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BACKGROUND FOR DESIGNING AN IMPROVED DIGITAL MUSICAL KEYBOARD INSTRUMENT

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CONTENTS

LIST OF SYMBOLS, ABBREVIATIONS AND TERMS

f	Frequency [Hz]
MIDI	Musical Instrument Digital Interface. Primitive serial bus for electrical communications.
12-TET	12-Tone Equal Temperament. Tuning in which the octave is divided into 12 equally spaced tones (logarithmically on frequency scale).
Octave	The difference between a frequency and twice the frequency.
Tone	Certain sound having constant properties.
Pitch	Certain frequency or frequency group, often on logarithmic scale.
Note	Certain pitch used in music or a notation for it.
Harmony	Ratio between frequencies of two pitches.
Chord	Combination of notes that form certain harmonies.
Tuning	Set of all available notes in an instrument.
Microtonal	Something is outside of 12-TET tuning system.
Isomorphic	Keyboard's harmonies recur in geometric patterns regardless of the point of observation.

1 INTRODUCTION

Playing and composing of music is easy with a computer nowadays. Ever increasingly, even the music studios are leaning on virtual software. With computer-based synthesizers, even acoustic instruments can be modeled with decent quality, so computer screen can easily be used at least to design whole musical pieces. Different synthesizers and effects can be played and adjusted *live* in real-time, or in non-real-time with time to spare. Computer aided music benefits from some kind of tool, an instrument, that works as an interface between human and computer. The most typical of this kind of an instrument is an electrical *keyboard*, which reminds the most of us of the black-and-white keyboard familiar from the piano. Thus, in musical sense, the piano keyboard, with its benefits and flaws, represents the state-of-the-art technology in integration between man and machine. Or is there room for improvement in the keyboard after all? What if learning to play the piano is wasted time, and playing can be learned more easily?

1.1 Background

As a result of a sauna evening the author ended up as a keyboardist in a freshly formed band. Experience from playing was non-existent, and the instrument was a cheap computer-attached MIDI-keyboard also. Soon it was clear that playing piano keyboard is not so easy and straightforward as one would think. Every chord and melody has 12 variations and fingerings depending on the pitch. Playing piano keyboard by ear requires knowing every variation, or the whole keyboard, which necessitates a lengthy and slavish practice. Therefore the learning curve of the piano keyboard is very steep. For example, a guitar does not have this kind of limitation, and even if it has its own weaknesses, they are justified by physics rather than artificial choices. The author of this work did not learn to play the piano keyboard properly in reasonable time, so surely there is something wrong with the instrument.

1.2 Objective and research questions

This work is a literature research, which explores the background for a new type of electrical keyboard instrument. This work surveys history of keyboard instruments and tunings and the mathematics hidden behind the music, and different options are considered from the basis of them. The aim is to create a comprehensive general level review on alternative implementations of keyboard instruments besides the piano keyboard.

The research aims to find answers to at least these questions:
- Why and how have we ended up with piano keyboard used today?
- What is the western *12-TET* tuning based on?
- What other alternatives is there for tuning and keyboard?

The purpose is also to prepare for a future research, that focuses on designing and implementation of a new kind of keyboard. Intention of the new instrument is to offer versatile electrical digital interface for computer aided music. It should at least have keys implemented with electrical switches and MIDI capabilities. This is how this work relates to electrical engineering.

The underlying motivation of this work is to own, and probably build, this kind of instrument, to learn to play and master it, and to develop as a musician.

1.3 Scope of thesis

This work explores background for future research in general level.

Terminology of music is covered as needed. It consists of a lot of historical vocabulary that is partly illogical in mathematical terms and can be misleading even to a reader familiarized with the subject.

Also, the purpose is not to cover mathematics in depth. Nonetheless, music and mathematics are closely related, and a lot of research has been made about the subject. For enthusiasts shall be recommended sources Campbell, 1987 and Rossing, 2007.

The type of the instrument to be investigated is restricted to a keyboard playable by fingers. As an interface between man and music, that is, between muscles and computer, this kind of keyboard is versatile and widely used.

Tuning to be studied is mainly the most used tuning, in which the octave is divided into 12 equally spaced pitches (notes). Other tunings are covered partly as needed, but their selection criteria or mathematical background is not covered in depth.

1.4 Structure of thesis

First chapter is introduction which covers the background of the thesis in general. The second chapter provides a brief overview of the history of keyboard instruments. Knowledge of history helps to get a more comprehensive understanding of the different development stages, as well as related ideas and compromises. The choices that led to current implementation should be known in order to to be able to learn from them and to see the direction of future development. In third chapter is discussion about the tuning of the pitches, as well as some mathematics of music. Mathematical methods are not discussed very thoroughly, but coherent designing of an instrument necessitates a brief overview on the topic. Fourth chapter deals with the layout of the keyboard, representing different options for it. The implementation of the mechanics is not detailed yet, and the subject is viewed mainly from the instrument's player's point of view. The fifth chapter presents the results and conclusions. There is pondering of causes for using different tuning systems and layouts, and starting points for a new kind of instrument. At the end there is conversation about the subject.

2 HISTORY OF KEYBOARD INSTRUMENTS

Pythagoras and his contemporaries performed acoustic tests with a monochord in around 500 BC. It had oblong sounding board, on top of which a string was stretched, and between them was put a movable bridge, with which the length or proportion of freely vibrating part of the string could be adjusted (the other half was damped). It was used as a musical aid for long – still in the Middle Ages – and it got more strings with time, after which it was called polychord. Illustration 1 represents the monochord. (Abrashev, Gadjev, 2000, p.149; Isacoff, 2001, p.34-51)

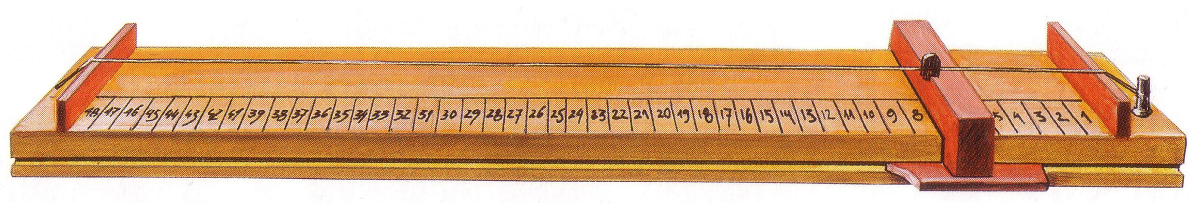


Illustration 1. Monochord (Abrashev, Gadjev, 2000, p.149).

According to his tests, Pythagoras created basis for musical theory that still has an important meaning in music. He invented the so-called Pythagorean tuning, in which the octave is divided into pitches in certain way. He chose 7 main pitches into the octave himself, but the mathematically similar divided-in-12 octave was also familiar to him and his ancient Greek contemporaries. Division of the octave into equal parts was also a familiar concept. (Abrashev, Gadjev, 2000, p.118; Isacoff, 2001, p.34-51, 133; Campbell, 1987, p.170-172.)

The first keyboard instrument is presumably the water organ hydraulis, which was invented by Ktesibios in Alexandria in 246 BC. It made use of organ pipes and air pump, and the air pressure was kept constant by means of water. It had plain row of keys that guided the valve pieces of the organ pipes. Illustration 2 represents the hydraulis. (Nordström, 1997, p.77; Wade-Matthews, 2000, p.230; Campbell, 1987, p.453.)

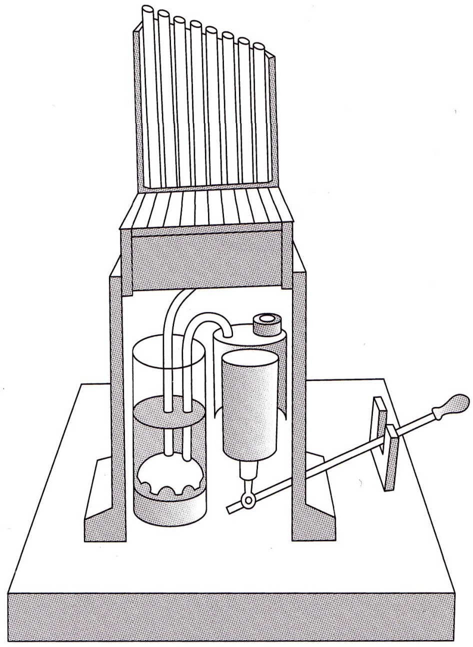


Illustration 2. Hydraulis (Abrashev, Gadjev, 2000, p.121).

In the 600s the church introduced organs. The first keyboards had 7 keys for each octave, that is, the present white keys. With more complex melodies and increased polyphony, the current black keys had to be added; first the lowered B (which in Germany became known as B, and the former B as H) and finally the rest of the 12 notes. As usual, the church resisted the use of harmonies and adding of the keys at first. Two notes six steps apart from each other was considered as especially bad harmony, "Diabolus in Musica" or Satan's interval, which is still avoided by the Catholic church in its rituals. (Rohrer, 2006; Merrick, 2011, p.38.) In the 1300s the keys were narrowed a bit, and the keyboard achieved its present form. (Wade-Matthews, 2000, p.216; Abrashev, Gadjev, 2000, p.189; Schulter, 2002.)

In the 1300s, simple key levers were added to the monochord (polychord), and with them the strings were hit in proper locations. Thus was born the clavichord. At first, every string was shared by several keys (fretted clavichord), but in later models in order to enable full polyphony, every key had their own respective string (fretless clavichord). Clavichord was popular in around 1500-1700's. It had fairly quiet voice, because a node was formed to the string in the point where the tangent hit it. Illustration 3 depicts the clavichord and its operational principle. (Abrashev, Gadjev, 2000, p.174.)

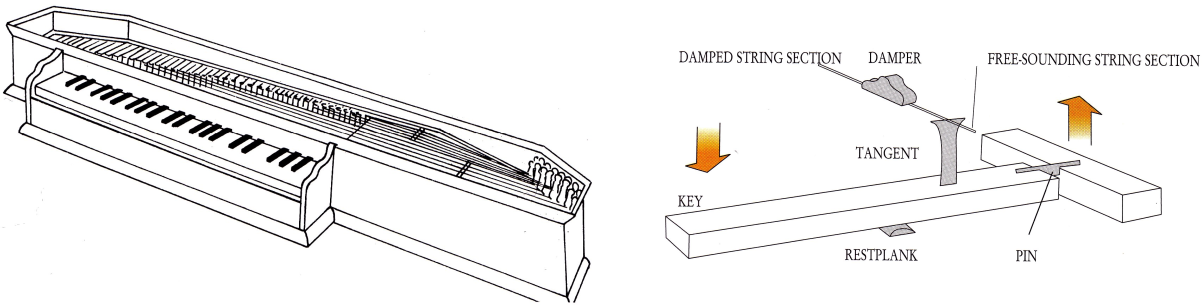


Illustration 3. Clavichord and its operational principle (edited: Nordström, 1997, p.75; Abrashev, Gadjev, 2000, p.174).

At the same time with clavichord also was born and flourished the harpsichord or cembalo, although they did not have a common origin. It had respective strings for each key, and at the end of each key lever there was a small claw, a plectrum, that plucked the string sounding, and muted it when the key was released. It had louder voice than clavichord, but the player could not affect its intensity. Later models had plural strings for each key, and they could be muted as wanted for example with a pedal. Illustration 4 depicts the harpsichord and its operational principle. (Abrashev, Gadjev, 2000, p.175.)

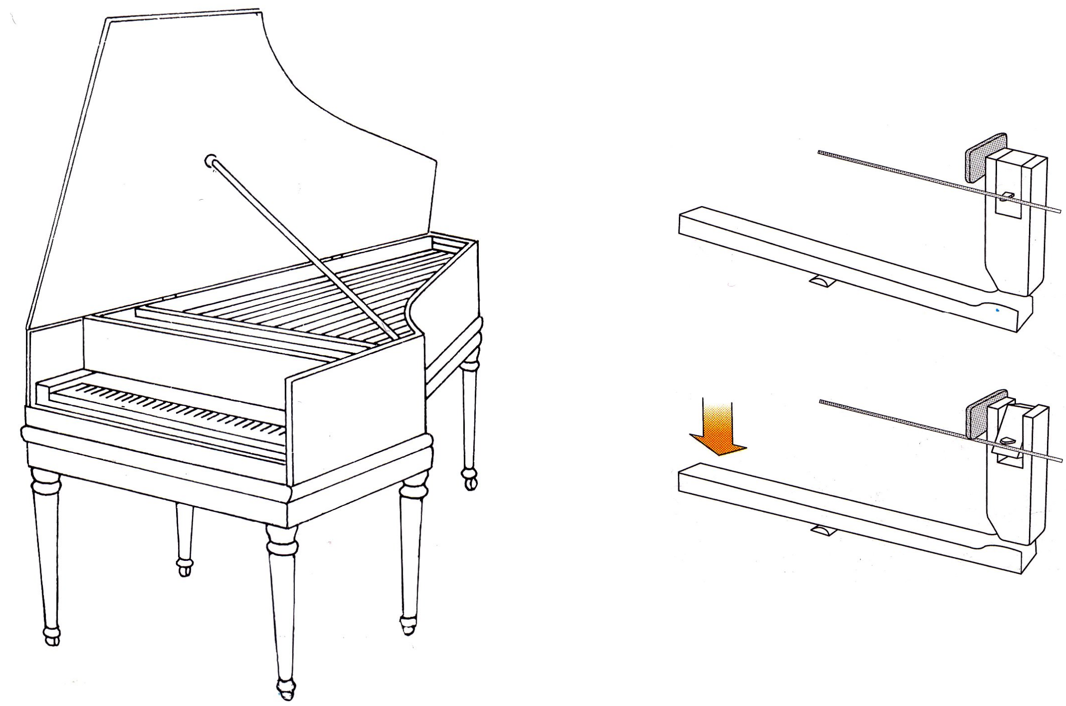


Illustration 4. Harpsichord alias cembalo and its operational principle (edited: Nordström, 1997, p.74; Abrashev, Gadjev, 2000, p.175).

In the 1400s, the triads became more common, which set some requirements for the tuning system (Isacoff, 2001, p.67). In the 1500s there was controversy about the best tuning system used in keyboard instruments. Offered were pythagorean tuning, just tuning, and meantone temperament. In addition to this jumble, equal temperament began to join in. Equal temperament was already used in lutes because of their mechanical construction. (Isacoff, 2001, p.126,158). Pythagorean tuning is not well suited for polyphonic music. For this reason, in the 1600s the meantone temperament was getting more common. (Campbell, 1987, s.176.) It is not suited for music that relies strongly on chords or melodies that vary their pitch either, so during the 1700s the equal temperament gained place. (Isacoff, 2001, p.247-251.)

In the beginning of the 1700s, the best features of clavichord and harpsichord were combined, and piano was created. Each key equals to a few strings, which are hit with a padded hammer. The strings can be damped or left sounding by means of pedals. After this the piano went through several small development phases. It was only in the 1850s that the sound of the piano started to resemble what it is today. Thicker, heavier strings were added in crossing shape, along with a strong iron frame to support them in order to achieve louder voice. Before that the thin, loose strings produced a lot of harmonic overtones, which were responsible for the characteristic little out-of-tune sounding voice of early pianos. The piano, like the harpsichord, has upright and horizontal versions. Horizontal piano is sometimes called a grand piano (or some derivative of German *flügel* in many other languages). Illustration 5 depicts the piano and its present hammer actuated operational principle. (Wade-Matthews, 2000, p.226-229.)

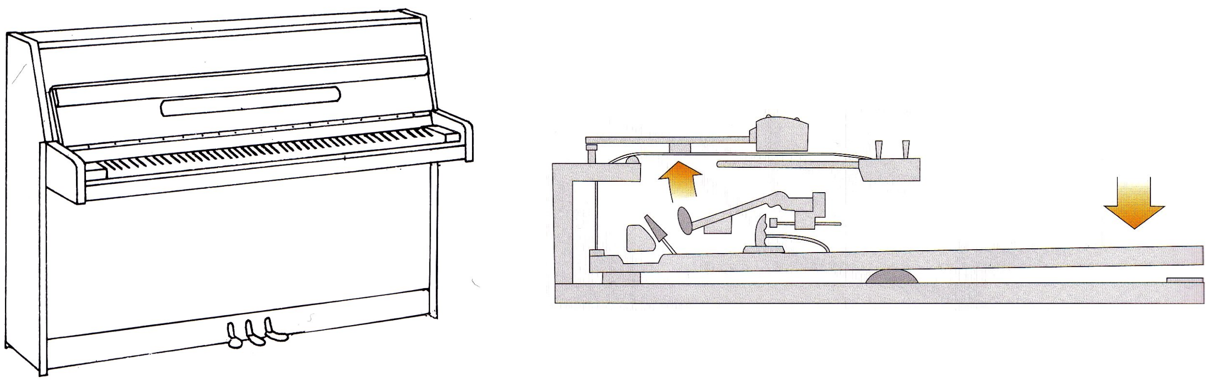


Illustration 5. Piano and its present hammer actuated operational principle (edited: Nordström, 1997, p.75; Abrashev, Gadjev, 2000, p.212).

During 1800s, the equal temperament still had some resistance (Campbell, 1987, p.181-182). In the early 1800s was introduced the accordion, which had a compact, isomorphic, 3-split keyboard on the other side. At the same time also appeared a concertina which resembles a small hand-held accordion, and in which a few different key layouts were tried out for space saving reasons. In the late 1800s, in 1882, Paul von Jankó introduced an alternative, equally divided keyboard layout for piano, but it did not catch on. A piano making use of this layout is depicted in Illustration 6. (Abrashev, Gadjev, 2000, s.227.)

Illustration 6. Piano using Jankó keyboard layout (Abrashev, Gadjev, 2000, p.227).

In the 1900s appeared electrical keyboard instruments. Hammond organ appeared in 1934, and was based on cogged wheel that spins in front of a magnet. Transistor-based synthesizer RCA Mark II appeared in 1959, and took up a wall of a small-sized room. In late 1960s were introduced the friendly-sized Moog synthesizers. In electrical keyboard instruments, the keys acted as switches that along with other control knobs directed the electricity in a certain way, and finally the electricity was transformed into sound in speaker. Illustration 7 depicts a Hammond organ and an early synthesizer from 1980s. (Wade-Matthews, 2000, p.240-241; Nordström, 1997, p.79)

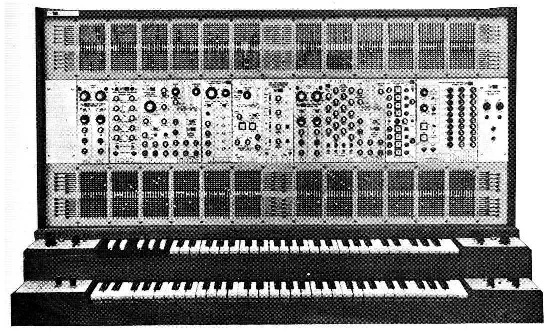
 

Illustration 7. Hammond organ (on the left) and a synthesizer from 1980s (on the right) (edited: Abrashev, Gadjev, 2000, s.254; Nordström, 1997, s.79).

Today, during the digital revolution, the keyboard instruments are mainly digital, and in general they can be connected to other musical equipment through MIDI (Musical Instrument Digital Interface) connection. There also exists plain keyboards that are equipped with MIDI. MIDI keyboards can be touch-sensitive, so MIDI can pass precise information of how the keys are pressed. Usually in electrical keyboards, the hammers and other mechanics have been replaced by switches, but in some keyboards there are dummy hammers for providing the tactile feedback of a piano keyboard. (Wade-Matthews, 2000, p.241.)

3 TUNING OF THE NOTES

Music has been a part of human culture since the dawn of time. Musical styles and instruments have varied with continents and eras. At the same time, different options for used notes and pitches are numerous. Yet culturally independent similarities exist. The human ear usually considers simple frequency ratios, or *harmonies*, as beautiful. They are also the easiest to distinguish from other soundscape.

3.1 Harmonies

Let us first examine the standing waves created in a vibrating string (illustration 8). The case could be for example one string tuned to a specific pitch in one note of a piano. The string is supported from both ends, so nodes form at the endpoints. In the simplest case, a half of the wavelength fits in between (illustration 8, uppermost wave). This frequency is the *fundamental frequency*. Whole-number multiples of the fundamental frequency can also form into the string. These are the *harmonics* of the fundamental frequency. Real-world instruments do not produce a discrete frequency; in addition to the fundamental frequency, also harmonics give color to the sound, as well as non-harmonic *partials* to some extent. This was the case for one pitch. (Campbell, 1987, p.19-22)

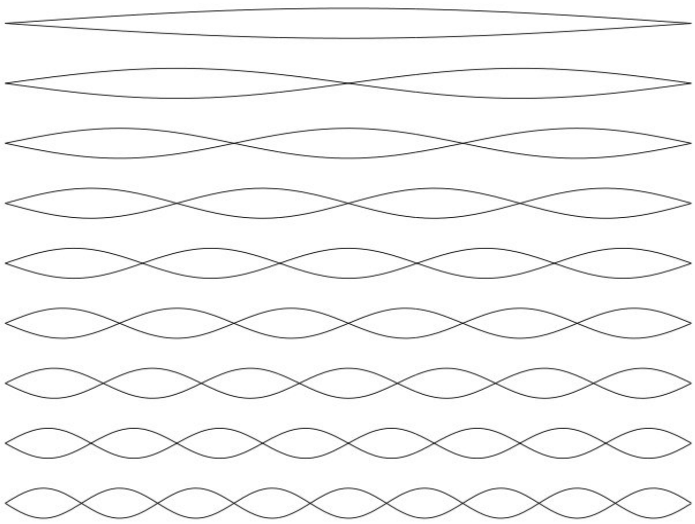


Illustration 8. Standing waves in a vibrating string. Fundamental frequency (uppermost) and its first 9 harmonics, including itself.

Next, let us look at the relation between two pitches of tones. Let the pitches be denoted by ratio X:Y, where X and Y are whole numbers. Order of the pitches is irrelevant here, so X:Y = Y:X. Thus, the simplest frequency ratios expressed by the smallest whole numbers are 1:1, 2:1, 3:2 and so on. Illustration 8 can again be of help in understanding the harmonies. For example the ratio 3:2 tells that in the same time, other pitch has three antinodes and the other has two. The ratio 2:1, that is, the simplest ratio, is often thought as being very close to ratio 1:1. Double the frequency is considered to be the same musical note as the original frequency, only played from higher pitch (Campbell, 1987, p.76). This is caused by the harmonics of the two frequencies being largely identical. The same scaling property applies to every pitch ratio that can be expressed with powers of two (formula 1):

$$f_0 = 2^n \cdot f_0 \qquad , n \in \mathbb{Z} \tag{1}$$

, where f_0 is the frequency of a note. According to this, frequencies of every note can be scaled into interval $[\, f_0 \,, 2f_0 \,[$. This interval in which the frequency doubles, is misleadingly called an *octave*. (This is due to historical reasons; playing of 1:2 harmony on a piano keyboard takes 8 white keys, including the endpoints, hence the numeral referring to 8.)

3.2 Pythagorean tuning

One division into notes that is pleasant to the human ear is thus achieved by applying the formerly mentioned simple harmonies of 2:1 and 3:2. Let us choose 3:2 as the frequency ratio of successive notes. So, the frequency of the next note is obtained by multiplying by with 3/2 (and the previous by dividing by 3/2). Let us choose 1:1 as the base frequency and take 12 steps from it. The resulting relative frequencies are shown in the first column of table 1. Like before, the frequencies ban be scaled into same octave. The results are shown as fractions in column 2 and as decimals in column 3. It is discovered that the frequencies of the first and last row are very close to each other. They can be considered as the same note with a small error. This discontinuity is expected, because 3/2 is not divisible by two (3 and 2 are prime numbers) and thus none of its whole number multiples. If more steps were taken, more notes would have alternative frequencies. The frequencies through one octave are shown in column 4 in order of magnitude. Frequency 1.00000 can also be interpreted as 2.00000. Column 5 shows the relative change from the frequency of the previous note, and also a suggestion for dividing the piano keyboard into black and white keys.

Table 1. Pythagorean tuning.

Relative frequencies (with 3/2 interval)	Relative frequencies in the same octave	Relative frequencies as decimals	Relative frequencies in order of magnitude	Relative change from previous frequency
1:1	1:1	1.00000	1.00000	1.05350
3:2	3:2	1.50000	1.06787	1.06787
9:4	9:8	1.12500	1.12500	1.05350
27:8	27:16	1.68750	1.20135	1.06787
81:16	81:64	1.26563	1.26563	1.05350
243:32	243:128	1.89844	1.35152	1.06787
729:64	729:512	1.42383	1.42383	1.05350
2187:128	2187:2048	1.06787	1.50000	1.05350
6561:256	6561:4096	1.60181	1.60181	1.06787
19683:512	19683:16384	1.20135	1.68750	1.05350
59049:1024	59049:32768	1.80203	1.80203	1.06787
177147:2048	177147:131072	1.35152	1.89844	1.05350
531441:4096	531441:524288	1.01364		

Let us look at the error in the point of discontinuity. As calculated before, proceeding 12 steps with frequency ratio 3:2 leads to the initial note, only 7 octaves higher. The same note is achievable with 7 steps of size of an octave. Different ways lead to slightly different frequencies, and their ratio (step in pitch) is presented in formula 2 (Kentner, 1979, p.36-39):

$$\left(\frac{3}{2}\right)^{12} : 2^7 = \frac{\left(\frac{3}{2}\right)^{12}}{2^7} = \frac{\left(\frac{531441}{4096}\right)}{128} = \frac{129.74634}{128} \approx 1.01364 \tag{2}$$

This can also be seen from column 3 of table 1 as the ratio of the last and the first line. In the last column, there is two different sizes of steps of pitch or relative changes of frequency. The difference between these two steps is the change of pitch caused by the error (proportion in formula 2); in other words the smaller step multiplied by the error gives the larger step. When proceeding from the initial note with the 3:2 frequency ratios, the step to the last note would be larger, whereas the 2:1 ratio dictates the last note being after shorter step. This error is known as the P*ythagorean comma* (Kentner, 1979, p.36-39). Comparing the size of the error to the size of the frequency change in steps of pitch (notes) obtained earlier we have (formula 3):

$$\begin{aligned} \frac{1.01364-1}{1.05350-1} &= \frac{0.01364}{0.05350} = 0.25502 \\ \frac{1.01364-1}{1.06787-1} &= \frac{0.01364}{0.06787} = 0.20102 \end{aligned} \tag{3}$$

Thus the size of the Pythagorean comma is approximately one quarter of a note. It is not much, but still audible.

The obtained octave is divided into 12 notes somewhat regularly, with slightly varied intervals. This tuning system is called the Pythagorean tuning. It is one of the oldest known tuning systems from the 500s BC. Pythagoras himself chose only the first 7 notes into an octave, that is, the white keys of a piano, after which the tuning developed through intermediate steps into that shown. The tuning produces well the frequency ratios 2:1, 3:2 and 4:3, but poorly the ratios 5:4 and 6:5. A serious problem is also the comma, which affects melodies crossing it and especially polyphonic chords. Because of the varying frequency ratios (note spacing), a certain melody can only be played from a certain pitch without distortion. On the other hand, it offers pure harmonies and the possibility of coloring a certain melody. (Campbell, 1987, p.170-182; Isacoff, 2001, p.48-51; Kentner, 1979, p.36-39; Barbour, 1951, p.1.)

3.3 12-Tone Equal Temperament

Considering a general purpose instrument, the previously mentioned point of discontinuity is highly disadvantageous, because melodies or chords passing it sound out-of-tune. Also, Pythagorean tuning does not produce proper higher harmonies. The matter can be improved by changing the spacing of the notes suitably, so that the harmonies improve and the discontinuities decrease. (Campbell, 1987, p.170-182.)

One solution proposal was *just diatonic scale*, in which the higher harmonies (5:4 and 6:5) were taken into account in the steps of the notes. However, the step sizes varied in it, which was remedied with *meantone temperament*. It was similar to Pythagorean tuning, but in it the relative changes of the frequencies were 1.118 and 1.070 instead of the previous 1.125 and 1.053. Nowadays is almost exclusively used a compromise solution where the octave is simply divided equally into 12 notes. This way the harmony 1:2 is perfect but the harmony

3:2 is not, and a small beat can be heard in it. In fact none of the other harmonies is perfect in it either, but many simple harmonies can be approximated with it, while more complex harmonies are left unachieved. (Campbell, 1987, p.170-182.)

Table 2 shows relative frequencies of the notes and their closest simple harmonies. The correspondences can be found for example by comparing sequentially to harmonic series, that is, 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7, 1/8, 3/8, 5/8, 7/8 and so on. There may be several close alternatives for a perfect harmony to be approximated. For example the third lowest harmony 16:9 could as well be 9:5, which is considerably simpler harmony. In this case however the inverse of the third note was chosen, because it can be thought as coming down three steps from the harmony 2:1. Equal spacing and periodicity with respect to octave help in perceiving and adopting the tuning. Furthermore, the number of the notes is relatively low, so the basic harmonies are somewhat easy to obtain. The frequency division is nonetheless adequately dense for approximating many historical or non-western styles of music. (Campbell, 1987, p.170-182.)

Table 2. 12-Tone Equal Temperament. Frequencies and their harmonies.

Relative frequency	Relative frequency as decimal	Closest simple harmony	Closest harmony as decimal	Relative deviation from harmony
$2^{0/12}$	1.00000	1:1	1.00000	0.0000
$2^{1/12}$	1.05946	1+1/15 = 16:15	1.06667	-0.0068
$2^{2/12}$	1.12246	1+1/8 = 9:8	1.12500	-0.0023
$2^{3/12}$	1.18921	1+1/5 = 6:5	1.20000	-0.0090
$2^{4/12}$	1.25992	1+1/4 = 5:4	1.25000	0.0079
$2^{5/12}$	1.33484	1+1/3 = 4:3	1.33333	0.0011
$2^{6/12}$	1.41421	1+2*1/5 = 7:5	1.40000	0.0102
$2^{7/12}$	1.49831	1+1/2 = 3:2	1.50000	-0.0011
$2^{8/12}$	1.58740	1+3*1/5 = 8:5	1.60000	-0.0079
$2^{9/12}$	1.68179	1+2*1/3 = 5:3	1.66667	0.0091
$2^{10/12}$	1.78180	1+7*1/9 = 16:9	1.77778	0.0023
$2^{11/12}$	1.88775	1+7/8 = 15:8	1.87500	0.0068
$2^{12/12}$	2.00000	2:1	2.00000	0.0000

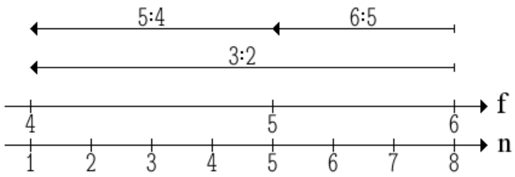
Along with popularization of keyboard instruments this tuning of evenly spaced 12 tones or pitches, also known as 12-Tone Equal Temperament (12-TET), has had its breakthrough in all western music. Almost every instrument is attempted to be tuned into it. Therefore a common standard has been agreed for the used frequencies. The frequencies in 12-TET can be calculated with formula 4, in which the base frequency n = 0 corresponds to the note A = 440 Hz (Huttunen, 1961, p.143).

$$f_n = 440\text{Hz} \cdot 2^{\frac{n}{12}} \qquad , n \in \mathbb{Z} \tag{4}$$

, where n is the ordinal of the note and f_n its corresponding frequency. At this point it is worth noticing that the common notation of musical notes has historical basis. Some of the markings and vocabulary are inherited from old instruments and other tuning systems predating the introduction of 12-TET, and are therefore poorly applicable. The subject is not to be covered too profoundly here, and the issues are brought up rather by mathematical means.

3.4 About chords

As stated previously, simple harmonies sound beautiful and are therefore the keystones of music. Let us examine the simplest harmonies, 1:1, 2:1, 3:2, 4:3, 5:4, 6:5, 7:6, 8:7, 9:8 and so on. Harmonies 3:2 and 4:3 can be interpreted as mirrored harmonies of each other because harmonies can be taken in both directions, so 4:3 = 3:4 = 3:2. Here the multiple of the simpler harmony was divided (or multiplied) out. Harmonies 7:6 and 8:7 have no proper approximation in 12-TET. Thus, the simplest (fundamental)harmonies in it are 1:1, 2:1, 3:2, 5:4, 6:5, 9:8 and 16:15. Harmonies 3:2, 5:4 and 6:5 have interesting feature, namely they all can be included in a *triad* consisting of three notes. This triad sounds massive and uniform, and it can be constructed in two ways that are called *major* and *minor*. Illustration 9 clarifies these frequency ratios. The illustration shows frequency f in logarithmic scale and the ordinal of the 12-TET note in corresponding linear scale. The frequency ratios expressed by the smallest possible whole numbers are obtained according to the formula 5. Thus in 12-TET, major triad is formed by taking a note, continuing four notes forward from it, and then three notes. Minor triad in turn is formed by continuing first three, and then four notes from the initial note.



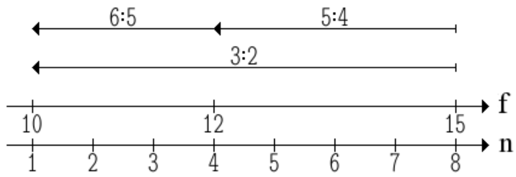


Illustration 9. Triads. Major on the left and minor on the right. *f* is frequency and *n* the ordinal of the note.

$$\begin{aligned} &Major: \quad \frac{1}{1}:\frac{5}{4}:\frac{3}{2} = \frac{4}{4}:\frac{5}{4}:\frac{6}{4} = 4:5:6 \\ &Minor: \quad \frac{1}{1}:\frac{6}{5}:\frac{3}{2} = \frac{10}{10}:\frac{12}{10}:\frac{15}{10} = 10:12:15 \end{aligned} \tag{5}$$

The triads and frequency ratios mentioned are important in diverse music, and they are the basis for many other chords. Therefore they must be taken into consideration when designing a keyboard layout.

3.5 Microtonality

Music that is based on notes lying outside of 12-TET tuning is called microtonal music. Several various microtonal tunings exist, with different motives or historical backgrounds for their use. Some of them are suitable for playing 12-TET notes. Computers have facilitated present-day microtonal music-making. Even though microtonal music offers new dimensions and cleaner harmonies, the 12-TET has stood its ground as the most used tuning system. It has moderate amount of notes, so it is easy to adopt, and the mechanical structure of the keyboard instrument using it is fairly simple. Further on in this document the referred tuning system is 12-TET unless otherwise stated. Microtonality is an interesting extension and it is worthy to pay attention to, but closer familiarization is beyond the scope of this work.

3.6 Conclusions about the tuning

A variety of exotic tunings exist, and they can be used to create exciting-sounding music. Nonetheless, for a general purpose keyboard instrument the 12-TET is a good choice. It is widely used, and for good reason. It gets close to the simplest harmonies with small amount of notes, which also simplifies the structure of the instrument. The hunt for the perfect pure harmonies is difficult anyway, because various multiples are always present due to the mathematic characteristics of music and nature overall. For example, into a string of a stringed instrument, there will form 2/1, 3/2 and other prime number expressible multiples of the fundamental frequency. There will be harmonics and also dissonant partials. They form regardless of the tuning, and they are out-of-tune with each other. In practice, a certain note does not mean a certain discrete frequency; the color of the sound or instrument voice derives from these diffuse sounds. This detuning is even wanted in synthesizers and different sound effects (for example chorus and various stereo effects). The more numerous and detuned oscillators play in a synthesizer, the stiffer and more spacious the soundscape is. Thinking the other way round, its is possible to synthesize a pure instrument voice, say, with a perfect 3:2 harmony, and then to use it with a modern synthesizer as a new instrument voice. The sound can be fine-tuned regardless of the tuning system.

Some electrical keyboard instruments have a feature of dynamic tuning that changes the frequency ratios of the 12-tone octave on-the-fly according to a certain base note. Tunings can also be freely constructed with computer later on, regardless of the physical structure of the (MIDI) keyboard instrument. Instruments with continuous pitch (opposite to discrete pitch) also exist, and they are suitable for any tuning system. However, they are quite demanding instruments to master. Even though technology allows microtonality in keyboards more easily nowadays, it is challenging for real-time playing (depending on the used division and mechanical structure). On the other hand, non-real-time computer-aided music-making does not even necessitate a separate instrument. A microtonal instrument serves best as an experimental instrument beside one with the 12-TET tuning. When thinking of an improved general-purpose keyboard instrument, the 12-TET tuning is a good starting point to begin with.

4 KEYBOARD LAYOUT

4.1 Piano layout

On a piano keyboard the pitch rises or increases when going from left to right. The keyboard is inherited from the early keyboard instruments from time before pianos or evenly spaced tuning system. It favors a certain *scale*, or a subset of notes of the tuning system that is used. This scale is set on the white keys, and the rest of the notes in the used tuning are placed on the black keys. Illustration 10 shows major triad on a piano keyboard going sequentially through all available pitches.

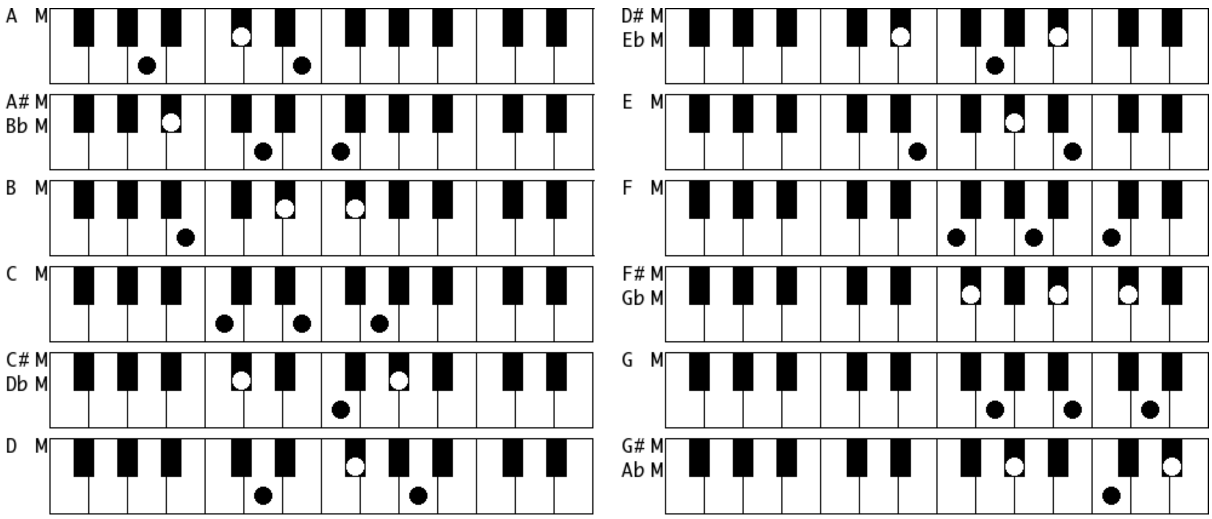


Illustration 10. Major triad on a piano keyboard going through all pitches. Used keys are marked with circles. Minor triad has its middle note one key lower (to the left).

The corresponding traditional character symbols of the chords are also marked in the picture. The letter of the chord is always the same as the letter of its first or lowest note. The letters go with the white keys, and the symbols for sharp (#) or flat (b) denote that the chord is to be moved one step higher or lower in pitch respectively. Last comes the name of the chord. Some notes (black keys) can be written both as a sharp of a lower note and a flat of a higher note. In fact, this is an exceptional property of 12-TET. For example, in the example of Pythagorean Tuning, two almost same frequencies were obtained for a single note. These frequencies can be considered as a sharp or flat versions of the same note. Their difference is much less than two different notes'. (Barbour, 1951, p.1.)

Illustration 10 also aims to bring up the fact, that the black keys are not located exactly in the middle of the white keys. It eases targeting the fingers to the keys in locations with no black keys, and helps locating the position of the fingers in relation to an octave. In uneven tunings prior to 12-TET, the chords depicted in illustration 10 did not have exactly the same frequency ratios, so their tactile identification was beneficial. The disadvantage is that occasionally the white keys need to be pressed from the sometimes narrow gap between the black keys.

Because single notes can be mirrored into other octaves, a single chord has several variations. Illustration 11 shows the different ways to play C major triad and C minor triad from the nearest keys. Three sequential keys marked with circles always form the same chord with a slight variation.

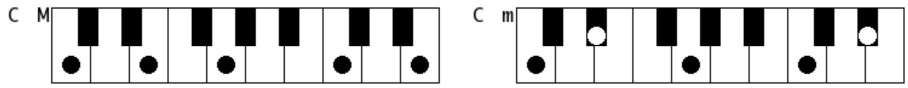


Illustration 11. Closest variations of major triad (on the left) and C minor triad (on the right) on a piano keyboard. M refers to major and m to minor.

4.2 Equal division into 2 rows (Jankó layout)

Because the tuning used nowadays is divided equally, it is natural to ask how the chords would behave on an equally divided keyboard. Let us first study a piano keyboard that is like it is not missing any black keys. Illustration 12a shows a major chord with four sequential pitches. Because the keyboard layout differs from a piano keyboard, it is not feasible to use the same musical notations. Now it is enough to refer to the notes with their ordinal numbers. As can be seen from the illustration, it is hard to stretch the thumbs one row higher than the rest of the fingers. Major chord is difficult for the left hand and minor chord for the right hand correspondingly. On the other hand the thumbs reach easily one row lower than the other fingers. Consequently, playing would be remarkably easier, if the black keys of the top row were also available on one row lower than the white keys. This leads to a three-row piano keyboard that is depicted in illustration 12b. It is observed that a certain chord can always be played with the same relative positioning of the fingers regardless of the pitch which is determined only by the location of the hand.

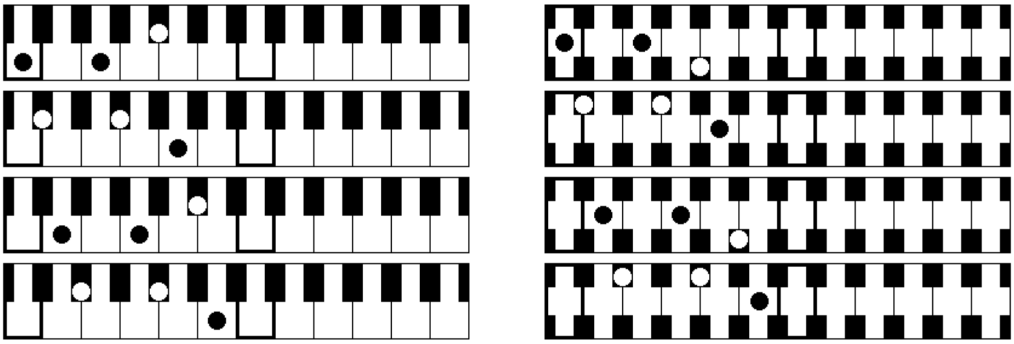


Illustration 12. Four sequential major triad chords with **a)** equally divided piano keyboard (left) and **b)** three-row equally divided piano keyboard (right). First note and its multiples are marked with thick borderlines.

Thus, three rows offer two-dimensional geometry in this 2-split keyboard layout. Keyboard in which a melody or a chord is always formed by a certain pattern is called *isomorphic* keyboard. More rows can be added in a previous manner thus generating a two-dimensional surface in which every second row is identical. The same note can be played from multiple locations, which enables the chords to be played in different ways at will, according to ergonomics or preference. Playing melodies will be easier, because the next key to be played is found in several points and it can be pressed with the most readily available finger. Also, same notes can be played with both hands, which enables rapid rhythms. In addition, perceiving of the notes becomes easier, because a certain change of pitch always corresponds to a certain geometric length on the keyboard (horizontally).

Illustration 13 shows a keyboard layout consisting of three two-row keyboards. A keyboard utilizing this kind of layout was first introduced by Paul von Jankó in 1882 and it is therefore called a Jankó keyboard layout. It is mathematically the simplest way to represent 12 keys in two dimensions, what will be discovered later on. The keys form a hexagonal structure or grid. The horizontal axis goes through every second note, and the diagonal axis through every note. Moving along the vertical axis the note does not change. Hexagonal arrangement is also the most compact possible arrangement for the keys. (Abrashev, Gadjev, 2000, p. 227.)

Illustration 13. Jankó keyboard layout and its 2-split arrangement. The 12 notes are numbered in order, and the first one is marked with thick borderlines.

Playing each chord in its basic form on a piano keyboard necessitates knowing 12 variations, whereas on an isomorphic keyboard one is sufficient. When taking into account the mirrored versions of the chords, the difference in variations is multiplied. However, it should be noted that there is a lot of similarity between the 12 variations of the piano keyboard. Illustration 14 shows the closest mirrored versions of the major and minor triads on the Jankó layout. A chord is always formed by three horizontally consecutive marked keys. Because every second row is identical, it is possible to choose any key on the vertical axis, depending on how it is the easiest to play.

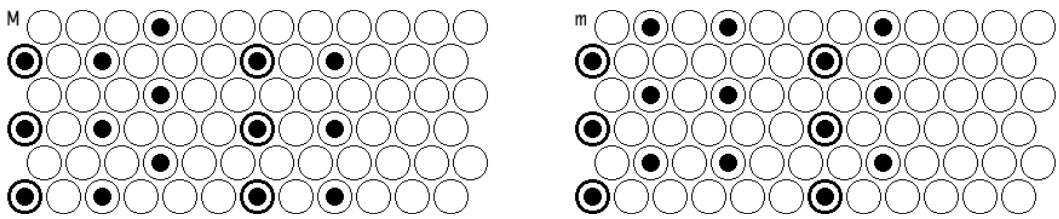


Illustration 14. Major triad chord (on the left) and minor triad chord (on the right) and their closest variations on Jankó keyboard layout. The first note is marked with thick borderlines.

There have been some keyboards based on the Jankó layout throughout the history, but at the moment of writing this document they are not available publicly. Sometimes piano manufacturers customize a Jankó keyboard on their pianos on a special request. A few prototypes of electrical MIDI keyboards have been made. Illustrations 15 and 16 show MIDI keyboard prototypes.

Illustration 15. Daskin Manufacturing Daskin 5, a prototype of a MIDI keyboard utilizing Jankó layout with 5 rows. (DASKIN Manufacturing, Inc., 2011)

Illustration 16. WholeTone Revolution, a prototype of a synthesizer utilizing Jankó layout. (WholeTone Technologies, 2011)

4.3 Equal division into 3 rows (button accordion layout)

Previously was covered the 2-split keyboard layout. The piano keyboard is also largely 2-split, that is, it has black and white rows of keys. Let us then examine a 3-split hexagonal keyboard layout. When the grid is filled sequentially into three rows in a similar manner to what was used with two rows, we obtain an arrangement shown in illustration 17. It is isomorphic likewise. The vertical axis and one diagonal axis go through every note in order, the other diagonal axis every second, and the horizontal axis every third note. The marked notes are identical vertically, and moving to the right their pitch increases by an octave.

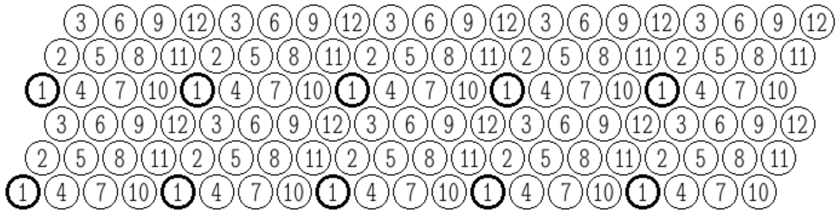


Illustration 17. Button accordion layout and its 3-split arrangement.

The 3-split keyboard layout is compact, and the fingers do not have to move much in any single dimension. Therefore it is used in a *chromatic* button accordion, although with a slight variation. For example the axes' angles and directions may vary. In addition, the accordion employs only five rows, whereas the illustration has six rows for demonstration. Here the chromatic refers to that all of the 12 notes are available. Remark: Due to the close relation between music and folklore the terminology may vary with geographic location. In many languages the word for *accordion* is *harmonica* or close to it. The operational principle and characteristic voice of these instruments resemble each other, so the context should be taken into account.

The major triad forms a pattern that consists of notes for example 1-5-8, and the corresponding minor consists of notes or keys 1-4-8. As can be seen, the 3-split keyboard no longer has the one-dimensional frequency axis covering the whole instrument. Therefore the pitch difference does not correspond to the geometric distance on the keyboard. However, a step in pitch of a certain size always means a step in a certain direction with certain distance.

4.4 Equal division into 4 rows (harmonic table)

Let us then examine a 4-split hexagonal keyboard layout. The grid is filled into four rows in the previously mentioned way. The obtained layout is shown in illustration 18. Here the horizontal axis goes through every fourth note and the vertical axis every second note. One diagonal axis goes through every note, and the other every third. The marked notes are vertically identical, and moving to the right their pitch increases by an octave. Consequently, the pitch steps from one to four can be described as steps on each of their own axes. When major triad is formed with notes 1-5-8 and minor triad with 1-4-8, the notes are next to each other and form arrow heads pointing into opposing directions.

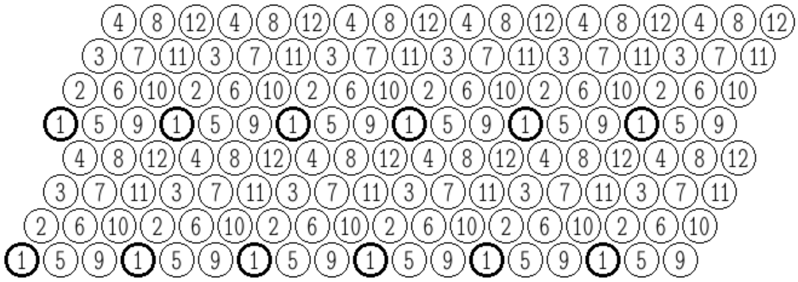


Illustration 18. 4-split keyboard layout.

The triads can be even more tight in the hexagonal grid if they are placed in triangles. Let us assume that the triads in illustration 18 are squeezed into triangles by bringing the end notes 1 and 8 next to each other. Now the diagonal axis that goes through every note stretches into like of the previous vertical axis, and the structure reorganizes into a slightly rotated hexagonal grid, in which the triads form triangles. When this newly formed grid is mirrored about this stretched axis (the direction of the axis with the end notes is inverted), and the whole grid is turned one-sixth of a circle clockwise, we obtain a layout shown in illustration 19. It is known as the *harmonic table*.

The horizontal axis goes through every note, the vertical every seventh (frequency ratios 3:2), one diagonal axis goes through every third (ratios 6:5) and the other diagonal axis every fourth (ratios 5:4). The marked notes are identical on an approximately horizontal line, and their pitch increases by an octave when going orthogonally upwards. Thus, major triad (notes 1-5-8) and minor triad (notes 1-4-8) form triangles pointing into opposing directions.

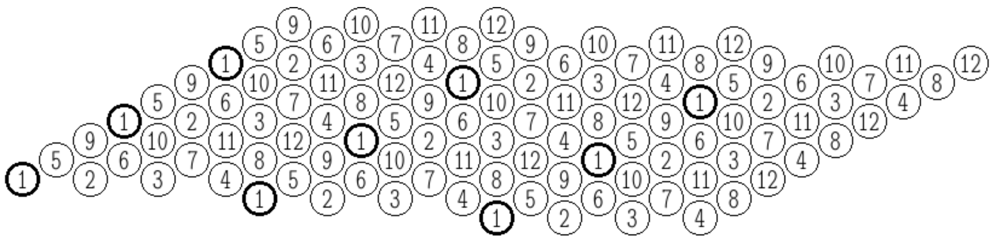


Illustration 19. Harmonic table and its 4-split arrangement.

The harmonies are easy to discover geometrically, hence the name. The illustrations 18 and 19 also show the different ways to divide the 12 notes into axes. The pitch changes do not correspond to geometric distances, but rather to different directions. There has been made a few instruments that make use of this keyboard layout. Illustration 20 shows probably the most well-known of these, the C-Thru-Music AXiS 64. (C-Thru-Music Ltd., 2013.)

Illustration 20. C-Thru-Music AXiS-64, a MIDI keyboard based on a harmonic table. (C-Thru-Music Ltd., 2013)

4.5 Modified division with 2 rows (Wicki-Hayden layout)

Illustration 21 shows a modified version of the two-row Jankó keyboard. The second row has been moved three steps to the left. The horizontal axis goes through every second note like in the Jankó layout. Other diagonal axis goes through every seventh note (frequency ratios 3:2) and the other every fifth (ratios 4:3). When moving up vertically the pitch increases by an octave. This layout was introduced independently by Kaspar Wicki and Brian Hayden unknowing of each other, and it is therefore called the Wicki-Hayden keyboard layout. Many times the melodies and chord progression base on every second note, so they are easy to play with little movement of the fingers. Also, the major and minor triads occur in fairly compact patterns. This layout is well suitable for playing single-handed, which is why it became common in concertina (an accordion-like handheld instrument). (Gaskins, 2003.) It is also used in some present-day single-handed keyboards. (Milne, Sethares, Plamondon. 2007).

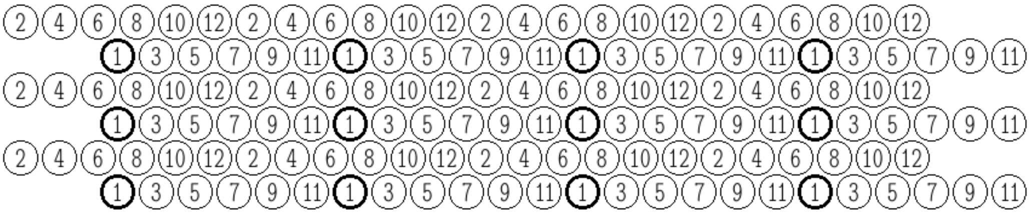


Illustration 21. Wicki-Hayden keyboard layout (edited: Gaskins, 2003).

4.6 Microtonal keyboard

The keyboard layouts presented earlier form a hexagonal grid surface, so their implementation base on some kind of hexagonal keyboard. This kind of keyboards are mainly available as specially manufactured or custom-tailored, and based on their price or features they are not aimed for typical keyboardists. Illustration 22 shows a programmable hexagonal keyboard. Its default keyboard layout is a slightly turned Jankó, but it suits well for a microtonal keyboard also. (Starr Labs, 2012.)

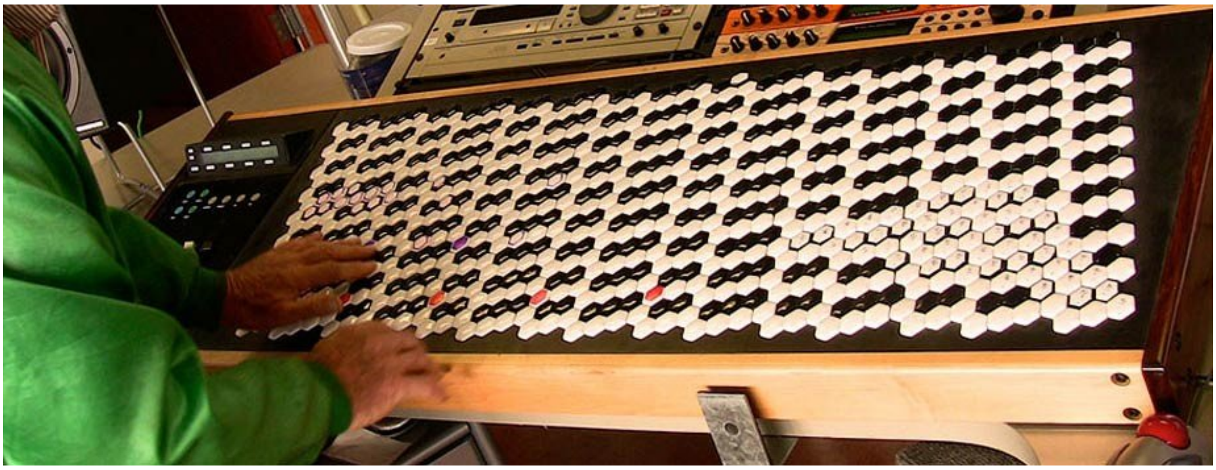


Illustration 22. Starr Labs Microzone U-990 (Starr Labs, 2012).

One obvious choice is to implement the instrument without distinct keys. Traditional instruments of continuous pitch are for example a violin, a fretless guitar, a trombone, or say, the human voice (singing). However, a keyboard provides polyphony easily. This kind of continuous pitch keyboard is nowadays possible to make electronically, which also enables easy changing of pitch steps. Illustration 23 shows a keyboard implemented this way. The instrument in question senses position of the fingers over three axes (vertical, horizontal and depth axis).

Illustration 23. Haken Audio Continuum fingerboard, a continuous pitch keyboard (Haken Audio, 2013).

4.7 Conclusions about the keyboard layout

The keyboard layouts shown previously have their pros and cons. The choice between them is largely a matter of opinion and familiarization, so it is difficult to define the best one. Now the object of special study is the Jankó keyboard layout.

Piano keyboard is common and cheap, and it suits quite well for playing one scale. Otherwise its playing capabilities (especially on equal tuning) are lacking. Its exceptional advantage is the tactile sensing of the position of the fingers within an octave.

Jankó keyboard is mathematically simple. Its horizontal axis embodies one-dimensional frequency, and the change in pitch corresponds to physical distance on the keyboard. The difference between the notes can be seen on the keyboard, felt with the fingers and heard with the ears, which makes the adoption easier. Several keys for a single note adds options for free fingers and free keys, which helps in playing and also in finding a pleasant and ergonomic finger position. It also helps two-handed playing of the same or nearby notes, which facilitates or enables fast rhythmic melodies and chord progression (like on a guitar). However, the playing requires moderate movement of the hands, and fingers of one hand can only reach a little over one octave in width.

The accordion keyboard is compact and has handy shape. It has multiple options to play melodies also.

If the shape of the keyboard or the mechanical implementation set no restrictions, the harmonic table may offer more logical axes. Harmonic table helps to perceive the harmonies geometrically. Chords and different scales form patterns, which are compact and fairly easy to memorize. Note changes in the direction of the main axes occur in a harmonic way, which may help in arranging harmonies and notes. The same notes repeat horizontally, which facilitates two-handed playing. However, in the harmonic table, the frequency is difficult to represent one-dimensionally, which may complicate playing of melodies.

Wicki-Hayden is a compact layout in which the fingers hardly have to move. Chords that go well with each other are close together like in the harmonic table, but melodies can be difficult to play. Each note has only one occurring instance, so the finger positions may sometimes be uncomfortable.

Microtonal instruments with continuous pitch offer possibility to play very diversely. The instrument itself does not limit harmonies or frequency ratios. However, all instruments of this kind share a common disadvantage, that the accuracy of the tuning relies on the motor skills of the player, so the playing requires intensive practice. Singing probably has the advantage; it is the most natural instrument from the perspective of human physiology and motor skills, but it can be challenging as well. Polyphony increases the difficulty level dramatically and decreases the selection of instruments omitting singing also. In practice, a versatile general-purpose instrument is based on some kind of keyboard. The implementation of a simple continuous keyboard may seem easy with current technology, but human physiology sets certain requirements. Creating a reasonable tactile feedback may be difficult, and the physiology of hands and fingers set the same limitations as in the previously mentioned keyboard layouts with distinct keys. Compatibility with other hardware also poses its own challenges.

5 RESULTS AND CONCLUSIONS

Nowadays music is strongly computer-aided, so a general-purpose instrument is actually just an interface between man and machine that is as versatile as possible. In practice, this interface is based on some kind of keyboard. The piano keyboard is ended up with due to historical reasons. It was born through a few intermediate steps and has remained unchanged for about 700 years, even though the used tuning system has changed in the meantime. Nowadays the 12-TET tuning system is used almost exclusively, and it is a compromise between the pure harmonies, the discontinuities resulting from them, and simplicity. Other tuning systems exist as well, but the 12-TET is so to say a safe bet and good to start with, even if intending to switch to a microtonal tuning later on. Choosing a keyboard is partly a matter of opinion. Different keyboards have been made, up to microtonal ones, but they have not gained favor or popularity, and their availability is poor in general. It seems to be worth it to make a keyboard for Jankó layout and 12-TET tuning. Compared to the piano keyboard it would be more versatile, more ergonomic and easier to learn, albeit similar. In a sense, the Jankó keyboard *repairs* the piano keyboard from which some keys have been *robbed*.

5.1 Starting point for a new keyboard instrument

The piano keyboard drags the burden of history with it. From it can be seen the 7-split octave from the Pythagorean times (the white keys) as well as the modifications made to it a couple of thousand years later (the black keys). It reflects the asymmetric tuning systems from ancient times and the compromise solutions of the hand-crafted mechanics. It served well in its time, but nowadays it is not necessary to use it specifically. In present keyboards it would be considerably more sensible to use for example the Jankó keyboard layout.

Nowadays music is common to all regardless of age or social status. It is diverse and polyphonic, and mainly based on the evenly spaced 12-TET tuning. Therefore it would be reasonable for the keyboard to be equally divided as well. The music theory and the mathematics lying behind it are understood better nowadays, and the related information is generally available. For example, the reader of this document does not have to be a pioneer musician or mathematician in order to understand the subject. The instruments are mainly personal, home-played, movable instruments, in which many settings and adjustments are made by oneself. The playing or its practice are no longer performed on shared instruments and thus according to the conditions set by them, for example concerning the keyboard layout. In practice, no mechanical implementation is an obstacle too hard to overcome with current manufacturing and serial production.

Jankó -keyboard offers many improvements over the traditional piano keyboard. The change in pitch corresponds to the physical distance on the keyboard, which makes playing and learning it easier. The piano keyboard on the other hand has holes where the black keys are missing from. In Jankó, chords and melodies can always be played in the mechanically same way regardless of the pitch, whereas the piano keyboard has 12 variations. It has multiple keys per note, so it is possible to choose the preferred or the most ergonomic way to play, whereas when playing the piano keyboard the fingers are frequently tangled. With Jankó it is possible to play the same notes with two hands, and it has narrower octave, so it enables to play in ways that are not possible with the piano keyboard. Jankó keyboard resembles the piano keyboard a lot, so switching to it does not require learning a completely new instrument.

Jankó keyboard also has its weaknesses. The worst is that for some reason it is not generally available, but in practice it has to be built by oneself. The mechanical implementation of it is not quite as easy as of the piano keyboard (although the difference is not significant in industrial manufacturing). With the traditional lever implementation several rows of keys leads to longer levers and more strict tolerances. The levers weight and take up space, especially if the rows are positioned on slightly different levels (in the manner familiar from the piano). In acoustic pianos this is not as big problem as in the mobile electrical keyboards. A better implementation for the keys would probably be separate fully electrical switches that would create a versatile interface to computer or other electrical instruments. Microtonal use would be possible also, with the mechanical limitations taken into account. However, it necessitates a special structure of the switches, that provides a pleasant tactile feedback to the player and also allows for registering the used force. Usually it is wanted that the keyboard instrument recognizes how fast the keys are pressed down (velocity feature) and how hard they are pressed while already in the lowest position (aftertouch feature). Considering the number of the keys, the structure of the switches may affect the price a lot. The hexagonal surface keyboards usually have a trade-off omitting at least the aftertouch feature. Of the devices introduced here, it is only found in both Jankó keyboards and the continuous pitch Haken Continuum. In piano keyboards the features usually exist.

5.2 Discussion

Due to historical reasons music is related with large amount of misleading vocabulary and cultural heritage, which slightly interfered with the literature research. Music and literacy are roughly as old. During certain time periods literacy and documentation were mainly responsibility of the church, which in part may have led to scarce or subjective information also. Matters like these should be taken into consideration when examining musical sources. The music theory could also be presented more straightforwardly. It would perhaps lower the threshold to start music as a hobby or to try more challenging harmonies and tuning systems. At the same time it would contribute to development of musical keyboards.

Instruments and their choice are associated with number of emotions and beliefs together with traditions, so the actual *right* solution is unlikely to be found. In my opinion, a keyboard instrument with Jankó layout is a superior aid for a music hobbyist, enthusiast or professional. It is however not intended to fully replace all other instruments.

I have let myself believe that the Jankó keyboard and its development were abandoned in haste and for wrong reasons. For example the statement that it was uncomfortable to play and the key pressure was uneven (Abrashev, Gadjev, 2000, p.227) does not in my opinion result from the layout of the keys, but mainly from the poor quality and finish of the mechanics. The design and craftsmanship of the prototypes were probably not so refined. Even if the implementation is a little more challenging, the present piano keys have quite some precision mechanics as well, and levers of roughly the same length. The point of time surely had its influence also; the quarrel about the tuning was still fresh in the memory (and the tuning problem is still lacking the conclusive solution). In practice, moving to Jankó keyboard meant moving to 12-TET, without the possibility of returning to the old tunings or even playing the old keyboard instruments. The mechanically more complex keyboard would surely have been tricky to retrofit into the existing instruments. Double keyboards did not fit into every instrument, or other keyboard was inconveniently placed. The price was definitely higher also, just as it is today. The few early adopters of the new keyboard were surely very brave and open-minded. Nowadays the switch is significantly easier.

There clearly is a cause for further research of Jankó keyboard and especially its implementation. More comprehensive research will not fit in the scope of this work.

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