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FAIR PRICING OF INDEX-LINKED BONDS:
CASE NORDEA ALL STARS EKSTRA 36/05

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1 INTRODUCTION

In recent years, different hybrid investment products have reached popularity. The idea of these products is to combine high returns of stock markets with the guaranteed returns of bank account or bonds. Therefore, products have embedded option that reduces downside risk for the investor.

Edleson and Cohn (1993) discuss that equity-linked products[1] were born in the 1980`s when just about everybody wished they would have been in the stock market because of its persistent upwards trek. Therefore, banks developed products that could offer investors some of the upward potential of bull market, with the minimal risk of capital losses. Referring to Niskanen and Niskanen (2003), equity-linked bonds are issued primarily when stock markets are expected to move up. Stoimenov and Wilkens (2005) argue that structured products offer a feature of facilitating positions in options without need to access to option exchanges. The advantage for private investor is that, it is cheaper to operate with these products than operate with corresponding single trades, considering transaction costs and commissions. However, despite large size and rapid growth of equity-linked instruments, the markets can hardly be described transparent.

The valuation is particularly important issue when examining these instruments and it is surprising that very little empirical research on the pricing has undertaken. Structured products are innovative instruments and they usually contain complicated conditions related to index compensation. Therefore, pricing each product is an individual action and examining the wide variety of bonds is very troublesome project and generalizing results are hard to reach. Hence, a relatively interesting and feasible study would be investigation of one individual instrument, its pricing behavior and the ability of theoretical models to price index-linked instruments.

[1] Depending on the nature of the structured instrument there are different terms used. *Equity-linked CD, equity-linked guaranteed investment certificate (ELGIC), equity-linked debt, equity-linked bond* are all slightly different structured products

The purpose of this thesis is to examine the following issues: First, how to price equity-linked bonds and to determine the fair price. We claim that in the primary market, index-linked bonds are overpriced, on average. In this study, equity-linked bond designates index-linked bond, in which the return is bounded to stock index performance. However, it should be noted that the structure examined in this study is not the only structure available, though it is very common in the markets. Second, we examine how to replicate the index-linked bond by combining zero-coupon bond and European call-option with theoretical option pricing method to determine the fair price of one structured product when issuers approximation of bid-ask spread is known. Finally, we describe, how do bond and option components of index-linked bond behave and furthermore, how sensitive the index-linked bond is to changes in option and bond prices, and how sensitive option price is to changes in volatility and bond price to changes in yield. Primary objective is to examine equity-linked instruments issued by corporations or commercial banks, though the fact is that there are a wide variety of those instruments issued by states and national banks in the world. Therefore, in the end of the thesis we study some benefits and costs that issuer banks could face.

The setting of this study is to deal with discounting the principal of zero-coupon bond and to count the value of Asian average price call option with the historical value of indices involved at the equity-linked bond. Equity data is gathered from the Standard & Poor's web site[2], benchmark rate and risk-free interest rate data are from Bank of Finland database and the data concerning equity-linked bond is from Nordea prospectus.

The thesis is organized as follows: Section 2 introduces some empirical study regarding equity-based instruments and describes theoretical background of structured bonds and option pricing methods. This section gives mainframe, how to evaluate the fair price of equity-linked instruments. Section 3 describes the data, research method and the features of particular index-linked bond under examination. Section 4 reports empirical results and the last chapter concludes.

[2] www.standarandpoors.com

2 THEORETICAL BACKGROUND

2.1 Features of Equity-Linked Bonds

Structured financial products combine the features of spot and futures markets (e.g., stocks, interest rate products and derivatives) and they promise tailor-made risk/return profiles for investors. Equity-linked bonds are usually issued by banks or other financial institutions and their return is linked to the return of equity-index during the maturity of the bond. The simplest form of instrument is that an investor pays an emission price that consists of principal and premium. At the end of the maturity, the issuer returns the principal and in addition to this he pays interest linked to the success of particular stock index. (Stoimenov & Wilkens, 2005)

However, there are usually a number of conditions included equity-linked instruments. Issuer can guarantee a minimum level for the principal regardless of index performance, index return can be determined as an average of stock index value at particular valuation days, index returns can be determined with return coefficient, return can be quoted in different currency than the principal or index can be basket index when it is formed of multiple indices. (Stoimenov & Wilkens, 2005)

Equity-linked bonds are not paid usually any coupons but investors get the whole principal and index return in the maturity day. Therefore bond forms exactly two different cash flows to investors: negative cash flow when investor pays an emission at $t = 0$ and positive cash flow when issuer returns the principal and the index return at $t = 1$. To investor, the equity-linked instrument, containing these previous conditions, offers an opportunity to invest in single or multiple foreign indices with limited downside risk. To underwriter, the instrument offers a good possibility to collect financing from the investors and to hedge their investments in instruments underlying the indices. (Stoimenov & Wilkens, 2005)

Equity-linked bond can be viewed usually as a combination of zero-coupon bond and long-term average-price call option though there are also different structures. However these components are not directly observable in the market. As a result, both bond and call option pricing models have to be used to estimate the components. Bond component valuation requires estimating the yield to maturity and option component valuation requires exercise price, the current value of the index, the time to maturity, the risk free rate and the volatility of the index. (Chen et al., 2001)

Turning to the issue of pricing, these securities have been traditionally priced by viewing the bond-like component stream and option-like component stream as distinct and pricing them separately. Typically, the bond-like component is priced using one of the most popular interest rate models available. The option-like component is typically priced with issuer's favorite option model, such as Black-Scholes Model in which the underlying asset is assumed to follow log-normal random walk. (Alobaidi & Mallier, 2002)

2.2 Zero-Coupon Bonds

It has become common to say that present value of certain cash flow is the size of that cash flow discounted appropriately by a discount factor. Zero-coupon bonds do not make any periodic coupon payments. Instead, the investor realizes interest by the difference between the maturity value and the purchase price. Zero-coupon bond price is the present value of the expected cash flows and thus the only cash flow is the maturity value. (Fabozzi, 2000: 59-60)

The interest rate or discount rate that an investor wants from investing in a bond is called required yield and it is determined by investigating the yields offered on comparable bonds in the market. Different yields can be used to calculate the present value of cash flows and the appropriate yield for each cash flow would then be based on a theoretical rate on a zero-coupon bond with a maturity equal to the time that cash flow will be received. (Fabozzi, 2000: 69)

There are different types of risks in zero-coupon bonds that investor is exposed to. However, one remarkable risk is eliminated in a zero-coupon investment, the reinvestment risk, because there is no coupon to reinvest. However, the biggest risk faced by an investor is market or interest rate risk. Changes in interest rates effect to price of zero-coupon bonds. Hence, when interest rate increases, the price of the bond decrease and vice versa. The most common used measure of this risk type is duration. Credit risk or default risk refers to the risk that the issuer of a fixed income security may default. Credit risk is gauged by quality ratings assigned by rating companies. Liquidity risk is the risk that the investor will have to sell the bond below its true value. The primary measure of liquidity is the size of the spread between the bid and the ask spread quoted by a dealer. For investor who plans to hold a bond until maturity and need not mark a position to market, liquidity risk is not a major concern. Other risks concerning zero-coupon bonds could be exchange risk, event risk or sector risk. (Fabozzi, 2000: 21-29)

2.3 Option features

When replicating the equity-linked bond, applicable call-option could have following features: 1) Call-options can be European options meaning they can be exercised only at the end of their maturity. 2) Call-options can be index options, therefore there is one or more stock indices as an underlying. If the underlying is the mean of multiple instruments, option is called basket-option. 3) Underlying stocks can pay dividends. 4) The value of the option is determined as an arithmetic mean value of particular valuation days. These options are called path-dependent or Asian options. 5) Stock index values can be quoted in different currency than fixed part, known as quanto-option.

Based on arbitrage pricing methods, we could determine the market price of the equity-linked instrument, if there were quoted prices for these options. However, this is not the case, thus option prices have to be determined with theoretical option pricing models.

2.3.1 The Definition of Volatility

The volatility of the stock is a measure of uncertainty about the returns provided by the stock. It can be defined as the standard deviation of the return provided by the stock in one year when the return is expressed using continuous compounding. There are several different ways to count volatility and their purpose of use varies. (Hull, 2003: 238)

Estimating volatility from historical data is a way to define volatility empirically and this is based on stock price observations at fixed intervals of time e.g., daily, weekly or monthly. More data generally lead to more accuracy, but volatility does change over time and data that are too old may not be relevant predicting future. An important issue whether time should be measured in calendar days or trading days when volatility is estimated. Many empirical researches have indicated, that days when the exchange is closed should be ignored. (Hull, 2003: 239) Historical volatility is also called the ex-post volatility.

In practice, traders usually work with implied volatility which is the volatility implied by option prices observed in the market. These are used to monitor the market's opinion about the volatility of a particular stock and they are calculated interpolating between actively and passively traded options (Hull, 2003: 250-251). Li (2005) finds that implied volatility is extensively used in financial markets research. This volatility is also the value of the volatility that, when employed in the Black-Scholes formula, results in a model price equal to the market price. However, Chen & Sears (1990) find that the ex-post volatility does a much better job than implied volatility in estimating the true volatility and the index-linked bond value, because the relative magnitudes of the ex-post approach are significantly smaller than their implied volatility counterparts, despite some pricing errors of the approach.

There are also some more sophisticated models to estimate the volatility, which are known as discrete-time models. The exponentially weighted moving average model (EWMA) is weighted model where the weights of the variances decrease exponentially as we move back through time. EWMA is designed to track changes in the volatility with relatively little amount of data. Another discrete-time model is known as GARCH (1,1) model proposed by Bollerslev in 1986 and practically EWMA is one special case of GARCH (1,1) model. (Hull, 2003: 374-375)

2.3.2 Black and Scholes Model

The Black-Scholes model was formulated in the early 1970's by Black, Scholes and Merton to determine the price of stock options. There are some basic assumptions concerning the model. First, it assumes that markets are efficient meaning that arbitrage opportunities do not exist. Second, the price of the underlying instrument is following a geometric Brownian motion and the volatility is constant. (Karoui, 1998)

The fundamental insight of Black-Scholes and Merton models is that under certain conditions an option's payoff can be exactly replicated by a particular dynamic investment strategy, involving only the underlying stock and riskless security (Campbell et al., 1997: 339). According to Li (2005), a useful property of the Black-Scholes option pricing model is that all model parameters except the volatility are directly observable from market data. This allows a market-based estimate of a stock's future volatility.

European call option Black and Scholes model can be extended to value options on futures contracts. The model was introduced by Fisher Black in 1976 and it is called Black 76 for futures contracts. The underlying assumption is that futures prices have the same log-normal property than it is assumed for stock prices. When pricing Asian options with Black and Scholes model we can apply the Black 76 for futures contracts by changing some parameters in the equation. (Hull, 2003: 278, 287)

2.3.3 Binomial Tree

A useful and very popular technique for pricing stock options involves constructing a *binomial tree*. This diagram represents different possible paths that might be followed by the stock price over the life of the option. However, one or two step binomial trees are very imprecise models of reality. A more realistic model is one that assumes stock price movements are composed of a much larger number of small binomial movements. This assumption underlies a widely used numerical procedure by Cox, Ross and Rubinstein. (Hull, 2003: 200, 392) Reynaerts et al. (2006) find that this model can be considered as a discrete time version of the Black-Scholes model. They also examine the suitability of CRR binomial tree to price Asian options arriving at a result that binomial method is very time consuming and complicated process to do that.

2.3.4 Index Options

Options on stock indices trade in both over-the-counter and exchange-traded markets. Some of the indices track movement of the stock market as a whole and others are based on the performance of a particular sector. (Hull, 2003:270)

Ackert and Tian (2000) discuss that index options have been examined using theoretical models, such as the Black-Scholes or the Cox et al. Binomial Option Pricing Model. Hull (2003) points that stock indices, currencies and futures contracts can, for the purposes of option evaluation, be considered as assets providing known yields. In the stock index, the relevant yield is the dividend yield on the stock portfolio underlying the index.

2.3.5 Exotic Options

Usually index-linked instruments are not the simplest ones containing different kinds of exotic options. For example, the features from Asian options and basket options are widely used to determine the refund of the index-linked bond.

Asian options are options where the payoff depends on the average price of the underlying asset during at least some part of the life of the option. Average price options are less expensive than regular options and are arguably more appropriate to meet some of the needs of corporate treasurers than regular options. If the underlying asset price, S, is assumed to be log-normally distributed and average price of S is a geometric average of the S's, analytic formulas are available for valuing European average price options. Asian options are used to make options more stable which leads to decreasing risk level and finally to lower price. (Hull, 2003:443)

However, according to Asbjörn & Jörgensen (1997) the type of averaging an Asian option may differ. The average may be based on discretely sampled prices or continuously sampled price observations. Averaging could be done also on arithmetic or geometric basis and the weighting of the observations in the average may be equal or flexible. Most of the previous literature has concentrated on studying European types of Asian options and American types of options have not received much attention.[3]

Options involving two or more risky assets are sometimes called as rainbow options. Possibly the most popular option of this category is a basket option. This is an option where the payoff is dependent on the value of a portfolio or basket of assets. The assets are usually either individual stocks or stock indices or currencies. A European basket option can be valued with Monte Carlo simulation, by assuming that the assets follow correlated geometric Brownian motion process. (Hull, 2003:446)

2.4 Earlier Studies

There is not very wide variation of studies concerning equity-linked bonds in consequence of novelty of the product supply. Stoimenov and Wilkens (2005) have examined the fairness of structured products in German markets. Their basic assumption is that all types of equity-linked instruments are, on average, in the primary market overpriced over their theoretical value and thus favoring the issuing institution. They use DAX stock prices to compare issuer prices in the primary and secondary markets and finally their result suggest that it is necessary to analyze prices carefully when trading equity-linked structured products.

Study related to relative pricing of enhanced CD products is made by Brooks (1996). He concludes that there is remarkable interest rate risk, and thus early withdrawals can increase the uncertainty. Milevsky and Kim (1997) compare the relative value of different

[3] Asbjörn & Jörgensen (1997) present the analytical solution for American-style Asian options in their working paper.

index-linked securities to investor in the Canadian markets. Their conclusion is that those securities become less attractive, the longer the investment horizon of the investor.

Roberts, Vijayraghavan and Aintablian (2002) report the announcement effects when a company issues a new index-linked bond in the Paris Bourse. Their event study results suggest that issuing increases the shareholder value and therefore index-linked bonds are overvalued in the primary markets at their issue day.

Wilkens et al. (2003) investigate also the pricing of structured products in the German markets. Their purpose is to compare quoted prices with the duplication strategies using exchange-traded options and to investigate average price differences dependent on product type, issuer and underlying. Their study finally reveals significant pricing differences of structured products which can be interpreted as being in favor of issuing corporation.

One of the earliest studies concerning index-linked bonds is made by Chen & Sears (1990). They investigate the valuation of Salomon Brothers' Standard and Poor's (S&P) 500 indexed note termed SPIN, which is a combination of a bond and call option on the S&P index. Their purpose is to present the variety of different variables that have influence on the price of the SPIN and to describe the valuation process. The results show that the price given by theoretical model observes substantially the market prices, though it has a little tendency to underestimate the actual SPIN prices. They find also that the meaning of the way how to count volatility is not remarkably significant. However, the pricing sensitivities in both, option and bond, components could become significant if there was critical error in yield and volatility values.

3 DATA AND RESEARCH METHOD

3.1 Data

The data for this research includes weekly closing bid-ask prices for the Nordea All Stars Ekstra 36/05 (Appendix A) for the particular quotation days from 9.12.2005 through 17.3.2006, including 14 quotations. Quotes are from Kauppalehti, which is the most well-known economical newspaper in Finland. From the bid-ask quotes we calculate and use mid-quotes. Ideally, there should be the sample period of sufficient length to provide evidence of behavior of the index-linked bond. However, data availability in Finnish equity-linked bond markets is not the best one and quotes are not updated on a high frequency basis thus our sample is to some extent limited.

Empirical investigation of this thesis is based on pricing equity-linked product in two parts. First, we analyze the present value of zero-coupon bond with yield, which is chosen to be the same as yield of Finnish Government benchmark bond. Then we determine the price of the option position, using annualized volatility with continuously compounded returns of daily closing prices of S&P All Stars Europe index (Appendix B) related to the period of 120 trading days. Prices of underlying index are from Standard & Poor`s. In order to assess the pricing of index option we use the most common theoretical option pricing models with some special properties. Risk free interest rates are extracted from continuously compounded Euribor 12 month interest rates.

3.2 Nordea All Stars Ekstra 36/05

Finnish equity-linked bond markets are relatively young and therefore there are only few issuers which are big commercial banks. First bonds were issued in 1994 and remarkable growth of demand has happened after 1997. Secondary markets are not yet very liquid and therefore Finnish instruments are usually warranted by issuers.

On 31.10.2005, Nordea All Stars Ekstra was issued by Nordea Bank of Finland to the public at par exclusive of underwriting fees, for a total of 35,000,000 €. The index-linked bond is a five-year zero-coupon bond with maturity date of 25.11.2010. At maturity it pays its holder: *(i)* The principal amount of 1,000 € per bond plus *(ii)* the excess of the S&P All Stars Europe index value over the initial value of the index times some predetermined multiplier if the change of the reference index is positive within maturity period.

There are several features of the call option component of the Nordea All Stars Ekstra: *(i)* the exercise price is fixed and equals 116.57; *(ii)* the multiplier is 1 and it represents the expected return multiplier; *(iii)* the option is European which means that it is not exercisable prior to maturity and; *(iv)* unlike exchange-traded options, Nordea rather than any particular exchange, is the guarantor of the option payoff. In essence, the Nordea All Stars Ekstra is a combination of a zero-coupon bond plus the long-term call option on the S&P All Stars Europe index.

There are also a few special properties concerning the Nordea All Stars Ekstra. First, the emission course is for about 105 % instead of 100 %. Second, the end-value of the index is calculated as an arithmetic mean of the particular valuation days, which are every third month's 10th day from the three last years of the maturity. The first valuation day is 10.11.2007 and the last 10.11.2010. However, it should be noted that the distribution of valuation days produces some difficulties in Asian option pricing process, because valuation days are distributed at the end of the maturity. Thus, we have to make an assumption that average is calculated from continuous observations on the asset price.

3.3 Estimating the Bond Value

The bond component of the index-linked bond is estimated using the simple zero-coupon bond pricing model:

$$\text{Bond Value} = \frac{q}{(1+r)^T} \times E, \qquad (1)$$

where q is guaranteed capital return percent for the principal, *E* is subscribed capital, *r* is annualized risk-free rate for time to maturity added to the issuer's risk premium and *T* is time to maturity of the bond in years.

All inputs are known except *r*, the yield. However, we have applied the yield from the Finnish Government benchmark bond to define the price of the zero-coupon bond. Though yield is not necessarily exactly the same as risk-free rate added the risk premium, it is very near it because the issuer is a bank and in Finland banks can be regarded almost as stable as the government. We have chosen the government bond which has loan period of 18.5.2005-15.9.2010 and it is very near the maturity of the index-linked bond which has loan period of 31.10.2005-25.11.2010. Input value of the rate is then the value of the government bond rate at the particular day that is under investigation.

3.4 Estimating the Option Value

In this study, we are pricing an average price Asian option, thus the index option price is valued using the applied Black and Scholes futures option pricing model. The Black and Scholes formula for the price at time zero of a futures option is as follows:

$$c = e^{-rT}(FN(d_1) - KN(d_2)), \tag{2}$$

where *F* is current value of the future or index, *K* is exercise price, *r* is risk free interest rate continuously compounded, *T* is time to expiration of the option in years and *N(.)* is cumulative normal density function. The d_1 and d_2 comes from the equations:

$$d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

where σ is yearly standard deviation of percentage changes in stock or index prices.

On each day that is under investigation, values for the above inputs must be determined to value the call option component. However, because the common feature of index-linked bonds is that their payoff depends on average price of the underlying index, the Black and Scholes model must be widened to consider Asian options. This can be done only by assuming that underlying asset price is log-normally distributed and average price of the underlying is a geometric average of the underlying. According to Hull (2003), the geometric average options can be treated like a regular option when the expected growth rate is set equal to 0.5 * (r - g - $\sigma^2 / 6$) rather than (r-g) and volatility is set equal to $\sigma / \sqrt{3}$ rather than σ. Therefore with these parameters the dividend yield equal to

$$r - \frac{1}{2}(r - g - \frac{\sigma^2}{6}),$$

where g is expected growth rate. Considering the newly issued Asian option that provides payoff at time *T* based on the arithmetic average between time zero and time *T*. Now it is possible to calculate the two first moments of the probability distribution assuming that distribution to be lognormal. These two moments M_1 and M_2 can be shown to be

$$M_1 = \frac{e^{(r-g)T} - 1}{(r-g)T} S_0$$

$$M_2 = \frac{2e^{[2(r-g)+\sigma^2]T} S_0^2}{(r-g+\sigma^2)(2r-2g+\sigma^2)T^2} + \frac{2S_0^2}{(r-g)T^2}\left(\frac{1}{2(r-g)+\sigma^2} - \frac{e^{(r-g)T}}{r-g+\sigma^2} \right)$$

If we assume that the average price is log-normal, we can regard an option on the average as like an option on a futures contract and use Black and Scholes equation (2) with

$F_0 = M_1$ and $\sigma^2 = \frac{1}{T}\ln\left(\frac{M_2}{M_1^2}\right)$

3.5 Constructing Index-Linked Bond

It is possible to create an investing strategy that replicates the index-linked instrument. This can be done by distributing an instrument to option-like component and to bond-like component and then combining these parts.

Table 1. An investment strategy replicating index-linked bond.

This table presents the cash flows to investor at moments of t=0 and t=T. Index-linked bond is constructed by combining fixed bond and call option. In the table q is guaranteed capital return percent for the bond principal, E is subscribed capital, r is annualized risk-free rate for time to maturity added the issuer's risk premium, T is time to maturity of the bond in years, S_T is the value of the index at the end of the period, S_0 is the exercise value of the index at t=0 and c is the value of the call option.

Instrument	t = 0	t=T
Fixed bond	$-\frac{q}{(1+r)^T}\times E$	$+q\times E$
Call option	$-\frac{E}{S_0}\times c$	$\frac{E}{S_0}\times(S_T - S_0), S_T \geq S_0$
Combination	$-E\times(\frac{c}{S_0}+\frac{q}{(1+r)^T})$	$+q\times E, S_T < q\times S_0$ $+\frac{S_T}{S_0}\times E, S_T \geq q\times S_0$

In Table 1 we have presented the way to calculate cash flows to index-linked bond at *t=0* by investing the sum $q * (1+r)^{-T} * E$ to fixed bond and by buying E / S_0 call options with end-value of the index as an underlying. At *t=T* the positive cash flows consist of principal and the positive difference between the index end-value and the exercise value. Table presents all cash flows to investor at moments *t=0* and *t=T*. Because the investment strategy presented in Table 1 replicates perfectly the negative cash flows paid by investor at the moment *t=0*, the price of the instrument is the combination of fixed bond and call option.

The price of equity-linked instrument can be written as follows:

$$P = C + \frac{q}{(1+r)^T} = CallOption + Bond \quad (3)$$

where P is the price of the index-linked bond, C is value of the call option, q is promised return percent for the principal, r is annualized risk-free rate for time to maturity added the issuer's risk premium and T is time to maturity of the bond.

It is possible to calculate the minimum value of the index-linked bond from the Equation (3). It is $P \geq q / (1+r)^T$ because index-linked bond guarantees the return percent *q* for the principal even when the option is worthless.

4 EMPIRICAL RESULTS

The empirical setting of this thesis is based on breaking securities down into their components. The first part is to determine the present value of zero-coupon bond by discounting the principal with appropriate interest rate and to determine the value of options to provide upside potential promised. Then we examine the price behavior of these components. In the second part, we will combine these two phases to get the wholesale value of the instrument and it is possible to compare significance of theoretical values and real values of the instrument to examine whether market price is fair or not. Finally, sensitivity analysis tests how changes in different variables effect to the price of index-linked bond.

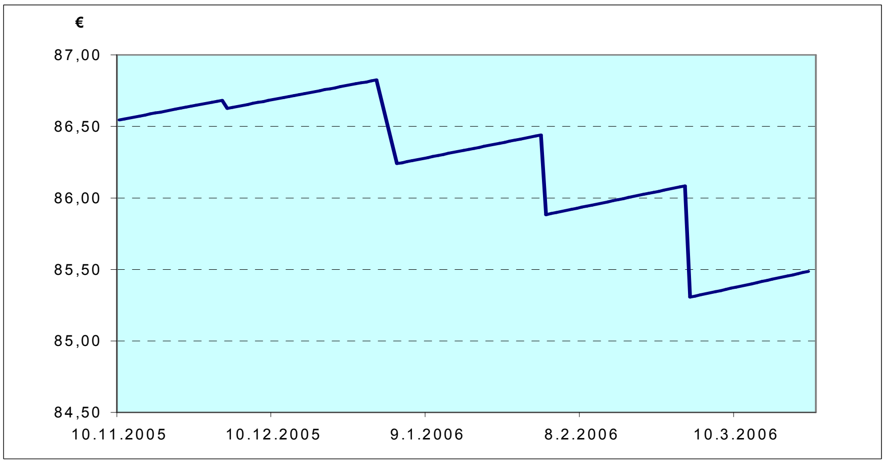
4.1 Price Behavior of Nordea All Stars Ekstra 36/05 Components

Figures 1 and 2 present the price behavior of the Nordea All Stars Ekstra option and bond components for the sample period. However, before turning to the results, it is instructive to note, that neither the bond nor option components are not directly observable in the market. What is observed is the sum of these components. As Figures 1 and 2 indicates, the implied option is the more volatile of the two index-linked bond components. Even though this higher volatility will result in the option component being more sensitive to pricing errors, Figure 1 indicates that the option component measures, on average, only little more than 15 % of the total index-linked bond value. Thus, pricing the bond component is also very important, since a pricing error in bond will make many times bigger impact on the index-linked bond value than the same option pricing error will have.

Figure 1. Price behavior of the option price for 10.11.2005 - 24.3.2006.

The most significant factors explaining the behavior of option component price are changes in the underlying index and changes in ex-post standard deviation. Figure 1 presents that option value has increased during the period being a consequence of about 11 % increase in the index value. Correspondingly, the bond price reductions are basically consequence of increase in monthly average discount rate. It is considerable that the increase in option price covers the decrease in bond price and leads to price increase in the index-linked bond.

Figure 2. Price behavior of the bond price* for 10.11.2005 - 24.3.2006.



*Bond prices are calculated using average monthly yield of the Finnish Government benchmark bond.

Figure 2 presents that the value of the zero-coupon part changes when the interest rate changes. Thus, the buyer of the structured bond has some common interest rate risk. However, this risk covers only the bond part of the product, not the whole product. It is also considerable that the significance of the interest rate risk can change when the relations of the component values change.

4.2 Pricing Results

Considering previous characteristics, it is suitable to refer to the relative pricing differences between theoretical values and real prices of the structures product.

$$\Delta P = \frac{P_{market} - P_{model}}{P_{model}} \quad (4)$$

Equation (4) entails a test of the pricing effectiveness of theoretical model of the index-linked bond. If the difference is negative, then the investor gets a better deal by buying the structured product than buying it separately in the underlying markets. If the difference is positive, then investor would achieve the same payoff at lower cost if he implemented the replicating strategy in the underlying markets. Table 2 presents statistical comparisons between the market prices and the theoretical values of the Nordea All Stars Ekstra. To allow for comparisons of prices across Euro levels that may differ significantly, the difference statistic is computed on a percentage, rather than absolute, basis. Significance is tested using *paired t-test.*

Table 2. Differences Between Nordea All Stars Ekstra Market and Model Prices

Instrument	Average Market Price	Average Model Price	Average Difference	Minimum Value
Nordea All Stars Ekstra[a]	105.97	102.32	3.56 % *	86.15
Option component	-	16.11	-	0
Bond component	-	86.15	-	86.15

[a] Market price of the Nordea All Stars Ekstra as well as model price in the Table is average price from the period 9.12.2005 – 17.3.2006. All model prices are calculated to concern the same period.
* Significantly different from 0 at the 5 % level.

On Table 2 we can see statistical comparisons between the market and theoretical values of the index-linked bond and theoretical values of the option and bond components separately. The market price represents observed price in the markets using mid-quotes between the known bid-ask spread. Model price is the value of the index-linked bond constructed by replicating the security with bond and option components and it is calculated from Equation (3). Average difference is calculated using Equation (4). Model and market price averages both are calculated using 14 observations from the given period. The emission course of the index-linked bond given by our model is also suitable to report being 98.84.

The results in Table 2 indicate that the model has a tendency to underestimate actual index-linked bond prices. Difference proves to be significant at the 5 % confidence level (Appendix C). It would be appear that pricing differences may occur primarily in the option component followed from complicated structure of the option component. It is also obvious that historical 120 day volatility might not be the most exact one to price this exceptional long maturity option. On the other hand there is evidence that ex-post volatility does a better job than implied volatility in estimating the true volatility.[4]

The bond value in Table 2 is calculated from Equation (1) and the option value from Equation (2) with Asian option pricing model. Minimum value indicates that when option component is worthless, the price of the index-linked bond is the value of the bond component because option value cannot be under 0. All in all, the conclusion based on values given by our theoretical model is that it would be rational for investor to implement the replicating strategy and buy a zero-coupon bond and call-option separately, assuming that they existed in the markets.

4.3 Sensitivity Analysis

Because the pricing results presented in Table 2 are influenced by the magnitude of the inputs required, it is instructive to examine the sensitivity of these results to changes in the input values. The risk of the index-linked bond is formed of the risk factors affecting the option and bond component. Table 3 presents sensitivity tests for the impacts of changes in the volatility on index-linked bond option component values and the impacts of changes in the yield on bond component values. These inputs represent the two primary unobservable factors, which are open to various interpretations, involved in pricing an index-linked bond. Practically, yield represents the interest rate risk and volatility the market risk. Thus, the market volatility raises the uncertainty which leads to increase in the option price.

[4] Chen K. & Sears S., 1990. Pricing the SPIN. *Financial Management*.

Table 3. Sensitivity analysis of the theoretical index-linked bond values

Table presents separately bond component sensitiveness to changes in annual yield, and option component sensitiveness to changes in annual volatility as well as to changes in the index value.

	Volatility (per year) [b]					Yield (per year) [c]				
Index value [a]	**0.09**	**0.105**	**0.12**	**0.135**	**0.15**	**2%**	**2,5%**	**3%**	**3,5%**	**4%**
130	18.90	20.10	21.34	22.64	23.97	90.57	88.39	86.26	84.20	82.19
125	15.44	16.73	18.05	19.40	20.76	90.57	88.39	86.26	84.20	82.19
120	12.27	13.63	14.99	16.36	17.74	90.57	88.39	86.26	84.20	82.19
115	9.45	10.82	12.20	13.56	14.93	90.57	88.39	86.26	84.20	82.19
110	7.00	8.34	9.68	11.00	12.35	90.57	88.39	86.26	84.20	82.19
105	4.96	6.21	7.47	8.74	7.93	90.57	88.39	86.26	84.20	82.19
100	2.10	3.02	4.01	5.05	6.12	90.57	88.39	86.26	84.20	82.19

[a] Average S&P All Stars Europe index value from 10.11.2005 to 24.3.2006 is 122.00.
[b] Average volatility of S&P All Stars Europe index is for about 11 % per year.
[c] Average Finnish Government 5-year benchmark bond yield from 10.11.2005 to 24.3.2006 is 3.10 %.

Both panels in Table 3 present the values of the option and bond components across various levels of the S&P Europe All Stars and selected input. First panel examines the option value across the different values of the volatility and the index and second panel examines the sensitivity of the bond value to changes in the index and the yield to maturity. The results in Table 3 illustrate that while changes in volatility can have significant impacts on the value of the option component, relatively smaller changes in the yield can have greater impacts on the bond value. For example, if index is selling for 120, and volatility changes from 0.12 to 0.135, increasing for about 12.5 %, the value of the option changes about 9 %. However, if the yield increases from 3 % to 3.5 %, increasing 17 %, the bond value changes about 2.5 %. It should be noted, that the changes in the index do not have an effect on the bond value, thus the primary factor in the bond price is yield. All in all, it would appear that the critical unknown variables used in estimating the index-linked bond value are the yield and the volatility and at the same time they are the biggest risk elements.

4.4 Evaluating the Costs and Benefits to Issuer

The issuer faces potential costs at the maturity day of the index-linked bond if options are in-the-money[5] while expiring. According to the terms of the issue, each bondholder receives the amount of options which is specified by the particular multiplier. In this case, investing to Nordea All Stars Ekstra, the multiplier is 1 for every 1,000 € in par value bonds held. With a total issue of 35,000,000 € there are 35,000 bonds and same amount of options. When the S&P All Stars Europe index is above 116.57 at maturity, which is the index value at bond departure day, Nordea must pay compensation for the investor. The higher the index moves at maturity, the more Nordea has to repay the present value of the capital received at the time of issuance. However, Nordea has index upside protection, which means that they are hedging the position with index derivatives in the derivative markets.

The principle of the index-linked bond is very similar with normal corporate bonds. It provides a specified amount of loan for issuer and a possibility to earn extra profit for an investor. Strongly simplified definition to index-linked bond is that its risk is located between bond and stock investment. On October 31, 2005 Nordea issued the Nordea All Stars Ekstra at par and received 35.000.000 exclusive of flotation costs. Nordea was therefore able to issue zero-coupon bonds with call options at par. Thus, the benefit of the bond to Nordea is measured by the value of the option, which represents the present value of the interest cost savings on the bond over the five-year life of the issue. The biggest profits for banks are formed of marginal which is difference between the principal paid by investor and the present value of that principal. However, the real profit is gained if bank is able to buy option structure with lower costs than the reached marginal is. The best situation is that if bank is able to hedge its position towards some of its open positions because then the marginal is doubled. It is also considerable that an Asian option structure guarantees the better marginal for bank because it is usually more profitable.

[5] In-the-money option is either a call option where the asset price is greater than the strike price or a put option where the asset price is less than the strike price.

5 CONCLUSIONS

The purpose of this thesis was to examine the properties and fair pricing of equity-linked instruments. We also concentrated on synthetic pricing of these instruments and finally we carried out some sensitivity analysis to express some interpretation and estimation issues related to pricing and risks. The objectives were acquired through theoretical and empirical examination of Nordea All Stars Ekstra 36/05 index-linked bond pricing process.

We can draw conclusions from the popularity of structured products that they offer new investment opportunities to an investor. However, at the same time it can be argued that popularity may be caused by inadequate understanding of risks and marginals of the bonds. Structured products can carry very complicated structures and conditions but after all, pricing is very often divided into pricing the option-like component and the bond-like component. Thus, different structures affect the way how to price option- and bond-components rationally. In the light of this study, it can be said that index-linked bonds provide an interesting possibility for investor to invest at the same time in reasonable safe instrument and in speculative upside potential providing instrument. However, different conditions and structures make it somewhat complex for investor to perceive the real potential. Therefore, determining the fair price is not necessarily very obvious process.

Our study suggests that index-linked bonds are priced a little bit over their theoretical value in Finland, which is in line with previous studies. However, it should be noted that this study examined only one product and we cannot make very strong generalizations. Our additional investigation examined the price behavior of bond and option components. It appeared that the most important single variables affecting the index-linked bond price are volatility for the option and yield for the bond. Despite the more volatile nature of the option component, the bond component forms the biggest part of the index-linked bond value and therefore discount yield have an obvious impact on the present value of the bond. Therefore, the interest rate risk is usually the biggest risk faced by an investor. Finally, we made a sensitivity analysis to become aware how the price of the particular index-linked bond changes when yield or volatility change and we suggest that sensitivity analysis is very sensible way to determine the risk level of the single index-linked bond to meet investor's risk aversion.

Index-linked bond examined in this study was chosen to be enough simple to make some conclusions about fair pricing. However, corporations have become innovative in the introduction of new financing instruments and that have also attracted the interest of academic researchers. For example, auction rate preferred stocks, puttable stocks, yield curve notes and index-linked linked bonds containing complex conditions have been under deeper estimation. All in all, these new corporate related securities are designed to attract new investors by providing additional potential payoffs that are not present in other securities, while at the same time covering some of the financing costs of the issuer. Therefore, it is not the easiest case for investor to determine which securities are priced fairly. Although, it may look like that index-linked bond does not carry a downside-risk, the value of the bond changes, and therefore the market risk is not limited to guaranteed capital.

For further research, we suggest to examine the pricing of index-linked bonds with wider variety of instruments and with different methods to count volatility. It would be also appropriate to test models that estimate the fair value of instruments that contain different basket options, quanto-options and more complicated structures. For example, Monte Carlo simulation could be suitable method to do this. From the bank point of view, it would be also useful to examine the construction of different option structures when hedging index-linked bond positions.

REFERENCES

Ackert L. F., Tian Y. S., 2000. Efficiency in index options markets and trading in stock baskets. *Journal of Banking and Finance*, 25 (2001), 1607-1634.

Alobaidi G., Mallier R., 2002. Pricing Equity-Linked Debt Using the Vasicek Model. *Acta Mathematica Universitatis Comenianae*,. LXXI, 2, 211-220. Canada, Peterborough: Trent University.

Asbjörn T. H., Jörgensen P. L., 1997. Analytical Valuation of American-style Asian Options. *Conference on Quantitative Methods in Finance*, Working paper.

Brooks R., 1996. Computing yields on Enhanced CDs. *Financial Services Review* , 5(1), 31-42.

Campbell J. Y., Lo A. W., MacKinlay C. A., 1997. The Econometrics of Financial Markets, 2nd Edition. Princeton: Princeton University Press.

Chen K., Sears S., 1990. Pricing the SPIN. *Financial Management*, Summer 1990.

Chen K. C., Taylor J. C., Wu L., 2001. Pricing Market Index Target-Term Securities. *Journal of Financial Management Research*, 14(1), 11-18.

Edleson M. E., 1993. Equity-Linked Products: Having your cake and eating it too. *Journal of Financial Planning*, January 1993.

Fabozzi F., 2000. The Handbook of Fixed Income Securities. New York: McGraw-Hill.

Hull J. C., 2003. Options, Futures and Other Derivatives, 5th Edition. New Jersey: Prentice Hall.

Karoui N. E., 1998. Robustness of the Black and Scholes Formula. Mathematical Finance, 8(2), 93-126.

Li S., 2005. A New Formula for Computing Implied Volatility. *Applied Mathematics and Computation*, 170(2005), 611-625.

Milevsky M. A., 1997. The Optimal Choice of Index-Linked GICs: Some Canadian Evidence. *Financial Services Review*, 6(4), 271-284.

Niskanen J., & Niskanen M., 2003. Yritysrahoitus. Helsinki: Edita.

Reynaerts H., 2006. Bounds for the price of a European-style Asian option in a binary tree model. European Journal of Operational Research, 168, 322-332.

Roberts G. S., Vijayraghavan V., Antablian S., 2002. Stock Index-Linked Debt and Shareholder Value: Evidence from the Paris Bourse*. European Financial Management*, 8(3), 339.

S & P All Stars Europe Index Methodology. [Standard & Poo's web page, retrieved April 9, 2006].
From:http://www2.standardandpoors.com/servlet/Satellite?pagename=sp/Page/IndicesBrowseMethodologyPg&r=1&l=EN&b=4&s=34&ig=357&i=159&si=&d=&xcd=SPALS&f=3

Stoimenov P. A., Wilkens S., 2005. Are structured products fairly priced? An analysis of the German market for equity-linked instruments. *Journal of Banking and Finance*, 29, 2971-2993.

Wilkens S., Erner C., Röder, K., 2003. The Pricing of Structured Products – An Investigation of the German Market. *University of Munster*, Working paper.

APPENDIX A: S&P All Stars Europe index description

The S&P All STARS Europe is designed to provide exposure to a basket of the highest ranked stocks in the S&P Europe 350 Index according to STARS. It is equally weighted and re-balanced semi-annually to take into account any changes in ranking. STARS was launched in 1987 for U.S. stocks and in 2002 for European and Asian stocks. It is a qualitative evaluation based on an analyst's determination of future appreciation potential of a specific common stock relative to its relevant S&P benchmark index based on a 12-month time horizon. The overarching investment philosophy driving the methodology is Growth at a Reasonable Price. Rankings range from 5 (Strong Buy) to 1 (Strong Sell). (S & P All Stars Europe Index Methodology)

On the Reference Date immediately preceding the relevant Rebalance Dates, to be eligible for selection for the S&P All STARS Europe Basket, a stock must comply with the following criteria (in which case, it will be an "eligible stock"):

1. It must be a member of the S&P Europe 350 Index

2. It must have a ranking of 4 STARS or higher

3. The issuer of the stock shall have a market capitalization equivalent at least to U.S. $3 billion.

4. The arithmetic average of the daily trading volume of the stock over the 6 months prior to the relevant Reference Date shall be equivalent at least to U.S. $5 million.

5. The average One-Month Historical Volatility of such stock is less than 80% in the six months prior to such Reference Date.

"Reference Date" means the first business day of the months of June and December. "Rebalance Dates" means the 10 Scheduled Trading Days prior to and including the third Friday of the months of June and December. The last of such Scheduled Trading Days shall be the "Last Rebalance Date", provided that if such day is a Disrupted Day the Last Rebalance Date shall be the next succeeding Scheduled Trading Day which is not a Disrupted Day. (S & P All Stars Europe Index Methodology)

APPENDIX B: Central conditions of Nordea All Stars Ekstra[6]

General information:

Issuer: Nordea Pankki Suomi Oyj

Capital of the loan: Max. 35,000,000 €

***Emission form*: Book-entry form**

***Privilege position*: Same as all unsecured commitments of the issuer**

Bond unit: 1,000 €

Minimum notation:1,000 €

Number of the loan: 36/2005

Emission day: 31.10.2005

Repayment day: 25.11.2010

Repayment: Same as principal

Interest: Index compensation. The capital does not accumulate any interest but at the repayment day there will be index compensation for every bond, which is 100 % of the increase in the reference index in terms and conditions of the bond. If the change in the reference index is negative or 0, there will be no compensation.

The change in the reference index is calculated by comparing the exercise value of the index to end-value of the index. Exercise value is the closing price of the index at the start day. End-value is a quarterly valued arithmetic mean of the closing prices of the index from three last years of the maturity.

Issuer's right to call the bond before maturity: No

Emission information

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[6] More accurate conditions are available in www.rahoitustarkastus.fi/NR/rdonlyres/ 2F9C5352-55D8-4D37-967C-5DD5151C2774/0/Nordea_jvk_36_05.pdf

Notation rights: Notation rights are not limited

Notation period: 31.10.2005-25.11.2005

Emission course: Changing, for about 105 %

Collateral: No

Notation commitments: No

Exchange listed: Yes

ISIN-code: FI0003019521

Tax at source: 28 %

Estimation of the accumulated capital amount and usage the capital: Maximum 35,000,000 € from which 0,1 % is emission costs. Bond is a part of the fund-raising of the issuer.

Index Compensation

Index compensation is calculated as follows:

Change in the reference index * the principal of the bond * 1.00 (expected return multiplier)

If the change in the reference index is negative or 0, there will be no compensation.

Reference index is taken from S&P All Stars Europe (Bloomberg: SPALSTEU)

Change in the reference index is calculated as follows: (A1-A0)/A0, where A1 is the *closing value* of the index and A0 is *the departure value* of the index.

Closing value is an arithmetic mean of closing prices from particular valuation days.

Departure value is the closing price of particular departure day, which is 29.11.2005.

Valuation days are every third month's 10^{th} day from the three last years of the maturity. The previous period takes from November 2007 to November 2010.

APPENDIX C: Paired t-test statistics

Paired samples t-test

	Mean	Std. deviation	Std. error mean	t	df	Sig. (2-tailed)
Real - Model	3.65	1.22	0.33	11.17	13	.000