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Lappeenranta **University of Technology**  
LUT School of Business

Bachelor's Thesis, Financial Management

**PREDICTIVE ABILITIES OF COMMON MODELS OF VOLATILITY – AN  
EMPIRICAL TEST WITH THE FINNISH MARKET DATA**

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## 1 INTRODUCTION

Aftermath of 2008-2012 financial crisis has shown that volatility estimations and ability to predict it is of utmost important to financial intermediates and all other actors in financial markets. Many a topic in finance and financial markets need accurate volatility estimates, e.g. derivatives pricings, value-at-risk and asset pricing to name a few. Also academic studies benefit from accurate volatility estimates; in case of Finland, empirical tests with Finnish market data add understanding about financial fundamentals and provide incentives to new lines of study.

A number of papers have studied predictive abilities of volatility models (see, e.g., Alberg, Shalit and Yosef (2008), Berglund, Hedvall and Liljeblom (1990), Curto, Pinto and Tavares (2009), Maukonen (2002), Walsh and Tsou (1998), Wilhelmsson (2006) and especially review from Poon and Granger (2003)). Using data for the period May 1988 to April 1999, Maukonen (2002) finds that future volatility can be modelled and more complex models (EWMA and GARCH) outperformed naïve ones. Results from Maukonen are parallel to Berglund et al (1990) in case of monthly GARCH performance and parallel to Walsh and Tsou (1998) in case of weekly EWMA.

However, these findings are far from conclusive. Furthermore, most of the previous studies have been conducted from international and U.S. data. There are only a few previous studies using Finnish market data to study predictive abilities of different volatility models. Studies of Berglund et al (1990) and Maukonen (2002) are conducted with data from the start of the 70's ending to 1987 and from 1988 to 1999, respectively. Many a change has happened in Finnish stock market in recent years and a new study with a newer data would be necessary. In addition, these studies have not tested whether predictive abilities of GARCH model can be improved by using different statistical distribution.

This bachelor thesis provides tests of volatility estimation using Finnish Stock Market Index (OMXHPI). Estimates are provided for daily performance. Study starts with a

naïve random walk process, then moves to Exponentially Weighted Moving Average (EWMA) and GARCH models and finally extend basic GARCH model to different statistical distribution (Student's  $t$  distribution) to test whether more complex models dominate simpler ones. In this paper, the study by Maukonen (2002) is used as a benchmark and is tested whether EWMA and GARCH still dominate even in 2000s. It can be expected that more complex models are better able to predict volatility and furthermore predictive abilities should rise when normal distribution is abandoned (Fama 1965, 42; Mandelbrot 1963, 395). From these insights rises the paper's main research problem and its sub problems. Paper's main research problem can be stated as follows:

*Are models that take account heteroskedasticity better than those with homoskedastic assumptions for predicting volatility on the Finnish stock market in period 2000-2013?*

And sub problems can be stated as follows:

*Is predictive ability of GARCH model increased when using Student's  $t$  distribution instead of normal distribution?*

*Are predictive abilities of applied models robust with in-sample when available data is divided to pre-financial crisis period (2000-2007) and financial crisis period (2008-2012)?*

The rest of the paper is organized as follows. Section 2 presents the theoretical framework of volatility and ways to forecast it as well as presenting applied models. Section 3 discusses the data, its properties and methodologies for forecasting and their evaluation. Section 4 presents the empirical results. Finally, section 5 concludes and gives suggestions to further studies.

## 2 THEORETICAL FRAMEWORK

### 2.1 Definition of volatility

This chapter gives an insight to the theoretical framework of volatility and ways to predict it. Applied models and mathematical formulations behind them are presented. Derivation of these models is beyond the scope of the work but references to additional sources are submitted.

Volatility (the second moment of return) is defined as a standard deviation of the return and is usually interpret as an uncertainty of the return of a single asset or portfolio (Poon and Granger 2003, 480). In general terms it can be stated that  $\sigma\sqrt{T}$  is equal to standard deviation of the total return earned in time T. To better understand volatility a return of an asset must first be introduced. Mathematically the return during  $i^{th}$  interval can be expressed as follows (Fama 1965, 45; RiskMetrics™ 1996, 46):

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right), \quad (1)$$

where  $i = 1, 2, \dots, n$ .

Logarithmic returns are used because this way stationary process is obtained and making predictions is possible (RiskMetrics™ 1996, 51-54). From the return a standard deviation (volatility) can be estimated. Using usual estimate  $s$  ( $\sigma$ , sigma) of the standard deviation of the  $u_i$  is

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2}, \quad (2)$$

where  $\bar{u}$  is the mean of the  $u_i$ .

### 2.2 Applied models

Models that are selected for testing are mostly from the study by Maukonen (2002) with an expansion of GARCH consisting different statistical distribution. Another

argument for the models could be the historical perspective of development of volatility models. Firstly, Fama (1965), Kendall (1953) and Roll (1984) showed that stock prices move randomly. However, as stated by RiskMetrics™ (1996, 78) random walk should only be used as a rough approximation of financial markets. This is one of the reasons why researchers have been looking for more sophisticated methods. One of these is EWMA and for a many the methodology developed by RiskMetrics™ (1996) is synonymous to EWMA<sup>1</sup>. However, these models assume constant variance and empirical evidence does not support this assumption<sup>2</sup>. More sophisticated models such as GARCH (see Engle 1982 for first presentation of model containing an autoregressive conditional heteroskedasticity and Bollerslev 1986 its generalization) are able to handle heteroskedasticity and thus should provide better estimation of volatility.

### 2.2.1 Random walk process

Random Walk Hypothesis<sup>3</sup> can be considered as a starting point to model financial markets. Lognormal random walk process from  $p_{t-1}$  to  $p_t$  can be expressed as follows (RiskMetrics™ 1996, 50):

$$p_t = \mu + p_{t-1} + \sigma \varepsilon_t \quad \varepsilon_t \sim IID N(0,1), \quad (3)$$

where  $\mu$  is the fixed parameter,  $p_{t-1}$  last period's lognormal price,  $\sigma$  is standard deviation of those returns and  $\varepsilon_t$  is a normally distributed random variable, which is identically and independently distributed. We assume  $\mu$  to be zero and this way model turns to more compact form:

$$p_t = p_{t-1} + \sigma \varepsilon_t \quad \varepsilon_t \sim IID N(0,1). \quad (4)$$

Standard deviation ( $\sigma$ ) is assumed constant and therefore next period volatility is estimated with lagged volatility:

$$\sigma_t^2 = \sigma_{t-1}^2. \quad (5)$$

---

<sup>1</sup> For the sake of simplicity rest of the study RiskMetrics™ and EWMA are used as a synonym. We will also address these consistently as EWMA.

<sup>2</sup> Discussion dates back to Mandelbrot (1963) and Fama (1965). Merton (1980) criticized lack of existing models' ability to take into account heteroskedasticity.

<sup>3</sup> See first concepts from Bachelier (1900) and Cootner (1964).

Above equations (4) and (5) assumes that price changes have constant variance (i.e.,  $\sigma$  will not change over time). As can be seen from Fama (1965, 35) and RiskMetrics™ (1996, 54) identically and independently distributed price change gives rise to this assumption. Identically distributed mean and variance can be described as unchanging over time or homoskedastic. Moreover, independence means that the values of returns are completely unrelated to each other or more formally:

$$\Pr(x_t = x | x_{t-1}, x_{t-2}, \dots) = \Pr(x_t = x),$$

where the term on the right hand side is the unconditional probability that the price change during time  $t$  will take the value  $x$  and the term on the left hand side is the conditional probability that the price change will take the value  $x$  (Fama 1965, 35).

## 2.2.2 Exponentially Weighted Moving Average

As stated above for the EWMA model in this study, a methodology from RiskMetrics™ is used. Firstly, exponentially weighted (standard deviation) volatility estimator can be stated as:

$$\sigma = \sqrt{(1 - \lambda) \sum_{t=1}^T \lambda^{t-1} (r_t - \bar{r})^2}, \quad (6)$$

where  $\lambda$  ( $0 < \lambda < 1$ ) and is often referred to as the *decay factor* (RiskMetrics™ 1996, 78). Decay factor as  $\lambda = 0.94$  is used for this thesis. This is estimated by RiskMetrics™ (1996) and own estimates are beyond the scope of this paper. However, for predictions estimator needs to be expressed a bit differently. For a one-step-ahead volatility estimation in RiskMetrics™ methodology is expressed as follows:

$$\sigma_{1,t+1|t}^2 = \lambda \sigma_{1,t|t-1}^2 + (1 - \lambda) r_{1,t}^2, \quad (7)$$

where subscript “ $t+1|t$ ” underscores the fact that volatility is time-dependent. Using above models from RiskMetrics™ (1996, 78-87) estimating a one-step-ahead volatility is possible.

### 2.2.3 Generalized Autoregressive Conditional Heteroskedasticity

Until this point models have assumed homoscedasticity and this assumption have showed to be inadequate in previous literature<sup>4</sup>. First model to allow variance to change over time is ARCH introduced by Engle (1982) and its generalization by Bollerslev (1986). This thesis concentrates to GARCH model presented by Bollerslev (1986) because it is more widely used mostly through its lag structure which is much more flexible. Firstly, GARCH (p,q) process from the theoretical point of view is presented and then showed GARCH (1,1) specification which is used in this thesis.

Theoretical parameterization of GARCH (p,q) process can be stated as follows (Bollerslev 1986, 310):

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \varepsilon_{t-j}^2 - \sum_{j=1}^p \beta_j v_{t-j} + v_t \quad (8)$$

$$v_t = \varepsilon_t^2 - h_t = (\eta_t^2 - 1)h_t, \quad (9)$$

where

$$\eta_t \sim IID N(0,1). \quad (10)$$

However, GARCH (p,q) can also be stated more practically oriented way, which is more easy to use for estimating purposes as shown by Bollerslev (1986, 309) and Maukonen (2002, 818). Formally:

$$u_t = \mu_0 + \mu_1 u_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t | \psi_{t-1} \sim N(0, h_t); \quad (11)$$

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-1} = \alpha_0 + A(L)\varepsilon_t^2 + B(L)h_t, \quad (12)$$

where

$$\begin{aligned} p &\geq 0, & q &> 0, & \alpha_0 &> 0, \\ \alpha_1 &\geq 0, & i &= 1, \dots, q, & \beta_1 &\geq 0, & i &= 1, \dots, p. \end{aligned}$$

GARCH (1,1) model can be obtained by setting  $p = 1$  and  $q = 1$ :

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<sup>4</sup> Once again see Fama (1965), Mandelbrot (1963) and Merton (1980) for the start of the discussion. Engle (1982) and Bollerslev (1986) showed first models to allow constant variance to change over time. For a models to handle asymmetric information see e.g. Nelson (1991) and Engle & Ng (1993)



$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}. \quad (13)$$

Above model is also used to predict a one-step-ahead estimate of volatility. However, for the GARCH process to be usable, conditional distribution function to  $\varepsilon_t$  need to be defined. With GARCH (1,1) process the normal distribution is used following Maukonen (2002) but in the second model distribution changes to Student's  $t$ <sup>5</sup>. Formally,  $\varepsilon_t \sim t(v)$ , where  $t(v)$  refers to the zero-mean  $t$  distribution with  $v$  degrees of freedom and scale parameter equal to one as shown by Curto et al. (2009, 313)<sup>6</sup>.

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<sup>5</sup> From several papers which have studied different distributions see e.g., Bollerslev (1987), Curto et al. (2009), Heracleous (2003), Poon & Granger (2003) and Wilhelmsson (2006).

<sup>6</sup> Derivation of GARCH with Student's  $t$  distribution in Appendix 1.

### 3 DATA AND METHODOLOGY

This chapter presents the data which is used for estimations and introduces testing methodology. Firstly, descriptive statistics is showed and some preliminary tests to analyze the data. Then, testing methodologies and different ways to rank models are submitted. These ranks are used to determine which model's predictive abilities are the best in this particular sample.

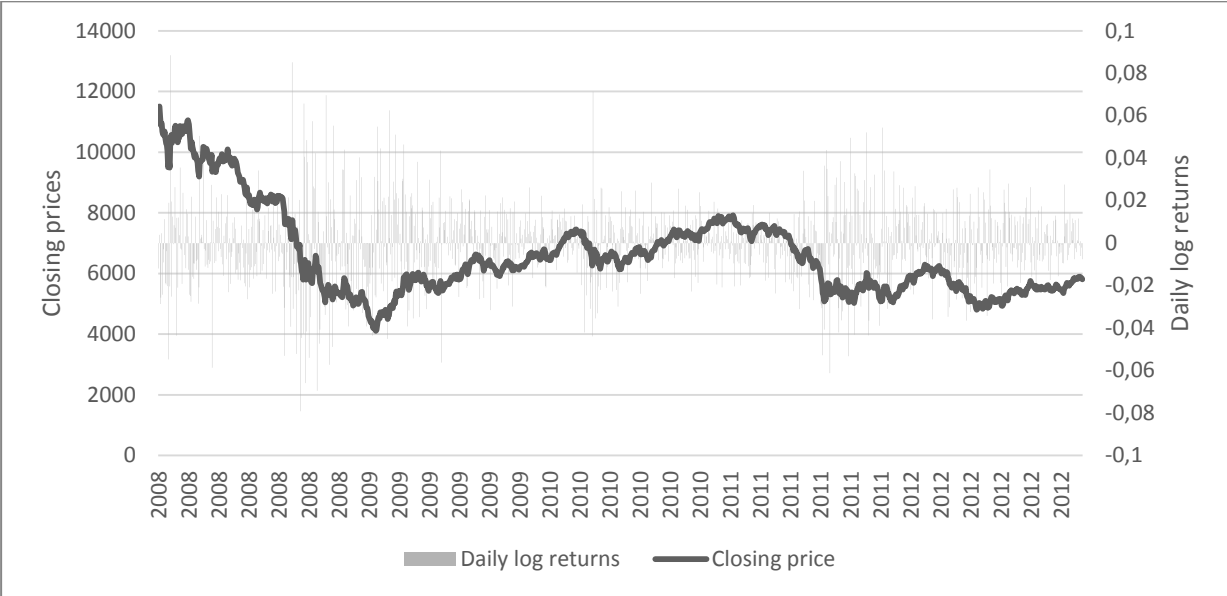
The OMX Helsinki (OMXHPI) is a share price index calculated on a daily basis. It consists of the 129 (as of 22 November 2013) traded stocks on the OMX Helsinki Stock Exchange which is a part of the OMX Group (Nasdaq OMX 2003). This particular index is used because of the liquidity problems and lack of the sufficient time-series with smaller indices. Logarithmic returns are calculated from the index as stated above and these returns are used throughout the paper. To introduce data, below in Figures 1 and 2 is presented daily logarithmic returns and closing prices of the index and in Table 1 basic descriptive statistics about the time-series.



**Figure 1. Daily logarithmic returns and closing prices of OMX Helsinki index from 3 January 2000 to 28 December 2007.**

As can be seen from Figure 1 above daily returns have fluctuated at start of the period greatly and then calmed down later years before the financial crisis. Fluctuations stem from the dot-com bubble which burst at the start of the new

millennium. This can also be seen from decline of index's closing prices in the same period when index dropped from its peak price of around 18 000 to a bit under 5 000. Fluctuations or volatility lessened from middle of the 2002 onwards and stayed moderate all through to end of 2007. Also growth trend of the index was positive and Finland, among other countries, enjoyed the time called Great Moderation<sup>7</sup>. However, from 2008 forward fluctuations of closing prices have increased and closing prices have lost their distinctive trend as showed in Figure 2.



**Figure 2. Daily logarithmic returns and closing prices of OMX Helsinki index from 2 January 2008 to 28 December 2012**

Figure 2 presents the above mentioned changes in volatility. Financial crisis increased fluctuations in returns and changes in closing prices. These were expected results and support theory of absence of constant volatility and increases trust that more complex models will have greater predictive abilities. Statistical properties of the time-series revealed anything exceptional which can be seen from Table 1 below.

<sup>7</sup> For a more discussion about Great Moderation see e.g., Bernanke (2004), Kim and Nelson (2004) and Stock and Watson (2002).

**Table 1. Descriptive statistics and preliminary tests for the OMX Helsinki index daily logarithmic percentage returns from 2000 to 2012**

Notes: J-B is the Jarque-Bera test to inspect normality of the series. The figures in parentheses are *p*-values for test statistics. B-G LM Test (1) is Breusch-Godfrey test for first lag. *P*-value in parentheses. Values which are statistically significant in 99 % confidence level are indicated with asterisk (\*).

	Daily Returns
<b>N</b>	3265
<b>Mean</b>	-0.03 %
<b>Median</b>	0.04 %
<b>Maximum</b>	14.56 %
<b>Minimum</b>	-17.42 %
<b>Standard deviation</b>	2.01
<b>Skewness</b>	-0.34
<b>Kurtosis</b>	9.54
<b>Jarque-Bera</b>	5886.505*
<b>Breusch-Godfrey LM Test (1)</b>	18.31*
<b>KPSS</b>	0.137*
<b>ARCH Heteroskedasticity</b>	37.98*

Table 1 shows descriptive statistics of the daily logarithmic returns. Daily returns were on average -0.03 per cent (annual mean -7.56 per cent), median was 0.04 per cent and daily standard deviation 2.01 (annual 31.91). However, daily returns have fluctuated between 14.56 to -17.42 per cent. Jarque-Bera test rejects the null hypothesis of normality and kurtosis is strongly positive. These observations are in line with the fact that daily returns in finance time-series have “fat tails”, i.e. too many large and small observation compared to normal distribution. Another result, which was well-documented and expected, was the rejection of null hypothesis concerning autocorrelation. Breusch-Godfrey test indicates a first order autocorrelation between errors and at the same time that there is a correlation between  $u_t$  and  $u_{t-1}$ . One required ability of the data is stationary in results which were tested with a unit root test (KPSS). KPSS was not able to reject null hypothesis ( $H_0$ : series is stationary) in results even though in index level series was non-stationary (unreported here). Finally, we tested whether residuals are homoscedastic or are volatility constant over time. As can be seen from Table 1 ARCH Heteroskedasticity Test rejected null hypothesis of homoskedastic residuals. As null hypothesis is rejected GARCH type models can be applied for the data.

Testing methodology follows the paper from Maukonen (2002). All four models generate a set of rolling one-step-ahead volatility estimates. Estimations are conducted in two different ways and with a different sample periods to test different predictive abilities as specified in research problems. The total sample of 3265 daily observations is firstly divided to two parts, first part is from beginning of 2000 to end of 2007 or pre-financial crisis era and second part is from start of 2008 to end of 2012 or financial crisis. Sample from 2000-2007 is used only for the model estimations and models' predictive abilities are tested on the second part. This way it is possible to test whether models which are estimated with pre-crisis data were able to predict increased volatilities of financial crisis. In the second case, the whole sample of 2000-2012 is used for model estimation. These newly estimated models are then used to estimate out of the sample volatilities (from 2 January to 29 October 2013, 210 observations) with longer time-series and at the same time with more information "loaded" to models. Using these two methodologies, 8 different estimations (four for the period from 2008 to 2012 and four for the period 2 January to 29 October 2013) are drawn for the comparison. To be capable of ranking predictive abilities of different models proper testing methods need to be defined.

With test of predictive abilities of models this thesis once again follows the paper by Maukonen (2002) and two alternative methods are used to rank different models. First method for evaluation is OLS regression-based efficiency test. A "true" volatility of time  $t$  as a depended variable and the estimation of volatility of time  $t$  as an independent variable is used:

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_t^{2*} + \varepsilon_t. \quad (14)$$

$R^2$  is used to assess models' efficiency and the model with the highest  $R^2$  will be best predictor. However, there have been controversies<sup>8</sup> between different authors about taking account biases in  $R^2$ . Because of this, two different methodologies are used for ranking criteria. For a second source of ranking, models will be used to calculate

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<sup>8</sup> Please see, Andersen and Bollerslev (1998), Pagan and Schwert (1990) and West and Cho (1995) for a necessary joint condition of  $(\beta_0, \beta_1) = (0,1)$  to be required. However, Taylor (1999) argues that previous is not necessary for volatility forecast to be unbiased.

four different error statistics<sup>9</sup>, namely RMSE, MAE, MAPE and HMSE<sup>10</sup>. Error statistics can be stated formally as follows:

$$RMSE = \sqrt{N^{-1} \sum_{t=1}^N (\sigma_t^2 - \sigma_{i,t}^{2*})^2}, \quad (15)$$

$$MAE = N^{-1} \sum_{t=1}^N |\sigma_t^2 - \sigma_{i,t}^{2*}|, \quad (16)$$

$$MAPE = N^{-1} \sum_{t=1}^N \left| \frac{\sigma_t^2 - \sigma_{i,t}^{2*}}{\sigma_t^2} \right|, \quad (17)$$

$$HMSE = N^{-1} \sum_{t=1}^N \left( \frac{\sigma_t^2 - \sigma_{i,t}^{2*}}{\sigma_{i,t}^{2*}} \right)^2. \quad (18)$$

Using these two approaches it can be determined whether more complex models are better to predict volatility, is it possible to estimate parameters from previous years' data to estimate volatility in crisis years and lastly test if change in distribution in GARCH model increases predictive ability.

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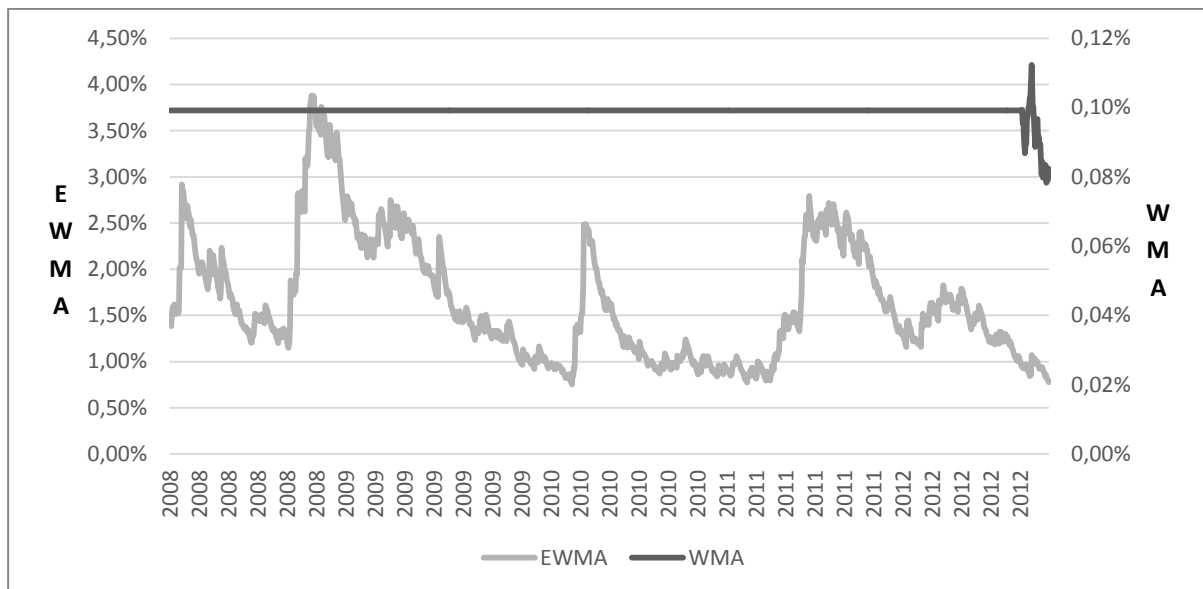
<sup>9</sup> Please see e.g., Bollerslev and Ghysels 1996, Diebold and Lopez 1996, Maukonen 2002 and Pagan and Schwert 1990 for more discussion about the error statistics.

<sup>10</sup> RMSE is an acronym from Root Mean Squared Error, MAE means Mean Absolute Error, MAPE stands for Mean Absolute Percentage Error and HMSE imply heteroskedasticity-adjusted MSE (Mean Squared Error).

## 4 RESULTS

Results are presented in two different tables. Table 2 show results when using error statistics, Table 3 show results from OLS regression. At first will be presented some general information to help to interpret the results.

Figure 3 plots equally and exponentially weighted moving averages of the daily volatility from the start of 2008 to the end of 2012. As can be seen from the figure EWMA estimates have fluctuated greatly as WMA (weighted moving average with constant weights) has been rather constant over time. It can be stated that EWMA is better estimation method for volatility as it describes changes in true volatility more accurately when compared to Figures 1 and 2 that shows great fluctuations of prices especially after 2008.



**Figure 3. Volatility estimates 2008-2012 using exponentially weighted moving average and weighted moving average.**

Figure 3 also indicates that models which do not expect constant volatility should possess better predictive abilities than models with constant variance as stated in chapter 2.2 above. This way it can be expected that GARCH models dominate in predictive abilities over random walk and EWMA.

Regression presented in (14) is estimated using OLS method with Newey and West (1987) heteroskedasticity and autocorrelation consistent covariance matrix estimator with lag=1. Reasons were similar to Maukonen (2002) as residuals were serial correlated with standard OLS estimation (reported under ARCH Heteroskedasticity Test and Breusch-Godfrey LM Test in Table 1). This procedure was first suggested by West and Cho (1995) and Taylor (1999) who pointed out that efficiency test of volatility estimations most likely will not fulfil assumptions underlying the OLS methodology. This way results are more reliable and can be interpreted with more confidence.

#### **4.1 Results from error statistics**

Models predictive ability has been ranked with using error statistics as described in chapter 3 above. Results can be seen from Table 2 (Panels A and B). First actual values of the different statistics is calculated then raised to 4<sup>th</sup> power to get numbers which are easier to handle with. Relative values are calculated from actual values to get a proper ranking criterion. This was done by multiplying each individual error with inverse of the highest error statistic observed. The smallest relative value tells which the best model in question is.



**Table 2. Results using symmetric and asymmetric error statistics**

The relative values, appear in brackets below the actual values, are computed by multiplying each individual error with inverse of the highest error observed. The smallest relative values are bolded indicating the smallest error, i.e. the best model judged with the statistic in question.

	RMSE (*10 <sup>4</sup> )	MAE (*10 <sup>4</sup> )	MAPE (*10 <sup>4</sup> )	HMSE (*10 <sup>4</sup> )
<b>Panel A: Estimation for period 2008-2012</b>				
<b>Random Walk</b>	0.127	0.004	0.249	0.007
	[0.109]	[0.109]	[0.109]	[0.019]
<b>EWMA</b>	0.121	0.003	0.237	0.008
	[0.104]	[0.104]	[0.104]	[0.019]
<b>GARCH</b>	0.071	0.002	0.140	0.002
	<b>[0.061]</b>	<b>[0.061]</b>	<b>[0.061]</b>	<b>[0.006]</b>
<b>t-GARCH</b>	1.168	0.033	2.290	0.398
	[1]	[1]	[1]	[1]
<b>Panel B: Estimation for period 2/1/2013-29/10/2013</b>				
<b>Random Walk</b>	0.008	0.001	0.072	0.000
	<b>[0.000]</b>	<b>[0.000]</b>	<b>[0.000]</b>	<b>[0.000]</b>
<b>EWMA</b>	0.264	0.018	2.347	0.128
	[0.038]	[0.038]	[0.038]	[0.009]
<b>GARCH</b>	0.047	0.003	0.421	0.004
	[0.007]	[0.007]	[0.007]	[0.000]
<b>t-GARCH</b>	6.854	0.473	60.978	15.014
	[1]	[1]	[1]	[1]

RMSE, MAE, MAPE and HMSE are explained in chapter 3 above.

As can be seen from the Panel A of the Table 2 GARCH model was ranked as the best model in estimation period from 2008 to 2012 and the second smallest relative values were in EWMA model. Panel A also shows that GARCH was ranked first in all error statistics but random walk was second best in HMSE together with EWMA. These results were quite expected from the theory and from previous studies. However, quite surprisingly *t*-GARCH was ranked to the worst model in all statistics and with a big difference to the others. Obviously, model's predictive abilities were not suitable for this data.

When predictive abilities were tested to out-of-sample data in 2013 (Panel B) random walk proved to be best ranked model in all statistics. However, in case of HMSE, which should take account heteroskedasticity, random walk and GARCH were as good predictors to three decimal places. GARCH was second best predictor also in all other categories. Once again the poorest predictive abilities were on *t*-GARCH model and again with a big difference.

Result from error statistics were rather anticipated expect from  $t$ -GARCH. It seems that normal distribution is still better representation of Finnish market data even though summary statistics showed excess kurtosis and small negative skewness.

## 4.2 Results from OLS regressions

Table 3 presents results of the OLS regression ( $R^2$ , parameter estimations and their standard errors and  $t$ -statistics) introduced in (14) above. As can be seen from Table 3 OLS regression gave very different explanation of the predictive abilities of the models. Using regression random walk and EWMA dominated and GARCH models were either poor predictors or they were not statistically significant or in many case both of these.  $R^2$  of the both random walk and EWMA were 98.5 per cents in 2008-2012 sample and 85 per cents in 2013 sample. At the same time  $R^2$  of the GARCH was 15.5 per cents in 2008-2012 and only 2 per cents in 2013 sample.  $t$ -GARCH fended even worse in both samples. From 2008 to 2012 it was able to predict only a fraction of per cent (0.01 %) and in sample of 2013 0.3 per cents. These confirms the results obtained from error statistics that  $t$ -GARCH has no predictive abilities in case of the Finnish market data used in this thesis. Poor abilities of the GARCH model were surprising because error statistics let to expect higher  $R^2$ . However, these results could be interpreted other way round and state that regression showed predictive abilities of GARCH model to be very close to those obtained by Maukonen (2002). He estimated  $R^2$  of GARCH to be 7.2 per cent. High  $R^2$  figures in the case of random walk and EWMA are based on the fact that “true” volatility was calculated with a volatility estimator in equation (6) as proposed by Hull (2010).

**Table 3. Results using OLS regression**

Regressions ( $\sigma_t^2 = \beta_0 + \beta_1 \sigma_t^{2*} + \varepsilon_t$ ) are estimated with OLS method using Newey and West (1987) heteroskedasticity and autocorrelation consistent covariance matrix estimator with lag=1. All figures in braces { } are adjusted correlation coefficients, while figures in brackets [ ] are standard errors and in parenthesis ( ) are p-values. Asterisk (\*) is added to those p-values that are statistically significant in 95 % confidence level and double asterisk (\*\*) to those which are statistically significant in 99 % confidence level.

	$R^2$	$\beta_0$	$H_0: \beta_0=0$	$\beta_1$	$H_0: \beta_1=1$
	{Adj. $R^2$ }	[std. error]	(p-value)	[std. error]	(p-value)
<b>Panel A: Estimation for period 2008-2012</b>					
<b>RW</b>	0.9842	0.0001	2.0457	0.9928	270.9442
	{0.9842}	[0.0000]	(0.0410)*	[0.0036]	(0.0000)**
<b>EWMA</b>	0.9842	0.0001	2.4748	1.0540	290.1205
	{0.9842}	[0.0001]	(0.0135)*	[0.0036]	(0.0000)**
<b>GARCH</b>	0.1546	0.0450	2.0243	-1.6020	-1.2551
	{0.1539}	[0.0222]	(0.0432)*	[1.2764]	(0.2097)
<b>t-GARCH</b>	0.0001	-0.0423	-0.4636	3.0503	0.6207
	{-0.0007}	[0.0913]	(0.6430)	[4.9140]	(0.5349)
<b>Panel B: Estimation for period 2/1/2013-29/10/2013</b>					
<b>RW</b>	0.8518	0.0007	3.0596	0.9238	40.0361
	{0.8511}	[0.9238]	(0.0025)**	[0.0231]	(0.0000)**
<b>EWMA</b>	0.8520	0.0007	3.0419	0.9834	40.4686
	{0.8513}	[0.0002]	(0.0027)**	[0.0243]	(0.0000)**
<b>GARCH</b>	0.0219	0.0079	2.1636	0.1209	0.3229
	{0.0172}	[0.0036]	(0.0316)*	[0.3745]	(0.7471)
<b>t-GARCH</b>	0.0036	-0.0142	-1.2581	1.2586	1.9744
	{-0.0012}	[0.0113]	(0.2098)	[0.0637]	(0.0497)*

Table 3 also presents parameter estimates of OLS presented in (14) and their standard errors. Additionally, t-test has been conducted to all parameters to see whether they are statistically significant or not. Intercepts were statistically significant with 95 percentages confidence level except t-GARCH which was not statistically significant. Random walk and EWMA parameters even were highly statistically significant in 2013 sample. Parameter estimates for dependent variable of random walk and EWMA were highly statistically significant in both samples and surprisingly t-GARCH was statistically significant in 2013 sample. GARCH model's dependent variable was not statistically significant in either tested sample. These results also indicate that random walk and EWMA were more efficient to predict volatility.

## 5 CONCLUSIONS

In this thesis predictive abilities of different volatility models in the Finnish market data has been analysed using two different testing methodologies. Applied models for testing were random walk process, Exponentially Weighted Moving Average (EWMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) with two different statistical distribution (normal and  $t$ -distribution). Data used to estimate volatility models was OMX Helsinki price index (OMXHPI) from the start of the 2000 to the 29<sup>th</sup> October 2013. Firstly, sample data from 2000 to 2007 was used to estimate the models and predictive abilities in the sample from start of 2008 to the end of 2012 were tested. In the later case the data from 2000 to 2012 was used in estimation process and models predictive abilities were tested to the period from 2<sup>nd</sup> January 2013 to 29<sup>th</sup> October 2013. First ranking methodology used was OLS regression with heteroskedasticity-consistent standard errors (Newey and West 1987 methodology) and for the second ranking methodology four different statistical errors (RMSE, MAE, MAPE, and HMSE) was used.

Main findings of the thesis are that volatilities seem to be predictable but in this data set assumptions based on theory were not fully fulfilled. The thesis showed mixed results whether models that account heteroskedasticity are better for predicting volatility. In the case of the error statistics in 2008-2012 sample GARCH model with the normal distribution dominated and EWMA was ranked as the second best. In the later sample random walk got the highest ranking closely followed by GARCH. However,  $t$ -GARCH model had the worst predictive abilities in both samples. These would indicate that models which account heteroskedasticity were better for volatility forecasting if tested with error statistics. When using OLS regression result where contrary even when heteroskedasticity-consistent standard errors were taken account. In the both samples, random walk and EWMA dominated over GARCH models. Additionally, either one of the GARCH models was not consistently statistically significant. To sum up research problems presented in introduction following can be stated: there is evidence that models that take account heteroskedasticity were better for volatility prediction but the evidence was ambiguous as stated above. This thesis found evidence that models predictive

abilities were robust when sample was divided to two parts (pre-financial crises 2000-2007 and financial crises 2008-2012). However, thesis showed that predictive abilities of GARCH model was not increased when different distribution was used. This might be the case of the particular data used in the thesis or wider phenomena.

The results of the thesis are useful for the academia and for the financial sector. As results were ambiguous more research, especially with the Finnish data, is needed. Results of the thesis opens also various new lines of study, e.g., comparison between historical volatility and implicit volatility, test whether poor predictive abilities of  $t$ -GARCH are vast or was it because of the data used. Benefits for the financial sector are in two-fold. Firstly, thesis showed that predicting volatility is possible even in the case of financial crises. Secondly, thesis showed that one does not necessarily need more complex GARCH models.

Future research on volatility could be deepening with different statistical distributions or conducting more empirical research by comparing predictive abilities in different countries. Another direction would be to study how investors can hedge portfolio's volatility. Yet additional line of study could be value-at-risk models in banking industry, for examples using of copulas to determine VaR for loan portfolios or determine market VaR when changes does not follow normal distribution.

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## APPENDICES

### Appendix 1. Derivation of GARCH with Student $t$ Distributions.

First we let conditional distribution for  $y_t$ ,  $t = 1, \dots, T$  to be standardized  $t$  with mean  $y_{t|t-1}$ , variance  $h_{t|t-1}$  and degrees of freedom  $\nu$ . Formally,

$$y_t = E(y_t|\psi_{t-1}) + \varepsilon_t = y_{t|t-1} + \varepsilon_t$$

$$\varepsilon_t|\psi_{t-1} \sim f_\nu(\varepsilon_t|\psi_{t-1})$$

$$= \Gamma\left(\frac{\nu+1}{2}\right) \Gamma\left(\frac{\nu}{2}\right)^{-1} \left((\nu-2)h_{t|t-1}\right)^{-1/2} \times \left(1 + \varepsilon_t^2 h_{t|t-1}^{-1} (\nu-2)^{-1}\right)^{-(\nu+1)/2}, \nu > 2 \quad (1)$$

Where  $\psi_{t-1}$  denotes all the available information until time  $t-1$  of the  $\sigma$ -field and  $f_\nu(\varepsilon_t|\psi_{t-1})$  is the conditional density function for  $\varepsilon_t$ . The  $t$ -distribution is symmetric around 0; the variance and the fourth moment are as follows (Kendall and Stuart 1969);

$$\text{Var}(\varepsilon_t|\psi_{t-1}) = h_{t|t-1}$$

$$E(\varepsilon_t^4|\psi_{t-1}) = 3(\nu-2)(\nu-4)^{-1}h_{t|t-1}^2, \nu > 4$$

It is well-known that for  $1/\nu \rightarrow 0$  the  $t$ -distribution approaches to normal distribution with variance  $h_{t|t-1}$ . However, when  $1/\nu > 0$ , the  $t$ -distribution has “fatter tails” than the corresponding normal distribution.

Now we take conditional mean  $y_{t|t-1}$  as constant and formulate returns as follows;

$$y_t = \mu + \varepsilon_t$$

And along with GARCH (p,q) model the conditional variance is

$$E(\varepsilon_t^2|\psi_{t-1}) = h_{t|t-1} = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j|t-1-j}$$

Where  $\omega > 0$ ,  $\alpha_1 \geq 0$ ,  $\beta_j \geq 0$ . The econometric model presented above with conditional  $t$ -distributed errors allows accounting for the observed leptokurtosis in financial time-series.