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Lappeenranta University of Technology
Faculty of Technology. LUT Energy
Lecture Note 7

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FUNDAMENTALS OF HEAT TRANSFER

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Preface

Following over 170+ pages and additional appendixes are formed based on content of Course: *Fundamentals of Heat Transfer*. Mainly this summarizes relevant parts on Book of *Fundamentals of Heat and Mass Transfer (Incropera),* but also other references introducing the same concepts are included. *Student's point of view* has been considered with following highlights:

- *Relevant topics are presented in a nutshell to provide fast digestion of principles of heat transfer.*
- *Appendixes include terminology dictionary.*
- *Totally 22 illustrating examples are connecting theory to practical applications and quantifying heat transfer to understandable forms as: temperatures, heat transfer rates, heat fluxes, resistances and etc.*
- *Most important Learning outcomes are presented for each topic separately.*

The Book, *Fundamentals of Heat and Mass Transfer (Incropera),* is certainly recommended for those going beyond basic knowledge of heat transfer. Lecture Notes consists of four primary content-wise objectives:

- *Give understanding to physical mechanisms of heat transfer,*
- *Present basic concepts and terminology relevant for conduction, convection and radiation*
- *Introduce thermal performance analysis methods for steady state and transient conduction systems.*
- *Provide fast-to-digest phenomenological understanding required for basic design of thermal models*

In first chapter basic concepts of heat transfer are introduced. *Conservation of Energy* or *'1st Law of Thermodynamics'* is presented as a general tool for heat transfer analysis. 3 different heat transfer modes: conduction, convection and radiation; are shortly introduced. Definitions for common concepts and variables related to thermal system analysis are summarized.

Stationary conduction chapter begins with introduction of *Fourier's Law* of heat conduction and its analogy to other physical experimental based Law's of nature. Thermal property characteristics of materials relevant for conduction heat transfer are summarized before introduction to *'General Heat Diffusion Equation'*. Differential energy balances in different coordinate systems are formed for 1-dimensional steady state conduction analysis. Also concepts of *'Thermal Resistance'* and *'Overall Heat Transfer Coefficient'* are presented and relevance as thermal circuit or system analysis tool is shown.

Heat transfer mechanisms and methods for heat transfer performance analysis relevant for structural heat transfer enhancement, more commonly known as '*Fins*', are introduced. A short summary of different designs and performance values is given as an ending of a Fin chapter.

'*Lumped Capacitance Method'* is presented as thermal performance analysis method of transient uniform temperature profile systems. Spatial effects in transient conduction are shown by *analytical approach*: forming and solving differential energy conservation equations in Cartesian, Cylindrical and Spherical coordinate systems. Importance of '*Boundary Conditions'* for thermal system performance and analytical solutions are highlighted.

Fundamentals of convection heat transfer include introduction to physical phenomenon involving velocity, thermal and concentration *boundary layer developments* are presented. Related flow dynamic and heat & mass transfer equations are derived from fundamental conservation laws to provide scientific basis for engineering heat transfer methods and tools for solving problems of convection. Definitions of *dimensionless parameters* and equations are presented and relevance of them is shown in differential conservation balance equations and in empirical convection correlations. All basic forms of convection are discussed. *Forced and free convection,* as well as *external and internal convection,* are separately handled. *Boiling and condensation* as special modes of convection are shortly introduced as a last, but not least meaningful, part of summarized fundamentals of convection heat transfer.

Types and thermal principles of most important industrial application of heat transfer, heat exchanger, are highlighted. Two main thermal performance and design methods, *Logarithmic mean temperature difference* and *efficiency – NTU* method, are presented by *thermal theory* based to *solution step* –wise approach with design charts applicable for common types of heat exchangers.

Physical principles and quantities of most complex, and thus perhaps most interesting, form of heat transfer, radiation, are introduced by means of *Stefan-Boltzmann's and Wien' displacement Laws* and *Black body radiation functions*. *Spectral radiation properties* of material surfaces are highlighted before giving finally two fundamental radiation heat transfer solution methods: (1) based on *Kirchoff's Law* and (2) *Radiation between two bodies.*

Wishing Thermal Balance for all Interested in a Quest of Engineering Wisdom in a Miraculous World of Heat Transfer,
Ari Vepsäläinen @ Kansainvälinen huippuyliopisto

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NOMENCLATURE

SYMBOLS

Roman

A	[m^2]	Area
B	[m]	Width
c	[J / kgK]	Specific Heat
C	[mol/ m^3]	Concentration
$\dot{C}$	[W / K]	Heat Capacity Rate
C^*	[-]	Ratio of Heat Capacity Rates in ε-Ntu Method.
C_f	[-]	Fanning Friction Factor (Friction Coefficient)
d	[m]	Diameter
d_h	[m]	Hydraulic Diameter
D	[m^2/s], [m]	Diffusivity, Diameter
e	[m]	Surface Roughness
E	[W/m^2]	Emissive Power
f	[-]	Darcy (Moody) Friction Factor
F	[-]	Correction Factor in LMTDMethod
g	[m^2/s]	Gravitational Acceleration
G	[kg / m^{2}s], [W / K]	Mass Velocity, Conductance
h	[W / m^{2}K]	Convection Heat Transfer Coefficient
h_m	[W/m^{2}K]	Convection Mass Transfer Coefficient
h_{fg}	[J/kg]	Latent Heat
j_H	[-]	Colburn *J*-Factor, $St\,Pr^{2/3}$
k	[W / mK]	Thermal Conductivity
K	[-]	Unit Resistance
L	[m]	Length
m	[kg]	Mass
N_A	[mol/s]	Convection Mass Transfer Rate
NTU	[-]	Number of Transfer Units, Dimensionless Conductance
Nu	[-]	Nusselt Number
p	[Pa]	Pressure
P	[-]	1. Temperature Effectiveness
	[W]	2. Power
Pr	[-]	Prandtl Number
q	[W]	Heat Transfer Rate
q''	[W / m^2]	Heat Flux
q_m	[kg / s]	Mass Flow Rate
q_V	[m^3 / s]	Volume Flow Rate
r	[m]	Radius
R	[K / W]	1. Thermal Resistance
R''	[m^{2}K / W]	Thermal Resistance per Area
	[m^{2}K / W]	1. R''_{Tc} Contact Resistance per Area
	[m^{2}K / W]	2. R''_F Fouling Resistance per Area
Re	[-]	Reynolds Number
s	[m]	1. Wall thickness, fin spacing
S	[m]	Spacing
St	[-]	Stanton Number
t	[m]	1. Thickness
	[s]	2. Time
T	[K]	Temperature
U	[W / m^{2}K]	Overall Heat Transfer Coefficient
v	[m^3 / kg]	Specific Volume
V	[m^3]	Volume
w	[m / s]	Flow Velocity
W	[W]	Work

Greek

α	[m^2 / s]	1. Thermal Diffusivity, $\alpha = K / \rho C_p$
	[-]	2. Absorptivity: 0-1
β	[K^{-1}]	Volumetric thermal expansion coefficient
	[m^2 / m^3]	Heat Transfer Area per Volume
δ	[m]	Boundary layer thickness
	[m]	Gap between plates
ε	[-]	1. Heat Exchanger Effectiveness
	[-]	2. Emissivity: 0-1
ρ	[kg / m^3]	Density
σ	[W/m^{2}K^4]	Stefan-Boltzmann constant
η_f	[-]	Fin Efficiency
η_o	[-]	Overall Surface Efficiency
η_p	[-]	Pump/Fan Efficiency
μ	[Pa s] or [kg/sm]	Dynamic Viscosity
ν	[m^2 / s]	Kinematic Viscosity
τ	[N/m^2]	1.Shear stress
	[s]	2. Time constant
θ	[ºC]	1. Temperature difference
	[rad]	2. Zenith angle

SUPERSCRIPTS

‘	Per length
‘’	Per area
‘’’	Per volume
*	Dimensionless

SUBSCRIPTS

avg	average
c	1. cold side of heat exchanger
	2. core of the heat exchanger
D	diagonal
e	entry to heat exchanger core
f	1. fouling
	2. fin
	3. fluid
ff	free-flow
fr	frontal
h	hot side of heat exchanger
i	1. inlet
	2. inside
L	longitudinal
lm	logarithmic mean
m	1. mass
	2. mean
	3. modified
max	maximum
min	minimum
o	1. outlet
	2. outside
p	isobaric
rad	radiation
s	surface
t	total
T	transverse
tb	tube-to-baffle
w	wall

1 INTRODUCTION

1.1 MODES OF HEAT TRANSFER

As simply described, fundamental engineering heat transfer knowledge consist know-how of evaluating rates three modes of heat transfer with specific conditions, properties and geometries, and further applying that to design and performance analysis of heat exchangers.

Table 1-1 *Three modes of heat transfer and their basic rate equations*

Conduction	**Convection**	**Radiation**
Heat transfer across medium.	Heat transfer between moving fluid and surface.	Heat transfer in form of electromagnetic waves emitted by surfaces at a finite temperature.
$q = -kA\frac{dT}{dx}$	$q = hA(T_S - T_\infty)$	$q = \varepsilon A\sigma\left(T_S^4 - T_\infty^4\right)$

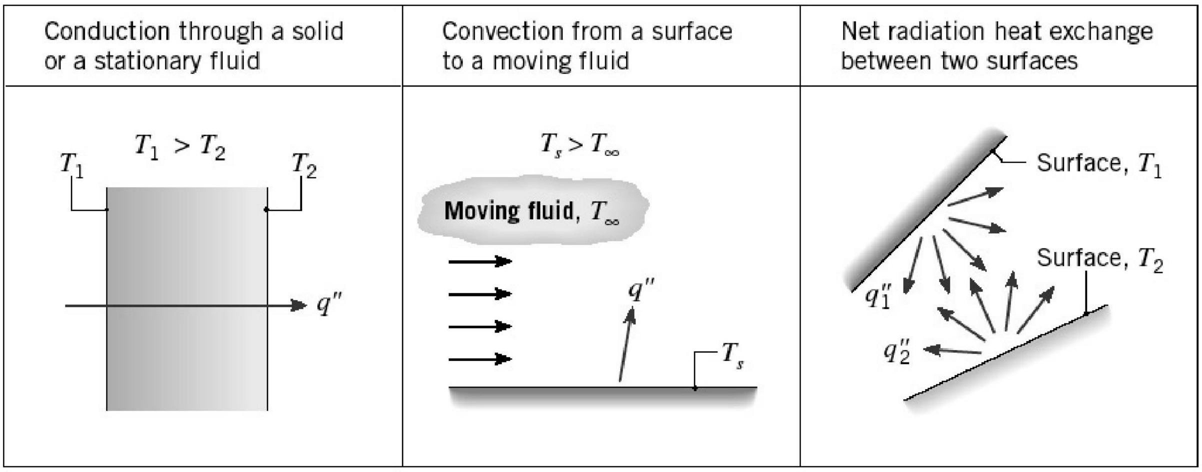


Figure 1-1 *Heat transfer modes: Conduction, Convection and Radiation*

1.1.1 Conduction

Conduction is *transfer of energy from more energetic particles to less energetic ones due to interaction between atomic and molecular particles.*

In *Solids* conduction is due to combination of

- (1) Vibrations of the molecules in a lattice and (2) Energy transport by free electrons

In *gases and liquids* conduction is

- (1) Collision and (2) Diffusion of molecules during their random motion.

Fourier's Law of Heat Conduction

$$q = -kA\frac{dT}{dx} \tag{1-1}$$

k	*Thermal conductivity [kJ/mK]*
A	*Area perpendicular to direction of heat transfer [m^2]*
dT/dx	*Temperature gradient [K/m]*

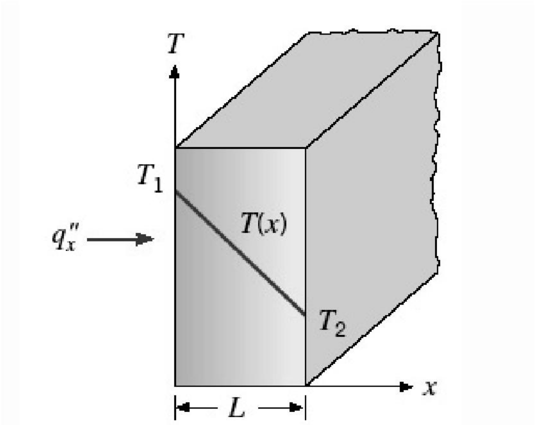


Figure 1-2 *One-dimensional heat transfer by conduction*

Thermal conductivity is rate of heat transfer through a unit thickness of material per unit area per unit temperature difference. Thermal conductivity is property of material and dependent on temperature (read chapter 2). Just to give the idea on range of conductivities: changing from insulation to metal thermal conductivity varies with 0.001-100 W/mK.

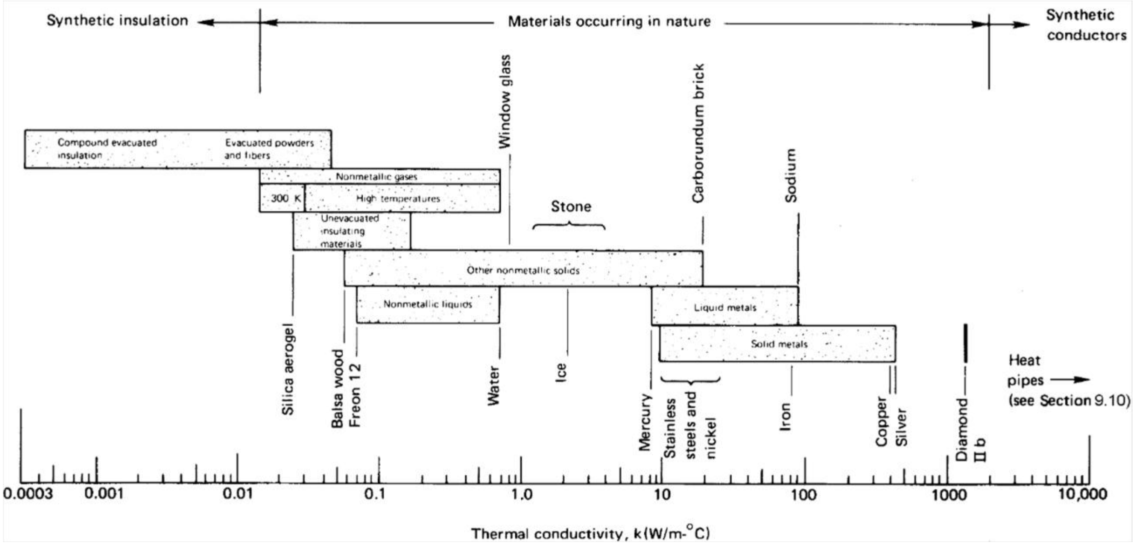


Figure 1-3 *Range of Conductivities*

1.1.2 Convection

Convection is classified according the nature of the flow:

- Free (Natural) Convection – Flow is induced by buoyancy forces, which are caused by temperature variations formed due to heat transfer in the fluid.
- Forced Convection – Flow is caused by external means: fan, pump, wind, etc.

Commonly, convection is sensible energy transfer of the fluid. Two special convection heat transfer cases associated with phase change between liquid and vapour states of fluid are (1) boiling and (2) condensation.

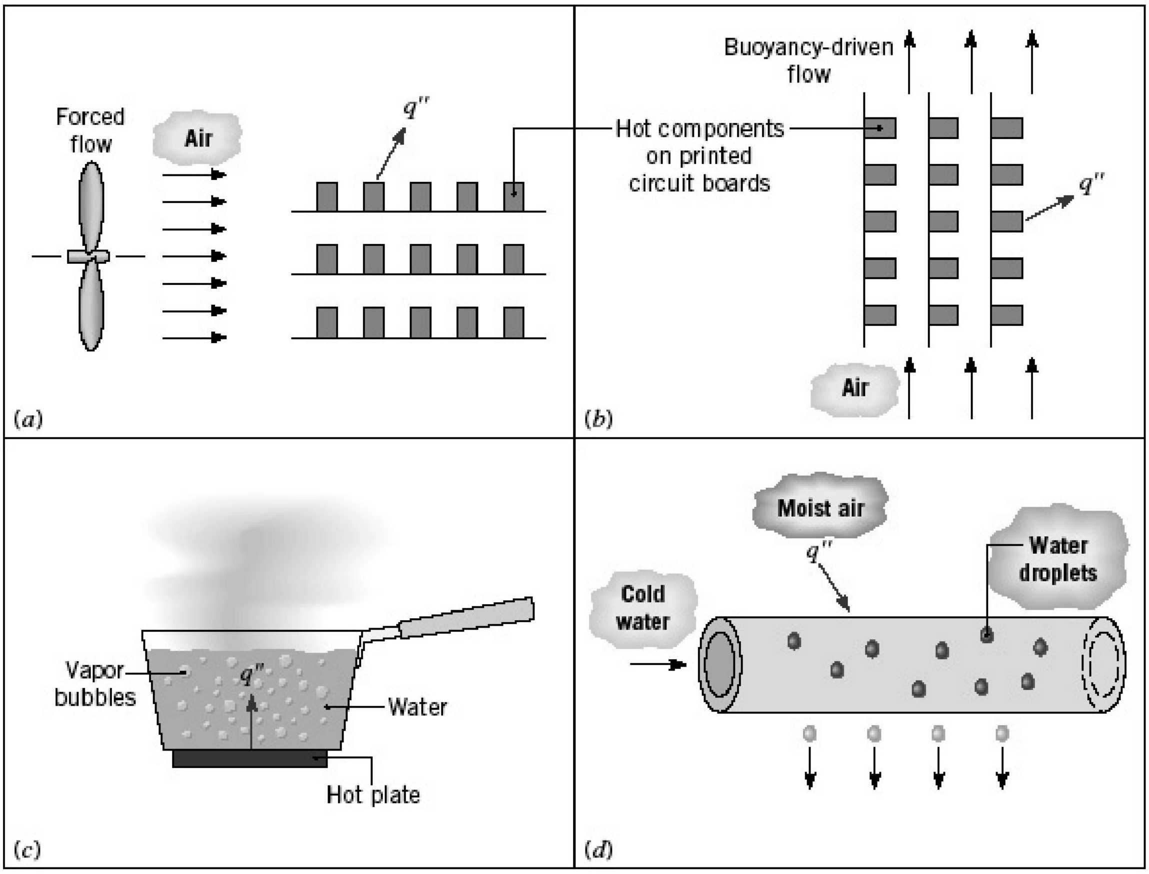


Figure 1-4 *Convection heat transfer classification: forced, free, (boiling, condensation)*

Convection heat transfer is combination of two mechanisms:

- Random molecular motion – diffusion (Conduction)
- And bulk/macroscopic fluid motion.

Two types of boundary layers are associated with fluid flow near surface and convective heat transfer. *Hydrodynamic Boundary Layer* is region of the fluid, where velocity varies from zero at surface to velocity u_∞ of bulk fluid flow. *Thermal Boundary Layer* is region of the fluid, where temperature varies from surface temperature to bulk fluid temperature. At the interface (surface) temperatures of solid and fluid are same and velocity of fluid is zero and heat is transferred only by random molecular motion (conduction).

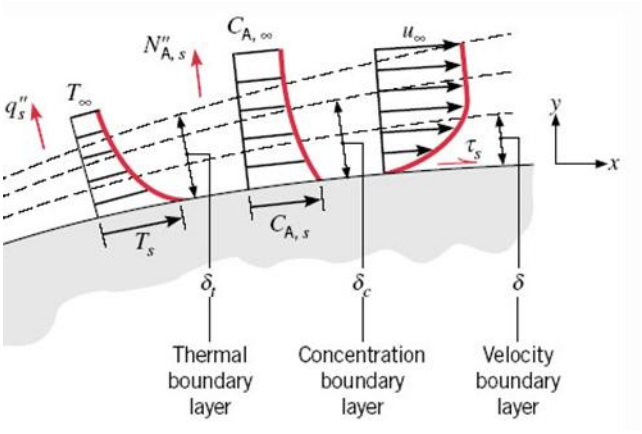


Figure 1-5 *Three Boundary layers: temperature, concentration and velocity boundary layers*

<u>Newton's Law of Cooling</u>
Convection heat transfer rate is expressed commonly as Newton's law of cooling:

$$q = hA(T_S - T_\infty) \tag{1-2}$$

h	*Convection heat transfer coefficient [W/Km^2]*
A	*Area perpendicular to heat transfer [m^2]*
T_S	*Surface temperature [K]*
T_∞	*Fluid temperature [K]*

Convection heat transfer coefficient is experimentally determined parameter and it is function of surface geometry, nature of fluid motion, properties of fluid and bulk fluid velocity.

1.1.3 Radiation

Radiation is energy emitted by matter that is at nonzero temperature in form of electromagnetic waves. Electromagnetic waves are caused by electronic configuration changes of atoms and molecules. Contrarily to conduction and convection, energy transfer due radiation is most efficient in vacuum. Even radiation is volumetric phenomenon, it is typically considered as transfer phenomenon between solid surfaces.

<u>Stefan-Boltzmann Law</u>
Surface that emits maximum rate of radiation is called *blackbody* or ideal radiator. Upper limit of emissive heat transfer is given by Stefan-Boltzmann law as

$$E_b = \sigma T_S^4 \tag{1-3}$$

E_b *= Emissive power of blackbody [W/m^2]*
σ *= Stefan-Boltzmann constant [W/m^{2}K^4]*
T_S *= Absolute temperature of the surface [K]*

Heat flux emitted by real surface is less than that of *blackbody* at the same temperature and is given as

$$E = \varepsilon \sigma T_S^4 \tag{1-4}$$

E *= Emissive power of real surface [W/m^2]*
ε *= Emissivity, $0 \le \varepsilon \ge 1$*

Emissivity depends strongly on material and finish of surface. Absorptivity α is another surface radiation property. Absorptivity is the fraction of the radiation energy incident on a surface that is absorbed and its value varies between 0 and 1. Blackbody is perfect absorber ($\alpha = 1$). Radiation incident on a surface from its surroundings is called irradiation, G, and rate, which irradiation is absorbed to surface is

$$G_{abs} = \alpha G \tag{1-5}$$

G_{abs} = *Absorbed radiation* [W/m^2]
α = *Absorptivity*, $0 \leq \alpha \geq 1$
G = *Irradiation* [W/m^2]

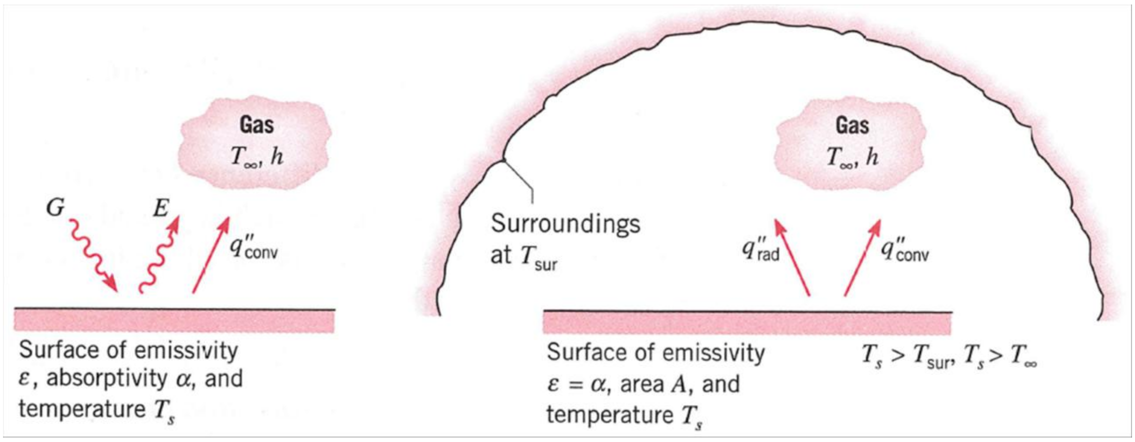


Figure 1-6 *Radiation heat transfer*

Kirchhoff's Law of Radiation

Emissivity and *absorptivity* of a surface are equal at the same temperature and wavelength. Usually, dependence on temperature and wavelength are ignored by approximating emissivity and absorptivity to be equals (a gray surface).

$$q'' = \varepsilon\sigma\left(T_S^4 - T_\infty^4\right) \tag{1-6}$$

q''	*Radiation heat flux* [W/m^2]
ε	*gray surface emissivity*
T_S	*Temperature at surface [K]*
T_∞	*Temperature at surroundings [K]*

1.2 CONSERVATION OF ENERGY

1st Law of Thermodynamics

The Law of conservation of energy over a time interval: “The increase in the amount of energy stored in a *control volume* must equal the amount of energy that enters the *control volume,* minus the amount of energy that leaves the *control volume*.” For *closed system* 1st law of thermodynamics over a time interval can be stated as

$$\Delta E_{st}^{tot} = Q - W \tag{1-7}$$

ΔE_{st}^{tot} = *Change in total energy stored in system*
Q = *Net heat transferred to system*
W = *Net work done by the system*

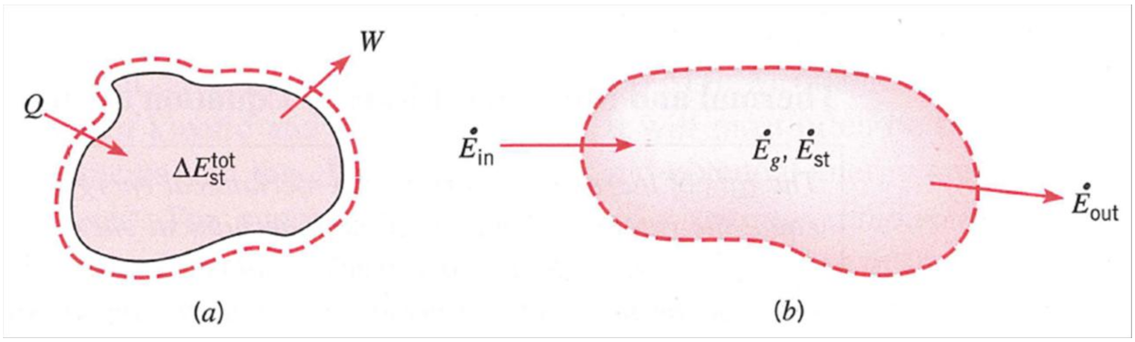


Figure 1-7 *Energy balance for a) a closed system over a time interval b) open system (control volume) at instant time given as rates.*

Total energy consists of mechanical energy, which is combination of kinetic and potential energies, and internal energy. While studying heat transfer, thermal energy is form of internal energy to be focused on. Statement of the first law of thermodynamics that is suitable for heat transfer analysis for control volume (also open system) can be given as *Thermal and mechanical energy equation*:

$$\dot{E}_{st} = \frac{d\dot{E}_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \tag{1-8}$$

E_{st} = *Stored thermal and mechanical energy*
$E_{in/out}$ = *Energy entering/leaving system*
E_g = *Thermal energy generation*

Energy storage and generation are *volumetric phenomena* and are usually proportional to magnitude of volume. For control volume, thermal energy generation can be chemical, electrical, electromagnetic or nuclear energy conversion. The inflow and outflow terms are *surface phenomena* and are generally proportional to surface area. Energy can be transferred across surface of control volume in forms of heat, work and mass containing thermal and mechanical energies.

For steady-state open system having no thermal energy generation, energy equation over control volume can be written as

$$\dot{m}(u_t + pv + \tfrac{1}{2}V^2 + gz)_{in} - \dot{m}(u_t + pv + \tfrac{1}{2}V^2 + gz)_{out} + q - \dot{W} = 0 \quad \textit{(1-9)}$$

u_t = *Specific internal energy*
pv = *Specific flow work*
$\tfrac{1}{2}V^2$ = *Specific kinetic energy*
gz = *Specific potential energy*

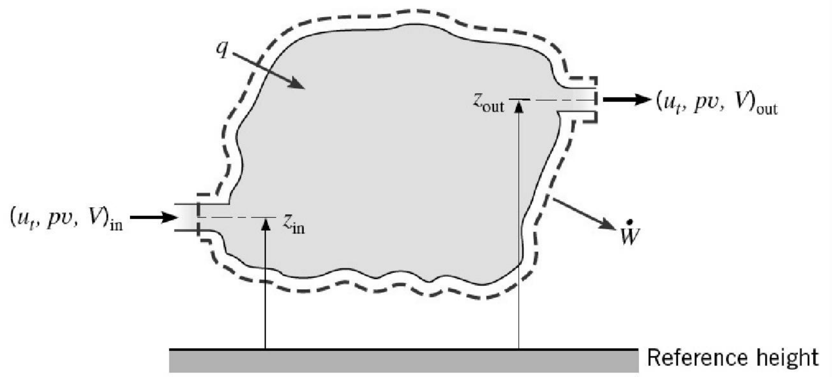


Figure 1-8 *Energy balance for a steady-flow, open system*

For Systems having negligible kinetic and potential energy changes and negligible work, this can be further reduced for ideal gases or incompressible liquids to *simplified steady-flow thermal energy equation*:

$$q = \dot{m}c_p(T_{out} - T_{in}) \quad (1\text{-}10)$$

1.2.1 Surface energy Balance

The most applied form of conservation of energy equation in heat transfer problems is surface energy balance

$$E_{In} = E_{Out} \quad (1\text{-}11)$$

, which in case of conduction and convection is

$$q_{conv} = q_{cond}\,. \quad (1\text{-}12)$$

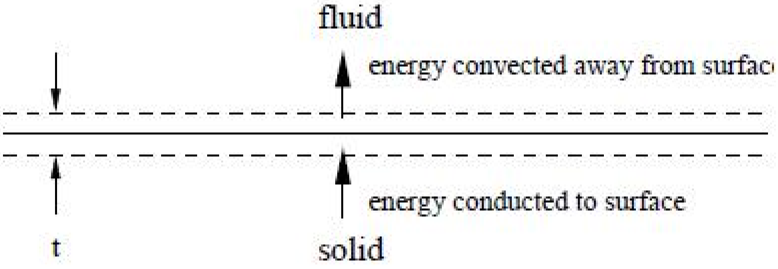


Figure 1-9 *Surface Energy Balance: Conduction – Convection (Wendl)*

1.3 APPLICATIONS

Fields of heat transfer applications are for example:

- Heat Exchangers: at power plants, etc.
- Cooling of Electronic equipments
- Buildings: insulations and air-conditioning
- Refrigeration
- Human body, etc.

Engineering Heat Transfer problems can be divided to two groups:

1) **Rating**: Determination of heat transfer rate for system having specified temperature difference.
2) **Sizing**: Determination of size of the system to transfer heat at specified rate for a specified temperature difference.

2 STATIONARY CONDUCTION

2.1 FOURIER'S LAW

1-Dimensional Steady-State form of *Fourier's Law* states for conduction heat transfer rate

$$q_x = -kA\frac{dT}{dx} \tag{2-1}$$

, and for conduction heat flux

$$q_x^{"} = -k\frac{dT}{dx}\,. \tag{2-2}$$

Direction of conduction heat flow is always normal to surface of constant temperature, *isothermal surface*. Generalization of conduction rate equation to 3-dimensional form gives

$$\bar{q}^{"} = -k\nabla T = -k\frac{\partial T}{\partial x}\hat{i} - k\frac{\partial T}{\partial y}\hat{j} - k\frac{\partial T}{\partial z}\hat{k} \tag{2-3}$$

2.1.1 Analogy of Fourier's, Ohm's and Fick's Laws

Origin of Fourier's Law is phenomenological meaning that it is derived from observer phenomenon rather than first principles of physics. Fourier Law has various numbers of important analogies: Ohm's and Fick's Laws being examples of electrical and mass transfer analogies.

Table 2-1 *Analogy of Fourier's, Ohm's and Fick's Laws*

Fourier's Law	Conduction heat flux	$\bar{q}^{"} = -k\nabla T$	Thermal conductivity k Temperature gradient ∇T
Ohm's Law	Electrical current density	$J^{"} = -\gamma\nabla V$	Electrical conductivity γ Voltage gradient V
Fick's Law	Mass diffusion flux	$\bar{q}^{"} = -\rho D\nabla m$	Diffusion coefficient D Mass concentration gradient ∇m

2.2 THERMAL PROPERTIES

Thermophysical properties of materials are classified to two categories: transport and thermodynamic properties. Thermal conductivity is transport property, and density and specific heat are common thermodynamic properties (Table 2-2).

Table 2-2 *Transport and thermodynamical material properties (Note analogy in transport properties)*

Transport		
Thermal conductivity	k	Heat Transfer, Conduction: $\bar{q}'' = -k\nabla T$
Diffusivity	D	Mass Transfer, Diffusion: $\bar{q}'' = -D\nabla m$
Viscosity	μ	Friction, shear stress: $\tau = \mu\nabla u$
Thermodynamic		
Density	ρ	
Specific heat	c_p	
Volumetric heat capacity	ρc_p	Describes the ability of system to store thermal energy

Thermal diffusivity $\alpha = k/\rho c_p$ [m^2/s] is ratio of heat conducted through the material to heat stored per unit volume.

2.2.1 Thermal Conductivity

As stated earlier in Fourier's Law, thermal conductivity is defined as

$$k = \frac{-q_x''}{(\partial T/\partial x)} \qquad \text{[W/mK]} \tag{2-4}$$

Conductivity depends on physical atomic and molecular structure of matter, which are related to state of matter.

Solids: Thermal conductivity for solids can be expressed as

$$k = k_e + k_l \tag{2-5}$$

$k_e \,\hat{=}$ *Conductivity associated with freely moving electrons*
$k_l \,\hat{=}$ *Conductivity related to vibration of lattice*

Structure of refractory materials is porous and their *effective thermal conductivity* is formed as a sum of different heat transfer modes: conduction, convection and radiation.

Liquids and gases: For fluids thermal energy is transported with molecular motion. As molecular spacing is much larger in case of liquid and gases compared to solids, thermal energy transport is less effective, therefore meaning smaller conductivities than that of solids. Similarly gases have generally smaller conductivities than liquids.

Typical conductivities and temperature dependence of materials are shown in Figure 2-1 figure.

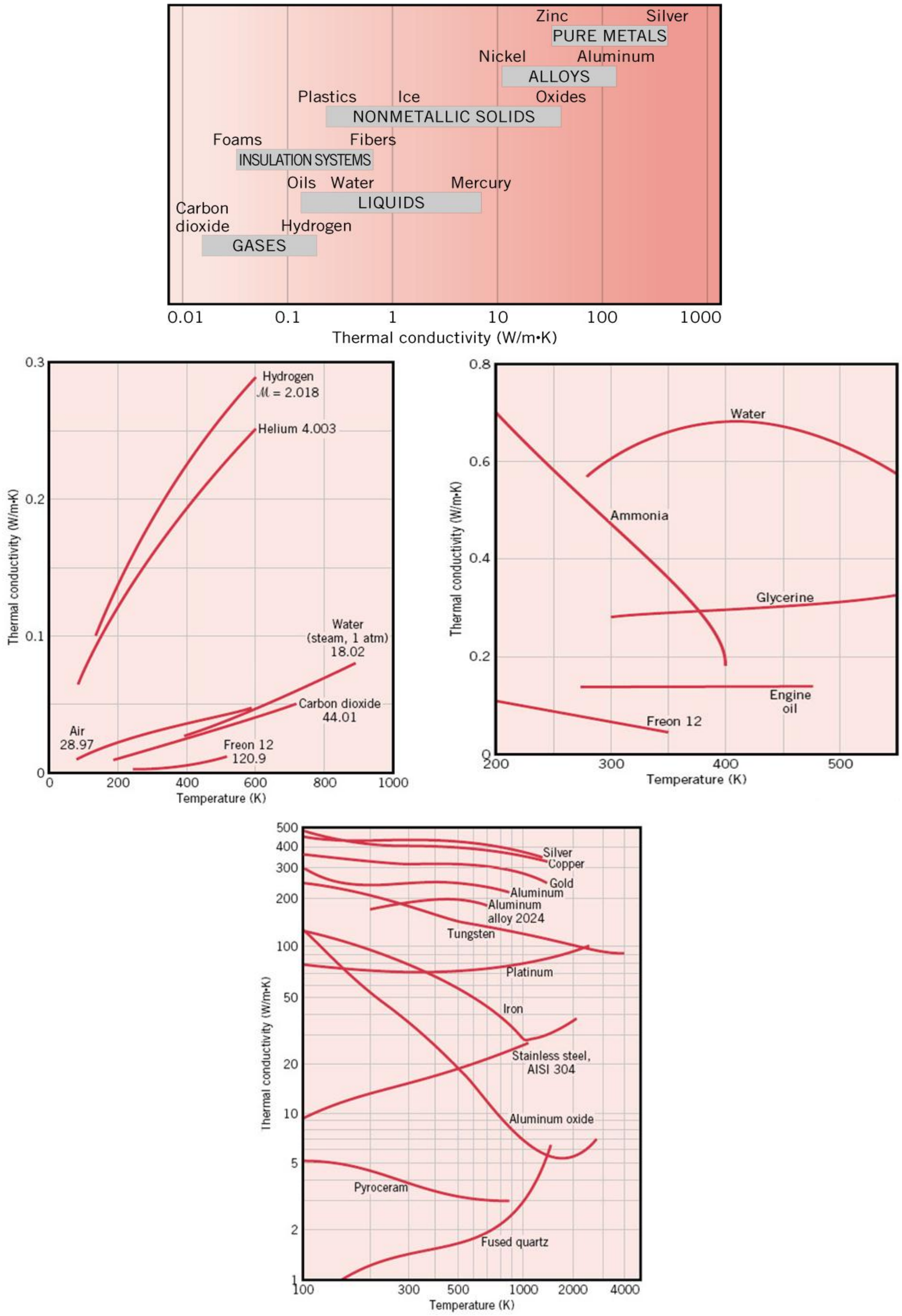
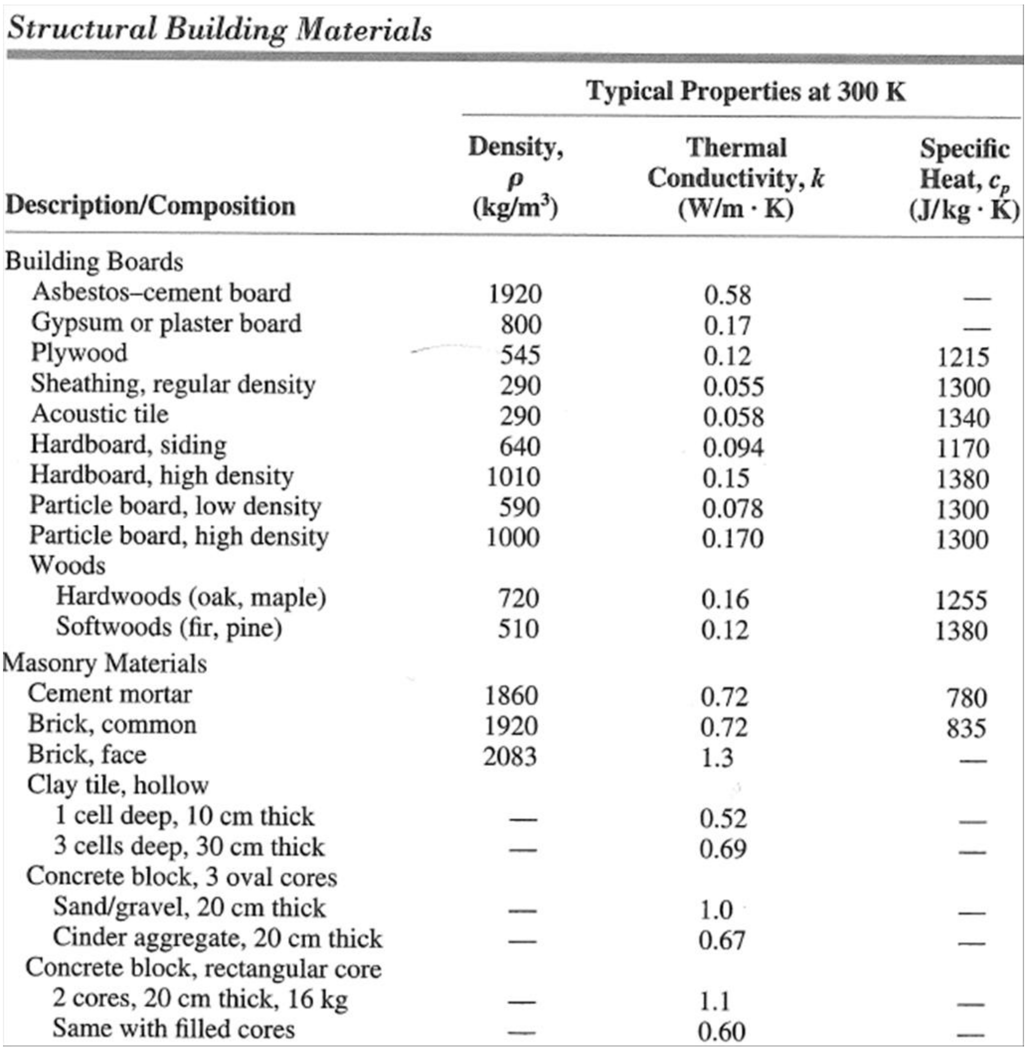


Figure 2-1 *Range and temperature dependency of thermal conductivities of various materials*

Table 2-3 *Thermal properties of building materials*



Structural Building Materials

Description/Composition	Typical Properties at 300 K Density, ρ (kg/m^3)	Thermal Conductivity, k (W/m · K)	Specific Heat, c_p (J/kg · K)
Building Boards			
Asbestos–cement board	1920	0.58	—
Gypsum or plaster board	800	0.17	—
Plywood	545	0.12	1215
Sheathing, regular density	290	0.055	1300
Acoustic tile	290	0.058	1340
Hardboard, siding	640	0.094	1170
Hardboard, high density	1010	0.15	1380
Particle board, low density	590	0.078	1300
Particle board, high density	1000	0.170	1300
Woods			
Hardwoods (oak, maple)	720	0.16	1255
Softwoods (fir, pine)	510	0.12	1380
Masonry Materials			
Cement mortar	1860	0.72	780
Brick, common	1920	0.72	835
Brick, face	2083	1.3	—
Clay tile, hollow			
1 cell deep, 10 cm thick	—	0.52	—
3 cells deep, 30 cm thick	—	0.69	—
Concrete block, 3 oval cores			
Sand/gravel, 20 cm thick	—	1.0	—
Cinder aggregate, 20 cm thick	—	0.67	—
Concrete block, rectangular core			
2 cores, 20 cm thick, 16 kg	—	1.1	—
Same with filled cores	—	0.60	—

2.3 GENERAL HEAT DIFFUSION EQUATION

Suitable statement of the first law of thermodynamics for heat transfer analysis for control volume, as described earlier, can be given as *Thermal and mechanical energy equation*:

$$\dot{E}_{st} = \frac{d\dot{E}_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{g} \qquad (2\text{-}6)$$

E_{st} = *Stored thermal and mechanical energy*
$E_{in/out}$ = *Energy entering/leaving system*
E_{g} = *Thermal energy generation*

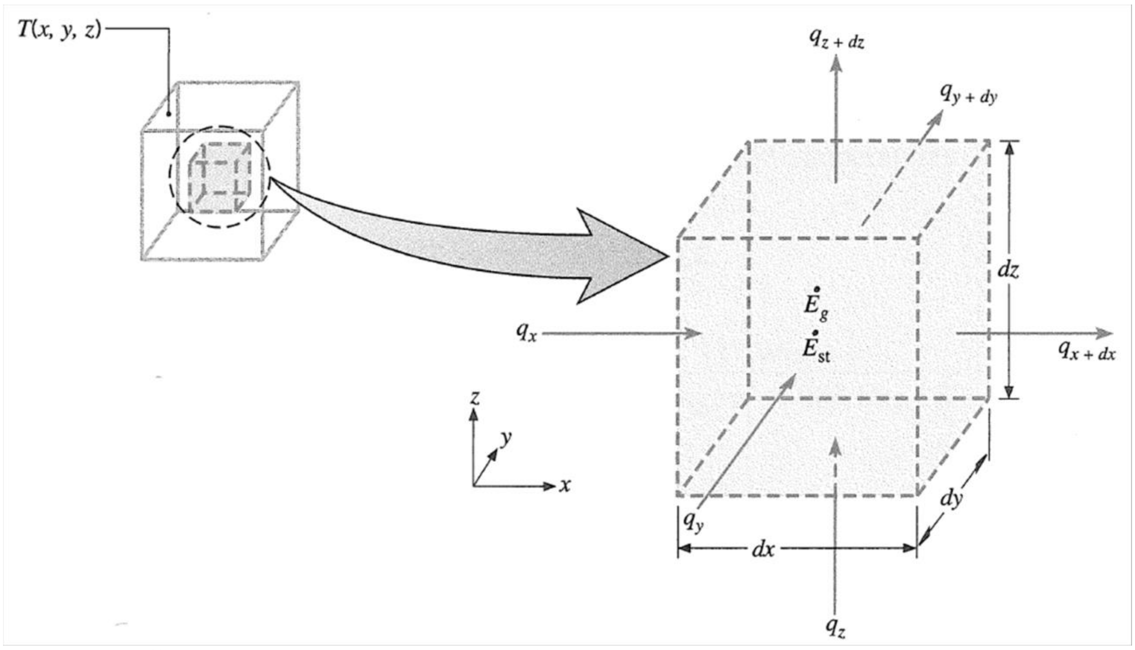


Figure 2-2 *Differential Cartesian control volume for conduction analysis*

The conduction heat rates at opposite surfaces can be given as Taylor series expansion:

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx \qquad (2\text{-}7)$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y}dy \qquad (2\text{-}8)$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}dz \qquad (2\text{-}9)$$

Energy source term can written as

$$\dot{E}_g = \dot{q}\, dx\, dy\, dz \qquad (2\text{-}10)$$

$\dot{q}$ = *Energy generation rate in unit volume W/*m^3

Change of thermal energy in control volume *with time* can be written as

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz \tag{2-11}$$

Writing energy equation by substituting conduction rates as energy flow terms, following form can be obtained:

$$q_x + q_y + q_z - \left(q_x + \frac{\partial q_x}{\partial x} dx \right) - \left(q_y + \frac{\partial q_y}{\partial y} dy \right) - \left(q_z + \frac{\partial q_z}{\partial z} dz \right) + \dot{q} dxdydz \quad = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

$$\Rightarrow \qquad -\frac{\partial q_x}{\partial x} dx - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz + \dot{q} dxdydz = \rho c_p \frac{\partial T}{\partial t} dxdydz \tag{2-12}$$

Conduction heat rates can be written according Fourier's Law:

$$q_x = -k\, dy\, dz \frac{\partial T}{\partial x} \tag{2-13}$$

$$q_y = -k\, dx\, dz \frac{\partial T}{\partial y} \tag{2-14}$$

$$q_z = -k\, dx\, dy \frac{\partial T}{\partial z} \tag{2-15}$$

By substituting conduction heat rates to energy equation the general form of *heat diffusion equation* (also *heat equation*) in Cartesian coordinates can be obtained

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right) - \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \tag{2-16}$$

Heat diffusion equation is simplified, if *thermal conductivity is constant* (isotropic conditions), and can be written as

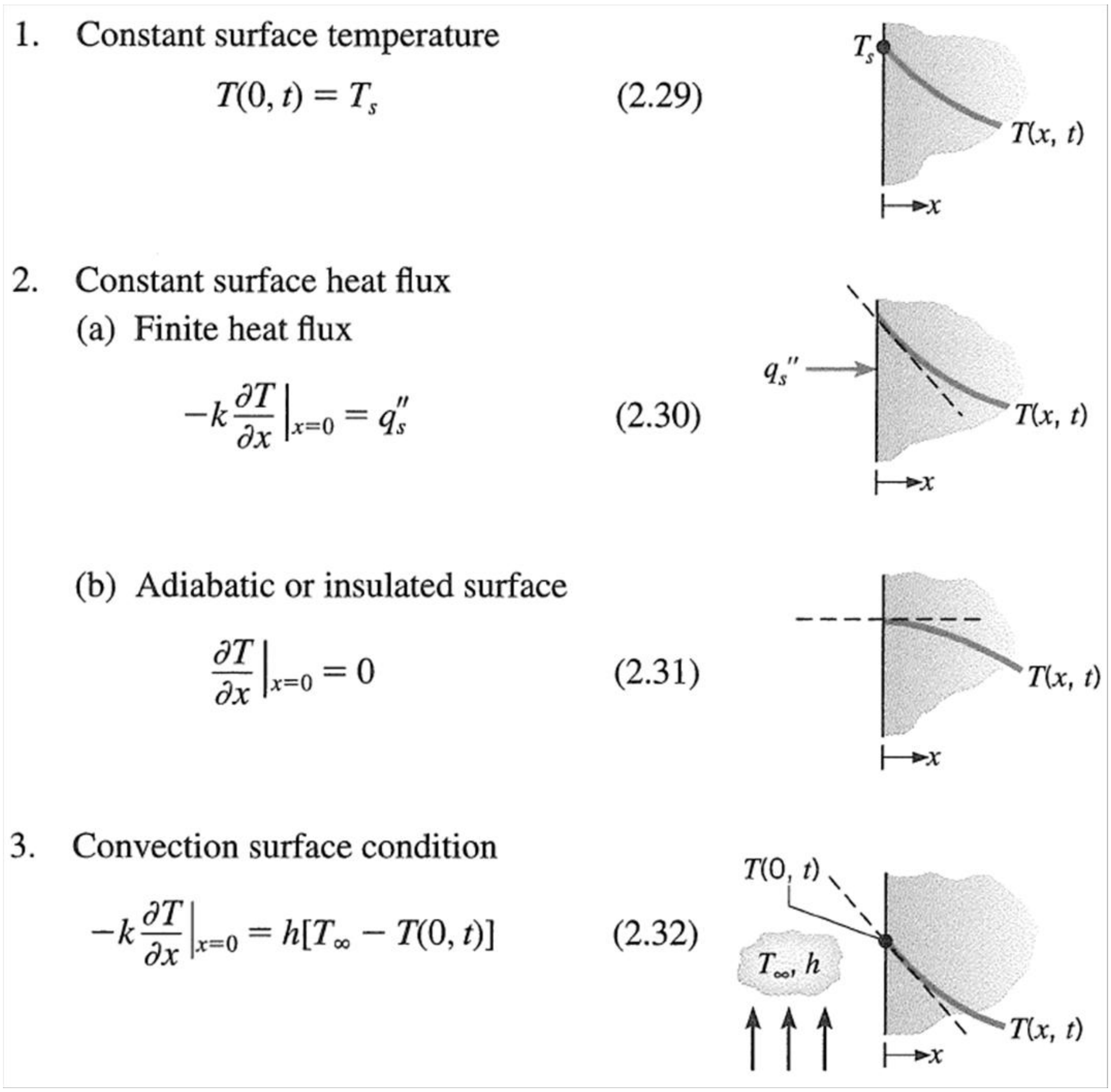
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{2-17}$$

Initial Condition

As heat equation is first order in time, only one condition, termed initial condition has to be specified. Typical condition is known temperature distribution T(x, y, z, t=0) at time t.

Boundary Conditions

For each coordinate (2^{nd} order) in heat equation two boundary conditions has to given to describe system. Typical boundary conditions are shown in Figure 2-3.



1. Constant surface temperature

$$T(0, t) = T_s \qquad (2.29)$$

2. Constant surface heat flux

(a) Finite heat flux

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \qquad (2.30)$$

(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (2.31)$$

3. Convection surface condition

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \qquad (2.32)$$

Figure 2-3 Boundary conditions for heat diffusion equation at the surface (Incropera)

2.4 1-DIMENSIONAL, STEADY-STATE CONDUCTION

General heat diffusion equation can written under steady-state, one-dimensional conditions with no heat generation as

$$\frac{d^2T}{dx^2} = \frac{d}{dx}\left(\frac{dT}{dx}\right) = 0 \tag{2-18}$$

One-dimensionality approximation is appreciable, when temperature gradient in one direction is significantly greater than in others, that is

$$\frac{\partial T}{\partial x} \succ\succ \frac{\partial T}{\partial y} \ , \ \frac{\partial T}{\partial x} \succ\succ \frac{\partial T}{\partial z} .$$

2.4.1 Plain Wall

Let us consider a plane wall (Figure 2-4), where heat is transferred via convection from hot fluid to one surface, via conduction across wall and via convection from other surface to cold fluid.

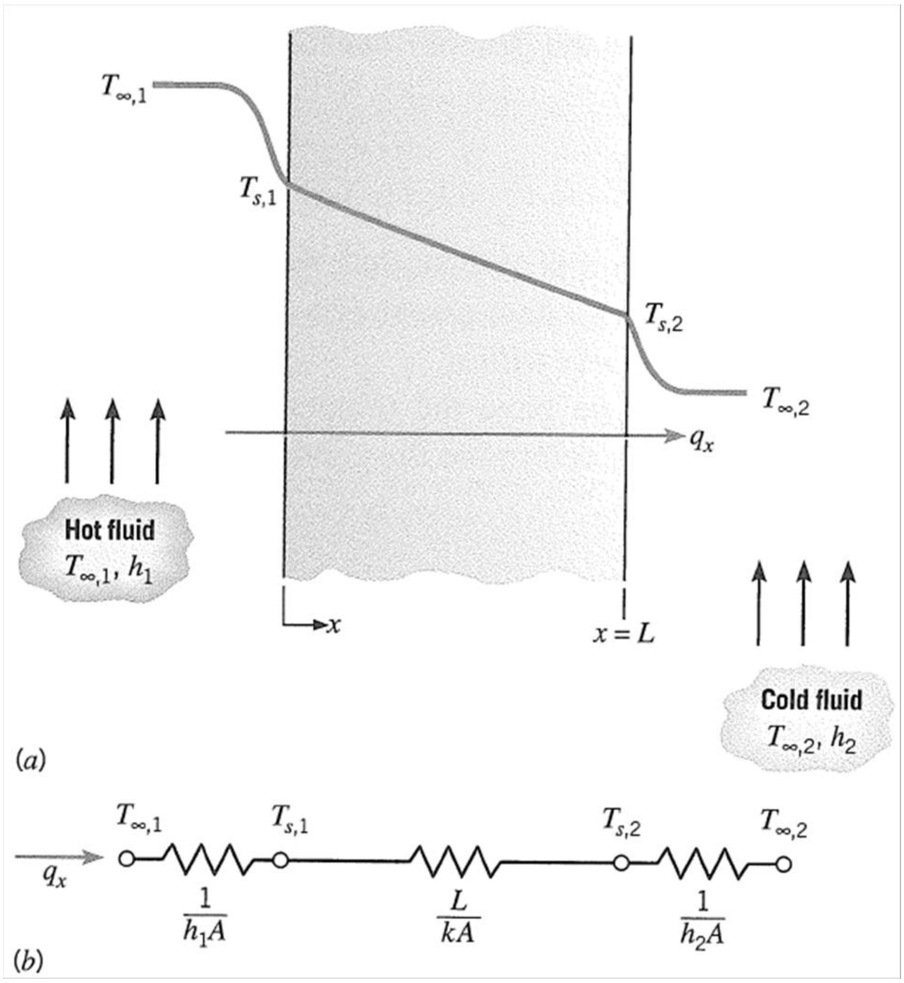


Figure 2-4 *a) Temperature distribution of plain wall b) equivalent thermal circuit*

For determining temperature distribution and conduction heat transfer rate across wall general heat diffusion equation is written for *one-dimensional conduction* and *constant thermal conductivity* as

$$\frac{d^2T}{dx^2} = \frac{d}{dx}\left(\frac{dT}{dx}\right) = 0 \qquad \text{(2-19)}$$

, which can be further solved as follows

$$\int d\left(\frac{dT}{dx}\right) = \int 0dx \quad \Rightarrow \quad \frac{dT}{dx} = \mathrm{C}_1 \quad \Rightarrow \int dT = \int C_1 dx$$

and gives *general solution:*

$$T(x) = C_1 x + C_2 \qquad \text{(2-20)}$$

Boundary Conditions: *in case both surface temperatures are known*

1. x = 0: T = $T_{s,1}$ $\quad \Rightarrow T_{s,1} = C_1 \cdot 0 + C_2$ eli $C_2 = T_{s,1}$
2. x = L: T = $T_{s,2}$ $\quad \Rightarrow T_{s,2} = C_1 L + T_{s,1}$ eli $C_1 = \dfrac{T_{s,2} - T_{s,1}}{L}$

From boundary conditions general solution forms temperature profile solution for plain wall:

$$T(x) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1} \qquad \text{(2-21)}$$

Conduction Heat Transfer Rate is given by Fourier's Law and by substituting solved boundary condition results following form is achieved:

$$q_x = -kA\frac{dT}{dx}$$

$$\frac{dT}{dx} = \frac{d}{dx}\left(\left(T_{s,2} - T_{s,1}\right)\frac{k}{L} + T_{s,1}\right) = \frac{T_{s,2} - T_{s,1}}{L}$$

$$q_x = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) \qquad \text{(2-22)}$$

2.4.2 Thermal Resistance & Overall Heat Transfer Coefficient

Thermal resistance for conduction in a plane wall (Figure 2-4) is determined as

$$R_{t,cond} \equiv \frac{L}{Ak}. \qquad \text{(2-23)}$$

Conduction heat transfer rate across plane wall is

$$q = \frac{Ak}{L}(T_{s,1} - T_{s,2}) = \frac{\Delta T}{R_{t,cond.}} \tag{2-24}$$

, which is analogical to definition of electrical current with relationship of voltage and electrical resistance

$$\mathrm{I} = \frac{\Delta U}{R}\ .$$

Thermal resistance for convection and *radiation* are defined similarly:

$$R_{t,conv.} \equiv \frac{1}{hA} = \frac{(T_s - T_\infty)}{q} \tag{2-25}$$

$$R_{t,rad} \equiv \frac{1}{h_r A} = \frac{(T_s - T_\infty)}{q_{rad}} \tag{2-26}$$

Total resistance can be calculated for *resistances in series* with

$$R_{t,tot} = \sum_i R_{t,i} \tag{2-27}$$

And for *resistances in parallel* with

$$\frac{I}{R_{t,tot}} = \sum_i \frac{I}{R_{t,i}} \qquad . \tag{2-28}$$

Conduction heat transfer rate in plane wall (Figure 2-4) can be expressed as

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{\sum_i R_{t,tot}} = UA(T_{\infty,1} - T_{\infty,2}) \tag{2-29}$$

And *Overall heat transfer coefficient* is defined as

$$U = \frac{1}{AR_{tot}}. \tag{2-30}$$

The Equivalent Thermal Circuit

Based on equivalent thermal circuits (Figure 2-4) for plain wall heat transfer rate can determined as

$$q_x = \frac{(T_{\infty,1} - T_{s,1})}{1/h_1 A} = \frac{(T_{s,1} - T_{s,2})}{L/kA} = \frac{(T_{s2} - T_{\infty,2})}{1/h_2 A} \tag{2-31}$$

2.4.3 Contact Heat Resistance

Surface roughness in composite systems forms additional heat resistance, which is called thermal contact resistance.

For *mating surface system* shown in Figure 2-5, *thermal contact resistance* is defined as

$$R''_{t,c} = \frac{T_A - T_B}{q''_x} \tag{2-32}$$

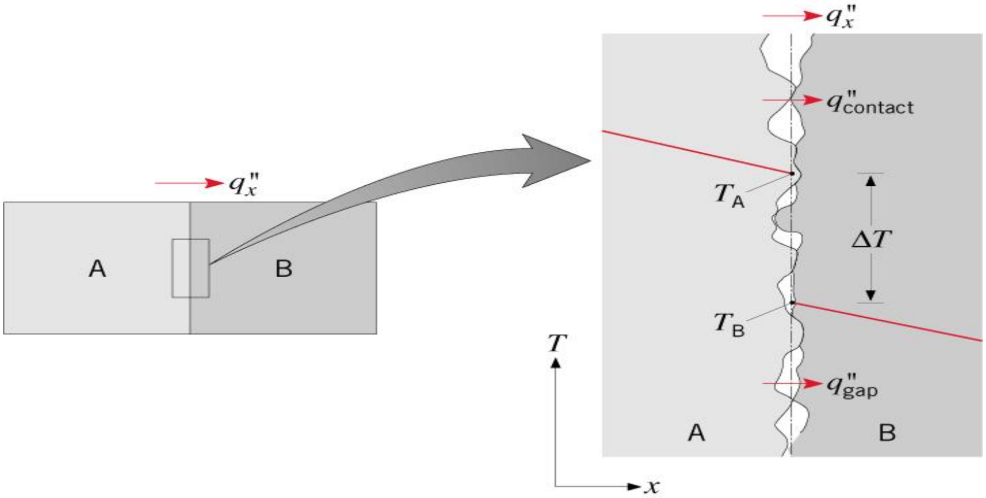


Figure 2-5 *Temperature drop due thermal contact resistance*

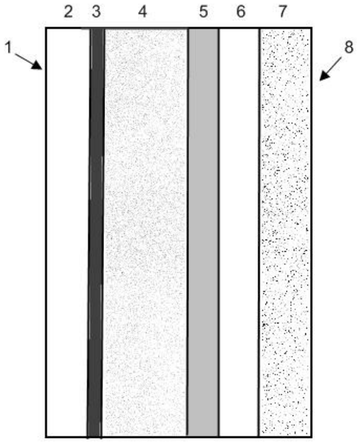
For smooth surfaces in contact with small characteristic gap width L, contact resistance can be approximated with relationship to interfacial gas conductivity as

$$R''_{t,c} \approx \frac{L}{k_{gas}} \tag{2-33}$$

Generally contact resistance is experimentally defined for different interfacial fluids, contacting materials and surface roughness.

Example I: ***Wall structure - Resistances in series***

A typical wall structure of a Finnish detached house is described on a picture below. Calculate the heat loss through the wall and the temperature distribution in the wall, when temperature outer side of the wall is -27 °C and inner side of the wall +20 °C.



Structure	Heat resistance
Inner surface	$R_1 = 1/(h_{\text{conv}} + h_{\text{w}}) = 0{,}15\ \text{m}^2\text{K/W}$
	$h_{\text{conv}} = 3\ \text{W/ m}^2\text{K}$
	$h_{\text{w}} = 3{,}7\ \text{W/ m}^2\text{K}$
Gypsum	$k = 0{,}23\ \text{W/ mK}$
	$s = 0{,}012\ \text{m}$
	$R_2 = s/k = 0{,}052\ \text{m}^2\text{K/W}$
Polyethen plastic	-
Insulator	$k = 0{,}045\ \text{W/ mK}$
	$s = 0{,}10\ \text{m}$
	$R_4 = s/k = 2{,}22\ \text{m}^2\text{K/W}$
Weather shield Board	$k = 0{,}09\ \text{W/ mK}$
	$s = 0{,}02\ \text{m}$
	$R_5 = s/k = 0{,}222\ \text{m}^2\text{K/W}$
Air gap	$R_6 = 0{,}2\ \text{m}^2\text{K/W}$
	$s = 0{,}015\ \text{m}$
Brick	$k = 0{,}07\ \text{W/ mK}$
	$s = 0{,}08\ \text{m}$
	$R_7 = s/k = 0{,}144\ \text{m}^2\text{K/W}$
Outer surface	$R_8 = 1/(h_{\text{conv}} + h_{\text{w}}) = 0{,}05\ \text{m}^2\text{K/W}$
	$h_{\text{conv}} = 15\ \text{W/ m}^2\text{K}$
	$h_{\text{w}} = 5\ \text{W/ m}^2\text{K}$

ASSUMPTIONS: From the surface (5) to the air and further from the air to the surface (6) heat is transferred mainly by convection. If we assume that the heat transfer coefficient for air is $h = 10\ \text{W/m}^2\text{K}$, the heat resistance R_6 is then $R_6 = 1/10 + 1/10 = 0{,}2\ \text{m}^2\text{K/W}$. Notice that the conduction schema (s/k) would give a noticeably different value for the heat resistance in the air gap.

At first we calculate the overall heat resistance, and after that heat loss power and finally the temperature variations.

$$R_{tot} = \sum_{i=1}^{8} R_i = (0{,}15 + 0{,}052 + 0 + 2{,}22 + 0{,}222 + 0{,}20 + 0{,}114 + 0{,}05)\ \text{m}^2\text{K/W}$$

$$= \underline{3{,}0\ \text{m}^2\text{K/W}}$$

Overall heat transfer coefficient: $U = \frac{1}{R_{tot}} = \underline{0{,}332\ \text{W/ m}^2\text{K}}$

Heat loss: $\phi = U\Delta T = 0{,}322\ \text{W/ m}^2\text{K} \cdot [20 - (-27)]\ \text{K} = \underline{15{,}6\ \text{W/m}^2}$

Temperature differences:

$$\Delta T_1 = \phi R_1 = 15{,}6\frac{\text{W}}{\text{m}^2} \cdot 0{,}15\frac{\text{m}^2\text{K}}{\text{W}} = \underline{2{,}3\ \text{K}}$$
$$\Delta T_2 = \phi R_2 = 15{,}6 \cdot 0{,}052 = \underline{0{,}8\ \text{K}}$$
$$\Delta T_3 = \phi R_3 = 15{,}6 \cdot 0 = \underline{0\ \text{K}}$$
$$\Delta T_4 = \phi R_4 = 15{,}6 \cdot 2{,}22 = \underline{34{,}6\ \text{K}}$$
$$\Delta T_5 = \phi R_5 = 15{,}6 \cdot 0{,}222 = \underline{3{,}5\ \text{K}}$$
$$\Delta T_6 = \phi R_6 = 15{,}6 \cdot 0{,}2 = \underline{3{,}1\ \text{K}}$$
$$\Delta T_7 = \phi R_7 = 15{,}6 \cdot 0{,}114 = \underline{1{,}7\ \text{K}}$$
$$\Delta T_8 = \phi R_8 = 15{,}6 \cdot 0{,}05 \cong \underline{1{,}0\ \text{K}}$$

Temperatures

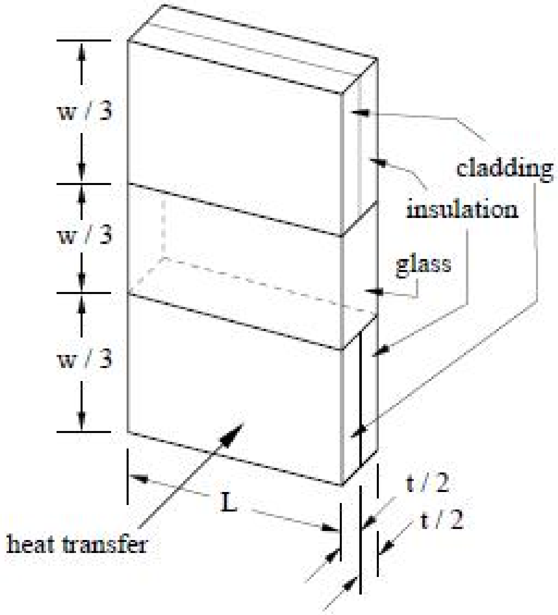
$$T_1 = \underline{17{,}7\ °\text{C}}\ (= T_{\text{i}})$$
$$T_2 = \underline{16{,}9\ °\text{C}}$$
$$T_3 = \underline{16{,}9\ °\text{C}}$$
$$T_4 = \underline{-17{,}7\ °\text{C}}$$
$$T_5 = \underline{-21{,}2\ °\text{C}}$$
$$T_6 = \underline{-24{,}3\ °\text{C}}$$
$$T_7 = \underline{-26{,}0\ °\text{C}}$$
$$T_8 = \underline{-27\ °\text{C}}\ (= T_{\text{o}})$$

In this example the convection heat transfer coefficients $h_{\text{conv}} = 3\ \text{W/m}^2\text{K}$ (inner surface) and $h_{\text{conv}} = 15\ \text{W/m}^2\text{K}$ (outer surface) are based on empirical and average values. At the outer side of the wall h_{conv} depends strongly on wind and varies after that.

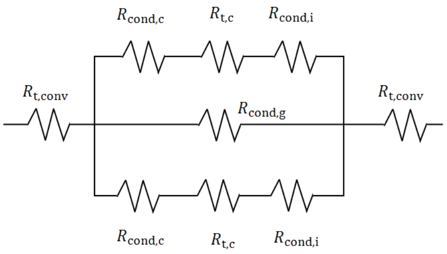
Example II: *Window - Circuit Analogy*

A manufacturer of pre-fabricated components for high-rise buildings is determining the thermal rating of a new panel design. The system consists of a header made up of an exterior cladding material having thermal conductivity k_c, and an inner insulator of conductivity k_i, window glass with conductivity k_g, and a footer of the same construction as the header. It can be assumed that heat transfer occurs one dimensionally through the panel. The panel height and width are w and L and the header, pane, and footer each have a height of w = 3. The total thickness is t, half of which is cladding and half of which is insulating material.

What would the thermal circuit for this system be assuming that there is a thermal contact resistance of Rc between the cladding and insulator components?



Thermal circuit



ANALYSIS: The thermal resistance at the outer side of the panel's surface (Eq. 2-25).

$$R_{\text{t,conv}} = \frac{1}{hA} = \frac{1}{hwL}$$

The conductive resistances for glass, cladding and insulation from Eq. 2-22.

$$R_{\text{cond,g}} = \frac{t}{k_{\text{g}} L w / 3}$$

$$R_{\text{cond,c}} = \frac{t/2}{k_{\text{c}} L w / 3}$$

$$R_{\text{cond,i}} = \frac{t/2}{k_{\text{i}} L w / 3}$$

The contact resistance between the cladding and insulation (Eq. 2-33).

$$R''_{\text{t,c}} = \frac{T_{\text{c}} - T_{\text{i}}}{q''_{\text{x}}}$$

2.4.4 The Cylinder

Cylindrical systems usually have temperature gradient in radial direction only and can therefore be treated as one-dimensional in spherical coordinate system. For one-dimensional, steady-state conditions with no heat generation radial conduction heat equation can be expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)=0 \tag{2-34}$$

By estimating constant thermal conductivity k and further differentiating as follows

$$\frac{kd\left(r\frac{dT}{dr}\right)}{dr}=0 \;|\cdot dr \qquad \Rightarrow \qquad \int d\left(r\frac{dT}{dr}\right)=\int 0dr$$

$$r\frac{dT}{dr}=C_1 \;|:\frac{r}{dr} \qquad \Rightarrow \qquad \int dT=\int C_1\frac{dr}{r}$$

The general solution can be obtained: $\boxed{T(r)=C_1\ln r+C_2}$ (2-35)

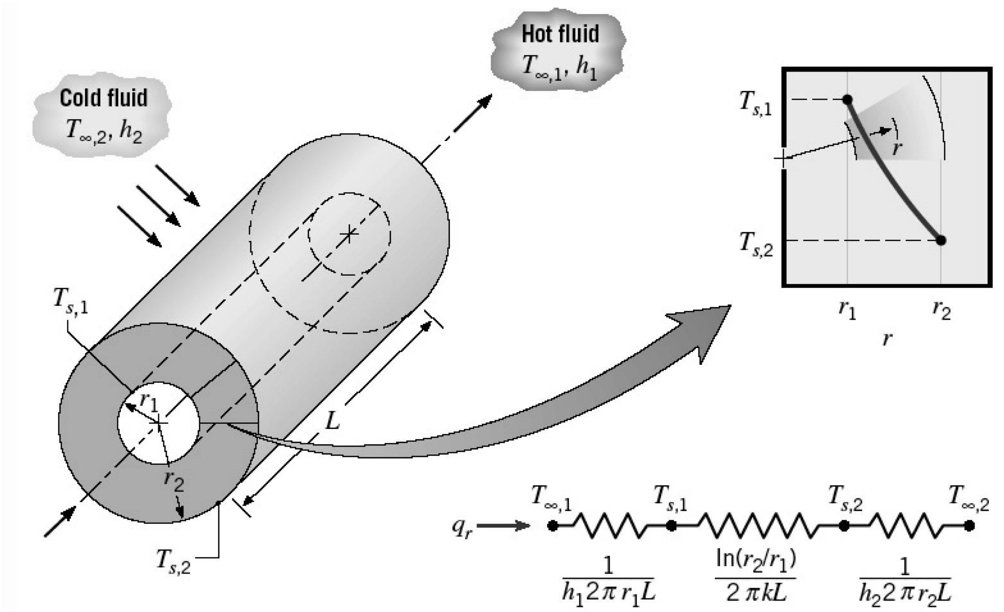


Figure 2-6 *Hollow cylinder: Temperature distribution and equivalent thermal circuit*

Boundary Conditions: for hollow cylinder shown in Figure 2-6

1. $r = r_1 : T = T_{s,1}$ => $T_{s,1} = C_1 \ln r_1 + C_2$
2. $r = r_2 : T = T_{s,2}$ => $T_{s,2} = C_1 \ln r_2 + C_2$

By solving constants as follows

$$\Rightarrow \mathrm{T}_{s,1}\text{-}\mathrm{T}_{s,2}\text{=}\ \mathrm{C}_1\ln r_1 - \mathrm{C}_1\ln r_2 \qquad \Rightarrow C_1 = \frac{T_{s,1}-T_{s,2}}{\ln r_1 - \ln r_2} = \frac{T_{s,1}-T_{s,2}}{\ln\left(\frac{r_1}{r_2}\right)}$$

$$\Rightarrow T_{s,1} = \frac{\left(\mathrm{T}_{s,1}\text{ - }\mathrm{T}_{s,,2}\right)}{\mathrm{In}\left(\frac{\mathrm{r}_1}{\mathrm{r}_2}\right)}\ln r_1 + C_2 \qquad \Rightarrow C_2 = T_{s,1} - \frac{\left(T_{s,1}-T_{s,2}\right)}{\ln\left(\frac{r_1}{r_2}\right)}\ln r_1$$

Solution for temperature distribution can be obtained:

$$T(r) = C_1\ln r + C_2 = \frac{\left(T_{s,1}-T_{s,2}\right)}{\ln\left(\frac{r_1}{r_2}\right)}\ln r + T_{s,1} - \frac{\left(T_{s,1}-T_{s,2}\right)}{\ln\left(\frac{r_1}{r_2}\right)}\ln r_1 \tag{2-36}$$

$$T(r) = \frac{\left(T_{s,1}-T_{s,2}\right)}{\ln\left(\frac{r_1}{r_2}\right)}\left[\ln r \ \ - \ln r_1\right] + T_{s,1}$$

$$T(r) = \frac{\left(T_{s,1}-T_{s,2}\right)\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_1}{r_2}\right)} + T_{s,1} \tag{2-37}$$

Conduction Heat Transfer Rate is given by Fourier’s Law and by substituting solved boundary conditions following form for conduction heat rate is achieved:

$$q_r = -kA\frac{dT}{dr}$$

$$\frac{dT}{dr} = \frac{d\left(T(r)\right)}{dr} = \frac{\left(T_{s,1}-T_{s,2}\right)}{r\ln\left(\frac{r_1}{r_2}\right)}$$

$$\Rightarrow q_r = \frac{-k2\pi rL\left(T_{s,1}-T_{s,2}\right)}{r\ln\frac{r_1}{r_2}} \tag{2-38}$$

Total thermal resistance for hollow cylinder can be defined from equivalent thermal circuit and radial heat transfer can be expressed as

$$R_{tot} = \frac{1}{A_1h_1} + \frac{\ln\left(r_2/r_1\right)}{2\pi Lk} + \frac{1}{A_2h_2} = \frac{1}{\pi D_1Lh_1} + \frac{\ln\left(D_2/D_1\right)}{2\pi Lk} + \frac{1}{\pi D_2Lh_2} \tag{2-39}$$

$$q_r = \frac{T_{\infty,1}-T_{\infty,2}}{R_{tot}}\ . \tag{2-40}$$

2.4.5 Conduction with Internal Heat Source

Let us consider steady-state, one-dimensional conduction in plane wall (Figure 2-7) with constant thermal conductivity having appropriate form of heat equation as

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \tag{2-41}$$

The general solution of heat equation is:

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \tag{2-42}$$

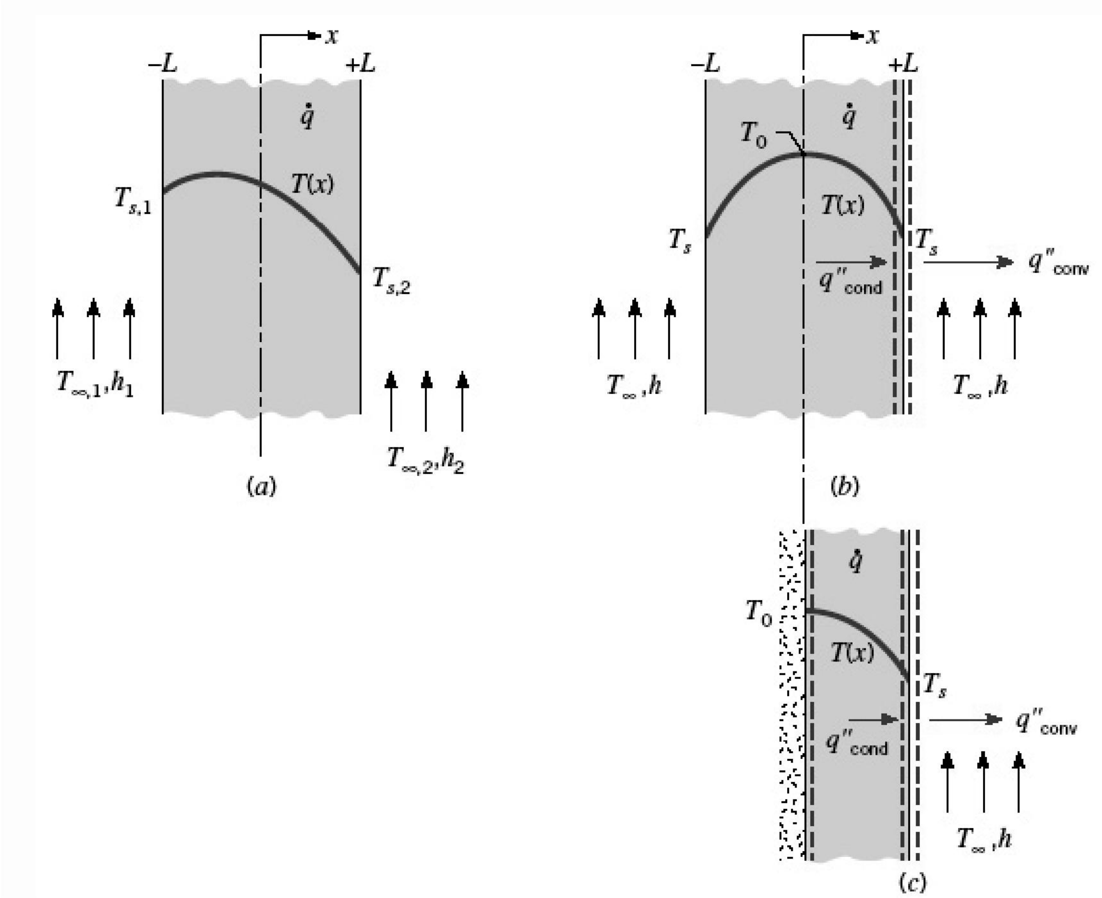


Figure 2-7 *Conduction in plane wall with uniform heat generation - Boundary conditions: a) asymmetrical b) Symmetrical c) adiabatic surface at mid plane.*

Boundary Conditions

General solution can be further solved for three common boundary conditions (a-c) as follows:

a) Asymmetrical case: known surface temperatures

$T(-L) = T_{s,1}$
$T(L) = T_{s,2}$

By solving integration constants temperature profile can be expressed as

$$T(x) = \frac{\dot{q}L}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{2-43}$$

b) Symmetrical case
Resulting temperature profile can be expressed as

$$T(x) = \frac{\dot{q}L}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s\,, \tag{2-44}$$

With maximum temperature at midline of wall, $x = 0$

$$T(0) = T_0 = \frac{\dot{q}L^2}{2k} + T_s \tag{2-45}$$

c) Adiabatic surface at midline
Symmetric temperature profile (b) results temperature gradient at midline to be *(dT/dx)* = 0, which means zero heat flux across midline. Thus temperature profile equation is same for (b) symmetric and (c) adiabatic surface at midline cases.

Surface temperature for cases (b) and (c) can be solved from energy equation. By considering that all thermal energy generated in wall is transferred to boundary via convection energy equation and surface temperature can be written as

$$\dot{E}_g = \dot{q}V = hA(T_s - T_\infty) \tag{2-46}$$

$$\Rightarrow T_s = T_\infty + \frac{\dot{q}V}{hA}\,. \tag{2-47}$$

2.5 TWO DIMENSIONAL STEADY-STATE CONDUCTION

2.5.1 Heat Diffusion Equation

In two dimensional conduction temperature distribution is characterized by two spatial coordinates T(x,y). Heat flux vector is characterized by two directional components, q_x’’ and q_y’’.

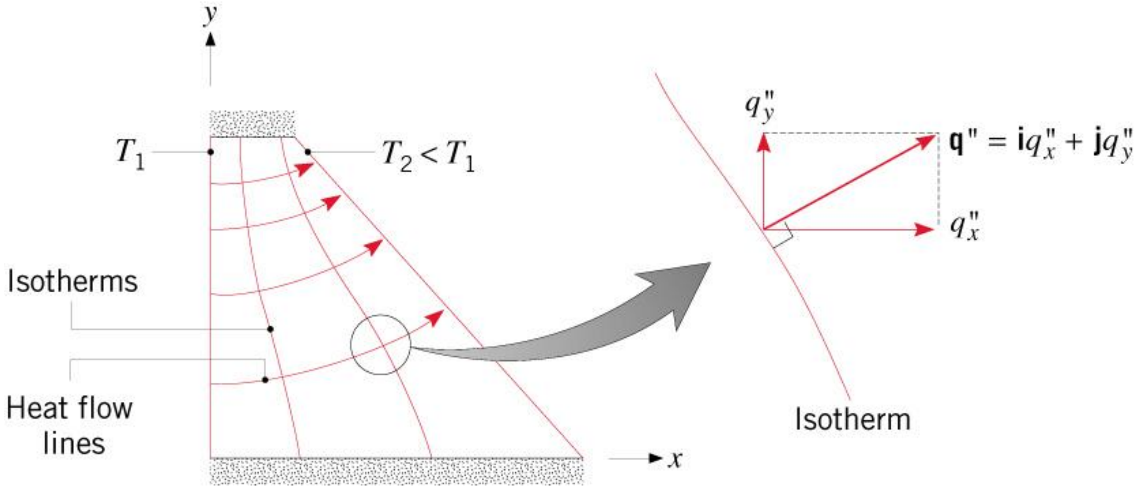


Figure 2-8Isotherms of two dimensional conduction

General form of heat diffusion equation in Cartesian coordinates was

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)-\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Assuming *steady-state*, two-dimensional conduction in a rectangular domain with *constant thermal conductivity* and *heat generation*, the heat equation is

$$\frac{d^2T}{dx^2}+\frac{d^2T}{dy^2}+\frac{\dot{q}}{k}=0 \tag{2-48}$$

Differential heat equation can be treated with different solution methods

1) **Exact/Analytical**: Separation of Variables
    - Limited to simple geometries and boundary conditions
    - ***Conduction shape factor is based on analytical solutions***

2) **Approximate/Graphical**: Flux Plotting
    - *Haven’t been considered here*
    - *limited value for quantitative considerations but a quick aid to establishing physical insights*

3) **Approximate/Numerical**: Finite-Difference/Finite Volume, Finite Element or Boundary Element Method.
    - *Haven't been considered here*
    - *Most useful approach and adaptable to any level of complexity*

2.5.2 Conduction Shape Factor

Two- or three-dimensional heat transfer in a medium bounded by two isothermal surfaces at T_1 and T_2 may be represented in terms of a conduction shape factor S. Heat transfer rate between these isotherms can be described with conduction resistance or with shape factor

$$q = \frac{(T_1 - T_2)}{R_{cond}} = Sk(T_1 - T_2) \tag{2-49}$$

thus, for conduction shape factor following relation can be written

$$S = \frac{1}{kR_{cond}} \tag{2-50}$$

Following tables give shape factors for two and three dimensional isotherm cases.

Table 2-4 Conduction Shape factors in three Coordinates

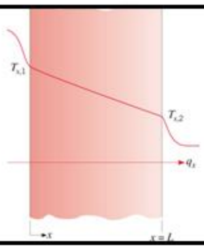
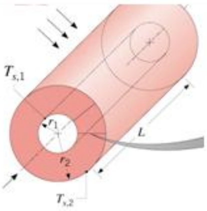
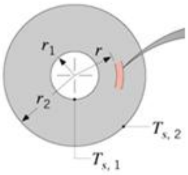
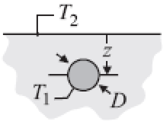
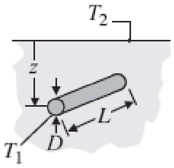
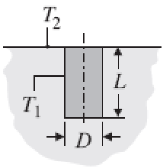
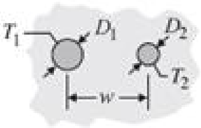
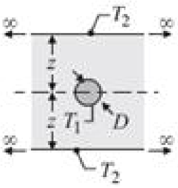
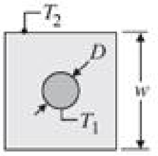
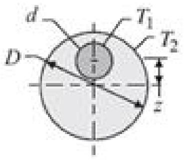
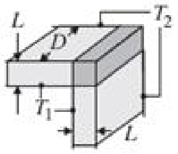
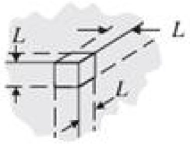
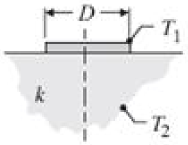
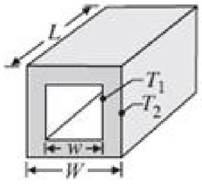
Wall	**Cylinder**	**Sphere**
		
$q = \frac{Ak}{L}(T_1 - T_s)$	$q_r = -\frac{k2\pi L}{\ln {r_1}/{r_2}}(T_1 - T_2)$	$q_r = \frac{4\pi k}{(1/r_1)-(1/r_2)}(T_1 - T_2)$
$S = \frac{A}{L}$	$S = -\frac{2\pi L}{\ln {r_1}/{r_2}}$	$S = \frac{4\pi}{(1/r_1)-(1/r_2)}$

Table 2-5 Two and three dimensional conduction shape factors

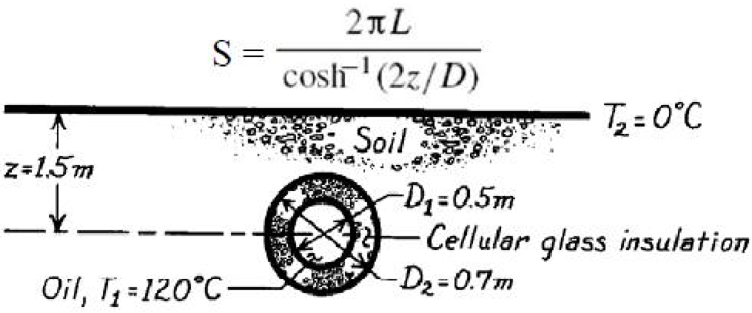
$[q = Sk(T_1 - T_2)]$

System	Schematic	Restrictions	Shape Factor
Case 1: Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Case 2: Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$
		$L \gg D$ $z > 3D/2$	$\dfrac{2\pi L}{\ln(4z/D)}$
Case 3: Vertical cylinder in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\ln(4L/D)}$
Case 4: Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1D_1}\right)}$
Case 5: Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$ $L \gg z$	$\dfrac{2\pi L}{\ln(8z/\pi D)}$
Case 6: Circular cylinder of length L centered in a square solid of equal length		$w > D$ $L \gg w$	$\dfrac{2\pi L}{\ln(1.08w/D)}$
Case 7: Eccentric circular cylinder of length L in a cylinder of equal length		$D > d$ $L \gg D$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)}$

System	Schematic	Restrictions	Shape Factor
Case 8: Conduction through the edge of adjoining walls		$D > L/5$	$0.54D$
Case 9: Conduction through corner of three walls with a temperature difference ΔT_{1-2} across the wall		$L \ll$ length and width of wall	$0.15L$
Case 10: Disk of diameter D and T_1 on a semi-infinite medium of thermal conductivity k and T_2		None	$2D$
Case 11: Square channel of length L		$\frac{W}{w} < 1.4$ $\frac{W}{w} > 1.4$	$\frac{2\pi L}{0.785 \ln(W/w)}$ $\frac{2\pi L}{0.930(W/w) - 0.050}$

Example III: ***Oil pipe under soil – Shape factor***

A long cylindrical pipe with diameter of $D_1 = 0{,}5$ is placed 1,5 m under the ground surface. The pipe is covered a cellular glass insulation with thickness of 0,1 m and is filled with oil with temperature of 120 °C. The temperature of the ground surface is 0 °C. Calculate the heat loss per unit length of the pipe.

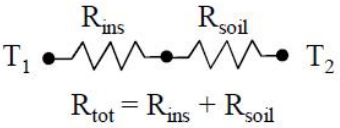


ASSUMPTIONS: (1) Temperature of oil is uniform at any instant, (2) Radiation exchange with the surroundings is negligible, (3) Constant properties

PROPERTIES: Table A.3, Soil (T = 300 K): k = 0,52 W/mK; Table A.3, Soil (T = 365 K): k = 0,069 W/mK;

ANALYSIS:

Conduction circuit:



The conduction resistances:

$$R_{\text{ins}} = \frac{\ln(D_2/D_1)}{2\pi L k_{\text{ins}}} = \frac{\ln(0{,}7\text{ m}/0{,}5\text{ m})}{2\pi L \cdot 0{,}069\text{ W/mK}} = \frac{0{,}776}{L}\text{ mK/W}$$

$$R_{\text{soil}} = \frac{1}{S k_{\text{ins}}} = \frac{\cosh^{-1}(2z/D_2)}{2\pi L k_{\text{soil}}} = \frac{\cosh^{-1}(2 \cdot 1{,}5\text{ m}/0{,}7\text{ m})}{2\pi L \cdot 0{,}52\text{ W/mK}} = \frac{0{,}653}{L}\text{ mK/W}$$

The heat transfer rate per unit length:

$$q = \frac{T_1 - T_2}{R_{\text{tot}}} = \frac{(120 - 0)°\text{C}}{\frac{1}{L}(0{,}776 + 0{,}653)\text{ W/mK}} = 84\text{ W/m} \times L$$

$$q' = 84\text{ W/m}$$

2.6 LEARNING OUTCOMES

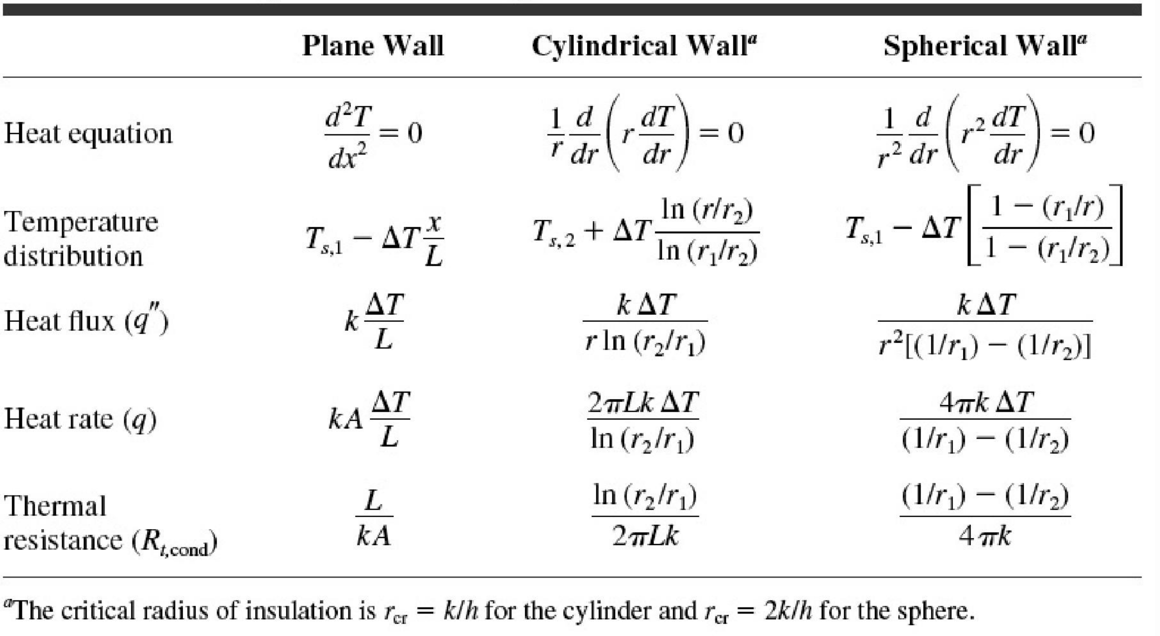
Chapter 2.4 consisted from following main concepts of one-dimensional, steady-state conduction (Table 2-6). Additionally very basic approach to 2-D conduction was presented as charts of shape factor.

Table 2-6 Learning Outcomes: *One-dimensional, steady-state conduction*

Level of Knowledge	**Concept**
	1D Conduction
Apply, *Understand,* *describe*	• *Fourier's Law* • *Thermal conductivity – temperature dependency* • *Heat equation solutions in different coordinates (Summarized in Table 2-7)* • *Circuit Analogy & heat resistances* • *1D conduction with internal heat sources*
	2D Conduction
Apply, *Understand,* *describe*	• *Shape factor*

Table 2-7 summarizes general heat equation solutions for one-dimensional, steady-state conduction heat transfer in different coordinates.[1]

Table 2-7 *1-dimensional, steady-state solutions for Heat Equation*



	Plane Wall	**Cylindrical Wall**[a]	**Spherical Wall**[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\text{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a] The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

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[1] *Note that with internal heat source heat equation has different boundary condition dependent solutions.*

3 FINS

Target of most frequent application of extended surfaces, fin, is to increase surface to enhance convection heat transfer between solid and fluid. Heat transfer from fin involves with combined conduction-convection effects. According to *Newton's law of cooling* convection heat transfer rate is

$$q = hA(T_S - T_\infty) \tag{3-1}$$

h = Convection heat transfer coefficient [W/Km^2]
A = Area perpendicular to direction of heat transfer [m^2]
T_S = Surface temperature [K]
T_∞ = Fluid temperature [K]

Generally, methods for increasing convective heat transfer include

- Increasing of temperature difference (T_s-T_∞)
- Enhancing convection heat transfer coefficient h by increasing fluid velocity
- Increase of heat transfer surface area A $\Rightarrow$ Fins

In practice, diversity of fin designs are only limited by imagination, cost of manufacturing and conditions involved in heat transfer application. Figure 3-1 shows designs of internally and externally finned tubes, and strip plate fins in plate-fin heat exchanger.

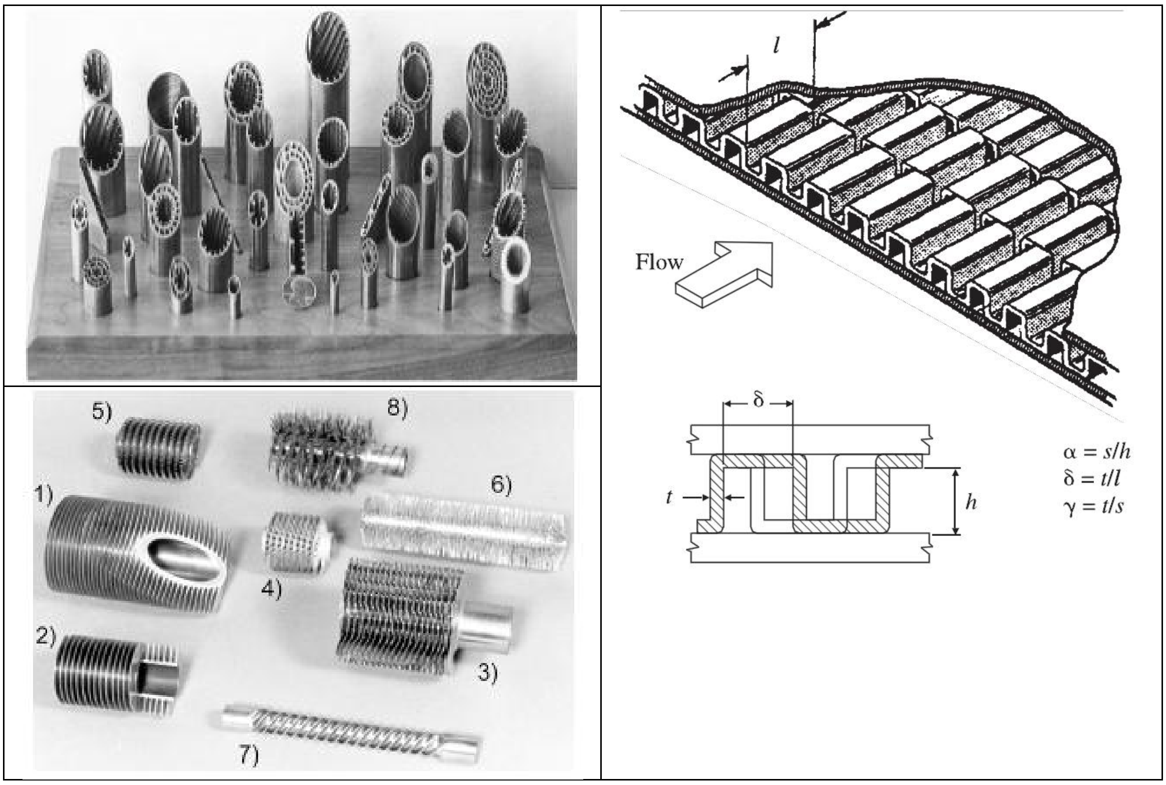


Figure 3-1 *(a) Types of internally and externally finned tubes, (b) rectangular offset strip plate fins (Bejan)*

3.1 FIN THEORY

To obtain general energy equation for extended surfaces energy balance over control volume has to be written. For steady-steady state and zero heat generation conditions with approximation of one-dimensional conduction in fin energy balance (Figure 3-2) can be expressed as

$$q_x = q_{x+dx} + dq_{conv} \tag{3-2}$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$\Rightarrow -\frac{\partial q_x}{\partial x} dx = dq_{conv} \tag{3-3}$$

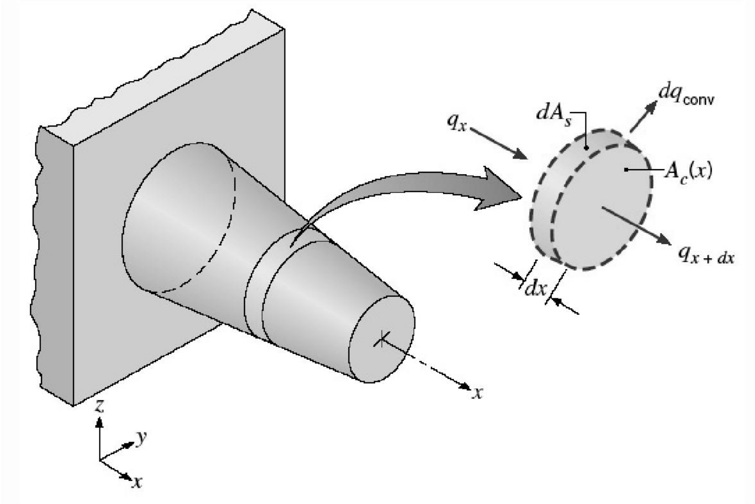


Figure 3-2 *Energy balance for control volume in Fin*

For a general case cross-sectional area is dependent on distance x from base of the fin as: $A_c = A_c(x)$. Conduction heat transfer rate at distance x from base of the fin is

$$q_x = -kA_c \frac{\partial T}{\partial x} \tag{3-4}$$

Convection from fin surface occurs within fin surface temperature $T(x) = T_s(x)$ and convective heat transfer rate from fin surface can be written as

$$dq_{conv} = hdA_s(T_s - T_\infty) \tag{3-5}$$

dA_s surface area of fin control volume

By substituting heat transfer rate equations to energy equation following general form of 1-dimensional conduction-convection in fin is achieved

$$\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) = \frac{h}{k}\left(T - T_\infty\right)\frac{dA_s}{dx} \tag{3-6}$$

$$\Rightarrow \frac{d^2T}{dx^2} + \frac{1}{A_c}\frac{dA_c}{dx}\frac{dT}{dx} - \frac{h}{kA_c}\frac{dA_s}{dx}\left(T - T_\infty\right) = 0 \tag{3-7}$$

3.2 FINS WITH UNIFORM CROSS-SECTION

In case of uniform fin cross-sectional area ($\frac{dA_c}{dx} = 0$), general fin heat equation can be written as

$$\frac{d^2T}{dx^2} - \frac{hP}{KA_c}\left(T - T_\infty\right) = 0 \tag{3-8}$$

P = Perimeter of Fin [m]

Temperature difference of fin and boundary, *excess temperature*, can written with

$$\theta = (T - T_\infty) \tag{3-9}$$

And resulting a simplified form of heat equation as

$$\frac{d\theta^2}{dx^2} - m^2\theta = 0 \tag{3-10}$$

$$m^2 = \frac{hP}{kA_c}$$

General solution for linear, homogenous 2nd order differential heat equation is

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{3-11}$$

Boundary Conditions

- **at the base of fin** (x=0):

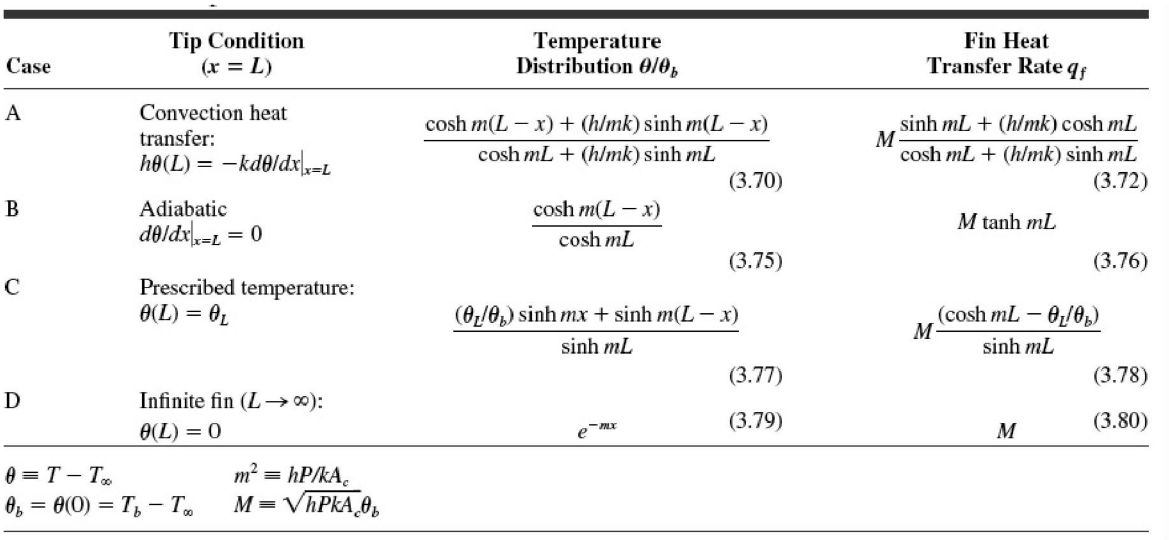
  $$\theta(0) = \theta_b = T_b - T_\infty = C_1 e^{m\cdot 0} + C_2 e^{-m\cdot 0}$$
  $$\Rightarrow \theta_b = C_1 + C_2$$

- **at fin tip (x=L):** four typical boundary conditions are

A) *Convection heat transfer*:	$h\theta(L) = -kd\theta / dx\|_{x=L}$
B) *Adiabatic – no heat transfer*:	$d\theta / dx\|_{x=L} = 0$
C) *Defined temperature*:	$\theta(L) = \theta_L$
D) *Infinite Fin*:	$(L \rightarrow \infty)$, $\theta(L) = 0$

Temperature profile and heat transfer rate solutions are presented in (Table 3-1).

Table 3-1 *Temperature distributions and fin heat transfer rate for uniform cross-section fins*



Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\rvert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\rvert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M \tanh mL$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.79)	M (3.80)

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

3.3 PERFORMANCE

Evaluation of enhance of heat transfer with fins due increased heat transfer area and parallel additional conduction resistance can be done with *fin effectiveness*. Fin effectiveness is defined as *ratio of the fin heat transfer rate to the heat rate that would exist without the fin* and is expressed as

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} \tag{3-12}$$

$A_{c,b}$ = Area of fin at base

θ_b = Temperature difference of fin base and boundary

With approximation of equivalent convection coefficient of finned and smooth non-finned base surfaces, estimation fin effectiveness for infinite fin (tip condition D) can be written as

$$\varepsilon_f = \frac{\sqrt{hPkA_c\theta_b}}{hA_c\,\theta_b} = \sqrt{\frac{Pk}{hA_c}} \tag{3-13}$$

From fin effectiveness approximation for infinite fin some heat transfer enhancement conclusions can be drawn

- Selection of high conductivity material increases fin heat transfer (aluminium, copper, etc.)
- Increase of ratio of perimeter and cross-sectional area by using thin closely spaced fins enhances heat transfer
- Fins are more effective in case of low convection coefficient meaning gas rather than liquid and in case of free convection.
- Generally, the use of fins is not justified unless effectiveness $\varepsilon_f \geq 2$.

Fin resistance, describing also fin performance, is defined by treating base and fluid temperature difference as driving potential for heat transfer as follows

$$R_{t,f} = \frac{\theta_b}{q_f} = \frac{T_b - T_\infty}{q_f} \tag{3-14}$$

Thermal resistance for non-finned surface can be written as

$$R_{t.b} = \frac{\theta_b}{q_b} = \frac{1}{hA_{c,b}} \tag{3-15}$$

In consequence fin effectiveness can be expressed as ratio of thermal resistance as

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}\,. \tag{3-16}$$

Fin efficiency is defined as ratio of real heat transfer from fin and heat transfer from fin having surface entirely at base temperature.

$$\eta_f = \frac{q_f}{q_{\max}} = \frac{q_f}{hA_f\theta_b} \tag{3-17}$$

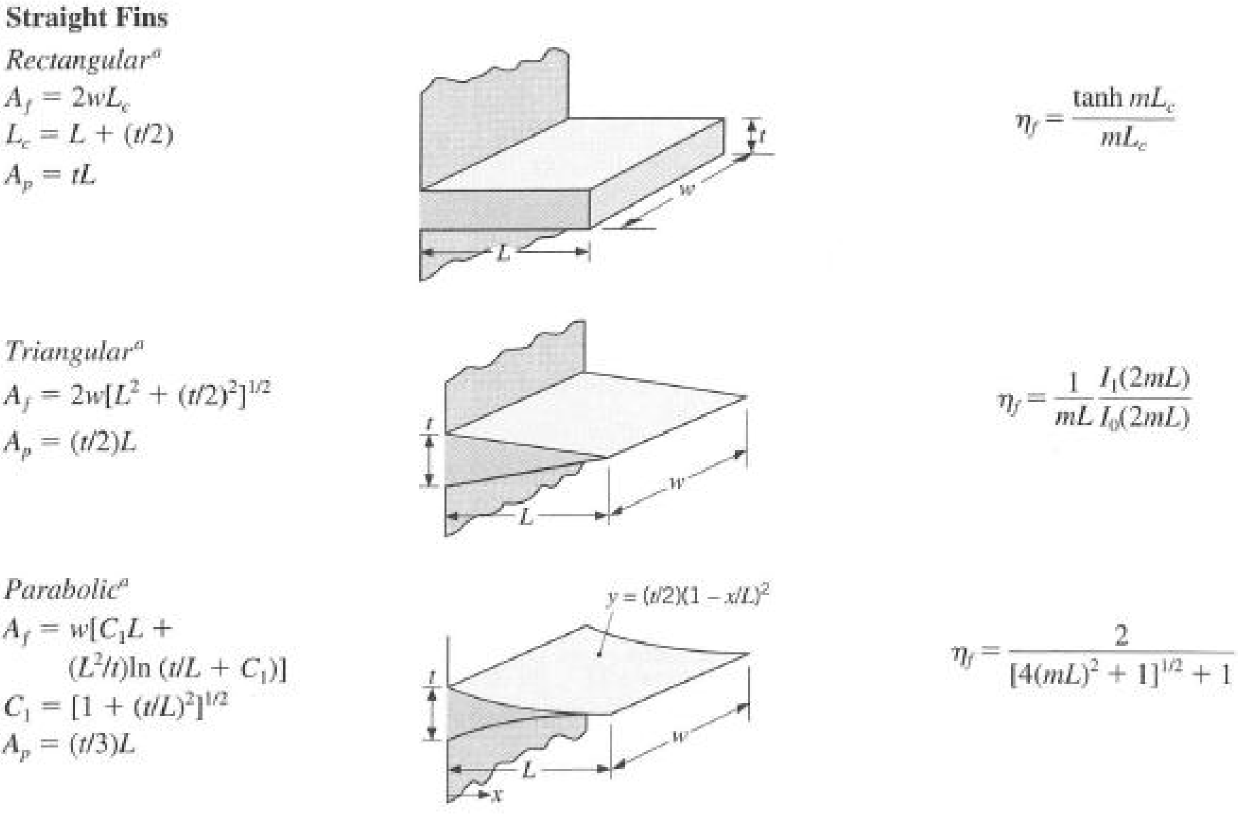


Figure 3-3 *Fin efficiencies of common fin shapes*

Overall Surface Efficiency characterizes thermal performance of combination of an array of fins and the attachment base surface and is defined as

$$\eta_0 = \frac{q_{tot}}{q_{\max}} = \frac{q_{tot}}{hA_{tot}\theta_b} = \frac{1}{\eta_0 hA_{tot}} \tag{3-18}$$

Total area of finned surface is calculated as

$$A_{tot} = NA_f + A_b \tag{3-19}$$

N	*Number of fins*
A_f	*Surface area of a single fin*
A_b	*Surface area of a base*

Total heat transfer rate from finned surface, fins and non-finned base surface, by assuming equivalent convection coefficients for finned and non-finned surfaces can be expressed as

$$q_{tot} = N\eta_f A_f h\theta_b + A_b h\theta_b \tag{3-20}$$

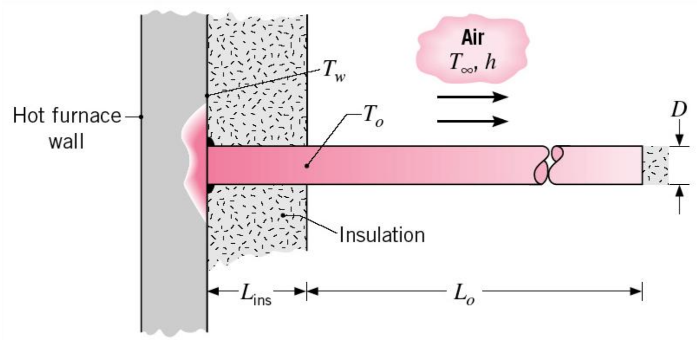
$$\eta_0 = 1 - \frac{NA_f}{A_{tot}}\left(1-\eta_f\right) \tag{3-21}$$

Example IV: ***Rod penetrating to Furnace – Fin theory***

A rod of diameter $D = 25$ mm and thermal conductivity $k = 60$ W/mK protrudes normally from a furnace wall that is at $T_w = 200$ °C and is covered by insulation of thickness $L_{ins} = 200$ mm. The rod is welded to the furnace wall and is used as a hanger for supporting instrumentation cables. To avoid damaging the cables, the temperature of the rod at its exposed surface, T_0, must be maintained below a specified operating limit of $T_{\max} = 100$ °C. The ambient air temperature is $T_\infty = 25$ °C, and the convection coefficient is $h = 15$ W/m^{2}K

a) Derive an expression for the exposed surface temperature T_0 as a function of the prescribed thermal and geometrical parameters. The rod has an exposed length L_0 and its tip is well insulated.

b) Will a rod with $L_0 = 200$ mm meet the specified operating limit? If not, what design parameters would you change? Consider another material, increasing the thickness of the insulation, and increasing the rod length. Also, consider how you might attach the base of the rod to the furnace wall as a means to reduce T_0.

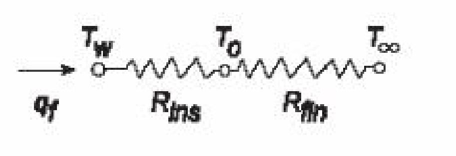


ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_0, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series and behaving as a fin with an adiabatic tip condition, R_{fin},

$$R_{\mathrm{ins}} = \frac{L_{\mathrm{ins}}}{kA_{\mathrm{c}}},$$

$$R_{\mathrm{fin}} = \frac{\theta_{\mathrm{b}}}{q_{\mathrm{f}}} = \frac{1}{\sqrt{hPkA_{\mathrm{c}}}\tanh mL_0}, \text{ (table 3-1)}$$



$$m = \sqrt{\frac{hP}{kA_{\mathrm{c}}}}, \qquad A_{\mathrm{c}} = \frac{\pi D^2}{4}, \qquad P = \pi D$$

From the thermal network,

$$q_{\mathrm{f}} = \frac{T_0 - T_\infty}{R_{\mathrm{fin}}} = \frac{T_{\mathrm{w}} - T_\infty}{R_{\mathrm{ins}} + R_{\mathrm{fin}}}$$

$$\underline{\Rightarrow T_0 = T_\infty + \frac{R_{\mathrm{fin}}}{R_{\mathrm{ins}} + R_{\mathrm{fin}}}(T_{\mathrm{w}} - T_\infty)}$$

(b) Substituting numerical values

$$A_{\mathrm{c}} = \frac{\pi(0{,}025\ \mathrm{m})^2}{4} = 4{,}909 \cdot 10^{-4}\ \mathrm{m}^2$$

$$P = \pi \cdot 0{,}025\ \mathrm{m} = 7{,}854 \cdot 10^{-2}\ \mathrm{m}^2$$

$$m = \sqrt{(15\ \mathrm{W/m^2K} \cdot 7{,}854 \cdot 10^{-2}\ \mathrm{m}^2)/(60\ \mathrm{W/mK} \cdot\ 4{,}909 \cdot 10^{-4}\ \mathrm{m}^2)} = 6{,}324\ \mathrm{m}^{-1}$$

$$\begin{aligned} hPkA_{\mathrm{c}} &= 15\ \mathrm{W/m^2K} \cdot 7{,}854 \cdot 10^{-2}\ \mathrm{m}^2 \cdot 60\ \mathrm{W/mK} \cdot\ 4{,}909 \cdot 10^{-4}\ \mathrm{m}^2 \\ &= 0{,}0347\ \mathrm{W^2/K^2} \end{aligned}$$

$$R_{\mathrm{ins}} = \frac{0{,}200\ \mathrm{m}}{60\ \mathrm{W/mK} \cdot 4{,}909 \cdot 10^{-4}\ \mathrm{m}^2} = 6{,}790\ \mathrm{K/W}$$

$$R_{\mathrm{fin}} = \frac{1}{\sqrt{0{,}0347\ \mathrm{W^2/K^2}} \cdot \sinh(6{,}324\ \mathrm{m}^{-1} \cdot 0{,}2\ \mathrm{m})} = 6{,}298\ \mathrm{K/W}$$

$$T_0 = 25\ ^\circ\mathrm{C} + \frac{6{,}298}{6{,}790 + 6{,}298}(200 - 25)\ ^\circ\mathrm{C}$$

$$\Rightarrow T_0 = 109\ ^\circ\mathrm{C} > T_{\max}$$

Design changes to reach the correct value for T_0:

1) *Decreasing A_c by using a tube instead of a rod.*
2) *Decreasing the thermal conductivity by changing rod's material (stainless steel, for example).*
3) *Introducing contact resistance between the wall and the rod by "tack welding"*
4) *Increasing the length of fin*
5) *Increasing the insulation thickness*
6) *Increasing the convection coefficient with air handling unit.*

3.4 LEARNING OUTCOMES

Besides understanding principles of fin theory three important concepts where described in *Fins Chapter* as stated in Table 3-2.

Table 3-2 *Learning Outcomes: Fins*

Level of Knowledge	**Concept**
Understand, *Describe*	• 1-dimensional conduction-convection characteristics of fin and related formulation of fin equation
Apply, *Understand,* *describe*	• Fin equation solutions for constant diameter fins: *temperatures and heat transfer rates* • Single fin performance: *effectiveness, resistance and efficiency* • Finned surface: *efficiency and heat transfer rates*

4 UNSTEADY-STATE CONDUCTION

Unsteady, or transient, heat transfer phenomenon arises commonly with changed boundary conditions. This chapter introduces solution methods for time dependent conduction problems.

4.1 THE LUMPED CAPACITANCE METHOD

For transient heat transfer processes, where ***temperature gradients in solid are negligible***, a simple approached called *Lumped Capacitance Method* can be utilized.

Let us consider a quenching of a hot metal forging that is initially at uniform temperature T_i and is cooled by immersing it into a pool containing liquid of lower temperature T_∞. Time dependent cooling of metal forging occurs due convection heat transfer and decreasing temperature difference of metal and liquid. In Lumped capacitance method spatially uniform temperature profile change of forging is assumed, which practically acceptable for solids having high thermal conductivity compared to convection or overall heat transfer coefficient between solid and boundary.

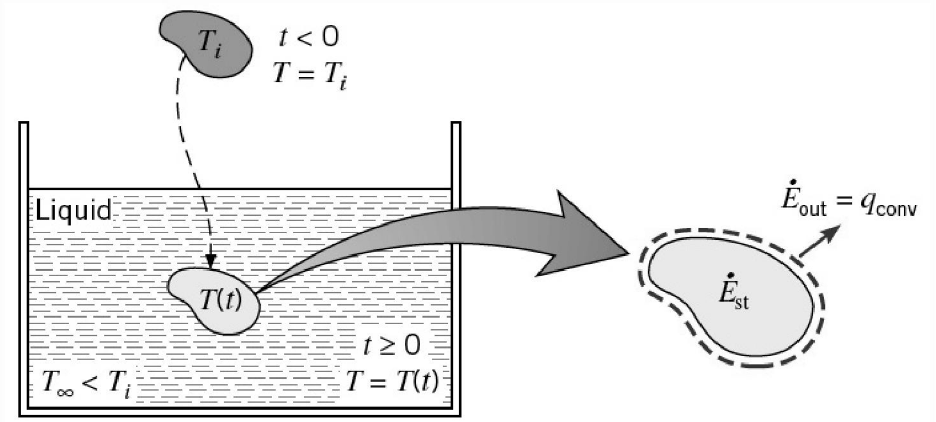


Figure 4-1 *Cooling of a hot metal forging*

With assumption of non-existing temperature gradient, Fourier's law of heat conduction is not available for solution. Energy equation over metal control volume can be expressed as

$$\dot{E}_{st} = -q_{conv} \tag{4-1}$$

$$\rho V c_p \frac{dT}{dt} = -hA_s(T - T_\infty)\,. \tag{4-2}$$

By notating temperature difference as $\theta = (T\text{-}T_\infty)$ with assumption of constant boundary temperature T_∞

$$\text{temperature gradient} \qquad \frac{dT}{dt} = \frac{d\theta}{dt} \tag{4-3}$$

energy equation $$\frac{\rho V c_p}{hA_s}\frac{d\theta}{dt} = -\theta \,. \qquad (4\text{-}4)$$

Integral solution with initial conditions at start time t = 0 and initial temperature $T(0) = T_\mathrm{i}$ for energy equation is

$$\frac{\rho V c_p}{hA_s}\ln\frac{\theta_i}{\theta} = t \qquad (4\text{-}5)$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p}t} \,. \qquad (4\text{-}6)$$

***Thermal Time Constant*:** By defining thermal time constant as follows

$$\tau_t = \left(\frac{1}{A_s h}\right)(\rho V c_p) = R_t C_t \qquad (4\text{-}7)$$

R_t	Convection heat transfer resistance
C_t	Lumped thermal capacitance

Time dependent solution for temperature can be expressed as

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau_t} \qquad (4\text{-}8)$$

Time dependent temperature behaviour of metal is characterized by exponentially decaying temperature difference between solid and liquid. Increase in either convection heat transfer resistance R_t or lumped thermal capacitance C_t will decrease temperature response of solids. (Figure 4-2)

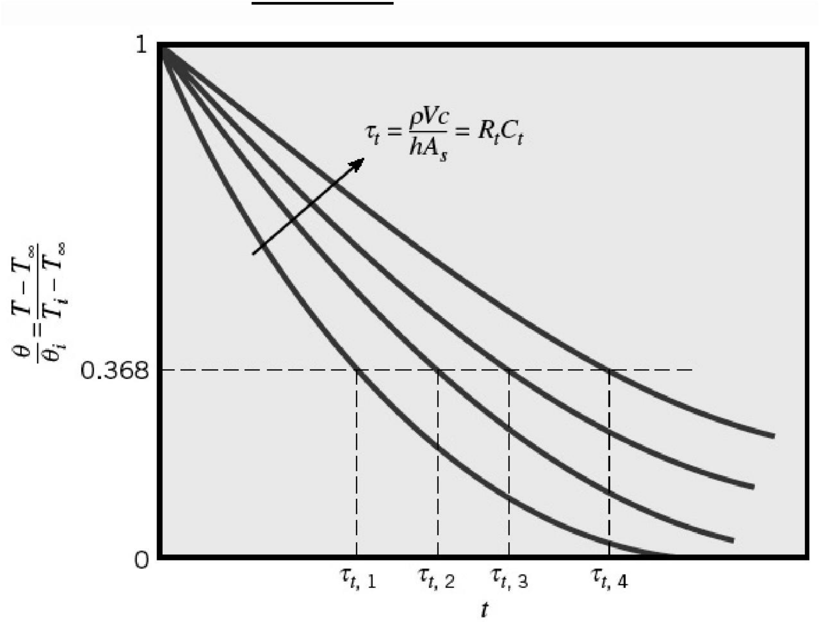


Figure 4-2 *Transient temperature response of lumped capacitance solids with different thermal time constants.*

Total heat transferred till time t can be solved by integrating convection heat transfer rate over time with substitution of time dependent temperature solution as follows

$$Q = \int_0^t q dt = hA_s \int_0^t \theta dt = (\rho V c_p)\theta_i \left(1 - \exp(-\frac{t}{\tau_t}) \right). \qquad (4\text{-}9)$$

4.1.1 Validity Criteria

Suitable and convenient validity criteria for use of simplest transient heat transfer problem solving method refer to assumption of uniform temperature profile of solid. Considering a steady-state conduction in plane wall (Figure 4-3) with constant temperature $T_{s,1}$ in other wall and other cooled by convection with fluid of temperature $T_\infty < T_{s,1}$, surface energy balance can be written as

$$\frac{kA}{L}(T_{s,1} - T_{s,2}) = hA(T_{s,2} - T_\infty) \qquad (4\text{-}10)$$

By restating energy equation, *Biot number* can be written

$$Bi \equiv \frac{hL}{k} = \frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_\infty} = \frac{(L/kA)}{(1/hA)} = \frac{R_{\text{cond}}}{R_{\text{conv}}}. \qquad (4\text{-}11)$$

Biot number is a dimensionless parameter describing the ratio of conduction in solid and convection resistances, and the ratio of temperature drop in solids to temperature difference with solid surface and liquid boundary. Figure 4-3 shows temperature profiles of solid plane wall with different Biot numbers and it can noted that with decreasing Biot number uniform temperature profile is approached.

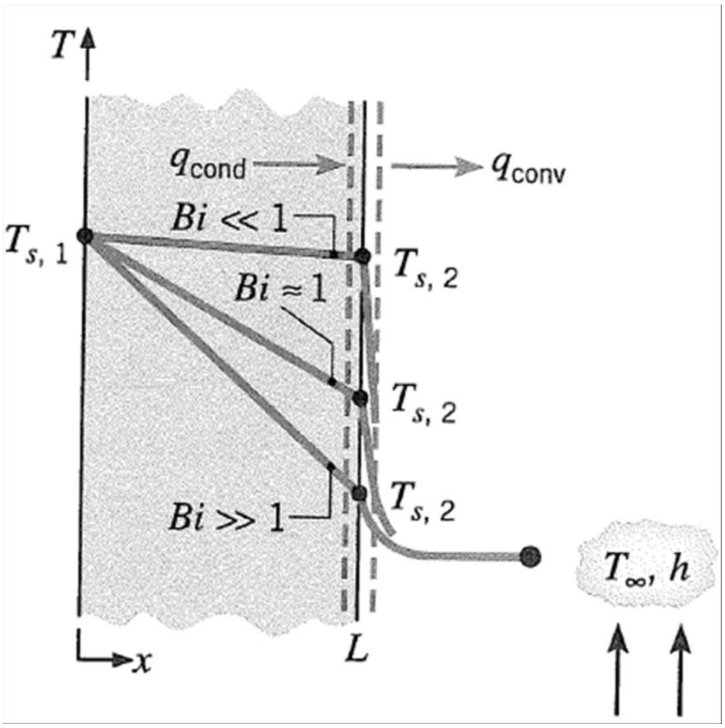


Figure 4-3 *Steady-state temperature distributions of plane wall with different Biot numbers*

Generally, Biot number $Bi << 1$ is considered as criteria for uniform temperature profile in solid during transient heat transfer. This condition can be interpreted as *“The conduction resistance within solid is much less compared to convection resistance across fluid boundary layer.”* Figure 4-4 presents strong effect of Biot number to solid temperature profile behavior with in a plane wall cooled symmetrically with convection on both surfaces.

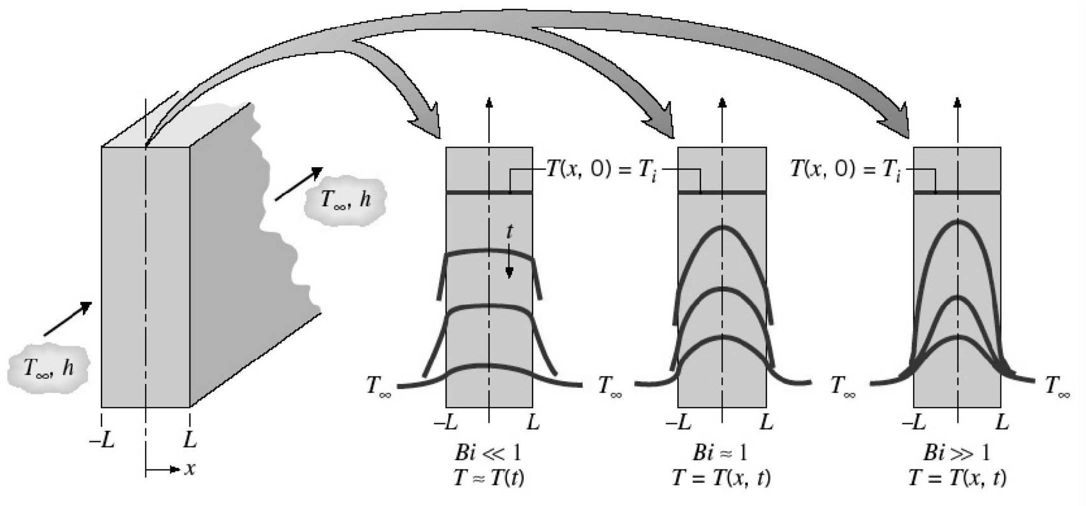


Figure 4-4 *Transient temperature distribution for different Bi Numbers in a plain wall*

Characteristic length is defined as ratio of solid volume to surface area and for the use of thermal calculations of complicated shapes

$$L_C = \frac{V}{A_S} \tag{4-12}$$

Fourier number, known also as *a dimensionless time*, characterizes together with Biot number transient conduction phenomenon. Fourier number is defined as

$$Fo = \frac{\alpha t}{L_C^2}. \tag{4-13}$$

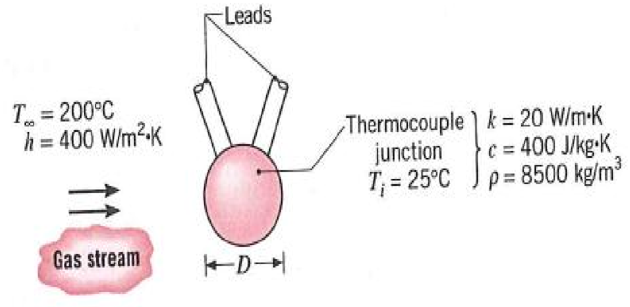
With Fourier and Biot numbers time dependent temperature equation can formulated as follows

$$Bi \cdot Fo = \frac{hL_C}{k}\frac{\alpha t}{L_C^2} = \frac{hA_S t}{\rho V c} \tag{4-14}$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p}t} = e^{-Bi \cdot Fo} \tag{4-15}$$

Example V: *A Thermocouple junction – Lumped Capacitance Method*

A thermocouple junction, which may be approximated as sphere, is to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is h = 400 W/m^{2}K and the junction thermophysical properties are k = 20 W/mK, c = 400 J/kgK and ρ = 8500 kg/m^3. Determine the junction diameter needed for the thermocouple to have a time constant of 1 s. If the junction is at 25 °C and is placed in a gas stream that is at 200 °C, how long will it take for the junction to reach 199 °C.



ASSUMPTIONS:

1. Temperature of junction is uniform at any instant
2. Radiation exchange with the surroundings is negligible
3. Losses by conduction through the leads are negligible
4. Constant properties

ANALYSIS:

1. From equation 4-7 and the fact that

$$A_s = \pi D^2 \qquad\qquad V = \pi D^3/6$$

for a sphere, it follows that

$$\tau_t = \frac{1}{h\pi D^2} \times \frac{\rho \pi D^3}{6} c$$

$$\Rightarrow D = \frac{6h\tau_t}{\rho c} = \frac{6 \cdot 400\ \mathrm{W/m^2K} \cdot 1\mathrm{s}}{8500 \frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 400\mathrm{J/kgK}} = 7{,}06 \cdot 10^{-4}\ \mathrm{m}$$

With $L_c = r_0/3$ it then follows from equation 4-11 that

$$Bi = \frac{h(r_o/3)}{k} = \frac{400\ \mathrm{W/m^2K} \cdot 3{,}53 \cdot 10^{-4}\ \mathrm{m}}{3 \cdot 20\ \mathrm{W/mK}} = 2{,}35 \cdot 10^{-3} \ll 1$$

Lumped capacitance method may be used to an excellent approximation.

2. From equation 4-5 the time required for the junction to reach T = 199 °C

$$t = \frac{\rho(\pi D^3/6)c}{h(\pi D^2)} \ln \frac{T_i - T_\infty}{\mathrm{T} - T_\infty} = \frac{\rho D c}{6h} \ln \frac{T_i - T_\infty}{\mathrm{T} - T_\infty}$$

$$t = \frac{8500 \frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 7{,}06 \cdot 10^{-4}\ \mathrm{m} \cdot 400\mathrm{J/kgK}}{6 \cdot 400\ \mathrm{W/m^2K}} \ln \frac{(25-200)\mathrm{K}}{(199-200)\mathrm{K}}$$

$$t = 5{,}2\ \mathrm{s} \approx 5\tau_t$$

4.2 1D TRANSIENT CONDUCTION WITH SPATIAL EFFECTS

Alternative approaches for lumped capacitance method have to be used with conduction problems having not negligible temperature profiles of solids. *Analytical solutions* can be derived for simple geometries from fundamental heat transfer equations and as a basis *general heat diffusion equation* can be written in one-dimensional form with no heat generation as

$$\frac{dT}{dt} = \alpha \nabla^2 T = \alpha \frac{\partial^2 T}{\partial x^2} \tag{4-16}$$

4.2.1 Dimensionless Governing Equations

By forming dimensionless forms of numbers and variables a total number of dependent variables in governing equations can be reduced. For the use in transient heat transfer problems a *dimensionless temperature difference* is defined as

$$\theta^* \equiv \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} \qquad , 0 \le \theta^* \le 1 \tag{4-17}$$

$T(x,t)$	Transient temperature profile
$T_i(x,0)$	Initial temperature profile

Similarly a *dimensionless spatial coordinate* for plane wall can be defined as

$$x^* \equiv \frac{x}{L} \qquad , -1 \le x^* \le 1 \tag{4-18}$$

L	Half thickness of the wall

Temperature, spatial and time derivates can be written in dimensionless form as

$$\begin{aligned} &\partial^2 T = (T_i - T_\infty)\partial^2 \theta^* \quad , \partial T = (T_i - T_\infty)\partial \theta^* \\ &\partial x^2 = L^2 \partial x^{*2} \\ &\partial t = \frac{L^2}{\alpha} \partial Fo \end{aligned} \tag{4-19}$$

<u>Dimensionless heat equation</u> can then be written as

$$\frac{(T_i - T_\infty)\partial^2 \theta^*}{L^2 \partial x^{*2}} = \frac{1}{\alpha}(T_i - T_\infty)\frac{\partial \theta^*}{\frac{L^2}{\alpha}\partial Fo} \tag{4-20}$$

$$\Rightarrow \frac{\partial^2 \theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial Fo} \tag{4-21}$$

4.2.2 Plain Wall

Let us consider a plain wall having uniform temperature profile, which is then suddenly exposed to convection conditions.

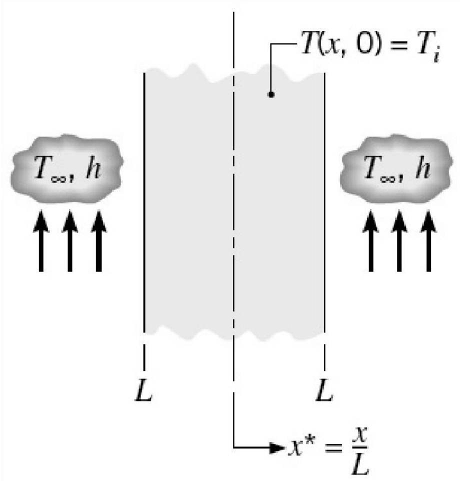


Figure 4-5 *Plain wall under convection conditions*

Initial Conditions

$\theta^*(x^*, Fo = 0) = 1$ *Dimensionless temperature difference in the beginning: wall temperature profile is initial temperature profile.*

Boundary Conditions

$\left.\frac{\partial\theta^*}{\partial x^*}\right|_{x^*=0} = 0$ *Heat transfer rate at midline of symmetrical wall is zero*

$\left.\frac{\partial\theta^*}{\partial x^*}\right|_{x^*=1} = -Bi\cdot\theta(x^* = 1, Fo)$ *Conduction heat transfer rate is equal to convection at wall.*

Solution of dimensionless transient heat equation can now be expressed with 3 dimensionless variables as $\theta^* = f(x^*, Fo, Bi)$, when ordinary form would require representation as $T = (x, t, T_i, T_\infty, L, k, \alpha, h)$.

Exact solution of dimensionless heat equation is

$$\theta^* = \sum_{n=1}^{\infty} C_n e^{-\zeta_n^2 Fo} * \cos(\zeta_n x^*)$$

$$Coefficient:\ C_n = \frac{4\sin\zeta_n}{2\zeta_n + \sin(2\zeta_n)} \quad (4\text{-}22)$$

$$Eigenvalue:\ \zeta_n \tan\zeta_n = Bi$$

Approximate Solution

Infinite series solution can be approximated with first term, if Fourier number > 0.2:

$$\boxed{\begin{array}{l} \theta^* = C_1 e^{-\zeta_1^2 Fo} * \cos\left(\zeta_1 x^*\right) \\ \\ Coefficient: \quad C_1 = \dfrac{4\sin\zeta_1}{2\zeta_1 + \sin\left(2\zeta_1\right)} \\ \\ Eigenvalue,\ 0 \le \zeta \le \pi: \quad \zeta_1 \tan\zeta_1 = Bi \end{array}} \tag{4-23}$$

Transient temperature profile can be solved from by formatting solution for temperature at midline of wall ($x^*=0$) as

$$\theta_0^* = \frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 e^{-\zeta_1^2 Fo} \tag{4-24}$$

$$\Rightarrow$$

$$\theta^* = \theta_0^* \cos\left(\zeta_1 x^*\right) \qquad \textit{(cos0=1)}$$

Heat transfer rate from other surface to boundary can be written based on energy balance as

$$Q = -\Delta E_{st} = \rho V c_p T_i - \int_v \rho c_p T(x,t) dV$$

$$\Rightarrow \tag{4-25}$$

$$Q = \rho V c_p \left(T_i - T_\infty\right)\left[1 - \frac{C_1}{L} e^{-\zeta_1^2 Fo} \frac{L}{\zeta_1} \sin\zeta_1\right] = \rho V c_p \left(T_i - T_\infty\right)\left[1 - \frac{\theta_0^* \sin\zeta_1}{\zeta_1}\right]$$

Maximum heat energy that could be transferred is

$$Q_0 = \rho V c p\left(T_i - T_\infty\right) \tag{4-26}$$

And relationship of energy transfer can be comprised as

$$\frac{Q}{Q_0} = 1 - \frac{\sin\zeta_1}{\zeta_1} Q_0^*. \tag{4-27}$$

4.2.3 Cylinder

General heat diffusion equation for cylindrical coordinates one-dimensional conditions with no heat generation are written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4-28}$$

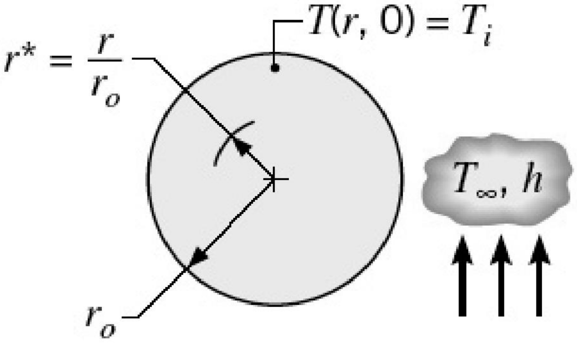


Figure 4-6 *Infinite Cylinder or Sphere*

One-dimensional conduction, responding infinite length, in case of cylinder is reasonable approximation, if length to radius ratio $L/r_0 \geq 10$.

Initial Conditions: Uniform initial temperature: $t = 0$, $T(r,t=0)=T_i$

Boundary Conditions: At surface conduction rate equals to convection heat transfer rate and at midpoint conduction rate is zero

$$r = r_0, \quad -k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = h\left(T\left(r = r_0, t\right) - T_\infty\right)$$

$$r = 0, \quad -k\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$$

Exact solution of dimensionless heat equation

$$\boxed{\begin{aligned}\theta^* &= \frac{T-T_\infty}{T_i-T_\infty} = \sum_{n=1}^{\infty} C_n e^{-\zeta_n^2 Fo} J_o\left(\zeta_n r^*\right)\\ C_n &= \frac{2}{\zeta_n}\cdot\frac{J_1(\zeta_n)}{J_o^2(\zeta_n)+J_1^2(\zeta_n)}\end{aligned}} \tag{4-29}$$

Quantities of Bessel functions of first kind: J_0 and J_1 are presented at ζ_n are positive roots of

$$\zeta_n \frac{J_1(\zeta_n)}{J_o(\zeta_n)} = Bi \tag{4-30}$$

And values are shown in Table 4-1 as function of Biot number. Dimensionless radius, heat transfer coefficient and time are defined as

$$r^* = \frac{r}{r_o}, \quad Bi = \frac{hr_o}{k}, \quad Fo = \frac{\alpha t}{r_o^2} \tag{4-31}$$

Approximate Solution

As in case of plain wall, infinite series solution can be approximated with first term, if Fourier number > 0.2. Relationship of transferred and maximum energy transfer can be expressed as

$$\frac{Q}{Q_0} = 1 - \frac{2\theta_0^*}{\zeta_1} J_1(\zeta_1) \qquad \text{(4-32)}$$

, where temperature of axis (r^*=0, $J_o(0)$=1) is calculated from

$$\theta_0^* = C_1 e^{-\zeta_1^2 Fo} \qquad \text{(4-33)}$$

4.2.4 Sphere

General heat diffusion equation for spherical coordinates under steady-state, one-dimensional conditions with no heat generation are written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{(4-34)}$$

Initial and boundary conditions are same than in case of cylinder:

Initial Conditions: Uniform initial temperature: t = 0, $T(r,t=0)=T_i$

Boundary Conditions: At surface conduction rate equals to convection heat transfer rate and at midpoint conduction rate is zero

$$r = r_0, \quad -k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = h\left(T(r = r_0, t) - T_\infty\right)$$

$$r = 0, \quad -k\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$$

Exact solution of dimensionless heat equation

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = \sum_{n=1}^{\infty} C_n e^{-\zeta_n^2 Fo} \cdot \frac{1}{\zeta_n r^*}\sin(\zeta_n r^*)$$

$$C_n = \frac{4[\sin\zeta_n - \zeta_n \cos\zeta_n]}{2\zeta_n - \sin(2\zeta_n)} \qquad \text{(4-35)}$$

Quantities of Bessel functions of first kind: J_0 and J_1 (Table 4-1), ζ_n are positive roots of

$$1 - \zeta_n \cot\zeta_n = Bi \qquad \text{(4-36)}$$

And values are shown in Table 4-1 as function of Biot number. Dimensionless radius, heat transfer coefficient and time are defined as

$$r^* = \frac{r}{r_o}, Bi = \frac{hr_o}{k}, Fo = \frac{\alpha t}{r_o^2} \tag{4-37}$$

Approximate Solution

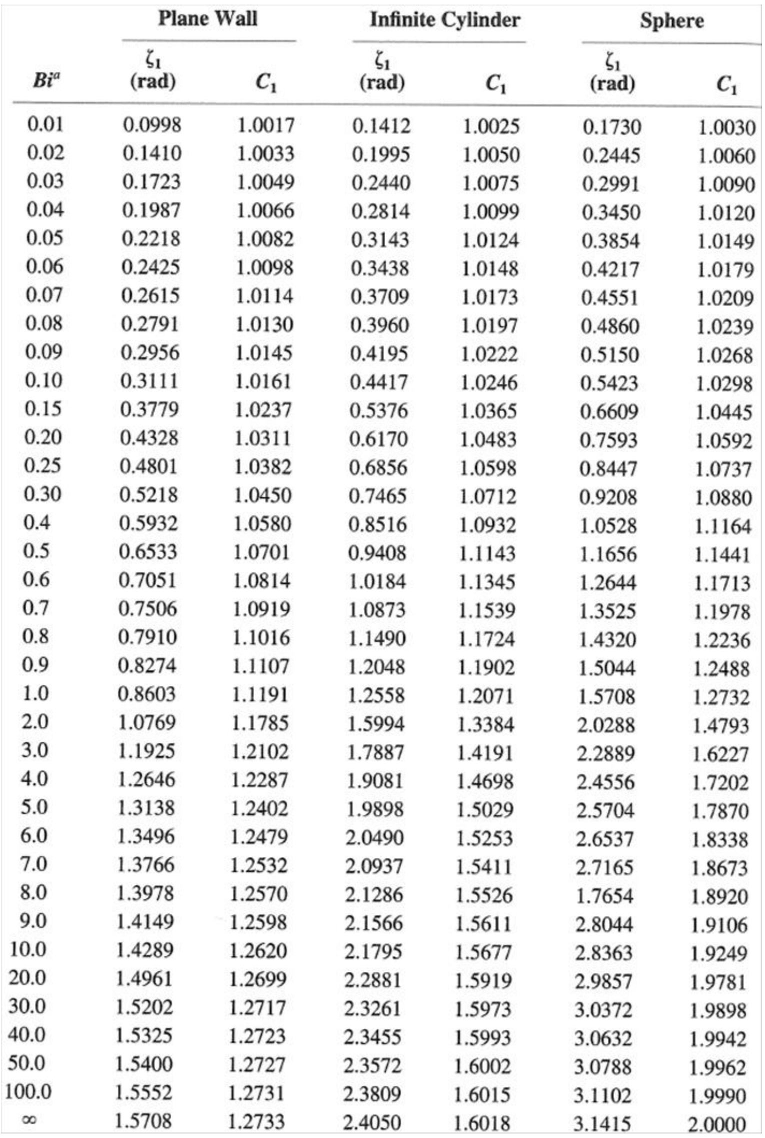
As in case of plain wall, infinite series solution can be approximated with first term, if Fourier number > 0.2. Relationship of transferred and maximum energy transfer can be expressed as

$$\frac{Q}{Q_0} = 1 - \frac{3\theta_0^*}{\zeta_1^3}\left(\sin\zeta_1 - \zeta_1\cos\zeta_1\right) \tag{4-38}$$

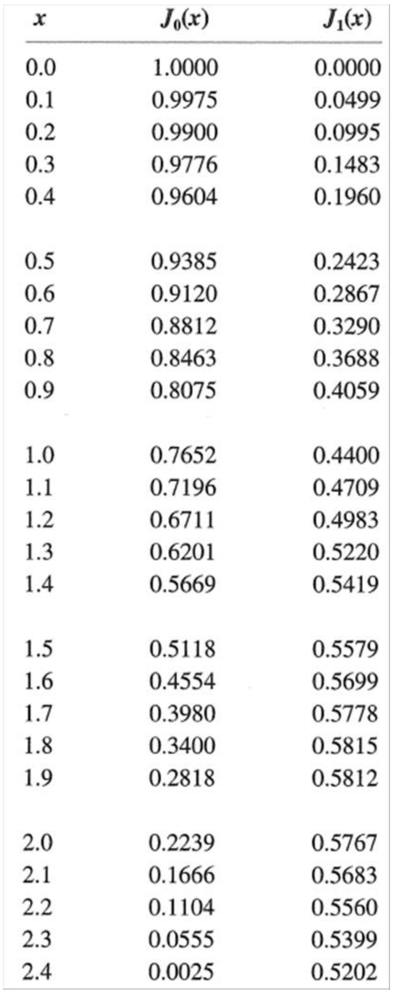
, where temperature of centre ($r^*=0$, $J_o(0)=1$) is calculated similarly to cylinder as

$$\theta_0^* = C_1 e^{-\zeta_1^2 Fo} \tag{4-39}$$

Table 4-1 *(a) One term approximation coefficients to series solution for transient 1D conduction and (b) Bessel Functions of the First Kind*



	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000



x	$J_0(x)$	$J_1(x)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202

4.2.5 Semi-Infinite Solid

By definition, Semi-infinite solid means a solid that extends to infinity in all but one direction. It is a practical approximation for heat transfer problems. *General heat diffusion equation* for semi-infinite solid can be written under steady-state, one-dimensional conditions with no heat generation as

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4-40}$$

Initial Conditions: Uniform initial temperature $T(x,t=0)=T_i$

Boundary Conditions: semi-infinity approximation: temperature at infinite distance from surface approaches initial uniform temperature $T(x\rightarrow \infty, t) = T_i$

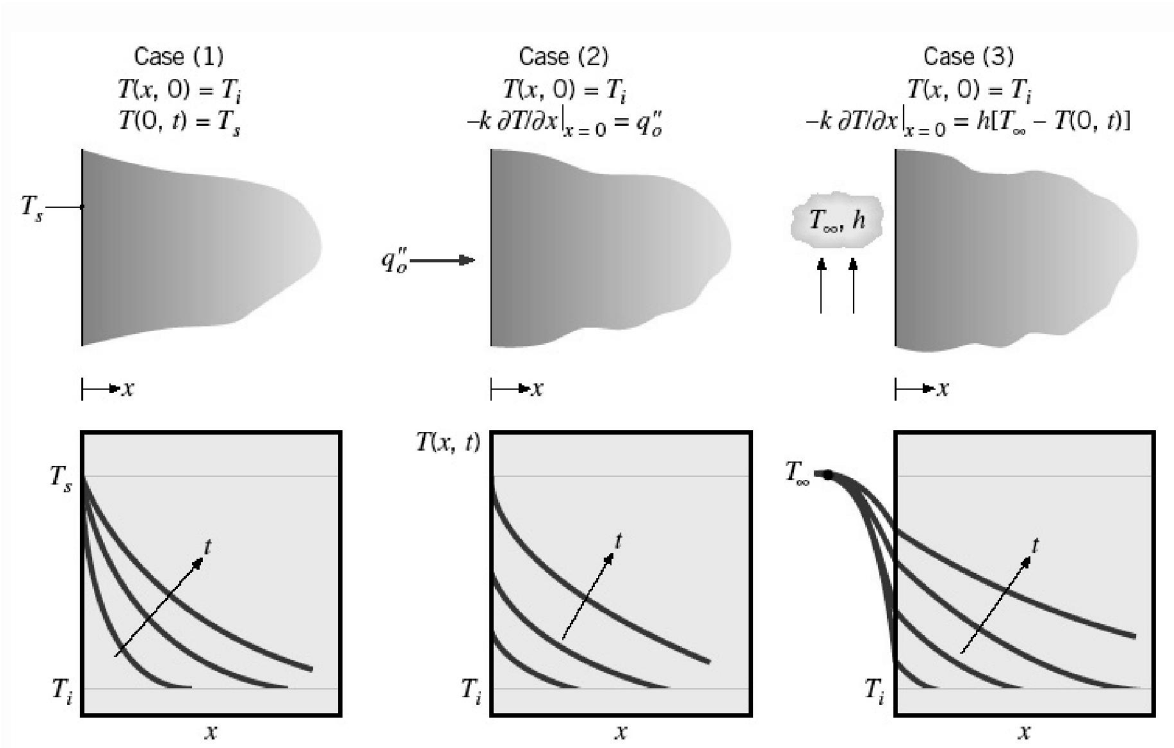


Figure 4-7 *Transient temperature profiles in semi-infinite solids with 3 surface conditions: constant temperature and heat flux, and surface convection.*

Analytical solutions of heat equation can be derived for three surface condition cases presented at Figure 4-7:

Case 1: Constant surface Temperature $T(x=0, t) = T_s$

$$\frac{T(x,t)-T_s}{T_i-T_s} = \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{4-41}$$

$$q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}} \tag{4-42}$$

Case 2: Constant surface heat flux $\left.-k\frac{\partial T}{\partial x}\right|_{x=0} = q_0''$

$$T(x,t)-T_i = \frac{2q_0''\sqrt{\alpha t/\pi}}{k}\cdot\exp\left(-\frac{x^2}{4\alpha t}\right)-\frac{q_0''x}{k}\cdot\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{4-43}$$

Case 3: Surface Convection $\left.-k\frac{\partial T}{\partial x}\right|_{x=0} = h\left(T(x=0)-T_\infty\right)$

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\cdot\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right) \tag{4-44}$$

Definition and values of Gaussian error function, *erf*, and complementary error function is presented at Table 4-2.

Table 4-2 *Gaussian Error functions*

w	erf w	w	erf w	w	erf w
0.00	0.00000	0.36	0.38933	1.04	0.85865
0.02	0.02256	0.38	0.40901	1.08	0.87333
0.04	0.04511	0.40	0.42839	1.12	0.88679
0.06	0.06762	0.44	0.46622	1.16	0.89910
0.08	0.09008	0.48	0.50275	1.20	0.91031
0.10	0.11246	0.52	0.53790	1.30	0.93401
0.12	0.13476	0.56	0.57162	1.40	0.95228
0.14	0.15695	0.60	0.60386	1.50	0.96611
0.16	0.17901	0.64	0.63459	1.60	0.97635
0.18	0.20094	0.68	0.66378	1.70	0.98379
0.20	0.22270	0.72	0.69143	1.80	0.98909
0.22	0.24430	0.76	0.71754	1.90	0.99279
0.24	0.26570	0.80	0.74210	2.00	0.99532
0.26	0.28690	0.84	0.76514	2.20	0.99814
0.28	0.30788	0.88	0.78669	2.40	0.99931
0.30	0.32863	0.92	0.80677	2.60	0.99976
0.32	0.34913	0.96	0.82542	2.80	0.99992
0.34	0.36936	1.00	0.84270	3.00	0.99998

[1] The Gaussian error function is defined as

$$\operatorname{erf} w = \frac{2}{\sqrt{\pi}} \int_0^w e^{-v^2}\, dv$$

The complementary error function is defined as

$$\operatorname{erfc} w \equiv 1 - \operatorname{erf} w$$

4.3 LEARNING OUTCOMES

Table 4-3 Learning Outcomes: *1-dimensional transient heat transfer*

Level of Knowledge	Concept
	Lumped Capacitance
Describe *Understand*	• *Convection – Heat storage* lumped capacitance energy equation
Apply *describe* *understand*	• Lumped capacitance *Time & temperature solutions + validity criteria* • Concepts: *time constant, Bi, Fo, characteristic length*
	Spatial Effects[2]
Describe *Understand*	• Form of heat equation • Concepts: *dimensionless temperature difference*
Apply *describe* *understand*	• One term approximate solutions: *for different geometries and boundary conditions*

---

[2] Not currently included to content of course: *Fundamentals of Heat Transfer*

5 FUNDAMENTALS OF CONVECTION

In principle, convection is energy transfer between a surface and a fluid flowing over the surface. Convection heat transfer is combination of two mechanisms*: random molecular motion (diffusion or conduction)* and *bulk/macroscopic fluid motion (advection)*. There are **two major objectives** in convection heat transfer analysis presented here:

- *Understanding the physical phenomenology associated with boundary layers underlying the convection heat transfer.*
- *Gain the knowhow to solve convection heat transfer problems with relevant knowledge of empirical friction and heat transfer coefficients.*

5.1 CLASSIFICATION OF CONVECTION HEAT TRANSFER

Convection is classified according the nature of the flow:

- Free (Natural) Convection – Flow is induced by *buoyancy forces*, which are caused by *temperature variations formed due to heat transfer* in the fluid.
- Forced Convection – Flow is caused by external means: fan, pump, wind, etc.
  - Convection is classified also as 1) *external* or 2) *interna*l, depending on whether fluid is flowing over a surface or inside a channel.
- Two special convection heat transfer cases associated with phase change between liquid and vapour states of fluid are *boiling* and *condensation*. Both processes may occur under either free or forced convection conditions.

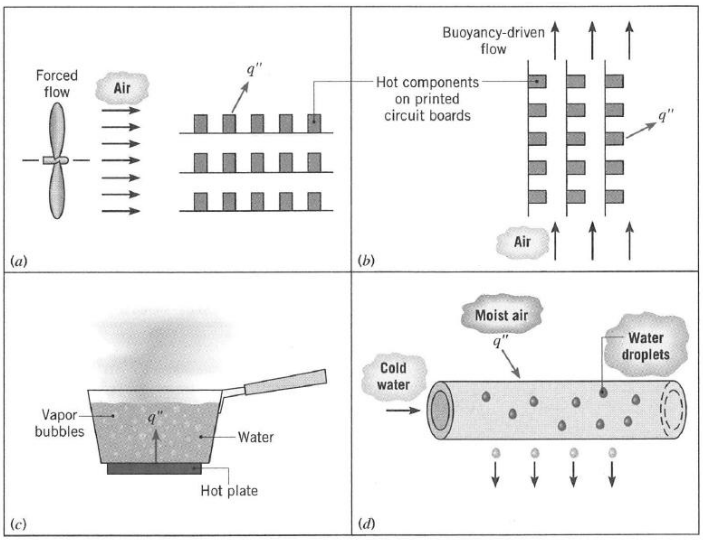


Figure 5-1 Convection heat transfer classification: forced, free, boiling and condensation

5.2 PROBLEM OF CONVECTION

Problem of convection is seen as determination of local or/and average convection coefficients. Local and total heat transfer rates are solved from rate equations, which are dependent on knowledge of local convection coefficients h or average convection coefficients $\bar{h}$.

Newton's Law of Cooling
Convection heat transfer rate is expressed commonly as *Newton's law of cooling*:

$$q = hA(T_S - T_\infty) \tag{5-1}$$

h = *Convection heat transfer coefficient [W/Km^2]*
A = *Area perpendicular to direction of heat transfer [m^2]*
T_S = *Surface temperature [K]*
T_∞ = *Fluid temperature [K]*

5.2.1 Variables effecting to Convection

Convection heat transfer rate and convection heat transfer coefficient are affected by

- fluid properties: density, thermal conductivity, dynamic viscosity, specific heat
- geometry and roughness of surface
- Mode of fluid flow: laminar or turbulent
- Mode of convection: free, forced, boiling or condensation

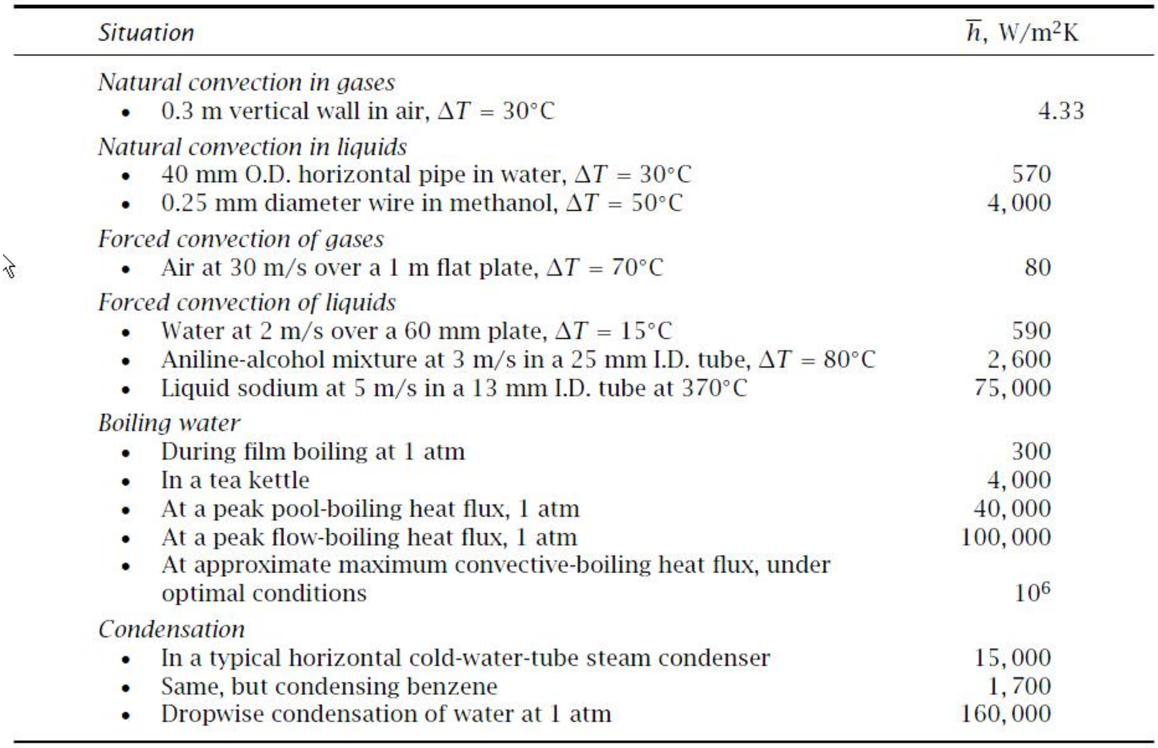
5.2.2 Convection heat transfer coefficient

Convection heat transfer coefficient is experimentally determined parameter and tables below show typical ranges for convection heat transfer coefficient.

Table 5-1 *Ranges of convection coefficients*

Type of Convection	**h [W/Km^2]**
Free convection of gases	2-25
Free convection of liquids	10-1000
Forced convection of gases	25-250
Forced convection of liquids	50-20 000
Boiling & condensation	2500-100 000

Table 5-2 *Convection heat transfer coefficients (Lienhard)*



Situation	$\overline{h}$, W/m^{2}K
Natural convection in gases	
• 0.3 m vertical wall in air, $\Delta T = 30°C$	4.33
Natural convection in liquids	
• 40 mm O.D. horizontal pipe in water, $\Delta T = 30°C$	570
• 0.25 mm diameter wire in methanol, $\Delta T = 50°C$	4,000
Forced convection of gases	
• Air at 30 m/s over a 1 m flat plate, $\Delta T = 70°C$	80
Forced convection of liquids	
• Water at 2 m/s over a 60 mm plate, $\Delta T = 15°C$	590
• Aniline-alcohol mixture at 3 m/s in a 25 mm I.D. tube, $\Delta T = 80°C$	2,600
• Liquid sodium at 5 m/s in a 13 mm I.D. tube at 370°C	75,000
Boiling water	
• During film boiling at 1 atm	300
• In a tea kettle	4,000
• At a peak pool-boiling heat flux, 1 atm	40,000
• At a peak flow-boiling heat flux, 1 atm	100,000
• At approximate maximum convective-boiling heat flux, under optimal conditions	10^6
Condensation	
• In a typical horizontal cold-water-tube steam condenser	15,000
• Same, but condensing benzene	1,700
• Dropwise condensation of water at 1 atm	160,000

Heat Transfer

Based on Newton's Law of cooling convection heat transfer rate can be formulated with *local convection heat transfer coefficient h* as follows

$$q'' = h(T_s - T_\infty)$$

$$q = \int_{A_s} h(T_s - T_\infty)dA_s = (T_s - T_\infty)\int_{A_s} hdA_s \tag{5-2}$$

Average convection coefficient is integral of local value over total surface area and with this total heat transfer rate based on average convection heat transfer coefficient can be then expressed as

$$\overline{h} \equiv \frac{1}{A_S}\int_{A_S} hdA_S \tag{5-3}$$

$$q = \overline{h}A_S(T_S - T_\infty) \tag{5-4}$$

In special case of *flat plate*, local convection coefficient varies only with distance x and thus average convection coefficient can be formulated as

$$\overline{h} \equiv \frac{1}{L}\int_0^L hdx \tag{5-5}$$

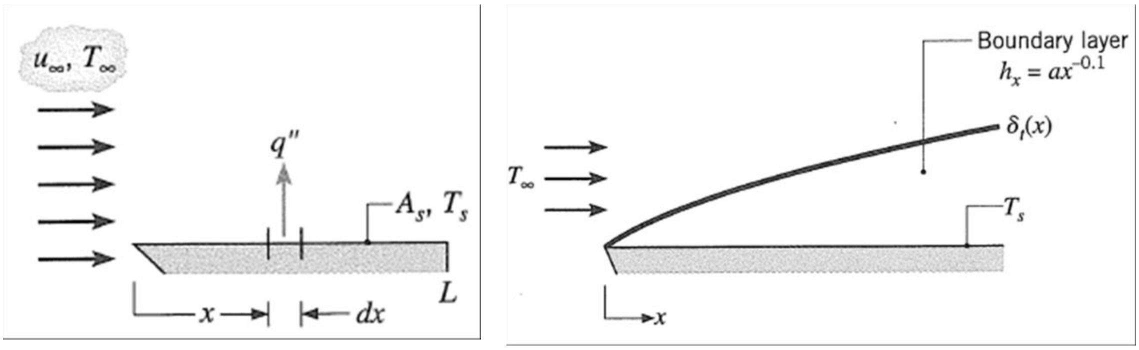


Figure 5-2 *Total and local convection heat transfer over flat plate*

Mass Transfer

Convection mass transfer from surface is analogical to convection heat transfer. Total convection mass transfer rate can be expressed with average convection heat

$$N_A = \bar{h}_m A_s \left(C_{A,s} - C_{A,\infty}\right) \quad \left[\frac{mol}{s}\right] \tag{5-6}$$

C_A *Concentration of species A* [$\frac{mol}{m^3}$]

, where average convection mass transfer coefficient is defined as

$$\bar{h}_m \equiv \frac{1}{A_s} \int_{A_s} h_m dA_s \tag{5-7}$$

For flat plate, average convection mass transfer coefficient can be written analytically to convection heat transfer coefficient as

$$\overline{h_m} \equiv \frac{1}{L} \int_0^L h_m dx \tag{5-8}$$

In special condition for ideal gases with saturation vapour pressure at surface, surface concentration can be written as

$$C_{A,s} = \frac{p_{A,sat}\left(T_S\right)}{RT_S} \tag{5-9}$$

$p_{A,sat}(T_S)$ *Saturation vapour pressure at surface temperature* T_s

5.3 LEARNING OUTCOMES

Table 5-3 Learning Outcomes: F*undamentals of Convection*

Level of Knowledge	Concept
Understand *Describe*	- *Convection modes* - *Newton's Law of Convection* - *Convection heat transfer coefficient: Range, Local or average? How it's formed?*
Apply *describe* *understand*	- *Newton's Law of Convection*

6 BOUNDARY LAYER THEORY

Formation and behaviour of boundary layers are fundamental phenomenon describing convection heat and mass transfer characteristics in fluid motion over solid surface. The fluid motion, flow, over a surface is divided to two different regions:

- *Boundary Layer, a thin fluid layer near surface, having large velocity, temperature or/and concentration gradients to opposite direction of flow and high shear stresses.*
- *Region outside of boundary layer having small shear stresses, thermal and mass transfer gradients.*

6.1 BOUNDARY LAYERS

6.1.1 Velocity Boundary Layer

Velocity Boundary Layer is region of the fluid, where velocity varies from zero at surface to velocity $0.99\,u_\infty$ of bulk fluid flow and *boundary layer thickness* can be written as

$$\delta \equiv y\left(u = 0{,}99u_\infty\right) \tag{6-1}$$

Velocity boundary layer and surface friction exists always in a flow over any surface. In a flow over a flat plate (Figure 1-5) boundary layer velocity profile changes with flow moving further on a plate. Inside the velocity boundary layer shear stress forces are large and form velocity gradients opposite to surface direction. A dimensionless measure of ratio of shear stress and velocity used in fluid mechanics is a *local friction coefficient*

$$C_f \equiv \frac{\tau_s}{\rho u_\infty^2/2} \tag{6-2}$$

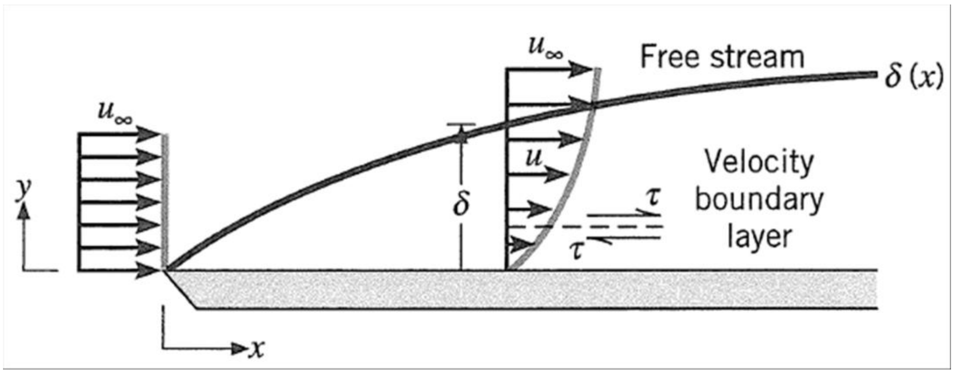


Figure 6-1 *Velocity boundary layer in a flow over a flat plate*

Shear stress for Newtonian fluids can be determined with velocity gradient as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{6-3}$$

μ *Dynamic viscosity*

6.1.2 Temperature Boundary Layer

Temperature Boundary Layer is region of the fluid, where temperature varies from surface temperature to bulk fluid temperature. At the interface (surface) temperatures of solid and fluid are same and velocity of fluid is zero and heat is transferred only by random molecular motion (conduction). Temperature boundary layer is characterized by temperature gradients and heat transfer.

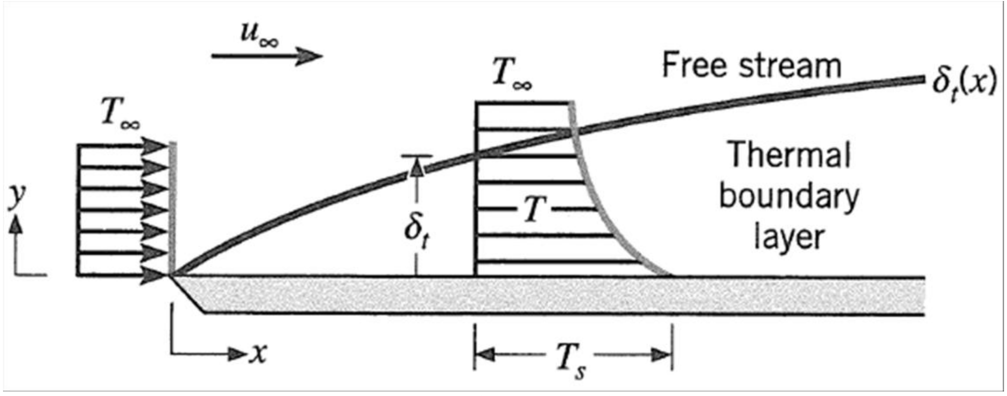


Figure 6-2 Temperature boundary layer in a flow over a flat plate

Development of temperature boundary layer is analogical to formation of velocity boundary layer. *Thickness of temperature boundary layer* is defined as

$$\delta_T \equiv y\left(\frac{T_S - T}{T_s - T_\infty} = 0{,}99\right) \qquad (6\text{-}4)$$

Within boundary layer in a case of flat plate temperature is function of both distances from the tip and height from plate: $T = T(y,x)$. Out of boundary layer temperature is bulk fluid temperature $T \approx T_\infty$.

Local surface heat flux at any distance from tip can be expressed according to Fourier's Law with temperature gradient and conductivity as

$$q_S^{''} = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (6\text{-}5)$$

Definition for convection heat flux is

$$q^{''} = h\left(T_S - T_\infty\right) \qquad (6\text{-}6)$$

At surface convection heat transfer is only conduction, ($q_S^{''} = q^{''}$), and thus a local convection heat transfer coefficient is

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_S - T_\infty} \qquad (6\text{-}7)$$

Thickness of temperature boundary layer is growing similarly with developing velocity boundary layer and consequently temperature gradients are decreasing.

6.1.3 Concentration Boundary Layer

Concentration Boundary Layer is similar to velocity and temperature boundary layers. It is characterized by concentration gradients and convection mass transfer rates. Thickness of concentration boundary layer is defined as

$$\delta_c \equiv y\left(\frac{C_{A,s}-C_A(y)}{C_{A,s}-C_{A,\infty}}=0{,}99\right) \tag{6-8}$$

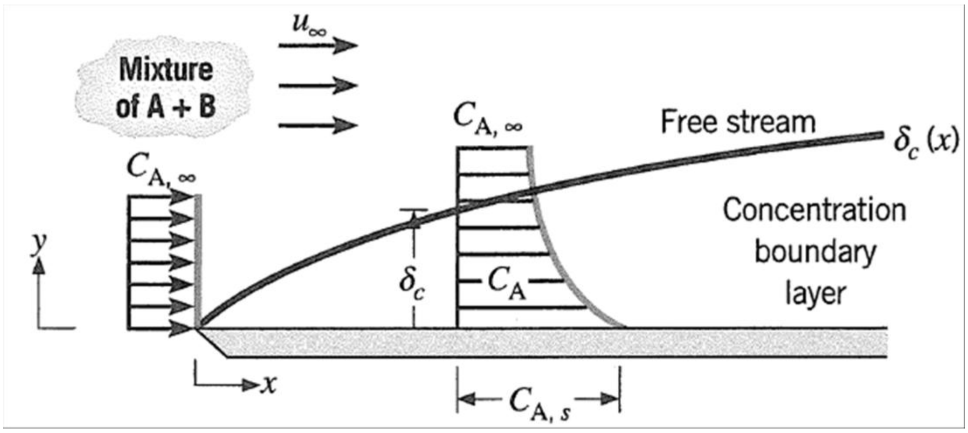


Figure 6-3 Concentration boundary layer in a flow over a flat plate

Molar flux by diffusion from surface any distance from tip of flat plate (Figure 6-3) can be written with *Fick's Law* as

$$N''_{A,s}=-D_{A,B}\left.\frac{\partial C_A}{\partial y}\right|_{y=0} \tag{6-9}$$

$D_{A,B}$ *Diffusion coefficient*

On the other hand, convective mass transfer is defined as

$$N''_{A,s}=h_m\left(C_{A,s}-C_{A,\infty}\right) \tag{6-10}$$

Convection mass transfer coefficient can thus be described analogically to heat transfer coefficient as

$$h_m=-\frac{D_{A,B}\left.\frac{\partial C_A}{\partial Y}\right|_{y=0}}{C_{A,s}-C_{A,\infty}} \tag{6-11}$$

6.1.4 Laminar and Turbulent Flows

Both surface friction and convection transfer rates depends strongly on flow condition and primarily weather the boundary layer is turbulent or laminar. Table 6-1 describes basic differences of laminar and turbulent flow conditions.

Table 6-1 *Laminar versus Turbulent flow*

Laminar	Turbulent
• *Streamlined and highly ordered*	• *Irregular and random 3D motion of parcels of fluid*
• *At steady-state conditions no time dependent fluctuation of velocity*	• *Highly fluctuating velocity components*
• *Momentum, temperature and concentration are "diffusing" through the boundary layer*	• *Chaotic fluctuation conditions result as higher order transfer of momentum, temperature and concentration*

Figure 6-4 shows boundary layer development in a flow on flat plat. Thickness of velocity boundary layer grows continuously with decreasing velocity gradients. Flow conditions are streamlined in *laminar region* until *transition zone* is reached. Conditions in transition zone fluctuate between laminar and turbulent flow. Turbulent boundary layer can be divided to

1) viscous sub-layer*: diffusion and linear velocity profile,*
2) buffer layer*: diffusion and turbulent mixing, and*
3) turbulent zone*: turbulent mixing dominating transport phenomenon.*

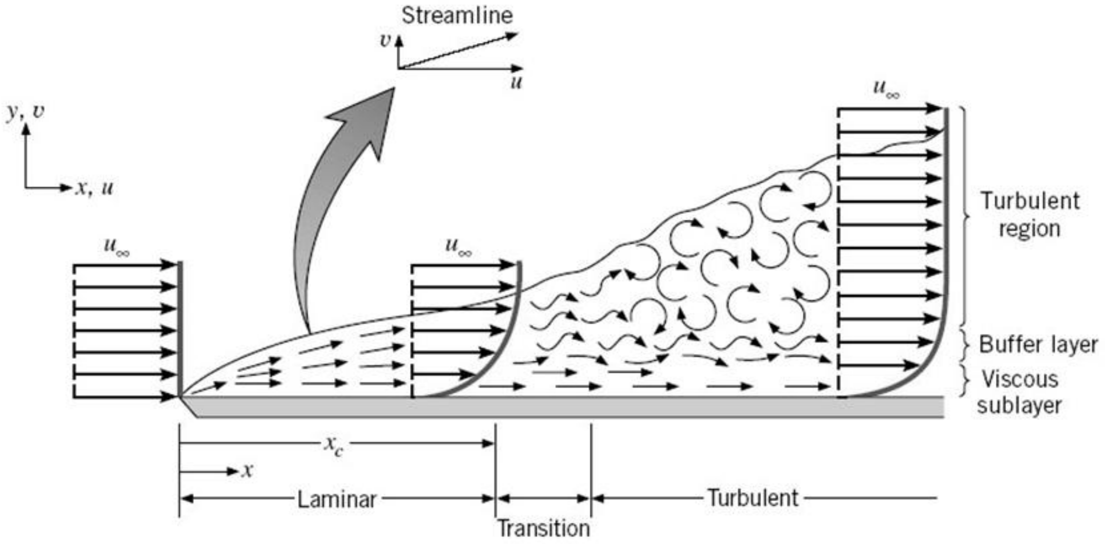


Figure 6-4 *Development of velocity boundary layer on flat plate*

Dimensionless grouping of parameters describing ratio of inertia to viscous forces is *Reynolds Number* and is defined for plate as

$$\mathrm{Re} = \frac{\rho u_\infty x}{\mu} \qquad \textit{(6-12)}$$

The condition for change from laminar to turbulent flow in case of flat plate is determined by *critical Reynolds number* $\mathrm{Re}_c \approx 5*10^5$, or $10^5 \prec \mathrm{Re}_c \prec 3*10^6$ depending on surface properties.

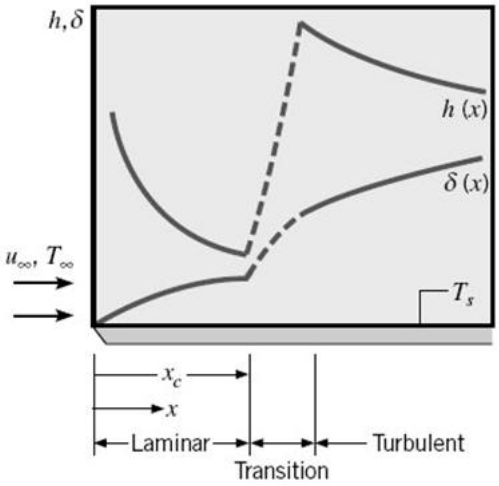


Figure 6-5 *Velocity boundary layer thickness and a local heat transfer coefficient for flow over a flat plate*

Figure 6.5 shows velocity boundary layer development with transition from laminar to turbulent regions in flow over flat plate. Local heat transfer coefficient is highly dependent on flow condition with its velocity gradients and transport properties. Thickening of boundary layer can be seen as decaying heat transfer at both laminar and turbulent regions, while a dramatic increase of heat transfer coefficient occurs in transition zone caused by enhanced turbulent transport conditions. Effect of turbulent conditions is similar in case of convection mass transfer from surface.

Example VI: ***Air flow over plate – Boundary conditions***

Consider airflow over a flat plate of length L = 1 m under conditions for which transition occurs at x_c = 0,5 m based on the critical Reynolds number, $Re_{x,c} = 5 \times 10^5$.

a) Evaluating the thermophysical properties of air at 350 K, determine the air velocity.,

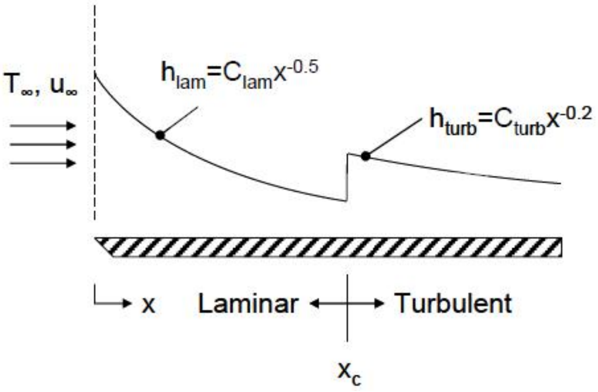
b) In the laminar and turbulent regions, the local convection coefficients are, respectively,

$$h_{\mathrm{lam}}(x) = C_{\mathrm{lam}} x^{-0,5} \text{ and } h_{\mathrm{turb}}(x) = C_{\mathrm{turb}} x^{-0,2}$$

Where, at T = 350 °C, $C_{\mathrm{lam}} = 8{,}845\ \mathrm{W/m^{3/2}\cdot K}$, and $C_{\mathrm{turb}} = 49{,}75\ \mathrm{W/m^{1,8}\cdot K}$. Develop an expression for the average convection coefficient, $\overline{h}_{\mathrm{lam}}(x)$, as a function of distance from the leading edge, x, for the laminar region, $0 \le x \le x_c$.

c) Develop an expression for the average convection coefficient $\overline{h}_{\mathrm{turb}}(x)$, as a function of distance from the leading edge, x, for the turbulent region, $x_c \le x \le L$.

d) On the same coordinates, plot the local and acerage convection coefficients, h_x, and $\overline{h}_x$, respectively, as a function of x for $0 \le x \le L$.



ASSUMPTIONS:
(1) Steady-state conditions,
(2) Constant properties

PROPERTIES:
Table A.4, air (T = 350 K):
k = 0,030 W/mK,
$\nu = 20{,}92 \times 10^{-6}\ \text{m}^2/\text{s}$,
Pr = 0,700.

ANALYSIS:

a) At 350 K with $x_{\text{c}} = 0{,}5\ \text{m}$,

$$\text{Re}_{\text{x,c}} = \frac{u_\infty x_{\text{c}}}{\nu}$$

$$\Rightarrow u_\infty = \frac{\text{Re}_{\text{x,c}}\nu}{x_{\text{c}}} = \frac{5 \cdot 10^5 \cdot 20{,}92 \cdot 10^{-6}\ \text{m}^2/\text{s}}{0{,}5\ \text{m}} = 20{,}9\ \text{m/s}$$

b) From Eq. 5-3, the average convection coefficient in the laminar region $0 \le x \le x_{\text{c}}$.

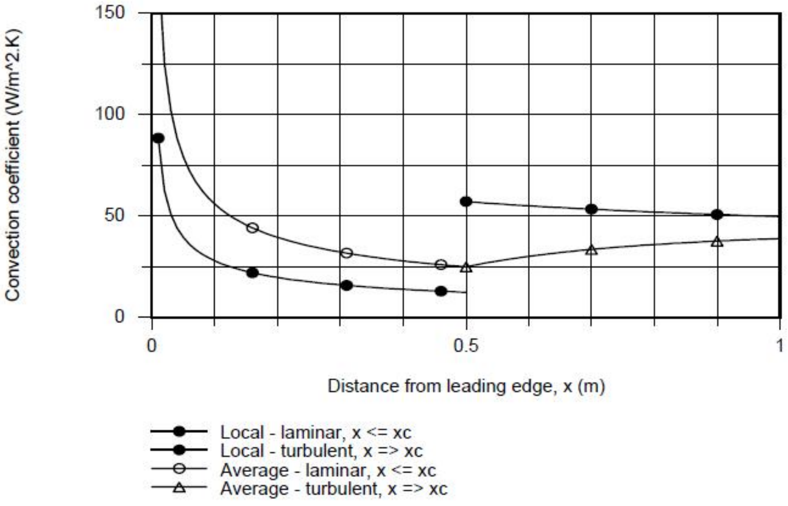
$$\overline{h}_{\text{lam}}(x) = \frac{1}{x}\int_0^x h_{\text{lam}}(x)\,dx = \frac{1}{x}C_{\text{lam}}\int_0^x x^{-0{,}5}\,dx = \frac{2}{x}C_{\text{lam}}x^{0{,}5} = 2C_{\text{lam}}x^{-0{,}5}$$

$$= 2h_{\text{lam}}(x) \qquad (1)$$

c) The average convection coefficient in the turbulent region $x_{\text{c}} \le x \le L$.

$$\overline{h}_{\text{turb}}(x) = \frac{1}{x}\left[\int_0^{x_{\text{c}}} h_{\text{lam}}(x)\,dx + \int_{x_{\text{c}}}^{x} h_{\text{turb}}(x)\,dx\right] =$$
$$\frac{1}{x}\left[2C_{\text{lam}}x^{0{,}5} + 1{,}25C_{\text{turb}}\left(x^{0.8} - x_{\text{c}}^{\ 0.8}\right)\right] \qquad (2)$$

d) The local and average coefficients, Eqs. (1) and (2) are plotted below as a function of x for the range $0 \le x \le L$.



6.2 BOUNDARY LAYER EQUATIONS

Fluid motion at boundary layer acts according to *fundamental laws of nature*: conservation of mass, energy and chemical species as well as momentum, Newton's 2nd law of motion, are valid for fluid flow characterization.

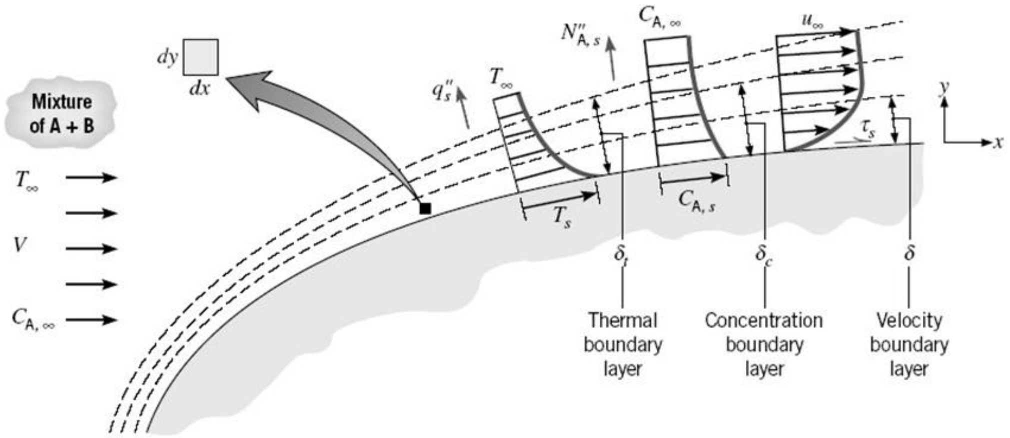


Figure 6-6 *Development of temperature, concentration and velocity boundary layers for curved surface*

Here conservation equations are presented for two-dimensional arbitrary surface with following approximations:

- Laminar flow & Steady-state conditions
- Constant fluid properties
- gradients to direction of Flow:

   1) Velocity $\frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}$

   2) temperature: $\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$

   3) pressure: $\frac{\partial p}{\partial x} \approx \frac{\partial p_\infty}{\partial x}$

Conservation of Mass

Overall *Continuity Equation* can be derived from writing mass balance for differential two-dimensional control volume as

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad (6\text{-}13)$$

For incompressible flow, meaning constant density, continuity equation reduces to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6-14}$$

Newton's Second Law of Motion

Momentum equation to x-coordinate direction can be reduced with assumptions made to

$$\underbrace{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}}_{\substack{\text{fluid motion x-momentum}\\ \text{leaving control volume}}} = \underbrace{-\frac{1}{\rho}\frac{\partial p_\infty}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}}_{\substack{\text{net pressure force}\\ +\text{ viscous shear stresses}}} \tag{6-15}$$

Conservation of Energy

Energy equation for boundary layer control volume may be expressed with conditions earlier presented as

$$\underbrace{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}}_{\substack{\text{net rate of thermal energy}\\ \text{leaving control system}\\ \text{with fluid motion}}} = \underbrace{\alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2}_{\substack{\text{net rates of y-direction conduction}\\ +\text{ viscous dissipation}}} \tag{6-16}$$

6.3 SIMILARITY

Similarity of conservation equations presented, applicable for low-speed, forced convection flows, can be further developed by forming dimensionless parameters and governing equations. Independent dimensionless spatial coordinates are defined as

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad ;\ L \text{ Characteristic length of surface}$$

Dimensionless dependent velocity components are defined as

$$u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad ;\ V \text{ Bulk flow velocity}$$

Dimensionless temperature and concentration differences can be expressed as

$$T^* = \frac{T-T_S}{T_\infty-T_S} \qquad\qquad C_A^{\ *} = \frac{C_A-C_{AS}}{C_{A\infty}-C_{AS}}$$

6.3.1 Dimensionless Groups and Equations

Table 6-2 shows dimensionless forms of conservation equations with three important dimensionless similarity parameters:

Reynolds number: $\mathrm{Re}_L = \dfrac{VL}{\nu} = \dfrac{\rho VL}{\mu}$

The ratio of inertia to viscous forces

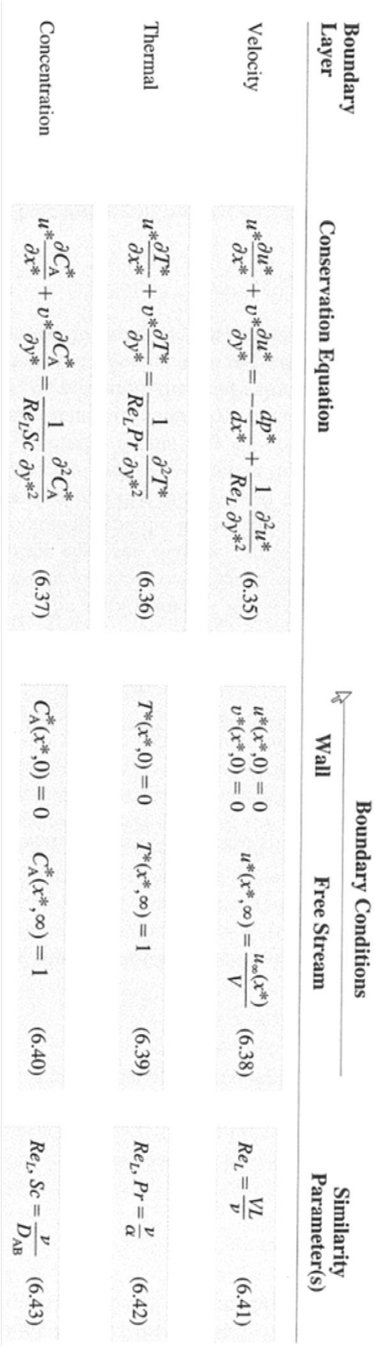
Prandtl number: $\Pr = \dfrac{\nu}{\alpha}$

The relative effectiveness of momentum and energy transport by diffusion in the velocity and thermal boundary layers

Schmidt number: $Sc = \dfrac{\nu}{D_{AB}}$

The relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers

Table 6-2 *Dimensionless boundary layer conservation equations and y-direction boundary conditions (Incropera)*



Boundary Layer	Conservation Equation	Boundary Conditions: Wall	Boundary Conditions: Free Stream	Similarity Parameter(s)
Velocity	$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ (6.35)	$u^*(x^*,0) = 0$ $v^*(x^*,0) = 0$	$u^*(x^*,\infty) = \frac{u_\infty(x^*)}{V}$ (6.38)	$Re_L = \frac{VL}{\nu}$ (6.41)
Thermal	$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$ (6.36)	$T^*(x^*,0) = 0$	$T^*(x^*,\infty) = 1$ (6.39)	$Re_L, Pr = \frac{\nu}{\alpha}$ (6.42)
Concentration	$u^*\frac{\partial C_A^*}{\partial x^*} + v^*\frac{\partial C_A^*}{\partial y^*} = \frac{1}{Re_L Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$ (6.37)	$C_A^*(x^*,0) = 0$	$C_A^*(x^*,\infty) = 1$ (6.40)	$Re_L, Sc = \frac{\nu}{D_{AB}}$ (6.43)

Following dimensionless parameters describing convection fluid dynamic, and convection heat and mass transfer functional dependences have been derived by similarity approach and can be used in formation of experimental correlations:

Friction Coefficient: $C_f = \frac{\tau_S}{\rho V^2/2} = \frac{2}{\mathrm{Re}_L} f\left(x^*, \mathrm{Re}_L\right)$

Dimensionless surface shear stress

Nusselt Number: $Nu = \frac{\bar{h}L}{k_f} = f\left(\mathrm{Re}_L, \mathrm{Pr}\right)$

Ratio of convection to conduction heat transfer

- If functional relation is known, convection heat transfer coefficient *h* can be determined.

Sherwood Number: $Sh = \frac{\overline{h_m}L}{D_{AB}} = f\left(\mathrm{Re}_L, Sc\right)$

Dimensionless concentration gradient

- If functional relation is known, convection mass transfer coefficient h_m can be determined for described geometry.

6.3.2 Boundary layer Thickness

Similarity relationships can be used for determining approximate relationships between boundary layer thicknesses as follows – (*in usual applications exponent n can be approximated with value 1/3*):

Ratio of velocity and temperature boundary layer thicknesses as

$$\frac{\delta}{\delta_t} \approx \left(\frac{\nu}{\alpha}\right)^n = \mathrm{Pr}^n \tag{6-17}$$

Ratio of velocity and concentration boundary layer thicknesses as

$$\frac{\delta}{\delta_c} \approx \left(\frac{\nu}{D_{AB}}\right)^{\mathrm{n}} = \mathrm{Sc}^n \tag{6-18}$$

Ratio of temperature and concentration boundary layer thicknesses as

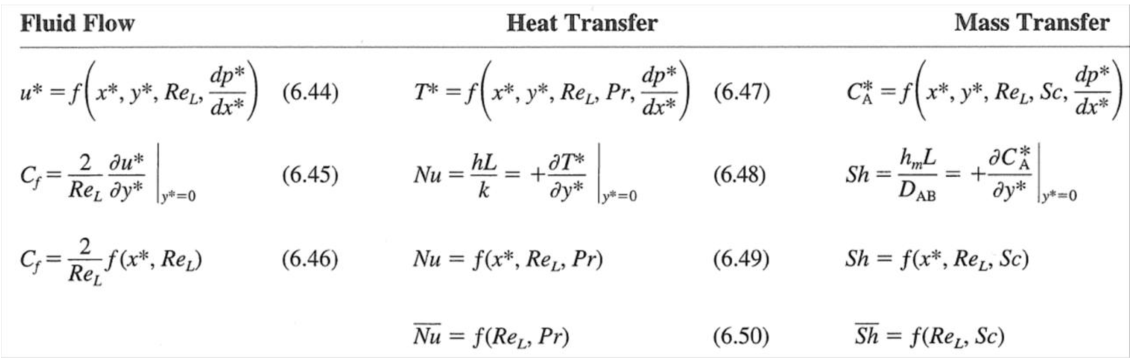
$$\frac{\delta_t}{\delta_c} \approx \left(\frac{\alpha}{D_{AB}}\right)^{\mathrm{n}} = \mathrm{Le}^n = \left(\frac{\mathrm{Sc}}{\mathrm{Pr}}\right)^n \tag{6-19}$$

Le Lewis number – *ratio of thermal and mass diffusivities*

6.4 ANALOGIES

As heat and mass transfer conservation equations are of the same form, analogous, and related to velocity field with dimensionless parameters: Reynolds, Prandtl and Schmidt numbers, the boundary layer temperature and concentration profiles are same form with similar boundary conditions.

Table 6-3 *Functional relations relevant to boundary layer analogies (Incropera)*



Fluid Flow		**Heat Transfer**		**Mass Transfer**
$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$
$C_f = \frac{2}{Re_L}\frac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.45)	$Nu = \frac{hL}{k} = +\frac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.48)	$Sh = \frac{h_m L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\Big\vert_{y^*=0}$
$C_f = \frac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$

6.4.1 Heat and mass transfer analogy

For particular geometry with similar boundary conditions following heat and mass transfer analogy can be written

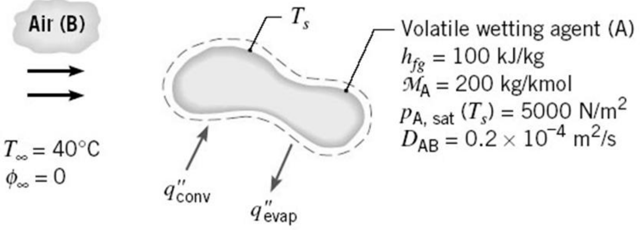
$$\frac{\mathrm{Nu}}{\mathrm{Pr}^n} = \frac{\mathrm{Sh}}{\mathrm{Sc}^n} \tag{6-20}$$

$$n \approx \tfrac{1}{3}$$ *Exponent value most applications*

This can be further developed to ratio of heat and mass transfer convection coefficients:

$$\frac{h}{h_m} = \frac{k}{D_{A,B} Le^n} = \rho c_p Le^{1-n} \tag{6-21}$$

Example VII: *Evaporative cooling of beverage bottle – Cooling Beer at Hot Beach*



Beer bottle inside a wet towel at hot beach conditions

ASSUMPTIONS:
(1) Heat and mass transfer analogy is applicable
(2) Vapor displays ideal gas behaviour,
(3) Radiation effects are negligible,
(4) Air properties may be evaluated at a mean boundary layer temperature assumed to be 300 K.

PROPERTIES: *Table A-4*: air (300 K): $\rho = 1{,}16\ \mathrm{kg/m^3}$ $C_\mathrm{p} = 1{,}007\ \mathrm{kJ/KgK}$ $\nu = 14{,}82 \cdot 10^{-6}\ \mathrm{m^2/s}$, $\alpha = 22{,}5 \cdot 10^{-6}\ \mathrm{m^2/s}$, $k = 26{,}3\ \cdot 10^{-3}\ \mathrm{W/mK}$

ANALYSIS:

1) Conservation of energy:

$$q''_{conv.} = q''_{evap}$$

$$h(T_\infty - T_s) = h_{fg} n''_A$$

2) Evaporative mass flux of water

$$n''_A = h_m(\rho_{A,sat}(T_s) - \rho_{A,\infty})$$

3) Heat and mass transfer analogy

$$\frac{h}{h_m} = \frac{k}{D_{A,B} L_e^{\,n}} = \rho c_p L_e^{\,1-n}$$,

Where $Le = \frac{\alpha}{D_{\mathrm{A,B}}}$

4) Bottle temperature difference to surroundings from equations above

$$T_\infty - T_s = h_{fg} \frac{D_{AB} Le^{1/3}}{k} \left(\rho_{A,sat}(T_s) - \rho_{A,\infty}\right)$$

5) Ideal gas law: $pV = nRT \left(n = \frac{m}{M},\ \rho = \frac{m}{V}\right)$ $\Rightarrow \rho = \frac{Mp}{RT}$

$$\rho_{\mathrm{A.sat}} = \frac{Mp_{\mathrm{A,sat}}}{RT_{\mathrm{A,sat}}}$$

By setting $\rho_{\mathrm{A},\infty} = 0$, an iterative equation is formed from eq. (4) and (5). Beginning the iteration with value $T_\mathrm{s} = 280\ \mathrm{K}$:

$$T_\mathrm{s} = T_\infty - \frac{h_\mathrm{fg} M}{R \rho C_\mathrm{p} Le^{2/3}} \cdot \left[\frac{p_\mathrm{A,sat}}{T_\mathrm{s}}\right]$$

$$T_\mathrm{s} = 313\ \mathrm{K} - \frac{100\ \mathrm{KJ/kg} \cdot 200 \frac{\mathrm{kg}}{\mathrm{kmol}}}{8{,}315 \frac{\mathrm{kJ}}{\mathrm{kmol} \cdot \mathrm{K}} \cdot 1{,}007\ \mathrm{kJ/KgK} \cdot 1{,}16\ \mathrm{kg/m^3} \cdot \left(\frac{22{,}5 \cdot 10^{-6}\ \mathrm{m^2/s}}{20 \cdot 10^{-6}\ \mathrm{m^2/s}}\right)^{2/3}} \cdot \left[\frac{5000\ \mathrm{N/m^2} \cdot 10^{-3} \mathrm{kJ/Nm}}{280\ \mathrm{K}}\right]$$

$$T_\mathrm{s} \approx 280\ \mathrm{K}$$

Comments:

1) *As such hot conditions, a 7°C Beer would be worth of its weight in gold?*

6.4.2 Reynolds Analogy

Table 6-3 shows similarity with velocity, thermal and concentration boundary layers and with similar boundary conditions all the engineering parameters combining equation can be formed as Reynolds Analogy. Validity of this analogy requires Pr and Sc ≈ 1 and *dp*/dx** ≈ 0. Modified Reynolds or Chilton-Colburn analogies containing correction functions have been developed for wider operating range as

Reynolds Analogy

$$C_f \frac{\mathrm{Re}_L}{2} = Nu = Sh \tag{6-22}$$

Chilton-Colburn Analogies

$$\frac{C_f}{2} = St\,\mathrm{Pr}^{2/3} = j_H \qquad 0.6 < \mathrm{Pr} < 60 \tag{6-23}$$

$$\frac{C_f}{2} = St_m\,\mathrm{Pr}^{2/3} = j_m \qquad 0.6 < Sc < 3000 \tag{6-24}$$

6.5 LEARNING OUTCOMES

Table 6-4 Learning Outcomes: *Boundary Layer Theory*

Level of Knowledge	Concept
Understand *Describe*	• *Development of Boundary Layers: Thermal, Velocity and Concentration* • *Transition from laminar to turbulent* • *Laminar Boundary layer Conservation equations: terms and their meaning* • *Concepts: Dimensionless Numbers: Re, Pr, Sc; Boundary layer thickness*
Apply ***understand*** *describe*	• *Heat and mass transfer analogy*

7 FORCED CONVECTION – EXTERNAL FLOW

In *forced convection* fluid is forced to flow over a surface by external means such as pump, fan or wind. In *external flow* boundary layers develop freely without disturbance of adjacent surfaces and at flow region out of boundary layer has negligible velocity, temperature and concentration gradients.

An engineering problem of forced convection at external flow is generally to find convection coefficients for described geometries. Here the principles and functional forms of convection correlations for low-speed forced convection flow are presented.

As discussed in boundary layer analogies chapter, local and average convection heat transfer coefficients have dimensionless functional form as

$$Nu_x = f(x^*, \mathrm{Re}_x, \mathrm{Pr}) \qquad \overline{Nu}_x = f(\mathrm{Re}_x, \mathrm{Pr})$$

And respective functional form for mass transfer coefficients as

$$Sh_x = f\left(x^*, \mathrm{Re}_{L,}\, Sc\right) \qquad \overline{Sh}_x = f\left(\mathrm{Re}_x, Sc\right)$$

There are two options to obtain convection coefficient functional relationships:

- Analytical approach – appropriate for simple geometries only
- Experimental or empirical approach – also complex geometries

7.1 EMPIRICAL CORRELATIONS

Experimental approach for determining convection coefficient for geometry, such as flat plate, could be done by electrically heating plate and simultaneously cooling it with convective fluid flow. Heating power would result as known convective heat flux, if insulation would be appropriate. With measurement of surface temperature, and fluid temperature and heat flux, convection coefficient can be calculated based on Newton's law of Cooling. From fluid properties and flow velocity descriptive dimensionless numbers: Reynolds, Prandtl and Nusselt numbers can be defined. As the functional dependence of Nusselt, Reynolds and Prandtl numbers is key question for determining general heat transfer coefficient correlation for described geometry, a correlation form to be fitted with test data may for example be chosen as

$$\overline{Nu_x} = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^n \tag{7-1}$$

By repeating measurements with variation of Nu, Pr and Re by changing for example convection flow velocity and temperature, characteristic dimensions of geometry and fluid, correlation parameters for convection coefficient can be solved (Figure 7-1). Correlation for dimensionless heat transfer coefficient, Nusselt number, obtained is valid for similar surface geometry and type of flow.

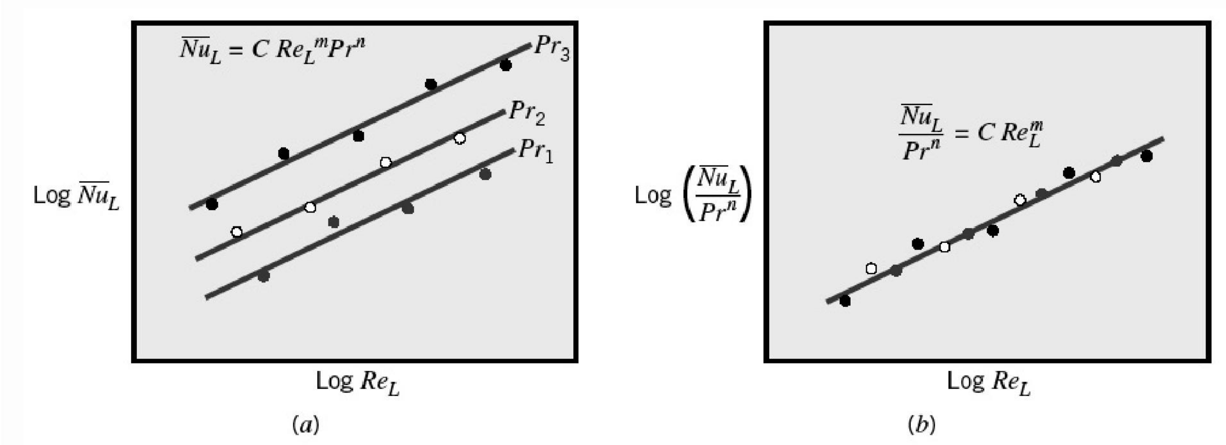


Figure 7-1 *Dimensionless fitting of convection heat transfer coefficient test data*

The effect of varying fluid properties with changing temperature at boundary layer can be accounted by calculating *film temperature*

$$T_f = \frac{T_s + T_\infty}{2} \tag{7-2}$$

Or by adding property correction to convection coefficient correlation as

$$\overline{Nu_x} = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^n\left(\mathrm{Pr}_\infty/\mathrm{Pr}_S\right)^r$$

or

$$\overline{Nu_x} = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^n\left(\mu_\infty/\mu_S\right)^r \tag{7-3}$$

7.2 ANALYTICAL SOLUTION FOR FLAT PLATE

Velocity component are presented with stream function ψ(x, y) as

$$u = \frac{\partial\psi}{\partial y} \qquad v = \frac{\partial\psi}{\partial x} \tag{7-4}$$

Similarity solution f and similarity variable η is then defined as

$$f(\eta) = \frac{\psi}{u_\infty\sqrt{vx/u_\infty}} \tag{7-5}$$

$$\eta = y\sqrt{u_\infty/vx} \tag{7-6}$$

7.2.1 Velocity Boundary Layer

By using similarity approach and differentiating velocity components momentum equation can be written as

$$2\frac{d^3 f}{d\eta^3} + f\,\frac{d^2 f}{d\eta^2} = 0 \tag{7-7}$$

Solutions for differential equation are presented in Table 7-1. It can noted that boundary layer thickness δ (u/u_∞=0.99) corresponds similarity value η=5 and thus boundary layer thickness in laminar flow over flat plate is

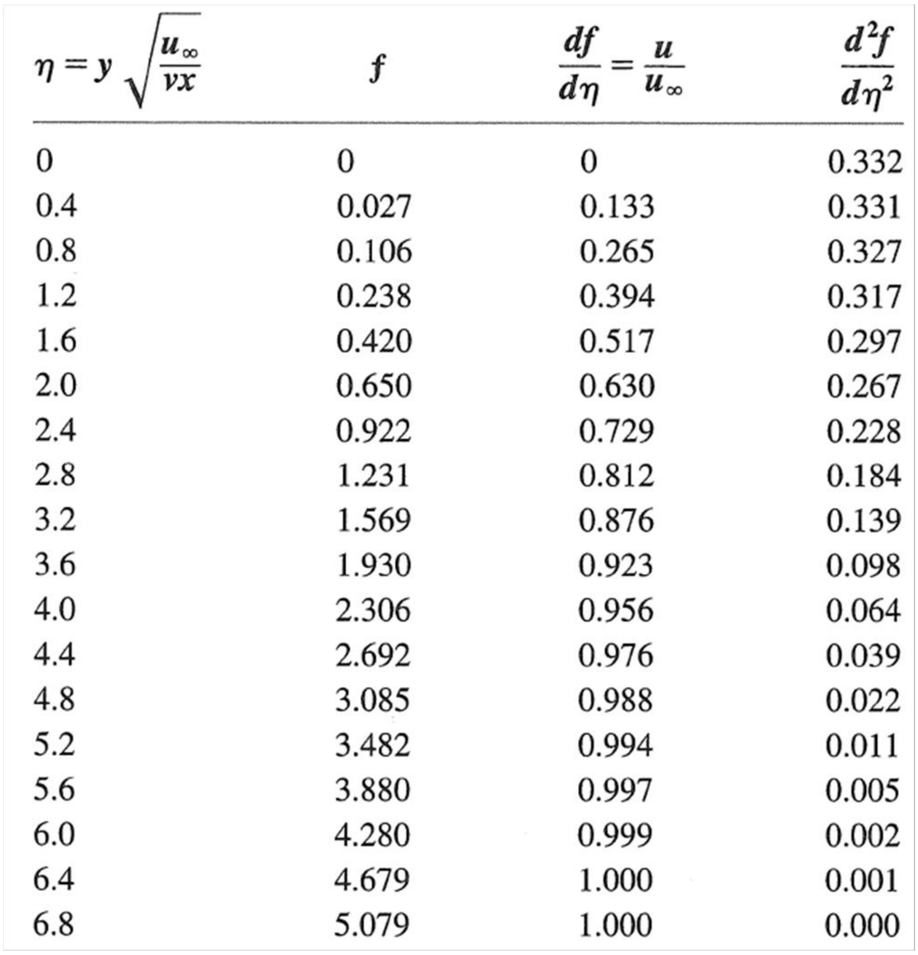
$$\delta = \frac{5x}{\sqrt{u_\infty / vx}} = \frac{5x}{\sqrt{\mathrm{Re}_x}} \tag{7-8}$$

Similarly wall shear stress and then local friction coefficient can be written as

$$\tau_S = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = 0.332 u_\infty \sqrt{\rho\mu u_\infty / x} \tag{7-9}$$

$$C_{f,x} = \frac{\tau_{s,x}}{\rho u_x^2 / 2} = 0.664\,\mathrm{Re}_x^{-1/2} \tag{7-10}$$

Table 7-1 *Analytical solutions of laminar flow over a flat plate*



$\eta = y\sqrt{\frac{u_\infty}{vx}}$	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.4	0.027	0.133	0.331
0.8	0.106	0.265	0.327
1.2	0.238	0.394	0.317
1.6	0.420	0.517	0.297
2.0	0.650	0.630	0.267
2.4	0.922	0.729	0.228
2.8	1.231	0.812	0.184
3.2	1.569	0.876	0.139
3.6	1.930	0.923	0.098
4.0	2.306	0.956	0.064
4.4	2.692	0.976	0.039
4.8	3.085	0.988	0.022
5.2	3.482	0.994	0.011
5.6	3.880	0.997	0.005
6.0	4.280	0.999	0.002
6.4	4.679	1.000	0.001
6.8	5.079	1.000	0.000

7.2.2 Thermal Boundary Layer

With dimensionless temperature $T^* = [(T-T_s)/(T-T_\infty)]$ energy equation can be written in similarity form as

$$\frac{d^2T^*}{d\eta^2} + \frac{\mathrm{Pr}}{2} f \frac{dT^*}{d\eta} = 0 \tag{7-11}$$

For Prandtl number Pr > 0.6, solution of differential energy equation gives correlation for temperature gradient as

$$\frac{dT^*}{d\eta} = 0.332\,\mathrm{Pr}^{1/3}$$

Local Nusselt number has consequently a form

$$Nu_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0.6 \tag{7-12}$$

7.3 CYLINDER IN A CROSS FLOW

In a flow across cylinder (Figure 7-2) maximum pressure ($p = p_{\max}$) is at *forward stagnation point*, which also has zero fluid velocity $u_\infty = 0$, and locates just in front of cylinder. Velocity u_∞ increases and pressure decreases with flow towards zenith. After change of pressure gradient to dp/dx > 0, velocity decelerates and velocity gradient at surface eventually becomes zero (Figure 7-3). As a result occur *boundary layer separation* and *wake* formation in downstream region.

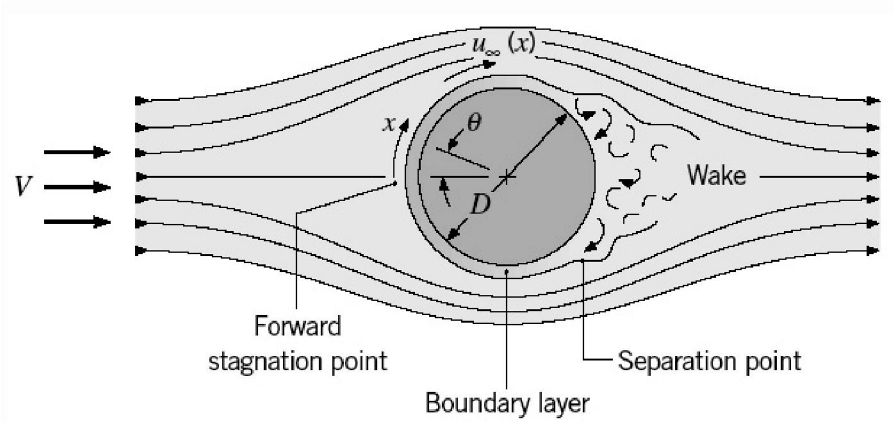


Figure 7-2 *Boundary layer formation and turbulent separation on a cylinder in a cross flow.*

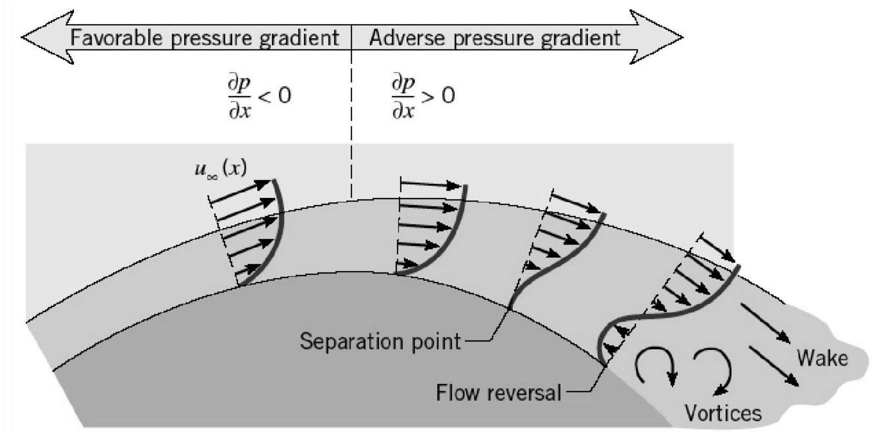


Figure 7-3 *Velocity profiles in turbulent wake formation in a cross flow over circular cylinder*

For a cylinder Reynolds number is defined as

$$\mathrm{Re}_L = \frac{VD}{\nu} = \frac{\rho VD}{\mu} \tag{7-13}$$

Figure 7-4 presents experimental results of convection coefficient variation with angular coordinate and Reynolds number in air cross flow over circular cylinder.

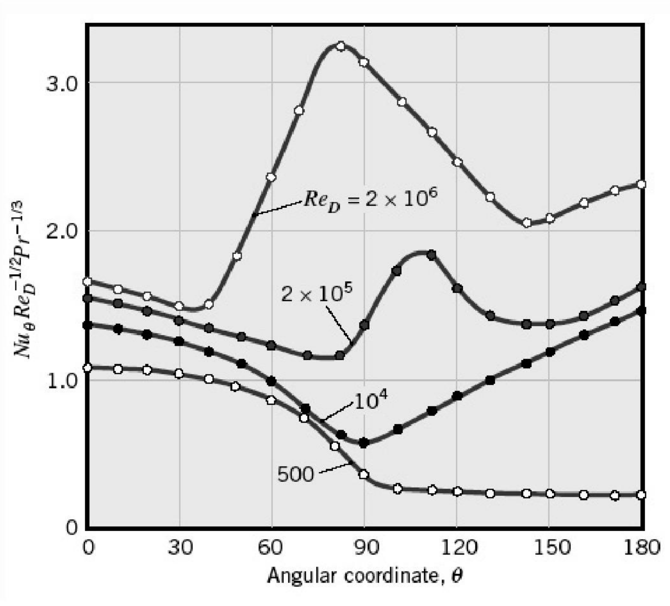


Figure 7-4 *Local convection heat transfer correlations for air cross flow over cylinder*

7.3.1 Empirical Convection Heat Transfer Correlations

Overall average convection heat transfer coefficient correlations are interesting in engineering applications. In following three correlations for cylinders in a cross flow are presented.[3]

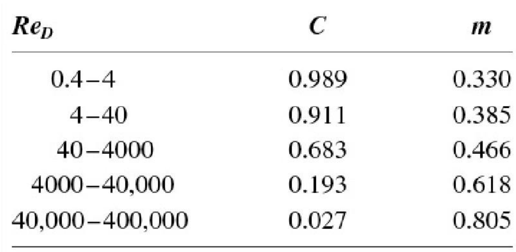
---

[3] All heat transfer correlations are applicable to mass transfer problems via analogy treatment: average Nusselt number is replaced by Sherwood number and Prandtl number by Schmidt number.

Hilpert's Correlation $$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0.7 \qquad (7\text{-}14)$$

Correlations coefficients are presented at Table 7-2 and properties are evaluated at film temperature.

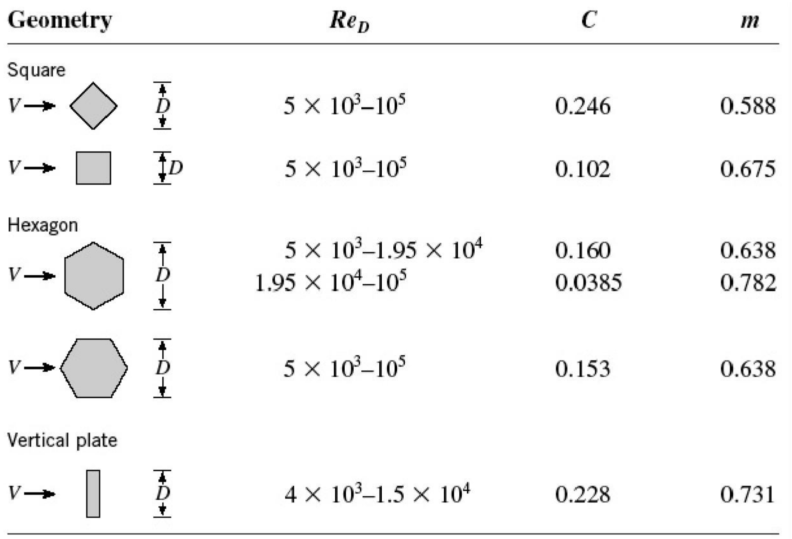
Table 7-2 *Correlation coefficients for Hilpert's Correlation*



Re_D	**C**	**m**
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

Convection correlation coefficients for other shapes than circular cylinders can be obtained from Table 7-3.

Table 7-3 *Correlation coefficients for Hilpert's Correlation for noncircular cylinders*



Geometry	**Re_D**	**C**	**m**
Square	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
	5×10^3–10^5	0.153	0.638
Vertical plate	4×10^3–1.5×10^4	0.228	0.731

Zukauskas $$\overline{Nu}_D = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4} \qquad (7\text{-}15)$$

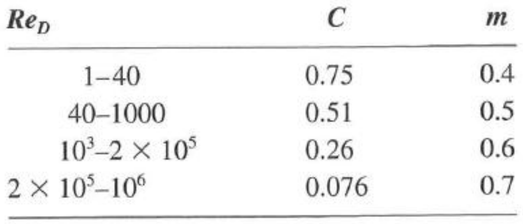
$$\mathrm{Pr} \leq 10 \quad \Rightarrow \quad n = 0.37$$
$$\mathrm{Pr} \geq 10 \quad \Rightarrow \quad n = 0.36$$

All the properties are calculated at bulk flow temperature t_∞ and correlation is valid for

$$0.7 \leq \mathrm{Pr} \leq 500$$
$$1 \leq \mathrm{Re}_D \leq 10^6$$

Correlation coefficients are presented at Table 7-4.

Table 7-4 *Correlation coefficients for Zukauskas correlation*



Re_D	C	m
1–40	0.75	0.4
40–1000	0.51	0.5
10^3–2×10^5	0.26	0.6
2×10^5–10^6	0.076	0.7

<u>Churchill & Bernstein</u>

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282000}\right)^{5/8}\right]^{4/5} \qquad (7\text{-}16)$$

All material properties should be evaluated at film temperature. Entire range of Reynolds numbers is covered and Pr > 0.2 is recommended.

7.4 SPHERE

Boundary layer phenomenon in flow over sphere is very much similar to those at cross flow over a circular cylinder. Convection heat transfer coefficient correlations is suggested by Whittaker as

$$\overline{Nu}_D = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4} \qquad (7\text{-}17)$$

All the properties are calculated at bulk flow temperature t_∞ except surface viscosity μ_s and correlation is valid for

$$0.71 \le \mathrm{Pr} \le 380$$
$$3.5 \le \mathrm{Re}_D \le 7.6 * 10^4$$
$$1.0 \le (\mu/\mu_s) \le 3.2$$

Another average convection heat transfer correlation for sphere is presented by Ranz & Marshall

$$\overline{Nu}_D = 2 + 0.6\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}$$

, where first term corresponds pure conduction to stationary medium as

$$\overline{Nu}_D = 2$$

7.5 FLOW ACROSS BANKS OF TUBES

Common heat exchanger design in industrial applications is a bank of tubes in a cross flow. Arrangement of tubes are either aligned or staggered. Dimensions of tube bank design are characterized by tube diameter D, longitudinal pitch S_T and transverse pitch S_L (Figure 7-5).

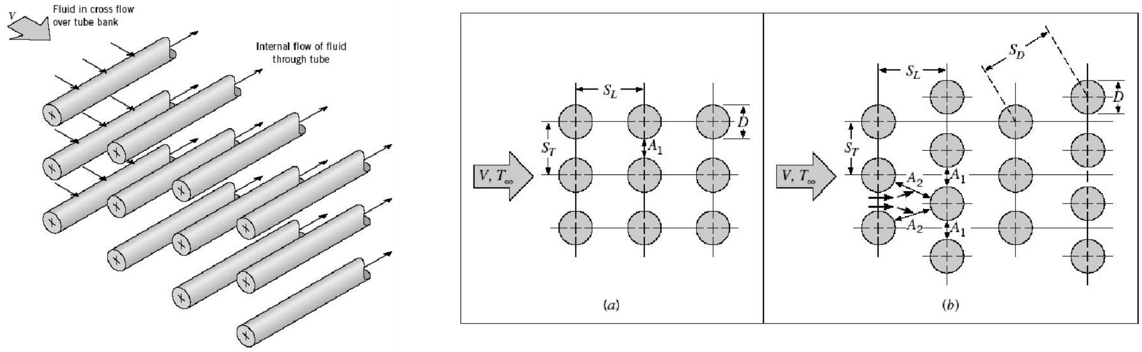


Figure 7-5 *Tube bank in a cross flow and arrangements of tubes: a) Aligned b) staggered tube arrangements in a bank*

Heat transfer coefficient depends on tube position in bank. Flow conditions and thus heat transfer coefficient in a first row of tubes are approximately equal to single tube in a cross flow. Heat transfer coefficient increases in next inner rows due to turbulent flow conditions caused by the first tube rows. Usually after fourth or fifth row heat transfer and flow conditions are stabilized and after this only minor change of heat transfer coefficient occurs.

Beyond the first tube aligned tubes are in wake turbulence flow caused by upstream tubes. In heat transfer point of view, aligned designs with S_L /S_T<0.7 are not recommended. In staggered designs heat transfer is enhanced by tortuous flow, particularly with small Reynolds numbers Re <100.

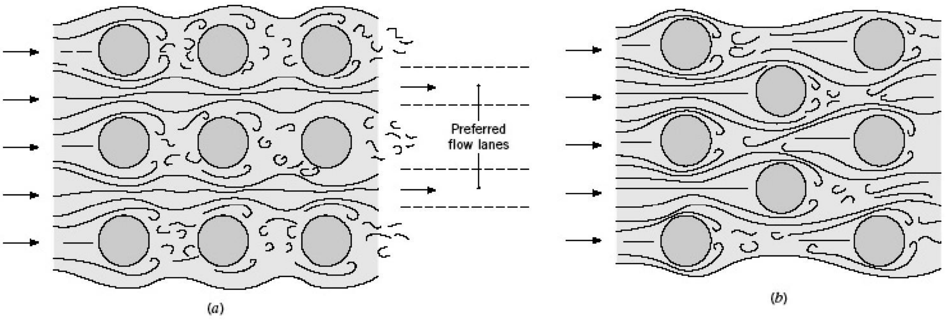


Figure 7-6 *Flow Conditions for a) aligned and b) staggered*

7.5.1 Grimison's Correlation

For air flow across tube bundles of 10 or more rows N_L, Grimison has suggested following correlation for dimensionless heat transfer coefficient

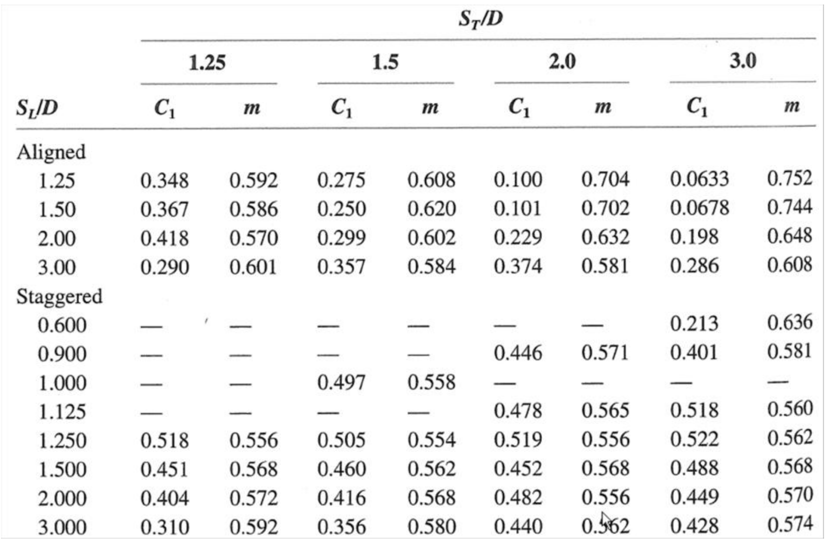
$$\overline{Nu}_D = C_1 \mathrm{Re}_D^{1/2} \tag{7-18}$$

$$NL \geq 10$$
$$2000 \leq \mathrm{Re}_D \leq 40000$$
$$\mathrm{Pr} = 0.7$$

Where correlation constants are shown in Table 7-5 and Reynolds number for cross flow is determined as

$$\mathrm{Re}_D = \frac{\rho V_{\max} D}{\mu} \tag{7-19}$$

Table 7-5 *Correlation constants for cross airflow over a tube bank*



	S_T/D							
	1.25		1.5		2.0		3.0	
S_L/D	C_1	m	C_1	m	C_1	m	C_1	m
Aligned								
1.25	0.348	0.592	0.275	0.608	0.100	0.704	0.0633	0.752
1.50	0.367	0.586	0.250	0.620	0.101	0.702	0.0678	0.744
2.00	0.418	0.570	0.299	0.602	0.229	0.632	0.198	0.648
3.00	0.290	0.601	0.357	0.584	0.374	0.581	0.286	0.608
Staggered								
0.600	—	—	—	—	—	—	0.213	0.636
0.900	—	—	—	—	0.446	0.571	0.401	0.581
1.000	—	—	0.497	0.558	—	—	—	—
1.125	—	—	—	—	0.478	0.565	0.518	0.560
1.250	0.518	0.556	0.505	0.554	0.519	0.556	0.522	0.562
1.500	0.451	0.568	0.460	0.562	0.452	0.568	0.488	0.568
2.000	0.404	0.572	0.416	0.568	0.482	0.556	0.449	0.570
3.000	0.310	0.592	0.356	0.580	0.440	0.562	0.428	0.574

All properties are evaluated in film temperature. Maximum fluid velocity is calculated at smallest cross-section:

$$V_{\max} = \frac{S_T}{S_T - D} V \qquad \textit{Aligned arrangement} \tag{7-20}$$

$$V_{\max} = \frac{S_T}{2(S_D - D)} V \qquad \textit{Staggered arrangement} \tag{7-21}$$

Heat transfer correlation is extended for other fluids as

$$\overline{Nu}_D = 1.13 C_1 \mathrm{Re}_D^{1/2} \mathrm{Pr}^{1/3} \tag{7-22}$$

$$NL \geq 10$$
$$2000 \leq \mathrm{Re}_D \leq 40000$$
$$\mathrm{Pr} \geq 0.7$$

7.5.2 Zukauskas Correlation

Another heat transfer correlation is recently suggested by Zukauskas for cross flow over tube banks as

$$\overline{Nu}_D = CC_2 \mathrm{Re}^{m}_{D,\max} \mathrm{Pr}^{0.36}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4} \tag{7-23}$$

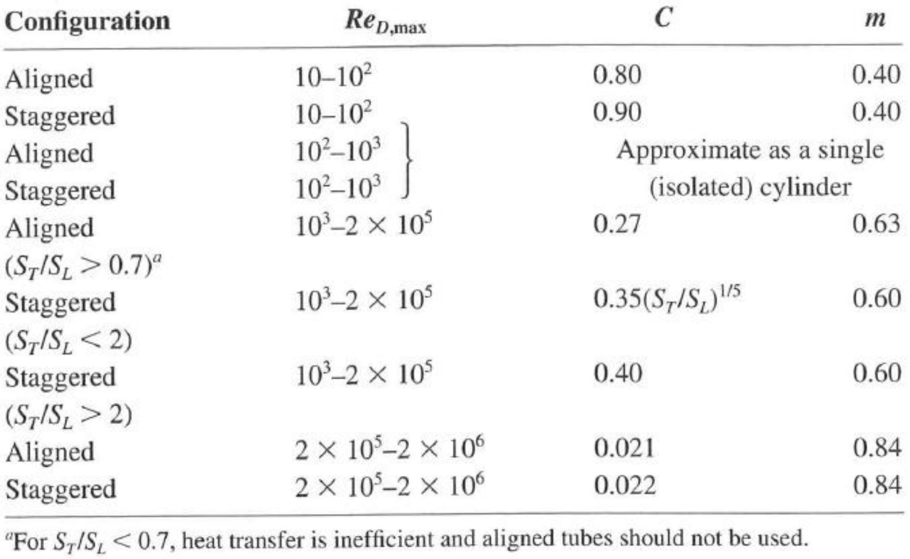
$$NL \geq 20$$

$$1000 \leq \mathrm{Re}_D \leq 2*10^6$$

$$0.7 \leq \mathrm{Pr} \leq 500$$

Constants are shown in Table 7-5 and all properties except $\mathrm{Pr_s}$ (*at surface tempetature*) are calculated in arithmetic mean temperature: $(T_i$-$T_o)/2$ of fluid inlet and outlet temperatures.

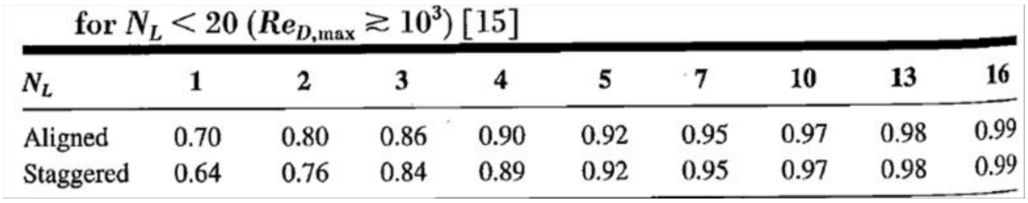
Table 7-6 *Zukauskas correlation constants*



Configuration	$Re_{D,\max}$	C	m
Aligned	10–10^2	0.80	0.40
Staggered	10–10^2	0.90	0.40
Aligned	10^2–10^3	Approximate as a single (isolated) cylinder	
Staggered	10^2–10^3		
Aligned $(S_T/S_L > 0.7)^a$	10^3–2×10^5	0.27	0.63
Staggered $(S_T/S_L < 2)$	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered $(S_T/S_L > 2)$	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.021	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

[a] For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

Table 7-7 *Correction factor C_2 for equation (7.23) (if number of rows < 20)*



for $N_L < 20$ $(Re_{D,\max} \gtrsim 10^3)$ [15]

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

7.5.3 Log-mean Temperature Difference

Heat transfer rate per unit length of tubes can be solved from

$$q' = N\bar{h}\pi D\Delta T_{lm} \tag{7-24}$$

N	*Number of tubes in bank*

Where appropriate form of temperature difference is *log-mean temperature difference* defined as

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_o}{T_s - T_i}\right)} \tag{7-25}$$

T_i	*Inlet fluid temperature*
T_s	*Outlet fluid temperature*

Fluid Outlet temperature may estimated from

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right) \tag{7-26}$$

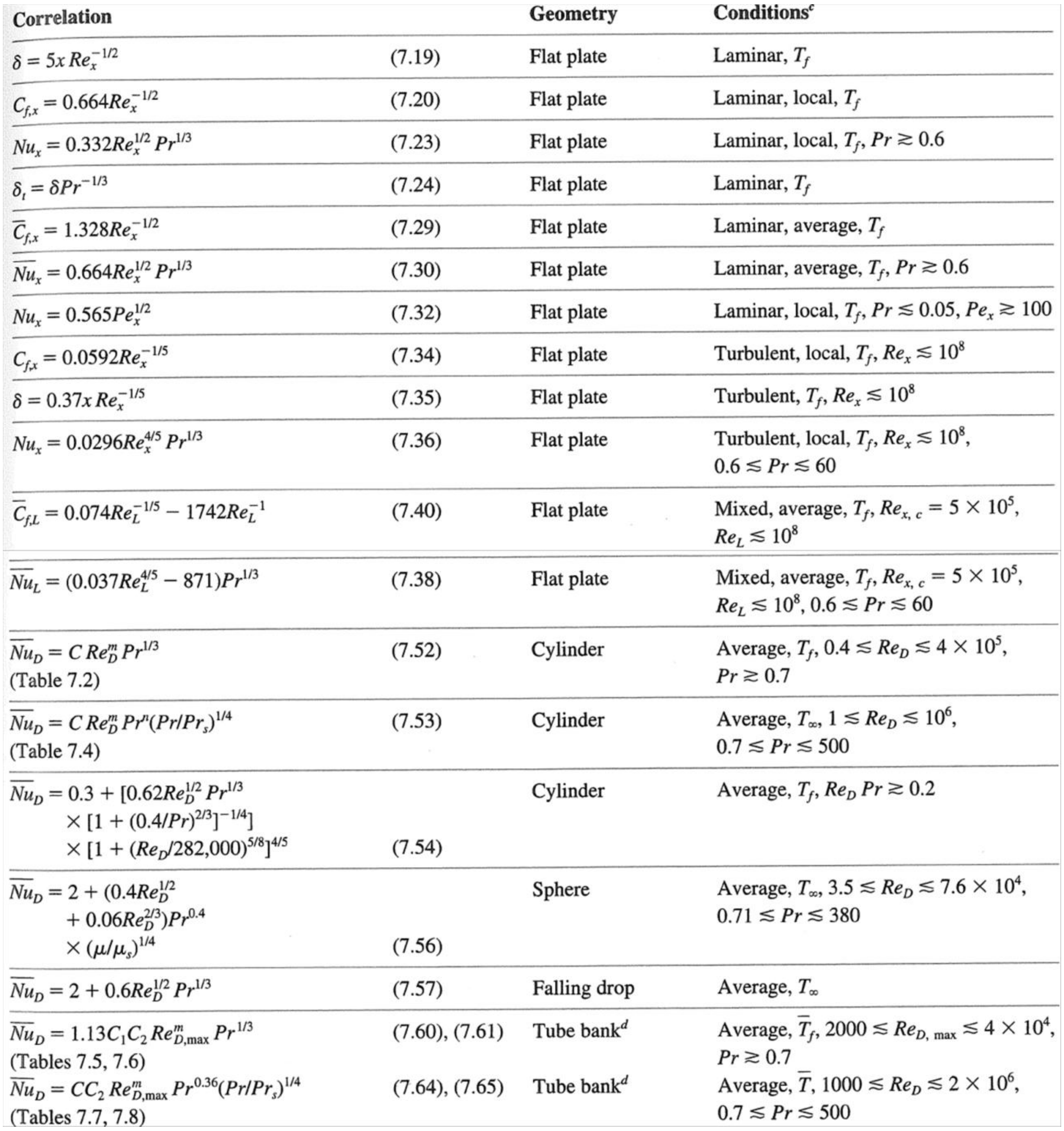
N_T	*number of tubes in transverse lane*

7.6 SELECTION OF CORRELATION

Following steps should be considered, while selecting correlation for specific application:

1) Flow conditions: Laminar or turbulent
2) Validity range: Re, Pr
3) Use or availability: Local / average
4) Geometry: plate, cylinder, sphere, tube bank, etc.
5) Reference temperature of fluid properties

Table 7-8 *Summary of convection correlations for external flow*

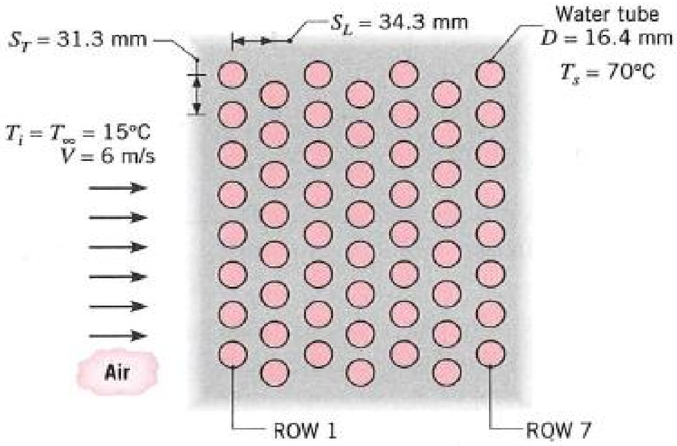


Correlation		Geometry	Conditions[c]
$\delta = 5x\,Re_x^{-1/2}$	(7.19)	Flat plate	Laminar, T_f
$C_{f,x} = 0.664 Re_x^{-1/2}$	(7.20)	Flat plate	Laminar, local, T_f
$Nu_x = 0.332 Re_x^{1/2}\,Pr^{1/3}$	(7.23)	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$
$\delta_t = \delta Pr^{-1/3}$	(7.24)	Flat plate	Laminar, T_f
$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$	(7.29)	Flat plate	Laminar, average, T_f
$\overline{Nu}_x = 0.664 Re_x^{1/2}\,Pr^{1/3}$	(7.30)	Flat plate	Laminar, average, T_f, $Pr \gtrsim 0.6$
$Nu_x = 0.565 Pe_x^{1/2}$	(7.32)	Flat plate	Laminar, local, T_f, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$
$C_{f,x} = 0.0592 Re_x^{-1/5}$	(7.34)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\,Re_x^{-1/5}$	(7.35)	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296 Re_x^{4/5}\,Pr^{1/3}$	(7.36)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$	(7.40)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$	(7.38)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3}$ (Table 7.2)	(7.52)	Cylinder	Average, T_f, $0.4 \lesssim Re_D \lesssim 4 \times 10^5$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C\,Re_D^m\,Pr^n (Pr/Pr_s)^{1/4}$ (Table 7.4)	(7.53)	Cylinder	Average, T_∞, $1 \lesssim Re_D \lesssim 10^6$, $0.7 \lesssim Pr \lesssim 500$
$\overline{Nu}_D = 0.3 + [0.62 Re_D^{1/2}\,Pr^{1/3}$ $\times [1 + (0.4/Pr)^{2/3}]^{-1/4}]$ $\times [1 + (Re_D/282{,}000)^{5/8}]^{4/5}$	(7.54)	Cylinder	Average, T_f, $Re_D\,Pr \gtrsim 0.2$
$\overline{Nu}_D = 2 + (0.4 Re_D^{1/2}$ $+ 0.06 Re_D^{2/3}) Pr^{0.4}$ $\times (\mu/\mu_s)^{1/4}$	(7.56)	Sphere	Average, T_∞, $3.5 \lesssim Re_D \lesssim 7.6 \times 10^4$, $0.71 \lesssim Pr \lesssim 380$
$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2}\,Pr^{1/3}$	(7.57)	Falling drop	Average, T_∞
$\overline{Nu}_D = 1.13 C_1 C_2\,Re_{D,\max}^m\,Pr^{1/3}$ (Tables 7.5, 7.6)	(7.60), (7.61)	Tube bank[d]	Average, $\overline{T}_f$, $2000 \lesssim Re_{D,\max} \lesssim 4 \times 10^4$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C C_2\,Re_{D,\max}^m\,Pr^{0.36} (Pr/Pr_s)^{1/4}$ (Tables 7.7, 7.8)	(7.64), (7.65)	Tube bank[d]	Average, $\overline{T}$, $1000 \lesssim Re_D \lesssim 2 \times 10^6$, $0.7 \lesssim Pr \lesssim 500$

Example VIII: ***Air flow over tube bank – Cross flow HX***

Air is used for cooling water in a cross flow tube bank. The tube bank is staggered and the air is forced to flow across the tube bank with velocity of $v = 6\ \mathrm{m/s}$. The air upstream temperature is 15 °C. The tube outside diameter is 16,4 mm and the longitudinal and transverse pitches are $S_\mathrm{L} = 34{,}3\ \mathrm{mm}$ and $S_\mathrm{T} = 31{,}3\ \mathrm{mm}$. There are seven rows of tubes in the airflow direction and eight tubs per row. Under typical operating conditions the cylinder surface temperature is at 70 °C.

a) Determine the air-side convection coefficient and the rate of heat transfer for the tube bundle.
b) What is the air-side pressure drop?



PROPERTIES: *Table A-4*:

air ($T_\infty = 15$ °C):
$\rho = 1{,}217\ \mathrm{kg/m^3}$
$C_\mathrm{p} = 1007\ \mathrm{J/KgK}$
$\nu = 14{,}82 \cdot 10^{-6}\ \mathrm{m^2/s}$,
$k = 0{,}0253\ \mathrm{W/mK}$,
$\mathrm{Pr} = 0{,}710$.

air ($T_\mathrm{s} = 70$ °C):
$\mathrm{Pr} = 0{,}701$. ($T_\mathrm{f} = 43$ °C):
$\nu = 17{,}4 \cdot 10^{-6}\ \mathrm{m^2/s}$,
$k = 0{,}0274\ \mathrm{W/mK}$,
$\mathrm{Pr} = 0{,}705$

ASSUMPTIONS:
(1) Steady-state conditions,
(2) Negligible radiation effects,
(3) Negligible effects of change in air temperature across tube bank on air properties.

ANALYSIS:

a) From equation 7-22 (Zukauskas correlation), the air-side Nusselt number is

$$\overline{Nu}_\mathrm{D} = C_2 C \mathrm{Re}_\mathrm{D,max}^m \mathrm{Pr}^{0{,}36} \left(\frac{\mathrm{Pr}}{\mathrm{Pr_s}}\right)^{1/4}$$

Since $S_\mathrm{D} = \sqrt{S_\mathrm{L}^2 + (S_\mathrm{T}/2)^2} = 37{,}7\ \mathrm{mm}$ is greater than $(S_\mathrm{T} + D)/2$, the maximum velocity occurs on the transverse plane A_1. Hence

$$V_\mathrm{max} = \frac{S_\mathrm{T}}{S_\mathrm{T} - D} V = \frac{31{,}3\ \mathrm{mm}}{(31{,}1 - 16{,}4)\ \mathrm{mm}} \cdot 6\ \mathrm{m/s} = 12{,}6\ \mathrm{m/s}$$

$$\mathrm{Re_{D,max}} = \frac{V_\mathrm{max} D}{\nu} = \frac{12{,}6 \frac{\mathrm{m}}{\mathrm{s}} \cdot 0{,}0164\ \mathrm{m}}{14{,}82 \cdot 10^{-6}\ \mathrm{m^2/s}} = 13{,}943$$

$$\frac{S_\mathrm{T}}{S_\mathrm{L}} = \frac{31{,}3\ \mathrm{mm}}{34{,}3\ \mathrm{mm}} = 0{,}91 < 2$$

It follows that from Tables 7-6

$$C = 0{,}35 \left(\frac{S_\mathrm{T}}{S_\mathrm{L}}\right)^{1/5} = 0{,}34 \qquad m = 0{,}6 \qquad C_2 = 0{,}95$$

$$\overline{Nu}_\mathrm{D} = 0{,}95 \cdot 0{,}34 \cdot 13{,}943^{0{,}6} \cdot 0{,}71^{0{,}36} \left(\frac{0{,}71}{0{,}701}\right)^{1/4} = 87{,}9$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 87{,}9 \cdot \frac{0{,}0253\ \mathrm{W/mK}}{0{,}0164} = 135{,}6\ \mathrm{W/m^2K}$$

From equation 7.67

$$T_s - T_o = (T_s - T_i) \cdot \exp\left(\frac{\pi D N \mathrm{h}}{\rho V N_T S_T C_p}\right)$$

$$T_s - T_o = 55\ ^\circ\mathrm{C} \cdot \exp\left(\frac{\pi \cdot 0{,}0164\ \mathrm{m} \cdot 56 \cdot 135{,}6\ \mathrm{W/m^2K}}{1{,}217 \frac{\mathrm{kg}}{\mathrm{m^3}} \cdot 6 \frac{\mathrm{m}}{\mathrm{s}} \cdot 8 \cdot 0{,}0313\ \mathrm{m} \cdot 1007\ \mathrm{J/KgK}}\right)$$

$$T_s - T_o = 44{,}5\ ^\circ\mathrm{C}$$

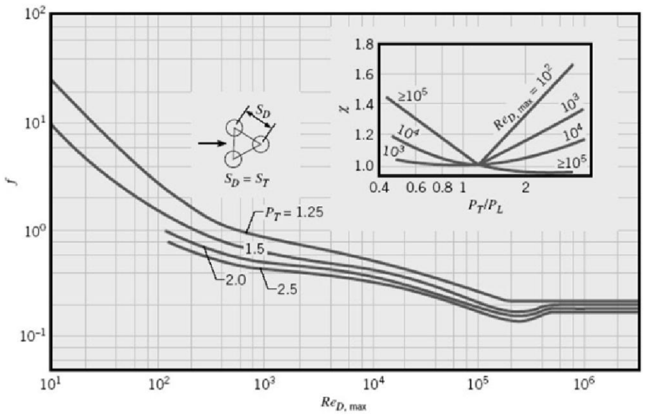
Heat flux is calculated from eq. 7-23 and 7-24

$$\Delta T_{\mathrm{lm}} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = \frac{55\ ^\circ\mathrm{C} - 44{,}5\ ^\circ\mathrm{C}}{\ln\left(\frac{55\ ^\circ\mathrm{C}}{44{,}5\ ^\circ\mathrm{C}}\right)} = 49{,}6\ ^\circ\mathrm{C}$$

$$q' = N\left(\bar{h}\pi D \Delta T_{\mathrm{lm}}\right) = 56\pi \cdot 135{,}6 \frac{\mathrm{W}}{\mathrm{m^2K}} \cdot 0{,}0164\ \mathrm{m} \cdot 49{,}6\ ^\circ\mathrm{C} = 19{,}4\ \mathrm{kW/m}$$

b) The pressure drop from equation

$$\Delta p = N_L \chi \left(\frac{\rho V_{\max}}{2}\right) f$$



From fig. $\mathrm{Re}_{D,\max} = 13{,}943\ P_T = (S_T/D) = 1{,}91$, and $\frac{P_T}{P_L} = 0{,}91$

$$\chi = 1{,}04$$

$$f = 0{,}35$$

$$\Delta p = 7 \cdot 1{,}04 \left(\frac{1{,}217\ \mathrm{kg/m^3} \cdot 12{,}6\ \mathrm{m/s}}{2}\right) \cdot 0{,}35 = 246 \frac{\mathrm{N}}{\mathrm{m^2}} = 2{,}46 \cdot 10^{-3}\ \mathrm{bars}$$

7.7 LEARNING OUTCOMES

Table 7-9 Learning Outcomes: *Forced Convection* - *External*

Level of Knowledge	Concept
Understand *Describe*	• *Principle of analytical solution* • *Development of Boundary Layers: cylinder* • *Basis for Selection of correlation*
Apply ***understand*** *describe*	• *Empirical Nu correlations: plate, cylinder, sphere & tube bank* • *Log-mean temperature difference – bank of tubes*

8 FORCED CONVECTION – INTERNAL FLOW

Unlike in external flow, boundary layer development is constrained by surface in internal flow. Within internal flow, *hydrodynamic entrance* or *fully developed* flow regions determine flow conditions besides laminar or turbulent flow type.

8.1 HYDRODYNAMICS

Let us study laminar flow in a circular tube, where fluid enters with uniform velocity. The viscous effect of tube walls will form boundary layer developing with increasing distance x from entrance. Simultaneously uniform velocity, *inviscid flow*, region shrinks until boundary layers merge at centreline of tube. Point of merging boundary layers is end of *hydrodynamic entrance region* and beginning of *fully developed* flow region.

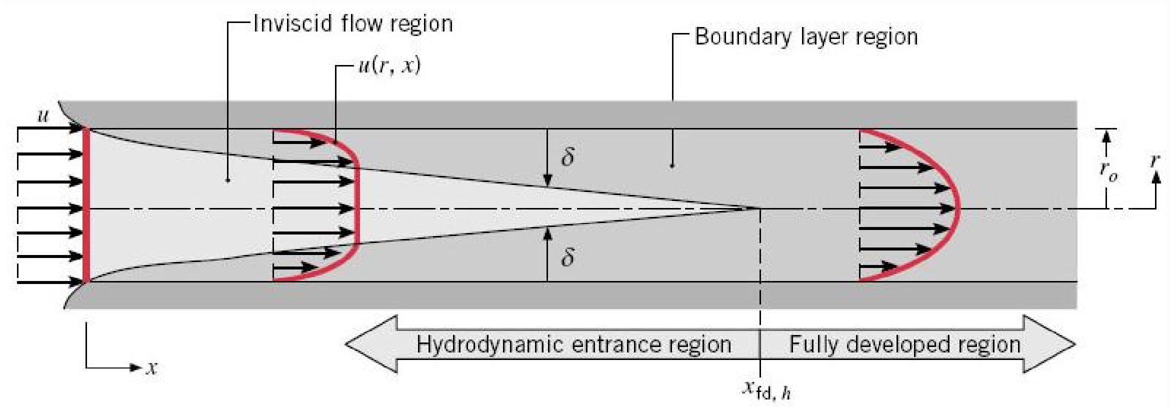


Figure 8-1 *Laminar hydrodynamic boundary layer development in tube flow*

Reynolds number for circular tube flow is defined as

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} \qquad (8\text{-}1)$$

u_m *Mean velocity in tube cross-section*

In a fully developed flow, critical Reynolds number corresponding start of turbulence flow behaviour is

$$\mathrm{Re}_{D,C} = 2300$$

Although larger Reynolds number (~10000) is needed for achieving fully turbulent conditions.
For laminar flow, one correlation form for *hydrodynamic entry length* is given as

$$\left(\frac{x_{fd,h}}{D}\right)_{lam} \approx 0.05\,\mathrm{Re}_D \qquad (8\text{-}2)$$

For turbulent flow, *hydrodynamic entry length* is not dependent on Reynolds number and a rough approximation is expressed as

$$10 \leq \left(\frac{x_{fd,h}}{D} \right)_{turb} \leq 60 \tag{8-3}$$

Mean velocity for incompressible flow can solved form

$$u_m = \frac{2}{r_o^2} \int_0^{r_o} u(r,x) r dr \tag{8-4}$$

And for laminar fully developed flow velocity profile is

$$\frac{u(r)}{u_m} = 2 \left[1 - \left(\frac{r}{r_0} \right)^2 \right] \tag{8-5}$$

Shape of laminar velocity profile in fully developed region is parabolic, while turbulent profile is more flat due to turbulent radial mixing. At entrance region velocity gradients are high near wall surface and decreases together with increasing boundary layer thickness towards fully developed region.

8.2 THERMAL PERFORMANCE

Let us now look to thermal behaviour of uniform temperature fluid flow entering to circular tube. When tube surface temperature is higher than fluid temperature, convection heat transfer develops a thermal boundary layer. *Thermally fully developed region* is eventually reached and the shape of temperature profile depends on whether boundary condition is

a) uniform surface temperature
or
b) constant surface heat flux.

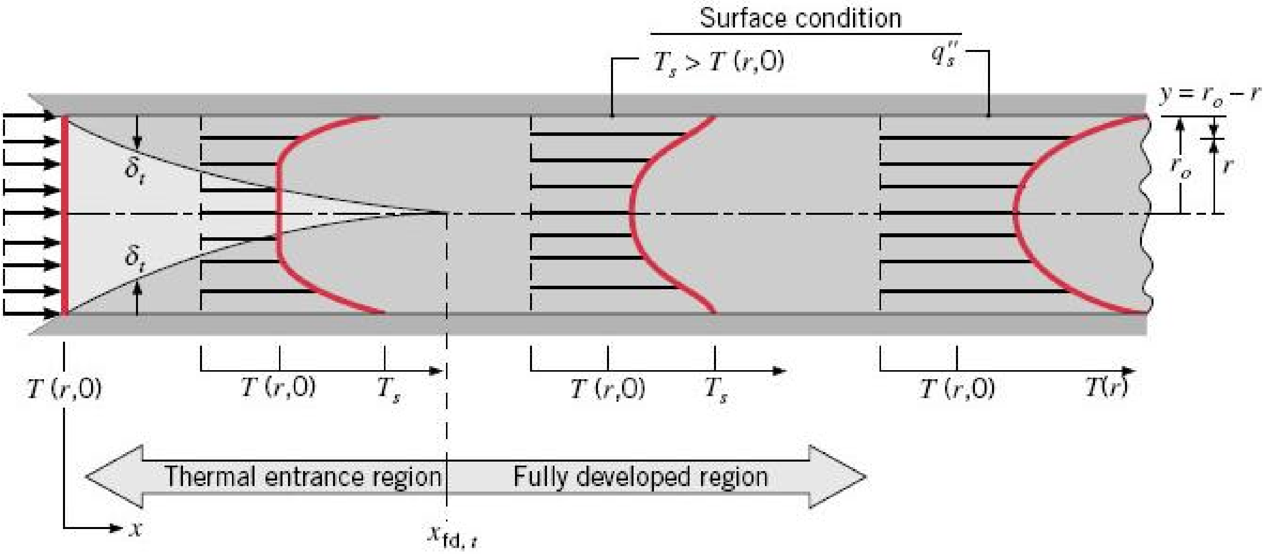


Figure 8-2 *Thermal boundary layer development*

For laminar flow, one correlation form for *thermal entry length* is given as

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} \approx 0.05 \mathrm{Re}_D \Pr \tag{8-6}$$

For extremely high Prandtl numbers (oils) a practical assumption is velocity profile as fully developed also throughout entrance region. For turbulent flows, high Reynolds number, conditions are independent from Pr and first rule of thumb approximation would be

$$\left(\frac{x_{fd,t}}{D}\right)_{turb} = 10 \tag{8-7}$$

Newton's law of cooling for internal flows has a form

$$q_s^{"} = h(T_S - T_m) \tag{8-8}$$

Where mean temperature is reference temperature and can be solved from

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} uTrdr \tag{8-9}$$

In thermally fully developed flow with constant properties local convection heat transfer coefficient is constant (Figure 8-3). Even mean temperature changes with distance from entry of tube, dimensionless temperature profile is independent of distance:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]_{fd,t} = 0 \tag{8-10}$$

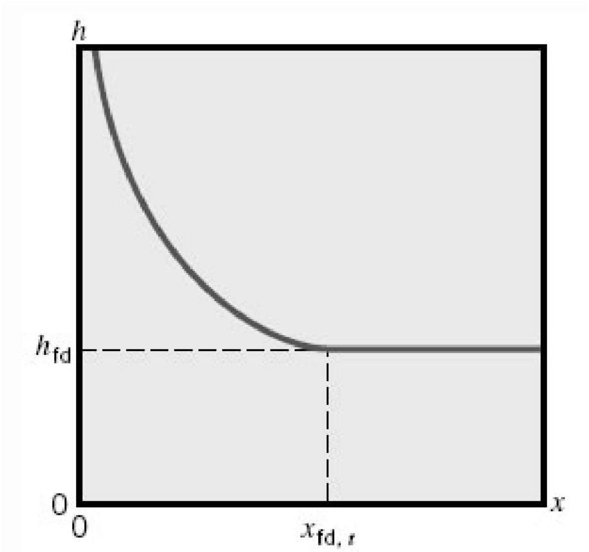
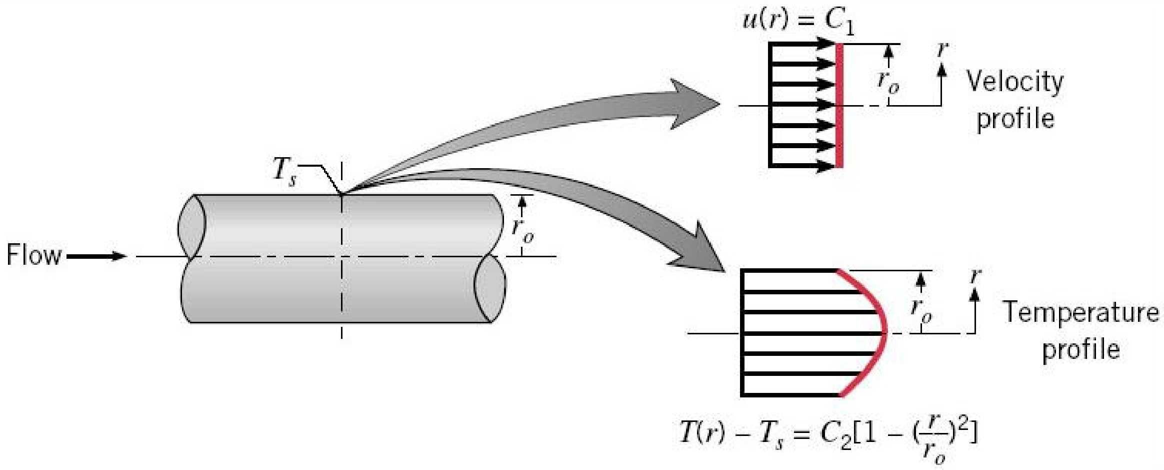


Figure 8-3 *Axial variation of convection heat transfer coefficient*

Example IX: ***Tube flow***

What are mean temperature and Nusselt number for tube flow with known velocity and temperature profiles?



ASSUMPTIONS: constant properties, incompressible

ANALYSIS: Mean temperature - Definition

$$T_m = \frac{2}{u_m r_0^{\ 2}}\int_0^{r_0} uTrdr = \frac{2C_1}{u_m r_0^{\ 2}}\int_0^{r_0}\left\{T_s + C_2\left[1-\left(\frac{r}{r_o}\right)^2\right]\right\}rdr$$

$$= \frac{2C_1}{u_m r_0^2}\left[T_s\frac{r^2}{2} + C_2\frac{r^2}{2} - \frac{C_2}{4}\frac{r^4}{r_0^2}\right]\Bigg|_0^{r_0}$$

$$= \frac{2C_1}{u_m r_0^2}\left(T_s\frac{r_0^{\ 2}}{2} + C_2\frac{r_0^{\ 2}}{2} - \frac{C_2}{4}r_0^{\ 2}\right)$$

$$= T_s + \frac{C_2}{2} \qquad \left(Note! \quad here\ C_1 = u_m\right)$$

Heat Flux – Fourier Law

$$q_s'' = k\frac{\partial T}{\partial r}\bigg|_{r=r_o} = -kC_2 2\frac{r}{r_o^2}\bigg|_{r=r_o} = -2C_2\frac{k}{r_o^2}\bigg|$$

Convection coefficient

$$h = \frac{q_s''}{T_s - T_m} = \frac{-2C_2(k/r_o)}{-C_2/2} = \frac{4k}{r_o}$$

Nusselt number

$$Nu = \frac{hD}{k} = \frac{(4k/r_o)*2r_o}{k} = 8$$

8.2.1 Energy Balance

Mean temperature variation in tube can be solved by energy balance approach. If steady-state convection flow occurs in tube with negligible viscous dissipation, incompressibility or ideal gas assumption with low pressure variation is reasonable, energy equation can be written to form

$$q_{conv} = \dot{m} c_p (T_{m,o} - T_{m,i}) \tag{8-11}$$

With constant heat flux boundary condition, mean temperature can be solved from energy equation as

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x \tag{8-12}$$

With constant surface temperature boundary condition, mean temperature has following relationship to flow distance x

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{P x \bar{h}}{\dot{m} c_p} \right) \tag{8-13}$$

$\bar{h}$ *Average convection coefficient from entry to distance x*

Surface temperature and flow mean temperatures in two common boundary conditions: constant surface heat flux and constant surface temperature; are shown in Figure 8-4.

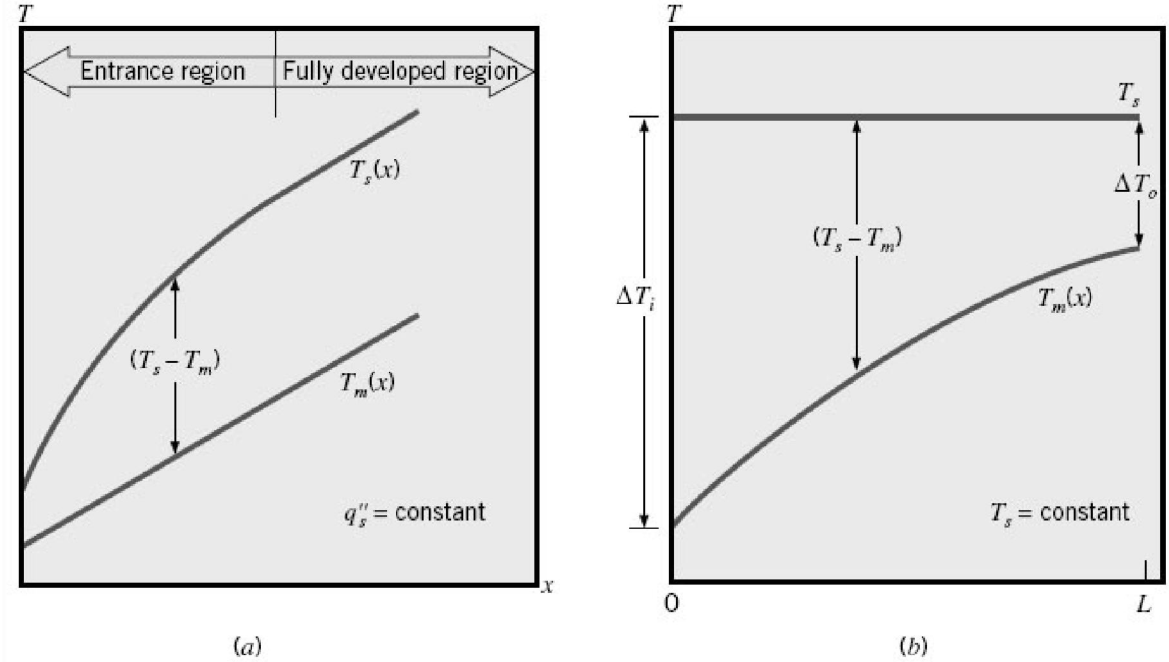
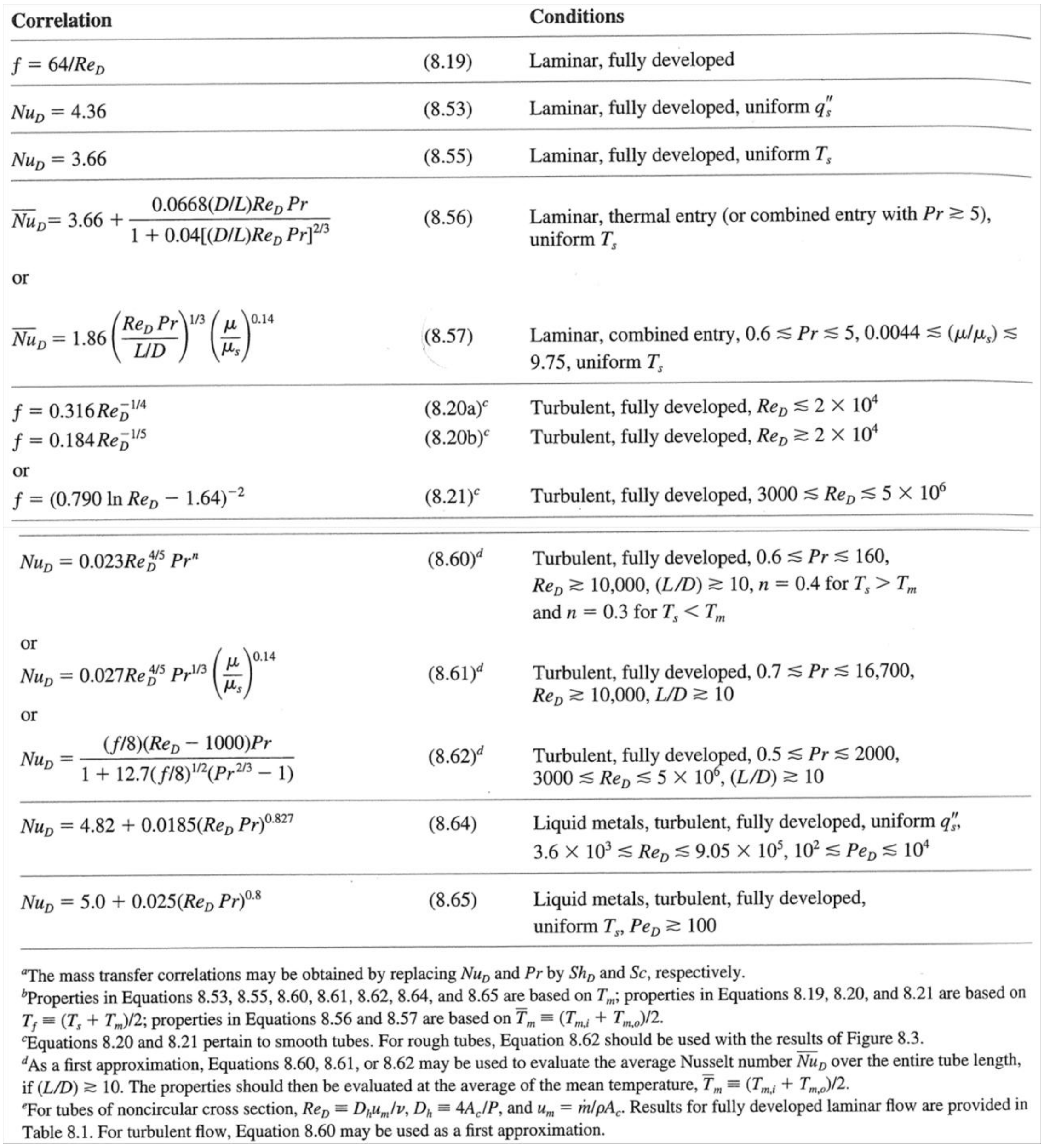


Figure 8-4 *Axial temperature variations for heat transfer in a tube a) constant surface heat flux b) constant surface temperature*

8.3 CONVECTION HEAT TRANSFER CORRELATIONS

Convection heat transfer correlations are summarized in table below and following chapter gives a brief introduction to principles beyond the equations.

Table 8-1 *Convection heat transfer correlations for flow in circular tube*



Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q''_s
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668(D/L)Re_D\,Pr}{1 + 0.04[(D/L)Re_D\,Pr]^{2/3}}$	(8.56)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s
or		
$\overline{Nu}_D = 1.86\left(\dfrac{Re_D\,Pr}{L/D}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.57)	Laminar, combined entry, $0.6 \lesssim Pr \lesssim 5$, $0.0044 \lesssim (\mu/\mu_s) \lesssim 9.75$, uniform T_s
$f = 0.316 Re_D^{-1/4}$	(8.20a)[c]	Turbulent, fully developed, $Re_D \lesssim 2 \times 10^4$
$f = 0.184 Re_D^{-1/5}$	(8.20b)[c]	Turbulent, fully developed, $Re_D \gtrsim 2 \times 10^4$
or		
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[c]	Turbulent, fully developed, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023 Re_D^{4/5}\,Pr^n$	(8.60)[d]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
or		
$Nu_D = 0.027 Re_D^{4/5}\,Pr^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[d]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
or		
$Nu_D = \dfrac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62)[d]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D\,Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q''_s, $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $10^2 \lesssim Pe_D \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D\,Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Pe_D \gtrsim 100$

[a] The mass transfer correlations may be obtained by replacing Nu_D and Pr by Sh_D and Sc, respectively.

[b] Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on T_m; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f \equiv (T_s + T_m)/2$; properties in Equations 8.56 and 8.57 are based on $\overline{T}_m \equiv (T_{m,i} + T_{m,o})/2$.

[c] Equations 8.20 and 8.21 pertain to smooth tubes. For rough tubes, Equation 8.62 should be used with the results of Figure 8.3.

[d] As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{T}_m \equiv (T_{m,i} + T_{m,o})/2$.

[e] For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

8.3.1 Laminar Flows

Energy balance for differential element in laminar, fully developed flow of incompressible, constant property fluid can be written with axial advection and radial conduction terms as

$$(d\dot{m})c_p \frac{\partial T}{\partial x}dx = q_r - \left(q_r + \frac{\partial q_r}{\partial r}dr\right) = -\frac{\partial q_r}{\partial r}dr \qquad \text{(8-14)}$$

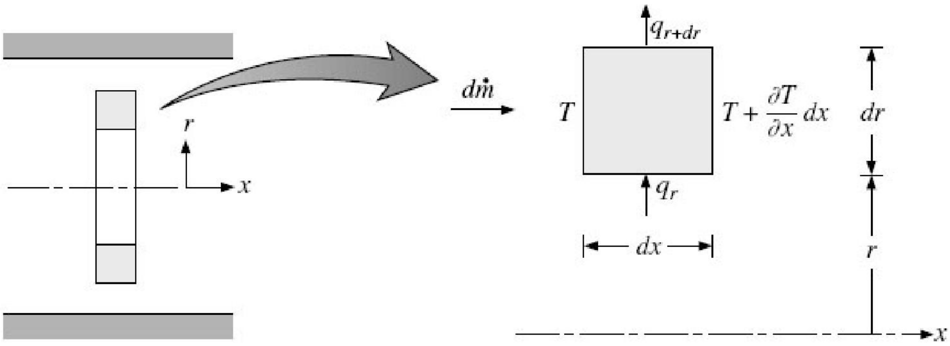


Figure 8-5 *Differential energy balance for in laminar, fully developed flow of incompressible, constant property fluid*

By substituting definitions of mass flow rate $d\dot{m} = \rho u 2\pi r dr$ and radial heat transfer rate $q_r = -k(\partial T/\partial r)2\pi r dx$, energy equation reduces to

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \qquad \text{(8-15)}$$

Constant heat flux - Fully developed conditions

For constant heat flux energy equation can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[1-\left(\frac{r}{r_o}\right)^2\right] \qquad \text{(8-16)}$$

Differential equation has a solution as constant Nusselt number

$$Nu_d = \frac{hD}{k} = 4.36$$

Constant surface temperature - Fully developed conditions

For constant heat flux energy equation can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[1-\left(\frac{r}{r_o}\right)^2\right]\frac{T_s - T}{T_s - T_m} \qquad \text{(8-17)}$$

Solution for above differential equation is a constant Nusselt number

$$Nu_d = 3.66$$

Conclusion: Nusselt number is constant and independent of Re, Pr and distance x for laminar, fully developed internal flow in circular tubes.

8.3.2 Entry Region

Figure 8-6 a presents Nusselt number solutions for *thermal entry length* with assumption of fully developed velocity profile, which would be in cases with unheated starting length or with large Prandtl number fluids, i.e. oils. Heat transfer coefficient for fully developed velocity profile is independent on viscosity and thus to change of Prandtl number.

For *simultaneous laminar velocity boundary layer and thermal boundary layer development* viscous forces, and *Prandtl* number, have effect to entry length heat transfer solution. Figure 8-6 a presents also combined entry length solution for Prandtl number 0.7, which is usually suitable in case of gas flows.

Correlation for average Nusselt number at entrance region for thermal entry length or low Prandtl number (Pr>5) problems, correlation has been presented as (Figure 8-6 b)

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)\,\mathrm{Re}_D\,\mathrm{Pr}}{1+0.04\left[(D/L)\,\mathrm{Re}_D\,\mathrm{Pr}\right]^{2/3}} \qquad (8\text{-}18)$$

For combined entry length problems Sieder and Tate correlation has form of

$$\overline{Nu}_D = 1.86\left(\frac{\mathrm{Re}_D\,\mathrm{Pr}}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad 0.60 < \mathrm{Pr} < 5 \qquad (8\text{-}19)$$

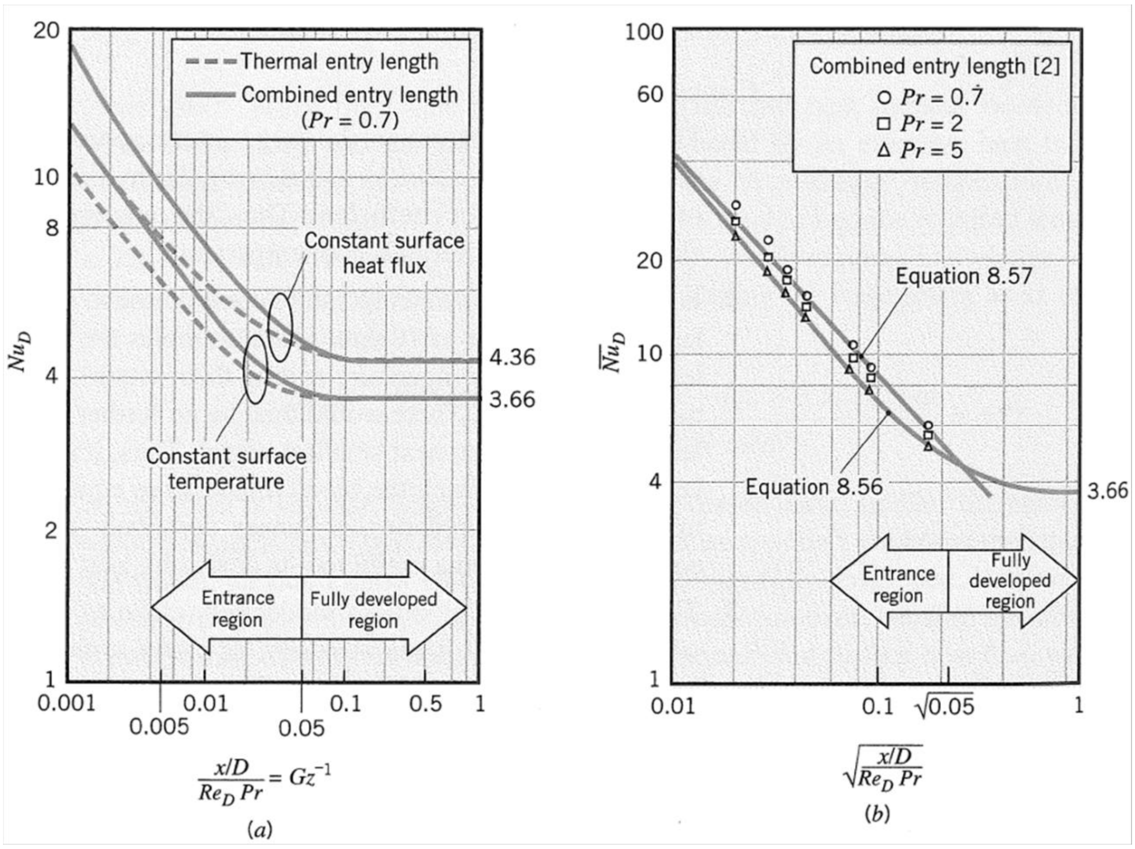


Figure 8-6 *Nusselt numbers entry length solutions for laminar flow in circular tube a) local Nusselt number solutions b) average Nusselt number correlations and test results*

8.3.3 Pressure drop and friction factor in fully developed flow

Moody friction factor is defined as

$$f = \frac{-(dp/dx)D}{\rho u_m^2/2} \tag{8-20}$$

Fanning friction coefficient is related to friction factor as

$$C_f = \frac{f}{4} = \frac{\tau_s}{\rho u_m^2/2} \tag{8-21}$$

Moody friction factor can be written for fully developed laminar flow as

$$f = \frac{64}{\mathrm{Re}_D} \tag{8-22}$$

Friction factors are presented at Table 8-1 for different flow regimes and in Figure 8-7. Pressure drop for fully developed flow between positions x_1 and x_2 is

$$\Delta p = -\int_{p_1}^{p_2} dp = f\,\frac{\rho u_m^2}{2D}(x_1 - x_2) \tag{8-23}$$

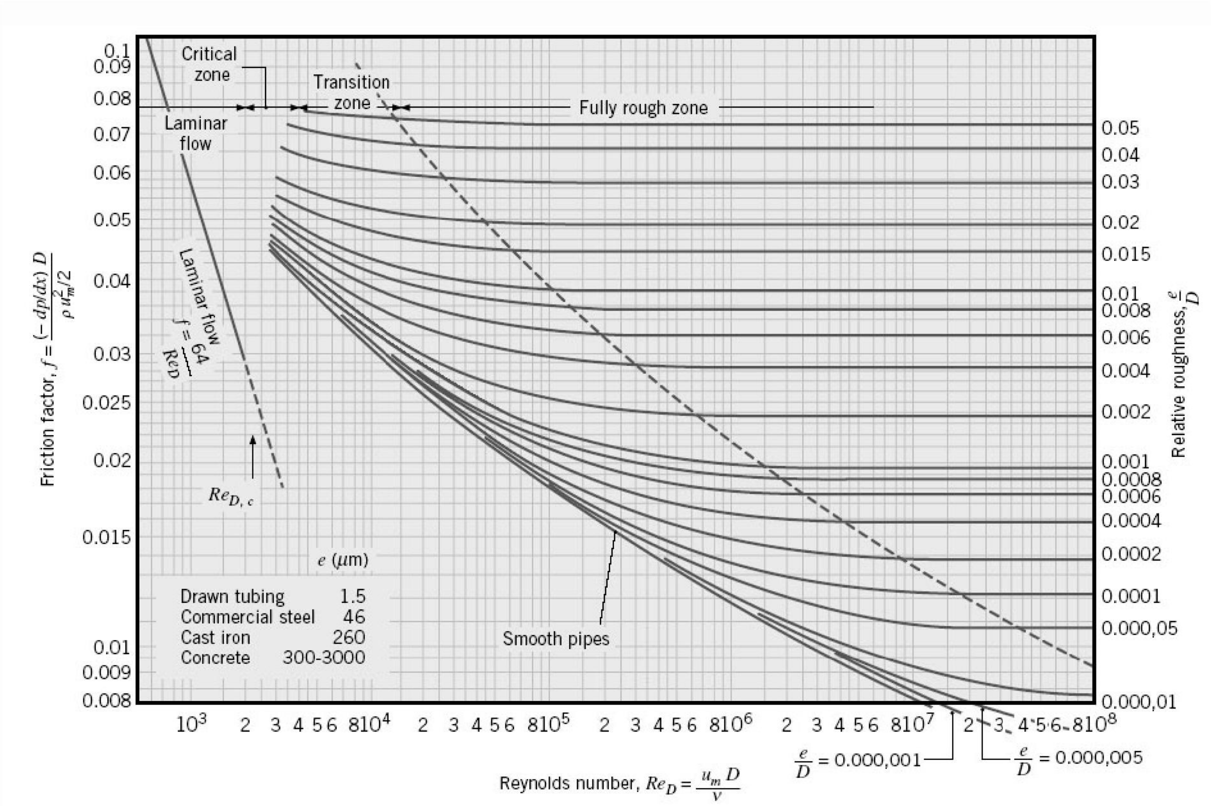


Figure 8-7 *Friction factor for fully developed flow in circular tube*

8.3.4 Turbulent Flows

Classical expression for solving local *Nusselt number* for fully developed turbulent flow in smooth circular tube is presented by *Colburn*.

Colburn Analogy

$$\frac{C_f}{2} = \frac{f}{8} = St\,\mathrm{Pr}^{2/3} = \frac{Nu_D}{\mathrm{Re}_D\,\mathrm{Pr}}\mathrm{Pr}^{2/3} \tag{8-24}$$

Colburn equation

$$Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{1/3} \tag{8-25}$$

Dittus-Boelter equations are suitable for moderate temperature differences of surface and mean fluid temperature, where properties are evaluated in mean temperature:

$$Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{0.4} \quad \text{(Heating)} \tag{8-26}$$

$$Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{0.3} \quad \textit{(Cooling)} \tag{8-27}$$

$$L/D \geq 10$$
$$\mathrm{Re}_D \geq 10000$$
$$0.6 \leq \mathrm{Pr} \leq 160$$

Temperature dependent property variations are taken account in correlation proposed by *Sieder and Tate* with viscosity ratio as

$$Nu_D = 0.023 \mathrm{Re}_D^{4/5} \mathrm{Pr}^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14} \tag{8-28}$$

For long tubes average dimensionless heat transfer coefficient is approximately same as in local fully developed region as entry length is relatively short (10<x/D<60):

$$\overline{Nu}_D \approx Nu_D \tag{8-29}$$

And for short tubes average *Nusselt number* can be evaluated with following expression

$$\frac{\overline{Nu}_D}{Nu_{D,fd}} = 1 + \frac{C}{(x/D)^m} \tag{8-30}$$

Coefficients C and m depends on dimensions and form of inlet and entry region as well as *Prandtl* and *Reynolds numbers*.

8.3.5 Noncircular Tubes

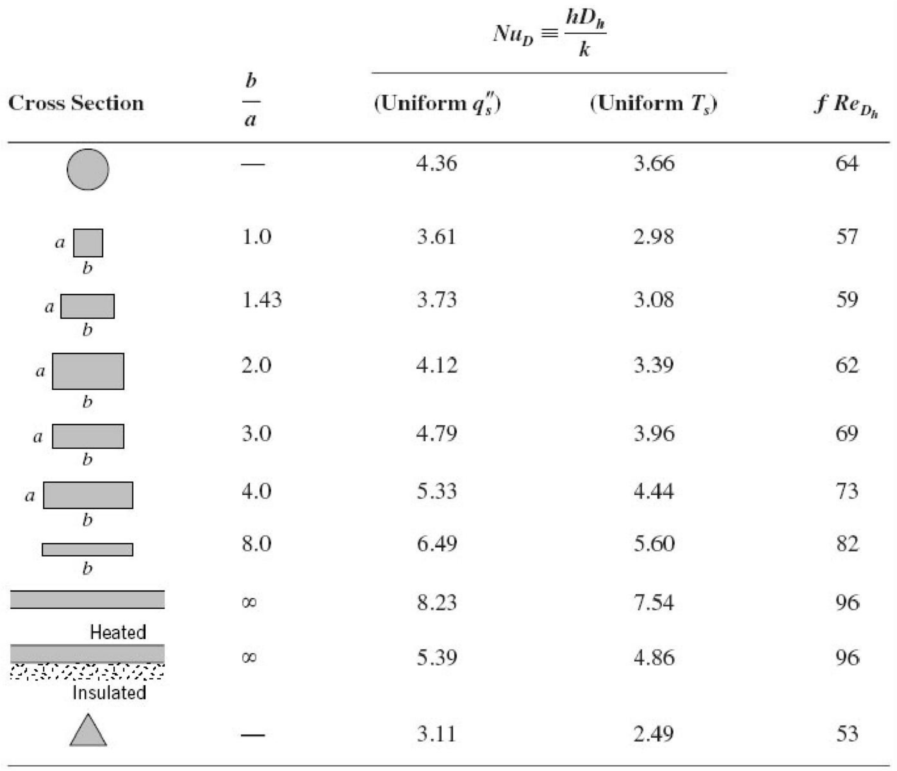
A simple way of approximating heat transfer for noncircular tubes is utilizing circular tube results with *hydraulic diameter* (effective diameter). Hydraulic diameter is defined as

$$D_h = \frac{4A_C}{P} \tag{8-31}$$

A_C	Cross-sectional area
P	Wetted Perimeter

For laminar flows, specifically in sharp tube with sharp corners, circular tube correlations are less accurate and Nusselt number correlations for fully developed conditions in laminar flow are listed in Table 8-2.

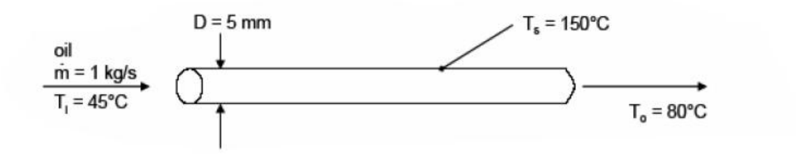
Table 8-2 *Nusselt numbers & friction factors for fully developed laminar tube flows in differing cross-section*



Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
a b	1.0	3.61	2.98	57
a b	1.43	3.73	3.08	59
a b	2.0	4.12	3.39	62
a b	3.0	4.79	3.96	69
a b	4.0	5.33	4.44	73
b	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

Example X: ***Heating engine oil – Tube flow with constant surface temperature***

Engine oil flows at a rate of 1 kg/s through a 5 mm diameter straight tube. The oil has an inlet temperature of 45 °C and it is desired to heat the oil to a mean temperature of 80 °C at the exit of the tube. The surface of the tube is maintained at 150 °C. **Determine the required length of the tube**. *Hint*; Calculate the Reynolds numbers at the entrance and exit of the tube before proceeding with your analysis.



ASSUMPTIONS:
(1) Steady-state,
(2) Incompressible flow,
(3) Negligible viscous dissipation.

PROPERTIES: Engine oil (T_i = 45 °C = 318 K): $\mu_\mathrm{i} = 16{,}3 \cdot 10^{-2}\,\mathrm{Ns/m^2}$; ($T_\mathrm{o}$ = 80 °C = 318 K): $\mu_\mathrm{o} = 3{,}25 \cdot 10^{-2}\,\mathrm{Ns/m^2}$

ANALYSIS: Reynolds numbers

$$\mathrm{Re}_\mathrm{Di} = \frac{4\dot{m}}{\pi D \mu_\mathrm{i}} = \frac{4 \cdot 1\,\mathrm{kg/s}}{\pi \cdot 0{,}005\,\mathrm{m} \cdot 16{,}3 \cdot 10^{-2}\,\mathrm{Ns/m^2}} = 1560, \text{ (laminar)}$$

$$\mathrm{Re}_\mathrm{Do} = \frac{4\dot{m}}{\pi D \mu_\mathrm{o}} = \frac{4 \cdot 1\,\mathrm{kg/s}}{\pi \cdot 0{,}005\,\mathrm{m} \cdot 3{,}25 \cdot 10^{-2}\,\mathrm{Ns/m^2}} = 7840, \text{ (turbulent)}$$

The transition occurs, when $\mathrm{Re_D} = 2300$, where

$$\mu_\mathrm{o} = \frac{4\dot{m}}{\pi D \mathrm{Re_D}} = \frac{4 \cdot 1\,\mathrm{kg/s}}{\pi \cdot 0{,}005\,\mathrm{m} \cdot 2300} = 11{,}1 \cdot 10^{-2}\,\mathrm{Ns/m^2}$$

From properties tables (Incr.Table A-5), this occurs at a transition temperature of $T_\mathrm{m,t} = 325\,\mathrm{K} = 52\,°\mathrm{C}$. Now we proceed to analyze separately the heat transfer in the laminar and turbulent regions.

PROPERTIES (Laminar region): Table A-5, $\left(T_\mathrm{m,l} = (45\,°\mathrm{C} + 52\,°\mathrm{C})/2 = 48{,}5\,°\mathrm{C} = 321{,}5\,\mathrm{K}\right)$: $C_\mathrm{p1} = 1999\,\mathrm{J/kgK}$, $\mu_1 = 13{,}2 \cdot 10^{-2}\,\mathrm{Ns/m^2}$, $k_\mathrm{l} = 0{,}143\,\mathrm{W/mK}$, $\mathrm{Pr_l} = 1851$.

$$\mathrm{Re_{Dl}} = \frac{4\dot{m}}{\pi D \mu_\mathrm{i}} = \frac{4 \cdot 1\,\mathrm{kg/s}}{\pi \cdot 0{,}005\,\mathrm{m} \cdot 13{,}2 \cdot 10^{-2}\,\mathrm{Ns/m^2}} = 1930$$

The hydrodynamic and thermal entry lengths are given by (Eq. 8-2, 8-6)

$$x_{fd,h} = 0.05\,\mathrm{Re}_D\, D = 0{,}05 \cdot 1930 \cdot 0{,}005\,\mathrm{m} = 0{,}48\,\mathrm{m}$$

$$x_{fd,t} = x_{fd,h}\,\mathrm{Pr_i} = 0{,}48\,\mathrm{m} \cdot 1851 = 890\,\mathrm{m}$$

Flow is hydrodynamically developed but thermally developing, (Eq. 8-18)

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)\mathrm{Re}_D\,\mathrm{Pr}}{1 + 0.04\left[(D/L)\mathrm{Re}_D\,\mathrm{Pr}\right]^{2/3}} = \overline{h}_1 D / k_1 \qquad (1)$$

To solve laminar region length L_1 we need also an equation for mean temperature variation, expressed between the inlet and the transition point. (Eq. 8-13)

$$\frac{T_\mathrm{s} - T_\mathrm{m}(x)}{T_\mathrm{s} - T_\mathrm{m,i}} = \exp\left(-\frac{\pi D x \bar{h}}{\dot{m}\, c_\mathrm{p}}\right)$$

$$\Rightarrow \bar{h}_1 L_1 = -\frac{\dot{m}\, c_\mathrm{p,l}}{\pi D}\ln\left(\frac{T_\mathrm{s} - T_\mathrm{m,t}}{T_\mathrm{s} - T_\mathrm{m,i}}\right) = -\frac{1\mathrm{kg/s}\cdot 1999\,\mathrm{J/kgK}}{\pi \cdot 0{,}005\mathrm{m}}\ln\left(\frac{150°\mathrm{C} - 52°\mathrm{C}}{150°\mathrm{C} - 45°\mathrm{C}}\right) \quad (2)$$

$$\bar{h}_1 L_1 = 8780\,\mathrm{W/mK}$$

The right values of $\overline{Nu}_\mathrm{D1}$, L_1 and $\bar{h}_1$ are solved by iterating Eqs (1) and (2). Beginning with the estimate $Nu_\mathrm{Di} = 3{,}66$ we find.

$\Rightarrow \overline{Nu}_\mathrm{D1} = 3{,}66$, $\bar{h}_1 = \frac{3{,}66 k_1}{D} = 105\,\mathrm{W/m^2K}$ (Eq. 1), $L_1 = 83{,}6\,\mathrm{m}$ (Eq. 2)

1st round

$\Rightarrow \overline{Nu}_\mathrm{D1} = 22{,}3$ (Eq. 1), $\bar{h}_1 = \frac{\overline{Nu}_\mathrm{Di} k_\mathrm{l}}{D} = 638\,\mathrm{W/m^2K}$ (Eq. 1), $L_1 = 13{,}8\,\mathrm{m}$ (Eq. 2)

2nd round

$\Rightarrow \overline{Nu}_\mathrm{D1} = 18{,}7$ (Eq. 1), $\bar{h}_1 = 535\,\mathrm{W/m^2K}$ (Eq. 1), $L_1 = 16{,}4\,\mathrm{m}$ (Eq. 2)

3rd round

⋮

$\Rightarrow \overline{Nu}_\mathrm{D1} = 16{,}9$ (Eq. 1), $\bar{h}_1 = 484\,\mathrm{W/m^2K}$ (Eq. 1), $L_1 = 18{,}1\,\mathrm{m}$ (Eq. 2)

10th round

$\Rightarrow \overline{Nu}_\mathrm{D1} = 16{,}9$ (Eq. 1), $\bar{h}_1 = 484\,\mathrm{W/m^2K}$ (Eq. 1), $L_1 = 18{,}1\,\mathrm{m}$ (Eq. 2)

11th round

PROPERTIES (Turbulent region): $T_\mathrm{m,2} = \frac{52\,°\mathrm{C} + 80\,°\mathrm{C}}{2} = 66\,°\mathrm{C} = 339\,\mathrm{K}$, $C_\mathrm{p2} = 2072\,\mathrm{J/kgK}$, $\mu_2 = 5{,}62 \cdot 10^{-2}\mathrm{Ns/m^2}$, $k_2 = 0{,}139\,\mathrm{W/mK}$, $\mathrm{Pr}_2 = 834$.

$$\mathrm{Re}_\mathrm{D2} = \frac{4\dot{m}}{\pi D \mu_\mathrm{i}} = 4530$$

We assume that the flow is fully-developed hydrodynamically and thermally and use Eq 8.62 from Table 8-1.

$$\overline{Nu}_\mathrm{D2} = \frac{(f/8)(\mathrm{Re}_\mathrm{D2} - 1000)\mathrm{Pr}_2}{1 + 12{,}7(f/8)^{1/2}\left(\mathrm{Pr}_2^{2/3} - 1\right)}$$

$$f = (0{,}790\,\ln\mathrm{Re}_\mathrm{D2} - 1{,}64)^{-2} = (0{,}790\,\ln\,(4530) - 1{,}64)^{-2} = 0{,}0398$$

$$\Rightarrow \overline{Nu}_\mathrm{D2} = \frac{(0{,}0398/8)(4530 - 1000)\cdot 834}{1 + 12{,}7(0{,}0398/8)^{1/2}(834^{2/3} - 1)} = 184$$

$$\Rightarrow \bar{h}_2 = \frac{\overline{Nu}_\mathrm{D2} k_2}{D} = 5120\,\mathrm{W/m^2K}$$

The required length L_2 can be found from (Eq. 8-13) expressed between the inlet and the transition point.

$$\Rightarrow L_2 = -\frac{\dot{m}\, c_{\mathrm{p},2}}{\pi D \bar{h}_2}\ln\left(\frac{T_\mathrm{s} - T_\mathrm{m,o}}{T_\mathrm{s} - T_\mathrm{m,t}}\right) = -\frac{1\,\mathrm{kg/s}\cdot 2072\,\mathrm{J/kgK}}{\pi \cdot 0{,}005\,\mathrm{m}\cdot 5120\,\mathrm{W/m^2K}}\ln\left(\frac{150°\mathrm{C} - 80°\mathrm{C}}{150°\mathrm{C} - 52°\mathrm{C}}\right) = 8{,}7\,\mathrm{m}$$

The total required length is $L = L_1 + L_2 = 18{,}1\,\mathrm{m} + 8{,}7\,\mathrm{m} = \underline{26{,}8\,\mathrm{m}}$

COMMENT: *If we had assumed the flow to be turbulent throughout,* ***we would have get higher average Nusselt number and significantly lower total length (L = 11,9 m)****. The variation of properties with temperature can be very important for some fluids such as oils.*

8.4 LEARNING OUTCOMES

Table 8-3 Learning Outcomes: *Forced Convection - Internal*

Level of Knowledge	Concept
Understand *Describe*	- *Hydrodynamic and thermal entry region: convection coefficient behaviour* - *Fully developed flow*
Apply ***understand*** *describe*	- *Empirical f & Nu correlations: turbulent/laminar, fully developed/entry effects* - *Log-mean temperature difference – fluid mean temperature* - *Concepts: hydraulic diameter*

9 FREE CONVECTION

Free or natural convection is formed due to buoyancy forces caused by density gradients and gravitational body forces that are proportional to density. In most cases density gradients are formed due to temperature gradients. Velocities in free convection cases are smaller than in forced convection, and thus resulting as much smaller heat transfer coefficients.

Importance of free convection is relevant in many multimode heat transfer effects, commonly with radiation, and free convection is preferred choice for cooling applications from operation cost minimization point of view.

<u>Applications</u>

- Cooling of electrical equipments
- Natural circulation boilers
- Cooling of pipelines
- Windows and radiators
- Condensers of refrigeration devices
- Flow dynamics of water systems and atmosphere

Free boundary flows are not bounded by surface and may occur in form of plumes or buoyant jets.

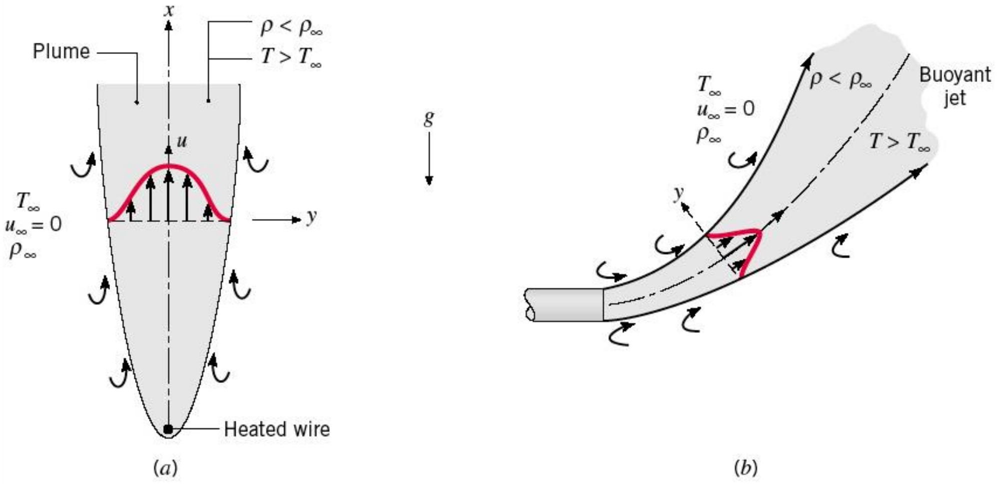


Figure 9-1 *Free boundary layer development a) plume above heated fire b) buoyant jet with from heat discharge*

Classical free convection flow bounded by surface is heated vertical wall. Boundary layer develops to surface of wall, when lighter heated fluid on wall starts to flow upwards. Velocity is zero both on wall and at quiescent fluid region, out of boundary layer area.

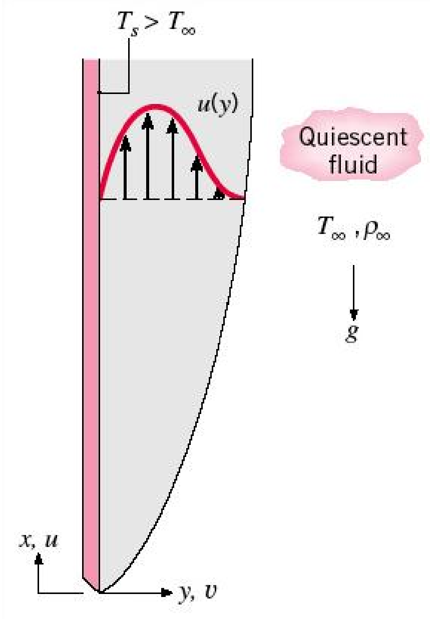


Figure 9-2 *Free convection boundary layer development on vertical heated plate*

9.1 ANALYTICAL SOLUTIONS FOR VERTICAL PLATE

Conservation principles are similar to those in forced convection. Mass, momentum and energy conservation equation set is valid for free convection boundary layer flows with negligible viscous effect and incompressible flow assumptions, (except in x-momentum equation density change resulting in buoyancy flow with viscous effect), are

Continuity
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (9\text{-}1)$$

X-momentum
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p_\infty}{\partial x}-g+\nu\frac{\partial^2 u}{\partial y^2} \qquad (9\text{-}2)$$

Energy
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (9\text{-}3)$$

By substituting pressure gradient $dp_\infty / dx = -\rho_\infty g$ in quiescent boundary region outside from boundary layer to x-momentum equation following momentum equation can be written

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\frac{\rho_\infty-\rho}{\rho}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (9\text{-}4)$$

If density differences causing free convection flow are only due to temperature differences, free convection flow can be related to fluid property, *volumetric thermal expansion coefficient*, which is defined as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \tag{9-5}$$

v *Specific volume [m^3/kg]*

Volumetric thermal expansion coefficient describes density change response to temperature change at constant pressure.

Boussinesq approximation states for density difference that

$$(\rho_\infty - \rho) \approx \rho\beta(T - T_\infty) \tag{9-6}$$

Giving a form for x-momentum conservation equation as

$$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}}_{Momentum\ of\ Convection} = \underbrace{g\beta(T - T_\infty)}_{Buoyancy\ force} + \underbrace{v\frac{\partial^2 u}{\partial y^2}}_{Viscous\ force} \tag{9-7}$$

For ideal gas ($pv = RT$) volumetric thermal expansion coefficient can be written as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = \frac{1}{\rho}\frac{p}{RT^2} = \frac{1}{T} \tag{9-8}$$

For liquids and non-ideal gases, temperature (&pressure) dependent correlations and property tables are available.

Similarity

By defining dimensionless parameters as

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad L = \text{Characteristic length of surface}$$

$$u^* = \frac{u}{u_0} \qquad v^* = \frac{v}{u_0} \qquad T^* = \frac{T - T_\infty}{T_S - T_\infty}$$

X-momentum conservation equation can be written in dimensionless form

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_S - T_\infty)}{u_0^2}T^* + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{9-9}$$

By choosing arbitrary reference velocity u_0 as

$$u_0 = \frac{g\beta(T_S - T_\infty)L^3}{v^2} \tag{9-10}$$

Reynolds number for free convection may be presented as

$$\mathrm{Re}_L = \left[g\beta(T_S - T_\infty)L^3 / v^2\right]^{1/2} \tag{9-11}$$

Grashof Number

Definition of *Grashof number*, dimensionless parameter relevant for free convection, is a square of Reynolds number described above. *Grashof number* describes ratio of buoyancy forces to viscous forces and is defined as

$$Gr = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu^2} \tag{9-12}$$

Grashof number in free convection flows corresponds to Reynolds number in forced convection flows. As Nusselt number is function of Reynolds and Prandtl numbers in forced convection cases, heat transfer coefficient *h(Gr)* is function of Grashof number in free convection problems. Convection domination mode assessment can be done with Grashof and Reynolds numbers as

Table 9-1 *Free or forced convection dominating*

$Gr/Re^2 \approx 1$	Mixed free and forced convection
$Gr/Re^2 << 1$	Forced convection
$Gr/Re^2 >> 1$	Free convection

9.1.1 Laminar Free Convection Solutions

Solutions for conservation equations for vertical heated plate can be presented in similarity parameter graphs (Figure 9-3). Boundary conditions for problem are

$$y = 0: \quad u = v = 0 \quad T = T_S$$
$$y \to \infty: \quad u \to 0 \quad T \to T_\infty$$

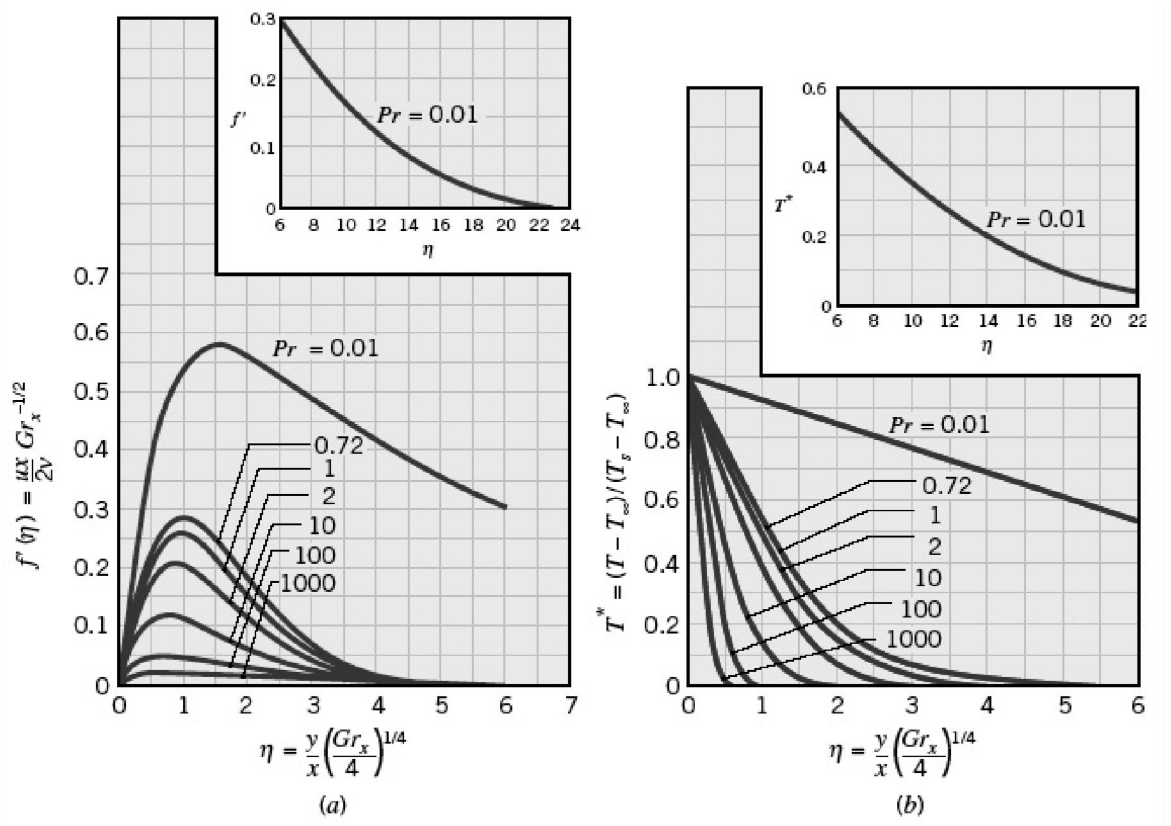


Figure 9-3 *Free convection boundary layer solutions for laminar flow on vertical surface a) velocity b) temperature profiles*

Local convection heat transfer coefficient h_x for laminar free convection on vertical plate is

$$Nu_x = \frac{h_x x}{k} = -\left(\frac{Gr_x}{4}\right)^{1/4} \left.\frac{dT^*}{d\eta}\right|_{\eta=0} = \left(\frac{Gr_x}{4}\right)^{1/4} g(\mathrm{Pr}) \tag{9-13}$$

$$g(\mathrm{Pr}) = \frac{0{,}75\mathrm{Pr}^{1/2}}{(0{,}609 + 1{,}221\mathrm{Pr}^{1/2} + 1{,}238\mathrm{Pr})^{1/4}}$$

Remembering that Prandtl number was defined as ratio of kinetic viscosity and thermal diffusivity as follows

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{\mu}{\rho}\frac{\rho c_p}{k} = \frac{\nu}{\alpha} \tag{9-14}$$

Average convection heat transfer coefficient $\bar{h}$ is obtained by integrating local value over wall length as

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{1/4} g(\mathrm{Pr}) = \frac{4}{3} Nu_L \tag{9-15}$$

9.1.2 Turbulence

As in forced convection, disturbances in flow may be amplified and turbulent flow may be induced in free convection flows. Similarly heat transfer is more effective in case of turbulent flow. Heat transfer coefficients for turbulent flows have to be defined from experimental correlations.

Rayleigh Number is used for correlating transition from laminar to turbulent boundary layer, which is dependent on ratio of buoyancy and viscous forces. Definition of *Rayleigh number* is product of *Grashof* and *Prandtl number* and is defined for vertical plates as

$$Ra = Gr\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} \tag{9-16}$$

In free convection transition from laminar to turbulent flow occurs at *Rayleigh number* exceeding approximately 10^9.

9.2 EMPIRICAL CORRELATIONS

Generally free convection heat transfer correlations are in form

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = CRa_L^n \tag{9-17}$$

n = $1/4$ *for laminar (Vertical plates: C=0.59)*
n = $1/3$ *for turbulent (Vertical plates: C=0.1)*

Fluid properties are defined in film temperature $T_f = (T_S - T_\infty)/2$.

9.2.1 Plates

Empirical correlation for *vertical plate* over entire range or Rayleigh number is proposed as

$$\overline{Nu}_L = \left\{0{,}825 + \frac{0{,}387\,Ra_L^{1/6}}{\left[1 + (0{,}492/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2 \tag{9-18}$$

Laminar flow, $10^4 \le Ra_L \le 10^9$
Turbulent flow, $10^9 \le Ra_L \le 10^{13}$

Average Nusselt number for *horizontal plates* can approximated with following correlations:

Heated surface up or cooled surface down

$$\overline{Nu}_L = 0{,}54Ra_L^{1/4} \qquad 10^4 \le Ra_L \le 10^7 \tag{9-19}$$

$$\overline{Nu}_L = 0{,}15Ra_L^{1/3} \qquad 10^7 \le Ra_L \le 10^{11} \tag{9-20}$$

Heated surface down or cooled surface up

$$\overline{Nu}_L = 0{,}27 Ra_L^{1/4} \qquad 10^5 \le Ra_L \le 10^{10} \qquad (9\text{-}21)$$

9.2.2 Long Horizontal Cylinder

Free convection boundary layer is formed across heated cylinder in quiescent fluid. Local Nusselt number depends on formation of laminar boundary layer as shown in Figure 9-4.

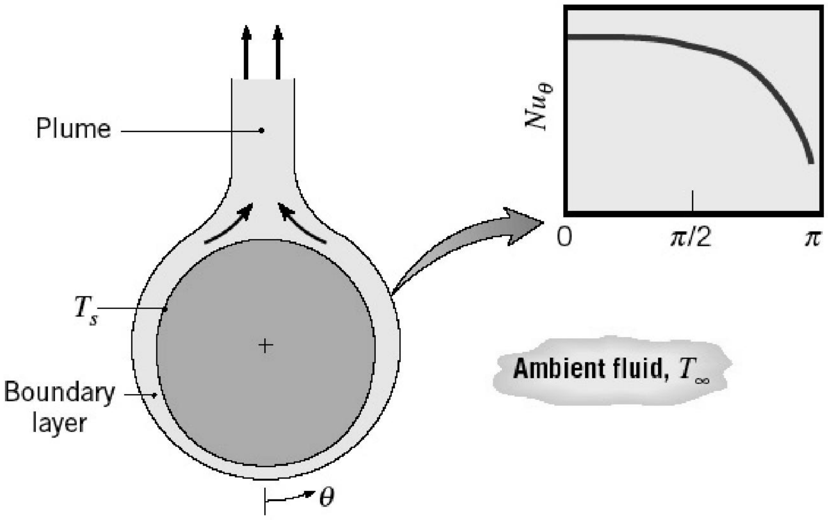


Figure 9-4 *boundary layer development and Nusselt number distribution on heated horizontal cylinder*

Correlation form for average Nusselt number is given by Churchill and Chu as

$$\overline{Nu}_D = \left\{ 0{,}60 + \frac{0{,}387 Ra_D^{1/6}}{\left[1 + (0{,}559/\mathrm{Pr})^{9/16}\right]^{8/27}} \right\}^2 \qquad \mathrm{Ra_D} \le 10^{12} \qquad (9\text{-}22)$$

9.2.3 Sphere

Average Nusselt number correlation for sphere is proposed by Churchill as

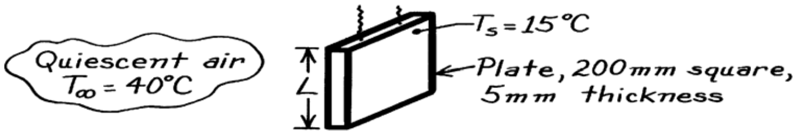
$$\overline{Nu}_D = 2 + \frac{0{,}589 Ra_D^{1/4}}{\left[1 + (0{,}469/\mathrm{Pr})^{9/16}\right]^{4/9}} \qquad (9\text{-}23)$$

$Pr \ge 0{,}7$
$Ra_D \le 10^{11}$

As Rayleigh number approaches zero, heat transfer coefficient approaches 2, which corresponds conduction heat transfer coefficient of sphere in stationary infinite medium.

Example XI: ***Analytical & Empirical Solutions– Free Convection over vertical plate***

A square aluminum plate 5 mm thick and 200 mm on a side is heated while vertically suspended in quiescent air at 40 °C. Determine the average heat transfer coefficient for the plate when its temperature is 15 °C by two methods: using results from the similarity solution to the boundary layer equations, and using results from an empirical correlation.



ASSUMPTIONS:
(1) Uniform plate surface temperature,
(2) Quiescent room air,
(3) Surface radiation exchange with surroundings negligible,
(4) Perfect gas behavior for air, $\beta = 1/T_f$.

PROPERTIES: *Table A-4,*
Air ($T_f = (T_s + T_\infty)/2 = (40 + 15)°C/2 = 300\,K$, 1 atm)
$\nu = 15{,}89 \cdot 10^{-6}\,m^2/s$, $k = 0{,}0263\,W/mK$, $\alpha = 22{,}5 \cdot 10^{-6}\,m^2/s$,
$Pr_2 = 0{,}707$.

ANALYSIS: Rayleigh number (Eq. 9-15)

$$Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha}$$

$$Ra_L = \frac{9{,}81\,m/s^2 \cdot (1/300K) \cdot (40-15)°C \cdot (0{,}2\,m)^3}{15{,}89 \cdot 10^{-6}\,m^2/s \cdot 22{,}5 \cdot 10^{-6}\,m^2/s} = 1{,}827 \cdot 10^7 < 10^9, \text{ (Laminar)}$$

Nusselt number (Uniform plate surface temperature, quiescent air) Eq. 9-14:

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{\frac{1}{4}} g(Pr) \qquad (1)$$

Interpolation of the Figure 9-3 (temperature gradient at the surface as a function of the Prandtl number) Eq. 9-12

$$g(Pr) = \frac{0{,}75 Pr^{0{,}5}}{(0{,}609 + 1{,}221 Pr^{0{,}5} + 1{,}238 Pr)^{0{,}25}} \qquad (2)$$

Substituting numerical values with with $Gr_L = Ra_L/Pr$

$$g(Pr) = \frac{0{,}75 \cdot 0{,}707 \cdot^{0{,}5}}{(0{,}609 + 1{,}221 \cdot 0{,}707^{0{,}5} + 1{,}238 \cdot 0{,}707)^{0{,}25}} = 0{,}501$$

$$\bar{h}_L = \frac{\overline{Nu}_L k}{L} = \frac{k}{L} \cdot \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{\frac{1}{4}} g(Pr) = \frac{0{,}0263\,W/mK}{0{,}20\,m} \cdot \frac{4}{3}\left(\frac{1{,}827 \cdot 10^7/0{,}707}{4}\right)^{\frac{1}{4}} \cdot 0{,}501$$

$$\bar{h}_L = 4{,}42\,W/m^2K$$

The appropriate empirical correlation for estimating $\bar{h}_L$ is given by Eq. 9-17

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left\{0{,}825 + \frac{0{,}387 Ra_L^{1/6}}{\left[1 + (0{,}492/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

$$\Rightarrow \bar{h}_L = \frac{k}{L}\left\{0{,}825 + \frac{0{,}387 Ra_L^{1/6}}{\left[1 + (0{,}492/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

$$\bar{h}_L = \frac{0{,}0263\,\mathrm{W/mK}}{0{,}2\,\mathrm{m}}\left\{0{,}825+\frac{0{,}387\cdot\left(1{,}827\cdot10^{7}\right)^{1/6}}{\left[1+\left(0{,}492/0{,}707\right)^{9/16}\right]^{8/27}}\right\}^{2} = 4{,}87\,\mathrm{W/m^2K}$$

Comments:

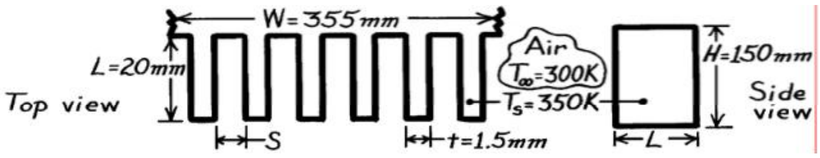
1) *In this case, both analytics and empirics, give a close to each other solution for free convection heat transfer coefficient*

Example XII: ***Free convection between Fins – Cooling of Parked Motor Cycle Engine***

Consider an array of vertical rectangular fins, which is to be used to cool an electronic device mounted in quiescent air at $T_\infty = 27\,°\mathrm{C}$ (1 atm). Each fin has $L = 20\,\mathrm{mm}$ and has $H = 150\,\mathrm{mm}$ and operates at an approximately uniform temperature of $T_s = 77\,°\mathrm{C}$

a) Viewing each fin surface as a vertical plate in an infinite quiescent medium, briefly describe why there exists an optimum fin spacing S. Using figure 9-3, estimate the optimum value of S for prescribed conditions.

b) For the optimum value of S and a fin thickness of $t = 1{,}5\,\mathrm{mm}$, estimate the rate of heat transfer from the fins for an array of width $W = 355\,\mathrm{mm}$.



3 ASSUMPTIONS:

(1) Fins are isothermal

(2) Radiation effects are negligible,

(3) Air is quiescent.

PROPERTIES: *Table A-4,*
Air ($T_\mathrm{f} = (T_\mathrm{s} + T_\infty)/2 = (27 + 77)°\mathrm{C}/2 = 52\,°\mathrm{C} = 325\,\mathrm{K}$):
$\nu = 18{,}41 \cdot 10^{-6}\,\mathrm{m^2/s}$, $k = 0{,}0282\,\mathrm{W/mK}$, $\mathrm{Pr}_2 = 0{,}703$.

ANALYSIS:

a) If fins are too close, boundary layers on adjoining surfaces will coalesce and heat transfer will decrease. If fins are too far apart, the surface area becomes too small and heat transfer decrease. $S_\mathrm{op} = 2 \cdot \delta_{\mathrm{x=H}}$. From Fig. 9-3, the edge of boundary layer corresponds to

$$\eta = \frac{\delta}{H}\left(\frac{\mathrm{Gr_H}}{4}\right)^{1/4} \approx 5$$

$$\mathrm{Gr_H} = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{9{,}81\,\mathrm{m/s^2} \cdot (1/325\,\mathrm{K}) \cdot (77-27)°\mathrm{C} \cdot (0{,}150\,\mathrm{m})^3}{(18{,}41 \cdot 10^{-6}\,\mathrm{m^2/s})^2}$$

$$= 1{,}5 \cdot 10^{7}$$

$$\delta(H) = \frac{5\cdot(0{,}15\,\mathrm{m})}{(1{,}5\cdot10^{7})^{1/4}} = 0{,}017\,\mathrm{m} = 17\,\mathrm{mm} \qquad \Rightarrow S_\mathrm{op} \approx 34\,\mathrm{mm}$$

b) The number of fins N can be found as

$$N = \frac{W}{S_\mathrm{op} + t} = \frac{355\,\mathrm{mm}}{(34 + 1{,}5)\,\mathrm{mm}} = 10$$

Heat transfer rate:

$$q_{conv} = \bar{h}A(T_\mathrm{s} - T_\infty) = 2N\bar{h}HL(T_\mathrm{s} - T_\infty)$$

The appropriate empirical correlation for estimating $\bar{h}_{\mathrm{L}}$ is given by Eq. 9-17

$$\overline{Nu}_{\mathrm{H}} = \frac{\bar{h}_{\mathrm{L}} H}{k} = \left\{ 0{,}825 + \frac{0{,}387\, Ra_L^{1/6}}{\left[1 + \left(0{,}492/\mathrm{Pr}\right)^{9/16}\right]^{8/27}} \right\}^2$$

$$\Rightarrow \bar{h}_{\mathrm{L}} = \frac{k}{H} \left\{ 0{,}825 + \frac{0{,}387\, Ra_L^{1/6}}{\left[1 + \left(0{,}492/\mathrm{Pr}\right)^{9/16}\right]^{8/27}} \right\}^2$$

$$\bar{h}_{\mathrm{L}} = \frac{0{,}0282\ \mathrm{W/mK}}{0{,}150\ \mathrm{m}} \left\{ 0{,}825 + \frac{0{,}387 \cdot \left(1{,}5 \cdot 10^7 \cdot 0{,}703\right)^{1/6}}{\left[1 + \left(0{,}492/0{,}703\right)^{9/16}\right]^{8/27}} \right\}^2 = 5{,}95\ \mathrm{W/m^2K}$$

$$q_{conv} = 2 \cdot 10 \cdot 5{,}95 \frac{\mathrm{W}}{\mathrm{m^2K}} \cdot 0{,}15\ \mathrm{m} \cdot 0{,}02\ \mathrm{m} \cdot (350 - 300)\ \mathrm{K} = 17{,}5\ \mathrm{W}$$

COMMENTS:

1) *Further reduction in S would compensate for the effect of fluid entrapment due to boundary layer merger.*
2) *From a more rigorous treatment* $S_{op} \approx 10\ mm$ *is obtained for the prescribed conditions – joining of boundary layers don't decrease heat transfer some much than increased heat transfer surface will enhance it –-> optimization problem*

9.3 LEARNING OUTCOMES

Table 9-2 Learning Outcomes: *Free Convection*

Level of Knowledge	Concept
Apply, *Understand, describe*	• *Similarity solutions for laminar free convection (Figure 9-3)* • *Concepts: Ra, Gr, Re and empirical Nu correlations, volumetric expansion* • *Distinguish free from forced convection: Gr limit*
Understand, describe	• *Boundary layer development in different geometries* • *Terms of conservation equations* • *transition from laminar to turbulent*

10 BOILING AND CONDENSATION

Boiling and condensation are *convection processes* occurring at solid-liquid or solid-vapour interface and are associated with *phase change of a fluid*. High heat transfer rates with small temperature differences are achieved due to latent heat of phase change. Two other important fluid properties effecting to boiling or condensation are liquid-vapour surface tension and density difference of two phases. Density difference of two phases induces a buoyancy force that causes fluid convective motion promoting heat transfer together with latent heat effect.

Applications

- Power Plants
  - Boiler: vaporization of pressurized liquid
  - Condenser: expanded vapour condensed back to feed water
- Refrigerators and heat pumps: evaporators and condensers
- Advanced electronic cooling
- Cooking: Boiling 3 min eggs & Nature: Rain, misty lake

10.1 PRINCIPLES OF BOILING

Evaporation at solid-liquid interface is termed as boiling. Boiling starts with formation of vapour bubbles, if surface temperature of solid rises over fluid saturation temperature corresponding liquid pressure. Newton's law of cooling for boiling heat transfer rate is

$$q_s'' = h(T_S - T_{sat}) = h\Delta T_e \qquad \textit{(10-1)}$$

ΔT_e	*Excess temperature*
$T_{sat}(p)$	*Fluid saturation temperature*

Boiling convection heat transfer is affected by the dynamics of formation, growth and movement of vapour bubbles and related liquid motion near surface.

10.1.1 Classification

Characteristic for *pool boiling* is relatively quiescent liquid, and bubble formation and free convection induced liquid motion near surface. *Forced convection boiling* is on the other hand characterized with externally induced fluid flow together with free convection and bubble-dynamics induced mixing.

Sub-cooled boiling means vapour formation in liquid having temperature below its saturation temperature. Boiling vapour formed at surface might later condense in liquid. In *saturated boiling* temperature of liquid slightly exceeds saturation temperature.

10.2 POOL BOILING

Pool boiling heat flux plotted against excess temperature is called boiling curve (Figure 10-1). Boiling curve reveals characteristics of different boiling regimes of pool boiling. Different boiling regimes are classified according excess temperature and each mode has its own bubble-fluid behaviour and heat transfer characteristics.

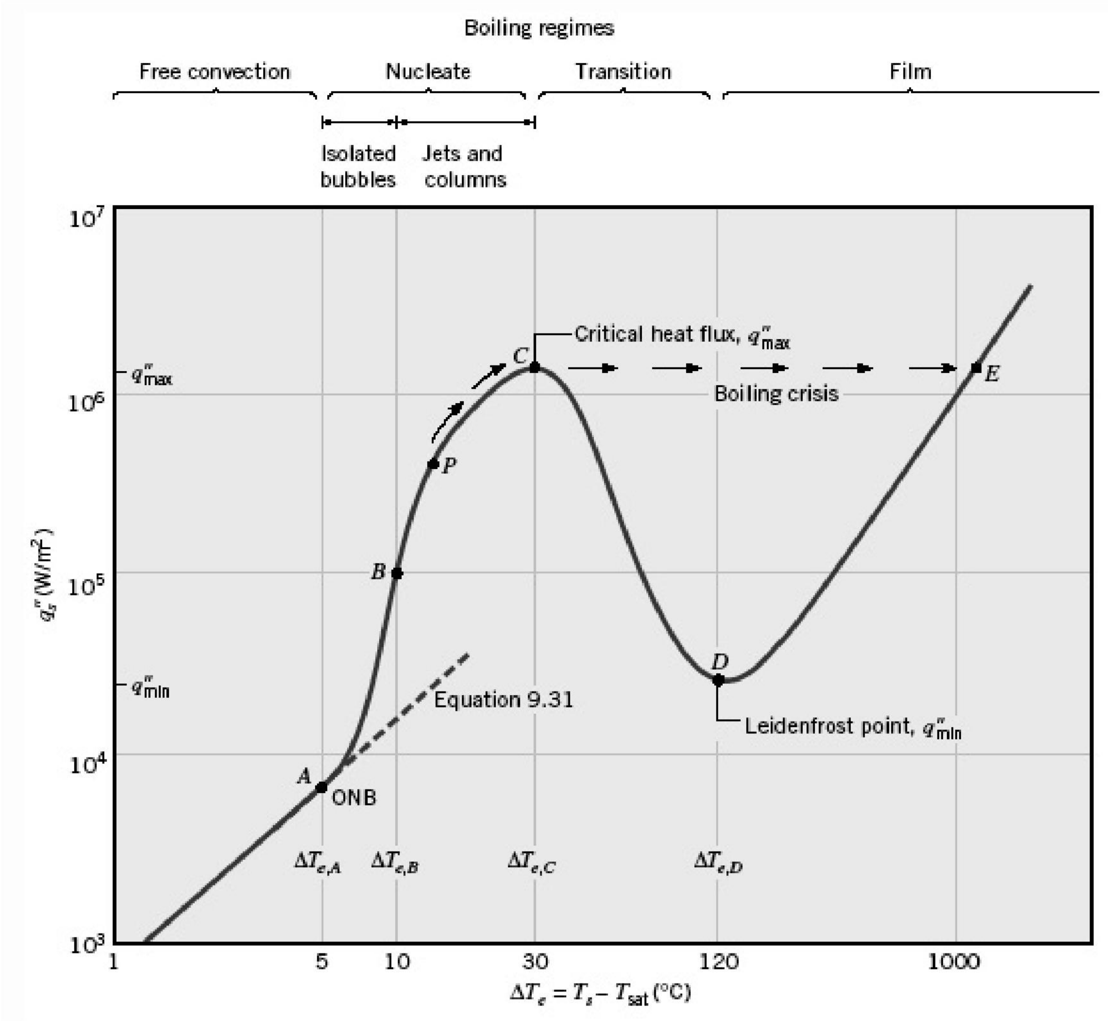


Figure 10-1 *Boiling curve for saturated water at atmospheric pressure*

10.2.1 Free Convection Boiling

Free convection boiling exists below ~5°C excess temperature (ΔT_e <5°C). In free convection boiling fluid motion is determined by free convection and thus free convection heat transfer correlations can be used in this boiling region.

10.2.2 Nucleate Boiling

Onset of nucleate boiling (ONB, ΔT_e =5°C) is starting point of nucleate boiling (Figure 10-1, point A), which continues until point of critical heat flux (point C, $\Delta T_{e,C}$ ≈30°C).

Region A-B: Isolated bubbles are form and rise from surface. Bubble separation from surface enhances fluid mixing near surface and increases both heat transfer coefficient and rate. Heat transfer is mostly from surface to fluid in boiling convective motion.

Region B-C: Vapour escapes as jets and bubbles and merges to slugs of vapour as bubble formation is further increased (Figure 10-2). Heat transfer coefficient increases until maximum point a, when it starts to decrease with excess temperature caused by increased vapour bubble formation on surface. Heat flux increases with excess temperature until critical heat flux ($\Delta T_{e,C} \approx 30°C$), end of nucleate boiling regime. Increased vapour formation prevents fluid to wet surface and thus local maximum heat flux is reached.

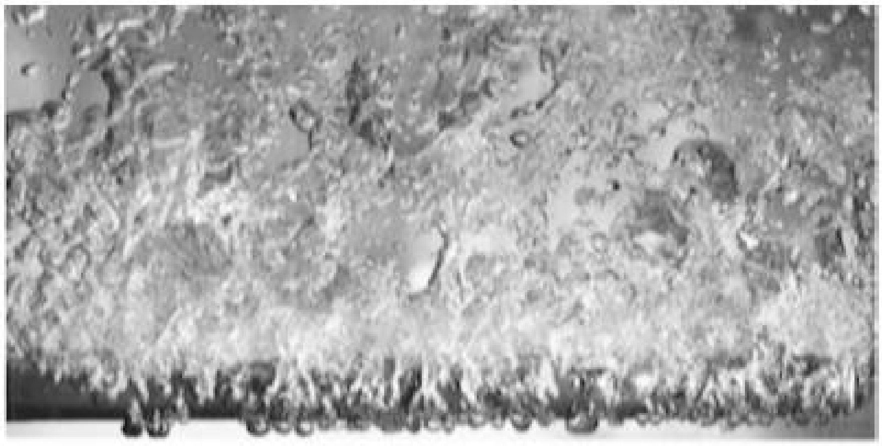


Figure 10-2 *Nucleate boiling with vapour jets*

10.2.3 Transition boiling

In transition region 30°C $< \Delta T_e <$120°C) vapour bubble formation is rapid and conditions at surface oscillates between nucleate and film boiling. Fraction of vapour film coverage increases with excess temperature. Heat transfer coefficient and heat transfer rate are decreasing caused by lower conductivity of vapour compared to liquid.

Figure 10-3 *Transition boiling*

10.2.4 Film Boiling

At *Leidenfrost* point (point D) minimum heat flux is reached and surface is covered totally by vapour film. Small water droplets are evaporated in vapour film. Heat transfer occurs by conduction and radiation through vapour blanket. Increase of heat transfer in film boiling region with excess temperature is enhanced by increase of surface radiation.

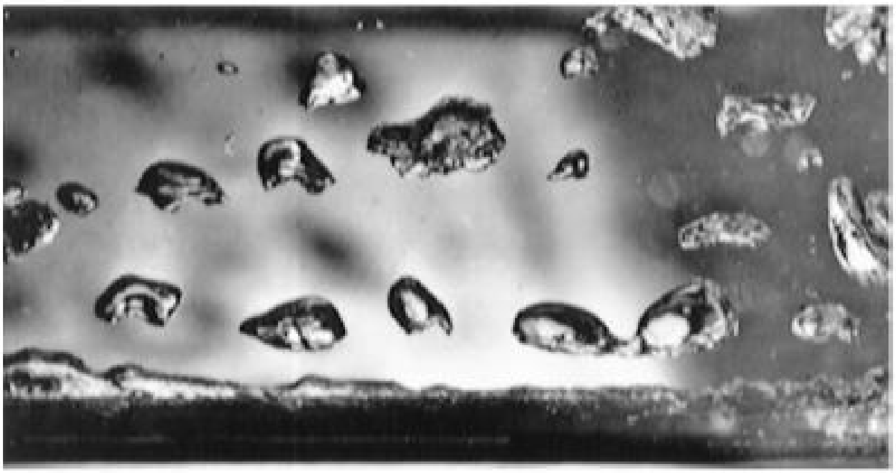


Figure 10-4 *Film boiling*

Increase of heat flux from point of maximum heat flux (point C) causes fast increase of surface temperature due to boiling crisis (or burnout point). Boiling crisis is a change of boiling regime from nucleate to film boiling with fast departure from boiling curve.

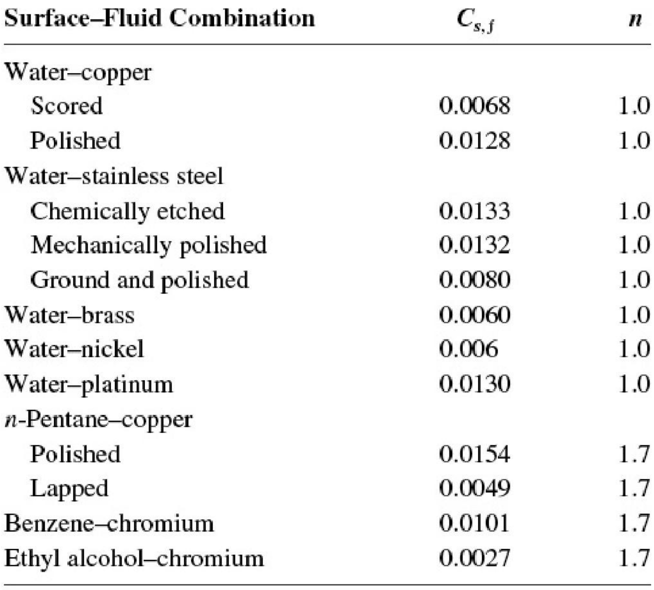
10.2.5 Pool Boiling Correlations

Nucleate Pool Boiling

Rohsenow correlation is widely used for determine nucleate boiling heat transfer rate

$$q_s^{"} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left(\frac{c_{p,l} \Delta T_e}{C_{S,f} h_{fg} \Pr_l^n} \right)^3 \qquad (10\text{-}2)$$

Table 10-1 *Coefficient values for Rohsenow correlation for various surface-liquid combinations*



Surface–Fluid Combination	$C_{s,f}$	n
Water–copper		
Scored	0.0068	1.0
Polished	0.0128	1.0
Water–stainless steel		
Chemically etched	0.0133	1.0
Mechanically polished	0.0132	1.0
Ground and polished	0.0080	1.0
Water–brass	0.0060	1.0
Water–nickel	0.006	1.0
Water–platinum	0.0130	1.0
n-Pentane–copper		
Polished	0.0154	1.7
Lapped	0.0049	1.7
Benzene–chromium	0.0101	1.7
Ethyl alcohol–chromium	0.0027	1.7

Rohsenow correlation applies only for clean surfaces and error marginal is ±100% when predicting heat fluxes.

Boiling phenomenon in nucleate region is similar to forced convection with fluid motion induced by rise of bubbles. Forced convection heat transfer correlations are generally in form of

$$\overline{Nu}_L = C_{fc} \mathrm{Re}_L^{m_{fc}} \mathrm{Pr}^{n_{fc}} \tag{10-3}$$

Heat fluxes

Prediction of critical, maximum, heat flux is important in sense of avoiding boiling crisis. Expression for maximum heat flux is presented as

$$q''_{\max} = C\ h_{fg}\rho_v \left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2}\right]^{1/4} \tag{10-4}$$

Leading constant C depends weekly on surface geometry. For spheres, horizontal cylinders and large finite heated surfaces value π/24=0.131 can be used. Critical heat flux has strong dependence to pressure due to surface tension and heat of vaporization.

Film Boiling

For film boiling on a cylinder or sphere heat transfer correlation obtained from laminar film condensation analogy is proposed as

$$Nu_D = \frac{\overline{h}_{conv}D}{k_v} = C\left[\frac{g(\rho_l - \rho_v)h'_{fg}}{v_v k_v (T_s - T_{sat})}\right]^{1/4} \tag{10-5}$$

C *Constant: 0.62 for cylinders, 0.67 for spheres*

Corrected latent heat accounting for the heat needed to maintain film temperature above saturation temperature may be approximated as

$$h'_{fg} = h_{fg} + 0.8c_{p,v}(T_S - T_{sat}) \tag{10-6}$$

At temperatures above 300°C radiation heat transfer has significant influence on total heat transfer. Effective radiation coefficient is expressed as

$$h_{rad} = \frac{\varepsilon\sigma(T_s^4 - T_{sat}^4)}{T_s - T_{sat}} \tag{10-7}$$

ε *Emissivity of solid*
σ *Stefan-Boltzmann constant*

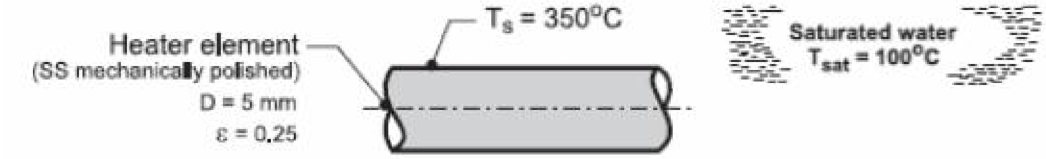
Total heat transfer coefficient for film boiling accounting convection and radiation parts is suggested by Bromley to be solved from

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\,\bar{h}^{1/3} \tag{10-8}$$

Example XIII: *Boiling water in Coffee Machine – Heater element*

A heater element of 5-mm diameter is maintained at a surface temperature of 350 °C when immersed horizontally in water under atmospheric pressure. The element sheath is stainless steel with a mechanically polished finish having an emissivity of 0.25.

a) Calculate the electrical power dissipation and the rate of vapor production per unit heater length.
b) If the heater were operated at the same power dispation rate in the nucleate boiling regime, what temperature would the surface achieve? Calculate the rate of wapor production per unit length for this operating condition.
c) Sketch the boiling curve and represent the two operating conditions of parts a) and b). Compare the results of your analysis. If the heater element is operated in the power controlled mode, explain how you would achieve these two operating conditions beginning with a cold element.



ASSUMPTIONS:
(1) Steady-state conditions,
(2) Water exposed to standard atmospheric pressure and uniform temperature

PROPERTIES: *Table A-6*, Saturated water, liquid (T_{sat} = 100 °C): ρ_l = 957,9 kg/m³, $C_{p,l}$ = 4217 J/kgK, μ_l = 279 · 10⁻⁶ Ns/m², Pr_l = 1,76, h_{fg} = 2257 kJ/kg, $h'_{fg} = h_{fg} + 0{,}80 \cdot C_{p,v}(T_s - T_{sat})$ = 2654 kJ/kg, σ = 58,9 · 10⁻³ N/m; Saturated water, vapour (T_{sat} = 100 °C): ρ_v = 0,5955 kg/m³, *Table A-4,* Water vapor ($T_f = (T_s + T_{sat})/2 \approx 500$ °C): ρ_v = 0,4405 kg/m³, $C_{p,v}$ = 4217 J/kgK, k_v = 0,0339 W/mK ν_v = 38,68 · 10⁻⁶ m²/s.

ANALYSIS:

a) Since ΔT_e > 120 °C, the element is operating in the film-boiling (FB) regime. The electrical power dissipation per unit length is

$$q'_s = \bar{h}(\pi D)(T_s - T_{sat}) \tag{1}$$

total heat transfer coefficient is

$$\overline{h}^{4/3} = \overline{h}_{\mathrm{conv}}^{4/3} + \overline{h}_{\mathrm{rad}}\overline{h} \qquad (2)$$

The convection coefficient, correlation eq. 10-5 ($C = 0{,}62$):

$$\frac{\overline{h}_{\mathrm{conv}}D}{k_{\mathrm{v}}} = C\left[\frac{g(\rho_{\mathrm{l}}-\rho_{\mathrm{v}})h'_{\mathrm{fg}}D^3}{\nu_{\mathrm{v}}k_{\mathrm{v}}(T_{\mathrm{s}}-T_{\mathrm{sat}})}\right]^{1/4}$$

$$\Rightarrow \overline{h}_{\mathrm{conv}} = \frac{0{,}0339\ \mathrm{W/mK}}{0{,}005\ \mathrm{m}}$$

$$\cdot\, 0{,}62 \cdot \left[\frac{9{,}81\ \mathrm{m/s^2}\cdot(957{,}9-0{,}5955)\ \mathrm{kg/\ m^3}\cdot 2654\ \mathrm{kJ/kg}\cdot(0{,}005\ \mathrm{m})^3}{38{,}68\cdot 10^{-6}\ \mathrm{m^2/s}\cdot 0{,}0339\ \mathrm{W/mK}\cdot(350-100)\mathrm{K}}\right]^{1/4}$$

$$\overline{h}_{\mathrm{conv}} = 225\frac{\mathrm{W}}{\mathrm{m^2K}}$$

The radiation coefficient, Eq. 10-7, where the total hemispherical emissivity $\varepsilon = 0{,}25$ and the Stefan-Bolzman constant $\sigma = 2{,}57\cdot 10^{-8}\ \mathrm{W/m^2K^4}$

$$\overline{h}_{\mathrm{rad}} = \frac{\varepsilon\sigma(T_{\mathrm{s}}^4 - T_{\mathrm{sat}}^4)}{T_{\mathrm{s}} - T_{\mathrm{sat}}}$$

$$\overline{h}_{\mathrm{rad}} = \frac{0{,}25\cdot 2{,}57\cdot 10^{-8}\ \mathrm{W/m^2K^4}\cdot(623^4-373^4)\ \mathrm{K^4}}{(623-373)\ \mathrm{K}} = 7{,}4\ \mathrm{W/m^2K}$$

Substituting numerical values into Eg. (1) and (2)

$$\overline{h} = 231\ \mathrm{W/m^2K}$$

$$q'_{\mathrm{s}} = 231\frac{\mathrm{W}}{\mathrm{m^2K}}\cdot(\pi\cdot 0{,}005\ \mathrm{m})(623-373)\ \mathrm{K} = 907\ \mathrm{W/m}$$

$$q''_{\mathrm{s}} = \frac{q'_{\mathrm{s}}}{\pi D} = \frac{907\ \mathrm{W/m}}{\pi\cdot 0{,}005\ \mathrm{m}} = 57{,}8\ \mathrm{kW/m^2}$$

The evaporation rate per unit length is

$$\mathrm{m}'_{\mathrm{b}} = \frac{q'_{\mathrm{s}}}{h_{\mathrm{fg}}} = 1{,}4\ \mathrm{kg/ms}$$

b) For the same heat flux, $q''_{\mathrm{s}} = 57{,}8\ \mathrm{kW/m^2}$, using the Rohsenow for the nucleate boiling (NB) regime.

$$q''_{\mathrm{s}} = \mu_{\mathrm{l}}h_{\mathrm{fg}}\left[\frac{g(\rho_{\mathrm{l}}-\rho_{\mathrm{v}})}{\sigma}\right]^{1/2}\left[\frac{C_{\mathrm{p,l}}\Delta T_{\mathrm{e}}}{C_{\mathrm{s,f}}h_{\mathrm{fg}}\mathrm{Pr}_{\mathrm{l}}^{\mathrm{n}}}\right]^3$$

Where, from Table 10-1, for stainless steel mechanically polished finish with water, $C_{\mathrm{s,f}} = 0{,}0132$ and $n = 1{,}0$

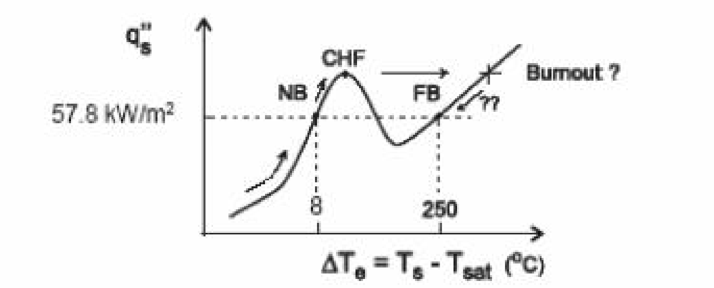
$$\Delta T_{\mathrm{e}} = \frac{C_{\mathrm{s,f}} h_{\mathrm{fg}} \mathrm{Pr}_{\mathrm{l}}^{\mathrm{n}} \left[\dfrac{q_{\mathrm{s}}''}{\mu_{\mathrm{l}} h_{\mathrm{fg}} \left[\dfrac{g(\rho_{\mathrm{l}} - \rho_{\mathrm{v}})}{\sigma} \right]^{\frac{1}{2}}} \right]^{\frac{1}{3}}}{C_{\mathrm{p,l}}} = 7{,}6\ \mathrm{K}$$

$$T_{\mathrm{s}} = \Delta T_{\mathrm{e}} + T_{\mathrm{sat}} = 107{,}6\ °\mathrm{C}$$

The evaporation rate per unit length is

$$\mathrm{m}_{\mathrm{b}}' = \frac{q_{\mathrm{s}}'}{h_{\mathrm{fg}}} = 1{,}4\ \mathrm{kg/ms}$$

The two operating conditions are shown on the boiling curve. The element can be operated at NB with the same heat flux $q_{\mathrm{s}}'' = 57{,}8\ \mathrm{kW/m^2}$, with a surface temperature of $T_{\mathrm{s}} = \Delta T_{\mathrm{e}} + T_{\mathrm{sat}} = 107{,}6\ °\mathrm{C}$



COMMENTS:

1) *If the element is cole, it would be brought to the NB condition following the arrow of the boiling curve. To reach the FB regime, the heat flux is increased to the critical heat flux (CHF). The burnout condition will then likely occur. If not, the FB point is reached by decreasing the heat flux.*

10.3 FORCED CONVECTION BOILING

In forced convection boiling fluid flow is due to combination of bulk fluid motion and buoyancy effects of vapour bubbles.

10.3.1 Two-Phase Flow

Internal, forced convection boiling is referred as two-phase flow and is associated with bubble growth at inner surface of heated tube through which liquid flows. The boiling flow goes through different two-phase flow regions.

1) Let us consider a vertical tube that is subjected to a constant surface heat flux, and where laminar sub-cooled liquid flow enters initially in ***single-phase forced convection*** mode.

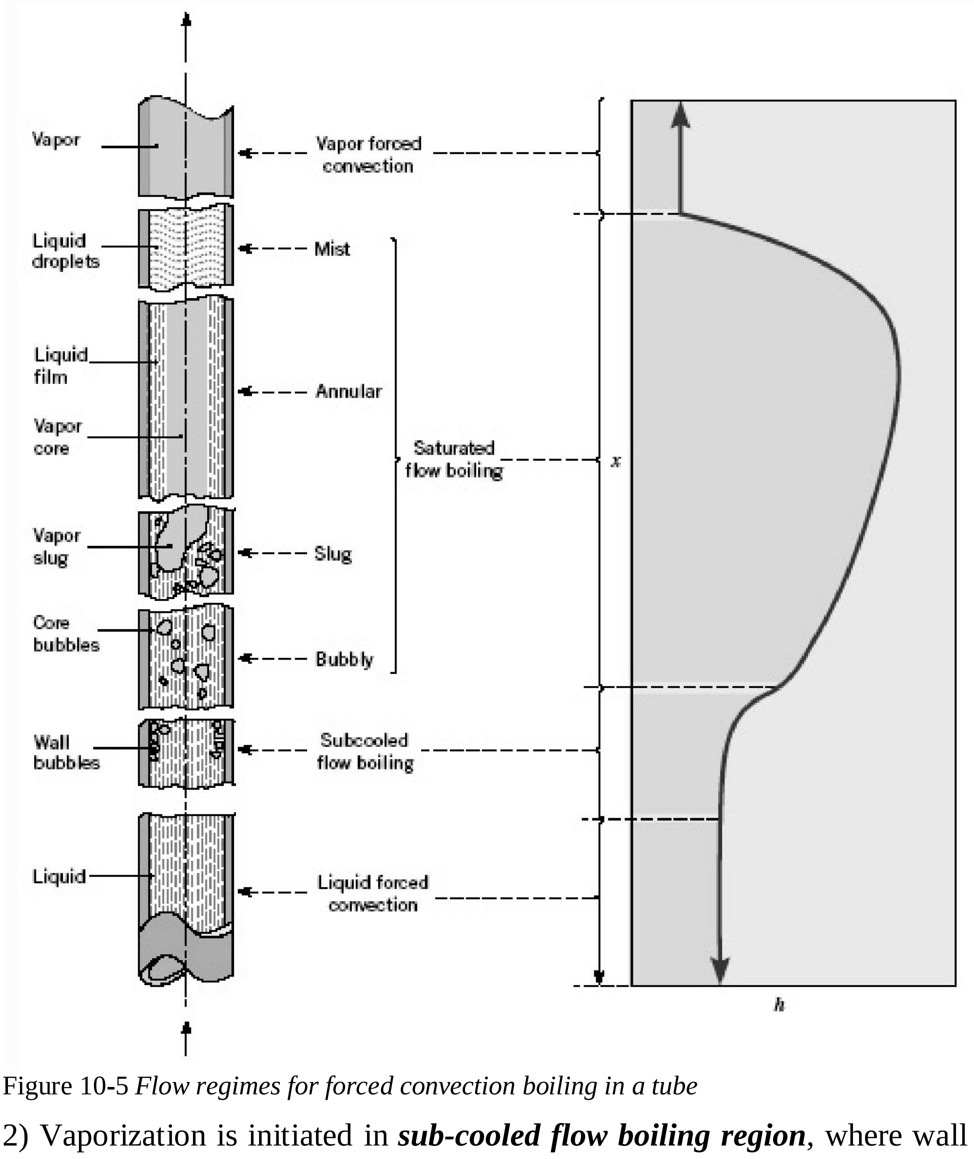


Figure 10-5 *Flow regimes for forced convection boiling in a tube*

2) Vaporization is initiated in ***sub-cooled flow boiling region***, where wall temperature exceeds saturation temperature. Radial temperature differences are significant as bubbles are adjacent to heated wall and sub-cooled liquid. Thickness of bubble region increases with downstream of flow and eventually core of liquid reaches saturation temperature of fluid, which is point of beginning of saturated flow boiling.

3) ***Saturated flow boiling***

In saturated flow boiling region mean velocity of fluid increases remarkably due to increase of vapour mass fraction.

- First stage is known as bubbly flow regime.
- 2nd stage, slug-flow regime has bubble coalescence forming vapour slugs.
- 3rd stage, annular-flow regime has liquid film on surface and vapour flowing at larger velocity in the core of tube
- Mist Regime: as liquid film dries eventually at transition zone, vapour carries remaining liquid in droplets with high velocity at core of tube.

4) ***Vapour forced convection***

When liquid droplets are completely evaporated flow regime changes to vapour forced convection. Here superheated vapour is in single-phase flow mode having significantly higher velocity than liquid entering to tube.

Figure 10-5 shows heat transfer coefficient behaviour at different flow regimes of two-phase flow. At sub-cooled boiling region heat transfer coefficient increase is around order of magnitude and smallest heat transfer coefficients exist at vapour forced convection regime. Following correlation has been defined for saturated flow boiling region in smooth circular tubes:

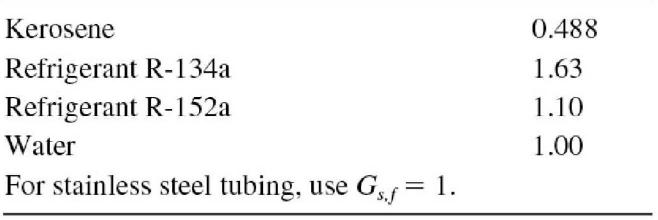
$$\frac{h}{h_{sp}} = 0.6683\left(\frac{\rho_l}{\rho_v}\right)^{0.1}\overline{X}^{0.16}(1-\overline{X})^{0.64} f(Fr) + 1058\left(\frac{q_S''}{\dot{m}'' h_{fg}}\right)^{0.7}(1-\overline{X})^{0.8} G_{s,f} \qquad (10\text{-}9)$$

Stratification parameter *f(Fr)* is unity for vertical tubes and horizontal tubes of $Fr<0.04$. For horizontal tubes having $Fr>0.04$ $f(Fr) = 2.63Fr^{0.3}$. Single-phase convection coefficient h_{sp} is associated with liquid forced convection and is defined at saturation temperature.

Following is valid for liquid phase *Froude number*

$$Fr = \left(\frac{q_s''}{\dot{m}'' h_{fg}}\right)^{2}\frac{1}{gD} \qquad (10\text{-}10)$$

Table 10-2 *Coefficient $G_{s,f}$ values for different surface-liquid combinations*



Fluid in Commercial Copper Tubing	$G_{s,f}$
Kerosene	0.488
Refrigerant R-134a	1.63
Refrigerant R-152a	1.10
Water	1.00
For stainless steel tubing, use $G_{s,f} = 1$.	

Mean vapour mass fraction can be solved from equation

$$\overline{X}(x) = \frac{q_S'' \pi D x}{\dot{m} h_{fg}} \tag{10-11}$$

x Distance from point, where vapour mass fraction exceeds zero

10.4 PRINCIPLES OF CONDENSATION

Reduction of vapour temperature below its saturation temperature results as condensation. In industrial applications condensation occurs commonly in contact between cool surface and vapour. Vapour releases it latent energy, which is transferred to cool surface and condensate is simultaneously formed.

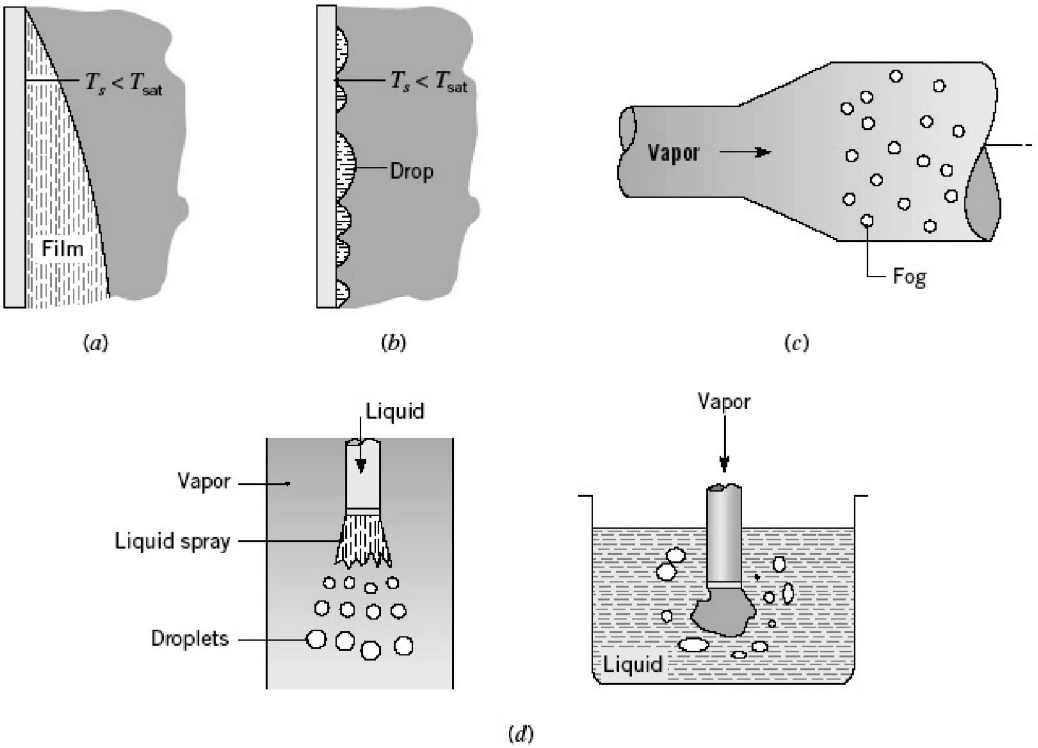


Figure 10-6 *Modes of Condensation a) film condensation b) droplet condensation on a surface c) homogenous condensation or fog formation d) Direct contact condensation*

10.4.1 Surface Condensation

Film Condensation

Condensate liquid forms a film flowing across cooling surface. Condensation latent heat conducts through liquid film to cooling surface from liquid film surface. Film condensation is most usual type of condensation in industrial equipments.

Drop-wise Condensation

Condensate forms droplets on cool surfaces cracks, pits and cavities. Size of the droplets covering typically 90% of surface varies from few micrometers to agglomerations visible to the naked eye. Drop-wise condensation mode conditions may be created with surface coatings that inhibit wetting.

10.4.2 Homogenous Condensation

Vapour condensates homogenously to liquid droplets with decreasing temperature. Liquid droplets are in form of fog. Temperature change can be caused for example velocity-pressure change of vapour.

10.4.3 Direct Condensation

Condensation of vapour occurs by cooling in a cold liquid, where vapour is sprayed. Other option is to spray liquid coolant to vapour, which causes vapour condensation to surface of liquid droplets.

10.4.4 Condensers

Condensate forms thermal resistance to heat transfer between vapour and cool surface. Condensate layer thickness increases to direction of flow and heat resistance increases with thickness. Thus short vertical surfaces or horizontal cylinders are preferred in design of condensers. Typical condenser is formed from horizontal tube bundles through which liquid coolant flows and around where vapour condensates.

For performance of condenser in terms of heat transfer rate and condensation, drop-wise condensation mode is preferable instead of film condensation operation. An order of magnitude higher heat transfer rates are achievable with drop-wise condensation compared to film condensation. In practice, condenser design calculations are done for film condensation mode, because difficulty of maintaining drop-wise condensation conditions.

10.5 FILM CONDENSATION

Total condensation heat transfer rate to cooling surface can be written in form of *Newton's Law of Cooling*:

$$q = \bar{h}_L A(T_{sat} - T_s) \tag{10-12}$$

Condensation mass flow rate can be then expressed with heat transfer rate as

$$\dot{m} = \frac{q}{h'_{fg}} = \frac{\bar{h}_L A(T_{sat} - T_s)}{h'_{fg}} \tag{10-13}$$

$$h'_{fg} = h_{fg}(1 + 0.68 Ja) \qquad \textit{Modified latent heat}$$

Dimensionless parameter, *Jakob Number*, describes the ratio of maximum sensible energy absorbed by the liquid to latent energy absorbed by liquid during condensation

$$Ja = \frac{c_p (T_{sat} - T_s)}{h_{fg}} \tag{10-14}$$

10.5.1 Laminar film condensation on a vertical plate

Let us consider laminar condensation film formation on a vertical plate. Film flows downward influenced by the gravity with increasing condensate film thickness and mass flow rate caused by continuous vapour condensation. Liquid-droplet interface is approximately at saturation temperature. Thermal boundary layer develops between cooling surface and vapour-liquid interface. Velocity gradient in both vapour and liquid is formed due shear stress forces. (Figure 10-7 a)

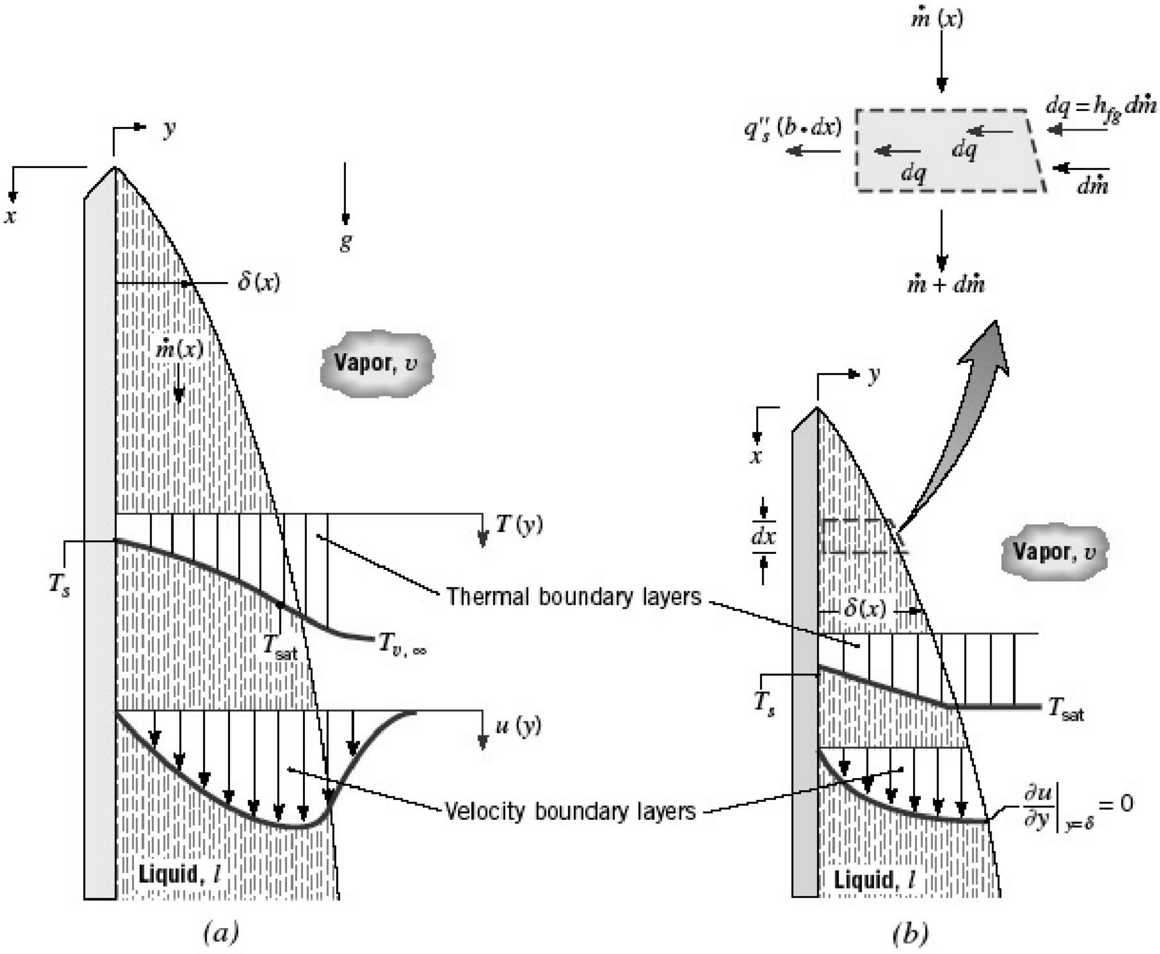


Figure 10-7 *laminar film condensation boundary layers on a vertical plate*

Analytical solution for laminar film condensation on a vertical plate is presented by Nusselt and it contains a set of approximations and assumptions (Figure 10-7):

- Constant properties in laminar flow
- Pure vapour at uniform saturation temperature
- Shear stress negligible in liquid-vapour interface: velocity gradient opposite to surface at film vapour interface is zero
- Momentum and energy transfer by advection of liquid in condensate film is assumed to be negligible, because of the low flow velocity of film: Heat transfer is only due to conduction from liquid-vapour surface to cooling surface and results a linear temperature profile in condensate film.

Newton’s second law of motion can be then written x- momentum equation as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=0=-\frac{1}{\rho}\frac{\partial p_\infty}{\partial x}+v\frac{\partial^2 u}{\partial y^2} \tag{10-15}$$

By stating pressure gradient $dp_\infty / dx = \rho g$, momentum equation reduces to

$$\frac{\partial^2 u}{\partial y^2}=-\frac{g}{\mu_l}(\rho_l-\rho_v) \tag{10-16}$$

Velocity profile can be obtained by integrating as

$$u(y)=\frac{g(\rho_l-\rho_v)\delta^2}{\mu_l}\left[\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \tag{10-17}$$

Condensate mass flow rate per unit width b is then

$$\Gamma(x)=\frac{\dot{m}}{b}=\int_0^{\delta(x)}\rho_l u(y)dy=\frac{g\rho_l(\rho_l-\rho_v)\delta^3}{3\mu_l} \tag{10-18}$$

For thickness of condensate layer following expression can be obtained

$$\delta(x)=\left[\frac{4k_l\mu_l(T_{sat}-T_s)x}{g\rho_l(\rho_l-\rho_v)h_{fg}^{'}}\right]^{1/4} \tag{10-19}$$

Local heat transfer coefficient can be written with liquid conductivity and condensate layer thickness as

$$h_x=\frac{k_l}{\delta}=\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3h_{fg}^{'}}{4\mu_l(T_{sat}-T_s)x}\right]^{1/4} \tag{10-20}$$

Average heat transfer coefficient can be solved by integrating over length of plate as

$$\bar{h}_L=0.943\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3h_{fg}^{'}}{\mu_l(T_{sat}-T_s)L}\right]^{1/4} \tag{10-21}$$

Average Nusselt number can thus be written as

$$\overline{Nu}_L=\frac{\bar{h}_L L}{k_l}=0.943\left[\frac{g\rho_l(\rho_l-\rho_v)h_{fg}^{'}L^3}{\mu_l k_l(T_{sat}-T_s)}\right]^{1/4} \tag{10-22}$$

10.5.2 Turbulent Film Condensation

Reynolds number of sliding water film on a vertical plate is defined as

$$\mathrm{Re}_\delta = \frac{4\Gamma}{\mu_l} = \frac{4\rho_l u_m \delta}{\mu_l} \tag{10-23}$$

Flow conditions in film condensation on vertical plate are laminar and wave free if $\mathrm{Re}_\delta \le 30$. Reynolds number for laminar wave free condensation film can be further modified by substituting laminar condensate film flow and thickness equations described above to Re_δ definition as

$$\mathrm{Re}_\delta = \frac{4\Gamma}{\mu_l} = \frac{4g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l^2} = 3.78\left[\frac{k_l L(T_{sat} - T_s)}{\mu_l h_{fg}^{'}(v_l^2/g)^{1/3}}\right]^{4/3} \tag{10-24}$$

By combining definition of Reynolds number and condensate mass flow rate equation for condensation heat transfer coefficient can be achieved as

$$\bar{h}_L = \frac{\mathrm{Re}_\delta\, \mu_l h_{fg}^{'}}{4L\left(T_{sat} - T_s\right)} \tag{10-25}$$

Transition to laminar and wavy film flow occurs after Reynolds number exceeds 30. Moreover turbulent conditions are reached with Reynolds number exceeding approximately 1800.

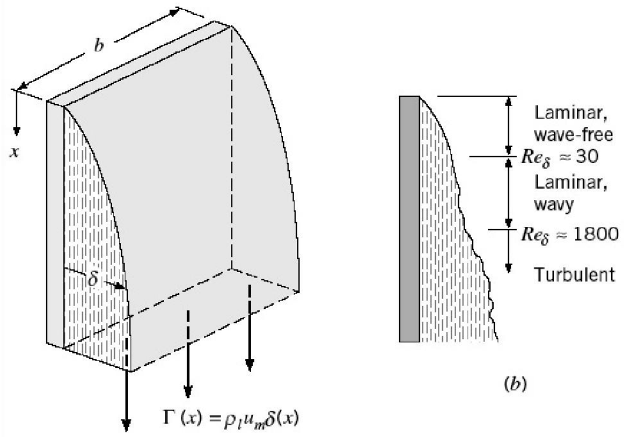


Figure 10-8 *Film condensation on vertical plate: a) condensate flow rate b) Flow regimes*

From following correlations Reynolds number can be solved in wavy laminar ($30 \le \mathrm{Re}_\delta \le 1800$) flow or at turbulent flow ($1800 \le \mathrm{Re}_\delta$):

$$\mathrm{Re}_\delta = \left[\frac{3.7 k_l L(T_{sat} - T_s)}{\mu_l h_{fg}^{'}(v_l^2/g)^{1/3}} + 4.8\right]^{0.82} \tag{10-26}$$

$$\mathrm{Re}_{\delta} = \left[\frac{0.069 k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2 / g)^{1/3}} \mathrm{Pr}_l^{0.5} - 151 \mathrm{Pr}_l^{0.5} + 253 \right]^{4/3} \qquad (10\text{-}27)$$

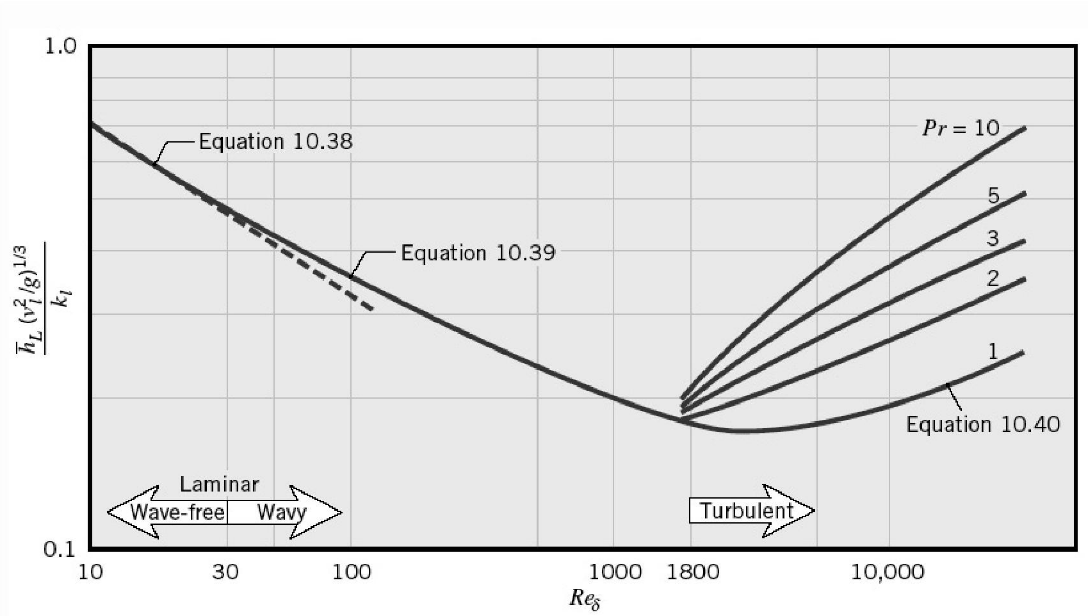


Figure 10-9 *Modified Nusselt number for condensation on a vertical plate*

10.5.3 Radial Systems

Film Condensation on Outer Surfaces

Typical condenser design tube arrangement is vertical tier of horizontal tubes (Figure 10-10d). Heat transfer coefficient on outer surface reduces from the first tube row to lower tube row levels caused by increased thickness of condensate film layer.

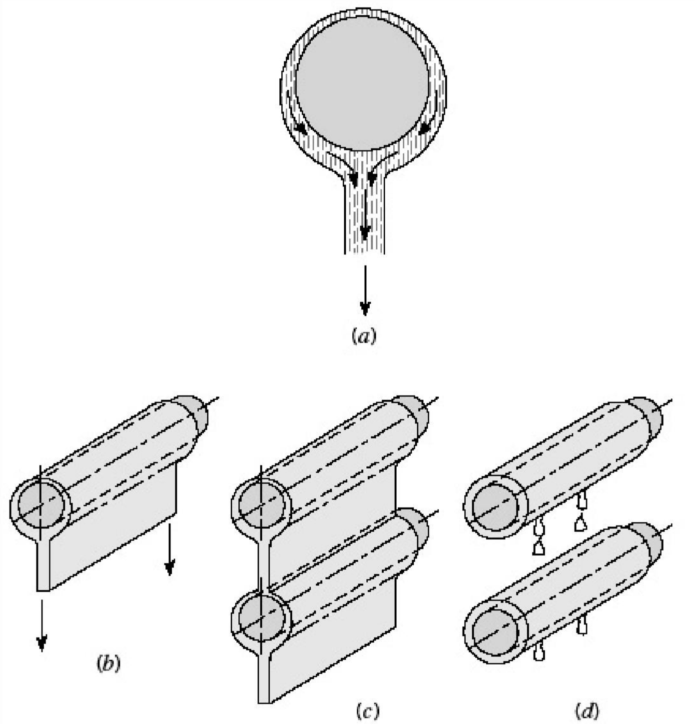


Figure 10-10 *Film condensation flow on a) sphere b) single horizontal tube c) vertical tier of horizontal tubes with continuous condensate sheet and d) with dripping condensate*

Extension of Nusselt analysis to sphere's and horizontal tubes outer surface laminar film condensation leads to solution of average heat transfer coefficient as

$$\bar{h}_D = C\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3 h'_{fg}}{\mu_l(T_{sat}-T_s)D}\right]^{1/4} \tag{10-28}$$

C = 0.826 Sphere
C = 0.729 Tube

Average heat transfer coefficient for vertical tier of N horizontal tubes can be solved from

$$\bar{h}_{D,N} = 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3 h'_{fg}}{N\mu_l(T_{sat}-T_s)D}\right]^{1/4} \tag{10-29}$$

Film Condensation in horizontal Tubes

Refrigeration and air-conditioner condensers typically have vapour condensing inside horizontal or vertical tubes. Film condensation conditions at inner surface of tube depend strongly on vapour velocity. With low velocities condensate flows from upper portion of the tube to the bottom, where it flows to longitudinal direction pushed by vapour flow shear forces.

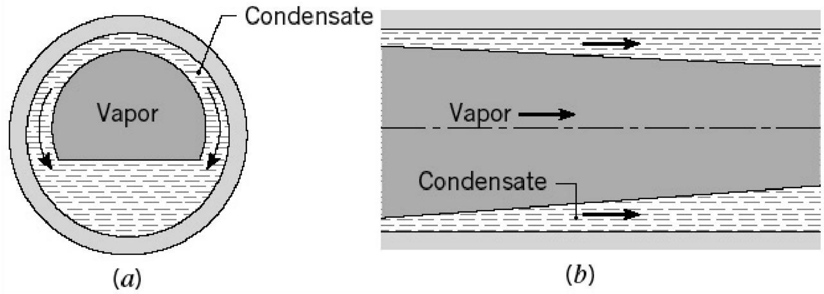


Figure 10-11 *Film condensation in a horizontal tube a) low vapour velocities b) large vapour velocities*

One low velocity criteria presented in form of Reynolds number is

$$\mathrm{Re}_{v,i} = \left(\frac{\rho_v u_{m,v} D}{\mu_v}\right) < 35000 \tag{10-30}$$

And related recommended average heat transfer coefficient is written as

$$\bar{h}_D = 0.555\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3 h'_{fg}}{\mu_l(T_{sat}-T_s)D}\right]^{1/4} \tag{10-31}$$

Where modified latent heat is determined as

$$h'_{fg} = h_{fg} + \frac{3}{8}c_{p,l}(T_{sat}-T_s) \tag{10-32}$$

10.6 DROP-WISE CONDENSATION

Drop-wise condensation is characterized by formation of droplets of varying size on surface and subsequent growth followed by slide down on cooling surface when they achieve critical size. Drop-wise condensation is one of the most efficient heat transfer mechanisms and an order of a magnitude higher heat transfer coefficients are typical compared to film condensation.

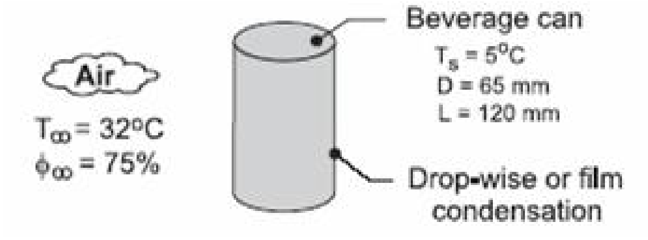
A simple heat transfer coefficient correlation for drop-wise condensation of steam on copper surfaces is presented as

$$h_{dropwise} = \begin{cases} 51100 + 2044 T_{sat} & 22°C < T_{sat} < 100°C \\ 255310 & T_{sat} > 100°C \end{cases} \tag{10-33}$$

In heat exchanger applications having enhancement of heat transfer by promoting drop-wise condensation, thermal resistance associated with other heat transfer mechanisms may remarkably larger.

Example XIV: ***Drop-wise condensation – Cool Koff Can in shower room***

A beverage can with a diameter of 65 mm and a height of 120 mm had a uniform temperature of 5 °C when it is removed from a refrigerator. The can is set on a table in a room with an ambient air temperature of 32 °C and a relative humidity of 75 %. Estimate the condensation rate assuming Drop-wise condensation.



3 ASSUMPTIONS:
(1) Condensation on top and bottom surface of can neglected,
(2) Negligible non-condensibles in water vapor-air,
(3) For film condensation, film thickness is small compared to diameter of can.

PROPERTIES: *Table A-6,*
Water vapor (T_∞ = 32 °C = 305 K): $p_{A,sat}$ = 0,04712 bar;
Water vapor ($p_A = \phi \cdot p_{A,sat}$ = 0,04712 bar) $T_{sat} \approx$ 300 K = 27 °C, h_{fg} = 2438 kJ/kg ;
Water liquid ($T_f = (T_s + T_{sat})/2$ = 289 K) $C_{p,l}$ = 4185 J/kgK.

From Eq. 10-13:

$$\dot{m} = \frac{q}{h_{fg}} = \frac{\overline{h}(\pi D L)(T_{sat} - T_s)}{h'_{fg}}$$

Where h'_{fg}, with Ja $= \frac{C_{p,l}(T_{sat}-T_s)}{h_{fg}}$,n (Eq. 10-14): $h'_{fg} = h_{fg}[1 + 0{,}68\,\text{Ja}]$

$$h'_{fg} = 2438\ \text{kJ/kg} \cdot \left[1 + 0{,}68 \cdot \frac{4185\ \text{J/kgK} \cdot (300 - 278)\text{K}}{2438\ \text{kJ/kg}}\right]$$

$$h'_{fg} = 2501\ \text{kJ/kg}$$

(T_{sat} is the saturation temperature of the water vapor in air at 32 °C having a relative humidity of $\phi_\infty = 0{,}75$.) For drop-wise condensation, the correlation of Eq. 10.33 with $T_{\mathrm{sat}} = 300\ \mathrm{K} = 27\ °\mathrm{C}$ yields.

$$\bar{\mathrm{h}} = \bar{\mathrm{h}}_{\mathrm{dc}} = 51104 + 2044\,T_{\mathrm{sat}}$$

$$22\ °\mathrm{C} < T_{\mathrm{sat}} \leq 100\ °\mathrm{C}$$

$\bar{\mathrm{h}}_{\mathrm{dc}}$, [W/m^{2}K]

T_{sat}, [°C]

$$\bar{\mathrm{h}}_{\mathrm{dc}} = 51104 + 2044\ 27 = 106292\ \mathrm{W/m^2K}$$

Hence, the condensation rate is

$$\dot{\mathrm{m}} = \frac{106292\,\frac{\mathrm{W}}{\mathrm{m^2K}} \cdot (\pi \cdot 0{,}065\ \mathrm{m} \cdot 0{,}125\ \mathrm{m})(27 - 5)\mathrm{K}}{2501\ \mathrm{kJ/kg}} = 0{,}0229\ \mathrm{kg/s}$$

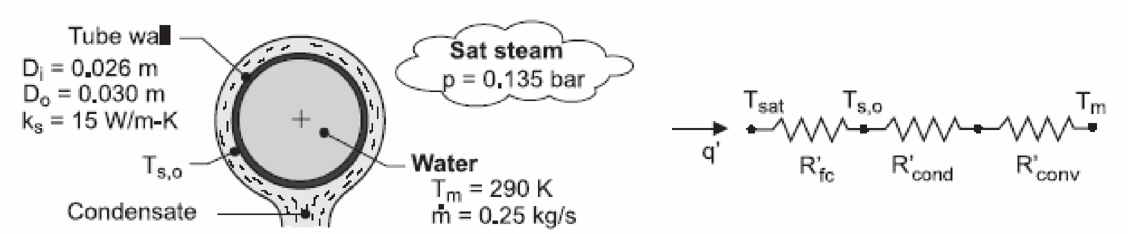
COMMENTS:

1) *With film condensation mode – cooling of can would take much more time, condensation rate circa 20 times less*

Example XV: *Condenser of a steam power plant*

The condenser of a steam power plant consists of AISI 302 stainless steel tubes ($k = 15\ \mathrm{W/mK}$), each of outer and inner diameters $D_{\mathrm{o}} = 30\ \mathrm{mm}$ and $D_{\mathrm{i}} = 26\ \mathrm{mm}$, respectively. Saturated steam at 0.135 bar condenses on the outer surface of a tube, while water at a mean temperature of $T_{\mathrm{m}} = 290\ \mathrm{K}$ is in fully developed flow through the tube.

a) For a water flow rate of $\dot{m} = 0.25\ \mathrm{kg/s}$, what is the outer surface temperature $T_{\mathrm{s,o}}$ of the tube and the rates of heat transfer and steam condensation per unit tube length? (As a first estimate, evaluate the properties of the liquid film at the saturation temperature.) If one wishes to increase the transfer rates, what is the limiting factor that should be addressed?
b) Explore the effect of the water flow rate on $T_{\mathrm{s,o}}$ and the rate of heat transfer per unit length.



ASSUMPTIONS: (1) Negligible concentration of noncondensible gases in the steam. (2) Uniform tube surface temperature, (3) Laminar film condensation, (4) Fully-developed internal flow, (5) Constant properties.

PROPERTIES: *Table A-6*, Water ($T_{\mathrm{m}} = 290\ \mathrm{K}$): $\mu = 0{,}00108\ \mathrm{Ns/m^2}$, $k = 0{,}598\ \mathrm{W/mK}$, $\mathrm{Pr} = 7{,}56$; Saturated water, vapour ($p = 0{,}135\ \mathrm{bar}$, $T_{\mathrm{sat}} = 325\ \mathrm{K}$): $\rho_{\mathrm{v}} = 0{,}0904\ \mathrm{kg/m^3}$, $h_{\mathrm{fg}} = 2378\ \mathrm{kJ/kg}$; Saturated water, liquid ($T_{\mathrm{f}} \approx T_{\mathrm{sat}}$): $\rho_{\mathrm{l}} = 987\mathrm{kg/m^3}$, $C_{\mathrm{p,l}} = 4182\ \mathrm{J/kgK}$, $\mu_{\mathrm{l}} = 528 \cdot 10^{-6}\ \mathrm{Ns/m^2}$, $k_{\mathrm{l}} = 0{,}645\ \mathrm{W/mK}$

ANALYSIS: From the thermal circuit, the heat rate may be expressed as

$$q' = \frac{T_{\mathrm{sat}} - T_{\mathrm{m}}}{R'_{\mathrm{fc}} + R'_{\mathrm{cond}} + R'_{\mathrm{conv}}} \tag{1}$$

Conductive heat resistance:

$$R'_{\mathrm{cond}} = \frac{\ln(D_{\mathrm{o}}/D_{\mathrm{i}})}{2\pi k_s} = \frac{\ln(0{,}030\ \mathrm{m}/0{,}026\ \mathrm{m})}{2\pi\cdot 15\ \mathrm{W/mK}} = 0{,}00152\ \mathrm{mK/W} \tag{2}$$

Convective heat resistance: $R'_{\mathrm{conv}} = (\pi D_{\mathrm{i}} h_{\mathrm{i}})^{-1}$ (3)

Reynolds number: $\mathrm{Re}_{\mathrm{D}} = \frac{4\dot{m}}{\pi D_{\mathrm{i}}\mu} = \frac{4\cdot 0{,}25\ \mathrm{kg/s}}{\pi\cdot 0{,}026\ \mathrm{m}\cdot 0{,}00108\ \mathrm{Ns/\ m^2}} = 11336$ (Turbulent)

Appropriate correlation for the Nusselt number is the Dittius-Boelter correlation, Eq. 8-23:

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = 0{,}023\mathrm{Re}_{\mathrm{D}}^{4/5}\mathrm{Pr}^{0{,}4}$$

$$\Rightarrow h_{\mathrm{i}} = \left(\frac{k}{D_{\mathrm{i}}}\right)0{,}023\mathrm{Re}_{\mathrm{D}}^{4/5}\mathrm{Pr}^{0{,}4}$$

Substituting numerical values to Eq. 3:

$$R'_{\mathrm{conv}} = \left(\pi k \cdot 0{,}023\mathrm{Re}_{\mathrm{D}}^{4/5}\mathrm{Pr}^{0{,}4}\right)^{-1}$$

$$= \left(\pi \cdot 0{,}598\ \mathrm{W/mK} \cdot 0{,}023 \cdot 11336^{4/5} \cdot 7{,}56^{0{,}4}\right)^{-1}$$

$$R'_{\mathrm{conv}} = 0{,}00588\ \mathrm{mK/W}$$

The resistance associated with the condensate film:

$$R'_{\mathrm{fc}} = (\pi D_{\mathrm{o}} h_{\mathrm{o}})^{-1} \tag{4}$$

Where the average convection coefficient for laminar film condensation on a horizontal (with $C = 0{,}729$) tube is:

$$\overline{\mathrm{h}}_{\mathrm{o}} = C\left[\frac{g\rho_{\mathrm{l}}(\rho_{\mathrm{l}}-\rho_{\mathrm{v}})k_{\mathrm{l}}^{3}h'_{\mathrm{fg}}}{\mu_{\mathrm{l}}(T_{\mathrm{sat}}-T_{\mathrm{s,o}})D}\right]^{1/4} \tag{5}$$

$$\overline{\mathrm{h}}_{\mathrm{o}} = 0{,}729\cdot\left[\frac{9{,}81\ \mathrm{m/s^2}\cdot 987\mathrm{kg/\ m^3}\cdot(987-0{,}0904)\ \mathrm{kg/\ m^3}\cdot(0{,}645\ \mathrm{W/mK})^3\cdot h'_{\mathrm{fg}}}{528\cdot 10^{-6}\ \mathrm{Ns/\ m^2}\cdot(325-T_{\mathrm{s,o}})\mathrm{K}\cdot 0{,}030\ \mathrm{m}}\right]^{1/4}$$

$$h'_{\mathrm{fg}} = h_{\mathrm{fg}} + 0{,}68\cdot C_{\mathrm{p,l}}\left(T_{\mathrm{sat}} - T_{\mathrm{s,o}}\right) \tag{6}$$

The unknown surface temperature may be determined from an additional rate equation, such as

$$q' = \frac{T_{\mathrm{s,o}}-T_{\mathrm{m}}}{R'_{\mathrm{cond}}+R'_{\mathrm{conv}}} \tag{7}$$

$$\Rightarrow T_{\mathrm{s,o}} = T_{\mathrm{m}} + q'\left(R'_{\mathrm{cond}} + R'_{\mathrm{conv}}\right)$$

Iteration from equations (4), (5), (6) and (7), with $T_{\mathrm{s,o}} = 300\ \mathrm{K}$ as a starting value, gives:

$h'_{\mathrm{fg}} =$ 2449 kJ/kg	$\overline{\mathrm{h}}_{\mathrm{o}} =$ 25,9 kJ/kg	R'_{fc} = 0,41 mK/W	q' = 4481 W/m	$T_{\mathrm{s,o}}$ = 323 K

10.7 LEARNING OUTCOMES

Table 10-3 Learning Outcomes: *Boiling & Condensation*

Level of Knowledge	**Concept**
	Boiling
Apply, Understand, *describe*	• *Distinguish pool boiling regimes* • *Concepts: Boiling curve, critical heat flux, Leidenfrost point, ONB, excess temperature* • *Heat transfer correlation under different pool boiling regimes* • *Stages of forced convection boiling*[4]
	Condensation
Understand, describe	• *Modes of condensation: film, droplet, homogeneous, direct* • *Film condensation flow regimes: laminar wave-free, Wavy laminar, turbulent*
Apply, Understand, *describe*	• *Heat transfer correlation h calculation: (1) under different film condensation regimes (2)at tube internal and external surfaces (3)at drop-wise condensation mode* • *Concepts: Jacob number, Newton's Law of cooling and condensation rate*

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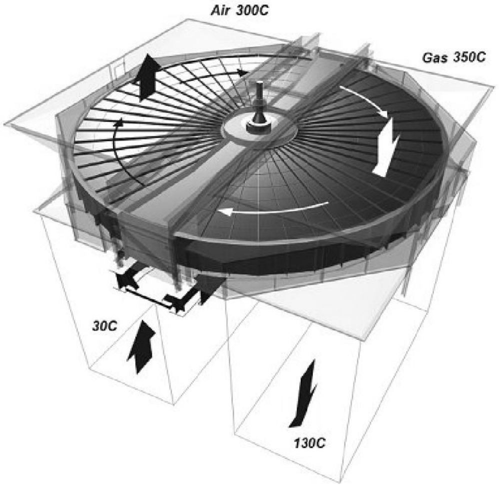
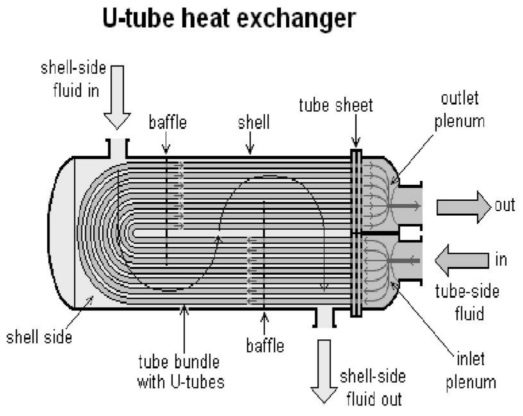
[4] Not currently included to content of Course: *Fundamentals of Heat Transfer*

11 HEAT EXCHANGERS

11.1 HEAT EXCHANGER TYPES

Heat Exchangers involve heat exchange between two fluids (T1 ≠ T2) separated by a solid wall and encompass a wide range of flow configurations. Classification is commonly done according to flow arrangement and type of construction. This chapter discusses only on regenerators, heat exchangers where heat transfer interlinked with cyclic movement of heat transfer surface between cold and hot streams – example configurations and definitions of heat exchangers are given in Table 11-1.

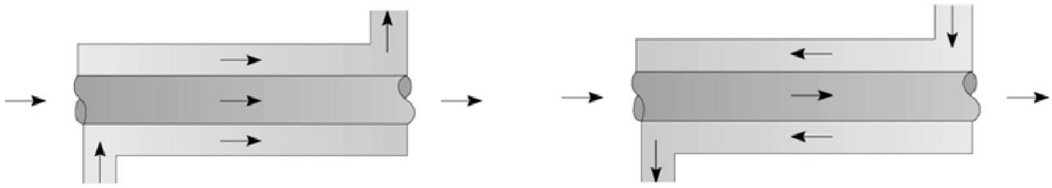
Table 11-1 *Recuperative air-preheater and regenerative shell-and-tube U-tube exchangers*

Recuperators *Heat exchangers where two fluids are separated by the heat transfer surface that normally has no moving parts*	**Regenerators** *Heat exchangers where the heat transfer is performed through a material that is alternately brought to contact with hot and cold streams, storing energy from the hot stream and releasing it into the cold one*
	

Introduction to thermal design of heat exchangers is presented in this lecture note and more comprehensive design guidelines can be found for example in Heat Exchanger Thermal Design Guide (*Saari J., 2010*)

11.1.1 Concentric tube heat exchanger

Most simple and for many small applications cheapest configuration is concentric heat exchangers, which can be constructed with two flow-arrangements: parallel and counterflow.



Parallel Flow (myötävirta) Counterflow (vastavirta)

Figure 11-1 *Parallel and counterflow concentric tube heat exchangers*

11.1.2 Cross-flow heat exchangers

Other flow-arrangement type is cross-flow heat exchangers (Figure 11-2), where hot fluid flow is perpendicular to cold one.

For cross-flow over the tubes, fluid motion, and hence mixing, in the transverse direction (y) is prevented by the finned tubes and in tube flow by tube walls. Cross-flow over tubes without fins is called unmixed flow. Heat exchanger performance is strongly influenced by mixing and heat exchange surface area. Plates can restrict mixing, but on the other hand provide additional surface and act as fins for tubes.

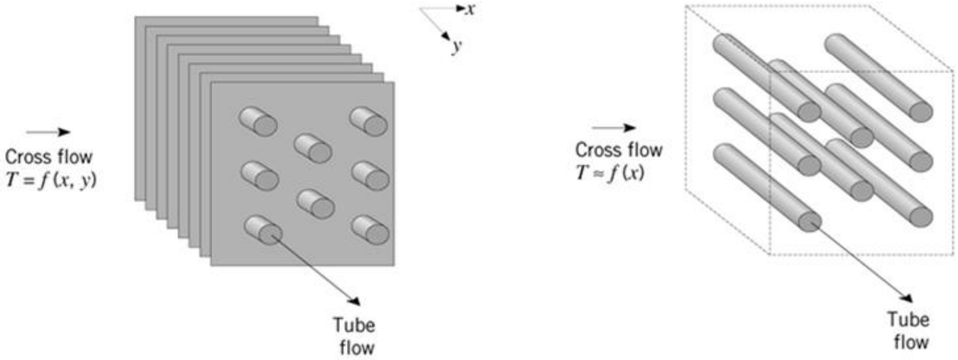


Figure 11-2 *Cross-flow HX's: (1) finned and both fluids unmixed (2) one fluid mixed and other unmixed*

11.1.3 Shell-and-tube heat exchanger

On flow-arrangement point-of-view, Shell-and-tube heat exchangers are mix of concentric and cross-flow heat exchangers (Figure 11-3): Baffles in shell side are used to establish a cross-flow and to induce turbulent mixing of the shell-side fluid, both of which enhance convection. Number of tube and shell passes are varied, e.g. Figure 1-1.

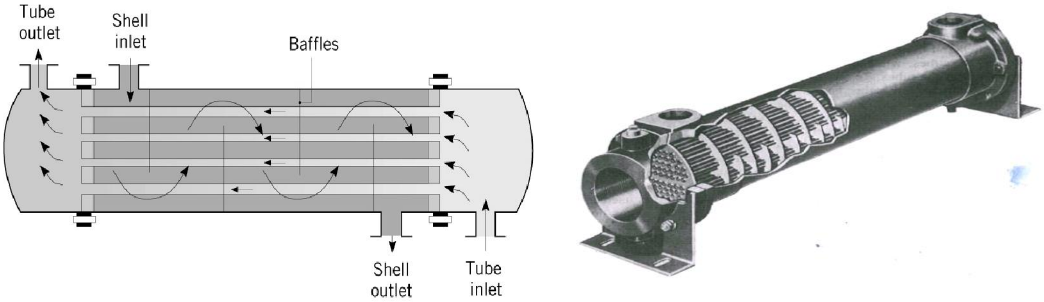


Figure 11-3 *Shell-and-tube heat exchanger with one shell pass and one tube pass*

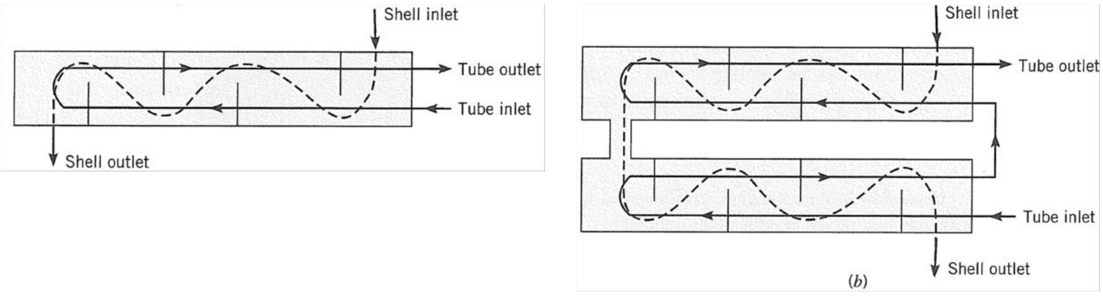


Figure 11-4 *Shell-and-tube heat exchangers: (1) one shell pass and two tube passes and (2) two shell passes and four tube passes*

11.1.4 Compact Heat Exchanger

Compact heat exchangers are characterized by large heat transfer surface areas per unit volume (>400 m^2/m^3), small flow passages ($D_h < 5mm$), and laminar flow. Commonly they are used for achieving large heat rates per unit volume, particularly when one or both fluids are gas.

Several surface configurations exist: (1) Flat or circular tubes, (2) plate or circular fins, (3) flow channels can be constructed from parallel plates, which may include fins or corrugation. Figure 11-5 shows examples of compact heat exchanger core structures.

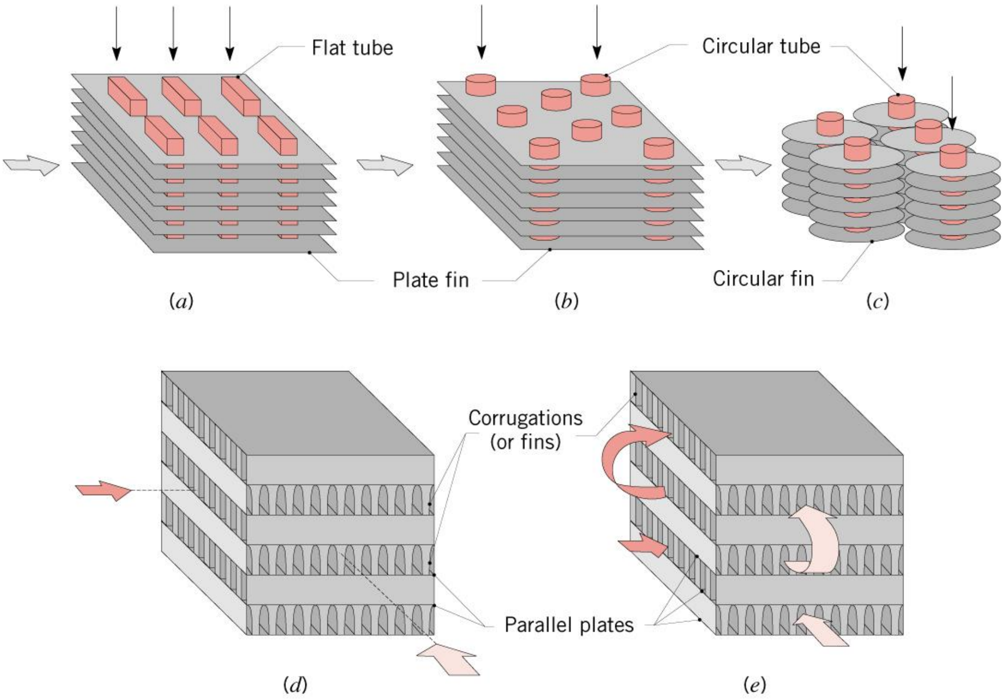


Figure 11-5 *Compact heat exchanger structures (a) Fin-tube flat tubes, continuous plate fins (b) Fin-tube: circular tubes, continuous plate fins (c) Fin-tube: circular tubes, circular fins (d) Plate-fin: single pass) (e) Plate-fin: multipass*

11.2 ENERGY BALANCE

Let us consider a parallel flow heat exchanger with cold and hot fluid streams (Figure 11-1). On the basis of 1st law of thermodynamics, the heat transfer rate q must equal the rate of heat lost by the hot fluid stream and gained by the cold fluid stream:

$$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = C_h(T_{h,i} - T_{h,o}) \tag{11-1}$$

$$q = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i}) = C_c(T_{c,o} - T_{c,i}) \tag{11-2}$$

$$C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}) \tag{11-3}$$

C = Heat capacity rates [W/K]

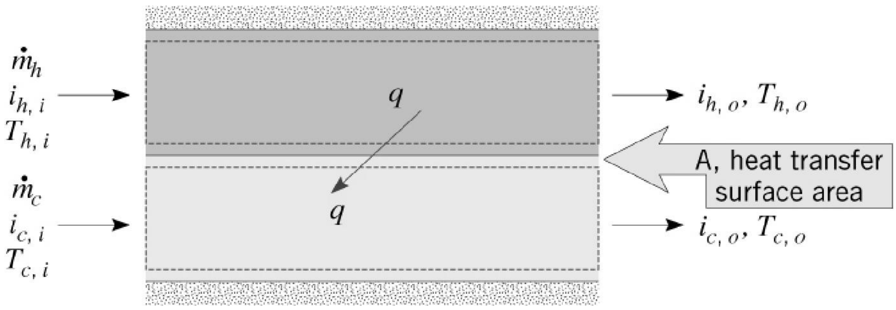


Figure 11-6 *Parallel flow of hot (h) and cold (c) fluid steams*

11.2.1 Overall Heat Transfer Coefficient

Heat transfer rate of above described heat exchanger can be also written with equation similar to Newton's Law of cooling

$$q = UA\Delta T_{\mathrm{lm}} \tag{11-4}$$

U	*Overall heat transfer coefficient*
A	*Heat transfer surface area*
ΔT_{lm}	*Logarithmic mean temperature difference*

The central variables in any heat exchanger analysis are the heat transfer rate *q* [*W*], heat transfer area *A* [m^2], heat capacity rates C [*W/K*], and the overall heat transfer coefficient *U*   . General definition for overall heat transfer coefficient U is

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} = \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h} \tag{11-5}$$

U	*Overall heat transfer coefficient*
c/h	*refers to cold/hot side*
R''_f	*Fouling resistance (see Table 11-2)*
R_w	*Wall conduction resistance*
η_o	*Fin surface efficiency*

For tubular, unfinned heat exchangers overall heat transfer coefficient U can be stated based on outer (o) and inner (i) convection and fouling resistances as

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{ln(D_o/D_i)}{2\pi Lk} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

Typical values of U [W / m^{2}K] *for tubular, non-finned heat exchangers (Saari, 2010):* *oils, ** water

Fluid 1			**Fluid 2**		
	gas, 1 bar	gas, ≥ 25 bar	liquid, high viscosity*	liquid, low viscosity**	phase change
gas, 1 bar	5…35	10...60	15…50	20…70	20…70
gas, ≥ 25 bar	10…60	100…400	100…400	150…500	200…500
liquid, high μ	15…50	100…400	100…400	200…500	200…900
liquid, low μ	20…70	150…500	200…500	400…1700	500…2000
phase change	20…70	200…500	200…900	500…2000	700…2500

Heat transfer surface area A means the contact area between one of the fluids, and the surface of the wall that separates the fluid[5]. Conductance of the heat exchanger is defined as

$$G = UA \qquad [W/K] \qquad (11\text{-}6)$$

Table 11-2 *Example fouling factors*

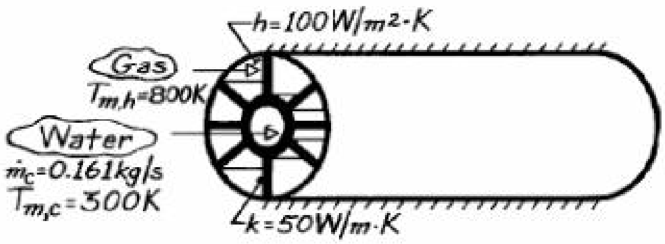
Fluid	R''_f ($m^2 \cdot K/W$)
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam (nonoil bearing)	0.0001

Table 11-3 *Range of overall heat transfer coefficients*

Fluid Combination	U ($W/m^2 \cdot K$)
Water to water	850–1700
Water to oil	110–350
Steam condenser (water in tubes)	1000–6000
Ammonia condenser (water in tubes)	800–1400
Alcohol condenser (water in tubes)	250–700
Finned-tube heat exchanger (water in tubes, air in cross flow)	25–50

Example XVI: *Overall Heat Transfer Coefficient – Annular Finned HX*

A heat recovery device involves transferring energy from the hot flue gases passing through an annular region to pressurized water flowing through the inner tube of the annulus. The inner tube has inner and outer diameters of 24 and 30 mm and is connected by eight struts to an insulated outer tube of 60-mm diameter. Each strut is 3 mm thick and is integrally fabricated with the inner tube from carbon steel (k = 50 W/mK).



$D_o = 60$ mm
$D_{i,1} = 24$ mm
$D_{i,2} = 30$ mm
$t = 3$ mm $= 0.003$m
$L = (60\text{-}30)/2$ mm $= 0.015$m

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[5] If the areas are different on each sides of the wall, the larger area is the one to be used in equation (1) as the heat transfer area. The areas are typically significantly different from each other in the case in tubular or extended-surface heat exchangers.

The velocity of water flow through the inner tube is 0,161 kg/s at 300 K while flue gases at 800 K flow through the annulus, maintaining a convection coefficient of 100 W/m^{2}K on both the struts and the outer surface of the inner tube. What is the rate of heat transfer per unit length of tube from gas to the water?

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction in strut, (4) Adiabatic outer surface conduction, (5) Negligible gas-side radiation, (6) Fully-developed internal flow, (7) Negligible fouling.

PROPERTIES: *Table A-6*, Water (300 K): $k = 0{,}613\ \mathrm{W/mK}$, $\mathrm{Pr} = 5{,}83$, $\mu = 885 \cdot 10^{-6}\ \mathrm{Ns/\ m^2}$.

ANALYSIS: The heat rate is

$$q = (UA)_\mathrm{C}(T_\mathrm{m,h} - T_\mathrm{m,c})$$

$$\frac{1}{(UA)_\mathrm{C}} = \frac{1}{(hA)_\mathrm{C}} + R_\mathrm{w} + \frac{1}{(\eta_o hA)_\mathrm{h}}$$

$$R_\mathrm{w} = \frac{\ln\,(D_\mathrm{i,2}/D_\mathrm{i,1})}{2\pi kl} = \frac{\ln\,(30/24)}{2\pi(50\ \mathrm{W/mK})\cdot 1\ \mathrm{m}} = 7{,}10 \cdot 10^{-4}\ \mathrm{K/W}$$

Reynolds number:

$$\mathrm{Re_D} = \frac{4\dot{m}}{\pi D_\mathrm{i,1}\mu} = \frac{4\cdot 0{,}161\ \mathrm{kg/s}}{\pi\cdot 0{,}024\ \mathrm{m}\cdot 885\cdot 10^{-6}\ \mathrm{Ns/\ m^2}} = 9990\ \text{(Turbulent)}$$

For turbulent flow Dittus-Boelter correlation (Eq. 8-22) gives

$$h_\mathrm{c} = \frac{k}{D_\mathrm{i,1}} Nu_\mathrm{D} = \frac{k}{D_\mathrm{i,1}} 0{,}023\,\mathrm{Re_D^{4/5}}\mathrm{Pr}^{0{,}4} = \frac{0{,}613\,\frac{\mathrm{W}}{\mathrm{mK}}}{0{,}024\ \mathrm{m}} \cdot 0{,}023 \cdot 9990^{4/5} \cdot 5{,}83^{0{,}4}$$

$$h_\mathrm{c} = 1883\ \mathrm{W/m^2K}$$

$$(hA)_\mathrm{c}^{-1} = (1883\ \mathrm{W/m^2K}\cdot\pi\cdot 0{,}024\ \mathrm{m}\cdot 1\ \mathrm{m})^{-1} = 7{,}043\cdot 10^{-3}\ \ \mathrm{K/W}$$

The fin efficiency is defined as (Eq. 3-21)

$$\eta_\mathrm{o} = 1 - \frac{NA_\mathrm{f}}{A_\mathrm{t}}(1-\eta_\mathrm{f})$$

$$A_\mathrm{f} = 2(Lw) = 2(0{,}015\ \mathrm{m}\cdot 1\ \mathrm{m}) = 0{,}03\ \mathrm{m^2}$$

$$A_\mathrm{t} = 8\cdot A_\mathrm{f} + \left(\pi D_\mathrm{i,2}\cdot l - Ntl\right) = 8\cdot 0{,}03\ \mathrm{m^2}\cdot(\pi\cdot 0{,}03\ \mathrm{m}\cdot 1\ \mathrm{m} - 8\cdot 0{,}003\ \mathrm{m}\cdot 1\ \mathrm{m})$$

$$A_\mathrm{t} = 0{,}31\ \mathrm{m^2}$$

Fin efficiency for rectangular fins (Figure 3-2)

$$\eta_\mathrm{f} = \frac{\tanh\,(mL)}{mL}$$

Where from table 3-1:

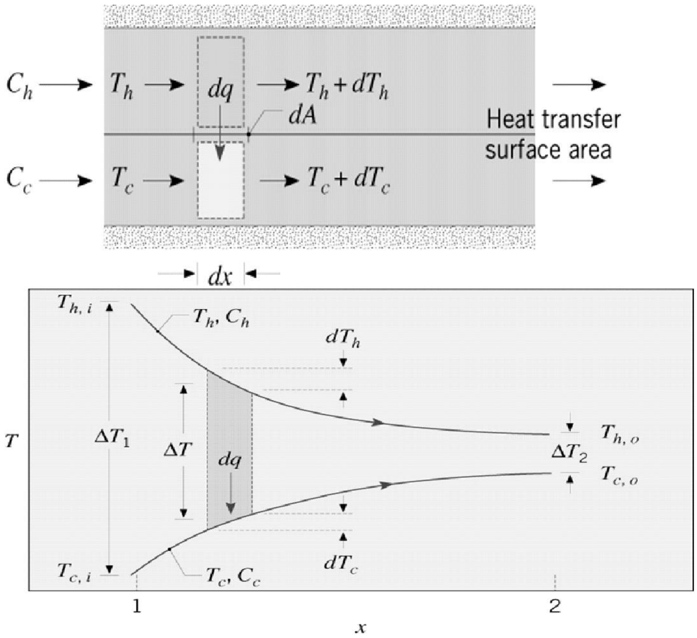
$$m = [2h/kt]^{1/2} = \left[(2\cdot 100\ \mathrm{W/m^2K})\Big/\left(50\,\frac{\mathrm{W}}{\mathrm{mK}}\cdot 0{,}003\ \mathrm{m}\right)\right]^{1/2} = 36{,}5\ \mathrm{m^{-1}}$$

11.2.2 Logarithmic temperature difference

Mean temperature difference (Figure 11-7) is accurately described by mean logarithmic temperature difference defined as

$$\Delta T_{\mathrm{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \qquad [K] \qquad (11\text{-}7)$$

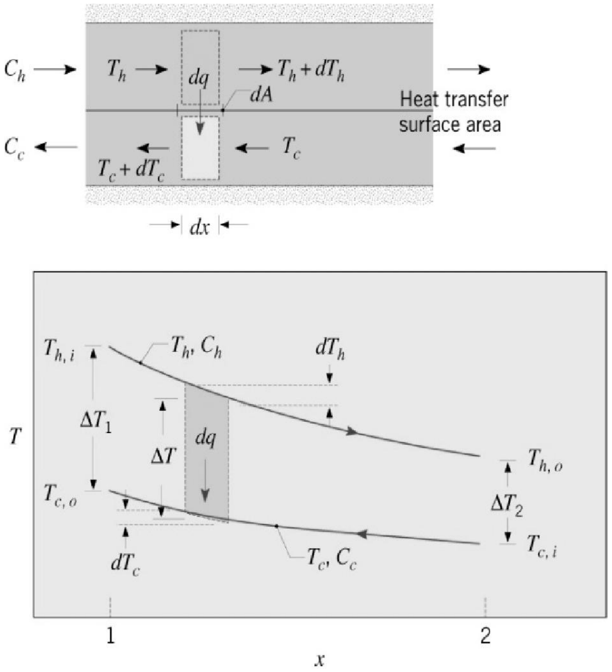
For parallel flow heat exchanger temperature profiles and logarithmic temperature difference are as follows



Parallel flow

Figure 11-7 *Logarithmic nature of temperature difference – parallel flow heat exchanger*

For parallel flow heat exchanger temperature profiles and logarithmic temperature difference are according



Counter flow

Figure 11-8 *Logarithmic nature of temperature difference – counter flow heat exchanger*

Other than pure parallel or counter flow arrangements correction factor for temperature difference have to be used as

$$\Delta T_{lm} = F\Delta T_{lm,CF} \tag{11-8}$$

Figure 11-9 and Figure 11-10 presents correction factors *F* for shell-and-tube and cross-flow heat exchangers according inlet and outlet temperatures.

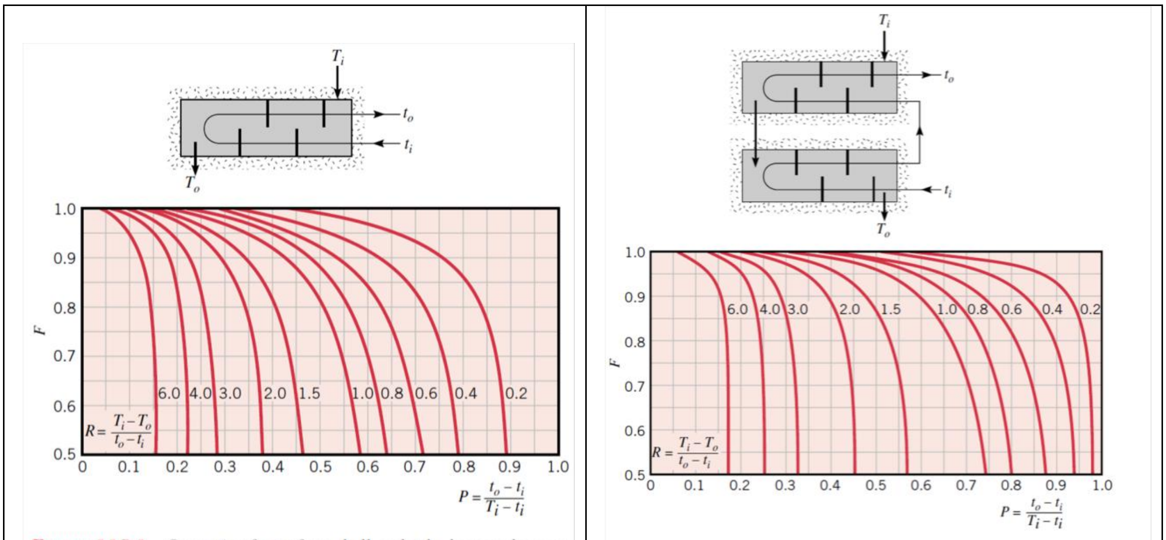


FIGURE 11S.1 Correction factor for a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes).

FIGURE 11S.2 Correction factor for a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes).

Figure 11-9 *Correction factors (F) for Shell-and-tube heat exchangers*

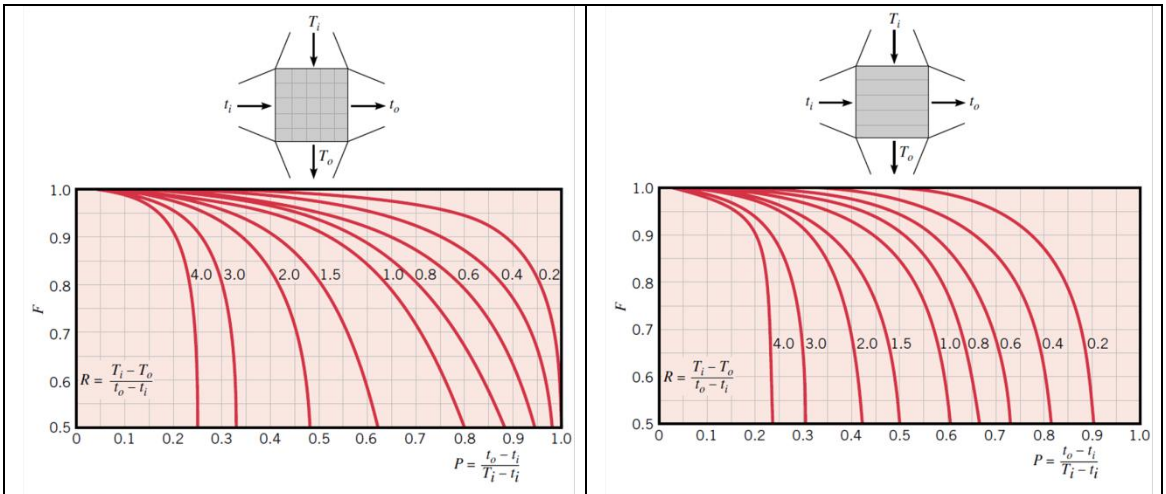


FIGURE 11S.3 Correction factor for a single-pass, cross-flow heat exchanger with both fluids unmixed.

FIGURE 11S.4 Correction factor for a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed.

Figure 11-10 *Correction factors (F) for Shell-and-tube heat exchangers*

11.3 LMTD METHOD

11.3.1 Solving a sizing problem

1) Out of heat transfer rate q and fluid outlet temperatures $T_{x,o}$, determine those that are unknown on the basis of those that are known → energy balance

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = C_h\left(T_{h,i} - T_{h,o}\right)$$
$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = C_c\left(T_{c,o} - T_{c,i}\right)$$

2) Calculate logarithmic temperature difference

$$\Delta T_{\mathrm{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

3) Determine F from graph or equation for the type of heat exchanger flow arrangement used (Figure 11-9, Figure 11-10)

$$\Delta T_{lm} = F\Delta T_{lm,CF}$$

4) Calculate required heat transfer area *A* from
   a. The heat transfer rate

$$q = U\,A\,\Delta T_{\mathrm{m}}$$

   b. and from the overall heat transfer coefficient *U*

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} = \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

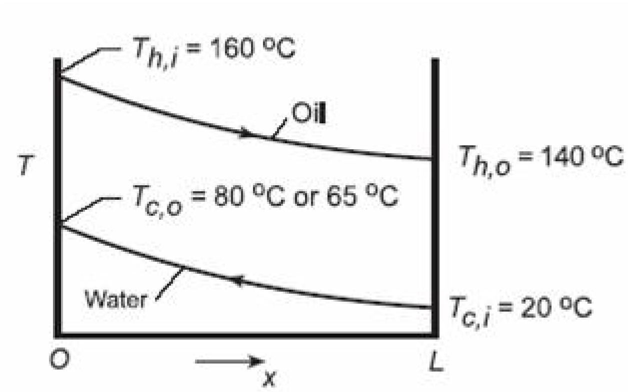
11.3.2 Performance Calculation

If the LMTD method is used in performance calculations for which both outlet temperatures must be determined from knowledge of the inlet temperatures, the solution procedure is iterative.

Example XVII: ***LMTD Method – Counter Flow HX with hot oil***

A counter flow, concentric tube heat exchanger is designed to heat water from 20 °C to 80 °C using hot oil, which is supplied to the annulus at 160 °C and discharged at 140 °C. The thin-walled inner tube has a diameter of $D_{\mathrm{i}} = 20\ \mathrm{mm}$, and the overall heat transfer coefficient is 500 W/m^{2}K. The design conditions calls for a total heat transfer rate of 3000 W.

a) What is the length of the heat exchanger?
b) After 3 years of operation, performance is degraded by fouling on the waterside of the exchanger, and the water outlet temperature is only 65 °C for the same fluid flow rates and inlet temperatures. What are the corresponding values of the heat transfer rate, the outlet temperature of the oil, the overall hat transfer coefficient, and the waterside fouling factor, $R''_{\mathrm{f,c}}$?



3 ASSUMPTIONS:
(1) Negligible heat loss to the surroundings
(2) Negligible tube wall conduction resistance
(3) Constant properties.

ANALYSIS:

a) Log mean temperature difference $\Delta T_{\mathrm{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln\,(\Delta T_2/\Delta T_1)}$

Where (for counter flow exchanger)

$$\Delta T_1 = T_{\mathrm{h,i}} - T_{\mathrm{c,o}} = (160 - 80)\ \text{°C} = 80\ \text{°C}$$

$$\Delta T_2 = T_{\mathrm{h,o}} - T_{\mathrm{c,i}} = (140 - 20)\text{°C} = 120\ \text{°C}$$

$$\Delta T_{\mathrm{lm}} = \frac{120\ \text{°C} - 80\ \text{°C}}{\ln\,(120\ \text{°C}/80\ \text{°C})} = 98{,}7\ \text{°C}$$

From

$$q = UA\Delta T_{\mathrm{lm}} = (\pi DL)U\Delta T_{\mathrm{lm}}$$

$$\Rightarrow L = \frac{q}{(\pi D)U\Delta T_{\mathrm{lm}}} = \frac{3000\ \mathrm{W}}{(\pi \cdot 0{,}020\ \mathrm{m}) \cdot 500\ \mathrm{W/m^2K} \cdot 98{,}7\text{°C}} = 0{,}968\ \mathrm{m}$$

b) With $q = C_{\mathrm{c}}(T_{\mathrm{c,o}} - T_{\mathrm{c,i}})$, the following ratio may be formed in terms of the design and 3 year conditions.

$$\frac{q}{q_3} = \frac{C_{\mathrm{c}}(T_{\mathrm{c,o}} - T_{\mathrm{c,i}})}{C_{\mathrm{c}}(T_{\mathrm{c,o}} - T_{\mathrm{c,i}})_3} = \frac{60\ \text{°C}}{45\ \text{°C}} = 1{,}333$$

Hence

$$q_3 = q/1{,}333 = 3000\ \mathrm{W}/1{,}333 = 2250\ \mathrm{W}$$

Having determined the ratio of heat rates, it follows that

$$\frac{q}{q_3} = \frac{C_{\mathrm{h}}(T_{\mathrm{h,i}} - T_{\mathrm{h,o}})}{C_{\mathrm{h}}(T_{\mathrm{h,i}} - T_{\mathrm{h,o}})_3} = \frac{20\ \text{°C}}{160\ \text{°C} - T_{\mathrm{h,o(3)}}} = 1{,}333$$

$$T_{\mathrm{h,o(3)}} = 160\ \text{°C} - \frac{20\ \text{°C}}{1{,}333} = 145\ \text{°C}$$

Log mean temperature difference is then

$$\Delta T_{\mathrm{lm}} = \frac{125\ \text{°C} - 95\ \text{°C}}{\ln\,(125\ \text{°C}/95\ \text{°C})} = 109{,}3\ \text{°C}$$

$$U_3 = \frac{q_3}{(\pi DL)\Delta T_{\text{lm}}} = 338\,\text{W/m}^2\text{K}$$

With $U = \left(\frac{1}{h_\text{i}} + \frac{1}{h_\text{o}}\right)^{-1}$ and $U_3 = \left(\frac{1}{h_\text{i}} + \frac{1}{h_\text{o}} + R''_{\text{f,c}}\right)^{-1}$,

$$R''_{\text{f,c}} = \frac{1}{U_3} - \frac{1}{U} = \left(\frac{1}{338} - \frac{1}{500}\right)\ \text{m}^2\text{K/W} = 9{,}59 \cdot 10^{-3}\ \text{m}^2\text{K/W}$$

COMMENTS:

1) *Would you wait for another three years – or plan maintenance clean-up?*

11.4 EFFECTIVENESS -NTU METHOD

The effectiveness-NTU method may be used without iteration for both design and performance calculations.

Heat exchanger effectiveness is defined as the ratio of actual heat transfer rate in the heat exchanger to the maximum heat transfer rate

$$\varepsilon = \frac{q}{q_{max}} \qquad (11\text{-}4)$$

Maximum heat transfer rate in heat exchanger is defined with temperature difference of inlet flows and minimum heat capacity rate as

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) \qquad (11\text{-}5)$$

$$C_{min} = \begin{Bmatrix} C_h\ if\ C_h < C_c \\ C_c\ if\ C_c < C_h \end{Bmatrix}$$

Hence, effectiveness can be also written with heat transfer rate definitions as follows

$$\varepsilon = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} \qquad (11\text{-}6)$$

Total heat duty of heat exchanger is then simply

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) \qquad (11\text{-}7)$$

11.4.1 How to obtain effectiveness? ε-NTU Relationships

Defined and correlated functional relationships between effectiveness ε, heat capacity ratio C_r and NTU are basis of ε-NTU method.

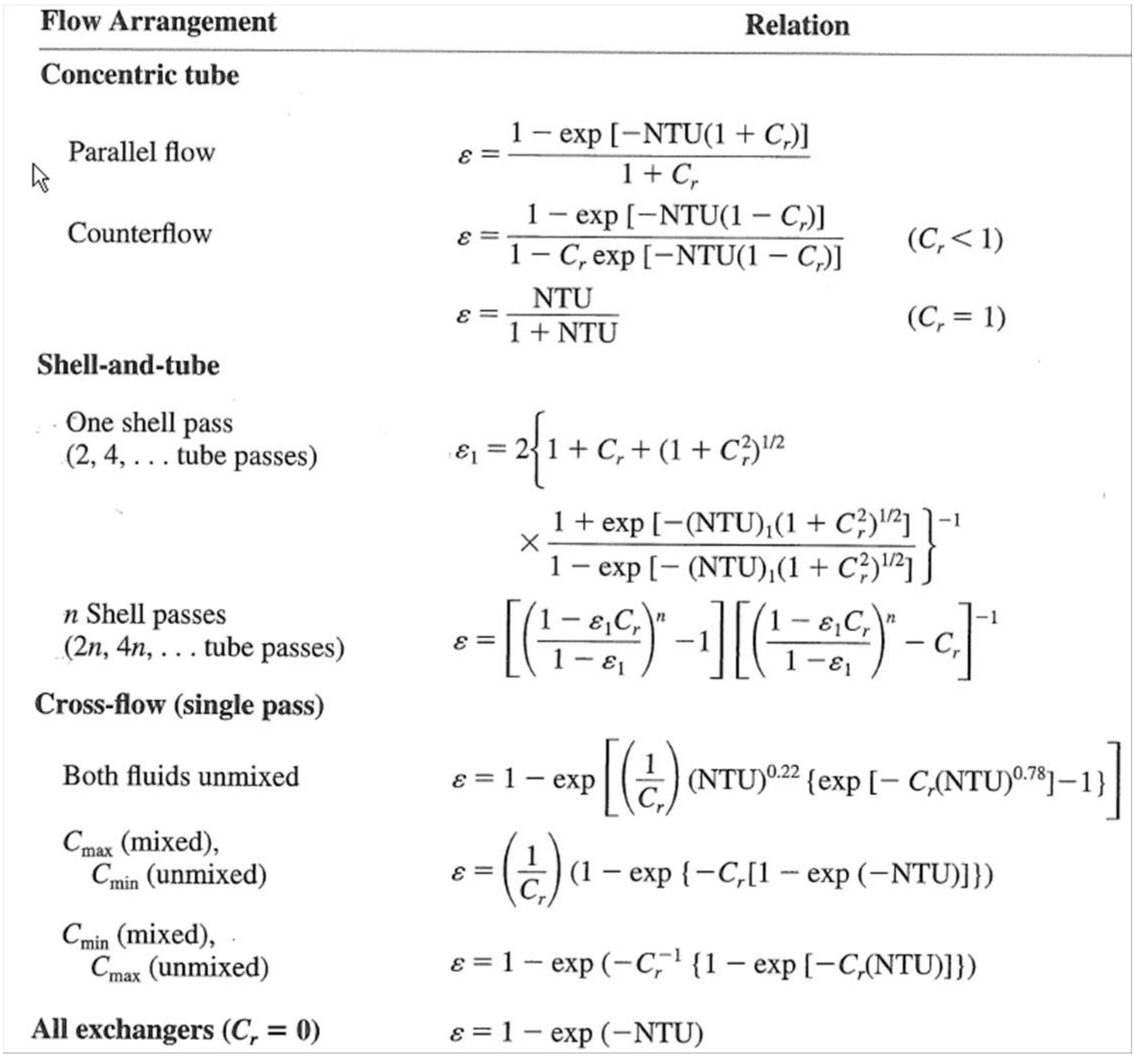
Effectiveness $\varepsilon = f(NTU, C_r)$

Heat capacity ratio $C_r = \frac{C_{min}}{C_{max}}$

Number of Transfer Units $$NTU = \frac{UA}{C_{min}}$$

Following tables shows effectiveness and NTU relations for different flow arrangements as correlations and charts. If one of the fluids experiences a phase change h, then the temperature of that fluid stream does not change, and C of that stream is infinity. Thus it follows that, any finite C of the other stream therefore becomes the C_{min}, and the heat capacity rate ratio will be 0.

Table 11-4 *Effectiveness relations*



Flow Arrangement	**Relation**
Concentric tube	
Parallel flow	$\varepsilon = \frac{1 - \exp[-\text{NTU}(1 + C_r)]}{1 + C_r}$
Counterflow	$\varepsilon = \frac{1 - \exp[-\text{NTU}(1 - C_r)]}{1 - C_r \exp[-\text{NTU}(1 - C_r)]}$ $(C_r < 1)$
	$\varepsilon = \frac{\text{NTU}}{1 + \text{NTU}}$ $(C_r = 1)$
Shell-and-tube	
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$
n Shell passes (2*n*, 4*n*, . . . tube passes)	$\varepsilon = \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$
Cross-flow (single pass)	
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_r}\right)(\text{NTU})^{0.22}\{\exp[-C_r(\text{NTU})^{0.78}] - 1\}\right]$
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\varepsilon = \left(\frac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\text{NTU})]\})$
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\text{NTU})]\})$
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\text{NTU})$

Table 11-5 *Effectiveness of crossflow of heat exchangers: (a) both mixed and (b) other mixed and other unmixed.*

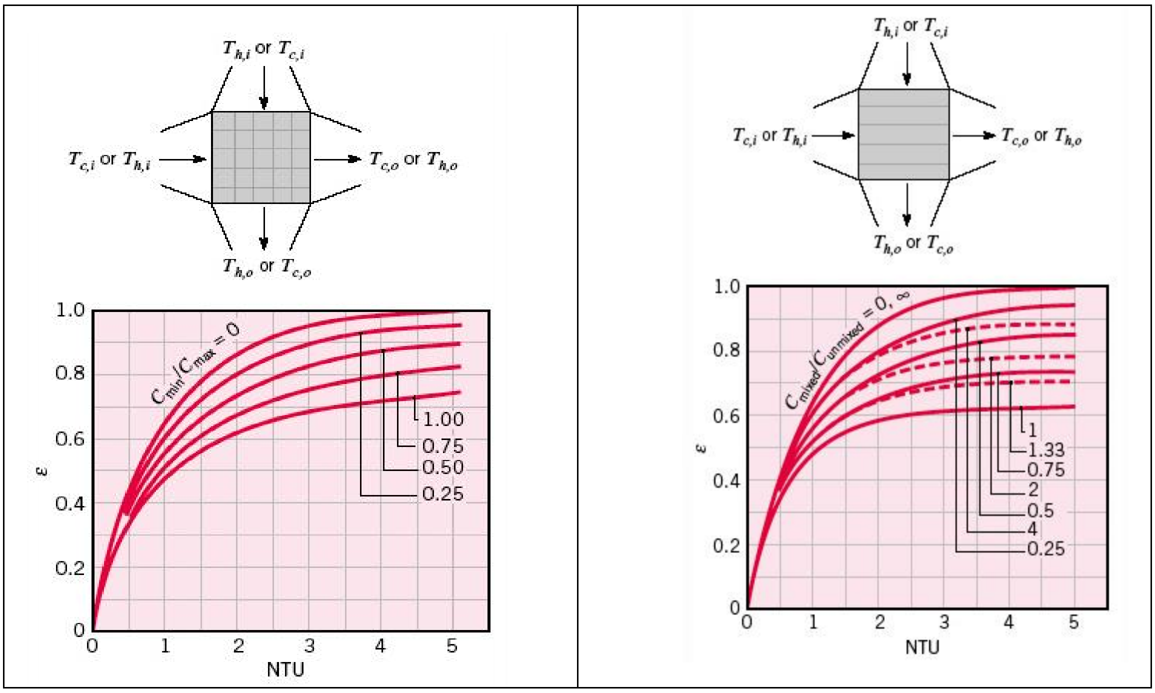
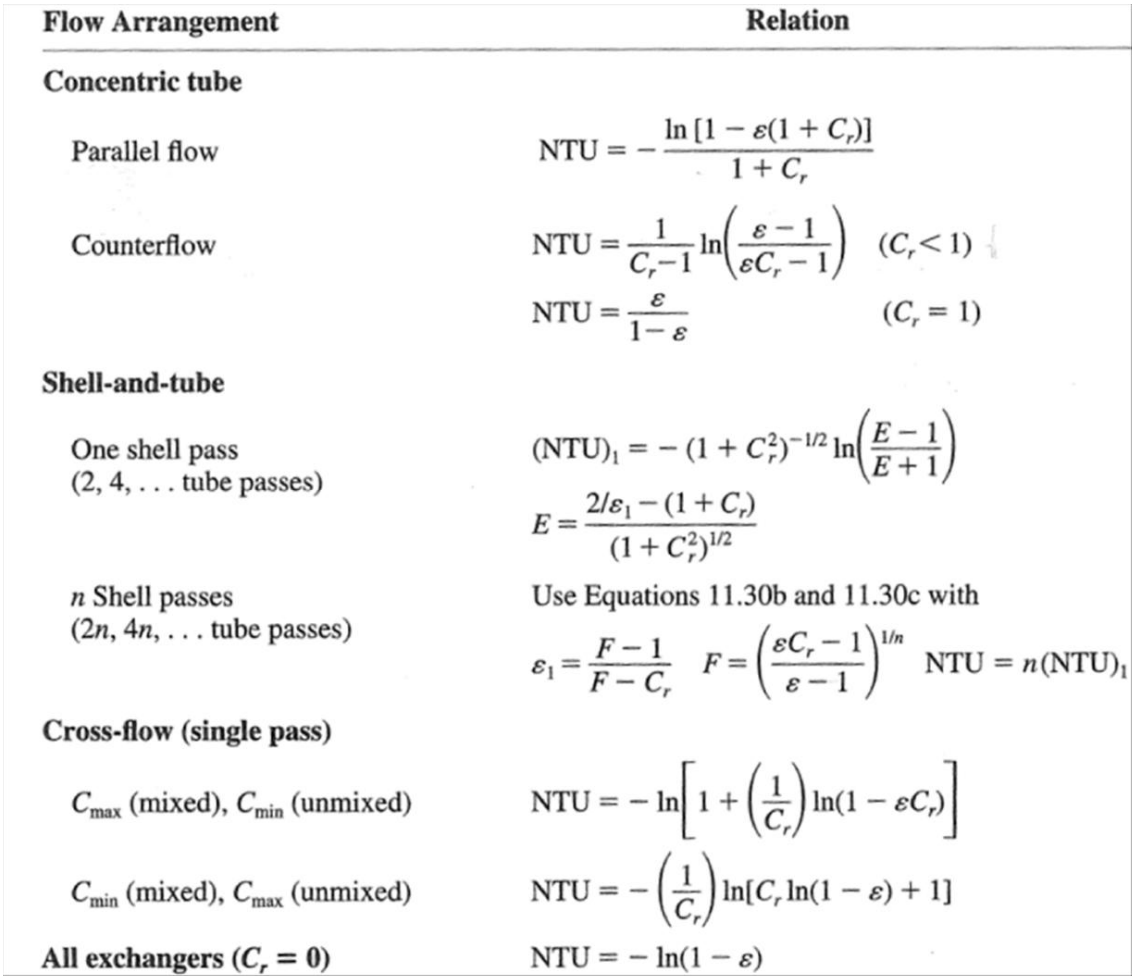
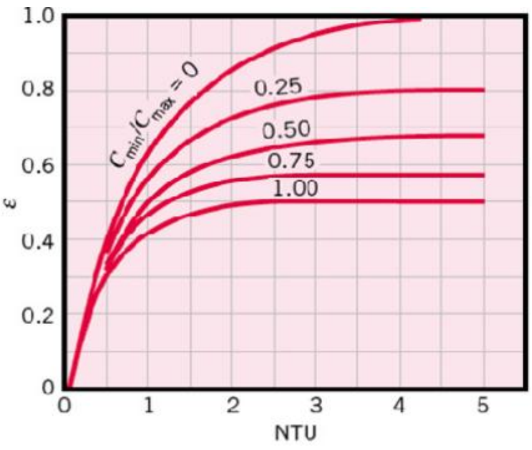
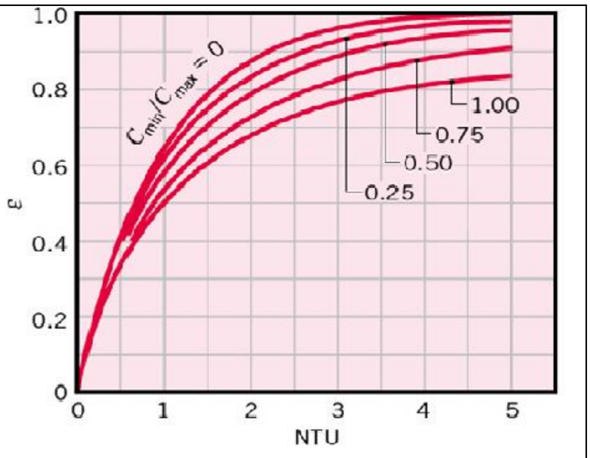
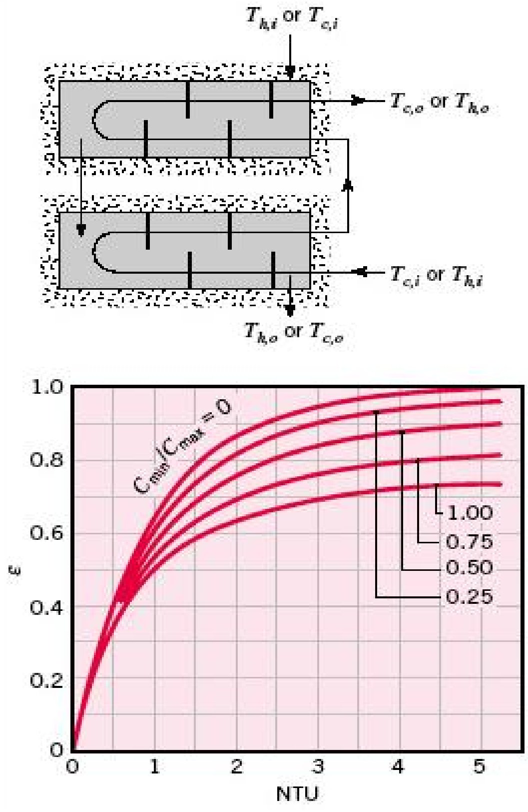
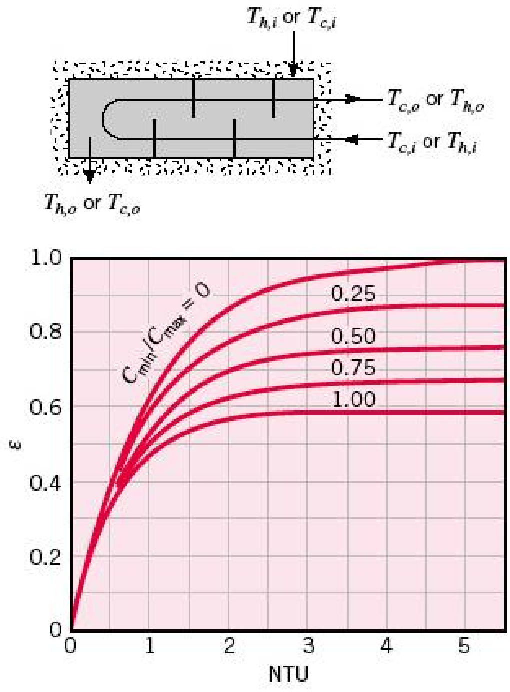


Table 11-6 *NTU Relations*



Flow Arrangement	**Relation**
Concentric tube	
Parallel flow	$\mathrm{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$
Counterflow	$\mathrm{NTU} = \dfrac{1}{C_r-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon C_r-1}\right) \quad (C_r < 1)$ $\mathrm{NTU} = \dfrac{\varepsilon}{1-\varepsilon} \quad (C_r = 1)$
Shell-and-tube	
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$ $E = \dfrac{2/\varepsilon_1-(1+C_r)}{(1+C_r^2)^{1/2}}$
n Shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r-1}{\varepsilon-1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$
Cross-flow (single pass)	
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\mathrm{NTU} = -\ln\left[1+\left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r\ln(1-\varepsilon)+1]$
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1-\varepsilon)$

Table 11-7 *Effectiveness of parallel, counterflow and two configurations of shell-and-tube heat exchangers*

 **FIGURE 11.10** Effectiveness of a parallel-flow heat exchanger (Equation 11.28).	 **FIGURE 11.11** Effectiveness of a counterflow heat exchanger (Equation 11.29).
	

11.4.2 Solving a sizing problem

Defining heat transfer surface area in design phase would follow four steps[6]:

1) Solve *Heat Capacity Ratio* C* and *Efficiency* ε

$$\varepsilon = \frac{q}{q_{max}}, \qquad q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right),\ C_r = \frac{C_{min}}{C_{max}}$$

2) Using ε-NTU relationships of figures (or equations) find *NTU*

$$NTU = f(\varepsilon, C_r)$$

3) Solve required *UA*

$$NTU = \frac{UA}{C_{min}}$$

4) Solve or estimate heat transfer area

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} = \frac{1}{(\eta_o h A)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o h A)_h}$$

11.4.3 Perf romance calculation

Steps for defining heat transfer rate and outlet temperatures in performance calculation would be following:

1) Solve *Overall heat transfer coefficient U* and *heat transfer area A*

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} = \frac{1}{(\eta_o h A)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o h A)_h}$$

2) Using ε-NTU *NTU*

$$NTU = \frac{UA}{C_{min}}$$

3) Solve efficiency of heat exchanger

$$\varepsilon = f(NTU, C_r)$$

4) Solve performance: heat transfer rate q and outlet temperatures

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) \qquad \varepsilon = \frac{C_h(T_{h,i}-T_{h,o})}{C_{min}(T_{h,i}-T_{c,i})} = \frac{C_c(T_{c,o}-T_{c,i})}{C_{min}(T_{h,i}-T_{c,i})}$$

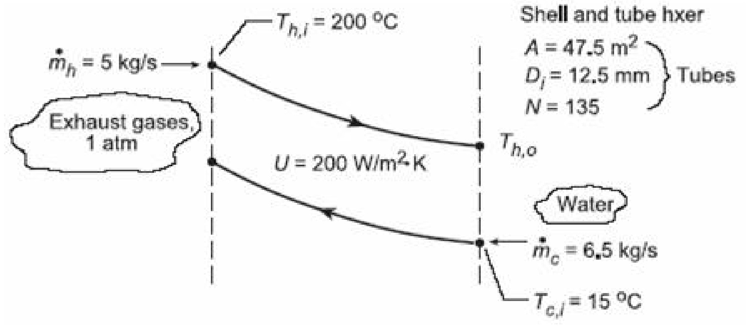
---

[6] Depending on known variables, steps could be in other order also

Example XVIII: *ε-NTU Method – Double Pass shell-and-tube HX*

A shell-and-tube heat exchanger consists of 135 thin-walled tubes in a double-pass arrangement, each of 12,5-mm diameter with a total surface area of 47,5 m^2. Water (the tube-side fluid) enters the heat exchanger at 15 °C and 6,5 kg/s and is heated by exhaust gas entering at 200 °C and 5 kg/s. The gas may be assumed to have the properties of atmospheric air, and the overall heat transfer coefficient is approximately 200 W/m^2K.

a) What are the gas and water outlet temperatures?
b) Assuming fully developed flow, what is the tube-side convection coefficient?
c) With all other conditions remaining the same, plot the effectiveness and fluid outlet temperatures as a function of the water flow rate over the range from 6 to 12 kg/s.
d) What gas inlet temperature is required for the exchanger to supply 10 kg/s of hot water at an outlet temperature of 42 °C, all other conditions remaining the same? What is the effectiveness for this operating condition?



ASSUMPTIONS:
(1) Negligible heat lost to surroundings,
(2) Fully developed conditions for internal flow of water in tubes
(3) Exhaust gas properties are those of air,
(4) The overall coefficient remains unchanged for the operating conditions examined.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c \approx 300\,\mathrm{K}$): $\rho = 997\,\mathrm{kg/m^3}$, $C_p = 4179\,\mathrm{J/kgK}$, $k = 0{,}613\,\mathrm{W/mK}$, $\mu = 855 \cdot 10^{-6}\,\mathrm{Ns/m^2}$, $\mathrm{Pr} = 5{,}83$; *Table A-4*, Air ($1\,\mathrm{atm}$, $\overline{T}_h = 400\,\mathrm{K}$): $\rho = 0{,}8711\,\mathrm{kg/m^3}$, $C_p = 1014\,\mathrm{J/kgK}$

ANALYSIS:

a) Using the ε-NTU method, first find the capacity rates, $C = \dot{m}\, C_p$

$$C_c = 6{,}5\frac{\mathrm{kg}}{\mathrm{s}} \cdot 4179\frac{\mathrm{J}}{\mathrm{kgK}} = 27{,}164\,\mathrm{W/K}$$

$$C_h = 5{,}0\ \frac{\mathrm{kg}}{\mathrm{s}} \cdot 1014\ \frac{\mathrm{J}}{\mathrm{kgK}} = 5{,}070\ \mathrm{W/K}$$

Recognizing that $C_h = C_{min}$ and $C_c = C_{max}$

$$\frac{C_{min}}{C_{max}} = \frac{C_h}{C_c} = \frac{5{,}070\ \mathrm{W/K}}{27{,}164\mathrm{W/K}} = 0{,}19$$

$$NTU = \frac{UA}{C_{min}} = \frac{200\,\mathrm{W/m^2K} \cdot 47{,}5\,\mathrm{m^2}}{5{,}070\ \mathrm{W/K}} = 5{,}070\,\mathrm{W/K}$$

From table 11-16 for the shell and tube exchanger, with $NTU = 5{,}070\,\mathrm{W/K}$ and $\frac{C_{min}}{C_{max}} = 0{,}19$

$$\varepsilon \approx 0{,}78$$

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{200\,°\mathrm{C} - T_{h,o}}{200\,°\mathrm{C} - 15\,°\mathrm{C}} = 0{,}78$$

$$\Rightarrow T_{\mathrm{h,o}} = 55{,}7\ °\mathrm{C}$$

From energy balances of the two fluids,

$$T_{\mathrm{c,o}} = T_{\mathrm{c,i}} + (C_{\mathrm{h}}/C_{\mathrm{c}})(T_{\mathrm{h,i}} - T_{\mathrm{h,o}}) = 15\ °\mathrm{C} + 0{,}19 \cdot (200 - 55{,}7)°\mathrm{C} = 42{,}4\ °\mathrm{C}$$

b) Reynolds number for estimating $\overline{h}_{\mathrm{i}}$ for water

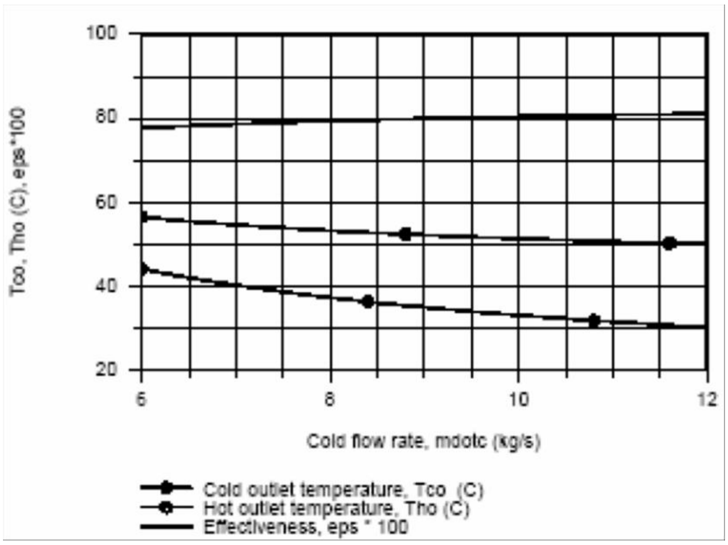
$$\mathrm{Re}_{\mathrm{D,i}} = \frac{4\dot{m}}{\pi D_{\mathrm{i}}\mu} = \frac{4\dot{m}_{\mathrm{c}}/N}{\pi D_{\mathrm{i}}\mu} = \frac{4\cdot 6{,}5\ \mathrm{kg/s}}{\pi\cdot 12{,}5\cdot 10^{-3}\ \mathrm{m}\cdot 855\cdot 10^{-6}\ \mathrm{Ns/\ m^2}\cdot 135} = 5736 \text{ (Turbulent)}$$

From Dittus-Boelter correlation (Eq. 8-22)

$$h_{\mathrm{i}} = \frac{k}{D_{\mathrm{i}}} Nu_{\mathrm{D}} = \frac{k}{D_{\mathrm{i}}} 0{,}023\,\mathrm{Re}_{\mathrm{D}}^{4/5}\mathrm{Pr}^{0{,}4} = \frac{0{,}613 \frac{\mathrm{W}}{\mathrm{mK}}}{12{,}5\cdot 10^{-3}\ \mathrm{m}} \cdot 0{,}023 \cdot 5736^{4/5} \cdot 5{,}83^{0{,}4}$$

$$h_{\mathrm{i}} = 2320\ \mathrm{W/m^2K}$$

c) Using the *IHT Heat Exchanger Tool, Shell and Tube, One-shell pass and N-tube passes,* and the prescribed properties, following the analysis of (a):



d) Using the above IHT model, the hot inlet temperature $T_{\mathrm{h,i}}$, required to provide $\dot{m}_{\mathrm{c}} = 10\ \mathrm{kg/s}$ with $T_{\mathrm{c,o}} = 42\ °\mathrm{C}$ and the effectiveness for this condition are

$T_{\mathrm{h,i}} = 74{,}4\ °\mathrm{C}$ $\qquad\qquad$ $\varepsilon = 0{,}55$

COMMENTS:

1) *for part (d) with m˙_h doubling, we should expect U to increase?*

11.5 LEARNING OUTCOMES

Table 11-8 Learning Outcomes: *Heat Exchangers*

Level of Knowledge	**Concept**
Understand *Describe*	• *Types and thermal principles: concentric, counter, parallel, cross flow, shell-and-tube, plate & compact, etc.* • *Concepts: log-mean temperature difference, overall heat transfer coefficient, energy balance*
Apply ***understand*** *describe*	• Two main thermal performance and design methods: Logarithmic mean temperature difference and efficiency – NTU method

12 RADIATION

Stefan-Boltzmann's law states heat flux emitted by black body

$$E_b = \sigma T_s^4 \qquad [W/m^2] \qquad \textit{(12-1)}$$

σ Stefan-Boltzmann-vakio 5,67 × $10^{-8}[W/m^2K^4]$
T_s *Temperature of black body* $[K]$

Radiation emitted by real surface can be written as

$$E = \varepsilon\sigma T_s^4 \qquad [W/m^2] \qquad \textit{(12-2)}$$

ε Emissivity $[-]$

Radiation net heat flux to real surface may be written according Kirchoff's law as follows

$$q''_{rad} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma(T_s^4 - T_{sur}^4) \qquad [W/m^2] \qquad \textit{(12-3)}$$

α Absorptivity $[-]$
$E_b(T_s)$ *Blackbody radiation flux* $[W/m^2]$
G *Irradiation = radiation flux from surroundings* $[W/m^2]$

Three equations above are very fundamental tools for simple engineering radiation calculations. They consists assumptions and thermal radiation concepts that are beneficial to understand for being aware of applicability of basic radiation equations and conditions requiring more detailed evaluation. Clarification to those radiation principles is given in this chapter.

12.1 PRINCIPLES

All bodies constantly emit energy by process of electromagnetic radiation. The intensity of energy flux depends upon (1) the temperature of the body and (2) the nature of its surface. Radiation has dual nature: (1) particles, which are photons or quanta (Planck) and (2) electromagnetic waves.

In all cases, radiation is characterized by a wavelength λ and frequency *f*, which are related through the speed at which radiation propagates in the medium of interest (according *Maxwell*):

$$\lambda = c/\nu \qquad [m\ ] \qquad \textit{(12-4)}$$

*c=2.998*10⁸ m/s speed of light in vacuum*

Thermal radiation is confined to the infrared, visible and ultraviolet regions of the spectrum of electromagnetic radiation (Figure 12-1). It is originated from oscillations and transitions of electrons of material at an absolute temperature T > 0K.

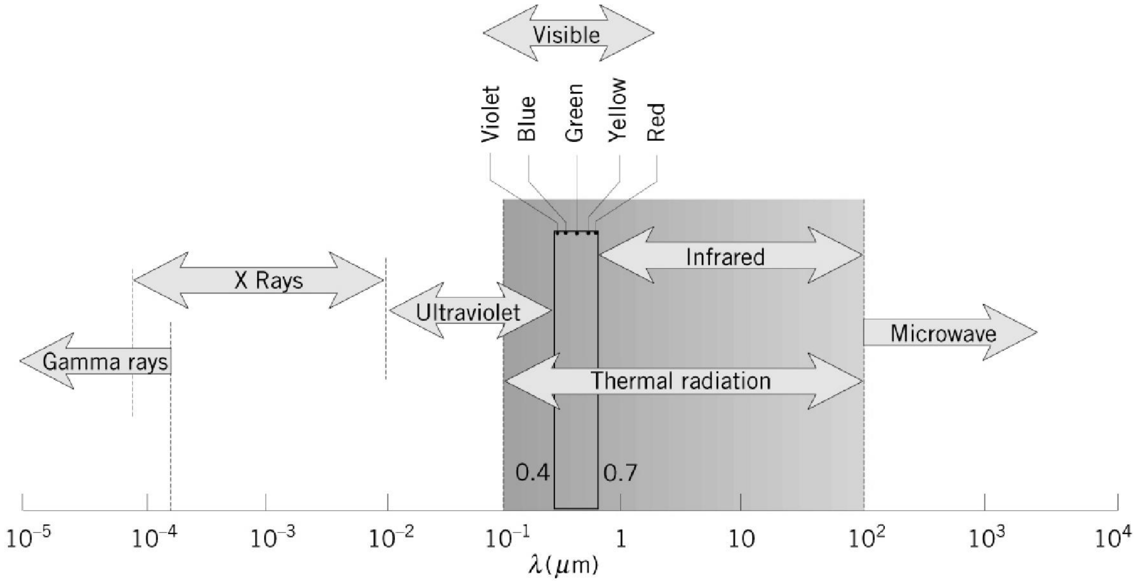


Figure 12-1 *Spectrum of electromagnetic radiation*

12.1.1 Blackbody

Black body is an idealization providing limits on radiation emission and absorption by matter.

1) A blackbody is an **Ideal emitter.**
    - For a prescribed temperature and wavelength, no surface can emit more radiation than a blackbody

2) A blackbody is a **diffuse emitter**.
    - Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction

3) A blackbody absorbs all incident radiation: the **ideal absorber**
    - Regardless of wavelength and direction

The **spectral (Planck) distribution of the blackbody emissive power** (determined theoretically and confirmed experimentally, Figure 12-2) is

$$E = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} \tag{12-5}$$

$C_1 = 3.742*10^8 W\mu m^4/m^2$ Radiation constant
$C_2 = 1.439*10^4 \mu mK$ Radiation constant

Wien's displacement law states relationship between wavelengths of maximum emissive power and temperature

$$\lambda_{max} T = C_3 \tag{12-6}$$

$C_3 = 2898\mu mK$ Radiation constant

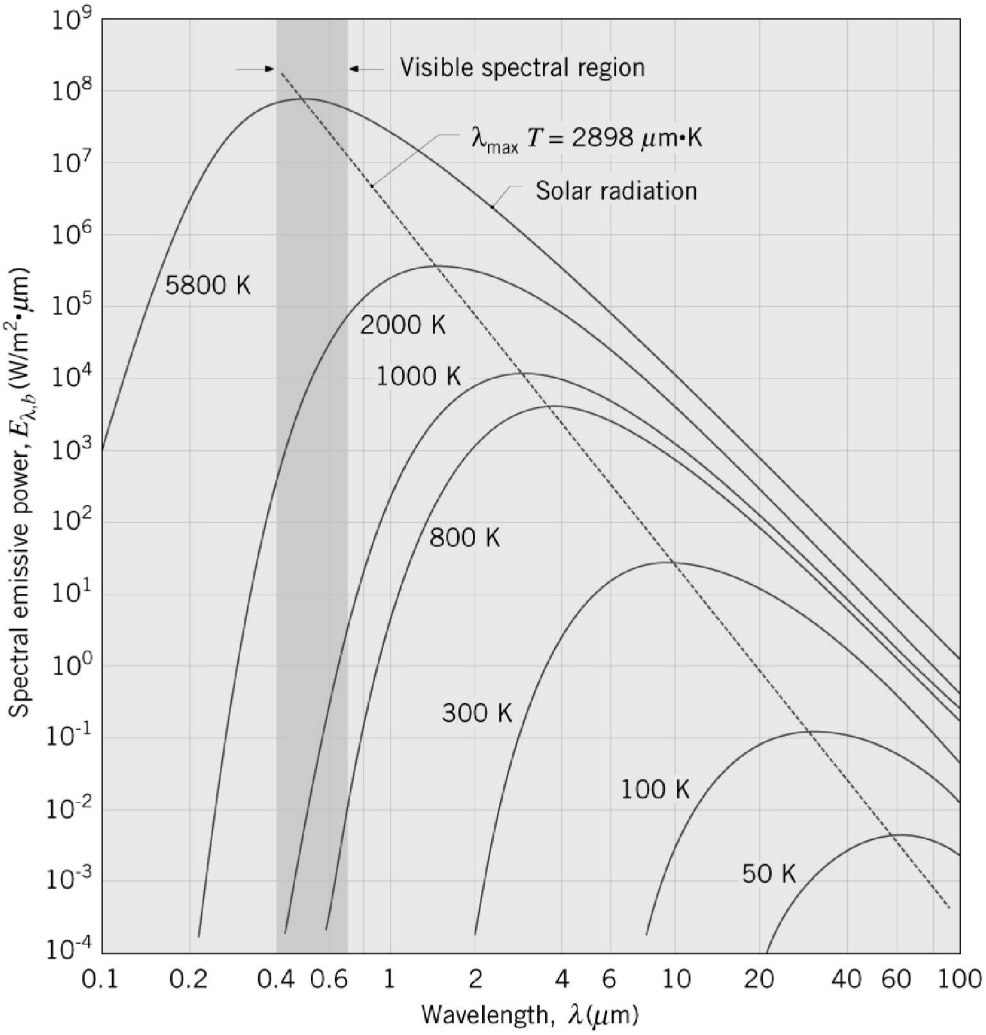


Figure 12-2 *Spectral emissive power of blackbody*

Some fundamental characteristics of spectral emissive power can be noted as

1) *At any wavelength magnitude of emitted radiation increases with increasing T*
2) *The fractional amount of total blackbody emission appearing at lower wavelengths increases with increasing T*
3) *T < 800K: emission is in the infrared region of the spectrum and is not visible to the eye*
4) *Emission from the sun may be approximated as that from a blackbody at 5800K, and significant fraction of radiation is in visible region of spectrum*

12.1.2 Stefan-Boltzmann Law & Band Emission

Stefan-Boltzmann Law is the total emissive power of a blackbody obtained by integrating the Planck distribution over all possible wavelengths

$$E_b = \int_0^\infty E_{b,\lambda}\, d\lambda = \sigma T^4 \tag{12-7}$$

$\sigma = 5.67 * 10^{8}\ W/m^2K^4$ *Stefan-Boltzmann constant*

Blackbody functions describe fraction of emission from blackbody at certain wavelength interval. Fraction of blackbody emission in interval from 0 to λ (Figure 12-3) is only function of λT as stated by

$$F_{0-\lambda} = \frac{\int_0^\lambda E_{b,\lambda} d\lambda}{\sigma T^4} = f(\lambda T) \tag{12-8}$$

Thus for interval from λ_1 to λ_2 fraction of blackbody emission is given by difference of blackbody functions as

$$F_{\lambda_1 \to \lambda_2} = \frac{\int_0^{\lambda_2} E_{b,\lambda} d\lambda - \int_0^{\lambda_1} E_{b,\lambda} d\lambda}{\sigma T^4} = F_{0\to\lambda_2} - F_{0\to\lambda_1} \tag{12-9}$$

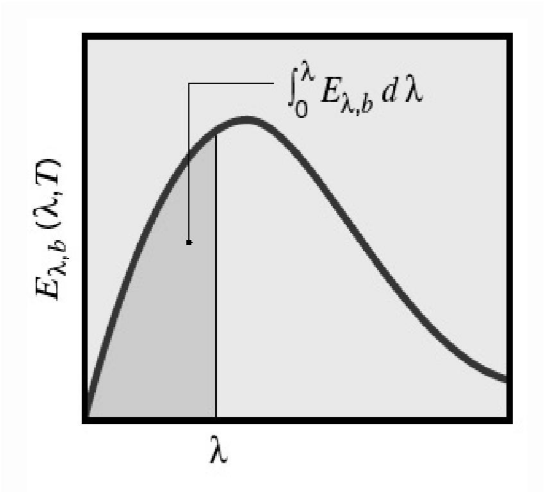


Figure 12-3 *Band emission – fraction of emission in described wavelength interval (/band): 0-λ*

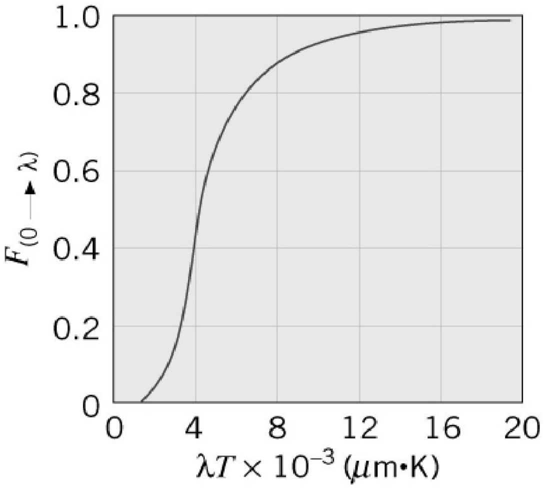
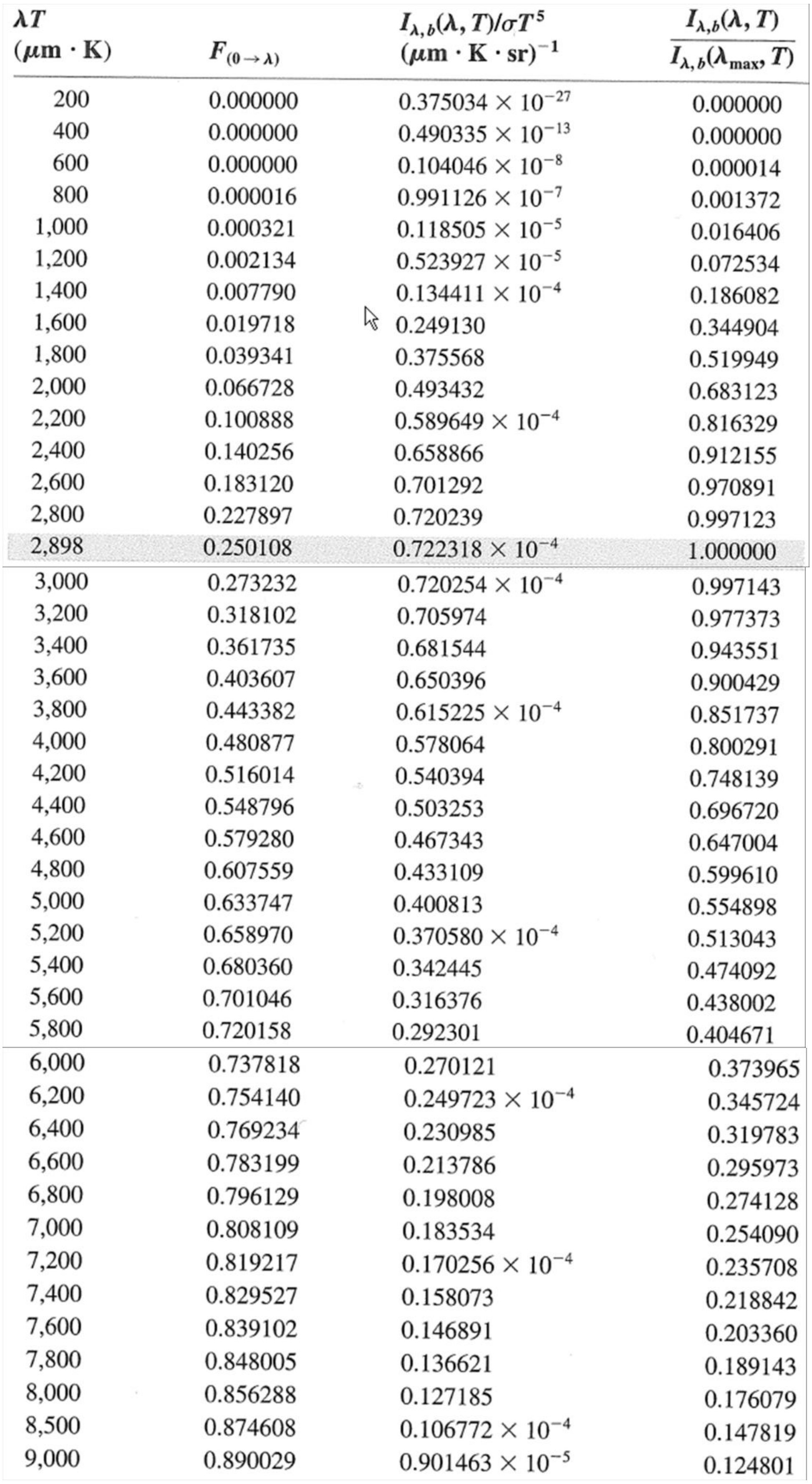


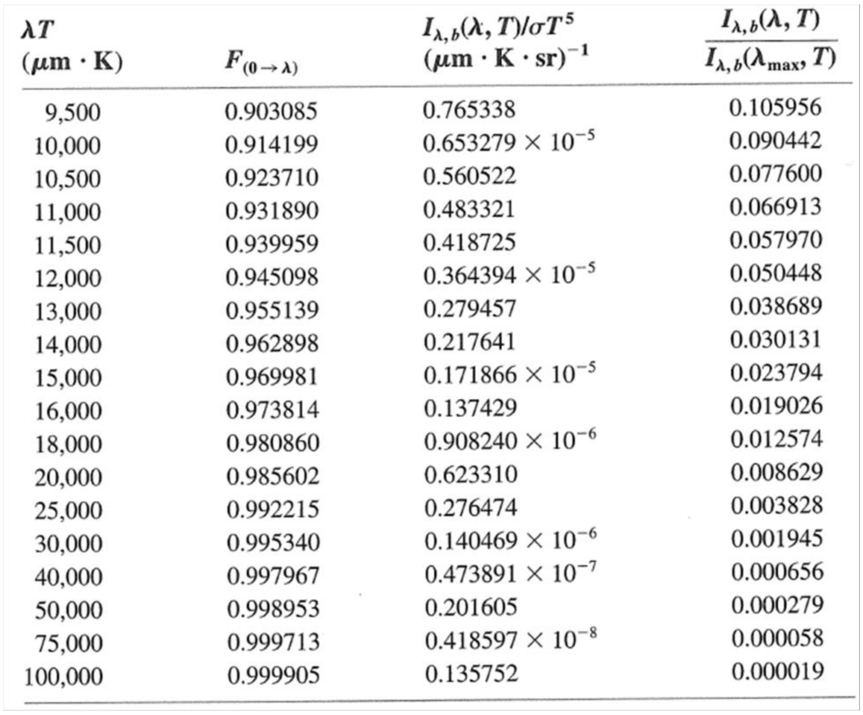
Figure 12-4 *Fraction of Blackbody emission function in spectral band (0-λ) as function of λT*

Table 12-1 gives values for Blackbody functions with λT.

Table 12-1 *Blackbody radiation functions*



λT (μm · K)	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{\text{max}}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801



λT ($\mu m \cdot K$)	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

12.1.3 Surface Response to Irradiation

Spectral irradiation, radiation incident to surface from surroundings with all wavelengths, may act with three different ways (Figure 12-5) while facing semitransparent surface

Reflection from the medium $G_{\lambda,ref} = \rho_\lambda G_\lambda$, ρ_λ *Spectral reflectivity*

Absorption within the medium $G_{\lambda,abs} = \alpha_\lambda G_\lambda$, α_λ *Spectral absorptivity*

Transmission through the medium $G_{\lambda,tr} = \tau_\lambda G_\lambda$, τ_λ *Spectral transmissivity*

Volumetric radiation balance for semitransparent medium is then spectrally

$$G_\lambda = G_{\lambda,ref} + G_{\lambda,abs} + G_{\lambda,tr} = \rho_\lambda G_\lambda + \alpha_\lambda G_\lambda + \tau_\lambda G_\lambda \tag{12-10}$$

This can be further reduced to sum of spectral radiation properties

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \tag{12-11}$$

For total, over wavelength distribution averaged properties, volumetric balance is similarly

$$G = G_{ref} + G_{abs} + G_{tr} = \rho G + \alpha G + \tau G \tag{12-12}$$

$$\rho + \alpha + \tau = 1 \tag{12-13}$$

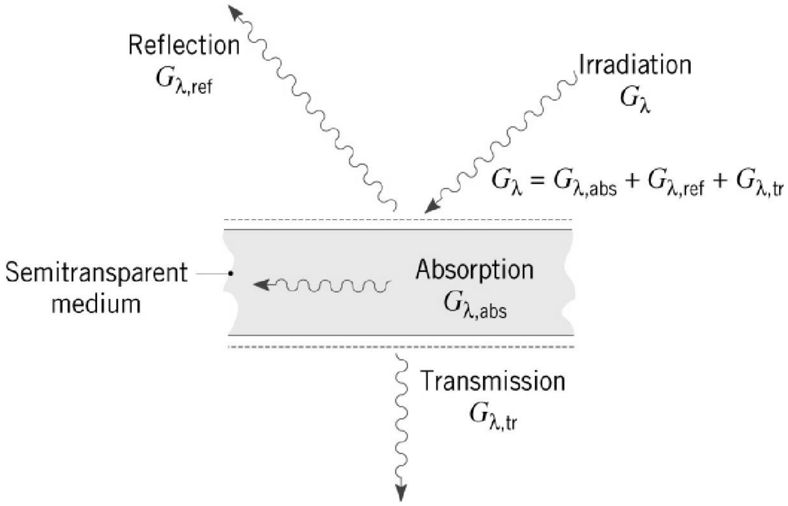


Figure 12-5 *Semitransparent surface response to irradiation*

For *opaque material*, material for which transmission is negligee ($G_{tr}=0$) and irradiation response is defined in fraction of micrometer from incident surface, irradiation is surface phenomenon and balances are written as

$$\mathrm{G} = G_{ref} + G_{abs} \tag{12-14}$$

$$\rho + \alpha = 1 \tag{12-15}$$

12.1.4 Kirchoff's Law

Kirchhoff's law equates the total, hemispherical emissivity of a surface to its total, hemispherical absorptivity:

$$\varepsilon = \alpha \tag{12-16}$$

Derivation of Kirchoff's law requires strict conditions associated to be fulfilled:

> *Irradiation of the surface corresponds to emission from a blackbody at the same temperature as the surface.*

With this formulation, net radiation balance (Figure 12-6) for opaque material can be stated as

$$q''_{rad} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma(T_s^4 - T_{sur}^4) \tag{12-17}$$

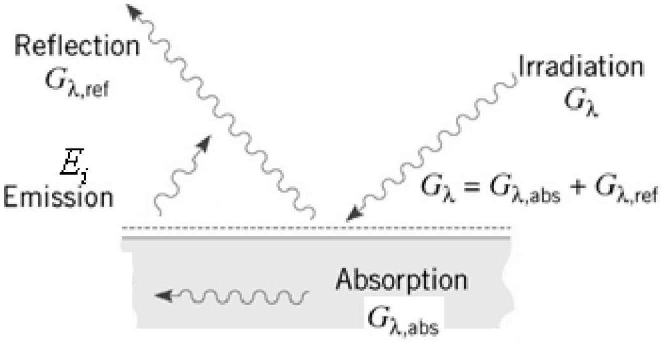


Figure 12-6 *Surface radiation balance for opaque material*

However, for spectral, directional properties formulation can be applied without restriction:

(12-18)

12.2 RADIATION QUANTITIES

12.2.1 Radiation Intensity & Emission

Let us first define Spectral Intensity

$$I_{\lambda,e}(\lambda,\theta,\Phi) = \frac{dq}{dA_1 cos\theta \cdot d\omega \cdot d\lambda} \qquad [W/m^2 \cdot sr \cdot \mu m] \tag{12-19}$$

rate at which radiant energy is emitted at wavelength λ in the (ϕ,θ) direction per Area of emitting surface normal to this direction, per unit solid angle about this direction, and per unit wavelength interval dλ about λ.

$dA_1 cos\theta$	Projected area $[m^2]$
q	Radiation heat transfer rate $[W]$
$d\omega$	Solid angle $[sr]$

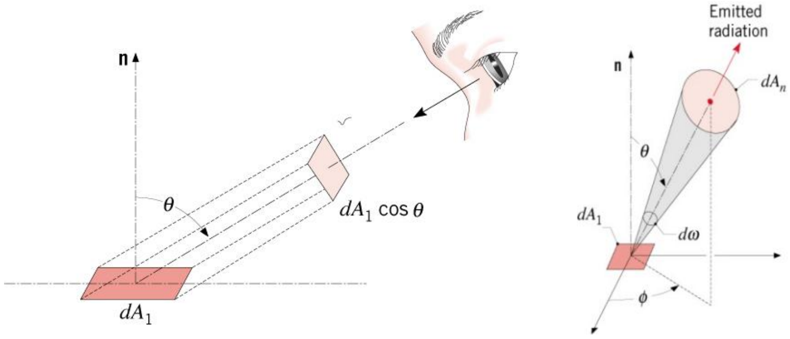


Figure 12-7 *(a) Projected Area dA_1 and (b) Solid angle*

Solid angle is defined as

$$d\omega = \frac{dA_n}{r^2} = \sin\theta d\theta d\Phi \qquad [sr] \tag{12-20}$$

For complete hemisphere integration results as

$$d\omega = \int_0^{2\pi}\int_0^{\pi/2} \sin\theta d\theta d\Phi = 2\pi \qquad [sr] \qquad (12\text{-}21)$$

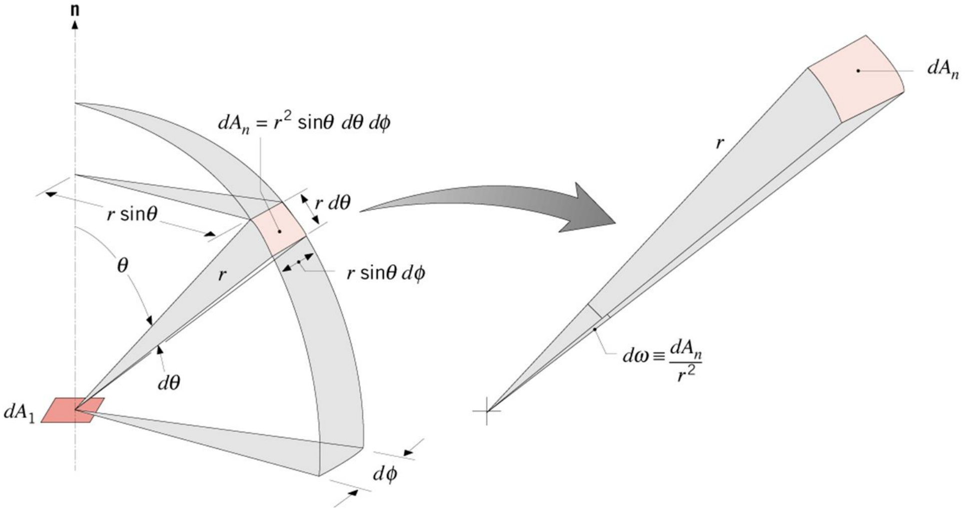


Figure 12-8 *Solid angle subtended by dA_n at point dA_1*

Good approximation for projected surface area is $d\omega = dA_1 cos\theta$ for those surfaces having radiation intensity independent from direction. These surfaces are called *diffuse*, and such radiation *isotropic*.

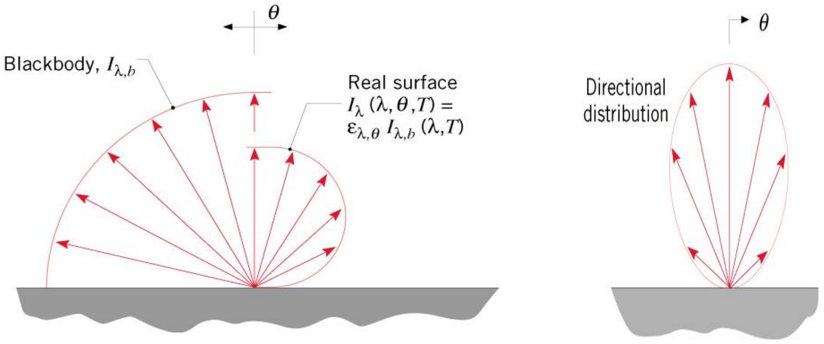


Figure 12-9 *(a) Diffuse radiation from Blackbody (left) and directional emission from real surface (right) (b) Directional distribution*

Total and spectral Emissive powers are

$$E = \frac{dq}{dA_1} \qquad [W/m^2] \qquad (12\text{-}22)$$

$$E_\lambda = \frac{dq}{dA_1 d\lambda} \qquad [W/m^2 m] \qquad (12\text{-}23)$$

And if intensity is independent on direction (diffuse surface)

$$E = \pi I_e \qquad [W/m^2] \qquad (12\text{-}24)$$

$$E_\lambda = \pi I_{\lambda,e} \qquad [\mathrm{W/m^2m}] \qquad (12\text{-}25)$$

12.2.2 From Irradiation and Emission to Radiosity

Irradiation is *radiation incident to surface from all directions* (Figure 12-10)

$$G = \frac{dq_{in}}{dA} = \pi I_i \tag{12-26}$$

If radiation is approximated to originate from large surroundings (*e.g. sky, furnace/ room walls*) acting like blackbody it can be written

$$G = G_{sur} = \sigma T_{sur}^4 \tag{12-27}$$

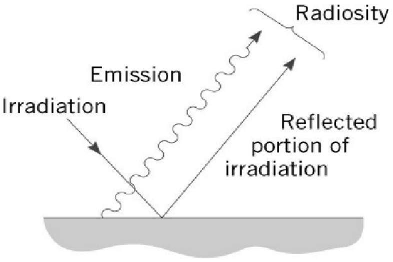


Figure 12-10 *Radiosity from surface*

The ***radiosity*** of an opaque surface accounts for *all of the radiation leaving the surface in all directions and may include contributions from both reflection and emission.* For radiosity it can be written as

$$J = \frac{dq_{out}}{dA} = \frac{dq_{emission} + dq_{reflection}}{dA} \tag{12-28}$$

If we consider these two terms, irradiation and radiosity, with definition of intensity, it can be written first for spectral irradiation

$$\boldsymbol{G_\lambda(\lambda)} = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\Phi)cos\theta \cdot sin\theta \cdot d\theta \cdot d\,\Phi \tag{12-29}$$

and total irradiation

$$G = \int_0^{\infty} G_\lambda(\lambda)d\lambda \tag{12-30}$$

Similarly for radiosity, first spectral

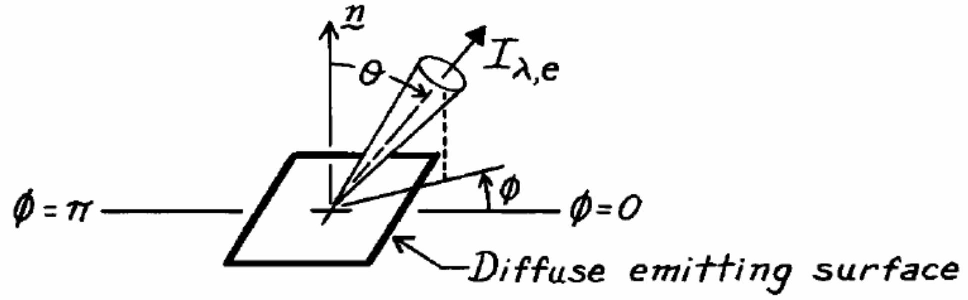
$$\boldsymbol{J_\lambda(\lambda)} = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\Phi)cos\theta \cdot sin\theta \cdot d\theta \cdot d\,\Phi \tag{12-31}$$

, and then total radiosity as

$$J = \int_0^{\infty} J_\lambda(\lambda)d\lambda \tag{12-32}$$

Example IXX: ***Diffuse emitter – Radiation quantities***

Determine the fraction of the total, hemispherical emissive power that leaves a diffuse surface in the directions $\pi/4 \leq \theta \leq \pi/2$ and $0 \leq \phi \leq \pi$.



ASSUMPTIONS: (1) Diffuse emitting surface.
ANALYSIS: the total, hemispherical emissive power is

$$E = \int_{\lambda=0}^{\infty} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi d\lambda$$

For a diffuse surface $I_e(\lambda, \theta, \varphi)$ is independent of direction, and as given by Eq. 12-27,

$$E = \pi I_e$$

The emissive power, which has directions prescribed by the limits on θ and φ, is

$$\Delta E = \int_{\lambda=0}^{\infty} I_{\lambda,e}(\lambda)d\lambda \left[\int_{\phi=0}^{\pi} d\phi\right]\left[\int_{\theta=\pi/4}^{\pi/2} \cos\theta\sin\theta\, d\theta\right]$$

$$\Delta E = I_{\lambda,e} \cdot [\phi]_0^{\pi} \cdot \left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} = I_{\lambda,e} 2\pi \left[\frac{\sin^2\pi/2}{2} - \frac{\sin^2\pi/4}{2}\right]$$

$$\Delta E = 0{,}25\pi I_{\lambda,e}$$

It follows that

$$\frac{\Delta E}{E} = \frac{0{,}25\pi I_{\lambda,e}}{\pi I_e} = 0{,}25$$

Example XX: ***Hot 5-mm-square – Radiation quantities***

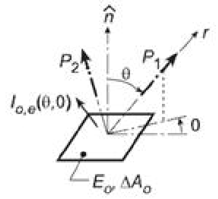
Consider a 5-mm-square, diffuse surface ΔA_o having a total emissive power of E_0 = 4000 W/m^2. The radiation field due to emission into the hemispherical space above the surface is diffuse, thereby providing a uniform intensity $I(\Theta,\phi)$. Moreover, if the space is a nonparticipating medium (non-absorbing, non-scattering, and non-emitting), the intensity is independent of radius for any (Θ,ϕ) direction. Hence intensities at any points P_1 and P_2 would be equal.

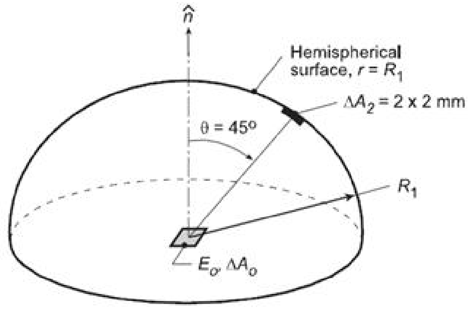
a) What is the rate at which radiant energy is emitted by ΔA_o, q_{emit}?
b) What is the intensity $I_{o,e}$ of the radiation field emitted from the surface ΔA_o?
c) Beginning with Equation $E_\lambda(\lambda) = I_{\lambda,e} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \cos\theta\sin\theta\, d\theta$ and presuming knowledge of the intensity $I_{o,e}$, obtain an expression for q_{emit}.
d) Consider the hemispherical surface located at $r = R_1$ = 0,5 m. Using the conservation of energy requirement, determine the rate at which radiant energy is incident on this surface due to emission from ΔA_o.

e) Using Equation 12.6 (Incr.),, determine the rate at which radiant energy leaving ΔA_o is intercepted by the small area ΔA_2 located in the direction (45°, ϕ) on the hemispherical surface. What is the irradiation on ΔA_2?

f) Repeat part (e) for the location (0°, ϕ). Are the irradiations at the two locations equal?

g) Using Equations 12.14, determine the irradiation $G1$ on the hemispherical surface at $r = R_1$.





3 ASSUMPTIONS:
(1) Diffuse surface, ΔA_o
(2) Medium above ΔA_o is almost non-participating,
(3) $R_1^2 \gg \Delta A_o, \Delta A_2$.

ANALYSIS:

a) The radiant power leaving ΔA_o by emission is

$$q_{\text{emit}} = E_o \cdot \Delta A_o = 4000 \frac{\text{W}}{\text{m}^2} (0{,}005 \text{ m} \times 0{,}005 \text{ m}) = 0{,}10 \text{ W}$$

b) The emitted intensity is $I_{o,e}$ and is independent of direction since ΔA_o is diffuse emitter.

$$I_{o,e} = \frac{E_o}{\pi} = 1333 \text{ W/m}^2 \cdot \text{sr}$$

The intensities at points P_1 and P_2 are also $I_{o,e}$ and the intensity in the directions shown in the schematic above will remain constant no matter how far the point is from the surface ΔA_o since the space is nonparticipating.

c) From knowledge of $I_{o,e}$, the radiant power leaving ΔA_o

$$q_{\text{emit}} = \int_h I_{o,e} \cdot \Delta A_o \cos\theta \sin\theta \, d\theta d\phi$$

$$q_{\text{emit}} = I_{o,e} \cdot \Delta A_o \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \cos\theta \sin\theta \, d\theta = \pi I_{o,e} \cdot \Delta A_o$$

$$q_{\text{emit}} = 0{,}10 \text{ W}$$

d) Defining control surfaces above ΔA_o and on A_1, the radiant power leaving ΔA_o must pass through A_1.

$$q_{1,\text{inc}} = E_o \cdot \Delta A_o = 0{,}10 \text{ W}$$

Average irradiation on the hemisphere, A_1

$$\bar{G}_1 = q_{1,\text{inc}}/A_1 = 63{,}7 \text{ mW/m}^2$$

e) From the radiant power leaving ΔA_o intercepted by ΔA_2, where $\Delta A_2 = 4 \times 10^{-6}$ m^2, located at (θ = 45 °, ϕ) as per schematic, follows that

$$q_{\Delta A_o \to \Delta A_2} = I_{o,e} \cdot \Delta A_o \cos\theta_2 \Delta w_{2-o} \qquad \text{,where } (\theta_2 = 45\ °\mathrm{C})$$

$$\Delta w_{2-o} = \Delta A_o \cos\theta_2 / \mathrm{R}_1^2 = 4 \cdot 10^{-6}\mathrm{m}^2 \cdot \frac{1}{(0{,}5\ \mathrm{m})^2} = 1{,}60 \cdot 10^{-5}\ \mathrm{sr}$$

Where $(\theta_2 = \theta^{\mathrm{o}})$, the direction normal to ΔA_2

$$q_{\Delta A_o \to \Delta A_2} = 1333 \frac{\mathrm{W}}{\mathrm{m}^2} \cdot \mathrm{sr} \cdot 25 \cdot 10^{-6}\ \mathrm{m}^2 \cdot \cos 45\ ° \cdot 1{,}60 \cdot 10^{-5}\ \mathrm{sr} = 3{,}77 \cdot 10^{-7}\ \mathrm{W}$$

From the definition of irradiation

$$G_2 = q_{\Delta A_o \to \Delta A_2} / \Delta A_2 = 93{,}3\ \mathrm{mW/m^2}$$

f) With ΔA_2, located at $(\Theta = 0, \phi)$ $\cos\theta_o = 1, \cos\theta_2 = 1$

g) Using Eg. 12.15 based upon Figure 12.10

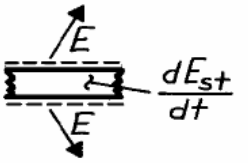
$$\bar{G}_1 = \int_{\mathrm{h}} I_{o,e} \cdot \mathrm{d}A_1\, \mathrm{d}w_{o-1} / A_1 = \pi I_{o,e} \cdot \Delta A_o / \Delta A_1 = 63{,}7\ \mathrm{mW/m^2}$$

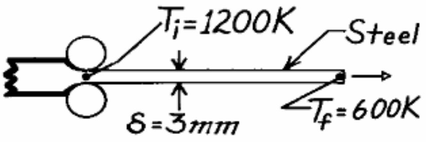
$$\mathrm{d}A_1 = R_1^2 \sin\theta\, d\theta d\phi \qquad \text{and} \qquad \mathrm{d}w_{o-1} = \Delta A_o \cos\theta / R_1^2$$

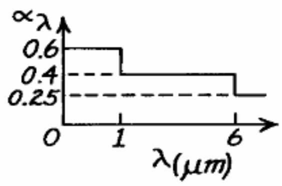
Example XXI: *Sheet steel – Radiation quantities*

Sheet steel emerging from the hot roll section of a steel mill has following properties: temperature of 1200 K, a thickness of δ = 3 mm, density of 7900 kg/m^3 and specific heat of 640 J/kgK.

a) What is the total hemispherical emissivity?
b) Accounting for emission from both sides of the sheet and neglecting heat transfer from the surroundings, determine the initial time rate of change of the sheet temperature $(dT/dt)_{\mathrm{i}}$.
c) As the steel cools, it oxidizes and its total, hemispherical emissivity increases. The correlation for the increasing emission is of the form$\varepsilon = \varepsilon_{1200}[1200\ \mathrm{K}/T\ (\mathrm{K})]$. How long it will take for the steel to cool from 1200 to 600 K?







ASSUMPTIONS: (1) Negligible conduction (in longitudinal direction), convection and radiation rom surroundings, (2) Negligible transverse temperature gradients.

PROPERTIES: Steel (given): $\rho = 7900\ \mathrm{kg/m^3}$, $c = 640\ \mathrm{J/kg \cdot K}$, $\varepsilon = 1200\varepsilon_{\mathrm{i}}/\mathrm{T\ (K)}$.

ANALYSIS:

a) The initial total hemispherical emissivity is

$$\varepsilon_{\mathrm{i}} = \int_0^\infty \varepsilon_\lambda [E_{\lambda \mathrm{b}}(1200\ \mathrm{K}) / E_{\mathrm{b}}(1200\ \mathrm{K})] \mathrm{d}\lambda$$

and integrating by parts using values from Table 12-1, find

$$\lambda T = 1200\ \mu\mathrm{m} \cdot \mathrm{K} \to F_{0-1\ \mu\mathrm{m}} = 0{,}002;\ \lambda T = 7200\ \mu\mathrm{m} \cdot \mathrm{K} \to F_{0-6\ \mu\mathrm{m}} = 0{,}819$$

$$\varepsilon_{\mathrm{i}} = 0{,}6 \cdot 0{,}002 + 0{,}4(0{,}819 - 0{,}002) + 0{,}25(1 - 0{,}819) = 0{,}373$$

b) From an energy balance on a unit surface area of strip (top and bottom),

$$-\dot{E}_{\text{out}} = \mathrm{d}E_{\text{st}}/\mathrm{dt} \qquad -2\varepsilon\sigma T^4 = \mathrm{d}(\rho\delta c T)/\mathrm{dt}$$

$$\left.\frac{\mathrm{dT}}{\mathrm{dt}}\right)_{\mathrm{i}} = -\frac{2\varepsilon_{\mathrm{i}}(1200/T)\sigma T^4}{\rho\delta c} = -\frac{2\cdot 0{,}373\cdot 5{,}67\cdot 10^{-8}\ \mathrm{W/m^2K^4}(1200\ \mathrm{K})^4}{7900\,\frac{\mathrm{kg}}{\mathrm{m^3}}\cdot 0{,}003\ \mathrm{m}\cdot\ 640\ \mathrm{J/kg\cdot K}}$$

$$= -5{,}78\ \mathrm{K/s}$$

c) From the energy balance,

$$\frac{\mathrm{dT}}{\mathrm{dt}} = \frac{2\varepsilon_{\mathrm{i}}(1200/T)\sigma T^4}{\rho\delta c},\ \int_{T_{\mathrm{i}}}^{T_{\mathrm{f}}}\frac{\mathrm{dT}}{T^3} = \frac{2400\varepsilon_{\mathrm{i}}\sigma}{\rho\delta c}\int_0^t \mathrm{dt},\ t = \frac{\rho\delta c}{4800\varepsilon_{\mathrm{i}}\sigma}\left(\frac{1}{T_{\mathrm{f}}^2}-\frac{1}{T_{\mathrm{i}}^2}\right)$$

$$t = \frac{7900\,\frac{\mathrm{kg}}{\mathrm{m^3}}\cdot 0{,}003\ \mathrm{m}\cdot\ 640\ \mathrm{J/kg\cdot K}}{4800\cdot 0{,}373\cdot 5{,}67\cdot 10^{-8}\ \mathrm{W/m^2K^4}}\left(\frac{1}{600^2}-\frac{1}{1200^2}\right)\mathrm{K^{-2}} = 311\ \mathrm{s}$$

COMMENTS: Initially, from Eq. 1.9(Incr.),, $h_{\mathrm{r}} \approx \varepsilon_{\mathrm{i}}\sigma T^3 = 36.6\ \mathrm{W/m^2\cdot K}$. Assuming a plate width of $W = 1\ \mathrm{m}$, the Rayleigh number may be evaluated from $Ra_{\mathrm{L}} = g\beta(T_{\mathrm{i}} - T_\infty)\,(W/2)^3/\nu\alpha$. Assuming $T_\infty = 300\ \mathrm{K}$ and evaluating properties at $T_{\mathrm{f}} = 750\ \mathrm{K}$, $Ra_{\mathrm{L}} = 1.8\cdot 10^8$. From Eq. 9.31(Incr.), $\mathrm{Nu_L} = 84$, giving $h = 9.2\ \mathrm{W/m^2\cdot K}$. Hence heat loss by radiation exceeds that associated with free convection. To check the validity of neglecting transverse temperature gradients, compute $\mathrm{Bi} = h(\delta/2)/k$. With $h = 36.6\ \mathrm{W/m^2{\cdot}K}$ and $k = 28\ \mathrm{W/m\cdot K}$, $\mathrm{Bi} = 0.002 << 1$. Hence the assumption is valid.

12.3 RADIATION PROPERTIES

12.3.1 Surface Emissivity

For most cases, reasonable approximation is that the hemispherical emissivity is equal to the normal emissivity $\varepsilon = \varepsilon_n$.

Ranges of normal emissivity for some metals and nonconductors are presented in Figure 12-11. Polished metals have very low emissivity, while emissivity increases strongly for unpolished and oxidized surfaces. Compared to metals nonconductors have higher total normal emissivity.

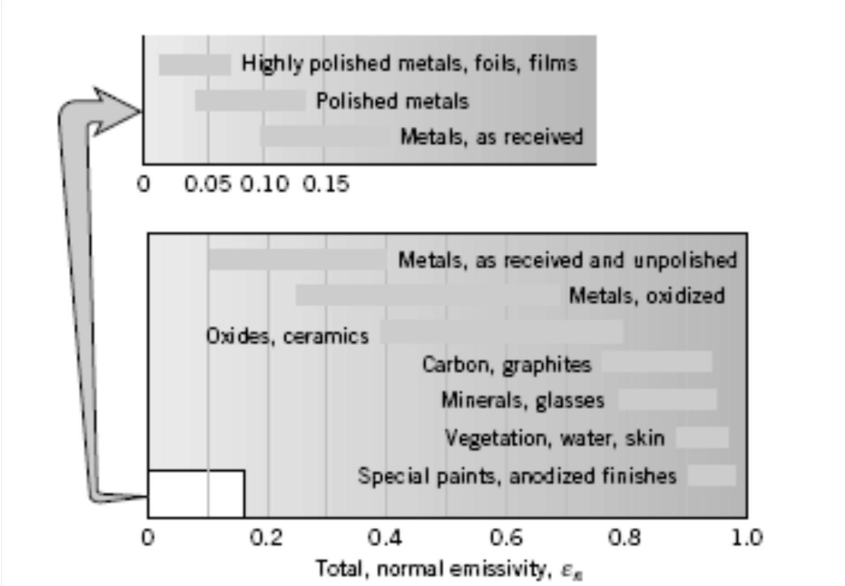


Figure 12-11 Total, normal emissivity

Materials have different spectral emissivity characteristics, e.g.: for metals emissivity decreases with increasing wavelength and nonmetals have they own spectral behavior curves.

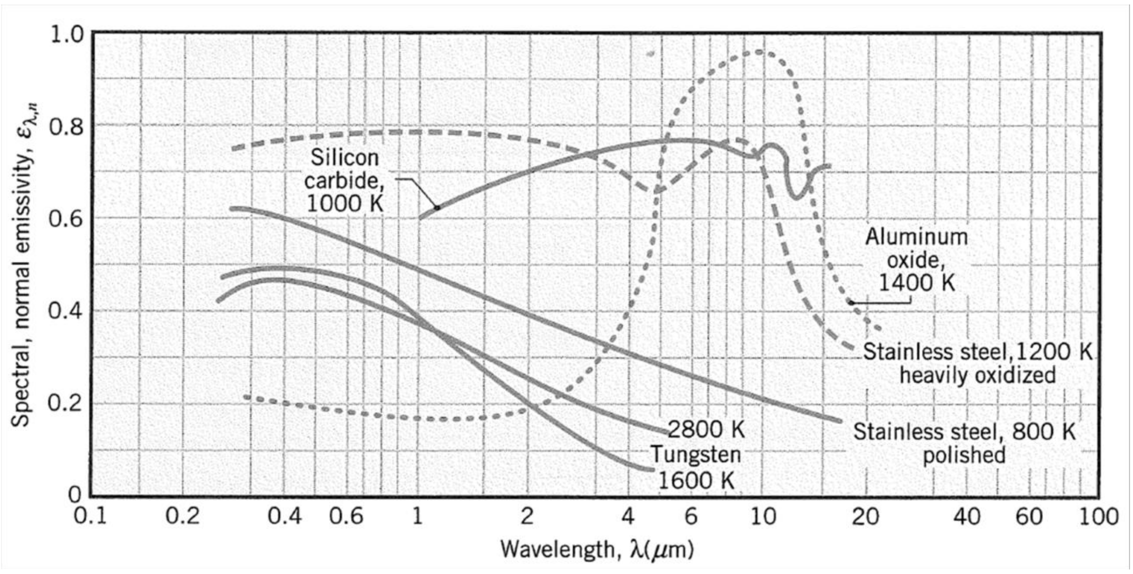


Figure 12-12 Spectral variation of emissivity

Radiation properties, and thus also emissivities, are dependent on temperature of material surface. Figure 12-13 shows variety of thermal emissivity behavior.

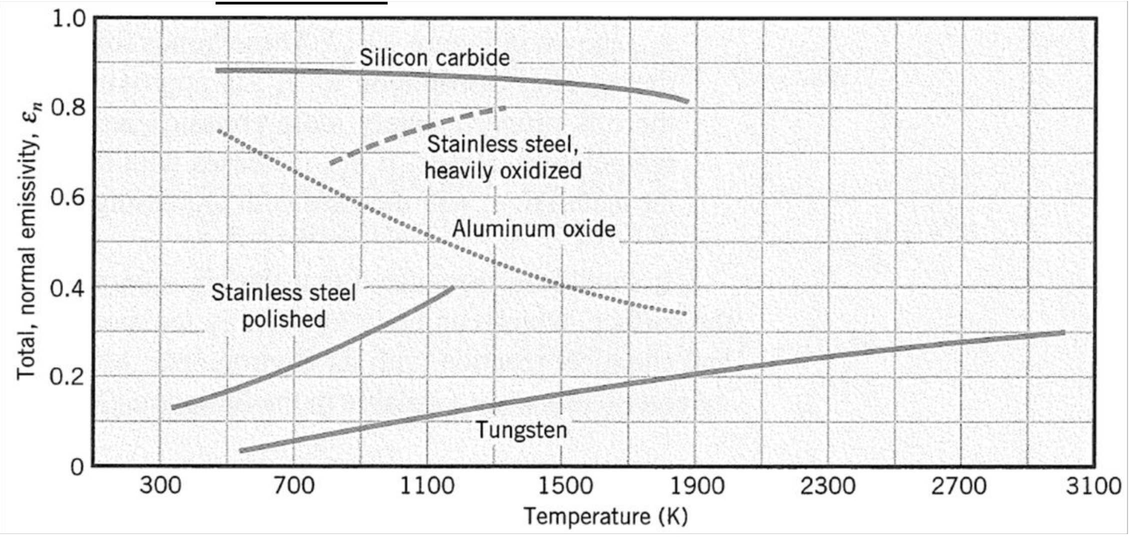


Figure 12-13 Examples of temperature dependence of total, normal emissivity

12.3.2 Surface Reflectivity and Absorptivity

Most of opaque materials presented in Figure 12-14 have a strong dependence of absortivity, and reflectivity (τ=1-α), on wavelength.

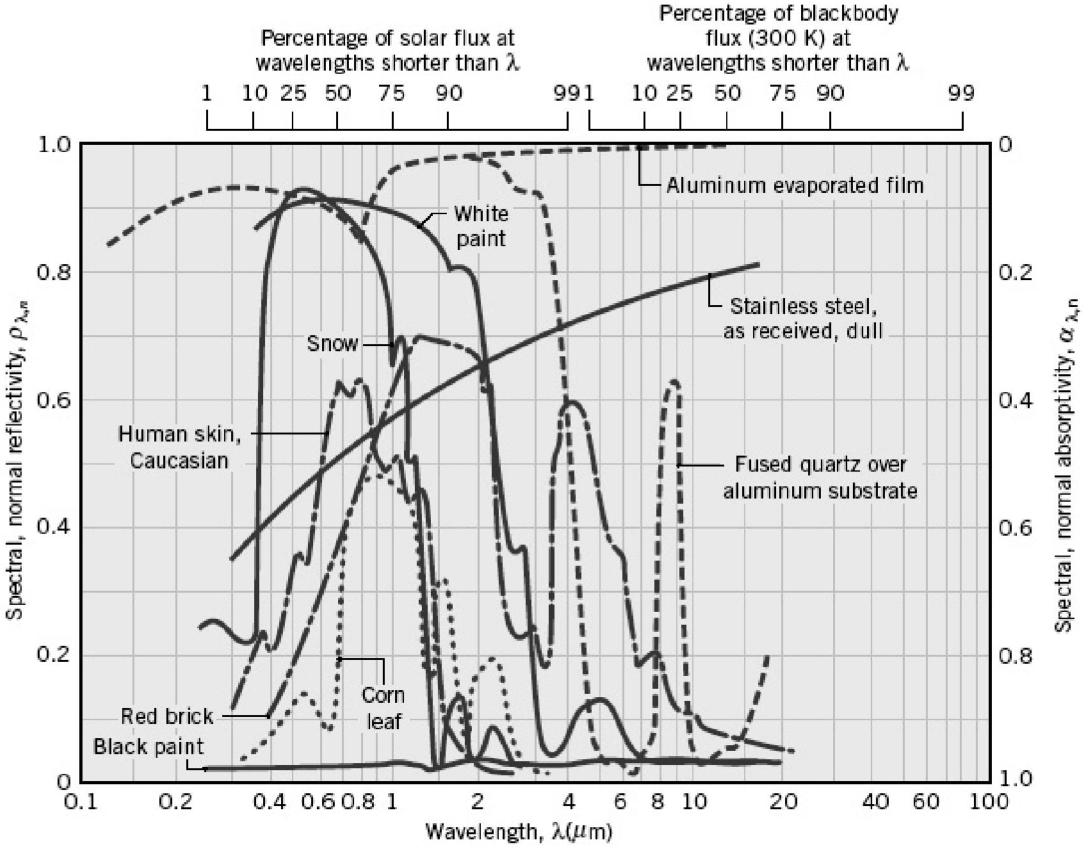


Figure 12-14 Spectral normal reflectivity and absorptivity

Surfaces may be divided to two idealized categorizes according the way they reflect radiation: (1) for *diffuse surfaces* reflected radiation is independent on reflection angle and on angle of incident radiation, (2) specular surfaces reflect radiation with equal angle to incident radiation (Figure 12-15).

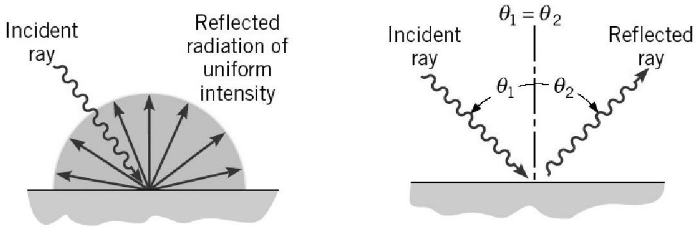


Figure 12-15 Diffuse and specular reflectivity surfaces

12.3.3 Surface Transmissivity

Figure 12-16 shows spectral transmissivity of few example materials.

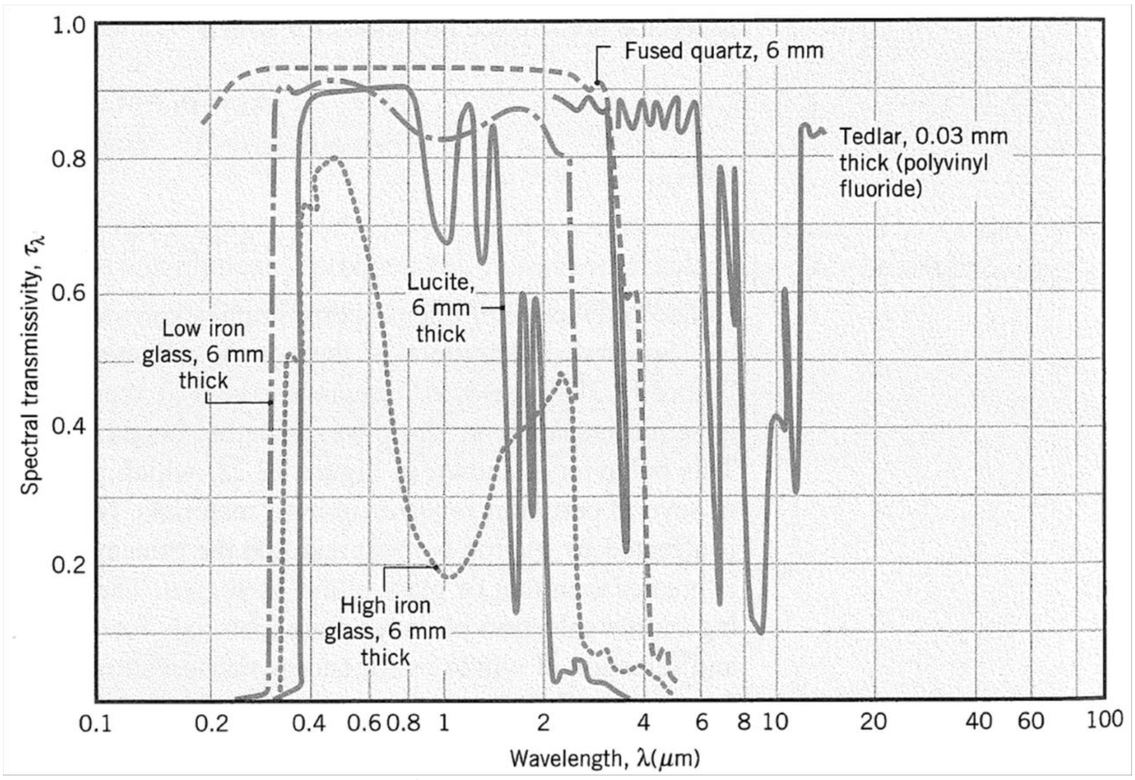


Figure 12-16 *Spectral transmissivity of selected materials*

12.4 RADIATION BETWEEN TWO SURFACES

12.4.1 The View Factor

View factor F_{ij} is defined as *fraction of the radiation leaving surface i that is intercepted by surface j.*

$$F_{ij} = \frac{q_{i\to j}}{A_i J_i} \tag{12-33}$$

$$q_{i\to j} = F_{ij} A_i J_i \tag{12-34}$$

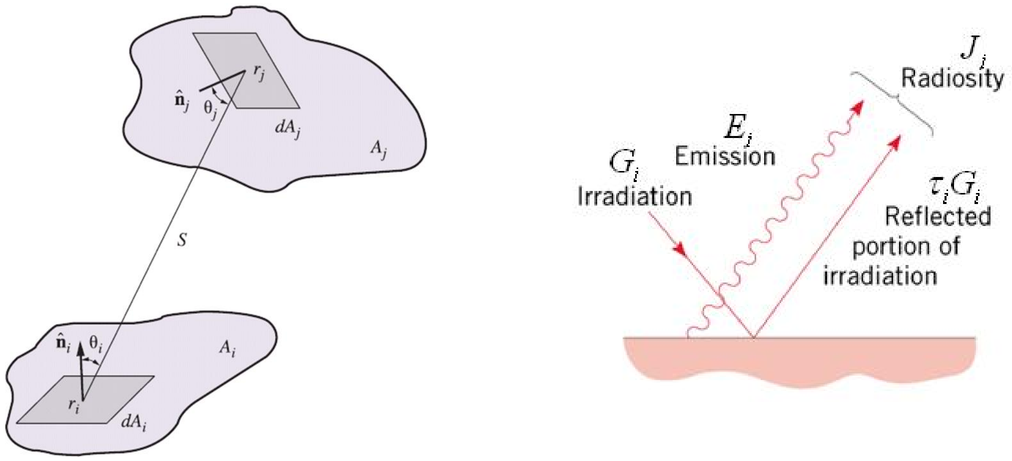
Summation rule for view factor is

$$\sum_{j=1}^{N} F_{ij} = 1 \qquad \textit{(closed system)} \qquad (12\text{-}35)$$

$$F_{i(j)} = \sum_{K=1}^{N} F_{ik} \qquad \textit{(between surfaces)} \qquad (12\text{-}36)$$

, and reciprocity relation

$$A_i F_{ij} = A_j F_{ji} \qquad (12\text{-}37)$$



12-17 (a) View factor associated with radiation exchange between two areas and (b) Radiosity J

For Blackbody A$_i$, ($J_{i=}$ E_{bi}) radiation intercepted by surface A$_j$ is

$$q_{ij} = A_i F_{ij} \sigma \left(T_i^4 - T_j^4\right) \qquad (12\text{-}38)$$

View factors are liste for example cases in Table 12-2.

12.4.2 Two Surface Enclosures

Between two surfaces a single expression for the rate of radiation transfer may be inferred from a network representation of the exchange (Figure 12-18) as

$$q_{1=}-q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}} \qquad (12\text{-}39)$$

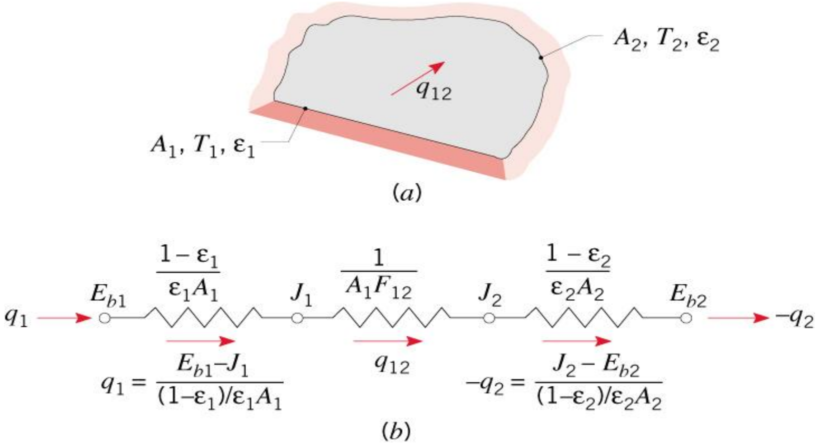


Figure 12-18 *Two surface enclosures*

This reduces for special condition for infinite parallel plates to form

$$q_1 = -q_2 = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 1} \qquad (12\text{-}40)$$

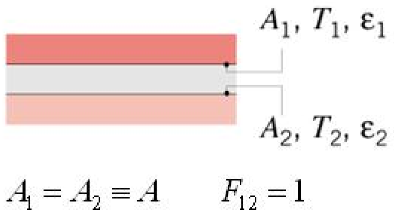
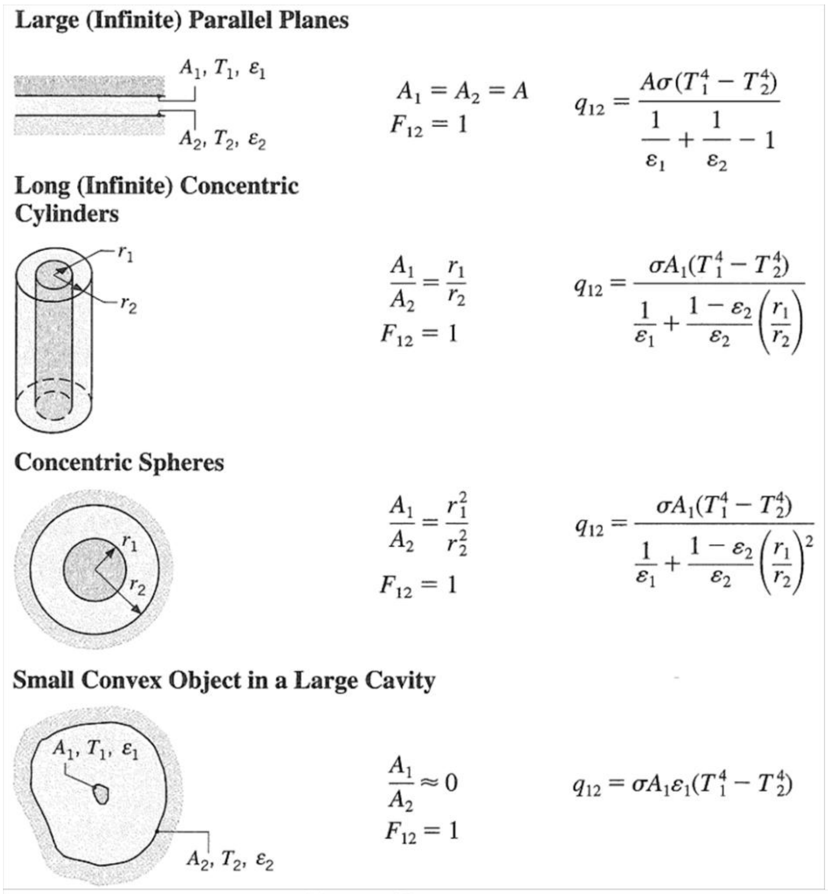


Figure 12-19 *Large (Infinite) Parallel Plates*

Table 12-2 *Example View Factors: 2- and 3-dimensional*



Large (Infinite) Parallel Planes

A_1, T_1, ε_1

A_2, T_2, ε_2

$$A_1 = A_2 = A \qquad F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

Long (Infinite) Concentric Cylinders

r_1

r_2

$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

Concentric Spheres

r_1

r_2

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \qquad F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$$

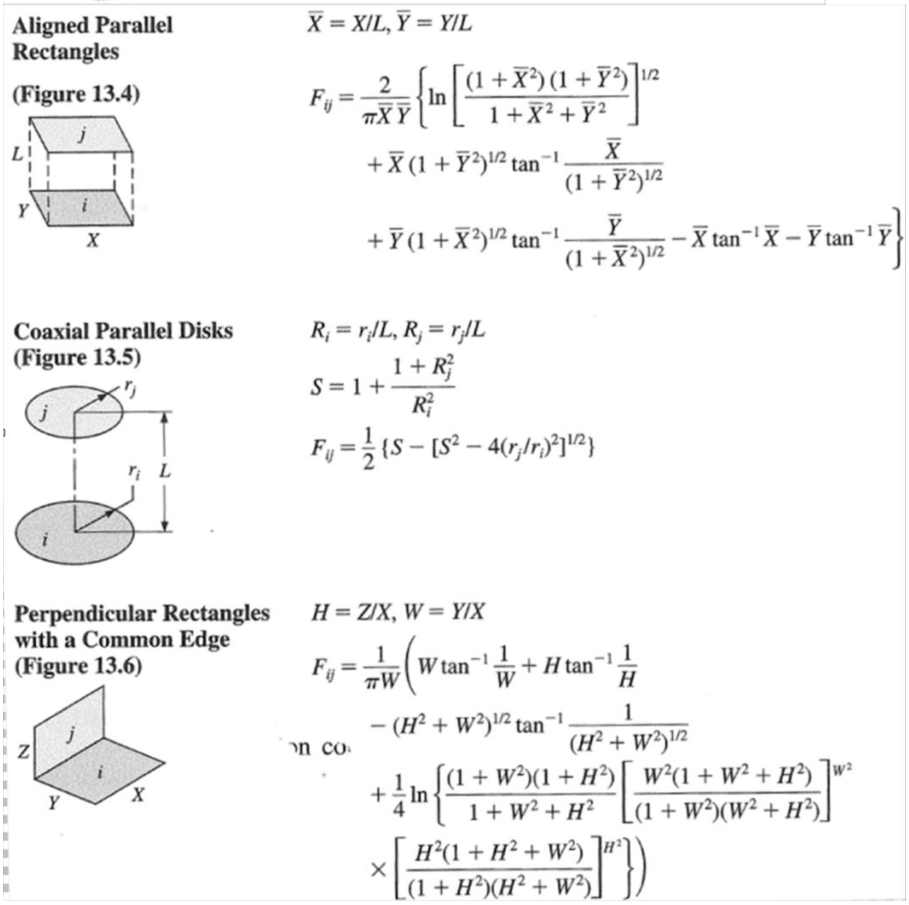
Small Convex Object in a Large Cavity

A_1, T_1, ε_1

A_2, T_2, ε_2

$$\frac{A_1}{A_2} \approx 0 \qquad F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$



Aligned Parallel Rectangles

(Figure 13.4)

$$\bar{X} = X/L, \bar{Y} = Y/L$$

$$F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$

Coaxial Parallel Disks
(Figure 13.5)

$$R_i = r_i/L, R_j = r_j/L$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$$

Perpendicular Rectangles with a Common Edge
(Figure 13.6)

$$H = Z/X, W = Y/X$$

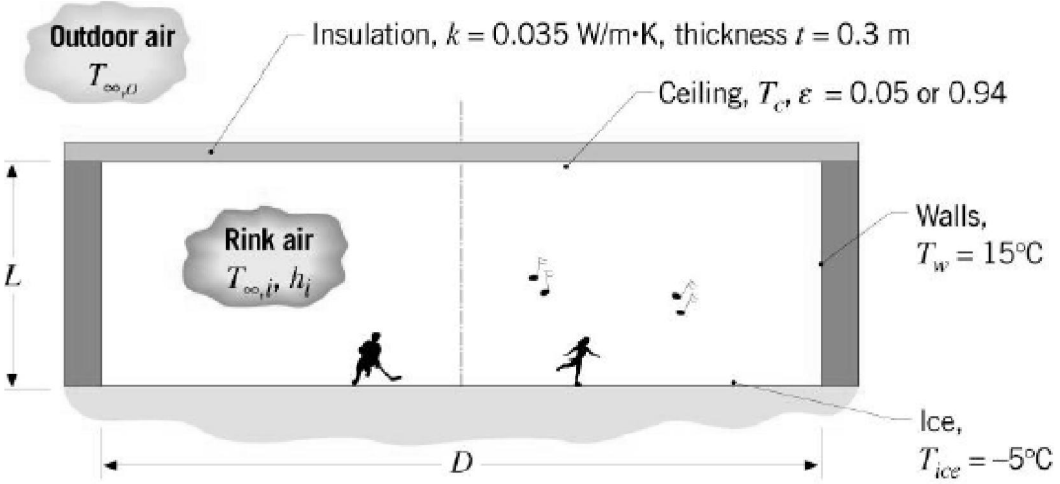
$$F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$$

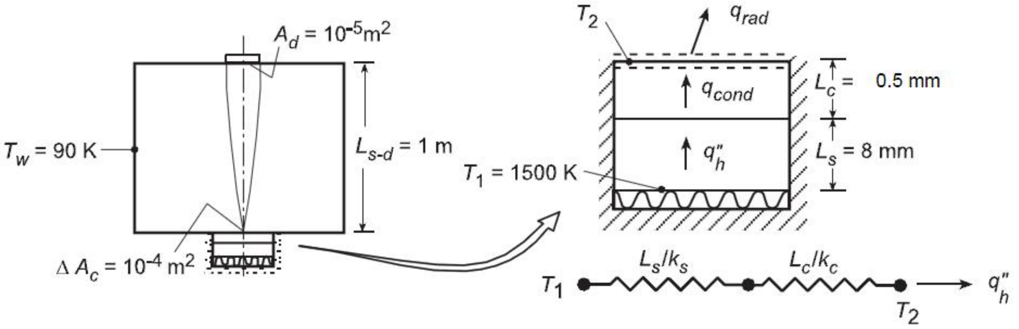
Example XXII: *Ice Hockey Hall – Radiation exchange between two bodies*

The assessment is to determine the ceiling irradiative properties for an ice rink in terms of ability to maintain surface temperature above the dew point. The rink has a diameter of D = 50 m and a height and a height of L = 10 m, and the temperatures of the ice and walls are -5 °C and 15 °C, respectively. The rink air temperature is 15 °C, relative humidity 70 %, and a convection coefficient of 5 W/m^{2}K characterizes conditions on the ceiling surface. The thickness and thermal conductivity of the ceiling insulation are 0.3 m and 0.035 W/m^{2}K, respectively, and the temperature of the outdoor air is -5 °C. Ceiling can be assumed as a diffuse-grey surface and the walls and ice may be approximated as blackbodies.

a) Consider a flat ceiling having an emissivity of 0,05 (highly reflective panels) or 0,94 (painted panels). Perform an energy balance on the ceiling to calculate the corresponding values of the ceiling temperature. Will condensation occur for either or both of the emissivities?

b) For each of the emissivities, calculate and plot the ceiling temperature as a function of the insulation thickness for . Identify conditions for which condensation will occur on the ceiling.





ASSUMPTIONS**:** (1) Rink comprised of the ice, walls and ceiling approximates a threesurface,diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Ice surface and walls are black, (4) Panels are diffuse-gray, and (5) Thermal resistance for convection on the outdoorside of the ceiling is negligible compared to the conduction resistance of the ceiling insulation.

PROPERTIES**:** *Psychrometric chart* (Atmospheric pressure; dry bulb temperature, Tdb = T□,i = 15□C; relative humidity, RH = 70%): Dew point temperature, Tdp = 9.4□C.

ANALYSIS**:** Applying an energy balance to the inner surface of the ceiling and treating all heat rates as energy *outflows*,

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0$$

$$-q_{\text{o}} - q_{\text{conv,c}} - q_{\text{rad,c}} = 0 \qquad (1)$$

$$q_{\text{o}} = \frac{(T_{\text{c}} - T_{\infty,\text{o}})}{R_{\text{cond}}} \qquad R_{\text{cond}} = \frac{t}{kA_{\text{c}}} \qquad (2,3)$$

$$q_{\mathrm{conv,c}} = h_{\mathrm{i}} A_{\mathrm{c}} \left(T_{\mathrm{c}} - T_{\infty,\mathrm{i}} \right) \tag{4}$$

$$q_{\mathrm{rad,c}} = \varepsilon E_{\mathrm{b}}(T_{\mathrm{c}}) A_{\mathrm{c}} - \alpha A_{\mathrm{w}} F_{\mathrm{wc}} E_{\mathrm{b}}(T_{\mathrm{w}}) - \alpha A_{\mathrm{i}} F_{\mathrm{ic}} E_{\mathrm{b}}(T_{\mathrm{i}}) \tag{5}$$

Since the ceiling panels are diffuse-gray, $\alpha = \varepsilon$. for parallel, coaxial disks

$$F_{\mathrm{ic}} = 0{,}672$$

From the summation rule applied to the ice (i) and the reciprocity rule,

$$F_{\mathrm{ic}} + F_{\mathrm{iw}} = 1 \qquad F_{\mathrm{iw}} = F_{\mathrm{cw}} \text{ (symmetry)}$$

$$F_{\mathrm{cw}} = 1 - F_{\mathrm{ic}}$$

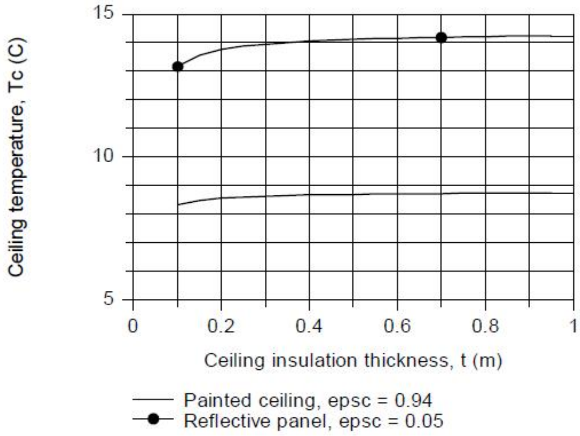
$$F_{\mathrm{cw}} = (A_{\mathrm{c}}/A_{\mathrm{w}}) F_{\mathrm{cw}} = (A_{\mathrm{c}}/A_{\mathrm{w}})(1 - F_{\mathrm{ic}}) = 0{,}410$$

where $A_{\mathrm{c}} = \pi D^2/4$ and $A_{\mathrm{w}} = \pi DL$
Using the foregoing energy balance, Eq. (1), and the rate equations, Eqs. (2-5), the ceiling temperature is calculated using radiative properties for the two panel types,

Ceiling panel	ε	T_{c} (°C)	
Reflective	0,05	14,0	
Paint	0,94	8,6	$T_{\mathrm{c}} < T_{\mathrm{dp}}$

Condensation will occur on the painted panel since $T_{\mathrm{c}} < T_{\mathrm{dp}}$.

(b) Applying the foregoing model for $0.1 \leq t \leq 1.0$ m, the following result is obtained



For the reflective panel (ε = 0.05), the ceiling surface temperature is considerably above dew point. Therefore, condensation will not occur for the range of insulation thicknesses. For the painted panel (ε = 0.94), the ceiling surface temperature is always below the dew point, and condensation occurs for the range of insulation thicknesses.

COMMENTS: **(***Icehockey or figure skating?)*

1) *From the analysis, recognize that radiative exchange between the ice and the ceiling has the dominant effect on the ceiling temperature. With the reflective panel, the rate is reduced nearly 20-fold relative to that for the painted panel. With the painted panel ceiling, condensation will occur for most of the conditions likely to exist in the rink.*

12.5 LEARNING OUTCOMES

Table 12-3 Learning Outcomes: *Radiation*

Level of Knowledge	Concept
Apply ***Understand*** *Describe*	• *Concepts: Stefan-Boltzmann's and Wien' displacement Laws and Black body radiation functions,* • *Surface/body properties: black body, gray, diffusive, opaque* • *Two radiation heat transfer solution methods: (1)* ***based on Kirchoff's Law*** *and (2)* ***Radiation between two bodies***
understand *describe*	• *Spectral radiation properties of material surfaces*

REFERENCES

Bejan A., and Kraus A. D., Heat Transfer Handbook, Hoboken, NJ: John Wiley & Sons, Inc, 2003, ISBN: 0-471-39015-1.

Holman, J.P., Heat Transfer. 6th edition, McGraw-Hill Book Company – Singapore. Singapore, 1989.

Incropera Frank P, DeWitt David, Fundamentals of Heat and Mass Transfer. 5th edition. John Wiley & Sons, Inc. United States, 2002.

Lampinen M. J., Assad El M., Kotiaho V., Lämmönsiirto-oppi, teknillinen korkeakoulu, Otaniemi 2008.

Lienhard J. H. IV, Lienhard J. H. V: A Heat Transfer Textbook, 3rd Edition. 2008.

Luttunen E., Lämmönsiirron yhtälöt ja taulukot, 2010.

Saari, J., Heat Exchanger Thermal Design Quide, LUT Energy Lecture Note 6, 2010.

Tynjälä T., Sarkomaa P., Lämmönsiirto Luennot, 1999 - 2000.

Yunus A. Çenel, Heat Transfer – A Practical Approach.

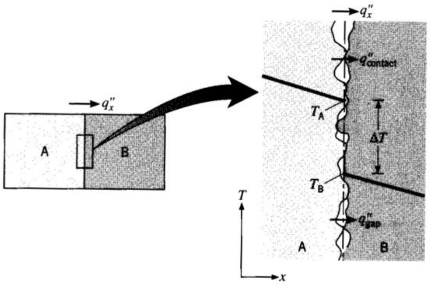
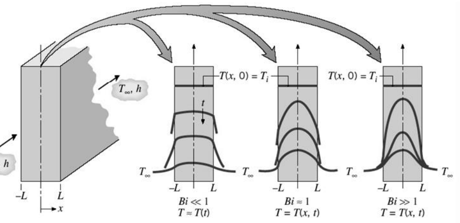
Wendl, Michael C., Fundamentals of Heat Transfer Theory and Applications: 1999, 2003 and version 2.1 on August 2005.

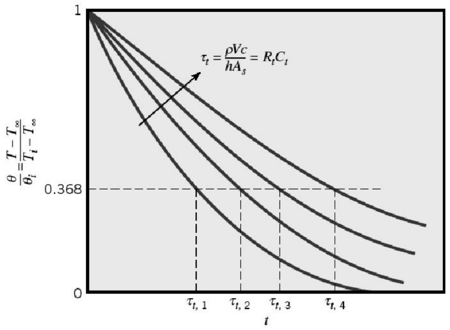
APPENDIX I: DIMENSIONLESS NUMBERS

Dimesionless Number	**Equation**	**Description**	***Parameters***
Biot number	$Bi = \frac{hL_c}{k}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance	h Heat Transfer Coefficient$[W/m^2K]$ $L_c \equiv V/A_s$ Characteristic Length $[m]$ k Thermal Conductivity $[W/mK]$
Fourier number	$Fo = \frac{\alpha t}{L_c^2}$	Ratio of the heat conduction rate to the rate of thermal storage in a solid. Dimensionless time.	α Thermal Diffusivity $[m^2/s]$ t Time $[s]$
Reynolds number	$Re = \frac{\rho u_\infty x}{\mu} = \frac{u_\infty x}{\nu}$	Ratio of the inertia and viscous forces	ρ Density $[kg/m^3]$ u_∞ Flow Velocity $[m/s]$ x Characteristic Length $[m]$ μ Dynamic Viscosity$[Ns/m^2]$ ν Kinematic Viscosity $[m^2/s]$
Prandtl number	$Pr = \frac{\nu}{\alpha}$	Ratio of momentum and thermal diffusivities	
Schmidt number	$Sc = \frac{\nu}{D_{AB}}$	Ratio of the momentum and mass diffusivities	D_{AB} Mass Diffusivity $[m^2/s]$
Nusselt number	$Nu = \frac{hL}{k}$	Ratio of convection to pure conduction heat transfer. Dimensionless temperature gradient.	
Sherwood number	$Sh \equiv \frac{h_m L}{D_{AB}}$	Dimensionless concentration gradient.	h_m Mass Transfer Coefficient $[m/s]$
Lewis number	$Le = \frac{Sc}{Pr}$	Ratio of the thermal and mass diffusivities	
	$\frac{Nu}{Pr^n} = \frac{Sh}{Sc^n}$	Mass and heat transfer analogy	
Stanton number	$St \equiv \frac{h}{\rho u_\infty c_p} = \frac{Nu}{RePr}$	Modified Nusselt number	c_p Specific Heat Capacity$[J/kgK]$
Grashof number	$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = Re_L^{1/2}$	Measure of the ratio of buoyancy forces to viscous forces	$g = 9{,}81\ m/s^2$ Acceleration of Gravity β Volumetric Expansion Rate $[1/K]$ T_s Surface Temperature $[K]$ T_∞ Flow Temperature$[K]$
Rayleigh number	$Ra = GrPr = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$	Product of the Grashof and Prandtl numbers	
Jacobi number	$Ja = \frac{c_{p,l}(T_{sat} - T_s)}{h_{fg}}$	Ratio of sensible to latent energy absorbed during liquid-vapor phase change.	h_{fg} Latent Heat of Vaporization $[J/kg]$ T_{sat} Saturated Temperature $[K]$

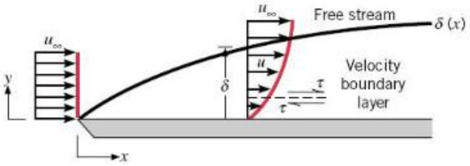
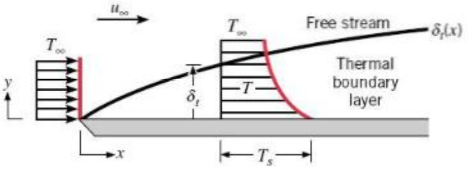
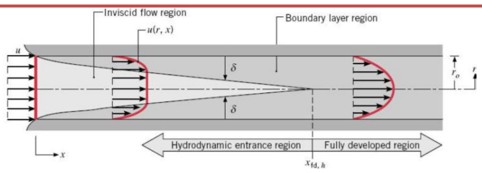
APPENDIX II: TERMINOLOGY

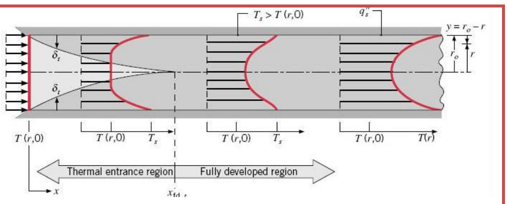
GENERAL		
Temperature ***Lämpötila***	*On the microscopic scale, temperature can be defined as the average energy in each degree of freedom in the particles in a system.* *0th law of thermodynamics: “Two bodies are in thermal equilibrium if both have same temperature even if they are not in contact”* Mikroskooppisella tasolla, lämpötila voidaan määrittää keskimääräiseksi energiaksi jokaisessa vapaus asteessa hiukkassysteemissä. 0th law of thermodynamics: “Two bodies are in thermal equilibrium if both have same temperature even if they are not in contact”	*T [K]* Scales: Kelvin, Celsius, Fahrenheit, thermodynamic, etc.
Heat **Lämpö**	*Form of energy that can be transferred between systems as a result of temperature difference* Energian muoto, joka siirtyy systeemien välillä lämpötilaeron johdosta.	*Q [J]*
Heat Transfer Rate **Lämpövirta**	*Amount of heat transferred per unit time* *2nd law of thermodynamics: Heat is transferred to direction of decreasing temperature. Driving force for heat transfer is temperature difference.* Siirtynyt lämpö aikaa kohden. Termodynamiikan 2. laki: Lämpö virtaa kohden alempaa lämpötilaa ja potentiaalina toimii lämpötilaero.	*q [W]*
Heat Flux ***Lämpövuo***	Heat transfer rate per unit area normal to direction of heat transfer Lämpövirta virtaa kohtisuoraa pinta-alaa kohden	*q” [W/m*^{2}*]*
Internal energy **Sisäenergia**	*“Sum of all microscopic forms of energy” & “Sum of kinetic and potential energies of molecules”* *(1) Sensible (thermal): transitional, rotational or vibration motion of molecules and atoms comprising matter.* *(2) Latent: related to intermolecular forces influencing to phase change* *(3) Chemical: energy stored to chemical bonds between atoms.* *(4) Nuclear: binding forces in nucleus.* “Kaikkien mikroskooppisten energioiden summa” & “molekyylien kineettisen ja potentiaalienergioiden summa” (1) Tuntuvat (terminen): siirtyvä, pyörivä, värähtelevä liike molekyyleillä ja atomeilla (2) Latentti: faasinmuutokseen liittyvä molekyylien sidosten energia (3) Kemiallinen: energia, joka on sitoutunut atomien välisiin kemiallisiin sidoksiin (4) Ydin: ydinhiukkasten sidosenergia	*U [J]*, unit mass basis: *u [kJ/kg]*
Heat Capacity **Lämpökapasiteetti**	*Energy required to raise temperature of a system by one degree* *Energia, joka vaaditaan nostamaan systeemin lämpötila yhden asteen*	*C=cm [J/K]*
Specific heat **Ominais-lämpökapasiteetti**	*Specific heat is energy required to raise temperature of a unit mass by one degree.* Energia, joka vaaditaan nostamaan yksikkö massan lämpötila yhden asteen.	*c =c(T,p) [J/kgK]*

	CONDUCTION	
Fourier's Law **Fourierin laki**	*Empirical conduction equation. Relation between the conduction heat transfer rate, cross section area and temperature gradient.* Kokeellinen johtumisyhtälö. Lämpövirran suhde poikkipinta-alaan ja lämpötilagradienttiin.	$q = -kA\dfrac{dT}{dx}$
Contact Heat Resistance **Kontaktilämpövastus**	*Surface roughness in composite systems forms additional heat resistance, which is called thermal contact resistance.* Kahden kiinteän aineen välillä oleva pinnan karheus aiheuttaa lämpövastuksen, jota nimetään kontaktilämpövastukseksi.	$R''_{t,c} = \dfrac{T_A - T_B}{q''_x}$ 
Thermal conductivity **Lämmönjohtuvuus**	*Thermal transport property for conduction. Depends on material and temperature.* Materiaaliominaisuus, joka kuvaa lämmönjohtumiskykyä – funktio materiaalista ja lämpötilasta.	$k = \dfrac{-q''_x}{(\partial T / \partial x)}$ *[W/mK]*
Thermal resistance **Terminen resistanssi**	*Thermal property or quantity describing Resistance to heat flux through medium or system* Materiaalin tai systeemin lämpövirran vastusta kuvaava suure	$R_{t,cond} \equiv \dfrac{L}{Ak}$ *[m²/K] (conduction, plane wall)* $R_{t,conv.} \equiv \dfrac{1}{hA} = \dfrac{(T_s - T_\infty)}{q}$ *(convection)* $R_{t,rad} \equiv \dfrac{1}{h_r A} = \dfrac{(T_s - T_\infty)}{q_{rad}}$ *(radiation)*
Overall heat transfer coefficient **Kokonaislämmönläpäisy-kerroin**	*Defines heat transfer coefficient of system, which can used with temperature difference and heat transfer area to determine heat transfer rate (analogy to Newton's Law of Cooling)* Määrittää lämmönsiirtokertoimen systeemille, jonka avulla lämpötilaeron ja lämmönsiirtopinta-alan kanssa lämpövirta määritetään (analogia Newtonin jäähtymislain kanssa)	$U = \dfrac{1}{AR_{tot}}$ *[W/m²]* $q = UA\Delta T$
Lumped capacitance method **Tasalämpötilamalli**	*A method used in unsteady state problems. Temperature of the solid, experiencing changes in its internal thermal energy, is spatially uniform at any instant during the transient process. Valid when* $Bi < 0.1$ Epästationaarisissa tapauksissa käytetty malli. Kiinteän aineen, joka kokee sisäenergian muutosta, lämpötilajakauma pysyy transientin aikana tasaisena. Validi kun $Bi < 0.1$	
A thermal time constant	*In transient conduction describes temperature respond rate of a medium to changes in its thermal environment.*	$\tau_t = \left(\dfrac{1}{A_s h}\right)(\rho V c_p) = R_t C_t$ *[s]*

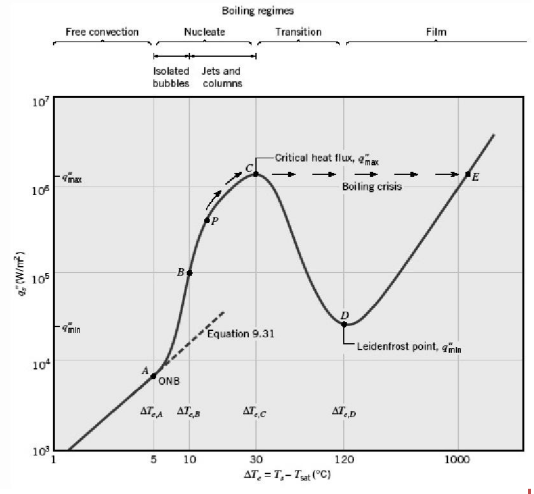
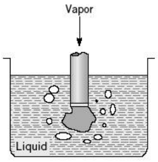
Terminen aikavakio	Kuvaa kiinteän aineen lämpötilanmuutoksen nopeusvastetta lämpötilan muutoksiin sen ympäristössä.	

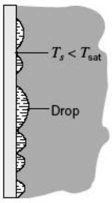
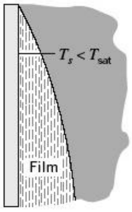
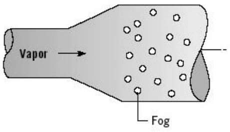
	FINS	
Fin Efficiency **Rivan hyötysuhde**	*The ratio of the fin heat transfer rate to the heat transfer rate that would exist if the entire fin surface were at the base temperature* Rivan lämpövirran suhde lämpövirtaan vastaavasta ideaalisesta rivasta, jossa rivan lämpövastus on nolla, eli ripa on samassa lämpötilassa kuin sen pohja.	$\eta_f = \frac{q_f}{q_{\max}} = \frac{q_f}{h A_f \theta_b}$
Fin Resistance **Ripavastus**	*Defined by treating base and fluid temperature difference as driving potential for heat transfer* Määritetään käsittelemällä rivan pohjan ja ympäristön lämpötilaeroa lämpövirran potentiaalina.	$R_{t,f} = \frac{\theta_b}{q_f} = \frac{T_b - T_\infty}{q_f}$
Fin Effectiveness **Rivan tehokkuus**	*The ratio of the fin heat transfer rate to the heat transfer rate that would exist without the fin* Rivan lämpövirran suhde lämpövirran suuruuteen ilman rivoitusta	
Overall Surface Efficiency **Ripapinnan kokonaishyötysuhde**	*Characterizes thermal performance of combination of an array of fins and the attachment base surface: ratio of fin surface heat transfer to heat transfer from same surface at base temperature* Kuvaa koko rivoituksen lämmönsiirtoa suhteessa ideaaliseen rivoitukseen, jossa rivan pinnat ovat pohjan lämpötilassa.	$\eta_0 = \frac{q_{tot}}{q_{\max}} = \frac{q_{tot}}{h A_{tot} \theta_b}$ *Total heat transfer rate from surface:* $q_{tot} = N \eta_f A_f h \theta_b + A_b h \theta_b$
Infinite fin **Äärettömän pitkä ripa**	*A fin, for which the temperature difference between fin tip and boundary is zero* Ripa, jolle lämpötilaero rivan pään ja ympäristön välillä on nolla.	0

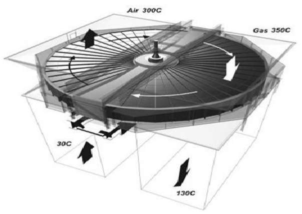
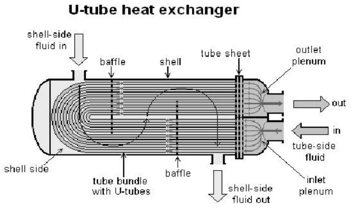
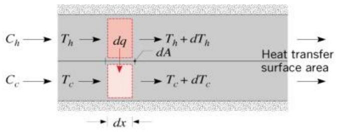
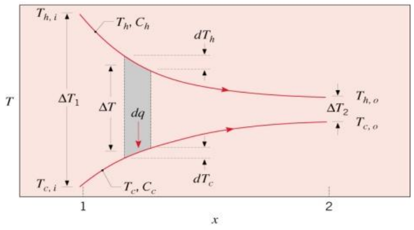
CONVECTION		
Newton's Convection Law	*Heat transfer rate in convection is proportional to the heat transfer area and the temperature difference between fluid and a surface.*	$q_{\text{conv}} = \bar{h}A(T_s - T_\infty)$
Newtonin konvektiolaki	Konvektiivisen lämmönsiirron lämpövirta on verrannollinen lämmönsiirtopinta-alaan, konvektiiviseen lämmönsiirtokertoimeen ja lämpötilaeroon virtaavan aineen ja pinnan välillä.	
Forced convection	*Convection in a Fluid, forced to flow over a surface*	
Pakotettu konvektio	Konvektiivinen lämmönsiirto fluidiin/sta, joka on pakotettu virtaamaan pinnan yli.	
Free convection	*Convection formed due to buoyancy forces caused by e.g. temperature dependent density gradients and gravitational body forces that are proportional to density.*	
Vapaa konvektio	Konvektio, joka aiheutuu nostevirtauksesta, jotka yleisimmin johtuvat lämpötilaeroista riippuvista tiheysgradienteista ja gravitaatiovoimista, jotka ovat verrannollisia tiheyteen.	
Velocity Boundary Layer	*Fluid velocity is reduced by fluid particles which make contact with the surface. Thickness of velocity boundary layer is defined as the distance which the flow velocity is 99 % of the free stream velocity.*	
Nopeusrajakerros	Pinnan karheudesta aiheutuva kitka hidastaa virtausta. Rajakerroksen paksuus määritellään kohtaan, jossa nopeus on hidastunut 99 %:iin vapaan virtauksen nopeudesta.	
Lämpötilan rajakerros	Syntyy, kun pinnan ja virtaavan fluidin välillä on lämpötilaero. Lämpötilan rajakerroksen paksuudella tarkoitetaan etäisyyttä pinnasta, jossa suhde $(T_s - T)/(T_\infty - T_s) = 0{,}99$.	
Thermal Boundary Layer	*Develops if the fluid free stream and surface temperature differ. Defined as the region of the fluid in which temperature gradients exist. Thickness of thermal boundary layer is defined as the distance for which the ratio* $(T_s - T)/(T_\infty - T_s) = 0{,}99$.	
Hydrodynamic Entrance	*The region from the inlet of a circular tube to the end of the inviscid flow region – until the touching point of boundary layers. Length depends on rate of turbulence.*	
Hydrodynaaminen sisääntuloalue	Alue putken sisääntulosta kitkattoman virtauksen loppuun saakka – rajakerroksien yhdistymispisteeseen. Sisääntuloalueen pituus riippuu virtauksen turbulenttisuudesta.	$\left(\frac{x_{\text{fd,h}}}{D}\right)_{\text{lam}} \approx 0{,}05\text{Re}_D$, *Laminar Flow* $10 \le \left(\frac{x_{\text{fd,h}}}{D}\right)_{\text{turb}} \le 60$, *Turbulent Flow*

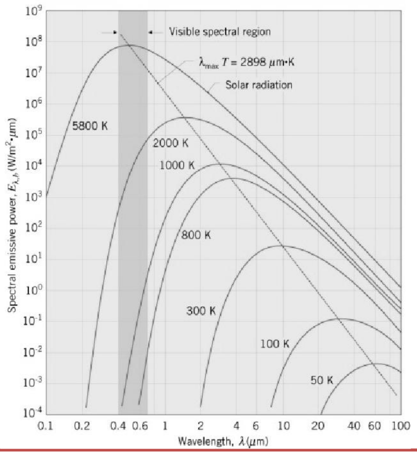
Thermal Entrance **Terminen sisääntuloalue**	*Region from an inlet of a circular tube to point of touch of boundary layers. Entry length depends turbulency and on Prandlt number of fluid* Alue putken sisääntulosta lämpötilarajakerrosten kohtaamiseen saakka. Pituus riippuu turbulenttisuuden lisäksi myös fluidin Prandtlin luvusta.	 $\left(\frac{x_{\mathrm{fd,t}}}{D}\right)_{\mathrm{lam}} \approx 0{,}05\mathrm{Re_D}\mathrm{Pr}$, *Laminar Flow* $\left(\frac{x_{\mathrm{fd,t}}}{D}\right)_{\mathrm{turb}} = 10$, *Turbulent Flow*
Reynolds analogy **Reynoldsin analogia**	*Describes the connections between heat and mass transfer in a convection heat transfer case with the dimensionless numbers Nu, Sh, (St, St$_m$), C$_f$, Re, Pr and Sc.* Esittää yhteyden aineen- ja lämmönsiirron välillä konvektiivisen lämmönsiirron tapauksessa dimensiottomien lukujen Nu, Sh, (St, St$_m$), C$_f$, Re, Pr ja Sc avulla.	$\frac{c_f}{2} = St = St_{\mathrm{m}}$ *Conditions:* $Pr = Sc \approx 1$ $dp^*/dx^* \approx 0$
Chilton-Colburn analogy **Chilton-Colburn analogia**	*Modified Reynolds analogy – wider validity range* Modifioitu Reynoldsin analogia – laajempi validiteetti	$\frac{c_f}{2} = StPr^{2/3} \equiv j_{\mathrm{h}}$ $0{,}6 < Pr < 60$ $\frac{c_f}{2} = St_{\mathrm{m}}Pr^{2/3} \equiv j_{\mathrm{h}}$ $0{,}6 < Sc < 60$,when dp^*/dx^*
Volumetric thermal expansion coefficient **Tilavuuden lämpölaajenemiskerroin**	*Density changes in response to a change in temperature at constant pressure* Ilmaisee tiheyserojen riippuvuutta lämpötila muutoksista vakiopaineessa.	$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_{\mathrm{p}}$ $\beta = \frac{1}{T}$ *(for ideal gas)* β *[K^{-1}]*
Convection heat transfer coefficient **Konvektiivinen lämmönsiirtokerroin**	*Heat transport property for convection. Dependent on fluid and flow properties, geometryl and temperature.* Lämmönsiirtokerroin konvektiolle. Riippuvainen fluidin ja virtauksen ominaisuuksista, geometriasta ja lämpötilasta	*h [W/m^{2}K]*
Convection mass transfer coefficient **Konvektion aineensiirtokerroin**	*Mass transport property for convection. Dependent on fluid and flow properties, geometryl and temperature.* Aineensiirtokerroin konvektiolle. Riippuvainen fluidin ja virtauksen ominaisuuksista, geometriasta ja lämpötilasta	*h$_m$ [m/s]*

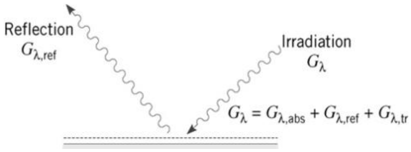
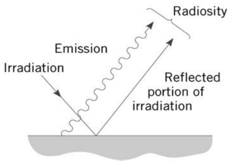
BOILING & CONDENSATION

Saturated Boiling	*Boiling, in which the temperature of the liquid has exceeded the saturation temperature.*	
Kylläinen kiehunta	Kiehunta, missä nesteen lämpötila on saavuttanut kiehumislämpötilan.	
Subcooled Boiling	*Boiling, in which the temperature of the liquid is below the saturation temperature.*	
Alijäähtynyt kiehunta	Kiehunta, missä nesteen lämpötila on alle kiehumislämpötilan.	
Modes of Pool Boiling **Allaskiehuntamuodot**		
Free convection	*Boiling without bubbles. ( )*	
Nucleate	*Boiling in bubbles. Divided in to regimes, isolated bubbles and jets. ( )*	
Transition	*Unstable regime of boiling. Rapid formation of bubbles forms a vapor film on the surface and the heat transfer rate will decrease. ( )*	
Film boiling	*The regime after the minimum heat flux (. The dominant form of heat transfer is radiation. The heat flux will increase.* *( )*	
Boiling Crises	*Temperature above the saturation temperature in which the heat flux reaches the burnout point and the heat flux is the critical heat flux (CHF). After that the boiling crises will occur and the heat transfer coefficient will decrease – over heating of material surface is possible.*	
Kiehumiskriisi	Lämpötila missä lämpövuo saavuttaa kriittisen lämpövuon (*CHF*). Tämän jälkeen kiehumiskriisi alkaa ja lämmönsiirto heikkenee – mahdollinen materiaalin ylikuumentuminen.	
Excess temperature	*The difference between the heating source temperature and saturation temperature*	$q''_s = h(T_S - T_{sat}) = h\Delta T_e$
Ylimäärä lämpötila	Kuumentavan lähteen ja kiehumispisteen välinen lämpötilaero.	ΔT_e *Excess temperature* $T_{sat}(p)$ *Fluid saturation temperature*
Direct Contact Condensation	*Vapour is condensed with direct contact with a cold liquid.*	
Kontaktilauhtuminen	Höyry lauhtuu kontaktissa kylmän nesteen kanssa	

Surface Condensation **Pintalauhtuminen**	*Condensation on a cold surface (Drop-wise & Film condensation)* Lauhtuminen kylmän seinämän pinnalla (pisara- ja filmilauhtuminen	
Drop-wise Condensation **Pisaralauhtuminen**	*Drops are formed on a cold surface in its cracks, pits and cavities. Since then the drops are grown and coalesced.- high heat transfer rates* Pisarat syntyvät kylmälle pinnalle sen pieniin säröihin ja kuoppiin. Muodostumisen jälkeen pisarat voivat kasvaa ja koaguloitua. – korkea lämmönsiirtokerroin	
Film condensation **Filmilauhtuminen**	*Condensation on the clean and smooth surface, The condensate liquid forms a film on the surface. The film flows downwards under the action of gravity.* Lauhtuminen puhtaalla ja tasaisella pinnalla. Lauhtunut neste muodostaa filmin pinnalle. Filmi valuu alaspäin gravitaation vaikutuksesta.	
Homogenous Condensation **Homogeeninen lauhtuminen**	*Vapor condenses out as droplets suspended in a gas phase to form a fog.* Höyry lauhtuu pisaroiksi jäähdytetystä höyrystä ja muodostaa sumua.	

HEAT EXCHANGERS		
Regenerative **Regeneratiivinen**	*Heat exchangers where the heat transfer is performed through a material that is alternately brought to contact with hot and cold streams, storing energy from the hot stream and releasing it into the cold one* *Lämmönsiirrin tyyppi, jossa kahta fluidia erottaa lämpöpinta, jossa tyypillisesti ei ole liikkuvia osia*	
Recuperative **Rekuperatiivinen**	*Heat exchangers where two fluids are separated by the heat transfer surface that normally has no moving parts* *Lämmönsiirrin tyyppi, jossa lämpö syklisesti varastoituu kuumasta virtauksesta lämpöpintaan ja myöhemmin vapautuu tästä lämmitettävään fluidiin*	
Effectiveness **Tehokkuus**	*The ratio of the actual heat transfer rate for a heat exchanger to the maximum possible heat transfer rate* Oikeasti lämmönsiirtimessä siirtyvän lämpövirran suhde maksimilämpövirtaan	$\varepsilon = \frac{q}{q_{\max}}$
Heat Capacity Ratio ***Lämpökapasiteettien suhde***	*The ratio of the minimum and maximum heat capacity rates.* Minimi ja maksimi lämpökapasiteettivirtojen suhde.	$C_r = \frac{C_{\min}}{C_{\max}}$
ε-NTU **ε-NTU**	*the effectiveness-NTU method* *(NTU, Number of Transfer Units)* tehokkuus – NTU menetelmä	$NTU \equiv \frac{UA}{C_{\min}}$ $\varepsilon = \frac{q}{q_{\max}}$ $\varepsilon = f(NTU, C_{\min}/C_{\max})$ $q = \varepsilon C_{\min}(T_{h,i} - T_{c,i})$
LMTD **LMTD**	*Log Mean Temperature Difference* *(Based by logarithm decay of the temperature difference in heat exchanger)* Logaritminen lämpötilaero (taustana logaritminen lämpötilaprofiilien käyttäytyminen lämmönsiirtimissä)	$\Delta T_{\mathrm{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$ $q = UA\Delta T_{\mathrm{lm}}$  

	RADIATION	
A Blackbody **Musta kappale**	*Absorbs and emits all incident radiation regardless of wavelength and direction. A blackbody is a diffuse emitter.* Absorboi ja emittoi kaiken säteilyn riippumatta aallonpituudesta ja suunnasta. Musta kappale on diffuusi säteilijä	
The Planck Distribution **Planckin jakauma**	*Spectral distribution of a blackbody emission.* Mustan kappaleen säteilyn spektrinen jakauma	
Wien's Displacement Law **Wienin siirtymälaki**	*The maximum wavelength of a blackbody dependence of temperature* Mustan kappaleen säteilyn maksimiaallonpituuden riippuvuus lämpötilasta	$\lambda_{max}T = C_3$ *C_3 = 2898μmK Radiation constant*
The Stefan-Bolzmann Law **Stefan-Bolzmannin laki**	*The total emissive power of a black body is directly proportional to the fourth power of the black body's temperature* Mustan kappaleen lähettämän säteilyn kokonaisteho on suoraan verrannollinen mustan kappaleen lämpötilan neljänteen potenssiin.	*Stefan-Boltzmann-vakio*
Band Emission **Aallonpituus kaistan lähettämä säteily**	*Radiation emission from a blackbody in the spectral band λ_1 to λ_2* Mustan kappaleen lähettämä säteily spektrisellä alueella λ_1:stä λ_2:een.	
Grey Surface **Harmaa pinta**	*Surface's spectral emissivity and absorptivity are independent of wavelength* Pinnan spektrinen emissiviteetti ja absorptiviteetti ovat riippumattomat aallonpituudesta.	
Opaque Surface **Opaakkipinta**	*A surface with no radiation transparency.* Pinta joka ei läpäise säteilyä.	

Diffuse Emitter **Diffuusi säteilijä**	*Intensity of emitted radiation is independent of direction* Lähetetyn säteilyn intensiteetti on riippumaton suunnasta.	$I_{\lambda,e}(\lambda,\theta,\phi) = I_{\lambda,e}(\lambda)$
Emissivity **Emissiviteetti**	*The real surface's ability to emit radiation in comparison to a blackbody at the same temperature.* Todellisen pinnan kyky lähettää säteilyä verrattuna mustaan kappaleeseen samassa lämpötilassa.	
The Total Hemispherical Emissivity **Puoliavaruudellinen kokonaisemissiviteetti**	*Emissivity over all possible directions and wavelengths* Emissiviteetti kaikkiin mahdollisiin suuntiin koko aallonpituusalueella.	
Spectral Emissivity **Spektrinen emissiviteetti**	*Emissivity at the given temperature as a function of wavelength* Emissiviteetti aallonpituuden funktiona annetussa lämpötilassa.	
The Total, Hemispherical Emission Power **Puoliavaruudellinen kokonaissäteily voimakkuus**	*Emission power at all possible wavelengths and in all possible directions.* Emissioteho koko aallonpituusalueella kaikkiin mahdollisiin suuntiin.	
Irradiation **Tulevan säteilyn voimakkuus**	*Represent the rate at which radiation is incident per unit area from all directions and at all wavelengths.* Kuvaa tulevan säteilyn tehoa pinta-alayksikköä kohti kaikista suunnista ja kaikilla aallonpituuksilla.	*G [W/m^2]* 
Radiosity **Lähtevän säteilyn voimakkuus**	*Represent all the radiant energy leaving a surface. Includes the reflected portion of the irradiation and the direct emission of a surface.* Kuvaa pinnalta lähtevää säteilytehoa, joka muodostuu heijastuneesta osuudesta tulevasta säteilystä ja pinnan suorasta säteily tehosta.	*J [W/m^2]* 
Radiation intensity **Säteilyintensiteetti**	*Rate of radiant energy propagation in a particular direction, per unit area normal to the direction, per unit solid angle about the direction* Osuus säteilytehosta tulevaa säteilyä kohtisuoraa olevaa pinta-alayksikköä ja avaruuskulmaa kohti.	*I [W/m^2·sr]*

Absorptivity, Transmissivity and Reflectivity **Absorptiokerroin, läpäisykerroin ja heijastuskerroin**	*Radiation properties describing surface response to fractions of irradiation divided to absorbed, transmitted and reflected radiation in a real surface.* Materiaalin säteilyominaiseeudet, jotka kuvaavat absorboituneen, läpäisseen ja heijastuneen säteilyn osuudet tulevasta säteilystä todellisella pinnalla.	$\alpha_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{abs}}(\lambda)}{G_\lambda(\lambda)}$ $\rho_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{ref}}(\lambda)}{G_\lambda(\lambda)}$ $\tau_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{tr}}(\lambda)}{G_\lambda(\lambda)}$ $\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$
Kirchhoff's Law of Radiation **Kirchhoffin laki**	*Equates the total, hemispherical emissivity of a surface to its total, hemispherical absorptivity.* määrittää puoliavaruudellisen kokonaisemissiviteetin yhtäsuureksi kokonaisabsorptiviteetin kanssa	$\varepsilon = \alpha$ Important derivation for opaque material radiation exchange with large surroundings: $q''_{rad} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma(T_s^4 - T_{sur}^4)$
View Factor **Näkyvyyskerroin**	*The fraction of the radiation leaving surface i that is intercepted by surface j* Osuus pinnalta *i* lähteneestä säteilystä, joka kohtaa pinnan *j*.	*, Reciprocity relation* *, Summation rule in the enclosure* *, Summation rule for surfaces*
Beer's Law[7] **Beerin laki**	*Exponential decay of monochromatic radiation intensity due to the absorption in the medium.* Monokromaattisen säteilyn intensiteetin eksponentiaalinen heikkeneminen väliaineessa tapahtuvan absorption vuoksi.	

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[7] *Not currently included to requirements of Course: Fundamentals of Heat Transfer*

APPENDIX III: THERMAL PROPERTIES

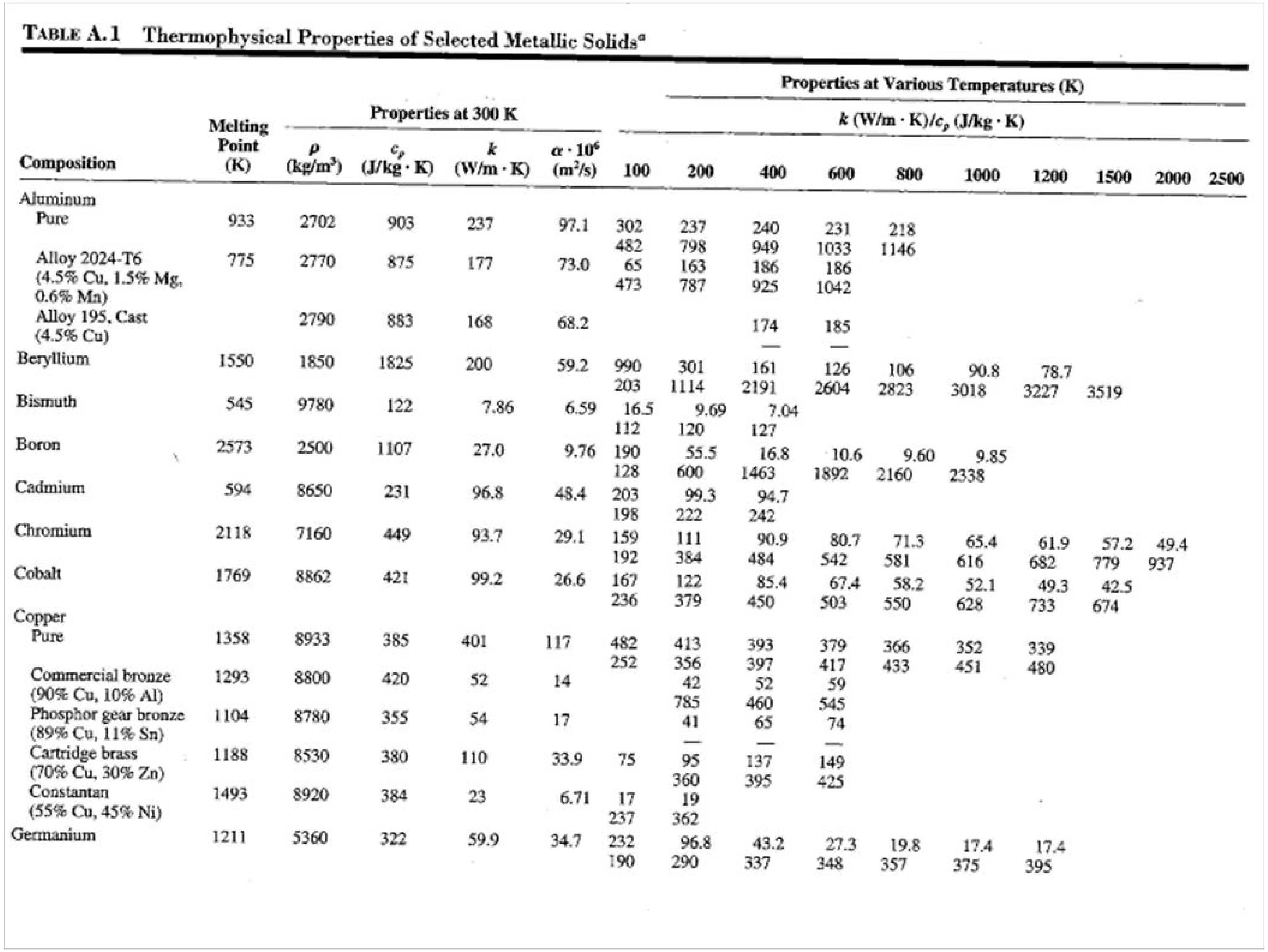


TABLE A.1 Thermophysical Properties of Selected Metallic Solids[a]

		Properties at 300 K				Properties at Various Temperatures (K)									
						k (W/m · K)/c_p (J/kg · K)									
Composition	Melting Point (K)	ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Aluminum															
Pure	933	2702	903	237	97.1	302	237	240	231	218					
						482	798	949	1033	1146					
Alloy 2024-T6 (4.5% Cu, 1.5% Mg, 0.6% Mn)	775	2770	875	177	73.0	65	163	186	186						
						473	787	925	1042						
Alloy 195, Cast (4.5% Cu)		2790	883	168	68.2			174	185						
								—	—						
Beryllium	1550	1850	1825	200	59.2	990	301	161	126	106	90.8	78.7			
						203	1114	2191	2604	2823	3018	3227	3519		
Bismuth	545	9780	122	7.86	6.59	16.5	9.69	7.04							
						112	120	127							
Boron	2573	2500	1107	27.0	9.76	190	55.5	16.8	10.6	9.60	9.85				
						128	600	1463	1892	2160	2338				
Cadmium	594	8650	231	96.8	48.4	203	99.3	94.7							
						198	222	242							
Chromium	2118	7160	449	93.7	29.1	159	111	90.9	80.7	71.3	65.4	61.9	57.2	49.4	
						192	384	484	542	581	616	682	779	937	
Cobalt	1769	8862	421	99.2	26.6	167	122	85.4	67.4	58.2	52.1	49.3	42.5		
						236	379	450	503	550	628	733	674		
Copper															
Pure	1358	8933	385	401	117	482	413	393	379	366	352	339			
						252	356	397	417	433	451	480			
Commercial bronze (90% Cu, 10% Al)	1293	8800	420	52	14		42	52	59						
							785	460	545						
Phosphor gear bronze (89% Cu, 11% Sn)	1104	8780	355	54	17		41	65	74						
							—	—	—						
Cartridge brass (70% Cu, 30% Zn)	1188	8530	380	110	33.9	75	95	137	149						
							360	395	425						
Constantan (55% Cu, 45% Ni)	1493	8920	384	23	6.71	17	19								
						237	362								
Germanium	1211	5360	322	59.9	34.7	232	96.8	43.2	27.3	19.8	17.4	17.4			
						190	290	337	348	357	375	395			

TABLE A.1 *Continued*

Composition	Melting Point (K)	Properties at 300 K				Properties at Various Temperatures (K) k (W/m · K)/c_p (J/kg · K)									
		ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Gold	1336	19300	129	317	127	327 109	323 124	311 131	298 135	284 140	270 145	255 155			
Iridium	2720	22500	130	147	50.3	172 90	153 122	144 133	138 138	132 144	126 153	120 161	111 172		
Iron															
Pure	1810	7870	447	80.2	23.1	134 216	94.0 384	69.5 490	54.7 574	43.3 680	32.8 975	28.3 609	32.1 654		
Armco (99.75% pure)		7870	447	72.7	20.7	95.6 215	80.6 384	65.7 490	53.1 574	42.2 680	32.3 975	28.7 609	31.4 654		
Carbon steels															
Plain carbon (Mn ≤ 1%, Si ≤ 0.1%)		7854	434	60.5	17.7			56.7 487	48.0 559	39.2 685	30.0 1169				
AISI 1010		7832	434	63.9	18.8			58.7 487	48.8 559	39.2 685	31.3 1168				
Carbon–silicon (Mn ≤ 1%, 0.1% < Si ≤ 0.6%)		7817	446	51.9	14.9			49.8 501	44.0 582	37.4 699	29.3 971				
Carbon–manganese–silicon (1% < Mn ≤ 1.65%, 0.1% < Si ≤ 0.6%)		8131	434	41.0	11.6			42.2 487	39.7 559	35.0 685	27.6 1090				
Chromium (low) steels															
½Cr–¼Mo–Si (0.18% C, 0.65% Cr, 0.23% Mo, 0.6% Si)		7822	444	37.7	10.9			38.2 492	36.7 575	33.3 688	26.9 969				
1 Cr–½Mo (0.16% C, 1% Cr, 0.54% Mo, 0.39% Si)		7858	442	42.3	12.2			42.0 492	39.1 575	34.5 688	27.4 969				
1 Cr–V (0.2% C, 1.02% Cr, 0.15% V)		7836	443	48.9	14.1			46.8 492	42.1 575	36.3 688	28.2 969				

Stainless steels															
AISI 302		8055	480	15.1	3.91			17.3 512	20.0 559	22.8 585	25.4 606				
AISI 304	1670	7900	477	14.9	3.95	9.2 272	12.6 402	16.6 515	19.8 557	22.6 582	25.4 611	28.0 640	31.7 682		
AISI 316		8238	468	13.4	3.48			15.2 504	18.3 550	21.3 576	24.2 602				
AISI 347		7978	480	14.2	3.71			15.8 513	18.9 559	21.9 585	24.7 606				
Lead	601	11340	129	35.3	24.1	39.7 118	36.7 125	34.0 132	31.4 142						
Magnesium	923	1740	1024	156	87.6	169 649	159 934	153 1074	149 1170	146 1267					
Molybdenum	2894	10240	251	138	53.7	179 141	143 224	134 261	126 275	118 285	112 295	105 308	98 330	90 380	86 459
Nickel															
Pure	1728	8900	444	90.7	23.0	164 232	107 383	80.2 485	65.6 592	67.6 530	71.8 562	76.2 594	82.6 616		
Nichrome (80% Ni, 20% Cr)	1672	8400	420	12	3.4			14 480	16 525	21 545					
Inconel X-750 (73% Ni, 15% Cr, 6.7% Fe)	1665	8510	439	11.7	3.1	8.7 —	10.3 372	13.5 473	17.0 510	20.5 546	24.0 626	27.6 —	33.0 —		
Niobium	2741	8570	265	53.7	23.6	55.2 188	52.6 249	55.2 274	58.2 283	61.3 292	64.4 301	67.5 310	72.1 324	79.1 347	
Palladium	1827	12020	244	71.8	24.5	76.5 168	71.6 227	73.6 251	79.7 261	86.9 271	94.2 281	102 291	110 307		
Platinum															
Pure	2045	21450	133	71.6	25.1	77.5 100	72.6 125	71.8 136	73.2 141	75.6 146	78.7 152	82.6 157	89.5 165	99.4 179	
Alloy 60Pt–40Rh (60% Pt, 40% Rh)	1800	16630	162	47	17.4			52 —	59 —	65 —	69 —	73 —	76 —		
Rhenium	3453	21100	136	47.9	16.7	58.9 97	51.0 127	46.1 139	44.2 145	44.1 151	44.6 156	45.7 162	47.8 171	51.9 186	
Rhodium	2236	12450	243	150	49.6	186 147	154 220	146 253	136 274	127 293	121 311	116 327	110 349	112 376	
Silicon	1685	2330	712	148	89.2	884 259	264 556	98.9 790	61.9 867	42.2 913	31.2 946	25.7 967	22.7 992		
Silver	1235	10500	235	429	174	444 187	430 225	425 239	412 250	396 262	379 277	361 292			
Tantalum	3269	16600	140	57.5	24.7	59.2 110	57.5 133	57.8 144	58.6 146	59.4 149	60.2 152	61.0 155	62.2 160	64.1 172	65.6 189
Thorium	2023	11700	118	54.0	39.1	59.8 99	54.6 112	54.5 124	55.8 134	56.9 145	56.9 156	58.7 167			
Tin	505	7310	227	66.6	40.1	85.2 188	73.3 215	62.2 243							

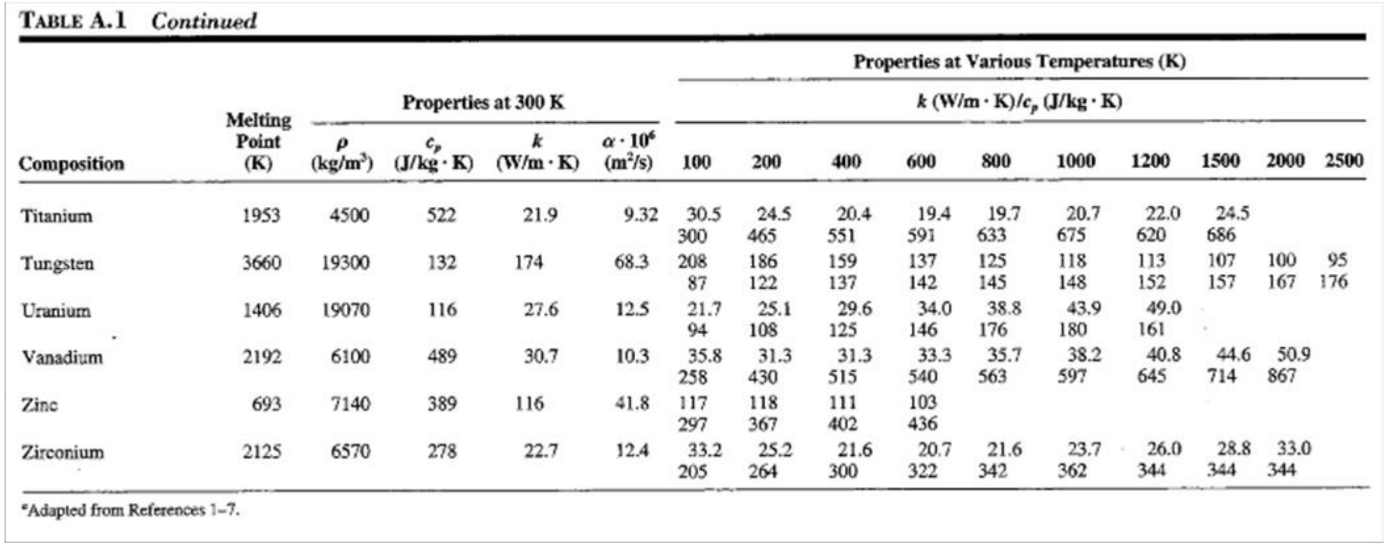


TABLE A.1 *Continued*

Composition	Melting Point (K)	Properties at 300 K				Properties at Various Temperatures (K)									
						k (W/m · K)/c_p (J/kg · K)									
		ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Titanium	1953	4500	522	21.9	9.32	30.5 300	24.5 465	20.4 551	19.4 591	19.7 633	20.7 675	22.0 620	24.5 686		
Tungsten	3660	19300	132	174	68.3	208 87	186 122	159 137	137 142	125 145	118 148	113 152	107 157	100 167	95 176
Uranium	1406	19070	116	27.6	12.5	21.7 94	25.1 108	29.6 125	34.0 146	38.8 176	43.9 180	49.0 161			
Vanadium	2192	6100	489	30.7	10.3	35.8 258	31.3 430	31.3 515	33.3 540	35.7 563	38.2 597	40.8 645	44.6 714	50.9 867	
Zinc	693	7140	389	116	41.8	117 297	118 367	111 402	103 436						
Zirconium	2125	6570	278	22.7	12.4	33.2 205	25.2 264	21.6 300	20.7 322	21.6 342	23.7 362	26.0 344	28.8 344	33.0 344	

[a] Adapted from References 1–7.

TABLE A.2 Thermophysical Properties of Selected Nonmetallic Solids[a]

		Properties at 300 K				Properties at Various Temperatures (K)									
						k (W/m · K)/c_p (J/kg · K)									
Composition	Melting Point (K)	ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Aluminum oxide, sapphire	2323	3970	765	46	15.1	450 —	82 —	32.4 940	18.9 1110	13.0 1180	10.5 1225				
Aluminum oxide, polycrystalline	2323	3970	765	36.0	11.9	133 —	55 —	26.4 940	15.8 1110	10.4 1180	7.85 1225	6.55 —	5.66 —	6.00 —	
Beryllium oxide	2725	3000	1030	272	88.0			196 1350	111 1690	70 1865	47 1975	33 2055	21.5 2145	15 2750	
Boron	2573	2500	1105	27.6	9.99	190 —	52.5 —	18.7 1490	11.3 1880	8.1 2135	6.3 2350	5.2 2555			
Boron fiber epoxy (30% vol) composite	590	2080													
k, ∥ to fibers				2.29		2.10	2.23	2.28							
k, ⊥ to fibers				0.59		0.37	0.49	0.60							
c_p			1122			364	757	1431							
Carbon															
Amorphous	1500	1950	—	1.60	—	0.67 —	1.18 —	1.89 —	2.19 —	2.37 —	2.53 —	2.84 —	3.48 —		
Diamond, type IIa insulator	—	3500	509	2300	—	10,000 21	4000 194	1540 853							
Graphite, pyrolytic	2273	2210													
k, ∥ to layers				1950		4970	3230	1390	892	667	534	448	357	262	
k, ⊥ to layers				5.70		16.8	9.23	4.09	2.68	2.01	1.60	1.34	1.08	0.81	
c_p			709			136	411	992	1406	1650	1793	1890	1974	2043	
Graphite fiber epoxy (25% vol) composite	450	1400													
k, heat flow ∥ to fibers				11.1		5.7	8.7	13.0							
k, heat flow ⊥ to fibers				0.87		0.46	0.68	1.1							
c_p			935			337	642	1216							
Pyroceram, Corning 9606	1623	2600	808	3.98	1.89	5.25 —	4.78 —	3.64 908	3.28 1038	3.08 1122	2.96 1197	2.87 1264	2.79 1498		

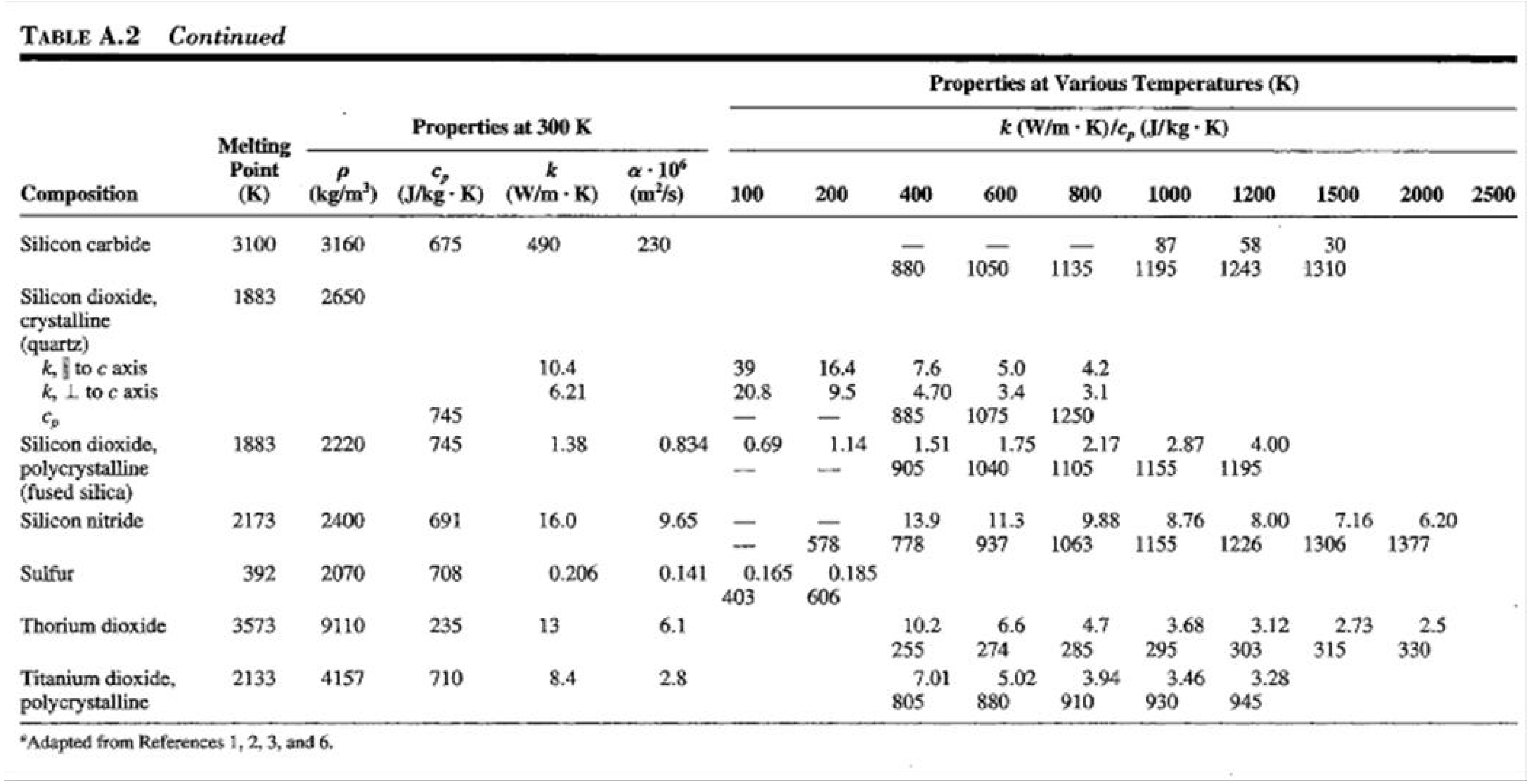


TABLE A.2 *Continued*

Composition	Melting Point (K)	Properties at 300 K				Properties at Various Temperatures (K): k (W/m · K)/c_p (J/kg · K)									
		ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Silicon carbide	3100	3160	675	490	230			— 880	— 1050	— 1135	87 1195	58 1243	30 1310		
Silicon dioxide, crystalline (quartz)	1883	2650													
k, ∥ to c axis				10.4		39	16.4	7.6	5.0	4.2					
k, ⊥ to c axis				6.21		20.8	9.5	4.70	3.4	3.1					
c_p			745			—	—	885	1075	1250					
Silicon dioxide, polycrystalline (fused silica)	1883	2220	745	1.38	0.834	0.69 —	1.14 —	1.51 905	1.75 1040	2.17 1105	2.87 1155	4.00 1195			
Silicon nitride	2173	2400	691	16.0	9.65	— —	— 578	13.9 778	11.3 937	9.88 1063	8.76 1155	8.00 1226	7.16 1306	6.20 1377	
Sulfur	392	2070	708	0.206	0.141	0.165 403	0.185 606								
Thorium dioxide	3573	9110	235	13	6.1			10.2 255	6.6 274	4.7 285	3.68 295	3.12 303	2.73 315	2.5 330	
Titanium dioxide, polycrystalline	2133	4157	710	8.4	2.8			7.01 805	5.02 880	3.94 910	3.46 930	3.28 945			

[a] Adapted from References 1, 2, 3, and 6.

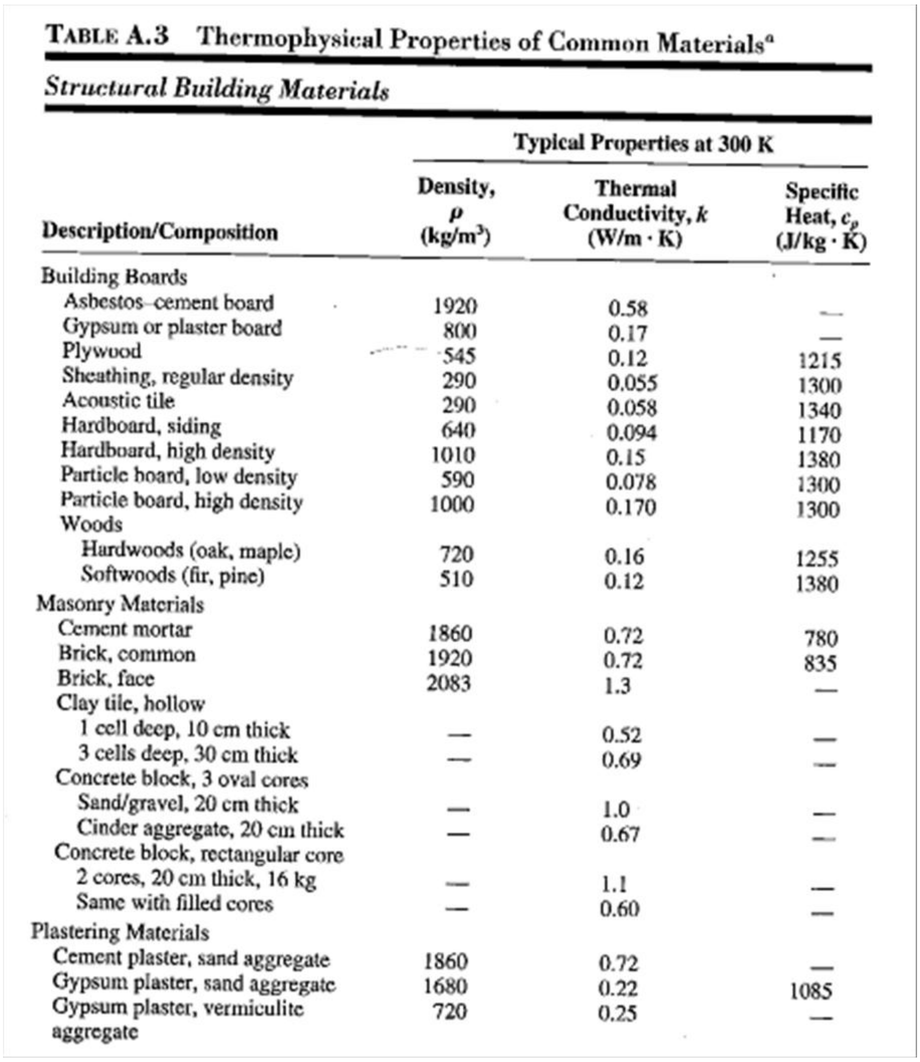


TABLE A.3 Thermophysical Properties of Common Materials[a]

Structural Building Materials

	Typical Properties at 300 K		
Description/Composition	Density, ρ (kg/m^3)	Thermal Conductivity, k (W/m · K)	Specific Heat, c_p (J/kg · K)
Building Boards			
Asbestos–cement board	1920	0.58	—
Gypsum or plaster board	800	0.17	—
Plywood	545	0.12	1215
Sheathing, regular density	290	0.055	1300
Acoustic tile	290	0.058	1340
Hardboard, siding	640	0.094	1170
Hardboard, high density	1010	0.15	1380
Particle board, low density	590	0.078	1300
Particle board, high density	1000	0.170	1300
Woods			
Hardwoods (oak, maple)	720	0.16	1255
Softwoods (fir, pine)	510	0.12	1380
Masonry Materials			
Cement mortar	1860	0.72	780
Brick, common	1920	0.72	835
Brick, face	2083	1.3	—
Clay tile, hollow			
1 cell deep, 10 cm thick	—	0.52	—
3 cells deep, 30 cm thick	—	0.69	—
Concrete block, 3 oval cores			
Sand/gravel, 20 cm thick	—	1.0	—
Cinder aggregate, 20 cm thick	—	0.67	—
Concrete block, rectangular core			
2 cores, 20 cm thick, 16 kg	—	1.1	—
Same with filled cores	—	0.60	—
Plastering Materials			
Cement plaster, sand aggregate	1860	0.72	—
Gypsum plaster, sand aggregate	1680	0.22	1085
Gypsum plaster, vermiculite aggregate	720	0.25	—

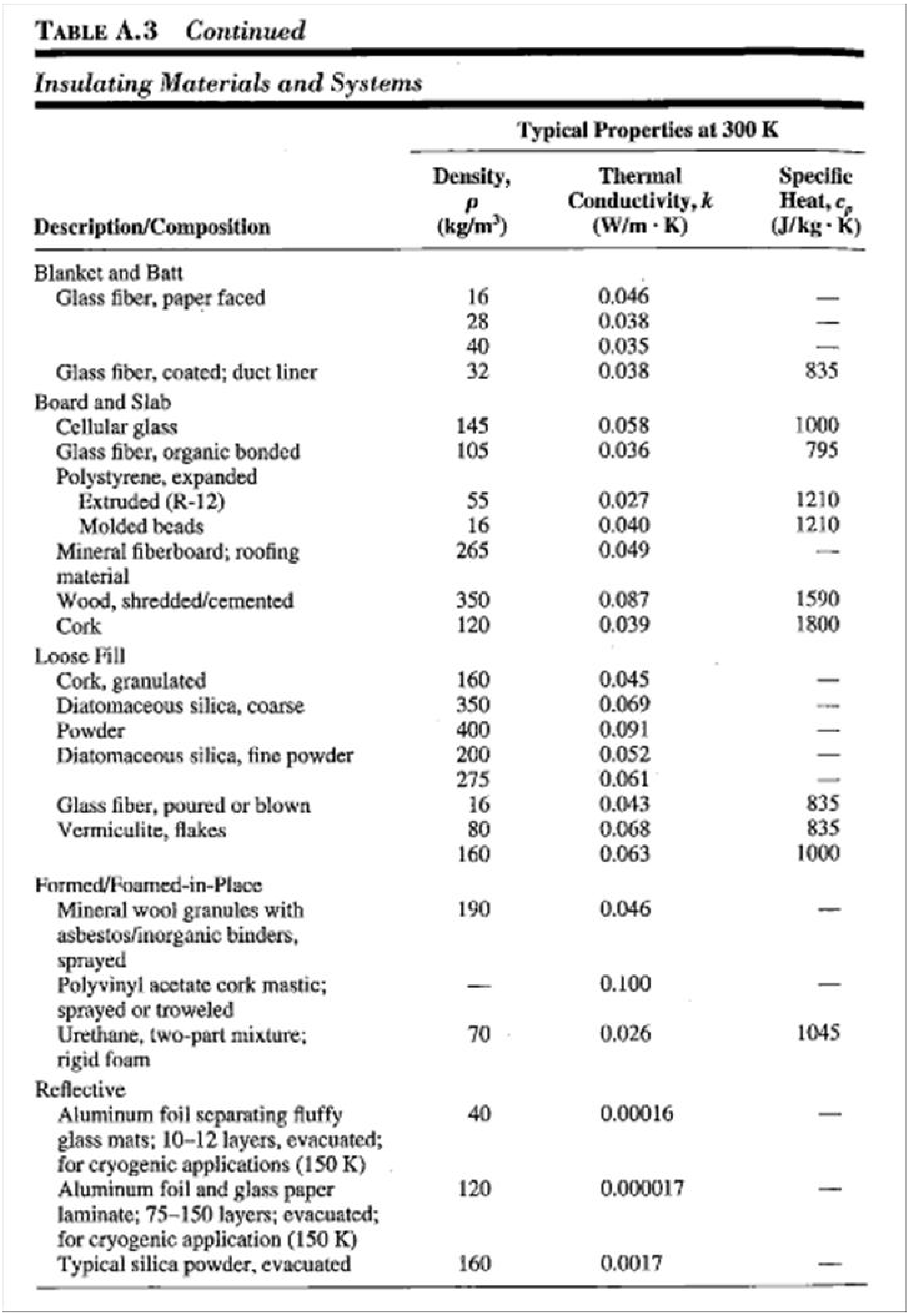


TABLE A.3 *Continued*

Insulating Materials and Systems

	Typical Properties at 300 K		
Description/Composition	**Density, ρ (kg/m^3)**	**Thermal Conductivity, k (W/m · K)**	**Specific Heat, c_p (J/kg · K)**
Blanket and Batt			
Glass fiber, paper faced	16	0.046	—
	28	0.038	—
	40	0.035	—
Glass fiber, coated; duct liner	32	0.038	835
Board and Slab			
Cellular glass	145	0.058	1000
Glass fiber, organic bonded	105	0.036	795
Polystyrene, expanded			
Extruded (R-12)	55	0.027	1210
Molded beads	16	0.040	1210
Mineral fiberboard; roofing material	265	0.049	—
Wood, shredded/cemented	350	0.087	1590
Cork	120	0.039	1800
Loose Fill			
Cork, granulated	160	0.045	—
Diatomaceous silica, coarse	350	0.069	—
Powder	400	0.091	—
Diatomaceous silica, fine powder	200	0.052	—
	275	0.061	—
Glass fiber, poured or blown	16	0.043	835
Vermiculite, flakes	80	0.068	835
	160	0.063	1000
Formed/Foamed-in-Place			
Mineral wool granules with asbestos/inorganic binders, sprayed	190	0.046	—
Polyvinyl acetate cork mastic; sprayed or troweled	—	0.100	—
Urethane, two-part mixture; rigid foam	70	0.026	1045
Reflective			
Aluminum foil separating fluffy glass mats; 10–12 layers, evacuated; for cryogenic applications (150 K)	40	0.00016	—
Aluminum foil and glass paper laminate; 75–150 layers; evacuated; for cryogenic application (150 K)	120	0.000017	—
Typical silica powder, evacuated	160	0.0017	—

TABLE A.3 *Continued*

Industrial Insulation

Description/ Composition	Maximum Service Temperature (K)	Typical Density (kg/m^3)	Typical Thermal Conductivity, k (W/m · K), at Various Temperatures (K)													
			200	215	230	240	255	270	285	300	310	365	420	530	645	750
Blankets																
Blanket, mineral fiber, metal reinforced	920	96–192									0.038	0.046	0.056	0.078		
	815	40–96									0.035	0.045	0.058	0.088		
Blanket, mineral fiber, glass; fine fiber, organic bonded	450	10				0.036	0.038	0.040	0.043	0.048	0.052	0.076				
		12				0.035	0.036	0.039	0.042	0.046	0.049	0.069				
		16				0.033	0.035	0.036	0.039	0.042	0.046	0.062				
		24				0.030	0.032	0.033	0.036	0.039	0.040	0.053				
		32				0.029	0.030	0.032	0.033	0.036	0.038	0.048				
		48				0.027	0.029	0.030	0.032	0.033	0.035	0.045				
Blanket, alumina–silica fiber	1530	48												0.071	0.105	0.150
		64												0.059	0.087	0.125
		96												0.052	0.076	0.100
		128												0.049	0.068	0.091
Felt, semirigid; organic bonded	480	50–125						0.035	0.036	0.038	0.039	0.051	0.063			
	730	50	0.023	0.025	0.026	0.027	0.029	0.030	0.032	0.033	0.035	0.051	0.079			
Felt, laminated; no binder	920	120											0.051	0.065	0.087	
Blocks, Boards, and Pipe Insulations																
Asbestos paper, laminated and corrugated																
4-ply	420	190								0.078	0.082	0.098				
6-ply	420	255								0.071	0.074	0.085				
8-ply	420	300								0.068	0.071	0.082				
Magnesia, 85%	590	185									0.051	0.055	0.061			
Calcium silicate	920	190									0.055	0.059	0.063	0.075	0.089	0.104

TABLE A.3 *Continued*

Industrial Insulation (Continued)

Description/ Composition	Maximum Service Temperature (K)	Typical Density (kg/m³)	Typical Thermal Conductivity, *k* (W/m · K), at Various Temperatures (K)													
			200	215	230	240	255	270	285	300	310	365	420	530	645	750
Cellular glass	700	145			0.046	0.048	0.051	0.052	0.055	0.058	0.062	0.069	0.079			
Diatomaceous	1145	345												0.092	0.098	0.104
silica	1310	385												0.101	0.100	0.115
Polystyrene, rigid																
Extruded (R-12)	350	56	0.023	0.023	0.022	0.023	0.023	0.025	0.026	0.027	0.029					
Extruded (R-12)	350	35	0.023	0.023	0.023	0.025	0.025	0.026	0.027	0.029						
Molded beads	350	16	0.026	0.029	0.030	0.033	0.035	0.036	0.038	0.040						
Rubber, rigid foamed	340	70						0.029	0.030	0.032	0.033					
Insulating Cement																
Mineral fiber (rock, slag or glass)																
With clay binder	1255	430									0.071	0.079	0.088	0.105	0.123	
With hydraulic setting binder	922	560									0.108	0.115	0.123	0.137		
Loose Fill																
Cellulose, wood or paper pulp	—	45							0.038	0.039	0.042					
Perlite, expanded	—	105	0.036	0.039	0.042	0.043	0.046	0.049	0.051	0.053	0.056					
Vermiculite, expanded	—	122			0.056	0.058	0.061	0.063	0.065	0.068	0.071					
		80			0.049	0.051	0.055	0.058	0.061	0.063	0.066					

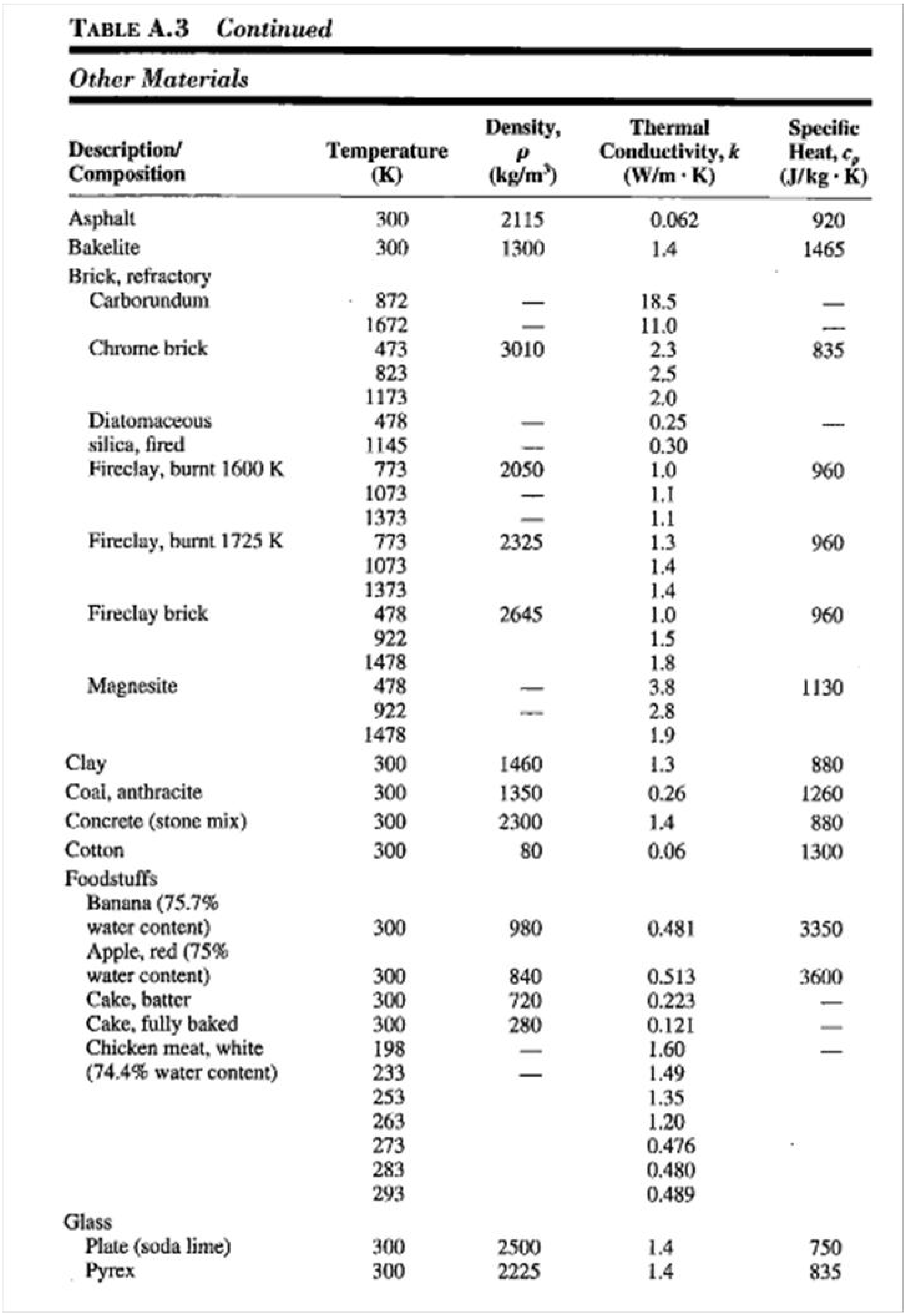


TABLE A.3 ***Continued***

Other Materials

Description/ Composition	Temperature (K)	Density, ρ (kg/m^3)	Thermal Conductivity, k (W/m · K)	Specific Heat, c_p (J/kg · K)
Asphalt	300	2115	0.062	920
Bakelite	300	1300	1.4	1465
Brick, refractory				
Carborundum	872	—	18.5	—
	1672	—	11.0	—
Chrome brick	473	3010	2.3	835
	823		2.5	
	1173		2.0	
Diatomaceous	478	—	0.25	—
silica, fired	1145	—	0.30	
Fireclay, burnt 1600 K	773	2050	1.0	960
	1073	—	1.1	
	1373	—	1.1	
Fireclay, burnt 1725 K	773	2325	1.3	960
	1073		1.4	
	1373		1.4	
Fireclay brick	478	2645	1.0	960
	922		1.5	
	1478		1.8	
Magnesite	478	—	3.8	1130
	922	—	2.8	
	1478		1.9	
Clay	300	1460	1.3	880
Coal, anthracite	300	1350	0.26	1260
Concrete (stone mix)	300	2300	1.4	880
Cotton	300	80	0.06	1300
Foodstuffs				
Banana (75.7% water content)	300	980	0.481	3350
Apple, red (75% water content)	300	840	0.513	3600
Cake, batter	300	720	0.223	—
Cake, fully baked	300	280	0.121	—
Chicken meat, white	198	—	1.60	—
(74.4% water content)	233	—	1.49	
	253		1.35	
	263		1.20	
	273		0.476	
	283		0.480	
	293		0.489	
Glass				
Plate (soda lime)	300	2500	1.4	750
Pyrex	300	2225	1.4	835

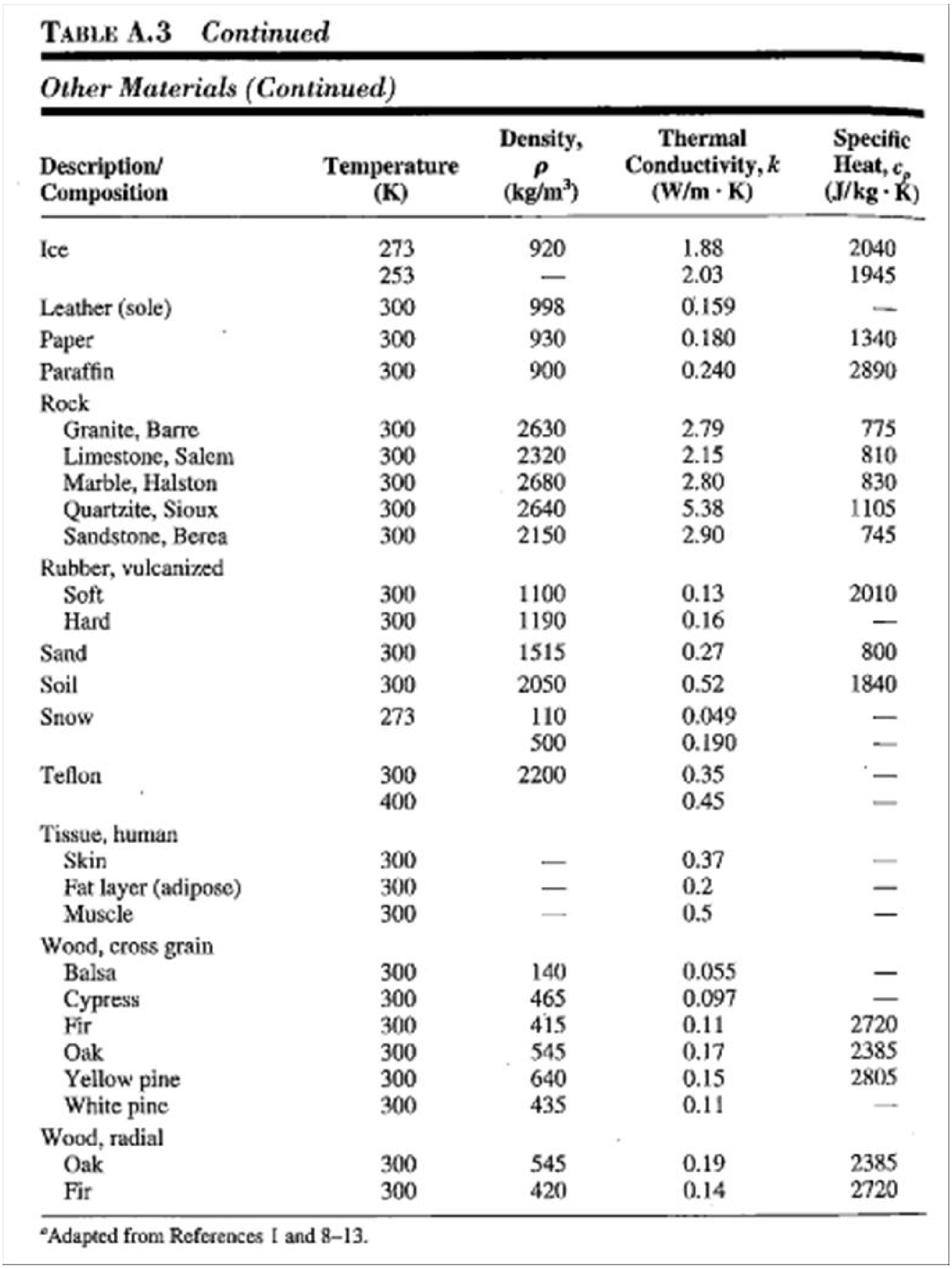


TABLE A.3 *Continued*

Other Materials (Continued)

Description/ Composition	Temperature (K)	Density, ρ (kg/m^3)	Thermal Conductivity, k (W/m · K)	Specific Heat, c_p (J/kg · K)
Ice	273	920	1.88	2040
	253	—	2.03	1945
Leather (sole)	300	998	0.159	—
Paper	300	930	0.180	1340
Paraffin	300	900	0.240	2890
Rock				
Granite, Barre	300	2630	2.79	775
Limestone, Salem	300	2320	2.15	810
Marble, Halston	300	2680	2.80	830
Quartzite, Sioux	300	2640	5.38	1105
Sandstone, Berea	300	2150	2.90	745
Rubber, vulcanized				
Soft	300	1100	0.13	2010
Hard	300	1190	0.16	—
Sand	300	1515	0.27	800
Soil	300	2050	0.52	1840
Snow	273	110	0.049	—
		500	0.190	—
Teflon	300	2200	0.35	—
	400		0.45	—
Tissue, human				
Skin	300	—	0.37	—
Fat layer (adipose)	300	—	0.2	—
Muscle	300	—	0.5	—
Wood, cross grain				
Balsa	300	140	0.055	—
Cypress	300	465	0.097	—
Fir	300	415	0.11	2720
Oak	300	545	0.17	2385
Yellow pine	300	640	0.15	2805
White pine	300	435	0.11	—
Wood, radial				
Oak	300	545	0.19	2385
Fir	300	420	0.14	2720

[a] Adapted from References 1 and 8–13.

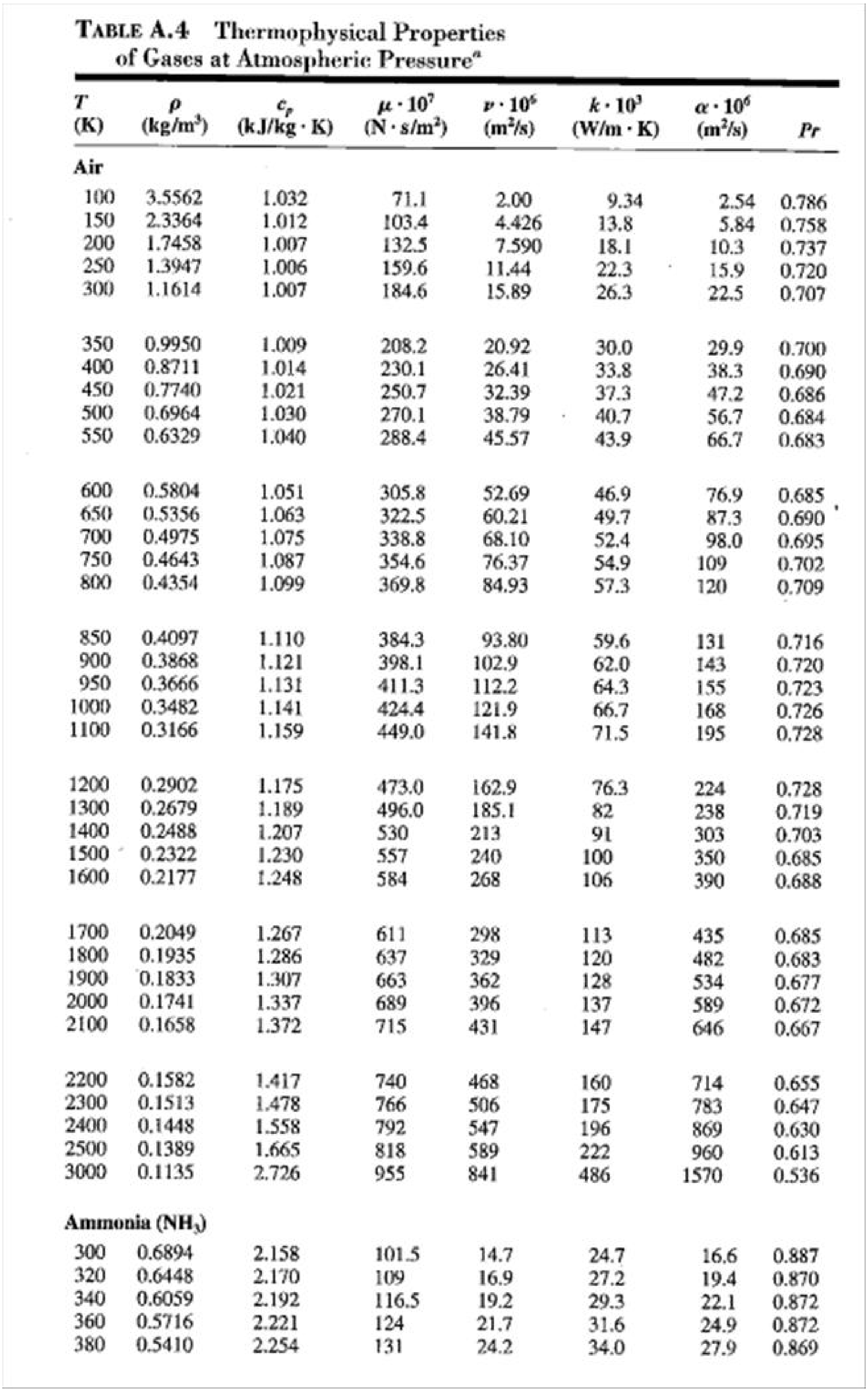


TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709
850	0.4097	1.110	384.3	93.80	59.6	131	0.716
900	0.3868	1.121	398.1	102.9	62.0	143	0.720
950	0.3666	1.131	411.3	112.2	64.3	155	0.723
1000	0.3482	1.141	424.4	121.9	66.7	168	0.726
1100	0.3166	1.159	449.0	141.8	71.5	195	0.728
1200	0.2902	1.175	473.0	162.9	76.3	224	0.728
1300	0.2679	1.189	496.0	185.1	82	238	0.719
1400	0.2488	1.207	530	213	91	303	0.703
1500	0.2322	1.230	557	240	100	350	0.685
1600	0.2177	1.248	584	268	106	390	0.688
1700	0.2049	1.267	611	298	113	435	0.685
1800	0.1935	1.286	637	329	120	482	0.683
1900	0.1833	1.307	663	362	128	534	0.677
2000	0.1741	1.337	689	396	137	589	0.672
2100	0.1658	1.372	715	431	147	646	0.667
2200	0.1582	1.417	740	468	160	714	0.655
2300	0.1513	1.478	766	506	175	783	0.647
2400	0.1448	1.558	792	547	196	869	0.630
2500	0.1389	1.665	818	589	222	960	0.613
3000	0.1135	2.726	955	841	486	1570	0.536
Ammonia (NH_3)							
300	0.6894	2.158	101.5	14.7	24.7	16.6	0.887
320	0.6448	2.170	109	16.9	27.2	19.4	0.870
340	0.6059	2.192	116.5	19.2	29.3	22.1	0.872
360	0.5716	2.221	124	21.7	31.6	24.9	0.872
380	0.5410	2.254	131	24.2	34.0	27.9	0.869

TABLE A.4 *Continued*

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Ammonia (NH_3) *(continued)*							
400	0.5136	2.287	138	26.9	37.0	31.5	0.853
420	0.4888	2.322	145	29.7	40.4	35.6	0.833
440	0.4664	2.357	152.5	32.7	43.5	39.6	0.826
460	0.4460	2.393	159	35.7	46.3	43.4	0.822
480	0.4273	2.430	166.5	39.0	49.2	47.4	0.822
500	0.4101	2.467	173	42.2	52.5	51.9	0.813
520	0.3942	2.504	180	45.7	54.5	55.2	0.827
540	0.3795	2.540	186.5	49.1	57.5	59.7	0.824
560	0.3708	2.577	193	52.0	60.6	63.4	0.827
580	0.3533	2.613	199.5	56.5	63.8	69.1	0.817
Carbon Dioxide (CO_2)							
280	1.9022	0.830	140	7.36	15.20	9.63	0.765
300	1.7730	0.851	149	8.40	16.55	11.0	0.766
320	1.6609	0.872	156	9.39	18.05	12.5	0.754
340	1.5618	0.891	165	10.6	19.70	14.2	0.746
360	1.4743	0.908	173	11.7	21.2	15.8	0.741
380	1.3961	0.926	181	13.0	22.75	17.6	0.737
400	1.3257	0.942	190	14.3	24.3	19.5	0.737
450	1.1782	0.981	210	17.8	28.3	24.5	0.728
500	1.0594	1.02	231	21.8	32.5	30.1	0.725
550	0.9625	1.05	251	26.1	36.6	36.2	0.721
600	0.8826	1.08	270	30.6	40.7	42.7	0.717
650	0.8143	1.10	288	35.4	44.5	49.7	0.712
700	0.7564	1.13	305	40.3	48.1	56.3	0.717
750	0.7057	1.15	321	45.5	51.7	63.7	0.714
800	0.6614	1.17	337	51.0	55.1	71.2	0.716
Carbon Monoxide (CO)							
200	1.6888	1.045	127	7.52	17.0	9.63	0.781
220	1.5341	1.044	137	8.93	19.0	11.9	0.753
240	1.4055	1.043	147	10.5	20.6	14.1	0.744
260	1.2967	1.043	157	12.1	22.1	16.3	0.741
280	1.2038	1.042	166	13.8	23.6	18.8	0.733
300	1.1233	1.043	175	15.6	25.0	21.3	0.730
320	1.0529	1.043	184	17.5	26.3	23.9	0.730
340	0.9909	1.044	193	19.5	27.8	26.9	0.725
360	0.9357	1.045	202	21.6	29.1	29.8	0.725
380	0.8864	1.047	210	23.7	30.5	32.9	0.729
400	0.8421	1.049	218	25.9	31.8	36.0	0.719
450	0.7483	1.055	237	31.7	35.0	44.3	0.714
500	0.67352	1.065	254	37.7	38.1	53.1	0.710
550	0.61226	1.076	271	44.3	41.1	62.4	0.710
600	0.56126	1.088	286	51.0	44.0	72.1	0.707

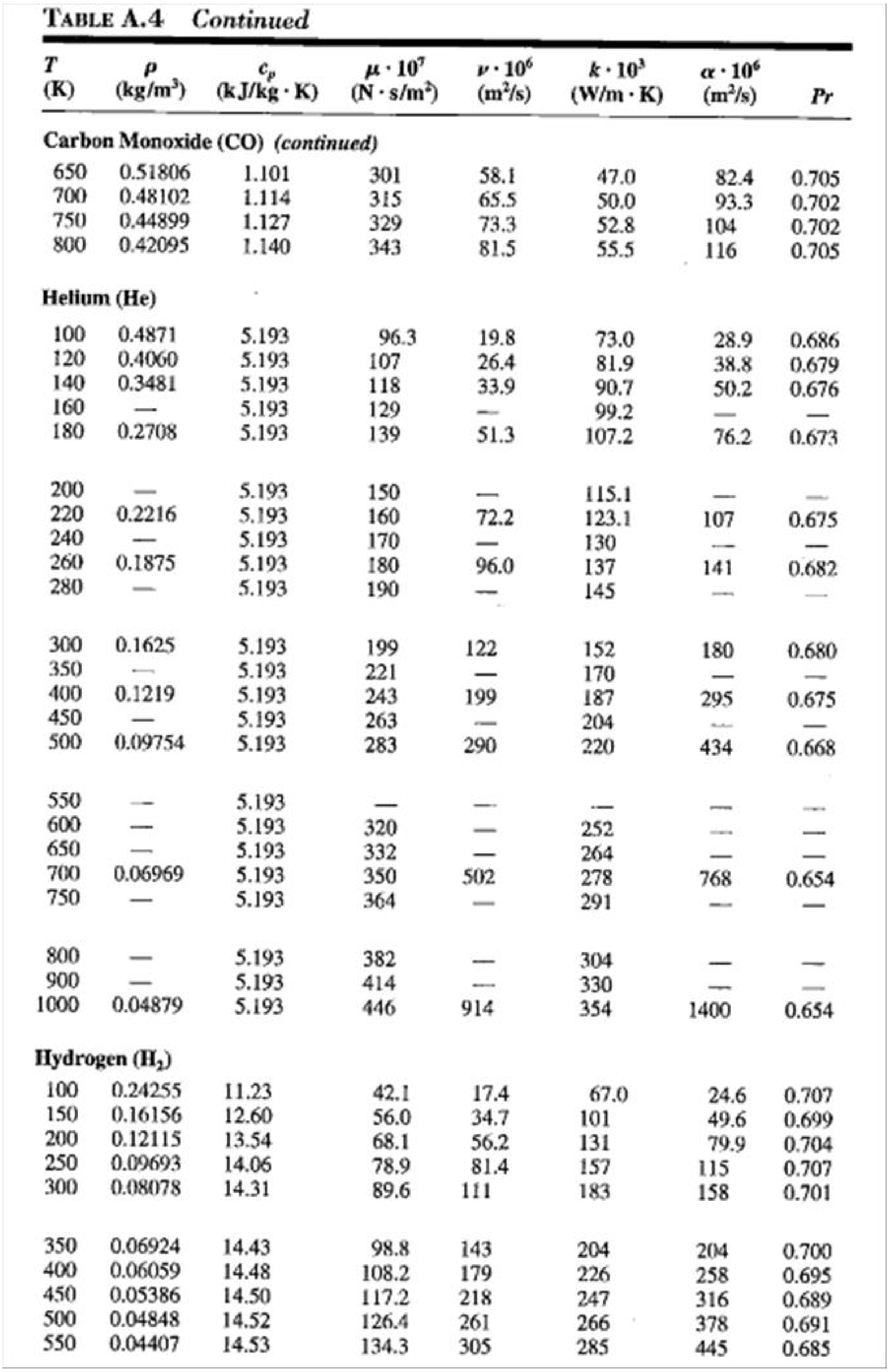


TABLE A.4 *Continued*

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Carbon Monoxide (CO) *(continued)*							
650	0.51806	1.101	301	58.1	47.0	82.4	0.705
700	0.48102	1.114	315	65.5	50.0	93.3	0.702
750	0.44899	1.127	329	73.3	52.8	104	0.702
800	0.42095	1.140	343	81.5	55.5	116	0.705
Helium (He)							
100	0.4871	5.193	96.3	19.8	73.0	28.9	0.686
120	0.4060	5.193	107	26.4	81.9	38.8	0.679
140	0.3481	5.193	118	33.9	90.7	50.2	0.676
160	—	5.193	129	—	99.2	—	—
180	0.2708	5.193	139	51.3	107.2	76.2	0.673
200	—	5.193	150	—	115.1	—	—
220	0.2216	5.193	160	72.2	123.1	107	0.675
240	—	5.193	170	—	130	—	—
260	0.1875	5.193	180	96.0	137	141	0.682
280	—	5.193	190	—	145	—	—
300	0.1625	5.193	199	122	152	180	0.680
350	—	5.193	221	—	170	—	—
400	0.1219	5.193	243	199	187	295	0.675
450	—	5.193	263	—	204	—	—
500	0.09754	5.193	283	290	220	434	0.668
550	—	5.193	—	—	—	—	—
600	—	5.193	320	—	252	—	—
650	—	5.193	332	—	264	—	—
700	0.06969	5.193	350	502	278	768	0.654
750	—	5.193	364	—	291	—	—
800	—	5.193	382	—	304	—	—
900	—	5.193	414	—	330	—	—
1000	0.04879	5.193	446	914	354	1400	0.654
Hydrogen (H_2)							
100	0.24255	11.23	42.1	17.4	67.0	24.6	0.707
150	0.16156	12.60	56.0	34.7	101	49.6	0.699
200	0.12115	13.54	68.1	56.2	131	79.9	0.704
250	0.09693	14.06	78.9	81.4	157	115	0.707
300	0.08078	14.31	89.6	111	183	158	0.701
350	0.06924	14.43	98.8	143	204	204	0.700
400	0.06059	14.48	108.2	179	226	258	0.695
450	0.05386	14.50	117.2	218	247	316	0.689
500	0.04848	14.52	126.4	261	266	378	0.691
550	0.04407	14.53	134.3	305	285	445	0.685

TABLE A.4 *Continued*

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Hydrogen (H_2) *(continued)*							
600	0.04040	14.55	142.4	352	305	519	0.678
700	0.03463	14.61	157.8	456	342	676	0.675
800	0.03030	14.70	172.4	569	378	849	0.670
900	0.02694	14.83	186.5	692	412	1030	0.671
1000	0.02424	14.99	201.3	830	448	1230	0.673
1100	0.02204	15.17	213.0	966	488	1460	0.662
1200	0.02020	15.37	226.2	1120	528	1700	0.659
1300	0.01865	15.59	238.5	1279	568	1955	0.655
1400	0.01732	15.81	250.7	1447	610	2230	0.650
1500	0.01616	16.02	262.7	1626	655	2530	0.643
1600	0.0152	16.28	273.7	1801	697	2815	0.639
1700	0.0143	16.58	284.9	1992	742	3130	0.637
1800	0.0135	16.96	296.1	2193	786	3435	0.639
1900	0.0128	17.49	307.2	2400	835	3730	0.643
2000	0.0121	18.25	318.2	2630	878	3975	0.661
Nitrogen (N_2)							
100	3.4388	1.070	68.8	2.00	9.58	2.60	0.768
150	2.2594	1.050	100.6	4.45	13.9	5.86	0.759
200	1.6883	1.043	129.2	7.65	18.3	10.4	0.736
250	1.3488	1.042	154.9	11.48	22.2	15.8	0.727
300	1.1233	1.041	178.2	15.86	25.9	22.1	0.716
350	0.9625	1.042	200.0	20.78	29.3	29.2	0.711
400	0.8425	1.045	220.4	26.16	32.7	37.1	0.704
450	0.7485	1.050	239.6	32.01	35.8	45.6	0.703
500	0.6739	1.056	257.7	38.24	38.9	54.7	0.700
550	0.6124	1.065	274.7	44.86	41.7	63.9	0.702
600	0.5615	1.075	290.8	51.79	44.6	73.9	0.701
700	0.4812	1.098	321.0	66.71	49.9	94.4	0.706
800	0.4211	1.22	349.1	82.90	54.8	116	0.715
900	0.3743	1.146	375.3	100.3	59.7	139	0.721
1000	0.3368	1.167	399.9	118.7	64.7	165	0.721
1100	0.3062	1.187	423.2	138.2	70.0	193	0.718
1200	0.2807	1.204	445.3	158.6	75.8	224	0.707
1300	0.2591	1.219	466.2	179.9	81.0	256	0.701
Oxygen (O_2)							
100	3.945	0.962	76.4	1.94	9.25	2.44	0.796
150	2.585	0.921	114.8	4.44	13.8	5.80	0.766
200	1.930	0.915	147.5	7.64	18.3	10.4	0.737
250	1.542	0.915	178.6	11.58	22.6	16.0	0.723
300	1.284	0.920	207.2	16.14	26.8	22.7	0.711

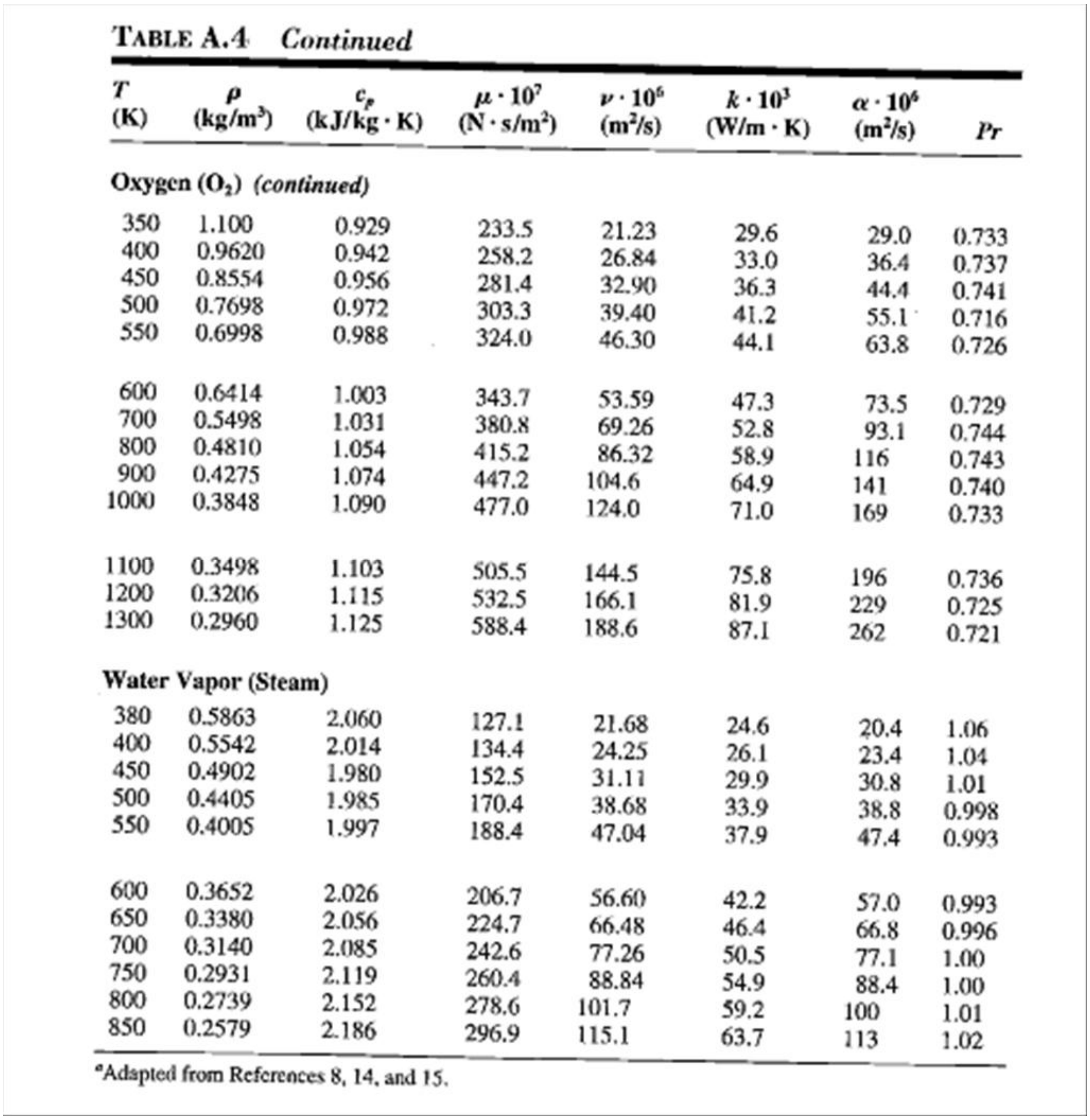


TABLE A.4 *Continued*

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Oxygen (O_2) *(continued)*							
350	1.100	0.929	233.5	21.23	29.6	29.0	0.733
400	0.9620	0.942	258.2	26.84	33.0	36.4	0.737
450	0.8554	0.956	281.4	32.90	36.3	44.4	0.741
500	0.7698	0.972	303.3	39.40	41.2	55.1	0.716
550	0.6998	0.988	324.0	46.30	44.1	63.8	0.726
600	0.6414	1.003	343.7	53.59	47.3	73.5	0.729
700	0.5498	1.031	380.8	69.26	52.8	93.1	0.744
800	0.4810	1.054	415.2	86.32	58.9	116	0.743
900	0.4275	1.074	447.2	104.6	64.9	141	0.740
1000	0.3848	1.090	477.0	124.0	71.0	169	0.733
1100	0.3498	1.103	505.5	144.5	75.8	196	0.736
1200	0.3206	1.115	532.5	166.1	81.9	229	0.725
1300	0.2960	1.125	588.4	188.6	87.1	262	0.721
Water Vapor (Steam)							
380	0.5863	2.060	127.1	21.68	24.6	20.4	1.06
400	0.5542	2.014	134.4	24.25	26.1	23.4	1.04
450	0.4902	1.980	152.5	31.11	29.9	30.8	1.01
500	0.4405	1.985	170.4	38.68	33.9	38.8	0.998
550	0.4005	1.997	188.4	47.04	37.9	47.4	0.993
600	0.3652	2.026	206.7	56.60	42.2	57.0	0.993
650	0.3380	2.056	224.7	66.48	46.4	66.8	0.996
700	0.3140	2.085	242.6	77.26	50.5	77.1	1.00
750	0.2931	2.119	260.4	88.84	54.9	88.4	1.00
800	0.2739	2.152	278.6	101.7	59.2	100	1.01
850	0.2579	2.186	296.9	115.1	63.7	113	1.02

[a]Adapted from References 8, 14, and 15.

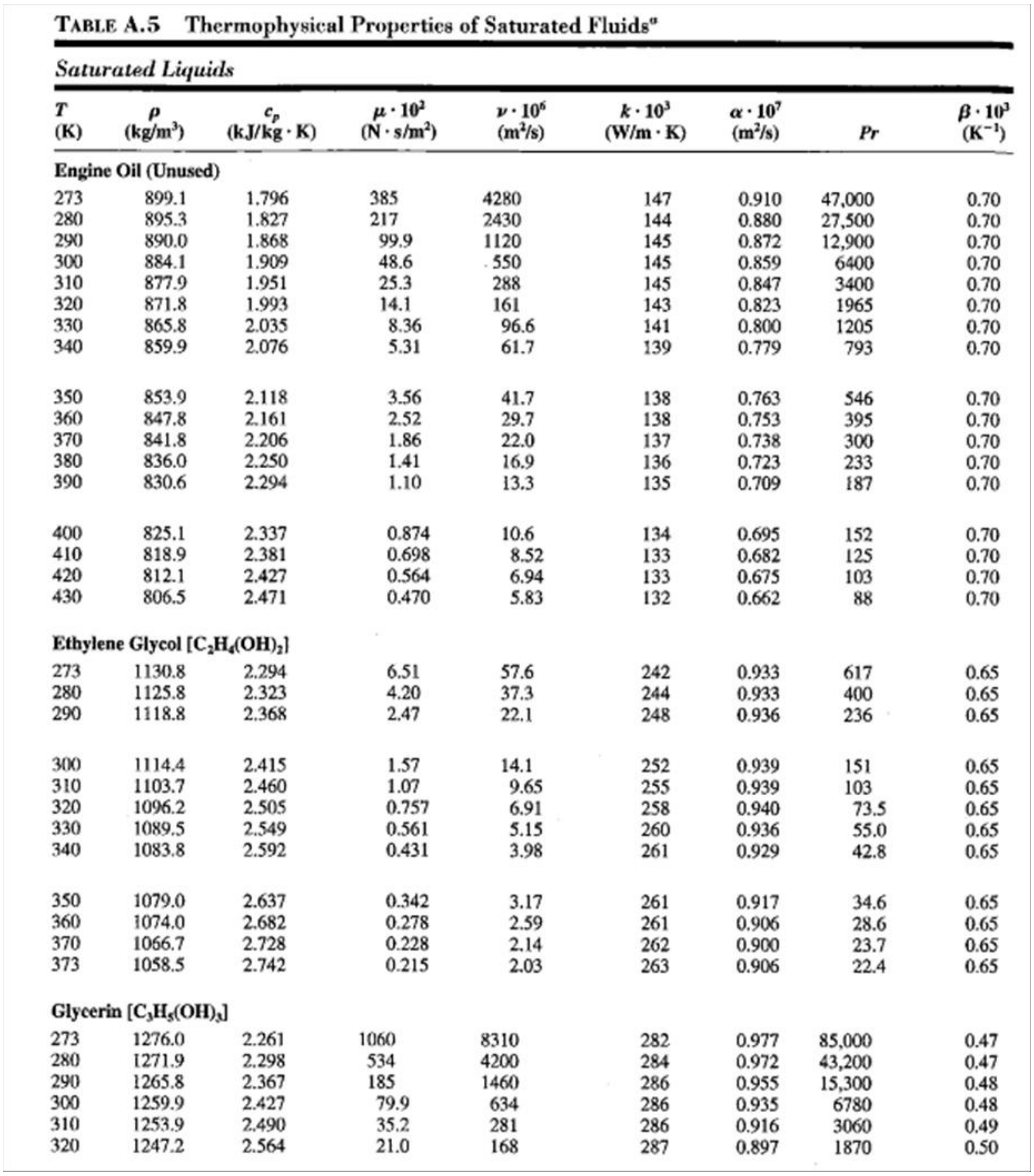


TABLE A.5 Thermophysical Properties of Saturated Fluids[a]

Saturated Liquids

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^2$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^7$ (m^2/s)	Pr	$\beta \cdot 10^3$ (K^{-1})
Engine Oil (Unused)								
273	899.1	1.796	385	4280	147	0.910	47,000	0.70
280	895.3	1.827	217	2430	144	0.880	27,500	0.70
290	890.0	1.868	99.9	1120	145	0.872	12,900	0.70
300	884.1	1.909	48.6	550	145	0.859	6400	0.70
310	877.9	1.951	25.3	288	145	0.847	3400	0.70
320	871.8	1.993	14.1	161	143	0.823	1965	0.70
330	865.8	2.035	8.36	96.6	141	0.800	1205	0.70
340	859.9	2.076	5.31	61.7	139	0.779	793	0.70
350	853.9	2.118	3.56	41.7	138	0.763	546	0.70
360	847.8	2.161	2.52	29.7	138	0.753	395	0.70
370	841.8	2.206	1.86	22.0	137	0.738	300	0.70
380	836.0	2.250	1.41	16.9	136	0.723	233	0.70
390	830.6	2.294	1.10	13.3	135	0.709	187	0.70
400	825.1	2.337	0.874	10.6	134	0.695	152	0.70
410	818.9	2.381	0.698	8.52	133	0.682	125	0.70
420	812.1	2.427	0.564	6.94	133	0.675	103	0.70
430	806.5	2.471	0.470	5.83	132	0.662	88	0.70
Ethylene Glycol [$C_2H_4(OH)_2$]								
273	1130.8	2.294	6.51	57.6	242	0.933	617	0.65
280	1125.8	2.323	4.20	37.3	244	0.933	400	0.65
290	1118.8	2.368	2.47	22.1	248	0.936	236	0.65
300	1114.4	2.415	1.57	14.1	252	0.939	151	0.65
310	1103.7	2.460	1.07	9.65	255	0.939	103	0.65
320	1096.2	2.505	0.757	6.91	258	0.940	73.5	0.65
330	1089.5	2.549	0.561	5.15	260	0.936	55.0	0.65
340	1083.8	2.592	0.431	3.98	261	0.929	42.8	0.65
350	1079.0	2.637	0.342	3.17	261	0.917	34.6	0.65
360	1074.0	2.682	0.278	2.59	261	0.906	28.6	0.65
370	1066.7	2.728	0.228	2.14	262	0.900	23.7	0.65
373	1058.5	2.742	0.215	2.03	263	0.906	22.4	0.65
Glycerin [$C_3H_5(OH)_3$]								
273	1276.0	2.261	1060	8310	282	0.977	85,000	0.47
280	1271.9	2.298	534	4200	284	0.972	43,200	0.47
290	1265.8	2.367	185	1460	286	0.955	15,300	0.48
300	1259.9	2.427	79.9	634	286	0.935	6780	0.48
310	1253.9	2.490	35.2	281	286	0.916	3060	0.49
320	1247.2	2.564	21.0	168	287	0.897	1870	0.50

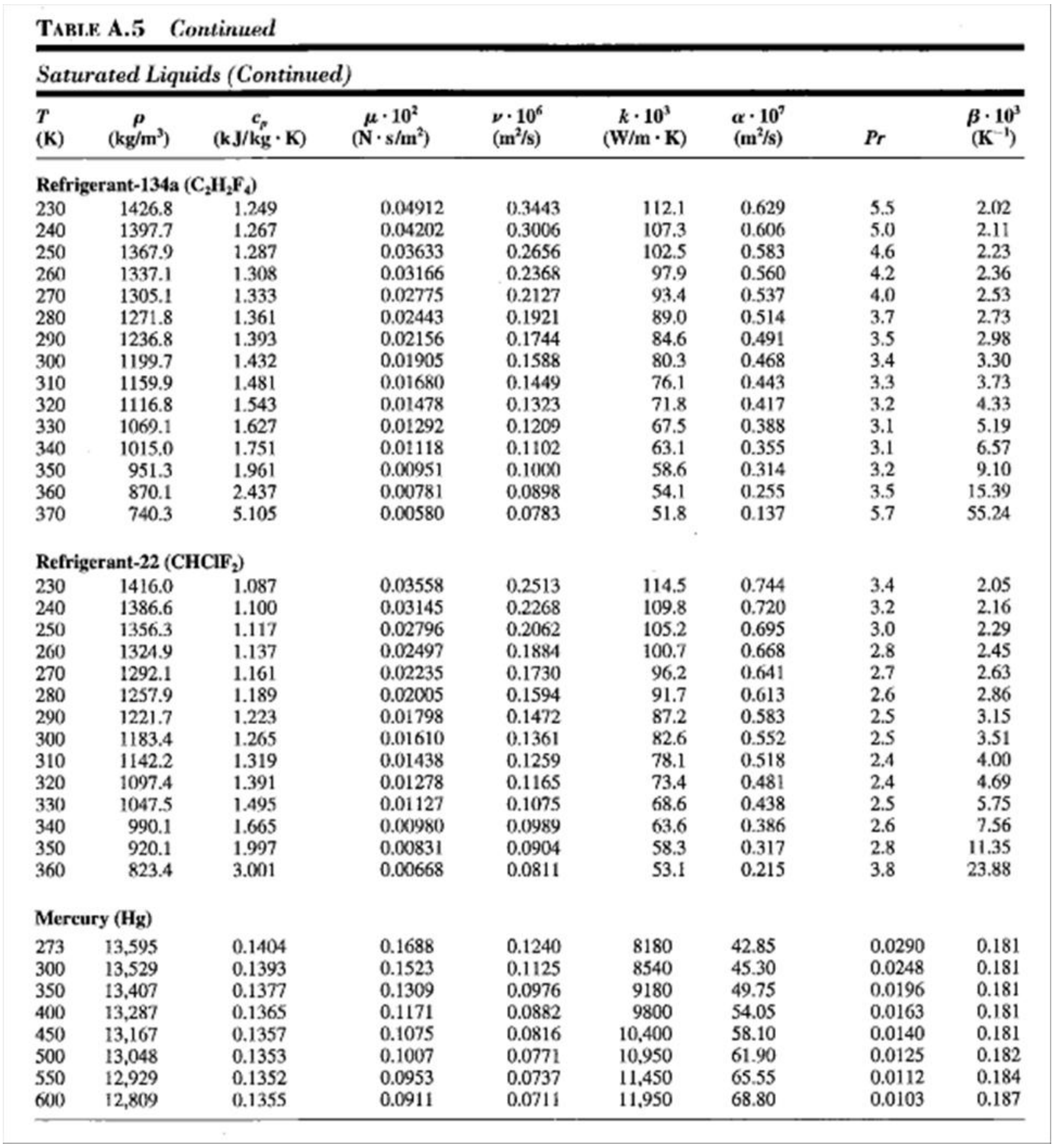


TABLE A.5 *Continued*

Saturated Liquids (Continued)

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^2$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^7$ (m^2/s)	Pr	$\beta \cdot 10^3$ (K^{-1})
Refrigerant-134a ($C_2H_2F_4$)								
230	1426.8	1.249	0.04912	0.3443	112.1	0.629	5.5	2.02
240	1397.7	1.267	0.04202	0.3006	107.3	0.606	5.0	2.11
250	1367.9	1.287	0.03633	0.2656	102.5	0.583	4.6	2.23
260	1337.1	1.308	0.03166	0.2368	97.9	0.560	4.2	2.36
270	1305.1	1.333	0.02775	0.2127	93.4	0.537	4.0	2.53
280	1271.8	1.361	0.02443	0.1921	89.0	0.514	3.7	2.73
290	1236.8	1.393	0.02156	0.1744	84.6	0.491	3.5	2.98
300	1199.7	1.432	0.01905	0.1588	80.3	0.468	3.4	3.30
310	1159.9	1.481	0.01680	0.1449	76.1	0.443	3.3	3.73
320	1116.8	1.543	0.01478	0.1323	71.8	0.417	3.2	4.33
330	1069.1	1.627	0.01292	0.1209	67.5	0.388	3.1	5.19
340	1015.0	1.751	0.01118	0.1102	63.1	0.355	3.1	6.57
350	951.3	1.961	0.00951	0.1000	58.6	0.314	3.2	9.10
360	870.1	2.437	0.00781	0.0898	54.1	0.255	3.5	15.39
370	740.3	5.105	0.00580	0.0783	51.8	0.137	5.7	55.24
Refrigerant-22 ($CHClF_2$)								
230	1416.0	1.087	0.03558	0.2513	114.5	0.744	3.4	2.05
240	1386.6	1.100	0.03145	0.2268	109.8	0.720	3.2	2.16
250	1356.3	1.117	0.02796	0.2062	105.2	0.695	3.0	2.29
260	1324.9	1.137	0.02497	0.1884	100.7	0.668	2.8	2.45
270	1292.1	1.161	0.02235	0.1730	96.2	0.641	2.7	2.63
280	1257.9	1.189	0.02005	0.1594	91.7	0.613	2.6	2.86
290	1221.7	1.223	0.01798	0.1472	87.2	0.583	2.5	3.15
300	1183.4	1.265	0.01610	0.1361	82.6	0.552	2.5	3.51
310	1142.2	1.319	0.01438	0.1259	78.1	0.518	2.4	4.00
320	1097.4	1.391	0.01278	0.1165	73.4	0.481	2.4	4.69
330	1047.5	1.495	0.01127	0.1075	68.6	0.438	2.5	5.75
340	990.1	1.665	0.00980	0.0989	63.6	0.386	2.6	7.56
350	920.1	1.997	0.00831	0.0904	58.3	0.317	2.8	11.35
360	823.4	3.001	0.00668	0.0811	53.1	0.215	3.8	23.88
Mercury (Hg)								
273	13,595	0.1404	0.1688	0.1240	8180	42.85	0.0290	0.181
300	13,529	0.1393	0.1523	0.1125	8540	45.30	0.0248	0.181
350	13,407	0.1377	0.1309	0.0976	9180	49.75	0.0196	0.181
400	13,287	0.1365	0.1171	0.0882	9800	54.05	0.0163	0.181
450	13,167	0.1357	0.1075	0.0816	10,400	58.10	0.0140	0.181
500	13,048	0.1353	0.1007	0.0771	10,950	61.90	0.0125	0.182
550	12,929	0.1352	0.0953	0.0737	11,450	65.55	0.0112	0.184
600	12,809	0.1355	0.0911	0.0711	11,950	68.80	0.0103	0.187

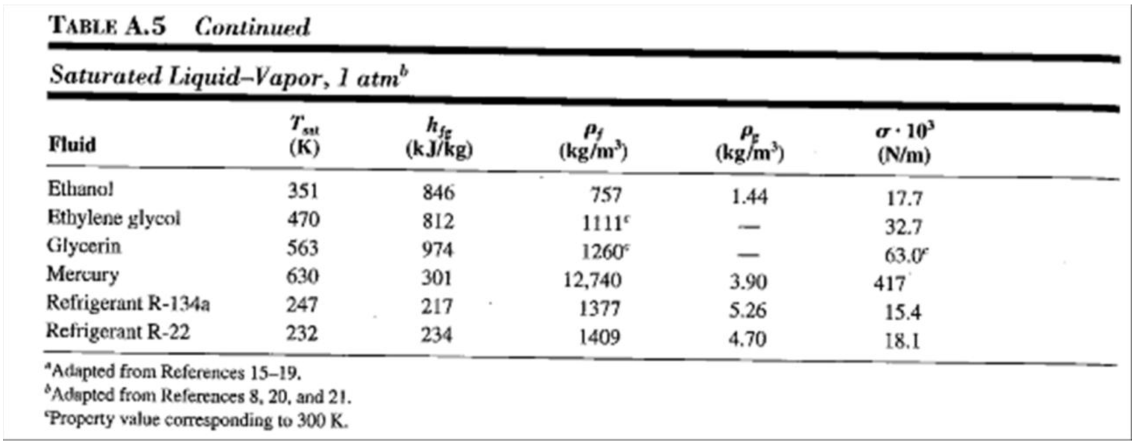


TABLE A.5 *Continued*

Saturated Liquid–Vapor, 1 atm[b]

Fluid	T_{sat} (K)	h_{fg} (kJ/kg)	ρ_f (kg/m^3)	ρ_g (kg/m^3)	$\sigma \cdot 10^3$ (N/m)
Ethanol	351	846	757	1.44	17.7
Ethylene glycol	470	812	1111[c]	—	32.7
Glycerin	563	974	1260[c]	—	63.0[c]
Mercury	630	301	12,740	3.90	417
Refrigerant R-134a	247	217	1377	5.26	15.4
Refrigerant R-22	232	234	1409	4.70	18.1

[a] Adapted from References 15–19.

[b] Adapted from References 8, 20, and 21.

[c] Property value corresponding to 300 K.

Table A.6 Thermophysical Properties of Saturated Water^a

Temperature, T (K)	Pressure, p (bars)^b	Specific Volume (m^3/kg)		Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K)		Viscosity (N · s/m^2)		Thermal Conductivity (W/m · K)		Prandtl Number		Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K^{-1})	Temperature, T (K)
		$v_f \cdot 10^3$	v_g		$c_{p,f}$	$c_{p,g}$	$\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	$k_f \cdot 10^3$	$k_g \cdot 10^3$	Pr_f	Pr_g			
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05	273.15
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74	275
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04	280
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1	285
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0	290
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5	295
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1	300
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6	305
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70.0	361.9	310
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4	315
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894	68.3	436.7	320
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2	325
330	0.1719	1.016	8.82	2366	4.184	1.911	489	10.29	650	21.7	3.15	0.908	66.6	504.0	330
335	0.2167	1.018	7.09	2354	4.186	1.920	453	10.49	656	22.0	2.88	0.916	65.8	535.5	335
340	0.2713	1.021	5.74	2342	4.188	1.930	420	10.69	660	22.3	2.66	0.925	64.9	566.0	340
345	0.3372	1.024	4.683	2329	4.191	1.941	389	10.89	668	22.6	2.45	0.933	64.1	595.4	345
350	0.4163	1.027	3.846	2317	4.195	1.954	365	11.09	668	23.0	2.29	0.942	63.2	624.2	350
355	0.5100	1.030	3.180	2304	4.199	1.968	343	11.29	671	23.3	2.14	0.951	62.3	652.3	355
360	0.6209	1.034	2.645	2291	4.203	1.983	324	11.49	674	23.7	2.02	0.960	61.4	697.9	360
365	0.7514	1.038	2.212	2278	4.209	1.999	306	11.69	677	24.1	1.91	0.969	60.5	707.1	365
370	0.9040	1.041	1.861	2265	4.214	2.017	289	11.89	679	24.5	1.80	0.978	59.5	728.7	370
373.15	1.0133	1.044	1.679	2257	4.217	2.029	279	12.02	680	24.8	1.76	0.984	58.9	750.1	373.15
375	1.0815	1.045	1.574	2252	4.220	2.036	274	12.09	681	24.9	1.70	0.987	58.6	761	375
380	1.2869	1.049	1.337	2239	4.226	2.057	260	12.29	683	25.4	1.61	0.999	57.6	788	380
385	1.5233	1.053	1.142	2225	4.232	2.080	248	12.49	685	25.8	1.53	1.004	56.6	814	385
390	1.794	1.058	0.980	2212	4.239	2.104	237	12.69	686	26.3	1.47	1.013	55.6	841	390
400	2.455	1.067	0.731	2183	4.256	2.158	217	13.05	688	27.2	1.34	1.033	53.6	896	400
410	3.302	1.077	0.553	2153	4.278	2.221	200	13.42	688	28.2	1.24	1.054	51.5	952	410
420	4.370	1.088	0.425	2123	4.302	2.291	185	13.79	688	29.8	1.16	1.075	49.4	1010	420
430	5.699	1.099	0.331	2091	4.331	2.369	173	14.14	685	30.4	1.09	1.10	47.2		430

TABLE A.6 *Continued*

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	Specific Volume (m³/kg) v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K) $c_{p,f}$	Specific Heat (kJ/kg · K) $c_{p,g}$	Viscosity (N · s/m²) $\mu_f \cdot 10^6$	Viscosity (N · s/m²) $\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k_f \cdot 10^3$	Thermal Conductivity (W/m · K) $k_g \cdot 10^3$	Prandtl Number Pr_f	Prandtl Number Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K⁻¹)	Temperature, T (K)
440	7.333	1.110	0.261	2059	4.36	2.46	162	14.50	682	31.7	1.04	1.12	45.1		440
450	9.319	1.123	0.208	2024	4.40	2.56	152	14.85	678	33.1	0.99	1.14	42.9		450
460	11.71	1.137	0.167	1989	4.44	2.68	143	15.19	673	34.6	0.95	1.17	40.7		460
470	14.55	1.152	0.136	1951	4.48	2.79	136	15.54	667	36.3	0.92	1.20	38.5		470
480	17.90	1.167	0.111	1912	4.53	2.94	129	15.88	660	38.1	0.89	1.23	36.2		480
490	21.83	1.184	0.0922	1870	4.59	3.10	124	16.23	651	40.1	0.87	1.25	33.9	—	490
500	26.40	1.203	0.0766	1825	4.66	3.27	118	16.59	642	42.3	0.86	1.28	31.6	—	500
510	31.66	1.222	0.0631	1779	4.74	3.47	113	16.95	631	44.7	0.85	1.31	29.3	—	510
520	37.70	1.244	0.0525	1730	4.84	3.70	108	17.33	621	47.5	0.84	1.35	26.9	—	520
530	44.58	1.268	0.0445	1679	4.95	3.96	104	17.72	608	50.6	0.85	1.39	24.5	—	530
540	52.38	1.294	0.0375	1622	5.08	4.27	101	18.1	594	54.0	0.86	1.43	22.1	—	540
550	61.19	1.323	0.0317	1564	5.24	4.64	97	18.6	580	58.3	0.87	1.47	19.7	—	550
560	71.08	1.355	0.0269	1499	5.43	5.09	94	19.1	563	63.7	0.90	1.52	17.3	—	560
570	82.16	1.392	0.0228	1429	5.68	5.67	91	19.7	548	76.7	0.94	1.59	15.0	—	570
580	94.51	1.433	0.0193	1353	6.00	6.40	88	20.4	528	76.7	0.99	1.68	12.8	—	580
590	108.3	1.482	0.0163	1274	6.41	7.35	84	21.5	513	84.1	1.05	1.84	10.5	—	590
600	123.5	1.541	0.0137	1176	7.00	8.75	81	22.7	497	92.9	1.14	2.15	8.4	—	600
610	137.3	1.612	0.0115	1068	7.85	11.1	77	24.1	467	103	1.30	2.60	6.3	—	610
620	159.1	1.705	0.0094	941	9.35	15.4	72	25.9	444	114	1.52	3.46	4.5	—	620
625	169.1	1.778	0.0085	858	10.6	18.3	70	27.0	430	121	1.65	4.20	3.5	—	625
630	179.7	1.856	0.0075	781	12.6	22.1	67	28.0	412	130	2.0	4.8	2.6	—	630
635	190.9	1.935	0.0066	683	16.4	27.6	64	30.0	392	141	2.7	6.0	1.5	—	635
640	202.7	2.075	0.0057	560	26	42	59	32.0	367	155	4.2	9.6	0.8	—	640
645	215.2	2.351	0.0045	361	90	—	54	37.0	331	178	12	26	0.1	—	645
647.3[c]	221.2	3.170	0.0032	0	∞	∞	45	45.0	238	238	∞	∞	0.0	—	647.3[c]

[a] Adapted from Reference 22.

[b] 1 bar = 10^5 N/m².

[c] Critical temperature.

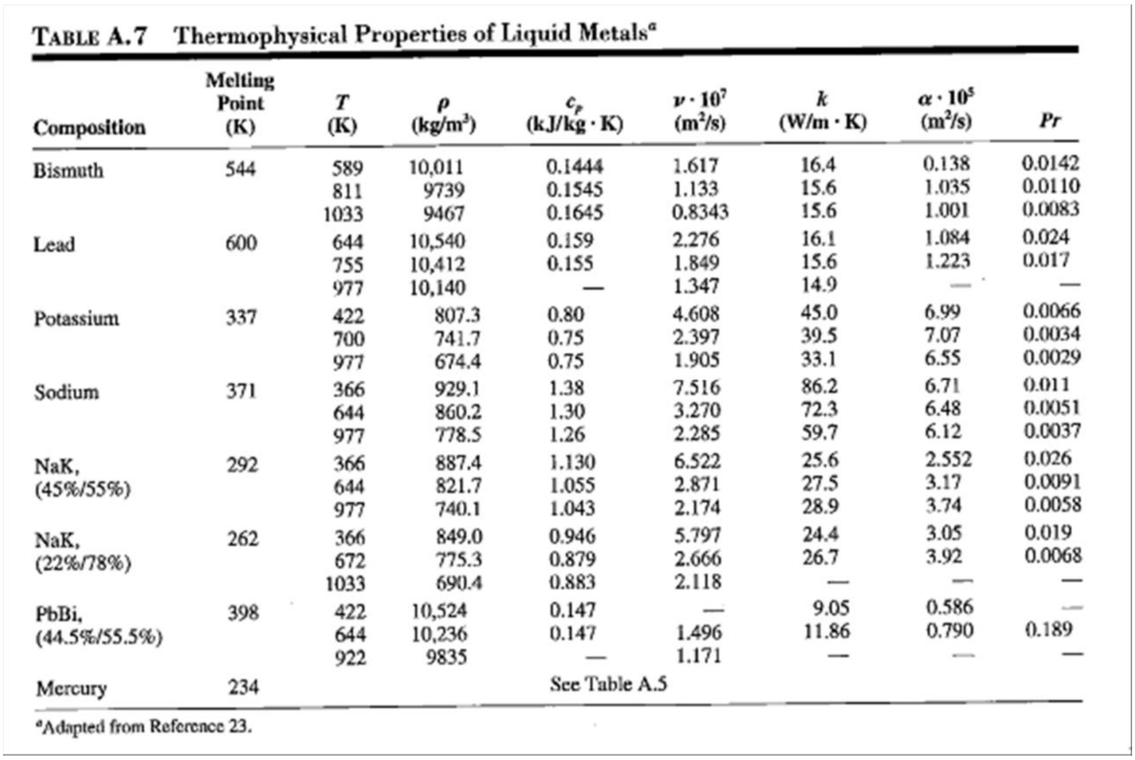
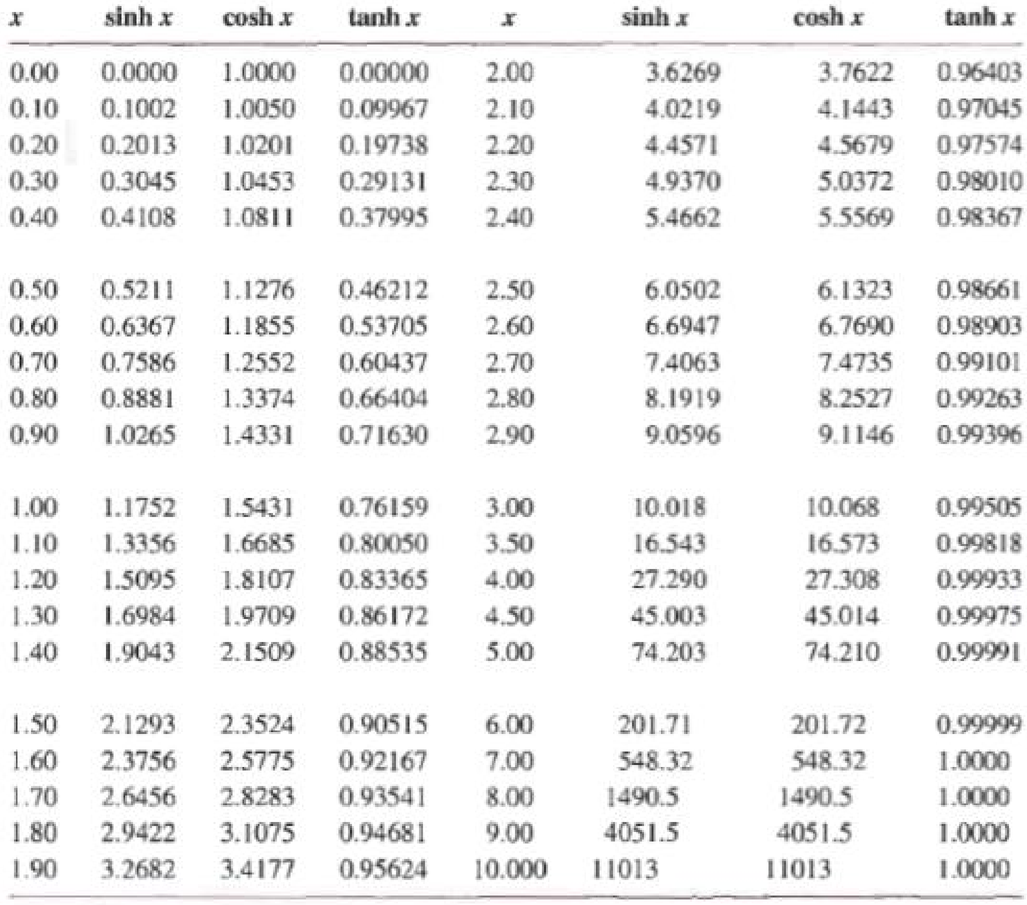


TABLE A.7 Thermophysical Properties of Liquid Metals[a]

Composition	Melting Point (K)	T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\nu \cdot 10^7$ (m^2/s)	k (W/m · K)	$\alpha \cdot 10^5$ (m^2/s)	Pr
Bismuth	544	589	10,011	0.1444	1.617	16.4	0.138	0.0142
		811	9739	0.1545	1.133	15.6	1.035	0.0110
		1033	9467	0.1645	0.8343	15.6	1.001	0.0083
Lead	600	644	10,540	0.159	2.276	16.1	1.084	0.024
		755	10,412	0.155	1.849	15.6	1.223	0.017
		977	10,140	—	1.347	14.9	—	—
Potassium	337	422	807.3	0.80	4.608	45.0	6.99	0.0066
		700	741.7	0.75	2.397	39.5	7.07	0.0034
		977	674.4	0.75	1.905	33.1	6.55	0.0029
Sodium	371	366	929.1	1.38	7.516	86.2	6.71	0.011
		644	860.2	1.30	3.270	72.3	6.48	0.0051
		977	778.5	1.26	2.285	59.7	6.12	0.0037
NaK, (45%/55%)	292	366	887.4	1.130	6.522	25.6	2.552	0.026
		644	821.7	1.055	2.871	27.5	3.17	0.0091
		977	740.1	1.043	2.174	28.9	3.74	0.0058
NaK, (22%/78%)	262	366	849.0	0.946	5.797	24.4	3.05	0.019
		672	775.3	0.879	2.666	26.7	3.92	0.0068
		1033	690.4	0.883	2.118	—	—	—
PbBi, (44.5%/55.5%)	398	422	10,524	0.147	—	9.05	0.586	—
		644	10,236	0.147	1.496	11.86	0.790	0.189
		922	9835	—	1.171	—	—	—
Mercury	234			See Table A.5				

[a] Adapted from Reference 23.



x	sinh x	cosh x	tanh x	x	sinh x	cosh x	tanh x
0.00	0.0000	1.0000	0.00000	2.00	3.6269	3.7622	0.96403
0.10	0.1002	1.0050	0.09967	2.10	4.0219	4.1443	0.97045
0.20	0.2013	1.0201	0.19738	2.20	4.4571	4.5679	0.97574
0.30	0.3045	1.0453	0.29131	2.30	4.9370	5.0372	0.98010
0.40	0.4108	1.0811	0.37995	2.40	5.4662	5.5569	0.98367
0.50	0.5211	1.1276	0.46212	2.50	6.0502	6.1323	0.98661
0.60	0.6367	1.1855	0.53705	2.60	6.6947	6.7690	0.98903
0.70	0.7586	1.2552	0.60437	2.70	7.4063	7.4735	0.99101
0.80	0.8881	1.3374	0.66404	2.80	8.1919	8.2527	0.99263
0.90	1.0265	1.4331	0.71630	2.90	9.0596	9.1146	0.99396
1.00	1.1752	1.5431	0.76159	3.00	10.018	10.068	0.99505
1.10	1.3356	1.6685	0.80050	3.50	16.543	16.573	0.99818
1.20	1.5095	1.8107	0.83365	4.00	27.290	27.308	0.99933
1.30	1.6984	1.9709	0.86172	4.50	45.003	45.014	0.99975
1.40	1.9043	2.1509	0.88535	5.00	74.203	74.210	0.99991
1.50	2.1293	2.3524	0.90515	6.00	201.71	201.72	0.99999
1.60	2.3756	2.5775	0.92167	7.00	548.32	548.32	1.0000
1.70	2.6456	2.8283	0.93541	8.00	1490.5	1490.5	1.0000
1.80	2.9422	3.1075	0.94681	9.00	4051.5	4051.5	1.0000
1.90	3.2682	3.4177	0.95624	10.000	11013	11013	1.0000

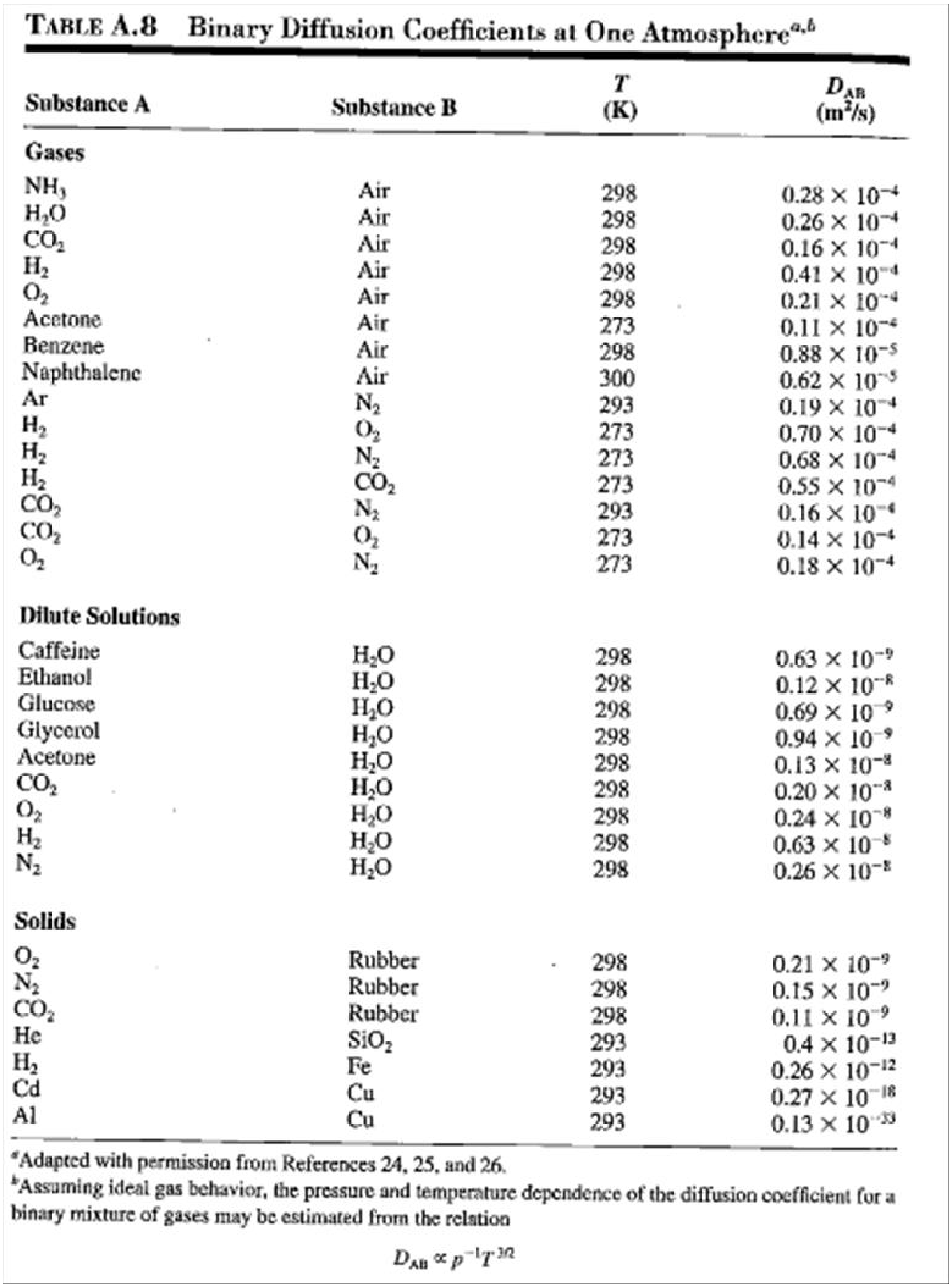


TABLE A.8 Binary Diffusion Coefficients at One Atmosphere[a,b]

Substance A	Substance B	T (K)	D_{AB} (m^2/s)
Gases			
NH_3	Air	298	0.28×10^{-4}
H_2O	Air	298	0.26×10^{-4}
CO_2	Air	298	0.16×10^{-4}
H_2	Air	298	0.41×10^{-4}
O_2	Air	298	0.21×10^{-4}
Acetone	Air	273	0.11×10^{-4}
Benzene	Air	298	0.88×10^{-5}
Naphthalene	Air	300	0.62×10^{-5}
Ar	N_2	293	0.19×10^{-4}
H_2	O_2	273	0.70×10^{-4}
H_2	N_2	273	0.68×10^{-4}
H_2	CO_2	273	0.55×10^{-4}
CO_2	N_2	293	0.16×10^{-4}
CO_2	O_2	273	0.14×10^{-4}
O_2	N_2	273	0.18×10^{-4}
Dilute Solutions			
Caffeine	H_2O	298	0.63×10^{-9}
Ethanol	H_2O	298	0.12×10^{-8}
Glucose	H_2O	298	0.69×10^{-9}
Glycerol	H_2O	298	0.94×10^{-9}
Acetone	H_2O	298	0.13×10^{-8}
CO_2	H_2O	298	0.20×10^{-8}
O_2	H_2O	298	0.24×10^{-8}
H_2	H_2O	298	0.63×10^{-8}
N_2	H_2O	298	0.26×10^{-8}
Solids			
O_2	Rubber	298	0.21×10^{-9}
N_2	Rubber	298	0.15×10^{-9}
CO_2	Rubber	298	0.11×10^{-9}
He	SiO_2	293	0.4×10^{-13}
H_2	Fe	293	0.26×10^{-12}
Cd	Cu	293	0.27×10^{-18}
Al	Cu	293	0.13×10^{-33}

[a] Adapted with permission from References 24, 25, and 26.

[b] Assuming ideal gas behavior, the pressure and temperature dependence of the diffusion coefficient for a binary mixture of gases may be estimated from the relation

$$D_{AB} \propto p^{-1} T^{3/2}$$

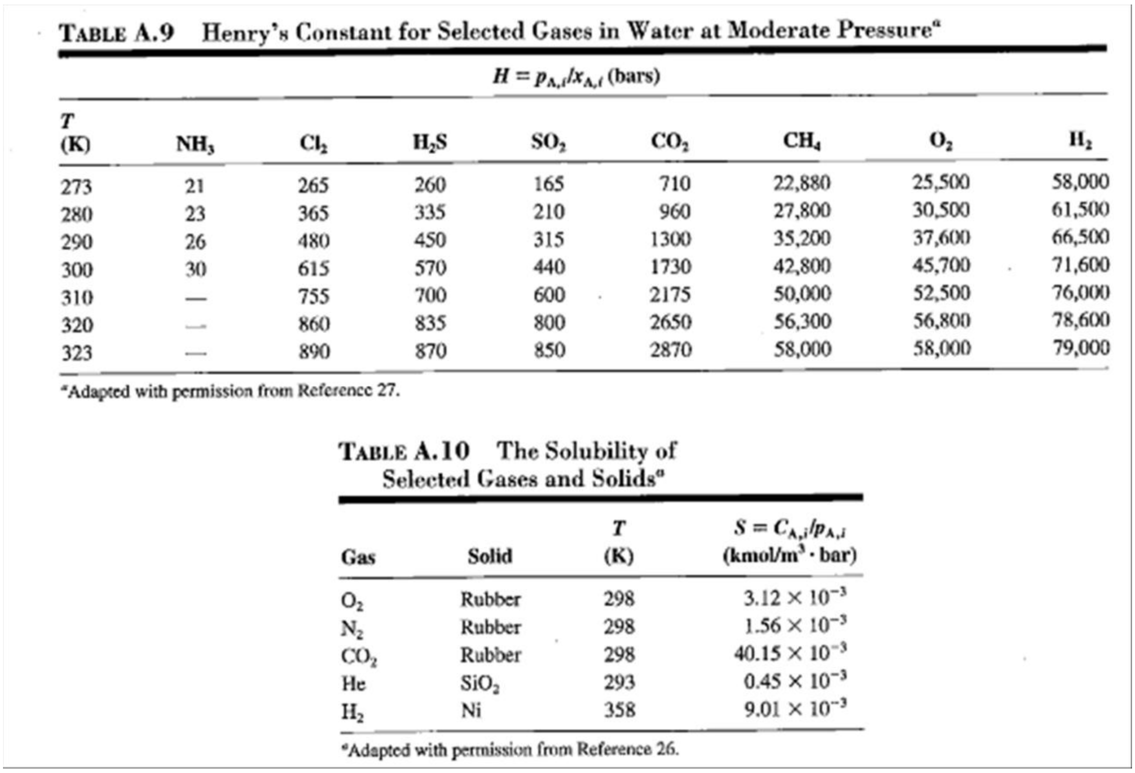


TABLE A.9 Henry's Constant for Selected Gases in Water at Moderate Pressure[a]

$H = p_{A,i}/x_{A,i}$ (bars)

T (K)	NH_3	Cl_2	H_2S	SO_2	CO_2	CH_4	O_2	H_2
273	21	265	260	165	710	22,880	25,500	58,000
280	23	365	335	210	960	27,800	30,500	61,500
290	26	480	450	315	1300	35,200	37,600	66,500
300	30	615	570	440	1730	42,800	45,700	71,600
310	—	755	700	600	2175	50,000	52,500	76,000
320	—	860	835	800	2650	56,300	56,800	78,600
323	—	890	870	850	2870	58,000	58,000	79,000

[a]Adapted with permission from Reference 27.

TABLE A.10 The Solubility of Selected Gases and Solids[a]

Gas	Solid	*T* (K)	$S = C_{A,i}/p_{A,i}$ (kmol/m^3 · bar)
O_2	Rubber	298	3.12×10^{-3}
N_2	Rubber	298	1.56×10^{-3}
CO_2	Rubber	298	40.15×10^{-3}
He	SiO_2	293	0.45×10^{-3}
H_2	Ni	358	9.01×10^{-3}

[a]Adapted with permission from Reference 26.

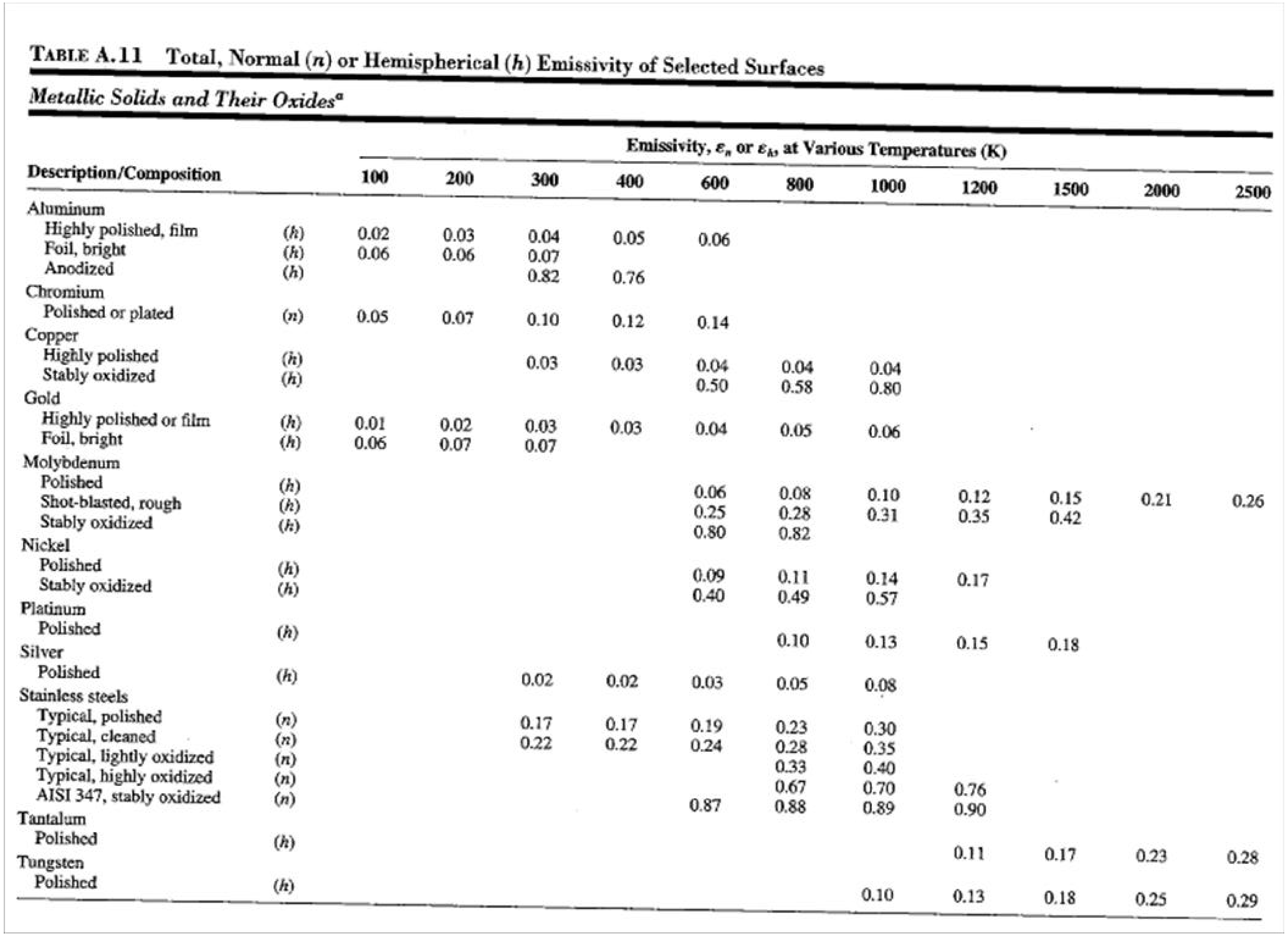


TABLE A.11 Total, Normal (n) or Hemispherical (h) Emissivity of Selected Surfaces

Metallic Solids and Their Oxides[a]

		Emissivity, ε_n or ε_h, at Various Temperatures (K)										
Description/Composition		**100**	**200**	**300**	**400**	**600**	**800**	**1000**	**1200**	**1500**	**2000**	**2500**
Aluminum												
Highly polished, film	(h)	0.02	0.03	0.04	0.05	0.06						
Foil, bright	(h)	0.06	0.06	0.07								
Anodized	(h)			0.82	0.76							
Chromium												
Polished or plated	(n)	0.05	0.07	0.10	0.12	0.14						
Copper												
Highly polished	(h)			0.03	0.03	0.04	0.04	0.04				
Stably oxidized	(h)					0.50	0.58	0.80				
Gold												
Highly polished or film	(h)	0.01	0.02	0.03	0.03	0.04	0.05	0.06				
Foil, bright	(h)	0.06	0.07	0.07								
Molybdenum												
Polished	(h)					0.06	0.08	0.10	0.12	0.15	0.21	0.26
Shot-blasted, rough	(h)					0.25	0.28	0.31	0.35	0.42		
Stably oxidized	(h)					0.80	0.82					
Nickel												
Polished	(h)					0.09	0.11	0.14	0.17			
Stably oxidized	(h)					0.40	0.49	0.57				
Platinum												
Polished	(h)						0.10	0.13	0.15	0.18		
Silver												
Polished	(h)			0.02	0.02	0.03	0.05	0.08				
Stainless steels												
Typical, polished	(n)			0.17	0.17	0.19	0.23	0.30				
Typical, cleaned	(n)			0.22	0.22	0.24	0.28	0.35				
Typical, lightly oxidized	(n)						0.33	0.40				
Typical, highly oxidized	(n)						0.67	0.70	0.76			
AISI 347, stably oxidized	(n)					0.87	0.88	0.89	0.90			
Tantalum												
Polished	(h)								0.11	0.17	0.23	0.28
Tungsten												
Polished	(h)							0.10	0.13	0.18	0.25	0.29

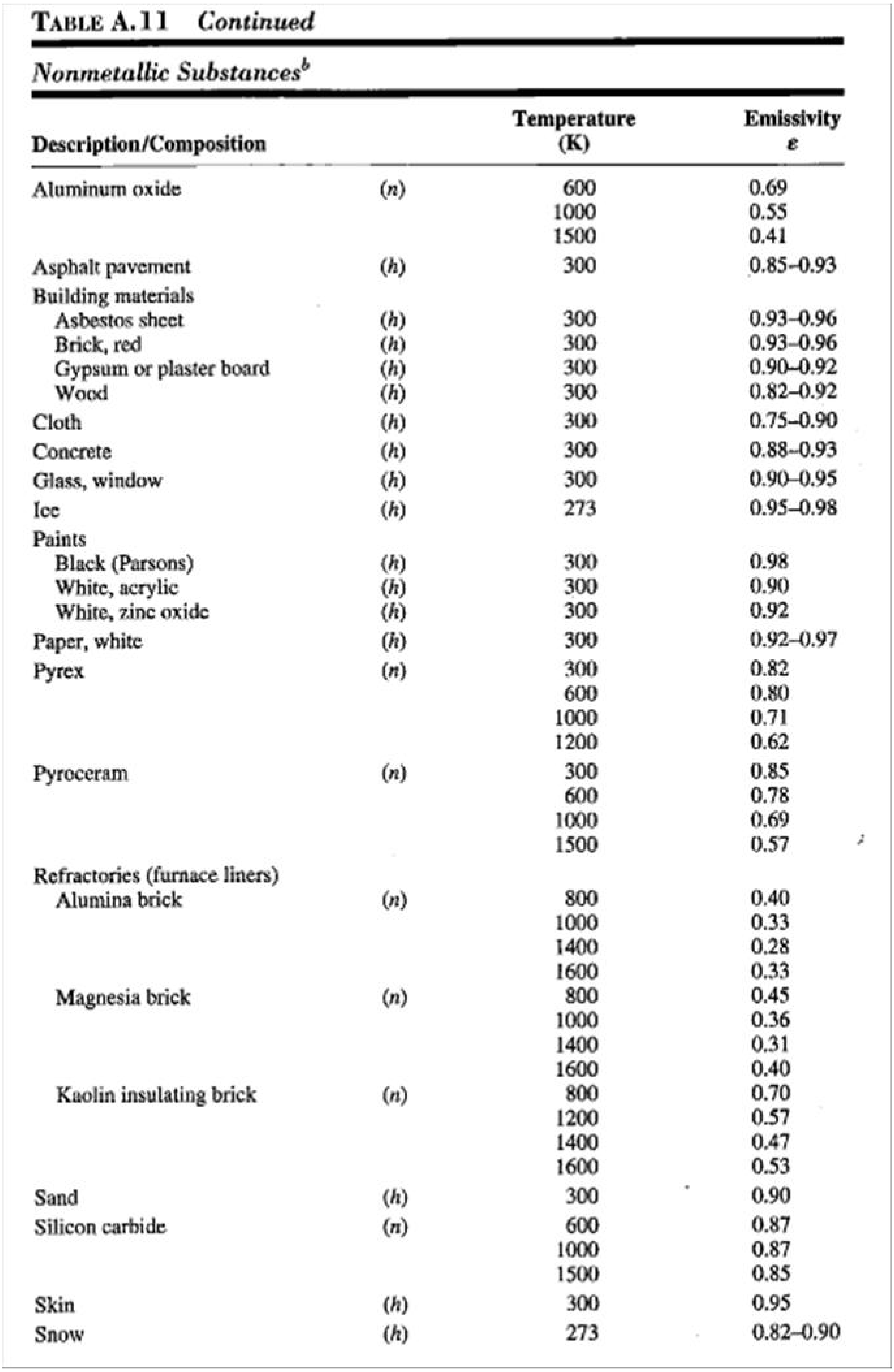


TABLE A.11 *Continued*

Nonmetallic Substances^b

Description/Composition		Temperature (K)	Emissivity ε
Aluminum oxide	(*n*)	600	0.69
		1000	0.55
		1500	0.41
Asphalt pavement	(*h*)	300	0.85–0.93
Building materials			
Asbestos sheet	(*h*)	300	0.93–0.96
Brick, red	(*h*)	300	0.93–0.96
Gypsum or plaster board	(*h*)	300	0.90–0.92
Wood	(*h*)	300	0.82–0.92
Cloth	(*h*)	300	0.75–0.90
Concrete	(*h*)	300	0.88–0.93
Glass, window	(*h*)	300	0.90–0.95
Ice	(*h*)	273	0.95–0.98
Paints			
Black (Parsons)	(*h*)	300	0.98
White, acrylic	(*h*)	300	0.90
White, zinc oxide	(*h*)	300	0.92
Paper, white	(*h*)	300	0.92–0.97
Pyrex	(*n*)	300	0.82
		600	0.80
		1000	0.71
		1200	0.62
Pyroceram	(*n*)	300	0.85
		600	0.78
		1000	0.69
		1500	0.57
Refractories (furnace liners)			
Alumina brick	(*n*)	800	0.40
		1000	0.33
		1400	0.28
		1600	0.33
Magnesia brick	(*n*)	800	0.45
		1000	0.36
		1400	0.31
		1600	0.40
Kaolin insulating brick	(*n*)	800	0.70
		1200	0.57
		1400	0.47
		1600	0.53
Sand	(*h*)	300	0.90
Silicon carbide	(*n*)	600	0.87
		1000	0.87
		1500	0.85
Skin	(*h*)	300	0.95
Snow	(*h*)	273	0.82–0.90

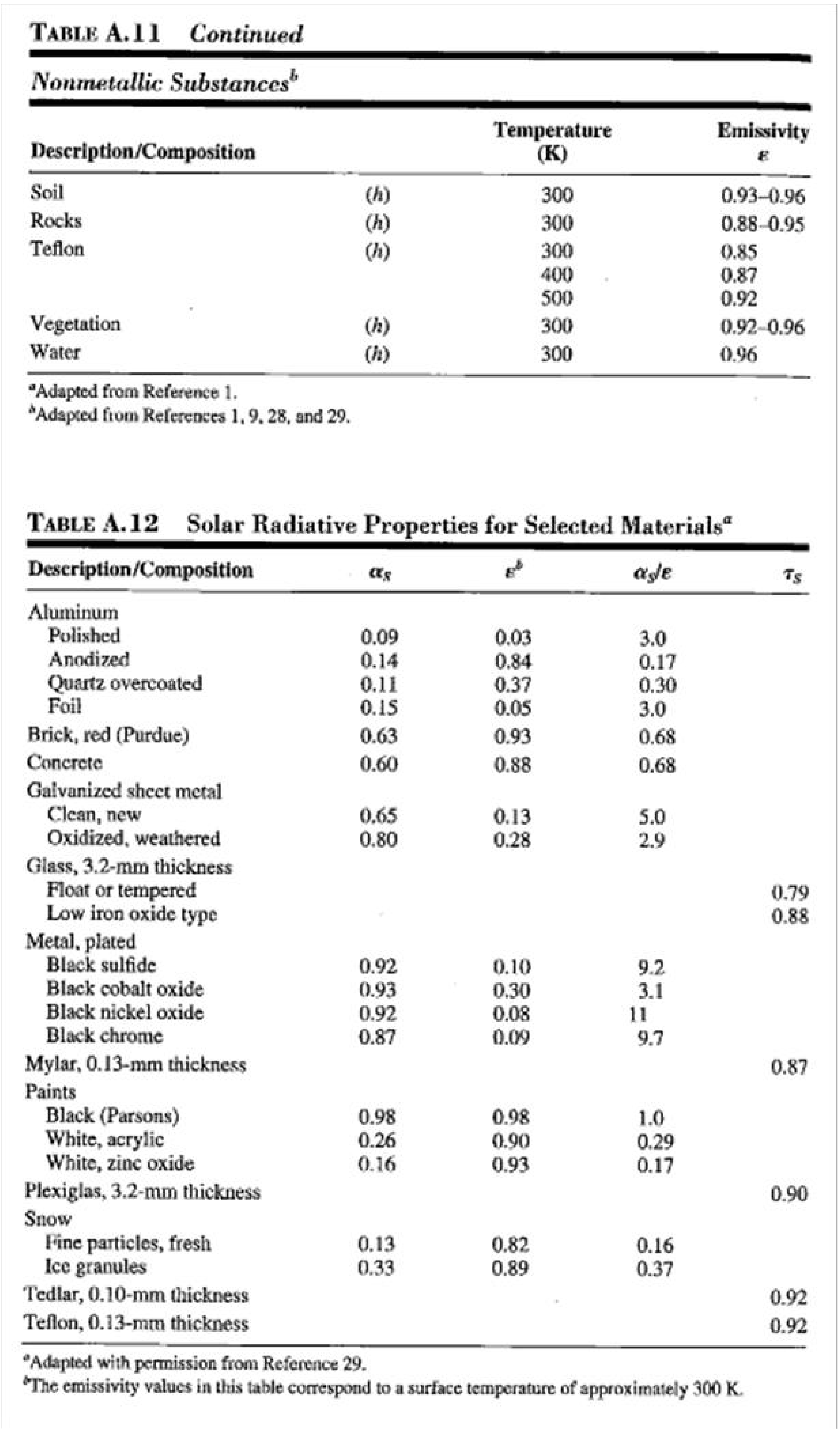


TABLE A.11 *Continued*

Nonmetallic Substances[b]

Description/Composition		Temperature (K)	Emissivity ε
Soil	(*h*)	300	0.93–0.96
Rocks	(*h*)	300	0.88–0.95
Teflon	(*h*)	300	0.85
		400	0.87
		500	0.92
Vegetation	(*h*)	300	0.92–0.96
Water	(*h*)	300	0.96

[a] Adapted from Reference 1.
[b] Adapted from References 1, 9, 28, and 29.

TABLE A.12 Solar Radiative Properties for Selected Materials[a]

Description/Composition	α_S	ε^b	α_S/ε	τ_S
Aluminum				
Polished	0.09	0.03	3.0	
Anodized	0.14	0.84	0.17	
Quartz overcoated	0.11	0.37	0.30	
Foil	0.15	0.05	3.0	
Brick, red (Purdue)	0.63	0.93	0.68	
Concrete	0.60	0.88	0.68	
Galvanized sheet metal				
Clean, new	0.65	0.13	5.0	
Oxidized, weathered	0.80	0.28	2.9	
Glass, 3.2-mm thickness				
Float or tempered				0.79
Low iron oxide type				0.88
Metal, plated				
Black sulfide	0.92	0.10	9.2	
Black cobalt oxide	0.93	0.30	3.1	
Black nickel oxide	0.92	0.08	11	
Black chrome	0.87	0.09	9.7	
Mylar, 0.13-mm thickness				0.87
Paints				
Black (Parsons)	0.98	0.98	1.0	
White, acrylic	0.26	0.90	0.29	
White, zinc oxide	0.16	0.93	0.17	
Plexiglas, 3.2-mm thickness				0.90
Snow				
Fine particles, fresh	0.13	0.82	0.16	
Ice granules	0.33	0.89	0.37	
Tedlar, 0.10-mm thickness				0.92
Teflon, 0.13-mm thickness				0.92

[a] Adapted with permission from Reference 29.
[b] The emissivity values in this table correspond to a surface temperature of approximately 300 K.

[i]

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[i] *'Practical training is an important adjunct to theoretical studies'* – Valuable Real life learning from unemployed philosopher and engineer