test verifying existence of ARCH effect (non-constant variance) and Ljung-Box test checking for serial autocorrelation in the time series. Therefore, a new information criteria function, called SLEIC, was suggested in publication **I** as follows

$$SLEIC = [SBIC \cdot (1 + \frac{\alpha}{2N}\sum_{i=1}^{N}(H_{1,i} + H_{2,i}))] \tag{3.6}$$

where

- $H_{1,i}$: vector of logical outputs for Ljung-Box test, $i = 1, 2, ..., 2L$,
- $H_{2,i}$: vector of logical outputs for Engel's test, $i = 1, 2, ..., 2L$,
- α: importance coefficient of Ljung-Box and Engel's tests,
- N: Number of lags analyzed by Engel's/Ljung-Box test.

To find an appropriate model for electricity price series, one has to maximize SLEIC function while varying orders p, q, r and m of GARCH(p, q) and ARMA(r, m) models, $max_{P,Q}\, SLEIC(res, k, H_1, H_2)$.

Due to ARMA stationarity requirement, the methodology presented above is applied not to original spot prices, but to price logarithmic returns. As presented in publication **I**, optimal ARMA-GARCH models are chosen for two data series, daily New England Pool

(NEPool) and weekly Nordic Power Pool (Nord Pool). Then these best models are then implemented in a Markov Chain Monte Carlo (MCMC) framework, where the model parameters are varied around the original values to create parameter and forecast predictive distributions. There are 5000 MCMC realizations for a 22-day forecast for NEPool and a 10-week forecast for Nord Pool. They are presented in Figures 3.15 and 3.16 respectively.

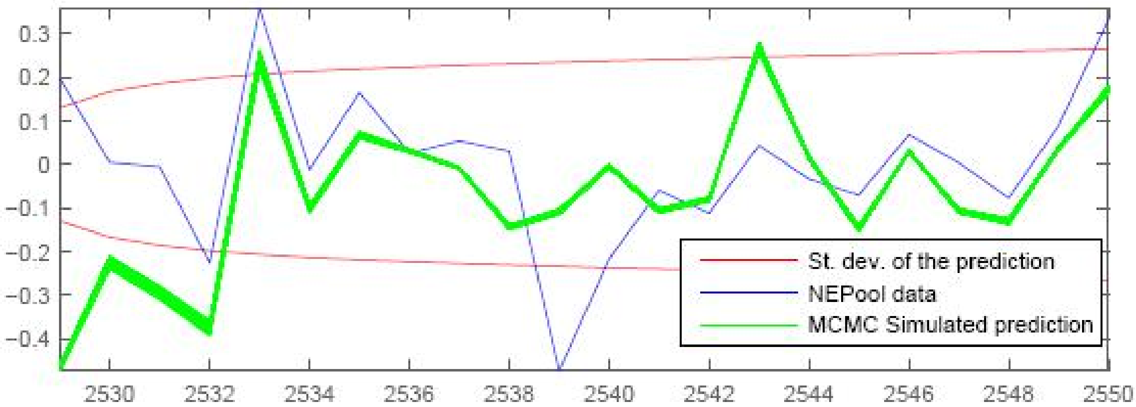

Figure 3.15: ARMA-GARCH simulation of 22 day forecast of NEPool price returns (source: publication **I**).

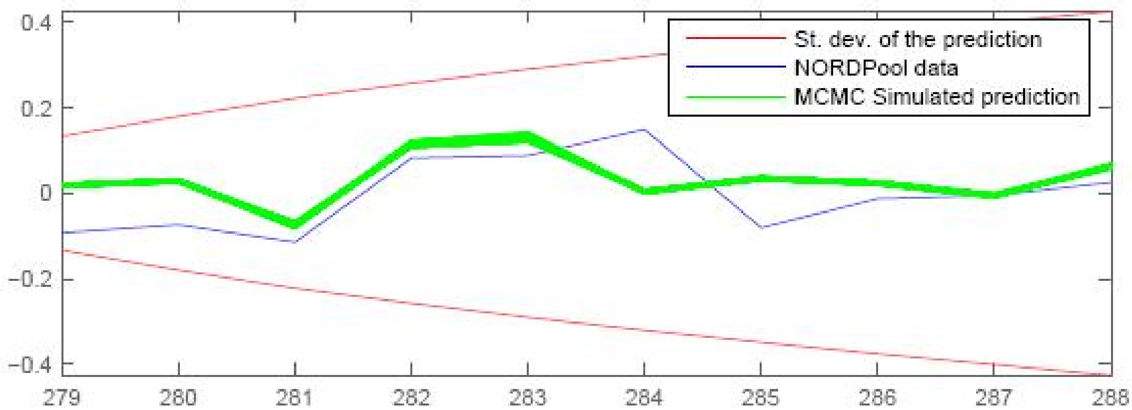

Figure 3.16: ARMA-GARCH simulation of 10 week forecast of Nord Pool price returns (source: publication **I**).

It has been shown that accurate prediction of spot prices with classical time series models is not possible. As discussed in publication **I**, basic time series models such as ARIMA can fit well the available data, but fail in forecasting, even when equipped with GARCH part. As visible in Figures 3.15 and 3.16, the MCMC simulation shows that predictive distribution (green) of ensemble forecast of the best models is not able to capture the true path (blue) of electricity spot price returns.

3.4 Stochastic differential equations – Ornstein-Uhlenbeck process with white and coloured noise

3.4.1 Stochastic processes

A group of models commonly applied in financial markets is represented by stochastic processes. A stochastic process with a measurable state space $(S, \mathcal{B})$ is a family (collection) of random variables $X_t, t \in T$ that is defined on the same probability space $(\Omega, \mathcal{F}, P)$, that is $X_t : (\Omega, \mathcal{F}) \to (E, \mathcal{B})$ for $t \in$ T. The process is called a discrete parameter process if $T \in \mathbb{N}$. If T is not discrete, the process is continuous. The time is represented as index t, and then X_t is the "position" or the "state" of the process at time t. The state space is $\mathbb{R}$ in most cases, and then the process is real-valued. There are also examples where S is a finite set, for instance a set of natural numbers or integers. Then the process is called a counting process. The mapping for every fixed $\omega \in \Omega$ on the parameter set T is called a realization or trajectory of the process.

A stochastic process $\{W_t, t \geq 0\}$ is known as Brownian Motion if it satisfies the following conditions:

i. $W_0 = 0$ almost surely,

ii. $(W_t)_{\mathbb{R}_+}$ is a process with independent increments,

iii. $W_t - W_s$ is normally distributed with $N(0, t-s)$, $(0 \leq s < t)$.

Wiener process $(W_t)_{\mathbb{R}_+}$ has expectation $E[W_t] = 0$ for all $t \in \mathbb{R}_+$ and covariance $K(s,t) = Cov[W_t, W_s] = \min\{s, t\}$ (Capasso and Bakstein, 2004).

Brownian Motion (also called Wiener process) is nowhere differentiable. It is also a common component of more elaborate stochastic models. Usually, stochastic processes are expressed in form of stochastic differential equations (SDE).

3.4.2 Numerical schemes for SDEs

Nowadays, mathematical modeling is done using numerical methods. Therefore, the choice of proper numerical schemes is very important also in the case of SDEs (Kloeden et al., 2003; Milstein, 1995). Here, four well-established approaches are briefly introduced.

EULER SCHEME

Consider a stochastic differential equation in the Itô form

$$dX_t = f(t, X_t)dt + g(t, X_t)dW_t, \quad X_{t_0} = x_0 \tag{3.7}$$

with the following solution

$$X_t = X_{t_0} + \int_{t_0}^{t} f(s, X_s)ds + \int_{t_0}^{t} g(s, X_s)dW_s \tag{3.8}$$

Then the solution can be approximated numerically with the Euler scheme

$$X_{n+1} = X_n + f(t_n, X_n)\Delta t_n + g(t_n, X_n)\Delta W_{t_n} \tag{3.9}$$

The Euler scheme is applicable only in the case of SDEs in Itô representation.

MILSTEIN SCHEME

The Milstein scheme is more accurate than the Euler scheme and can be used whenever the partial derivative of g over x is available. Then it takes the form

$$X_{n+1} = X_n + f(t_n, X_n)\Delta t_n + g(t_n, X_n)\Delta W_{t_n} + \frac{1}{2}g(t_n, X_n)\frac{\partial g}{\partial x}(\Delta W_{t_n}^2 - \Delta t_n) \quad (3.10)$$

HEUN SCHEME

The Heun scheme is often called the improved Euler scheme as it evaluates both the f and g functions at the current point as well as at the estimated succeeding point, and the results of both functions are averaged to get the definite rate of change. The formulation is as follows

$$X^*_{n+1} = X_n + f(t_n, X_n)\Delta t_n + g(t_n, X_n)\Delta W_{t_n} \quad (3.11)$$

$$X_{n+1} = X_n + \frac{1}{2}[f(t_n, X_n) + f(t_{n+1}, X^*_{n+1})]\Delta t_n + \frac{1}{2}[g(t_n, X_n) + g(t_{n+1}, X^*_{n+1})]\Delta W_{t_n} \quad (3.12)$$

In this approach, first a prediction of the next value of X is computed. Then this prediction is corrected and the final approximation of X_{n+1} is obtained.

RUNGE-KUTTA SCHEME

The Runge-Kutta method used for solving ordinary differential equations can also be transformed and applied to SDEs.

$$\begin{array}{ll}
F_0 = f(X_i, t_i), & G_0 = g(X_i, t_i), \\
X_i^{(0)} = X_i + \frac{1}{2}F_0\Delta t_i + \frac{1}{2}G_0\Delta W_i, & \\
F_1 = f(X_i^{(0)}, t_i + \frac{1}{2}\Delta t_i), & G_1 = g(X_i^{(0)}, t_i + \frac{1}{2}\Delta t_i), \\
X_i^{(1)} = X_i + \frac{1}{2}F_1\Delta t_i + \frac{1}{2}G_1\Delta W_i, & \\
F_2 = f(X_i^{(1)}, t_i + \frac{1}{2}\Delta t_i), & G_2 = g(X_i^{(1)}, t_i + \frac{1}{2}\Delta t_i), \\
X_i^{(2)} = X_i + \frac{1}{2}F_2\Delta t_i + \frac{1}{2}G_2\Delta W_i, & \\
F_3 = f(X_i^{(2)}, t_i + \Delta t_i), & G_3 = g(X_i^{(2)}, t_i + \Delta t_i),
\end{array}$$

$$X_{i+1} = X_i + \frac{1}{6}(F_0 + 2F_1 + 2F_2 + F_3)\Delta t_i + \frac{1}{6}(G_0 + 2G_1 + 2G_2 + G_3)\Delta W_i$$

The SDE studies presented in this work are solved with the use of the Euler scheme.

3.4.3 Maximum likelihood estimation of process parameters

Maximum likelihood is a well known statistical method of estimation of model parameters. Given a sample of n identically and independently distributed (i.i.d.) observations $x_1, x_2, \ldots, x_n$ from distribution $f(\cdot|\theta)$, one can specify the the joint density function for all observations

$$f(x_1, x_2, \ldots, x_n|\theta) = f(x_1|\theta) \cdot f(x_2|\theta) \cdot \ldots \cdot f(x_n|\theta) \quad (3.13)$$

In the maximum likelihood approach, the x_i values are given as observations, and treated as fixed parameters in the distribution function. It is the parameter θ that is optimized to maximize the joint distribution function. Therefore, the likelihood function is understood in the following sense

$$\mathcal{L}(\theta|x_1, x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta) \tag{3.14}$$

Very often it is possible to find an explicit form of the likelihood function and maximize it analytically. Then, in many cases, it is easier to maximize the logarithm of the likelihood function, called the log-likelihood.

$$\ln \mathcal{L}(\theta|x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \ln f(x_i|\theta) \tag{3.15}$$

This is allowed due to the fact that logarithm is a monotone transformation.

3.4.4 Ornstein-Uhlenbeck processes

A natural alternative to Box-Jenkins time series models are models based on stochastic differential equations (SDE). The most common SDE model used for modelling electricity spot prices is the mean-reverting Ornstein-Uhlenbeck process $\{W_t, t \geq 0\}$ being the solution of the following stochastic differential equation (3.16). The initial value X_0 of the process $\{X_t, t \geq 0\}$ is a given random variable, possibly a constant, independent of the Wiener process $\{W_t, t \geq 0\}$.

$$dX_t = k(X^* - X_t)dt + \sigma dW_t \tag{3.16}$$

Here the price change dX_t is directed towards the mean reversion level X^* and being pulled towards it with rate $k > 0$. The Wiener process increment dW_t generated at each time t guarantees model stochasticity and $\sigma > 0$ is the volatility or average magnitude, per square-root time, of the random fluctuations. The parameter estimation for the model is most commonly done using maximum likelihood estimation which can be easily derived from SDE solution and the assumption that the process is normally distributed (Øksendal, 1995).

The basic mean reverting model, due to the constant character of mean reversion level and rate, cannot be applied to prices directly. It is most commonly used for logarithmic price returns. As presented in Figure 3.17, the ARMA-GARCH model residuals are not normal.

Like ARMA-GARCH models, standard OU models do not reproduce spot price dynamics very well. The process spikes too often and the magnitudes of spikes are too low, as depicted in Figure 3.18.

3.4.5 Ornstein-Uhlenbeck process with coloured noise

As the classical Ornstein-Uhlenbeck process fails to capture electricity spot price characteristics, there is a possibility to replace white noise in that process (the Wiener increments)

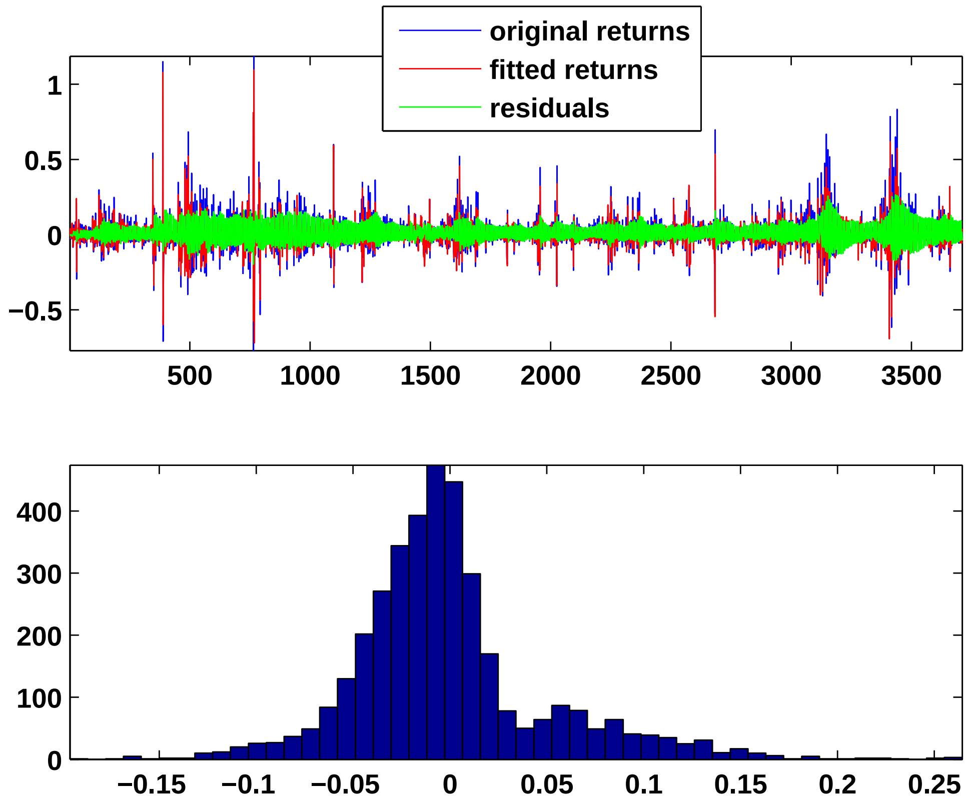

Figure 3.17: ARMA-GARCH simulation and model residuals for Nord Pool daily price (source: Naeem (2009)).

with coloured noise. A coloured noise process $\{\zeta_t, t \geq 0\}$ produces a sequence of correlated random variables $\zeta(t_1), \zeta(t_2), \ldots,$ each having the same standard deviation. Coloured noise is a Gaussian process. Moreover, it can be completely described by its mean and covariance functions (Arnold, 1974). The scalar exponential coloured noise process is given in the form of a linear SDE, specifically the Ornstein-Uhlenbeck process as follows

$$d\zeta(t) = -\frac{1}{\tau}\zeta(t)dt + \alpha dW_t \tag{3.17}$$

whose solution is

$$\zeta(t) = \zeta(0)e^{\frac{-t}{\tau}} + \alpha\int_0^t e^{-\frac{(t-s)}{\tau}}dW_s \tag{3.18}$$

where τ is the correlation time for coloured noise and α is the diffusion constant. The parameter τ indicates the time over which the process is significantly correlated in time. In case of daily electricity spot prices, τ can take value 7 due to the price weekly periodicity. W_t is a standard Wiener process with $dW_t \sim N(0, dt)$ for an infinitesimal time interval dt. For $t > s$, the scalar exponential coloured noise process in Equation (3.18) has the following mean, variance and autocovariance

- $\mathbb{E}[\zeta(t)] = \zeta(0)e^{\frac{-t}{\tau}}$

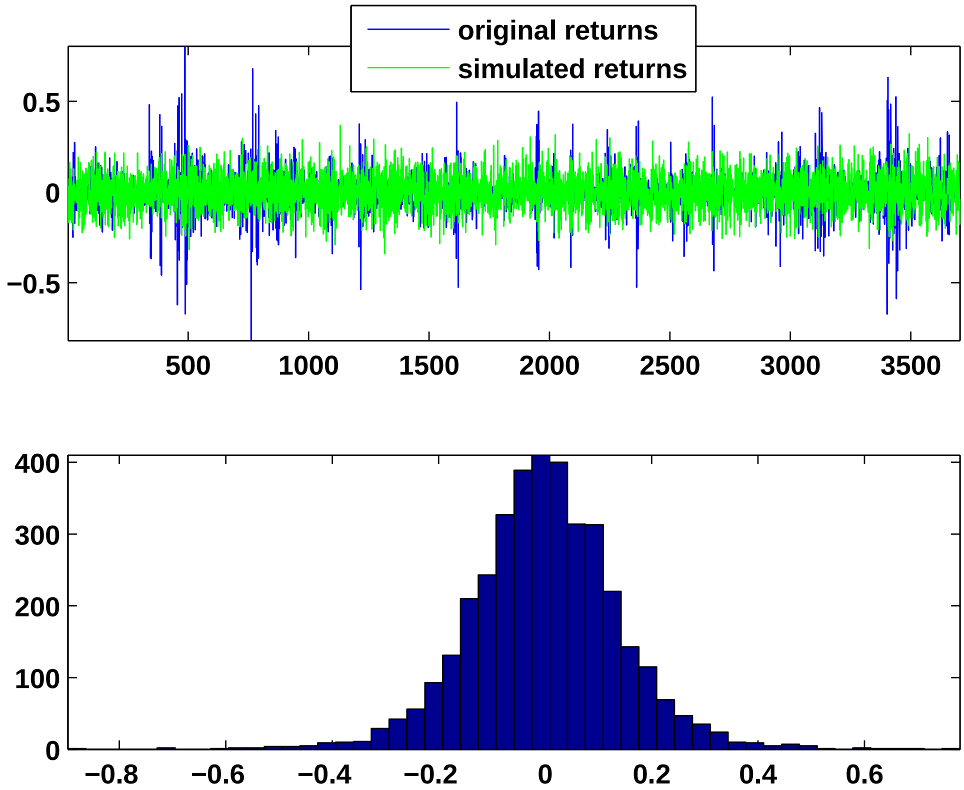

Figure 3.18: Ornstein-Uhlenbeck simulation and model residuals for Nord Pool daily price (source: Naeem (2009)).

- $Var[\zeta(t)] = \frac{\alpha^2\tau}{2}(1 - e^{-\frac{2t}{\tau}})$
- $Cov[\zeta(t), \zeta(s)] = \frac{\alpha^2\tau}{2}e^{-\frac{|t-s|}{\tau}}$

Knowing that prices have strong weekly periodicity, which also affects autocorrelation, it can be assumed that the colour of the noise would carry a similar feature. Coloured noise approach allows us to treat the spike asymmetry, that is sudden 1-day jump and longer 2-4-day relaxation, which was mentioned in Section 3.1.2.

As presented by Mtunya (2010), all the relevant process parameters are estimated from the real Nord Pool data with use of maximum likelihood methodology. Then the simulation of log-prices is done as presented in Figure 3.19, and the original prices are reconstructed as shown in Figure 3.20.

Analogical fit and simulation is done on the detrended and deseasonalized (as described in Section 3.2.2) series. The resulting price estimate is plotted in Figure 3.21.

Relevant statistics of the original and simulated prices are collected in Table 3.2.

Even though the simulation reproduces the true price statistics reasonably, the trajectory of simulated price is far from the original. Also, despite the ability to produce prominent

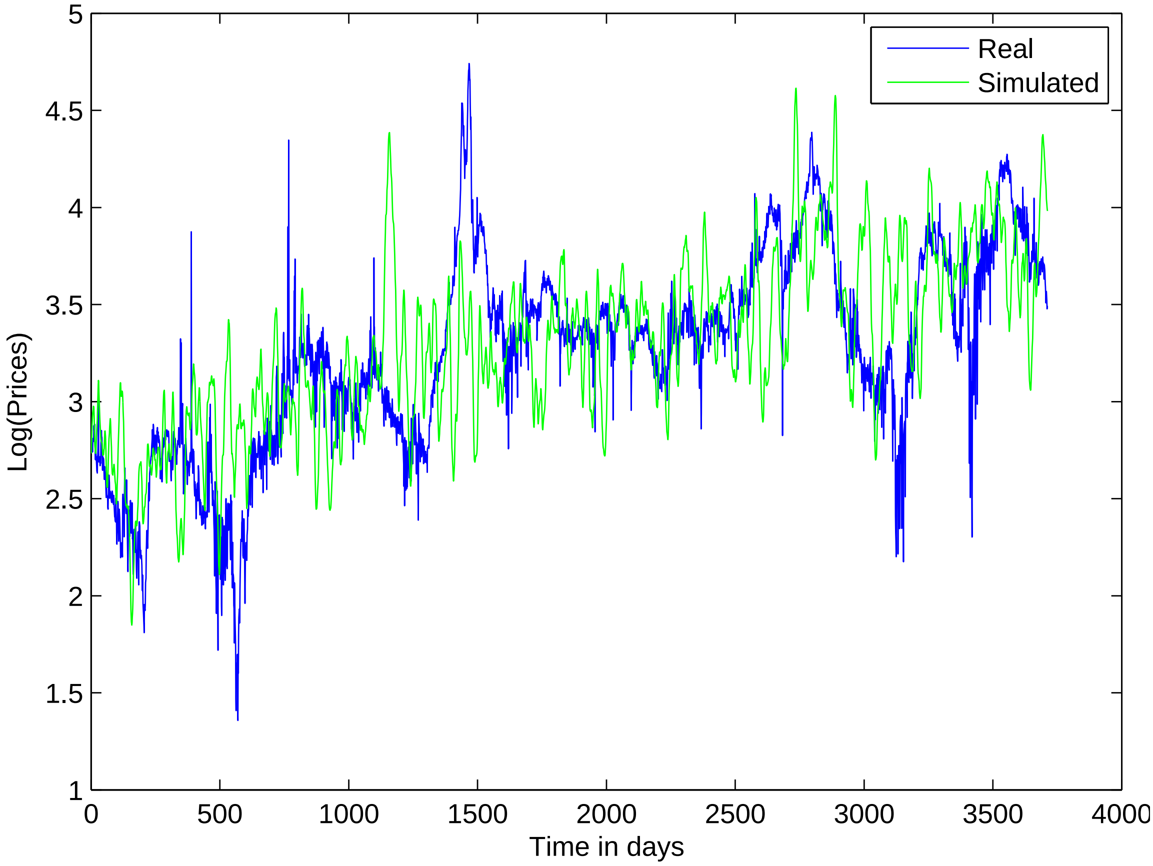

Figure 3.19: Coloured noise mean-reverting simulation for Nord Pool daily spot log-prices and prices (source: Mtunya (2010)).

Table 3.2: Real (original) pure prices data vs Simulated data (source: Mtunya (2010)).

	Real pure prices	Simulated pure prices
Mean	0.72864	0.83423
Std. Dev	7.47424	8.94825
Skewness	0.92309	0.95938
Kurtosis	6.97564	5.51393
Minimum	-30.46631	-19.69480
Maximum	46.23567	49.76592

spikes, they are not as sharp as the real ones. Often their relaxation takes more time than expected.

3.5 Multiple mean-reverting jump diffusion process

When enriching a classical mean reverting process with a jump process, one can produce spikes, but their mean reversion has to be treated separately from the base mean process

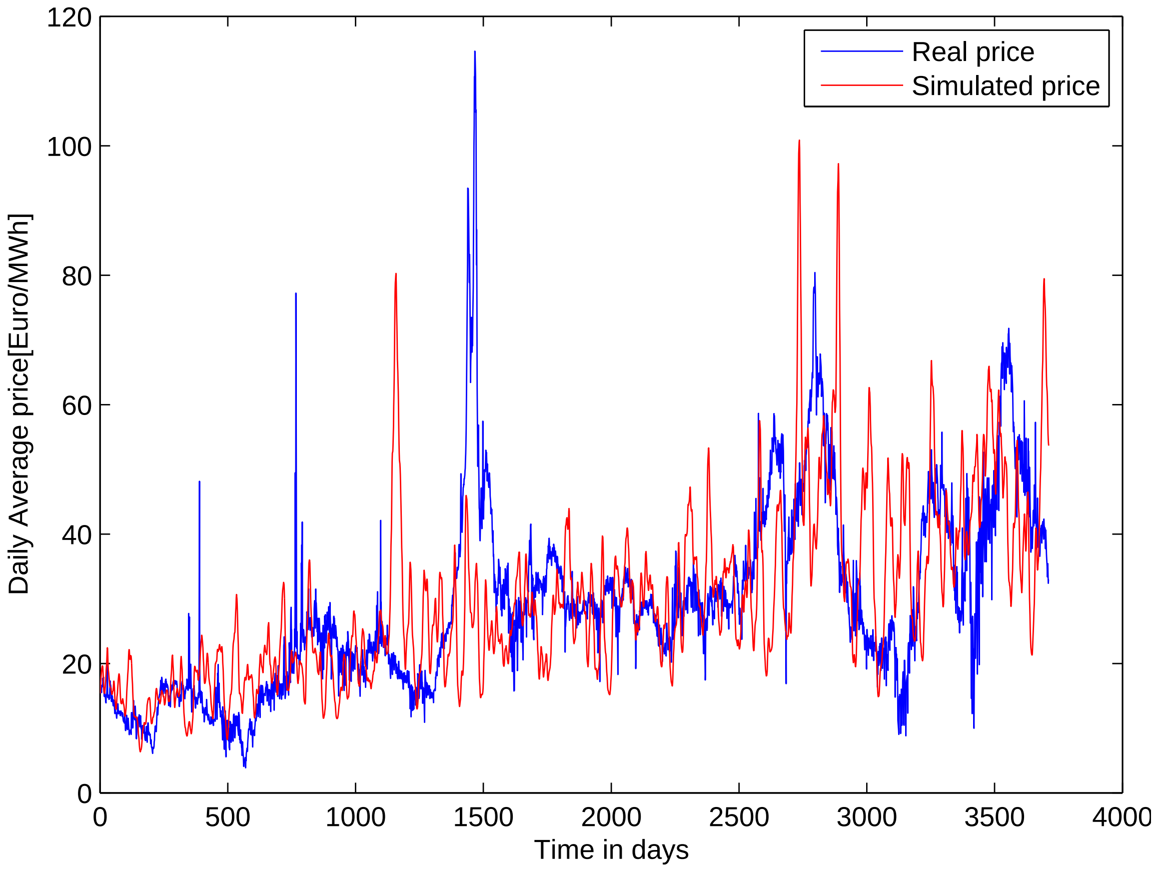

Figure 3.20: Coloured noise mean-reverting simulation for Nord Pool daily spot log-prices and prices (source: Mtunya (2010)).

(Nampala, 2010). When working with Nord Pool hourly spot prices, it is important to remember, that even though the prices are settled for each hour of the day, the auctions are actually run only once per day. Thus there is little hour-to-hour relation in the prices, but rather a day-to-day similarity of particular prices at hours of consecutive days. It was found that if a spike occurs in a specific trading hour, the same hour of the next day will still be less relaxed than any other hour of the same day. Secondly, the jumps added on top of the base mean reverting process should not be independent from the base process. We can thus assume separate "panic models" with a separate model triggered by some condition that depends on the base process. This would result in a two-regime model. As publication **III** proves, spikes are on average more likely to occur from higher price levels than from lower ones. A similar analogy was found when modeling the Irish electricity market uplift price (see publication **II**). Finally, since the regular constant mean reversion is inadequate for modeling spot prices, one can construct it as a moving mean level based on some driving factors.

As suggested in publication **III**, consider Equation (3.19) as an extension of an Ornstein-Uhlenbeck process with an additional jump component (in the form of compound Poisson process) and including a second drift of the spike regime.

$$dM_t = \gamma(t, M_t)dt + \sigma_t dW_t + J(t, M_t)dN_t \tag{3.19}$$

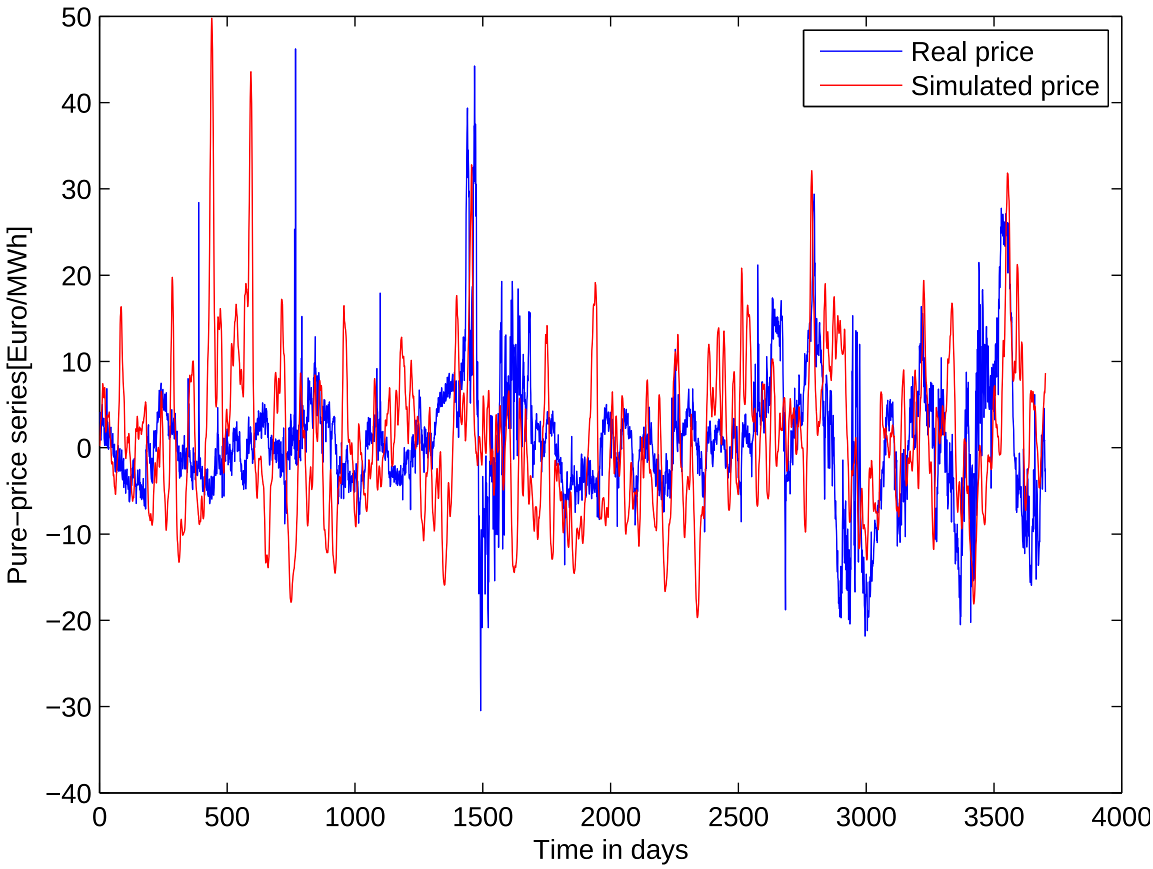

Figure 3.21: Coloured noise mean-reverting simulation for Nord Pool detrended and deseasonalized system prices (source: Mtunya (2010)).

where, given a price threshold M^* beyond which the prices are regarded spiky

$$\gamma(X) = \begin{cases} \alpha(X^* - X_t), & \forall_{X \leq M^*} \\ \beta(Y^* - X_t), & \forall_{X > M^*} \end{cases} \tag{3.20}$$

Hence, if the prices that surpass the threshold are denoted Y, the final simulation equation can be constructed as presented below with the following components: Equation (3.21) representing the regular non-spiky regime mean reversion and Wiener increments, Equation (3.22) and Equation (3.23) standing for mean reversion provided that there was a spike 24 or 48 hours before respectively and, finally, Equation (3.24) representing the jump components necessary to create proper price changes.

$$\begin{aligned} \Delta X(t_k) = \alpha(X^*_{t_k} - X(t_{k-24}))\Delta t \mid_{(v(t_{k-24})=0)} + \sigma(t_k)\Delta W(t_k) && (3.21)\\ + \beta_1(X^*_{t_k} - Y(t_{k-24}))\Delta t \mid_{(v(t_{k-24})=1)} && (3.22)\\ + \beta_2(X^*_{t_k} - Y(t_{k-24}))\Delta t \mid_{(v(t_{k-48})=1)} && (3.23)\\ + J(t_{k-48})v(t_{k-48}) + J(t_{k-24})v(t_{k-24}) + J(t_k)v(t_k) && (3.24) \end{aligned}$$

where

- X_{t_k} is the spot price in time t_k,

- $\Delta X(t_k)$ is the simulated price change,
- α is the mean reversion rate for non-spiky regime,
- $X^*_{t_k}$ is the time-dependent mean reversion level,
- v_{t_k} is the binary indicator of a spike event at time t_k,
- $\sigma(t_k)$ is the standard deviation for Wiener increments,
- $\Delta W(t_k)$ is the Wiener process increment,
- β_1 is the mean reversion rate between a spike and the price 24 hours later,
- β_2 is the mean reversion rate between the price 24 hours after the spike and 48 hours after the spike,
- $J(t_k)$ is the jump size in time t_k.

The simulation in Matlab is performed with an assumption of a time varying mean price level dependent on the temperatures (known as one of the price drivers in the Nordic countries). Figure 3.22 presents a simulation of the former. For the price level depending on temperatures in Finland, a moving regression of two months is used. That is, for each next day's price the horizon of last 60 days is used to build a regression model explaining prices by temperatures. From that model the next price is projected. The results in comparison with the original Finnish spot price are shown in Figure 3.22.

Besides the general visual investigation, it is necessary to perform a more detailed statistical comparison of the generated prices and spikes with respect to the true series. Table 3.3 provides such an assessment. It can be easily seen that the mean value as well as the standard deviation of prices are very close. Also skewness differs not much from the true one, and kurtosis is as significantly high as the original one. When it comes to spikes, the distribution parameters are all very close.

Table 3.3: Basic statistics for Finland area spot price and price spikes (source: publication **III**).

	number	mean	std	skewness	kurtosis
orig prices	52608	23.5990	13.0982	3.6055	35.2557
sim prices	52608	23.8024	13.5725	4.3122	49.2521
orig spikes	240	29.1706	39.9169	4.0015	22.4366
sim spikes	222	29.3215	39.7939	4.1183	23.4566

The results by general graphical as well as more detailed statistical comparison prove to resemble well the true data behavior. Both price and spike parameters do not differ significantly from their counterparts. The small differences observed could be possibly still decreased by putting more emphasis on intraday price behavior, i.e. enriching the simulation with the probability structure of particular times of day that may be more spiky than

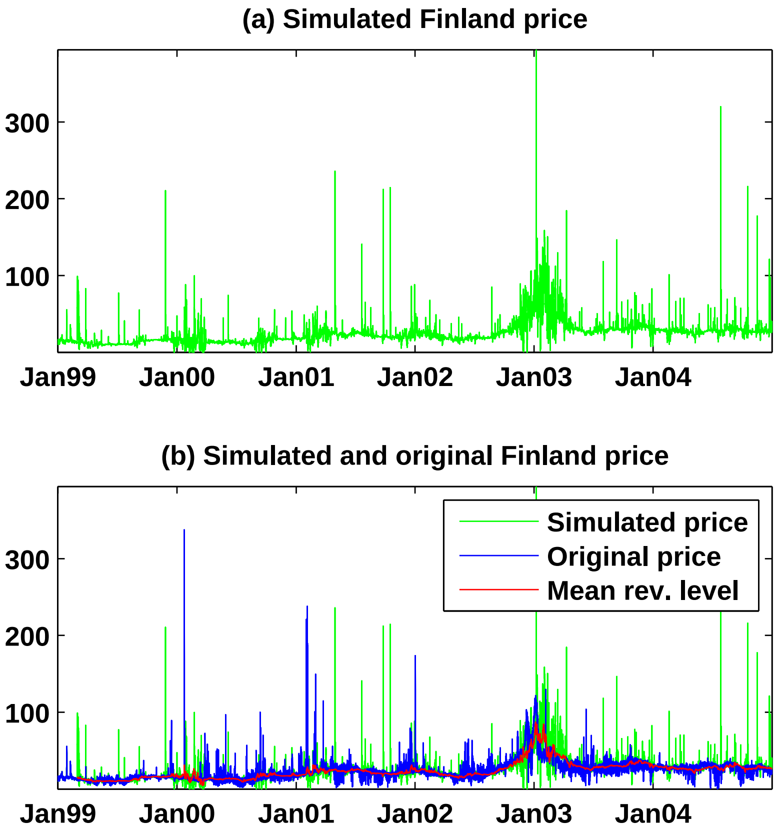

Figure 3.22: Finland price simulated with the multiple mean reversion model with non-constant mean reversion level and the original price (source: publication **III**).

others. Also, the regular price path has a strong 24-hour periodic structure, which was not fully captured here.

Finally, the model performance should be verified by an out-of-sample simulation. The parameters of the model described in the previous sections are estimated once again, now for a smaller data set – a learning set of 2000 days. Then, based on the estimates the simulation is run again and the outcome is compared with the original prices as presented in Figure 3.23.

It can be seen that the underlying mean level follows the original trend, but one can notice some weaknesses in the simulation. Firstly, the model does not include any component driving behaviour of prices in particular hours. That is, the dependence of prices within specific trading periods of consecutive days was considered in case of spiky regime, but no methodology was employed that would make the prices more likely to be higher in peak hours and lower otherwise. Therefore, the original data shows a lot more variance due to not only that type of seasonality, but the weekly periodicity as well. Also, the simulation generated some very high price spikes even when they were not occurring in the original price series. The cause of this is that the past five years used for model calibration were more spiky at this particular price level, while the out-of-sample simulation falls into

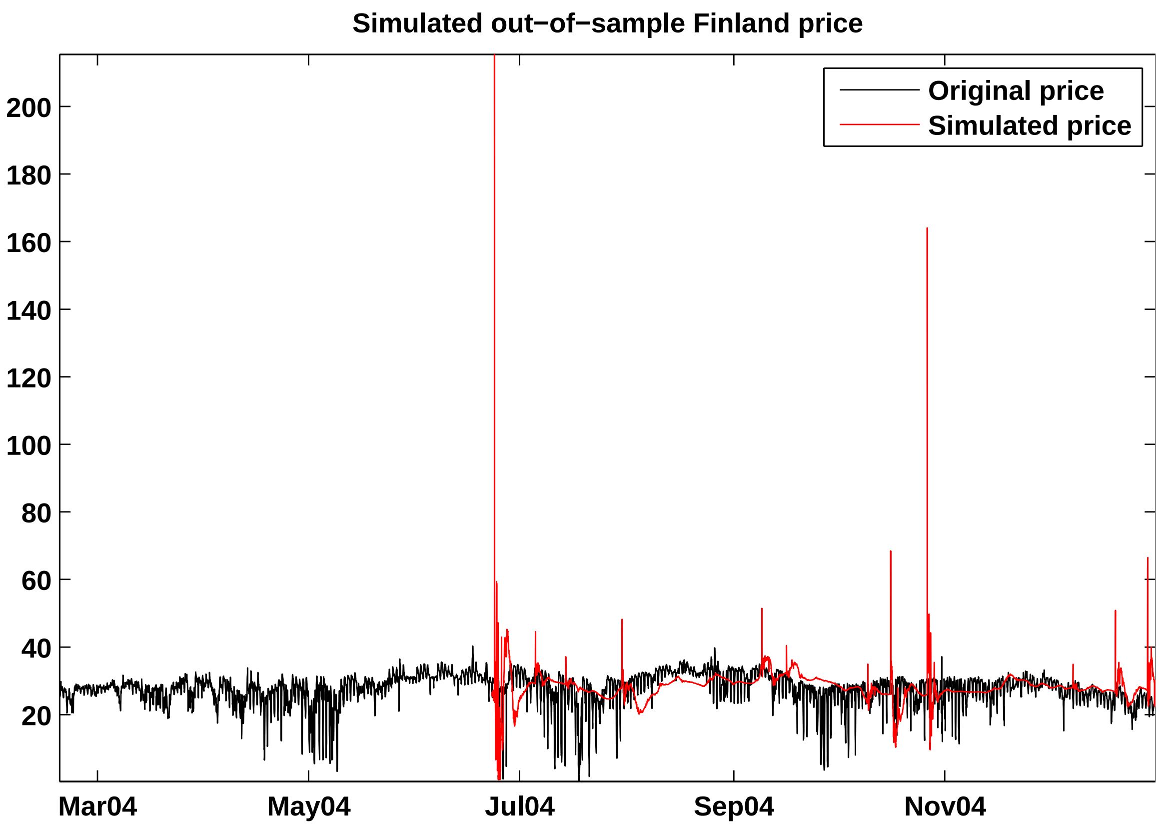

Figure 3.23: Out-of-sample multiple mean reversion simulation (source: publication **III**).

a more stable period, both economically and with respect to hydro-storage in the Nordic electricity market. This demonstrates that the spike process based on price level only is not always sufficient, as the simulation lacks general economical information. That is, the simulation may jump upwards at a time which in reality is very stable in terms of price level and rather causes downward spikes. Moreover, the spikes in the simulation are generated through a separate jump process. Even though the spikes occurrence is stochastically dependent on price level, it does not originate in the real price dynamics.

3.6 Deterministic indicators for 2-regime models

Price spikes are a challenging feature for models of electricity spot prices. Price values during spikes are so different from the base price series, that some researchers assume that spot prices operate in two regimes – regular and non-regular (spiky). After each spike, market specialists are usually able to find a reason that caused it in hindsight. Among those triggers one could mention sudden temperature changes, transmission capacity saturation or power plant or transmission line outage. This would mean that potential two-regime models should have some triggering indicators of when the prices switch between regimes. The following sections present an analysis of the influence of transmission capacity on spike occurrence for two case studies, Nordic and New Zealand's markets. Other triggering conditions are studied as well.

3.6.1 Nord Pool

This section verifies the existence of two regimes in spot prices in the case of two Nordic countries, as discussed by Murara (2010). Finland and Sweden are of interest here because a considerable amount of electricity which is used in these two countries comes from Norway. A part of this imported power is transmitted within Sweden and a part is forwarded to Finland. Thus, in the case of any congestions in the power grid between Sweden and Finland, one would expect the Finnish area price to rise. The aim of this study is to separate electricity spot prices into two regimes using different criteria, and compare the correlations between those regimes. Three different ways to identify two regimes are analyzed. One of the regimes is always the regular regime. The other one, generally called the non-regular regime, is given three different names, referring each to a different specification of non-regularity: the *spiky regime*, the *capacity-limited regime* and the *split regime*.

The first one of them, called the spiky regime, refers to all prices that are mathematically considered as spikes. In particular, these are observations that surpass the local mean level by more than twice the standard deviation (Jabłońska, 2008). This definition allows us to account for local trends in electricity prices and for the fact that not all high prices are actually spikes. Sometimes the price level is higher in general without specific outstanding spikes. The analysis window for spike extraction (the neighborhood that defines locality) can be decided on the basis of available data set.

The capacity-limited regime means prices that occur on days when transmission capacity from Sweden to Finland saturates. Here saturation is defined as transmission reaching over 90% of the capacity limit available on a given day. Finally, the split regime refers to prices on those days when Finnish and Swedish prices differ by more than 4 euros. It is expected that there regimes would be highly correlated with one another. That is, transmission limitations cause area spot prices to diverge. The following sections test this hypothesis. The time instants of active non-regular regimes are identified and a binary variable is created. It assigns a value 0 to the regular regime and a value 1 to the non-regular regime.

CORRELATIONS

When talking about correlation of binary variables it is not as informative to use the classical Pearson correlation, as in case of continuous variables. The reason to this is that the classical correlation tells about linear dependence between two variables, whereas the scatter plot of two binary variables will cover only the four corners of a unit square, simply with a different number of hits in each of the vertices. However, the classical correlation coefficients are computed in order to anyhow get a sufficient view on regime dependencies. The results are presented in Table 3.4. There are three groups of numbers (in different rows) which come from the following observation space criteria:

coef1 values are found from the whole data set, *coef2* is considered in a space of observations where for a given pair of regimes, at least one of those two is active and,

coef3 is analogical to the second one, but the space of considered time instances is now consisting of days when at least one of the three non-regular regimes is active, or ON.

Table 3.4: Classical Pearson correlation coefficients for occurrences of ON states of regimes with respect to the three criteria (source: (Murara, 2010)).

	r1-r2	r1-r3	r2-r3
coef1	0.1798	0.1379	0.0401
coef2	-0.6937	-0.8276	-0.8691
coef3	-0.2697	-0.0489	-0.6456

The correlations calculated on the whole data space are slightly positive (coef1), but that is mostly due to the many simultaneous OFF states in all regime criteria. That is why it is important to verify correlations on observation space limited to days when only either of the non-regular regimes in a given pair is active. The correlation seems to be strongly negative for all three pairs (coef2), not positive as expected. Finally, when for each pair one considers the space indicated by all three regimes (coef3), the values still remain negative, though lower in magnitudes than in the second case. Hence, we trust the classical correlation approach, there is a negative correlation between price splitting and transmission capacity saturation, as well as between reaching transmission limit and spike occurrence.

In the following figures the scatter plots of Finnish and Swedish price are presented. They are coloured by regular and non-regular regime occurrence, for split, capacity-limited and spiky regimes in Figures 3.24, 3.25 and 3.26, respectively. Figure 3.24 shows that prices in Finland and Sweden can split in both directions equally frequently. Also, there are many instances when prices are equal. In Figure 3.25 the right panel shows a scatter plot of prices when transmission capacity is saturated, and the left panel shows the regular regime instances. There seems to be a visible pattern, that when transmission capacity saturates, many Finnish prices are above the Swedish level. However, there are also many cases when the prices do not split, despite the saturation. Moreover, in the left panel it can be observed that Finnish price can split also when there is no capacity saturation. From Figure 3.26 it can be seen that many of the spikes in Finland occur simultaneously with spikes in Sweden.

In Figure 3.27, the time instances of all active non-regular regimes are plotted.

CO-OCCURRENCES

As the regime variables considered are binary, not only Pearson correlation is computed, but also *co-occurrence*. Co-occurrence is understood as proportions of time instances of different non-regular regimes being active simultaneously, in relation to the total space count.

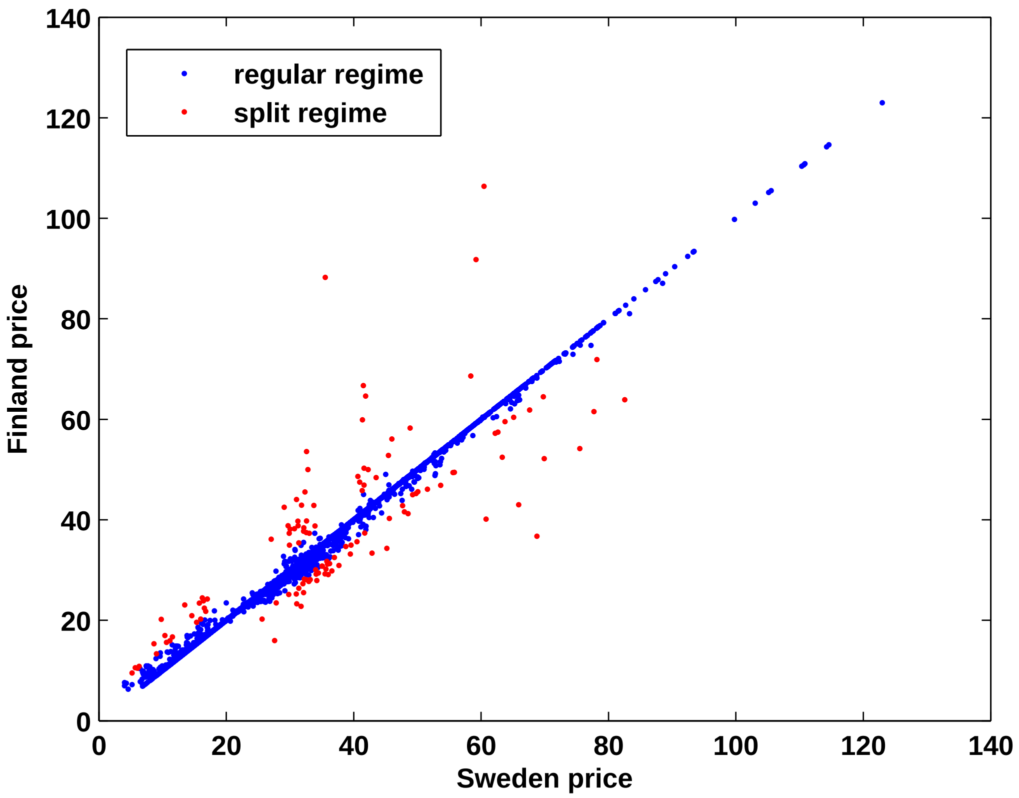

Figure 3.24: Prices scatter plot for regimes split by price difference (source: Murara (2010)).

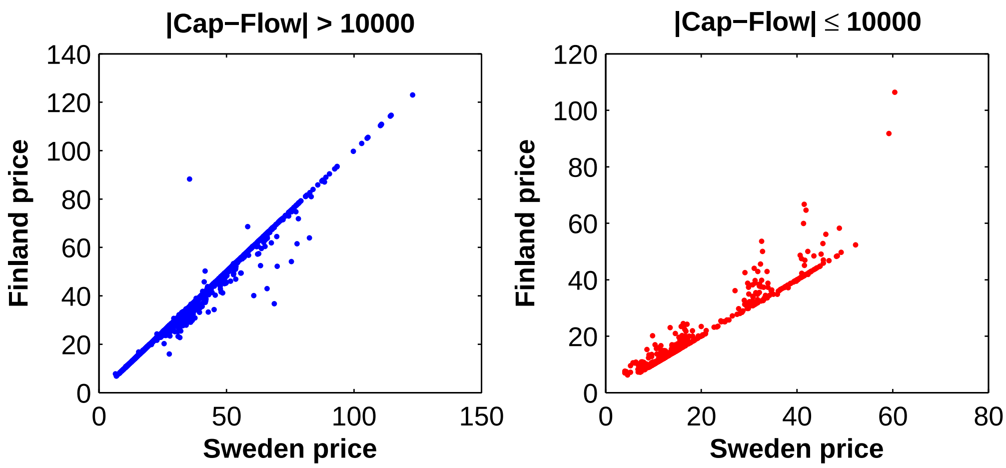

Figure 3.25: Price scatter plots for regimes split by capacity-flow difference (source: Murara (2010)).

- Correlation *coocl* is computed as a ratio between cases when both/all regimes are concordant with respect to the total number of observations

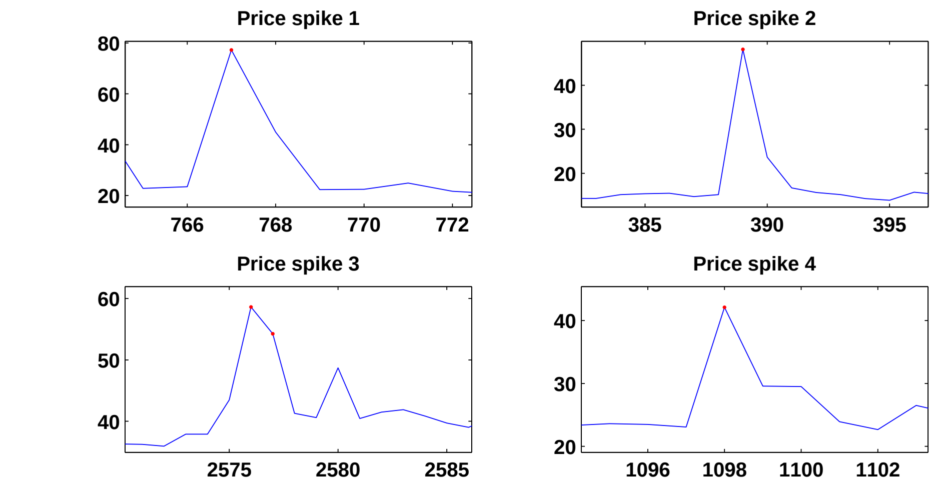

Figure 3.26: Prices scatter plot for regimes split by spike occurrence in Finland (source: Murara (2010)).

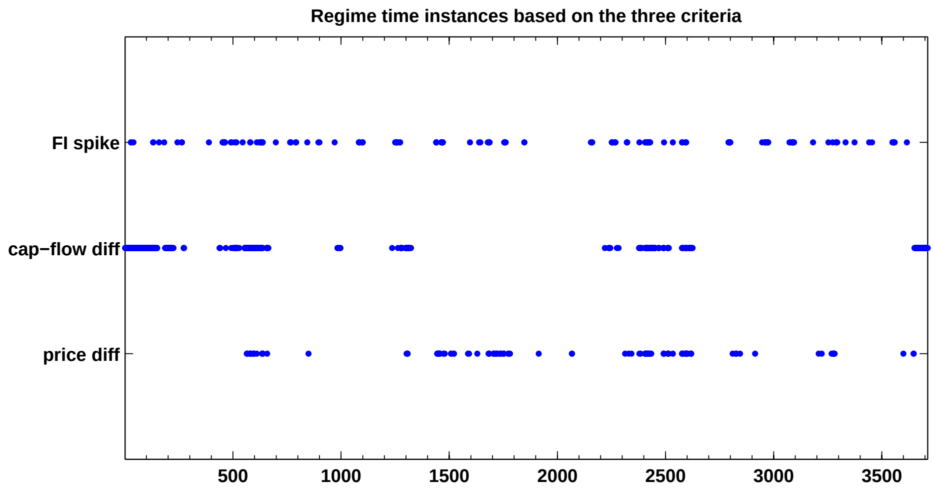

Figure 3.27: Time instances of non-regular regime ON with respect to different criteria (source: Murara (2010)).

$$cooc1_{r1-r2} = \frac{N(r1=r2)}{N},$$
$$cooc1_{r1-r3} = \frac{N(r1=r3)}{N},$$
$$cooc1_{r2-r3} = \frac{N(r2=r3)}{N},$$
$$cooc1_{r1-r2-r3} = \frac{N(r1=r2=r3)}{N}$$

- Correlation *cooc2* is computed as a ratio between cases when both/all non-regular regimes are ON with respect to the number of time instances when either of them is ON. Thus here the majority of the cases when both/all regimes are OFF are neglected

$cooc2_{r1-r2} = \frac{N(r1=1 \wedge r2=1)}{N(r1=1 \vee r2=1)}$,

$cooc2_{r1-r3} = \frac{N(r1=1 \wedge r3=1)}{N(r1=1 \vee r3=1)}$,

$cooc2_{r2-r3} = \frac{N(r2=1 \wedge r3=1)}{N(r2=1 \vee r3=1)}$,

$cooc2_{r1-r2-r3} = \frac{N(r1=1 \wedge r2=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$

- Correlation *cooc3* is analogous to *cooc1*, but now in relation to the number of all cases

  $cooc3_{r1-r2} = \frac{N(r1=1 \wedge r2=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$,

  $cooc3_{r1-r3} = \frac{N(r1=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$,

  $cooc3_{r2-r3} = \frac{N(r2=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$,

  $cooc3_{r1-r2-r3} = \frac{N(r1=1 \wedge r2=1 \wedge r3=1)}{N(r1=1 \vee r2=1 \vee r3=1)}$

where

r1 – split regime,

r2 – capacity-limited regime,

r3 – spiky regime,

1 – non-regular regime takes value 1 when it is ON,

0 – non-regular regime takes value 0 when it is OFF,

N(c) – number of observations satisfying condition (c), and

N – total number of observations = 3712.

The comparison of time points when any of the non-regular regimes is active gives a low co-occurrence percent. *E.g.* for *cooc1*, the results are high because of many simultaneous OFF cases for all regimes (regime 2 with price criterion had 123 ON instances, with cap-flow criterion it had 480 ON cases and with spikes in Finland there were 161 occurrences, all are out of 3712 observations, so with many simultaneous zeroes we have high *cooc1* values). In Table 3.5 there is also confirmation of the previously computed Pearson correlation estimates – negative correlations result in very low co-occurrence rates. They stay around 0.1 or below for all pairs of criteria.

It therefore seems that even though some indicators are often claimed as reasons for spike occurrence, none of them is a reliable predictor for forecasting future spikes. This has also been verified for other potential spike indicators (Baya et al., 2009), such as power line outages, extreme temperature changes or rainfall level.

Table 3.5: Co-occurrences which correspond to the three criteria (source: Murara (2010)).

	$r1-r2$	$r1-r3$	$r2-r3$	$r1-r2-r3$
cooc2	0.8677	0.9364	0.8440	0.8241
cooc1	0.1024	0.0923	0.0508	0.0297
cooc3	0.0832	0.0357	0.0461	0.0297

3.6.2 New Zealand

Although most work described in this dissertation was based on Nord Pool prices, some analyzes were also run on other available data sets. One of then comes from New Zealand.

New Zealand electricity market is from many points of view a very interesting example for spot prices analysis. Its electricity sector is principally (70%) based on renewable energy sources such as hydropower, geothermal power and a steadily increasing wind energy which makes New Zealand one of the most sustainable countries in the world when it comes to energy generation. On the other hand, its electricity demand is growing by an average of 2.4% per year since 1974 and by 1.7% over 1997 - 2007.

New Zealand is characterized by an unbalanced geographical demand-supply relation. The highest electric power production takes place on the South Island whereas the highest demand comes from the more populated and industrialized North Island. Moreover, the electricity market in New Zealand is not pooled. The main participants are seven generators/retailers who trade at 244 nodes across the transmission grid. This analysis uses 11 nodes spread all along both South and North Island, as presented in Figure 3.28.

Six of the chosen nodes are located near powerplants – in particular, Benmore, Tuai and Whakamaru that represent the hydro power generation, while Huntly, Otahuhu and Stratford are based on geothermal generation. The remaining 5 nodes are only splitting substations.

Even though each of the trading nodes obtains half-hourly prices separately, the general shape of all daily price series follows the same path, which can be seen in Figure 3.29. Only a few exceptions are noticeable, when a particular node's price shows a spike, while the other ones do not.

The nodal prices in New Zealand are as volatile and spiky as the prices in other markets, like Nord Pool. Therefore, similar hypotheses of regime indicators can be studied as for the Nordic market. The non-regular regimes here are defined in a similar manner to the Nordic case, that is the spiky regime and the capacity-limited regime. These two are compared for all 11 nodal price series, in order to verify correlations between the criteria.

Spikes extracted from the data can be combined with the occurrence of transmission grid binding constraints. Here, a congestion is defined as a situation when the flow of electricity on a transmission path equals the physical limit of that path. That limit is known, based on the electrical resistance in transmission lines. Figures 3.30 and 3.31 present spikes from North and South Island, respectively, plotted against constraints which occur in all analysed eleven nodes. The blue colour represents limits from North Island and red dots are the ones

Figure 3.28: Location of the 11 nodes in the New Zealand grid used in the analysis.

from South Island. It can be clearly seen that there are definitely less spikes than there are transmission constraint time instances.

It can be seen from Table 3.6 that the co-occurrence ratios are not significantly different from zero. That means that there are very few cases when the transmission constraints are the actual reason for price spikes. Some other factors, in addition to transmission capacity saturation were analyzed in (Baya et al., 2009), and the results were similar to Nord Pool conclusions. Some factors seem to explain spikes' presence, but their capability to forecast future spike occurrence is poor.

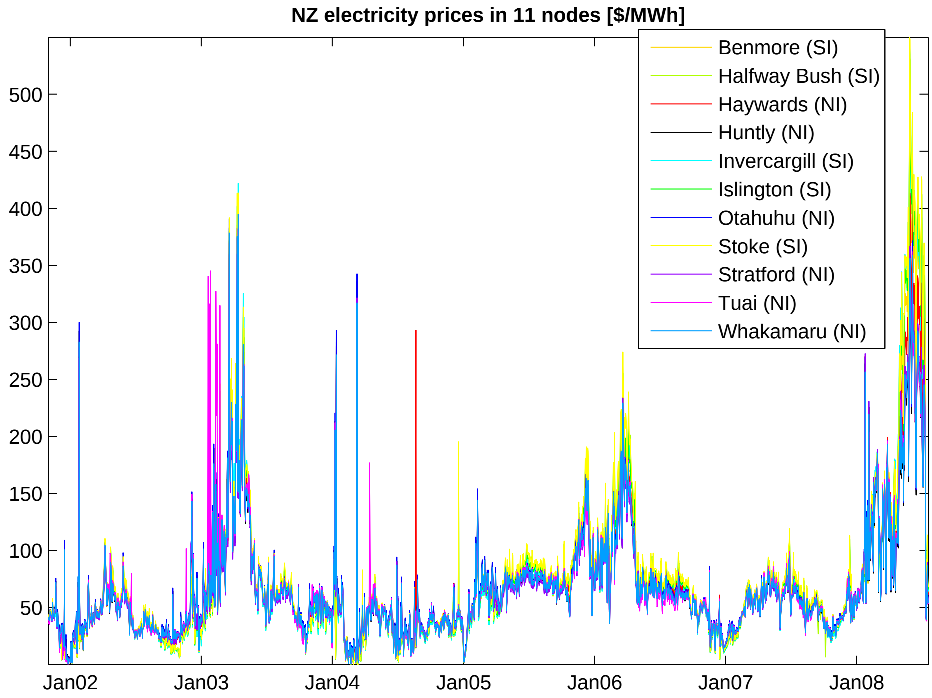

Figure 3.29: New Zealand electricity prices from chosen 11 nodes.

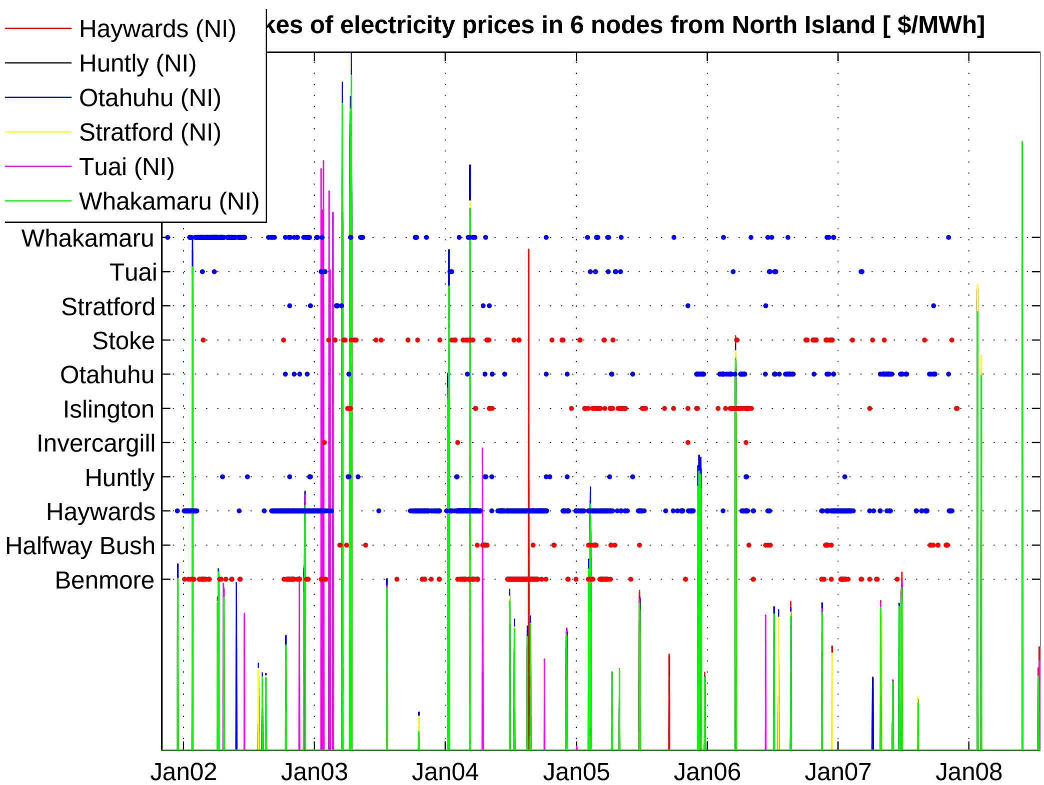

Figure 3.30: Spikes of electricity prices in 6 nodes from North Island.

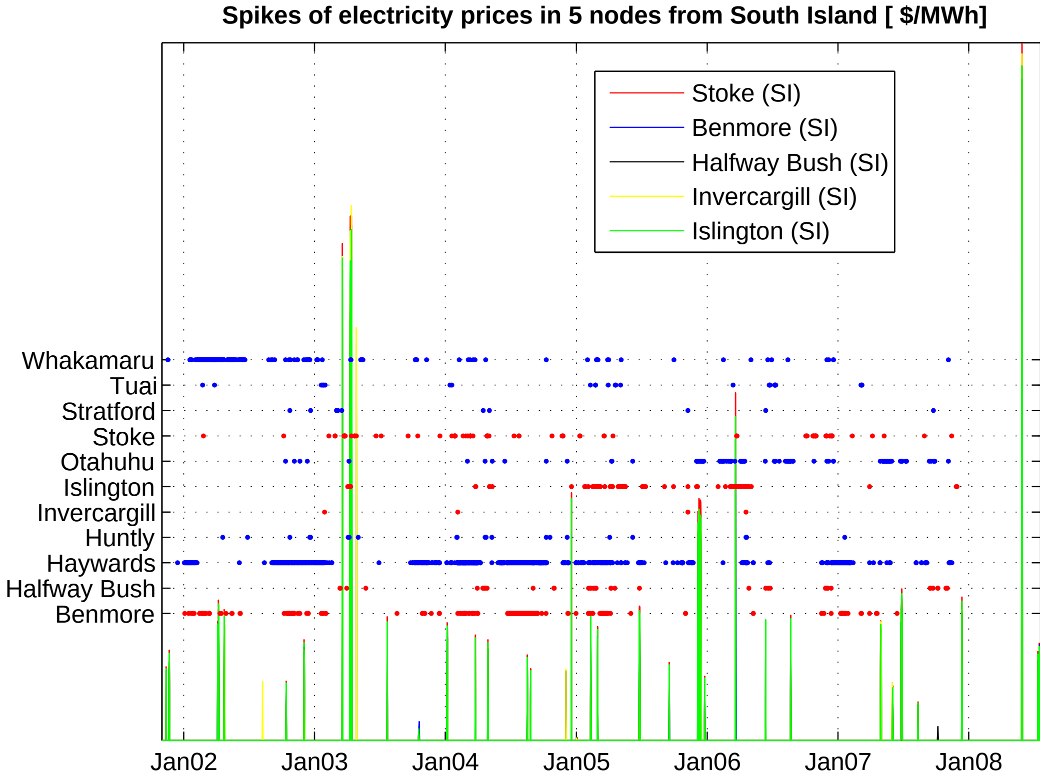

Figure 3.31: Spikes of electricity prices in 5 nodes from South Island.

Table 3.6: Co-occurrence measures of spiky and capacity-limited regimes for 11 New Zealand nodes.

Node	Co-occurrence
BEN	0.0024
HWB	0.0012
HAY	0.0090
HLY	0
INV	0
ISL	0.0016
OTA	0.0024
STK	0.0012
SFD	0
TUI	0.0020
WKM	0.0016

CHAPTER IV

The missing link – human psychology

The Great Recession of 2008-2009 has prompted a lot of new analysis of why markets sometimes become overrated and then come melting down at a critical point. This chapter presents some suggestions for possible reasons standing behind permanent failure of econometric models in predicting the emergence and scale of financial crises. The discussion features also criticism of the classical Efficient Market Hypothesis in Section 4.1 and references to supposed forces driving traders actions in Section 4.2.

4.1 The Efficient Market Hypothesis vs. economy meltdowns

Financial crises are not unfamiliar to any market in the world. Whether the market crashes by itself or gets influenced by a more general meltdown, these events are always painful to market participants and cause a lot of rethinking after they happen, especially whether they could be predicted and avoided. For instance, the world's largest market by capitalization, the New York Stock Exchange (NYSE) has itself suffered a number of crashes and mini-crashes within its over two-century history. These include the Black Thursday crash of the NYSE on October 24, 1929, the Black Monday (1987), Friday the 13th mini-crash on October 13, 1989, and October 27, 1997 mini-crash. Among possible reasons to those and other crises specialists name automated trading, overvaluation, illiquidity, and *market psychology*. However, the first in the list can not explain all the meltdowns, as crises have been present long before computerized trading.

That latest crash of September 2008 has gained a lot of attention after the worldwide economic crisis that precipitated from it, as finance scientists have been unable to identify any rational econometric triggers to the crash. Especially, many blame this failure on the classical assumption that markets follow the *Efficient Market Hypothesis* (EMH). EMH tells that financial markets are *information-efficient*. That is, traders cannot permanently benefit from market investments when they have the public information available about the companies' performance, the economic situation, etc. In other words, any investment has equal chances of good or bad performance.

The Efficient Market Hypothesis is usually referred to in three main versions: *weak*, *semi-strong*, and *strong*. Weak EMH asserts that prices of all assets (stocks, bonds, etc.) already reflect all past publicly available information. Semi-strong EMH claims that prices are balanced in reflecting both all publicly available information and changing to reflect new public information. Finally, strong EMH adds the assumption that prices instantly reflect even hidden or *insider* information. Reality shows evidence for and against the weak and semi-strong EMHs, whereas there exists powerful evidence against strong EMH.

Nowadays, some researchers claim that improvement of information exchange systems has influenced price efficiency positively (Wagener et al., 2010). That is, since the news are available faster, traders can react to them immediately. Historically, EMH has been closely related to mathematical modeling of markets with random walk and martingale models. Such models basically assume that predicting outcomes of a stochastic variable (such as the stock price), cannot be based in the long term on its own historical performance. Therefore, following any even risk-adjusted investment strategies based on stock values cannot bring excess returns. But later on researchers have started to criticize these claims, as in the long run stock prices do reveal a form of sinusoidal pattern and thus cannot be treated as completely random. In theory, when following an investment strategy like 10 best 12-month performers and 10 worst 12-month performers, and choosing those for the next 12 months, one should earn statistically the same return from both. But a recent study from the London Business School (Dimson et al., 2008) shows that this is not the case. Researchers considered British market's best and worst performing stocks starting from year 1900, and they calculated the return from buying those, holding them and rebalancing the portfolios every month. The shocking outcome of the study was that if one has invested only £1 in each of those stocks in the beginning, the investments would have turned into £2.3m in the case of the best performers and into only £49 for the worst ones. Later similar results have been obtained in 18 out of 19 other market studies.

The truth seems to be that rational public information has little to say when human psychology comes into play. Different emotional forces are these days commonly referred to as animal spirits, or Keynes' forces, and are further described in Section 4.2.

4.2 Animal spirits – Keynes' forces

When thinking of animals, one can easily imagine a chain of predators and preys. In nature, a weak individual often falls victim to a predator as its focus on its own hunger (greed) prevents it from noticing emerging danger. On the other hand, an animal may not get enough food if it is too protective (fear), and therefore will fall weaker and eventually die. This situation that seems to reflect Darwinian evolutionary theory very well may not be that far from what is observed in financial markets. According to prof. Andrew Lo from MIT, investors can be compared to competing species when they adapt as risk profiles shift and this makes them resemble real animal behavior (Lo, 2004).

The term *animal spirits* appeared in literature already in 1936. Such emotions as confidence and trust were proposed by John Maynard Keynes to be driving humans in their actions in financial markets (Keynes, 1936). The discipline of behavioral finance has emerged over at least the last 15 years as the means to capture such effects. However, as can be read from Marchionatti (1999), over these years many specialists have not wanted to accept the importance of psychology as one of major drivers of economy. To convince sceptics, some authors have tried formalizing Keynes' forces mathematically, for example in terms of catastrophe theory (Harris, 1979). Others have focused on individual ideas, such as risk-aversion (Kupiec and Sharpe, 1991). The latter is easy to describe mathematically through a measure of satisfaction called utility functions (Marshall, 2010). Some have found an inverse relation between consumer and business confidence and national unemployment rate (Middleton, 1996).

Nevertheless, a strong focus on animal spirits is recently prompted largely by the Great Recession of 2008-2009. Akerlof and Shiller published their book discussing trading psychology (Akerlof and Shiller, 2009) in 2009. In their main points they state that conventional economic modeling only accounts for quantifiable facts, whereas real financial dynamics is strongly dependent on the irrational, emotional and often intuitive human decisions. Even though the explanation is brought down to an individual, one has to remember that even government decisions still have human factors behind them and, therefore, economies fall globally (Kling, 2009). Surely, Akerlof and Shiller gave a very useful viewpoint, especially after the Great Recession of 2008-2009.

Other authors have also underlined the importance of trust and confidence (Tonkiss, 2009) or rational expectations (Kurz, 2010) as crucial forces pulling markets towards or away from economic crises. The image of steely-cold investors appears to be just a myth, as the investment profiles as well as risk aversion levels of individual investors tend to have a lot to do with simple human mood. Happy investors have a lot more self-confidence and trust in their own skills to beat the market.

Ahead of all other factors, *money illusion* is the first one to blame when seeking reasons of economy booms and busts. Simply, people tend to ignore the main financial indicator that affects the value and prices of anything, that is inflation. Especially, an event in anyone's life like buying one's own house, stores the corresponding price in memory a lot better than prices of any other products of a given period. Years later, when all (not only housing) prices raise, investors still perceive feasible increase in real estate values only, and that gives an exaggerated impression of the incredible profitability of housing investment potential.

Within the last few years a new methodology, called multi-agent modeling, has opened up the possibility for modeling animal spirits. Examples of multi-agent macroeconomic models can be found in Grauwe (2011). In these models agents adaptively learn from their mistakes. Some models are specifically catering for transaction taxes, or the aforementioned greed and risk aversion (Demary, 2011). It seems that few of the attempts so far, however, has targeted electricity spot price modeling so far, even though a number of them targeted the functioning of electricity markets (Tesfatsion, 2011). The main contribution of this dissertation is to present a number of ensemble models for modeling spikes in electricity prices, and to discuss their applicability to other financial markets. The methodological basis and construction of such models is presented in the following Chapter 5.

CHAPTER V

Ensemble models for electricity spot market dynamics

This chapter presents the author's main scientific contribution to modeling electricity spot prices. It introduces an approach that is aimed at joining existing econometric and stochastic models with a novel approach that describes the electricity market as a population of interacting individuals. It is intended to reflect the real psychology standing behind their behavior. It is done with reference to physical analogies of market dynamics (see Section 5.1). A number of models with alternative settings are also presented.

5.1 Physics of financial markets and prices

This section presents possible physical analogies to the financial world that could well be used for modeling the missing aspects of market trading psychology.

5.1.1 Population dynamics

Whether one considers stocks or any commodity, their price formation is always a process involving a group of traders, not a single one. Therefore, an intuitive analogy would be to treat traders as individuals of a bigger population. In case of animals, one way to characterize a population (a swarm, a flock, etc.) is through distance measures between the individuals, or through their density (as a function of distance). For instance, a column of an ant colony will always travel as a whole, based on a common target spatial density. When speaking of traders, one can treat their space as a range of prices, and their distance would simply be differences between the individuals' bids.

Moreover, as already discussed, people, like other species of animals, have animal spirits. These, in financial markets, mean mostly fear and greed, influenced by investors' common trading biases, such as herding, overconfidence or short-term thinking. Therefore, there is enough motivation to consider describing traders' behavior with models used in mathematical biology. Such an example is the Capasso-Bianchi system of stochastic differential equations in a general form (5.1), used for modelling animal population dynamics (see Morale et al. (2005)) or price herding (see Bianchi et al. (2003), Capasso et al. (2005)). In this model, the movement of each particle k in the total population of N individuals is based on the location of each individual with respect to the whole population $f(X_t^k)$, as well as on its local interaction with the closest neighbors $h(k, \mathbf{X}_t)$.

$$dX_N^k(t) = [f(X_t^k) + h(k, \mathbf{X}_t)]dt + \sigma dW^k(t), \quad \text{for } k = 1, \ldots, N. \tag{5.1}$$

The target population density may be driven by information of the external environment. This, in the ant colony example mentioned, can be *e.g.* terrain type. Obviously, on a perfectly flat surface the individuals can keep bigger distances, as they have no risk of losing sight of their neighbors. But whenever the colony enters irregular terrain, like tall grass, its individuals will keep closer to each other in their local neighborhood. Therefore, the model can be rewritten into a different general form (5.2), where the environmental influence on the kth particle's location is characterized through the potential $\nabla U(X_N^k(t))$, and the local interactions balance as a potential between aggregative G and repulsive V forces. The randomness of the whole system is still maintained through keeping the Wiener process increment $dW^k(t)$.

$$dX_N^k(t) = [\gamma_1 \nabla U(X_N^k(t)) + \gamma_2(\nabla(G - V_N) * X_N)(X_N^k(t))]dt \; + \sigma dW^k(t), \tag{5.2}$$

for $k = 1, \ldots, N$.

Coming back to the financial market analogy, the population is a group of traders in the spot market, and the measure of their distance is the price. Traders do observe the prices of the market and thus create the general price path, which can also be understood as the global (in *macroscale*) population formation. However, there is a limit to overcrowding (in *microscale*) which in power trading can be interpreted as the physical impossibility of two market participants to buy the same asset. Therefore, there is enough motivation to employ models proposed by Morale et al. (2005) in mathematical biology into financial market modelling, including electricity spot markets. There, each individual price path simulated from the model represents a particle, and ensemble of simulations provides coupling between the participants (in *mesoscale*). The movement of each particle can be driven by external information coming from the environment, expressed via suitable potentials.

The idea of joining mathematical biology with financial time series modeling has already been proposed by Capasso et al. (2003), Bianchi et al. (2003) and Capasso et al. (2005). The authors studied a phenomenon called *price herding* with the example of the Italian car market. This was motivated by the fact that usually prices of a family of goods belonging to the same segment strongly interact among themselves. The model had the following form

$$\mathrm{d}X_N^k = X_N^k \left(s_k \alpha + \tfrac{1}{N} \textstyle\sum_{j=1, j \neq k}^N \left(\tfrac{1}{A_{kj}} \left(\tfrac{I_j}{I_k} \right)^{\beta_{kj}} \cdot \nabla \mathrm{K}_a (X_N^k - X_N^j) \right) \right) \mathrm{d}t \tag{5.3}$$

$$+ \sigma_k \bar{X}^k \mathrm{d}W_k \tag{5.4}$$

where $\nabla \mathrm{K}_a(X_N^k - X_N^j)$ is the aggregation kernel for the price herding. This model accounts for different sensitivity of given prices with respect to their market share $\frac{I_j}{I_k}$, as well as market sensitivity to inflation through $s_k \alpha$. The model was estimated and analyzed for 8 car brands over years 1991-1999. The estimation was done with use of *maximum likelihood* and *maximum a posteriori* methods. The prediction results looked very promising as the forecasts were in big part staying within the estimated confidence levels. However, as the authors admitted, the fitted model was not able to produce jumps in the process which are an important characteristic of the real prices.

$$dX_t^k = \gamma_t[(X_t^* - X_t^k) + (f(k, \mathbf{X}_t) - X_t^k)]dt + \sigma_t dW_t^k + {}^+J_t^k dN_t + {}^-J_t^k dN_t \quad (5.5)$$

5.1.2 Can the price be a liquid?

Here we study another, different and yet not that distinct, physical analogy of markets, i.e. treating the market as a liquid.

Analogies between turbulent flow and market information

In recent years a new discipline called *econophysics* has emerged. Its role is to investigate analogies between financial markets and well known physical systems (Mantegna and Stanley, 2000). It has been shown that price changes do not follow normal distribution. Therefore, the heat equation, whose solution is Gaussian, is not sufficient to reproduce market dynamics.

Financial data have been found to reveal features similar to turbulent flow of a liquid. In particular, Voit (2001) compare the energy turbulence to the role of information in the market. Moreover, Müller et al. (1997) discuss two basic types of traders in financial markets, that is the short- and long-term traders. The former follow the high frequency trading and react instantaneously to sudden price changes. The latter only focus on the long-term evolution of the price, ignoring the fluctuations in short time scale. The information flow between the time scales is asymmetric and the flow from long to short time scale is claimed to be information cascade, analogical to energy cascade in turbulent flow. Finally, what is a spatial distance in fluids becomes the time delay in the markets. The full list of analogies between between hydrodynamic turbulence and financial markets is presented in Table 5.1.

Table 5.1: Analogies between between hydrodynamic turbulence and financial markets (source: Voit (2001)).

Hydrodynamic turbulence	Financial markets
Energy	Information
Spatial distance	Time delay
Laminar periods interrupted by turbulent bursts (intermittency)	Clusters of low and high volatility
Energy cascade in space hierarchy	Information cascade in time hierarchy
Advection of the particles	Traders' movement towards the higher price
$\langle(\Delta u)^n\rangle \propto (\Delta r)^{\xi n}$	$\langle(\Delta x)^n\rangle \propto (\Delta t)^{\xi n}$

MOMENTUM IN FINANCIAL MARKETS

Since the 1980s researchers have been repeatedly noticing that, on average, stocks performing well keep doing so over some further time. As the best-performers strategy example mentioned in Section 4.1 shows, the Efficient Market Hypothesis seems to fail, due to the presence of so called *momentum* in financial markets. This phenomenon is not reserved for stock markets only. Similar behaviour, or analogically understood features, can be observed in any commodity markets. Even though there is a number of happenings that can be understood as market anomalies, the momentum effect is too strong to be classified that way. Simply, there seems to be little coincidence in this value accumulation.

In many funds the managers are rewarded for good performance and for beating the market. Thus they must be holding the most popular and valuable stocks. When they perform well, clients invest even more money, which again go into the same investments and additionally boosts shares that are already performing well. Simply, investors are buying stocks just because their price has risen. This is the essence of the momentum effect.

THE BURGERS' EQUATION

A physical analogy to the momentum phenomenon can be found in fluid dynamics. Burgers' equation (5.6) is a one-dimensional form of the Navier-Stokes equations without the pressure term and volume forces. It is widely used in various areas of applied mathematics, such as modeling of fluid dynamics and traffic flow (Cole, 1951; Hopf, 1950).

$$u_t + \alpha u u_x + \alpha u_{xx} = f(x,t) \tag{5.6}$$

When thinking of understanding markets and fluids, the price could represent one dimensional measurement of fluid pressure along a periodic domain. This characterization is not far from stock market reality. Worldwide trading takes place in a periodical domain of the earth, when an exchange closing in one time zone may be in the same time opening for another trading day in other geographically distant stock exchanges. The information circulates in a periodical fashion around the world.

In this light, it is not surprising to see the results of one recent study of Burgers' equation, which has shown an interesting simulation of fluid pressure measurements (in a single point) that very closely resemble electricity spot price realizations (Yang and McDonough, 2002). Therefore, one can consider implementing the Burgers' equation with parameters estimated from the electricity spot price series. Looking at the philosophy behind this, in the prices (and financial markets in general) one really finds correspondence to actual physical phenomena. In particular, for the Burgers' equation in the form (5.6), we can discern the following analogies:

u stands for the price,

$f(x,t)$ describes the fundamentals (of a periodic character),

αu_{xx} is the diffusion term that is related to the fact that the spot market tends to reach an equilibrium price,

u_x is the spread between any given day's average and most common bids, *i.e.* the mean and the mode of the traders' prices on a given day,

uu_x is the momentum term that expresses traders' movement towards the most common price. This effect is magnified at higher prices.

In particular, the momentum effect should occur when a sufficiently big subgroup of the whole population has significantly different behavior (external information) that deviates from the total population mean. This has been noticed in studies related to animal and human spatial dynamics, with in a large group of people that were asked to move randomly around a big hall. They started to follow individuals that has specific direction orders whenever that small subgroup was reaching 5% of the total population. Again, in terms of prices this could be understood as considerable departure of population price mode from its price mean. Then the rest of individuals may follow that trend and unexpectedly amplify that deviation to a scale of a prominent price spike.

5.2 An ensemble mean-reverting jump diffusion model

The simplest idea of following neighbors in the case of traders can be expressed as turning towards the mean value of closest prices. Therefore, the first modest implementation of the idea of using animal dynamics for modeling the traders' prices comes as a combination of the mean-reverting jump-diffusion model presented in publication **III** with the system (5.1). In this study individual spot price traders are represented as an ensemble. Price realizations of all of them are described with a system of stochastic differential equations. This is called a *Lagrangian representation*, that is each individual particle is followed in its movement separately. As it was mentioned in (Morale et al., 2005), due to computational complexity, this approach makes sense for small or medium-sized populations. To reflect reality, the ensemble size is set to 330, as to ensure statistical significance of the results with a sufficient level of randomness. Numerical simulations have shown that this and the following models make sense for any ensemble size $n \geq 3$. However, it is the higher ensemble values $n \geq 100$ that provide the right balance between the low and medium range variations.

In particular, each of those differential equations has the form (5.7)

$$dX_t^k = \gamma_t[(X_t^* - X_t^k) + (f(k, \mathbf{X}_t) - X_t^k)]dt + \sigma_t dW_t^k + {}^{+}J_t^k dN_t + {}^{-}J_t^k dN_t, \quad (5.7)$$

for $k = 1, \ldots, N$, where

- X_t^k is the price of trader k at time t,
- X_t^* is the global price reversion level at time t,
- γ_t is the mean reversion rate at time t,
- $\mathbf{X}_t$ is the vector of all traders' prices at time t,
- $f(k, \mathbf{X}_t)$ is a function describing local interaction of trader k with his neighbors (small range of individuals from vector $\mathbf{X}_t$),

– W_t^k is the Wiener process value for trader k at time t,

– σ_t is the standard deviation for Wiener increment at time t,

– ${}^+J_t^k$ is the positive jump for trader k at time t,

– ${}^-J_t^k$ is the negative jump for trader k at time t,

– N_t is the count process for jumps at time t.

In this model we follow the global mean reversion level X_t^* and rate γ_t in a moving fashion, with half a year historical horizon (182 days). The local interaction $f(k, \mathbf{X}_t)$ is based on following the mean value of neighbors within a price range equal to 10% of the total price range. The jump processes ${}^+J$ and ${}^-J$ are dependent on current price level at each time t, as it was found that electricity spot price is more likely to spike from higher levels than from lower (see Nampala (2010) and publication **III**). The model parameters of the mean reverting part are estimated with use of Maximum Likelihood (MLE) approach. The log-likelihood function for Ornstein-Uhlenbeck process can be found from Øksendal (1995). The probabilities of jumps are generated from Poisson distribution based on probability of spike occurrence from specific price levels. The jump sizes are sampled from empirical distribution of the original prices.

An important remark has to be made about this and the following models with respect to the price herding model proposed by Capasso et al. (2003), Bianchi et al. (2003) and Capasso et al. (2005). All the ensemble members are assumed to have the same process coefficients, that is the parameters are time- but not individual-dependent. Therefore, they are estimated from a single system price process, and it is the ensemble mode that is claimed to reconstruct the original price dynamics.

Therefore, spikes generated by the jumps are reflecting a *panic* reaction of traders in an uncertain environment, on both positive and negative sides. We would claim that these jump processes originate in human *fear and greed* emotions.

In this model, the interpretation of trading psychology factors is be as follows:

γ_t, (X_t^*) stands for short-term thinking. As discussed in previous chapters, traders tend to forget distant historical events and tend to relate their investment performance only to the latest events.

$f(k, \mathbf{X}_t)$ represents herding. Market participants may trust either their own skills, or those of others that might have better information available. Standing out from the crowd is not common.

${}^+J_t^k$, ${}^-J_t^k$ reflect a panic reaction of traders in an uncertain environment, on both positive and negative side. Whenever prices enter thin ice, that is a relatively high level, traders become anxious and may panic and put extreme bids on the table. We would claim that these bids, emerging in the form of jump processes, originate in human *fear and greed* emotions.

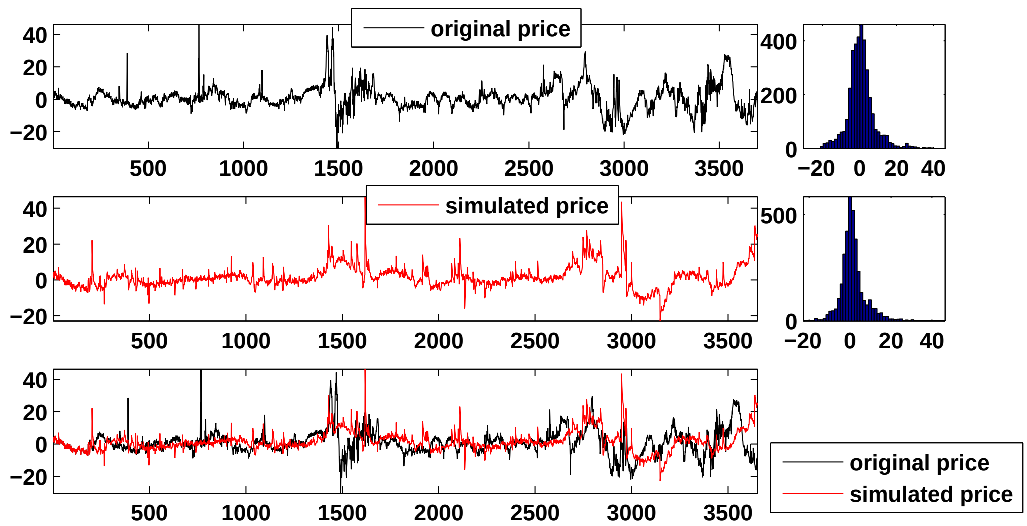

Figure 5.1: Ensemble simulation: global reversion to moving mean level with moving rate, and local to neighbors' mean (source: publication **V**).

In Figure 5.1, the original price and an example simulated trajectory are presented together with their respective histograms. It can be seen that the simulation nicely follows the original data, both in the long term and in the appearance of spikes.

The original and simulated histograms are similar. However, is is necessary to quantify the difference as well. Therefore, Table 5.2 collects comparison of the basic statistics for original pure trading prices and the mean ensemble values. These are mean, standard deviation and five consecutive central moments scaled with proper power of standard deviation, i.e. following the equation (5.8).

$$m_k(X) = \frac{\mathrm{E}[X - \mathrm{E}(X)]^k}{\sigma^k}, \tag{5.8}$$

where X is the considered random variable, k is the moment order and σ is the standard deviation of variable X. From Table 5.2 we observe that especially skewness and kurtosis are having values very close to the real ones. The 5th moment is still comparable. Only 6th moment starts to be higher for the ensemble than the original data by the factor of nearly 1.5 and the 7th moment by the factor of 2.

To complement this analysis we employ one more comparison measure, i.e. the probability of the series to exceed specific levels (PoE measure). The original pure price series and each simulated ensemble series are sliced into ten intervals each, from their respective minimum to maximum values. Then the percentage of observations is calculated, which is falling above each slice threshold. These probabilities are illustrated in Figure 5.2, together with the respective mean value and standard deviation limits of the original price.

Clearly, the real data's probabilities fall within the envelope of the whole ensemble. That confirms the statistical accuracy and robustness of our approach. Simulation results prove that the approach reconstructs many statistical features of the real spot price trading. These

Table 5.2: Original and ensemble statistics: global reversion to a moving mean level with a moving rate, and local to neighbors' mean (source: publication **V**).

	Original	Ensamble
Means	0.7286	1.6948
St dev	7.4742	6.1412
Skewness	0.9231	0.9445
hline Kurtosis	6.9756	6.9135
5th moment	18.8175	22.0969
6th moment	104.9133	150.4392
7th moment	423.4505	832.3929

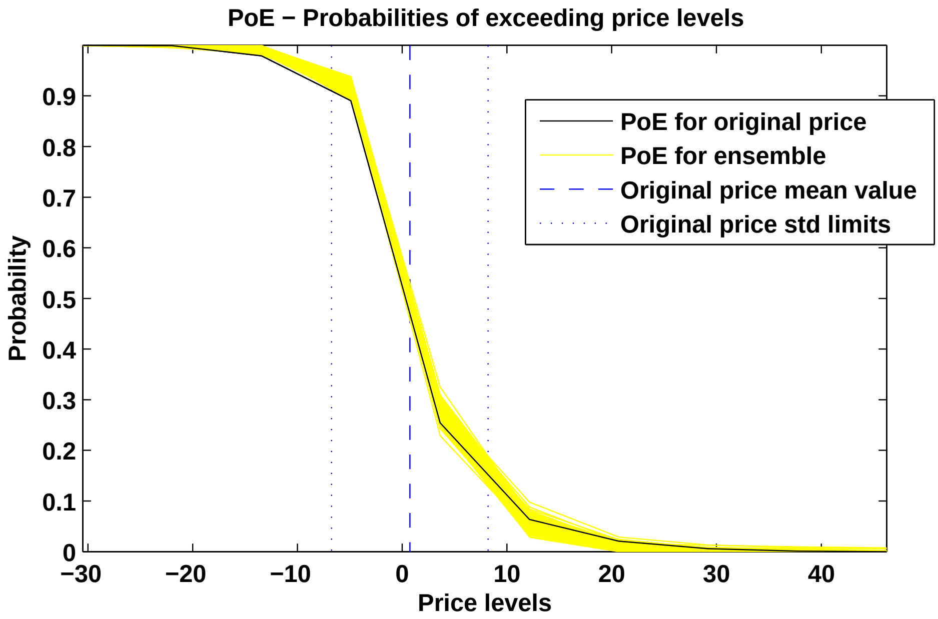

Figure 5.2: Original pure price and ensemble probabilities of exceeding specific levels (source: publication **V**).

are measured by comparing distribution histograms of the original and simulated series, statistical central moments up to the 7th one, as well as a measure showing probability of the prices exceeding specific levels. All these show remarkable resemblance. However, a major weakness of the model comes from the fact that the spikes are still generated through a superimposed jump process. Even though spikes are formally dependent on the price process itself, their generation still has a degree of randomness in forcing the jumps instead of true price dynamics.

5.3 Ensemble simulation with Burgers'-type interaction

In Section 5.2, it has been shown that the proposed model can successfully reproduce a number of statistical features of electricity spot prices. However, the spikes, which are of highest interest in this study, were generated through separate jump processes dependent on the price level. As discussed in publication **V**, we argue that price spikes originate in human psychology and get magnified through the market momentum. Therefore, they should occur based on price dynamics alone. Thus in the following model the jump processes are eliminated. Also, the mean-based local interaction $f(k, \mathbf{X}_t)$ is replaced by global interaction as a Burgers' type momentum component $h(k, \mathbf{X}_t)$. Here, the notation u_x in the Burgers' equation is understood as a mean distance of ensemble members from the mode of the ensemble. That is,

$$u_x = \frac{1}{n}\sum_{i=1}^{n}(u_i - u_{mode}) = \frac{1}{n}\sum_{i=1}^{n} u_i - \frac{1}{n}\sum_{i=1}^{n} u_{mode} = \bar{u} - u_{mode} \tag{5.9}$$

This interpretation corresponds to a Bayesian view of market dynamics where the mode represents the state estimate. In classical Fisherian models, the mean renders the state estimate.

Hence, the model takes the form

$$dX_t^k = [\gamma_t(X_t^* - X_t^k) + \theta_t(h(k, \mathbf{X}_t) - X_t^k)]dt + \sigma_t dW_t^k, \tag{5.10}$$

where $h(k, \mathbf{X}_t) = \mathbf{M}(\mathbf{X}_t) \cdot [\mathbf{E}(\mathbf{X}_t) - \mathbf{M}(\mathbf{X}_t)]$ and $\mathbf{M}(X)$ stands for the mode of a random variable X. Also, θ_t represents the strength of that global interaction at time t and is allowed to be different from the mean field force γ. All the remaining notations are the same as in model (5.7). This model preserves the short term thinking bias in the form of the moving mean reversion level X_t^*.

5.3.1 Parameter estimation

The model estimation is also done by MLE. Following the solution of a mean reverting process we obtain the forms (5.11) and (5.12) for process mean value and variance, respectively.

$$\mathbf{E}(\mathbf{X}_t) = [\gamma X^* + \theta\mathbf{M}(\mathbf{E}(P_t)(\mathbf{E}(P_t) - \mathbf{M}(P_t)))] \cdot \frac{1}{\gamma + \theta} \cdot (1 - e^{-(\gamma+\theta)t}) \tag{5.11}$$

$$\mathbf{Var}(\mathbf{X}_t) = \frac{\sigma^2}{2(\gamma + \theta)} \cdot (1 - e^{-2(\gamma+\theta)t}) \tag{5.12}$$

NORMAL DISTRIBUTION

At first, normality is assumed for simplicity. Then the population mean $\mathbf{E}(P_t)$ is expected to equal the population mode $\mathbf{M}(P_t)$, and thus the log-likelihood function takes the form (5.13).

$$\mathcal{L}(\mathbf{X}, X^*, \gamma, \theta, \sigma) = n \ln\left(\frac{1}{\sqrt{2\pi\mathbf{Var}(\mathbf{X}_t)}}\right) - \sum \frac{(X_i - \mathbf{E}(\mathbf{X}_t))^2}{\mathbf{Var}(\mathbf{X}_t)} \tag{5.13}$$

PARAMETRIC DISTRIBUTION – TUKEY g-AND-h TRANSFORMATIONS

In real life data examples, distributions are often not normal. Some of the popular algorithms to deal with non-normal data sets are g-and-h transformations. By applying different values of g and h one can reshape a standard normal distribution to any other shapes with a given target skewness and kurtosis value, respectively. The main limitation with these transformations is that the resulting distributions do not have explicit probability density functions (pdfs), nor cumulative distribution functions (cdfs). Instead, these distributions are defined in terms of a normal percent point function

$$G(p, g, h) = A + B \cdot \left(e^{gZ_p} - 1\right) \frac{e^{hZ_p^2}}{2g}, \quad 0 < p < 1 \tag{5.14}$$

For $g = 0$ and $h = 0$, the g-and-h distribution reduces to standard normal distribution.

The density function can be found by computing the numerical derivative of the cdf. The optimal values for parameters g and h can be estimated by a least squares fit from original data histogram. An example fit for the pure price series is presented in Figure 5.3. The data is first rescaled to have a mean value equal to zero and a variance equal to 1. The optimal g-and-h distribution values are found as $g = 0.5783$ and $h = 0.6854$. Clearly, the proposed parametric distribution is a lot more reasonable for electricity price data than the standard normal distribution.

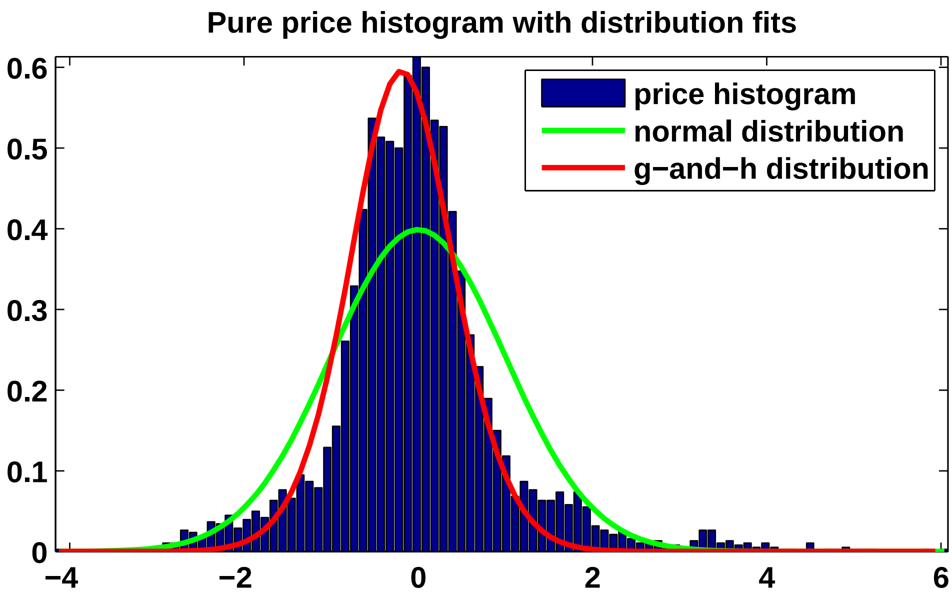

Figure 5.3: Pure price histogram with normal and g-and-h distribution fits.

Numerical MLE for g-and-h distributions means maximization of joint distribution function of n i.i.d. random variables

$$L = \prod_{i=1}^{n} f(x, \theta) \tag{5.15}$$

with respect to the parameter vector θ. Here, f is a numerical derivative of the parametric cumulative distribution function (5.14).

WELL-POSEDNESS OF THE SDE SYSTEM

Since the topology and dimension of the problem's domain are not known, this dissertation does not discuss the mathematical properties of the proposed system of stochastic differential equations. The Burgers' equation is a one-dimensional Navier-Stokes momentum equation. Since the well-posedness or ill-posedness of the latter has not been proven to either direction, it is unlikely to yield easily even in the case presented in this work. Since the Efficient Market Hypothesis essentially translates to the heat equation, the direct problem of which is obviously well-posed, the very changes that are introduced here to market dynamics are likely to push the corresponding SDE from well-posedness to chaoticity. Therefore, the main focus of this work is to analyze via numerical solutions how well the proposed equations succeed in reproducing the real market time series properties, especially those that seemingly contradict standard assumptions of neoclassical economic theory, in particular the Efficient Market Hypothesis.

5.3.2 Simulation results

The simulation results for the model, considering the assumption of data normality, can be seen in Figure 5.4. The general price level follows the original data. The main advantage of the current approach is the accurate simulation of spikes, even though the model does not have any jump component. Thanks to interactions of the individuals the model reproduces the price spikes based on the pure price dynamics. One can see that spikes in the simulation are not as frequent as in original data.

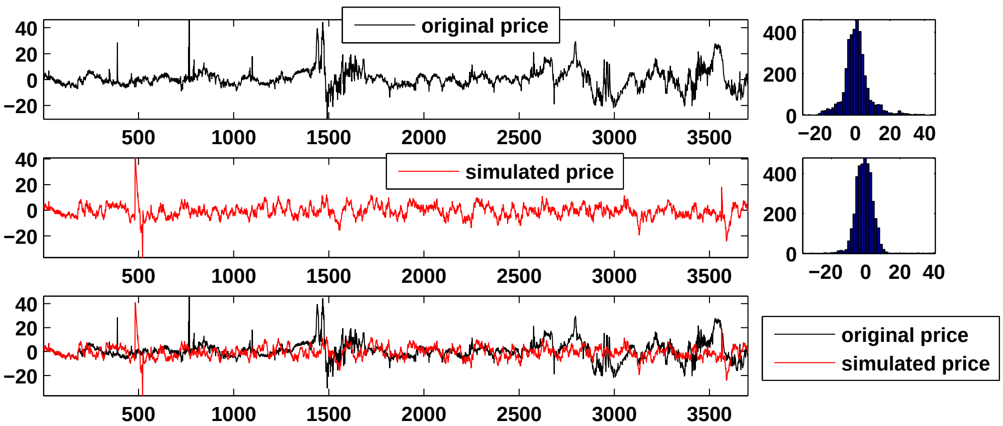

Figure 5.4: Ensemble simulation: global reversion to moving mean level with a moving rate, and Burgers'-type local interaction (source: publication **V**).

Table 5.3 collects the statistics for the original price and the ensemble. The values of central moments are not exactly reproduced, but can be accepted as the values are relatively close to the real data. Especially, the simulated kurtosis value is close to the true one.

Analogically to simulation of model (5.7), also here the measure of price probabilities to exceed specific price levels is employed. Figure 5.5 illustrates it for the original price as

Table 5.3: Original and ensemble statistics: global reversion to moving mean level with moving rate and Burgers' type interaction (source: publication **V**).

	Original	Ensemble
Means	0.72	-0.43
St dev	7.47	5.01
Skewness	0.92	0.35
Kurtosis	6.97	7.68
5th moment	18.81	24.40
6th moment	104.91	242.69
7th moment	423.45	380.57

well as the ensemble. The ensemble envelope does not cover well the original data in the range of positive extreme values, but the results are promising.

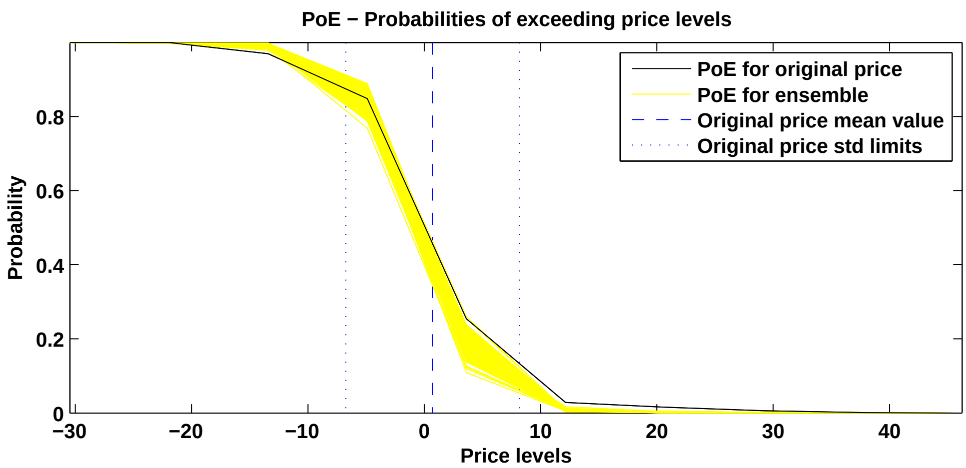

Figure 5.5: Original pure price and ensemble probabilities of exceeding specific levels (source: publication **V**).

An intuitive explanation for the not completely adequate reconstruction of price statistics can be the normality assumption for MLE. Therefore, the results with g-and-h parametric distribution are also presented (see Figure 5.6). This model is able to spike as in the previous case, but the simulation is still missing the medium range variations. That is clearly visible from the histogram, where the original prices are more common to have values around levels of -20 and 20.

Also, the plot of the whole ensemble of histograms confirms that the medium range variations are not well captured. That would be automatically reflected in the PoE measure results and central moments' values.

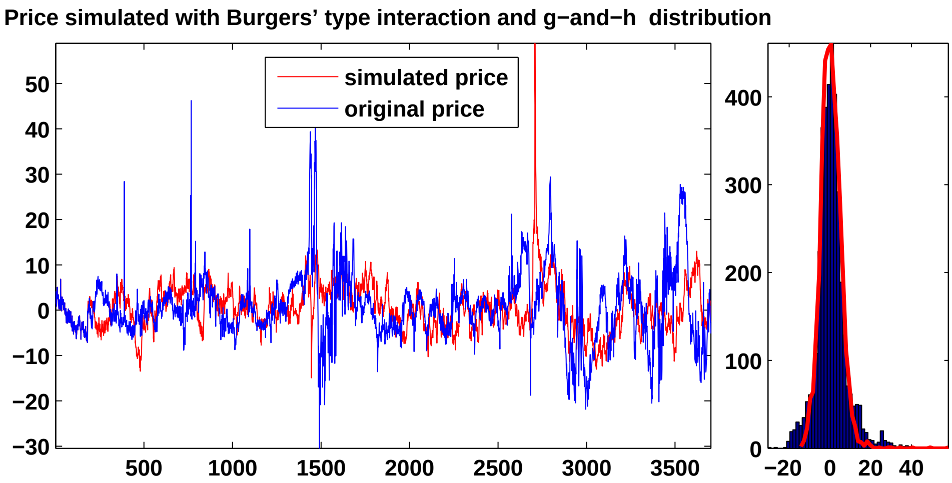

Figure 5.6: Original and simulated price with Burgers' type interaction and g-and-h distribution for maximum likelihood parameter estimation.

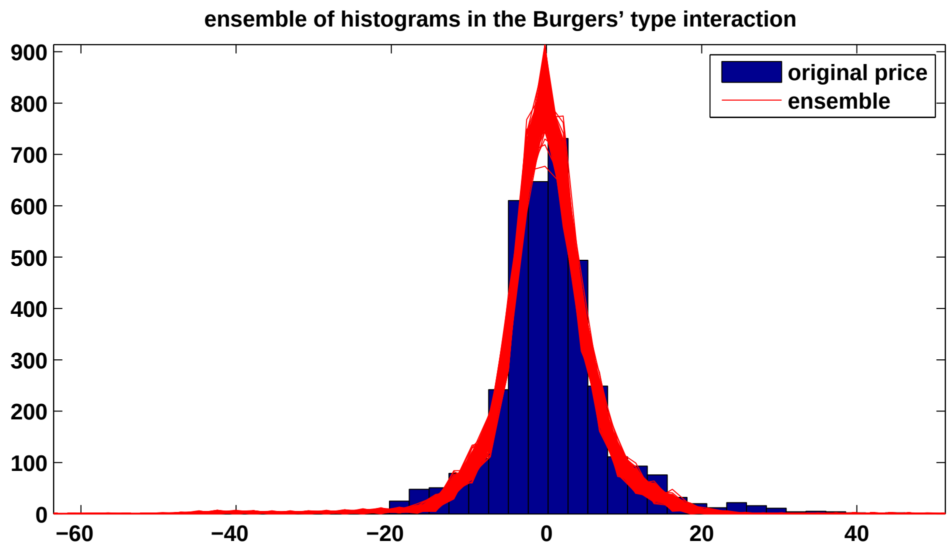

Figure 5.7: Original pure price histogram and the ensemble histograms.

5.4 A Capasso-Bianchi type model for electricity spot market price

This section aims at combining all the previous models presented in this chapter. The main goal is to account for the main components of the Capasso-Bianchi population dynamics model, that is:

Underlying field is expressed in terms of a mean reversion level, estimated from the real data. It ensures that even if prices spike, they will come back to the usual level.

Global interaction is the term that allows the prices to escape from the underlying mean level, magnifying the fact that most of the population members bid distantly from the population mean value. It is formed as the Burgers' momentum term.

Local interaction means that each trader at each time point sees the closes $p\%$ of the total population and turns towards the most distant price from that neighborhood. It can be interpreted as a tendency of traders to believe that others (especially those bidding most distantly) have better information available.

Hence, the model takes the form

$$dX_t^k = [\gamma_t(X_t^* - X_t^k) + \theta_t(h(k, \mathbf{X}_t) - X_t^k) + \xi_t(g(k, \mathbf{X}_t) - X_t^k)]dt + \sigma_t dW_t^k, \quad (5.16)$$

where the new component $g(k, \mathbf{X}_t)$ represents now the maximally distant member of k-th trader's neighborhood, formed by the closest $p\%$ of the population. That is

$$g(k, \mathbf{X}_t) = \max_{k \in I}\{\mathbf{X}_t^k - \mathbf{X}_t\}, \quad \text{where} \quad I = \{k|_{X^k \in N_{p\%}^k}\} \quad (5.17)$$

and $N_{p\%}^k$ means the neighborhood of the k-th individual formed by the closest $p\%$ of the population.

This model preserves all the aforementioned trading biases. There remains a question of an adequate p value to be used as the percent of the population in local interaction. The aim is to find an optimum at the interval $p \in [1, 20]\%$. The goodness of fit is measured in two ways. One is the absolute sum of differences between the four main parameters of the real data and the ensemble distribution, that is the mean, standard deviation, skewness and kurtosis. The second criterion considers the minimal discrepancy between the shapes of histograms of the real data and the ensemble mode. Results of the latter are presented in Figure 5.8. As we can see, the ensemble mode histogram fits the real data best for the p values close to 5%. Lower values of p make the simulation more leptokurtic, whereas the higher values widen the histogram and skew it to the right.

The exact value is found by searching the minimum of the absolute error as follows

$$AE = \sum_{i=1}^{4} |par_i^d - par_i^e| \quad (5.18)$$

where par_i is the i-th parameter out of the following four: mean, standard deviation, skewness and kurtosis, and d, e denote and real data and the ensemble, respectively. As can be seen from the results for different p values collected in Table 5.4, the value of 5% is found at the optimal.

Simulation results can be seen in Figure 5.9. The generated trajectory reproduces the main pure trading price characteristics. In particular, it holds a moving mean reversion level, it allows medium range variations and, as most important, it spikes prominently.

The simulation statistics are collected in Table 5.5. One can see a relatively close resemblance of the mean, standard deviation, skewness and kurtosis values. Also the higher central moments of the generated data are of the same order as the original values.

Analogically to the previous cases, the exceedance probability is verified for the ensemble. Same as before, each of the simulated ensemble series is sliced into ten intervals each, from its respective minimum to maximum value. Then the percentage of observations is calculated, which is falling above each slice threshold. These probabilities are illustrated

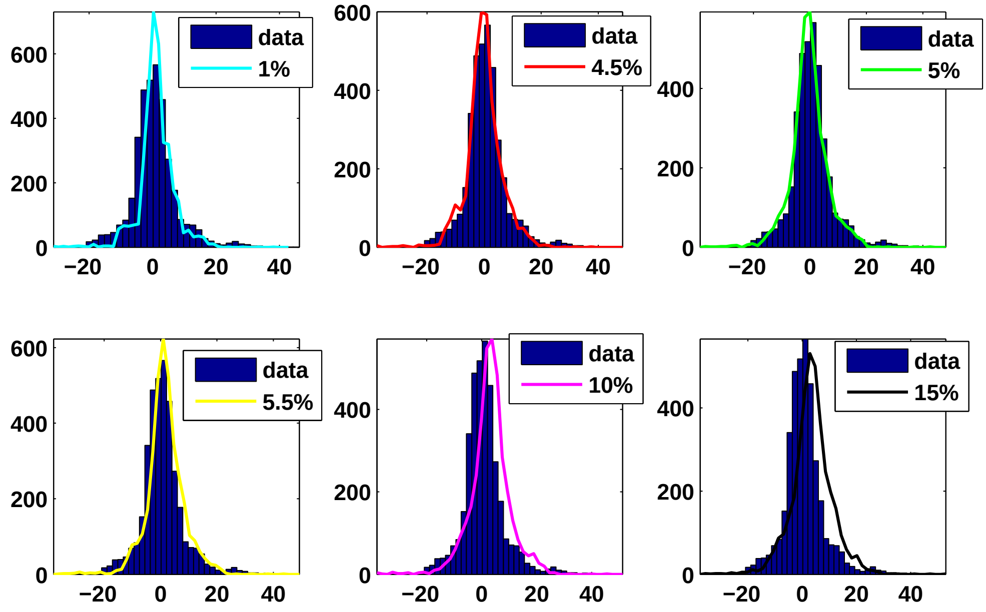

Figure 5.8: Ensemble mode histograms for the simulation with a global mean reversion underlying field, local extreme value interaction and global Burgers' type interaction, with different percents of population acting in the local interaction.

Table 5.4: Absolute error between the parameters of the histograms of the real data and the ensemble with respect to the percent of population active in the local interaction.

p	AE
4.5%	10.81
4.6%	9.43
4.7%	7.22
4.8%	5.45
4.9%	4.90
5.0%	**1.72**
5.1%	3.67
5.2%	5.21
5.3%	6.89
5.4%	8.77
5.5%	10.71

in Figure 5.10, together with the respective mean value and standard deviation limits of the original price. The ensemble is now well capturing the true price on both extreme value sides, as well as in the center. It presents a great improvement with respect to the model not having the local interaction (see Figure 5.5).

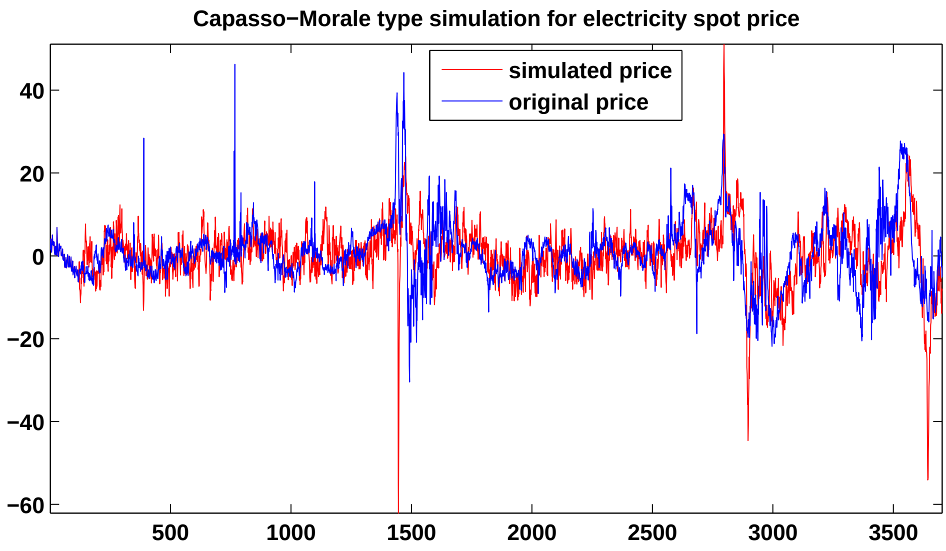

Figure 5.9: Electricity spot price simulation with a global mean reversion underlying field, local extreme value interaction and global Burgers' type interaction.

Table 5.5: Original and ensemble statistics: global reversion to a moving mean level with a moving rate, local extreme value interaction and Burgers' type global interaction.

	Original	Ensemble
Means	0.72	-0.05
St dev	7.47	7.14
Skewness	0.92	0.82
Kurtosis	6.97	7.59
5th moment	18.81	36.38
6th moment	104.91	390.69
7th moment	423.45	841.42

Finally, also the ensemble of histograms is plotted against the original data histogram as depicted in Figure 5.11. Clearly, not only the medium range variations are now captured by the ensemble members, but the general distribution shapes are very close. In Figure 5.12, a sample of ensemble histograms is chosen to show how differently they behave in the medium range.

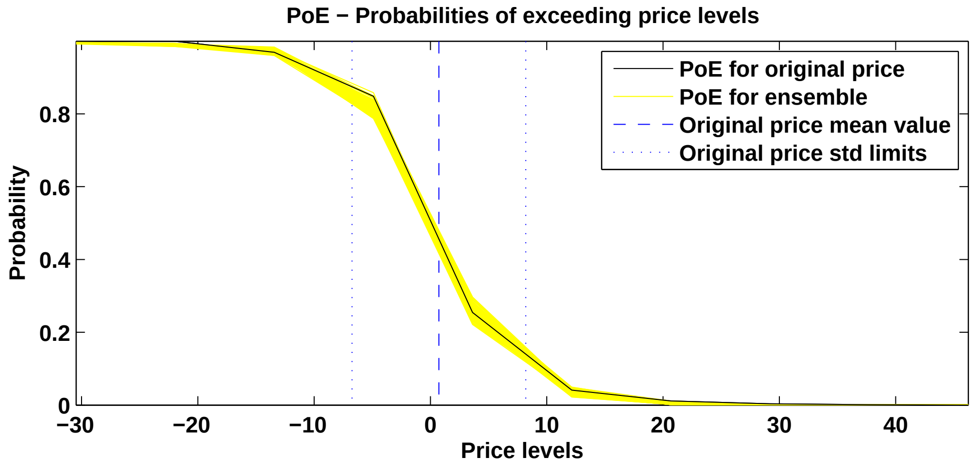

Figure 5.10: Original pure price histogram and the ensemble histograms for the simulation with a global mean reversion underlying field, local extreme value interaction and global Burgers' type interaction.

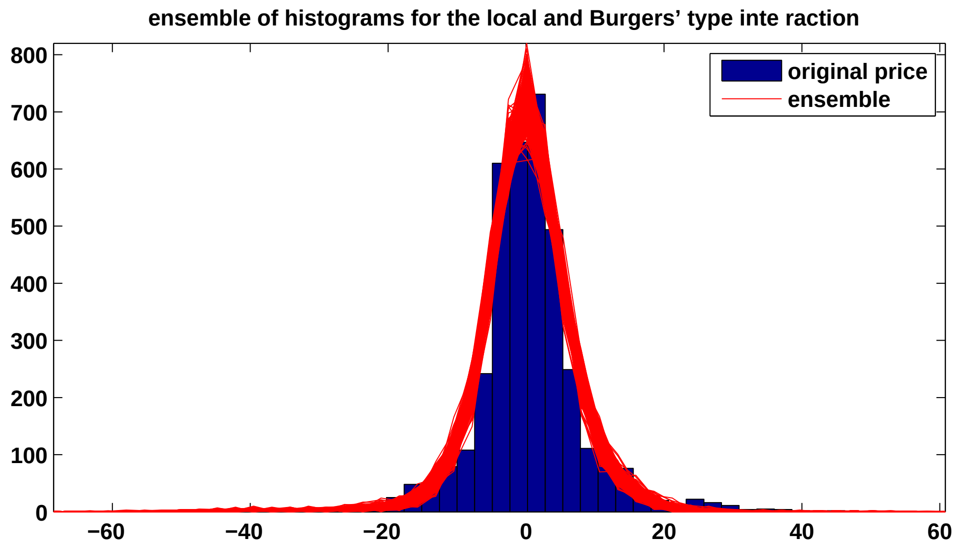

Figure 5.11: Original pure price histogram and the ensemble histograms for the simulation with a global mean reversion underlying field, local extreme value interaction and global Burgers' type interaction.

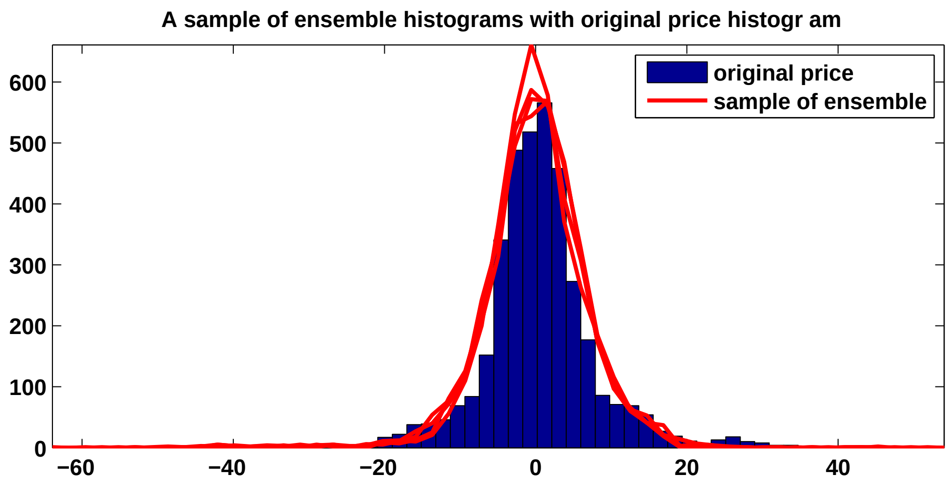

Figure 5.12: A sample of ensemble histograms.

CHAPTER VI

Discussion and suggestions for future work

The main purpose of this thesis was to present a novel family of models reconstructing dynamics of electricity spot market price behavior. As the currently existing approaches persistently fail in predicting price spikes, the author has argued that they stemmed from psychology of the traders and from market dynamics, rather than from deterministic factors.

The work started with a wide review of different classical and more elaborate approaches used in the subject. The author discussed a possible influence of deterministic factors on electricity spot prices. Among those, air temperature, electricity demand and water hydrological storage levels were mentioned. A multiple moving regression model was used to deseasonalize the original data and form a pure trading price. This series was claimed to be reflecting the true nondeterministic dynamics of the market. The pure trading series was also investigated with respect to introduction of European Emissions Allowance trading. It was discovered that since the start of Emissions Trading Scheme, the spot price dynamics changed significantly.

Next, the classical ARMA-GARCH models and mean reverting stochastic differential equations were presented as common tools for modeling electricity spot prices. The author discussed a number of their weaknesses. For instance, the classical time series models can be applied to price returns only, as they require data stationarity. Moreover, even when ARMA and GARCH components have optimally chosen orders, their predictive distribution fails to capture the real data behavior. The other common approach is based on a mean reverting stochastic process. A pure Ornstein-Uhlenbeck process fails to reproduce prominent spikes, as it is, by definition, a Gaussian process. Therefore, it cannot reconstruct the specific non-regular leptokurtic distribution of the price series. This model can be enriched with a jump component. However, it has been shown that mean reversion of spikes has to be treated differently than the base process, with separately estimated reversion rate. Moreover, the jump process can only be generated in a probabilistic way, not having the right occurrence times of the spikes. One of the studies has shown an observation that spikes are more probable to occur from more extreme price levels. Therefore, they should rather appear from price dynamics rather than through a separate jump process.

Following the aforementioned conclusions, the author suggested two possible reasons for the failure of currently used econometric models in predicting spot price spikes. These are the existence of market momentum, violating the EMH assumptions, and the animal spirits of market participants. The former means that markets navigate towards higher prices, which every now and then gets magnified to the scale of an extreme event. The latter indicates that what happens in financial markets is not just a result of cold calculation and econometrics. Behind every financial decision there is a human standing and, therefore, his emotions as well.

To incorporate the suggested reasons in ensemble modeling, the author referred to methodology proposed in mathematical biology for modeling dynamics of animal populations. As traders in a market form a population and, as already mentioned, have animal spirits, a system of stochastic differential equations was used, where the measure of distance between traders was a price. The individuals were allowed to interact in three scales. The macroscale was driving the direction of the whole population. The microscale dealt with each individual separately. Finally, the mesoscale allowed interactions of each individual with its closest neighborhood. The second goal was to incorporate the idea of market momentum. A physical analogy to this phenomenon can be found in fluid dynamics. In particular, the Burgers' equation contains the component reflecting the liquid carrying its own momentum. Therefore, a Burgers' type interaction was implemented in the model proposed in this work. In this way, whenever the majority of the ensemble members (mode) was bidding sufficiently far from the population mean, that difference was getting magnified to a price spike.

The simulation results showed that the proposed model was able to reproduce a number of electricity spot price statistics. Firstly, the process was spiking even though no jump component was included in the model. Secondly, the basic trading biases were incorporated, that is short term thinking and herding. The former allowed to follow a global moving mean reversion level reflecting the underlying field of the population. The latter formed local interactions between individuals and created a proper level of small and medium range variations. Moreover, the general statistics of the ensemble were able to reproduce a number of parameters of the original data distribution. These included the mean, variance, skewness and kurtosis. Also, the orders of higher central moments were comparable with the original values, and the probabilities of the ensemble crossing specified price levels were able to capture the respective probabilities of the real data.

It seems that the latest assumptions about human psychology and market momentum ruling market dynamics may be true. A possibility for extension of this study would be to estimate the model using real bid and offer data from the spot market, accounting for different types and sizes of generators and consumers, as it was done in the case of the Italian car market by Capasso et al. (2003), Bianchi et al. (2003) and Capasso et al. (2005). Then the forces driving the ensemble members would be individual-dependent. Moreover, the comparison of statistical distributions of the real and simulated data could be improved with use of f-divergence and blended f-divergence measures.

Nevertheless, the outcomes of this work open new possibilities for modeling not only electricity spot prices, but other commodity markets as well. The proposed model should be studied for other price series to verify, whether it can also reproduce extreme events in other financial markets. Finally, the numerical scheme used in this work could be considered as a base for a numerical method for solving the Burgers' equation.

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Part II: Publications

PUBL. I

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Reliability of ARMA and GARCH models of electricity spot market prices

Piotr Ptak, Matylda Jabłońska, Dominique Habimana and Tuomo Kauranne *

Lappeenranta University of Technology - Dept of Mathematics
P.O.Box 20, 53851 Lappeenranta - Finland

Abstract. Electricity spot market price is notoriously difficult to predict because of the high variability of its volatility that results in prominent price spikes, interlaced with more Gaussian behavior. Such varying volatility has prompted researchers to use GARCH modeling to forecast spot prices. In this article, we study the reliability of an optimally chosen GARCH and its accompanying ARMA model of two electricity spot market price time series using a Markov Chain Monte Carlo (MCMC) method. The MCMC method is used to estimate the parameters of the ARMA-GARCH model. It appears based on this analysis that even an optimally chosen ARMA-GARCH model is not sufficient to explain the behavior of electricity spot market price.

1 Introduction to electricity spot markets

Nordic power suppliers generated around 397,6 TWh last year, 40% of which came from Sweden, 35% from Norway, 16% from Finland and remaining 9% from Denmark. Most energy producers try to keep flexibility between different energy sources, mostly to diversify raw materials price risk. Table 1 presents repartition of electric energy origins among the Scandinavian countries.

Table 1: Different types of energy sources in Scandinavia.

Country	Hydropower	Nuclear Power	Other thermal sources (coal, gas)	Other renewable sources (wind)
Norway	99%		1%	
Finland	20%	33%	47%	
Denmark			81%	19%
Sweden	46%	42%	12%	

Electricity spot markets have been studied widely over the last twenty years due to the complex structure of electricity price time series [1]. Electricity prices on real-time markets are both highly volatile and difficult to predict. However, ongoing analyses of spot markets are conducted in order to make markets as close to perfect as possible. The main obstacle is that techniques of calculating electricity prices differ significantly in different countries. Nevertheless, the aim is to set the prices based on day-ahead and hour-ahead orders, so that the balance between supply and demand is met.

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In spite of being highly volatile, electricity prices have some visible statistical features. Firstly, they are highly correlated with temperature and hydrological conditions - the higher is the precipitation, the cheaper is electricity. Secondly, the prices are extremely dependent on demand. When power generation is below the adequate level, prices rise. This forces buyers to consume less and suppliers to increase production. When supply is sufficient, prices drop, resulting in lower power generation and ordinary consumption levels.

Spot markets are exchange markets where the exchange of takes place within up to two working days after striking a deal. This characterizes equally share, bond, currency and commodity exchanges. Electricity trading is one of the most significant spot markets. However, there is one main feature which distinguishes electricity from other types of exchangeable stock. Usually differences between demand and supply can be managed by storage capacity. Unfortunately, electricity is something that cannot be kept in a warehouse. In this manner, spot trading provides a possibility of almost permanent balance between supply and demand.

2 Spot trading on NORDPool/NEPool

In 1996 the first international electric power exchange was set up. The main goal was to create a common Nordic market with a guarantee of strong competition between suppliers in the area. That was possible due to a wide diversity of Scandinavian energy sources: hydropower (Norway, Sweden, Finland), nuclear power (Sweden, Finland), thermal power (Sweden, Finland, Denmark) and significantly increasing wind power (Denmark). Nowadays, Nordic Power Exchange (NORDPool) is owned by two Scandinavian grid companies: Norwegian Statnett SF and Swedish Affärsverket Svenska Kraftnät, 50 per cent of shares for each.

The part of NORDPool's activity, that we are interested in, is Elspot - the spot floor, collecting next day's demands for electric power for each of the 24 hours of the following day and set the system prices for that day. A strict daily schedule is obligatory for all market participants. It covers receiving buy/sell offers from participants, system prices' calculation, data verification and discussion on probable participants' concerns and, finally, next day's prices publication by the exchange.

The New England Power Pool (NEPool) was formed in 1971 as a six-state region electricity coordinator. Though it is a corporation (not a stock exchange) its most important role is to provide spot market trading, which will match electric power supply and demand. Similarly to NORDPool, hour-ahead and day-ahead orders are used in estimating the system prices, which should be a compromise between buyers' and sellers' expectations. Moreover, the Pools are of a not-for-profit character. Their goal is to work out electricity prices in order to match demand and supply. In addition they have strict policies forbidding any professional connections between employees and companies trading in the Pools.

Both Pools provide an interesting data set for mathematical modelling. Their unique features emerge from the impossibility of storing electric energy.

3 Time series forecasting

Classical Box-Jenkins time series methods have been extended by many new features in the hope of making them apply to time series with more complex behavior. Typical Box-Jenkins methods [2], such as ARMA and ARIMA forecasting, are based on an underlying assumption of ergodicity over some time scale, and on linear dynamics.

In practice, these assumptions are hard to verify and one often resorts to empirical trial and error in finding a suitable model and hoping that the residuals it leaves do not display any significant structure.

More recently, it has become computationally possible to study the validity of such assumptions by Monte Carlo simulation. A particularly appropriate variant is the Markov Chain Monte Carlo method that can be used to study the covariance of model parameters as well as the robustness of its forecasts by treating ARMA and GARCH model parameters as samples from some distribution.

3.1 GARCH models built upon ARMA models

In a classical time series approach, one of the biggest challenges is to provide a mathematical explanation of changing volatility in the data. Since returns of electricity price data shows heteroscedasticity, i.e. volatility that varies in time, we use (Generalized) Autoregressive Conditionally Heteroscedastic (G)ARCH fitting [2, 3]. These types of models are widely used for time series that have variance varying with time. Financial data sets are often characterized by so-called variance clustering [2, 4], which means noticeable periods of higher and lower disturbances in the series.

An ARCH or GARCH model is used to complement an underlying ARMA model. An ARMA model is just a GARCH model that assumes homoschedasticity, i.e. a constant variance. A GARCH model is therefore applied to the residual left by the ARMA model.

An autoregressive conditional heteroscedasticity model represents the variance of a current error term as a function of variances of error terms at previous time periods. ARCH simply describes the error variance by the square of error at a previous period.

In general, an ARCH(Q) model is represented as follows:

$$u_t = C + \sigma_t v_t$$

$$\sigma_t^2 = K + \alpha_1 u_{t-1}^2 + \ldots + \alpha_Q u_{t-Q}^2$$

where:

- C is a constant in error term
- $v_t \sim N(0, 1)$

- u_t are the return residuals (differences between the base ARMA model and original returns)
- σ_t^2 is the variance of residuals in time step t
- u_{t-i}^2 is the squared error term from i-th lag

A model is called generalized autoregressive and conditionally heteroscedastic (GARCH), if a second autoregressive moving average model (ARMA model) is used to represent error variance. A GARCH(P,Q) model is given by:

$$u_t = C + \sigma_t v_t$$

$$\sigma_t^2 = K + \alpha_1 u_{t-1}^2 + \ldots \alpha_Q u_{t-Q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_P \sigma_{t-P}^2$$

where:

- σ_{t-i}^2 is the variance from i-th lag

Moreover, except the conditional variances estimated in the model for every time step t, there is an unconditional variance of the series which can be expressed by the following formulae:

$$\sigma^2 = \frac{K}{1 - \sum_{i=1}^{Q} \alpha_i - \sum_{i=1}^{P} \beta_i}$$

The conditional standard deviation forecast changes from period to period and approaches the unconditional standard deviation. In the case of stationary ARCH/GARCH forecasting, predicted magnitudes for conditional variances always converge to the unconditional ones. Moreover, for estimation in heteroscedastic models a maximum likelihood method (unlike to ARMA methods) needs to be employed instead of ordinary least squares.

4 MCMC for time series

Markov Chain Monte Carlo (MCMC) techniques are numerical computation methods that can be used to estimate unknown parameters of ARMA(P,Q)-GARCH(P,Q) models which will be constructed for both NORDPool and NEPool spot markets. These techniques can be extended up to several estimates in any given model. MCMC techniques are also used to construct the distributions of unknown parameters based on random variables generated from specific well known distributions, as described in a Bayesian formulation of any problem [5]. MC methods are used to sample random numbers from different probability distributions.

When one wants to study a particular problem, an MCMC method is constructed in such way that it generates a random sample from given distributions. In general, the prior distribution contains the prior knowledge about the unknown parameters given any model. A good selection of the prior distribution results in the best parameters known to be more probable than others. In Markov Chain Monte Carlo methods, the main idea is to create a Markov Chain using random sampling so that the created chain has the posterior distribution as its unique stationary distribution, i.e. the MCMC methods create ergodic Markov Chains meaning that the process will end up in having the same stationary distribution independent of the initial distribution.

4.1 Random Walk Metropolis Algorithm

It has been shown that with too wide a proposal distribution many of the candidate points are rejected and the chain stagnates for long periods and the target distribution is reached slowly. On the other hand, when the proposal distribution is too narrow, the acceptance ratio is high but a representative sample of the target distribution is achieved slowly. A very practical way for solving this issue takes the previously simulated value into account when the proposal is constructed.

Step 1: Initialization

- Choose θ_0, then set $\theta_{old} = \theta_0$
- Choose the covariance matrix C
- Choose the length of the chain M, and set $i = 1$

Step 2: Acceptance step (Metropolis step)

- Choose sample θ_{old} from $\mathrm{N}(\theta_{old}, C)$ and u from $\mathrm{U}[0, 1]$
- Calculate $\mathrm{SS}_{\theta old}$ and $\mathrm{SS}_{\theta new}$
- If $\mathrm{SS}_{\theta new} < \mathrm{SS}_{\theta old}$ or $u < e^{-\frac{1}{2\sigma^2}(\mathrm{SS}_{\theta new} - \mathrm{SS}_{\theta old})}$, set $\theta_i = \theta_{new}$. Else set $\theta_i = \theta_{old}$
- if $i < M$, set $i = i + 1$ and go to step 1. Else, stop the algorithm [5].

Where

- θ_0 is a vector of initial parameter values of the model;
- θ_{old} is a vector of the previous sampled parameter values;
- θ_{new} is a vector of new sampled parameter values;
- M is the length of the chain;
- i is the number of iterations;
- u is the random value;

- $SS_{\theta_{old}}$ is the total sum of squares of previous sampled parameter values;
- $SS_{\theta_{new}}$ is the total sum of squares of new sampled parameter values.

In the algorithm the proposal width is the covariance matrix C of the Gaussian proposal distribution, or variance in one dimensional case. The problem of how to choose a proposal distribution is now transformed into the problem of choosing the covariance matrix C so that the sampling is efficient. In general, this is done by choosing a fixed covariance matrix by hand, by using some heuristic or "trial and error" strategy. But recently, some new techniques based on modifications of the Metropolis algorithm have been introduced in order to update the covariance matrix, like adaptive proposal (AP) and adaptive Metropolis (AM) [6].

4.2 Initialization of MCMC

When the Random Walk Metropolis algorithm with a Gaussian proposal distribution is used, the covariance matrix should be defined. It is important to choose the starting point θ_0 for the convergence rate. In a nonlinear model the starting point for an MCMC implementation is

$$\theta_0 = \min_{\theta} \sum_{i=1}^{n} \left(y_i - f\left(x_i, 0\right)\right)^2$$

where

- i is measurement index;
- θ is a vector of unknown parameter values;
- y_i represents the measurement vector;
- x_i represents the control variable.

The covariance matrix of the Gaussian proposal can be chosen by trial and error. However, it is useful to use the covariance approximation obtained from linearization. This means that the model is linearized and then the formula from linear theory

$$\widehat{C} = \sigma^2 \left(X^T X\right)^{-1}$$

is used. Where X is a vector of all control variables in the model.

In the case of NORDPool and NEPool time series we use MCMC techniques to sample the parameter values of ARMA(P,Q)-GARCH(P,Q) models based on the estimated parameter values of constructed models as inputs of MCMC methodology. Finally, we compare the standard errors associated to the estimated parameters with MCMC errors and test the reliability of forecasts by comparing MCMC simulated predictions to original data.

5 Estimating NORDPool/NEPool return series

Estimation and Forecasting procedures are based on two sets of data. First set comprises a total of 289 weekly points of historical spot prices for NORDPool. NEPool data set of daily prices lasts over 2551 days, so nearly 7 years. Use of GARCH technique requires returns as an input data. Both original time series and returns for NORDPool and NEPool are shown in Figure 1.

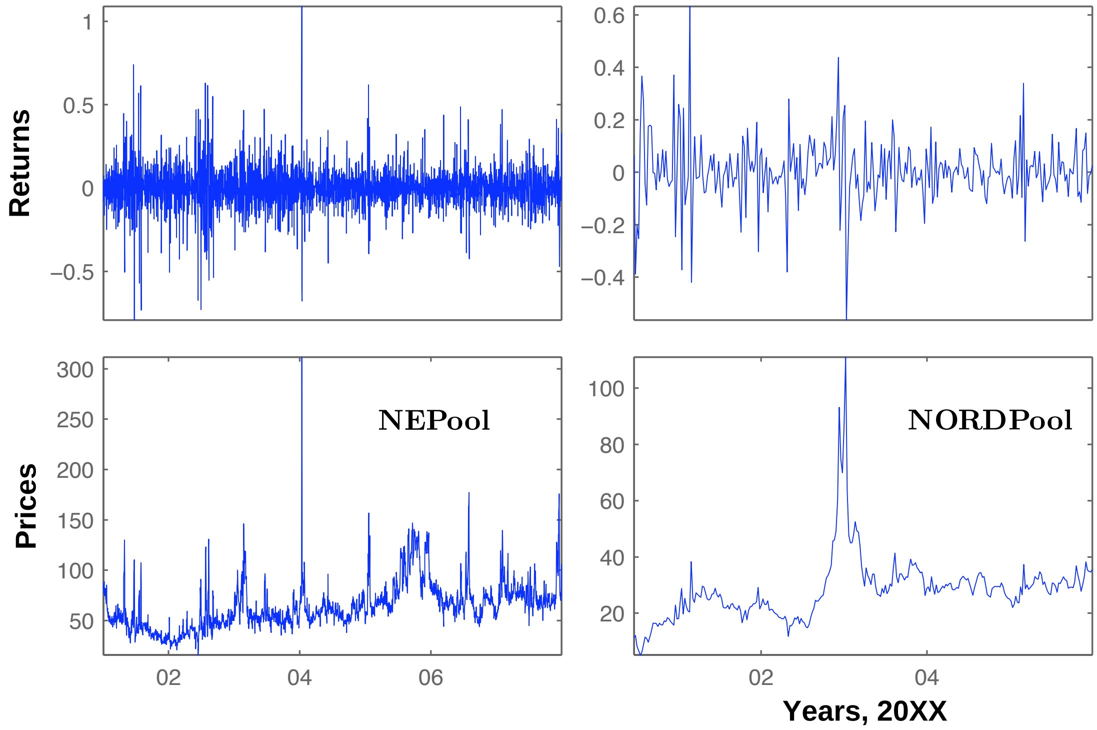

Fig. 1: Original series and its returns.

We can see that both sets are build of clusters with different variation of amplitude. Peaks are common components of energy spot prices. Due to their appearances, such signals are difficult to estimate by basic mathematical tools.

Peaks are undesired because of their non-differentiable nature. Use of Stochastic Differential Equations is impossible and one has to address this problem with methods of discrete type.

5.1 Identifying GARCH coefficients

First step is to examine autocorrelation and partial autocorrelation functions of the given data sets. These functions are depicted in Figure 2. As we can see both correlation and partial correlation at different lags are not very high and reach -0.17 for NEPool data set.

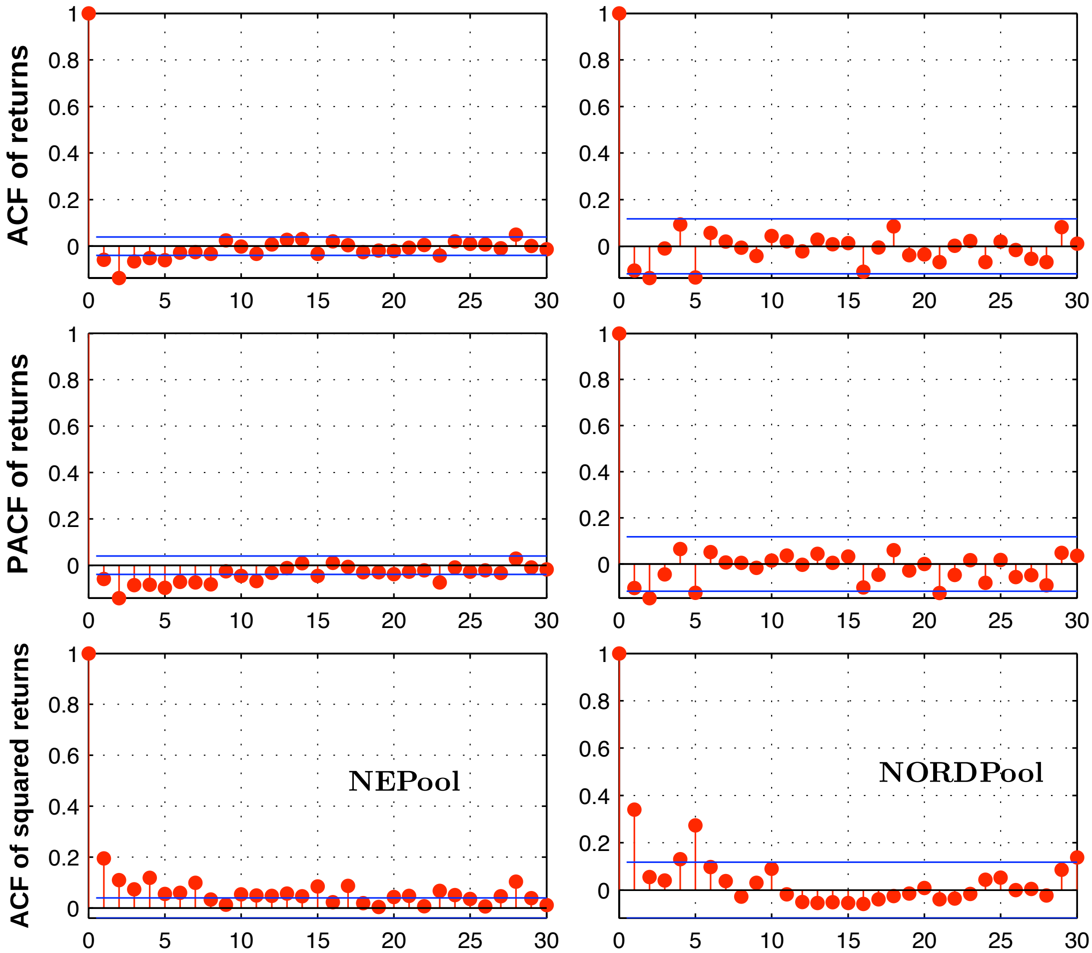

Fig. 2: Significantly higher correlation for squared returns.

Decision on type of model adequate for data comes partially from two tests: Engle's hypothesis test [7] for presence of ARCH/GARCH effects and Ljung-Box Q-statistic lack-of-fit hypothesis test [8]. The former examines a signal for a presence of GARCH components. The later checks if a signal includes ARMA effects.

Ljung-Box test verifies if there is a significant serial correlation in the raw returns for NORDpool and NEPool tested for 1 to 20 lags of the ACF at the 5% level of significance. The same test for squared returns indicates that both NORDPool and NEPool contain significant serial correlation.

Engel's test for the raw returns of NORDPool and NEPool rejects hypothesis that both series do not contain ARCH effect at the 5% level of significance. Squared returns of NORDPool do not include ARCH effect whereas squared returns of NEPool indicate presence of this effect.

Therefore, the presence of heteroscedasticity for NEPool indicates that GARCH modeling is appropriate.

5.2 Model fitting

This section describes a way to find a good GARCH model for the NEPool data. It also describes a criteria function build on Schwarz's Bayesian information criteria (SBIC), see [2]. Engel's and Ljung-Box tests give an output in a binary form, 1 or 0. Here, zero indicates lack of GARCH/ARMA effect in the series, while one indicates its presence. The SBIC is formulated as follows:

$$\mathrm{SBIC} = \log\left(\sigma_{res}^2\right) + \frac{k}{T} \cdot \log(T)$$

where:

σ_{res}^2	variance of residuals between returns and its fitted model
k	number of parameters of GARCH model
T	length of tested time series

We suggest a new information criteria function, called SLEIC:

$$\mathrm{SLEIC} = \left[\mathrm{SBIC} \cdot \left(1 + \frac{\alpha}{2L} \sum_{i=1}^{L} (H_{1,i} + H_{2,i})\right)\right]^{-1}$$

where:

SLEIC	information criteria function based on Schwartz-Bayesian information criteria, Ljung-Box test and Engel's test
$H_{1,i}$	vector of logical outputs for Ljung-Box test, $i = 1, 2, ..., 2L$
$H_{2,i}$	vector of logical outputs for Engel's test, $i = 1, 2, ..., 2L$
α	importance coefficient of Ljung-Box and Engel's tests
L	number of lags analyzed by Engel's/Ljung-Box tests

To find an appropriate model for both Pools, we maximize SLEIC function while varying orders P, Q, R and M of GARCH(P,Q) and ARMA(R,M) models.

$$\max_{P,Q} \mathrm{SLEIC}(res, k, H_1, H_2)$$

Figure 3 depicts the information criteria level (SLEIC) with respect to model complexity. Level of information criteria for NEPool returns is higher that for the NORDPool ones. It is due to lack of ARCH effect within squared returns of NORDPool series, i.e. no heteroscedasticity. Chosen models for NEPool and NORDPool are ARMA(1,1) GARCH(2,1) and GARCH(2,1), respectively.

The difference in the shapes of the SLEIC values for NordPool and NEPool is likely a result of the different length of the time series. Our NordPool series only contains 250 values, whereas the NEPool comprises 2500 values. NEPool data can therefore be modelled reliably by many more GARCH models than the sparse NordPool data set we have. This fact is reflected also in the higher values of the SLEIC function for NEPool

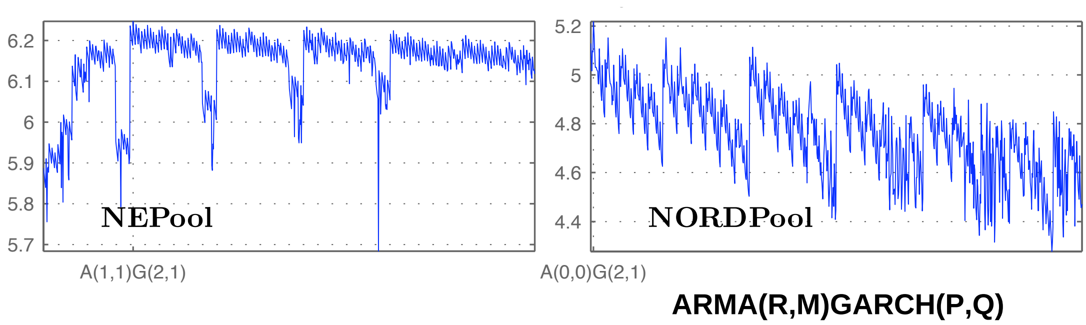

Fig. 3: Information criteria SLEIC with subject to realizations of different GARCH models.

SLEIC level analysis was performed for all possible ARMA and GARCH models up to ARMA(5,5) and GARCH(5,5), which results in 850 realizations. Explicit formulas for optimal models are:

NEPool

$$y_t = -1.206 \cdot 10^{-4} + 0.6844 \cdot y_{t-1} - 0.9096 \cdot \varepsilon_{t-1} + \varepsilon_t$$
$$\sigma_t^2 = 9.7011 \cdot 10^{-4} + 0.2758 \cdot \sigma_{t-1}^2 + 0.4713 \cdot \sigma_{t-2}^2 + 0.1943 \cdot \varepsilon_{t-1}^2$$

NORDPool

$$y_t = 8.345 \cdot 10^{-3} + \varepsilon_t$$
$$\sigma_t^2 = 2.623 \cdot 10^{-3} + 0.373 \cdot \sigma_{t-2}^2 + 0.516 \cdot \varepsilon_{t-1}^2$$

5.3 Post-estimation analysis

To examine chosen models both tests from Section 5.1 should be applied to residuals resulting from difference between returns and series of fitted model.

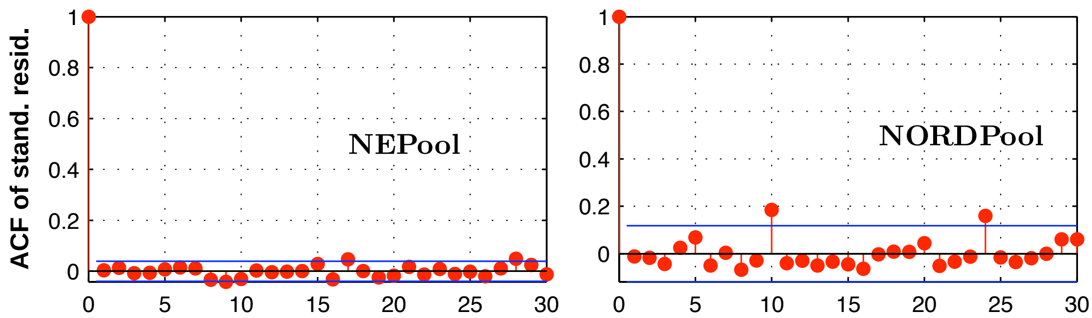

Fig. 4: Autocorrelation of standardized residuals.

Here by standardized residuals we mean the innovations divided by their conditional standard deviation. Tests for presence of GARCH/ARMA effects show that neither of standardized residuals of Pool series contains these effects.

6 Results: statistics and reliability of forecasts

6.1 Scatter plots and histograms of the sampled parameters

Since we found the most appropriate models and estimated their parameters, it is advisable to perform verification of the estimates reliability. Employing the MCMC methodology, we state the initial parameter values $\theta_{0,ne}$ as a vector of the estimated coefficients from ARMA(1,1)-GARCH(2,1) model for NEPool expressed as

$$\theta_{0,ne} = [\psi_{0,ne}\, \phi_{0,ne}\, C_{ne}\, K_{ne}\, \alpha_{1,ne}\, \beta_{1,ne}\, \beta_{2,ne}]^T$$

where

- $y_t = \lambda_{0,ne} + \lambda_{1,ne} y_{n-1} + \varepsilon_t$
- $\epsilon_t = C_{ne} + \sigma_t v_t E$
- $\sigma_t^2 = K_{ne} + \alpha_{1,ne}\epsilon_{t-1}^2 + \beta_{1,ne}\sigma_{t-1}^2 + \beta_{2,ne}\sigma_{t-2}^2$

and from GARCH(2,1) model for NORDPool as

$$\theta_{0,no} = [C_{no}\, K_{no}\, \alpha_{1,no}\, \beta_{1,no}\, \beta_{2,no}]^T$$

where

- $y_t = \lambda_{0,no} + \varepsilon_t$
- $\varepsilon = C_{no} + \sigma_t v_t$
- $\sigma_t^2 = K_{no} + \alpha_{1,no}\varepsilon_{t-1}^2 + \beta_{1,no}\sigma_{t-1}^2 + \beta_{2,no}\sigma_{t-2}^2$

Since the prior distribution for the unknown parameters θ is assumed to be Gaussian, it is treated as an extra sum of squares, then,

$$\mathrm{SS}_{new} = \sum_{i=1}^{p} \left(\frac{\theta_i - \mu_i}{v_i} \right)^2$$

where

- μ_i is the average value of the sampled parameter values at iteration i;
- v_i is standard deviation of the sampled parameter values at iteration i;
- $\theta_i \sim \mathrm{N}(\mu_i, v_i^2)$, that is, independent prior specification for θ.

After generating parameter chains with a length of 5000, we study their pair wise joint distributions, to reveal possible correlation between estimated parameters. We find that correlation coefficients for NEPool model vary from -0.9 to 0.47. This fact shows a significant level of correlation. Similarly, we analyze

NORDPool estimates and obtain coefficients with a range from approximately -0.72 to 0.44. Given results violate MCMC assumptions that require model parameters to be uncorrelated. This violation is a sign of non-ergodicity present in the residuals of GARCH models.

A next step is to study the parameters' histograms to verify the character of their distributions. These do not follow MCMC theory either – distributions of some parameters appear to be non-Gaussian for both models. This can be easily seen from Figures 5 and 6 presenting histograms and pair wise scatter plots for NEPool and NORDPool respectively.

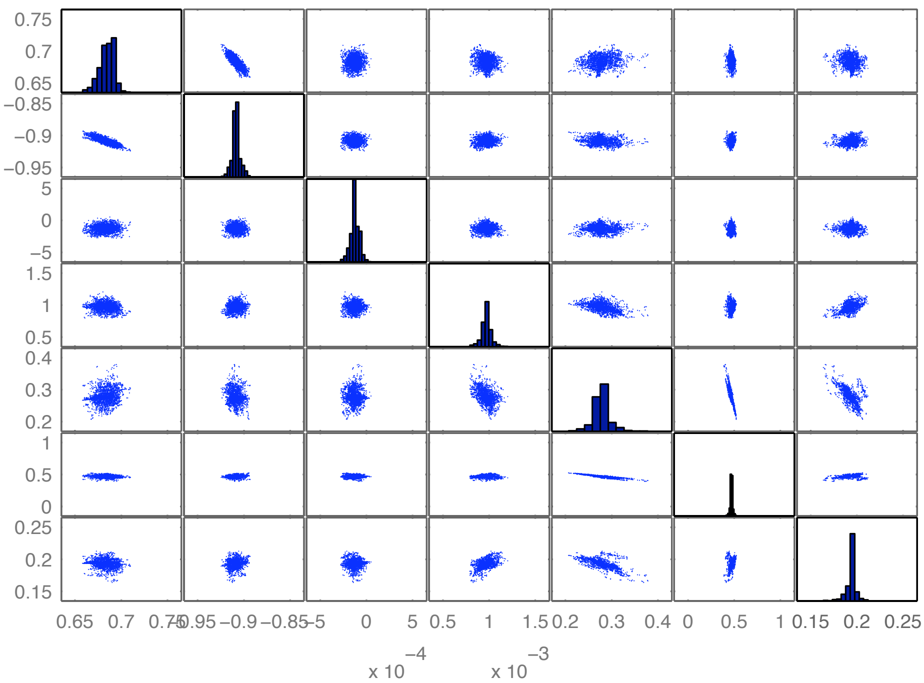

Fig. 5: Pair wise scatter plots for NEPool model parameters.

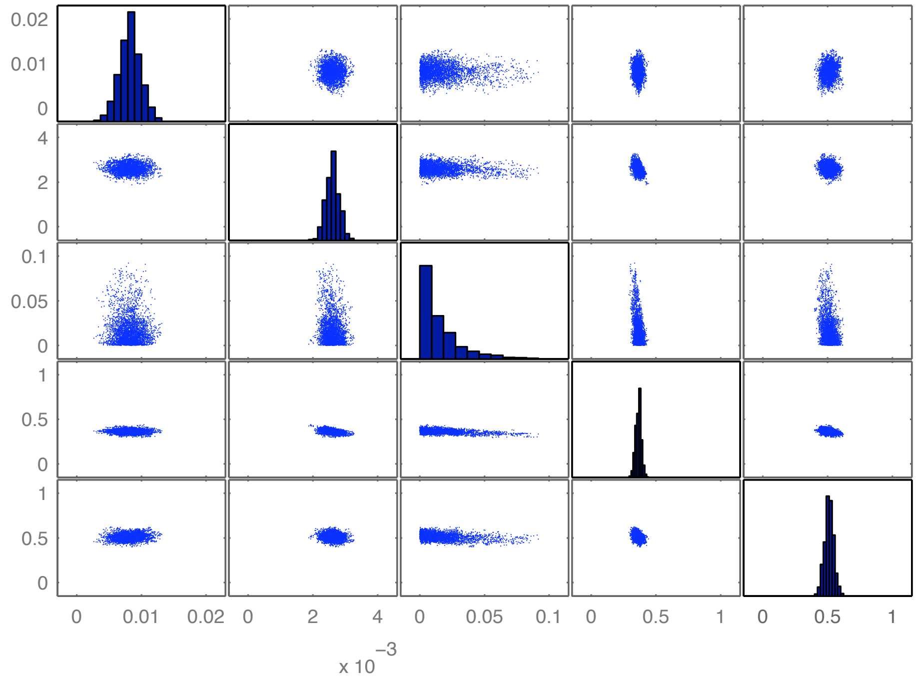

Fig. 6: Pair wise scatter plots for NORDPool model parameters.

One reason to the non-Gaussian distribution reflected by parameter covariance is the constraint of non-negativity imposed upon most parameters, which were bounding ranges of prior distributions.

6.2 Predictive distributions of sampled price returns

MCMC methods are based on random sampling and result in empirical distributions for unknown parameters. Moreover, it is possible to sample values for model prediction at different points and construct a distribution also for the response curves of the model, called 'predictive distributions', which give the information related to uncertainties in unknown parameters.

In case of NEPool spot market, a predictive distribution was constructed based on the sampled values for model prediction in terms of price returns, where 22 values were predicted as shown in Figure 7.

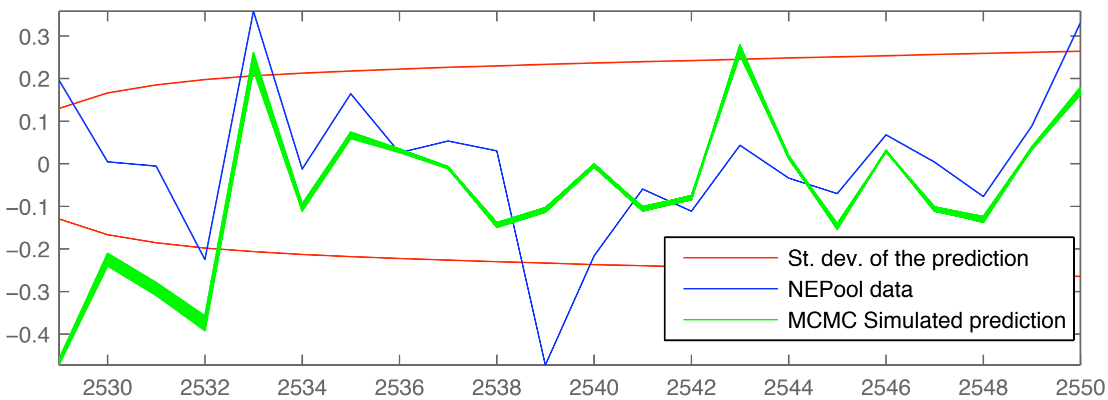

Fig. 7: Predictive distribution of price returns for NEPool series.

Figure 7 shows that the predictive distribution for the price returns will most likely lie inside the calculated bounds. However, we can see that the longer the forecasting horizon is, the more uncertainty predicted values have. On the other hand, the posterior distribution of the forecast is concentrated around the initial prediction. Figure 7 indicates that ARMA(1,1) GARCH(2,1) model for NEPool can be used for forecasting returns, but only in a short-term horizon ahead. This conclusion stems from comparison of random variations of the predictive distribution of returns and the original return series.

In case of NORDPool spot market, 10 values were predicted from the sampled returns. Analogically, comparison of predictive distribution for portfolio returns and original returns indicates that a GARCH(2,1) model for NORDPool can also be used for forecasting the returns for a short-term horizon, as shown in Figure 8.

On the other hand, the fact that the true time series does not lie within the posterior distribution of GARCH forecasts means that there must be some essential feature in electricity spot price time series not captured by the GARCH paradigm, and by implication not by any ARMA model either.

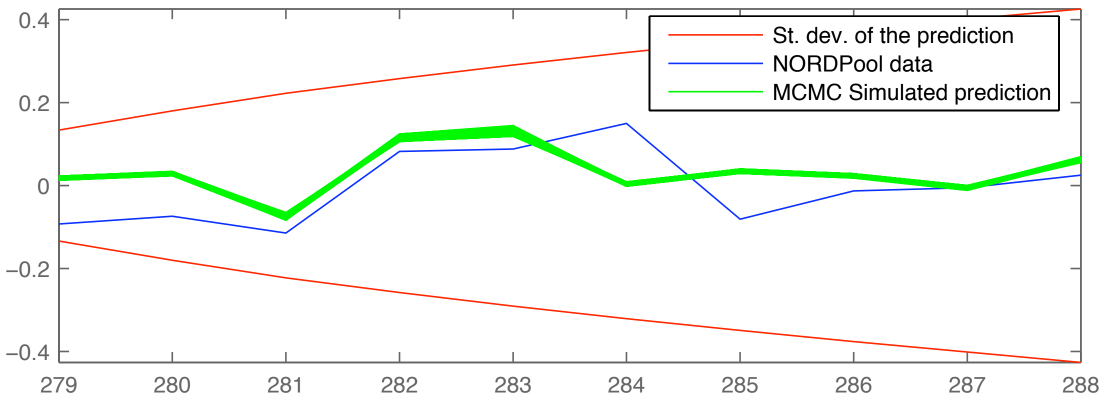

Fig. 8: Predictive distribution of price returns for NEPool series.

In summary, even though some MCMC assumptions were violated, shapes of predictive distributions for model coefficients confirmed the initial prediction of their values. They also indicated that both estimated models may work reasonably in short-term forecasting.

7 Conclusions

We have identified ARMA and corresponding GARCH forecast models for two time series of electricity spot market prices, the Nordic NordPool and the U. S. NEPool. Models for both series are statistically optimal within a wide spectrum of ARMA and GARCH orders. Both the size of the data sets, and the behavior of the two time series are quite different, even if both series display prominent spikes.

GARCH models assume that a time series can be modeled by a linear model with the sole assumption that its variance may depend on past variance history. We have tested the validity of this assumption by carrying out a Markov Chain Monte Carlo (MCMC) analysis on the parameters of such optimally identified GARCH models.

The results of the MCMC analysis indicate that although the models are able to forecast the future behavior of spot market prices with some skill, the models are not well identifiable. This is shown in the non-Gaussian structure of model parameter covariance, and also in the escape shown by the true spot price from the confidence envelope provide by MCMC sampling of model parameters.

Such results indicate that the behavior of electricity spot price is not captured by just adding the assumption of heteroschedasticity - there must be something deeper at play. In fact, other research groups have come to the same conclusion by different means, such as Bottazzi, Sapio and Secchi [9]. They study the Subbotin family of distributions and similarly identify that NordPool time series needs at least two different distributions to capture its dynamics.

Indeed, it appears as if the price time series would obey two different dynamics. The first of these is a relatively regular "elastic" behavior, when the market is efficient with supply and demand that balance each other. The second one occurs when some event pushes the market to a "seller's market" that allows spot prices to surge because non-elastic demand temporarily exceeds potential supply. Such a dual market nature would call for at least two different models to be used simultaneously. The reliability of such a dual model setup can, on the other hand, be analyzed using an appropriate modification of the MCMC paradigm, the so-called Reversible Jump MCMC (RJ-MCMC), as proposed by Laine et al [6].

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Stochastic simulation of the Uplift process for the Irish Electricity Market

Matylda Jabłońska * Arno Mayrhofer † James P. Gleeson ‡

Abstract. In the Irish electricity market participants declare their true marginal costs and therefore the Shadow Price alone does not guarantee that generators will recover their fixed running costs. The so-called uplift complements the price and ensures that the generators recover their total costs. The aim of this paper is to review purely stochastic features of the uplift and make an attempt to simulate a new process reconstructing the original data characteristics. We propose two alternative algorithms basing on the uplift wait-jump structure as well as daily and annual seasonality. Presented results show that this kind of reconstruction is possible up to a quantitatively comparable degree.

Keywords. Electricity Price, Irish All Island Market for Electricity, Uplift, Stochastic Simulation, Financial Time Series

1 Introduction

Electricity prices are as popular in research studies as any other financial time series. However, due to the main feature that differentiates electricity from other commodities, i.e. its non-storability, the electricity spot prices are one of the most challenging types of time series in terms of simulation and forecasting. Moreover, it has already been shown by many authors, e.g. [8], that their behaviour cannot be fully captured by classical time series models.

Methodologies for electricity price calculation may vary among different markets. For instance, in the Irish All Island Market for Electricity the System Marginal Price (SMP) calculated on a half-hourly basis with use of Market Scheduling and Pricing (MSP) Software consists of two components. The first one, *Shadow Price*, represents the marginal cost per 1 MW of power necessary to meet

*Corresponding author. Lappeenranta University of Technology, Department of Mathematics and Physics, Skinnarilankatu 34, 53850 Lappeenranta, Finland, tel. (+358) 40 53 13041, fax (+358) 5 621 2898, matylda.jablonska@lut.fi

†University of Vienna, Department of Mathematics, Dr.-Karl-Lueger-Ring 1, 1090 Vienna, Austria, arnom@amconception.de

‡Department of Mathematics and Statistics, University of Limerick, Ireland, james.gleeson@ul.ie

demand in a particular half-hour trading period. It is considered as within an unconstrained schedule, i.e. with no power transmission congestions. What complements the half-hourly SMP values is the so-called *Uplift*, which, added on top of the Shadow Price, makes sure that all the generators recover their total costs, including any expenses associated with start up and no-load costs.

The problem of models for uplift calculation has already been addressed in various studies, with example of [6], [5] and [2]. Thus the process values are obtained from software tools, which solve complicated optimization programmes with constraints based on knowledge of generation and demand. However, an interesting question emerges – whether the uplift process can be described and simulated as an individual stochastic process, with no background or constricting variables. This issue has been posed by Bord Gáis company at the 70^{th} European Study Group with Industry (hosted by Mathematics Applications Consortium for Science and Industry) and provided the basis for this work (see [4]). Even though there is no open market in uplift itself, having a stochastic model of the process is useful both to Bord Gáis and to other industry participants. Such a model can be used for VaR style analysis of the risk inherent in a book of power contracts, as well as being used to set prices for wholesale customers.

In this article we analyze the Irish uplift time series as a pure stochastic process, making note of its statistical features and proposing reasonable simulation approaches. The data set covers 451 days of half-hourly observations, which gives a decent background for a reliable statistical study. The aim of the simulations is to synthetically reconstruct a series which visually behaves similarly to the original uplift series and shows comparable statistical parameters (mean, standard deviation, skewness, kurtosis) and autocorrelation structure. The first simulation attempts are built on uplift features like jump waiting, jumps and zero-reversion. The other approach depends on probabilities for uplift price levels and constant plateaus for specific trading periods as well as seasonal components. All the proposed algorithms show that it is possible to reconstruct a non-negative process consisting of plateaus and jumps, but we also verify that the last proposed approach gives the best reconstruction of the uplift intra-day structure.

The paper is organized as follows: Section 3 presents a simulation approach and its results based on uplift jump waiting, jumps and zero-reversion features. Section 4 contains simulation algorithms based on uplift behaviour conditional on time of the day and presents respective results. Finally, Section 5 concludes and gives suggestions for future work.

2 Introduction to uplift calculation and study motivation

In the Irish Electricity Market, the System Marginal Price for each half-hourly trading period (SMP_h) consists of two components. The first one, Shadow Price (SP_h) representing the marginal price of electricity per MWh in each half-hourly trading period based on the information provided by the generators and the uplift (UP_h) represents the correction applied retrospectively to the shadow prices to ensure the fixed running costs recovery for all generators.

Every day the uplift process values are determined (see [9] for more details) by the Single Electricity Market Operator (SEMO) by solving a quadratic program that minimizes both uplift revenues (the Cost objective) and the Shadow Price distortion (the Profile objective).

$$\min_{UP_h,\, h=1,\ldots,48} F(UP_h) \equiv \underbrace{\alpha \sum_h \left[(SP_h + UP_h) \sum_g Q_{gh} \right]}_{\text{Cost objective}} + \underbrace{\beta \sum_h UP_h^2}_{\text{Profile objective}}$$

subject to

$$\begin{aligned} \textstyle\sum_h [(SP_h + UP_h) Q_{gh}] \geq CR_g \quad & \text{for all } g = 1, 2, \ldots, G \\ UP_h \geq 0 \quad & \text{for all } h = 1, 2, \ldots, 48 \end{aligned}$$

where $F(UP_h)$ represents the uplift function, and Q_{gh} means quantity of electricity produced by generator g in half-hourly trading period h. Parameters α and β stand for importance of the uplift Cost objective and importance of the uplift Profile objective respectively. CR_g is the total cost of running for generator g, given by

$$CR_g = \sum_h [Q_{gh} C_u + NLC_g \mathbb{I}_{Q_{gh}>0}] + ST_g$$

where C_u is the variable fuel cost per unit, NLC_g is the no-load cost of generator g representing the generator's expenditure when operating in stand-by mode and not producing electricity, and ST_g is the start-up cost of turning on a generator g that stays switched off as long as no production takes place. These costs will be considered constant for all half-hourly trading periods h for all days t. The first listed constraint ensures that each generator g recovers its costs CR_g and the second one certifies that all uplift values stay positive.

Also, there have been some alternatives of objective functions and constraints studied as can be found from [4]. Thus we can see that methodology of uplift calculation is well established. However, as typical for highly volatile electricity price markets, there appears a need for a more statistical analysis of the uplift process. The prices are determined on a day-to-day basis, whereas risk models require long-term view on the price behaviour. Companies tend to analyze risks and plan preferably for the whole year. But exact generation and consumption quantities can not be predicted for such a long time horizon. And this is where stochastic analysis comes to play a significant role.

As soon as one is able to investigate statistical features driving a given process it is possible, by using Monte Carlo simulations, to get more information on the distribution of the uplift prices. We assume that this knowledge would leave that general patterns of the process unchanged, and would rather give a better view on process behaviour and would support electricity companies in risk analysis (including probability of outstanding values/spikes occurrence) and contracting prices for the customers.

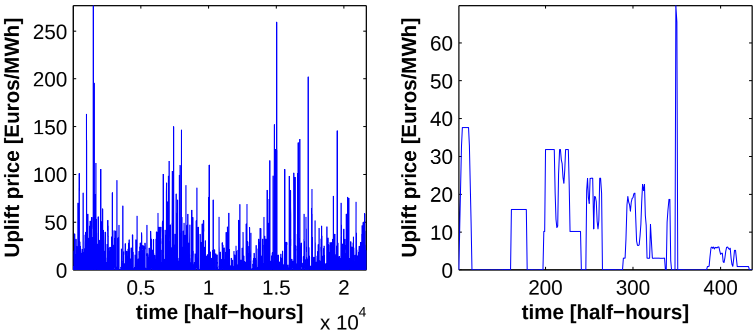

Figure 1: *Uplift half-hourly over 451 days (left panel) and 7 days (right panel).*

3 Uplift as a jump-waiting and jumping / zero-reverting process

3.1 Properties

The data set covers 451 days of half-hourly observations. Figure 1 (left panel) presents the original half-hourly uplift data series. The definition of uplift as a complement of shadow price states its first important feature, i.e. non-negativity. Moreover, from Figure 1 (right panel) we can see that the process has a clear step structure, i.e. there are plateaus and jumps. The presented analyzes will be performed on horizon equal precisely to one year.

Usually, when dealing with 'easy' and predictable financial or economical time series, we first think of a classical time series approach, i.e. using Autoregressive Moving Average (ARMA) models (see [3]), optionally extended by a Generalized Autoregressive Conditionally Heteroscedastic (GARCH) approach (see [1]). In case of uplift a first visual investigation tells us that ARMA models are not applicable here. However, we do use a piece of the classical theory. Since we expect the price series to be strongly seasonal (prices depend on demand which is seasonal), we use the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) to investigate the uplift periodicity. Figure 2 presents the respective results. As expected, we clearly see the humps in the ACF repeating with 48-lag regularity. Moreover, they are slightly locally maximal for every 336-th lag (48 half-hours times 7 week days), showing the weekly periodicity as well. This weekly periodicity is not as significant as the daily periodicity. To reproduce the 48 half-hour periodicity in the simulation we will base the current observation on the one that occurred 48 periods earlier, with a regression-estimated coefficient.

As we have already seen from the visual representation of the uplift data, we know that the process has a step (up and down) structure. Therefore, consistent with typical approaches (see [7]), we decide to use the Poisson distribution to model the plateaus sizes in the uplift series. Moreover, as can be seen from Figure 1, the jump waiting times are not distributed uniformly, but

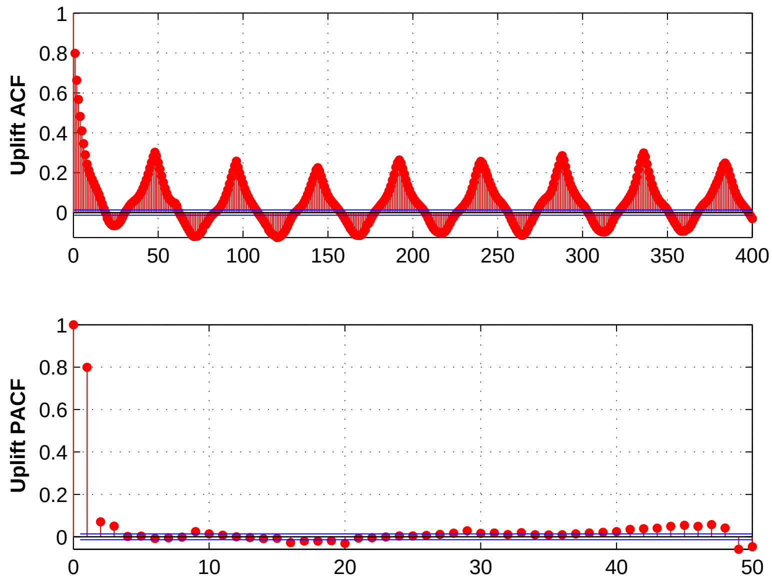

Figure 2: *ACF and PACF of uplift series.*

rather depend on the current price level. In particular, the zero level constant parts of the process are considerably longer, whereas when the process reaches relatively high values, it jumps down almost immediately. Therefore, we predetermined 4 different heuristic price levels for which we used different values of the Poisson parameter λ, estimated as an average length of plateau within the specified uplift level. They are as follows:

- $\lambda = 20$ for $U_h = 0$
- $\lambda = 8$ for $0 < U_h \leq 40$
- $\lambda = 1$ for $40 < U_h \leq 100$
- $\lambda = 0.05$ for $U_h > 100$

Also, we state that the probability whether the process jumps up or down at a given time point depends on the current price level, i.e. the higher the current uplift is, the more likely it is to jump down. Moreover, if the current price crosses a considerably high price level (also defined heuristically), the process will continue jumping down until it reaches zero. This tends to occur after a small number of half hours. Figure 3 presents normalized histograms for jumps up and down with respect to different price levels.

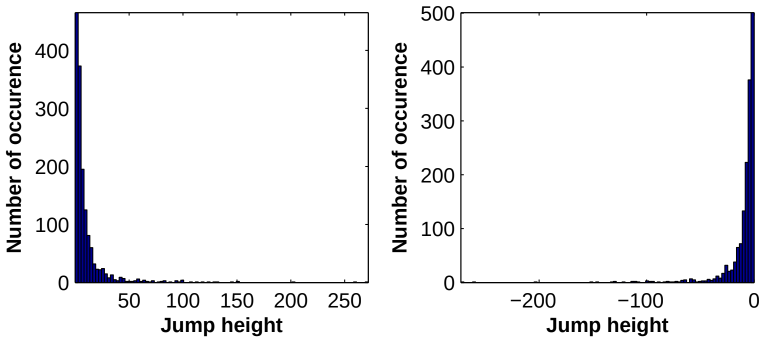

Figure 3: *Histograms for occurrences of jumps up (left panel) and down (right panel) depending on current uplift level.*

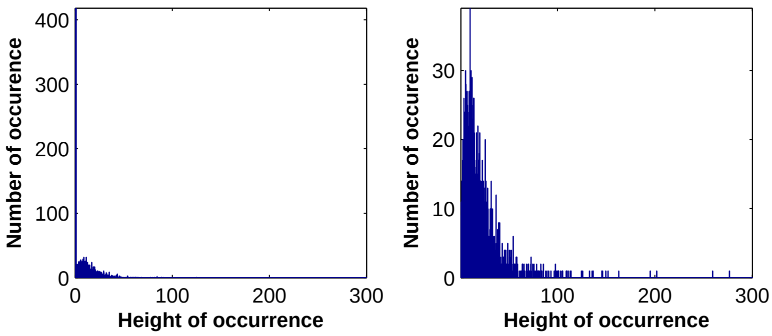

Figure 4: *Empirical histograms for uplift jump heights up (left panel) and down (right panel).*

Along with the up/down jump probabilities there comes a need for the study of jump heights. We identify the empirical unconditional distributions of jump magnitudes both up and down (see Figure 4) and use those later for jump height simulations.

As we can see from the plots, the histograms seem to be of exponential shape. For the purpose of sampling, we build empirical cumulative distribution functions and use those for random number generation, rather than estimate possible exponential distribution parameters.

We noticed that in the real data it is very likely that the prices are in most cases constant (usually zero) in the night hours. This characteristic is accounted for in the final process formulation as follows: after having the base process simulated, we set the night hours uplift to zero, with uniformly distributed probability of zero level being from 6 to 10 hours long.

3.2 Model and simulation algorithm

The review and discussion of Section 3.1 supplies insights about the structure and specific statistical features of the uplift price process. Using this information, we build a simulation algorithm implementing particular types of series behaviour as follows:

- non-negativity and zero-reversion
- strong 48 half-hourly periodicity
- constant price steps with length depending on price level
- jumps up and down with direction depending on price level
- probability of uplift staying constant or in particular zero being higher for night hours.

Based on the features we can formulate the uplift model as follows:

$$U_h = \begin{cases} J_u, & U_{h-1} = 0 \\ J_d, & U_{h-1} > M \\ \beta U_{h-48} + J_u \cdot v_u + J_d \cdot v_d, & \text{otherwise} \end{cases}$$

$$U_{h+1:h+w|_{U=u}} = U_h$$

where

- U_h and U_{h-1} is the uplift for trading period h and $h-1$ respectively

- J_u is a jump up with empirical distribution of upward jumps

- J_d is a jump down with empirical distribution of downward jumps

- M is a heuristic *large* uplift threshold

- β is the estimated regression coefficient for the 48-half-hourly periodicity

- U_{h-48} is the uplift for trading period $h-48$, i.e. the respective half-hourly trading period on the previous day

- v_u is a binary-distributed variable for jumps up

- v_d is a binary-distributed variable for jumps down, and $v_d = 1 - v_u$

- $U_{h+1:h+w}$ is the uplift for w consecutive trading periods

- $w|_{U=u}$ is the Poisson-distributed process waiting time conditional on the uplift level U.

This model strongly underlines the daily periodicity of uplift behaviour, as well as the fact that process waiting times are not the same for different price levels. Also the fact whether the next process move goes upwards or downwards is related to the current uplift status. This behaviour is expected to be related to particular daily electricity consumption patterns – highest in the peak morning and afternoon hours, and lowest at night.

Having identified the main features and defined the model we can build an algorithm for process simulation. The aim of this simulation is not to precisely reconstruct the real uplift series, but rather to synthetically produce a process that quantitatively behaves similarly to the original data in the long and short term horizon. We write down the simulation algorithm in a form of pseudo-code as follows:

1. compute regression coefficient for U_h and U_{h+48} dependence, where U_h is the uplift value in moment h
2. set the first 48 simulation values as the first 48 observations from the real uplift
3. initiate the Poisson parameters for 4 price thresholds
4. generate jump waiting time based on the current price level
5. set current time point as sum of last time point and the new generated waiting time
6. set the uplift values within the waiting time equal to the previous uplift value
7. then
   a. if the last price value after the last jump is higher than a predetermined threshold, force only jumps down until the uplift reaches level zero; if jumps down make uplift go below zero, set the last price to zero
   b. else, if the last price value after the last jump equals zero, force only jump up by a magnitude generated from empirical distribution
   c. else, based on the current price level sample whether the process jumps up or down and then sample the jump magnitude
   d. add the sampled jump magnitude to the price value that occurred 48 periods ago, multiplied by the earlier estimated regression coefficient
   e. move the current time point by one step ahead
8. return to point 4 while the current time point does not exceed the target process length to be simulated
9. set the night hours equal to zero with uniformly distributed probability of zero level being from 6 to 10 hours long.

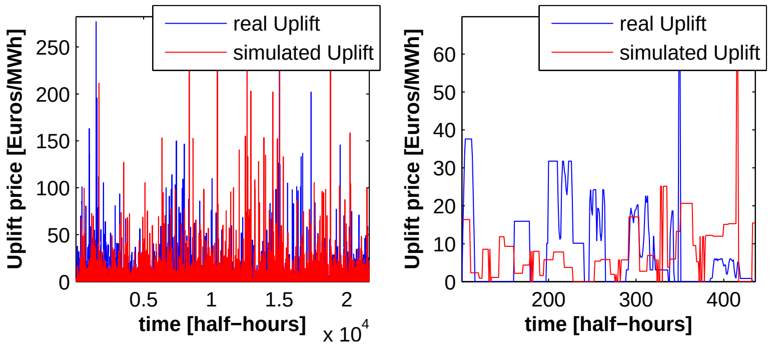

Figure 5: *Uplift half-hourly over 451 days (left panel) and 7 days (right panel).*

3.3 Results

The simulation was run using the algorithm presented in Section 3.2 for a sample as long as the real data set to get an insight of whether not only the short but also long term reconstruction gives any reasonable results. Figure 5 (left panel) presents the whole simulated realization which is quantitatively comparable with the original uplift series. Figure 5 (right panel) presents a slice of 7 days from the simulation, confirming that the general behaviour of the synthetic process is comparable with the original data.

We can see that our simulation is also able to produce values significantly standing out from the process mean, as it is for the real data. An additional aim of the simulation was to restore ACF and PACF structures similar to those of the real data, which is displayed in Figure 6. The autocorrelations of the original series was showing clear half-hourly periodicity (significant humps at every 48th lag) – we managed to reconstruct that feature up to a certain degree. The humps for the simulated series ACF are skewed. Also, there are no significant humps on the negative side, whereas this was the case for the real data. The PACFs of both original and synthetic data look comparable.

Moreover, we compare the real and simulated uplift distribution parameters to verify differences between (features like) mean value, standard deviation, skewness and kurtosis. Table 1 collects the mentioned figures for 5 different simulation runs against the original uplift parameters. We can see that the mean values of the generated series are comparable with the real data. So is skewness of ran simulations. Kurtosis results seem to present some controversy – even though simulation does not seem robust with respect to this parameter, the original kurtosis falls within the standard deviation (STD Sim) neighbourhood around the kurtosis' mean value of the five simulations (Mean Sim). Finally, standard deviation of the produced series remains regularly too low with respect to true uplift.

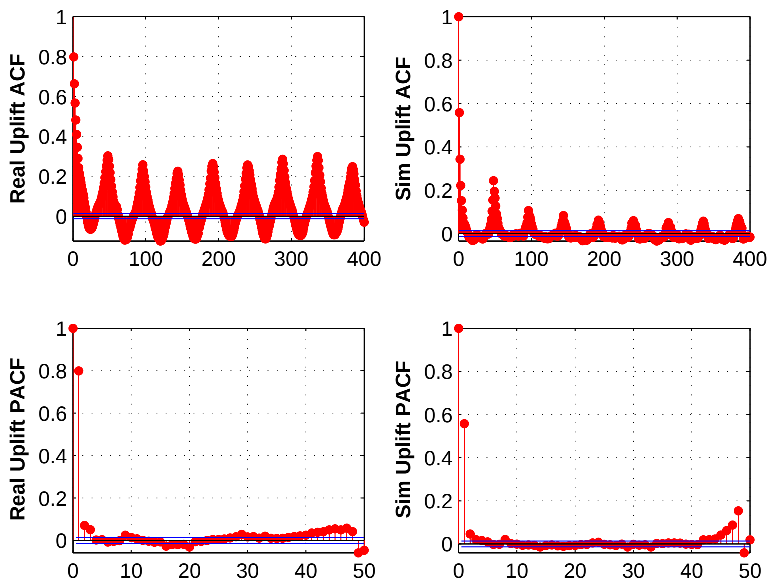

Figure 6: *ACF and PACF of the real and simulated uplift process.*

Table 1: *Real and simulated uplift distribution parameters.*

	mean	st. dev.	skewness	kurtosis
Real uplift	7.28	13.08	4.72	51.16
Simulation 1	7.36	10.35	4.60	56.17
Simulation 2	7.13	9.72	3.84	46.93
Simulation 3	7.01	10.01	4.65	61.33
Simulation 4	7.39	10.56	5.86	98.42
Simulation 5	6.91	10.05	4.45	49.05
Mean Sim	7.16	10.14	4.68	60.38
STD Sim	0.21	0.32	0.73	22.13

Finally, we use one more technique to verify statistical properties of the real and simulated series, that is we compare probabilities of exceedance for different level prices. In particular, we split the prices in slices by every 20 Euro for the range from 0 to 140 and verify what is the frequency of prices crossing the given thresholds. This gives a view on chances of uplift reaching particular elevations, including the highest spiky observations.

Figure 7 presents results for eight different price levels, from 0 to 140 Euro split by every 20 Euro. The probabilities are computed for 100 independent uplift simulations. We can see that for all levels except $thr = 0$ the simulated probabilities are about two to three times smaller than the observed ones. However, we do not notice that in the overall process mean estimates compared with the original parameter, as the simulation is on the other side more likely to give the price non-zero values.

We can see that the proposed algorithm reproduces quite a lot of the original data behaviour. Most important parameters fall into reasonable neighbourhood of the ones for real uplift. Also, the general quantitative look of the process for both long and short term horizon seems to resemble the real uplift structure up to a significant level. The simulated time series reproduces some of the 48-half-hourly periodicity as well.

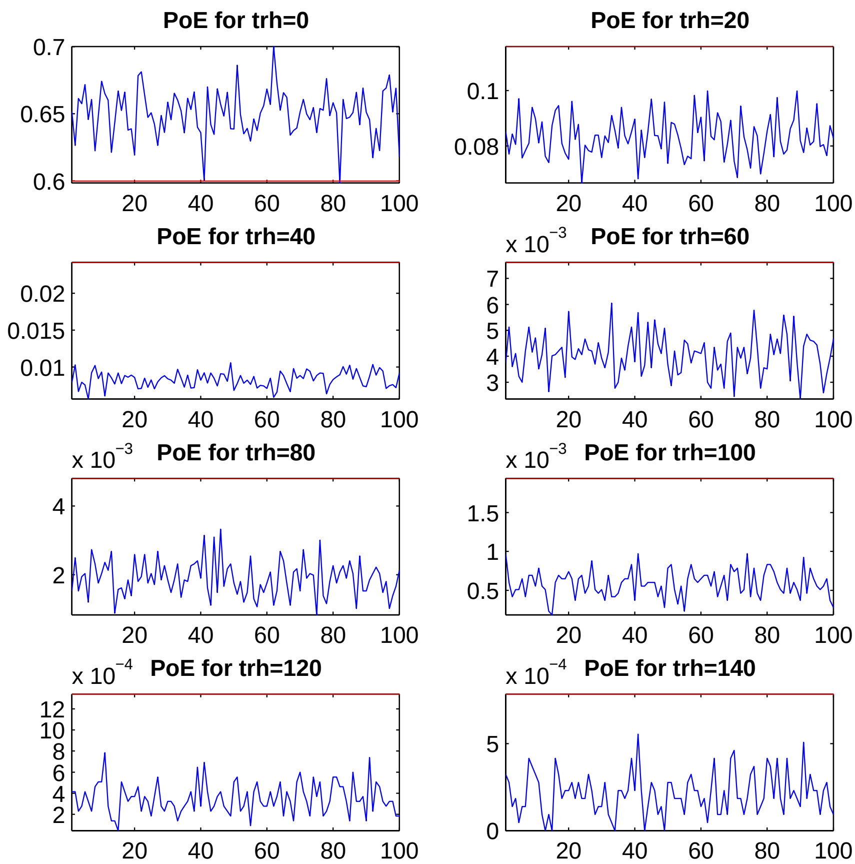

Figure 7: *Probabilities of uplift exceeding certain price levels.*

4 Uplift as a seasonal process depending on time of the day

The simulation discussed in the previous section has a few disadvantages like heuristic parameters for the plateau length distribution. Furthermore, as an examination of the plateau lengths shows they are not actually Poisson distributed but rather follow a more difficult distribution. Considering uplift only on a specific time of day was the initial idea for a new simulation with the characteristics described below.

4.1 Properties

For a suitable reproduction of the data it proved vital to split the problem into two parts:

1. uplift is constant
2. uplift price depends on time of day and month.

In the following sections these two properties shall be examined closely and described in a way so that they can be used for the simulation.

4.1.1 Constant uplift

Two observations can be made by looking at the data set when it comes to constant uplift. The first observation is the dependence of the constant parts on the current time of day. It is much more likely that uplift is constant during the night and the early morning (00:00-07:00). The second observation is, that uplift being constant is related to the current uplift, i.e. low uplift is more likely to be constant. These two observations will now be verified and will prove sufficient in describing the constant plateaus.

The probability P_{tod}, i.e. the probability of uplift being constant depending on the current time of day, can be seen in Figure 8. The uplift is reset each day at 06:00 and is given in half-hourly intervals. Thus the following figures with time of day dependence will always be from 06:00 to 05:30. A clear periodicity can be seen and during times where electricity demand changes most, less constant parts are observed. To include the reset at 06:00 a new uplift price will be calculated every time according to the properties discussed in the next section.

To verify the other observation, the probability P_{U}, i.e. the probability of constant uplift depending on the uplift, is displayed in Figure 9. Due to the small amount of data available only $U \in [0, 10]$ can be considered accurate and an exponential is used for fitting.

It can be argued that this might not be the best approach since P_{tod} and P_{U} are not actually independent. Indeed as will be shown later, this might be a problem that needs to be taken into consideration. However due to the small amount of data available no other approach could be found.

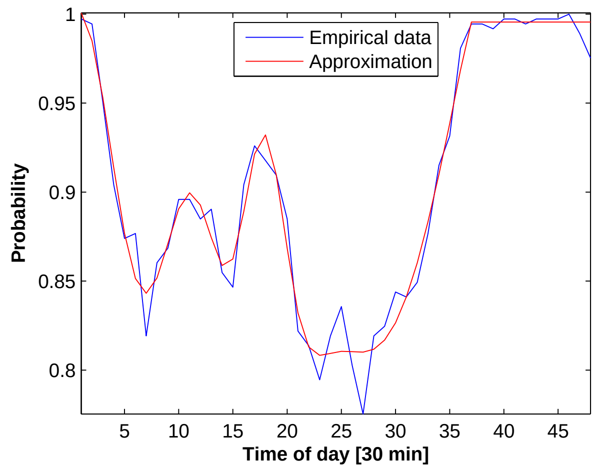

Figure 8: *Probability of constant uplift depending on time of day.*

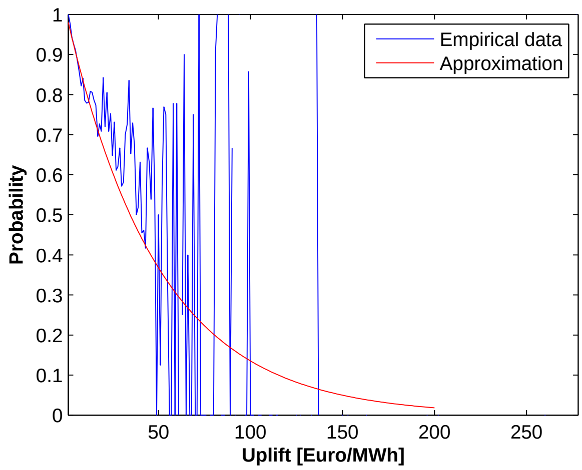

Figure 9: *Probability of constant uplift depending on uplift.*

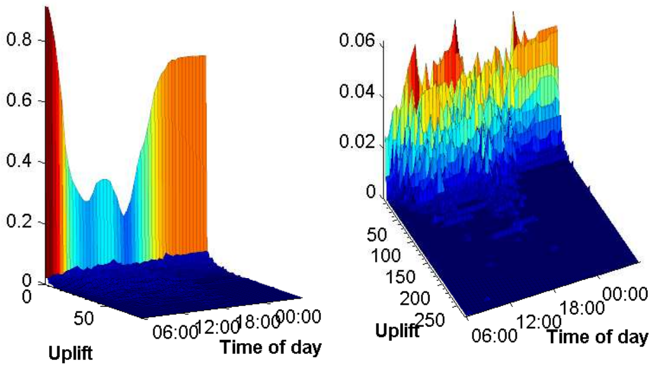

Figure 10: *Uplift histogram with entire data.*

In the simulation we will then use these probabilities to determine the probability of constant uplift as

$$P(U_i = U_{i-1}) = P_{\text{tod}}(i\%48) * P_{\text{U}}(U_{i-1}) \tag{1}$$

where % is the modulo operator.

4.1.2 Periodicities in uplift prices

At first an uplift histogram was considered separately for each time of day. The resulting graph can be seen in Figure 10. The plot on the left hand side shows the full histogram, emphasizing the probability of uplift being zero. The right panel neglects zero uplift to demonstrate daily periodicities of uplift greater than zero.

A first version of the simulation used this data, but the results showed a low mean (by a factor of about two). When considering a smoothed picture of the uplift, cf. Figure 11, annual seasonality shows up. Although only 1 year and 3 months of data was available it appeared necessary to include those changes. One large difference between summer and winter is the uplift around 12:00. This inhibited large uplifts around that time and thus reduced the mean of the simulation.

Due to that the same histogram as above was considered, this time for each month separately. It can be seen from Figure 12 (and in every other month) that there is a clear discontinuity between uplift being zero and non-zero, i.e. the most probable uplift value is zero, with much lower probabilities for non-zero values. Leading to the necessity of treating those two cases separately.

For $U > 0$ this resulted in approximately 30 data points for each histogram at a specific time of day. Figure 13 presents the one for May at 12:00. The bin size used for the histogram is 1. Even though the distribution can be suspected to be of Poisson type, due to a small number of data points it is not possible to have complete certainty and, therefore, the empirical distributions are

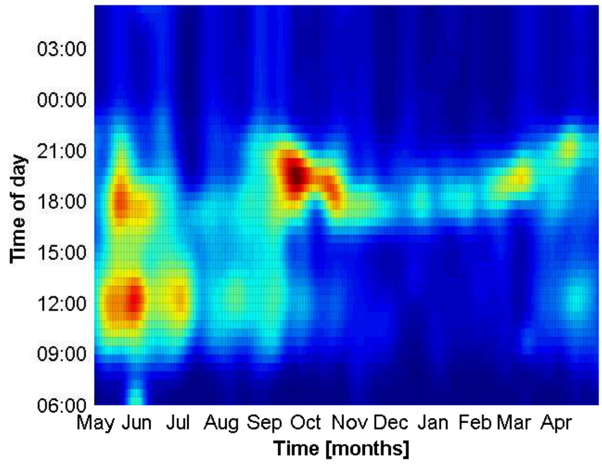

Figure 11: *Uplift (smoothed in horizontal direction).*

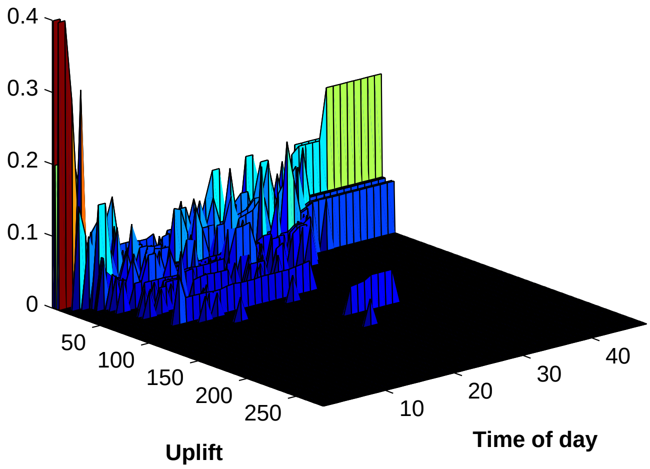

Figure 12: *Uplift histogram for May*

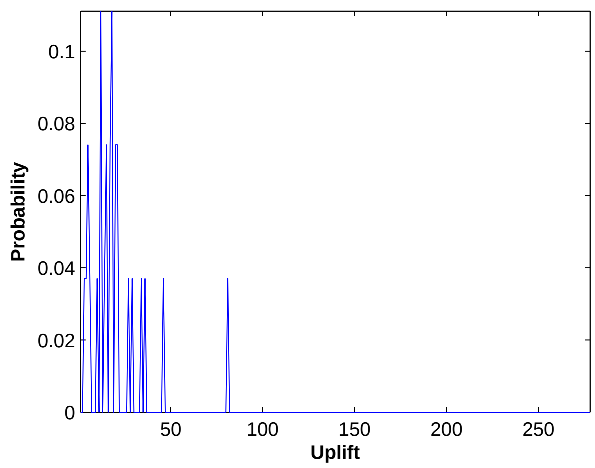

Figure 13: *Uplift histogram for May at 12:00.*

advised to be used for the simulation. Again a clear daily periodicity can be observed, as well as differences between distinct months.

The probability $P(U_i = 0)$ shows inverse proportionality to the mean of the $U > 0$ histogram and maybe it is possible to identify a correlation between those two parameters in the future. As noted in the previous section the discontinuity between 05:30 and 06:00 can be also seen from $P(U_i = 0)$.

4.2 Model and simulation algorithm

With the properties mentioned and characterized above it is now possible to formulate a new probability-based model for the uplift process

$$U_h = U_{h-1} \cdot v_1 + 0 \cdot v_2 + (\widetilde{U} + \beta U_{h-48}) \cdot (1 - v_1) \cdot (1 - v_2)$$

where

- U_h is the uplift for trading period h
- v_1 is a binary-distributed variable with success probability $P_{\mathrm{tod}}(tod) * P_{\mathrm{U}}(U_{i-1})$, where $P_{\mathrm{tod}}(tod)$ is the probability of uplift being constant for given time of the day tod and $P_{\mathrm{U}}(U_{i-1})$ is the probability of uplift being constant provided that the uplift on the previous trading period is equal U_{i-1}
- v_2 is a binary-distributed variable with success probability $P(U_i = 0)(m, tod)$ which stands for likelihood of uplift being zero provided that the trading period h falls into month m and time of the day tod

- $\widetilde{U}(m, tod)$ comes from empirical distribution of uplift values strictly greater than zero $U_h > 0$ for specific month m and time of the day tod

- β is the estimated regression coefficient for the 48-half-hourly periodicity.

Having the model formulated a new approach to the simulation can be constructed. The algorithm, implemented in Matlab, can be described as follows:

1. create regression coefficient reg for U_h and U_{h+48} dependence

2. use the first 48 half-hours of the real data for initiation

3. start the loop to create as much data as there is available from the real data

4. calculate current time tod and month m

5. if random number r_1 is smaller than $P_{\text{tod}}(tod) * P_{\text{U}}(U_{i-1})$ and $tod \neq 06:00$ set $U_i = U_{i-1}$

6. else if random number r_2 is smaller than $P(U_i = 0)(m, tod)$ set $U_i = 0$

7. else the $U > 0$ histogram is used for the specific tod and m to find a random variable $\widetilde{U}(m, tod)$ distributed accordingly (via the inverse cumulative distribution function), then $U_i = \widetilde{U} + \beta U_{i-48}$.

The model and respective algorithm give credit not only to the above mentioned daily (48-half-hourly) periodicity, but also to monthly seasonal patterns. It emphasizes the fact that electricity price behaviour in weather-dependent countries tends to have different behaviour in different months of the year.

4.3 Results

The first simulations were conducted without considering the monthly changes. As a result the values for mean, standard deviation and kurtosis were too low. However, it showed that the constant parts correlated very well with the original data, since it was possible to see the typical structures which showed up at low uplift. After improving the model by including the monthly dependence the performance improved drastically. In Figure 14 (left panel) simulated uplift for one year and three months can be seen. The seasonality and high spikes show up similar to those of the real data. The appearing structures can be further examined in Figure 14 (right panel) where data for a single week is extracted.

As mentioned in Section 3.3 another important feature of the uplift data is the characteristic ACF and PACF. For the new simulation these two graphs can be seen in Figure 15. Compared to the first model the negative parts of the ACF can now also be captured while the positive parts show similar good accordance.

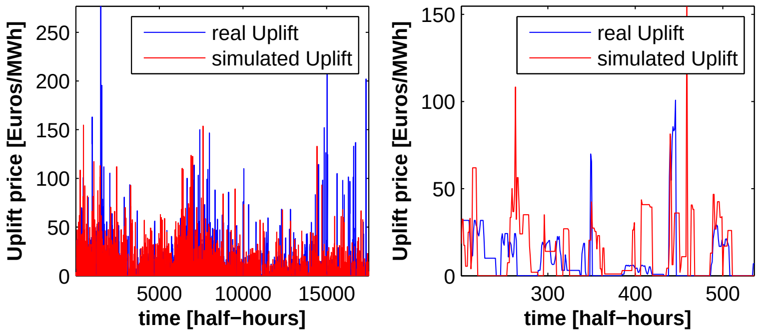

Figure 14: *Comparison of real and simulated uplift over (left panel) 451 days and (right panel) 7 days.*

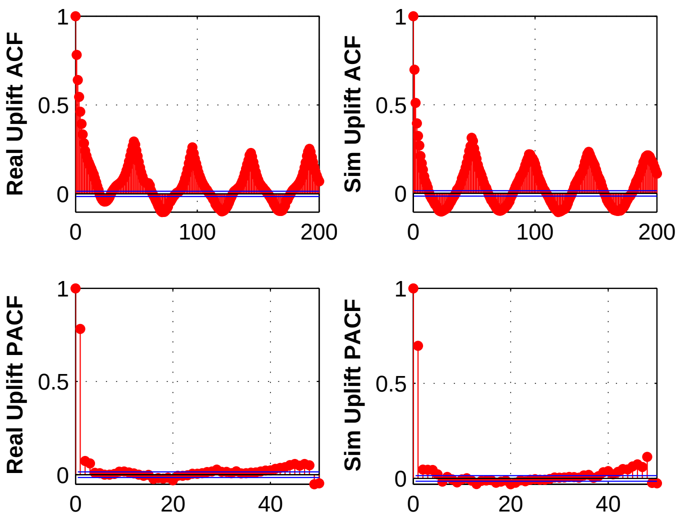

Figure 15: *Comparison of ACF and PACF for real and simulated uplift.*

Table 2: *Real and simulated uplift distribution parameters.*

	mean	st. dev.	skewness	kurtosis
Real uplift	7.28	13.08	4.73	51.16
Simulation 1	5.76	11.49	3.56	25.43
Simulation 2	5.61	11.74	3.94	35.05
Simulation 3	5.39	11.47	4.36	41.70
Simulation 4	5.50	11.37	3.72	27.69
Simulation 5	5.54	11.78	4.74	53.44
Mean Sim	5.36	11.57	4.06	33.66
STD Sim	0.34	0.18	0.48	11.36

Comparing the four characteristic values, mean, standard deviation, skewness and kurtosis (see Table 2), for the real data with five simulations it is possible to see that most values are close together. However, all the parameters are generally too low with respect to the true estimates. It can be expected that this issue becomes improved as soon as more data is available. As mentioned before, due to a short data horizon available, there were only 30 or 60 prices available for determining the distributions for each trading period within different months separately. As long as there is from 3 to five years of data available, we expect significant improvement for the second algorithm's results. Another possible explanation for the slightly lower mean is the constant uplift model. As noted earlier two probabilities P_{tod} and P_{U} are multiplied although they are not independent. This results in comparably shorter plateaus at middle and especially high uplift and since constant parts are followed by a jump, usually to a lower price, this lowers the mean as well as the standard deviation.

Table 2 also shows the strong fluctuations in kurtosis, due to the coarse distributions for uplift prices ($U > 0$) this is not surprising. Note that these fluctuations were also visible in the previous simulation (cf. Section 3.3).

Analog to the first proposed methodology, for these results we again verify the uplift probabilities of exceedance for specific price levels split by each 20 Euro, from zero up to 140 Euro, again collecting the outcome for 100 independent stochastic simulations. As Figure 16 shows, the second approach reduces probability differences with respect to the original ones for some price levels. On the other hand, the likelihood for level zero is now also lower than the true value and this fact is reflected in the significant difference between mean values of the original and simulated prices. Despite the differences between the true and simulated probabilities, we can see that all independent runs result in similar levels of likelihood, proving that the simulation algorithm brings robust outputs.

As mentioned in Section 3.1, the studies, i.e. all distributions and parameter analyses were run on exactly one year of data. Therefore, we use a part of the remaining observations to verify

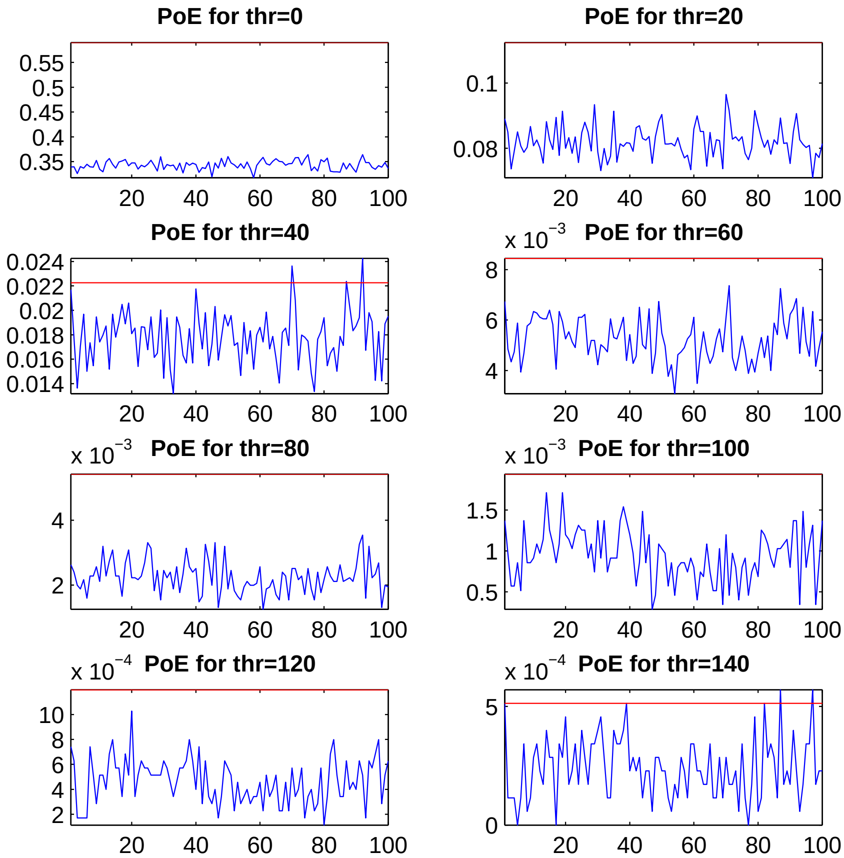

Figure 16: *Probabilities of uplift exceeding certain price levels.*

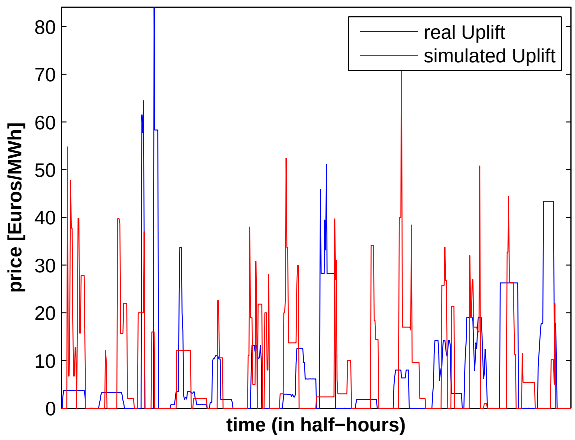

Figure 17: *Out-of-sample simulation for algorithm 2.*

the simulation performance on out-of-sample data. As the second algorithm was more convenient for reconstruction of intra-day specific uplift behaviour, we pick this one for the final comparison. Figure 17 presents simulation of the two weeks following the one-year part of data used for estimation.

Clearly, the general process intra-day behaviour is similar to the true path. Moreover, the simulation is able to reproduce both high and close to zero values, analogical to the real uplift characteristics. Table 3 collects basic statistics for the real and five times independently simulated out-of-sample data. The differences are acceptably low. Note that for all parameters the original values fall into the Mean Sim $\pm$ STD Sim intervals.

Table 3: *Real and simulated uplift distribution parameters on out-of-sample 2-week horizon.*

	mean	st. dev.	skewness	kurtosis
Real uplift	6.88	11.80	2.59	10.74
Simulation 1	6.72	10.93	2.04	8.01
Simulation 2	7.63	13.18	1.98	7.14
Simulation 3	4.70	11.29	3.80	23.62
Simulation 4	8.35	16.76	3.34	19.68
Simulation 5	8.02	16.02	1.31	4.53
Mean Sim	7.08	13.64	2.49	12.60
STD Sim	1.47	2.67	1.04	8.48

5 Conclusion and suggestions for future work

The aim of this study was to reconstruct behaviour of the uplift process coming from the Irish All Island Market for Electricity based only on the process itself. For this purpose we reviewed different statistical features of the process and proposed two alternative simulation algorithms. The suggested methodologies were able to reconstruct the real uplift behaviour up to different degree levels.

The first approach was based on finding respective distributions for process jump waiting times as well as jump direction and sizes, dependent on the given price level. The method did manage to produce a plateau-step structure similar to the original uplift path. It also returned main distribution parameters quantitatively comparable with the real ones, except for kurtosis. Also standard deviation seem to regularly differ from the original one by approximately 30%. Moreover, this approach failed to sufficiently reconstruct the autocorrelation and partial autocorrelation seasonal structure.

The aim of the second simulation was to eliminate certain heuristic parameters but eventually a different approach could be found by utilising other uplift characteristics. It proved of importance to include annual seasonality as well as the dependence on the current time of day. Although empirical distributions are used for most of the simulation it can be suspected that it might be possible to find analytical expressions as soon as more data is available. The simulation provided promising results which were approximately 25% too low. Two possible explanations for this phenomena have been given. Amongst them is the mathematically incorrect treatment of dependent probabilities. With two or three years more data it might be possible to also find annual seasonality in P_{tod} which can be exploited to eliminate the P_{U} term.

Finally, we verified the results of the proposed algorithms by comparing the uplift probabilities of exceeding certain, evenly distributed price levels. Even though we could notice differences between the desired and simulated likelihood outcomes, the results were robust over a number of independent simulations. Moreover, as each simulation brings independent uplift paths, the methodology could be employed within Monte Carlo framework, where a combined simulation would result in a broader view on the probability distributions describing the price behaviour. Doing this is recommended as soon as more data (at least 2 years) is available, and the seasonal probability components can be estimated more reliably.

This article studied an electricity price series different from those most commonly known (like in Scandinavian or New Zealand markets). It made it more challenging, since popular ARMA-GARCH or mean-reverting jump diffusion models could not be used. Nevertheless, we did obtain very good results with still much room for future improvement.

Acknowledgements

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PUBL. III

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PUBL. IV

Jabłońska, M., Viljainen, S., Partanen, J., and Kauranne, T. (2010) The Impact of Emissions Trading on Electricity Spot Market Price Behavior. Submitted to *International Journal of Energy Sector Management*.

The Impact of Emissions Trading on Electricity Spot Market Price Behavior

Matylda Jabłońska, Satu Viljainen, Jarmo Partanen, and Tuomo Kauranne

***Abstract*—Under the Kyoto protocol, emissions trading was imposed upon the Nordic Nord Pool Spot market in 2005. We seek to characterize the impact of emissions trading on electricity spot market price behavior by statistically comparing the prices before and after emissions trading was introduced. The analysis is based on the skill of regression models in explaining price behavior before and after 2005. It turns out that regression models based on background variables such as temperature, water reservoir levels, and even the price of emission rights themselves lose much of their skill from 2005 onwards. The histogram of the residual time series of an optimally calibrated regression model demonstrates a considerably more 'fat-tailed' behavior after 2005. This may be a sign that the increased medium- and long-term uncertainty brought about by emissions trading has introduced a strong 'psychological' component into price behavior, increasing its volatility and making it prone to more frequent price spikes.**

***Index Terms*—Electricity Spot Price, Emissions Trading Scheme, Multiple Regression Model**

I. Introduction

NORD Pool Spot's system price acts as the reference price for many financial instruments: futures, forwards, and options, as well as for the Nordic OTC/bilateral wholesale market, and it is used by electricity distributors as the basis for quoting prices to end consumers. Therefore, understanding the price micro-behavior and its short- and long-term trends is of high importance for different parties. It is already known that electricity spot prices are the most volatile of all financial time series, reaching a volatility value of up to 50%. Despite this challenge, both the short- and long-term mean levels of the price display some regularity, as do most electricity markets worldwide, and this can be partly explained with the use of background variables through multiple regression models.

A new factor was introduced to Nord Pool Spot pricing in 2005, namely trading of carbon dioxide emission allowances. It is known to have influenced the prices; however, there has been so far no mathematical evidence of how strong and of what character that influence is. Most research related to the Emission Trading Scheme (ETS) done so far has covered identified benefits and failures of introducing the scheme. Also its influence on industries of particular countries has been studied [1], but the total impact of emissions trading on spot prices has received little attention. For instance, Kara *et al.* in [2] have analyzed the effects of ETS on Finnish industries, and Sousa *et al.* in [3] for the Iberian electricity market. Some more mathematically based studies have focused on short-term relations between emission price changes and electricity price responses to it [4], or the influence of ETS news on the spot price in the case of the Australian market [5]. Keppler and Mansanet-Bataller have found that gas and carbon prices have some influence on emissions prices and that this effect carries on to electricity prices [6]. Finally, knowing that electricity prices in overall rose after the beginning of the ETS, Bonacina and Gulli tested the impact of market power by studying whether the influence of allowances on spot prices would have been stronger or weaker under market power than under perfect competition [7]. There have been attempts to propose options for market policy makers, and to bring the prices back from the ETS-caused lifted level to the previous one [8].

The aim of this paper is to statistically verify how the electricity spot price behavior has changed since the beginning of emissions trading. As the prices are known to be highly seasonal and dependent on specific driving factors, we set the foundations of our methodology in classical time series theory and multiple regression modeling. The residual time series from a properly designed regression model can be split in time with respect to the date when the European emission allowance (EUA) trading started, and the behavior of the two series compared through an extensive statistical analysis.

The paper is organized as follows. Section II-A presents the characteristics of the Nordic electricity market, the principles of its system price formation, and a review of researchers' approaches to modeling electricity prices. Section II-B briefly revises the emissions trading scheme and its controversies. Section III presents the data, the mathematical framework of our study, and the results. Finally, Section IV

M. Jabłońska is with the Department of Mathematics and Physics, Lappeenranta University of Technology, Finland, P.O.Box 20, FI-53851 Lappeenranta, Finland, e-mail: matylda.jablonska[at]lut.fi

S. Viljainen and J. Partanen are with LUT Energy, Lappeenranta University of Technology, Finland

T. Kauranne is with the Department of Mathematics and Physics, Lappeenranta University of Technology, Finland

presents the conclusions.

II. Framework

In this section, we present the market framework of Nord Pool Spot price formation, as well as the general mechanism and controversies standing behind the European carbon dioxide emissions trading.

A. Nordic electricity spot market

The deregulated Nordic electricity market is characterized as an 'energy-only' market with a single, uniform market clearing price. Geographically, the market is composed of Norway, Sweden, Denmark, Finland, and Estonia. When set up in 1996, it became the first international electric power exchange [9]. Its initial goal was to establish a common Nordic market that would guarantee strong competition between suppliers in the area. At present 317 companies from 20 countries trade on the exchange. Offering both day-ahead and intraday trades to its participants, Nord Pool is also the largest electricity market in the world.

1) Price formation in Nord Pool: A marginal pricing scheme is applied to the price formation in the Nordic electricity spot market. The market clearing price is found at the intersection of the supply and demand curves that are formulated in the day-ahead spot markets for each hour of the following day, based on the supply offers of electricity generators and the demand bids of electricity retailers and large electricity users. Generators' offers reflect the marginal costs of producing electricity, whereas the demand bids indicate the buyers' willingness to pay. The spot market is organized by the power exchange Nord Pool Spot. The trading cycle is characterized as a closed auction and it takes place once a day. The power exchange contributes to balancing the supply and demand in the short and long term. It provides incentives to use the power plants in the right merit order and enables the efficient use of the generation plants located across the market area. The market price formed at the power exchange also acts as a reference price in the bilateral electricity trading that takes place outside the power exchange.

According to the logic of marginal pricing, the generator with the highest marginal costs needed to satisfy the demand defines the market clearing price. All the employed generators are paid the same market price. Generators that are called to operate are always guaranteed to receive enough money to cover their variable costs. For the generator at the margin, the compensation will be exactly equal to its variable costs. For the other generators, the obtained revenues also cover some of their fixed costs. The principles of price formation are illustrated in Fig. 1.

In addition to the spot market revenues, the generators may also earn money by operating in the regulating power markets. In the Nordic electricity market, the regulating power markets are organized for reliability reasons by the national transmission system operators. Demand resources may also participate in the regulating power markets.

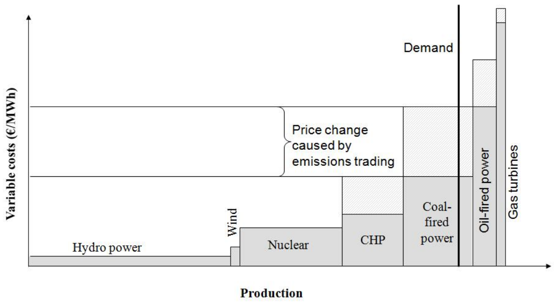

Fig. 1. Principles of marginal price formation in the Nordic electricity market.

The fact that the Nordic electricity market is an 'energy-only' market means that the revenues earned by the generators in the electricity spot market suffice to cover the short-term marginal costs as well as the long-term, 'going-forward' costs of the electricity generation plants. Generators' offers are not subject to offer capping. In shortage situations, prices are allowed to peak and the demand's willingness to pay for electricity settles the market price. During these shortage hours, generators are able to earn profits on their fixed costs. Separate capacity markets are not considered necessary as the energy market alone, by default, provides the generators with adequate revenues that facilitate new entry and allow keeping the existing power plants in operation.

2) Electricity spot price modeling and forecasting: It is well understood that being able to forecast electricity spot and forward prices is of high importance in both long and short term. Most recent studies focus on seeking the best methods for day-ahead price forecasting, as the spot price's high volatility and prominent spikes are the basic risk factors for market participants. Their main reason lies in the competitive character of the deregulated markets, as a big number of traders can significantly lower the mean price level, but will also make it more volatile at the same time [10].

Due to the spot price nature, we know that accurate prediction with classical time series models is not possible. Most recently proposed approaches are based on background deterministic variables known to be influencing electricity prices, such as load [11], production type [12], [13], temperatures [10], and other different climatic factors [14]. To reduce the electricity price forecasting errors, one can also account for the known types of spot price periodicity. Among those, we consider seasonal weather influence [15], as well as weekday effects [16].

As no perfect model for short term spot price forecasting has been found so far, it is crucial from the risk-management point-of-view to know at least confidence intervals of the computed predictions [17]. Also, being able to model long-term price trajectories is equally important. The latter has been proposed through, for example, a price duration curve approach [18]. Moreover, on top of all forecasting efforts, we should be aware of any possible economic impacts of electricity market price forecasting errors [19]. Thus each new better model should always be revised in an on-going fashion because, as we discuss in the following sections, the influence of price driving factors, as well as new economic situation and policies, can significantly change model parameters and, therefore, its forecasting performance.

B. Emissions trading

1) Emissions allowances and trading: Emissions trading is a market-based methodology used to control pollution by providing economic incentives to achieve reductions in the emissions of pollutants [20]. It is agreed that a central authority of a country sets a limit (also known as a cap) on the amount of a pollutant that can be emitted. The total agreed limit is allocated or sold to all the country's emitting industrial sites in the form of emissions allowances. Firms are obliged to hold a number of permits (or credits) equivalent to their emissions. It is often the case that some of the allowance holders emit more than allocated, and thus need to increase their emission permits by buying credits from those who use less of their allowances. This process is called emissions trading. In effect, the buyer is paying a charge for polluting, while the seller is being rewarded for having reduced emissions. This way, it is expected that those who can reduce emissions most cheaply will do so, achieving pollution reduction at the lowest overall cost to society [21].

2) Allowance price and trading controversies: The emission allowances were allocated to the actors after the pre-Kyoto-period had actually started. In Finland this took place around February 2005, and in some countries it did not happen until 2006. Thus, at the beginning there was actually no market for emissions, and consequently, no price for emissions. Moreover, some actors knew better than others what would and especially should happen to prices; that is, they would rise.

There was a lot of uncertainty in the amount of allocated emissions. At the beginning there was no general knowledge of whether there are enough permits allocated to cover all emissions. When the EU published the result of 2005 showing that there were plenty of emissions for every actor, in April 2006 the prices decreased immediately to about 20–25%, and at the end of the pre-Kyoto period, emission allowances became temporarily worthless. Overall, emissions trading can be seen to have introduced a substantial amount of medium- to long-term uncertainty to the electricity markets. In this paper, we have set out to characterize the consequences of this uncertainty on the spot market prices by statistical modeling.

III. Data, methodology, and results

THIS section presents our regression modeling approach to describe the varying mean level of electricity spot prices. The focus of this paper is on the system price that is obtained from the total supply and demand curves in the Nordic market area. We first consider the most influential factors driving the price (part III-A); further, we list the specific characteristics of our proposed model (part III-C) and, finally, we study the model residuals split with respect to the Kyoto protocol enforcement date, 16 Feb 2005 (part III-D).

A. Factors driving electricity spot prices

Electricity spot price is not a purely stochastic process. It also has long- and short-term mean levels, which can, at least partly, be explained by background variables that are used to build the regression models. The type and level of explanatory power in such supporting data is very much dependent on individual markets.

Prices in the Nordic electricity market are characterized as being highly volatile. This follows partly from the fact that prices are allowed to peak when the market is short. Another thing that contributes to the high volatility is the large variations in the demand and supply of electricity. For instance, temperature strongly affects the demand; in total, the demand varies between 50–100%. A similar phenomenon can be seen in many northern regions, such as Russia and North America. We have also learnt that demand is a strong factor in influencing the local trend in the spot price of electricity. In specific markets, the demand is significantly correlated with the local temperature. On the other hand, demand is lower over the weekends and during the nights.

On the supply side, markets with high thermal or gas-based production will have their price variations caused mainly by the changes in fossil fuel prices and the prices of European emission allowances (EUA). For strongly hydro-based markets, the water reservoir levels will rule the price trends. Most energy producers aim to maintain a balance between different energy sources, in order to ameliorate the risk in the price of raw materials to produce electricity. Table I presents a repartition of electric energy production methods among the Scandinavian countries. We can see that water reservoir levels, especially those from Norway and Sweden, can be of great significance to the availability of cheap power, and hence to the level of spot prices.

TABLE I
DIFFERENT TYPES OF ENERGY SOURCES IN SCANDINAVIA (2007).

Country	Hydropower	Nuclear Power	Other thermal sources	Other renewable sources
Norway	99%		1%	
Finland	20%	33%	47%	
Denmark			81%	19%
Sweden	46%	42%	12%	

Historically, being a hydro-dominated market, Nord Pool has shown that the deviations of water levels from normal have been reflected in the electricity spot prices. However, the introduction of the emission trading of the EU changed the dynamics of the market, as depicted in Fig. 2. Here, we plot the normalized time series of both the Nordic system price and the deviation of Scandinavian hydrological situation from normal. Specifically, we calculate it as the difference between the mean value indicated as the average between minimal and maximal possible hydro storage over the last 10-year history and the hydrological situation in a given week.

A discussion on the high deviation in winter 2002 is carried out later on in Section III-D. Otherwise, we can see that until the beginning of year 2005, the direction of price mean level was highly correlated with the hydrological storage deviation, whereas from that point on, the time series follow increasingly divergent trajectories.

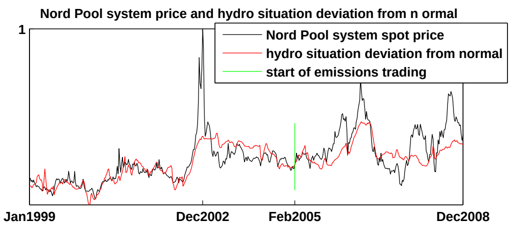

Fig. 2. Normalized Nord Pool system price with respect to deviation of hydrological situation from normal.

Normally, in a good hydro year, the electricity spot price, on average, is slightly below the marginal cost of a coal-fired power plant, including the cost of emissions. In a bad hydro year, on the other hand, the electricity spot price is a little over the marginal cost of a coal-fired power plant, including the cost of emissions. Another example of a hydro-dependent market is New Zealand. One of the crucial aspects of that region is that most of electricity production takes place in the south of the country (South Island), whereas the highest demand is in the densely inhabited northern part of the North Island.

Summarizing the aforementioned factors, we have pursued a regression analysis for Nord Pool using temperatures and the water reservoir level as background variables for the study. We do not use demand to avoid collinearity between explanatory variables (as mentioned, demand is very highly correlated with temperature). However, to take into account the weekday pattern in demand and thereby prices, we use classical time series decomposition to remove weekly and seasonal periodicities.

B. *Estimation of the trend and seasonal components*

Daily spot prices are known to have two main types of periodicity: weekly (demand related) [16] and annual (weather related) [15]. Therefore, the first step is to work on the price time series decomposition, to deseasonalize and detrend the series so as to leave the indeterministic part for modeling purposes. This operation was performed in two steps. First, the prices were detrended and deseasonalized with the use of classical additive decomposition methodology.

Depending on data character, one can consider two main options for a decomposition model (see [22], [23], [24]):

$$X_t = T_t \times S_t \times I_t \quad \text{(multiplicative model)} \tag{1}$$

or

$$X_t = T_t + S_t + I_t \quad \text{(additive model)} \tag{2}$$

where X_t is the original data, T_t stands for the trend, and S_t and I_t for the seasonal and irregular components, respectively.

There are no proven 'automatic' techniques to identify trend components in the time series data. However, as long as the trend is monotonous (consistently increasing or decreasing), this part of the data analysis is fairly straightforward. Many monotonous time series data can be adequately approximated by a linear function. If there is a clear monotonous nonlinear component, the data first needs to be transformed to remove the nonlinearity. Usually a logarithmic, exponential, or (less often) polynomial function can be used. In the case of our data, we use a linear trend estimated by the least-squares method to obtain the trend line $T_t = at + b$. When the trend line is identified, it is subtracted from the original data. The remaining series is used for the seasonal component. The number of seasonal indices equals the data seasonality order. In our case these are 7 and 365 days. Finally, the random (irregular) component can be isolated by subtracting from the seasonally adjusted series (additive models) or dividing the adjusted series by the trend-cycle component (multiplicative models).

C. *Regression with explanatory variables*

Now, consider the linear time series regression model

$$Y_t = \beta_0 + \beta_1 x_{1t} + \ldots + \beta_k x_{kt} + \epsilon_t = x_t'\beta + \epsilon_t, t = 1, \ldots, T \tag{3}$$

where $x_t = (1, x_{1t}, \ldots, x_{kt})'$ of size $(k+1)\times 1$ is the vector of explanatory variables, $\beta = (\beta_0, \beta_1, \ldots, \beta_k)'$ of size $(k+1)\times 1$ is the vector of coefficients, and ϵ_t is a random error term. Note that the dimension $k+1$ comes from the fact that besides differently valued explanatory variables, we also allow a constant term in the model. In matrix form the model is expressed as

$$\mathbf{Y} = \mathbf{X}\beta + \epsilon \tag{4}$$

where $\mathbf{Y}$ and ϵ are $(T\times 1)$ vectors and $\mathbf{X}$ is a $(T \times (k+1))$ matrix. The standard assumptions of the time series regression model are:

1) the linear model (Equation (3)) is correctly specified,
2) y_t, x_t are jointly stationary and ergodic,
3) the regressors x_t are predetermined: $E[X_{is}\epsilon_t] = 0$ for all $s \le t$ and $i = 1, \ldots, k$,
4) $E[x_x x_t'] = \Sigma_{xx}$ is of full rank $k+1$, and
5) $x_t\epsilon_t$ is an uncorrelated process with a finite $(k+1)\times(k+1)$ covariance matrix $E[\epsilon_t^2 x_t x_t'] = S = \sigma^2\Sigma_{xx}$.

The second assumption rules out trending regressors, the third rules out endogenous regressors but allows lagged dependent variables, the fourth avoids redundant regressors or exact multicollinearity, and the fifth implies that the error term is a serially uncorrelated process with constant unconditional variance σ^2. In the time series regression model, the regressors x_t are random and the error term ϵ_t is not assumed to be normally distributed.

As we already discussed in Section III-A, particular factors have significant influence on electricity spot price behavior. Therefore, the second step after the classical approach above is to use the obtained detrended and deseasonalized price series as the dependent variable in a regression model. Before estimating the desired model, the explanatory variables are initially detrended and also deseasonalized to have them treated analogically to the prices. Further, to get the best regression fit we have to make sure that the independents are properly aligned with the dependent variable in time. For that purpose, the crosscorrelations between the time series were studied. As a result, we find that prices should be lagged with respect to water reservoir levels by 10–11 days, which is connected with the hydro generators' 1–2 week ahead planning. The temperature variable does not need any time shift. Thus, we conclude that the day-ahead temperature forecasts known to market participants on the bidding day are good estimates of the actual temperature measurements occurring on the delivery day.

When having the dependent and explanatory variables properly aligned, we estimate the least-squares-optimal regression model. However, the fit is not done globally on the whole data set at once, but in a moving regression fashion, where every day a half-a-year history is used to project the resulting price for the given moment. This is because we are aware that the prices still have some more local trends, dependent on other variables not available for the study. As evidence of this effect, we illustrate in Fig. 3 how the moving regression parameter estimates for temperature and hydrological storage level differ over the whole modeling horizon. The figure shows a considerable variation in the values of parameters estimated for both variables over ten years of Nord Pool Market history. It is remarkable that the sign of their value changes frequently.

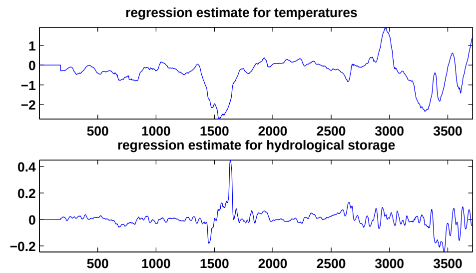

Fig. 3. Moving regression coefficients for explanatory variables.

Finally, we construct the resulting residual series, which can be assumed to be a pure market series representing electricity trading characteristics only. The result presented in Fig. 4 plots the original system price, the fit, and the resulting residual series.

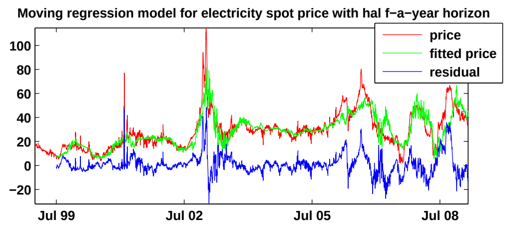

Fig. 4. Moving regression fit for Nord Pool system price with half-a-year horizon window.

D. Discussion on the results

Even a quick look at the resulting residual series reveals that our simple regression model is not sufficient to explain completely the nature of electricity spot prices. Especially, it is not able to capture the high volatility changes and even less the price spikes. The results leave some space for discussion on the explanatory variables used in the regression model. One could argue that there could be some more, for instance, economical information used, such as prices of fossil fuels (very influential on thermal power generation).

Moreover, there is one more interesting phenomenon visible in the regression fit. Along the time axis, the regression fit seems to follow local price mean values fairly well except for two particular periods. One is the fall-winter time of 2002–2003. Even the regression parameter estimates from Fig. 3 show a sudden, significant change in that period when hydrological situation overrules the usual strong domination of temperature.

We know that there was a chain of events that influenced spot prices in that time. Summer 2002 was warm and exceptionally dry, which resulted in water reservoirs being reduced below the average level in summer and autumn 2002. Consequently, prices began to rise. In autumn 2002, a public discussion arose whether there was enough production capacity and reserve capacity in the case of peak demand, if the coming winter was to be cold. Indeed, winter came early and was severely cold. The fear of the lack of sufficient capacity in that situation (low water reservoirs and high demand) turned the market wild. Prices took a giant leap up, possibly by speculation on the high value of low hydro resources. Finally, the situation came back to normal when players realized that there was enough capacity after all and water reservoirs would easily recover (there was a lot of snow in the mountains of Norway and Sweden), which indeed happened in March–April 2003.

The second period of a poor regression fit starts in year 2005 and continues until the end of the data horizon. This year is the time when carbon dioxide emissions trading has been enforced, which for instance in Finland started in February 2005. Therefore, we can split the computed residual series into two periods: before 15 Feb 2005 (from this point on referred to as period 1) and after 16 Feb 2005 (period 2). Before proceeding, we also eliminate the time interval from 11 Oct 2002 to 17 Mar 2003 from period 1 because of the above-mentioned hydro-based market speculation. This is done to avoid bias in the residual statistics caused by this known exceptional event.

Having the residual series split, we plot the two separate periods with relevant statistics to compare how the regression fit differs between the plots for period 1 and period 2, presented in Figs. 5 and 6, respectively. We can see that not only the series trajectories but also their distributions are significantly different. From the respective histograms (upper right plots) in Figs. 5 and 6 we can see that the residuals in period 1 are a lot more regularly distributed. The shape is not completely Gaussian when compared with the theoretical normal distribution (red line), but as we already mentioned, normality of the error term is not a requirement in regression models. The corresponding distribution in period 2 is a lot more irregular, displaying a three-modal pattern.

Next we have verified the presence of serial autocorrelation in the residuals. For both periods, we can see from the autocorrelation (ACF) and partial autocorrelation (PACF) functions (bottom plots) in Figs. 5 and 6 that neither of the series has independently distributed uncorrelated residuals, which supports our previous statement that the constructed regression may still be missing some economy-based explanatory variables. However, the most important remark is that the character of autocorrelation differs between the two periods.

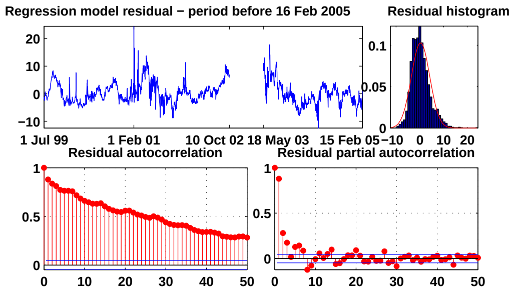

Fig. 5. Residual price series for the period from 1 Jul 1999 to 15 Feb 2005, with accompanying statistics.

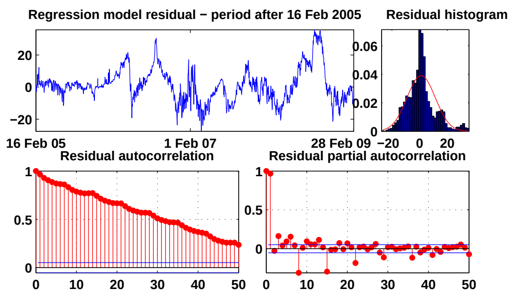

Fig. 6. Residual price series for the period from 16 Feb 2005 to 28 Feb 2009, with accompanying statistics.

Since the regression model was constructed with a moving window, we investigate how the window size actually influences both residual statistics in the two periods, and their general goodness of fit. Therefore, Table II collects numerical results for the following horizons: 14, 91, 182, and 365 days. For these we compute the respective mean, standard deviation, skewness, and kurtosis values. Also, we add information about the Durbin-Watson test statistic. Its values always fall into the interval [0,4], and values above 2 would indicate that the residuals are independent and identically distributed. However, no such values occur.

The most important observations from the statistics are as follows:

- in most cases the period 1 mean is closer to the expected zero value than the period 2 mean,
- for all horizons the standard deviation of residuals in period 2 is higher than in period 1,
- the residuals in period 1 are always more skewed than the residuals in period 2 (in absolute value, regardless of the skewness direction),

TABLE II
MOVING REGRESSION FIT STATISTICS FOR DIFFERENT HORIZON WINDOWS (IN DAYS [D]).

	14d	91d	182d	365d
mean1	0.0811	-0.0230	**0.3249**	-0.1451
mean2	0.0292	0.0602	**1.1203**	1.7088
st.dev.1	1.1136	2.8608	**3.8823**	6.3526
st.dev.2	1.5061	7.2856	**10.2174**	14.3043
skewness1	1.4835	0.7241	**0.8253**	-0.0386
skewness2	-0.3575	0.3951	**0.7106**	-0.3640
kurtosis1	17.9119	6.3997	**4.6813**	4.9378
kurtosis2	10.0958	4.6494	**4.0480**	3.0940
D-W1	1.0992	0.3926	**0.2406**	0.1026
D-W2	0.6785	0.1270	**0.0694**	0.0288

- for all window sizes, kurtosis (distribution peakedness) is persistently higher for period 1 than in period 2; this means that in period 1 the regression is able to capture most of the regular mean level price behavior but, as expected, fails in explaining high price changes and spikes.

The Durbin-Watson test statistic shows that even though we face an undesired serial autocorrelation for both periods (as already understood from the ACF and PACF plots), this situation is always substantially worse for period 2.

In Table II we have boldfaced the column that collects statistical values for our original horizon of 182 days. We can see that for this case the skewness and kurtosis of both distributions are close to each other. But from the histograms we could already notice that the distributions were actually very different. Therefore, we employ one more comparison measure – the probability of the series to exceed specific levels. We slice both period 1 and period 2 residual series into ten intervals each, from their respective minimum to maximum values. Then we calculate the percentage of observations falling above each slice threshold. These probabilities have been illustrated in Fig. 7, together with the respective mean values and standard deviation limits of each period.

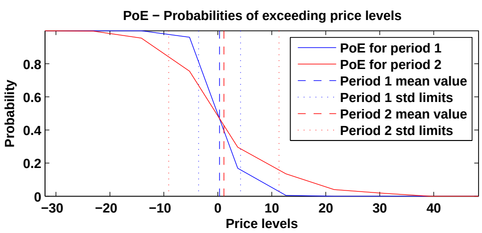

Fig. 7. Residual price series probabilities of exceeding specific price levels, with the time horizon split into before (period 1) and after (period 2) 16 Feb 2005.

We can see that the shapes of the probability lines differ, and that residuals in period 1 are approximately equally likely to fall below and above the mean level, whereas for period 2 almost 60% of values stay below the mean. The residuals of period 2 display a more "fat-tailed" pattern than those for period 1.

E. Regression model with the EUA prices included

The various types of statistical comparisons carried out have shown that even though regression models can never be sufficient to describe electricity spot prices, our given specific example would have performed a lot better in the time before the beginning of emission trading. Perhaps this means that in the case of using regression for explaining price mean levels, allowance prices should become an additional explanatory variable in the model. Therefore, we have repeated exactly the same methodology but now enriching the regression model by an additional variable, that is, prices of emission allowances. The resulting fit is presented in Fig. 8.

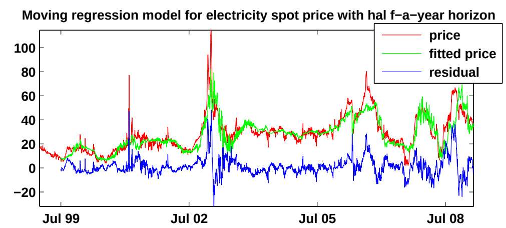

Fig. 8. Moving regression fit for the Nord Pool system price with a half-a-year horizon window, with Emission Allowance prices included in the regression model.

The fit does not change for period 1 because there were no emissions traded and, therefore, no prices to be included in the model. However, the regression seems to follow better the original price from period 2. Let us analyze the character of the residuals after 16 Feb 2005 as plotted in Fig. 9. Even though the enriched model seems to damp some of the variation compared with Fig. 6, the shape of the histogram and the respective autocorrelation levels are still significantly different from the period 1 statistics, as depicted previously in Fig. 5. The residual standard deviation before February 2005 remains clearly below half of the ETS period value. Also, the histogram is still asymmetric and quite non-regular.

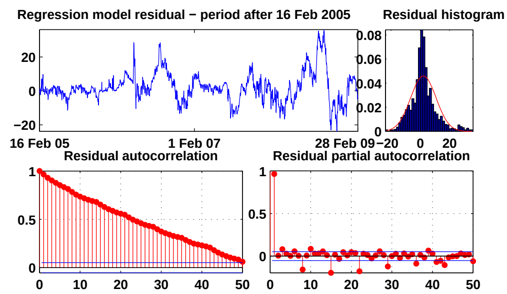

Fig. 9. Residual price series for the period from 16 Feb 2005 to 28 Feb 2009, with its accompanying statistics, with emission allowance prices included in the regression model.

Moreover, the autocorrelation and partial autocorrelation remain similar to those of the first fit for period 2 rather for period 1. Finally, we analogically illustrate in Fig. 10 the probabilities of residuals from period 2 to cross specific levels. The curves still differ significantly in a similar fashion as before allowances were included in the model. Also, the period 2 mean level did not get any closer to zero, and its standard deviation is persistently twice as high when compared with period 1.

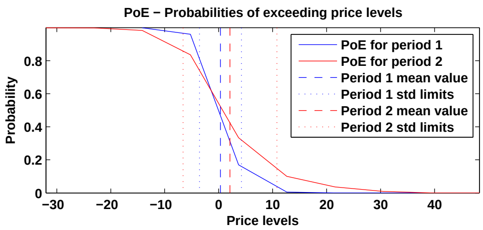

Fig. 10. Residual price series probabilities of exceeding specific price levels, with the time horizon split into before (period 1) and after (period 2) 16 Feb 2005, with emission allowance prices included in the regression model.

From the presented results, we conclude that since the emissions trading started, the spot prices have changed fundamentally, showing a much more irregular behavior in short- and long-term mean levels than before.

IV. Conclusion

It has been known already, and spelled out by market analysts, that carbon dioxide emissions trading highly influenced electricity spot prices. However, so far there has been little mathematical evidence in the matter. Therefore, the aim of this paper was to statistically analyze and show how significantly the behavior of electricity spot prices has changed since the beginning of emissions trading. As the prices are known to be highly seasonal and dependent on specific driving factors, we have set the foundations of our methodology in classical time series theory and multiple regression modeling. The data used in this study came from the Nordic market. It included the Nord Pool spot price itself and a number of explanatory variables such as electricity consumption, mean regional temperature, hydrological storage level, and emission allowance prices, all covering a horizon of over ten years from 1 January 1999 till 28 February 2009.

We started our study from a discussion on possible factors driving electricity spot prices in the Nordic market. This was done with respect to the geographical location and the production source profile. Next, we have cleared the available data from the trend and known deterministic periodicities. Then the detrended and deseasonalized variables were used in a regression model, explaining the spot price varying mean level. However, an important feature of the regression is that it was run in a moving fashion, with a specified regression window of six months. This let us avoid biasing the regression parameter estimates towards one single mean value, as the influence of particular factors has changed considerably over the years.

From the regression fit we computed its difference from the real price time series. The resulting residuals were then split in two parts with respect to the date when the emissions trading started in Finland, that is, 16 Feb 2005. Our results based on a residual distribution study and an autocorrelation analysis showed that there is a fundamental statistical difference between the fitting skills of the model before and after the above-mentioned date. We have also compared probabilities for both parts of the residual series when exceeding specific levels, which proved the same.

Finally, following the conclusion, we have enriched the regression model with an additional variable – the emission allowance price. It appeared that the residual series after February 2005 still remained significantly different and a lot more irregularly distributed than the one before. Clearly, a regression model defined uniformly for the whole 1999–2008 period loses its fitting skills from the beginning of February 2005. This leads us to the conclusion that spot markets have adopted distinctly different dynamics since emission trading started, and the influence of EUAs is deeper than a simple added-on-top relation. It has introduced a stronger psychological component to the price, as witnessed by the fat-tailed distribution of the residual time series.

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Matylda Jabłońska is a doctoral student studying for D.Sc. in Applied Mathematics at Lappeenranta University of Technology (LUT), focusing on electricity spot market price modeling. She completed her first M.Sc. (2007) in Financial and Actuarial Mathematics from Wrocław University of Technology, Poland, and the second M.Sc. (2008) in Technomathematics from LUT, Finland. Her work experience includes also two years of work as Data and Finance Analyst for ALM Department in bank centre of Bank Zachodni WBK, Poland.

Satu Viljainen is a professor of electricity and energy markets at Lappeenranta University of Technology, Finland. Her present research interests include the functioning of electricity markets, economic regulation of electricity networks, and electricity market legislation. She gained her M.Sc. in electrical engineering and D.Sc. (Tech.) from Lappeenranta University of Technology in 2001 and 2005, respectively.

Jarmo Partanen received his D.Sc. (Tech.) degree in electrical engineering from Tampere University of Technology in 1991. He has been a professor at Lappeenranta University of Technology (LUT) since 1994. At present Prof. Partanen is the Head of the Institute of Energy Technology at LUT. His main fields of interest are smart grids, electricity market, RES and regulation of electricity distribution business.

Tuomo Kauranne is an Associate Professor of Applied Mathematics at Lappeenranta University of Technology, Finland. His current research interests include modelling financial time series, with a particular emphasis on including psychological factors, such as Keynes' 'Animal Spirits', in the models. Dr. Kauranne also chairs Arbonaut Ltd., a Finnish technology company that carries out operational vegetation risk analyses of power transmission lines, in collaboration with Utility Risk Management Corporation LLC based in Stowe, VT.

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Multi-agent stochastic simulation for the electricity spot market price

Matylda Jabłońska and Tuomo Kauranne

Abstract The Great Recession of 2008-2009 has dented public confidence in econometrics quite significantly, as few econometric models were able to predict it. Since then, many economists have turned to looking at the psychology of markets in more detail. While some see these events as a sign that economics is an art, rather than a science, multi-agent modelling represents a compromise between these two worlds. In this article, we try to reintroduce stochastic processes to the heart of econometrics, but now equipped with the capability of simulating human emotions. This is done by representing several of Keynes' Animal Spirits with terms in ensemble methods for stochastic differerential equations. These terms are derived from similarities between fluid dynamics and collective market behavior. As our test market, we use the price series of the Nordic electricity spot market Nordpool.

1 Introduction

After the Great Recession of 2008-2009, many mathematical and econometric models used in economy have received a lot of criticism, since they were not able to predict the emergence of the asset bubble in the U. S. housing market. As a result of this, econometricians have increasingly turned towards seeking explanations to what happened in the psychological element in market traders' actions. This has repeatedly brought up the idea of emotions that influence human economic behavior. These emotions are also known as *animal spirits* and were originally introduced by John Maynard Keynes in his 1936 book [15].

Not only global economic upheavals display such behavior. A different type of extreme event can be observed in deregulated electricity spot markets which are known to be one of the most volatile financial markets. This distinctive phenomenon

Matylda Jabłońska and Tuomo Kauranne
Lappeenranta University of Technology, Department of Mathematics and Physics, P.O.Box 20, 53850 Lappeenranta, Finland, e-mail: matylda.jablonska@lut.fi, e-mail: tuomo.kauranne@lut.fi

is the appearance of price spikes, i.e. sudden price changes to values up to dozens of times higher within only an hour, and again falling back to the previous level within a couple of hours or days. After each spike, market specialists are able to find a reason that caused it in hindsight. But few of those reasons are reliable predictors for future spikes [4, 25]. Nor has any econometric model shown any skill in forecasting those sudden price changes, to these authors' knowledge.

In the current study, we investigate the possible origin of price spikes in animal spirits that rule the behavior of all traders. Our modelling attempt of electricity spot markets is not directly based on any notion of intelligent – albeit emotional – agents. Instead, we have sought to equip classical Ornstein-Uhlenbeck type mean-reverting econometric models with new non-linear terms that emulate the impact of a distinct animal spirit each. Although the motivation for these models is not grounded in the psychology of intelligent agents, they represent very similar collective behavior to that of models based on intelligent agents. So does the real price history of electricity on the NordPool Spot market, the largest electricity spot market in the world.

This modelling approach is not limited to only spot markets for electricity, where prominent spikes can partly be understood as a consequence of the non-storability of electricity. After suitable normalizing transformations, identical models simulate accurately the behavior of other commodity markets. We have conducted such a study also on the oil spot market for Brent crude. In both cases, the free parameters in the terms of the models are calibrated with a Bayesian Maximum Likelihood principle from the real time series. After this step, the resulting simulated price series reproduces the distribution of the real price series almost exactly up to the sixth statistical moment.

The article is organized as follows. Sect. 2 presents theoretical aspects used in our proposal. Sect. 3 describes briefly the data set, as well as the two proposed models and their results. Sect. 4 concludes.

2 Theoretical framework

2.1 Electricity spot market price

As we already mentioned, electricity spot markets and their prices are very distinctive due to electricity non-storability. Therefore, defining appropriate models that would relevantly forecast spot prices is very challenging in both short and long term. Some researchers took trials in analyzing specific auction theories to investigate the electricity market power. The two most common ones are supply function equilibrium and multi-unit independent private value [27]. Some claim that information available to power traders is asymmetric on both auction sides which violates the goal of lowering volatility and marginal price [1]. An idea of modelling several competitive traders in an electricity market as a coupled system of mathematical programs with equilibrium constraints was proposed in [9], however, without explicit numerical results.

Depending on type of electricity market, there have been different attempts based in classical time series theory [28], methodologies utilizing price periodicities [37, 21], basic and more elaborate stochastic mean reversion models [10], wait-jump structures [13] or regime switching approaches [35]. What is certain is that a lot of electricity price local trends and part of the volatility can be explained by historical information on factors known to be driving the prices [31]. Nevertheless, even though we know how important specific patterns are in the electricity prices [20], the most challenging part of modeling their dynamics lies in the non-explained part which we claim to be highly influence by traders' psychology. Therefore, as presented later in Sect. 3.1, we calibrate our simulation based on a time series from which we remove all known deterministic factors.

2.2 Animal spirits in financial markets

The term *animal spirits* appeared in literature already in 1936, introduced by Keynes [15]. However, as we can read from [22], many specialists did not want to accept importance of psychology as one of major economy drivers. An attempt to formalize Keynes' forces can be found in a work related to catastrophe theory [8]. Some researchers focused on the idea of risk-aversion [17]. Others have found an inverse relation between consumer and business confidence and national unemployment rate [23].

The issue gained notoriety when Akerlof and Shiller published their book discussing the trading psychology [2]. They claim that real financial dynamics is strongly based in irrational, emotional and often intuitive decisions by human agents. Even if agent-based modelling builds upon individual psychology, also government decisions still have human factors behind and, therefore, economies fall globally [16].

Other authors underline the importance of trust and confidence [33] or rational expectations [18] as crucial forces pulling markets towards or away from economic crisis. Multi-agent models have become a popular method to address human emotions, and they have been applied to macroeconomy [7], where agents adaptively learn from their mistakes. Other models specifically cater for transaction taxes, greed and risk aversion in [6]. A number of studies apply agent-based models to electricity markets [32]. However, they have focused on the technical aspect of meeting demand and supply. Such models often work well only for regular price evolution. We argue that price spikes originate in human psychology. In this study, we present an ensemble model that accounts for some of the animal spirits in the spot markets.

2.3 Capasso-Morale-type population dynamics

We know, that people, as other animal species, have animal spirits. These, in financial market mean mostly fear and greed, influenced then by our collective trading biases: herding, overconfidence and short-term thinking. The type of model we have adopted are based on the Capasso-Morale system of stochastic differential equations (1), used so far for modelling animal population dynamics (see [24]). Its basic equation has the form

$$dX_N^k(t) = [\gamma_1 \nabla U(X_N^k(t)) + \gamma_2(\nabla(G - V_N) * X_N)(X_N^k(t))]dt + \sigma dW^k(t), \qquad (1)$$

for $k = 1, \ldots, N$. The equation describes physical herding of animal populations.

In our case, the population is a group of traders in the spot market, and a measure of their spatial distance is the price. Traders do observe one another and thus create a mean price path, which could be also understood as the global (in *macroscale*) population formation. However, there is also a limit for overcrowding (in *microscale*) which in power trading could be interpreted as physical impossibility of two market participants to buy the same asset of electricity. Each individual price path simulated from the model proposed in this paper represents a single trader, and the simulation of the ensemble would provide coupling between the participants (in *mesoscale*). The movement of each particle is driven by an external information coming from the environment, expressed via suitable potentials.

2.3.1 Momentum in financial markets

Since the 1980s researchers have been repeatedly noticing that, on average, stocks performing well keep doing so over some further time. It is called the momentum effect. Similar behaviour can be observed in any commodity markets. In many funds the managers are rewarded for good performance and for beating the market. Thus they must be holding the most popular and rapidly appreciating stocks. When they perform well, clients invest even more money, which again goes into the same investments and boosts shares that have already performed well even further. Simply put, investors are buying stocks just because their price has risen. This is the essence of the momentum effect.

A physical analogy to the momentum phenomenon can be found in fluid dynamics. The Burgers' equation (2) is a one-dimensional form of the Navier-Stokes equations without the pressure term and volume forces. It is widely used in various areas of applied mathematics, such as modeling of fluid dynamics and traffic flow [5, 11].

$$u_t + \theta u u_x + \alpha u_{xx} = f(x,t) \qquad (2)$$

To build an analogy between markets and fluids, the price represents some one dimensional measurement of fluid, such as pressure, along a periodic domain. This characterization is not far from stock market reality. Worldwide trading takes place in a periodical domain of the earth. With one exchange closing in one time zone, another one is opening for another trading day on the next continent. The information circulates in a periodic fashion around the world.

In the Burgers' equation (2) u stands for the price, $f(x,t)$ describes the fundamentals (often of a periodic character), αu_{xx} is the diffusion term that is related to the fact that the spot market tends to reach an equilibrium price, u_x is the spread between any given day's average and most common bids, i.e. the mean and the mode of the bid distribution, uu_x is the momentum term that expresses traders' tendency to move towards the most common price. This effect is magnified at higher prices.

The momentum effect should occur when a sufficiently big subgroup of the whole population has significantly different behavior (external information) that deviates from the total population mean. This has been noticed in studies related to animal and human spatial dynamics, when a large group of people is asked to keep moving randomly around a big hall. When a sub-group of five per cent or more are silently told to move towards a given target, the whole population will follow. Again, in terms of prices this could be understood as considerable departure of the mode of the population price distribution from its mean price. Then the rest of the individuals may follow that trend and this effect unexpectedly amplifies that deviation to a scale of a prominent price spike.

3 Multi-agent simulations for electricity spot market

3.1 Data

The data used for this study is the daily system price from the Nord Pool electricity spot market covering a horizon of over 10 years from January 1999 to February 2009. However, for model and simulation calibration we do not take the original prices, but its detrended and deseasonalized version. We motivate it by the fact that electricity spot prices are known to be highly periodical seasonally [37], as well as weekly [21]. Some of these effects may be removed by regression models, based on high correlation of electricity prices with specific background variables, such as load [34], production type [3, 12], temperatures [29], and other different climatic factors [19].

Therefore, we first remove specific types of periodicity with use of classical time series additive decomposition, and then build a regression model employing available explanatory variables, that is temperatures and water reservoir levels. Moreover, the regression is not run globally but in a moving fashion with half a year horizon (182 days). This is motivated by the fact that except for obvious periodicities, there are also other cycles (like economic) driving electricity prices. Also, influence of specific factors on the prices changes over years.

The fit and resulting residual series, also claimed to be a pure trading price series reflecting more clearly the electricity spot market dynamics, are presented in Fig. 1.

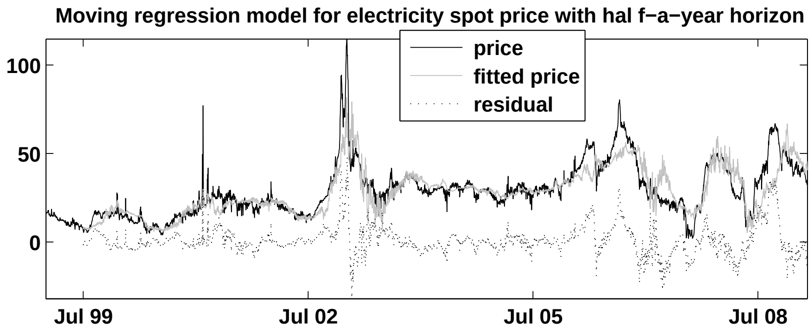

Fig. 1 Moving regression fit for Nord Pool system price with half-a-year horizon window.

3.2 Mean-reverting jump diffusion ensemble simulation

In this study we propose to represent individual spot price traders as an ensemble. Price realizations of all of them are described with a system of stochastic differential equations (*Lagrangian representation*). As it was mentioned in [24], this approach makes sense for small or medium-sized populations. To reflect reality, we set the ensemble size to 300, because currently the Nord Pool market has approximately 330 participants.

In particular, each of those differential equations has form (3)

$$dX_t^k = \gamma_t[(X_t^* - X_t^k) + (f(k,\mathbf{X}_t) - X_t^k)]dt + \sigma_t dW_t^k + {}^+J_t^k dN_t + {}^-J_t^k dN_t, \quad (3)$$

for $k = 1,\dots,N$, where X_t^k is the price of trader k at time t, X_t^* is the global price reversion level at time t, γ_t is the mean reversion rate at time t, $\mathbf{X}_t$ is the vector of all traders' prices at time t, $f(k,\mathbf{X}_t)$ is a function describing local interaction of trader k with his neighbors (small range of individuals from vector $\mathbf{X}_t$), W_t^k is the Wiener process value for trader k at time t, σ_t is the standard deviation for Wiener increment at time t, ${}^+J_t^k$ is the positive jump for trader k at time t, ${}^-J_t^k$ is the negative jump for trader k at time t, N_t is the count process for jumps at time t.

The model parameters of the mean reverting part are estimated with use of Maximum Likelihood (MLE) approach. The log-likelihood function for Ornstein-Uhlenbeck process can be found from [26]. The probabilities of jumps are generated from Poisson distribution based on probability of spike occurrence from specific price levels. The jump sizes are estimated from empirical distribution of the original prices.

In this model we follow the global mean reversion level X_t^* and rate γ_t in a moving fashion with half a year historical horizon (182 days). This feature represents *short-term thinking*, that is one of the main trading biases characterizing market participants. The local interaction $f(k,\mathbf{X}_t)$ is based on following the mean value of neighbors within price range equal to 10% of the total price range, and it stands for the *herding* bias. The jump processes ^+J and ^-J are dependent on current price level at each time t, as we know that electricity spot price is more likely to spike from higher levels than from lower [14]. Therefore, spikes generated by the jumps are reflecting *panic* reaction of traders in the uncertain environment, on both positive and negative side. We could claim that they originate from human *fear and greed* emotions.

In Fig. 2 we can see the original price and example simulated trajectory (for one out of 300 traders) together with their respective histograms. We can see that the simulation nicely follows the original data, both in the long term and in the appearance of spikes.

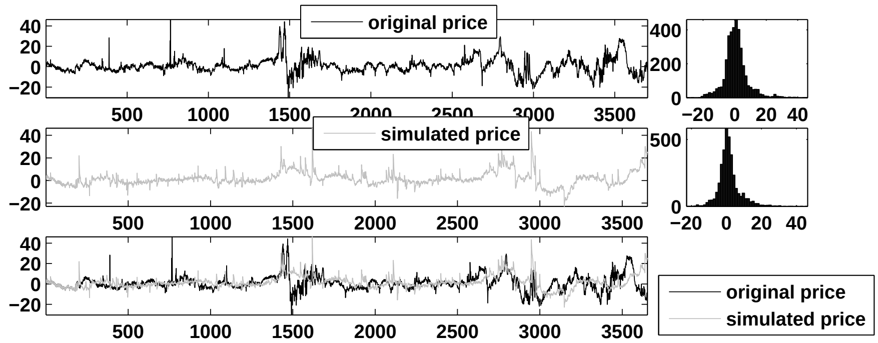

Fig. 2 Ensemble simulation: global reversion to moving mean level with moving rate, and local to neighbors' mean.

The original and simulated histograms are similar. However, we want to quantify the difference as well. Therefore, Table 1 collects comparison of the basic statistics for original pure trading prices and the mean ensemble values. These are mean, standard deviation and five consecutive central moments. We observe that especially skewness and kurtosis have values very close to one another. The 5th moment is still comparable. Only 6th moment starts to be higher for the ensemble than the original data by the factor of nearly 1.5 and the 7th moment by the factor of 2.

To complement the whole analysis we employ one more comparison measure, i.e. the probability of the series to exceed specific levels. We slice both the original pure price series and each simulated ensemble series into ten intervals each, from their respective minimum to maximum values. Then we calculate the percentage of observations falling above each slice threshold. These probabilities are illustrated in Fig. 3, together with the respective mean value and standard deviation limits of the original price.

Table 1 Original and ensemble statistics: global reversion to moving mean level with moving rate, and local to neighbors' mean.

	Original	Ensamble
Means	0.72	1.69
St dev	7.47	6.14
Skewness	0.92	0.94
Kurtosis	6.97	6.91
5th moment	18.81	22.09
6th moment	104.91	150.43
7th moment	423.45	832.39

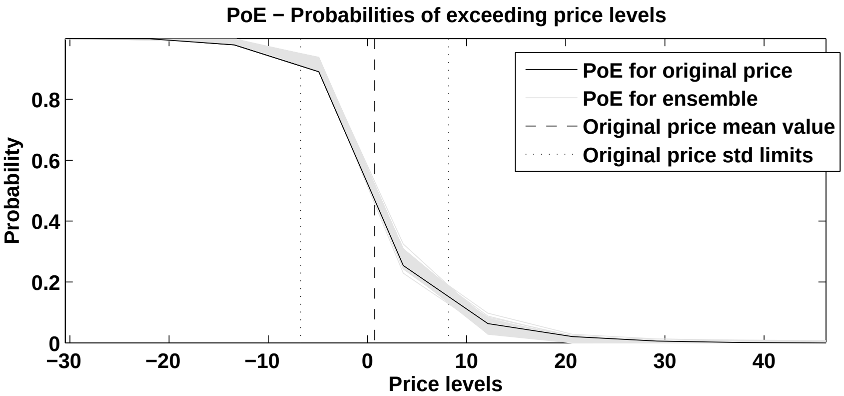

Fig. 3 Original pure price and ensemble probabilities of exceeding specific levels.

Clearly, the real data's probabilities fall within the envelope of the whole ensemble. That confirms statistical accuracy and robustness of our approach.

3.3 Ensemble simulation with Burgers'-type interaction

The ensemble model proposed in Sect. 3.2 reproduces the real price dynamics very well. However, it has a weakness in that the jump components are just superimposed on the base mean-reverting process. They should rather be based directly on price dynamics. We propose to eliminate the jump processes from the model (3) and replace the mean-based local interactions $f(k,\mathbf{X}_t)$ with a Burgers'-type momentum component $h(k,\mathbf{X}_t)$. Thus the model takes the form

$$dX_t^k = [\gamma_t(X_t^* - X_t^k) + \theta_t(h(k,\mathbf{X}_t) - X_t^k)]dt + \sigma_t dW_t^k, \tag{4}$$

where $h(k,\mathbf{X}_t) = \mathbf{E}(\mathbf{X}_t)\cdot[\mathbf{E}(\mathbf{X}_t) - \mathbf{M}(\mathbf{X}_t)]$ and $\mathbf{M}(X)$ stands for the mode of a random variable X. Also, θ_t represents the strength of that local interaction at time t.

The model estimation is also done by MLE. Following the solution of a mean reverting process we get that the process has mean value and variance in form of (5) and (6).

$$\mathbf{E}(\mathbf{X}_t) = [\gamma X^* + \theta \mathbf{E}(\mathbf{E}(P_t)(\mathbf{E}(P_t) - \mathbf{M}(P_t)))] \cdot \frac{1}{\gamma+\theta} \cdot (1 - e^{-(\gamma+\theta)t}) \tag{5}$$

$$\mathbf{Var}(\mathbf{X}_t) = \frac{\sigma^2}{2(\gamma+\theta)} \cdot (1 - e^{-2(\gamma+\theta)t}) \tag{6}$$

When assuming for simplicity that the process is normally distributed, the population mean $\mathbf{E}(P_t)$ is expected to equal the population mode $\mathbf{M}(P_t)$, and thus the log-likelihood function takes form (7).

$$\mathscr{L}(\mathbf{X}, X^*, \gamma, \theta, \sigma) = n \ln \left(\frac{1}{\sqrt{2\pi \mathbf{Var}(\mathbf{X}_t)}} \right) - \sum \frac{(X_i - \mathbf{E}(\mathbf{X}_t))^2}{\mathbf{Var}(\mathbf{X}_t)} \tag{7}$$

The simulation results for this model can be seen in Fig. 4. The general price level follows the original data. Moreover, the simulation spikes, even though the model does not have any jump component. Thanks to interactions of the individuals, the price spikes are based on the pure price dynamics. One can see that spikes in the simulation are not as frequent as in original data. We can blame here the normality assumption for MLE. Future work could consider some numerical methods for MLE of parametric distributions like g-and-h.

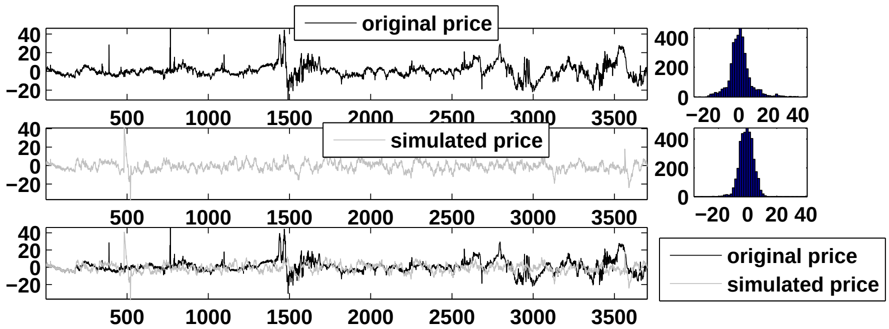

Fig. 4 Ensemble simulation: global reversion to moving mean level with moving rate, and Burgers'-type local interaction.

Table 2 collects the statistics for the original price and the ensemble. The values of central moments are not as close to the original ones as the first model was able to produce, but the distribution of the simulated price is leptokurtic on a level similar to that of the true price.

When we measure the probability of the price to exceed a specific price level, see Fig. 5, it is visible that the ensemble envelope does not cover the original data in the range of positive extreme values well, but the results are still promising. The model

Table 2 Original and ensemble statistics: global reversion to moving mean level with moving rate, and Burgers'-type local interaction.

	Original	Ensamble
Means	0.72	-0.43
St dev	7.47	5.01
Skewness	0.92	0.35
Kurtosis	6.97	7.68
5th moment	18.81	24.40
6th moment	104.91	242.69
7th moment	423.45	380.57

can probably improved by changing the MLE function or by enriching the model with suitable potential functions representing the economic situation.

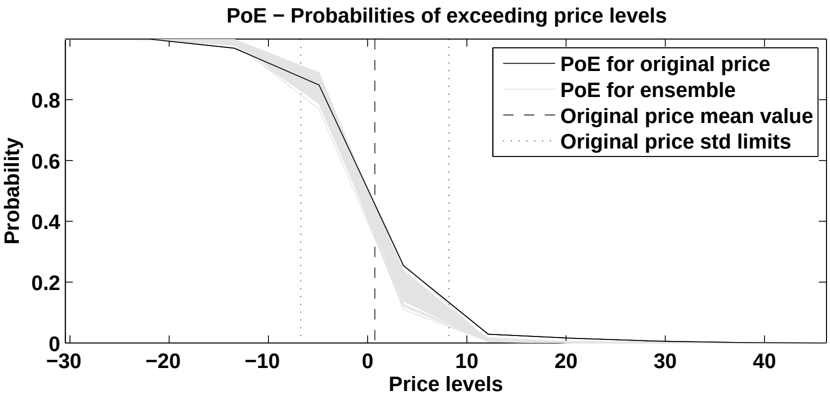

Fig. 5 Original pure price and ensemble probabilities of exceeding specific levels.

4 Conclusions

In this work we have presented two possible multi-agent models that simulate bids of electricity spot market participants. The study was carried out on data originally coming from Nord Pool market. However, we have removed any known deterministic factors from the available price time series, so that we are left only with data reflecting best the true spot market dynamics.

The multi-agent models that we proposed in this paper were based on a Capasso-Morale-type population dynamics Lagrangian approach, where movement of each individual is described by a separate stochastic differential equation. However, these agents keep interacting with each other at each time instant, on both local and global basis. Our model caters for the most common trading biases, i.e. short-term thinking and herding. Also, we have included terms that represent panic that originates from market uncertainty. Finally, the second model has eliminated the need for a separate jump component in the simulation. Instead, it uses an interaction term that has been borrowed from fluid dynamics that represents market momentum.

Simulation results presented in our paper prove that our approaches reproduces well many statistical features of the real spot price time series. This was measured by comparing distribution histograms of the original and simulated series, through statistical central moments up to the 6th order, as well as by the probability of the prices to exceed a specific level. All these showed remarkable resemblance. Also, the second simulation was able to reproduce price spikes based only on price dynamics and ensemble behavior.

As to suggestions for future work, we hope to improve the multi-agent model still by employing more elaborate functions for the local interaction of traders, as well as by the inclusion of potentials that would represent market information available to the traders. Also, the MLE assumptions of model (4) should be revised.

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